A SIMULATION MODEL OF BELL CANADA

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The Canadian Department of Communications (DOC), contracted the Institute of Applied Economic Research (IAER), of Concordia University to build a simulation model of Bell Canada taking into account its productive and financial characteristics.

The work was done at the IAER during the period from June 1st, 1977 to March 31st, 1978, by the following team of researchers:

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FORWARD

Sir George Williams University and L'Ecole des Hautes Etudes Commerciales affiliated to the Université de Montréal jointly established on June 2nd,1969 the International Institute of Quantitative Economics (I.I.Q.E.) to initiate original research and promote international scientific collaboration in the field of quantitative economics.

A major reorganization of the I.I.Q.E. took place in April 1976 resulting in the adoption of a new policy statement and set of objectives as well as the renaming of the I.I.Q.E. to the Institute of Applied Economic Research (I.A.E.R.). Consequently, the I.A.E.R. located at the Sir George Williams Campus, has been established as Concordia University's institute for programs of socio-economic research and training related to both the developing world and Canada.

Nations both rich and poor, individually and collectively share many common domestic and international problems, which contribute to the growing threat of global deterioration. Prominent among these problems are the need for economic development of less developed countries and the need for readjustments in the economic policies of industrialized societies. Recognition of the importance of these problems should lead institutions and interested individuals to apply existing socio-economic knowledge to their solution.

The I.A.E.R. believes that a major step towards finding acceptable solutions to the above problems is domestic and international cooperation. To this end, the I.A.E.R. utilizes the most modern methods of scientific analysis available, as well as the services of internationally recognized experts in the relevant fields in:

1) initiating, organizing and implementing major economic research projects, at both international and Canadian levels, occasionally in collaboration with other research institutes and interested specialists;

2) organizing seminars and conferences on specific economic issues of particular international and Canadian interest;

3) serving as a link between Concordia University and the Canadian private sector with the objective of increasing the latter's awareness of, participation in and support for applied economic research.

The I.A.E.R. believes that it has a necessary and useful role to play in both Canada and the developing world, particularly Latin America and Francophone Africa, given the accumulated experience and expertise of its research staff.

Professor V. Corbo
Director

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CHAPTER I

INTRODUCTION

The purpose of this study is to build an econometric model of Bell Canada to be used for policy simulations. A model is developed, estimated,and then validated for the period 1952-1976. The validation is done by performing extensive simulations with models with alternative objective functions. We find that within the sample period the best tracking is obtained for a model that takes as given the production of each output of Bell (Local, Telephone Message Toll and Other Toll Services), and minimizes the cost of production subject to a regulatory and a technological constraint. This model is also used for forecasting Bell Canada's capital requirements, labor requirements, "raw materials" requirements and profit levels for the period 1977-1981. These forecasts are performed under different assumptions with respect to future prices for Bell services.

In Chapter II we develop and estimate demand equations for Local Services, Telephone Message Toll Services and Other Toll Services. These equations are estimated individually and then as a system of seemingly unrelated equations. We conclude form these estimations that Local services is price inelastic and Telephone Message Toll and Other Toll Services are price elastic.

In Chapter III we estimate a one output production function for

Bell Canada. We start with a general translog function and then after a sequence of tests we conclude that a constant returns Cobb-Douglas function cannot be rejected from our data. After estimating a constant returns to scale Cobb-Douglas production function, we find that the high collinearity between capital and "raw materials" in the sample (over 99%) does not allow to identify the separate effect of these two inputs in production. Then, we estimate the production function from the side conditions for cost minimization. From this estimation we can obtain significant coefficients for the separate effect of the different inputs in production: Labor, "Raw Materials" and Capital. We further find from this estimation that the results are not statistically different form the ones obtained from the direct estimation of the Cobb-Douglas function, although, they permit us a much more precise estimation of the parameters of the function.

In Chapter IV we estimate production possibility frontiers. After an extensive testing of alternative functional forms, we finish with a frontier that can be represented by a translog constant elasticity of transformation function for outputs and a Cobb-Douglas function for inputs.

In Chapter V we analyse two one-output models of Bell Canada. The first is a cost minimization model and the second is a profit maximization one. We simulate the cost minimization model and conclude that it describes the input choice of Bell quite accurately.

In Chapter VI we analyse two-multiple output models. The first is a cost minimization one and the second a profit maximization one. The cost minimization model is validated for the sample period and found to describe Bell behavior very closely. In contract, the profit maximization model was found to have a cross-over for the quantity of non-local services which could not be accounted for. In this chapter we used the cost minimization model to simulate the effect of eliminating the regulatory constraint. From this simulation a strong Averch-Johnson effect is found.

In Chapter VII, we use the multiple output cost minimization model of Chapter VI to simulate the effect on factor inputs and profit levels of alternative price strategies for Bell Canada services. These simulations are done for the period 1977-1981. From these simulations we conclude that the most sensitive variable to the alternative price strategies is the capital input. When we simulated the effect of the price increases that were requested by Bell, we found that as a result of the increase profits Bell was forced, because of the rate of return constraint, to drastically increase its capital input. A similar level of output can be produced with substantially less capital at somewhat lower prices than those requested by Bell while still maintaining the same rate of return on capital.

Finally in Appendix A we present the data base used in our models and in Appendix B we study the possible monopsony power of Bell Canada in their purchase of financial instruments. We conclude in this appendix that there is no evidence of monopsony power.

CHAPTER II

DEMAND FOR TELEPHONE SERVICES: LOCAL, TOLL AND OTHER SERVICES

I Introduction

In this chapter we re-estimate the demand equations of the previous year project. There are four main differences between the present estimates and the one from last year's project.

First, the sample period has been extended to 1976 and actual instead of extrapolated values have been used for the years 1973 to 1976. This was made possible by the new information made available through the CRTC Exhibit of January 1977 as well as by direct information for 1976 provided by Bell to DOC.

Second, we consider an alternative model which was not used last time, that is the "habit formation model", which has been used successfully used by others for services which have a habit formation element like Local telephone services and Toll messages.

Third, we allow for contemporaneous correlation in the disturbances across equations and estimate all equations simultaneously using Zellner's seemingly unrelated procedure. In this way we obtain more efficient estimates.

Fourth, we re-define the quantity and price variables by using Divisia quantity and price indexes.

In the estimation of demand equations, two alternative specifications are considered: 1) a double log equation and 2) a "habit formation" equation.

II The Models

In the double log formulation the demand equation is given by:

$$(1)\quad \ln SO_{it} = \alpha_o + \alpha_1 \ln \frac{P_{1t}}{PD_t} + \alpha_2 \ln \frac{P_{2t}}{PD_t} + \alpha_3 \ln \frac{P_{3t}}{PD_t} + \alpha_4 \ln YD_t$$

III The Data

Before proceeding to analyse the estimated results we will describe the data used.

a) Quantity Demanded

We work with three outputs: Local, Telephone Message Toll and Other Toll services. For Local services the quantity demanded is measured as the revenue from these types of services at 1967 prices. In the case of Telephone Message Toll services, the quantity demanded is measured as a divisia quantity index of the three types of toll services. That is, Intra-Bell Telephone Message Toll Service, Trans-Canada Telephone Message Toll Service and the Canada-US and Overseas Telephone Message Toll Service. Each of these services is measured as the revenue from each type of services (including uncollectables) at 1967 prices.

The Other Toll category was measured as the revenue from this type of service at 1967 price. The Telephone Message Toll divisia quantity index was normalized to the 1967 dollar revenues from these services.* The source of information for the revenue figures was provided by Bell to DOC.

b) The Price of Each Telephone Service

For local services, the price index is taken directly from Bell data. For Telephone Message Toll services, the price index is defined as the ratio of the current dollar revenues from these types

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* That is, the scale of the computed quantity index was defined in such a way that the value of this index for 1967 was equal to the dollar revenue from this service in 1967.

of services and the normalized divisia quantity index of this service. For Other Toll services, the data was taken directly from Bell Exhibit.

c) The Real Income Variable

The demand equations that we estimate are aggregated for Business and Household. This is caused by the inexistence, up to now, of disaggregated data on the public domain. Thus, the income variable that we require is a variable related to the over-all level of economic activity in the Quebec-Ontario region. Indeed, for the income variable we used a Divisia Quantity Index of the Gross Provincial Products at 1967 prices of Quebec and Ontario. Where the price indexes used were the consumer price indexes of Montreal and Toronto respectively.

d) The Price Deflator

The price deflator used in our computation is defined as the ratio of the current dollar Gross Provincial Product of both provinces and the normalized divisia quantity index of Gross Provincial Products.

e) Other Exogenous Variables

We also study if there is a shift in the demand for Telephone Message Toll services caused by advertising and/or Post Office strikes. For this purpose the following variables are defined:

(i) Advertising expenditure by Bell Canada divided by the price deflator defined above.

(ii) Sum of the Advertising, Commercial and Marketing expenditures deflated by the price deflator.

(iii) An index of strikes in the Post Office. This index is defined as the ratio of the man-hour striked in each year to the employment in man-hours for that year. Since the post office annual report provides employment information only about full-time and part-time employees, they are considered respectively as 250 and 75 days work per year, then multiplying the number of persons by days worked, we obtain the man days worked in a year.

2.4 The Empirical Results

We start by analysing the results of the double log model with and without correction for auto-correlation. Furthermore, we use two estimation procedures: estimation equation by equation and estimation by Zellner's seemingly unrelated regression procedure.

In Table 2.1 we present the results for the estimation without correction for auto-correlation and using only the own price variable in each equation. All the results from this table indicate strong auto-correlation in the disturbances. Therefore, the computed t-values are meaningless and no statistical inference can be based on computed value of R^2. Thus, we proceed to the results corrected for auto-correlation that appear in Table 2.2. From this table we observe that after correction for auto-correlation only the disturbances of the equation for local services are still auto-correlated. What is disturbing from these results is the very high value for the auto-regressive coefficient (RHO) which is close to one in all cases but Telephone Message Toll. This is an indication that something very systematic has been left out of our equations. Furthermore, the own price elasticity of Toll services is not significantly different from minus one, a result difficult to accept.

In Tables 2.3 and 2.4 we allow for cross price effects by introducing the prices of the other two services in each equation. Now each equation has the same set of regressors, therefore, for the case of RHO equals to zero,the estimation equation by equation and the estimation by Zellner's procedure yield the same results. Thus, only the results of the estimation equation by equation are presented for Table 2.3.

Again, as before, there is positive auto-correlation in the disturbances of the equations estimated by OLSQ (Table 2.3). When a correlation for auto-correlation is performed (Table 2.4), the value of RHO is very close to one in all equations but Telephone Message Toll. Furthermore, RHO is the most significant coefficient in the model. Thus, again we conclude that some systematic variable has been left out.

We proceed now to the estimation of the habit formation model given by equation (2) above.

In Table 2.5, we present the results for the Habit Formation model with correction for autocorrelation. Again, we present the results obtained from the estimation equation by equation (top of the tables) and the ones from the Zellner's procedure.

From the results of the Zellner's estimation procedure, we observe that RHO is significant only in the local service equation. Furthermore, the estimated value of RHO is only around .5 in this case. For the other toll equation the price elasticity is positive. Thus, the habit formation model does not apply to this equation. This result is not surprising given the type of service considered.

The long run price elasticities of Local and Telephone Message Toll are -.574 and -.993. On the other hand, the long run income elasticities are 1.049 and 1.321 respectively.

In Table 2.6, we estimate the habit formation model allowing for cross price elasticities. One major problem with these results is the high collinearity among the price variables. Thus, the estimated values of the own price elasticities become very unreliable; this is specially so for local services. The own price elasticity of other toll services is still positive, although now is not significant. Thus, we go back to the "Habit Formation Model" without cross price elasticities for local and Telephone Message Toll and we exclude the joint variable from the Other Toll equation. Furthermore, we also work now with variables in per-capita form. Thus, to complete our estimations we re-estimate the model of Table 2.5 in per-capita form excluding the lagged endogenous variable in the equation for other toll and we introduce advertising as a regressor in the equation for Telephone Message Toll.*

The final results for our demand equation appear in Table 2.7. Total advertising expenditures per capita have a positive and significant effect on the demand for Telephone Message Toll services. The price elasticities computed from these equations are -.754 for Local, -1.123 for Telephone Message Toll and -1.351 for Other Toll services.

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* We also included a variable for Post-Office strikes in the equation for Telephone Message Toll. This variable had a positive coefficient as expected but it was not significant. Of course, this could be due to a problem of time aggregation. The effect of Post-Office strikes in the demand for toll services could affect the monthly and/or quarterly demand equations but they do not show up in the annual demand equation.

TABLE 2.1

DOUBLE LOG-DEMAND MODEL*

	Constant	$\ln \frac{P_{it}}{PD_t}$	$\ln YD_t$		D.W.	R^2
a) OLSQ EQUATION BY EQUATION						
Local	-10.003 (-10.56)	.332 1.33	1.495 (16.77)		.7076	.9897
Telephone Message Toll	-10.512 (-11.65)	-.642 (-3.45)	1.483 (17.64)		.8741	.9966
Other Toll	-47.453 (-10.51)	3.467 (3.51)	4.775 (11.29)		.7056	.9799
Total	-10.21 (-10.34)	.061 (.24)	1.564 (16.90)		.7187	.9921
b) ZELLNER'S PROCEDURE						
Local	-10.580 (-33.89)	.493 (6.81)	1.549 (52.69)		.8044	.9895
Telephone Message Toll	-11.075 (-36.91)	-.524 (-9.04)	1.535 (54.85)		.9270	.9965
Other Toll	-51.985 (-32.21)	4.487 (13.31)	5.199 (34.37)		.8829	.9789

* D.W. is the Durbin-Watson Statistic, R^2 is the multiple determination coefficient and the terms in parenthesis are t-values computed under the null hypothesis that the true value of the respective coefficient is zero.

TABLE 2.2

DOUBLE LOG MODEL: CORRECTED FOR AUTO-CORRELATION*

	Constant	$\ln \frac{P_{it}}{PD_t}$	$\ln YD_t$	RHO	D.W.	R^2
	a) MAXIMUM LIKELIHOOD EQUATION BY EQUATION					
Local	7.935 (4.48)	-.248 (-2.67)	.121 (1.60)	.981 (166.29)	1.1475	.9997
Telephone Message Toll	-7.913 (-5.53)	-1.025 (-4.76)	1.241 (9.35)	.675 (4.10)	2.0654	.9979
Other Toll	-10.966 (-2.91)	-1.205 (-2.43)	1.408 (4.25)	.874 (23.35)	2.0265	.9986
Total	8.502 (3.28)	-.452 -3.89)	.181 (1.94)	.984 (139.28)	2.1107	.9996
	b) ZELLNER'S PROCEDURE					
Local	4.837 (9.25)	-.357 (-8.91)	.268 (7.92)	.971 (277.16	1.2287	.9996
Telephone Message Toll	-7.487 (-11.29)	-1.046 (-11.21)	1.202 (19.54)	.715 (11.47)	2.0784	.9979
Other Toll	-13.919 (-8.92)	-1.465 (-6.71)	1.662 (11.65)	.829 (47.99)	1.9361	.9985

* See Note to Table 2.1

TABLE 2.3

DOUBLE LOG MODEL WITH CROSS PRICE ELASTICITIES*

	OLSQ EQUATION BY EQUATION							
	Constant	$\ln \frac{P_{1t}}{PD_t}$	$\ln \frac{P_{2t}}{PD_t}$	$\ln \frac{P_{3t}}{PD_t}$	$\ln YD_t$		D.W.	R^2
Local	-8.330 (-5.13)	1.152 (1.67)	-.054 (-.11)	-.981 (-1.30)	1.340 (8.87)		.7236	.9906
Telephone Message Toll	-9.704 (-8.06)	.302 (.59)	-.443 (-1.22)	-.637 (-1.14)	1.406 (12.56)		.8155	.9968
Other Toll	-42.628 (-7.82)	3.306 (1.43)	-1.930 (-1.18)	1.748 (.69)	4.33 (8.55)		.8474	.9821

* See Note to Table 2.1

TABLE 2.4

DOUBLE LOG MODEL WITH CROSS PRICE ELASTICITIES: CORRECTED FOR AUTO-CORRELATION*

	Constant	$\ln \frac{P_{1t}}{PD_t}$	$\ln \frac{P_{2t}}{PD_t}$	$\ln \frac{P_{3t}}{PD_t}$	$\ln YD_t$	RHO	D.W.	R^2
MAXIMUM LIKELIHOOD EQUATION BY EQUATION								
Local	6.394 (4.60)	-.162 (-1.12)	.114 (1.23)	-.345 (-2.09)	.162 (2.22)	.974 (135.04)	1.0993	.9997
Telephone Message Toll	-4.828 (-3.71)	.188 (0.44)	-.317 (-1.21)	-1.489 (-3.34)	.951 (7.86)	.683 (6.32)	2.1656	.9990
Other Toll	-10.596 (-2.62)	-1.535 (-1.95)	.399 (0.80)	.072 (0.08)	1.387 (4.00)	.894 (26.16)	2.2079	.9988
ZELLNER'S PROCEDURE								
Local	6.096 (11.90)	-.128 (-1.80)	.111 (2.44)	-.388 (-5.2)	.178 (5.74)	.973 (325.50	1.0648	.9997
Telephone Message Toll	-5.373 (-8.51)	.623 (3.10)	-.372 (-2.81)	-1.780 (-7.86)	1.002 (17.04)	.516 (10.00)	1.8959	.9988
Other Toll	-15.795 (-10.12)	-.857 (-1.79)	.371 (1.25)	-1.184 (-2.37)	1.826 (12.66)	.794 46.79	1.5735	.9983

* See note to Table 2.1

TABLE 2.5

HABIT FORMATION MODEL: CORRECTED FOR AUTO-CORRELATION*

	Constant	$\ln \frac{P_{it}}{PD_t}$	$\ln YD_t$	$\ln SO_{it-1}$	RHO	D.W.	R^2
a) MAXIMUM LIKELIHOOD EQUATION BY EQUATION							
Local	-.929 (-2.59)	-.070 (-1.51)	.187 (3.43)	.832 (19.76)	.305 (1.46)	2.1731	.9998
Telephone Message Toll	-4.493 (-3.94)	-.473 (-3.19)	.679 (4.42)	.493 (4.59)	.120 (.54)	2.1094	.9990
Other Toll	-7.577 (-2.23)	.444 (1.07)	.798 (2.36)	.774 (9.87)	.116 (.47)	2.0912	.9979
Total	-1.041 (-1.98)	-.108 (-1.42)	.206 (2.68)	.839 (15.44)	-.008 (-.53)	1.883	.9996
b) ZELLNER'S PROCEDURE							
Local	-1.008 (-5.82)	-.117 (-4.37)	.214 (7.79)	.796 (33.78)	.437 (5.06)	2.1920	.9998
Telephone Message Toll	-3.913 (-8.04)	-.449 (-8.28)	.597 (9.25)	.548 (12.81)	-.082 (-.89)	1.9033	.9989
Other Toll	-6.705 (-4.05)	.485 (2.38)	.705 (4.28)	.808 (22.40)	.110 (1.02)	2.1115	.9979

* See Note to Table 2.1

TABLE 2.6

HABIT FORMATION MODEL WITH CROSS PRICE ELASTICITIES: CORRECTED FOR AUTO-CORRELATION*

a) MAXIMUM LIKELIHOOD EQUATION BY EQUATION

	Constant	$\ln \frac{P_{1t}}{PD_t}$	$\ln \frac{P_{2t}}{PD_t}$	$\ln \frac{P_{3t}}{PD_t}$	$\ln YD_t$	$\ln SO_{it-1}$	RHO	D.W.	R^2
Local	-.800 (-2.05)	-.034 (-0.26)	.078 (0.97)	-.364 (-2.08)	.238 (4.22)	.718 (10.59)	.588 (3.94)	2.2827	.9998
Telephone Message Toll	-4.276 (-3.91)	.570 (1.90)	-.434 (-2.20)	-1.092 (-2.33)	.755 (5.03)	.296 (2.12)	.206 (1.00)	2.1929	.9993
Other Toll	-7.07 (-2.26)	.107 (0.11)	-.692 (-1.38)	1.063 (1.00)	.748 (2.38)	.789 (9.03)	-.044 (-.16)	2.1283	.9981

b) ZELLNER'S PROCEDURE

	Constant	$\ln \frac{P_{1t}}{PD_t}$	$\ln \frac{P_{2t}}{PD_t}$	$\ln \frac{P_{3t}}{PD_t}$	$\ln YD_t$	$\ln SO_{it-1}$	RHO	D.W.	R^2
Local	-.717 (-3.87)	-.104 (-1.93)	.127 (3.82)	-.411 (-6.06)	.286 (11.32)	.617 (19.04)	.749 (19.57)	2.2078	.9998
Telephone Message Toll	-4.533 (-9.43)	.492 (3.63)	-.467 (-5.31)	-.969 (-4.80)	.783 (11.88)	.287 (4.80)	.140 (1.87)	2.0283	.9992
Other Toll	-6.967 (-5.05)	.374 (0.89)	-.686 (-3.00)	.782 (1.74)	.740 (5.39)	.784 (22.06)	-.086 (-0.85)	2.0433	.9981

* See Note to Table 2.1

Table 2.7

HABIT FORMATION MODEL: Corrected for Auto-correlation with Advertising Expenditures in Demand for Toll*

ZELLNER'S PROCEDURE

	Constant	$\ln \frac{P_{1t}}{PD_t}$	$\ln \frac{P_{2t}}{PD_t}$	$\ln \frac{P_{3t}}{PD_t}$	$\ln \frac{YD_t}{N_t}$	$\ln \frac{SO_{it-1}}{N_{t-1}}$	$\ln \frac{ADVT_t}{N_t}$	RHO	D.W.	R^2
Local	-.934 (-8.14)	-.153 (-4.93)			.219 (7.44)	.797 (33.76)		.393 (5.68)	1.7629	.9985
Telephone Message Toll	-2.956 (-11.42)		-.577 (-9.23)		.649 (8.50)	.486 (11.53)	.075 (2.58)		1.5440	.9979
Other Toll	-7.347 (-23.81)			-1.351 (-5.80)	1.541 (8.71)			.856 (43.29)	2.0808	.9979

* - See Note to Table 2.1
- Advertising includes marketing - commercial and advertising expenditures
- The dependent variable in these equations are in per-capita form.

where SO_{it} is the quantity demanded of the service i (1 local, 2 toll, 3 other services) in period t, P_{it} is the price of service i in period t, PD_t is a price deflator for period t and YD_t is real income.

The second model used is of the habit formation type, and is based on the assumption that the demand for a type of telephone service is a function of income, prices and a state variable S_t proportional to last period's demand, and representing the stock of accumulated telephone habits.

Thus, the "habit formation" model is given by the following pair of equations:

$$\ln SO_{it} = \beta_o+\beta_1 \ln \frac{P_{1t}}{PD_t} + \beta_2 \ln \frac{P_{2t}}{PD_t} +\beta_3 \ln \frac{P_{3t}}{PD_t} +\beta_4 \ln YD_t +\beta_5 \ln S_t$$

with:

$$\ln S_t = \Theta \ln SO_{it-1}$$

Replacing the second equation in the first we obtain:

$$(2) \quad \ln SO_{it} = \beta_o+\beta_1 \ln \frac{P_{1t}}{PD_t} +\beta_2 \ln \frac{P_{2t}}{PD_t}+\beta_3 \ln \frac{P_{3t}}{PD_t} +\beta_4 \ln YD_t+\beta_5\Theta \ln SO_{it-1}$$

A priori, we expect $\beta_1 < 0$, $\beta_4 > 0$ and $\beta_5 \Theta > 0$. Due to the presence of a lagged endogenous variable on the right-hand side of this equation, ordinary least squares would yield inconsistent estimates if the disturbances of this equation are auto-correlated. We therefore, in our estimation, begin by assuming a first-order auto-regressive process for the disturbances, and use the maximum likelihood estimation procedure to estimate simultaneously the coefficient of the auto-regressive process and the coefficients of the equation by means of a non-linear algorithm.

CHAPTER III

A PRODUCTION FUNCTION FOR BELL CANADA: THE ONE OUTPUT CASE

I. Introduction

In the study of the technology of Bell Canada, we choose to start with a general form of production function which can be considered as a production function by itself or as a second order approximation to any production function. Where the approximation is made about a point in which the logarithms of each of the inputs are made equal to zero. This form of production function is the Transcendental Logarithmic Function (translog)[1/].

1/ See Christensen, L.R., D.W. Jorgenson and L.J. Lau, "Conjugate duality and the transcendental logarithmic production function"(abstract), Econometrica, 39,4, 255-256, 1971.

Other references on translog production functions include:

a- E. Berndt and L. Christensen, The Translog function and the substitution of equipment, structures, and labour in U.S. manufacturing 1929-1968, Journal of Econometrics 1,1, 81-113, 1973.

b- Vittorio Corbo and Patricio Meller,"The Translog production function: some evidence from establishment data." Mimeo, July 1977, our presentation here follows close the one in in Corbo and Meller, see also:

c- Vittorio Corbo *et al*, Rate Adjustment Guidelines for Regulated Industries: A Model for Bell Canada, IAER, May 1976.

In the estimation of production models the standard hypothesis is that the function belongs to a restricted class which satisfies some *a priori* restrictions of the technology. The production functions most frequently used are the Cobb-Douglas, the CES, and the translog, the last being a recent development (Christensen, Jorgenson, and Lau, 1971). The Cobb-Douglas production function restricts all Allen partial elasticities of substitution to be equal to one. The CES function restricts the above elasticities to be constant and equal for any pair of inputs and for all points in input space. In addition, both the Cobb-Douglas and the CES functions assume strong separability. The translog function, on the other hand, does not assume strong separability; moreover, it does not restrict the values of the elasticity of substitution at any point in input space.

The estimation of translog function has become very popular lately for the flexibility that it provides (E. Berndt and L.R. Christensen, 1973; E. Berndt and L.R. Christensen, 1974; E. Berndt and D. Wood, 1975; D. Humphrey and J.R. Moroney, 1975). All these studies use a translog function with three inputs having nine regressors besides the constant.[1] To avoid multicollinearity problems in small samples, the usual estimation procedure has been to work with side conditions for profit

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[1] In general an n input translog function has $2n+1+\frac{n(n-1)}{2}$ parameters.

maximization in competitive product and factor markets. With this procedure, the parameters of the associated translog function are estimated from a system of semi-logarithmic equations with one equation for each input. Each of these equations gives the cost share of an input as a linear function of the logs of each of the inputs. The difficulty that arises with this approach is that it is impossible to know if the parameters that one is estimating are those of a translog function, or a spurious set resulting from misspecification introduced by the use of untested and incorrect assumptions. In the case of Bell Canada, this problem is exorbitant due to the presence of regulations.

In this chapter we use time series data on output and factor inputs to estimate directly, i.e. without using side conditions a translog function for Bell Canada. Then, we compare the results from direct estimation with the ones obtained from the estimation of a simultaneous cost minimizing model subject to a regulatory constraint. We find the results are not statistically different.

II. The Model and Principal Hypotheses

The unconstrained transcendental logarithmic translog production function for one output and three inputs with symmetry imposed ($\gamma_{sk} = \gamma_{ks}$), allowing for Hicks-neutral technical change, can be written as:

$$(1) \quad \ln Q_t = \alpha_0 + \alpha_1 \ln L_t + \alpha_2 \ln M_t + \alpha_3 \ln K_t + 1/2\ \gamma_{11} (\ln L_t)^2 + \gamma_{12} (\ln L_t)(\ln M_t) + \gamma_{13} (\ln L_t)(\ln K_t) + 1/2\ \gamma_{22} (\ln M_t)^2 + \gamma_{23} (\ln M_t)(\ln K_t) + 1/2\ \gamma_{33} (\ln K_t)^2 + \beta .\ D_t$$

where Q_t: is the total revenue minus indirect taxes in millions of 1967 dollars, constructed as a divisia quantity index of: Local Services, Intra-Bell, Trans Canada and adjacent members, United States and overseas and other toll. The variables was normalized to make the average equal to one.

L_t: Weighted man-hours where the weights are the relative hourly wage rate of the different labour categories in 1967. The variable is normalized as above.

M_t: Intermediate inputs ("raw materials" for short), measured as a divisia quantity index of Cost of materials, services, rent and supplies, uncollectables, plus indirect taxes, all of them in constant 1967 dollars. The variable is also normalized.

K_t: Net capital stock in millions of 1967 dollars, normalized as above.

D_t: Percentage of calls Direct Distance Dialed.

The hypothesis of constant returns to scale can be tested directly from (1). Constant returns to scale imply the following restrictions on the parameters of this function for sector i (E. Berndt and L. Christensen, 1973, p. 84).

(i) $\sum_{k=1}^{3} \alpha_k = 1$

(ii) $\sum_{k=1}^{3} \gamma_{sk} = 0 \quad s=1,2,3$

(iii) $\sum_{s=1}^{3} \gamma_{sk} = 0 \quad k=1,2,3$

(iv) $\sum_{s=1}^{3} \sum_{k=1}^{3} \gamma_{sk} = 0$

With symmetry imposed *a priori*, restrictions (iii) and (iv) are not independent of (i) and (ii). Therefore, we test for constant returns to scale in model (1) by imposing constraints (i) and (ii) on the parameters.

A production function is considered to be well-behaved if it has positive marginal products for each input (monotonicity) and if it is quasi-concave. The translog function does not satisfy these restrictions globally. Still, if we can find wide enough regions in input space (including the observed input combination) where these restrictions are satisfied, we can consider the translog function as well-behaved for relevant input combinations. To do this, monotonicity and quasi-concavity of the estimated translog function must be checked at every data point in the sample. For details of how to check for this see Appendix.

The translog function does not assume separability: rather, it must be tested. In the case of three inputs, three types of weak separability may exist: the weak separability of L and M from K (denoted LM-K), L and K from M (denoted LK-M), and M and K from L (denoted MK-L). In the case of the translog function of equation (1), these separability conditions are fullfilled globally if and only if (E. Berndt and L. Christensen, 1973, p. 102):

(2) LM-K: (i) $\alpha_1\ \gamma_{23} - \alpha_2\ \gamma_{13} = 0$

(ii) $\gamma_{11}\gamma_{23} - \gamma_{12}\gamma_{13} = 0$

(iii) $\gamma_{12}\gamma_{23} - \gamma_{22}\gamma_{13} = 0$

(3) LK-M: (i) $\alpha_1\ \gamma_{23} - \alpha_3\ \gamma_{12} = 0$

(ii) $\gamma_{11}\gamma_{23} - \gamma_{13}\gamma_{12} = 0$

(iii) $\gamma_{13}\gamma_{23} - \gamma_{33}\gamma_{12} = 0$

(4) MK-L: (i) $\alpha_2\ \gamma_{13} - \alpha_3\ \gamma_{12} = 0$

(ii) $\gamma_{22}\gamma_{13} - \gamma_{23}\gamma_{12} = 0$

(iii) $\gamma_{23}\gamma_{13} - \gamma_{33}\gamma_{12} = 0$

If we impose constant returns to scale (CRTS) then in each of the set of conditions { (2) , (3) and (4) } only one of equations (ii) and (iii) is independent.

The linear restrictions $\gamma_{13}=\gamma_{23}=0$ satisfy (2), the conditions for global separability LM-K In the same way $\gamma_{23}=\gamma_{12}=0$ satisfy the set of restrictions (3) and $\gamma_{13}=\gamma_{12}=0$ satisfy restrictions (4). All the global separability conditions are satisfied simultaneously if and only if $\gamma_{13}=\gamma_{12}=\gamma_{23}=0$ and, in the CRTS case, the function is Cobb-Douglas.[2]

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[2] If we do not restrict the translog function to exhibit CRTS then the restricted translog function will include terms with the square of the logs of each input and therefore can not be a Cobb-Douglas function.

If we substitute the CRTS restrictions in (2) and (3) above then a set of nonlinear separability conditions can be derived (E. Berndt and L. Christensen, 1973, p. 91). A summary of these conditions is reproduced below:

TABLE 3.1

PARAMETER RESTRICTIONS FOR GLOBAL FUNCTIONAL SEPARABILITY

Separability Type	Linear restrictions fulfilling separability	Non-linear restrictions fulfilling separability	
		General case	CRTS *
LM-K	$\gamma_{13}=\gamma_{23}=0$	$\alpha_1\gamma_{23}-\alpha_2\gamma_{13}=0$ $\gamma_{11}\gamma_{23}-\gamma_{12}\gamma_{13}=0$ $\gamma_{12}\gamma_{23}-\gamma_{22}\gamma_{13}=0$	$\gamma_{33}=\gamma_{23}^2/\gamma_{22}$ $\alpha_3=1+(\alpha_2\gamma_{23}/\gamma_{22})$ $(\sigma_{13}=\sigma_{23}\neq 1)$
LK-M	$\gamma_{12}=\gamma_{23}=0$	$\alpha_1\gamma_{23}-\alpha_3\gamma_{12}=0$ $\gamma_{11}\gamma_{23}-\gamma_{12}\gamma_{13}=0$ $\gamma_{13}\gamma_{23}-\gamma_{33}\gamma_{12}=0$	$\gamma_{33}=\gamma_{23}^2/\gamma_{22}$ $\alpha_3=(\alpha_2-1)\gamma_{23}/\gamma_{22}$ $(\sigma_{12}=\sigma_{23}\neq 1)$
MK-L	$\gamma_{13}=\gamma_{12}=0$	$\alpha_2\gamma_{13}-\alpha_3\gamma_{12}=0$ $\gamma_{22}\gamma_{13}-\gamma_{23}\gamma_{12}=0$ $\gamma_{23}\gamma_{13}-\gamma_{33}\gamma_{12}=0$	$\gamma_{33}=\gamma_{23}^2/\gamma_{22}$ $\alpha_3=\alpha_2\gamma_{23}/\gamma_{22}$ $(\alpha_{12}=\alpha_{13}\neq 1)$

* In addition to the restrictions for CRTS presented above.

It can also be shown that if one set of non-linear separability restrictions holds, then neither of the other two can be satisfied (E. Berndt and L.R. Christensen, 1973).

One of the difficulties with the tests for weak separability in a translog function is that they require the aggregator function to be linear in the logs. Thus the tests presented above are a joint test of weak separability and a linear logarithmic aggregator. Under the translog specification of technology the joint character of the tests makes them inseparable and the tests are biased in favor of rejecting the hypothesis of weak separability (see Blackorby, Primont and Russell, 1977).

In our testing of the translog model we use a set of nested hypothesis. We use a 1% significance level for each test. There are a total of eight such tests. Therefore, the overall significance is approximately eight per cent. The tests are performed by using a sequence of F-tests. Obviously these F-tests are asymptotically equivalent to maximum-likelihood-ratio tests.

III. Statistical Results

When we estimated the translog function (1), correcting for auto-correlation, we obtain the following results:

TABLE 3.2: UNCONSTRAINED ONE OUTPUT TRANSLOG PRODUCTION FUNCTION

	Estimated Coefficient	t- Statistic
α_0	-.1080	-.719
α_1	.3646	2.453
α_2	.1217	1.003
α_3	1.2071	4.056
γ_{11}	5.2367	1.086
γ_{22}	-.7157	-.237
γ_{33}	.7384	.402
γ_{12}	-2.6178	-.789
γ_{13}	1.5492	.632
γ_{23}	.4561	.198
β	-.0854	-.215
ρ	.5565	2.229
R^2	.9997	
DW	2.2920	
SSR	.00230830	

Following our testing procedure we test now for CRTS. The results from the estimation of equation (1) subject to the CRTS restrictions (i) and (ii), follow:

TABLE 3.3: TRANSLOG PRODUCTION FUNCTION SUBJECT TO CRTS

	Estimated Value	t-Statistic
α_0	-.4176	-9.513
α_1	.3288	3.029
α_2	.0156	.143
α_3	.6556	5.284
γ_{11}	.4466	1.463
γ_{22}	-2.6545	-1.194
γ_{33}	-1.2527	-.909
γ_{12}	.4775	.854
γ_{13}	-.9245	-2.024
γ_{23}	2.1767	1.260
β	.7385	6.979
ρ	.6775	3.800
R^2	.9995	
DW	2.0143	
SSR	.00365904	

In order to test for the existence of constant returns to scale, we perform a Chow test. For this purpose we compute the F-statistic using the sum of squares of the residuals (SSR) from Tables 3.2 and 3.3. The F-statistic can be written as:

$$F = \frac{\dfrac{SSR^{TLCRTS} - SSR^{TL}}{4}}{\dfrac{SSR^{TL}}{24-12}}$$

where SSR^{TLCRTS} = the sum of squares of the residuals for the translog subject to CRTS (from Table 3.3)

SSR^{TL} = the sum of squares of the residuals for the translog (unconstrained, from Table 3.2).

Our null hypothesis is the presence of CRTS. If the null hypothesis is true, the above statistic is distributed as F with 4 and 12 degrees of freedom.

The computed F value is 1.7631, the 5% F (4,12) is 3.26, therefore, we cannot reject the hypothesis of constant returns to scale.

Using CRTS as the maintained hypothesis, next we test for complete global separability, i.e. whether or not the function is of the Cobb-Douglas type, (that is, with $\gamma_{11} = \gamma_{22} = \gamma_{33} = \gamma_{12} = \gamma_{13} = \gamma_{23} = 0$).

The results of estimating a one-output CRTS Cobb-Douglas production function follows:

TABLE 3.4: COBB-DOUGLAS PRODUCTION FUNCTION WITH CRTS

	Estimated Value	t-statistic
α_0	-.5029	-16.037
α_1	.5771	9.051
α_2	.0403	.401
α_3	.3826	5.261
β	.9889	15.285
ρ	.5121	2.845
R^2	.9993	
DW	1.7934	
SSR	.00563706	

Using the results of Table 3.4, now we run a test to see whether or not a Cobb-Douglas technology can be rejected. - The test is

$$F = \frac{\dfrac{SSR^{CDCRTS} - SSR^{TLCRTS}}{6}}{\dfrac{SSR^{TLCRTS}}{24-8}}$$

The computed F is 1.4416, the 5% F (6,16) is 2.74, therefore we cannot reject the Cobb-Douglas specification of technology.

One of the problems left with the estimated Cobb-Douglas CRTS function is that the coefficient of raw materials is not significantly different from zero at a 1% level. This result is due in part to the high collinearity in the sample, between K and M. In effect, the correlation coefficient between these two variables is .996. Thus, the identification of the separate effect of K and M in production is hopeless without imposing some other constraints in the estimation. Now we will estimate our own output production function using side conditions for cost minimization.

Thus we assume that Bell Canada minimizes cost subject to a technology constraint (a three input Cobb-Douglas function) and a regulatory constraint, i.e.

$$\text{Min} \qquad C = wL + mM + vK$$

$$\text{subject to} \qquad Q = A[L^{\alpha_1} M^{\alpha_2} K^{\alpha_3}]^r e^{\beta \cdot D}$$

$$\text{and} \qquad PQ = wL + mM + sK$$

where $\alpha_1 + \alpha_2 + \alpha_3 = 1.0$ and r is the degree of homogeneity of the production function; C is total cost; w is price of labor services, m is price of raw materials, v is price of capital services. P is price of output; s is the allowed price of capital service.

The minimization of cost subject to the above constraints yields the Lagrangean

$$\phi = wL+mM+vK+\lambda_1(Q-A[L^{\alpha_1}M^{\alpha_2}K^{\alpha_3}]^r e^{\beta \cdot D}) + \lambda_2(PQ-wL-mM-sK).$$

This leads to the following first order conditions

$$\frac{\partial \phi}{\partial L} = (1 - \lambda_2)w - \lambda_1 r\alpha_1 \frac{Q}{L} = 0 \qquad (2)$$

$$\frac{\partial \phi}{\partial M} = (1 - \lambda_2)m - \lambda_1 r\alpha_2 \frac{Q}{M} = 0 \qquad (3)$$

$$\frac{\partial \phi}{\partial K} = v - \lambda_2 s - \lambda_1 r\alpha_3 \frac{Q}{K} = 0 \qquad (4)$$

$$\frac{\partial \phi}{\partial \lambda_1} = Q - A[L^{\alpha_1} M^{\alpha_2} K^{\alpha_3}]^r e^{\beta \cdot D} = 0 \qquad (5)$$

$$\frac{\partial \phi}{\partial \lambda_2} = PQ - wL - mM - sK = 0 \qquad (6)$$

w is measured as the ratio labor payments in current dollars and the number of weighted man hours. m is measured as the ratio of current dollars value of "raw materials" cost and the divisia quantity index of raw materials. v is computed as indicated in Appendix I. P is computed as the ratio of total revenues net of taxes plus uncollectables and the divisia quantity index of output defined above. s is defined as the solution to equation (4).

From equations (2) and (3) we obtain:

$$\frac{\alpha_2}{\alpha_1} = \frac{mM}{wL} \qquad (7)$$

and from (2) and (4) we obtain

$$\frac{\alpha_3}{\alpha_1} = \frac{(v - \lambda_2 s)K}{(1 - \lambda_2)wL} \qquad (8)$$

We proceed to estimate (7) and (8) with the restriction that $\alpha_1+\alpha_2+\alpha_3 = 1.0$. In the estimation we take λ_2 as a constant although in fact it is a variable. Thus, we are estimating some kind of an average value for λ_2 from the joint estimation of these two equations.

The results of the joint estimation of (7) and (8) are:

Coefficient	Estimated Value	t-statistic
α_1	.457	25.11
α_2	.304	25.17
α_3	.239	8.85
λ_2	.588	20.36

We observe from these estimated coefficients that all are statistically significant. Furthermore, λ_2 has a very low standard error and is less than one as required in the Averch-Johnson model.

The value of λ_2 is assumed constant only for the estimation of the production function. When we simulate a complete model of Bell Canada, λ_2 is taken as a variable. To complete the estimation of the production function we replace the estimated values of α_1, α_2 and α_3 in equation (5) above obtaining:

$$Q = A[L^{.457} M^{.304} K^{.239}]^r e^{\beta \cdot D}$$

Then we estimate A, r and β from the regression

$$\ell n\ Q = \ell n\ A + r\ \ell n\ \text{INPUT} + \beta D \qquad (9)$$

where $\text{INPUT} = L^{.457} M^{.304} K^{.239}$.

The estimated values from the regression are:

Coefficient	Estimated Value	t-statistic
A	2.0687	4.355
r	1.0275	21.366
β	.8908	21.368

$R^2 = .999$ DW = 1.52 T = 24

Following we test for CPTS in this function. The null hypothesis of CRTS (r = 1.0) can not be rejected. So we re-estimate (9) subject to CRTS. The estimated values from this regression are:

Coefficient	Estimated Value	t-statistic
A	2.359	161.211
β	.914	77.486

$R^2 = .999$ DW = 1.48 T = 24

Here we are working with original units for the output and the inputs.

Our final production function for the one output case is given by:

$$Q_t = 2.359\ L_t^{.457}\ M_t^{.304}\ K_t^{.239}\ e^{.914D_t} \qquad (10)$$

To complete our testing we test if the set of coefficients estimated in Table 3.4 are statistically different from the values taken by the parameter in equation (10) above. The computed F-value is: 1.25, the 1% F(6,19) is 3.94, therefore the results from Table 3.4 and from equation (10) are not statistically different.

Thus, in the one output case, we can not reject the null hypothesis that the relation between output and factor inputs can be described by a Cobb-Douglas CRTS production function with neutral technical change.

Appendix to Chapter III

The translog function does not satisfy monotonicity neither quasi-concavity globally, therefore, this condition should be checked for an estimated translog function at every data point.

Monotonicity requires $\partial Q/\partial L > 0$, $\partial Q/\partial M > 0$ and $\partial Q/\partial K > 0$; differentiating the translog function we find:

$$F1_j \equiv \frac{\partial Q_j}{\partial L_j} = \frac{Q_j}{L_j} (\alpha_1 + \gamma_{11} \ln L_j + \gamma_{12} \ln M_j + \gamma_{13} \ln K_j)$$

$$F2_j \equiv \frac{\partial Q_j}{\partial M_j} = \frac{Q_j}{M_j} (\alpha_2 + \gamma_{12} \ln L_j + \gamma_{22} \ln M_j + \gamma_{23} \ln K_j)$$

$$F3_j \equiv \frac{\partial Q_j}{\partial K_j} = \frac{Q_j}{K_j} (\alpha_3 + \gamma_{13} \ln L_j + \gamma_{23} \ln M_j + \gamma_{33} \ln K_j)$$

Using these expressions, we compute the relevant partial derivatives, given a set of parameter values, for each sample point of input and output values, in order to check for monotonicity.

A function is strictly quasi-concave (strictly convex isoquants) if the bordered Hessian matrix is negative definite. In the case of three inputs, this requires the bordered principal minors to be positive and negative respectively (see Takayama 1974, p.123).

Differentiating the partial derivatives computed above we obtain expressions of the following form:

$$F11_j \equiv \frac{\partial^2 Q_j}{\partial L_j^2} = \frac{Q_j}{L_j}\left[\gamma_{11} + F1_j \frac{L_j}{Q_j}\left(F1_j \frac{L_j}{Q_j} - 1\right)\right]$$

$$F13_j \equiv \frac{\partial^2 Q_j}{\partial L_j \; \partial K_j} = \frac{Q_j}{L_j \; K_j}\left[\gamma_{13} + F1_j \frac{L_j}{Q_j} F3 \frac{K_j}{Q_j}\right]$$

Similar expressions can be derived for the other inputs. The bordered Hessian matrix is given by:

$$\dot{\bar{H}}_j = \begin{bmatrix} 0 & F1_j & F2_j & F3_j \\ F1_j & F11_j & F12_j & F13_j \\ F2_j & F21_j & F22_j & F23_j \\ F3_j & F31_j & F32_j & F33_j \end{bmatrix}$$

The bordered principal minors of this matrix are computed for every data point on factor inputs.

One of the most important characteristics of a technology is the elasticity of substitution. The Allen elasticity of substitution between L_j and K_j (Allen, 1938, p. 504) is given by:

$$\sigma_{13}^{j} = \frac{F1_j L_j + F2_j M_j + F3_j K_j}{L_j K_j} \left(|R13_j| / |\overline{H}_j| \right)$$

Where $|R13_j|$ is the cofactor of $F13_j$ in $\overline{H}_j$. Analogous expressions can be derived for σ_{11}^{j} σ_{22}^{j} σ_{33}^{j} σ_{12}^{j} and σ_{23}^{j}. These elasticities of substitution must also be computed at every data point.

CHAPTER IV

A PRODUCTION FUNCTION FOR BELL CANADA: THE MULTIPLE OUTPUT CASE

In this chapter, we disaggregate the output variable of Chapter III and proceed to estimate a multiple output production frontier. We did extensive work on the estimation of a translog production frontier, but due to the high collinearity among the variables, we were unsuccessful.[1] Then, we proceeded as in Klein (1947) and Hasenkamp (1976) assuming that the production possibility frontiers are acceptable in inputs and in outputs.

I. The Estimation of an Input Function

Thus, in our study of multiple output production frontiers, we start with the following equality.

$$f(\underline{y}) = h(L,M,K) \qquad (1)$$

which states that a composite of outputs given by $f(\underline{y})$, is produced by some combination of inputs, given by the function $h(L,M,K)$.

The vector $\underline{y}$ has the components (y_1, y_2, y_3) if we are dealing with a three output production frontier; and $(y_1, \overline{y}_2)$ if our function is a two-output one.

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[1] L.R. Christensen, D.W. Jorgenson and L.J. Lau. "Transcendental Logarithmic Production Frontiers". The Review of Economics and Statistics. 55 (February 1973) pp. 28-45

Our outputs are thus the following:

y_1 = revenue from local calls, in constant 1967 dollars.

y_2 = toll calls (excluding Other Toll), calculated as a Divisia quantity index of Intra-Bell, Trans-Canada and US and Overseas Telephone Message Toll Services.

y_3 = revenue from Other Toll, in constant 1967 dollars.

These are the outputs that we use in our three-output production frontier. For the two-output case, we use y_1 and $\bar{y}_2$, where $\bar{y}_2$ is defined as a Divisia aggregator of y_2 and y_3.
Direct estimation of this production frontier is not possible due to the high collinearity among the variables in equation (1). Thus here we estimate the production frontier using side conditions for cost minimization as in Chapter III. We assume that the firm is minimizing cost subject to her production function and to the regulatory constraint; i.e.

$$\text{Min} \quad C = wL+mM+vK$$

subject to: $$f(\underline{y}) = h(L,M,K) \qquad (2)$$

and $$p_1y_1 + p_2y_2 + p_3y_3 = wL+mM+sK \qquad (3)$$

Where C is total cost, L is labour measured in weighted man hours, M is raw materials and K is net capital stock. w, m,v are their market prices. p_1,p_2,p_3 are the prices of the three outputs: y_1 y_2 and y_3. For the solution of this minimization problem, we form the Lagrangean ψ :

$$\psi = wL + mM + vK - \lambda_1 \left[f(\underline{y}) - h(L,M,K) \right] - \lambda_2 \left[p_1 y_1 + p_2 y_2 + p_3 y_3 - wL - mM - sK \right] \quad (4)$$

first order conditions for a minimum of costs are given by (2) and (3) above and the following equations:

$$\frac{\partial \psi}{\partial L} = (1-\lambda_2)\ w - \lambda_1 \ \frac{\partial h}{\partial L} = 0 \quad (5)$$

$$\frac{\partial \psi}{\partial M} = (1-\lambda_2)\ m - \lambda_1 \ \frac{\partial h}{\partial M} = 0 \quad (6)$$

$$\frac{\partial \psi}{\partial K} = v - \lambda_2 \ s - \lambda_1 \ \frac{\partial h}{\partial K} = 0 \quad (7)$$

Following our findings of Chapter III, we specify the function h to be a Cobb-Douglas "input" function:

$$h(L,M,K) = \alpha_o \ e^{\beta D_t} \ (L^{\alpha_1} \ M^{\alpha_2} \ K^{\alpha_3})^r, \text{ with } \alpha_1 + \alpha_2 + \alpha_3 = 1.0 \quad (8)$$

thus $\frac{\partial h}{\partial L} = \frac{r\alpha_1 h}{L}$, $\frac{\partial h}{\partial M} = \frac{r\alpha_2 h}{M}$, $\frac{\partial h}{\partial K} = \frac{r\alpha_3 h}{K}$ which

we replace into equations (5), (6), (7).

From equations (5) and (6), after substituting for $\frac{\partial h}{\partial L}$ and $\frac{\partial h}{\partial M}$ we obtain:

$$\frac{\alpha_2}{\alpha_1} = \frac{mM}{wL} \quad (9)$$

and from equations (5) and (7), after substituting for $\frac{\partial h}{\partial L}$ and $\frac{\partial h}{\partial K}$ we obtain:

$$\frac{\alpha_3}{\alpha_1} = \frac{K}{wL} \frac{\left(v - \lambda_2 \ s\right)}{(1-\lambda_2)} \quad (10)$$

As in Chapter III, we estimate $\alpha_1, \alpha_2, \alpha_3$ and λ_2 from equations (9) and (10) plus the restriction that

$$\alpha_1 + \alpha_2 + \alpha_3 = 1 \qquad (11)$$

Here λ_2 is taken as constant to be estimated with α_1, α_2 and α_3. This is done in order to estimate the α's.

In the simulation part, λ_2 is estimated as a variable coming from the solution of the system of equations.

The results of estimating equations (9), (10), and (11) simultaneously were presented in Chapter III, we reproduce them here:

	t-statistic
$\alpha_1 = .457271$	25.113
$\alpha_2 = .303920$	25.171
$\alpha_3 = .238809$	8.850
$\lambda_2 = .588044$	20.357

Using these estimated α's, we computed a time series that we call $INPUT_t$, as follows:

$$INPUT_t = L_t^{.457271} M_t^{.30392} K_t^{.238809}$$

The variable INPUT is the one used in the estimation of the multiple output production frontiers.

II The Estimation Production Frontiers:

In this section, we describe the estimation of the functions $f(\underline{y})$ that satisfy the following relation:

of the one that will be used in our simulations.

$$f(\underline{y}) = \alpha_0 e^{\beta . D_t} \left[L^{\alpha_1} \; M^{\alpha_2} \; K^{\alpha_3} \right]^r \text{ with } \alpha_1 + \alpha_2 + \alpha_2 = 1$$

i.e. $$f(\underline{y}) = \alpha_0 \, e^{\beta . D_t} \; INPUT_t^{\;r}$$

where D_t is number of calls direct distance dialed and r is the returns to scale coefficient. We go on to present the different functional forms that we tried.

(1) The Powell and Gruen CET output function

Powell and Gruen,[2] following the work of Uzawa,[3] proposed the following Constant-Elasticity-of-Transformation (PG-CET) output function.

$$f(\underline{y}) = (\Sigma \beta_i y_i^{\alpha})^{1/\alpha}$$

if $\beta_i > 0$ and $\alpha > 1$ then the function is convex.

For the two-output case, the function to estimate becomes

$$\alpha_0 e^{\beta . D_t} \; INPUT_t^{\;r} = \beta_1 \, y_1^{\alpha} + \beta_2 \, \overline{y}_2^{\alpha}$$

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2 Powell A.A. and F.H.G. Gruen, "The Constant Elasticity of Transformation Production Frontier and Linear Supply System." International Economic Review. 9 (October 1968), 315-328

3 Uzawa H. "Production Functions with Constant Elasticities of Substitution." Review of Economic Studies. 29 (October 1962), 291-299

See also G. Hasenkamp. "A Study of Multiple-output Production Functions: Klein's railroad study revisited." Journal of Econometrics, Vol. 4, No. 3 (August 1976) pp. 253-262.

Taking logs for estimation purposes, the function is

$$\ln \text{INPUT}_t = \frac{1}{r.\alpha} \log (\beta_1 Y_1^{\alpha} + \beta_2 \overline{Y}_2^{\alpha}) - \frac{1}{r} \log \alpha_0 - \frac{\beta . D_t}{r},$$

Estimating the above equation using a non-linear method, we obtain the following

	Estimated Coefficient	t-statistic
r	1.1596	2.985
α	-8.6242	-.058
β_1	.9998	61.955
β_2	.0002	.175
α_0	.6461	.546
β	.6439	1.631
LLF	32.5062	
R^2	.9630	
DW	.1121	

This result is not satisfactory, mainly because α is less than one. Extensive experimentation was carried out with this functional form, but it proved quite difficult to handle, especially due to the high degree of non-linearity that the formula has.

We estimated the PG-CET function for three outputs. As a normalization rule for identification, we imposed the condition that the sum of the β's be equal to one. We incorporated this restriction in our estimating formula as follows.

$$\ell n \text{ INPUT} = \frac{1}{r \cdot \alpha} \log (\beta_1 Y_1^{\alpha} + \beta_2 Y_2^{\alpha} + (1-\beta_1-\beta_2) Y_3^{\alpha}) - \frac{1}{r} \log \alpha_0 - \frac{\beta \cdot D_t}{r}$$

Estimating the above formula with a non-linear method, we obtain:

Coefficients	Estimated Values	t-statistic
r	1.6701	5.478
α	-1.9621	-2.078
β_1	.97325	19.728
β_2	.02649	.543
β_3	.00026	*
α_0	.0528	.673
β	.3486	1.546

LLF	69.2964
R^2	.9983
DW	2.2636
SSR	.00327249

From the estimated values we have $\alpha<1$, then the function that we have obtained is non-convex.

* The t-value can be calculated from the variance-covariance matrix or estimating again using $(1-\beta_1-\beta_3)$ instead of β_2.

(2) The Diewert Function

W.E. Diewert[4] defines a functional form for the production frontier as follows:

Let B be a symmetrix matrix with the following properties:

(i) B is an M by M positive semi-definite symmetric matrix

(ii) there exists a vector $\underline{y}^*$ with each component positive such that $B\underline{y}^* \geqslant \underline{0}$ (where $\underline{0}$ is a M dimensional vector of zeros), and

(iii) if $\underline{y} > \underline{0}$ and $B\underline{y} \geqslant \underline{0}$ then $\underline{y}'\ B\underline{y} > 0$

If B satisfied the above conditions, then the following quadratic square rooted, homogeneous of degree one production frontier is defined:

$$F(\underline{y}) \equiv (\underline{y}'B\underline{y})^{\frac{1}{2}} \text{ for } \underline{y} \geqslant \underline{0} \text{ such that } B\underline{y} \geqslant \underline{0}$$

If B is either positive definite or semidefinite matrix, the transformation curves are convex sets.[5]

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[4] Diewert W.E. "Functional Forms for Revenue and Factor Requirements Functions." International Economic Review, Vol. 15, No. 1, Feb. 1974, p. 119-130.

[5] For the mathematical foundations of the relationships among matrices and convexity and concavity, see

A. Benavie. Mathematical Techniques for Economic Analysis (Prentice-Hall, 1972)

and Rockafellar, R.T., Convex Analysis (Princeton University Press, 1970).

We used the Diewert functional form to estimate the production function for the two output case. The equation to be estimated can be written as

$$\left[[y_1 \bar{y}_2] \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{12} & \beta_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ \bar{y}_2 \end{bmatrix} \right]^{\frac{1}{2}} = \alpha_o e^{\beta D_t} \text{INPUT}^r$$

which can be estimated using logs as follows:

$$\ell n \text{ INPUT} = \frac{1}{2r} \ell n \left(\beta_{11} y_1^2 + 2\beta_{12} y_1 \bar{y}_2 + \beta_{22} \bar{y}_2^2\right) - \frac{1}{r} \ell n \, \alpha_o \frac{\beta}{r} D_t$$

From the estimation, using $\alpha_o = 1.0$ as a normalization rule, we obtained the following results:

Cofficients	Estimated Values	t-statistics
r	1.115	22.424
β_{11}	.5366	1.174
β_{12}	1.1709	1.169
β_{22}	-1.4842	-1.158
β	.6992	8.963
LLF	65.6482	
R^2	.9976	
DW	2.1194	

From our results above, it can be seen that the matrix B is not positive semi-definite, since $\hat{\beta}_{22}$ is negative.

(3) Translogarithmic Production Frontiers: The two-output case

We have already presented the translog functional form in our section on one-output (several inputs) production functions. In the two-output case, the $f(\underline{y})$ takes the following form:

$$f(\underline{y})=\beta_1 \ln y_1+\beta_2 \ln \overline{y}_2+\tfrac{1}{2}\delta_{11}(\ln y_1)^2+\tfrac{1}{2}\delta_{22}(\ln \overline{y}_2)^2+\delta_{12}(\ln y_1)(\ln \overline{y}_2)$$

where y_1 and $\overline{y}_2$ are normalized variables and the $\underline{y}$ is a Divisia index of y_2 and y_3. Equating $h(x)= f(y)$ and introducing $\beta_1+\beta_2= 1$ as a normalization rule, the equation to estimate becomes:

$$\ln \text{INPUT}_t=\frac{1}{r}\left(\beta_1 \ln y_1+\beta_2 \ln \overline{y}_2+\tfrac{1}{2}\delta_{11}(\ln y_1)^2+\tfrac{1}{2}\delta_{22}(\ln \overline{y}_2)^2+\delta_{12}(\ln y_1)(\ln \overline{y}_2)\right)-\frac{1}{r}\ln \alpha_0-\frac{\beta . D_t}{r}$$

Following are the results of estimating the above two-output translog production frontier (with $\beta_1+\beta_2=1$)

Coefficients	Estimated Values	t-statistics
r	1.145	6.026
β_1	.99429	2.174
β_2	.00571	.012
δ_{11}	-2.7706	-.349
δ_{22}	-1.3592	-.361
δ_{12}	1.9104	.349
α_0	.00196	.947
β	.6256	3.176
LLF	65.0287	
R^2	.9977	
DW	2.2230	
SSR	.00441072	

Finally, we restrict the two-output production frontier to exhibit Constant Elasticity of Transformation; which implied the following restrictions on the parameters (symmetry imposed)

$$\beta_1 + \beta_2 = 1 \qquad \text{(a)}$$

$$\delta_{11} + \delta_{12} = 0 \qquad \text{(b)}$$

$$\delta_{12} + \delta_{22} = 0 \qquad \text{(c)}$$

Restriction (a) was used before as normalization rule; restrictions (b) and (c) imply

$$\delta_{11} = \delta_{22} = -\delta_{12}$$

So that our two-output CET translog function becomes for estimation:

$$\ell n\ (\text{INPUT}_t) = \frac{1}{r}\ (\beta_1 \ell n y_1 + \beta_2 \ell n \overline{y}_2 + \tfrac{1}{2}\delta_{11}((\ell n y_1)^2 + (\ell n \overline{y}_2)^2)$$

$$-\delta_{11}(\ell n y_1)\ (\ell n \overline{y}_2) - \frac{1}{r}\ \ell n\ \alpha_0 - \frac{\beta . D_t}{r}$$

subject to $\beta_1 + \beta_2 = 1$

Estimating the previous equation we obtain:

Coefficients	Estimated Values	t-statistics
r	1.1330	27.366
β_1	.9337	9.858
β_2	.06628	.700
$\delta_{11}=\delta_{22}=-\delta_{12}$	-.4086	-1.942
α_0	.00203	4.839
β	.68344	9.570
LLF	69.5282	
R^2	.9981	
DW	2.2174	

When we tested for a CET translog frontier, the computed F was .1658, the 1% F(2,17) is 6.11. Thus, we cannot reject the null hypothesis. We have also found that the function is not convex, therefore its level sets are not convex sets. We also estimated three output translog frontiers, but there was always too much collinearity to allow for precise estimation of individual coefficients (for details see appendix to this chapter).

Thus, in the multiple output simulation model the two-output CET frontier is used.

APPENDIX ONE TO CHAPTER IV

Notes on the results of the three output translog production function (see Table at the end of the Appendix)

Column (1): This is an unrestricted translogarithmic production function. The only restriction imposed is $\beta_1 + \beta_2 + \beta_3 = 1$, as a normalization rule. For this equation it was very difficult to obtain a convergence of the non-linear estimation procedure used. This was probably due to the large number of parameters involved. Notice that the convergence that was obtained is quite far from the rest of our results, thus one would not put too much faith on the coefficients obtained. This result, however, can be used for comparison (mainly of the logs of the likelihood function (LLF), and the sums of square residuals (SSR). The t-values beneath the estimated values for r correspond to the hypothesis that $r \neq 1$, while the other t-values correspond to the hypotheses that the corresponding coefficient is different from zero.

Column (2): Here we restrict $\delta_{33} = 0$. Notice that now the estimated parameters, specially for r and β, are much more plausible while their t-values improved as well.

Column (3): Here we have $\delta_{33} = \delta_{22} = 0$. Notice that from column (2) on the results for r and β are quite stable, indicating the presence of increasing returns to scale as usually we cannot reject the hypothesis that $r > 1$.

Column (4): $\delta_{33} = \delta_{22} = \delta_{11} = 0$.

Column (5): $\delta_{33} = \delta_{22} = \delta_{11} = \delta_{23} = 0$.

Column (6): $\delta_{33} = \delta_{22} = \delta_{11} = \delta_{23} = \delta_{13} = 0$.

Column (7): $\delta_{12}=\delta_{13}=\delta_{23}=0$. That is, the function is completely globally separable (TLGS). To test whether we cannot reject the hypothesis of complete global separability, we use the following statistic:

$$F= \frac{\dfrac{SSR^{TLGS} - SSR^{TL}}{3}}{\dfrac{SSR^{TL}}{25-(11-3)}} = \frac{\dfrac{.00327748 - .00310466}{3}}{\dfrac{.00310466}{25-8}} = .4639$$

Column (8): In addition to $\beta_1+\beta_2+\beta_3=1$ we impose here r=1.4. This is similar to column one except for the additional restriction on r. This regression was run since we had problems converging to reasonable values in Column (1)

Column (9): This regression imposes the translog function to have Constant Elasticity of Transformation (TL-CET). CET requires the following restrictions on the parameters (with symmetry imposed).

$$\beta_1+\beta_2+\beta_3 = 1 \qquad \text{(a)}$$

$$\delta_{11}+\delta_{12}+\delta_{13} = 0 \qquad \text{(b)}$$

$$\delta_{12}+\delta_{22}+\delta_{23} = 0 \qquad \text{(c)}$$

$$\delta_{13}+\delta_{23}+\delta_{33} = 0 \qquad \text{(d)}$$

Testing for the existence of CET we accept it, as our calculated F statistic is .532 while the $F_{(3, 17)} = 5.18$

Column (10): This is a Cobb-Douglas on outputs production function. Here $\delta_{11}=\delta_{22}=\delta_{33}=\delta_{12}=\delta_{13}=\delta_{23}=0$. Also, as noted before $\beta_1+\beta_2+\beta_3= 1$, this is a CET production function. It is also completely globally separable since the CET restrictions (b), (c), (d) together with the restriction of $\delta_{12}=\delta_{13}=\delta_{23}= 0$ imply $\delta_{11}=\delta_{22}=\delta_{33}=\delta_{12}=\delta_{13}=\delta_{23}= 0$.

We apply an F test to test whether, having accepted CET (Column (9)), we can accept (cannot reject) the hypothesis that the function is Cobb-Douglas. The calculated F statistic is:

$$F = \frac{\dfrac{SSR^{CDCET} - SSR^{TLCET}}{3}}{\dfrac{SSR^{TLCET}}{25-(11-3)}} = \frac{\dfrac{.00373699 - .00366699}{3}}{\dfrac{.00366699}{17}} = .1082$$

Thus we cannot reject the hypothesis that the function is a CET - Cobb-Douglas. As noted before, we began allowing for convexity of the level sets (product transformation curves). However, we end up with a function that has non-convex level sets (convex to the origin production possibilities frontiers), as illustrated below in the space (y_1,y_2)

Here the firm, if it could, would produce only y_2. This fact, together with the observed presence of increasing returns to scale, constitutes quite a strong case for regulation of the firm in question. Similar results were obtained by G. Hansenkamp in his study of the american railroads.

$$\ln \text{INPUT}_t = \frac{1}{r}(\beta_1 \ln y_1 + \beta_2 \ln y_2 + \beta_3 \ln y_3 + \frac{1}{2}\delta_{11}(\ln y_1)^2 + \frac{1}{2}\delta_{22}(\ln y_2)^2 + \frac{1}{2}\delta_{33}(\ln y_3)^2 + \delta_{12}(\ln y_1)(\ln y_2) + \delta_{13}(\ln y_1)(\ln y_3) + \delta_{23}(\ln y_2)(\ln y_3)$$

Subject to: $\beta_1+\beta_2+\beta_3 = 1$; for additional restrictions see the following pages.

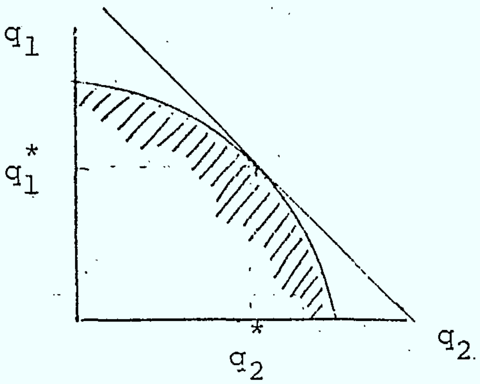
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
r	8.493 (.153)	1.478 (1.060)	1.464 (1.103)	1.406 (1.855)	1.439 (1.978)	1.459 (3.025)	1.443 (2.039)	1.4 ---	1.4718 (1.639)	1.37[illegible]8 (3.013)
β_1	-5.702 (-.123)	.898 (1.418)	.974 (2.474)	1.030 (5.336)	.925 (6.621)	.920 (6.927)	.992 (5.862)	.972 (2.180)	.7929 (4.923)	.8371 (7.2[illegible]0)
β_2	3.038 (.137)	-.100 (-.200)	-.161 (-.529)	-.196 (-.964)	-.153 (-.775)	-.144 (-.796)	-.173 (-.855)	-.153 (-.381)	.0406 (.264)	-.0[illegible]39 (.5[illegible]7)
β_3	3.664 (.104)	.202 (.873)	.187 (.928)	.166 (1.229)	.208 (1.856)	.224 (2.011)	.181 (1.276)	.181 (1.120)	.1665 (1.025)	.1[illegible]99 (2.4[illegible])
δ_{11}	120.972 (.149)	2.395 (.219)	.916 (.170)				.449 (.934)	1.290 (.158)	.1368 (.103)	
δ_{22}	.574 (.020)	.790 (.162)					-.0865 (-.432)	.484 (.105)	-.1210 (-.053)	
δ_{33}	7.662 (.145)						-.0270 (-.472)	.0415 (.093)	-.0147 (-.107)	
δ_{12}	-29.184 (-.145)	-1.263 (-.175)	-.164 (-.078)	.190 (.869)	.061 (.491)	.075 (1.310)		-.548 (-.088)	-.0153 (-.009)	
δ_{13}	-33.662 (-.145)	-.121 (-.096)	-.092 (-.078)	.106 (.735)	.0106 (.127)			-.025 (-.022)	-.1215 (-.241)	
δ_{23}	11.495 (.143)	.046 (.045)	.014 (.016)	-.141 (-.760)				-.107 (-.199)	.1363 (.228)	
α_0	.0000 (.004)	.0003 (.419)	.0003 (.454)	.0005 (.897)	.0004 (.888)	.0004 (1.245)	.0004 (.911)	.0005 (6.174)	.00034 (.684)	.0[illegible]058 (1.5[illegible]8)
β	4.095 (.175)	.726 (2.565)	.6898 (4.095)	.682 (4.490)	.732 (5.459)	.719 (9.311)	.681 (4.063)	.731 (2.408)	.6749 (3.249)	.6[illegible]34 (10.0[illegible]7)
LLF	69.7004	69.9429	70.7274	71.4608	71.7706	72.4355	71.4502	69.9217	70.0465	71.8416
R	.9987	.9986	.9986	.9986	.9986	.9986	.9986	.9986	.9984	.9984
DW	2.6396	2.6788	2.6779	2.6961	2.7062	2.6880	2.6822	2.7028	2.4486	2.3992
SSR	.00310466	.00326252	.00326832	.00327469	.00338246	.00338541	.00327748	.00326805	.00366699	.00373699

APPENDIX TWO TO CHAPTER IV

A Note on Concavity and Convexity of Production Functions

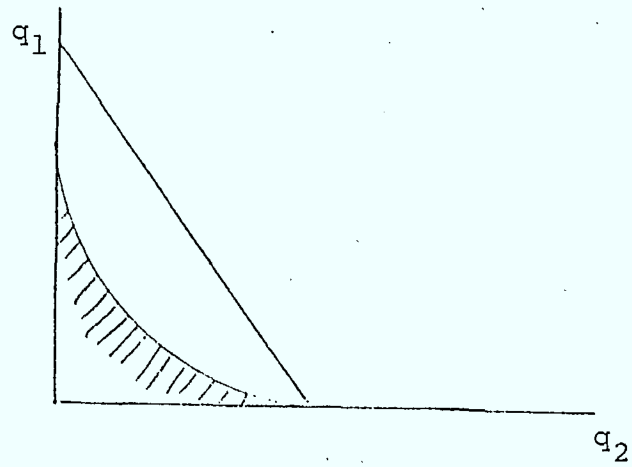
Before going on to the estimation of multiple output production functions, we describe some of their characteristics.

Multiple output production functions are usually assumed to have convex (concave to the origin) level sets or transformation curves, calculated for a given input vector. Convex level sets are required to allow for a competitive equilibrium where all the outputs are produced.

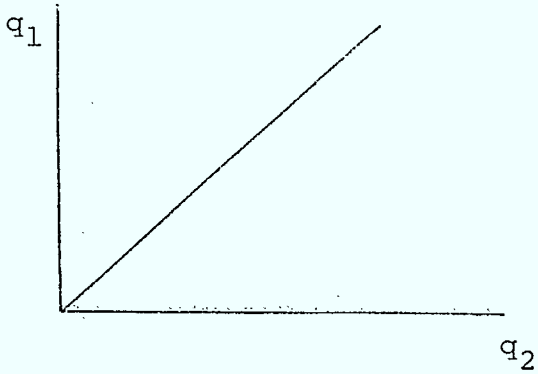


A convex (i.e. concave to the origin) production possibility frontier allows for an equilibrium with both q_1^* and q_2^* greater than zero.

On the other hand, if the production possibility frontier among the outputs are convex to the origin (non-convex sets) then in general, only one output will be produced, unless the industry (or the firm) is forced, say by regulation, to produce non-zero quantities of all the outputs.



Finally, the following cannot be a production possibility frontier, because it would mean that both outputs could be increased indefinitely with a given level of inputs.



For an output function to have convex level sets, it is necessary for the function to be quasi-convex.

Conversely, quasi-concave functions have non-convex level sets.

L.R. Klein proposed a multiple (two) output production function of the form

$$y_1 y_2^{\delta} = A\, x_1^{\alpha_1}\, x_2^{\alpha_2}\, x_3^{\alpha_3}$$

where α_j , δ and A are parameters. This is a Cobb-Douglas in output, Cobb-Douglas in input (CD-CD).

The trouble with the CD in output function is that it is not convex, as pointed out by Nerlove**. What one needs are functional forms, that allow for convexity (and non-convexity).

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* Klein L.R. 1947. The use of cross-section data in econometrics with application to a study of production of railroad services in the United States, mimeo (National Bureau of Economic Research, Wash. D.C.)

** Marc Nerlove, 1965. Estimation and identification of Cobb-Douglas production functions (North-Holland, Amsterdam).

We are going to use functions that are flexible enough to be either convex or non-convex, and see what the results tell us about the production characteristics of Bell Canada.

CHAPTER V

A SIMULATION MODEL OF BELL CANADA: ONE OUTPUT PRODUCTION FUNCTION

In the development of a one output production function model of Bell, we proceed in two stages. First, we develop a model in which output is taken as exogenous and the firm is assumed to minimize cost. The advantage of this model is that it allows us to study the Averch-Johnson (A-J) effect directly. Furthermore, it does not require knowledge of price elasticities and therefore the results are robust to the specification of demand functions used. In a second stage, we develop a model where the firm is assumed to maximize its profits subject to the one output production technology and the regulatory constraint. For the profit maximization case we need demand equations. In Chapter II, we found that the demand for local services was price inelastic. Thus, in the profit maximization case we take the quantity of local services as exogenous and we solve for the optimal quantity of non-local services (Telephone Message Toll and Other Toll Services).

I The Cost Minimization Model

The one output production function cost minimization model was already discussed when we estimated the production function in Chapter III. Here we will renumber and reproduce the equations for easy reference:

$$(1-\lambda_2)\ w - \lambda_1 r\alpha_1 \ \frac{Q}{L} = 0 \qquad (1)$$

$$(1-\lambda_2)\ m - \lambda_1 r\alpha_2 \ \frac{Q}{M} = 0 \qquad (2)$$

$$v-\lambda_2 \ \ s - \lambda_1 r\alpha_3 \ \frac{Q}{K} = 0 \qquad (3)$$

$$Q - A \left[L^{\alpha_1} M^{\alpha_2} K^{\alpha_3}\right]^r e^{\beta D} = 0 \qquad (4)$$

$$PQ+R6 - wL-wM-sK = 0 \qquad (5)$$

where

Q= Output, Divisia quantity index of local services, Intra-Bell, Trans Canada, US and Overseas and Other Toll Services.

R6 = Miscellaneous revenue in current dollars

w= Price Index of labor services

m= Divisia Price index of raw materials

v= Price Index of capital services

L= Quantity of labor

M= Divisia Quantity Index of raw materials

K= Quantity of capital

s= "allowed price of capital services"

λ_1= Lagrangian multiplier of the technology constraint

λ_2= Lagrangian multiplier of the technology constraint

The estimated value of the parameters, obtained in Chapter III, are:

$r=1$, $\alpha_1= .457$, $\alpha_2= .304$, $\alpha_3=.239$, $A=2.359$ and $\beta=.914$.

Equations (1) to (5) form a system of five equations in five endogenous variables: $L,M,K,\lambda_1,\lambda_2$. The exogenous variables of this system of equations are P,Q,R6,w,m,v and s.

We solve this system of equations first. Then, we study how it described the behavior of Bell in choosing inputs for a given output and then finally, we compare the results with the one obtained in the case of no regulation. That is, we solve the above model imposing $\lambda_2=0$.

From the comparison of the solution with $\lambda_2=0$ and the one for λ_2 free, we obtain a measure of the effects of regulation on the choice of factor inputs (The A-J effect).

The solution of the system of equations (1) to (5) appears in Tables 5.1 and 5.2. To facilitate comparisons, we have printed next to the simulated values of the endogenous variables, their historical values.

Table 5.1

One Output Cost Minimization Model with Regulatory Constraint: L y K

	L	LS	K	KS
1953	46.1000	37.0932	690.400	828.869
1954	48.2000	39.7972	764.900	905.348
1955	51.9000	43.3918	871.300	1025.86
1956	55.7000	48.7287	989.900	1123.77
1957	57.8000	53.3416	1127.10	1215.05
1958	57.6000	52.9961	1280.00	1378.43
1959	56.5000	54.5758	1429.50	1466.83
1960	54.6000	52.3975	1579.10	1624.23
1961	52.4000	50.6060	1721.90	1760.06
1962	52.3000	53.6496	1860.10	1831.17
1963	53.5000	52.9579	2004.40	2016.48
1964	54.4000	52.9471	2150.40	2182.32
1965	55.8000	54.4612	2283.60	2312.91
1966	57.5000	57.5359	2431.20	2430.33
1967	56.6000	60.3887	2585.60	2496.74
1968	55.5000	60.1141	2734.00	2620.61
1969	56.6000	62.4323	2886.00	2732.25
1970	57.8000	62.2822	3054.80	2932.78
1971	58.1000	61.1178	3190.40	3099.93
1972	57.5000	60.6382	3334.90	3242.23
1973	60.4000	64.4057	3494.00	3365.62
1974	63.9000	69.3636	3653.50	3478.21
1975	64.1000	72.3551	3808.90	3583.90
1976	67.3000	72.1101	3978.90	3814.96
	1	2	3	4

Table 5.2

One Output Cost Minimization Model with Regulatory Constraint:

	M	MS	LAM1S	LAM2S
1953	51.8164	60.0111	.199587	.705545
1954	57.4713	66.1281	.142833	.753365
1955	65.7304	74.7874	.260744	.639761
1956	75.8391	82.8537	.224100	.696750
1957	78.8253	92.7082	.126543	.835457
1958	86.5843	95.8538	.192338	.743094
1959	91.9356	101.428	.354787	.524710
1960	97.8976	102.220	.231662	.674455
1961	103.770	104.065	.295588	.564164
1962	110.847	113.481	.348971	.485506
1963	116.983	113.788	.262265	.597502
1964	118.208	113.873	.303801	.510372
1965	128.536	116.913	.245349	.552625
1966	136.274	125.274	.172893	.713152
1967	137.920	136.561	.225054	.646601
1968	144.717	142.714	.256882	.555789
1969	168.965	153.884	.255216	.605409
1970	168.853	162.413	.358394	.455079
1971	195.550	169.436	.343761	.486231
1972	194.922	181.385	.308969	.563202
1973	209.050	189.380	.305848	.573031
1974	209.669	193.722	.321449	.592741
1975	207.925	208.686	.395089	.525923
1976	225.593	208.037	.419635	.504252
	1	2	3	4

We observe, from these tables that the model described extremely well the input levels of Bell Canada, for the whole 1952 - 1976 period. Furthermore, λ_2 the Lagrangian multiplier of the regulatory constraint is always less than v/s. The condition $\lambda_2<v/s$ is needed for the firm to face downward sloping iso-cost lines. Thus, we have found that a cost minimization model described very well, for a given output, the input levels chosen by Bell Canada.

Using as benchwork the simulated values of Tables 5.1 and 5.2 we proceed now to solve our system of equations with the restriction $\lambda_2=0$. That is, we solve for the cost minimizing input mix assuming no regulation. The results of this simulation appear in Tables 5.3 and 5.4.

From the results of Tables 5.3 and 5.4 we observe the standard A-J effect. When regulatory constraint is eliminated labor increases (17.9% in 1976), capital decreases substantially (40.9% in 1976), and " raw materials " increases (17.9% in 1976).

In chapter VI we will compare these results with the ones obtained for the multiple output production cost.

Table 5.3

One Output Cost Minimization Model without Regulatory Constraint: L and K

	LB	LS	KB	KS
1953	37.0932	45.0915	828.869	444.814
1954	39.7972	48.4651	905.348	483.027
1955	43.3918	50.9225	1025.86	615.950
1956	48.7287	56.8582	1123.77	687.198
1957	53.3416	63.0303	1215.06	713.735
1958	52.9961	62.4450	1378.43	817.064
1959	54.5758	63.3512	1456.83	911.555
1960	52.3975	63.7900	1624.23	867.555
1961	50.6060	61.1040	1760.06	965.102
1962	53.6496	63.2717	1831.13	1083.32
1963	52.9579	64.9967	2016.48	1049.42
1964	52.9471	64.7522	2182.32	1148.93
1965	54.4612	68.5128	2312.91	1112.74
1966	57.5359	73.7498	2430.38	1101.51
1967	60.3887	75.4167	2496.74	1229.54
1968	60.1141	74.2974	2620.61	1334.01
1969	62.4323	76.3251	2732.29	1440.09
1970	62.2822	73.9474	2932.78	1695.77
1971	61.1178	72.8728	3059.92	1769.44
1972	60.3382	72.9311	3242.23	1771.94
1973	64.4057	77.9976	3365.62	1928.22
1974	69.3636	82.9075	3473.21	1967.08
1975	72.3551	84.8712	3533.90	2125.15
1976	72.1101	85.0347	3814.96	2255.62
	1	2	3	4

Note B indicates benchwork values (simulated values from Table 5.1) and S indicates simulated values.

Table 5.4

One Output Cost Minimization Model without Regulatory Constraint:

	MB	MS	LAM1B	LAM1S
1953	66.0111	72.9525	.199587	.835313
1954	56.1281	80.5335	.142833	.862644
1955	74.7874	87.7681	.260744	.849415
1956	82.8537	96.6780	.224100	.867996
1957	93.7082	109.552	.126543	.908723
1958	95.8598	112.946	.192338	.882143
1959	101.428	117.738	.354787	.856486
1960	102.220	124.447	.231662	.866434
1961	104.065	125.655	.295588	.818899
1962	113.481	133.835	.348971	.801959
1963	113.788	139.656	.262265	.800511
1964	113.873	139.263	.303801	.758810
1965	116.913	147.079	.245349	.756545
1966	125.274	160.580	.172893	.786399
1967	136.561	170.547	.225054	.795299
1968	142.714	176.387	.256882	.785456
1969	153.684	188.130	.255216	.790710
1970	162.413	192.833	[illegible]	.780859
1971	169.436	202.025	.343761	.797733
1972	181.385	219.243	.300669	.854974
1973	189.880	229.347	.305848	.877771
1974	193.722	231.549	.321449	.920865
1975	208.686	244.786	.395089	.977545
1976	208.037	245.325	.419635	.998193

II. The Profit Maximization Model

As we saw in chapter two, the demand for local services is price inelastic. Thus, the marginal revenue from local services is negative. This important feature of the operation of Bell Canada has to be incorporated into a profit maximization model of a carrier. In the development of the one output production function model, we have a composite output which is a quantity index of local and non-local services.[1] Furthermore, the quantity of local services provided by Bell is considered as exogenous. That is, firm's decisions about changes in total output are carried out only through changes in non-local services (i.e. toll and other services).

The one output characteristic of the model is given by the specification of technology where labor, raw materials and capital inputs are combined through a translog production function to produce a composite commodity.

The second main characteristic of the model is that the production of non-local services and the input level - labor hiring, raw materials use and capital use - are computed simultaneously from the side conditions - for profit maximization.

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[1] To facilitate comparisons within the results of Chapter VI, miscellaneous revenues are left out of the production function, but they are taken as an exogenous variable and included in the measure of revenue.

In the model that we use the firm is supposed to maximize profits subject to technology and a rate of return constraint. The firm produces a composite output (Q) which is the sum of local services in constant dollars (Q_L) and a divisia quantity index of non-local services (Q_{NL}). Output is produced with: labor (L), raw materials (M) and capital (K). We assume that the firm hire factors at fixed prices. Thus, our model can be formulated in the following way:

$$\text{Max Profits} = P_L\, Q_L + P_{NL}\, Q_{NL} - wL - mM - vK \qquad (6)$$

subject to a technology constraint:

$$F[(Q_L + Q_{NL}),\ L,M,K] = 0 \qquad (7)$$

and to a regulatory constraint

$$P_L Q_L + P_{NL}\, Q_{NL} + R6 - wL - mM = sK \qquad (8)$$

where the new variables introduced are

P_L = Price Index of local services, 1967 = 1.0

P_{NL} = Divisia Price Index of non local services, 1967 = 1.0

Q_L = Quantity of local services, in 1967 dollars

Q_{NL} = Divisia Quantity index of non-local services claimed as $Q - Q_L$ base = 1967 value.

We assume that the firm chooses Q_{NL}, L, M, and K to maximize the level of profit. On the other hand Q_L is exogenous. The Lagrangian for this problem can be written as:

$$\Omega = P_L Q_L + P_{NL} Q_{NL} - wL - mM - vK$$
$$-\mu_1 [P_L Q_L + P_{NL} Q_{NL} + R6 - wL - mM - sK]$$
$$-\mu_2 [F([Q_L + Q_{NL}], L, M, K)]$$

The first order conditions for this problem are given by:

$$\frac{\partial \Omega}{\partial Q_{NL}} = P_{NL} [1 + \frac{1}{\eta_{NL}}] (1-\mu_1 - \mu_2 \frac{\partial F}{\partial (Q_L + Q_{NL})} = 0 \qquad (9)$$

$$\frac{\partial \Omega}{\partial M} = -(1-\mu_1) w - \mu_2 \frac{\partial F}{\partial} = 0 \qquad (10)$$

$$\frac{\partial \Omega}{\partial} = -(1-\mu_1) m - \mu_2 \frac{\partial F}{\partial} = 0 \qquad (11)$$

$$\frac{\partial \Omega}{\partial K} = - v + \mu_1 s - \mu_2 \frac{\partial F}{\partial} = 0 \qquad (12)$$

$$P_L Q_L + P_{NL} Q_{NL} = wL + mM + sK \qquad (13)$$

$$F[(Q_L + Q_{NL}), L, M, K] = 0 \qquad (14)$$

Where μ_1 and μ_2 are lagrangian multipliers and η_{NL} is the price elasticity of demand for non-local services.

Adding to these first order conditions the demand equation for non-local services we obtain a system of seven equations in seven unknowns: Q_{NL}, P_{NL}, L, M, K, μ_1 and μ_2.

We can get rid of μ_2 by working with equation (9) to (12): Thus,

$$(7) \quad \frac{\partial (Q_1 + Q_n)}{\partial L} = - \frac{\frac{\partial F}{\partial L}}{\frac{\partial F}{\partial (Q_L + Q_{NL})}} = \frac{w}{P_{NL} (1 + \frac{1}{\eta_{NL}})} \qquad (15)$$

$$(8)\quad \frac{\partial(\Omega_L+\Omega_{NL})}{\partial M} = -\frac{\frac{\partial F}{\partial M}}{\frac{\partial F}{\partial(\Omega_L+\Omega_{NL})}} = \frac{m}{P_{NL}\left[1+\frac{1}{\eta_{NL}}\right]} \qquad (16)$$

$$(9)\quad \frac{\partial(\Omega_L+\Omega_{NL})}{\partial K} = -\frac{\frac{\partial F}{\partial K}}{\frac{\partial F}{\partial(\Omega_L+\Omega_{NL})}} = \frac{v-\mu_1 s}{(1-\mu_1)P_{NL}\left[1+\frac{1}{\eta_{NL}}\right]} \qquad (17)$$

The system of equations (13), (14), (15), (16), (17) and the demand equation for non-local services conform a system of six equations in six unknowns: Ω_{NL}, P_{NL}, L, M, K and μ_1.

CHAPTER VI.

A SIMULATION MODEL OF BELL CANADA: TWO OUTPUT PRODUCTION FRONTIER

In this chapter we extend the model of Chapter V by disaggregating the one output production function into a two output production frontier. This chapter is divided into two sections. In the first section we present and simulate a cost minimization model and in the second section we present and simulate a profit maximizing model.

1 The Cost Minimization Model

We already discussed in Chapter IV, in the context of the estimation of a production frontier, a cost minimization model of Bell Canada. Here, for easy reference, we will renumber and reproduce the equation.

If a firm minimizes cost subject to a production frontier and a regulatory constraint, then its choice of inputs for a given vector of outputs is restricted to the following set of equations:

$$(1-\lambda_2)\ w - \lambda_1 \frac{r\alpha_1 h}{L} = 0 \qquad (1)$$

$$(1-\lambda_2)\ m - \lambda_1 \frac{r\alpha_2 h}{M} = 0 \qquad (2)$$

$$v - \lambda_2 s - \lambda_1 \frac{r\alpha_3 h}{K} = 0 \qquad (3)$$

$$\beta_1 \ln \frac{y_1}{a_1} + \beta_2 \ln \frac{y_2}{a_2} + \tfrac{1}{2}\delta_{11}\left(\ln \frac{y_1}{a_1}^2 + \ln \frac{y_2}{a_2}^2 - \ln \frac{y_1}{a_1} \ln \frac{y_2}{a_2}\right) - (\ln \alpha_0 + \beta D$$

$$+ r(\alpha_1 \ln L + \alpha_2 \ln M + \alpha_3 \ln K)) = 0 \qquad (4)$$

$$P_1 y_1 + R6 = wL + mM + sK \qquad (5)$$

with $h = \alpha_0 E^{\beta D}[L^{\alpha_1} M^{\alpha_2} K^{\alpha_3}]^r$, as defined in Chapter IV.

where the new symbols introduced are:

$\bar{y}_2$ = Divisia quantity index of the four Toll Services (three telephone message tolls and other toll)

$\bar{P}_2$ = Current dollar value of Toll services divided by y_2

λ_1 = Lagrangian multiplier of the technology constraint

λ_2 = Lagrangian multiplier of the regulatory constraint

a_1 = Mean value of y_1 in the sample

a_2 = Mean value of $\bar{y}_2$ in the sample

The estimated value of the parameters, obtained in Chapter IV and V, are:

$r=1.133$, $\alpha_1=.457$, $\alpha_2=.304$, $\alpha_3=.239$, $\alpha_o=.00203$,
$\beta=.6834$ and $\delta_{11}=-.4086$

Given the vector of outputs, equations (1) to (5) form a system of five equations in five unknowns. The unknowns are L, M, K, λ_1 and λ_2. The exogenous variables of this system are $P_1, y_1, \bar{P}_2, \bar{y}_2$, R6, w,m,v, and s.

To validate our cost minimization model we proceed now to simulate it for the period 1952 - 1976. The results of the simulations appear in Tables 6.1 and 6.2. From these results we see that the multiple-output cost minimization model tracks even better than the one output cost minimization model. This is especially so for for the capital stock variable. In general, the results are very

Table 6.1

Multiple Output Cost Minimization Model with Regulatory Constraint: L and K

	L	LS	K	KS
1953	46.1000	40.3607	690.400	693.490
1954	48.2000	42.1228	764.900	782.767
1955	51.9000	44.7161	871.300	922.357
1956	55.7000	48.8559	999.900	1067.05
1957	57.8000	53.3560	1127.10	1109.85
1958	57.6000	52.2772	1280.00	1331.20
1959	56.5000	53.8220	1429.50	1425.32
1960	54.6000	52.4848	1579.10	1551.08
1961	52.4000	51.5696	1721.90	1723.91
1962	52.3000	54.0441	1860.10	1759.37
1963	53.5000	53.9254	2004.40	2002.52
1964	54.4000	53.7973	2150.40	2180.66
1965	55.8000	54.9961	2283.60	2371.38
1966	57.5000	57.6582	2431.20	2502.34
1967	56.6000	60.3921	2585.60	2505.98
1968	55.5000	60.1114	2734.00	2634.50
1969	56.6000	60.1805	2886.00	2939.25
1970	57.8000	61.3093	3054.80	3021.55
1971	58.1000	60.4178	3190.40	3322.56
1972	57.5000	62.7551	3334.90	3208.59
1973	60.4000	64.0937	3494.00	3524.71
1974	63.9000	67.7918	3653.50	3684.78
1975	64.1000	69.7097	3808.90	3674.76
1976	67.3000	68.2913	3978.90	4169.47
	1	2	3	4

Table 6.2

Multiple Output Cost Minimization Model with Regulatory Constraint: M, and

	M	MS	LAM1S	LAM21
1953	51.8164	65.2978	144.291	.650675
1954	57.4713	69.9927	97.4876	.765554
1955	65.7304	77.0699	165.072	.600444
1956	75.8391	83.0701	129.680	.665209
1957	78.8253	92.7336	76.0244	.821433
1958	86.5843	94.5535	106.854	.739883
1959	91.9356	100.027	195.896	.520351
1960	97.8976	102.390	130.040	.670531
1961	103.770	106.047	169.929	.555737
1962	110.847	114.316	198.929	.480403
1963	116.983	115.866	150.400	.593355
1964	118.208	115.708	175.067	.507172
1965	128.536	118.061	139.846	.594366
1966	136.274	125.540	97.9748	.721117
1967	137.920	136.569	129.665	.647142
1968	144.717	142.707	149.441	.596653
1969	168.965	148.334	139.599	.623130
1970	168.853	159.876	206.110	.465179
1971	195.550	167.495	194.245	.503457
1972	194.922	188.651	191.460	.552605
1973	209.050	188.462	178.174	.587666
1974	209.669	189.332	184.155	.599206
1975	207.925	201.056	229.063	.544446
1976	225.593	197.020	236.192	.535679
	1	2	3	4

close to the ones obtained for the one output case.

Following use the simulated values of Tables 6.1 and 6.2 as benchwork and simulate the effect on factor inputs of eliminating the regulatory constraint. The results appear in Tables 6.3 and 6.4. From these tables we observe again the strong A-J effect on input mix.

Thus we conclude the study of the cost minimization simulation model with the observation that it describes very well the input mix choice of Bell Canada. This model is used as chapter VII to forecast the input choice and profit levels under different scenarios with respect to output prices.

Table 6.3

Multiple Output Cost Minimization Model without Regulatory Constraint: K and L

	LB	LS	KB	KS
1953	40.3607	46.0797	693.490	454.562
1954	42.1228	48.8782	782.767	487.206
1955	44.7161	50.7948	922.357	614.405
1956	48.8559	56.2709	1067.05	680.100
1957	53.3560	61.6944	1109.85	698.609
1958	52.2772	61.2867	1331.20	801.903
1959	53.8220	62.2557	1425.22	896.166
1960	52.4948	63.5571	1591.08	864.389
1961	51.5696	61.6812	1723.91	974.219
1962	54.0441	63.3594	1799.27	1083.82
1963	53.9254	65.7901	2002.62	1062.43
1964	53.7973	65.5302	2180.66	1162.73
1965	54.9961	69.4415	2371.88	1127.82
1966	57.6592	74.3855	2502.34	1111.06
1967	60.3921	75.4864	2505.98	1230.67
1968	60.1114	74.3888	2634.50	1335.65
1969	60.1805	75.5199	2938.25	1424.69
1970	61.3093	73.5887	3021.55	1688.54
1971	60.4178	73.4430	3322.56	1783.23
1972	62.7551	74.9573	3208.59	1821.17
1973	64.0937	78.5705	3524.71	1841.65
1974	67.7918	82.6322	3684.78	1960.54
1975	69.7097	83.2724	3674.76	2085.12
1976	68.2913	83.3345	4165.47	2210.52
	1	2	3	4

Table 6.4

Multiple Output Cost Minimization Model without Regulatory Constraint: M and 2

	MB	MS	LAM1S	LAM12
1953 .	65.2978	74.5514	144.291	471.580
1954 .	66.9927	81.2199	97.4876	482.706
1955 .	77.0699	87.5479	165.072	469.531
1956 .	83.0701	95.6794	129.680	474.476
1957 .	92.7336	107.230	76.0244	492.273
1958 .	94.5535	110.851	106.854	479.735
1959 .	100.027	115.702	195.896	472.440
1960 .	102.390	123.993	130.040	478.031
1961 .	106.047	126.842	169.929	457.493
1962 .	114.316	134.021	198.929	448.842
1963 .	115.866	141.361	150.400	451.243
1964 .	115.702	140.937	175.067	432.701
1965 .	118.061	149.073	139.846	435.249
1966 .	125.540	161.964	97.9748	453.227
1967 .	136.569	170.705	129.665	459.312
1968 .	142.707	176.604	149.441	458.534
1969 .	148.334	186.145	139.599	465.064
1970 .	159.876	191.898	206.110	462.739
1971 .	167.495	203.606	194.245	475.530
1972 .	188.651	225.334	191.460	511.154
1973 .	188.462	231.032	178.174	529.712
1974 .	189.332	230.780	184.155	560.200
1975 .	201.056	240.174	229.063	600.653
1976 .	197.020	240.420	236.192	620.733
	1	2	3	4

II The Profit Maximization Model

The model that we use in this section is an extension of the one in Chapter V, where the only difference lies in the specification of technology. There we worked with a one output production function, here with a production possibility frontier.

In this case the Lagrangian is written as:

$$\Omega = P_1 y_1 + \overline{P}_2 \overline{y}_2 + R6 - wL - mM - vK - \mu_1 [H(y_1, \overline{y}_2, L, M, K)] - \mu_2 [P_1 y_1 + \overline{P}_2 \overline{y}_2 + R6 - wL - mM - sK]$$

where

$H(y_1, \overline{y}_2, L, M, K) = 0$ is the equation (4) of the previous section.

The first order conditions for the maximization of Ω yields equations (4) and (5) from section 1, plus the following equations:

$$\frac{\partial \Omega}{\partial \overline{y}_2} = (1-\mu_2)\overline{P}_2 \left[1+\frac{1}{\eta_2}\right] - \frac{\mu_1}{y_2}\left[-\beta_2 + \delta_{11} \ln \frac{\overline{y}_2}{a_2} - \delta_{11} \frac{y_1}{a_1}\right] = 0 \qquad (6)$$

$$\frac{\partial \Omega}{\partial L} = (1-\mu_2)\, w + \mu_1 \frac{r \alpha_1 \alpha_0 e^{\beta D} [L^{\alpha_1} M^{\alpha_2} K^{\alpha_3}]^r}{L} = 0 \qquad (7)$$

$$\frac{\partial \Omega}{\partial M} = (1-\mu_2)\, w + \mu_1 \frac{r \alpha_2 \alpha_0 e^{\beta D} [L^{\alpha_1} M^{\alpha_2} K^{\alpha_3}]^r}{M} = 0 \qquad (8)$$

$$\frac{\partial \Omega}{\partial K} = (1-\mu_2)\, w + \mu_1 \frac{r \alpha_3 \alpha_0 e^{\beta D} [L^{\alpha_1} M^{\alpha_2} K^{\alpha_3}]^r}{K} = 0 \qquad (9)$$

where:

η_2 = Price elasticity of the four Toll services

μ_1 = Lagrangian Multiplier of the technology constraints

μ_2 = Lagrangian Multiplier of the regulatory constraints

Equations (4) to (9) conform a system of six equations in six unknowns. The unknowns are $\overline{y}_2$, L, M, K, μ_1 and μ_2.

The only new parameter introduced here is η_2 which is computed as the weighted average of the Telephone Message Toll Service price-elasticity and the Other Toll price-elasticity of Table 2.7 in Chapter II.

When this model was simulated, we obtained the results shown in Tables 6.5, 6.6 and 6.7. The model tracks quite well for labor, it underestimated capital in all but six years and it tracks very close labor. For "raw materials", they are over-estimated at the beginning of the sample and underestimated at the end. For μ_2 the estimated value is always lower than $\frac{s}{v}$ as required by the theory of regulation. For non-local services (QNLC) we observe that the profit maximizing model overestimates the values up to 1964 and then underestimated the observed values thereafter. We cannot find a rationale for this cross-over effect. Finally in Tables 6.8, 6.9, and 6.10 we present the results of the multiple output simulation model without

Table 6.5

Multiple Output Profit Maximization Model with Regulatory Constraint: L and K

		L	LS	K	KS
1953	.	46.1000	44.8254	690.400	620.767
1954	.	48.2000	46.7192	764.900	700.642
1955	.	51.9000	48.2131	871.300	860.037
1956	.	55.7000	52.0036	989.900	1013.33
1957	.	57.8000	57.1175	1127.10	1039.83
1958	.	57.6000	55.9986	1280.00	1260.46
1959	.	56.5000	57.0005	1429.50	1375.66
1960	.	54.6000	55.5345	1579.10	1546.14
1961	.	52.4000	54.5015	1721.90	1678.74
1962	.	52.3000	55.7127	1860.10	1780.73
1963	.	53.5000	55.3744	2004.40	1986.78
1964	.	54.4000	54.1042	2150.40	2178.11
1965	.	55.8000	54.7500	2283.60	2371.51
1966	.	57.5000	57.1837	2431.20	2499.23
1967	.	56.6000	59.7188	2585.60	2498.35
1968	.	55.5000	59.4653	2734.00	2590.20
1969	.	56.6000	59.0627	2886.00	2895.93
1970	.	57.9000	60.1073	3054.80	2971.22
1971	.	58.1000	59.2822	3190.40	3278.27
1972	.	57.5000	61.7393	3334.90	3137.13
1973	.	60.4000	63.3526	3494.00	3347.96
1974	.	63.9000	67.1051	3653.50	3486.53
1975	.	64.1000	69.6325	3808.90	3405.30
1976	.	67.3000	68.0751	3978.90	3857.31
		1	2	3	4

Table 6.6

Multiple Output Profit Maximization Model with Regulatory Constraint: M, M_1 and M_2

		M	MS	MU1S	MU2S
1953	.	51.8164	72.5214	191.276	.557166
1954	.	57.4713	77.6308	129.319	.702670
1955	.	65.7304	83.0975	196.129	.539304
1956	.	75.8391	88.4224	147.834	.649216
1957	.	78.8253	99.2717	90.8309	.792379
1958	.	86.5843	101.285	122.008	.709049
1959	.	91.9356	105.934	212.183	.489507
1960	.	97.8976	108.340	138.832	.653795
1961	.	103.770	112.076	180.142	.536040
1962	.	110.347	117.845	204.669	.469191
1963	.	116.983	118.980	153.750	.586754
1964	.	118.208	116.362	175.740	.505829
1965	.	128.536	117.533	139.547	.594934
1966	.	136.274	124.507	97.6596	.721811
1967	.	137.920	135.046	129.126	.646333
1968	.	144.717	141.173	150.279	.595650
1969	.	169.965	145.579	139.542	.623945
1970	.	158.853	156.741	205.214	.466026
1971	.	195.550	164.347	193.997	.504550
1972	.	194.922	185.597	192.780	.551207
1973	.	209.050	188.301	160.686	.628984
1974	.	209.669	187.414	190.736	.550532
1975	.	207.925	200.833	243.105	.526260
1976	.	225.593	196.396	249.028	.520381
		1	2	3	4

Table 6.7

Multiple Output Profit Maximization Model with Regulatory Constraint: QLOC and QNLC

	QLOC	QLOCS	QNLC	QNLCS
1952	137.000	137.000	60.2226	87.1209
1953	148.000	148.000	65.3967	94.1476
1954	162.900	162.900	75.9085	102.261
1955	181.700	181.700	86.9111	113.313
1956	200.600	200.600	95.7637	126.244
1957	216.600	216.600	101.404	135.287
1958	233.600	233.600	111.098	143.941
1959	250.900	250.900	118.180	153.152
1960	269.500	269.500	126.863	163.592
1961	289.600	289.600	148.318	174.719
1962	308.700	308.700	159.473	184.261
1963	325.000	325.000	181.857	188.230
1964	350.800	350.800	205.324	199.117
1965	380.700	380.700	231.300	217.031
1966	410.000	410.000	259.000	235.306
1967	437.600	437.600	286.828	250.379
1968	472.800	472.800	329.563	265.688
1969	512.400	512.400	358.981	285.311
1970	546.600	546.600	375.229	305.654
1971	589.600	589.600	423.877	334.297
1972	635.800	635.800	492.635	359.040
1973	690.300	690.300	560.212	384.761
1974	746.200	746.200	637.795	418.378
1975	792.200	792.200	692.265	434.553
	1	2	3	4

regulation. To compare results we have used again as benchwork the simulated values of Tables 6.5 to 6.7. The simulated values for 1953 and 1975 correspond to a local convergence, therefore they should be left out. When analysing the results of these tables, we observe again a strong reduction in capital requirements. On the other hand, the profit maximizing level of non-local services has increased slightly.

Table 6.8

Multiple Output Profit Maximization Model without Regulatory Constraint: L and K

	LB	LS	KB	KS
1953	44.8254	38.2785	620.767	402.940
1954	46.7192	51.5765	700.642	514.102
1955	48.2131	52.9782	860.037	640.815
1956	52.0036	58.3967	1013.33	705.793
1957	57.1175	64.0856	1039.83	725.686
1958	55.9986	63.8767	1260.46	835.798
1959	57.0005	64.6318	1375.66	930.389
1960	55.5345	66.1094	1546.14	899.098
1961	54.5015	64.1293	1678.74	1012.89
1962	55.7127	64.8866	1780.73	1109.94
1963	55.3744	67.2856	1936.78	1086.58
1964	54.1042	66.1449	2178.11	1173.64
1965	54.7500	69.6854	2371.51	1131.76
1956	57.1837	74.4288	2498.23	1111.65
1967	59.7188	75.2619	2498.35	1227.01
1968	59.4653	73.9308	2590.20	1327.43
1969	59.0627	74.7318	2895.93	1410.02
1970	60.1073	72.6618	2971.22	1667.27
1971	59.2822	72.6375	3278.27	1763.72
1972	61.7393	74.0451	3137.13	1799.00
1973	63.3526	77.4162	3347.56	1814.59
1974	67.1051	81.4446	3486.53	1932.37
1975	69.6325	82.8550	3405.30	1817.01
1976	68.0751	82.1699	3957.31	2179.63
	1	2	3	4

Table 6.9

Multiple Output Profit Maximization Model without Regulatory Constraint: M, M_1 and M_2

		MB	MS	MU1S	MU2S
1953	.	72.5214	57.0932	191.276	31.9882
1954	.	77.6308	85.7036	129.319	479.408
1955	.	83.0975	91.3112	196.129	467.017
1956	.	88.4224	99.2940	147.834	472.236
1957	.	99.2717	111.396	90.8309	489.890
1958	.	101.285	115.536	122.008	477.210
1959	.	105.934	120.118	212.183	470.188
1960	.	108.340	128.972	138.832	475.636
1961	.	112.076	131.876	180.142	455.227
1962	.	117.845	137.251	204.669	447.480
1963	.	118.990	144.574	153.750	449.951
1964	.	115.362	142.259	175.740	432.186
1965	.	117.533	149.596	139.547	435.054
1966	.	124.507	162.058	97.6596	453.194
1967	.	135.046	170.197	129.126	459.487
1968	.	141.173	175.517	150.279	458.895
1969	.	145.579	184.202	139.542	465.687
1970	.	156.741	189.481	206.214	463.488
1971	.	164.347	201.373	193.997	476.199
1972	.	185.597	222.592	192.780	511.953
1973	.	198.301	227.638	160.686	530.714
1974	.	187.414	227.464	190.736	561.236
1975	.	200.833	238.915	243.105	618.986
1976	.	196.396	237.060	249.028	621.848
		1	2	3	4

Table 6.10

Multiple Output Profit Maximization Model without Regulatory Constraint: QLOC and QNLOC

	QLOCB	QLOCS	QNLCB	QNLCS
1953	137.000	137.000	87.1209	33.9653
1954	148.000	148.000	94.1476	95.9297
1955	162.900	162.900	102.261	104.325
1956	181.700	181.700	113.313	116.047
1957	200.600	200.600	126.244	129.147
1958	216.600	216.600	135.287	138.947
1959	233.600	233.600	143.941	147.886
1960	250.900	250.900	153.152	159.097
1961	269.500	269.500	163.592	169.674
1962	289.600	289.600	174.719	180.971
1963	303.700	308.700	184.261	192.902
1964	325.000	325.000	188.230	198.307
1965	350.800	350.800	199.117	212.532
1966	380.700	390.700	217.031	232.741
1967	410.000	410.000	235.306	250.094
1968	437.600	437.600	250.379	265.399
1969	472.800	472.800	265.688	283.786
1970	512.400	512.400	285.311	301.625
1971	546.600	546.600	305.654	324.071
1972	589.600	589.600	334.257	351.704
1973	635.800	635.800	353.040	374.649
1974	690.300	690.300	384.761	407.130
1975	746.200	746.200	413.378	355.516
1976	792.200	792.200	434.553	460.269
	1	2	3	4

CHAPTER VII

POLICY SIMULATIONS WITH THE COST MINIMIZATION MODEL

In this chapter we use the cost minimization model of the first section of Chapter VI, to simulate the effect on factor inputs and profit levels of alternative future price regimes for telephone services.

We perform simulations for the period 1977-1981. For the simulation exercises we need, besides the price of the outputs, forcasts for the price of factor inputs, and the exogenous variables of the output demand equations. These last variables are: Gross Provincial Product of Quebec, Gross Provincial Product of Ontario, Retail Prices of Quebec, Retail Prices of Ontario, Population of both Provinces, and Bell Canada Advertising Expenditures.

The forecasts for Gross Provincial Products at constant prices, were obtained for Quebec from Bureau De La Statistique Du Quebec (1977), and for Ontario from Sawyer, J.A. et al. (1978). For retail prices we used the forecasted rate of growth of the implicit price index of personal expenditures on consumers goods and services from Sawyer, J.A. et al. (1978). For Population of Ontario we also used Sawyer, J.A. et al. (1978). For Population of Quebec we used Office de Planification et de Developpement du Quebec (1977). The Advertising expenditures, in constant dollars, were forecasted using the average rate of growth of the last five years.

In the case of input prices we used mixed autoregressive moving average (ARMA) models. Finally, our technological change indicator (D) was forecasted assuming exponential growth at a rate equal to the average of the period 1952-1976.

Finally, for the "allowed price of capital services" (s), we assume that it keeps the same ratio to the price of capital services as in the period 1967-1974. That is, we assume that the ratio of the allowed total return on capital to the cost of capital services of the above period is maintained in the period 1977-1981. This assumption implies the following relation $s = 1.4338\ v$. Miscellaneous revenues were taken as 2.5% of total revenues (this was the last five years average).

Now we have all the elements to perform our simulations.

I. Simulating with Constant 1976 Nominal Prices of Service

In this simulation, we assume that the nominal 1976 price of each telephone service does not change in the whole period.

The results of this simulation, appear in Table 7.1. In order to facilitate comparisons we also present the simulated values of each of the endogenous variables for the period 1972-1976. The model converged only for the first three years of the simulation period (1977 to 1979) and those are the results reported here.

As we see from these results, the higher demand for local and non-local services (due to the increase in income and the decrease in real prices of services) caused a small increase in factor inputs. In contrast, the rate of profit in total revenue (PROFIT/TRS) goes from 16.9% in 1976 to 13.5% in 1979.

Table 7.1

Simulation No. 1: Constant 1976 Nominal Prices

	LS	KS	MS	LAM1S	LAM2S
1972	62.6802	3220.63	188.426	190.756	.553718
1973	64.0219	3537.33	188.251	177.612	.588506
1974	67.7404	3693.70	189.189	183.745	.599996
1975	69.7924	3660.90	201.294	229.899	.543327
1976	68.5486	4119.78	197.762	238.641	.532625
1977	69.4673	4423.40	202.881	237.454	.561127
1978	72.5517	4483.96	212.214	268.924	.540393
1979	76.7698	4413.66	224.091	319.673	.503107

	ALOCS	AMLCS	PROFIT	TRS
1972	589.609	423.877	162.640	1122.09
1973	635.800	492.635	187.304	1281.42
1974	690.300	560.212	197.360	1447.49
1975	746.200	637.795	248.210	1672.74
1976	792.200	692.265	322.276	1904.36
1977	853.292	794.736	321.234	2113.26
1978	926.324	914.736	342.318	2360.81
1979	1009.19	1046.49	354.784	2634.77

II. Simulating with Requested Price Increases

In this simulation, we assume that the requested prices by Bell will be implemented starting in 1978. Thus, in our simulation we have assumed an 11% increase in Toll prices and a 23.0% increase in Local services. The results of the simulation appear in Table 7.2. When comparing these with the ones of Table 7.1, we observe that output levels are smaller in Table 7.2. This result, is due to the lower quantity demanded of services caused by the higher price of services. The increase in prices more than compensate for the decrease in quantity and we end up with higher revenues in Table 7.2 than in Table 7.1. To achieve the same "allowed price of capital" with the higher revenues, capital stock has to increase substantially in this simulation experiment. Profits are also substantially higher when compared with the observed 1976 values and with the ones from Table 7.1. In terms of profit share of total revenue, the share is 16.9% in 1976 and it is 17.7% in 1978. Thereafter, we have frozen the nominal prices of telephone services and thus the profit share in 1982 is down to 14.6%.

III. Simulating with Price Increases equal to the Increase in Consumer Prices

In this simulation we keep constant the real price of telephone services at their 1976 level. Output levels are now even lower than in Table 7.1, this is due to the higher real prices used

Table 7.2

Simulation No. 2: Requested Price

	LS	KS	MS	LAM1S	LAM2S
1972	62.6802	3220.83	188.426	190.756	.553718
1973	64.0219	3537.33	188.251	177.612	.588508
1974	67.7404	3693.70	189.188	183.745	.599896
1975	69.7924	3660.90	201.294	229.899	.543327
1976	68.5486	4119.78	197.762	238.641	.532625
1977	61.2976	5925.04	179.021	186.880	.619634
1978	62.4556	5938.61	182.682	206.626	.611339
1979	64.2063	5950.24	187.418	231.196	.600545
1980	66.4979	5959.24	192.896	261.555	.586801
1981	69.3241	5958.60	199.143	299.810	.568940

	QLOCS	QNLCS	PROFIT	TRS
1972	589.600	423.877	162.640	1122.08
1973	635.800	492.635	197.304	1281.42
1974	690.300	560.212	197.360	1447.49
1975	746.200	637.795	248.210	1672.74
1976	792.200	692.265	382.276	1904.36
1977	826.740	734.565	430.286	2350.04
1978	875.179	826.656	453.328	2556.93
1979	933.604	935.293	478.299	2803.74
1980	1002.17	1064.70	505.303	3096.10
1981	1081.71	1219.41	533.881	3441.80

for this simulation. In constrast, total revenues are higher now than in Tables 7.1 and 7.2. One of the most important characteristics of these simulation results are the substantial increases in capital requirements when compared with the results of Tables 7.1 and 7.2. Thus, to achieve the same "allowed price of capital services", with higher total revenues, Bell will need substantially higher capital requirements. The profit share in this simulation goes from 16.9% in 1976 to 16.6% in 1979 and 16.3% in 1981. Thus, most of the changes are not in profit but in the capital intensity of production.

Table 7.3

Simulation No. 3. Constant Real Prices

	LS	KS	MS	LAM1S	LAM2S
1972	62.6802	3220.83	189.426	190.756	.553718
1973	64.0219	3537.33	188.251	177.612	.588508
1974	67.7404	3693.70	189.189	183.745	.599896
1975	69.7924	3660.90	201.294	229.899	.543327
1976	68.5486	4119.78	197.762	238.641	.532625
1977	66.0410	5029.11	192.874	212.089	.591307
1978	66.2400	5502.45	193.752	221.817	.594639
1979	66.9268	5871.40	195.359	237.689	.593196
1980	67.8784	6194.38	196.900	257.901	.589503
1981	68.9440	6519.29	198.051	281.523	.584790

	QLQCS	QNLCS	PROFIT	TRS
1972	589.600	423.377	162.640	1122.08
1973	635.800	492.635	197.304	1281.42
1974	690.300	560.212	197.360	1447.49
1975	746.200	637.795	248.210	1672.74
1976	792.200	692.265	322.276	1904.36
1977	843.321	750.121	365.222	2206.79
1978	899.087	813.294	420.034	2511.03
1979	956.685	875.515	471.961	2834.60
1980	1016.95	943.733	525.241	3191.61
1981	1090.75	1020.79	584.117	3598.65

APPENDIX A

DATA BANK

LOC	Local Services Revenues
INTRA	Intra Bell Telephone Message Toll Revenues in constant 1967 dollar.
TRANS	Trans Canada and Adjacent Members Telephone Message Toll Revenues in constant 1967 dollar.
USO	United States and Overseas Telephone Message Toll Revenues in constant 1967 dollar.
OTR	Other Toll Revenues in constant 1967 dollar.
Q6	Total of Directory Advertising and Miscellaneous Revenues in constant 1967 dollar.
P1	Price index for Local Revenues.
P2	Price index for INTRA
P3	Price index for TRANS
P4	Price index for USO
P5	Price index for OTR
P6	Price index for Q6
QL	Local Services Revenues in constant 1967 dollar
PLOC	Price index of Local Services Revenues
TOLL	Non Local Services Revenues in constant 1967 dollar
PT	Price index of Non Local Services Revenues.
QTOL	Telephone Toll Services Revenues in constant 1967 dollar.
PTOL	Price index for QTOL
QUE	Gross Provincial Product of Quebec in current dollar
ONT	Gross Provincial Product of Ontario in current dollar
YD	Sum of Provincial Product of Quebec and Ontario in constant 1967 dollar.

MTL	Consumer Price index of Montreal
TOR	Consumer Price index of Toronto
CPI	Computed Consumer Price index of Montreal and Toronto
POPONT	Population of Ontario
POPQUE	Population of Quebec
POP	Sum of Population in Quebec and Ontario.
L	Weighted man hours
MM	Raw material
K	Net capital in constant 1967 dollar
W	Wage Rate
M	Factor Price of Raw Material
U	Factor Price of Capital
ADVT	Total of Advertising, Commercial and Marketing
DDD	Direct Distance Dialing
INPUT	Input as Defined in Chapter IV
LTD	Long-term debts
CE	Common Equity
PE	Preferred Equity
IB	Rate of Return on Long-term Debt
RTCE	Rate of Return on Common Equity
WK16	Factor Price of Capital
PK	Plant Price Index
D	Depreciation Rate
Thetan	The capital gains parameter
U	Corporate Income Tax Rate
RRG	Rate of return on Long-term Government Bonds

RRB	Rate of Return on 10 Industries Bonds
CC1	The Cost of Capital

YEAR	LOC	INTRA	TRANS	USO	OTR	Q6
1952	126.400	45.2000	2.10000	6.10000	1.70000	14.9185
1953	137.000	48.3000	2.40000	6.90000	2.30000	16.9351
1954	148.000	51.7000	2.60000	7.90000	2.90000	19.5181
1955	162.900	57.5000	4.80000	8.80000	4.30000	19.3296
1956	181.700	64.0000	5.70000	10.4000	6.30000	19.3061
1957	200.600	68.2000	6.50000	12.9000	7.80000	22.2211
1958	216.600	70.1000	7.50000	14.2000	9.30000	25.4251
1959	233.600	75.4000	8.70000	16.3000	10.5000	27.1982
1960	250.900	78.8000	9.50000	17.3000	12.5000	28.7949
1961	269.500	84.9000	10.6000	16.5000	14.7000	30.6263
1962	289.600	100.100	12.1000	17.9000	18.0000	32.5632
1963	308.700	104.400	13.4000	19.9000	21.6000	31.9770
1964	325.000	112.500	14.8000	24.3000	30.2000	32.2175
1965	350.800	125.300	16.4000	28.7000	34.9000	33.2632
1966	380.700	137.000	19.6000	34.7000	40.0000	34.4291
1967	410.000	152.800	22.1000	39.0000	45.1000	36.6000
1968	437.600	164.700	25.3000	42.7000	54.1000	38.8764
1969	472.800	187.200	29.3000	49.6000	63.4000	41.7777
1970	512.400	198.700	32.0000	55.6000	72.8000	45.2106
1971	546.600	203.700	35.0000	59.8000	77.3000	48.5206
1972	589.600	220.900	42.6000	71.3000	90.9000	21.3584
1973	635.800	246.900	51.6000	89.8000	108.000	22.1889
1974	690.300	277.200	64.3000	104.200	119.700	22.3092
1975	746.200	308.900	76.9000	120.800	138.200	25.2199
1976	792.200	332.400	81.6000	129.000	156.700	29.3519

YEAR	P1	P2	P3	P4	P5	P6
1952	.924000	1.06050	1.09190	.944600	.976100	.741000
1953	.933000	1.06050	1.12260	.944600	1.00140	.740000
1954	.933000	1.06050	1.14100	.944600	1.01670	.752000
1955	.933000	1.06050	1.14100	.944600	1.01670	.756000
1956	.933000	1.06050	1.14100	.938300	1.01670	.784000
1957	.933000	1.06050	1.14100	.914500	1.01670	.801000
1958	.939000	1.07260	1.14100	.914500	1.01670	.812000
1959	1.00000	1.13310	1.13640	.914500	1.01670	.829000
1960	1.00000	1.13310	1.12690	1.00440	1.01670	.839000
1961	1.00000	1.11810	1.09560	1.02340	1.01670	.843000
1962	1.00000	1.04320	1.05920	1.02340	1.01790	.855000
1963	1.00000	1.04320	1.04100	1.02340	1.01920	.870000
1964	1.00000	1.04320	1.03140	1.02340	1.01800	.892000
1965	1.00000	1.04320	1.02180	1.02340	1.01390	.921000
1966	1.00000	1.00720	1.00360	1.02340	1.00060	.962000
1967	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1968	1.00000	.987800	.999000	1.00000	.999000	1.03300
1969	1.00000	.992200	.996500	1.00470	1.01660	1.07800
1970	1.00000	1.10930	.996500	1.00630	1.01600	1.12800
1971	1.03900	1.13410	.996500	1.00630	1.04000	1.16400
1972	1.06800	1.15790	.996200	1.00630	1.04570	1.22200
1973	1.09800	1.19250	.994500	1.00630	1.07360	1.33400
1974	1.12200	1.21350	.994500	1.00630	1.10680	1.53300
1975	1.17700	1.24160	1.05400	1.06780	1.15840	1.70500
1976	1.25000	1.30130	1.13740	1.14160	1.24460	1.86700

YEAR	PLOC	QL	TOLL	PT	QTOL	PTOL
1952	.924000	126.400	55.4459	1.03973	53.4674	1.04717
1953	.933000	137.000	60.2226	1.04176	57.6434	1.04841
1954	.933000	148.000	65.3867	1.04311	62.1980	1.04918
1955	.933000	162.900	75.9085	1.04257	71.3007	1.04863
1956	.933000	181.700	86.9111	1.04174	80.2934	1.04783
1957	.933000	200.600	95.7637	1.03870	87.6373	1.04453
1958	.939000	216.600	101.404	1.04718	91.7897	1.05385
1959	1.00000	233.600	111.098	1.08827	100.271	1.09931
1960	1.00000	250.900	118.180	1.10068	105.439	1.11315
1961	1.00000	269.500	126.863	1.09072	112.009	1.10193
1962	1.00000	289.600	148.318	1.03751	130.178	1.04134
1963	1.00000	308.700	159.473	1.03616	137.768	1.03961
1964	1.00000	325.000	181.857	1.03508	151.631	1.03866
1965	1.00000	350.800	205.324	1.03362	170.413	1.03773
1966	1.00000	380.700	231.300	1.00818	191.299	1.00978
1967	1.00000	410.000	259.000	1.00000	213.900	1.00000
1968	1.00000	437.600	286.828	.992622	232.709	.991217
1969	1.00000	472.800	329.563	.998969	266.123	.994918
1970	1.00000	512.400	358.981	1.06474	286.207	1.07704
1971	1.03900	546.600	375.229	1.08324	298.020	1.09412
1972	1.06800	589.600	423.877	1.09707	333.273	1.11010
1973	1.09800	635.800	492.635	1.12062	385.109	1.13243
1974	1.12200	690.300	560.212	1.13826	440.917	1.14576
1975	1.17700	746.200	637.795	1.18167	500.113	1.18688
1976	1.25000	792.200	692.265	1.25336	536.231	1.25437

YEAR	QUE	ONT	YD	MTL	TOR	CPI
1952	6295.00	9939.00	20664.0	.796000	.779000	.785619
1953	6690.00	10668.0	22251.3	.788000	.775000	.780089
1954	6837.00	10469.0	21979.3	.791000	.785000	.787378
1955	7350.00	11479.0	23846.4	.792000	.788000	.789596
1956	8260.00	12911.0	26436.7	.802000	.800000	.800818
1957	8688.00	14190.0	27605.9	.825000	.831000	.828736
1958	8917.00	14474.0	27458.0	.850000	.853000	.851882
1959	9526.00	15265.0	28942.5	.859000	.855000	.856560
1960	10055.0	15750.0	29818.5	.866000	.865000	.865401
1961	10570.0	16481.0	31009.4	.876000	.870000	.872348
1962	11461.0	17835.0	33248.4	.886000	.878000	.881126
1963	12092.0	19046.0	34747.3	.901000	.893000	.896126
1964	13405.0	20907.0	37674.9	.915000	.908000	.910739
1965	14724.0	22948.0	40439.2	.934000	.930000	.931571
1966	16310.0	25686.0	43406.8	.962000	.971000	.967499
1967	17651.0	27916.0	45567.0	1.00000	1.00000	1.00000
1968	18863.0	30636.0	47757.7	1.03400	1.03800	1.03646
1969	20602.0	34054.0	50841.2	1.06700	1.08000	1.07503
1970	22031.0	36276.0	52988.7	1.08800	1.10800	1.10037
1971	23760.0	39956.0	56914.5	1.10900	1.12600	1.11950
1972	26428.0	44890.0	61237.2	1.15100	1.17300	1.16462
1973	30097.0	51492.0	65599.8	1.22700	1.25400	1.24374
1974	34927.0	59576.0	68650.2	1.36300	1.38500	1.37659
1975	39010.0	65300.0	67656.3	1.51200	1.56000	1.54176
1976	44668.0	75000.0	72403.4	1.61400	1.67660	1.65280

YEAR	POPONT	POPQUE	POP
1952	4788.00	4174.00	8962.00
1953	4941.00	4269.00	9210.00
1954	5115.00	4388.00	9503.00
1955	5266.00	4517.00	9783.00
1956	5405.00	4628.00	10033.0
1957	5636.00	4769.00	10405.0
1958	5821.00	4904.00	10725.0
1959	5969.00	5024.00	10993.0
1960	6111.00	5142.00	11253.0
1961	6236.00	5259.00	11495.0
1962	6351.00	5371.00	11722.0
1963	6481.00	5481.00	11962.0
1964	6631.00	5584.00	12215.0
1965	6788.00	5685.00	12473.0
1966	6961.00	5781.00	12742.0
1967	7127.00	5864.00	12991.0
1968	7262.00	5928.00	13190.0
1969	7385.00	5985.00	13370.0
1970	7551.00	6013.00	13564.0
1971	7703.00	6028.00	13731.0
1972	7824.00	6050.00	13874.0
1973	7939.00	6081.00	14020.0
1974	8094.00	6134.00	14228.0
1975	8226.00	6188.00	14414.0
1976	8331.00	6243.00	14574.0

YEAR	L	MM	K	W	M	V
1952	44.9000	48.1783	626.600	1.67773	.741074	.635233E-01
1953	46.1000	51.8164	690.400	1.80152	.740074	.953645E-01
1954	48.2000	57.4713	764.900	1.88029	.752073	.985060E-01
1955	51.9000	65.7304	871.300	1.96069	.756074	.846447E-01
1956	55.7000	75.8391	989.900	2.00592	.784081	.866673E-01
1957	57.8000	78.8253	1127.10	2.09481	.801046	.966026E-01
1958	57.6000	86.5843	1280.00	2.20990	.812045	.881951E-01
1959	56.5000	91.9356	1429.50	2.31823	.829044	.840941E-01
1960	54.6000	97.8976	1579.10	2.46282	.839040	.945627E-01
1961	52.4000	103.770	1721.90	2.60840	.843038	.862381E-01
1962	52.3000	110.847	1860.10	2.72122	.855038	.830704E-01
1963	53.5000	116.983	2004.40	2.81271	.870037	.909535E-01
1964	54.4000	118.208	2150.40	2.88658	.892038	.849520E-01
1965	55.8000	128.536	2283.60	2.97491	.921037	.956500E-01
1966	57.5000	136.274	2431.20	3.15165	.962034	.110192
1967	56.6000	137.920	2585.60	3.40247	1.00000	.108983
1968	55.5000	144.717	2734.00	3.68991	1.03301	.107317
1969	56.6000	168.965	2886.00	3.99788	1.07801	.110649
1970	57.8000	168.853	3054.80	4.42526	1.12788	.100710
1971	58.1000	195.550	3190.40	4.85577	1.16413	.104430
1972	57.5000	194.922	3334.90	5.52783	1.22215	.118812
1973	60.4000	209.050	3494.00	5.90199	1.33404	.131491
1974	63.9000	209.669	3653.50	6.44194	1.53303	.141787
1975	64.1000	207.925	3808.90	7.39906	1.70504	.154311
1976	67.3000	225.593	3978.90	8.10431	1.86704	.159550

YEAR	ADVT	DDD	INPUT
1952.	9.38328	0.	86.0841
1953.	9.50069	0.	91.1645
1954.	9.58994	0.	98.3947
1955.	9.72286	0.	109.368
1956.	9.86251	.600000E-02	121.630
1957.	9.97002	.130000E-01	129.107
1958.	9.98898	.530000E-01	136.725
1959.	10.0139	.910000E-01	141.706
1960.	10.0596	.159000	145.617
1961	10.1050	.224000	148.495
1962.	10.1640	.263000	154.187
1963.	10.2113	.311000	161.218
1964.	10.2628	.373000	165.727
1965.	10.3338	.433000	174.473
1966.	10.4512	.471000	182.768
1967.	10.4979	.507000	184.815
1968	10.5278	.568000	188.355
1969	10.6375	.623000	201.806
1970	10.7397	.678000	206.494
1971	10.8760	.721000	218.683
1972	11.0367	.760000	219.747
1973	11.1895	.819620	232.148
1974	11.3485	.877000	240.974
1975	11.4727	.938380	243.114
1976.	11.6464	1.00000	257.500

YEAR	LTD	CE	PE	IB	RTCE
1952	149.960	306.428	0.	.288485E-01	.100096
1953	182.160	377.106	0.	.255001E-01	.136716
1954	216.275	387.612	0.	.237496E-01	.138269
1955	220.238	461.902	0.	.294255E-01	.980272E-01
1956	252.344	536.006	0.	.285097E-01	.874509E-01
1957	289.293	622.315	0.	.274718E-01	.836254E-01
1958	354.450	631.362	0.	.257872E-01	.689195E-01
1959	379.293	734.400	0.	.276026E-01	.530910E-01
1960	458.194	751.245	0.	.266550E-01	.754369E-01
1961	474.736	848.160	0.	.309491E-01	.591574E-01
1962	527.832	956.839	0.	.347714E-01	.510528E-01
1963	597.616	981.212	0.	.359380E-01	.663416E-01
1964	616.730	1100.00	0.	.386980E-01	.528271E-01
1965	668.942	1139.03	0.	.392558E-01	.669158E-01
1966	808.774	1331.78	0.	.358636E-01	.926608E-01
1967	932.566	1380.24	0.	.337274E-01	.955757E-01
1968	1062.83	1428.37	0.	.355299E-01	.988582E-01
1969	1161.24	1480.33	0.	.408498E-01	.987052E-01
1970	1300.51	1539.93	93.9970	.431741E-01	.795485E-01
1971	1512.81	1581.67	197.997	.445630E-01	.916916E-01
1972	1684.72	1640.67	197.991	.449929E-01	.103911
1973	1868.78	1705.81	248.988	.486475E-01	.116986
1974	2254.63	1769.54	332.002	.492490E-01	.119362
1975	2614.06	2030.95	343.211	.519629E-01	.125999
1976	2909.72	2136.30	376.997	.539624E-01	.120295

YEAR	WK16	PK	D	THETAN	U
1952	.635233E-01	.869000	.461246E-01	.450229E-01	.512685
1953	.953645E-01	.851000	.466798E-01	.322475E-01	.458296
1954	.985060E-01	.843000	.454312E-01	.211412E-01	.453566
1955	.846447E-01	.841000	.431880E-01	.131653E-01	.434968
1956	.866673E-01	.854000	.433628E-01	.528116E-02	.432968
1957	.966026E-01	.859000	.515580E-01	.307461E-02	.436111
1958	.881951E-01	.864000	.519818E-01	.197053E-02	.428099
1959	.840941E-01	.864000	.542014E-01	.130720E-02	.469802
1960	.945627E-01	.869000	.543015E-01	.141188E-02	.473053
1961	.862381E-01	.865000	.550178E-01	.270497E-02	.486337
1962	.830704E-01	.873000	.557486E-01	.189293E-02	.484833
1963	.909535E-01	.883000	.576802E-01	.411598E-02	.481151
1964	.849520E-01	.879000	.588702E-01	.614123E-02	.483993
1965	.956500E-01	.894000	.603759E-01	.320110E-02	.482855
1966	.110192	.936000	.612440E-01	.642582E-02	.478934
1967	.108983	1.00000	.616697E-01	.161961E-01	.466347
1968	.107317	1.04900	.623696E-01	.293889E-01	.472282
1969	.110649	1.10000	.642247E-01	.350534E-01	.477334
1970	.100710	1.17300	.641633E-01	.399121E-01	.487045
1971	.104430	1.24300	.633784E-01	.475052E-01	.453299
1972	.118812	1.32600	.670594E-01	.518836E-01	.433526
1973	.131491	1.41000	.697810E-01	.577366E-01	.460416
1974	.141787	1.57800	.692587E-01	.607250E-01	.465115
1975	.154311	1.72700	.734134E-01	.753196E-01	.449893
1976	.159550	1.836^0	.747486E-01	.801905E-01	.437794

YEAR	RRG	RRB	CC1
1952	.356300E-01	.432000E-01	.346512E-01
1953	.370500E-01	.442000E-01	.364918E-01
1954	.317600E-01	.390000E-01	.317757E-01
1955	.313700E-01	.370000E-01	.311131E-01
1956	.362500E-01	.438000E-01	.369222E-01
1957	.411300E-01	.528000E-01	.446118E-01
1958	.411200E-01	.492000E-01	.407499E-01
1959	.507400E-01	.570000E-01	.467838E-01
1960	.518500E-01	.576000E-01	.461438E-01
1961	.504600E-01	.552000E-01	.444099E-01
1962	.511300E-01	.552000E-01	.445747E-01
1963	.508800E-01	.546000E-01	.435710E-01
1964	.518300E-01	.554000E-01	.446601E-01
1965	.520800E-01	.566000E-01	.453713E-01
1966	.569000E-01	.640000E-01	.512792E-01
1967	.593700E-01	.692000E-01	.551057E-01
1968	.674600E-01	.776000E-01	.609204E-01
1969	.758400E-01	.864000E-01	.674168E-01
1970	.791300E-01	.922000E-01	.715865E-01
1971	.694800E-01	.828000E-01	.653792E-01
1972	.723200E-01	.828000E-01	.658105E-01
1973	.756100E-01	.847000E-01	.661565E-01
1974	.890300E-01	.101700	.787326E-01
1975	.903500E-01	.107600	.845338E-01
1976	.917600E-01	.107200	.849476E-01

APPENDIX B

BELL CANADA: FINANCIAL ASPECTS

In our simulation models of Chapters V and VI the factor price of capital was taken as exogenous to the firm. Of course if the firm has monopsony power in factor markets than marginal cost of factors instead of average prices should be used. For this chapter, we investigate the relation between the cost of the different capital sources (equity and debt) and the characteristics of the firm (especially debt and equity levels). The results of this chapter could have two uses. First, as a test of the possible monopsony power of Bell Canada in capital markets. Second, to find a stable relation for forecasting the cost of capital faced by Bell.

I Introduction

Although there are many financial instruments, we worked with the three major broad classes as follows i) Long term debt ii) Common equity iii) Preferred equity.
As described in IAER (1977), we specify rates of return equations which (the inverse investors demand function) depend on the value of debt, equity, income and the rate of returns to alternative assets (from the investor's point of view). One immediate empirical problem was the fact that Bell Canada started issuing preferred equity in 1970. To overcome this difficulty,

the rate of return on equity, was defined to be the weighted average of rates of return on the two types of equities common and preferred. That is,

$$re_t = rc_t \frac{(\text{Value of Common Equity})_t}{(\text{Value of Equity})_t} + rp_t \frac{(\text{Value of Preferred Equity})}{(\text{Value of Equity})_y}$$

where rc_t and rp_t are rates of return on common and preferred equity respectively.

The equations that we estimated are of the general form:

$$r_{bt} = F(P_{bt}B_t, P_{et}E_t, r_{at}, Y_t) \qquad (1)$$

$$r_{et} = G(P_{bt}B_t, P_{et}E_t, r_{at}, Y_t) \qquad (2)$$

where r_{bt} and r_{et} are rates of return on debt and equity respectively, F and G are functional forms, P_{bt} and P_{et} are the prices of debt and equity, r_{at} is the rate of returns to alternative assets and Y_t is the income.

II Data

The income and variable was already defined in Chapter II. The rates of return on different financial instruments are defined as follows:

i) rate of return on debt was defined as equals to the ratio of interest payments on the long term debt to the value of the long term debt.

ii) rate of return on preferred equity. Since there is not a unique form of preferred equity, we defined the rate of return on each type of preferred equity as the dividend per share divided by the price of the respective share. Then the aggregate rate of return on preferred equity is defined as a weighted average of the rates of return of all the different issues of preferred stock outstanding.

iii) the rate of return on the common stock, was computed using the Discounted Cash Flow method. In this method, the required return is defined as the descount rate which equates the present value of the dividends plus the expected capital appreciation by investors in common share to the market value of the shares. Following this method, we express the required rate of return as the summation of the ratio of the dividend per common share declared over the average market price of common share plus an expectation variable. In our case, we used for the expectation variable a distributed lag of the growth rate of earnings per common share. This last variable is found in the company s annual reports, thus,

$$g_t = \frac{ER_t - ER_{t-1}}{ER_{t-1}}$$

where ER stands for earnings per common share and g_t is the rate of growth. Then, the expectations variable was defined as an eight years distributed lag on g_t.

The final variable used in estimation is the rate of return on alternative assets. Two different rates of return were used for this purpose. First, the rate of return on 10 industries as it is calculated by McLeod, Young and Wein; Second, the rate of return on long-term corporate bond as it is reported in the Bank of Canada Review.

III Statistical Results

Two types of functional forms are used: linear and non-linear We start with the lines case. The equations in this case are given by:

$$r_{bt} = \alpha_o+\alpha_1 P_{bt}B_t+\alpha_2 P_{et}E_t+\alpha_3 r_{at}+\alpha_4 Y_t \quad (3)$$

$$r_{et} =\beta_o+\beta_1 P_{bt}B_t+\beta_2 P_{et}E_t+\beta_3 r_{at}+\beta_4 Y_t \quad (4)$$

Where the value of r_{at} is chosen in some regression as the rate of return on long-term government bonds and in others as the 10 industries bond rate. The results of these linear regressions were always contrary to *apriori* expectations. That is, the coefficient of debt in the equation for the rate of return on debt was always negative, when statistically significant. We also found, most of the time, a very low Durbin-Watson statistic. After correcting for autocorrelation, none of the alternative assets had a significant coefficient.

The second set of equations that we estimated were of the double-log form: that is,

$$\ell n\,(r_{bt}) = \alpha_o + \alpha_1 \;\ell n\,(P_{bt}B_t) + \alpha_2 \;\ell n\,(P_{et}E_t) + \alpha_3 \;\ell n\,(r_{at}) + \alpha_4 \;\ell n\,(Y_t) \quad (5)$$

$$\ell n\,(r_{et}) = \beta_o + \beta_1 \;\ell n(P_{bt}B_t) + \beta_2 \;\ell n\,(P_{et}E_t) + \beta_3 \;\ell n\,(r_{at}) + \beta_4 \;\ell n(Y_t) \quad (6)$$

The results of the estimation of these equations appear in Table B.1. The OLSQ estimation of (5) did not provide satisfactory results since autocorrelation was present as indicated by a very low Durbin-Watson statistic in all the equations (Table B.1). Therefore, the same equations were re-estimated by correcting for first order and using the maximum likelihood as iterative procedure. The estimation was first performed for each equation separately and then corresponding pairs of Tables B.2 and B.4 were estimated simultaneously through the Zellner's seemingly unrelated procedure.

Table B.1

Rate of Return on Debt - Double Log Model*

Alternative	Constant	Long-Term Debt	Equity	Alternative	Income	D.W.	R^2
Name	-5.450 (-15.44)	.2305 (1.41)	.0841 (.41)			.8883	.8480
Name	-13.3156 (-9.36)	-.3690 (-2.22)	.1213 (.76)		1.0953 (5.64)	.8416	.9071
Government Bond	-5.0894 (-5.12)	.1918 (1.01)	.0970 (.47)	.0672 (.39)		.8614	.8485
Government Bond	-13.2240 (-8.04)	.3769 (-2.09)	.1242 (.77)	.0151 (.11)	1.094 (5.62)	.8423	.9071
Corporate Bond	-5.1694 -6.44)	.1907 (.99)	.1047 (.50)	.0580 (.39)		.8712	.8485
Corporate Bond	-14.803 (-8.48)	-.3017 (-1.78)	.0636 (.40)	-.1713 (-1.42)	1.1870 (5.90)	.8699	.9106

* t-values are presented in the parenthesis.

Table B.2

Rate of Return on Debt-Double Log Model*

Alternatives	Constant	Long-Term Debt	Equity	Alternatives	Income	RHO	D.W.	R^2
			Estimation of Single Equation					
None	-7.2678 (-13.41)	-.2960 (-1.67)	.8436 (3.78)			.5578 (4.60)	1.3911	.9423
None	-11.4998 (-4.44)	.4418 (-2.12)	.5343 (2.03)		.6962 (1.69)	.4929 (2.90)	1.3015	.9505
Government Bond	-8.6748 (-5.87)	-.2700 (-1.53)	.9341 (3.88)	-.2089 (-1.02)		.5497 (4.77)	1.3940	.9453
Government Bond	-12.968 (-4.45)	-.4125 (-2.08)	.6210 (2.29)	-.2163 (-1.09)	.7002 (1.77)	.4892 (3.14)	1.2386	.9538
Corporate Bond	-9.4463 (-7.06)	-.2586 (-1.55)	.9930 (4.40)	-.3212 (-1.77)	(5.70)	.57791	1.3673	.9503
Corporate Bond	-14.033 (-5.20)	-.4060 (-2.23)	.6385 (2.52)	-.3315 (-1.96)	.7580 (2.02)	.4990 (3.60)	1.2071	.9594
			Zellner's Procedure					
None	-7.2422 (-21.29)	-.2978 (-2.60)	.8433 (5.83)			.5323 (6.86)	1.3509	.9421
None	-10.5553 (-6.35)	.4347 (-3.40)	.6634 (3.83)		.5168 (1.92)	.5723 (6.15)	1.4200	.9492
Government Bond	-8.6666 (-9.35)	-.2703 (-2.43)	.9333 (6.16)	-.2088 (-1.61)		.5434 (7.49)	1.3847	.9453
Government Bond	-12.2381 (-6.70)	-.4109 (-3.47)	.8055 (4.65)	-.2606 (-2.13)	.4958 (1.97)	.5727 (7.18)	1.3610	.9518
Corporate Bond	-9.456 (-11.22)	.2581 (-2.46)	.9935 (7.00)	.3223 (-2.82)		.5797 (9.09)	1.3700	.9503
Corporate Bond	-13.272 (-7.96)	-.3987 (-3.62)	.7825 (4.81)	-.3510 (-3.36)	.5808 (2.44)	.5748 (7.91)	1.3000	.9582

* t-values are presented in parenthesis.

Table B.3

Rate of Return on Equity-Double Log Model*

Alternatives	Constant	Long-Term Debt	Equity	Alternatives	Income	D.W.	R^2
None	1.4767 (1.72)	2.1978 (5.55)	-2.6637 (-5.41)			.7481	.3886
None	-19.9633 (-6.32)	.5636 (1.53)	-2.5622 (-7.27)		2.9856 (6.92)	1.3745	.6879
Government Bond	-.4606 (-.19)	2.405 (5.22)	-2.733 (-5.52)	-.3609 (-.86)		.8266	.3976
Government Bond	-23.033 (-6.50)	.8273 (2.13)	-2.658 (-7.67)	-.5055 (-1.73)	3.0352 (7.23)	1.4379	.7055
Corporate Bond	4.1669 (2.19)	1.8161 (3.98)	-2.4661 (-4.96)	.5563 (1.57)		.7029	.4147
Corporate Bond	-20.1613 (-5.11)	.5726 (1.49)	-2.5699 (-7.06)	-.0228 (-.08)	2.9978 (6.58)	1.3773	.6879

* t-values are presented in the parenthesis.

Table B.4

Rate of Return on Equity-Double Log Model*

Alternatives	Constant	Long-Term Debt	Equity	Alternatives	Income	RHO	D.W.	R^2
				Estimation of Single Equation				
None	-21.962 (-1.99)	2.373 (3.23)	-.1287 (-.19)			.9536 (43.07)	1.9491	.7424
None	-21.5157 (-4.28)	.7236 (1.37)	-3.0778 (-5.24)		3.3726 (4.60)	.1634 (.83)	1.5157	.7623
Government Bond	-24.765 (-2.15)	2.44 (3.27)	.0723 (.097)	-.365 (-.80)		.9503 (42.32)	2.0945	.7509
Government Bond	-22.32 (-3.79)	.7671 (1.34)	-3.062 (-5.06)	-.1303 (-.26)	3.375 (4.50)	.1569 (.73)	1.5075	.7632
Corporate Bond	-24.2038 (-1.97)	2.4552 (3.17)	.0190 (.02)	-.2034 (-.44)		.9520 43.32	2.0331	.7451
Corporate Bond	-18.898	.6580	-3.062	.2967	3.2335	.1852	1.5111	.7680
				Zellner's Procedure				
None	-27.7684 (-3.81)	2.7604 (5.83)	.0676 (.15)			.9583 (85.08)	1.9744	.7388
None	-20.0372 (-6.24)	.9460 (2.88)	-3.243 (-8.69)		3.202 (6.78)	.2035 (1.76)	1.5588	.7597
Government Bond	-26.3938 (-3.62)	2.556 (5.43)	.1272 (.27)	-.3611 (-1.26)		.9521 (72.79)	2.1027	.7506
Government Bond	-20.1427 (-5.51)	.99098 (2.87)	-3.309 (-8.85)	-.0091 (-.03)	3.225 (6.92)	.1922 (1.64)	1.5610	.7590
Corporate Bond	-23.675 (-3.07)	2.4201 (4.97)	-.0002 (0)	-.2016 (-.70)		.9514 (67.17)	2.0297	.7450
Corporate Bond	-17.422 (-4.43)	.8321 (2.52)	-3.255 (-8.69)	.3737 (1.36)	3.132 (6.48)	.2159 (1.93)	1.5389	.7657

* t-values are presented in the parenthesis.

After correcting for autocorrelation and also using Zellner s seemingly unrelated procedure, still there is one major problem in these estimations. That is, we would expect a positive relation between a given financial instrument and its own site of return, but in most of our results this is not so. We expected to have a positive sign for α_1 and negative for α_2 and in the case of equity $\beta_1<O$, $\beta_2>O$. But the results in Tables (2.1) - (2.4) and also the results of simple linear estimation indicat-d reverse results. We also estimated equation in which debt and equity values were deflated by the price of capital goods. Thus we reestimated equations (5) and (6) as follows

$$\ell n\, r_{bt}=\alpha_o+\alpha_1 \quad \ell n\left(\frac{P_{bt}B_t}{P_{kt}}\right)+\alpha_2 \quad \ell n\left(\frac{P_{et}E_t}{P_{kt}}\right)+\alpha_3 \quad \ell n\, r_{at}+\alpha_4 \quad \ell n \quad (Y) \qquad (7)$$

$$\ell n\,(r_{et})=\beta_o+\beta_1 \quad \ell n\left(\frac{P_{bt}B_t}{P_{kt}}\right)+\beta_2 \quad \ell n\left(\frac{P_{et}E_t}{P_{kt}}\right)+\beta_3 \quad \ell n\, r_{at}+\beta_4 \quad \ell n \quad (Y) \qquad (8)$$

But, the coefficient of debt in (7) still appears to be negative and in the same equation, equity had a positi-e sign. The other equation (8) did not perform as well. These results did not change when equations (7) and (8) were run in simple linear form. One of the causes for these poor results could be the strong collinearity between debt and equity. To check for this, we reestimate (7) and (8) leaving out the value of the alternative asset. Finally we estimated equations (7) and (8) using only the corresponding financial asset in each equation and dropping the income terms. The results were the following:

$$\ell n\,(r_{bt}) = \underset{(-7.78)}{-5.889} + \underset{(5.63)}{.3378}\left[\ell n\left(\frac{P_{bt}B_t}{P_{kt}}\right) - \underset{(5.93)}{.563}\;\ell n\left(\frac{P_{pt-1}B_{t-1}}{P_{kt-1}}\right)\right]$$

$$- \underset{(-1.83)}{.116}\left[\ell n\,(r_{at}) - .563\;\ell n\,(r_{at-1})\right] - .563\;\ell n\,(r_{at-1})$$

$R^2 = .9072$

D.W. = 1.9572

Where r_{at} is the rate of return of corporate bonds.

$$\ell n\,(r_{et}) = \underset{(-3.06)}{-7.527} + \underset{(2.36)}{.544}\left[\ell n\left(\frac{P_{et}E_t}{P_{kt}}\right) - \underset{(10.96)}{.764}\;\ell n\left(\frac{P_{et-1}E_{t-1}}{P_{kt-1}}\right)\right]$$

$$- \underset{(-1.17)}{.429}\left[\ell n\,(r_{at}) - .764\;\ell n\,(r_{at-1})\right] - .764\;\ell n\,(r_{et-1})$$

$R^2 = .6102$

D.W. = 2.0478

Where r_{at} is the rate of return of government bonds.

Now, the financial asset has the expected sign but the rate of return on alternative asset still has a sign contrary to expectations.

Thus, we conclude this section with the observation that there is no strong evidence of monopsony power in capital interest.

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