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IAER Special Study No. 79-002
Contract No. 02SU.36100-8-9515
DOC File No. 5420-7
DGCE Document No. 116

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FOREWORD

The Institute of Applied Economic Research (IAER), successor institution of the International Institute of Quantitative Economics founded in 1969, has been active in its present form since April 1976. The IAER has firmly established itself as Concordia University's Institute for programmes of socio-economic research and training related to both the developing world and Canada.

The IAER envisages the most fundamental problems of economic and social development in the developing world to be:efficient use of scarce economic resources; creation of employment opportunities; overpopulation; food availability and the development of the rural sector; equitable distribution of income; development of an indigenous research capability and planning of educational systems; and, the social implications of alternative development strategies. These problems require new kinds of international collaboration between the developed and developing countries.

For the industrialized countries, such as Canada, the IAER sees some of the major problems of economic and social development to be: management of natural resources, especially energy; preservation of the environment; improvement and management of urban public services; regional economic disparities; inflation and unemployment; and the development of socially acceptable income policies. These problems require improved forms of collaboration at the national level among universities, the public, government institutions and the private sector.

The IAER, through international and Canadian collaboration, attempts to make a contribution to the solution of some of these problems. In order to begin effectively the task of conceptualizing, defining and analysing these fundamental problems, the IAER utilizes the most modern methods of scientific analysis available, as well as the services of recognized experts in the relevant fields, who participate as Senior Research Advisors and Research Associates.

The IAER's contribution to the solution of some of these major problems, referred to in the preceding statement, takes the form of:

1) initiating, organizing and implementing major economic research projects, at both international and Canadian levels, occasionally in collaboration with other research institutes and interested specialists;

2) organizing seminars and conferences on specific economic issues of particular international and Canadian interest; and

3) serving as a link between Concordia University and the Canadian private sector with the objective of increasing the latter's awareness of participation in, and support for applied economic research.

The IAER, given its expertise and experience, believes that it has a useful and necessary role to play both in the developing world and in Canada.

Professor Vittorio Corbo
Director

CHAPTER V

CHAPTER VI

The Canadian Department of Communications (DOC), contracted the Institute of Applied Economic Research (IAER), of Concordia University under contract # 02SU.36100-8-9515 to continue work on the building of a simulation model of Bell Canada. The models take into account the productive and financial characteristics of the carrier. The work was done at the IAER during the period from July 1st, 1978 to March 31st, 1979, by the following team of researchers:

PROJECT DIRECTOR:	Professor Vittorio Corbo
RESEARCH ASSOCIATES:	Professor Jon Breslaw
	Professor Jean-Marie Dufour
RESEARCHER:	José M. Vrljicak
RESEARCH ADVISOR:	Professor Robert S. Pindyck

Professor B. Smith who undertook concurrently a project with Professor V. Corbo on "Economies of Scale and Economies of Scope in Bell Canada", also collaborated in this project. The multiple output cost model used in this study draws heavily on his research with Professor Corbo.

We would like to thank the members of the DOC for their cooperation, and for the beneficial discussions with us while carrying out this study. Also we would like to thank M. Daskalakis who assisted in the estimation of the demand model, I. Rakita who was helpful in the initial estimation of the financial model, Melanie Neufield who provided secretarial assistance throughout this project and Esther Massa and Johanne Yelle for their typing.

in particular) may have objectives other than profit maximization, or at least objectives in addition to profit maximization. Even if the rate of return constraint still applies, alternative objectives of the firm may result in different inefficiencies or in no inefficiency at all. Second, regulatory agencies themselves usually have mixed objectives. While in the long run rate of return targets usually dominate in the determination of allowed prices, because of political constraints regulatory agencies in the short run may find themselves regulating prices, to some extent independently of the resulting rate of return. Furthermore, many regulated companies produce more than one product, and an objective of the regulatory agency may be to regulate the relative prices of these products, so as to cross-subsidize one product at the expense of another. As we will see, these alternative objectives of the regulatory agency may again lead to different kinds of inefficiencies, or to no inefficiency at all.

If one is interested in measuring the regulation-induced inefficiency for a firm such as Bell Canada, these issues become extremely important. Let us examine them in somewhat more detail.

Consider first the implications of alternative objectives for the firm. As mentioned above, objectives other than that of profit maximization can result in something quite different from the standard Averch-Johnson over-capitalization. As an example of this, if the objective of the firm is revenue maximization instead of profit maximization, then the rate of return constraint will result in under-capitalization rather than over-capitalization, i.e.

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Now let us consider the objectives of the regulatory agency. Most studies of regulation-induced inefficiency are based on the assumption that the regulatory objective is a rate of return for the firm that is in some sense "fair". Indeed, there is little doubt that in the long run a fair rate of return is the dominant objective in the regulation of monopolistic firms (although as we will see, even in the long run other objectives may also be important). However, even for a monopolistic firm that produces only a single output, in the short run there may be political or institutional constraints that prevent the application of rate of return objectives. Regulatory lag is of course one example of this, where the regulatory agency is simply not able to respond instantaneously to changing (increasing or decreasing) costs by adjusting price to keep the rate of return fixed. In some cases, however, even where the time required for a rate review is not a problem, political constraints on the regulatory agency may prevent the agency from adjusting prices in a way necessary to achieve a desired rate of return. In this case we could say that the objective of the regulatory agency is the price itself, rather than the rate of return. But all our observations on the behavior of the firm in which our estimations are based refer to short run points. Thus, in our empirical implementation of the model we assume price regulation for monopoly services. (Although we stress again that such an objective is likely to apply only over the short term).

Many regulated firms, and Bell Canada in particular, produce more than one output, and regulatory objectives often involve the cross-subsidization of outputs. This means that in addition to

CHAPTER I

INTRODUCTION

One of the fundamental questions involved in the regulation of a monopoly is the inefficiency introduced by the regulatory constraint. In their classic paper, Averch and Johnson showed that for a two input case (labor and capital) a monopoly operating under rate of return regulation would tend to over-capitalize, i.e. would use a capital-labor ratio greater than the cost-minimizing capital-labor ratio. This Averch-Johnson result has become the basis for most of the analyses of regulation-induced inefficiency. A number of economist have dealt with ways of extending the Averch-Johnson analysis and making it more applicable to realistic problems, but in general these have consisted of variations on the basic theme of determining the effect on a profit-maximizing monopoly of a rate of return constraint.[1]

Most attempts to measure the size of the inefficiency resulting from regulation have been based on this basic Averch-Johnson model. Typically, one attempts to measure the extent and cost of over-capitalization that should result if the firm tries to maximize profit under the rate of return constraint. There are a number of problems, however, with this basic framework for measuring inefficiency. First of all, the objective of the firm may not be profit maximization. There is now considerable evidence from the industrial organization literature that many firms (and large ones

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[1] As an example of a variation on this theme, a number of economists have attenpted to determine the effects of regulatory lag, i.e. the lags resulting between the time a price change is requested or the actual rate of return changes and the time that the price change is actually enacted by the Regulatory Commission. One can show that in some cases regulatory lag will diminish the extent of over-capitalization that would otherwise result.

constraint which depends on the parameters of the production frontier. But these same parameters are precisely the ones that we need to estimate.

From a comparison of the allowed and actual rate of return we concluded that a rate of return regulation was in effect only after 1966. Thus, we tried to estimate the production frontier parameterizing the Lagrangian multiplier of the regulatory constraint by imposing a value of zero up to 1966 and then allowing it to take two or three sets of values for the period 1967 to 1976. These values were taken as parameters to be estimated jointly with the other parameters of the production frontier. This procedure introduces some biases in our estimates because the multiplier of the constraint is indeed a variable but we treat it as a parameter in our estimations. Attempts of this sort were not successful with respect to stability of the estimated values of λ.

The second problem arises when the model is simulated; if we treat the multiplier (λ) as a variable in the simulation it is very difficult to obtain good tracking for the different variables involved due to the fact that in the estimations it is treated as a parameter. Thus, the values of the residuals in the estimation of the equations affect λ directly; λ capturing all the random errors. Hence, in our final model, short term regulation enters only through the price of local and telephone message toll services.

What is important here is to recognize that the regulation of prices can critically alter the nature and extent of regulation-induced inefficiency. The effect on inefficiency will depend on the structure of production of the firm.

the exact opposite of the standard Averch-Johnson effect.[1] In fact, it appears that revenue maximization is at least one of the managerial objectives of a number of larger firms, and this is sometimes used to explain the fact that empirical studies often fail to show evidence of over-capitalization in the regulated firm operating under a rate of return constraint.

The firm might operate with still other objectives. In our work we have considered profit maximization to be the primary objective of the firm subject to regulated prices for the monopoly services, local service and telephone message toll, and a production frontier. However, it may be that the firm does not maximize or minimize at all, but rather <u>satisfices</u>. There is also considerable evidence for satisficing behavior in large firms, where managers operate under the restriction of some minimum acceptable level of profit (rather that maximize profit), and possibly miminum and maximum acceptable levels of other variables. We have not attempted to determine the effects of satisficing behavior on the efficiency of the firm (not to our knowledge has anyone else), but clearly we cannot be confident that over-capitalization will occur as it did before if the firm is engaging in some kind of satisficing behavior.

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[1] To see that the revenue maximizing firm under-capitalizes when faced with a rate of return constraint, just solve the constrained maximization problem

$$\text{MAX } PQ \text{ s.t. } PQ - wL \leq sK$$

with s, the allowed rate of return, greater than the cost of capital r. It is easily shown that the solution to this problem yields $MP_K/s = MP_L/w$, where MP_K and MP_L are the marginal products of capital and labor. But cost is minimized when $MP_K/r = MP_L/w$. Since $s > r$, the firm is therefore under-capitalized.

where f is an aggregator function for outputs and h is an aggregator function for inputs. The problem here is that the restriction of separability in outputs and inputs implies that there is no difference in the capital-intensities of the different outputs, so that a priori the relative price constraint would have no impact on the extent of inefficiency. Because of this problem, we have now estimated a production possibility frontier and a multiple output cost function for Bell Canada that is unrestricted, i.e. that is not a priori separable in outputs and inputs.

In Table 1 we summarize the effects of regulation for various objectives of the regulatory agency and for various objectives of the firm.

its rate of return objective, the regulatory agency might also wish to set <u>relative prices</u>, i.e. the ratio of the price of one output to the price of another output. Note that as long as the <u>levels</u> of the individual prices are not an objective of the regulatory agency, (but only the ratios of prices), then this is consistent with a rate of return target as an additional objective.

The regulation of Bell Canada is a good example of where relative prices are a regulatory objective in addition to the rate of return. In our work we have look at Bell Canada as a firm producing two major outputs, regulated telephone services, (local telephone services and telephone message toll) and other toll services. It has been argued that one of the objectives of the regulation of Bell Canada has been to subsidize local telephone service at the expense of long distance and other toll services. In our model this has been introduced by taking the price of local services and telephone message toll as fixed at the discretion of the regulatory authority. Then we assume that Bell can choose the price of other toll services as to maximize profits. Then the weighted price of local and message toll services can be compared to their marginal cost to analyze the extent of cross-subsidy. We have also experimented extensively with the introduction of a separate rate of return constraint but our results have not been successful. Two type of difficulties have arisen. The first is that to obtain accurate estimates of the coefficients of capital in a general translog production frontier as well as for the cost frontier, we need a side condition for capital. In the presence of a regulatory constraint, the side conditions for capital involve the Lagrangian multiplier (λ) of the regulatory

This report has six chapters and one Appendix. In Chapter II we estimate demand equations for telephone services. The demand equations that we estimate are of three types: Flexible functional forms, choice between linear in the logs and linear in the variables models and habit formation model. High price collinearity makes it very difficult to study cross-price effects. Also the lack of disaggregate data into business and residential does not allow the estimation of separate demand equations for these two types of services. A priori one would expect different demand functions for both types of users. For residential demand, one would use models of consumption and for business demand, models of demand for intermediate inputs.

In Chapter III we present the structure of a simulation of the real structure of Bell Canada. By real structure we mean the determination of factor inputs: Labor, Capital and Raw materials. Two alternative models of the real structure of Bell are presented. They differ in the specification of the underlying production technology. In the first model the production technology is characterized by a general two-output three-input translog production function. In the second, the technology is implicit in a general two-output, three-input translog cost function.

In Chapter IV, we validate the model of the real structure of Bell Canada. We validate first the demand model by itself, then the production model by itself and finally both models

Let us take Bell Canada as an example. Assuming that the firm's objective is to maximize its profits subject to the rate of return constraint, the resulting over-capitalization may be either exacerbated or eliminated by the price constraint. To see this, suppose first that regulated services are much more capital-intensive than other toll service. In this case, subsidizing the regulated service through the relative price constraint will reinforce the over-capitalization that results from the rate of return constraint, since the firm will produce more of the regulated service, and hence use more capital than it would have had it not faced the relative price constraint. If, on the other hand, non-regulated service is more capital-intensive than regulated service, the relative price constraint will work in just the opposite direction and will reduce the extent of over-capitalization. In fact, if the relative price constraint and the difference in capital-intensities are strong enough, the result could even be under-capitalization as the relative price effect overwhelms the rate of return effect.

We thus see that the determination of regulation-induced inefficiency depends critically on the structure of production, and this is exactly the reason that we have recently attempted to estimate unrestricted production and cost structures for Bell Canada. Recall that in the IAER report of March 31, 1978, production functions for Bell Canada were estimated that were *a priori* restricted to be separable in outputs and inputs, i.e. were of the form

$$F(y_1,y_2,x_1,x_2,x_3) = f(y_1,y_2) - h(x_1,x_2,x_3) = 0$$

TABLE 1

EFFECTS OF REGULATION

OBJECTIVE OF FIRM

		PROFIT MAX.	REVENUE MAX.	COST MIN.
	Rate of Return	Over-capitalization (standard Averch-Johnson case)	Under-capitalization	Over-capitalization if cost is minimized subject to largest output satisfying rate-of-return constraint.
Objective of Regulatory Agency (i.e. Constrained Variables)	Rate of Return and Relative Prices	Relative price constraint could reinforce over-capitalization, or could lead to less use of capital. Depends on the structure of production.	Relative price constraint could reinforce under-capitalization, or could lead to more use of capital. Depends on the structure of production.	Could lead to over-capitalization or under-capitalization. Depends on the structure of production.
	Prices	If price ceiling is not linked to a rate of return, no bias in single-product case. In multi-product case can lead to over-or under-capitalization, depending on structure of production.	Same as under profit maximization.	Same as under profit maximization.

the study remained plagued by a high degree of collinearity among the price variables.

The main conclusions are the following. First, and not surprisingly, the flexible functional forms (Translog and Generalized Leontief) do not happen to be very useful in the present context. The likely reason for this is that such models are best fit to describe final demand systems (consumer or households), while we have here mixed Household-Business data. Second, the Box-Cox analysis suggests the double-log model after all remains here a good approximation. Third, a careful analysis of the error structure (considering autoregressive processes up to order 3 on the errors) suggests the errors follow an autoregressive process of order 1 (and not higher). Fourth, the "habit formation" model does not produce good results. Fifth, it still appears very difficult to estimate cross-price elasticities because of the multicollinearity between the price variables. Sixth, a quite satisfactory set of demand equations (Table 3.11) is obtained by suppressing the cross-price effects and taking into account (via dummy variables) certain discontinuities in the behaviour of prices (which may reflect differing regulatory behaviours). Seventh, there is no basis, from these results, for stating there is strong substitutability between Telephone Message Toll and Other Toll services (if anything, they rather suggest the existence of some form of complementarity between these two types of services).

together. From this validation exercise we find that both models predict the actual values of the demand levels and factor demand quite accurately.

In Chapter V, we present a financial model of Bell Canada which is also integrated with a model that reproduces the income statement of the company. The model is estimated and then validated within the sample period. Also we simulate the financial and income model taking as inputs the simulated values coming from the demand and cost functions.

In Chapter VI, we perform forecasts for the period 1977-1983 under the following two assumptions about price regimes. Firstly, we assume constant 1979 nominal prices; secondly, we assume constant 1979 real prices. Forecasts of the income and financial models are run under both price regimes.

$$(2.3)\quad y_i\,(\underline{P}/E) = \frac{P_i^{-1}\quad E[\alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln(P_j/E)]}{\sum_{k=1}^{N} \alpha_k + \sum_{k=1}^{N} \sum_{m=1}^{N} \gamma_{km} \ln(P_m/E)}, \quad i = 1,\ldots, N$$

where the restrictions $\sum_{i=1}^{N} \alpha_i = 1, \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} = 0$

are imposed in order to identify the parameters. Let us now assume all the consumers have the same utility functions and differ only via their incomes; then, the demands (per capita) may be conveniently reexpressed in budget share form:

$$(2.4)\quad m_i \equiv \frac{P_i y_i^*}{E^*} = \frac{\alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln P_j - \sum_{j=1}^{N} \gamma_{ij} \lambda \ln E^*}{\sum_{k=1}^{N} \alpha_k + \sum_{k=1}^{N} \sum_{m=1}^{N} \gamma_{km} \ln P_m}, \quad i = 1,\ldots, N$$

where y_i^* is consumption (per capita) of good i, E^* is income per capita, $\lambda \equiv \{ \int E \ln E\ \phi\ (E)\} / E^* \ln E^*$ and $\phi(E)$ is the distribution of income (in probability density form). Note, furthermore, that these demands are homothetic if

$\sum_{j=1}^{N} \gamma_{ij} = 0, \ i = 1,\ldots, N.$

The Generalized Leontief reciprocal indirect utility function is defined by:

$$(2.5)\quad h(\underline{v}) = \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij}\, v_i^{1/2} v_j^{1/2} + \sum_{i=1}^{N} b_{oi} \ln v_i + b_{oo}$$

where $b_{ij} = b_{ji}$, $\sum_{i=1}^{N} b_{oi} = 0$. The resulting system of demand functions has the form:

$$(2.6)\quad m_i = \frac{\sum_{j=1}^{N} b_{ij}\, P_i^{1/2}\, P_j^{1/2} + b_{oi} \alpha E^*}{\sum_{k=1}^{N} \sum_{m=1}^{N} b_{km} P_k^{1/2}\, P_m^{1/2}}$$

where $\alpha \equiv \int E^2\ \phi(E) dE/(E^*)^2$ and the restriction

CHAPTER II

THE DEMAND MODEL

2.1 Introduction

In order to model the demand for the different telephone services (Local, Telephone Message Toll and Other Toll), we basically considered three different approaches. The first one consisted in using so-called "flexible functional forms" based on explicit assumptions concerning the utility functions of consumers. The second approach was a choice of functional form based on the Box-Cox transformation, the general form analyzed including as special cases the linear demand model and the double-log model. The third approach consisted in updating and trying to improve (in particular, via a more careful consideration of the error structure) the double-log model used in our previous study (IAER, 1978), in both its simple form and the "habit formation" version (with demands and income in per capita form).

As pointed out in the previous IAER(1978) study the main way to improve the results obtained at this stage would have been the access to disaggregate business-residential information on telephone services. There are good theoretical reasons for thinking these may have different behaviours and the difference is of great interest with respect to regulation issues. Unfortunately, such data did not become available for the present study. Nevertheless, and despite this problem, it is important to note the data we were using here had been appreciably revised (for the last 6 or 7 years) and an updating of the previous results was in order. Of course, as pointed out in IAER (1978), as pointed out above, a separability assumption is needed; otherwise all we can hope is that the resulting functional forms will prove to be useful local approximations.

B) Double-Log Models and Box-Cox Transformation

In the double log formulation, the demand equations have the form:

$$(2.7) \qquad \ln SO_{it} = \alpha_{oi} + \alpha_{1i} \ln \frac{P_{1t}}{PD_t} + \alpha_{2i} \ln \frac{P_{2t}}{PD_t} + \alpha_{3i} \ln \frac{P_{3t}}{PD_t} + \alpha_{4i} \ln \frac{YD_t}{POP_t} + u_{it}$$

where SO_{it} is the quantity demanded (per capita) of service i in period t, P_{it} is the price of service i in period t, PD_t is a price deflator for period t, YD_t is real income, POP_t is the population of Quebec and Ontario and u_{it} is a random disturbance. Regarding the disturbances, we will assume they are either independant (normal) or autocorrelated according to an autoregressive scheme. In the last case, the most standard model consists in assuming the u_t's follow an AR(1) process (autoregressive process of order 1): $u_t = \rho u_{t-1} + \varepsilon_t$, where $\varepsilon_t \overset{ind}{\sim} N[0,\sigma^2]$. We also consider the possibility that they follow autoregressive processes of higher order such as AR(2), AR(3), etc. A priori, we expect $\alpha_{ii}<0$ and $\alpha_{4i}>0$ (for equation i).

The double-log model has the great advantage of being relatively easy to interpret and estimate. Nevertheless, it implies constant income and price elasticities which may seem too rigid. An elegant way of assessing whether the double-log model is appropriate is to consider the Box-Cox (1964) transformation:

2.2 The Models

A) Flexible Functional Forms

An attractive way of modelling a demand system is to specify flexible functional forms for the indirect utility function of consumers and derive the corresponding demand functions. The functional forms are called "flexible" in the sense that the various (own and cross) price elasticities as well as the income elasticities can vary and are not constrained a priori at a base point. Such an approach is adopted, in particular, by Christensen, Jorgenson and Lau (1975), Christensen and Manser (1977) and Berndt, Darrough and Diewert (1977).

More specifically, we consider here two different functional form for the (reciprocal) indirect utility functions: Translog and Generalized Leontief. Then, given h(v) the reciprocal indirect utility function of a consumer, where $\underline{v} = \frac{\underline{P}}{E}$, $\underline{P}$ is the vector of prices and E is income, the system of demand functions of this consumer is easily obtained via Roy's identity (1942, 1947):

$$(2.1) \qquad \underline{y}\ (v) = \frac{\nabla h(\underline{v})}{\underline{v}'\nabla h(\underline{v})}$$

The translog reciprocal indirect utility function is defined by:

$$(2.2) \qquad \ln h(\underline{v}) = \alpha_o + \sum_{i=1}^{N} \alpha_i \ln v_i + \frac{1}{2}\sum_{i=1}^{N} \sum_{i=1}^{N} \gamma_{ij} \ln v_i \ \ln v_j$$

where $\gamma_{ij} = \gamma_{ji}$, for all i, j, and N is the number of goods. Using Roy's identity, we obtain the following system of demand functions:

inconsistent estimates if the disturbances of this equation are auto-correlated. We therefore, in our estimation, begin by assuming a first-order auto-regressive process for the disturbances, and use the maximum likelihood estimation procedure to estimate simultaneously the coefficient of the auto-regressive process and the coefficients of the equation by means of a non-linear algorithm.

$\sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij} = 1$ is imposed in order to identify the parameters. Homotheticity will hold if $b_{oi} = 0$, $i = 1,\ldots, N$.

In order to estimate the systems (2.4) and (2.6), we interpret each budget share equation as the expectation of m_i given E^*, P_1, $P_2,\ldots, P_N$. Let the vector of error terms at time t be $\underline{\varepsilon}(t) = (\varepsilon_1(t), \varepsilon_2(t),\ldots, \varepsilon_N(t))'$. Then, assuming the error vectors are independent (across time) with covariance matrix Ω, the parameters may be estimated by a non-linear procedure (maximum likelihood). Furthermore, different hypotheses (like homotheticity) may be tested using likelihood ratio tests.

Now, in order to be able to apply this approach to our problem, we consider the three different telephone services under study constitute three goods while all other consumer goods can be aggregated into one single good. (This, of course, involves an assumption of separability.) We are thus implicitly considering a system of four goods (N=4) and four demand equations. Nevertheless, since the shares must sum to 1, it is sufficient to estimate only three of the four demand relationships. We thus do not need a measure of the output of the fourth (aggregate) good but only a price index for it.

From the above developments, it is easy to see that such an approach requires data at the level of final demand (consumer or households), in opposition to intermediate demand (e.g. business demand). Thus, in the case of demand for telephone services, disaggregated business-residential information is in principle necessary for the approach to be applicable. Furthermore, of services and the normalized Divisia quantity index of this service. For Other Toll services, the data were taken directly from Bell Exhibit.

C) The Real Income Variable (YD_t)

The demand equations that we estimate are aggregated for Business and Household. This is caused by the non-existence, up to now, of disaggregated data in the public domain. Thus, the income variable that we require is a variable related to the overall level of economic activity in the Quebec-Ontario region. Indeed, for the income variable we used the sum of Gross Provincial Products of Quebec and Ontario at 1967 prices (deflating both Provincial Products with the consumer price index for Canada).

The price deflator used in our computations is the consumer price index for Canada (1967=1).

D) Other Variables

For the flexible functional forms (Translog and Generalized Leontief), we use as price index of the aggregate good (the fourth good) a Divisia price index of the consumer price indices of Montreal and Toronto, with arithmetic weights based on the Gross Provincial Products of Quebec and Ontario. For the λ and α variables we employ the estimates obtained by Berndt, Darrough and Diewert (1977) for Canada as a whole.* Note also that the variable

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* For the period 1972-75, we had to extrapolate (setting the values for this period equal to the value of 1971). Since these two series were exhibiting very little variability, this approximation is not likely to have been of much importance.

$$(2.8) \quad SO_{it}^{(\lambda_i)} = \gamma_{oi}+\gamma_{1i}P_{1t}^{(\lambda_i)} +\gamma_{2i} P_{2t}^{(\lambda_i)} + \gamma_{3i} P_{3t}^{(\lambda_i)} +\gamma_{4i} \frac{YD_t}{POP_t} + u_{it}$$

where

$$SO_{it}^{(\lambda_i)} = (SO_{it}^{(\lambda_i)} - 1)/\lambda_i, \; P_{it}^{(\lambda_i)} = (P_{it}^{(\lambda_i)} - 1)/\lambda_i,$$

$$YD_t^{(\lambda)} = [(\frac{YD_t}{POP_t})^{(\lambda_i)} - 1]/\lambda_i$$

When $\lambda_i = 0$, (8) reduces to the double-log model while $\lambda_i = 1$ gives a linear demand model, and a wide variety of alternative functional forms may be considered by changing the value of λ_i. Clearly we can then assess whether the double-log model is consistent with the data by testing $\lambda_i = 0$.

The "habit formation" model is a modification of the double-log model based on the assumption that the demand for a type of telephone service is a function of income, prices and a state variable S_{it} proportional to last period's demand, and representing the stock of accumulated telephone habits. It is given by the following pair of equations:

$$\ln SO_{it} = \beta_{oi} +\beta_{1i}\ln\frac{P_{1t}}{PD_t} +\beta_{2i}\ln\frac{P_{2t}}{PD_t} +\beta_{3i}\ln\frac{P_{3t}}{PD_t} +\beta_{4i}\ln \frac{YD_T}{POP_t} + \beta_{5i}\ln S_{it} + u_{it} \qquad i= 1,2,3$$

with:

$$\ln S_{it} = \Theta_i \ln SO_{i,t-1}$$

Replacing the second equation in the first, we obtain:

$$(2.9) \quad \ln SO_{it} = \beta_{oi} +\beta_{1i}\ln\frac{P_{1t}}{PD_t}+ \beta_{2i} \ln\frac{P_{2t}}{PD_t} +\beta_{3i}\ln\frac{P_{3t}}{PD_t} +\beta_{4i}\ln \frac{YD_t}{POP_t} + \beta_{5i}\Theta_i \ln SO_{i,t-1} + u_{it} \qquad i=1,2,3$$

A priori, we expect $\beta_{ii} < 0$, $\beta_{4i} > 0$, and $\beta_{5i}0_i > 0$ (for equation i). Due to the presence of a lagged endogenous variable on the right hand side of this equation, ordinary least squares would yield

2.4 The Empirical Results

The results of our estimations relating to the flexible functional forms, based on Translog (TLOG) and Generalized Leontief (GL) reciprocal indirect utility functions, are reported in Tables 2.1 to 2.4. More precisely, the results for the estimation of nonhomothetic, nonsymmetric versions of the TLOG and GL forms are in Table 2.1; then, in Table 2.2, symmetry is imposed (a requirement which coming from the general theory of demand), and, in Table 2.3, both symmetry and homotheticity are imposed. The likelihood ratio test statistics for comparing these three versions for the two functional forms considered are given in Table 2.4. Quite noticeable is the fact that symmetry is rejected, a conclusion which is at odds with the standard theory of demand. We can note also that the free (nonhomothetic, nonsymmetric) versions of both functional forms produce (despite 18 coefficients in each case) pretty bad fits (R^2) for the demand of local telephone services and implied demand elasticities are in several cases very unreasonable (i.e. the demand for local telephone services appears to be very elastic). Such deceiving results are not, in fact, too surprising given that such an approach has a sound theoretical basis only when applied to final demands data (and not aggregate business-residential data like the ones we have here). Another important aspect is the strong non-linearity of the estimation problem which leads to very important computational costs. In view of these observations we decided to switch to a simpler and hopefully more robust approach.

2.3 The Data

Before proceeding to analyse the results of the estimations, we will describe the data used.

A) Quantity Demanded

We work with three outputs: Local, Telephone Message Toll and Other Toll services. For Local services the quantity demanded is measured as the revenue from these types of services at 1967 prices. In the case of Telephone Message Toll services, the quantity demanded is measured as a Divisia quantity index with arithmetic weights of the three types of toll services, that is: Intra-Bell Telephone Message Toll Services, Trans-Canada Telephone Message Toll Services, U.S. and Overseas Telephone Message Toll Services. Each of these services is measured as the revenue from them (including uncollectables) at 1967 prices.

The Other Toll category was measured as the revenue from this type of service at 1967 prices. The Telephone Message Toll Divisia quantity index was normalized to the 1967 dollar revenues from these services.*

To obtain the per capita quantities, the series above are divided by the population of Quebec and Ontario (POP_t).

B) The Price of Each Telephone Service

For local services, the price index is taken directly from Bell data. For Telephone Message Toll services, the price index is defined as the ratio of the current dollar revenues from these

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* That is, the scale of the computed quantity index was defined in such a way that the value of this index for 1967 was equal to the dollar revenue from this service in 1967.

TABLE 2.2

PARAMETER ESTIMATES OF TWO FUNCTIONAL FORMS

NON-HOMOTHETIC AND SYMMETRIC *

TRANSLOG		GENERALIZED LEONTIEF	
PARAMETER	ESTIMATE	PARAMETER	ESTIMATE
α1	.00615 (4.035)	β01	.00098 (-3.705)
α2	.00254 (4.150)	β02	-.00030 (-2.849)
α3	-.00083 (-3.898)	β03	.00006 (-2.849)
γ11	.00763 (10.252)	β11	.00178 (.864)
γ21	-.00432 (-11.171)	β21	-.01019 (-12.460)
γ31	(-1.446)	β31	-.00080 (-1.518)
γ41	-.00511 (-3.019)	β41	.02288 (7.779)
γ22	.00184 (6.668)	β22	-.00207 (-2.894)
γ32	-.00118 (-6.481)	β32	-.00256 (-7.615)
γ42	.00193 (2.897)	β42	.02099 (17.459)
γ33	-.00067 (-2.879)	β33	-.00253 (-5.940)
γ43	.00081 (3.547)	β43	.00667 (14.486)
SMPL= 25		SMPL=25	
EQ1 SSR= .108050E-04 R^2 = .2344 DW = .2532		EQ1 SSR= .8654E-05 R^2 = .3868 DW = .2051	
EQ2 SSR= .17106E-05 R^2 = .7984 DW = .4331		EQ2 SSR= .14024-05 R^2 = .8347 DW = .5953	
EQ3 SSR= .19919E-06 R^2 = .9635 DW = .6972		EQ3 SSR= .19048E-06 R^2 = .9651 DW = .7394	
Log of Likelihood Function=560.872		Log of Likelihood Function=556.339	

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* t-values must be divided by $\sqrt{3}$

E (income per capita) is simply measured by the sum of the Gross Provincial Products (in current dollars) of Quebec and Ontario divided by the population of the two provinces, while the shares m_i (i=1,2,3) are obtained by dividing revenues (in current dollars) for each service by the sum of the Gross Provincial Products (in current dollars).

TABLE 2.4

LIKELIHOOD RATIO TEST RESULTS FOR TWO FUNCTIONAL FORMS

TEST STATISTIC

TEST	TRANSLOG	GENERALIZED LEONTIEF	NO. OF RESTRICTIONS	.01 CHI-SQUARE CRITICAL VALUE
Ho: Symmetry Hl: Free	18.34	28.962	18-12=6	16.811
Ho: Symmetry and Homotheticity Hl: Symmetry	27.914	16.760	12-9=3	11.344
Ho: Symmetry and Homotheticity Hl: Free	45.710	43.610	18-9=9	21.666

TABLE 2.1

PARAMETER ESTIMATES OF TWO FUNCTIONAL FORMS

FREE: NON-HOMOTHETIC NON-SYMMETRIC*

TRANSLOG		GENERALIZED LEONTIEF	
PARAMETER	ESTIMATE	PARAMETER	ESTIMATE
$\alpha 1$	.01119 (8.842)	$\beta 01$	.00056 (-3.134)
$\alpha 2$	.00392 (7.517)	$\beta 02$	.00001 (.122)
$\alpha 3$	-.00061 -2.295	$\beta 03$	.00016 (3.997)
$\gamma 11$	.05292 (6.034)	$\beta 11$	.105726 (6.104)
$\gamma 12$	.02059 (-3.222)	$\beta 12$	-.06097 (-6.082)
$\gamma 13$	.01632 (-2.099)	$\beta 13$	-.01328 (-.949)
$\gamma 14$	-.01443 (-5.961)	$\beta 14$	-.01962 (-4.281)
$\gamma 21$	.01739 (3.93)	$\beta 21$	.00056 (-3.134)
$\gamma 31$	.00482 (5.818)	$\beta 31$	.01309 (8.223)
$\gamma 41$	2.5336 (2.618)	$\beta 41$	7.1633 (3.460)
$\gamma 22$	-.00653 (-1.946)	$\beta 22$	.02858 (-5.728)
$\gamma 32$	-.00297 (-4.244)	$\beta 32$	-.00876 (-8.023)
$\gamma 42$	-1.4777 (-2.198)	$\beta 42$	-5.3909 (-5.039)
$\gamma 23$	-.00736 (-1.937)	$\beta 23$	-.00591 (-.867)
$\gamma 33$	-.00231 (-3.222)	$\beta 33$	.00394 (-3.272)
$\gamma 43$	.02275 (.029)	$\beta 43$	1.5223 (.943)
$\gamma 24$	-.00416 (-2.7861	$\beta 24$	.00381 (-1.357)
$\gamma 34$	-.00082 (-1.906)	$\beta 34$	.00013 (-.172)

SMPL=25	SMPL=25
EQ1	EQ1
SSR= .676047E-05	SSR= .558023E-05
R^2 = .5210	R^2 = .6046
DW = .5344	DW = .5881
EQ2	EQ2
SSR= .100832E-05	SSR= .86757E-06
R^2 = .8812	R^2 = .8977
DW = .6069	DW = .6068
EQ3	EQ3
SSR= .16307E-06	SSR= .178023E-06
R^2 = .9701	R^2 = .9673
DW =1.1772	DW =1.1490
Log of Likelihood Function=570.042	Log of Likelihood Function=570.814

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* t-values must be divided by $\sqrt{3}$

TABLE 2.5

BOX-COX MODEL

MAXIMUM LIKELIHOOD EQUATION BY EQUATION*

	Constant	$\tilde{P}_{it}^{(\lambda)}$	$\tilde{P}_{2t}^{(\lambda)}$	$\tilde{P}_{3t}^{(\lambda)}$	$YD_t^{(\lambda)}$	λ	D.W.	R^2
Local	-1.213 (-87.278)	.0274 (1.058)	.00140 (.070)	-.0788 (-2.983)	.203 (3.216)	.8	.50	.9892
Telephone Message Toll	-2.695 (-33.355)	.0198 (.147)	-.106 (-.960)	-.335 (-2.395)	.224 (4.275)	.3	.64	.9944
Other Toll	-1.673 (-114.942)	.801 (.321)	-.0107 (-.531)	-.0660 (-2.505)	.344 (4.501)	.6	1.00	.9941

* $\tilde{P}_{it} = P_{it}/PD_t$, $\tilde{P}_{it}^{(\lambda)} = (\tilde{P}_{it}^{(\lambda)}-1)/\lambda$, i = 1,2,3, and $YD_t^{(\lambda)} = (YD_t^{(\lambda)} - 1)/\lambda$.

D.W. is the Durlin-Watson statistic, R^2 is the multiple determination coefficient and the terms in parentheses are the t-statistics for testing the null hypothesis that the time value of the respective coefficient is zero. In this table the t-statistics are computed conditionally on the obtained value of λ

TABLE 2.3

PARAMETER ESTIMATES OF TWO FUNCTIONAL FORMS

HOMOTHETIC AND SYMMETRIC *

TRANSLOG		GENERALIZED LEONTIEF	
PARAMETER	ESTIMATE	PARAMETER	ESTIMATE
$\alpha 1$	.00886 (102.328)		
$\alpha 2$	.00467 (135.369)		
$\alpha 3$	.00101 (62.437)		
$\gamma 11$	.00485 (5.567)	$\beta 11$	.00648 (3.871)
$\gamma 12$	-.00389 (-10.571)	$\beta 12$	-.00970 (-12.065)
$\gamma 13$	.00075 (1.764)	$\beta 13$	-.00049 (-.742)
$\gamma 22$	.00164 (6.548)	$\beta 14$	.01268 (10.994)
$\gamma 23$	-.00178 (-8.061)	$\beta 22$	-.00062 (-1.159)
$\gamma 33$	-.00139 (-4.193)	$\beta 23$	-.00269 (-6.640)
		$\beta 24$	.01778 (38.515)
		$\beta 33$	.00298 (-5.743)
		$\beta 34$	.00721 (33.795)
SMPL=25		SMPL=25	
EQ1 SSR= .12257E-04 R^2 = .1315 DW = .1762		EQ1 SSR= .10460E-04 R^2 = .2588 DW = .2036	
EQ2 SSR= .18545E-05 R^2 = .7814 DW = .3692		EQ2 SSR= .15806-05 R^2 = .8137 DW = .4974	
EQ3 SSR= .30981E-06 R^2 = .9932 DW = .5292		EQ3 SSR= .18547E-06 R^2 = .9660 DW = .8057	
Log of Likelihood Function=546.915		Log of Likelihood Function=547.959	

* t-values must be divided by $\sqrt{3}$

TABLE 2.7

DOUBLE-LOG MODEL

ORDINARY LEAST SQUARES: EQUATION BY EQUATION

	Constant	$\ell n \frac{P_{1t}}{PD_t}$	$\ell n \frac{P_{2t}}{PD_t}$	$\ell n \frac{P_{3t}}{PD_t}$	$\ell n \frac{YD_t}{POP_t}$	D.W.	R^2	L^*
Local	-5.045 (-11.967)	2.160 (2.961)	-.506 (-.882)	-1.678 (-2.356)	1.288 (3.858)	.68	.9789	33.425
Telephone Message Toll	-6.008 (-17.279)	1.220 (2.027)	-.756 (-1.597)	-1.212 (-2.063)	1.531 (5.562)	.76	.9926	38.238
Other Toll	-13.102 (-7.097)	9.331 (2.920)	-3.479 (-1.384)	-2.430 (-.779)	6.002 (4.107)	.8222	.9550	-3.496

* L= LOG OF LIKELIHOOD FUNCTION

A natural choice here consists in using double-log demand functions. Now, since constant elasticities may seem too rigid, we tested this functional form using the classical approach of Box and Cox (1964). The results of the estimation of model (2.8) by non-linear least squares (maximum likelihood, non corrected for autocorrelation), using for λ a grid between -2.0 and 2.0, are given in Table 2.5. Since there are clear signs of serial dependence in the residuals (and this could affect appreciably all significance tests) we corrected for autocorrelation (assuming the errors follow an autoregressive process of order 1). The results are presented in Table 2.6. We see that, for the Telephone Message Toll and Other Toll equations, the hypothesis that $\lambda = 0$ cannot be rejected (at level .05). In the Local equation, λ appears significant; nevertheless, one can check easily that the relationship there obtained (with $\lambda = -.8$), implies absurdly big price and income elasticities. Consequently, we retain the double-log model $(\lambda = 0)$ as a reasonable approximation in this context.

In Table 2.7 we present the results for the estimation of the double-log model (equation (2.7)) without correction for autocorrelation. All the results from this table indicate strong evidence of autocorrelated disturbances. We thus proceed and correct for autocorrelation by first assuming the errors follow an AR(1) process. In Table 2.8, we find the results of the estimation equation by equation. One can note that the own price elasticity of the demand for Other Toll services is positive while the demand for Telephone Message Toll appears inelastic, two pretty unacceptable results. In Table 2.9, Zellner's

TABLE 2.9

DOUBLE-LOG MODEL: AR (1) ERRORS

$u_t=\rho u_{t-1}+\varepsilon_t$

ZELLNER'S PROCEDURE

	Constant	$\ell n \frac{P_{1t}}{PD_t}$	$\ell n \frac{P_{2t}}{PD_t}$	$\ell n \frac{P_{3t}}{PD_t}$	$\ell n \frac{YD_t}{POP_t}$	ρ	D.W.	R^2
Local	-.891 (-.548)	-.0706 (-.519)	.00950 (.106)	-.230 (-1.773)	.166 2.690	.985 (127.359)	1.10	.9995
Telephone Message Toll	-5.121 (-26.536)	.559 (1.482)	-.504 (-2.032)	-1.666 (-5.038)	.810 (5.311)	.614 (7.773)	2.05	.9983
Other Toll	-7.770 (-16.529)	-1.049 (-1.199)	.487 (.872)	-1.181 (-1.468)	1.735 (5.096)	.831 (30.472)	1.70	.9979

LOG OF LIKELIHOOD FUNCTION= 185.180

TABLE 2.6

BOX-COX MODEL: CORRECTED FOR AUTO CORRELATION

MAXIMUM LIKELIHOOD: EQUATION BY EQUATION*

	Constant	$\tilde{P}_{1t}^{(\lambda)}$	$\tilde{P}_{2t}^{(\lambda)}$	$\tilde{P}_{3t}^{(\lambda)}$	$YD_t^{(\lambda)}$	λ	ρ	D.W.	R^2	LR** $\lambda=0$
Local	-23.906 (-10.420)	-.585 (-.201)	.943 (.511)	-7.571 (-3.137)	12.412 (4.605)	-.8	.907 (10.535)	1.58	.9995	7.5
Telephone Message Toll	-16.264 (-10.889)	.995 (.429)	-2.107 (-1.401)	-4.284 (-2.216)	6.609 (4.662)	-.4	.890 (9.540)	2.41	.9983	1.0
Other Toll	-10.730 (-12.189)	-3.227 (-2.074)	1.270 (1.240)	.526 (.404)	3.329 (4.799)	-.1	.883 (9.216)	2.21	.9985	.4

* t- Statistics (in parentheses) conditional on λ

** Likelihood ratio for testing λ=0. Critical value (.05) = 3.84

that the second autocorrelation coefficient comes out significant only in the Local equation (in both Tables 2.10 and 2.11

In Tables 2.12 and 2.13 we present the results for the estimation of the double-log model when the disturbances are assumed to follow an AR(3) process. Again, estimating either equation by equation or with Zellner's procedure, we obtain implausible own price elasticities; the elasticity of the demand for Message Toll with respect to the price of Other Toll remains negative (and significant).

Thus, from the results in Tables 2.10 to 2.13, we conclude there is no strong basis for including more than one autocorrelation coefficient in the error structure. On the other side, Table 2.9 suggests it is important to include one.

The results for the habit formation version of the double-log model (with AR(1) errors) are given in Tables 2.14 (equation by equation) and 2.15 (Zellner's procedure). Both methods of estimation give pretty implausible results. The own price elasticities for Local and Other Toll services are positive (although not significant); the demand for Telephone Message Toll appears inelastic and its cross-price elasticity with respect to the price of Other Toll still is negative; furthermore, the coefficient of the lagged dependent variable in the Other Toll equation is negative (although not significant).

Therefore, none of the above systems of demand equations appear satisfactory, the most plausible one being probably in Table 2.9. Further, in this last Table, all the cross-price elasticities (except the elasticity of Telephone Message Toll with respect to the price of Other Toll) appear non-significant.

TABLE 2.8

DOUBLE-LOG MODEL: AR(1) ERRORS

$$u_t = \rho u_{t-1} + \varepsilon_t$$

MAXIMUM LIKELIHOOD: EQUATION BY EQUATION

	Constant	$\ln \frac{P_{1t}}{PD_t}$	$\ln \frac{P_{2t}}{PD_t}$	$\ln \frac{P_{3t}}{PD_t}$	$\ln \frac{YD_t}{POP_t}$	ρ	D.W.	R^2	L
Local	-1.137 (-.700)	-.0833 (-.609)	.0109 (.121)	-.233 (-1.674)	.168 (2.611)	.984 (110.286)	1.11	.9995	80.689
Telephone Message Toll	-5.103 (-26.715)	.339 (.862)	-.459 (-1.843)	-1.500 (-4.356)	.780 (5.331)	.686 (8.076)	2.17	.9984	58.228
Other Toll	-6.735 (-8.874)	-1.750 (-2.384)	.660 (1.392)	.287 (.357)	1.446 (4.438)	.910 (36.931)	2.32	.9985	41.502

LOG OF (JOINT) LIKELIHOOD FUNCTION= 180.419

TABLE 2.11

DOUBLE-LOG MODEL: AR(2) ERRORS

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t$$

ZELLNER'S PROCEDURE

	Constant	$\ell n \frac{P_{1t}}{PD_t}$	$\ell n \frac{P_{2t}}{PD_t}$	$\ell n \frac{P_{3t}}{PD_t}$	$\ell n \frac{YD_t}{POP_t}$	ρ_1	ρ_2	D.W.	R^2
Local	-.118 (-.035)	.000484 (.005)	-.0651 (-.994)	-.197 (-1.584)	.138 (2.701)	1.442 (9.574)	-.449 (-3.016)	2.06	.9995
Telephone Message Toll	.0746 (.006)	-.229 (-.684)	-.452 (-2.022)	-.134 (-.400)	.522 (3.297)	.665 (3.664)	.324 (1.782)	2.14	.9981
Other Toll	-7.184 (-11.000)	-2.142 (-3.075)	.701 (1.576)	.840 (1.120)	1.735 (4.935)	.671 (3.170)	.209 (1.074)	2.33	.9984

LOG OF LIKELIHOOD FUNCTION = 181.649

procedure was used (assuming the ε_t's across equations are contemporaneously correlated) and the three equations estimated jointly. We get, in this way, more efficient estimates. Now the own price elasticities all have the expected signs; nevertheless the own price elasticities for Local and Other Toll services do not come out significant (at level .05) and the demand for Telephone Message Toll services appears inelastic (which seems difficult to believe). All the cross-price elasticities came out non-significant except for the elasticity of Telephone Message Toll with respect to the price of Other Toll; this last number is negative indicating Telephone Message Toll is a complement to Other Toll (a somewhat surprising outcome). It may be noted also that the autocorrelation coefficients all come out highly significant.

We also considered the possibility that the disturbances follow higher order autoregressive processes. In Tables 2.10 (equation by equation) and 2.11 (Zellner's procedure), we present the results of the estimation when the disturbances are assumed to follow an AR(2) process. Again, the estimation equation by equation produces positive own price elasticities for the Local and Other Toll equations (although these are not significant) and the demand for Telephone Message Toll services appears inelastic. We observe the same sign pattern when using Zellner's procedure. The elasticity of the demand for Telephone Message Toll with respect to the price of Other Toll still comes out negative (although not significant). The results in Table 2.9 clearly appear more plausible. In this respect, we should note also

TABLE 2.13

DOUBLE - LOG MODEL: AR(3) MODEL

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \rho_3 u_{t-3} + \varepsilon_t$$

ZELLNER'S PROCEDURE

Constant	$\ln \frac{P_{1t}}{PD_t}$	$\ln \frac{P_{2t}}{PD_t}$	$\ln \frac{P_{3t}}{PD_t}$	$\ln \frac{YD_t}{POP_t}$	ρ_1	ρ_2	ρ_3	D.W.	R^2
-1.062 (-.373)	.0182 (.182)	-.0115 (-.176)	-.393 (-3.117)	.212 (3.702)	1.237 (7.438)	-.0338 (-.121)	-.211 (-1.189)	1.80	.9995
-5.283 (-25.275)	.142 (.373)	-.441 (-1.951)	-1.027 (-3.436)	.965 (6.240)	.595 (3.293)	-.251 (-1.393)	.362 (2.408)	2.13	.9983
-7.458 (-14.297)	-2.164 (-3.083)	.618 (1.433)	.580 (.869)	1.753 (5.274)	.561 (2.866)	.0877 (.389)	.172 (1.063)	2.11	.9982

LOG OF LIKELIHOOD FUNCTION = 178.763

TABLE 2.10

DOUBLE-LOG MODEL: AR(2) ERRORS

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t$$

MAXIMUM LIKELIHOOD EQUATION BY EQUATION

	Constant	$\ell n \frac{P_{1t}}{PD_t}$	$\ell n \frac{P_{2t}}{PD_t}$	$\ell n \frac{P_{3t}}{PD_t}$	$\ell n \frac{YD_t}{POP_t}$	ρ_1	ρ_2	D.W.	R^2	L
Local	-.567 (-.192)	.00290 (.025)	-.0300 (-.414)	(-.271) (-2.014)	.159 (2.895)	1.462 (7.830)	-.469 (-2.553)	2.15	.9996	79.621
Telephone Message Toll	(-1.498) (-.197)	-.316 (-.823)	-.342 (-1.365)	-.204 (-.536)	.575 (3.340)	.687 (3.067)	.299 (1.336)	2.17	.9981	54.764
Other Toll	-7.060 (-10.285)	-2.087 (-3.010)	.622 (1.402)	.934 (1.238)	1.698 (4.795)	.630 (2.816)	.251 (1.216)	2.28	.9984	41.244

LOG OF JOINT LIKELIHOOD FUNCTION = 175.629

TABLE 2.15

DOUBLE-LOG HABIT FORMATION MODEL: AR(1) ERRORS

$$u_t = \rho u_{t-1} + \varepsilon_t$$

ZELLNER'S PROCEDURE

	Constant	$\ell n \frac{P_{1t}}{PD_t}$	$\ell n \frac{P_{2t}}{PD_t}$	$\ell n \frac{P_{3t}}{PD_t}$	$\ell n \frac{YD_t}{POP_t}$	$\ell n SO_{i,t-1}$	ρ	D.W.	R^2
Local	-1.075 (-5.043)	.245 (2.151)	-.0332 (-.478)	-.359 (-2.637)	.223 (4.809)	.757 (14.396)	.376 (2.736)	2.31	.9997
Telephone Message Toll	-3.495 (-6.171)	.847 (3.766)	-.554 (-3.337)	-1.045 (-3.217)	.723 (5.847)	.360 (3.152)	.0696 .412	2.18	.9988
Other Toll	-8.590 (-8.387)	-2.281 (-3.362)	.671 (1.634)	.494 (.694)	1.751 (5.480)	-.280 (-2.017)	.903 (48.744)	2.20	.9985

LOG OF LIKELIHOOD FUNCTION = 194.293

TABLE 2.12

DOUBLE-LOG MODEL: AR(3) ERRORS

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \rho_3 u_{t-3} + \varepsilon_t$$

MAXIMUM LIKELIHOOD EQUATION BY EQUATION

	Constant	$\ln \frac{P_{1t}}{PD_t}$	$\ln \frac{P_{2t}}{PD_t}$	$\ln \frac{P_{3t}}{PD_t}$	$\ln \frac{YD_t}{POP_t}$	ρ_1	ρ_2	ρ_3	D.W.	R^2	L
Local	12.325 (.069)	.0266 (.246)	-.0359 (-.521)	-.343 (-2.522)	.222 (3.953)	1.308 (6.573)	.0274 (.078)	-.336 (-1.552)	1.96	.9995	76.589
Telephone Message Toll	-5.246 (-23.109)	.483 (1.148)	-.542 (-2.156)	-1.424 (-4.079)	.914 (5.060)	.500 (2.065)	-.167 (-.763)	.160 (.899)	2.04	.9984	55.166
Other Toll	-6.814 (-11.667)	-2.422 (-3.755)	.499 (1.288)	1.391 (2.281)	1.570 (5.338)	.399 (1.976)	.0546 (.240)	.375 (2.038)	2.27	.9985	41.457

LOG OF JOINT LIKELIHOOD FUNCTION= 173.212

TABLE 2.16

DIAGONAL DOUBLE-LOG MODEL WITH DUMMIES

MAXIMUM LIKELIHOOD: EQUATION BY EQUATION

	Constant	$\ell n \frac{P_{1t}}{PD_t}$	$\ell n \frac{P_{2t}}{PD_t}$	$\ell n \frac{P_{3t}}{PD_t}$	$\ell n \frac{YD_t}{POP_t}$	D^{*}_{1t}	D^{*}_{2t}	ρ	D.W.	R^2	L
Local	1.335 (.283)	-.196 (-2.093)			.153 (2.139)			.991 (123.421)	1.22	.9994	77.216
Telephone Message Toll	-5.056 (-26.453)		-1.441 (-9.850)		.686 (4.454)	.0895 (3.441)	.117 (4.883)	.703 (6.745)	2.57	.9988	58.075
Other Toll	-6.915 (-9.264)			-.942 (-1.630)	1.413 (3.781)			.893 (27.579)	2.07	.9981	36.822

LOG OF (JOINT) LIKELIHOOD FUNCTION = 172.113

* D_{1t} = 1, for t = 1959 - 1976

= 0, otherwise

** D_{2t} = 1, for t = 1970 - 1976

0, otherwise

TABLE 2.14

DOUBLE-LOG HABIT FORMATION MODEL: AR(1) ERRORS

$$u_t=\rho u_{t-1}+\varepsilon_t$$

MAXIMUM LIKELIHOOD: EQUATION BY EQUATION

	Constant	$\ln \frac{P_{1t}}{PD_t}$	$\ln \frac{P_{2t}}{PD_t}$	$\ln \frac{P_{3t}}{PD_t}$	$\ln \frac{YD_t}{POP_t}$	$\ln SO_{i,t-1}$	ρ	D.W.	R^2	L
Local	-1.054 (-4.626)	.203 (1.754)	-.0104 (-.147)	-.319 (-2.242)	.233 (4.911)	.767 (13.508)	.416 (2.635)	2.41	.9997	84.779
Telephone Message Toll	-3.551 (-5.937)	.822 (3.394)	-.581 (-3.150)	-1.063 (-2.981)	.707 (5.233)	.341 (2.770)	.189 (.988)	2.33	.9989	60.704
Other Toll	-8.767 (-7.680)	-2.494 (-3.352)	.866 (1.950)	.676 (.905)	1.874 (5.481)	-.298 (-1.893)	.907 (51.208)	2.25	.9985	42.038

LOG OF (JOINT) LIKELIHOOD FUNCTION= 187.521

close to 1). Then, using Zellner's procedure (Table 2.17), all the income and price elasticities appear significant and have the expected signs; furthermore, both Telephone Message Toll and Other Toll services appear to be price-elastic. The dummy variables in Telephone Message Toll equation are highly significant, showing the importance of taking into account the discontinuities observed. We thus incline to consider the demand system in Table 2.17 as being the most satisfactory with the existing aggregate data.

Another interesting observation which comes out of this set of experiments is that there is no basis for considering that Telephone Message Toll services constitutes a strong substitute for Other Toll services. In the most satisfactory set of results (Table 2.17), the elasticity of the demand for Telephone Message Toll services is zero; in all other Tables, it comes out negative (suggesting complementarity, if one is prepared to believe this is possible[1]).

---

[1] The fact that residential and business demands are not separated may again be of importance in the interpretation of such a result.

TABLE 2.17

DIAGONAL DOUBLE-LOG MODEL WITH DUMMIES

MAXIMUM LIKELIHOOD: ZELLNER'S PROCEDURE

	Constant	$\ell n \frac{P_{1t}}{PD_t}$	$\ell n \frac{P_{2t}}{PD_t}$	$\ell n \frac{P_{3t}}{PD_t}$	$\ell n \frac{YD_t}{POP_t}$	D_{1t}	D_{2t}	ρ	D.W.	R^2
Local	2.412 (.456)	-.155 (-2.317)			.167 (2.737)			.992 (157.558)	1.17	.9994
Telephone Message Toll	-5.190) (-33.544)		-1.401 (-12.154)		.775 (6.106)	.114 (7.285)	.0991 (6.239)	.576 (7.156)	2.43	.9986
Other Toll	-7.865 (-18.193)			-1.720 (-4.789)	1.785 (5.505)			.812 (33.698)	1.70	.9976

LOG OF LIKELIHOOD FUNCTION = 190.890.

We postulate that Bell canada maximizes profits subject to a translog production frontier and fixed regulated prices for local and telephone message toll. Given a demand equation, the assumption of fixed prices for regulated services is equivalent to the assumption of fixed quantity of regulated service.

The problem can be stated as:

Maximize

(1) $$\Pi = P_1\, y_1\, (P_1) + P_2 y_2 - wL - mM - vK$$

subject to:

(2) $$\begin{aligned} 0 = \ln(F+1) = {} & \ln A_0 + B_1 \ln\hat{L} + B_2 \ln\hat{M} + B_3 \ln\hat{K} + B_4 \ln\hat{TC} \\ & + \tfrac{1}{2}H_{11}(\ln\hat{L})^2 + \tfrac{1}{2}H_{22}(\ln\hat{M})^2 + \tfrac{1}{2}H_{33}(\ln\hat{K})^2 \\ & + \tfrac{1}{2}H_{44}(\ln\hat{TC})^2 + H_{12}\ln\hat{L}\,\ln\hat{M} + H_{13}\ln\hat{L}\,\ln\hat{K} \\ & + H_{14}\ln\hat{L}\,\ln\hat{TC} + H_{23}\ln\hat{M}\,\ln\hat{K} + H_{24}\ln\hat{M}\,\ln\hat{TC} + H_{34}\ln\hat{K}\ln\hat{TC} \\ & + J_{11}\ln\hat{y}_1\ln\hat{L} + J_{12}\ln\hat{y}_1\ln\hat{M} + J_{13}\ln\hat{y}_1\ln\hat{K} \\ & + J_{14}\ln\hat{y}_1\ln\hat{TC} + J_{21}\ln\hat{y}_2\ln\hat{L} + J_{22}\ln\hat{y}_2\ln\hat{M} \\ & + J_{23}\ln\hat{y}_2\ln\hat{K} + J_{24}\ln\hat{y}_2\ln\hat{TC} + A_1\ln\hat{y}_1 \\ & + A_2\ln\hat{y}_2 + \tfrac{1}{2}G_{11}(\ln\hat{y}_1)^2 + \tfrac{1}{2}G_{22}(\ln\hat{y}_2)^2 \\ & + G_{12}\ln\hat{y}_1 \cdot \ln\hat{y}_2 \end{aligned}$$

(3) $y_1 = \overline{y}_1$ or $(P_1 = \overline{P}_1)$

where

y_1= Quantity of regulated services, divisia quantity index of local service, Intra-Bell message toll service, Trans Canada message toll service and U.S. and Overseas message toll service.

P_1= Price of regulated services, revenues from regulated services, divided by y_1, index 1967 = 1.00

y_2= Other toll services in 1967 prices.

P_2 = Price of other toll services, Index 1967 = 1.00

L = Labor in weighted man hours, with 1967 weights

CHAPTER III

THE TECHNOLOGICAL STRUCTURE OF BELL CANADA

3.1 Introduction

In our work we take the objective of the firm to be profit maximization subject to a regulatory constraint in the form of fixed prices for local services and for message toll services. As far as the objectives of the regulatory agency are concerned, we assume that in the short run they are to fix the prices of local and message toll services.

In this chapter two alternative characterization of technologies are developed. In the first one, the technology of Bell Canada is described through a multiple output production function. In the second, it is described by multiple output cost function.

3.2 The Profit Maximization Model with a Production Frontier

We assume that Bell sells three outputs: local services, message toll services and other toll services. It uses in its production three factors of production: labor, raw materials and capital. In the production side we consider two outputs: regulated output (y_1), which is a divisia quantity index of local services and message toll services; and other toll services (y_2). These two outputs are produced by a general translog production frontier.[1]

---

[1] L.R. Christensen, D.W. Jorgenson and L.J. Lau "Transcendental Logarithmic Production Frontiers," The Review of Economics and Statistics, 55 (February 1973), pp. 28-45.

The production possibility frontier, equation 2, is a general translog function which does not restrict a priori the type of technical change.

To solve the problem of maximizing (1) subject to (2) and (3), we set up the following Lagrangian:

$$\Psi = P_1Y_1 + P_2Y_2 - wL - mM - vK - \mu_1[F(Y_1,Y_2,L,M,K,TC)]$$

$$-\mu_2[Y_1 - \bar{Y}_1 (P_1)]$$

First order conditions for the maximum of profits are given by (2) and (3) above and the following equations:

$$\frac{\partial \Psi}{\partial L} = -w - \mu_1 \frac{\partial F}{\partial L} = 0 \qquad (4)$$

$$\frac{\partial \Psi}{\partial M} = -m - \mu_1 \frac{\partial F}{\partial M} = 0 \qquad (5)$$

$$\frac{\partial \Psi}{\partial K} = -v - \mu_1 \frac{\partial F}{\partial K} = 0 \qquad (6)$$

$$\frac{\partial \Psi}{\partial Y_2} = P_2[1+\eta_2] - \mu_1 \frac{\partial F}{\partial Y_2} = 0 \qquad (7)$$

Where η_2 is the reciprocal of the price elasticity of the demand for other toll services.

The system of equations (2) to (7) is a system of 6 equations in six unknowns: Y_1, Y_2, L, M, K, μ_1.

To complete the specification of this model we need to specify the demand equations and the production frontier. The estimation of the demand equations was discussed in Chapter II above. Here we will discuss the estimation of the production possibility frontier.[1]

Dividing (5) by (4) we obtain:

$$\frac{\frac{\partial F}{\partial M}}{\frac{\partial F}{\partial L}} = \frac{m}{w}$$

---

[1] For further analysis on this production possibility frontier see B. Smith and V. Corbo "Economies of Scale and Economies of Scope in Bell Canada". IAER, 1979.

M = "Raw Materials", defined as the cost of materials, services, rent and supplies, uncollectables and indirect taxes not allocated to labor and capital, all of them in millions of 1967 dollars.

K = Net capital stock, in millions of 1967 dollars

TC = Technology indicator. Throughout this report a considerable amount of effort was directed towards constructing a conceptually sound index of technical change from the available data. Although it is felt that a more representative index could not be constructed from the available data, it is quite likely true that more information on the technical aspects of the production process would lead to a better index.

In the index constructed an attempt was made to take account of improvements in the type of capital improved, the spread of these improvements throughout the Bell Canada System and the importance of the improvements for local and toll services. The formula for the index is:

$$TC = FNEW[\tau\ PDPH + (1-\tau)\ ACCESS]; \qquad \tau = \frac{Q_L}{Q_L+Q_T}\ ;$$

where FNEW is the factor of capital improvement defined as 1 plus the percent of main stations switched by crossbar, ESS and SP1;

ACCESS is the percentage of telephones with access to direct dialing;

PDPH is the percent of dial phones;

Q_L, Q_T are respectively local and toll output aggregates.

w = Wage rate

m = Unit cost of raw materials

v = Unit cost of capital services

$\hat{X} = \frac{X}{\bar{X}}$, where $\bar{X}$ is the mean of X ($X = L, M, K, TC, y_1$ and y_2)

For estimation purposes, we add a random error to each of the equations. We further assume that the random errors are contemporaneously correlated and thus the four equations are estimated as a multivariate equation system using Zellner's seemingly unrelated estimation method.

The results of the estimation appear in Tables 3.1 and 3.2.

From the results obtained, we see that most of the coefficients are statistically significant. Also, the estimated translog function is monotonic and quasi-concave on factor inputs for every data point. Also the output frontier has a negative slope.[1]

The model presented up to now does not include a regulatory constraint. We computed for the sampling period an allowed rate of return on capital which was compared with the actual rate of return on capital. From the comparison of these two series we observe that up to 1967 the actual return on capital was substantially above the allowed rate of return. Thus, we conclude that the regulation of earnings for share was not enforced up to that year. From 1967 on, when an explicit rate of return regulation was introduced, the allowed rate and the actual rate were much closer. Thus, for the period 1967-1976 we introduced a separate regulatory constraint in our model.

When a regulatory constraint is added to our optimization problem the only change introduced is in equation (9). The change is that the numerator in the right-hand side of the equation has to be multiplied by $(1 - DR * \lambda)$ and a new term has to be added which is equal to $\frac{K * (DR * \lambda * S)}{W * L}$ where the new symbols are:

---

[1] For details see Smith and Corbo, <u>op. cit.</u>, Appendix B.

[redacted] Expressions [redacted] for $\frac{[redacted]}{\partial M}$ and $\frac{[redacted]}{\partial L}$ ca[redacted]e [redacted]air[redacted]fr[redacted](2)[redacted]Th[redacted]

we obtain:

$$\frac{mM}{wL} = \frac{M\frac{\partial F}{\partial M}}{L\frac{\partial F}{\partial L}} = \frac{B_2 + H_{12}\ \ell n\hat{L} + H_{22}\ \ell n\hat{M} + H_{23}\ell n\hat{K} + H_{24}\ \ell n\hat{TC} + J_{12}\ \ell n\hat{y}_1 + J_{22}\ \ell n\hat{y}_2}{B_1 + H_{11}\ \ell n\hat{L} + H_{12}\ \ell n\hat{M} + H_{13}\ell n\hat{K} + H_{14}\ \ell n\hat{TC} + J_{11}\ \ell n\hat{y}_1 + J_{21}\ \ell n\hat{y}_2} \qquad (8)$$

similarly, dividing (6) by (4) we obtain:

$$\frac{\frac{\partial F}{\partial K}}{\frac{\partial F}{\partial L}} = \frac{v}{w}$$

therefore:

$$\frac{vK}{wL} = \frac{[B_3 + H_{13}\ \ell n\hat{L} + H_{23}\ \ell n\hat{M} + H_{33}\ \ell n\hat{K} + H_{34}\ \ell n\hat{TC} + J_{13}\ \ell n\hat{y}_1 + J_{23}\ \ell n\hat{y}_2]}{B_1 + H_{11}\ \ell n\hat{L} + H_{12}\ \ell n\hat{M} + H_{13}\ \ell n\hat{K} + H_{14}\ \ell n\hat{TC} + J_{11}\ \ell n\hat{y}_1 + J_{21}\ \ell n\hat{y}_2} \qquad (9)$$

dividing (7) by (4) we obtain:

$$\frac{\frac{\partial F}{\partial y_2}}{\frac{\partial F}{\partial L}} = \frac{P_2[1+\eta_2]}{w}$$

which can be written as:

$$\frac{-P_2 y_2}{wL} = \frac{A_2 + J_{21}\ell n\hat{L} + J_{22}\ \ell n\hat{M} + J_{23}\ \ell n\hat{K} + J_{24}\ \ell n\hat{TC} + G_{12}\ \ell n\hat{y}_1 + G_{22}\ \ell n\hat{y}_2}{[B_1 + H_{11}\ell n\hat{L} + H_{12}\ \ell n\hat{M} + H_{13}\ \ell n\hat{K} + H_{14}\ \ell n\hat{TC} + J_{11}\ \ell n\hat{y}_1 + J_{21}\ \ell n\hat{y}_2]\ (1+\eta_2)} \qquad (10)$$

We estimate our general translog production frontier by estimating simultaneously equations (2), (8), (9) and (10), using for η_2 its value estimated in the previous chapter i.e. η_2=.5815

For the estimation, we use the following normalization: $\Sigma A_i = -1$. This normalization is needed to estimate an implicit production frontier.

TABLE 3.1

GENERAL TRANSLOG PRODUCTION FRONTIER

PARAMETER	ESTIMATED COEFFICIENT	T-STATISTIC
A_0	1.1336*	153.64
B_1	.3290*	10.61
B_2	.1824*	10.27
B_3	.4738*	10.07
B_4	.9974*	9.22
H_{11}	-.1661*	-2.60
H_{22}	.0890*	3.89
H_{33}	-.1944*	-2.93
H_{44}	-8.3065*	-3.93
H_{12}	-.0735*	-2.68
H_{13}	-.3011*	-4.47
H_{14}	.0200	.13
H_{23}	-.1499*	-4.54
H_{24}	.1338	1.74
H_{34}	.2731	1.72
J_{11}	.2078*	2.88
J_{12}	-.0157	-.41
J_{13}	.2369*	2.39
J_{14}	4.3912*	4.70
J_{21}	.0267*	4.82
J_{22}	.0188*	5.86
J_{23}	.0377*	5.56
J_{24}	-.0099	-1.03
A_1	-.9654*	-266.62
A_2	-.0346*	-19.11
G_{11}	-2.2618*	-5.37
G_{22}	-.0258*	-7.89
G_{12}	-.0116	-1.60

An asterisk next to a coefficient indicates that the coefficient is significant at a 5% level or less.

- DR is a dummy variable that takes a value of zero up to 1966 and one from 1967 to 1976.
- λ is the Lagrangean multiplier of the regulatory constraint.
- S is the allowed price of capital services.

The Lagrangean multiplier (λ) is a variable which should be less than one. As a way of estimating its average value over the sample we introduced it as a parameter to be estimated jointly with the other parameters of the production frontier. When we performed the estimation the point estimate for λ was -.013 and its t-value -.315, and thus we cannot obtain a reliable average value of λ from our sample. As it was discussed in our introductory chapter, in the short run regulation takes the form of the fixing of a price structure. Thus, in the rest of the model no explicit rate of return regulation is introduced. Rather regulation is introduced through the price of the outputs.

Now we proceed with a further analysis of the estimated production frontier of Tables 3.1 and 3.2.

As a further property of the technology we can study if the translog frontier exhibits constant returns to scale.

Constant returns to scale implies, besides the normalization rule introduced above, the following additional parameter restrictions.

$$
\begin{aligned}
&B_1 + B_2 + B_3 && = 1 \\
&G_{11}+G_{12}+J_{11}+J_{12}+J_{13} && = 0 \\
&G_{12}+G_{22}+J_{21}+J_{22}+J_{23} && = 0 \\
&J_{11}+J_{21}+H_{11}+H_{12}+H_{13} && = 0 \\
&J_{12}+J_{22}+H_{12}+H_{22}+H_{23} && = 0 \\
&J_{13}+J_{23}+H_{13}+H_{23}+H_{33} && = 0 \\
&J_{14}+J_{24}+H_{14}+H_{24}+H_{34} && = 0 \\
&\tfrac{1}{2}G_{11}+G_{12}+J_{11}+J_{12}+J_{13}+\tfrac{1}{2}G_{22}+J_{21}+J_{22}+J_{23}+\tfrac{1}{2}H_{11} && \\
&+H_{12}+H_{13}+\tfrac{1}{2}H_{22}+H_{23}+\tfrac{1}{2}H_{33} && = 0
\end{aligned}
$$

The last restriction is implied by the second through sixth restriction.

We also can test for separability between outputs and inputs. If the function is separable in outputs and inputs, then we can work with an aggregate output index. Separability between outputs and inputs requires $J_{11} = J_{12} = J_{13} = J_{21} = J_{22} = J_{23} = 0$.

Our testing indicated that both of these hypotheses are rejected.[1]

Using the estimated translog frontier, equations (2), (3), (8), (9) and (10) conform a system of 5 equations in five unknowns y_1, y_2, L, M and K.

---

[1] For details of these tests see Smith and Corbo, <u>op. cit.</u>, Part IV.

TABLE 3.2

SUMMARY STATISTICS OF INDIVIDUAL EQUATIONS

Equation 2 (Prod. Frontier)

R^2 = *

D-W = 1.475

SSe = 0.0142

Equation 8 (Materials)

R^2 = .980

D-W = 1.328

SSe = .0128

Equation 9 (Capital)

R^2 = .994

D-W = 1.398

SSe = .0307

Equation 10 (Other Toll)

R^2 = .998

D-W = 2.119

SSe = .0048

* R^2 is not computed for this equation because the dependent variable and its mean are zero.

3.3 A Profit Maximization Model with a Cost Frontier

In the model developed in the previous section the technology of Bell Canada was characterized by a multiple output production frontier. In this section we introduce an alternative characterization of technology. McFadden (1970) introduced the joint cost function which is dual to the production frontier. All the properties of the underlying technology can be studied from the joint cost function. In the previous section, the translog production frontier was used to describe input and output choices of Bell Canada. In this section, a translog cost function is introduced as an alternative characterization of technology. The results of the two approaches are then compared.

Before proceeding, it should be noted that translog cost and production functions are not self-dual and thus could yield different properties of the technology. Indeed, using aggregate macro data it was found by Burgess (1975) and Appelbaum (1978) that in the one output case the translog cost frontier and the translog production frontier did yield contradictory results with respect to the properties of the underlying technology. The introduction of the joint cost model allows a direct test of the extent to which the production results are so that in characterizing the Bell production process.

It is assumed that Bell Canada maximizes profits, given by equation (1) above, subject to a translog cost function and a fixed quantity of regulated services (equation (3)).

The joint cost function can be written in general form as: $C=C(w,m,v,TC,y_i,y_2)$. In the translog case of the cost function it takes the following form:

In this model it can be shown that the marginal cost of regulated services is given by $\mu_1 \frac{\partial F}{\partial y_1}$ and the marginal cost of other toll services is given by $\mu_1 \frac{\partial F}{\partial y_2}$. After we have found the values for y_1, y_2, L, M and K the value of μ_1 can be obtained from any of the equations (4) to (7).

restrictions must be introduced.

The joint cost function (11) is obtained from the problem of minimizing costs for a given output vector. Thus, some further relations can be derived based on this property of the cost function. Indeed, Hall (1973) suggested estimating the parameters of the joint cost function indirectly from behavioral relations implied by economic theory.

From cost minimization subject to a production frontier we can obtain the following behavioral relations based on Sheppard's lemma:

$$L = \frac{\partial C}{\partial w} \qquad (13)$$

$$M = \frac{\partial C}{\partial m} \qquad (14)$$

$$K = \frac{\partial C}{\partial v} \qquad (15)$$

Diewert (1974) suggested that more efficient parameter estimates could be obtained from the simultaneous estimation of the joint cost function and the side conditions. In a fashion similar to the estimations of the joint production frontier, we estimate simultaneously the function and the side conditions taking (11), (12), (13), (14) and (15) as a multivariate system of equations.

We can rewrite equations (12) to (15) in terms of the parameters of the translog cost function as follows:

$$\frac{P_2 y_2}{C} = \frac{1}{1+\eta_2} \, [C_2 + C_{2w} \, \ell n\hat{w} + C_{2m} \, \ell n\hat{m} + C_{V} \, \ell n\hat{v} + C_{2T} \, \ell n\widehat{TC} + C_{12} \, \ell n\hat{y}_1 + C_{22} \, \ell n\hat{y}_2] \qquad (12)'$$

$$
\begin{aligned}
\ell n\hat{C} = {} & C_o + C_w \ \ell n\hat{w} + C_m \ \ell n\hat{m} + C_v \ \ell n\hat{v} + C_T \ \ell n\widehat{TC} \\
& + \tfrac{1}{2}C_{ww} \ (\ell n\hat{w})^2 + C_{wm} \ \ell n\hat{w} \ \ell n\hat{m} + C_{wv} \ \ell n\hat{w} \ \ell n\hat{v} + C_{wT} \ell m\hat{w} \ \ell n\widehat{TC} \\
& + \tfrac{1}{2}C_{mm} \ (\ell n\hat{m})^2 + C_{mv} \ \ell n\hat{m} \ \ell n\hat{v} + C_{mT} \ \ell n\hat{m} \ \ell n\widehat{TC} \\
& + \tfrac{1}{2}C_{vv} \ (\ell n\hat{v})^2 + C_{vT} \ \ell n\hat{v} \ \ell n\widehat{TC} + \tfrac{1}{2}C_{TT} \ (\ell n\widehat{TC})^2 \qquad (11) \\
& + C_1 \ \ell n\hat{y}_1 + C_2 \ \ell n\hat{y}_2 + \tfrac{1}{2}C_{11}(\ell n\hat{y}_1)^2 + C_{12} \ \ell n\hat{y}_1 \ell n\hat{y}_2 \\
& + \tfrac{1}{2}C_{22} \ (\ell n\hat{y}_2)^2 + C_{1w} \ \ell n\hat{y}_1 \ \ell n\hat{w} + C_{1m} \ \ell n\hat{y}_1 \ \ell n\hat{m} \\
& + C_{1v} \ \ell n\hat{y}_1 \ \ell m\hat{v} + C_{1T} \ \ell n\hat{y}_1 \ \ell n\widehat{TC} + C_{2w} \ \ell n\hat{y}_2 \ \ell n\hat{w} \\
& + C_{2m} \ \ell n\hat{y}_2 \ \ell n\hat{m} + C_{2v} \ \ell n\hat{y}_2 \ \ell n\hat{v} + C_{2T} \ \ell n\hat{y}_2 \ \ell n\widehat{TC}
\end{aligned}
$$

where the new symbols introduced are:

C = Total cost in millions of current dollars,

$C = wL + mM + vK$

$\hat{X} = \frac{X}{\bar{X}}$ where $\bar{X}$ is the mean of X and

$X = C, w, m, v, TC, y_1$ and y_2

The profit maximizing problem can be stated in terms of the following Lagrangian

$$\phi = P_1 y_1 + P_2 y_2 - C(w,m,v,TC,y_1,y_2) - \Theta[y_1 - \bar{y}_1(P_1)]$$

First order conditions for the maximization of profits are given by:

$$\frac{\partial\phi}{\partial y_2} = P_2 \ [1+\eta_2] - \frac{\partial C}{\partial y_2} = 0 \qquad (12)$$

and equation (2):

From the inverse demand equations for y_2 we have $P_2 = f(y_2)$. Thus, these two equations provide a system of two equations in two unknowns y_1, y_2. Given the large number of parameters, in order to estimate the cost function with more precision further

The last of these restrictions is implied by the 2nd, 3rd and 4th.

For estimation purposes, a disturbance is added to the equations. It is expected that the disturbances on the multivariate equation system to be contemporaneously correlated. Homogeneity of degree one in factor prices implies that the dependent variable for equations (13)', (14)' and (15)' sum identically to unity for every data point. This implies that disturbances for these equations add identically to zero for every data point. The fact that each dependent variable is expressed as a share implies that these restrictions are fulfilled in the estimation. In this case, the covariance matrix of disturbances for the multivariate system will be singular. Thus, for this system of equations, Zellner's seemingly unrelated estimation procedure cannot be used to obtain efficient estimates. It is known that this problem can be solved by deleting one of the share equations and estimating the other two jointly with the cost function using Zellner's procedure. If the Zellner procedure is iterated until convergence is achieved, then the resulting parameter estimates are independent of the cost share equation deleted before estimation (Berndt and Christensen (1973) , Oberhofer and Kmenta (1974) and Brown <u>et al</u> (1976)). Thus, homogeneity of degree one in factor prices is imposed and equations (11),12)', (13)' and (15)' are estimated using the iterative-Zellner procedure. The parameters of equation (14)' are then retrieved, using the other parameter estimates and the homogeneity restrictions. The results of the estimation appear in Table 3.3.

A translog cost frontier is not *a priori* restricted to be

$$\frac{wL}{C} = C_w + C_{ww}\ \ell n\hat{w} + C_{wm}\ \ell n\hat{m} + C_{wv}\ \ell n\hat{v} + C_{wT}\ \ell n\widehat{TC}$$
$$+ C_{1w}\ \ell n\hat{y}_1 + C_{2w}\ \ell n\hat{y}_2 \qquad (13)'$$

$$\frac{mM}{C} = C_m + C_{wm}\ \ell n\hat{w} + C_{mm}\ \ell n\hat{m} + C_{mv}\ \ell n\hat{v} + C_{mT}\ \ell n\widehat{TC}$$
$$+ C_{1m}\ \ell n\hat{y}_1 + C_{2m}\ \ell n\hat{y}_2 \qquad (14)'$$

$$\frac{vK}{C} = C_v + C_{wv}\ \ell n\hat{w} + C_{mv}\ \ell n\hat{m} + C_{wv}\ \ell n\hat{v} + C_{vT}\ \ell n\widehat{TC}$$
$$+ C_{1v}\ \ell n\hat{y}_1 + C_{2v}\ \ell n\hat{y}_2 \qquad (15)'$$

Equations (11), (12)', (13)', (14)' and (15)' constitute a multivariate system of equations which can be used to estimate the joint cost function. Moreover, not all parameters in this system are free. The cost function must be homogeneous of degree one in factor prices. Necessary and sufficient conditions for homogeneity of degree one are given by the following restrictions on the parameters of the cost function:

$$C_{wT} + C_m + C_v = 1$$

$$C_{ww} + C_{wm} + C_{wv} = 0$$

$$C_{wm} + C_{mm} + C_{mv} = 0$$

$$C_{mv} + C_{mv} + C_{vv} = 0$$

$$C_{wT} + C_{mT} + C_{vT} = 0$$

$$C_{1w} + C_{1m} + C_{1v} = 0$$

$$C_{2w} + C_{2m} + C_{2v} = 0$$

$$\tfrac{1}{2}C_{ww} + C_{wm} + C_{wv} + \tfrac{1}{2}C_{mm}$$
$$+ C_{mv} + \tfrac{1}{2}C_{vv} = 0$$

TABLE 3.3

GENERAL TRANSLOG JOINT COST FUNCTION

PARAMETER	ESTIMATED COEFFICIENT	T-STATISTIC
C_0	.0148*	2.821
C_w	.3205*	104.792
C_m	.1900*	93.930
C_v	.4894*	147.142
C_T	-.4889*	-6.360
C_{ww}	-.1068*	-3.453
C_{wm}	.0405	1.994
C_{wv}	.0663*	2.691
C_{wT}	-.1890*	-4.966
C_{mm}	.0496*	2.220
C_{mv}	-.0917*	-5.742
C_{mT}	-.0506*	-2.473
C_{vv}	.0239	.849
C_{vT}	.2396*	5.650
C_{TT}	-.4344	.293
C_1	.8537*	15.503
C_2	.0292*	34.805
C_{11}	.0395	.049
C_{12}	-.0234	-4.061
C_{22}	.0144*	7.419
C_{1w}	.1342*	5.307
C_{1m}	.0208	1.314
C_{1v}	-.1550*	-5.370
C_{1T}	-.1589	-.146
C_{2w}	-.0321*	-5.652
C_{2m}	.0023	.526
C_{2v}	.0298*	4.714
C_{2T}	.0269*	3.683

An asterisk next to a coefficient indicates that the coefficient is significant at a 5% level or less.

globally monotonic in factor inputs nor to be concave in input prices. For the cost minimization problem solution to be optimal the estimated cost function must be concave and positive monotone in factor prices (Diewert (1974)). Thus, as in the production case, these properties are locally verified at every data point.

The results from the estimation of the translog frontier are presented in Tables 3.3 and 3.4 below.

The estimated translog cost frontier of Table 3.3 is indeed monotone and concave in input prices. Also, the underlying output frontier is negatively sloped.[1]

---

[1] For details on all these properties and the respective tests see B. Smith and V. Corbo "Economies of Scale and Economies of Scope in Bell Canada", IAER, 1979.

Using the translog cost function it is possible to test if the underlying production frontier exhibits constant returns to scale.

Constant returns to scale implies the following additional restrictions in the parameters of the cost function.

$C_1 + C_2 = 1; \quad C_{1W} + C_{2W} = 0; \quad C_{1V} + C_{2V} = 0; \quad C_{1T} + C_{2T} = 0;$

$C_{11} + C_{12} = 0$ and $C_{12} + C_{22} = 0$

When the translog cost model was tested for constant returns to scale, the null hypothesis was rejected in favor of increasing returns to scale.[1]

It is also possible to test whether the associated production frontier is group separable in inputs and outputs. This is equivalent to a test for the appropriatness of using an output aggregator for the analysis of technology. Group separability between inputs and outputs require:

$$C_{1W} = C_{2W} = C_{1m} = C_{2m} = C_{1V} = C_{2V} = 0$$

Using the joint cost function, the test for group separability is also rejected. Thus, as in the previous section, the conclusion is drawn that the technology of Bell Canada is a non-separable and exhibits non-constant returns to scale.

The estimated translog joint cost function of equation (11) plus equations (12)', (13)', (14)' and (15)' form a system of five equations in five unknowns y_2, L, M, K and C which can be used as an alternative model of the real structure of Bell Canada.

The main advantage of using this model is that for fixed

---

[1] For details of this test see Smith and Corbo (1979).

[2] For details see Smith and Corbo (1979).

TABLE 3.4

SUMMARY STATISTICS OF INDIVIDUAL EQUATIONS

Equation 11

R^2 = .9997

D-W = 1.505

SSe = .0125

Equation 12'

R^2 = .985

D-W = .993

SSe = .0014

$\hat{\rho}$ = .123
(1.420)

Equation 13'

R^2 = .982

D-W = .937

SSe = .0081

Equation 15'

R^2 = .979

D-W = 1.251

SSe = .0094

output prices the system of equation is linear in the variables and thus the model can be simulated easily.

A further comparison between the results of these two models can be a complished by comparing the estimated marginal costs from both models. For the translog model the marginal cost of regulated services is given by $\frac{\partial C}{\partial y_1}$ and the one for other by $\frac{\partial C}{\partial y_2}$. When these marginal costs are compared to the ones obtained from the production frontier the results are very close again.

In subsequent chapters the joint cost model is used as a characterization of the real structure of Bell Canada.

4.1 Validation of the Demand Model

For the validation of the demand model we perform a dynamic simulation of the demand model of Chapter II. In this simulation we take as given the value of the right hand side variables, with the exception of the lagged values of the endogenous variables which are solved from the equations.

The results of these simulations appear in Tables 4.1 to 4.3.

CHAPTER IV

A SIMULATION MODEL OF BELL CANADA: THE REAL STRUCTURE

In this chapter we develop a model of the real structure of Bell Canada. For a given vector of prices, this model is block triangular. The first block is the demand block. The second block is the one formed by the conditions of profit maximization for a given vector of prices, which in this special case is the same as that for cost minimization for a given vector of output. For this purpose we use the multiple output cost model presented in Chapter III above. The advantage of using the side order conditions starting from a cost function instead of a production frontier is that the former are linear in the inputs and the latter are not.

We validate first the demand equations by themselves and the factor requirements equations by themselves. Then, for the validation of the model of the real structure of Bell we use its block triangularity property and proceed in two stages. Initially, we simulate output levels for local services (y_{11}), message toll services (y_{12}) and other toll services (y_2). Then, on the second stage, with the simulated values of $y_1 = y_{11} + y_{12}$ and y_2 we solve equations (11), (13)', (14)' and (15)' of Chapter III.

Table 4.2

Demand for Telephone Message Toll Services

		y_{12}	y^S_{12}
1952	.	53.4674	53.4674
1953	.	57.6434	60.5561
1954	.	62.1980	63.5914
1955	.	71.3007	70.2055
1956	.	80.2934	79.2289
1957	.	87.6373	87.5376
1958	.	91.7897	90.4590
1959	.	100.271	101.462
1960	.	105.439	104.342
1961	.	112.009	111.099
1962	.	130.178	129.581
1963	.	137.768	138.441
1964	.	151.631	151.861
1965	.	170.413	166.742
1966	.	191.299	193.778
1967	.	213.900	214.815
1968	.	232.709	239.035
1969	.	266.123	264.282
1970	.	286.207	281.080
1971	.	298.020	300.708
1972	.	333.273	331.990
1973	.	385.109	377.409
1974	.	440.917	451.056
1975	.	500.113	496.209
1976	.	536.231	531.968

```
          COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
          **********************************************
```

ACTUAL AND PREDICTED VARIABLES... y_{12} y^S_{12}

```
CORRELATION COEFFICIENT =     .9997
             (SQUARED =     .9994

ROOT-MEAN-SQUARED ERROR =     3.541

MEAN ABSOLUTE ERROR =         2.517

MEAN ERROR =                  .2014

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        1.003

THEIL"S INEQUALITY COEFFICIENT =                        .7165E-02

FRACTION OF ERROR DUE TO BIAS =                         .3235E-02

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =          .1902E-01

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =       .9777


ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                      .1575E-01
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =       .9810
```

Table 4.1

Demand for Local Services

		Y_{11}	Y_{11}^S
1952	.	126.400	126.400
1953	.	137.000	137.497
1954	.	148.000	148.473
1955	.	162.900	162.307
1956	.	181.700	177.747
1957	.	200.600	195.072
1958	.	216.600	210.761
1959	.	233.600	225.974
1960	.	250.900	243.569
1961	.	269.500	262.319
1962	.	289.600	283.530
1963	.	308.700	305.532
1964	.	325.000	331.265
1965	.	350.800	358.799
1966	.	380.700	389.551
1967	.	410.000	420.170
1968	.	437.600	451.764
1969	.	471.400	485.528
1970	.	504.300	518.537
1971	.	538.000	552.656
1972	.	579.800	591.098
1973	.	625.500	634.759
1974	.	679.400	686.916
1975	.	734.300	731.269
1976	.	779.700	779.378

COMPARISON OF ACTUAL AND PREDICTED TIME SERIES

****

ACTUAL AND PREDICTED VARIABLES... Y_{11} Y_{11}^S

CORRELATION COEFFICIENT = .9994
(SQUARED = .9988

ROOT-MEAN-SQUARED ERROR = 8.181

MEAN ABSOLUTE ERROR = 6.806

MEAN ERROR = -2.755

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED = .9794

THEIL"S INEQUALITY COEFFICIENT = .9693E-02

FRACTION OF ERROR DUE TO BIAS = .1134

FRACTION OF ERROR DUE TO DIFFERENT VARIATION = .2291

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION = .6576

ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
COEFFICIENT FROM UNITY = .2424
FRACTION OF ERROR DUE TO RESIDUAL VARIANCE = .6442

(1) Dynamic Simulation of Demand for Local Services

We start with demand for local services. The comparison of actual (y_{11}) and simulated (y^S_{11}) values are presented in Table 4.1 below. From this table we observe that the equation tracks quite well the evolution of the dependent variable. Analyzing the summary statistics at the bottom of Table 4.3 we see that the regression coefficient of actual on predicted values is very close to one. Furthermore, over half of Theil's inequality coefficient is due to a residual variance.

(2) Dynamic Simulation of Demand for Message Telephone Toll Services

In Table 4.2 below, we compare the actual (y_{12}) and simulated values (y^S_{12}) of message telephone toll services. The tracking of this equation is also quite good. The regression coefficient of actual on predicted values is .9997. Furthermore, 98.1% of Theil's inequality coefficient is due just to a residual variance and therefore the fraction of error due to bias is close to zero.

(3) Dynamic Simulation of Demand for Other Toll Services

The actual and simulated values of Other Toll services, y_2 and y^S_2 respectively appear in Table 4.3 below. Again, as for the other demand equations, it performs quite well. The regression coefficient of actual or predicted values is close to one. There is a small fraction of error due to bias and a large one due to residual variance. That is, there are no systematic differences between the actual and predicted series.

Thus, we conclude from these results that the demand system performs quite well during the sampling period.

Table 4.3

Demand for Other Toll Services

		y_2	y_2^S
1952	.	1.70000	1.70000
1953	.	2.30000	2.52761
1954	.	2.90000	3.15445
1955	.	4.30000	4.49103
1956	.	6.30000	6.51384
1957	.	7.80000	8.42120
1958	.	9.30000	9.63924
1959	.	10.5000	11.5631
1960	.	12.5000	13.1983
1961	.	14.7000	15.0720
1962	.	18.0000	18.0059
1963	.	21.6000	20.5643
1964	.	30.2000	24.9204
1965	.	34.9000	29.9570
1966	.	40.0000	37.3461
1967	.	45.1000	43.2652
1968	.	54.1000	50.5073
1969	.	63.4000	58.1351
1970	.	72.8000	65.0932
1971	.	77.3000	73.0552
1972	.	90.9000	88.1361
1973	.	108.000	107.263
1974	.	119.800	134.335
1975	.	138.500	147.204
1976	.	156.700	161.067

```
COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
**********************************************
```

ACTUAL AND PREDICTED VARIABLES... y_2 y_2^S

```
CORRELATION COEFFICIENT =     .9961
               (SQUARED =     .9921

ROOT-MEAN-SQUARED ERROR =     4.466

MEAN ABSOLUTE ERROR =         2.866

MEAN ERROR =                  .3386

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =      .9591

THEIL"S INEQUALITY COEFFICIENT =                     .3443E-01

FRACTION OF ERROR DUE TO BIAS =                      .5748E-02

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =       .1526

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =    .8416


ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                   .1855
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =    .8088

          END OF PHASE 2
```

TABLE 4.4

LABOR AND CAPITAL REQUIREMENTS

ACTUAL VALUES FOR OUTPUTS

	LSIM	L	KSIM	K
1952	46.9170	44.9000	608.121	626.600
1953	46.4085	46.1000	695.576	690.400
1954	47.7120	48.2000	770.852	764.900
1955	50.4460	51.9000	880.027	871.300
1956	54.9936	55.7000	1007.04	989.900
1957	58.6197	57.8000	1122.03	1127.10
1958	56.7741	57.6000	1272.99	1280.00
1959	58.2536	56.5000	1389.60	1429.50
1960	54.4490	54.6000	1560.95	1579.10
1961	52.6701	52.4000	1708.69	1721.90
1962	55.3104	52.3000	1860.95	1860.10
1963	54.7731	53.5000	1982.38	2004.40
1964	52.2808	54.4000	2157.21	2150.40
1965	54.8694	55.8000	2292.28	2283.60
1966	56.8125	57.5000	2449.40	2431.20
1967	59.3066	56.6000	2607.53	2585.60
1968	56.3841	55.5000	2741.76	2734.00
1969	57.5321	56.6000	2897.98	2886.00
1970	58.2862	57.8000	3026.26	3054.80
1971	55.5021	58.1000	3181.83	3190.40
1972	54.5513	57.5000	3368.28	3334.90
1973	59.7096	60.4000	3528.38	3494.00
1974	64.6732	63.9000	3638.92	3653.50
1975	66.2245	64.1000	3830.80	3808.90
1976	65.8168	67.3000	3937.36	3978.90

4.2 Validation of the Factor Requirements Model

In this section we validate the factor requirements model presented in Section 3.3 of Chapter III. Given a vector of outputs, equations (11), (13)', (14)' and (15)' of that chapter are solved for L,K,M, and C. We start validating the model making the output levels equal to their historical values. The result of the simulations appear in Tables 4.4 to 4.7. We see from these results that the factor requirements model tracks quite well. Indeed, for the three inputs the actual and predicted values are very close. This is especially so for capital, a variable which is of particular interest to the Telecommunication authorities.

Furthermore, the tracking for all the variables is especially good for the latter part of the sample. From the statistical comparison of actual and predicted values we find that for the four variables the correlation between actual and predicted values is over 95%. In fact, the respective correlation coefficients are 95.22% for labor, 99.98% for capital, 99.52% for raw materials and 99.97% for total cost. The tracking for labor is substantially better than the one obtained in IAER (1978). It is also seen that for the four variables most of the differences between actual and predicted series can be attributed to the residual variance.

4.3 Validation of the Complete Real Model

In this section we simulate the model used in the previous section with simulated instead of actual values for the demand variables. Thus, we simulate the factor requirements model conditional on the values obtained in Section 4.1 for the demand

TABLE 4.6

MATERIAL REQUIREMENTS AND TOTAL COST:

ACTUAL VALUES OF OUTPUTS

```
                MSIM              M                 COSTSIM             COST

        ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

1952    +      41.1896          42.4608            175.963             175.496
1953    +      46.8473          45.9759            190.802             189.063
1954    +      51.5086          51.1042            206.739             206.761
1955    +      59.0541          58.3350            229.621             231.105
1956    +      65.2915          67.9400            260.215             262.056
1957    +      71.6270          69.9111            294.974             292.383
1958    +      75.9623          77.1386            316.618             320.120
1959    +      81.1379          82.0535            349.160             350.012
1960    +      84.9778          86.2575            370.203             373.553
1961    +      90.4103          91.1128            394.382             395.652
1962    +      99.1715          98.0741            433.607             424.319
1963    +      102.326          103.402            458.776             458.487
1964    +      103.579          104.337            478.401             484.499
1965    +      109.273          113.569            519.284             525.065
1966    +      117.385          118.468            579.651             580.788
1967    +      127.554          116.547            651.079             628.030
1968    +      131.497          122.307            705.478             691.652
1969    +      140.094          143.302            793.853             791.828
1970    +      149.665          144.569            903.723             900.246
1971    +      158.143          168.413            964.670             990.847
1972    +      171.760          173.292            1110.02             1122.67
1973    +      180.239          186.739            1286.88             1293.03
1974    +      183.154          186.361            1513.82             1516.85
1975    +      194.525          185.056            1789.98             1752.27
1976    +      197.305          199.898            1989.31             2017.83
```

TABLE 4.5

COMPARISON OF ACTUAL AND PREDICTED VALUES OF LABOR AND CAPITAL : ACTUAL VALUES OF OUTPUT

```
ACTUAL AND PREDICTED VARIABLES...      L                LSIM

CORRELATION COEFFICIENT =    .9522
                (SQUARED =    .9066

ROOT-MEAN-SQUARED ERROR =    1.556

MEAN ABSOLUTE ERROR =        1.298

MEAN ERROR =                -.9105E-01

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =       .9526

THEIL"S INEQUALITY COEFFICIENT =                      .1385E-01

FRACTION OF ERROR DUE TO BIAS =                       .3426E-02

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =        .2264E-05

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =     .9966


ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                    .2339E-01
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =     .9732

ACTUAL AND PREDICTED VARIABLES...      K                KSIM

CORRELATION COEFFICIENT =    .9998
                (SQUARED =    .9996

ROOT-MEAN-SQUARED ERROR =    20.17

MEAN ABSOLUTE ERROR =        16.80

MEAN ERROR =                 .5677

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =       .9974

THEIL'S INEQUALITY COEFFICIENT =                      .4175E-02

FRACTION OF ERROR DUE TO BIAS =                       .7923E-03

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =        .1516E-01

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =     .9840


ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                    .1760E-01
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =     .9816
```

variables. The results of the simulations appear in Tables 4.8 to 4.11. We can see from these results that the complete real model also tracks very well the actual values of the real variables. For the three inputs, over 95% of the differences between the actual and predicted series is due just to residual variance.

Thus, our model of the real structure of Bell Canada tracks extremely well the demand for outputs and the factor requirements. Now we move on to analyse the financial model in Chapter 5 and then in Chapter 6 the whole model is used for forecasting purposes.

TABLE 4.7

COMPARISON OF ACTUAL AND PREDICTED VALUES OF MATERIALS AND TOTAL COST: ACTUAL VALUES OF OUTPUTS

```
ACTUAL AND PREDICTED VARIABLES...       M                 MSIM

CORRELATION COEFFICIENT =     .9952
               (SQUARED =     .9904

ROOT-MEAN-SQUARED ERROR =     4.641

MEAN ABSOLUTE ERROR =         3.283

MEAN ERROR =                  .1179

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =       1.001

THEIL"S INEQUALITY COEFFICIENT =                      .1889E-01

FRACTION OF ERROR DUE TO BIAS =                       .6448E-03

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =        .3739E-02

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =     .9956


ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                    .1481E-03
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =     .9992

ACTUAL AND PREDICTED VARIABLES...       COST              COSTSIM

CORRELATION COEFFICIENT =     .9997
               (SQUARED =     .9994

ROOT-MEAN-SQUARED ERROR =     12.75

MEAN ABSOLUTE ERROR =         7.853

MEAN ERROR =                  .2958

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =       1.002

THEIL"S INEQUALITY COEFFICIENT =                      .7572E-02

FRACTION OF ERROR DUE TO BIAS =                       .5380E-03

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =        .7479E-02

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =     .9920


ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                    .5442E-02
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =     .9940
```

TABLE 4.9

COMPARISON OF ACTUAL AND PREDICTED VALUES OF LABOR AND CAPITAL: ENDOGENOUS OUTPUTS

```
ACTUAL AND PREDICTED VARIABLES...     L              LSIM

CORRELATION COEFFICIENT =    .9315
                (SQUARED =    .8677

ROOT-MEAN-SQUARED ERROR =    1.829

MEAN ABSOLUTE ERROR =        1.431

MEAN ERROR =                 .1072

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =      1.002

THEIL'S INEQUALITY COEFFICIENT =                     .1632E-01

FRACTION OF ERROR DUE TO BIAS =                      .3431E-02

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =       .3718E-01

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =    .9594

ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                   .2340E-04
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =    .9965

ACTUAL AND PREDICTED VARIABLES...     K              KSIM

CORRELATION COEFFICIENT =    .9997
                (SQUARED =    .9993

ROOT-MEAN-SQUARED ERROR =    27.87

MEAN ABSOLUTE ERROR =        20.19

MEAN ERROR =                 6.062

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =      1.004

THEIL'S INEQUALITY COEFFICIENT =                     .5779E-02

FRACTION OF ERROR DUE TO BIAS =                      .4730E-01

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =       .3180E-01

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =    .9209

ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                   .2752E-01
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =    .9252
```

TABLE 4.8

LABOR AND CAPITAL REQUIREMENTS: ENDOGENOUS OUTPUTS

		LSIM	L	KSIM	K
1952	.	47.0025	44.9000	608.515	626.600
1953	.	47.0738	46.1000	708.775	690.400
1954	.	47.6023	48.2000	770.971	764.900
1955	.	49.7772	51.9000	871.860	871.300
1956	.	54.0757	55.7000	999.365	989.900
1957	.	58.3009	57.8000	1127.68	1127.10
1958	.	56.7353	57.6000	1267.69	1280.00
1959	.	58.1739	56.5000	1402.50	1429.50
1960	.	54.4955	54.6000	1556.17	1579.10
1961	.	52.9456	52.4000	1707.54	1721.90
1962	.	55.7201	52.3000	1861.22	1860.10
1963	.	55.7003	53.5000	1981.62	2004.40
1964	.	54.2388	54.4000	2140.36	2150.40
1965	.	54.5543	55.8000	2288.37	2283.60
1966	.	57.1475	57.5000	2475.02	2431.20
1967	.	59.2351	56.6000	2611.71	2585.60
1968	.	57.4879	55.5000	2748.62	2734.00
1969	.	56.9197	56.6000	2872.69	2886.00
1970	.	57.3698	57.8000	2988.82	3054.80
1971	.	55.0876	58.1000	3188.55	3190.40
1972	.	53.2076	57.5000	3341.96	3334.90
1973	.	58.1048	60.4000	3498.44	3494.00
1974	.	64.0903	63.9000	3693.07	3653.50
1975	.	64.0880	64.1000	3760.41	3808.90
1976	.	65.1868	67.3000	3907.90	3978.90

TABLE 4.11

COMPARISON OF ACTUAL AND PREDICTED VALUES OF MATERIALS AND COSTS: ENDOGENOUS OUTPUTS

```
ACTUAL AND PREDICTED VARIABLES...        M                  MSIM

CORRELATION COEFFICIENT =     .9950
               (SQUARED =     .9901

ROOT-MEAN-SQUARED ERROR =     4.838

MEAN ABSOLUTE ERROR =         3.383

MEAN ERROR =                  .5858

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =      1.019

THEIL"S INEQUALITY COEFFICIENT =                     .1975E-01

FRACTION OF ERROR DUE TO BIAS =                      .1466E-01

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =       .5193E-01

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =    .9334

ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                   .3210E-01
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =    .9532

ACTUAL AND PREDICTED VARIABLES...        COST               COSTSIM

CORRELATION COEFFICIENT =     .9997
               (SQUARED =     .9994

ROOT-MEAN-SQUARED ERROR =     15.11

MEAN ABSOLUTE ERROR =         9.984

MEAN ERROR =                  3.827

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =      1.016

THEIL"S INEQUALITY COEFFICIENT =                     .9012E-02

FRACTION OF ERROR DUE TO BIAS =                      .6410E-01

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =       .2725

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =    .6634

ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                   .2619
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =    .6740
```

TABLE 4.10

MATERIAL REQUIREMENTS AND TOTAL COST:

ENDOGENOUS OUTPUTS

	MSIM	M	COSTSIM	COST
1952	41.2526	42.4608	176.197	175.496
1953	47.6510	45.9759	193.969	189.063
1954	51.4308	51.1042	206.484	206.761
1955	58.3138	58.3350	226.948	231.105
1956	64.3697	67.9400	256.890	262.056
1957	71.5151	69.9111	294.784	292.383
1958	75.7941	77.1386	315.860	320.120
1959	81.4007	82.0535	350.546	350.012
1960	84.8974	86.2575	369.750	373.553
1961	90.6526	91.1128	395.191	395.652
1962	99.5986	98.0741	435.124	424.319
1963	103.272	103.402	462.146	458.487
1964	105.202	104.337	483.712	484.499
1965	108.841	113.569	517.508	525.065
1966	118.413	118.468	584.690	580.788
1967	127.583	116.547	651.373	628.030
1968	132.983	122.307	712.055	691.652
1969	138.655	143.302	786.242	791.828
1970	147.442	144.569	891.245	900.246
1971	157.767	168.413	963.250	990.847
1972	168.967	173.292	1094.40	1122.67
1973	177.036	186.739	1267.11	1293.03
1974	183.952	186.361	1523.17	1516.85
1975	189.444	185.056	1747.55	1752.27
1976	195.544	199.898	1972.75	2017.83

AIB = Average interest on bonds.

ARE = Average rate of return on equity.

Equations (1) and (2) have the same set of regressors, therefore, in this case Zellner's seemingly unrelated regression procedure coincides with ordinary least squares. Furthermore, by definition, for each observation RADEBT plus RAEQUI equals RAVAK and this imposes constraints on the parameters and random errors (U_t^1 and U_t^2) of equations (1) and (2).

The restrictions on the parameters are:

$$a_1 + a_2 = 0,\ b_1 + b_2 = 0,\ c_1 + c_2 = 1 \qquad \text{and}$$

The restriction in the random errors of both equations is $U_t^1 + U_t^2 = 0$ for all t. These restrictions on the parameters and on the random errors of equations (1) and (2) are satisfied when the equations are estimated by ordinary least squares. Thus, we estimated these equations by non-linear least squares, correcting for auto-correlation, obtaining the following results:

$$RADEBT_t = \underset{(-1.85)}{-67.76} (AIB_t/ARE_t) + \underset{(11.05)}{.4764} RAVAK_t$$

$$RAEQUI_t = \underset{(1.85)}{67.76} (AIB_t/ARE_t) + \underset{(11.05)}{.5236} RAVAK_t$$

$$\rho = 1 \quad R^2 = .996 \quad DW = 1.31 \quad T = 24$$

The above equations are estimated as first differences, forcing ρ to be equal to one, since the estimated ρ was in fact very close to one when was free. In this case, a_1 and a_2 are each equal to zero.

CHAPTER V

A FINANCIAL AND INCOME STATEMENT MODEL OF BELL CANADA

5.1 The Demand for Financial Instruments

In this section we develop a model to link the requirements of economic capital with the financing requirements for this capital. The model consists of two demand equations, one for real long term debt and one for real equity and one equation that links net economic capital with net book value of capital. Also, we have an equation to explain the number of preferred equity.

A. The Demand for Real Long Term Debt and Real Equity

We specify demand equations in which real long term debt and real equity are linear functions of the relative cost of debt and of equity and the sum of real debt and real equity.

The demand equations are written as:

$$RADEBT_t = a_1 + b_1 (AIB_t/ARE_t) + c_1 \; RAVAK_t + U_t^1 \qquad (1)$$

$$RAEQUI_t = a_2 + b_2 (AIB_t/ARE_t) + c_2 \; RAVAK_t + U_t^2 \qquad (2)$$

where

RADEBT = Average long term debt in 1967 prices. The price deflator used is the price index of Telephone plant.

REQUI = Average total equity (preferred plus common stock) in 1967 prices. Price deflator used was the same as for RADEBT.

RAVAK = RADEBT + RAEQUI. That is Net Average Accounting Real Value of Plant and Equipment.

From the estimation of this equation we obtained the following results:

$$RAVAK_t = -306277 + \underset{(1.627)}{.920648}\ K_t + \underset{(.56)}{158.52} \cdot TIME$$

$$\rho = 1.03926 \quad R^2 = .9956 \quad DW = 2.05 \quad T = 24$$

and where TIME = 1952, ...

This completes the section on Bell's demand for average long term debt and for average equity capital, as well as on the relationship between real average accounting value of plant and equipment (RAVAK) and economic capital (K). We go on to study in detail the relationships behind the firm's income statement.

From these regression estimates, we observe that all signs are in accordance with a priori expectations. In equation (2), we have estimated a demand function for the aggregation of common equity and preferred equity. This was done because Bell started to issue preferred equity only in 1970, thus, we have a limited number of data points to estimate the demand for average real preferred equity (RAPE). In our Income statement model we need a prediction of the dividend paid on preferred equity. For this we need to predict RAPE. We use an autoregressive relationship, for this purpose, where $RAPE_t$ is related to $RAPE_{t-1}$ as follows:

$$RAPE_t = z_1 + z_2 \cdot RAPE_{t-1} \tag{3}$$

We obtained the following results when we estimated equation (3)

$$RAPE_t = \underset{(4.6)}{70.875} + \underset{(5.77)}{.6266} \cdot RAPE_{t-1}$$

$$R^2 = .8926 \quad DW = 2.4183 \quad T = 6$$

To close our financial model we need to link RAVAK with the net economic capital.

B. The Relation between RAVAK and K

What we need now is a link between RAVAK and net economic capital (K). For this purpose we introduce a simple relationship between RAVAK and economic capital and Time, as follows

$$RAVAK_t = d_o + d_1 \cdot K_t + d_2 \cdot TIME \tag{4}$$

where:

RTOE = Total operating expenses in 1967 dollars

RNKCAD = Non capital costs and depreciation in 1967 dollars

The equation was estimated by using non-linear least squares and the following results were obtained:

$$RTOE_t = \underset{(3.84)}{101.43} + \underset{(19.34)}{.7468}\ RNKCAD_t$$

$$\rho = \underset{(12.3)}{.844} \qquad R^2 = .9982 \qquad DW = 1.51 \qquad T = 24$$

RTOE and RNKCAD are both expressed in millions of dollars.

C. Interest Charges:

The interest charges incurred, depend on the amount of short- and long-term debt outstanding. For Bell, short-term debt represents on the average, something less than 1% of total debt and consequently, can be omitted from the succeeding regression equation without a significant reduction in the explanatory capabilities of the model.

The regression equation linking interest charges and long-term debt is deflated by using the consumer price index. We postulate a relation linking real interest charges and long-term debt as follows:

$$RINT_t = r_o + r_1\ RADEBT_t + U_t \tag{6}$$

However, due to significant residual auto-correlation in the above equation, we modify equation (6) by including a lagged endogenous

5.2 Bell Income Statement Items

A natural extension of our modelling of Bell is to build a sub-model to reproduce the income statements of the Company. The model that we develop translates the economic variables of our financial and economic model into the accounting items included in the income statement of the Company.

Here we present the model according to each item that appears on the Company's income statement.

A. Operating Revenues:

Total operating revenues (TORE) are obtained as the sum of revenues for local services, toll message services, other toll services and miscellaneous revenues. All these with the exception of miscellaneous revenues are obtained from the demand module given the prices of the respective services. Miscellaneous revenues and uncollectables are treated as exogenous.

B. Operating Expenses:

The operating expenses in the income statement of the Company does not include the cost of equity and debt capital. We relate operating expenses to the aggregate of labour costs, raw materials costs and depreciation costs. Both variables are expressed in 1967 prices by using the consumer price index as deflator.

The equation to be estimated can be written as:

$$RTOE_t = \beta_o + \beta_1 \; RNKCAD_t + U_t \qquad (5)$$

When equation (4) was estimated by using ordinary least squares, the following results were obtained:

$$RINCTAX = \underset{(-.59)}{-2.17} + \underset{(22.37)}{.4712} \; RTXBASE$$

$$\underset{}{\rho} = \underset{(2.48)}{.4875} \qquad R^2 = .9913 \qquad DW = 1.40 \qquad T = 24$$

E. <u>Preferred Dividends</u>:

The preferred dividends paid by Bell to its shareholders depends, of course, on the amount of preferred equity that the Company holds. Bell started to use preferred equity as a financing instrument in 1970 and, therefore, estimating the regression equation for preferred dividends was done using only seven data points.

The relevant regression equation can be written in the following manner:

$$RDIVPR_t = e_o + e_1 \; RAPE_t + U_t \tag{9}$$

where:

RDIVPR = Preferred Dividends paid to shareholders, deflated by the consumer price index

RAPE = The amount of average preferred equity deflated again by the consumer price index

When equation (9) was estimated using ordinary least squares, the following results were obtained:

$$RDIVPR_t = \underset{(-1.59)}{-2.58} + \underset{(8.83)}{.0959} \; RAPE_t$$

$$R^2 = .9397 \qquad DW = .784 \qquad T = 7$$

variable on the right hand side which results from the implementation of a partial adjustment model.

The resulting regression can then be written in the form:

$$RINT_t = r_o + r_1 \; RADEBT_t + r_2 \; RINT_{t-1} + U_t \qquad (7)$$

When equation (7) is estimated by ordinary least squares, the results are given as:

$$RINT_t = \underset{(0.52)}{-.552} + \underset{(3.2)}{.01307} \; RADEBT + \underset{(14.4)}{.8351} \; RINT_{t-1}$$

$$R^2 = .996 \qquad DW = 2.38 \qquad T = 24$$

D. Income Tax:

The income tax paid by the Company depends on its tax base. The tax base is given by the difference between total operating revenues plus other income and total operating expenses and interest charges, that is:

RTAXBASE = Total operating revenue-Total operating expenses + Other income - Interest charges, in 1967 dollars.

The relevant regression equation can then be written as:

$$RINCTAX_t = \gamma_o + \gamma_1 \; RTAXBASE_t + U_t \qquad (8)$$

where

RINCTAX = Income tax paid by Bell and deflated by the consumer price index

RTAXBASE = The net income before taking extraordinary items into account. The price deflator used was the same as for INCTAX.

The above constituted the income statement model in current (nominal) dollars. The model in constant (1967) dollars is as follows:

(SR.1) $\mathrm{RTOREX} = \mathrm{RSERVIX} + \mathrm{RMISNETX}$

(SR.2) $\mathrm{RTOES} = 101.43 + .7468\ \mathrm{RNKCADX} - .844\ (101.43 + .7468\ \mathrm{RNKCADX}_{t-1} - \mathrm{RTOES}_{t-1})$

(SR.3) $\mathrm{RNORS} = \mathrm{RTOREX} - \mathrm{RTOES}$

(SR.4) $\mathrm{RIBUIS} = \mathrm{RNORS} + \mathrm{ROTHIX}$

(SR.5) $\mathrm{RINTS} = -.552 + .0131\ \mathrm{RADEBTS} + .8351\ \mathrm{RINTS}_{t-1}$

(SR.6) $\mathrm{TAXBASES} = \mathrm{RIBUIS} - \mathrm{RINTS}$

(SR.7) $\mathrm{RINCTAXS} = -2.17 + .4712\ \mathrm{RTXBASES} - .4875\ (-2.17 + .4712\ \mathrm{RTXBASES}_{t-1} - \mathrm{RINCTAXS}_{t-1})$

(SR.8) $\mathrm{RIBEIS} = \mathrm{RTAXBASES} - \mathrm{RINTXS}$

(SR.9) $\mathrm{RN19S} = \mathrm{RIBEIS} + \mathrm{REXTRIX}$

(SR.10) $\mathrm{RDIVPRS} = -2.58 + .0959.\mathrm{RAPES}$

(SR.11) $\mathrm{RNI21S} = \mathrm{RN19S} - \mathrm{RDIVPRS}$

Both models ((SN.1) to (SN.11) and (SR.1) to (SR.11)) reproduce the income statement in current and constant dollars. Additionally, we need to following relationships:

(A.1) $\mathrm{RAVAKS} = -306277 + .920648\ \mathrm{KX} + 158.52\ \mathrm{TIMEX} - 1.03926\ (-306277 + .920648\ \mathrm{KX}_{t-1} + 158.52\ \mathrm{TIMEX}_{t-1} - \mathrm{RAVAKS}_{t-1})$

(A.2) $\mathrm{RADEBTS} = -67.76.(\mathrm{AIBX}/\mathrm{AREX}) + .4764\ \mathrm{RAVAKS}$

(A.3) $\mathrm{RAEQUIS} = 67.76.\ (\mathrm{AIBX}/\mathrm{AREX}) + .5326\ \mathrm{RAVAKS}$

(A.4) $\mathrm{RTAXBASES} = \mathrm{TAXBASES}/\mathrm{CPIX}$

F. Other Income Statement Items:

Two other income statement components that were not modelled are Extraordinary Items and Other Income. Due to its variability and insignificant effect on bottom line totals, Extraordinary Items can be regarded as being the noise inherent in the modelling of any large scale system. Alternatively, we can treat it as exogenous. Other income can also be regarded as exogenously determined.

5.3 The Income Statement Model

With the behavioural equations presented above and a set of exogenous variables (i.e. exogenous to the whole model or determined in other parts of our complete model of Bell Canada) we can solve for a set of endogenous variables to reproduce the income statement of the company. The income statement model is as follows, where variables ending in X are exogenous (except the tax variables) and the ones ending in S are endogenous (thus simulated).

(SN.1) TOREX = SERVIX + MISNETX

(SN.2) TOES = RTOES • CPIX

(SN.3) NORS = TOREX - TOES

(SN.4) IBUIS = NORS + OTHIX

(SN.5) INTS = RINTS • CPIX

(SN.6) TAXBASES = IBUIS - INTS

(SN.7) INCTAXS = RINCTAXS • CPIX

(SN.8) IBEIS = TAXBASES - INCTAXS

(SN.9) N19S = IBEIS + EXTRIX

(SN.10) DIVPRS= RDIVPRS • CPIX

(SN.11) NI2IS = NI9S - DIVPRS

where MISNETX = MISCUR + DIRCUR - UNCOL

B. The Endogenous Variables

There are four variables for each item; an R at the beginning means in constant dollars, while an S at the end means simulated values, for example:

TOE = total operating expenses

TOES = simulated value of TOE

RTOE = real TOE

RTOES = simulated RTOE

The other nominal variables are:

NOR = Net operating revenues

IBUI = Income before underlisted items (i.e. before interest charges)

INT = Interest charges

TAXBASE = Income before income taxes and extraordinary item, equal to the tax base

INCTAX = Income taxes

IBEI = Income before extraordinary item

EXTRI = Extraordinary item

NI19 = Net income after extraordinary item

DIVPR = Dividends on Preferred Shares

NI21 = Net income applicable to common shares after extraordinary item

A. The Exogenous Variables

The exogenous variables in the model are:

CPIX = consumer price index in Canada, 1967 = 1

TOREX = SERVIX + MISNETX = $y_1 + y_2 + y_3$ + MISCUR + UNCOLX + DIRCUR service revenue (local, toll and other toll) plus miscellaneous revenues in current dollars, plus directory assistance also in current dollars.

RTOREX = TOREX/CPIX

OTHIX = Other income

ROTHIX = OTHIX/CPIX

EXTRIX = Extraordinary items (from income statement), treated as income (i.e. extraordinary expenses are negative income)

KX = economic capital required, this variable is determined by other modules

LX = labour input, determined elsewhere

MX = raw materials input, determined elsewhere

RNKCADX = MX + AAA·LX + KX·DECX = Real non capital costs and depreciation coefficient of actual (RAVAK) on predicted (RAVAKS) is quite close to unity, being 1.033. We can see also that the bias of the simulated variable is small.

We go on now to see how well our income statement model predicts the variables. In this model, total operating revenues (TOE) are exogenous. The first endogenous variable is total operating expenses, which was analysed in equation (5) above. In Table 5.5 we have the actual and predicted values of the real and nominal series for total operating expenses (TOE). We can observe that the model predicts quite closely the variable in question , the regression coefficient being .9791. The bias is small, since 0.2 percent of the error is due to bias. In Table 5.6 we have the values for Net Operating Revenues, which is the difference between Total Operating Expenses. We can see that this variable simulates quite well. The regression coefficient is 1.018 and the bias is also small, 0.2 percent.

The next endogenous variable in the Income Statement model is Income Before Underlisted Items (IBUI) which is defined as follows IBUI - NOR - OTHIX where OTHIX is Other Income, treated here as exogenous. In Table 5.7 we have the analysis of the variable Income Before Underlisted Items (IBUI), as usual in real and current dollars. The fitting is here also quite reasonable, the actual values being on the average 1.017 times the simulated values. Here also the bias small, being as before 0.2 percent of the error.

5.4 Validation of the Financial and Income Statement Model

As a validation of our financial and income statement model we have run simulations of the model, for both current and constant values, assuming that the values of the exogenous variables are equal to the historical values. The results of this validation are presented in the following tables.

In Table 5.1 we have the real value of long-term debt, its simulated value and the same variables in current dollars. All the comparisons in this chapter are done with the real variables, although the current dollars variables are also presented. We can observe that the tracking for real average long-term debt (RADEBTS) is quite good. Similar results can be observed in Table 5.2, where the equity (common and preferred) variables are presented.

In the debt series, actual values are on the average 1.136 times the simulated values, while for the equity, actual values are .9464 times the simulated values. In both cases the fraction of error due to bias is quite small.

In Table 5.3 we present the validation of equation (3), the equation used to obtain the predicted value of preferred equity. Here also we can see that the predicted values follow quite closely the actual ones, both in real and in nominal terms; actual values are .9877 times the simulated values, for the real variables.

In Table 5.4 we have the actual and predicted values of AVAK, both in real and in nominal terms. This corresponds to equation (4) of this chapter. The simulated values are quite close to the actual ones and the correlation coefficient is high, as it is in all the comparisons made in this validation. The regression

In Table 5.12 we have the results of the item labelled N19, which is Net Income After Extraordinary Item. Extraodrinary item (EXTRIX) is taken as exogenous in our model and is a variable usually quite small and erratic. RN19 follows thus quite closely the endogenous variable that preceeded it, namely, Income Before Extraodrinary Item (IBEI) which was analysed in the previous paragraph.

Since 1970 Bell Canada issued Preferred Equity. Accordingly, we have next study Dividends on Preferred Shares (DIVPR). In Table 5.14 we have the actual and predicted values for DIVPR in real and nominal terms. This variable is endogenous and the equation to predict it is equation (9) of the text. We can see that the simulated values are relatively close to the actual values of the variable. The regression coefficient of actual values on predicted series is .9662 and the fraction of error due to bias is small at a half percent.

The final item in Bell Canada's income statement is called Net Income Applicable to Common Shares After Extraodrinary Item. We have labelled this variable NI21. In Table 5.14 we present the analysis of the behaviour of this item. We can see that the predicted values are reasonable close to the actual values of the variable, the regression coefficient being .9914. The bias is quite small, since the fraction of error due to bias is 0.3 percent.

In Table 5.15 we present the actual and simulated values of the return on average total capital for Bell Canada, where the actual rate is defined as:

$$\text{RETURN} = \frac{\text{INT} + \text{N19}}{\text{AVAK}}$$

Next, we have the Interest Charges (INT), which are generated within our model as we saw in equation (6). In Table 5.8 we have the analysis of the results on this variable. The prediction is also quite good, the regression coefficient of actual (RINT) on predicted (RINTS) is 1.018. For this variable the fraction of error due to bias is 28.4 percent.

The Tax Base (TAXBASE) serves to predict the amount of income taxes paid by the company, and in our model is an endogenous variable. In Table 5.9 we analyse the results of the tax base. We can observe that the predicted values are reasonable close to the actual ones, the actual values being 99.2 percent of the simulated values. Here the percentage of error due to bias is quite small being 5.9 percent.

Having predicted the Tax Base, we go on to simulate the amount of income taxes paid (INCTAX). In Table 5.10 we have the results of the validation of equation (8) of our model. We can see that the results are reasonably good in general, although for some years (eg 1976) the behaviour is erratic. The actual values are on the average 99.8 percent of the simulated values and the bias here is also small, the fraction of error due to bias being 7.9 percent.

Next, we have Income Before Extraordinary Item (IBEI), for which the results are in Table 5.11. This variable is the result of the difference between TAXBASE and INCTAX, thus its behaviour is the result of the behaviour of the variables just named. Again, we can observe that IBEI is predicted by the model in a closed fashion, the bias being also small.

TABLE 5.1

VALIDATION: LONG-TERM DEBT

```
              RADEBT           RADEBTS          ADEBT            ADEBTS

        .....................................................................

1952    .     223.867          223.867          194.540          194.540
1953    .     266.049          273.734          226.408          232.948
1954    .     294.091          320.250          247.919          269.970
1955    .     318.174          370.886          267.584          311.915
1956    .     356.528          428.198          304.475          365.681
1957    .     401.496          492.186          344.885          422.788
1958    .     445.487          555.843          384.901          480.249
1959    .     522.498          608.671          451.438          525.892
1960    .     594.539          684.632          516.654          594.945
1961    .     660.266          728.781          571.130          630.395
1962    .     710.936          770.944          620.647          673.034
1963    .     759.720          836.548          670.833          738.671
1964    .     810.106          871.710          712.083          766.233
1965    .     848.406          924.207          758.475          826.241
1966    .     940.657          980.352          880.455          917.609
1967    .     1022.47          1021.75          1022.47          1021.75
1968    .     1078.52          1056.21          1131.37          1107.97
1969    .     1159.27          1086.56          1275.20          1195.21
1970    .     1145.90          1111.10          1344.14          1303.32
1971    .     1178.04          1131.73          1464.30          1406.74
1972    .     1229.88          1149.89          1630.82          1524.75
1973    .     1267.79          1169.88          1787.58          1649.54
1974    .     1225.09          1181.34          1933.19          1864.16
1975    .     1280.00          1185.81          2210.56          2047.90
1976    .     1291.62          1189.11          2371.41          2183.20
           COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
           ******************************************

  ACTUAL AND PREDICTED VARIABLES...      RADEBT          RADEBTS

  CORRELATION COEFFICIENT =     .9902
                 (SQUARED =     .9805

  ROOT-MEAN-SQUARED ERROR =     68.22

  MEAN ABSOLUTE ERROR =         60.53

  MEAN ERROR =                 -12.91

  REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =       1.136

  THEIL'S INEQUALITY COEFFICIENT =                      .3888E-01

  FRACTION OF ERROR DUE TO BIAS =                       .3583E-01

  FRACTION OF ERROR DUE TO DIFFERENT VARIATION =        .4741

  FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =     .4901

  ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
       FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
            COEFFICIENT FROM UNITY =                    .4045
       FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =     .5597
```

and the simulated values of the rate of return is:

$$\text{RETURNS} = \frac{\text{INTS} + \text{N19S}}{\text{AVAKS}}$$

We also compare the simulated values with the actual ones. As can be observed, the simulated values follow the actual ones in a satisfactory manner. The regression coefficient of actual return on predicted return is .9219, while the fraction of error due to bias is insignificant.

To complete the information on the validation of our financial and income statement model, we present as Table 5.16 the values of the exogenous variables that enter into it.

We close here the discussion of the validation of our financial and income statement model. It can be fairly said that, as a whole, the model tracks quite well. Next we go on to simulate our Income Statement model taking as exogenous the variables simulated previously in this study.

TABLE 5.2

VALIDATION: EQUITY

```
                RAEQUI        RAEQUIS        AEQUI         AEQUIS

         ...............................................................

1952    .      332.673       332.673       289.093       289.093
1953    .      384.274       375.438       327.017       319.498
1954    .      452.921       426.513       381.812       359.550
1955    .      523.952       497.751       440.644       418.609
1956    .      584.137       568.262       498.853       485.296
1957    .      675.136       643.730       579.942       552.964
1958    .      725.273       728.275       626.636       629.230
1959    .      816.153       814.589       705.156       703.805
1960    .      855.593       871.698       743.510       757.506
1961    .      947.358       947.952       819.465       819.979
1962    .      1009.82       1015.30       881.569       886.357
1963    .      1096.93       1057.91       968.590       934.130
1964    .      1204.52       1125.33       1058.78       989.161
1965    .      1251.63       1156.16       1118.96       1033.60
1966    .      1273.21       1188.83       1191.72       1112.74
1967    .      1355.29       1234.44       1355.29       1234.44
1968    .      1337.12       1273.07       1402.64       1335.45
1969    .      1322.06       1310.42       1454.27       1441.46
1970    .      1357.26       1359.98       1592.06       1595.26
1971    .      1369.46       1373.47       1702.24       1707.22
1972    .      1360.88       1387.85       1804.52       1840.29
1973    .      1331.98       1403.65       1878.09       1979.15
1974    .      1264.70       1417.80       1995.70       2237.29
1975    .      1274.42       1424.18       2200.93       2459.56
1976    .      1321.29       1433.76       2425.89       2632.39
          COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
          **********************************************

   ACTUAL AND PREDICTED VARIABLES...     RAEQUI       RAEQUIS

   CORRELATION COEFFICIENT =     .9833
                  (SQUARED =     .9669

   ROOT-MEAN-SQUARED ERROR =     66.42

   MEAN ABSOLUTE ERROR =         46.03

   MEAN ERROR =                  2.360

   REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =      .9464

   THEIL'S INEQUALITY COEFFICIENT =                     .3085E-01

   FRACTION OF ERROR DUE TO BIAS =                      .1263E-02

   FRACTION OF ERROR DUE TO DIFFERENT VARIATION =       .4188E-01

   FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =    .9569


   ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
       FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
            COEFFICIENT FROM UNITY =                    .8546E-01
       FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =     .9133
```

TABLE 5.3

VALIDATION: PREFERRED EQUITY

```
                    RAPE           RAPES          APE            APES

          +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

1970      +         70.1185        70.1185        82.2490        82.2490
1971      +         113.969        114.815        141.664        142.715
1972      +         149.316        142.824        197.993        189.385
1973      +         149.364        160.377        210.603        226.131
1974      +         162.086        171.376        255.772        270.431
1975      +         181.864        178.269        314.079        307.870
1976      +         186.718        182.588        342.814        335.232



                 COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
                 **********************************************

  ACTUAL AND PREDICTED VARIABLES...     RAPE           RAPES

  CORRELATION COEFFICIENT =     .9863
                (SQUARED =     .9728

  ROOT-MEAN-SQUARED ERROR =     6.329

  MEAN ABSOLUTE ERROR =         5.052

  MEAN ERROR =                 -.9902

  REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        .9877

  THEIL'S INEQUALITY COEFFICIENT =                       .2108E-01

  FRACTION OF ERROR DUE TO BIAS =                        .2447E-01

  FRACTION OF ERROR DUE TO DIFFERENT VARIATION =         .6967E-04

  FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =      .9755

  ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
       FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
            COEFFICIENT FROM UNITY =                     .5394E-02
       FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .9701
```

TABLE 5.4

VALIDATION: FINANCIAL CAPITAL

```
                RAVAK          RAVAKS          AVAK           AVAKS

         *************************************************************

1952   .     556.540        556.540        483.633        483.633
1953   .     650.323        649.172        553.425        552.445
1954   .     747.012        746.762        629.731        629.521
1955   .     842.126        868.637        708.228        730.524
1956   .     940.665        996.460        803.328        850.977
1957   .     1076.63        1135.92        924.827        975.752
1958   .     1170.76        1284.12        1011.54        1109.48
1959   .     1338.65        1423.26        1156.59        1229.70
1960   .     1450.13        1556.33        1260.16        1352.45
1961   .     1607.62        1676.73        1390.60        1450.37
1962   .     1720.75        1786.24        1502.22        1559.39
1963   .     1856.65        1894.45        1639.42        1672.80
1964   .     2014.63        1997.04        1770.86        1755.39
1965   .     2100.04        2080.36        1877.44        1859.84
1966   .     2213.87        2169.18        2072.18        2030.35
1967   .     2377.76        2256.19        2377.76        2256.19
1968   .     2415.65        2329.29        2534.01        2443.42
1969   .     2481.33        2396.98        2729.47        2636.68
1970   .     2503.15        2471.08        2936.20        2898.58
1971   .     2547.50        2505.20        3166.55        3113.97
1972   .     2590.75        2537.73        3435.34        3365.04
1973   .     2599.77        2573.54        3665.67        3628.69
1974   .     2489.79        2599.14        3928.88        4101.45
1975   .     2554.42        2609.99        4411.49        4507.46
1976   .     2612.90        2622.87        4797.29        4815.59

          COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
          **********************************************

  ACTUAL AND PREDICTED VARIABLES...      RAVAK          RAVAKS

  CORRELATION COEFFICIENT =     .9965
                (SQUARED =     .9930

  ROOT-MEAN-SQUARED ERROR =     64.07

  MEAN ABSOLUTE ERROR =         52.89

  MEAN ERROR =                 -10.55

  REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        1.033

  THEIL"S INEQUALITY COEFFICIENT =                       .1642E-01

  FRACTION OF ERROR DUE TO BIAS =                        .2712E-01

  FRACTION OF ERROR DUE TO DIFFERENT VARIATION =         .1531

  FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =      .8198


  ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
       FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
            COEFFICIENT FROM UNITY =                     .1244
       FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .8485
```

TABLE 5.5

VALIDATION: TOTAL OPERATING EXPENSES

```
                    RTOE           RTOES          TOE            TOES

              ****************************************************************

1952    .       224.616        224.616        133.156        133.156
1953    .       237.034        241.001        146.347        148.797
1954    .       253.786        259.673        161.456        165.201
1955    .       278.729        283.076        182.033        184.872
1956    .       306.308        309.344        206.091        208.133
1957    .       332.496        330.221        232.304        230.714
1958    .       346.888        344.863        252.205        250.733
1959    .       359.000        357.011        270.758        269.258
1960    .       365.836        363.496        287.120        285.284
1961    .       372.495        369.687        301.350        299.079
1962    .       387.378        383.231        322.017        318.570
1963    .       408.106        400.885        347.296        341.151
1964    .       423.729        412.911        366.487        357.131
1965    .       447.490        432.980        397.631        384.737
1966    .       464.943        451.471        436.585        423.935
1967    .       465.943        456.742        465.943        456.742
1968    .       474.719        468.461        502.783        496.155
1969    .       512.398        499.761        574.881        560.704
1970    .       516.891        514.039        623.932        620.489
1971    .       538.865        541.438        691.963        695.267
1972    .       547.543        563.043        763.736        785.356
1973    .       584.874        597.653        872.980        892.053
1974    .       602.680        622.327        1007.26        1040.09
1975    .       629.001        641.927        1171.62        1195.70
1976    .       678.559        680.414        1367.68        1371.41

              COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
              **********************************************

   ACTUAL AND PREDICTED VARIABLES...      RTOE           RTOES

   CORRELATION COEFFICIENT =     .9977
                 (SQUARED =      .9954

   ROOT-MEAN-SQUARED ERROR =     8.813

   MEAN ABSOLUTE ERROR =         7.003

   MEAN ERROR =                  .4014

   REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        .9791

   THEIL"S INEQUALITY COEFFICIENT =                       .9840E-02

   FRACTION OF ERROR DUE TO BIAS =                        .2075E-02

   FRACTION OF ERROR DUE TO DIFFERENT VARIATION =         .7023E-01

   FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =      .9277


   ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
        FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
             COEFFICIENT FROM UNITY =                     .8854E-01
        FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .9094
```

TABLE 5.6

VALIDATION: NET OPERATING REVENUES

```
                    RNOR           RNORS           NOR            NORS

           .............................................................

  1952   .        11.3367        11.3367        51.2420        51.2420
  1953   .        23.7093        19.7424        55.6153        53.1661
  1954   .        28.1734        22.2868        57.9186        54.1736
  1955   .        35.1047        30.7578        62.8669        60.0280
  1956   .        39.6606        36.6248        67.8844        65.8418
  1957   .        38.7257        41.0012        70.6815        72.2713
  1958   .        45.4251        47.4503        76.6123        78.0848
  1959   .        85.4244        87.4136        105.846        107.347
  1960   .        106.123        108.463        117.728        119.564
  1961   .        128.324        131.131        132.307        134.578
  1962   .        150.103        154.250        148.978        152.425
  1963   .        155.463        162.684        155.681        161.826
  1964   .        173.597        184.415        176.285        185.641
  1965   .        189.666        204.177        195.330        208.223
  1966   .        203.280        216.751        208.462        221.112
  1967   .        236.093        245.293        236.093        245.293
  1968   .        254.263        260.520        255.695        262.323
  1969   .        261.681        274.317        267.209        281.386
  1970   .        316.638        319.490        312.704        316.147
  1971   .        342.386        339.813        326.824        323.521
  1972   .        381.355        365.855        361.680        340.060
  1973   .        393.876        381.097        402.225        383.151
  1974   .        393.885        374.238        432.866        400.030
  1975   .        411.416        398.490        494.246        470.169
  1976   .        427.482        425.627        536.248        532.510

                 COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
                 **********************************************

     ACTUAL AND PREDICTED VARIABLES...      RNOR           RNORS

     CORRELATION COEFFICIENT =      .9981
                    (SQUARED =      .9962

     ROOT-MEAN-SQUARED ERROR =      8.813

     MEAN ABSOLUTE ERROR =          7.003

     MEAN ERROR =                  -.4014

     REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =      1.018

     THEIL'S INEQUALITY COEFFICIENT =                     .1860E-01

     FRACTION OF ERROR DUE TO BIAS =                      .2075E-02

     FRACTION OF ERROR DUE TO DIFFERENT VARIATION =       .9189E-01

     FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =    .9060


     ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
          FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
               COEFFICIENT FROM UNITY =                   .7488E-01
          FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =    .9230
```

TABLE 5.7

VALIDATION: INCOME BEFORE UNDERLISTED ITEMS

	RIBUI	RIBUIS	IBUI	IBUIS
1952	14.1071	14.1071	53.4070	53.4070
1953	27.0685	23.1016	58.2172	55.7680
1954	33.1871	27.3005	61.8194	58.0745
1955	40.1407	35.7937	66.7967	63.9578
1956	46.6268	43.5909	73.4010	71.3584
1957	47.3214	49.5969	77.6972	79.2870
1958	53.5495	55.5748	83.4218	84.8943
1959	94.4804	96.4697	113.521	115.021
1960	114.256	116.595	124.704	126.540
1961	136.022	138.829	138.972	141.244
1962	158.585	162.732	156.411	159.858
1963	164.888	172.109	164.093	170.238
1964	183.969	194.787	185.710	195.067
1965	200.084	214.594	205.025	217.918
1966	215.709	229.180	220.460	233.110
1967	256.135	265.336	256.135	265.336
1968	275.333	281.590	277.618	284.245
1969	282.345	294.982	289.689	303.866
1970	338.517	341.370	337.290	340.733
1971	368.152	365.579	356.611	353.307
1972	407.935	392.435	393.883	372.263
1973	424.023	411.244	441.503	422.429
1974	424.769	405.122	477.496	444.660
1975	444.729	431.803	547.585	523.508
1976	465.374	463.519	601.475	597.737

```
            COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
            **********************************************

 ACTUAL AND PREDICTED VARIABLES...          RIBUI          RIBUIS

 CORRELATION COEFFICIENT =     .9984
                (SQUARED =     .9967

 ROOT-MEAN-SQUARED ERROR =     8.813

 MEAN ABSOLUTE ERROR =         7.003

 MEAN ERROR =                 -.4014

 REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =          1.017

 THEIL'S INEQUALITY COEFFICIENT =                        .1727E-01

 FRACTION OF ERROR DUE TO BIAS =                         .2075E-02

 FRACTION OF ERROR DUE TO DIFFERENT VARIATION =          .9728E-01

 FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =       .9006

 ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
      FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
           COEFFICIENT FROM UNITY =                      .8098E-01
      FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =       .9169
```

TABLE 5.8

VALIDATION: INTEREST CHARGES

```
                    RINT          RINTS          INT            INTS

          ***********************************************************

1952    .   11.9627       11.9627        7.09169        7.09169
1953    .   14.0158       13.0166        8.65349        8.03658
1954    .   15.0479       14.5047        9.57331        9.22775
1955    .   15.6208       16.4095       10.2016        10.7167
1956    .   17.4872       18.7495       11.7658        12.6151
1957    .   19.7361       21.5402       13.7890        15.0494
1958    .   21.1877       24.7029       15.4046        17.9603
1959    .   24.7692       28.0348       18.6809        21.1439
1960    .   29.5005       31.8103       23.1530        24.9658
1961    .   32.9553       35.5405       26.6610        28.7524
1962    .   35.7106       39.2068       29.6853        32.5915
1963    .   38.1518       43.1262       32.4670        36.7001
1964    .   40.5309       46.8590       35.0555        40.5288
1965    .   42.4407       50.6626       37.7120        45.0178
1966    .   46.8253       54.5730       43.9694        51.2446
1967    .   52.7498       58.3799       52.7498        58.3799
1968    .   57.5662       62.0096       60.9693        65.6754
1969    .   64.3147       65.4374       72.1574        73.4171
1970    .   64.2015       68.6209       77.4968        82.8314
1971    .   67.9022       71.5492       87.1941        91.8773
1972    .   73.0459       74.2320      101.888        103.542
1973    .   78.2563       76.7338      116.805        114.532
1974    .   78.7935       78.9729      131.687        131.987
1975    .   86.3994       80.9012      160.934        150.693
1976    .   87.9584       82.5546      177.285        166.394

            COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
            **********************************************

ACTUAL AND PREDICTED VARIABLES...      RINT           RINTS

CORRELATION COEFFICIENT =     .9906
                (SQUARED =    .9812

ROOT-MEAN-SQUARED ERROR =     3.976

MEAN ABSOLUTE ERROR =         3.236

MEAN ERROR =                 -2.118

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        1.018

THEIL"S INEQUALITY COEFFICIENT =                       .3847E-01

FRACTION OF ERROR DUE TO BIAS =                        .2839

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =         .2783E-01

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =      .6883


ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                     .1189E-01
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .7042
```

TABLE 5.9

VALIDATION: TAX BASE

```
                   RTAXBASE        RTAXBASES       TAXBASE         TAXBASES

              * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

1952      .       78.1278         78.1278         46.3154         46.3154
1953      .       80.2766         77.3089         49.5637         47.7314
1954      .       82.1237         76.7803         52.2461         48.8467
1955      .       86.6587         81.5230         56.5950         53.2410
1956      .       91.6070         87.3088         61.6352         58.7433
1957      .       91.4715         91.9430         63.9082         64.2376
1958      .       93.5524         92.0625         68.0173         66.9340
1959      .       125.748         124.472         94.8396         93.8769
1960      .       129.392         129.421         101.551         101.574
1961      .       138.827         139.049         112.312         112.491
1962      .       152.448         153.099         126.726         127.267
1963      .       154.673         156.919         131.626         133.538
1964      .       174.185         178.675         150.655         154.538
1965      .       188.292         194.581         167.313         172.901
1966      .       187.954         193.678         176.490         181.865
1967      .       203.385         206.956         203.385         206.956
1968      .       204.555         206.370         216.648         218.570
1969      .       193.888         205.402         217.531         230.449
1970      .       215.223         213.656         259.793         257.901
1971      .       209.808         203.588         269.417         261.430
1972      .       209.339         192.653         291.996         268.721
1973      .       217.540         206.283         324.698         307.897
1974      .       206.910         187.084         345.809         312.672
1975      .       207.579         200.151         386.651         372.816
1976      .       210.458         214.007         424.190         431.343

              COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
              **********************************************

   ACTUAL AND PREDICTED VARIABLES...      RTAXBASE       RTAXBASES

   CORRELATION COEFFICIENT =      .9911
                  (SQUARED =      .9823

   ROOT-MEAN-SQUARED ERROR =      7.043

   MEAN ABSOLUTE ERROR =          4.963

   MEAN ERROR =                   1.717

   REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =       .9921

   THEIL"S INEQUALITY COEFFICIENT =                      .2138E-01

   FRACTION OF ERROR DUE TO BIAS =                       .5943E-01

   FRACTION OF ERROR DUE TO DIFFERENT VARIATION =        .4871E-04

   FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =     .9405


   ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
        FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
             COEFFICIENT FROM UNITY =                     .3320E-02
        FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .9372
```

TABLE 5.10

VALIDATION: INCOME TAXES

```
                 RINCTAX        RINCTAXS        INCTAX         INCTAXS

          ..............................................................

1952   .        40.0546         40.0546         23.7450         23.7450
1953   .        36.7907         36.8944         22.7150         22.7790
1954   .        37.2484         35.2925         23.6970         22.4527
1955   .        37.6937         36.8677         24.6170         24.0775
1956   .        39.6628         39.2723         26.6860         26.4233
1957   .        39.8917         41.2991         27.8710         28.8543
1958   .        40.0495         41.2789         29.1180         30.0119
1959   .        59.0771         56.5128         44.5560         42.6220
1960   .        61.2092         58.8268         48.0390         46.1692
1961   .        67.5163         63.3543         54.6210         51.2539
1962   .        73.9119         69.9702         61.4410         58.1643
1963   .        74.4212         71.7683         63.3320         61.0744
1964   .        82.2745         82.0184         71.1600         70.9385
1965   .        88.3353         89.5124         78.4930         79.5390
1966   .        86.9470         89.0869         81.6440         83.6534
1967   .        91.5640         95.3432         91.5640         95.3432
1968   .        92.8339         95.0669         98.3220         100.687
1969   .        92.5493         94.6109         103.835         106.148
1970   .        104.823         98.5002         126.531         118.898
1971   .        95.1054         93.7562         122.126         120.393
1972   .        90.7349         88.6039         126.561         123.589
1973   .        100.140         95.0260         149.468         141.835
1974   .        96.2140         85.9795         160.802         143.697
1975   .        93.3701         92.1365         173.918         171.620
1976   .        92.1312         98.6653         185.696         198.866

            COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
            **********************************************

   ACTUAL AND PREDICTED VARIABLES...      RINCTAX        RINCTAXS

   CORRELATION COEFFICIENT =     .9898
                  (SQUARED =     .9797

   ROOT-MEAN-SQUARED ERROR =     3.529

   MEAN ABSOLUTE ERROR =         2.647

   MEAN ERROR =                  .9941

   REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        .9977

   THEIL'S INEQUALITY COEFFICIENT =                       .2326E-01

   FRACTION OF ERROR DUE TO BIAS =                        .7935E-01

   FRACTION OF ERROR DUE TO DIFFERENT VARIATION =         .2866E-02

   FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =      .9178


   ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
        FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
             COEFFICIENT FROM UNITY =                     .2321E-03
        FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .9204
```

TABLE 5.11

VALIDATION: INCOME BEFORE EXTRAORDINARY ITEM

	RIBEI	RIBEIS	IBEI	IBEIS
1952	38.0731	38.0731	22.5704	22.5704
1953	43.4859	40.4145	26.8487	24.9524
1954	44.8753	41.4878	28.5491	26.3940
1955	48.9650	44.6553	31.9780	29.1635
1956	51.9442	48.0365	34.9492	32.3200
1957	51.5799	50.6439	36.0372	35.3832
1958	53.5029	50.7835	38.8993	36.9222
1959	66.6713	67.9592	50.2836	51.2549
1960	68.1825	70.5947	53.5119	55.4051
1961	71.3105	75.6947	57.6905	61.2373
1962	78.5360	83.1287	65.2849	69.1026
1963	80.2519	85.1512	68.2939	72.4632
1964	91.9109	96.6567	79.4946	83.5993
1965	99.9570	105.068	88.8198	93.3616
1966	101.007	104.591	94.8463	98.2120
1967	111.821	111.613	111.821	111.613
1968	111.722	111.303	118.326	117.883
1969	101.339	110.791	113.696	124.301
1970	110.400	115.156	133.262	139.003
1971	114.702	109.832	147.291	141.036
1972	118.604	104.050	165.435	145.133
1973	117.400	111.257	175.230	166.061
1974	110.696	101.104	185.007	168.975
1975	114.209	108.014	212.733	201.195
1976	118.326	115.341	238.494	232.477

```
              COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
              **********************************************

ACTUAL AND PREDICTED VARIABLES...           RIBEI          RIBEIS

CORRELATION COEFFICIENT =       .9817
               (SQUARED =       .9638

ROOT-MEAN-SQUARED ERROR =       5.371

MEAN ABSOLUTE ERROR =           4.341

MEAN ERROR =                    .7229

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =          .9867

THEIL"S INEQUALITY COEFFICIENT =                         .3021E-01

FRACTION OF ERROR DUE TO BIAS =                          .1812E-01

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =           .6723E-03

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =        .9812


ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                       .4758E-02
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =        .9771
```

TABLE 5.12

VALIDATION: NET INCOME AFTER EXTRAODRINARY ITEM

	RN19	RN19S	N19	N19S
1952	38.0731	38.0731	22.5704	22.5704
1953	43.4859	40.4145	26.8487	24.9524
1954	44.8753	41.4878	28.5491	26.3940
1955	48.9650	44.6553	31.9780	29.1635
1956	51.9442	48.0365	34.9492	32.3200
1957	51.5799	50.6439	36.0372	35.3832
1958	53.5029	50.7835	38.8993	36.9222
1959	66.6713	67.9592	50.2836	51.2549
1960	68.1825	70.5947	53.5119	55.4051
1961	71.3105	75.6947	57.6905	61.2373
1962	78.5360	83.1287	65.2849	69.1026
1963	80.2519	85.1512	68.2939	72.4632
1964	91.9109	96.6567	79.4946	83.5993
1965	99.9570	105.068	88.8198	93.3616
1966	101.007	104.591	94.8463	98.2120
1967	111.821	111.613	111.821	111.613
1968	111.722	111.303	118.326	117.883
1969	101.339	110.791	113.696	124.301
1970	110.400	115.156	133.262	139.003
1971	114.702	109.832	147.291	141.036
1972	117.855	103.300	164.526	144.225
1973	121.542	115.399	180.626	171.458
1974	110.696	101.104	185.007	168.975
1975	172.040	165.846	305.331	293.793
1976	118.326	115.341	238.494	232.477

```
        COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
        **********************************************

ACTUAL AND PREDICTED VARIABLES...      RN19          RN19S

CORRELATION COEFFICIENT =      .9865
               (SQUARED =      .9731

ROOT-MEAN-SQUARED ERROR =      5.371

MEAN ABSOLUTE ERROR =          4.341

MEAN ERROR =                   .7229

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =       1.003

THEIL'S INEQUALITY COEFFICIENT =                      .2899E-01

FRACTION OF ERROR DUE TO BIAS =                       .1812E-01

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =        .9700E-02

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =     .9722


ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                    .2778E-03
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =     .9816
```

TABLE 5.13

VALIDATION: DIVIDENDS ON PREFERRED SHARES

```
                RDIVPR           RDIVPRS          DIVPR            DIVPRS

             ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

1970     +      5.07836          4.14822          5.70655          4.66135
1971     +      8.08757          8.43597          9.34979          9.75257
1972     +      10.7956          11.1229          13.0796          13.4761
1973     +      10.7607          12.8067          14.0200          16.6858
1974     +      12.1754          13.8619          17.5944          20.0316
1975     +      15.5166          14.5231          24.8445          23.2538
1976     +      16.7580          14.9375          28.8470          25.7132



                    COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
                    **********************************************

     ACTUAL AND PREDICTED VARIABLES...      RDIVPR         RDIVPRS

     CORRELATION COEFFICIENT =     .9353
                   (SQUARED =     .8748

     ROOT-MEAN-SQUARED ERROR =     1.332

     MEAN ABSOLUTE ERROR =         1.165

     MEAN ERROR =                 -.9488E-01

     REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =       .9662

     THEIL"S INEQUALITY COEFFICIENT =                      .5580E-01

     FRACTION OF ERROR DUE TO BIAS =                       .5072E-02

     FRACTION OF ERROR DUE TO DIFFERENT VARIATION =        .8029E-02

     FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =     .9869

     ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
          FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
               COEFFICIENT FROM UNITY =                    .8461E-02
          FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =     .9865
```

TABLE 5.14

VALIDATION: NET INCOME APPLICABLE TO COMMON SHARES AFTER EXTRAORDINARY ITEM

```
                 RNI21           RNI21S          NI21            NI21S

        ................................................................................

1952    .     38.0731         40.6508         22.5704         24.5848
1953    .     43.4859         42.9922         26.8487         26.9490
1954    .     44.8753         44.0654         28.5491         28.3995
1955    .     48.9650         47.2330         31.9780         31.1750
1956    .     51.9442         50.6142         34.9492         34.3613
1957    .     51.5799         53.2216         36.0372         37.4871
1958    .     53.5029         53.3612         38.8993         39.0827
1959    .     66.6713         70.5369         50.2836         53.4392
1960    .     68.1825         73.1724         53.5119         57.6162
1961    .     71.3105         78.2723         57.6905         63.4693
1962    .     78.5360         85.7064         65.2849         71.3615
1963    .     80.2519         87.7288         68.2939         74.7637
1964    .     91.9109         99.2344         79.4946         85.9416
1965    .     99.9570         107.646         88.8198         95.7604
1966    .     101.007         107.169         94.8463         100.700
1967    .     111.821         114.190         111.821         114.190
1968    .     111.722         113.880         118.326         120.565
1969    .     101.339         113.369         113.696         127.105
1970    .     105.321         111.007         127.556         134.341
1971    .     106.615         101.396         137.941         131.284
1972    .     107.059         92.1771         151.447         130.749
1973    .     110.781         102.592         166.606         154.772
1974    .     98.5211         87.2424         167.412         148.944
1975    .     156.524         151.323         280.486         270.539
1976    .     101.568         100.404         209.647         206.764

              COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
              **********************************************

   ACTUAL AND PREDICTED VARIABLES...      RNI21          RNI21S

   CORRELATION COEFFICIENT =     .9759
                (SQUARED =     .9523

   ROOT-MEAN-SQUARED ERROR =     6.388

   MEAN ABSOLUTE ERROR =         5.142

   MEAN ERROR =                 -1.106

   REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        .9914

   THEIL'S INEQUALITY COEFFICIENT =                       .3576E-01

   FRACTION OF ERROR DUE TO BIAS =                        .3000E-01

   FRACTION OF ERROR DUE TO DIFFERENT VARIATION =         .4972E-02

   FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =      .9650


   ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
        FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
             COEFFICIENT FROM UNITY =                     .1464E-02
        FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .9685
```

TABLE 5.15

VALIDATION: RETURN ON TOTAL CAPITAL

```
                       RETURN          RETURNS

              ................................

   1952     .   .613317E-01    .613317E-01
   1953     .   .641500E-01    .597145E-01
   1954     .   .605377E-01    .565855E-01
   1955     .   .595566E-01    .545912E-01
   1956     .   .581518E-01    .528041E-01
   1957     .   .538762E-01    .516859E-01
   1958     .   .536845E-01    .494669E-01
   1959     .   .596272E-01    .588753E-01
   1960     .   .608372E-01    .594261E-01
   1961     .   .606585E-01    .620459E-01
   1962     .   .632201E-01    .652140E-01
   1963     .   .614612E-01    .652578E-01
   1964     .   .646862E-01    .707124E-01
   1965     .   .673961E-01    .744037E-01
   1966     .   .669902E-01    .736111E-01
   1967     .   .692127E-01    .753450E-01
   1968     .   .707556E-01    .751234E-01
   1969     .   .680915E-01    .749877E-01
   1970     .   .717795E-01    .765322E-01
   1971     .   .740507E-01    .747965E-01
   1972     .   .775510E-01    .736298E-01
   1973     .   .811396E-01    .788136E-01
   1974     .   .806066E-01    .733795E-01
   1975     .   .105693        .986111E-01
   1976     .   .866696E-01    .828292E-01

        COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
        **********************************************

ACTUAL AND PREDICTED VARIABLES...     RETURN       RETURNS

CORRELATION COEFFICIENT =    .9168
             (SQUARED =    .8405

ROOT-MEAN-SQUARED ERROR =    .4598E-02

MEAN ABSOLUTE ERROR =        .4056E-02

MEAN ERROR =                 .7765E-04

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        .9219

THEIL'S INEQUALITY COEFFICIENT =                        .3334E-01

FRACTION OF ERROR DUE TO BIAS =                         .2852E-03

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =          .1805E-03

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =       .9995

ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                      .3649E-01
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =       .9632
```

TABLE 5.16

VALIDATION: EXOGENOUS VARIABLES

	CPIX	AIB	ARE	EXTRIX	OTHIX	PK
1952	.781503	.364536E-01	.100096	0.	2.16508	.869000
1953	.774566	.382208E-01	.136716	0.	2.60191	.851000
1954	.778035	.386147E-01	.138269	0.	3.90086	.843000
1955	.780347	.381249E-01	.980272E-01	0.	3.92977	.841000
1956	.791908	.386429E-01	.874509E-01	0.	5.51658	.854000
1957	.816185	.399814E-01	.836254E-01	0.	7.01565	.859000
1958	.838150	.400221E-01	.689195E-01	0.	6.80954	.864000
1959	.847399	.413810E-01	.530910E-01	0.	7.67412	.864000
1960	.857803	.448133E-01	.754369E-01	0.	6.97594	.869000
1961	.865896	.466811E-01	.591574E-01	0.	6.66574	.865000
1962	.876301	.478296E-01	.510528E-01	0.	7.43304	.873000
1963	.892486	.483980E-01	.663416E-01	0.	8.41198	.883000
1964	.908671	.492296E-01	.528271E-01	0.	9.42519	.879000
1965	.930636	.497208E-01	.669158E-01	0.	9.69512	.894000
1966	.965318	.499394E-01	.926608E-01	0.	11.9980	.936000
1967	1.00000	.515905E-01	.955757E-01	0.	20.0424	1.00000
1968	1.04046	.538898E-01	.988582E-01	0.	21.9225	1.04900
1969	1.08786	.565852E-01	.987052E-01	0.	22.4798	1.10000
1970	1.12370	.576553E-01	.787531E-01	0.	24.5856	1.17300
1971	1.15607	.595465E-01	.892037E-01	0.	29.7866	1.24300
1972	1.21156	.624763E-01	.994274E-01	-.908062	32.2029	1.32600
1973	1.30289	.653424E-01	.111699	5.39629	39.2780	1.41000
1974	1.44509	.681193E-01	.114310	0.	44.6299	1.57800
1975	1.60116	.728023E-01	.120085	92.5974	53.3394	1.72700
1976	1.72139	.747596E-01	.115339	0.	65.2271	1.83600

TABLE 5.16 (cont'd)

VALIDATION: EXOGENOUS VARIABLES

	DECX	MISCUR	TOREX	UNCOL	WM	WLCOR
1952	.566550E-01	3.20000	184.398	.357880	.741074	1.69303
1953	.566338E-01	3.80000	201.963	.386131	.740074	1.81627
1954	.558243E-01	4.30000	219.374	.509337	.752073	1.89562
1955	.532538E-01	3.50000	244.900	.557241	.756074	1.97639
1956	.529346E-01	2.20000	273.975	.663928	.784081	2.02233
1957	.582912E-01	2.50000	302.986	.904820	.801046	2.11185
1958	.579688E-01	2.70000	328.818	1.12770	.812045	2.22591
1959	.598111E-01	2.90000	376.605	1.36030	.829044	2.33527
1960	.590843E-01	3.10000	404.848	1.59602	.839040	2.48667
1961	.590046E-01	3.90000	433.657	1.66511	.843038	2.63203
1962	.595667E-01	4.50000	470.995	1.92528	.855038	2.74388
1963	.613650E-01	5.20000	502.977	2.25200	.870037	2.83471
1964	.620350E-01	5.10000	542.772	2.24179	.892038	2.90667
1965	.634087E-01	5.40000	592.961	2.80132	.921037	2.99505
1966	.648240E-01	5.80000	645.047	3.15379	.962034	3.21050
1967	.652460E-01	6.40000	702.035	3.52003	1.00000	3.46077
1968	.664228E-01	7.00000	758.478	3.32413	1.03301	3.75600
1969	.686071E-01	8.60000	842.090	4.06018	1.07801	4.07078
1970	.691044E-01	9.60000	936.636	6.13177	1.12788	4.50005
1971	.697091E-01	28.3000	1018.79	4.62179	1.16413	4.94918
1972	.743950E-01	34.8000	1125.42	3.96555	1.22215	5.64473
1973	.774471E-01	29.6000	1275.20	4.60584	1.33404	6.02575
1974	.798139E-01	34.2000	1440.12	6.20400	1.53303	6.61296
1975	.835412E-01	43.0000	1665.87	8.97532	1.70504	7.57488
1976	.863555E-01	54.8000	1903.92	8.80481	1.86704	8.33328

VALIDATION OF THE COMPLETE MODEL

The validation performed in this section consists of running our Income Statement model with the simulated values for the services coming from the demand equations and the simulated values for labour, capital and raw materials coming from the cost model. When compared with the validation done in the previous section, it can be observed that for Average Preferred Equity (APE) the simulated results are identical with the one of the validation section. This is so because in our model the variable does not depend on either the demand for services or the input levels.

In this validation the main variables that change are Total Operating Revenues which were taken before as exogenous and now are still exogenous to this Income Statement model but are the result of a simulation of the demand equations. We call this variable TORES and its real counterpart RTORES. (See Table 5.21). In general, and as can be expected, the simulations done in this section do not follow as closely the actual values. However, we believe the results to be quite satisfactory, especially taking into account that we are simulating over a 25 years period and due to the nature of the model and the way the simulation was run, the errors tend to accumulate. The results of this simulation appear in Tables 5.17 through 5.32.

TABLE 5.17

SIMULATION: LONG-TERM DEBT

```
                RADEBT        RADEBTS        ADEBT         ADEBTS

           ..........................................................

1952     .      223.867       223.867        194.540       194.540
1953     .      266.049       290.038        226.408       246.822
1954     .      294.091       331.480        247.919       279.437
1955     .      318.174       380.034        267.584       319.609
1956     .      356.528       441.599        304.475       377.127
1957     .      401.496       502.052        344.885       431.264
1958     .      445.487       560.430        384.901       484.214
1959     .      522.498       607.203        451.438       524.627
1960     .      594.539       685.355        516.654       595.576
1961     .      660.266       733.683        571.130       634.640
1962     .      710.936       783.073        620.647       683.627
1963     .      759.720       838.648        670.833       740.530
1964     .      810.106       879.869        712.083       773.410
1965     .      848.406       939.353        758.475       839.786
1966     .      940.657       1013.14        880.455       948.299
1967     .      1022.47       1047.29        1022.47       1047.30
1968     .      1078.52       1077.27        1131.37       1130.06
1969     .      1159.27       1095.93        1275.20       1205.53
1970     .      1145.90       1097.96        1344.14       1287.91
1971     .      1178.04       1147.34        1464.30       1426.15
1972     .      1229.88       1170.04        1630.82       1551.48
1973     .      1267.79       1189.55        1787.58       1677.27
1974     .      1225.09       1217.11        1933.19       1920.60
1975     .      1280.00       1183.66        2210.56       2044.19
1976     .      1291.62       1177.83        2371.41       2162.50

           COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
           ********************************************

ACTUAL AND PREDICTED VARIABLES...      RADEBT        RADEBTS

CORRELATION COEFFICIENT =     .9892
                  (SQUARED =     .9786

ROOT-MEAN-SQUARED ERROR =     71.24

MEAN ABSOLUTE ERROR =         63.25

MEAN ERROR =                 -23.30

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        1.128

THEIL'S INEQUALITY COEFFICIENT =                       .4036E-01

FRACTION OF ERROR DUE TO BIAS =                        .1070

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =         .3972

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =      .4959


ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                     .3307
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .5623
```

TABLE 5.18

SIMULATION: EQUITY

```
               RAEQUI          RAEQUIS          AEQUI           AEQUIS

        ..............................................................

1952    .     332.673          332.673         289.093         289.093
1953    .     384.274          393.354         327.017         334.744
1954    .     452.921          438.853         381.812         369.953
1955    .     523.952          507.804         440.644         427.063
1956    .     584.137          582.990         498.853         497.874
1957    .     675.136          654.573         579.942         562.278
1958    .     725.273          733.318         626.636         633.586
1959    .     816.153          812.980         705.156         702.414
1960    .     855.593          872.496         743.510         758.199
1961    .     947.358          953.344         819.465         824.642
1962    .     1009.82          1028.63         881.569         897.997
1963    .     1096.93          1060.22         968.590         936.172
1964    .     1204.52          1134.30         1058.78         997.047
1965    .     1251.63          1172.80         1118.96         1048.49
1966    .     1273.21          1224.86         1191.72         1146.47
1967    .     1355.29          1262.51         1355.29         1262.51
1968    .     1337.12          1296.21         1402.64         1359.72
1969    .     1322.06          1320.72         1454.27         1452.80
1970    .     1357.26          1345.54         1592.06         1578.32
1971    .     1369.46          1390.62         1702.24         1728.54
1972    .     1360.88          1409.99         1804.52         1869.65
1973    .     1331.98          1425.27         1878.09         2009.63
1974    .     1264.70          1457.10         1995.70         2299.31
1975    .     1274.42          1421.82         2200.93         2455.49
1976    .     1321.29          1421.37         2425.89         2609.64

              COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
              **********************************************

     ACTUAL AND PREDICTED VARIABLES...      RAEQUI         RAEQUIS

     CORRELATION COEFFICIENT =     .9844
                   (SQUARED =      .9690

     ROOT-MEAN-SQUARED ERROR =     65.51

     MEAN ABSOLUTE ERROR =         43.93

     MEAN ERROR =                 -9.053

     REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        .9422

     THEIL'S INEQUALITY COEFFICIENT =                       .3027E-01

     FRACTION OF ERROR DUE TO BIAS =                        .1910E-01

     FRACTION OF ERROR DUE TO DIFFERENT VARIATION =         .5683E-01

     FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =      .9241


     ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
          FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
               COEFFICIENT FROM UNITY =                     .1033
          FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .8776
```

TABLE 5.19

SIMULATION: PREFERRED EQUITY

```
                 RAPE             RAPES            APE              APES

          ..........................................................................

1970   .       70.1185          70.1185          82.2490          82.2490
1971   .       113.969          114.815          141.664          142.715
1972   .       149.316          142.824          197.993          189.385
1973   .       149.364          160.377          210.603          226.131
1974   .       162.086          171.376          255.772          270.431
1975   .       181.864          178.269          314.079          307.870
1976   .       186.718          182.588          342.814          335.232




                 COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
                 **********************************************

     ACTUAL AND PREDICTED VARIABLES...        RAPE            RAPES

     CORRELATION COEFFICIENT =     .9863
                   (SQUARED =      .9728

     ROOT-MEAN-SQUARED ERROR =     6.329

     MEAN ABSOLUTE ERROR =         5.052

     MEAN ERROR =                 -.9902

     REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =         .9877

     THEIL"S INEQUALITY COEFFICIENT =                        .2108E-01

     FRACTION OF ERROR DUE TO BIAS =                         .2447E-01

     FRACTION OF ERROR DUE TO DIFFERENT VARIATION =          .6967E-04

     FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =       .9755

     ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
          FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
               COEFFICIENT FROM UNITY =                      .5394E-02
          FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =       .9701
```

TABLE 5.20

SIMULATION: FINANCIAL CAPITAL

	RAVAK	RAVAKS	AVAK	AVAKS
1952	556.540	556.540	483.633	483.633
1953	650.323	683.391	553.425	581.566
1954	747.012	770.332	629.731	649.390
1955	842.126	887.838	708.228	746.672
1956	940.665	1024.59	803.328	875.000
1957	1076.63	1156.63	924.827	993.541
1958	1170.76	1293.75	1011.54	1117.80
1959	1338.65	1420.19	1156.59	1227.04
1960	1450.13	1557.85	1260.16	1353.78
1961	1607.62	1687.03	1390.60	1459.28
1962	1720.75	1811.71	1502.22	1581.62
1963	1856.65	1898.87	1639.42	1676.70
1964	2014.63	2014.17	1770.86	1770.46
1965	2100.04	2112.16	1877.44	1888.27
1966	2213.87	2238.00	2072.18	2094.77
1967	2377.76	2309.81	2377.76	2309.81
1968	2415.65	2373.48	2534.01	2489.78
1969	2481.33	2416.66	2729.47	2658.32
1970	2503.15	2443.51	2936.20	2866.24
1971	2547.50	2537.96	3166.55	3154.69
1972	2590.75	2580.04	3435.34	3421.13
1973	2599.77	2614.82	3665.67	3686.90
1974	2489.79	2674.21	3928.88	4219.91
1975	2554.42	2605.49	4411.49	4499.68
1976	2612.90	2599.20	4797.29	4772.13

```
                COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
                **********************************************

   ACTUAL AND PREDICTED VARIABLES...      RAVAK          RAVAKS

   CORRELATION COEFFICIENT =      .9966
                      (SQUARED =      .9932

   ROOT-MEAN-SQUARED ERROR =     69.00

   MEAN ABSOLUTE ERROR =         53.86

   MEAN ERROR =                 -32.35

   REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        1.027

   THEIL'S INEQUALITY COEFFICIENT =                       .1759E-01

   FRACTION OF ERROR DUE TO BIAS =                        .2199

   FRACTION OF ERROR DUE TO DIFFERENT VARIATION =         .9238E-01

   FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =      .6878

   ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
        FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
             COEFFICIENT FROM UNITY =                     .7250E-01
        FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .7076
```

TABLE 5.21

SIMULATION: TOTAL OPERATING REVENUES

```
              RTOREX          RTORES          TOREX           TORES

        .......................................................................

1952   .     235.953         236.064         184.398         184.485
1953   .     260.743         265.591         201.963         205.718
1954   .     281.960         284.745         219.374         221.542
1955   .     313.834         311.734         244.900         243.261
1956   .     345.969         340.084         273.975         269.315
1957   .     371.222         365.597         302.986         298.395
1958   .     392.313         384.546         328.818         322.307
1959   .     444.424         438.174         376.605         371.308
1960   .     471.959         462.973         404.848         397.140
1961   .     500.818         491.747         433.657         425.801
1962   .     537.481         529.922         470.995         464.371
1963   .     563.569         559.520         502.977         499.364
1964   .     597.325         598.373         542.772         543.724
1965   .     637.156         636.343         592.961         592.204
1966   .     668.223         677.122         645.047         653.638
1967   .     702.035         711.330         702.035         711.330
1968   .     728.982         745.180         758.478         775.332
1969   .     774.079         780.581         842.090         849.163
1970   .     833.529         834.628         936.636         937.871
1971   .     881.251         893.088         1018.79         1032.47
1972   .     928.898         935.549         1125.42         1133.47
1973   .     978.751         979.310         1275.20         1275.93
1974   .     996.565         1021.77         1440.12         1476.55
1975   .     1040.42         1041.52         1665.87         1667.64
1976   .     1106.04         1105.83         1903.92         1903.57
              COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
              **********************************************

    ACTUAL AND PREDICTED VARIABLES...       RTOREX          RTORES

    CORRELATION COEFFICIENT =      .9996
                  (SQUARED =       .9993

    ROOT-MEAN-SQUARED ERROR =      8.325

    MEAN ABSOLUTE ERROR =          6.178

    MEAN ERROR =                  -1.513

    REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        .9840

    THEIL'S INEQUALITY COEFFICIENT =                       .6146E-02

    FRACTION OF ERROR DUE TO BIAS =                        .3305E-01

    FRACTION OF ERROR DUE TO DIFFERENT VARIATION =         .2474

    FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =      .7195


    ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
         FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
              COEFFICIENT FROM UNITY =                     .2588
         FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .7081
```

TABLE 5.22

SIMULATION: TOTAL OPERATING EXPENSES

	RTOE	RTOES	TOE	TOES
1952	224.616	224.616	133.156	132.010
1953	237.034	242.367	146.347	149.772
1954	253.786	255.942	161.456	163.114
1955	278.729	275.331	182.033	180.416
1956	306.308	300.942	206.091	202.669
1957	332.496	331.125	232.304	231.362
1958	346.888	339.729	252.205	247.039
1959	359.000	358.494	270.758	269.706
1960	365.836	360.229	287.120	282.558
1961	372.495	369.302	301.350	298.615
1962	387.378	392.568	322.017	325.927
1963	408.106	404.848	347.296	344.242
1964	423.729	412.183	366.487	356.529
1965	447.490	426.039	397.631	378.520
1966	464.943	452.290	436.585	424.708
1967	465.943	472.771	465.943	472.771
1968	474.719	482.048	502.783	510.444
1969	512.398	496.224	574.881	557.000
1970	516.891	511.489	623.932	617.217
1971	538.865	525.454	691.963	674.522
1972	547.543	548.983	763.736	763.193
1973	584.874	584.624	872.980	872.188
1974	602.680	623.288	1007.26	1041.87
1975	629.001	642.071	1171.62	1195.83
1976	678.559	667.056	1367.68	1343.51

```
                COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
                **********************************************

   ACTUAL AND PREDICTED VARIABLES...      RTOE            RTOES

   CORRELATION COEFFICIENT =     .9972
                 (SQUARED =     .9945

   ROOT-MEAN-SQUARED ERROR =     9.509

   MEAN ABSOLUTE ERROR =         7.368

   MEAN ERROR =                  2.412

   REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =         .9920

   THEIL'S INEQUALITY COEFFICIENT =                        .1064E-01

   FRACTION OF ERROR DUE TO BIAS =                         .6432E-01

   FRACTION OF ERROR DUE TO DIFFERENT VARIATION =          .4665E-02

   FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =       .9310


   ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
       FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
            COEFFICIENT FROM UNITY =                       .1082E-01
       FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =        .9249
```

TABLE 5.23

SIMULATION: NET OPERATING REVENUE

```
                    RNOR            RNORS           NOR             NORS

               .+.......................................................

 1952     .     11.3367         11.3367         51.2420         52.3877
 1953     .     23.7093         18.3764         55.6153         52.1907
 1954     .     28.1734         26.0179         57.9186         56.2601
 1955     .     35.1047         38.5028         62.8669         64.4840
 1956     .     39.6606         45.0268         67.8844         71.3062
 1957     .     38.7257         40.0967         70.6815         71.6234
 1958     .     45.4251         52.5841         76.6123         81.7790
 1959     .     85.4244         85.9301         105.846         106.899
 1960     .     106.123         111.731         117.728         122.290
 1961     .     128.324         131.516         132.307         135.042
 1962     .     150.103         144.913         148.978         145.068
 1963     .     155.463         158.721         155.681         158.735
 1964     .     173.597         185.143         176.285         186.243
 1965     .     189.666         211.117         195.330         214.440
 1966     .     203.280         215.933         208.462         220.339
 1967     .     236.093         229.264         236.093         229.264
 1968     .     254.263         246.934         255.695         248.034
 1969     .     261.681         277.855         267.209         285.090
 1970     .     316.638         322.040         312.704         319.420
 1971     .     342.386         355.797         326.824         344.266
 1972     .     381.355         379.915         361.680         362.224
 1973     .     393.876         394.126         402.225         403.017
 1974     .     393.885         373.277         432.866         398.249
 1975     .     411.416         398.345         494.246         470.038
 1976     .     427.482         438.985         536.248         560.419

               COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
               **********************************************

 ACTUAL AND PREDICTED VARIABLES...        RNOR           RNORS

 CORRELATION COEFFICIENT =     .9978
                (SQUARED =     .9956

 ROOT-MEAN-SQUARED ERROR =     9.509

 MEAN ABSOLUTE ERROR =         7.368

 MEAN ERROR =                 -2.412

 REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        1.003

 THEIL'S INEQUALITY COEFFICIENT =                       .1995E-01

 FRACTION OF ERROR DUE TO BIAS =                        .6432E-01

 FRACTION OF ERROR DUE TO DIFFERENT VARIATION =         .6427E-02

 FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =      .9292


 ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
      FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
           COEFFICIENT FROM UNITY =                     .2304E-02
      FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .9334
```

TABLE 5.24

SIMULATION: INCOME BEFORE UNDERLISTED ITEMS

	RIBUI	RIBUIS	IBUI	IBUIS
1952	14.1071	14.1071	53.4070	54.5528
1953	27.0685	21.7355	58.2172	54.7927
1954	33.1871	31.0316	61.8194	60.1609
1955	40.1407	43.5388	66.7967	68.4138
1956	46.6268	51.9930	73.4010	76.8228
1957	47.3214	48.6924	77.6972	78.6391
1958	53.5495	60.7085	83.4218	88.5885
1959	94.4804	94.9862	113.521	114.573
1960	114.256	119.863	124.704	129.266
1961	136.022	139.214	138.972	141.708
1962	158.585	153.395	156.411	152.501
1963	164.888	168.146	164.093	167.147
1964	183.969	195.515	185.710	195.669
1965	200.084	221.535	205.025	224.136
1966	215.709	228.362	220.460	232.337
1967	256.135	249.306	256.135	249.306
1968	275.333	268.004	277.618	269.957
1969	282.345	298.519	289.689	307.570
1970	338.517	343.919	337.290	344.005
1971	368.152	381.562	356.611	374.052
1972	407.935	406.494	393.883	394.427
1973	424.023	424.273	441.503	442.295
1974	424.769	404.161	477.496	442.879
1975	444.729	431.659	547.585	523.377
1976	465.374	476.877	601.475	625.646

COMPARISON OF ACTUAL AND PREDICTED TIME SERIES

```
ACTUAL AND PREDICTED VARIABLES...     RIBUI        RIBUIS

CORRELATION COEFFICIENT =      .9981
               (SQUARED =      .9962

ROOT-MEAN-SQUARED ERROR =      9.509

MEAN ABSOLUTE ERROR =          7.368

MEAN ERROR =                  -2.412

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =      1.004

THEIL'S INEQUALITY COEFFICIENT =                     .1853E-01

FRACTION OF ERROR DUE TO BIAS =                      .6432E-01

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =       .7573E-02

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =    .9281


ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                   .3254E-02
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =    .9324
```

TABLE 5.25

SIMULATION: INTEREST CHARGES

```
                RINT           RINTS          INT            INTS

          ••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••

1952    .       11.9627        11.9627        7.09169        7.03067
1953    .       14.0158        13.2297        8.65349        8.17537
1954    .       15.0479        14.8295        9.57331        9.45100
1955    .       15.6208        16.8003        10.2016        11.0087
1956    .       17.4872        19.2511        11.7658        12.9646
1957    .       19.7361        22.0880        13.7890        15.4332
1958    .       21.1877        25.2203        15.4046        18.3393
1959    .       24.7692        28.4477        18.6809        21.4020
1960    .       29.5005        32.1646        23.1530        25.2294
1961    .       32.9553        35.9004        26.6610        29.0288
1962    .       35.7106        39.6659        29.6853        32.9323
1963    .       38.1518        43.5371        32.4670        37.0196
1964    .       40.5309        47.3088        35.0555        40.9211
1965    .       42.4407        51.2363        37.7120        45.5215
1966    .       46.8253        55.4807        43.9694        52.0973
1967    .       52.7498        59.4718        52.7498        59.4718
1968    .       57.5662        63.1966        60.9693        66.9193
1969    .       64.3147        66.5513        72.1574        74.7022
1970    .       64.2015        69.3793        77.4968        83.7203
1971    .       67.9022        72.3865        87.1941        92.9221
1972    .       73.0459        75.1946        101.888        104.535
1973    .       78.2563        77.7948        116.805        116.060
1974    .       78.7935        80.3264        131.687        134.272
1975    .       86.3994        82.0034        160.934        152.728
1976    .       87.9584        83.3276        177.285        167.829

                 COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
                 **********************************************

      ACTUAL AND PREDICTED VARIABLES...      RINT           RINTS

      CORRELATION COEFFICIENT =     .9904
                   (SQUARED =     .9810

      ROOT-MEAN-SQUARED ERROR =     4.363

      MEAN ABSOLUTE ERROR =         3.624

      MEAN ERROR =                 -2.785

      REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        1.006

      THEIL'S INEQUALITY COEFFICIENT =                       .4193E-01

      FRACTION OF ERROR DUE TO BIAS =                        .4075

      FRACTION OF ERROR DUE TO DIFFERENT VARIATION =         .7144E-02

      FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =      .5854


      ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
           FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
                COEFFICIENT FROM UNITY =                     .9762E-03
           FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .5916
```

TABLE 5.26

SIMULATION: TAX BASE

```
                   RTAXBASE        RTAXBASES        TAXBASE         TAXBASES

          ..........................................................................

  1952      .      78.1278         80.8591          46.3154         47.5221
  1953      .      80.2766         75.4378          49.5637         46.6173
  1954      .      82.1237         79.5687          52.2461         50.7099
  1955      .      86.6587         87.6056          56.5950         57.4050
  1956      .      91.6070         94.8227          61.6352         63.8582
  1957      .      91.4715         90.4601          63.9082         63.2059
  1958      .      93.5524         96.6072          68.0173         70.2492
  1959      .      125.748         123.843          94.8396         93.1709
  1960      .      129.392         132.635          101.551         104.037
  1961      .      138.827         139.352          112.312         112.679
  1962      .      152.448         144.017          126.726         119.569
  1963      .      154.673         153.037          131.626         130.127
  1964      .      174.185         178.904          150.655         154.748
  1965      .      188.292         201.037          167.313         178.614
  1966      .      187.954         191.946          176.490         180.240
  1967      .      203.385         189.835          203.385         189.835
  1968      .      204.555         191.742          216.648         203.037
  1969      .      193.888         207.459          217.531         232.868
  1970      .      215.223         215.699          259.793         260.285
  1971      .      209.808         219.001          269.417         281.130
  1972      .      209.339         208.526          291.996         289.892
  1973      .      217.540         218.674          324.698         326.234
  1974      .      206.910         184.621          345.809         308.607
  1975      .      207.579         199.010          386.651         370.649
  1976      .      210.458         227.308          424.190         457.817

                 COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
                 **********************************************

     ACTUAL AND PREDICTED VARIABLES...      RTAXBASE        RTAXBASES

     CORRELATION COEFFICIENT =    .9863
                   (SQUARED =    .9729

     ROOT-MEAN-SQUARED ERROR =    8.534

     MEAN ABSOLUTE ERROR =        6.192

     MEAN ERROR =                 .8061E-01

     REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =         .9772

     THEIL'S INEQUALITY COEFFICIENT =                        .2577E-01

     FRACTION OF ERROR DUE TO BIAS =                         .8923E-04

     FRACTION OF ERROR DUE TO DIFFERENT VARIATION =          .3143E-02

     FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =       .9968


     ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
          FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
               COEFFICIENT FROM UNITY =                       .1909E-01
          FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =        .9808
```

TABLE 5.27

SIMULATION: INCOME TAX

```
                RINCTAX         RINCTAXS         INCTAX           INCTAXS

        ...........................................................................

1952    .       40.0546         40.0546          23.7450          23.5407
1953    .       36.7907         35.3852          22.7150          21.8665
1954    .       37.2484         36.3004          23.6970          23.1346
1955    .       37.6937         39.5846          24.6170          25.9385
1956    .       39.6628         42.7400          26.6860          28.7832
1957    .       39.8917         40.5649          27.8710          28.3433
1958    .       40.0495         43.4031          29.1180          31.5611
1959    .       59.0771         56.2081          44.5560          42.2870
1960    .       61.2092         60.3367          48.0390          47.3272
1961    .       67.5163         63.4950          54.6210          51.3415
1962    .       73.9119         65.6900          61.4410          54.5386
1963    .       74.4212         69.9383          63.3320          59.4686
1964    .       82.2745         82.1258          71.1600          71.0369
1965    .       88.3353         92.5545          78.4930          82.2313
1966    .       86.9470         88.2705          81.6440          82.8875
1967    .       91.5640         87.2758          91.5640          87.2758
1968    .       92.8339         88.1746          98.3220          93.3687
1969    .       92.5493         95.5801          103.835          107.286
1970    .       104.823         99.4628          126.531          120.022
1971    .       95.1054         101.019          122.126          129.677
1972    .       90.7349         96.0830          126.561          133.574
1973    .       100.140         100.864          149.468          150.477
1974    .       96.2140         84.8188          160.802          141.781
1975    .       93.3701         91.5992          173.918          170.600
1976    .       92.1312         104.933          185.696          211.343
                COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
                **********************************************

    ACTUAL AND PREDICTED VARIABLES...      RINCTAX        RINCTAXS

    CORRELATION COEFFICIENT =     .9790
                   (SQUARED =     .9585

    ROOT-MEAN-SQUARED ERROR =     4.895

    MEAN ABSOLUTE ERROR =         3.712

    MEAN ERROR =                  .3235

    REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =      .9706

    THEIL"S INEQUALITY COEFFICIENT =                     .3210E-01

    FRACTION OF ERROR DUE TO BIAS =                      .4368E-02

    FRACTION OF ERROR DUE TO DIFFERENT VARIATION =       .1802E-02

    FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =    .9938

    ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
         FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
              COEFFICIENT FROM UNITY =                   .2075E-01
         FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =    .9749
```

TABLE 5.28

SIMULATION: INCOME BEFORE EXTRAORDINARY ITEM

```
                  RIBEI          RIBEIS          IBEI          IBEIS

         ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

 1952    +      38.0731        40.8045        22.5704        23.9814
 1953    +      43.4859        40.0526        26.8487        24.7508
 1954    +      44.8753        43.2682        28.5491        27.5753
 1955    +      48.9650        48.0210        31.9780        31.4666
 1956    +      51.9442        52.0826        34.9492        35.0750
 1957    +      51.5799        49.8952        36.0372        34.8625
 1958    +      53.5029        53.2041        38.8993        38.6881
 1959    +      66.6713        67.6352        50.2836        50.8839
 1960    +      68.1825        72.2981        53.5119        56.7095
 1961    +      71.3105        75.8569        57.6905        61.3372
 1962    +      78.5360        78.3272        65.2849        65.0305
 1963    +      80.2519        83.0985        68.2939        70.6586
 1964    +      91.9109        96.7779        79.4946        83.7106
 1965    +      99.9570        108.483        88.8198        96.3828
 1966    +      101.007        103.675        94.8463        97.3527
 1967    +      111.821        102.559        111.821        102.559
 1968    +      111.722        103.568        118.326        109.669
 1969    +      101.339        111.879        113.696        125.581
 1970    +      110.400        116.236        133.262        140.263
 1971    +      114.702        117.982        147.291        151.453
 1972    +      118.604        112.443        165.435        156.318
 1973    +      117.400        117.809        175.230        175.757
 1974    +      110.696        99.8017        185.007        166.826
 1975    +      114.209        107.411        212.733        200.049
 1976    +      118.326        122.375        238.494        246.474

               COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
               **********************************************

   ACTUAL AND PREDICTED VARIABLES...     RIBEI          RIBEIS

   CORRELATION COEFFICIENT =     .9820
                (SQUARED =     .9642

   ROOT-MEAN-SQUARED ERROR =     5.305

   MEAN ABSOLUTE ERROR =         4.199

   MEAN ERROR =                 -.2429

   REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        .9828

   THEIL'S INEQUALITY COEFFICIENT =                       .2968E-01

   FRACTION OF ERROR DUE TO BIAS =                        .2097E-02

   FRACTION OF ERROR DUE TO DIFFERENT VARIATION =         .2075E-04

   FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =      .9979


   ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
        FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
             COEFFICIENT FROM UNITY =                     .8169E-02
        FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .9897
```

TABLE 5.29

SIMULATION: NET INCOME AFTER EXTRAORDINARY ITEM

```
                 RN19          RN19S         N19           N19S

         ................................................................

1952    .    38.0731       40.8045       22.5704       23.9814
1953    .    43.4859       40.0526       26.8487       24.7508
1954    .    44.8753       43.2682       28.5491       27.5753
1955    .    48.9650       48.0210       31.9780       31.4666
1956    .    51.9442       52.0826       34.9492       35.0750
1957    .    51.5799       49.8952       36.0372       34.8625
1958    .    53.5029       53.2041       38.8993       38.6881
1959    .    66.6713       67.6352       50.2836       50.8839
1960    .    68.1825       72.2981       53.5119       56.7095
1961    .    71.3105       75.8569       57.6905       61.3372
1962    .    78.5360       78.3272       65.2849       65.0305
1963    .    80.2519       83.0985       68.2939       70.6586
1964    .    91.9109       96.7779       79.4946       83.7106
1965    .    99.9570       108.483       88.8198       96.3828
1966    .    101.007       103.675       94.8463       97.3527
1967    .    111.821       102.559       111.821       102.559
1968    .    111.722       103.568       118.326       109.669
1969    .    101.339       111.879       113.696       125.581
1970    .    110.400       116.236       133.262       140.263
1971    .    114.702       117.982       147.291       151.453
1972    .    117.855       111.694       164.526       155.410
1973    .    121.542       121.951       180.626       181.153
1974    .    110.696       99.8017       185.007       166.826
1975    .    172.040       165.243       305.331       292.646
1976    .    118.326       122.375       238.494       246.474


             COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
             **********************************************

     ACTUAL AND PREDICTED VARIABLES...      RN19          RN19S

     CORRELATION COEFFICIENT =      .9866
                    (SQUARED =      .9733

     ROOT-MEAN-SQUARED ERROR =      5.305

     MEAN ABSOLUTE ERROR =          4.199

     MEAN ERROR =                  -.2429

     REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        1.003

     THEIL'S INEQUALITY COEFFICIENT =                       .2849E-01

     FRACTION OF ERROR DUE TO BIAS =                        .2097E-02

     FRACTION OF ERROR DUE TO DIFFERENT VARIATION =         .9621E-02

     FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =      .9883

     ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
          FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
               COEFFICIENT FROM UNITY =                     .2545E-03
          FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .9976
```

TABLE 5.30

SIMULATION: DIVIDENDS ON PREFERRED SHARES

```
                    RDIVPR          RDIVPRS          DIVPR           DIVPRS

             ..............................................................

1970    .          5.07836         4.14822         5.70655         4.66135
1971    .          8.08757         8.43597         9.34979         9.75257
1972    .          10.7956         11.1229         13.0796         13.4761
1973    .          10.7607         12.8067         14.0200         16.6858
1974    .          12.1754         13.8619         17.5944         20.0316
1975    .          15.5166         14.5231         24.8445         23.2538
1976    .          16.7580         14.9375         28.8470         25.7132



              COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
              **********************************************

ACTUAL AND PREDICTED VARIABLES...     RDIVPR        RDIVPRS

CORRELATION COEFFICIENT =     .9353
                (SQUARED =     .8748

ROOT-MEAN-SQUARED ERROR =     1.332

MEAN ABSOLUTE ERROR =         1.165

MEAN ERROR =                 -.9488E-01

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =     .9662

THEIL"S INEQUALITY COEFFICIENT =                    .5580E-01

FRACTION OF ERROR DUE TO BIAS =                     .5072E-02

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =      .8029E-02

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =   .9869

ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                  .8461E-02
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =   .9865
```

TABLE 5.31

SIMULATION: NET INCOME APPLICABLE TO COMMON SHARES AFTER EXTRAORDINARY ITEM

```
               RNI21          RNI21S         NI21           NI21S

         ...............................................................

1952     .     38.0731        43.3822        22.5704        25.9959
1953     .     43.4859        42.6303        26.8487        26.7474
1954     .     44.8753        45.8459        28.5491        29.5808
1955     .     48.9650        50.5987        31.9780        33.4781
1956     .     51.9442        54.6603        34.9492        37.1163
1957     .     51.5799        52.4729        36.0372        36.9664
1958     .     53.5029        55.7818        38.8993        40.8485
1959     .     66.6713        70.2128        50.2836        53.0683
1960     .     68.1825        74.8757        53.5119        58.9207
1961     .     71.3105        78.4346        57.6905        63.5693
1962     .     78.5360        80.9049        65.2849        67.2894
1963     .     80.2519        85.6762        68.2939        72.9592
1964     .     91.9109        99.3556        79.4946        86.0529
1965     .     99.9570        111.060        88.8198        98.7816
1966     .     101.007        106.253        94.8463        99.8409
1967     .     111.821        105.137        111.821        105.137
1968     .     111.722        106.145        118.326        112.351
1969     .     101.339        114.456        113.696        128.386
1970     .     105.321        112.088        127.556        135.601
1971     .     106.615        109.546        137.941        141.700
1972     .     107.059        100.571        151.447        141.933
1973     .     110.781        109.144        166.606        164.468
1974     .     98.5211        85.9398        167.412        146.795
1975     .     156.524        150.720        280.486        269.393
1976     .     101.568        107.438        209.647        220.761

            COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
            *********************************************

   ACTUAL AND PREDICTED VARIABLES...       RNI21          RNI21S

   CORRELATION COEFFICIENT =      .9790
                 (SQUARED =       .9585

   ROOT-MEAN-SQUARED ERROR =      6.225

   MEAN ABSOLUTE ERROR =          5.242

   MEAN ERROR =                  -2.072

   REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =        1.010

   THEIL'S INEQUALITY COEFFICIENT =                       .3470E-01

   FRACTION OF ERROR DUE TO BIAS =                        .1108

   FRACTION OF ERROR DUE TO DIFFERENT VARIATION =         .2004E-01

   FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =      .8691

   ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
        FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
             COEFFICIENT FROM UNITY =                     .1992E-02
        FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =      .8872
```

TABLE 5.32

SIMULATION: RETURN ON TOTAL CAPITAL

```
                        RETURN          RETURNS

              ******************************

  1952    .     .613317E-01      .641232E-01
  1953    .     .641500E-01      .566163E-01
  1954    .     .605377E-01      .570170E-01
  1955    .     .595566E-01      .568862E-01
  1956    .     .581518E-01      .549024E-01
  1957    .     .538762E-01      .506227E-01
  1958    .     .536845E-01      .510175E-01
  1959    .     .596272E-01      .589108E-01
  1960    .     .608372E-01      .605263E-01
  1961    .     .606585E-01      .619250E-01
  1962    .     .632201E-01      .619382E-01
  1963    .     .614612E-01      .642203E-01
  1964    .     .646862E-01      .703952E-01
  1965    .     .673961E-01      .751503E-01
  1966    .     .669902E-01      .713445E-01
  1967    .     .692127E-01      .701489E-01
  1968    .     .707556E-01      .709251E-01
  1969    .     .680915E-01      .753421E-01
  1970    .     .717795E-01      .781454E-01
  1971    .     .740507E-01      .774642E-01
  1972    .     .775510E-01      .759822E-01
  1973    .     .811396E-01      .806135E-01
  1974    .     .806066E-01      .713518E-01
  1975    .     .105693          .989791E-01
  1976    .     .866696E-01      .868170E-01

          COMPARISON OF ACTUAL AND PREDICTED TIME SERIES
          **********************************************

ACTUAL AND PREDICTED VARIABLES...     RETURN          RETURNS

CORRELATION COEFFICIENT =     .9264
                (SQUARED =     .8582

ROOT-MEAN-SQUARED ERROR =     .4357E-02

MEAN ABSOLUTE ERROR =         .3447E-02

MEAN ERROR =                  .1401E-04

REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =       .9179

THEIL"S INEQUALITY COEFFICIENT =                      .3157E-01

FRACTION OF ERROR DUE TO BIAS =                       .1034E-04

FRACTION OF ERROR DUE TO DIFFERENT VARIATION =        .5798E-03

FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =     .9994


ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)
     FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION
          COEFFICIENT FROM UNITY =                    .4621E-01
     FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =     .9538
```

CHAPTER VI

MODEL FORECASTS

In this chapter we produce forecasts for the period 1976-1983 with the real models of Chapter IV and the financial and income model of Chapter V. In these forecasts the main endogenous variables are forecasted under two alternative future price regimes for telephone services.

For the forecasts, we need besides the price of the outputs, forecasts for the price of factor inputs and the other exogenous variables of the real and financial model. The forecasts used for all these variables is discussed in the Appendix.

1. Forecast with Constant 1979 Nominal Prices of Services

In this simulation, we assume that the nominal 1979 estimated price of each telephone service does not change in the whole period. We have actual prices for 1978 and the 1979 prices are estimated using the allowed price increases of the last rate case. In this set of forecasts we have assumed a 10.5% price increase in Local Services, a 6.3% increase in the price of Toll services and a 15% increase in the price of Other Toll services. The lower increase in the price of Toll services reflects the fact that the authorized price increase affects only Intra-Bell Traffic.

In Tables 6.1 and 6.2 we present the demand forecasts with constant nominal prices. Actual values are used for demand for Toll services up to 1978 and for Local and Other Toll services up to 1977. Furthermore, the 1978 values for the latter two services were obtained using actual value of revenues and an estimate of the price index of these services. Thus, in effect the forecasts with the demand equations are done for the period 1978-1983. For purposes of comparison we present also the actual values of

TABLE 6.1

SIMULATED VALUES OF LOCAL AND TOLL SERVICES:

CONSTANT 1979 NOMINAL PRICES

	QLOC (y_{11})	PLOC	REVLOC
1972	579.800	1.08600	629.663
1973	625.500	1.11600	698.058
1974	679.400	1.14000	774.516
1975	734.300	1.19600	878.223
1976	779.700	1.27000	990.219
1977	820.500	1.35000	1107.68
1978	900.300	1.40300	1263.12
1979	954.387	1.49200	1423.94
1980	1021.38	1.49200	1523.89
1981	1092.95	1.49200	1630.68
1982	1168.67	1.49200	1743.65
1983	1247.72	1.49200	1861.60

	QTOL (y_{12})	PTOL	REVTOL
1972	333.273	1.11010	369.967
1973	385.109	1.13243	436.110
1974	440.917	1.14576	505.185
1975	500.113	1.18688	593.573
1976	536.231	1.25437	672.630
1977	579.637	1.28853	746.878
1978	644.922	1.35489	873.798
1979	697.868	1.36967	955.851
1980	774.268	1.36967	1060.49
1981	863.539	1.36967	1182.76
1982	963.212	1.36967	1319.28
1983	1069.60	1.36967	1465.00

TABLE 6.2

SIMULATED VALUES OF OTHER TOLL SERVICES: CONSTANT 1979 NOMINAL PRICES

	QOTH (y_2)	POTH	REVOTH
1972	90.9000	1.04570	95.0541
1973	108.000	1.07360	115.949
1974	119.800	1.10670	132.583
1975	138.500	1.15540	160.023
1976	156.700	1.24470	195.044
1977	171.300	1.30520	223.581
1978	202.200	1.37860	278.753
1979	202.631	1.50100	304.149
1980	235.799	1.50100	353.934
1981	275.311	1.50100	413.241
1982	320.211	1.50100	480.636
1983	368.004	1.50100	552.373

the variables for the period 1972-1976. From these demand forecasts we observe a substantial increase in Other Toll services, an increase in Toll services and a smaller increase in demand for Local services. The results for Other Toll services reflect the high price elasticity of the demand for Other Toll services and the forecasted decrease in real prices of the services. This decrease is implied by the constant nominal 1979 prices for the service and the increasing Consumer Price Index.

In Table 6.3 we present the forecast for labour, raw materials and capital requirements. Associated with the growth in output from Tables 6.1 to 6.2 and the evolution of input prices we forecast that labour requirements will be almost constant, and an increase in capital and material requirements.

With these forecasted values for the real variables and using the block triangular property of the complete model, we now forecast the financial variables.

In Tables 6.4, 6.5 and 6.6, we present the forecasts of the financial and income model for the variables in both real and in current dollars. In Table 6.4 we have the forecast of the (Average) Preferred Equity (APE), Total Equity (AEQUI), Long Term Debt (ADEBT), the sum of Equity and Debt (AVAK), Total Operating Revenues (TORE) and Total Operating Expenses (TOE). In Table 6.5 presented are the forecasts for Net Operating Revenues (NOR), Income Before Underlisted Items (IBUI), Interest Charges (INT), Tax Base (TAXBASES), Income Tax (INCTAX) and Income Before Extraodrinary Item (IBEI).

In Table 6.6 we have the forecasts for Net Income After Extraordinary Item (NIG), Dividends on Preferred Shares (DIVPR), Net

TABLE 6.3

SIMULATED VALUES OF LABOR, MATERIAL AND CAPITAL REQUIREMENTS: CONSTANT 1979 NOMINAL PRICES

		L	M	K
1972	.	53.2076	168.967	3341.96
1973	.	58.1048	177.036	3498.44
1974	.	64.0903	183.952	3693.07
1975	.	64.0880	189.444	3760.41
1976	.	65.1868	195.544	3907.90
1977	.	63.2571	197.440	4030.76
1978	.	66.4313	206.887	4185.11
1979	.	66.3384	209.311	4225.20
1980	.	66.3121	215.802	4381.77
1981	.	66.3493	222.938	4544.68
1982	.	66.1344	230.126	4708.38
1983	.	65.6065	236.680	4856.32

TABLE 6.4

FINANCIAL MODEL: SIMULATION WITH CONSTANT 1979 NOMINAL PRICES

	RAPES	APES	RTORES	TORES
1976	186.718	342.814	1105.83	1903.57
1977	187.883	386.430	1147.95	2134.00
1978	188.613	422.527	1248.22	2477.67
1979	189.070	460.806	1301.07	2751.04
1980	189.357	501.595	1339.32	3011.52
1981	189.537	545.199	1385.08	3304.99
1982	189.649	591.907	1434.93	3627.97
1983	189.720	642.014	1482.82	3968.47

	RAEQUIS	AEQUIS	RAVAKS	AVAKS
1976	1321.29	2425.89	2612.90	4797.29
1977	1539.86	3167.12	3030.14	6232.27
1978	1592.81	3568.17	3131.27	7014.61
1979	1606.56	3915.54	3157.54	7695.61
1980	1660.27	4397.95	3260.12	8635.85
1981	1716.15	4936.47	3366.85	9684.69
1982	1772.30	5531.46	3474.11	10842.9
1983	1823.05	6169.22	3571.03	12084.4

	RADEBTS	ADEBTS	RTOES	TOES
1976	1291.62	2371.41	678.559	1367.68
1977	1490.28	3065.15	674.192	1496.24
1978	1538.46	3446.43	704.230	1706.72
1979	1550.98	3780.07	713.119	1883.13
1980	1599.85	4237.91	733.246	2104.80
1981	1650.70	4748.22	754.741	2352.81
1982	1701.80	5311.43	775.981	2624.28
1983	1747.98	5915.20	795.118	2914.70

TABLE 6.5

FINANCIAL MODEL: SIMULATION WITH CONSTANT 1979 NOMINAL PRICES

	RNORS	NORS	RIBUIS	IBUIS
1976	427.275	535.892	465.167	601.119
1977	473.763	637.761	502.253	690.723
1978	543.985	770.951	572.595	827.739
1979	587.949	867.911	618.509	932.530
1980	606.070	906.724	639.005	980.779
1981	630.343	952.177	665.768	1036.71
1982	658.952	1003.69	697.015	1099.93
1983	687.702	1053.77	728.673	1163.43

	RINTS	INTS	RTAXBASES	TAXBASES
1976	87.9584	177.285	210.281	423.833
1977	92.3854	205.032	218.848	485.691
1978	96.7124	234.385	244.831	593.354
1979	100.489	265.362	252.648	667.168
1980	104.283	299.346	237.390	681.433
1981	108.115	337.035	224.442	699.670
1982	111.984	378.716	213.258	721.216
1983	115.818	424.560	201.560	738.867

	RINCTAXS	INCTAXS	RIBEIS	IBEIS
1976	92.1312	185.696	118.149	238.137
1977	98.6165	218.861	120.231	266.830
1978	112.053	271.565	132.777	321.790
1979	116.319	307.164	136.329	360.004
1980	109.413	314.073	127.977	367.360
1981	103.451	322.495	120.991	377.176
1982	98.2486	332.265	115.010	388.950
1983	92.7693	340.068	108.791	398.799

TABLE 6.6

FINANCIAL MODEL: SIMULATION WITH CONSTANT 1979 NOMINAL PRICES

	RN19S	N19S	RDIVPRS	DIVPRS
1976	118.149	238.137	15.3336	26.3951
1977	130.990	286.830	15.4454	28.7123
1978	142.853	341.790	15.5154	30.7976
1979	145.788	380.004	15.5593	32.8994
1980	136.871	387.360	15.5868	35.0478
1981	129.373	397.176	15.6040	37.2332
1982	122.920	408.950	15.6148	39.4793
1983	116.264	418.799	15.6216	41.8081

	RNI21S	NI21S	RETURNS
1976	102.816	211.742	.865952E-01
1977	115.545	258.118	.789218E-01
1978	127.338	310.992	.821393E-01
1979	130.229	347.105	.838617E-01
1980	121.284	352.312	.795180E-01
1981	113.769	359.942	.758115E-01
1982	107.305	369.471	.726436E-01
1983	100.642	376.991	.697890E-01

Income Applicable to Common Shares After Extraordinary Item (NI21) and the Percent Return on Average Total Capital (RETURNS), where

$$\text{RETURNS} = \frac{\text{INTS} + \text{N19S}}{\text{AVAKS}}$$

As expected, the rate of return on total capital achieves a peak in 1979 and then it decreases towards the end of the sample period but achieving a value close to the one in 1976.

It should be noted that for the forecast of RAVAKS we used a different relationship than the one described and used in Chapter V, the reason being that the forecasted values of RAVAKS obtained with the relationship of Chapter V were unreasonably low, and increasingly so, due to the unstable autoregressive process involved. Thus, in the forecasts we used the simple relationship:

$$\text{RAVAK}_t = d_o + d_1 \; K_t$$

which when estimated gave

$$\text{RAVAK}_t = \underset{(4.3)}{389.256} + \underset{(17.3)}{.6552} \; . \; K_Y$$

$$R^2 = .929$$

2. <u>Forecast with Constant 1979 Real Prices of Services</u>

In this set of forecasts we keep the price of telephone services constant at their 1979 level.

In Tables 6.7 and 6.8 we present the demand forecasts. If we compare them with the ones from Tables 6.1 and 6.2, we see that these values are lower. This is caused by the higher real prices used in this forecast. Thus, via the price elasticities we obtain a lower estimate for the quantity demanded. In Table

TABLE 6.7

SIMULATED VALUES OF LOCAL AND TOLL SERVICES: CONSTANT 1979 REAL PRICES

	QLOC	PLOC	REVLOC
1972	579.800	1.08600	629.663
1973	625.500	1.11600	698.058
1974	679.400	1.14000	774.516
1975	734.300	1.19600	878.223
1976	779.700	1.27000	990.219
1977	820.500	1.35000	1107.68
1978	900.300	1.40300	1263.12
1979	954.387	1.49200	1423.94
1980	1011.67	1.58663	1605.15
1981	1072.64	1.68370	1806.00
1982	1136.69	1.78404	2027.90
1983	1202.92	1.88845	2271.65

	QTOL	PTOL	REVTOL
1972	333.273	1.11010	369.967
1973	385.109	1.13243	436.110
1974	440.917	1.14576	505.185
1975	500.113	1.18688	593.573
1976	536.231	1.25437	672.630
1977	579.637	1.28853	746.878
1978	644.922	1.35489	873.798
1979	697.868	1.36967	955.851
1980	710.346	1.45654	1034.65
1981	728.991	1.54565	1126.77
1982	749.785	1.63776	1227.97
1983	768.815	1.73362	1332.83

TABLE 6.8

SIMULATED VALUES OF OTHER TOLL SERVICES: CONSTANT 1979 REAL PRICES

	QOTH	POTH	REVOTH
1972	90.9000	1.04570	95.0541
1973	108.000	1.07360	115.949
1974	119.800	1.10670	132.583
1975	138.500	1.15540	160.023
1976	156.700	1.24470	195.044
1977	171.300	1.30520	223.581
1978	202.200	1.37860	278.753
1979	202.631	1.50100	304.149
1980	212.136	1.59620	338.611
1981	223.637	1.69386	378.810
1982	235.464	1.79480	422.611
1983	245.392	1.89984	466.207

6.9 we present the forecasts for the factor inputs. As a consequence of lower output values of this set of simulations, we end up with smaller factor requirements.

The forecasts for the financial and income model under a regime of real 1979 prices are presented in Tables 6.10, 6.11 and 6.12, while in Table 6.13, we have the exogenous forecasted variables for this financial model. Under this regime, the rate of return increases to reach 10.6% in 1983, after a forecasted return in 1977 of 7.9%

TABLE 6.9

SIMULATED VALUES OF LABOR, MATERIAL AND CAPITAL REQUIREMENTS: CONSTANT 1979 REAL PRICES

		L	M	K
1972	.	53.2076	168.967	3341.96
1973	.	58.1048	177.036	3498.44
1974	.	64.0903	183.952	3693.07
1975	.	64.0880	189.444	3760.41
1976	.	65.1868	195.544	3907.90
1977	.	63.2571	197.440	4030.76
1978	.	66.4313	206.887	4185.11
1979	.	66.3384	209.311	4225.20
1980	.	63.3370	206.753	4225.89
1981	.	60.5956	204.891	4231.45
1982	.	57.7805	203.030	4234.80
1983	.	54.8675	200.612	4222.09

TABLE 6.10

FINANCIAL MODEL: SIMULATION WITH CONSTANT 1979 REAL PRICES

	RAPES	APES	RTORES	TORES
1976	186.718	342.814	1105.83	1903.57
1977	187.883	386.430	1147.97	2134.04
1978	188.613	422.527	1248.22	2477.67
1979	189.070	460.806	1301.07	2751.04
1980	189.357	501.595	1357.14	3051.60
1981	189.537	545.199	1420.66	3389.88
1982	189.649	591.907	1488.29	3762.88
1983	189.720	642.014	1554.46	4160.19

	RAEQUIS	AEQUIS	RAVAKS	AVAKS
1976	1321.29	2425.89	2612.90	4797.29
1977	1539.86	3167.12	3030.14	6232.27
1978	1592.81	3568.17	3131.27	7014.61
1979	1606.56	3915.54	3157.54	7695.61
1980	1606.80	4256.30	3157.99	8365.32
1981	1608.70	4627.40	3161.63	9094.37
1982	1609.85	5024.44	3163.82	9874.49
1983	1605.49	5433.00	3155.50	10678.2

	RADEBTS	ADEBTS	RTOES	TOES
1976	1291.62	2371.41	678.559	1367.68
1977	1490.28	3065.15	674.192	1496.24
1978	1538.46	3446.43	704.230	1706.72
1979	1550.98	3780.07	713.120	1883.13
1980	1551.19	4109.01	708.103	2031.76
1981	1552.93	4466.97	704.578	2194.51
1982	1553.97	4850.04	700.686	2366.44
1983	1550.01	5245.24	694.951	2542.80

TABLE 6.11

FINANCIAL MODEL: SIMULATION WITH CONSTANT 1979 REAL PRICES

	RNORS	NORS	RIBUIS	IBUIS
1976	427.275	535.892	465.167	601.119
1977	473.781	637.796	502.271	690.757
1978	543.985	770.951	572.595	827.739
1979	587.948	867.909	618.509	932.529
1980	649.038	1019.84	681.972	1093.90
1981	716.083	1195.36	751.508	1279.89
1982	787.604	1396.44	825.668	1492.67
1983	859.505	1617.39	900.477	1727.04

	RINTS	INTS	RTAXBASES	TAXBASES
1976	87.9584	177.285	210.281	423.833
1977	92.3854	205.032	218.863	485.725
1978	96.7124	234.385	244.831	593.354
1979	100.489	265.362	252.648	667.166
1980	103.646	297.393	277.596	796.506
1981	106.306	331.105	304.621	948.788
1982	108.540	366.575	333.430	1126.10
1983	110.354	403.782	361.649	1323.26

	RINCTAXS	INCTAXS	RIBEIS	IBEIS
1976	92.1312	185.696	118.149	238.137
1977	98.6238	218.877	120.239	266.848
1978	112.053	271.565	132.777	321.790
1979	116.319	307.163	136.329	360.004
1980	128.358	368.298	149.238	428.208
1981	141.231	439.884	163.391	508.905
1982	154.872	523.054	178.557	603.046
1983	168.202	615.445	193.447	707.816

TABLE 6.12

FINANCIAL MODEL: SIMILATION WITH CONSTANT 1979 REAL PRICES

	RN19S	N19S	RDIVPRS	DIVPRS
1976	118.149	238.137	15.3336	26.3951
1977	130.998	286.848	15.4454	28.7123
1978	142.853	341.790	15.5154	30.7976
1979	145.788	380.004	15.5593	32.8994
1980	158.132	448.208	15.5868	35.0478
1981	171.773	528.905	15.6040	37.2332
1982	186.468	623.046	15.6148	39.4793
1983	200.920	727.816	15.6216	41.8081

	RNI21S	NI21S	RETURNS
1976	102.816	211.742	.865952E-01
1977	115.553	258.136	.789247E-01
1978	127.338	310.992	.821393E-01
1979	130.228	347.104	.838616E-01
1980	142.546	413.161	.891301E-01
1981	156.169	491.672	.945651E-01
1982	170.853	583.567	.100220
1983	185.298	686.008	.105972

TABLE 6.13

FINANCIAL MODEL: EXOGENOUS VARIABLES FOR FORECAST

	CPIX	AIBARE	EXTRIX	PK
1976	1.72139	.648175	.181899E-11	1.83600
1977	1.85896	.612739	20.0000	2.05676
1978	1.98497	.612739	20.0000	2.24018
1979	2.11445	.612739	20.0000	2.43722
1980	2.24855	.612739	20.0000	2.64894
1981	2.38613	.612739	20.0000	2.87648
1982	2.52832	.612739	20.0000	3.12106
1983	2.67630	.612739	20.0000	3.38401

	DECX	MISCUR	UNCOL
1976	.863555E-01	54.8000	8.80481
1977	.877334E-01	64.9000	9.00000
1978	.891112E-01	71.0000	9.00000
1979	.904891E-01	77.1000	10.0000
1980	.918670E-01	83.2000	10.0000
1981	.932448E-01	89.3000	11.0000
1982	.946227E-01	95.4000	11.0000
1983	.960006E-01	101.500	12.0000

4

APPENDIX

FORECASTING OF THE EXOGENEOUS VARIABLES

In order to perform the simulation, we had to forecast a number of exogeneous variables. The forecasts for Gross Provincial Products at constant prices for the period 1978-1983 were obtained from the Bureau de la Statistique de Québec (1979), and for Ontario from Sawyer, J.A. et al (1978). For the Population of Ontario, we also used Sawyer, J.A. et al (1978), while for the Population of Québec we used Office de Planification et de Développement du Québec (1977).

For the other variables, we used mixed autoregressive integrated moving average (ARIMA) processes. Following the methodology of Box and Jenkins (1971), the processes were first identified using sample autocorrelations and partial autocorrelations, then estimated and finally forecasted (1977-86). The processes adopted for each variable are indicated in Table A with the sample period used for the estimation of each of them. Note that, for some of the series, which did not exhibit a homogeneous behaviour, we had to use shorter estimation periods.

TABLE A

Variables	Logarithm Taken	Process[1]	Estimation Period
Canadian Consumer Price Index (CPI)	x	ARIMA (0,1,0)	1952-76
Depreciation Rate (DEP)		ARIMA (0,1,0)	1952-76
Miscellaneous Revenue (MISCUR)		ARIMA (0,1,0)	1971-76
Other Income (OTHI)	x	ARIMA (2,1,0)	1952-76
Price of Capital Goods (PK)	x	ARIMA (2,1,0)	1952-76
Allowed Price of Capital Services (SK)	x	ARIMA (2,1,0)	1952-76
Technology Indicator (TC)	x	ARIMA (1,1,0)	1952-76
Rental Price of Capital (v)	x	ARIMA (1,1,0)	1952-76
Wage Rate (w)	x	ARIMA (1,1,0)	1965-76
Price of Raw Materials (m)	x	ARIMA (1,1,0)	1952-76

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[1] In the description ARIMA (p,d,q), p indicates the degree of autoregressive part, d the degree of differentiating and q the degree of the moving average part.

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