HERMES PROJECT

REPORT ON THE PRELIMINARY PHASE

PREPARED FOR THE

NATIONAL TELECOMMUNICATIONS BRANCH
DEPARTMENT OF COMMUNICATIONS

Dec 1971

BY

LABORATOIRE D'ECONOMETRIE
UNIVERSITE LAVAL
AND
SORES INC., MONTREAL

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SUMMARY

During the preliminary phase of the HERMES project, a model was developed for planning interregional telecommunications network capacity expansions from the initial state at minimum capital cost levels given certain configurations of demand changes and other constraints. The present version of this model (called HERMES I) is operational, based on simplified facility expansion cost functions and on abstracted representation of the Canadian interregional network.

The model consist of two parts. The first part (consisting of the CADUCEE software) analyses the telecommunications network, the related capacity expansion cost functions and the specified demand increases, and identifies admissible facility assignment chains and the bounds on capacity expansions. Thus this part of the model permits to eliminate all so called dominated chains, which cannot be part of the solution under any circumstances and also the dominated capacity expansion increments. The most important procedure of this part of the model is a generalization of the shortest chain algorithm for non-directed networks, based on Bellman's principle of optimality.

The second part of the model (consisting of the TRANCHE software) identifies the minimum cost expansion program, using as its main procedure a mixed integer linear programming algorithm of the branch-and-bound type.

The principal output of the model consist of the values of capacity expansion activities constituting the minimum cost capacity expansion program.

To use the full potentiel of the models of the HERMES series, it is recommanded that a demand model be concurrently developped which would provide forecasts of different patterns of increases of demand for telecommunications facilities in Canada. Although there is no established methodology applicable to the problem a few approaches can be suggested. It is expected that one of these approaches would play the main role in the procedure adopted, namely a model based on the structure of activity analysis.

The HERMES 1 is the first one of a series, and future developments will involve the following steps:

- Improvements of the methodology already developped, including the software and its applications to more detailed networks.

- The introduction of additional factors affecting the planning of network expansions

- The linking of the network capacity expansion models with demand models

- The introduction of dynamic considerations: capacity expansion planning over time.

As a guide to the reader, the following remarks concerning the contents of the various parts of the report may be useful.

- Chapters 1, 2, 5, and 6 will provide a general overview of the HERMES I model, as well as an exposé on possible approaches for a demand model and a description of recommanded extensions and further developments for the models of the HERMES series. These 4 chapters give a fairly complete presentation, in semi-technical terms, of the work carried-out and the results obtained during the preliminary phase of the project.

- Chapter 4 presents some of the results that were obtained while using the model to solve specific problems submitted by D.O.C. personnel.

- Chapter 3 is a very detailed and technical presentation of the HERMES I model. Combined with the reduced example of the appendix, which takes the reader through almost every step of the calculations, it is intended to provide a complete understanding of the various mechanisms used in the model.

FOREWORD

The overall objectives of the Preliminary Phase of the HERMES project were:

a) To develop a mathematical model for planning interregional telecommunications network capacity expansions from the initial state at minimum capital cost levels, given certain hypothetical configurations of demand changes and other constraints.

b) To develop a methodology for realistic demand forecasts for interregional telecommunications.

c) To carry out preliminary work on methodological development of an integrated approach to demand forecasting and/or simulation and the planning of interregional network capacity changes.

According to the terms of reference the preliminary phase was to be divided in two parts. The first part was to lead to an interim report containing the following:

i) Detailed formulation of the first version of the model.

ii) Operational but probably inefficient software relating to (i).

iii) Two sets of data recommendations:

- data needed to construct the model;
- data needed to operate the model.

In turn, the second part was to lead to a Final Report, due on January 31, 1972 and containing the following:

i) Translation of the reduced model into an operational model; detailed formulation of the model; its functioning.

ii) Preliminary results of the model, forecasts and simulation.

iii) Conclusions: recommendations concerning subsequent phases.

In concurrence with the National Telecommunications Branch, it was decided to consolidate the contents of both documents into the present report, which is submitted at an earlier date than was originally anticipated. The scope of the interim report, submitted on October 26, 1971, was accordingly reduced to a very brief activity report.

The HERMES project is a joint enterprise carried out by:

- The National Telecommunications Branch of the Department of Communications
- Economics Department, Carleton University
- Laboratoire d'économétrie, Université Laval
- Sorès Inc., Montréal.

The following persons collaborated in the activities leading to this report:

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1. PROLEGOMENON

Project HERMES is a joint enterprise carried out by a quadripartite team, whose composition is detailed in the Foreword. From the beginning it has been considered that such an arrangement was a necessary condition of the success of the project. It has to be realized however that there are inevitable delays and costs involved in coordinating the working of such a team. It will also be noted that each member's contribution carries a special flavour. And this is how it ought to be. The present text, for instance, is clearly different from what it would have been, had it been written as an internal departmental document, as a study ordered from a consulting firm working alone, or again, as a straight academic exercise.

A number of serious difficulties are inherent in the very nature of the project. Telecommunications are in a state of rapid expansion and undergoing profound changes. These will continue at an increasing rate. Trends observed in the past are not, by themselves, a very reliable guide. The economics of the telecommunications are rendered particularly difficult by the special characteristics of the "products" and of the "processes" involved. On the one hand, these "products" are highly individualized and defy any simple measurement procedures. On the other hand, the "processes" are more often than not characterized by the importance of joint costs, of decreasing average costs, of substitution possibilities and of often very wide ranges of technical alternatives. And then, there are particular difficulties in arriving at operational definitions of both the cost and the demand concepts. All this is further complicated by the very special institutional and corporate structure of the telecommunications industry.

On the methodological side, precedents are few and the ones that exist concern certain aspects only of the problems with which we are concerned in this study. The conceptual effort concerned with the difficulties mentioned in the preceding paragraph is thus inevitably to a large extent exploratory.

The challenge of the HERMES project lies in the necessity to combine some quite fundamental and advanced concepts of economic theory and of mathematical programming with the very down-to-earth realities of the Canadian interregional telecommunications network. The results are not of universal applicability, at least not in the form in which they are being worked out. In a sense, the HERMES project is "custom made". However, one can envisage its modifications and extensions which may go a long way in increasing the transferability of these results. The basic methodology and the overall conceptualisation of the relevant software are quite general. Applications to more detailed studies of regional networks or of specialized networks within Canada are first to come to mind. Adaptations to the special characteristics of the telecommunications planning problems in other countries are certainly possible.

It was decided from the beginning that the HERMES project would proceed by stages. The results contained in this report are based on real data, but severely simplified or "rationalized". Thus, they are to be considered to some extent as experimental. The HERMES model itself is in a fairly advanced stage of development, and thus can efficiently handle a much more detailed and realistic data base. A large part of the work of future phases will be devoted to developing such a data base. Proceeding by stages means that the models developed in the course of the project must be perfectible, that provisions must be made for accommodating and taking advantage of new data, for increasing the scope and the complexity of the relations these models contain and for improving their performance. This perfectibility requirement evidently imposes certain additional burdens on the conceptualization of these models and on the development of computing and other procedures necessary to operate them.

It was decided not to approach telecommunications carriers for additional information, statistical and other, until the project is in a more advanced stage of development. It is clear however that their cooperation will be essential if the project is to yield truly operational results.

Some government departments have been approached and are being kept informed of the progress of the project. No input from these other departments has been used to obtain the results reported on here. In the future stages of the project, however, it is expected that such inputs will become important.

2. NATURE OF THE MODEL

2.1 General Remarks

Although based on real, though of course simplified and aggregated, data and although fully operational within its terms of reference, the HERMES I model is just the first one of a series. The development of the methodology, including the corresponding software, was done to a very large extent and at a considerable additional effort and expense in view of future refinements and extensions. Already the description of the model and its functioning contained in this and in the following chapters goes a good deal beyond the fairly narrow objectives of the Preliminary Phase. It is not until Chapter 6 of this Report however that the full potentialities of the model in view of future developments are discussed at length.

The HERMES I model is formulated with reference to the Canadian telecommunications network. However, the methodology developed is perfectly general and applicable to the telecommunications network of any country, or group of countries. It will be noted that the methodology in question covers all the essential steps from the conceptualization and economic analysis, through mathematical formulation of the model to the development of the relevant software. It is evident however that the implantation of this methodology in a foreign context would still require a considerable effort of data collection and data integration which could not be undertaken except in close collaboration with the specialists of the country, or the countries, concerned.

The basic function of the HERMES I model is to find optimal telecommunications capacity expansion programs, given specified increases in demand. Demand for telecommunications facilities, and changes in this demand, are specified in a manner exogenous to the model. In subsequent phases of the project a series of HORAE models will be developed whose function will be to provide these specifications and thus to work in tandem with models of the HERMES series.

For the purposes of the HERMES I model demand is defined as the number of circuits or channels deemed necessary at a given level of quality of service between any two demand points. It will be noted that it is thus the demand for facilities (transmission facilities only in the case of HERMES I, transmission and nodal facilities in subsequent versions) that is taken into account here, and not demand for communications as such.

By optimal capacity expansion program is meant a program (which may be not unique) which minimizes the total capital cost of capacity expansion. In subsequent models of the HERMES series, operating costs will undoubtedly also be introduced.

The model is formulated with reference to the Canadian interregional telecommunications network, as established by the Department of Communications officers. Evidently, the real network with all its details could not be incorporated into the model and a simplified version had to be used. The simplified version resembles the real network with respect to its geographic pattern. It includes 98 demand points, of which 19 are portals to foreign nodes.

It will be understood that, as a general rule, a number of alternative facility assignment chains are available to satisfy the demand existing between any two points. The number of these chains may, in fact, be extremely large. Thus, preliminary calculations indicate that, already in the simplified network referred to above, the number of possible facility assignment chains between Montreal and Vancouver is well over 30,000. To evaluate and compare such a number of alternatives within the context of mixed integer programming is well beyond the possibilities of even the most powerful computers. The first major mathematical problem to be solved was thus to find a procedure to identify the facility assignment chains which are candidates for inclusion in an optimal capacity expansion program - these are called admissible chains - and to eliminate the chains which are dominated, in the sense that although possible, they are so inefficient that they could never appear in any reasonable capacity expansion program and certainly not in the optimal program. In this way, the problems submitted to the model could be reduced to manageable dimensions without sacrificing any relevant information and, also, maintaining the original level of detail. Such a procedure was formulated, and made operational. It is described in Sections 2.3 and 3.3. below. Apart from supplying intermediate results essential for the functioning of the model, this procedure yields certain valuable by-products of interest in the overall appreciations of the telecommunications network concerned and in the evaluation of its proposed extensions.

The other mathematical difficulty was the importance of decreasing costs and of joint costs - it is the capacity expansion costs which are meant here. As it is well known, the presence of such costs complicates very considerably the search for optimal solutions and, in addition, makes sensitivity analyses of the results more laborious. It is, of course, the indivisibilities characteristic of telecommunication facilities which are responsible for these difficulties. The techniques employed to overcome these difficulties are those of mixed integer linear programming. They have been made operational and adapted to the special needs of the HERMES project. They remain however more complex and laborious than the technique of continuous programming applicable in other contexts. The use of mixed integer programming to handle real life problems is still a fairly recent development.

HERMES I is an optimizing model. However, its searching for optimum solutions is always done within a framework of hypotheses: hypotheses concerning the demand pattern, concerning capacity expansion costs and, of course, concerning the structure of the telecommunications network itself, including a given state of network capacity. It is not designed to supply "once for all" solutions. Its proper role is that of a simulating device supplying conditional solutions to certain ranges of problems and tracing the likely consequences of alternative hypotheses.

It is essential to realize that the presence of decreasing costs and of joint costs makes for the successive solutions of the optimizing procedure being non-additive. This may be illustrated as follows. Supposing a certain demand increase for a given pair of points is specified and the corresponding minimum cost capacity expansion program identified. Then another pair of points is taken up and again a minimum cost capacity expansion program is identified, designed to satisfy the demand increase specified for the second pair of points. Now, if the same demand increases for both these pairs of points are considered simultaneously, the corresponding minimum cost capacity expansion program will not, in general, be the sum of the two capacity expansion programs found previously and the cost of this overall program will, in general, be less than the sum of the costs of the two original programs. This is so because the two demand increases may well be assigned to share certain facilities and thus to take advantage of decreasing average costs. This consideration is of paramount importance in the planning of communication networks and is reflected in certain fundamental characteristics of the HERMES I model.

The limitations of the HERMES I model are inherent in its formulation and will become apparent as its description unfolds. Although it is elaborate and detailed by the usual standards, it is no more than a simplified version of just a segment of the relevant reality. Its principal limitation is perhaps its static character. To refine the degree of detail and to increase the number of variables is time-consuming and expensive, but does not call for major revisions of the methodology originally developed. To introduce dynamic considerations, so that capacity expansion programs spread over time can be taken into account and optimized, calls for some major conceptual revisions, a fairly fundamental reformulation of the methodology, and a new conceptualization of the relevant software. It is evident, however, that such a dynamization will have to appear somewhere along the line of development of the HERMES series of models, given the extremely rapid growth of demand for telecommunications and given the particular nature of capacity expansion cost functions for telecommunications.

To sum up, the model HERMES I is certainly perfectible in the sense of having a considerable potential for future refinements and extensions, including the construction of dynamic versions. However, its present version already is fully operational, based on real, though simplified, data and supported by a system of software which, although functioning and reasonably efficient, is also capable of considerable further development.

The remainder of this and the following chapter deal with the HERMES I model, leaving to Chapters 5 and 6 the question of its possible extensions and future developments.

2.2 Structure of the model

The HERMES I model consists of two parts. The first part serves to analyse the telecommunications network, the related capacity expansion cost functions and the specified demand increases. Its principal output is the identification of admissible facility assignment chains and of the upper bounds on capacity expansions. It also yields certain by-products of interest. The second part serves to identify minimum cost capacity expansion programs. It uses as its principal input the principal output of the first part.

The role of the first part of the HERMES I model may be briefly described as follows.

The national interregional telecommunications system is represented by a non-directed network in which there can be at most one link between any two nodes. In due course the present simplified network will be replaced by a more complicated one, allowing for more than one link between any two geographic nodes and for more than one nodal facility at any given geographic point, to take account of different facility systems, of the existence of distinct telecommunications carriers, and of other considerations. However, the device of dummy nodes and links related to them allows the representation of this more complicated network by a so-called "enlarged network" whose formal structure is strictly analogous to the network discussed here, so that the methodology presented below applies equally well to the more complicated network.

Capital capacity expansion cost functions are defined for every link (and node: geographic, or a dummy node) in the interval between the existing capacity (assumed to be equal to the present level of utilization) and an upper bound on capacity expansion, set at a maximum of 30 service channels , although in the actual applications of the model, the upper bounds are set at lower levels, depending on the specified pattern of demand increases. In fact, successive revisions of the upper bounds on capacity expansions play an important role in the first part of the HERMES I model and contribute significantly to its efficiency.

The second part of the HERMES I model uses a mixed integer linear programming formulation (TRANCHE) to choose the minimum cost capacity expansion program. This kind of formulation, instead of a much easier continuous programming formulation, is made necessary by the presence of decreasing average costs (due to the fact that the total cost functions are step functions) and to the presence of joint costs: demand concerning two or more different pairs of demand points may be accommodated by the same transmission, or nodal, facility.

In the mixed integer linear programming formulation, every facility assignment chain between every pair of points considered in any given problem gives rise to an activity, i.e. to a variable (facility assignment activity) and all possible capacity expansion increments also give rise to activities which, in addition, must be represented by integer variables. Even in a network of moderate size the number of possible facility assignment chains and of possible capacity expansion increments may be extremely large. The handling of tens of thousands of variables in mixed integer linear programming problems is not a practical proposition.

It is therefore essential to eliminate in advance facility assignment chains which cannot, under any of the circumstances considered, form part of a minimum cost capacity expansion program. It is also important to eliminate in advance the capacity expansion increments which cannot, under any of the circumstances considered, form part of a minimum cost capacity expansion program designed to satisfy a specified pattern of demand increases. These are the capacity expansion increments which lie beyond the upper bounds on capacity expansions. Such facility assignment chains and such capacity expansion increments are called dominated. Facility assignment chains and capacity expansion increments which are not dominated are called admissible.

The main function of the first part of the HERMES I model is to identify the admissible facility assignment chains and the admissible capacity expansion increments (this in fact means identifying the upper bounds on capacity expansions), and thus to make the second part of the model operational, apart from yielding certain interesting by-products. This is done with the help of the computer program CADUCEE with its principal subroutines BØRNE and DØMINØ.

The concept of the lower and upper bounds on incremental capacity expansion costs plays an essential role in the first part of the HERMES I model. These bounds depend on the initial state of the network (the installed and assumed fully utilized capacity on every link and node of the network) and on the upper bounds on capacity expansions. These, in turn, depend on the pattern of specified demand increases.

The efficiency of the first part of the HERMES I model depends in a vital way on the lower and upper bounds on incremental capacity expansion costs on the links (and nodes) of the network being as close together as possible. This is why an iterative procedure is incorporated into the CADUCEE program whose purpose is to confront repeatedly the specified pattern of demand increases with the structure of the network and with the capacity expansion cost functions to narrow, step-by-step, the intervals between the lower and the upper bounds in question. This iterative procedure, which is used only if the problem submitted to the model involves demand increases involving more than one pair of demand points, makes use of the concepts of the "maximum contemplated demand increase" and "maximum relevant demand increase" defined and discussed below.

To sum up, the principal function of the first part of the HERMES I model is to identify the relevant variables for the second part of the model.

The main procedure of the second part of the HERMES I model is mixed integer linear programming. The capacity expansion programs are identified with reference to the initial state of the network. The model uses the computer program TRANCHE. The heart of this program is a standard mixed integer linear programming algorithm of the branch-and-bound type. This algorithm is surrounded by fairly elaborate procedures for the efficient handling of inputs, on the one hand, and for the outputting of results on the other. The inputs into TRANCHE are, in the first place, the principal outputs of the first part: the admissible facility assignment chains and the upper bounds on capacity expansions. In addition, TRANCHE requires as inputs the complete specifications of capacity expansion cost functions, functions which have already been, to some extent and for a different purpose, utilized in the first part of the model. The output of TRANCHE is the identification of the minimum cost capacity expansion program corresponding to the specified pattern of demand increases. Such a program takes the form of a list of transmission capacity expansions (and, in future versions, of capacity expansions of nodal facilities) and their costs. The corresponding facility assignment chains are also identified, chiefly in order to demonstrate that the specified demand increases are in fact satisfied by the expansion program concerned.

The structure of the model and the sequence of the main groups of operations involved in its functioning is illustrated by the flow charts of Figures 1 and 2.

It is to be noted that the HERMES I model works in units of 1 service channel for both demand increase specifications and for the measurement of capacity and of capacity expansions. In future models of the HERMES series, finer, and not necessarily uniform, units may be used, without any fundamental change of the methodology proposed here, though at the price of a heavier data organization and computing effort.

As shown in Figure 1, CADUCEE starts with the network characteristics and the problem specified as pairs of points between which a demand increase is given. From this is calculated the "maximum contemplated demand increase." This is the maximum number of service channels which could be added to one link to meet the specified increase and can be illustrated as follows. Suppose we specify an increased demand of 2 channels between A and B and 2 between C and D. On any link X, the maximum possible number of channels which we might have to add to meet these demands is 4 channels. The bounds are then calculated based on this number, the "maximum contemplated increase". The software then identifies the minimum cost chains between the specified points and produces the DØMINØ tables. Admissible nodes and admissible chains are then identified for each pair of points specified. It should be noted from the flow chart that output of these data and tables is optional. There is also an option of resorting to specific exclusion rules to eliminate chains which might be otherwise admissible.

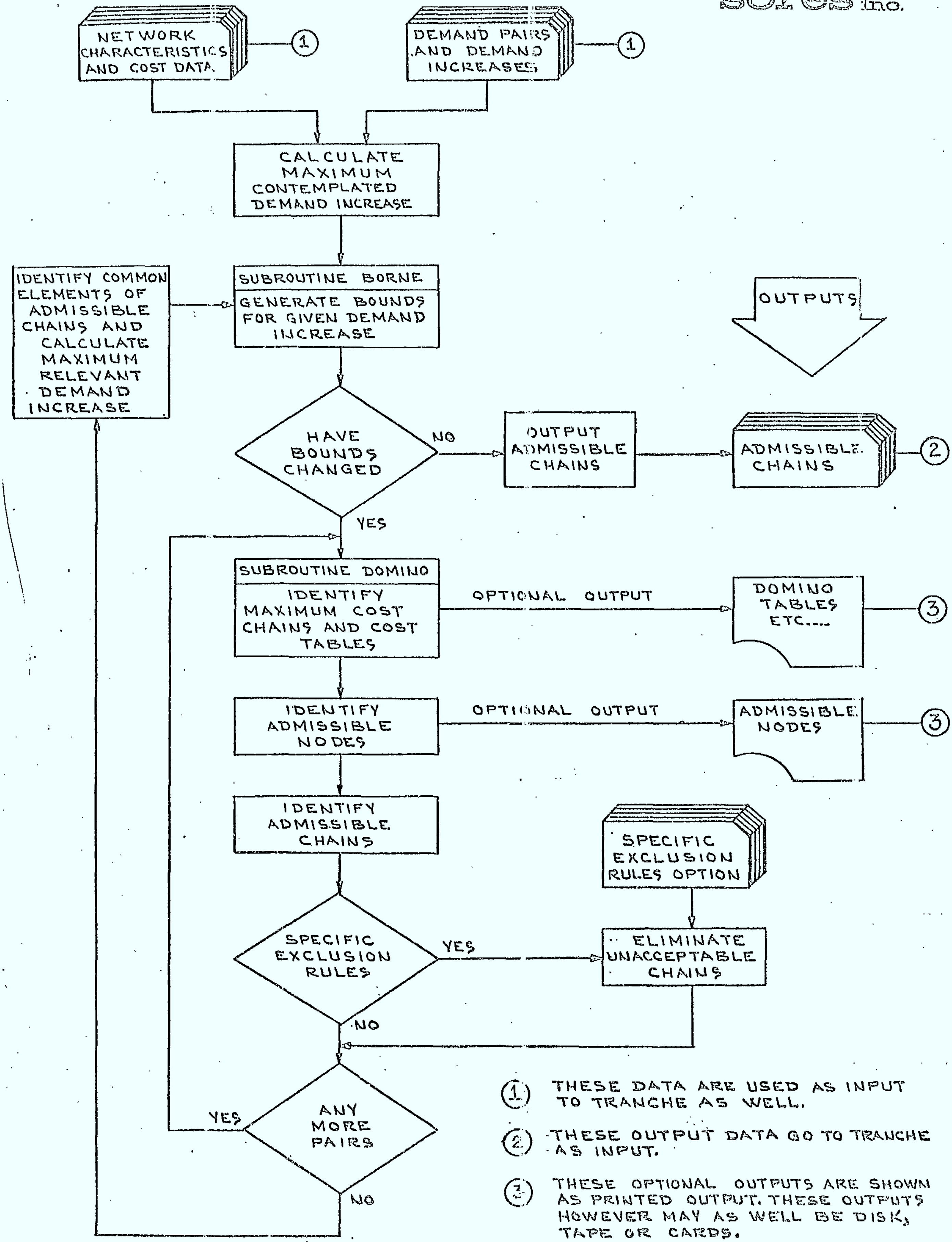


Figure 1: HERMES I model - General Flow Chart - CADUCEE

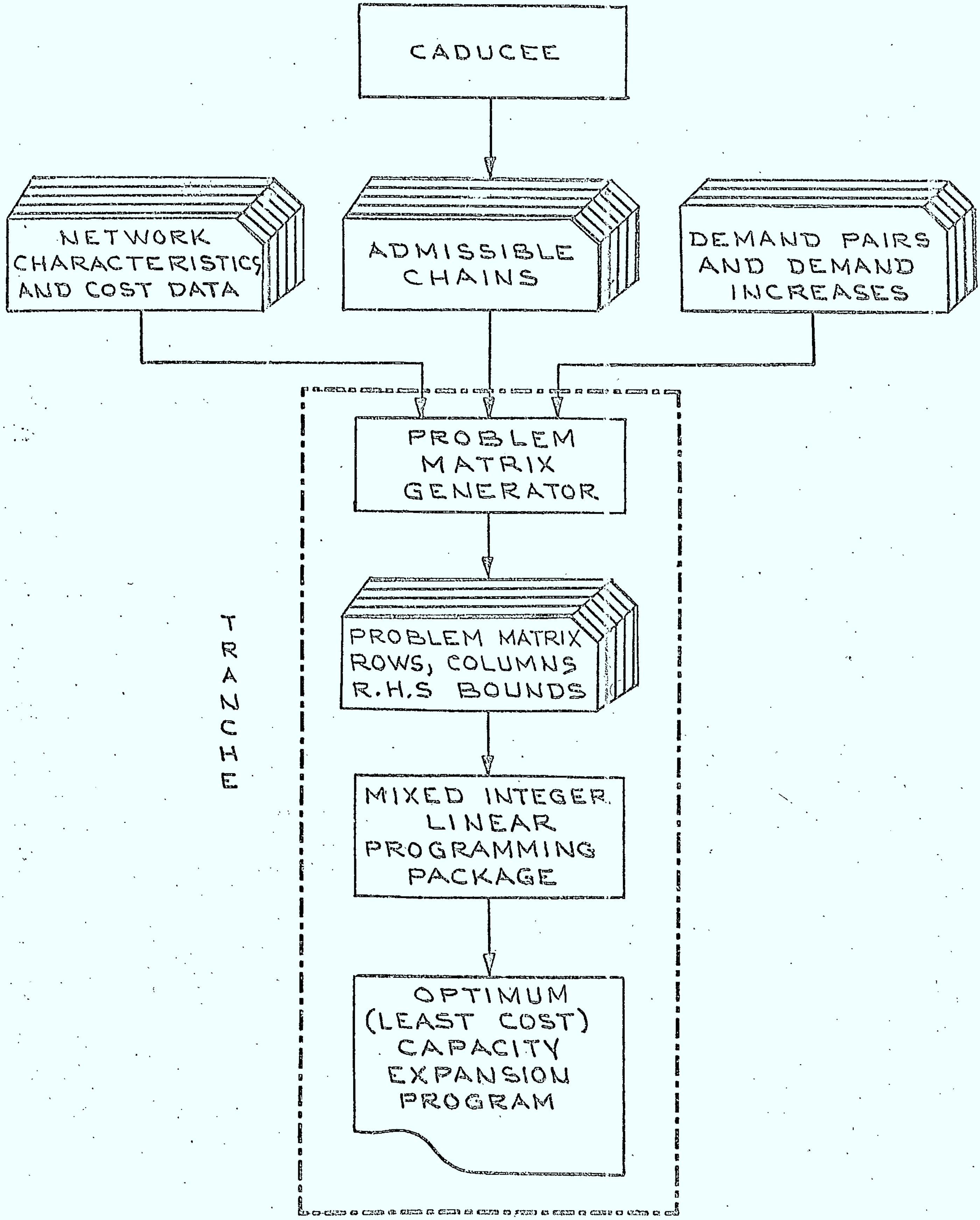


Figure 2: HERMES I model - General flow chart (cont'd)- TRANCHE

Once all pairs specified have been examined, the software then tightens the bounds by identifying common elements of chains and calculating "maximum relevant demand increase". Again to illustrate using the above example, suppose that the chains between A and B use links X, Y and Z and those between C and D use links K, L and X. Thus the "maximum relevant demand increase" for K, L, Y and Z ~~if~~ is 2, whereas for link X it is 4. The bounds are revised on this basis and if they have changed since the last iteration, the process of identifying new chains and nodes is repeated. If the bounds have not changed, they are as tight as the software can make them and no further iteration is undertaken. The admissible chains at this point are punched out for input to TRANCHE.

As shown in Figure 2, TRANCHE begins with the same network characteristics and cost data as CADUCEE. In addition, the CADUCEE output is fed into the computer. Using these data, the Problem Matrix Generator develops all of the specific formats required by the Mixed Integer Linear Programming package which solves the problem and finds the minimum cost facilities expansion program.

2.3 Formulation and analysis of the network, cost functions and demand: CADUCEE

2.3.1 Inputs

The computer program CADUCEE which corresponds to the first part of the HERMES I model requires the following three groups of inputs.

i) A representation of the telecommunications network by a non-directed graph having the property that there can be only one link between any two nodes: a proposed extension of the model, described in some detail in Chapter 6 of this Report, will allow for more than one link between any two geographic nodes, corresponding to more than one transmission facility between these nodes. The extension proposed involves the construction of enlarged networks in which appear the so-called dummy nodes and which allow for the existence of more than one facility between any two geographic nodes while respecting the formal requirement of the existence of at most one link between any two nodes (geographic, or dummy). Although, of course, the network used in the model is a simplification of the real telecommunications network, the degree of detail taken into account may be fairly large, the model being able to handle networks with several hundred nodes. The network used to obtain the first results of the Preliminary Phase contained 98 nodes. It is described in some detail in Section 3.2 and illustrated in Figure 3 . The network is not specified once for all. In successive utilizations of the model, the network can be easily modified. In particular, certain parts of it of particular interest in any given application may be specified in greater detail. Also, non-existent but contemplated links may be introduced to evaluate their potential role in planned capacity expansion programs.

ii) The capacity expansion cost functions on all the elements of the network involved. In the HERMES I model, the elements involved are the links corresponding to transmission facilities. In future extensions, cost functions of nodal facility capacity expansions will be introduced and treated in a manner analogous to that described here for the transmission facilities. These are capital costs. Operating costs are not taken into account in the HERMES I model. More complete discussions of the cost concepts relevant to HERMES I and to other models of this series appear in several places in this report, and in particular in Sub-section 3.2.2. It will be noted that in every utilization of the model, cost functions have to be specified for all the elements of the network, in the HERMES I model for all the links, though of course, for parts of the network which are not of main interest, it may suffice to have first estimates indicating the orders of magnitude.

In the CADUCEE program, the cost functions are not used as such but serve to calculate the lower and upper bounds on incremental capacity expansion costs. The bounds in question may be revised several times in the course of any given utilization of the model. These revisions are part of the mechanism of the computer program and are described in the following Sub-section and in Chapter 3. However, the concept of the lower and upper bounds on incremental capacity expansion costs and its relation to the cost functions will be taken up here. Capacity expansion is measured in discrete units of 1 service channel. Capacity expansion cost functions are step functions. For transmission facilities the possible increments are $1,000, $3,000, $5,000 or $9,000 per mile. For the nodal facilities, of course, the possible increments will be in the total capacity expansion costs, and not expressed on per mile basis.

For each link (and node) the lower bound and the upper bound is established for the incremental unit (i.e. per 1 service channel) cost of capacity expansion. For links this cost relates to a unit increase in capacity over the whole length of the link.

The lower and upper bounds on incremental capacity expansion costs are calculated as the lowest and the highest, respectively, capacity transmission cost per 1 service channel, within the interval between the initial state and the maximum contemplated demand increase, or the maximum relevand demand increase, depending on which stage of the algorithm the calculation is being made. For links, these bounds are always one of the amounts of $1,000, $3,000, $5,000 or $9,000 multiplied by the mileage. It will be noted that if the interval in question is equal to 1 service channel only, the lower and upper bounds necessarily coincide.

It will be observed that the lower and upper bounds refer to incremental capacity expansion costs per 1 service channel. The lower bound corresponds to the most favourable and the upper bound to the least favourable configuration of facility assignments throughout the network, insofar as the given link (or node) is concerned. The actual costs are likely to be higher than the lower bound multiplied by the number of service channels installed and lower than the upper bound multiplied by the number of service channels installed. The setting of the lower and upper bounds on incremental capacity expansion costs serves two distinct purposes in the model. In the first place, the knowledge of these bounds is necessary for the identification of admissible facility assignment chains. In the second place, the interaction of the specified demand increases with the lower and upper bounds on admissible assignment chains yields the identification of admissible capacity expansion increments.

It will be recalled that facility assignment chains and capacity expansion increments are represented by variables in the second part of the model. The only way to make this second part of the model operational is to keep the numbers of variables down to manageable dimensions. This is done by considering the admissible chains and the admissible capacity increments only.

iii) Specified pattern of demand increases. In any given utilization of the model, demand increases can be specified for one, or for any number of pairs of demand points, which correspond to the nodes of the network. As pointed out earlier, if demand increases are specified for more than one pair of demand points, it is essential to treat them simultaneously and not sequentially.

In the HERMES I model, demand is always taken to be two-way demand, and all transmission (and nodal) facilities are assumed to be two-way facilities. This does not mean, of course, that all demand is necessarily demand for instantaneous two-way communications.

2.3.2 Mechanism

The purpose of the first part of the model is to identify the admissible chains and the admissible capacity expansion increments and thus to eliminate the dominated chains and the dominated capacity expansion increments.

This is achieved by a progressive evaluation and elimination procedure. This procedure may be applied to one, or to any number of, pairs of demand points to be treated simultaneously within a given problem.

The results are not invariant with respect to:

- the smallest discrete (lump) increment in demand and in transmission (and in nodal) facilities; it is assumed here to be 1 service channel;
- the initial state of the network; the installed, and assumed to be fully utilized, capacity;
- the specified pattern of demand increases concerning all the pairs of points to be treated in a given problem.

The procedure is a generalization of the shortest chain algorithm for non-directed networks and is based on Bellman's "principle of optimality". The generalization proposed here consists in taking account of the fact that on each link (node) there is a lower bound and an upper bound on the incremental capacity expansion cost, instead of a single incremental cost coefficient, the actual value depending on the facility requirements which could be assigned to this link (node) to accommodate the demands between the demand points involved in any given problem.

For any pair of the demand points considered, a complete chain is a chain connecting these two points, called NØRG and NDEST respectively, and an incomplete chain is a chain connecting NØRG to any point other than NDEST.

The procedure progressively eliminates complete, or incomplete, chains which are dominated by other chains. A complete, or an incomplete, chain is dominated if the sum of the lower bounds of the incremental capacity expansion costs of its links (and nodes) is higher than the sum of the upper bounds of the incremental capacity expansion costs of the links (and nodes) of some other chain connecting the same pair of nodes. A dominated chain cannot form part of any minimum cost capacity expansion program. A complete chain is dominated if any incomplete chain it contains is dominated. It will be recalled that an admissible chain is a chain which is not dominated.

If, in any given problem, one pair of demand points only is being considered, these lower and upper bounds have to be set once only. If more than one pair of demand points are to be treated simultaneously, the lower and upper bounds set at the beginning of the problem are successively revised, the number of revisions being at most equal to the number of pairs of demand points considered, less one. The effect of these revisions is to bring the lower and upper bounds closer together, or to leave them unchanged. It will be recalled that the power of the procedure depends on the lower and upper bounds being as close together as possible, while insuring that no admissible chains nor admissible capacity expansion increments are eliminated.

At the beginning of solving any given problem a "maximum contemplated demand increase" is calculated. This number is the same for all the links (and nodes) of the network. It is equal to the sum (expressed as a number of service channels) of the demand increases specified for all the pairs of demand points to be treated simultaneously in this problem.

In subsequent revisions of the lower and upper bounds the concept of the "maximum contemplated demand increase" is replaced by the concept of the "maximum relevant demand increase". This number will not, in general, be the same for all the links (and nodes) of the network. For any given link (or node) this number is equal to the sum of the demand increases specified for the pairs of demand points whose admissible chains (i.e. admissible chains connecting them) pass through this link (or node). It will be noted that, for any link (or node), a pair of demand points must be included in the calculation of the "maximum relevant demand increase" even if only one of its admissible chains passes through this link or node.

Given the capacity expansion cost functions, the lower and upper bounds on incremental capacity expansion costs depend on the interval between the initial state, that is the installed (and assumed fully utilized) capacity and the upper bound on capacity expansion which equals either the "maximum contemplated demand increase" or the "maximum relevant demand increase", depending at which stage of the problem the calculation is being made. Capacity expansion increments which are outside this interval are considered dominated. It will thus be seen that the successive revisions of the lower and upper bounds have at the same time the effect of progressively reducing the list of admissible capacity expansion increments.

Since capacity expansion costs on any element of the network are independent of the capacity expansion costs on any other element, the lower bound for a chain is the sum of the lower bounds on its elements and the upper bound for a chain is the sum of the upper bounds on its element. The enlarged network proposed for future models of the HERMES series preserves the principle of the independence of costs on the elements of the network. Hence the procedure described here will be equally well applicable to such enlarged networks.

The lower and upper bounds are calculated and then revised by the subroutine BØRNE. The program CADUCEE identifies the admissible facility assignment chains for each pair of demand points in turn. Once all the pairs have been treated, it calls the subroutine BØRNE to revise the lower and upper bounds and again treats all the pairs of points concerned. The procedure stops when no further revisions of the bounds are possible.

Every time the main program CADUCEE is used for any pair of demand points, it requires the following information:

- the specification of the two nodes of the pair of points concerned: NØRG and NDEST

- the lower and upper bounds on the capacity expansion costs on all the links (and nodes) of the network, calculated with reference to the initial state and with reference to the "maximum contemplated demand increase" or to the "maximum relevant demand increase" on each link (and node), as the case may be.

The procedure starts by calculating the costs of minimum cost chains, under different cost assumptions between NØRG and NDEST and every other node of the network. This is done with the help of the subroutine DØMINØ of the program CADUCEE.

DØMINØ calculates four tables, although in fact three of them only are required in further calculations:

- the cost of the minimum cost chain, costs being set at their upper bounds, from NØRG to every other node of the network;

- the cost of the minimum cost chain, costs being set at their upper bounds, from NDEST to every other node of the network (this table is not required in further calculations);

- the cost of the minimum cost chain, costs being set at their lower bounds, from NØRG to every other node of the network;

- the cost of the minimum cost chain, costs being set at their lower bounds, from NDEST to every other node of the network.

It will be noted that the costs of chains which appear in the above tables are per 1 service channel. However, since they are upper (lower) bound costs, upper (lower) bound costs for capacity expansions for 2, 3, etc. service channels are simply the corresponding multiple of the bounds for 1 service channel - up to the "maximum contemplated demand increase" or up to the "maximum relevant demand increase", as the case may be.

It will also be noted that the numbers contained in the above four tables depend on the initial state of every link (and node) of the network and on the "maximum contemplated demand increase" or the "maximum relevant demand increase", depending at which stage of the procedure they were calculated.

The above four tables having been calculated by the subroutine DØMINØ, the main program CADUCEE takes over and eliminates all the dominated nodes. A node is dominated if all possible chains connecting NØRG and NDEST through this node are dominated.

For every node of the network a comparison is made between:

- the sum of the costs of the two minimum cost chains connecting this node with NØRG and with NDEST respectively, costs being set at their lower bounds;

- the cost of the minimum cost chain connecting NØRG and NDEST, costs being set at their upper bounds.

If the first term of this comparison is greater than the second term, the nodes concerned is eliminated as being a dominated node. Two of the four DØMINØ tables are used in this operation.

All the links connecting a dominated node with any other node, whether dominated or not, are eliminated.

It may be noted that when the procedure is used for one or more pairs of geographically close points, the elimination of dominated nodes will provide a non-arbitrary delineation of the geographic region relevant to the question of capacity expansion to accommodate an increase in demand between the pairs of nodes concerned.

CADUCEE starts by identifying chains having one link, called "chains of length 1" starting from NØRG. It uses these to identify chains "of length 2", etc. Every time a chain meets a node, a comparison is made between:

- the cost of the incomplete chain concerned - costs of its links being set at their lower bounds;

- the cost of the minimum cost chain connecting NØRG to the node concerned, costs being set at their upper bounds. This information is contained in one of the four DØMINØ tables.

If the first term of this comparison is greater than the second term, the incomplete chain concerned is eliminated as a dominated incomplete chain. All complete chains containing this incomplete chain are also implicitly eliminated.

The procedure stops when no further incomplete or complete admissible chains can be identified.

The program CADUCEE then takes up another pair of demand points, calls the subroutine DØMINØ and repeats the procedure described above, starting with the calculation of the DØMINØ tables.

Once all the pairs of points concerned have been treated, the subroutine BØRNE is called to revise the lower and upper bounds and again all the pairs of demand points are treated in turn. This revision starts by sorting out all the admissible chains by the pairs of demand points to which they relate. Then, for each link (or node) of the network, the pairs of demand points are identified whose admissible chains (one or more) pass through this link (or node). The "maximum relevant demand increase" for each link (or node) is then calculated as the sum of the demand increases for all the pairs of demand points so identified. These "maximum relevant demand increases" are then used to recalculate the lower and upper bounds on incremental capacity expansion costs and also as a by-product, to identify the admissible capacity expansion increments. The procedure stops when no further revisions of bounds are possible.

Once the admissible chains have been identified by the above procedure for all the pairs of demand points concerned, they may be further tested to eliminate those among them which violate the specific elimination rules which reflect the institutional and technical peculiarities of the Canadian national interregional telecommunications system. The taking into account of these specific rules is an option in the CADUCEE program: option "specific exclusion rules".

2.3.3 Outputs

CADUCEE outputs all the admissible complete chains connecting the demand points of all the pairs considered. These chains are identified (as sequences of nodes and as sequences of links). It also outputs the upper bounds on capacity expansion on every link (and node) of the network and thus, implicitly, identifies the admissible capacity expansion increments.

All the chains which have been eliminated are dominated chains, that is chains that cannot appear in the minimum cost capacity expansion program with the given initial state and the specified pattern of demand increases. There is therefore no danger of missing the optimum solution by submitting to the program TRANCHE only the chains identified as admissible. However, if the option "specific exclusion rules" is used, it may happen that some admissible chains are eliminated.

The program contains an option to print (punch on cards, transfer to disk, etc.) the four DØMINØ tables. If this option is used, CADUCEE will also identify (as sequences of nodes and as sequences of links) the two corresponding minimum cost chains between points of each of the pairs of points considered, and output them. This information is not required in further calculations, but may be of interest by itself.

Among other uses, these additional outputs provide information relevant to the evaluation of possible new links between nodes not at present directly connected.

2.4 Identification of minimum cost capacity expansion programs: TRANCHE

2.4.1 Activities and constraints

The procedure of the identification of minimum cost expansion programs is formulated in terms of Activity Analysis. The elements of the procedure are:

- variables, usually referred to as "activities"; the procedure chooses the optimum set of values of these variables;
- constraints which these variables have to respect, individually or, more often, in sub-sets;
- the objective functions where the variables of the problem appear as arguments and whose value is to be optimized (minimized or maximized).

The procedure involves two types of activities.

- The facility assignment activities:

  These are the facility assignment chains of the preceding section. They correspond to the allocation of demand to different chains of links (and by implication to the corresponding sequences of nodal facilities). These activities are somewhat evocative of the routing of traffic between any pair of demand points, however they do not in fact represent the routing of traffic. They merely represent sets of facilities which may serve the demand between any two demand points. These activities (variables) are non-negative and continuous. However, in the HERMES I model, demand is always specified in discrete units of one service service channel and so is capacity and capacity expansions. As a consequence, the variables concerned will always take integer values. But they do not have to be declared as integer variables, which would have the effect of rendering the computations unnecessarily cumbersome. Their levels indicate the capacity of the corresponding chains allocated to satisfy the demand between the points concerned. There is one such variable for every admissible facility assignment chain between the points of every pair of points.

- The capacity expansion activities.

  These are non-negative integer variables representing discrete additions of transmission capacity. In future version of the model, when nodal capacity expansion is also taken into account, another set of nodal capacity expansion activities will have to be introduced.

An investment or capacity expansion activity corresponds to the building of an indivisible facility or of a block of equipment for the purpose of increasing the capacity of an element of the network: a link (or a node). It may thus concern a transmission facility or a block of transmission equipment (or a nodal facility or a block of nodal equipment). The level of an investment activity represents the number of facilities or of blocks of equipment installed. It is thus a non-negative integer. If an investment decision is of the "yes or no" kind, the corresponding investment variable will be a 0-1 variable. If the installation of a facility or of a block of equipment of a given kind may be repeated a certain number of times, the corresponding investment activity is a non-negative integer variable. In practive, it will always have a known upper bound.

It is to be noted that the actual transmission of messages does not appear as an activity anywhere in the model.

It will also be noted that the static nature of the model puts rather severe conditions on the interpretation of the activities (variables) as defined above. Thus, the levels of the facility assignment activities represent unchanging and permanent claims on the transmission (and nodal) facilities all along their respective chains. Once a change in demand is specified, demand is assumed to remain at this level indefinitely. The capacity expansion activities, on the other hand, are to be interpreted as "once for all" activities. The capacity expansions concerned are to be undertaken immediately and the equilibrium, described by an optimal solution of the model, will not be attained until all these new investments have been implemented to meet the new pattern of demand.

The first group of constraints ensures that all demand increases are satisfied: the sum of all the facility assignment activities between any two demand points must be equal to the demand increase between these two points.

The second group of constraints ensures that the existing and new capacity on every element of the network is at least sufficient to handle all the demand allocated to this element. In the HERMES I model, which is concerned exclusively with transmission facilities, links are the only elements considered. When, in future versions, nodal facilities also are taken into account, analogous constraints will have to be defined for the nodes of the network. It is clear that, since they appear in the same constraints, the allocation of demand variables and the expansion of capacity variables must be expressed in the same units. It is in these constraints that the integer variables appear reflecting the indivisibility (or "lumpiness") of investment decisions concerning transmission facilities.

The third group of constraints in which appear the same integer variables ensures the precedence of capacity expansions on any given element of the network. The presence of decreasing unit costs makes it necessary to exclude the possibility of getting "solutions" where an addition to a facility is included in the capacity expansion program while the original facility itself is not.

If capacity expansion on every relevant element of the network is allowed, the presence of the three above groups of constraints cannot give rise to a situation where there are no feasible solutions. On may, however, consider situations where restrictions are imposed on capacity expansions. They would be either restrictions on individual elements of the network or global restrictions on the whole capacity expansion program. One might, for instance, allow capacity expansion on certain specified links only, or fix fairly low upper bounds on allowable capacity expansion on certain links. Global restrictions may, for instance, take the form of a constraint on the total capital outlay. In these cases, the corresponding mathematical problem may have no feasible solutions.

In future models of the HERMES series, the concept of the annual capital charges may be used. This concept will involve taking into account the expected life, interest charges, and annual maintenance costs.

It will be recalled that the investment or capacity expansion activities represent discrete additions of transmission (and, in later versions, also of nodal) facilities. Each of these activities has a cost associated with it. Thus, the procedure works in terms of total costs. However, the shape of these total cost functions is such that the underlying average costs are decreasing over the intervals corresponding to discrete capacity expansions. In any solution including the optimal, some of these activities will appear with positive values, the others will appear with zero values. The function to minimize is the sum of the costs of the capacity expansion activities, the activities appearing at zero levels making, of course, no contribution to the total cost.

2.4.2 Mixed integer linear programming procedure.

The procedure which constitutes the core of the program TRANCHE is that of mixed integer linear programming.

The basic conceptual difficulty one faces here stems from the indivisibilities which characterize telecommunication facilities and equipment. These indivisibilities give rise to the phenomena of joint costs and of decreasing average costs which necessitate the introduction of what are known in the economic programming parlance as conditional constraints. It is the presence of these constraints which calls for the introduction of integer variables.

Geometrically, the presence of conditional constraints means that the admissible region over which the cost function is to be minimized is not convex, as ~~if~~ is the case in the ordinary linear or quadratic programming. The usual methods of solution which rely heavily on the concept of the supporting plane are no longer applicable and the combinatorial character of the problem has to be faced directly.

This not only makes for much heavier computations than those required to solve ordinary programming problems of comparable size, but also means that sensitivity analyses, once an optimal solution is found, are much more difficult. The usual parametrization procedures are no longer applicable and sensitivity analyses become essentially combinatorial problems involving heavy computations. It thus becomes more than ever necessary to have a close collaboration with the users of the model who are best qualified to indicate the precise nature of the sensitivity analyses they are interested in.

The TRANCHE program uses a "branch and bound" algorithm for solving mixed integer linear programming problems. The principle of this algorithm is described in sub-section 3.1.4. It is essential to have an extremely efficient computer program, and a powerful computer since the volume of calculations is often several hundred times greater than in the case of continuous linear programming problems of comparable dimensions. It is also essential, of course, to reduce the number of variables and to fix the lowest possible upper bounds on the integer variables. This is achieved by CADUCEE.

2.4.3 Solutions.

A solution of the model gives in the first place the values of the facility assignment activities associated with the minimum cost capacity expansion program. These are obtained as a by-product of the principal output of the second part of the HERMES I model.

This principal output consists of the values of capacity expansion activities constituting the minimum cost capacity expansion program.

Finally, the procedure gives the total cost associated with this capacity expansion program and its breakdown by the elements of the network where capacity expansion is indicated. The actual cost of transmitting messages does not appear anywhere in the model.

HERMES I is not a model which is formulated once for all and intended to turn out a unique optimal capacity expansion program . It is destined to be used repeatedly with different sets of capacity expansion minimum cost functions, with modifications of the original network and, of course, for different patterns of demand increases. Any model, however elaborate, is a brutal simplification of reality. It would be to misunderstand completely the nature of the project HERMES to expect to get out of it a once-for-all straightforward answer to the question: what ought to be the program of capacity expansion of the interregional telecommunications network.

3. THE OPERATIONAL FORMULATION OF THE HERMES I MODEL

3.1 The Mathematics of the Model

3.1.1 The nature of the problem and the tools available

In Chapter 2, we have discussed the structure of the model as being twofold: a search for a set of admissible chains and a mixed integer linear program to establish the optimum capacity expansion.

The first problem is treated with some of the tools of graph theory and the notion of dominated chains. The second is handled by a mathematical programming formulation requiring integer variables to express the mathematical representation of indivisible phenomena like the addition of indivisible quanta of capacity and ordering of these capacity additions.

The presence of integer variables mean that the admissible region over which the cost function is to be minimized is not convex, as it is the case in the ordinary linear or quadratic programming. The usual methods of solution which rely heavily on the concept of the supporting hyperplane are no longer applicable and combinatorial methods using clever partial enumerations of the admissible points are used. The resulting computing time increases drastically. Moreover, a sensitivity analysis from a given optimal solution is much more difficult. Parametrization procedures are impossible due to the integer variables and the computation of shadow prices indicating the marginal contribution of relaxing certain constraints is no longer applicable. The sensitivity analysis must be conducted through combinatorial variations of sets of parameter values and recomputation of new optimal solutions.

In order to express the problem, we will start with a few definitions.

The network is represented by a set N of indices for the nodes with typical element called j and a set L of indices for the links with typical element called l. Some elements of the network represent capacity expansion increments. The set of indices labeling these elements will be called K and a typical index will be designated by k.

For each $k \in K$, a capacity expansion cost function is defined from the initial capacity to an upper limit which is sufficient for all the problems to be submitted to the model. This cost function is a step function. The domain of this function is a set of integers from 0 to the above mentioned upper limit; but, the steps having different possible sizes, we must define a set T (k) of <u>ordered capacity expansion activities</u> whose typical notation will be t. The first activity starts from the initial state and has a possibility of having its level increased, one unit of capacity at a time, as long as the cost increments stay the same. As soon as the cost increments change, we define a second activity and so on. It should be noted that we must exhaust the first activity before the second starts and so on for the following ones. We will denote y (k;t) the integer which is the level of the capacity expansion activity t of k, and c (k;t) the cost increment associated with the same activity.

For each pair i of elements of N, there exists a set of chains, that is, a sequence of links going from NØRG, the first element of the pair, to NDEST, the last one. We denote R (i) as this set of chains and r its typical element. With each chain r a <u>facility assignment activity</u> can be defined; its value is a non-negative number x (i;r) which is the level of facilities required to satisfy demand between the elements of the pair i of demand points. We could also define a unit cost c (i;r) associated with x (i;r) but it would not have a clear meaning in the actual state of the expansion problem.

The typical problem is to choose the capacity expansion configuration in the network which minimizes the cost of facility expansion to meet a given level of demand between pairs of demand points. Let D denote the set of indices labeling the pairs of demand points relevant for the problem and let i be such a pair and d(i) the demand.

The mixed integer programming problem is the following:

$$\underset{x,y}{\text{Minimize}} \quad z = \sum_{k\in K} \sum_{t\in T(k)} c(k;t)\, y\,(k;t)$$

Subject to

1) for all (i;r), $\quad x\,(i;r) \geqslant 0$
   for all (k;t) $\quad y\,(k;t) \in \{0,1,2,\ldots\}$

2) demand constraints

$$\sum_{r\in R(i)} x(i;r) = d\,(i),\ i \in D$$

3) capacity constraints

$$\sum_{i\in D} \sum_{r\in R(i)} \delta(r;k)\, x\,(i;r) - \sum_{t\in T(k)} y\,(k;t) \leqslant 0,\ k \in K$$

where $\delta\,(r;k)$ takes the value 1 if the chain r uses the element k and 0 otherwise.

4) sequencing constraints imposing the order of the capacity expansion activities for each element of K.

5) bounding constraints

$$y\ (k;t) \leqslant \bar{y}\ (k;t), \text{ for all } (k;t)$$

As outlined in Chapter 2, this general formulation is impractical when it comes to solving a problem on the computer since, for a given pair i, R(i) could have several tens of thousands of elements. We must find a way of reducing the size of the problem without loosing anything. The notion of admissibility and its negation, the notion of domination, provides the method of reducing the problem to the level where a solution becomes practical.

3.1.2 Analysis of the network

We are looking for a sufficient condition such that, when a facility assignment activity satisfies it, we are sure the corresponding variable enters the solution with a zero value, i.e., it is absent from the optimal capacity expansion program. The theorem of optimality in dynamic programming which says, loosely stated, that any subset of choices extracted from an optimal sequence of choices must also be optimal, provides the keystone of the methodology. For example, if the best way of going from the point A to point F is the sequence of points A, B, C, D, E, F, then the subset B, C, D, must be the best way of going from B to D, otherwise the first sequence would not be optimal.

Before developing this idea, it should be noted that, for a given problem, the set of demand pairs of points and the associated demands d(i) permit a first reduction in the number of capacity expansion activities for each capacitated element. In effect, the worst which can happen is that each such element will be required to satisfy all the demands. We define therefore the maximum contemplated demand:

$$\max\ CD = \sum_{i \in D} d(i)$$

Consequently, the set T(k) of capacity expansion activities for the network element k is now such that:

$$\sum_{t \in T(k)} \bar{y}\ (k;t) = \max\ CD, \text{ for all } k \in K.$$

Let us define a few more helpful concepts:

max (k) is the upper bound cost of a unit increase for the argument of the capacity expansion cost function of the k^{th} element (link or node) in the interval (0, max CD);

$$\max(k) = \underset{t \in T(k)}{\mathrm{MAX}}\ c\ (k;t).$$

min (k) is defined similarly

$$\min(k) = \underset{t \in T(k)}{\mathrm{MIN}}\ c\ (k;t)$$

Lmax (i) is the least cost with respect to all the chains having the element of the pair i as end points, the cost on each capacitated element k being at its max (k), that is

$$\mathrm{Lmax}\ (i) = \underset{r \in R(i)}{\mathrm{MAX}} \sum_{k \in K} \delta(k;r) \max(k),$$

where $\delta(k;r) = 1$ if the element k belongs to the chain r and 0 otherwise.

Smin (i;r) is the summation of the costs at their respective lower bound for all k elements of the chain r between the pair of nodes i, that is

$$\mathrm{Smin}\ (i;r) = \sum_{k \in K} \delta(k;r) \min(k)$$

Now we are ready to state a sufficient condition which will allow us to reduce the number of facility assignment activities.

A Sufficient condition

Proposition 1:

Whatever $n = \sum_{t \in R(k)} y\ (k;t)$, the number of units of added capacity on an element k, n being in the interval (0, max CD), then

$$\min(k) \leqslant c\ (k;t) \leqslant \max(k)$$

and it follows that

$$\sum_{t \in T(k)} \min(k)\ y\ (k;t) \leqslant \sum_{t \in T(k)} c(k;t)\ y\ (k;t) \leqslant \sum_{t \in T(k)} \max(k)\ y\ (k;t)$$

and it follows again, for a given chain r which has been assigned the same number n of units of facilities on each of its elements, that:

$$\sum_k\sum_t \delta(k;r) \min(k)\, y(k;t) \leq \sum_k\sum_t \delta(k;r)\, c(k;t)\, y(k;t) \leq \sum_k\sum_t \delta(k;r) \max(k)\, y(k;t)$$

and, substituting according to our definitions:

$$S\min(i;r)\, n \leq \sum_k\sum_t \delta(k;r)\, c(k;t)\, y(k;t) \leq \sum_k \delta(k;r) \max(k)\, n.$$

Proposition 2:

Consider the chain r for the pair i, and suppose there exists a sub-chain u of r between the pair i* such that:

a) $L\max(i^*) < S\min(i^*;u)$,

then, all the n facility units assigned to the chain r could be transferred to another chain which would differ from r only as far as the sub-chain u is concerned, the sub-chain u* corresponding to Lmax (i*) replacing u.

(To be sure, the transfer is possible if the maximum contemplated expansion on each k of the new sub-chain is large enough; this is guaranteed, whatever the already assigned facilities, if we have chosen the maximum contemplated demand as the maximum contemplated expansion for the elements of the new sub-chain).

b) What is now important to note is the necessary decrease in total expansion cost associated with that transfer. This follows from Proposition 1 and Proposition 2 b since for the non common parts of the two chains that is u and u*, we have:

$$\sum_k\sum_t \delta(k;u^*) c(k;t) y(k;t) \leq L\max(i^*) n < S\min(i^*;u) n \leq \sum_k\sum_t \delta(k;u) c(k;t) y(k;t)$$

with $\sum_t y(k;t) = n$ for both chains u* and u;

it is clear that a transfer of n units from u to u* decreases the total cost, therefore the level of the facility assignment activity corresponding to the old chain r must be zero.

Definition of an admissible chain:

A chain for which there does not exist a sub-chain which satisfies the condition a) is called an admissible chain.

Definition of a dominated chain:

It is a chain which is not admissible.

In fact, in the CADUCEE software we use a more restrictive notion of domination since we employ the notion of the incomplete chain, that is a sub-chain from the origin of the chain, rather than the concept of a sub-chain which includes the other one. This results in some dominated chains, in this more restrictive sense, remaining in the optimizing program with a loss of efficiency which may be of some importance if large demand increases are being considered. CADUCEE II will take care of this modification

This concept of the search for admissible chains reduces drastically the size of the problem. The search is based on the concept of "maximum contemplated demand" from which the max (k) and min (k) was computed. From a first iteration giving the admissible chains, we could now tentatively try to reduce further the number of capacity expansion activities on certain capacitated elements. This is done with the concept of "maximum relevant demand" which replaces the "maximum contemplated demand". To fix an upper bound on the numbers of possible units of expansion on an element, we add only the demands for the pairs of points whose admissible chains use that particular element and the search for new dominated chains starts again. Finally, when there is no further change in the maximum relevant demands, we are ready to state the reduced optimizing problem. However, before leaving the subject, we will explain briefly the way in which the least cost Lmax(i) is computed in the subroutine DØMINØ.

Suppose we have a network and the costs which are fixed numbers associated with the links. (If costs are associated with the nodes also, we define dummy links). The problem is to find the cost of a least cost chain between an origin NØRG and a destination NDEST. We are interested in the cost and not in the chain.

Notice first that since the graph is finite, the least cost is the cost of a chain which has at most L-1 links, L-1 being the "length" of a chain using each link without repetition. We will now reason by induction.

Let LCOST (NØRG,S) denote the least cost we are looking for and LCOST(J,S;X≤x) the least cost for the chains between J and S among the costs of the chains smaller than or equal to X as far as the number of links is concerned. Lest COST(J,S) be the cost of the link J,S. If that links does not exist, COST(J,S) = ∞; if J = S, COST (J,J) = 0. We will find, using the same procedure, LCOST(NØRG,S) for all S.

Among the chains of length ≤ 1 from NØRG, we have, for all S:

$$LCOST(NØRG,S) \leq COST(NØRG,S)$$

Among the chains of length ≤ 2 from NØRG, we have, for all S:

$$LCOST(NØRG,S) \leq LCOST(NØRG;S;X\leq 2) =$$

$$\min\left\{\min_J\left[COST(NØRG,J) + COST(J,S)\right]; \min\left[COST(NØRG,S)\right]\right\}$$

Among the chains of length ≤ 3 from NØRG, we have for all S:

$$LCOST(NØRG,S) \leq LCOST(NØRG,S;X\leq 3) =$$

$$\min\left\{\min_J\left[LCOST(NØRG,J;\ X\leq x-1) + COST(J,S)\right];\ \min\left[LCOST(NØRG,S;\ X\leq x-1)\right]\right\}$$

and so on ...

$$LCOST(NØRG,S) \leq LCOST(NØRG,S;X\leq x) =$$

$$\min\left\{\min_J\left[LCOST(NØRG,J;\ X\leq x-1) + COST(J,S)\ ;\ \min\left[LCOST(NORG,S;\ X\leq x-1)\right]\right\}$$

In fact, it is not efficient to reach for $X \leq L-1$ because of the following proposition:

> If, for all S, $LCOST(NØRG,S;X\leq x) = LCOST(NØRG,S;X\leq x-1)$ there is no need to go further since having added another link to any chain having a length $\leq x-1$ has not changed the least upper bounds of the cost from NØRG to any destination S. We can write
>
> $$LCOST(NØRG,S) = LCOST(NØRG,S;X\leq x-1)$$

3.1.3 The formal structure of the optimizing model

In this section, the formal structure found in subroutine TRANCHE is outlined. Call TA(k) the set of admissible ordered capacity expansion activities of element k of the network and call RA(i) the set of admissible facility assignment activities for the pair of demand points i when the maximum relevant demands do not change anymore.

The reduced mixed integer linear programming problem is:

$$\underset{x,y}{\text{minimize}} \quad z = \sum_{k \in K} \sum_{t \in TA(k)} c(k;t)\, y(k;t)$$

subject to:

1) for all $(i;r)$, $x(i;r) \geqslant 0$

for all $(k;t)$, $y(k;t) \in \{0, 1, 2, \ldots\}$

2) demand constraints:

$$\sum_{r \in RA(i)} x(i;r) = d(i), \quad i \in D$$

3) capacity constraints:

$$\sum_{i \in D} \sum_{r \in RA(i)} \delta(r;k)\, x(i;r) - \sum_{t \in TA(k)} y(k;t) \leqslant 0, \quad k \in K$$

where $\delta(r;k)$ takes the value 1 if the chain r uses the elements k, and 0 otherwise.

4) sequencing constraints:

For a typical link k of the network, let the cost function of k be like the one described in Section 2.3. We will define the set TA(k) of admissible ordered capacity expansion activities the following way. Call $y(k;1)$ the level of the first activity, $y(k;1)$ takes the value 0 or 1 giving therefore a big jump in expansion cost for $y(k;1) = 1$; call $y(k;2)$ the level of the second activity, $y(k;2)$ takes the value 0, 1, 2, 3 or 4 giving a sequence of equal small jumps in the cost; call $y(k;3)$ the level of the third activity, $y(k;3)$ takes the values 0 or 1 giving a middle size jump; finally, suppose the maximum relevant demand increase for k is 8, the final activity $y(k;4)$ takes the values 0, 1 or 2 for two equal small jumps in the cost. Note that effectively the sum of the upper bound is

$$\bar{y}(k;1) + \bar{y}(k;2) + \bar{y}(k;3) + \bar{y}(k;4) = 1 + 4 + 1 + 2 = 8.$$

This guarantees that we can meet the maximum relevant demand increase with our expansion activities on k.

We must also sequence the expansion activities in a given order. This is done the following way:

a) $4y(k;1) \geqslant y(k;2)$ or equivalently $4y(k;1) - y(k;2) \geqslant 0$

This constraint guarantees that $y(k;2)$ will not be greater than 0 before $y(k;1) = 1$ and moreover, $y(k;2)$ will not reach a value greater than 4. As well, there are separate bounding constraints.

b) $y(k;1) + y(k;2) \geqslant \left[\bar{y}(k;1) + \bar{y}(k;2)\right] y(k;3)$ or

$y(k;1) + y(k;2) - \left[1 + 4\right] y(k;3) \geqslant 0$

This constraint forces $y(k;3)$ to wait at 0 until $y(k;1)$ and $y(k;2)$ take respectively the values 1 and 4.

If a non-zero initial state is present, for example initial $y(k;1) = 1$ and initial $y(k;2) = 2$, that is, three channels already installed, we write

b') $y(k;1) + \left[\text{initial state of } y(k;2)\right] + y(k;2) - (1 + 4)\, y(k;3) \geqslant 0$

and the bounding constraints will indicate that $y(k;1) = 1$ and $y(k;2)$ can take only the value 0, 1 or 2.

The last sequencing constraint is similar to the first.

c) $2y(k;3) - y(k;4) \geqslant 0$

In brief, the three preceding constraints assure that we climb up the cost function in the right order. No capacity expansion activity will be chosen before the preceding activity has reached its upper bound.

5) bounding constraints:

These constraints delimit the domain of variation of each integer variable, permitting in particular the assignment of initial states to these variables.

3.1.4 Principle of Branch and Bound Algorithm

The mixed integer programming algorithm described in this section is classified as a "tree search" method. The particular type of tree search algorithm is known as branch and bound.

The features of the branch and bound algorithm are : (i) it is easy to understand, (ii) it is easy to program on a computer, (iii) the upper bound on the number of steps needed in the algorithm increases exponentially as the size of the problem increases.

Consider a pure integer program:

$$\min Z = cy \text{ subject to } Ay \geqslant b,\ y \geqslant 0,\ \text{integers}.$$

If each component of y is bounded from above by an integer M, then there are $(M + 1)^n$ possible solutions y, where n is the number of variables. We could test each of these solutions with the minimum (maximum) value of the objective function as the optimal solution. Since the number of solutions is usually very large, the algorithm tries to avoid inspection of solutions which are dominated by solutions already inspected.

We first solve the integer program as a linear program. If all variables $y_i \geqslant 0$ and all are integer, then y is clearly the optimal solution to the integer program since the integer constraints were ignored in obtaining the solution. If a particular component $y_k = [y_k] + f_k$, where $0 < f_k < 1$, then we solve two linear programs, one with the additional constraint $y_k = [y_k]$ and one with the additional constraint $y_k = [y_k] + 1$. If one of the two still does not give integer solutions then two more linear programs are solved, etc.

All the solutions obtained in this way can be partially ordered as a tree with the root of the tree representing the linear programming solution obtained without any additional integer constraints. When a solution y_0 does not satisfy the integer constraints, it branches to two others y_1 and y_2. The solution y_0 is called the "predecessor" of y_1 and y_2, and y_1 and y_2 are called the "successors" of y_0.

If the successors to y_1 and y_2 are all infeasible, then we have to branch again from y_0. A node may have more than two successors. A node is called a terminal node if it has no successors; this definition implies that a terminal node represents a feasible or infeasible integer solution. The idea of the branch and bound method lies in the following two facts:

- Because the predecessor has fewer constraints than the successors and additional constraints cannot improve the value of the objective function, the optimum value of a successor is always larger than or equal to the optimum value of the predecessor.

- If two integer feasible solutions have the same predecessor, then the optimum value of the first solution is less than the optimum value of the second. That is, the further away the value of the solution is from the linear programming solution, the worse is the resulting value of the objective function.

During the computation of the branch and bound method, we keep the optimum value Z^* of the best integer feasible solution found so far. If a node with a non-integer solution has an optimum value worse than Z^* then all the successors of that node must have optimum values worse than Z^*. There is therefore no point in branching from that node and the branch is abandoned.

We proceed in this fashion until we find that terminal node which represents the optimal integer feasible solution; that is, all other branches have been abandoned as having optimal values greater than the Z^*.

3.2 Assembling the Model Inputs from the Data Base

We have already discussed the nature and structure of the HERMES I model and the nature of the problems to be solved. In order to solve these problems, the data inputs to the model required specification.

Specifically, data was required on the network to determine the physical characteristics of the problem area, on cost to give the model the information necessary to optimize, on demand such that the problem could be specified, and finally, on specific exclusion rules or routing rules for facility assignment chains so that the problem remained manageable.

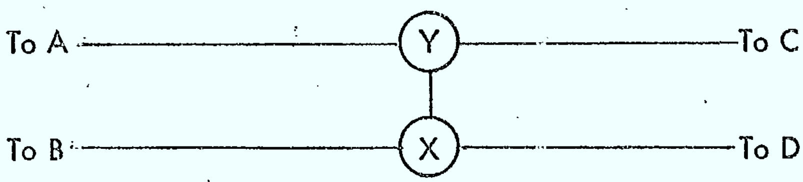
3.2.1 The network

The HERMES I model is constructed around a simplified representation of the Canadian Telecommunications inter-toll facilities network. This initial network consists of geographic points between which demand for telecommunication facilities is specified. Connecting these points or nodes are links representing existing or contemplated facilities.

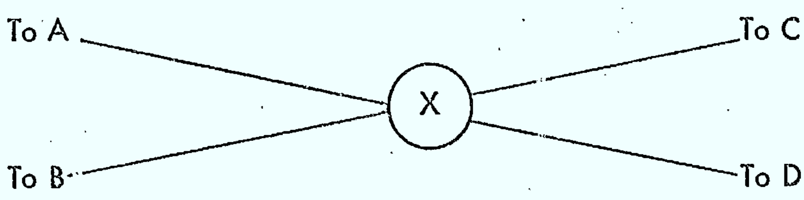
This network was arrived at after considerable discussion between Sorès and the Department of Telecommunications and was agreed to by both parties as being to a degree representative of the real network, yet abstracted to a sufficient degree to allow it to be handled already at the preliminary stage of the project.

The first level of simplification or abstraction agreed upon was that the network representation should show only the major links between major facility demand points and eliminate for the time being all intermediate facility demand points and sub-networks surrounding such points, intermediate and major.

The second level of abstraction was that, in some cases, major links were "moved" to show them passing through a major facility demand point when in fact this point was bypassed but connected. For example, if the actual network (at the first level of abstraction) was as follows about points X and Y;



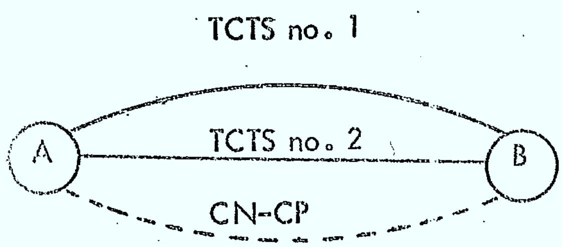
This could be represented in the model network as:



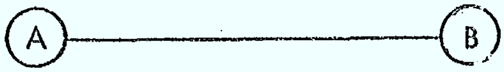
The third level of abstraction was that no differentiation was made between carriers.

The fourth level was that, where more than one facility existed between two points of the simplified network, these were shown as a single facility.

Thus, for example, if the network were as follows between points A and B:



This would be represented in the computer as:



The final abstract network which was developed consists of 98 nodes and 143 links (See Figure 3).

3.2.2 Cost functions

The cost of installation of new facilities were developed by the Department of Communications, consistant with the level of abstraction of the network.

Rather than establish the engineering cost figures for facilities (a task which would have been impossible given the time constraints) some general cost functions were developed. These cost functions, generally speaking, relate new facilities costs to the length in miles of the facility, the type of facility (light and heavy routes), and to whether new routes, new system of existing routes, or additional microwave channels on existing systems are involved. Some allowance was made for becoming more route-specific by introducing special categories of facilities where costs were specified a priori, by allowing for a "difficulty factor" on some routes, and by using nodal as opposed to link costs for transborder facilities.

There were basically two cost functions:

- Investment Cost function-Heavy Routes (Figure 4)

- Investment Cost function-Nodal facilities to U.S.A. (Figure 5)

The cost categories for links are as follows:

<u>Expansion categories microwave</u>

1) Heavy route

   a) New route
   b) New system - existing route
   c) New channels on existing system

2) Light route

   a) New route
   b) New system - existing route
   c) New channels on existing system

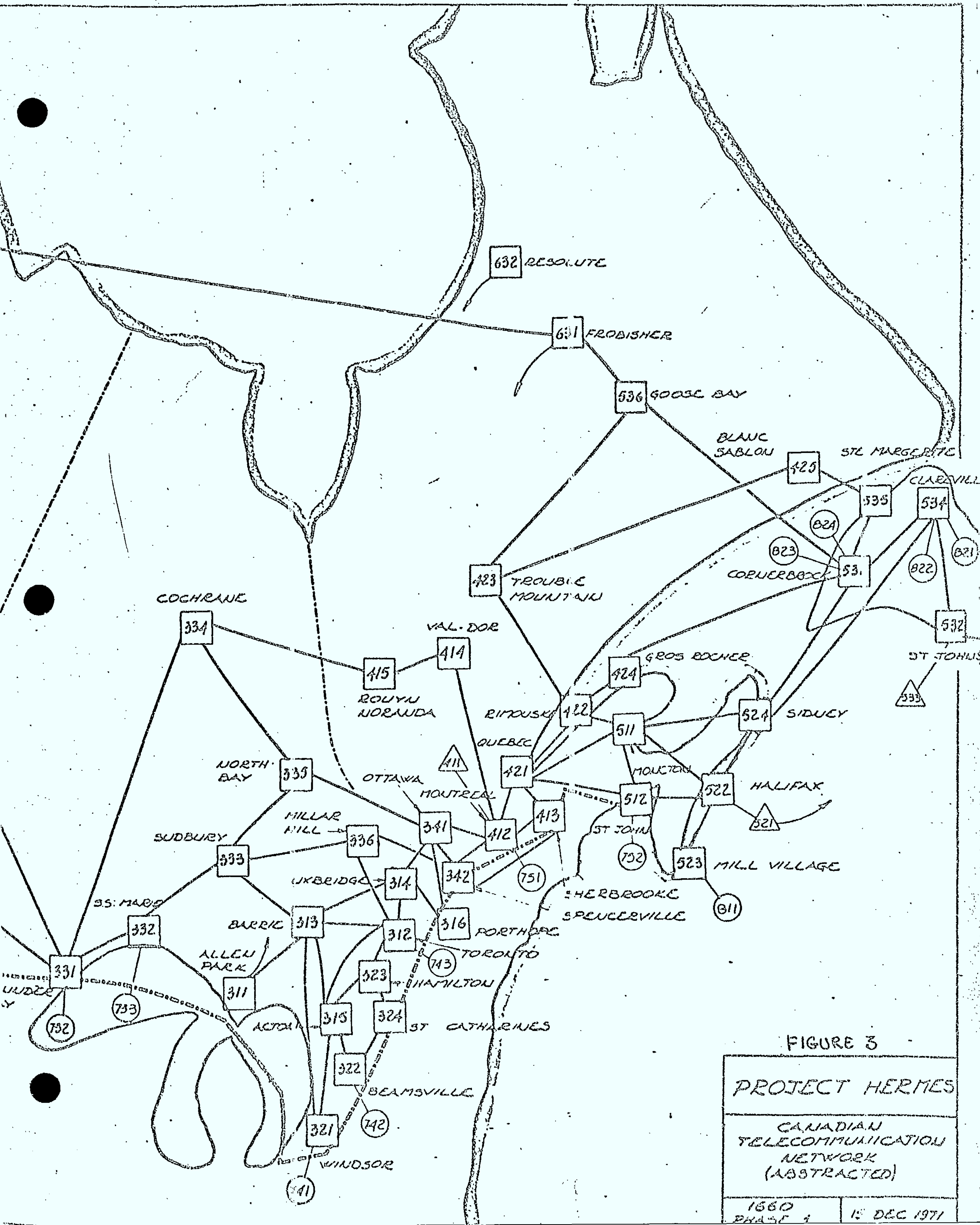
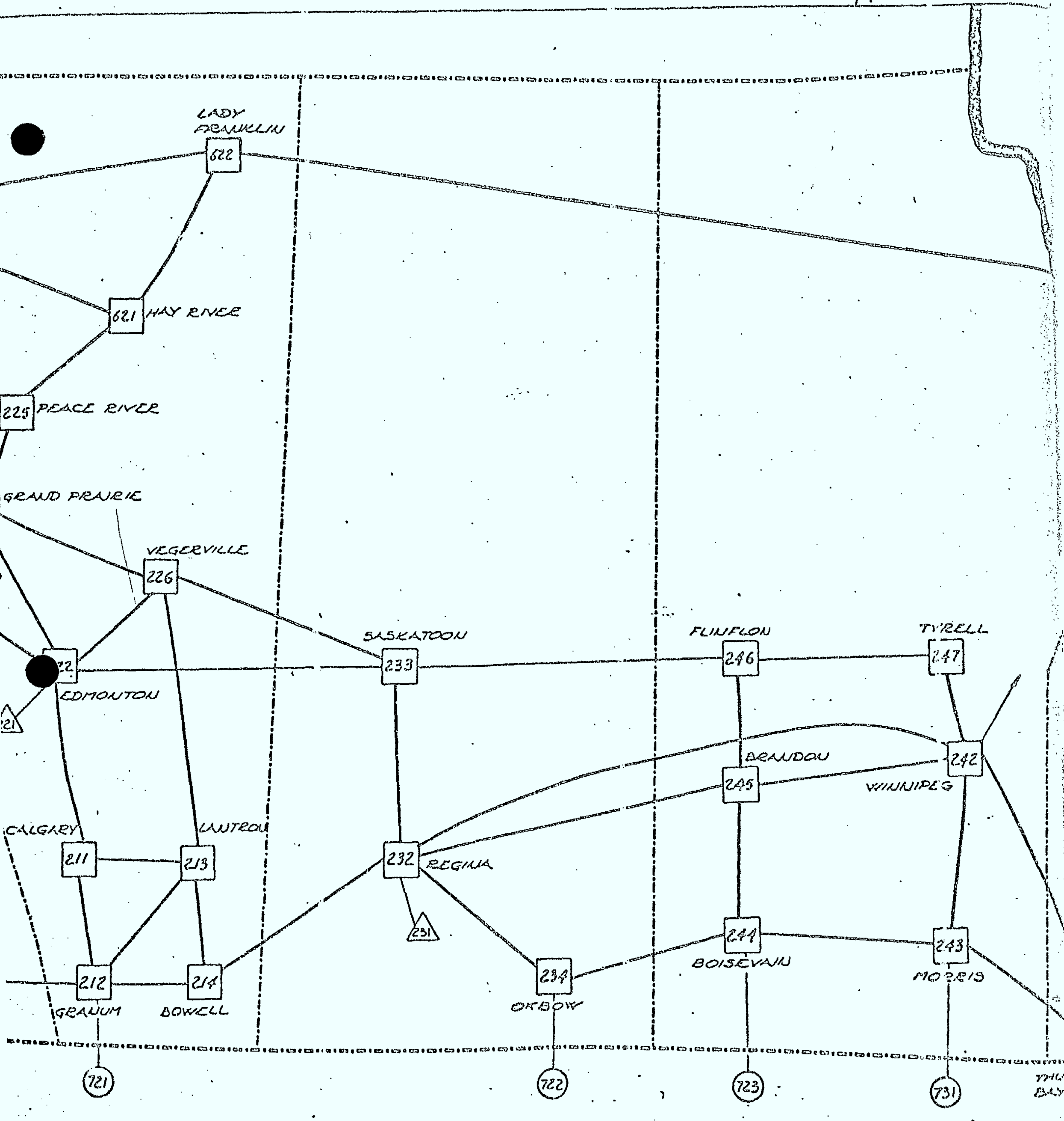


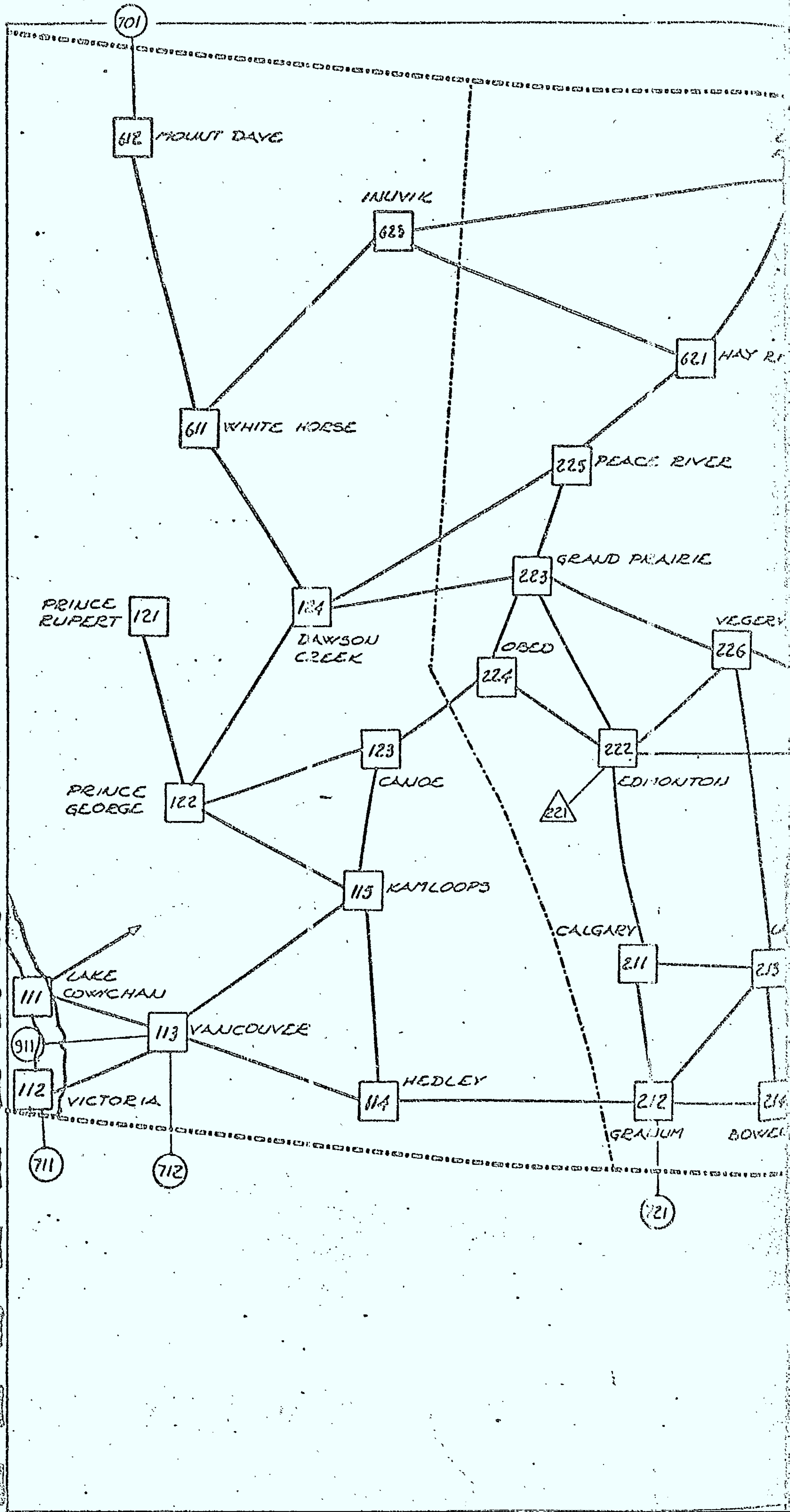
FIGURE 3

PROJECT HERMES

CANADIAN TELECOMMUNICATION NETWORK (ABSTRACTED)

1660 PHASE 1 | 15 DEC 1971


LADY FRANKLIN
622
621
HAY RIVER
225
PEACE RIVER
GRAND PRAIRIE
VEGERVILLE
226
EDMONTON
SASKATOON
233
FLINFLON
246
TYRELL
247
242
WINNIPEG
BRANDON
245
CALGARY
211
213
LANTROU
232
REGINA
231
212
GRANUM
214
BOWELL
234
OXBOW
244
BOISEVAIN
243
MORRIS
721
722
723
731



701
612 MOUNT DAVE
INUVIK
623
621 HAY RI
611 WHITE HORSE
225 PEACE RIVER
GRAND PRAIRIE
223
PRINCE RUPERT
121
124
DAWSON CREEK
OBED
224
226
123
CANOE
222
EDMONTON
221
PRINCE GEORGE
122
115 KAMLOOPS
CALGARY
211
213
LAKE COWICHAN
111
113 VANCOUVER
911
112
VICTORIA
HEDLEY
114
212
GRAHUM
711
712
721

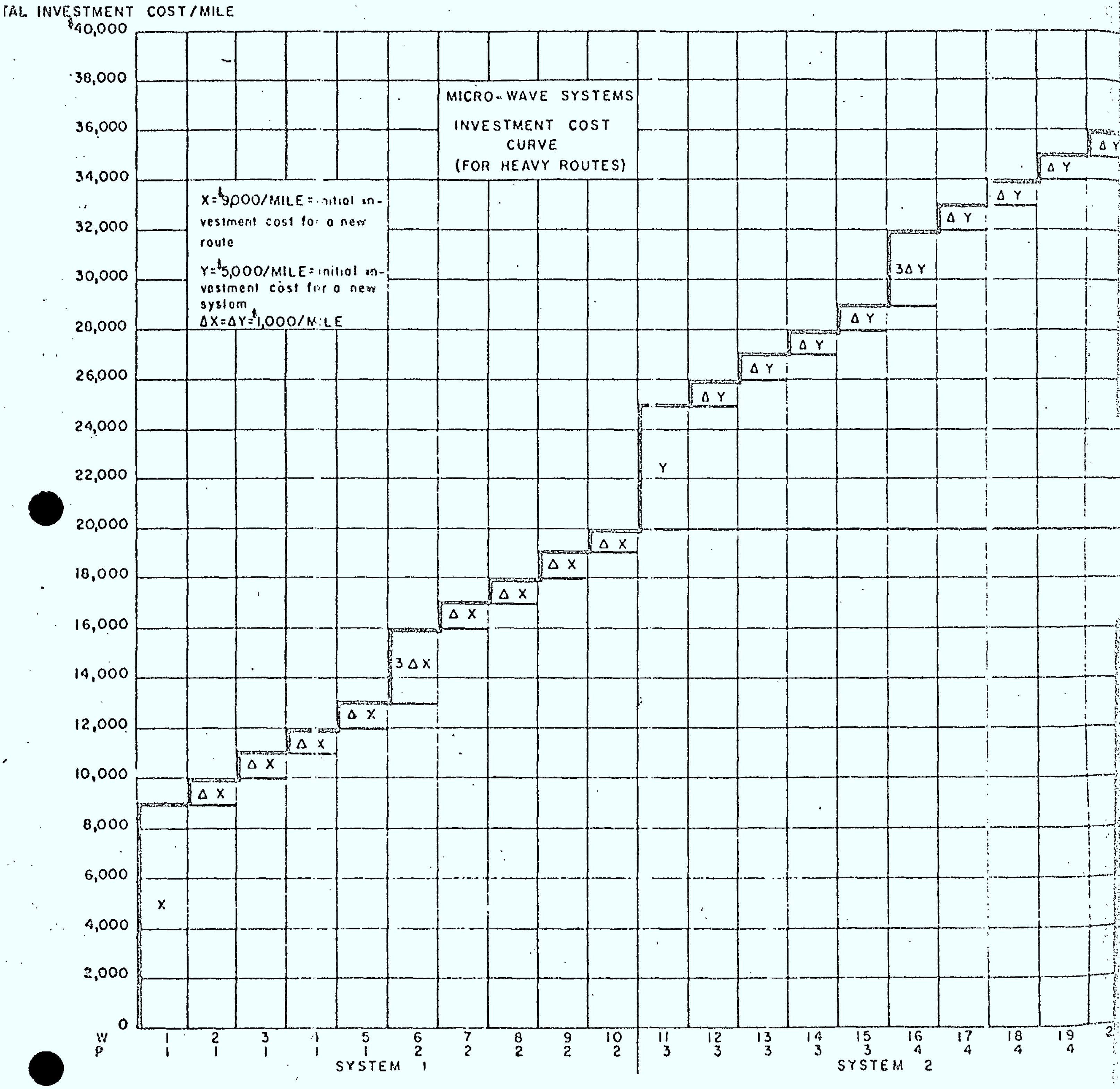


Figure 4 - Investment cost functions - Heavy routes

Figure 5 - Investment cost functions - Nodal facilities to U.S.A.

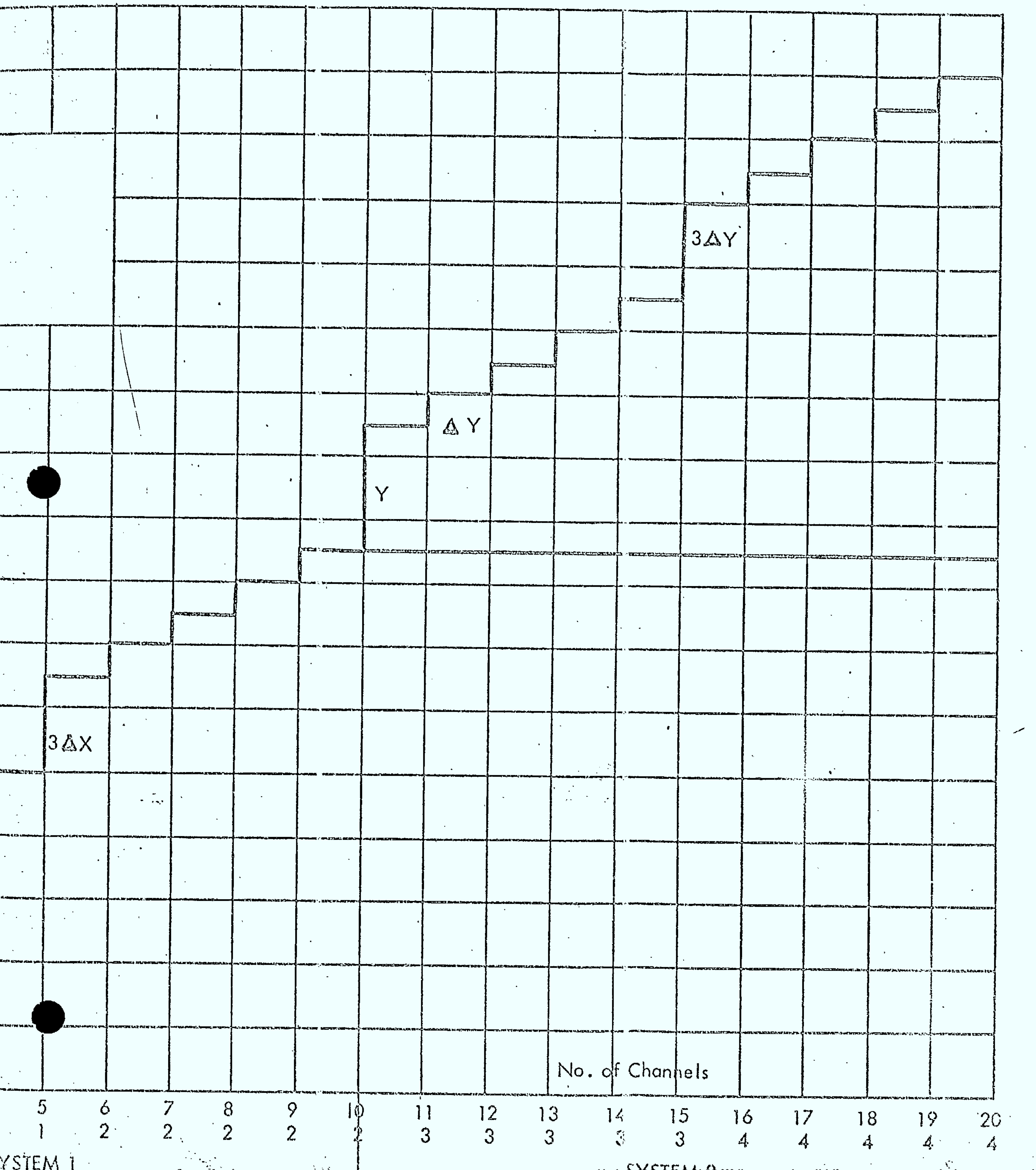
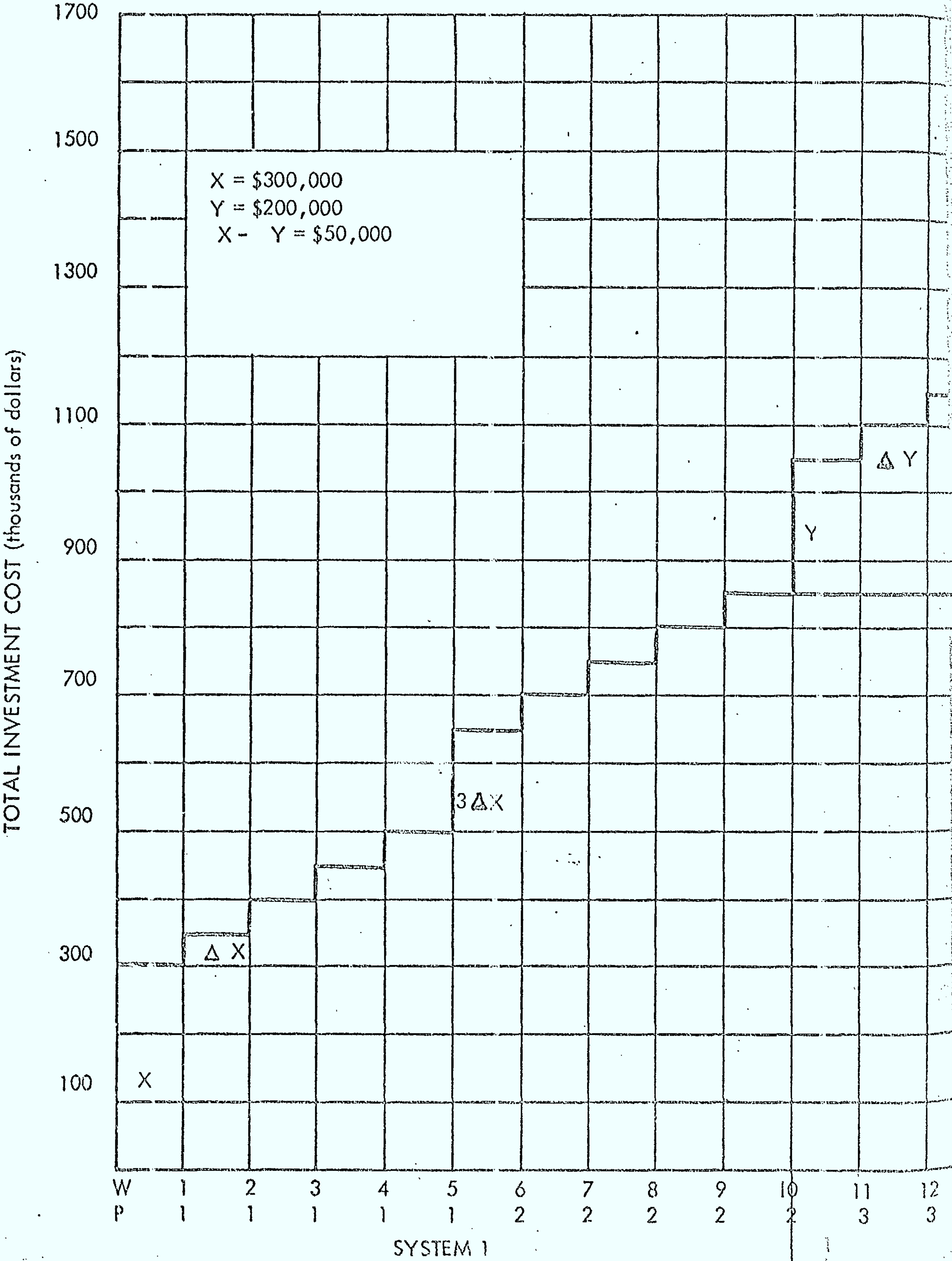


Figure 5 - Investment cost functions - Nodal



3) Specifics - any link that cannot be costed under 1 or 2 above.

4) Terminal equipment only on links to Continental U.S.A.

3.2.3 Demand

Demand as referred to in this chapter means demand for facilities expressed in service channels. In the formulation of the model, existing demand was assumed to equal supply, that is, installed capacity.

Problems are posed by choosing a pair of points and specifying a level of facility requirement between these points. Since in the network supply and demand are equal, there is no "slack" in the installed facilities and the demand must be met by creating new facilities.

Note: In actual practice, the "demand" will include a percentage of spare facilities for a number of reasons.

3.2.4 Specific exclusion rules

In order to make problem solving possible using the model, a method had to be developed to reduce the number of possible routes by which one node could be connected with another. For example, the number of possible routes between Montreal and Vancouver which the model would have to consider exceeds 30,000. All but a few hundred of these obviously should not be considered and should be eliminated.

In order to do this, exclusion rules were developed by the Department of Communications and introduced as an potion into the HERMES I models. These rules were divided into two classes: general and specific.

The more general exclusion rules are applicable to all areas and involve the following:

- A node may appear only once in any facility assignment chain.

- Crossing of an interregional boundary more than twice on any chain is not permitted.

- The satellite may be used only once in any chain.

As well as the above, there are many specific rules which apply to one area only. For example:

- Chains originating and terminating in B.C. may not involve points east of Alberta.

- Chains originating west of Thunder Bay may not pass through Rouyn-Noranda.

There are many such rules whose function is to make the size of the problem manageable.

3.3 Identification of Admissible Facilities Assignment Chains and of Upper Bounds on Capacity Expansion: CADUCEE

The objective of the CADUCEE software is to identify non-dominated nodes and admissible chains for specified levels of demand expressed in service channels between any number of pairs of demand points in the network and to identify the upper bounds on capacity expansion. The following is a detailed discussion of the program developed for this purpose (See also Figure 1).

In order to solve any particular problem, we first read from punched cards the number of pairs of demand points to be considered (NCØUP). We then read the network node numbers of each pair and the level of demand specified.

The subroutine BØRNE is then called which reads in the cost functions for facilities expansion and the data on each link of the network. These data consist of the link number, the origin and destination of the link (the nodes connected), the length in miles of the link, a code identifying the proper cost function for facilities expansion on this link, the number of channels already installed and the maximum number which can be added. For each link, the subroutine calculates the marginal cost of adding each service channel up to the level of the maximum contemplated demand increase. Once this has been done for all of the network links, the subroutine then calculates the upper and lower bounds costs for the specified level of demand increase, the "maximum contemplated demand increase".

Note: When the subroutine is called in subsequent iterations, the bounds are recalculated without recalculating the marginal costs and the specified demand increase is the "maximum relevant demand increase".

The program then begins two iterative procedures or "loops", one within the other. In the inner loop (NGT), the calculations are carried out for each pair of specified demand points in turn. For each pair (NØRG and NDEST) the minimum expansion costs are calculated at upper and at lower bounds from NØRG to every other point and from NDEST to every other point using the DØMINØ subroutine. At the option of the user, the four DØMINØ tables may be output on cards, tape, disk or printer. Dominated nodes are then eliminated and non-dominated nodes are printed out. Admissible chains are then identified and printed out. These chains are identified by means of their component link numbers.

Once the inner loop is terminated, that is the above calculations have been carried out for all pairs of specified demand points, the common elements of admissible chains are identified and the "maximum relevant demand increase" is calculated for each link of the network. The BØRNE subroutine is recalled and the upper and lower bounds are recalculated, based on this "maximum relevant demand increase". The program then re-enters the inner loop and repeats the calculations based on the new upper and lower bounds. This overall process constitutes the outer loop by which non-dominated nodes and admissible chains are identified by successive iteration.

At maximum, this outer loop is repeated a number of times corresponding to the number of pairs of demand points in the problem. However, in most cases, the number of iterations will be less and a test is made to stop the procedure when no further changes in maximum relevant demand are occurring from one iteration to the next.

The data cards of the program are set out in the following manner:

First card:

col.	1 2 3 4 5	6 7 8 9 10
	NCØUP	IMPR

NCØUP : number of pairs of points of demand
IMPR : code permitting the option of printing the cost tables ($IMPR > 0$) or not printing them ($IMPR = 0$)

Second group of cards:

col.	1 2 3 4 5	6 7 8 9 10	11 12 13 14 15
	NØRG	NDEST	NDEM

NØRG : node of origin number
NDEST : node of destination number
NDEM : value of demand for this pair

There are as many cards in this group as there are pairs of points of demand.

Third group of cards:

col. 1 2 3 4 5 | 6 7 8 9 10 | 11 12 13 14 15 | 16 17 18 19 20

X_1 ΔX Y ΔY

These are the costs of the cost function; there are four cards, one for each link category.

Fourth group of cards:

col. 1 2 3 | 4 5 6 | 7 8 9 | 10 11 12 | 13 14 ... 35 36 | 37 | 38 ... 43 | 44 ... 49

A Ø D MI LD(I) LD(12) CT x_1 Δx

A: - link number

Ø, D: - numbers of nodes joined by this link

MI: - length of link (in miles)

LD (I): - information concerning existing facilities, i.e. if X_1 exists, how many ΔX_1 exist, how many are permitted? Does X_2 exist, etc.? respectively, up to how may ΔY are permitted? Thus, I = 12.

CT: - link category (for the choice of the cost function)

x_1: Δx_1, y, Δy: same meaning as for X_1, ΔX_1, Y, ΔY.
The costs given by the variables starting with a small letter take priority over the costs given by the variables starting with a capital letter if these variables (small letters) have a value different from 0. It is thus possible to specify a cost function different from the categories of functions predicted.

This group is composed of as many cards as there are links.

The output of the CADUCEE software is variable at the option of the user. In all cases, however, the first output is the specifications of the problem, that is:

- The number of pairs of points in the problem.

- The network node numbers of each pair of points and the demand increase specified for each pair.

- The data read by the BØRNE subroutine, that is, the cost data and facilities characteristics for each link.

At the user's option, the four cost tables calculated in the DØMINØ subroutine at each iteration and for all pairs of demand points may be output on the printer or on disk, tape or punched cards.

At each iteration and for each pair of demand points, the non-dominated nodes and the admissible chains are printed.

In this first version of CADUCEE, it is possible at the option of the user to verify whether or not the admissible chains found by the methods described above are also admissible in terms of any set of specific rules for expansion of facilities. The Department of Communications has specified such rules for the network. In the software routines developed to do this, the specific rules apply not only to the end points of the chains found but also to the incomplete chains which make up the admissible chains. The cards necessary to exercise this option are as follows:

First card:

col.	1 2 3 4 5	6 7 8 0 10	11 ... 15	16 ... 20	21 ... 25	26 ... 30
	NSØM	NX	ND_1	ND_2	IMPR	ISPEC

NSØM : number of nodes in the network
NX : total number of specific rules
ND_1 : number of specific rules of the type "admissible nodes"
(ND_2-ND_1): number of specific rules of the type "non-admissible nodes"

Second card:

col.	11 12 13 14 15	16 ... 20
	NØRG	NDEST

Third group of cards:

col.	1 2 3	4 5 6	7 8 9 10 11 12
	Ø	D	C.B.S.

Ø : numbers of nodes describing a link
C.B.S. : upper bound cost

As many cards are read as there are links in the network.

Fourth group of cards:

These cards provide the information concerning the specific rules. The rules are divided into three categories:

- admissible nodes
- non-admissible nodes
- chains of the type "disjunctive constraints"

The data on these cards do not change unless modifications to the specific rules are made.

Fifth group of cards:

col. 1 2 3 4 5 6 7 ... 12

Ø D C.B.I.

C.B.I. : lower bound cost

This group is similar to the third group.

3.4 Solving the Minimum Cost Capacity Expansion Problem: TRANCHE

The software described under the general title TRANCHE consists of two parts. These can be described as the problem matrix generator and the mixed integer linear programming package. They are related to each other and to CADUCEE as shown in the flow-chart of Section 2.2 (see Figure 2).

3.4.1 Formulation of the mixed integer linear problem

Linear programming is a technique for treating problems involving complicated arrays of interacting variables. In mixed integer linear programming we add a further complexity in that some of the variables must take on only integer values. In our discussion of the formulation of such a problem, however, we can treat all variables in the same manner as long as we bear in mind that the stipulated variables must in the solution be integer.

In our problem, we are concerned with elements of four kinds: demand, capacity, the order in which facilities are built, and the cost. We will therefore introduce three types of inequalities or rows in the problem matrix and the objective function, or cost row, to handle them.

The variables with which we are dealing or the columns of the problem matrix are of two types: facility assignment variables and capacity expansion variables.

The rows and columns of the problem matrix are related by entries in the body of the matrix called coefficients.

The form of the matrix is as follows:

			CHAIN			FACILITIES								
			A B	B C	etc.	AB 1	2	3	4	BC 1	2	3	4	
I	Demand	A to B	1											≥ Demand AB
		B to C		1										≥ Demand BC
		etc.												etc.
II	Capacity	Link AB	1			-1	-1	-1	-1					≤ 0
		Link BC		1						-1	-1	-1	-1	≤ 0
		etc.												etc.
III	Expansion order	AB 1								9	-1			≥ 0
		2								1	1	-5		≥ 0
		3										9	-1	≥ 0
IV	Cost					$	$	$	$	$	$	$	$	

ROWS: As we have already stated, there are four types of rows related to demand, supply or capacity, expansion order and cost.

The demand rows are shown in the problem matrix above identified as I. The inequalities take the form

$$a_{ij} \geqslant X$$

where

- a is the coefficient (1 in most cases)
- i is the row associated with demand between the pair in question
- j is the route column associated with the demand in question
- X is the value of demand in service channels between the pair in question.

The capacity rows are identified as II in the problem matrix. The inequalities take the form:

$$a_{nj} + \sum_{\forall k} b_{nk} \leqslant 0$$

where

- a is the coefficient relating demand with capacity
- n is the row associated with capacity on the link in question
- j is the column associated with the route in question
- b is the coefficient relating capacity with facilities between the pair in question. Note this coefficient is negative.
- k is the column associated with a particular stage of facilities development between the pair in question. Note that k assumes a range of values as there are several stages of development of facilities between any pair.

The order in which facilities are built requires several rows for the facilities between each pair. One less row is required than the number of stages of development of facilities between the pair. The inequality is of the form:

$$a_{ij} + b_{ik} \geqslant 0$$

where

- a is a positive coefficient
- i is the row associated with a particular stage of development of facilities between a pair
- j is the column associated with the stage of development in question
- b is a negative coefficient
- k is the column associated with the next stage of development of facilities between the pair in question.

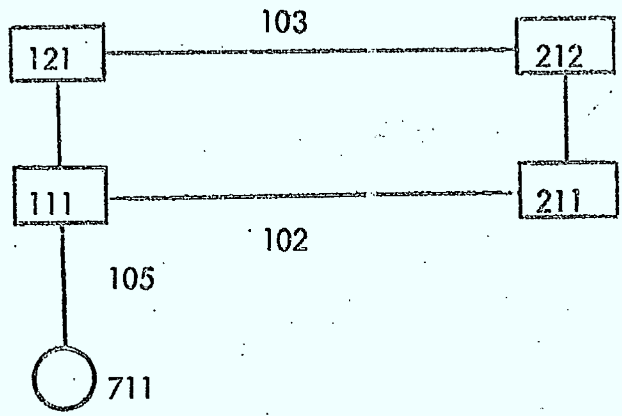
The objective function which we seek to minimize is shown in the matrix above as IV. We are seeking to minimize:

$$\sum_{\forall j} c_j$$

where c is the cost coefficient of a particular column j and the sum is taken over all values of j.

3.4.2 Setting up and solving a problem

For purposes of clarity in outlining the processes and operation of the setting up and solution of a problem using the model, we shall confine the discussion in this section to a highly simplified network consisting of five nodes and five links as follows:



The problem is that we wish to introduce an additional 15 service channels between 111 and 212. It should be noted that the network shown and the problem posed is trivial and is in no way intended to show the power and versatility of the software developed. Rather it was chosen so that:

- It effectively shows the relationships among the variables, the coding structure, etc., without being unnecessarily burdensome to read.

- The problem posed can be easily solved by hand calculation and/or a little common sense.

The coding structure of the nodes and links follows the same scheme as in any larger network.

NODES: - The first digit represents the Region.

The regions are:		
	B.C.	1
	Prairies	2
	Ontario	3
	Quebec	4
	Maritimes	5
	North	6
	U.S.	7
	East Coast Foreign	8
	West Coast Foreign	9

- The second digit represents the sub-region. This can be a North-South and/or Provincial split as required.

- The third digit is assigned sequentially within sub-region.

LINKS: Links are numbered sequentially from 101 upward.

The input data for the sample network were conceived in the same format and level of detail as provided for the HERMES I model. The data are as follows:

From Node No.	To Node No.	Mileage	Installed capacity	Maximum capacity	Expansion category	Notes
111	121	125	1	3	2	
111	211	150	4	10	1	
121	212	60	2	10	1	
211	212	40	0	10	3	First channel cost = $570,000
111	711	-	1	10	4	

Note: The Expansion Category column is used to determine which of the cost functions applies to the addition of facilities on the link.

In order to get these data into the computer, they are punched on cards. A standard form was developed for this and is shown completed with the data for our sample network, in Figure 6.

A word of explanation on the column headings is perhaps in order. The first four headings are obvious in their meaning. The next twelve can be divided into four groups, X1, X2, Y1 and Y2 corresponding to the four major steps in the cost curves.

The first step is the installation of the first channel on a new chain. This is what column X1 indicates. If this step has been taken, that is, if there is some installed capacity, the value of this column will be one.

Following this step, up to four additional channels may be added without encountering the next step. This is what column IDX1 indicates; the number of installed channels between the first and second step. The total number of channels which can be installed in step one is shown in column LX1. This is usually five.

Figure 6

Computer Input Form
Sample Network

LINK	FR.	TO	MILES	X 1	I D X 1	L X 1	X 2	I D X 2	L X 2	Y 1	I D Y 1	L Y 1	Y 2	I D Y 2	L Y 2	E X P	$X	$ΔX	$Y	$ΔY	
101	111	121	125	0	0	5	0	0	5							2					*
102	111	211	150	1	3	5	0	0	5	0	0	5	0	0	5	1					
103	121	212	60	1	1	5	0	0	5							1					
104	211	212	40	0	0	5	0	0	5							3	570				+
105	111	711	1	1	0	5	0	0	5							4					

* Link 101 is a light route. Installed capacity is therefore assumed to be zero.

\+ Link 104 is a special route having a cost for the first channel of $570,000.

Note: In this example, expansion does not go beyond 10 channels per link except for link 102.

The next three columns X2, IDX2 and LX2 are the same as the above but for the second step. The next six columns are for steps three and four.

The last four columns on the form are for special cost values which are required for expansion category three.

The deck of cards produced from these data is shown in the overall flow chart (in section 2.2) as Link Data.

The Link Data deck of cards is fed into the computer to undergo what is called the Problem Matrix Generator phase. In this phase, the raw data are used to generate all of the rows and columns of the "basic" matrix required for the optimization phase as well as the values of "coefficients" which make up the matrix. This matrix is shown in Figure 7.

As outlined in the section on problem formulation, there are three types of rows or "constraints" used in this formulation. The first type is the demand rows. These are prefixed by "D" in the matrix in Figure 7 and take the form DIJ: where I and J are three-digit node numbers. Thus D111 121 is demand between nodes 111 and 121. In our example network, this demand must be greater than or equal to zero, the installed capacity on this link. It should be noted that, wherever we refer to demand we mean demand for facilities in service channels.

The second type of row is capacity. These are of the form CI where I is a three-digit link number. Thus C101 in our example means the capacity of link 101.

The third type of row represents the sequencing constraints on the addition of facilities. These take the form NMI where N facility must be built before M facility on link I. Thus, in our example, X1 DX1101 states that X1, the first channel, must be installed before DX1, the four remaining channels in step 1, on link 101.

There are two types of columns in the formulation shown. The first type are the route columns. These are of the form RIJK, where R is the prefix for chain I is the chain number assigned serially, and J and K are the node numbers connected by this chain. Thus R1 111 112 is chain number 1 between nodes 111 and 112.

An important point should be noted here.

- In the basic matrix, only one chain exists between each pair of nodes, corresponding to the direct link between those nodes.

The second type of column represents the facilities on a link. These take the form MI, where M is the facility installed or to be considered for addition and I is the link number. Thus, in our example, X1 101 represents facility X1, which we know is the first channel of a new chain and 101 is the link number.

D X_2 10^3	Y_1 10^3	I D Y_1 10^3	D Y_1 10^3	Y_2 10^3	I D Y_2 10^3	D Y_2 10^3		X_1 10^4	I D X_1 10^4	D X_1 10^4	X_2 10^4	I D X_2 10^4	D X_2 10^4		X_1 10^5	I D X_1 10^5	D X_1 10^5	X_2 10^5	I D X_2 10^5	D X_2 10^5		SIGN	R.H.S
																						≥	0
																						≥	4
																						≥	2
																						≥	0
																						≥	1
																						≤	0
																						≤	0
-1	-1	-1	-1	-1	-1	-1																≤	0
								-1	-1	-1	-1	-1	-1									≤	0
															-1	-1	-1	-1	-1	-1		≤	0
																						≥	0
																						≥	0
																						≥	0
																						≥	0
																						≥	0
																						≥	0
																						≥	0
																						≥	0
-1																						≥	0
1	-5																					≥	0
	9		-1																			≥	0
	1	1	1	-5																		≥	0
				9		-1																≥	0
								9		-1												≥	0
								1	1	1	-5											≥	0
											9		-1									≥	0
															9		-1					≥	0
															1	1	1	-5				≥	0
																		9		-1		≥	0
ΔX 10^3	10^3	0	ΔY 10^3	$3\Delta Y$ 10^3	0	ΔY 10^3		500,000	0	ΔX 10^4	$3\Delta X$ 10^4	0	ΔX 10^4		0	0	ΔX 10^5	$3\Delta X$ 10^5	0	ΔX 10^5		NA	MINIMIZE
0 4	0 1	0 0	0 4	0 1	0 0	0 4																	

	R1 $^{1}1_{1}$ $^{1}2_{1}$	R1 $^{1}1_{1}$ $^{2}1_{1}$	R1 $^{1}2_{1}$ $^{2}1_{2}$	R1 $^{2}1_{1}$ $^{2}1_{2}$	R1 $^{1}1_{1}$ $^{7}1_{1}$	X_1 $^{1}0_{1}$	I $^{D}X_1$ $^{1}0_{1}$	$^{D}X_1$ $^{1}0_{1}$	X_2 $^{1}0_{1}$	I $^{D}X_2$ $^{1}0_{1}$	$^{D}X_2$ $^{1}0_{1}$	X_1 $^{1}0_{2}$	I $^{D}X_1$ $^{1}0_{2}$	$^{D}X_1$ $^{1}0_{2}$	X_2 $^{1}0_{2}$	I $^{D}X_2$ $^{1}0_{2}$	$^{D}X_2$ $^{1}0_{2}$	X_1 $^{1}0_{3}$	I $^{D}X_1$ $^{1}0_{3}$	$^{D}X_1$ $^{1}0_{3}$	X_2 $^{1}0_{3}$	I $^{D}X_2$ $^{1}0_{3}$	$^{D}X_2$ $^{1}0_{3}$
	1																						
		1																					
			1																				
				1																			
					1																		
	1					-1	-1	-1	-1	-1	-1												
		1										-1	-1	-1	-1	-1	-1						
			1															-1	-1	-1	-1	-1	-1
				1																			
					1																		
1						9		-1															
1						1	1	1	-5														
1									9		-1												
2												9		-1									
2												1	1	1	-5								
02															9		-1						
03																		9		-1			
03																		1	1	1	-5		
03																					9		-1
03																					1	1	1
03																							
03																							
03																							
04																							
04																							
04																							
05																							
05																							
05																							
						X $^{1}0_{1}$	0	ΔX $^{1}0_{1}$	$^{3}\Delta X$ $^{1}0_{1}$	0	ΔX $^{1}0_{1}$	0	0	ΔX $^{1}0_{2}$	$^{3}\Delta X$ $^{1}0_{2}$	0	ΔX $^{1}0_{2}$	0	0	ΔX $^{1}0_{3}$	$^{3}\Delta X$ $^{1}0_{3}$	0	ΔX $^{1}0_{3}$
UP						0	0	0	0	0	0	1	1	0	0	0	0	1	3	0	0	0	0
LO						1	0	4	1	0	4	1	1	3	1	0	4	1	3	1	1	0	4

Figure 7. Basic Problem Matrix

	R1 111 121	R1 111 211	R1 121 212	R1 211 212	R1 111 711		X1 101	I DX1 101	DX1 101	X2 101	I DX2 101	
D 111 121	1											
D 111 211		1										
D 121 212			1									
D 211 212				1								
D 111 711					1							
C 101	1						-1	-1	-1	-1	-1	
C 102		1										
C 103			1									
C 104				1								
C 105					1							
X1 DX1 101							9		-1			
X1 X2 101							1	1	1	-5		
X2 DX2 101										9		
X1 DXL 102												
X1 X2 102												
X2 DX2 102												
X1 DX1 103												
X1 X2 103												
X2 DX2 103												
X2 Y1 103												
Y1 DY1 103												
Y1 Y2 103												
Y2 DY2 103												
X1 DX1 104												
X1 X2 104												
X2 DX2 104												
X1 DX1 105												
X1 X2 105												
X2 DX2 105												
COST							X 101	0	ΔX 101	3ΔX 101	0	[illegible]
BOUNDS UP							0	0	0	0	0	[illegible]
LO							1	0	4	1	0	[illegible]

The right hand side of the basic matrix is identified as "R.H.S." in Figure 7. The right hand side shows the limit of the value of a row in the final solution. The relationship of the row to the R.H.S. is shown in the "Sign" column. For example, row D111 211 (the demand between node 111 and node 211) is specified to be greater than or equal to four.

The bounds serve the same function for the columns that the R.H.S. serve for the rows; that is, they specify the limits of the values of a column in the final solution. The bounds, however, unlike the R.H.S., express both an upper and lower limit. Where this upper and lower limit are equal, the value of the column is fixed. Where bounds are not specified, the column value may be anything.

The rows, columns, right hand side, and bounds are related and interact through the entries shown in the body of the matrix. Let us take a very simple example of this to show the principle. Suppose we have the following matrix:

	R1 301 302	X1 125	IDX1 125	DX1 125	Sign	RHS
D301 302	1				≥	2
C 125	1	-1	-1	-1	≤	0
X1DX1 125		9		-1	≥	0
Cost		0	0	10	N.A.	Min.
Bounds Up		1	1	0		
Lo		1	1	3		

This represents a case where the demand on Link 125 between 301 and 302 is two channels and the installed capacity is two channels.

Demand is shown in the first row of the matrix as being required to be greater than or equal to 2, the R.H.S. The interrelationship between demand and capacity comes about through the two "coefficients" in column R 1 301 302. One must bear in mind that all columns (and therefore all coefficients in a column) are initially multiplied by zero and are not "active" unless the multiplier is changed to satisfy a requirement, so that the corresponding variables do not appear in the current solution.

There is only one coefficient in the D 301 302 row and it is equal to one. Therefore, if the relationship expressed by the sign and the R.H.S. is to hold true, this coefficient must be multiplied by two (at minimum). To do this, we must multiply the column R 1 301 302 by 2. This, then, makes the coefficient (in this column) of C 125 equal to 2 as well.

The relationship in row C 125 must now be checked. The requirement is that C 125 must be less than or equal to 0. In order to meet this requirement, we must bring more columns into play. We see that the coefficients of column X 1 125, IDX 1 125 and DX 1 125 in row C 125 are all equal to minus one. We could thus satisfy the requirement by multiplying one of them by two; or two of them by one; or in fact, any combination of one or more of them by any number.

An examination of the bounds, however, determines our course of action. We see that the bounds on column X1 125 and IDX1 125 are both one. This means that both of these rows must be multiplied by one and our requirement is satisfied. This is in fact the manner in which existing capacity is introduced at no cost. By adjusting the bounds, the proper columns are called up in the solution, that is, they are multiplied by a number other than zero.

We have therefore multiplied columns X1 125 and IDX1 125 by one. What effect does this have on other rows? Column X1 125 has a coefficient in row X1 DX1 125 equal to 9. Since 9 is greater than or equal to zero, the requirement of this row, the condition is satisfied.

Since there are no more rows, all conditions are satisfied and we can compute the cost. The columns which are now non-zero are R1 301 302, X1 125 and ID X1 125. R1 301 302 has no coefficient in the cost row and therefore we do not consider it. The coefficient of X1 125 and ID 125 in the cost row are equal to zero and therefore the cost of this solution is zero.

This far, we have been concerned only with the setting up of the "basic" matrix. However, the basic matrix does not represent a problem. The posing of a problem comes from the output of CADUCEE in the form of the list of Admissible Chains, the pairs between which new demand is specified and the level of this new demand.

In our very simple problem, the output of CADUCEE would be the two admissible chains

111 - 121 - 212

and 111 - 211 - 212

We now have our two chains and a stipulated level of demand between 111 and 212 and we must get this into the problem matrix. To do this, the problem matrix generator must add more rows and columns.

First, in order to deal with the demand, it must add a demand row. This row will be D 111 212 and it must be greater than or equal to 15 as per our problem. Capacity will have to be satisfied by adding capacity to existing links, so we do not have to add any capacity rows.

In order to relate demand with capacity, however, two new facility assignment columns must be added. These will be, using our already established convention, R1 111 212 and R2 111 212. The changed portion of the basic matrix showing the added rows and columns and the coefficients through which they interact with the other elements is shown as follows:

							ADD			
		R1 111 121	R1 111 211	R1 121 212	R1 211 212	R1 111 711	R1 111 212	R2 111 212	Sign	R.H.S.
	D 111 121	1							≥	0
	D 111 211		1						≥	4
	D 121 212			1					≥	2
	D 211 212				1				≥	0
	D 111 711					1			≥	1
Add.	D 11 212						1	1	≥	15
	C 101	1					1		≤	0
	C 102		1					1	≤	0
	C 103			1			1		≤	0
	C 104				1			1	≤	0
	C 105					1			≤	0

The circled coefficients have been added. Note that since demand can be satisfied between 111 and 212 by either one of the two new chains, a coefficient is added in both chain columns. Note as well that chain R1 111 212 uses capacity on link 101 and 103 and, therefore, is shown to effect both of these rows. In the same way, route 2 effects row C 102 and C 104. The problem can now be solved.

Using the problem matrix, the minimum cost solution or expansion program is sought subject to all the constraints. The problem matrix is fed into the Mixed Integer Linear Programming Package.

The first solution sought is the continuous or linear programming solution. In this solution, the requirement that the variables take only integer values is not adhered to. Once this solution is found by iteration, the branch and bound method described earlier is used to find the optimal mixed-integer solution.

4. FIRST RESULTS

4.1 Trial Problems

Once the CADUCEE and TRANCHE models had been made operational, it was deemed desirable to test them on the network, solving realistic problems. The trial problem selected was:

> Determine the optimal capacity expansion program for increased demand, as follows:
>
> Montreal-Toronto 2 service channels
> Toronto-Halifax 2 service channels

An additional problem was posed to prove the effectiveness of CADUCEE. This was:

> Determine the non-dominated links and nodes for the following increased demand:
>
> Montreal-Vancouver 2 service channels

The results of these problems were presented to the Department of Communications at a meeting on November 1st, 1971.

4.2 Input Data

The cost and capacity data for each link are calculated and input into the computer. The total cost for a channel addition is obtained by multiplying the distance for the link by the appropriate unit cost of the category. The unit cost data used is shown in Table 4-1. Both CADUCEE and TRANCHE rely on this data as basic input.

TABLE 4-1

COST CATEGORY DATA

Category No.	Unit costs			
	X	DX	Y	DY
1	9	1	5	1
2	5	1	0	0
3	9	1	5	1
4	300	50	200	50
5	200	50	0	0

Units: Categories 1, 2 and 3: $1,000/mile/channel
Categories 4 and 5: $1,000 per channel

4.3 Toronto-Montreal-Halifax Problem

4.3.1 CADUCEE

A demand increase of 2 service channels is hypothesized between Montreal-Toronto, and Toronto-Halifax. Thus, the initial maximum contemplated demand increase will be 4 service channels and the maximum relevant demand increase in parts of the network (i.e. somewhere East of Montreal, where Toronto-Montreal demand would not be routed) will be 2 service channels.

a) Dominated nodes:

The CADUCEE portion was run initially including all nodes and links in the network. That is to say there were 95 nodes, 124 links and some 10,000 chains. After two iterations, for 4 channels and then for 2 channels where applicable, the network had been reduced to 25 nodes and 37 links through the determination of non-dominated nodes.

The non-dominated nodes are listed in Table 4-2 for each demand pair, and illustrated in Figure 8 for the combined problem. The region of interest is defined by these nodes and links. The minimum cost routes from origin to destination at lower and upper bounds were computed and are given in Tables 4-3 and 4-4, respectively.

At this stage, also, data was produced on the cost of the minimum cost chain between each node and the origin and destination. These are the DØMINØ tables and are shown in Table 4-5.

b) Admissible chains:

In the second stage of CADUCEE, all possible chains, costed at lower bounds, were compared with the least cost chain costed at upper bounds, to select the admissible chains. At the end of this stage, the problem had been reduced to 15 nodes, 23 links and 47 chains. Of the 47 chains, 40 were for the Toronto-Halifax demand, 7 for Toronto-Montreal. These chains are listed in Table 4-6.

It is to be noted that for 4 service channels between Toronto and Halifax, there would be 306 chains rather than the 40 obtained with only two channels.

The links from which the admissible chains are formed, and the associated nodes, are shown in Figure 9.

TABLE 4-2

Trial Problem - Non-Dominated Nodes
Toronto-Montreal (2 service channels); Toronto-Halifax (2 service channels)

TORONTO-MONTREAL (4 service channels)

Node No.	Name
312	Toronto
313	Barrie
314	Uxbridge
315	Acton
316	Port Hope
322	Beamsville
323	Hamilton
336	Millar Hill
341	Ottawa
342	Spencerville
412	Montreal

TORONTO-HALIFAX (4 or 2 service channels)

Node No.	Name
311	Allen Park
312	Toronto
313	Barrie
314	Uxbridge
315	Acton
316	Port Hope
322	Beamsville
323	Hamilton
324	St. Catharines
333	Sudbury
335	North Bay
336	Millar Hill
341	Ottawa
342	Spencerville
412	Montreal
413	Sherbrooke
421	Quebec
422	Rimouski
423	Trouble Mountain
424	Gros Rocher
511	Moncton
512	St. John
522	Halifax
523	Mill Village
524	Sydney

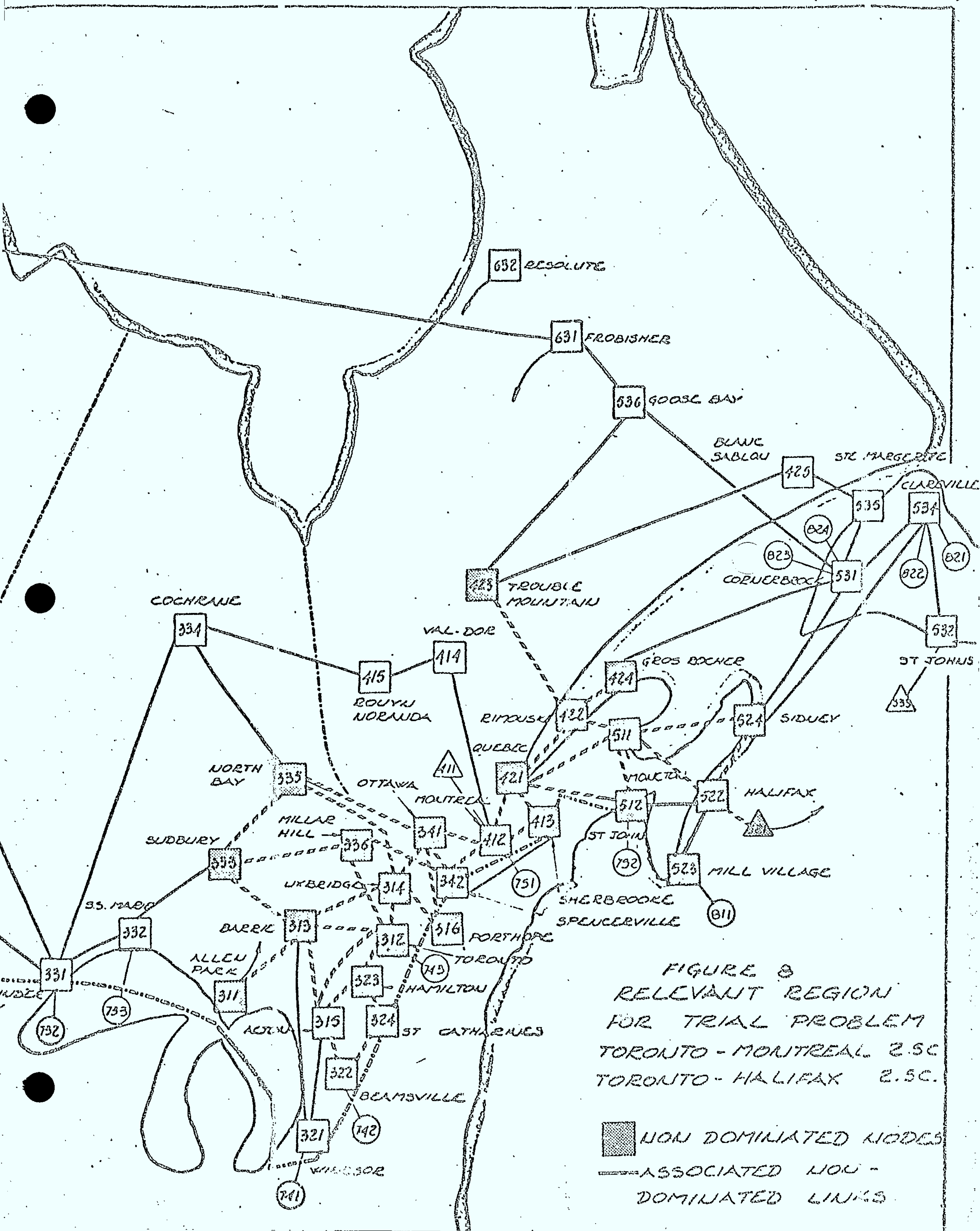
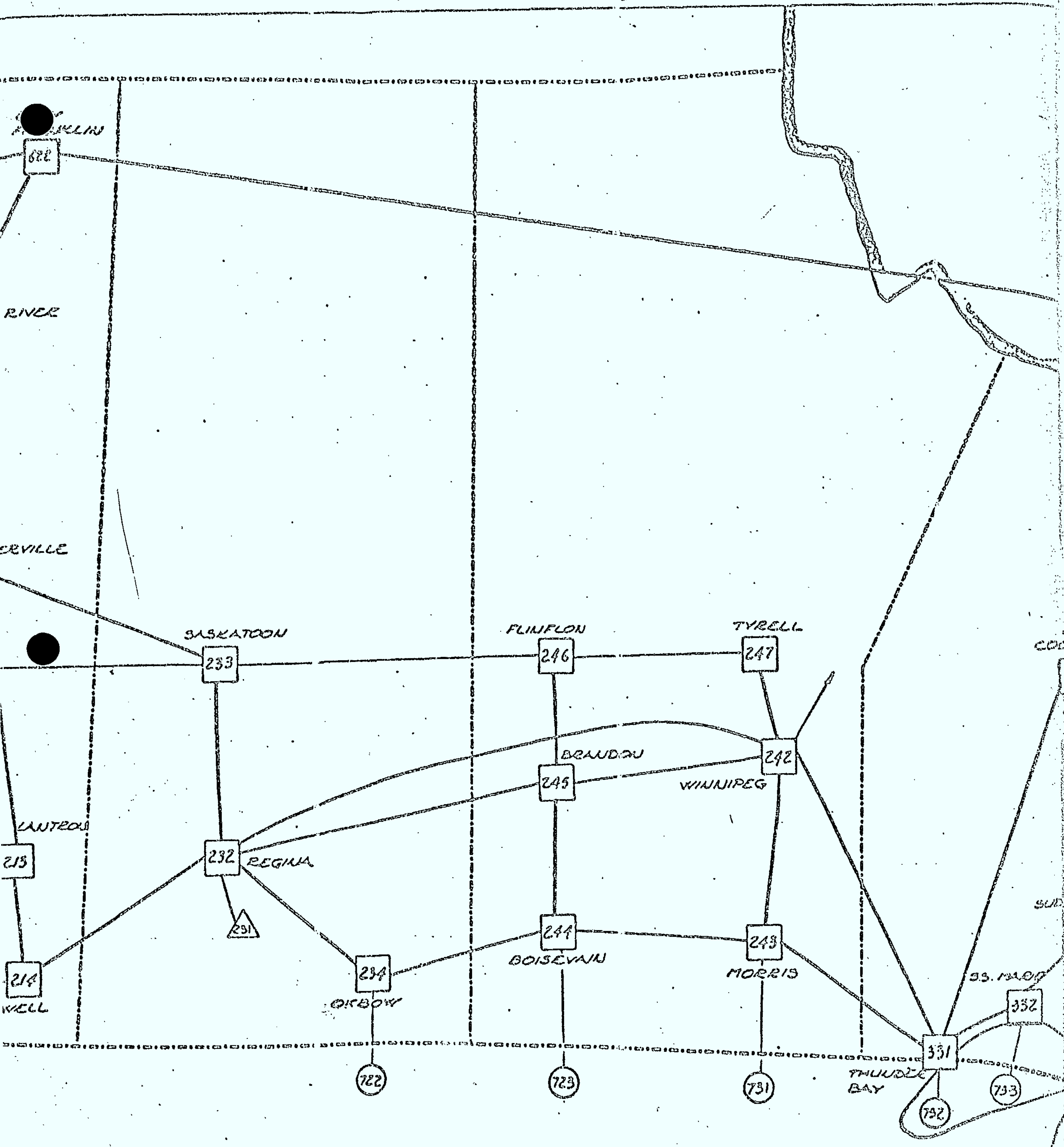
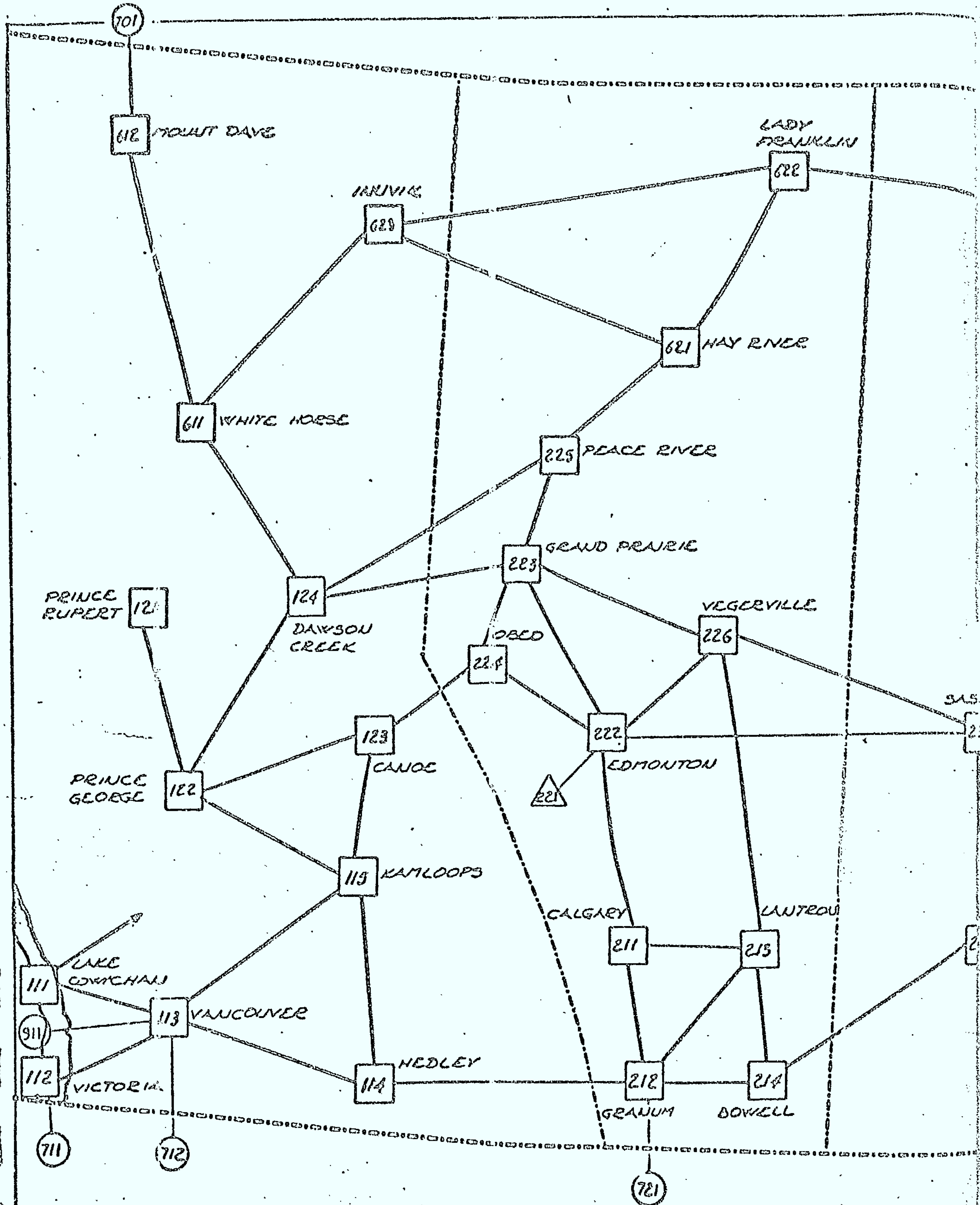


FIGURE 8
RELEVANT REGION FOR TRIAL PROBLEM
TORONTO - MONTREAL 2.5C
TORONTO - HALIFAX 2.5C.


622
RIVER
SASKATOON
233
FLINFLON
246
TYRELL
247
BRANDON
245
242
WINNIPEG
213
232
REGINA
291
214
234
OXBOW
244
BOISEVAIN
243
MORRIS
722
723
731
331
THUNDER
BAY
332
732
733



701
612 MOUNT DAVE
INUVIK
623
LADY FRANKLIN
622
621 HAY RIVER
611 WHITE HORSE
225 PEACE RIVER
223 GRAND PRAIRIE
PRINCE RUPERT
124
DAWSON CREEK
VEGERVILLE
226
224
OBED
123
CANOE
222
EDMONTON
221
PRINCE GEORGE
122
115 KAMLOOPS
CALGARY
211
215
111
LAKE COWICHAN
911
113 VANCOUVER
112
VICTORIA
114
HEDLEY
212
GRANUM
214
DOWELL
711
712
721

TABLE 4-3

Trial Problem - Minimum Cost Chain (at lower bounds)

a) TORONTO-HALIFAX (4 or 2 service channels)

Node No.	Name
312	Toronto
314	Uxbridge
341	Ottawa
412	Montreal
421	Quebec
512	St. Johns
522	Halifax

Minimum cost (at lower bounds): $1,030,000.

b) TORONTO-MONTREAL (4 service channels)

Node No.	Name
312	Toronto
314	Uxbridge
341	Ottawa
412	Montreal

Minimum cost (at lower bounds): $310,000.

TABLE 4-4

Trial Problem - Minimum Cost Chain (at upper bounds)

a) TORONTO-HALIFAX (4 or 2 service channels)

Node No.	Name
312	Toronto
336	Millar Hill
342	Spencerville
412	Montreal
421	Quebec
422	Rimouski
511	Moncton
522	Halifax

Minimum cost (at upper bounds): $1,480,000.

b) TORONTO-MONTREAL (4 service channels)

Node No.	Name
312	Toronto
336	Millar Hill
342	Spencerville
412	Montreal

Minimum cost (at upper bounds): $440,000.

TABLE 4-5

An Example of DØMINØ Tables
for Trial Problem

TORONTO-MONTREAL (4 s.c.)
TORONTO-HALIFAX (4 or 2 s.c.)

Node No.	Node Name	Minimum costs ($1,000)			
		At Upper Bound		At Lower Bound	
		From Toronto	From Montreal	From Toronto	From Montreal
113	Vancouver	2 980	3 100	2 580	2 650
312	Toronto	0	440	0	310
341	Ottawa	380	150	200	110
412	Montreal	440	0	310	0
522	Halifax	2 610	2 355	1 029	719
532	St. Johns	4 385	4 130	1 417	1 107
		From Toronto	From Halifax	From Toronto	From Halifax
113	Vancouver	2 935	4 115	2 580	3 370
312	Toronto	0	1 480	0	1 030
341	Ottawa	380	1 190	200	830
412	Montreal	440	1 040	310	720
522	Halifax	1 480	0	1 030	0
532	St. Johns	3 100	1 980	1 420	600

TABLE 4-6

Admissible Chains for Trial Problems

Toronto-Montreal (2 s.c.)
Toronto-Halifax (2 s.c.)

a) Toronto (312) to Montreal (412) (4 s.c.)

No. of Links	From	To	NODES IN CHAINS					
3	312	412	312	314	342	412		
3	312	412	312	336	342	412		
4	312	412	312	313	314	341	412	
4	312	412	312	314	316	341	412	
4	312	412	312	314	341	342	412	
5	312	412	312	313	314	341	342	412
5	312	412	312	314	316	341	342	412

TABLE 4-6 cont'd

(b) Toronto – Halifax (cont'd)

No. of Links	From	To	NODES IN CHAINS
8	312	522	312 314 341 342 412 421 422 511 522
8	312	522	312 314 341 342 412 421 511 512 522
8	312	522	312 314 341 342 412 421 512 511 522
8	312	522	312 314 341 412 413 421 422 511 522
8	312	522	312 314 341 412 413 421 512 511 522
8	312	522	312 314 341 412 421 422 511 512 522
8	312	522	312 314 341 412 421 422 511 524 522
8	312	522	312 314 341 412 421 512 511 524 522
8	312	522	312 336 342 412 421 422 511 512 522
9	312	522	312 313 314 341 342 412 421 422 511 522
9	312	522	312 313 314 341 342 412 421 512 511 522
9	312	522	312 313 314 341 412 421 422 511 512 522
9	312	522	312 314 316 341 342 412 421 422 511 522
9	312	522	312 314 316 341 342 412 421 512 511 522
9	312	522	312 314 316 341 412 421 422 511 512 522
9	312	522	312 314 341 342 412 421 422 511 512 522
9	312	522	312 314 341 342 412 421 422 511 524 522
9	312	522	312 314 341 412 413 421 422 511 512 522
10	312	522	312 313 314 341 342 412 421 422 511 512 522
10	312	522	312 314 316 341 342 412 421 422 511 512 522

TABLE 4-6 cont'd

(b) Toronto (312) to Halifax (522) - (4 or 2 s.c.)

No. of links	From	To	NODES IN CHAINS
6	312	522	312 314 341 412 421 511 522
6	312	522	312 314 341 412 421 512 522
6	312	522	312 336 342 412 421 512 522
7	312	522	312 313 314 341 412 421 511 522
7	312	522	312 313 314 341 412 421 512 522
7	312	522	312 314 316 341 412 421 512 522
7	312	522	312 314 341 342 412 421 511 522
7	312	522	312 314 341 342 412 421 512 522
7	312	522	312 314 341 412 413 421 512 522
7	312	522	312 314 341 412 421 422 511 522
7	312	522	312 314 341 412 421 511 512 522
7	312	522	312 314 341 412 421 512 511 522
7	312	522	312 336 342 412 421 422 511 522
8	312	522	312 313 314 341 342 412 421 512 522
8	312	522	312 313 314 341 412 421 422 511 522
8	312	522	312 313 314 341 412 421 511 512 522
8	312	522	312 313 314 341 412 421 512 511 522
8	312	522	312 314 316 341 342 412 421 512 522
8	312	522	312 314 316 341 412 421 422 511 522
8	312	522	312 314 316 341 412 421 512 511 522

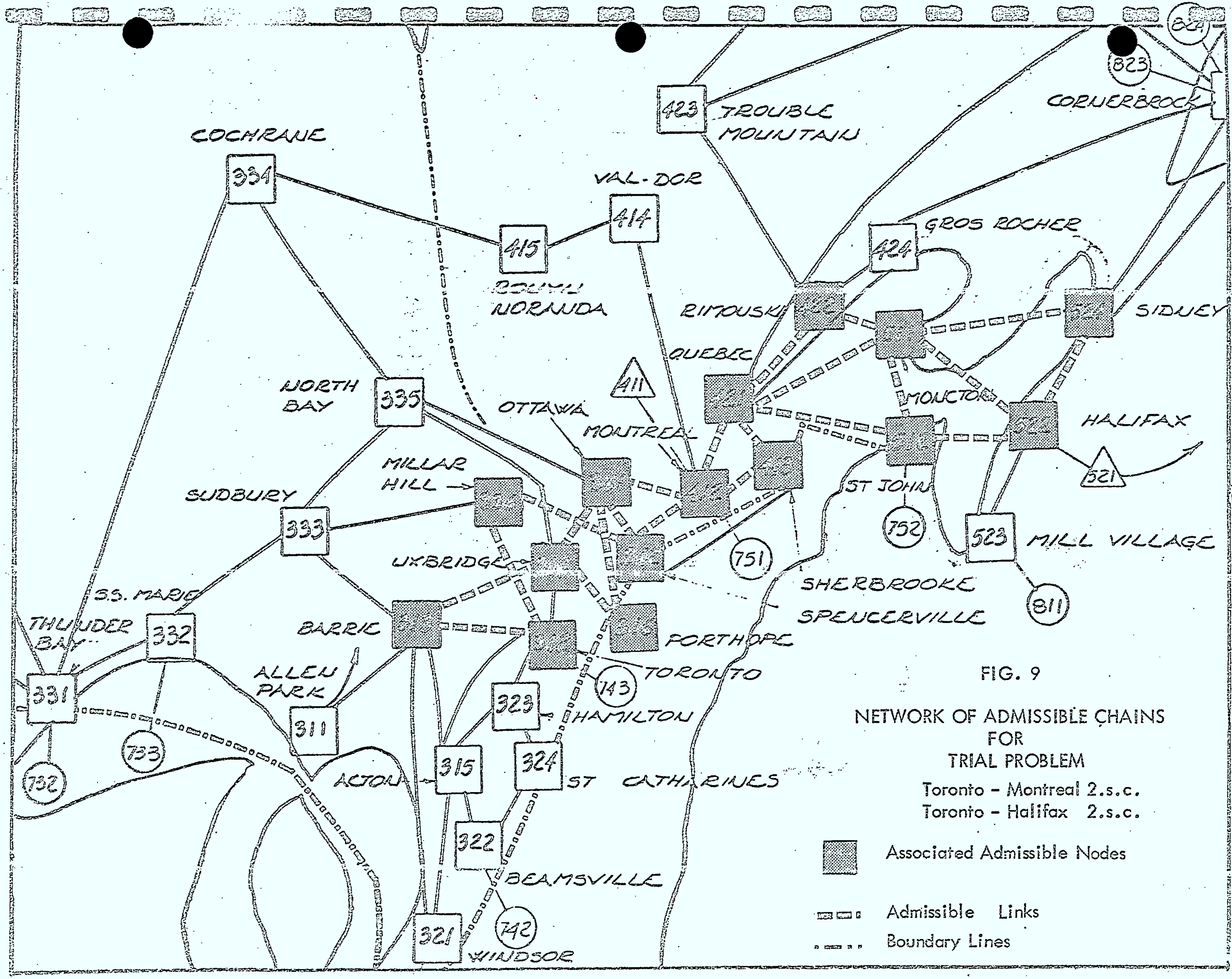


FIG. 9

NETWORK OF ADMISSIBLE CHAINS
FOR
TRIAL PROBLEM

Toronto - Montreal 2.s.c.
Toronto - Halifax 2.s.c.

Associated Admissible Nodes

Admissible Links

Boundary Lines

4.3.2 TRANCHE

a) Continuous solution

The problem is now in suitable form for the TRANCHE model.

As is described in Chapter 3, the model first seeks an optimal solution assuming that all variables are continuous. The resulting facility expansion program is given in Table 4-7. This "solution", which has no real meaning, was "costed" at $2,982,000.

The significance of this continuous solution is that any integer solution will cost as much or more. This is then a Lower Bound on expansion cost.

TABLE 4-7

Continuous Solution
Capacity Expansion for Trial Problem

Nodes				Additional capacity
From		To		
No.	Name	No.	Name	
312	Toronto	314	Uxbridge	4.0
314	Uxbridge	341	Ottawa	4.0
341	Ottawa	412	Montreal	4.0
412	Montreal	421	Quebec	2.0
421	Quebec	422	Rimouski	1.0
421	Quebec	512	St. John	1.0
422	Rimouski	511	Moncton	1.0
511	Moncton	522	Halifax	1.0
512	St. John	522	Halifax	1.0

b) Integer solutions

Having found an optimal continuous solution, the model proceeds to seek integer solutions and, ultimately, the optimal integer solution.

The results of the integer solution search are summarized in Table 4-8. The optimal solution was found to cost $3,600,000 and is summarized in Table 4-9, and Figure 10.

INTEGER NODES

21	28	31	32	33
4740.0000	3920.0000	3630.0000	3600.0000	3600.0000 (optimal)
INTEGER	INTEGER	INTEGER	INTEGER	INTEGER
.	.	.	.	1.0000
.	.	.	.	.
.	.	.	.	.
1.0000	3.0000	3.0000	3.0000	3.0000
.	1.0000	1.0000	1.0000	.
.	1.0000	1.0000	.	.
.	.	.	.	.
.	.	.	.	.
3.0000	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	1.0000
.	.	.	.	.
.	.	.	.	.
1.0000	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	1.0000	1.0000	1.0000	1.0000
.	3.0000	3.0000	3.0000	3.0000
1.0000	.	.	.	.
.	.	.	.	.
.	.	.	.	.
3.0000	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	1.0000	1.0000	1.0000
.	.	.	.	.
.	.	.	.	.
1.0000	3.0000	3.0000	3.0000	3.0000
.	1.0000	.	.	.
.	1.0000	.	.	.
3.0000	.	1.0000	1.0000	1.0000
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

Toronto - Montreal (2.s.c.)
Toronto - Halifax (2.s.c.)

INTE

NODE		15	21	2
FUNCTIONAL ($1.000)		5955.0000	4740.0000	3920.
ESTIMATION		INTEGER	INTEGER	INTE
(nodes)	(link)			
312-313	DX1158	.	.	.
	X2158	.	.	.
	DX2158	.	.	.
312-314	DX1159	3.0000	1.0000	3.00
	X2159	1.0000	.	1.00
	DX2159	1.0000	.	1.00
	Y1159	.	.	.
	DY1159	.	.	.
312-336	DX1162	.	3.0000	.
	X2162	.	.	.
	DX2162	.	.	.
313-314	DX1164	.	.	.
	X2164	.	.	.
	DX2164	.	.	.
314-316	DX1168	.	1.0000	.
	X2168	.	.	.
	DX2168	.	.	.
314-341	X2170	1.0000	.	1.00
	DX2170	3.0000	.	3.00
	DX1173	.	1.0000	.
316-341	X2173	.	.	.
	DX2173	.	.	.
	DX1183	.	3.0000	.
336-342	X2188	.	.	.
	DX2188	.	.	.
	DX1189	.	.	.
341-342	X2189	.	.	.
	DX2189	.	.	.
	DX2190	3.0000	1.0000	3.00
341-412	Y1190	1.0000	.	1.00
	DY1190	1.0000	.	1.00
	DX1191	.	3.0000	.
342-412	X2191	.	.	.
	DX2191	.	.	.
	DX1194	.	.	.
412-413	X2194	.	.	.
	DX2194	.	.	.
	Y1194	.	.	.

.	.	.	.	.
2.0000	2.0000	2.0000	2.0000	2.0000
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
1.0000	2.0000	2.0000	2.0000	2.0000
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
1.0000	.	.	.	.
.	.	.	.	.
.	.	.	.	.
1.0000	2.0000	2.0000	2.0000	2.0000
.	.	.	.	.
1.0000	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	1.0000	1.0000	1.0000	1.0000
.	1.0000	1.0000	1.0000	1.0000
.	.	.	.	.
.	.	.	.	.
1.0000	.	.	.	.
1.0000	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	1.0000	1.0000	1.0000	1.0000
.	.	.	.	.
.	.	.	.	.

	DY1194	.	.
412-421	DX1196	2.0000	2.0000
	X2196	.	.
	DX2196	.	.
	DX1198	.	.
	X2198	.	.
421-413	DX2198	.	.
	Y1198	.	.
	DY1198	.	.
	DX1200	.	1.0000
421-422	X2200	.	.
	DX2200	.	.
	X1244	.	.
421-511	DX1244	.	.
	X2244	.	.
	DX2244	.	.
421-512	DX1201	1.0000	1.0000
	X2201	1.0000	.
	DX2201	1.0000	.
422-511	DX1204	.	1.0000
	X2204	.	.
	DX1209	.	1.0000
511-512	X2209	.	.
	DX2209	.	.
	Y1209	.	.
	DY1209	.	.
511-522	DX1210	.	.
	DX1211	.	.
511-524	X2211	.	.
	DX2211	.	.
	X1212	1.0000	1.0000
512-522	DX1212	1.0000	1.0000
	X2212	.	.
	DX2212	.	.
	DX1217	.	.
	X2217	.	.
524-522	DX2217	.	.

TABLE 4-9

Optimal Capacity Expansion Program
for Trial Problem

Montreal-Toronto: 2 s.c.
Toronto-Halifax: 2 s.c.

From		To		Additional capacity
No.	Name	No.	Name	
312	Toronto	313	Barrie	1
313	Barrie	314	Uxbridge	1
312	Toronto	314	Uxbridge	3
314	Uxbridge	341	Ottawa	4
341	Ottawa	342	Spencerville	1
342	Spencerville	412	Montreal	1
341	Ottawa	412	Montreal	3
412	Montreal	421	Quebec	2
421	Quebec	422	Rimouski	2
422	Rimouski	511	Moncton	2
511	Moncton	524	Sydney	1
524	Sydney	522	Halifax	1
511	Moncton	522	Halifax	1

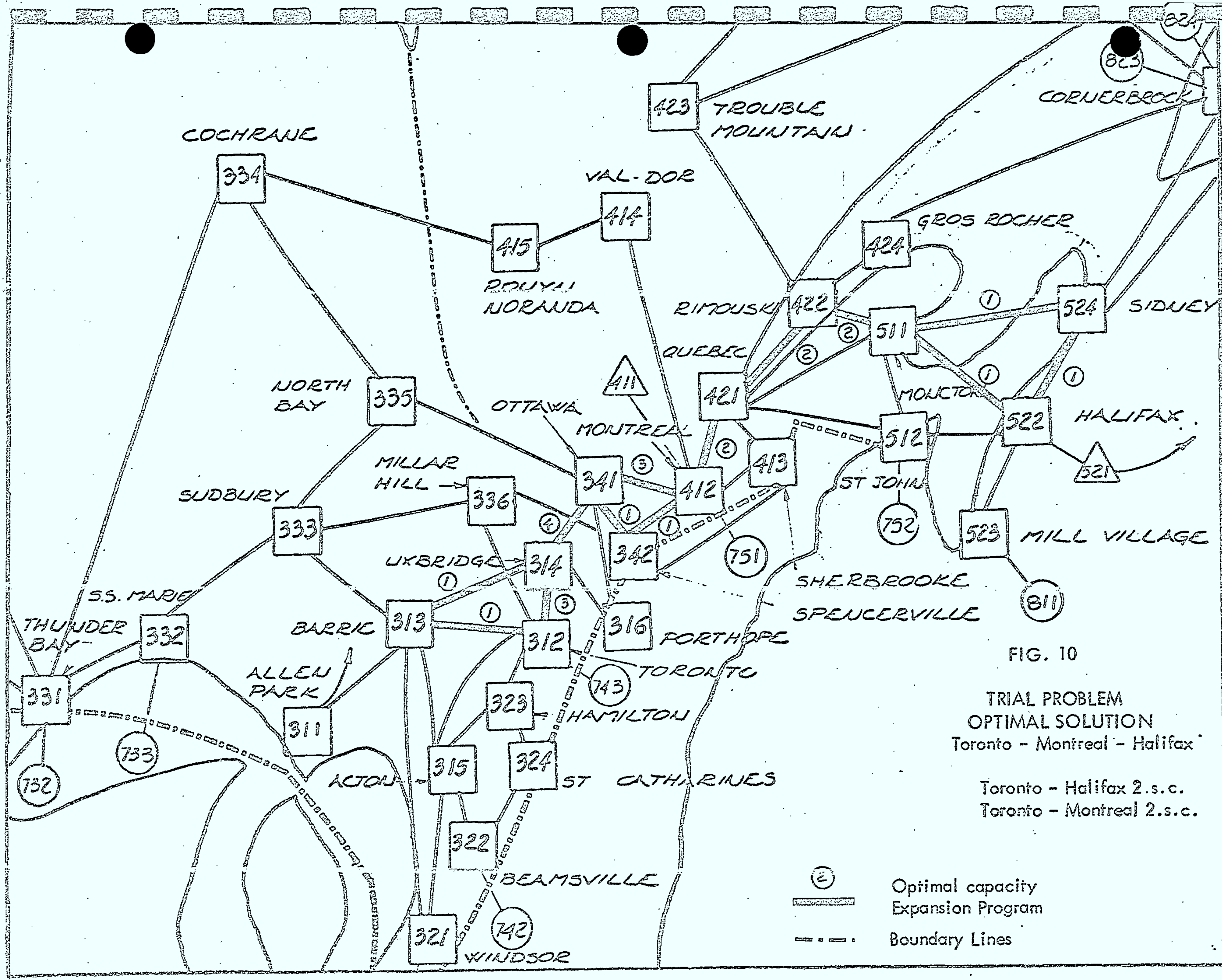


FIG. 10

TRIAL PROBLEM
OPTIMAL SOLUTION
Toronto - Montreal - Halifax

Toronto - Halifax 2.s.c.
Toronto - Montreal 2.s.c.

c) Explanation of output

Since the integer solution search procedure may be of interest, Table 4-8 is explained below.

The "Node" row is an internal counter only. The "Functional" row is the value or "cost" of each integer solution. Thus the first solution had a cost of $5,955,000.

The search then proceeded by searching for integer solutions which were better than the previous one. When no other solution could be found that was better, the solution was declared to be optimal. (The "Estimation" row merely shows that the solutions were integer.)

Below these three rows are a listing of the values of integer variables. The left hand column is a list of the variables, using their code discussed in Chapter 3. The key numbers are the last three digits, e.g. 158, 159, which define the links. The number of the nodal pair defining each link has been added, beside the link variables, for clarity of exposition.

It is to be noted that the final solution is the optimal one and is the same as that given in Table 4-9.

4.3.3 Analysis of results.

The optimal expansion cost was found to be $3,600,000, requiring capacity addition on 13 of 23 links.

One question that arises is the cost of by-passing Rimouski and adding a new link from Quebec City to Moncton. From the input cost data, it can be seen that the cost of installing the first channel will be $5,130,000 and, for the second channel, $570,000. Thus, the two channels can be installed for $6,300,000.

The cost of by-passing Rimouski will then be the cost of the Quebec-Moncton link less the cost of the Quebec-Rimouski link ($400,000) and of the Rimouski-Moncton link ($600,000), that is, $1,000,000. This comes to a marginal cost of $5,300,000.

4.4 Montreal-Vancouver Problem

4.4.1 The problem

A demand increase of 2 service channels is hypothesized between Montreal-Vancouver. Two runs were made on this problem to prove the power of the CADUCEE program.

In the first run, no a priori exclusion rules were employed. In the second, these rules were introduced, and had some effect on reduction in Ontario and Quebec.

In discussing the results, the current output format of the CADUCEE software will also be illustrated.

4.4.2 The solution

a) Basic network

The basic network consists of 124 links, 95 nodes and well over 30,000 possible chains.

The first output from CADUCEE is the translation of nodes numbers into the internal numbering system. This is shown in Table 4-10. This can be referred to in studying subsequent tables.

b) Least cost chain

The Least Cost Chains at upper and lower bounds are shown in Table 4-11.

c) Dominated nodes

At the end of the DØMINØ stage, when dominated nodes had been eliminated, the network had been reduced to 41 nodes and 64 links. The untranslated list of non-dominated nodes is shown in Table 4-12 and illustrated in Figure 11.

The DØMINØ tables for this problem are shown in Table 4-13.

d) Admissible chains

It is in the search procedure of admissible chains that the exclusion rules are considered. Without exclusion rules, there were 338 admissible chains, comprising 36 nodes and 57 links. The links and nodes constituting the admissible chains are illustrated in Figure 11.

TABLE 4-10

Conversation Table for
Montreal - Vancouver
2. s.c.

Network node number			Internal node no.
1	1	1	1
1	1	3	2
6	6	6	3
1	1	2	4
1	1	4	5
1	1	5	6
2	1	2	7
1	2	2	8
1	2	3	9
1	2	1	10
1	2	4	11
2	2	4	12
2	2	3	13
2	2	5	14
6	1	1	15
2	1	1	16
2	2	2	17
2	1	3	18
2	1	4	19
2	2	6	20
2	3	2	21
2	3	3	22
6	2	1	23
2	3	4	24
2	4	2	25
2	4	5	26
2	4	6	27
2	4	4	28
2	4	3	29
2	4	7	30
3	3	1	31
3	1	1	32
3	1	3	33
3	1	2	34
3	1	4	35

Network node number			Internal node number
3	1	5	36
3	2	3	37
3	3	6	38
3	2	1	39
3	3	3	40
3	1	6	41
3	3	5	42
3	4	1	43
3	2	2	44
3	2	4	45
3	3	2	46
3	3	4	47
4	1	5	48
3	4	2	49
4	1	2	50
4	1	3	51
4	1	4	52
4	2	1	53
4	2	2	54
5	1	1	55
5	1	2	56
4	2	3	57
4	2	4	58
4	2	5	59
5	3	6	60
5	3	1	61
5	3	5	62
5	2	2	63
5	2	4	64
5	2	1	65
5	2	3	66
5	3	4	67
5	3	2	68
6	3	1	69
6	3	2	70

TABLE 4-11

Least Cost Chains
for
Montreal - Vancouver
2.s.c.

a) Cost at upper bounds

CRIG.	DEST.	VALEUR
2	6	260
6	9	415
9	12	520
12	17	695
17	22	1080
22	21	1240
21	25	1640
25	31	2030
31	47	2450
47	42	2680
42	43	2925
43	50	3035

b) Cost at lower bounds

CRIG.	DEST.	VALEUR
2	5	130
5	7	515
7	19	650
19	21	945
21	25	1345
25	31	1735
31	47	2155
47	48	2290
48	52	2320
52	50	2650

TABLE 4-12

Non-dominated nodes

for
Montreal-Vancouver
2.s.c.

1	2	4	5	6	7	9	12	16	17	18
19	20	21	22	24	25	26	28	29	31	33
34	35	36	37	38	40	41	42	43	44	45
46	47	48	49	50	51	52	53			

TABLE 4-13

DOMINO tables for

Montreal - Vancouver
2. s.c.

Origin: Vancouver Destination: Montreal

Internal node number	minimum cost			
	at upper bound		at lower bound	
	From origin	From destination	From origin	From destination
1	540	3575	60	2710
2	0	3035	0	2650
3	5040	5090	4560	4920
4	65	3100	65	2715
5	380	2895	130	2520
6	260	2775	250	2600
7	985	2480	515	2135
8	535	2710	525	2605
9	415	2620	405	2445
10	870	3045	860	2940
11	710	2535	700	2445
12	520	2515	510	2340
13	705	2455	695	2365
14	775	2525	765	2435
15	1460	3285	1450	3195
16	895	2540	605	2155
17	695	2340	685	2165
18	1060	2405	590	2065
19	1125	2470	650	2000
20	965	2195	745	2105
21	1240	1795	945	1705
22	1080	1955	985	1865
23	2400	4150	1090	2760

TABLE 4-13 (cont'd)

DOMINO tables for

Montreal - Vancouver
2.s.c.

Oringin: Vancouver Destination: Montreal

Internal node number	minimum cost			
	at upper bound		at lower bound	
	From origin	From destination	From origin	From destination
24	1390	1860	1095	1770
25	1640	1395	1345	1305
26	2020	1845	1205	1455
27	2610	2365	1285	1795
28	1590	1660	1295	1570
29	1695	1390	1400	1300
30	2010	1765	1405	1675
31	2030	1005	1735	915
32	2965	590	2670	420
33	2865	490	2570	320
34	2875	440	2580	310
35	2845	470	2550	280
36	2935	500	2615	345
37	2915	480	2620	350
38	2820	280	2525	280
39	3135	700	2795	545
40	2690	410	2395	410
41	2890	515	2595	310
42	2680	355	2385	355
43	2925	110	2630	110
44	2965	530	2645	375
45	2945	510	2650	380
46	2480	620	2185	620

TABLE 4-13 (cont'd)

DOMINO tables for

Montreal - Vancouver
2.s.c.

Origin: Vancouver Destination: Montreal

Internal node number	minimum cost			
	at upper bound		at lower bound	
	From origin	From destination	From origin	From destination
47	2450	585	2155	495
48	2585	720	2290	360
49	2970	105	2675	105
50	3035	0	2650	0
51	3130	95	2745	95
52	2735	870	2320	330
53	3200	165	2815	165
54	3400	365	3015	365
55	3700	665	3315	665
56	3820	785	3200	550
57	3565	530	3180	530
58	3750	715	3085	435
59	4815	1780	3430	780
60	5065	2030	3480	830
61	4720	1685	3435	785
62	4985	1950	3464	814
63	4075	1040	3370	720
64	3940	905	3555	905
65	4120	1085	3375	725
66	4925	1890	3540	890
67	5470	2435	3685	1035
68	5695	2660	3760	1110
69	7290	7340	6810	7170
70	7290	7340	6810	7170

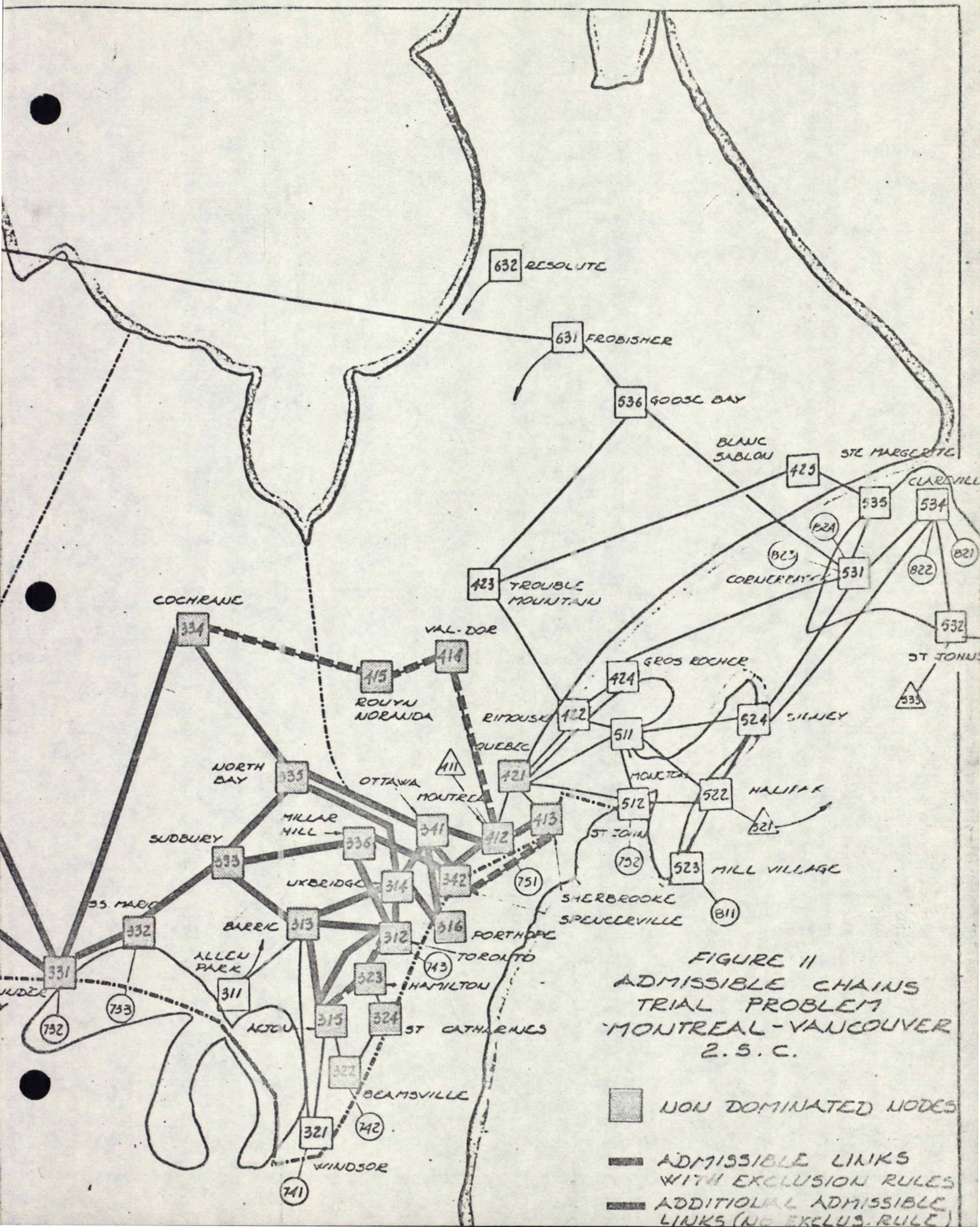
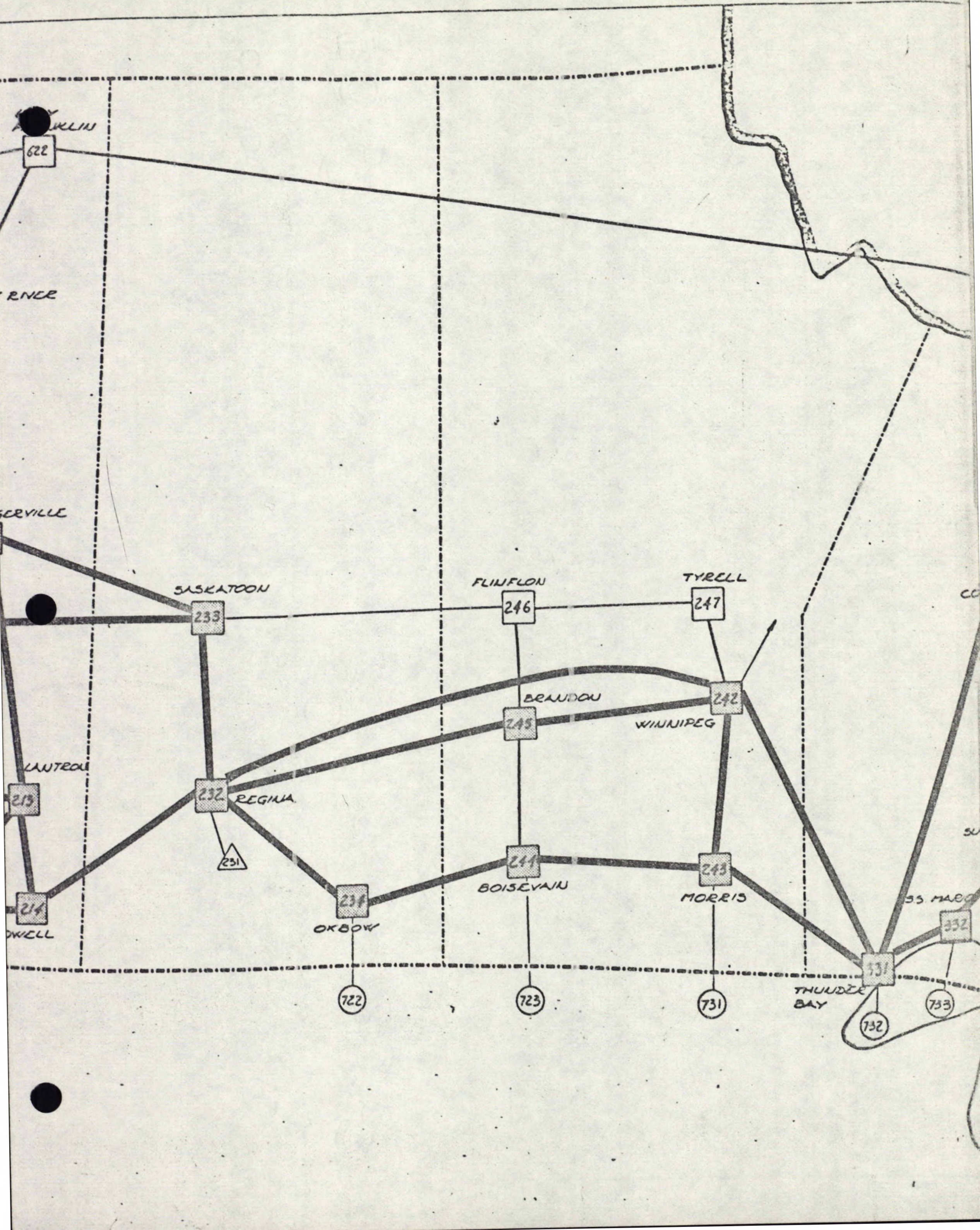
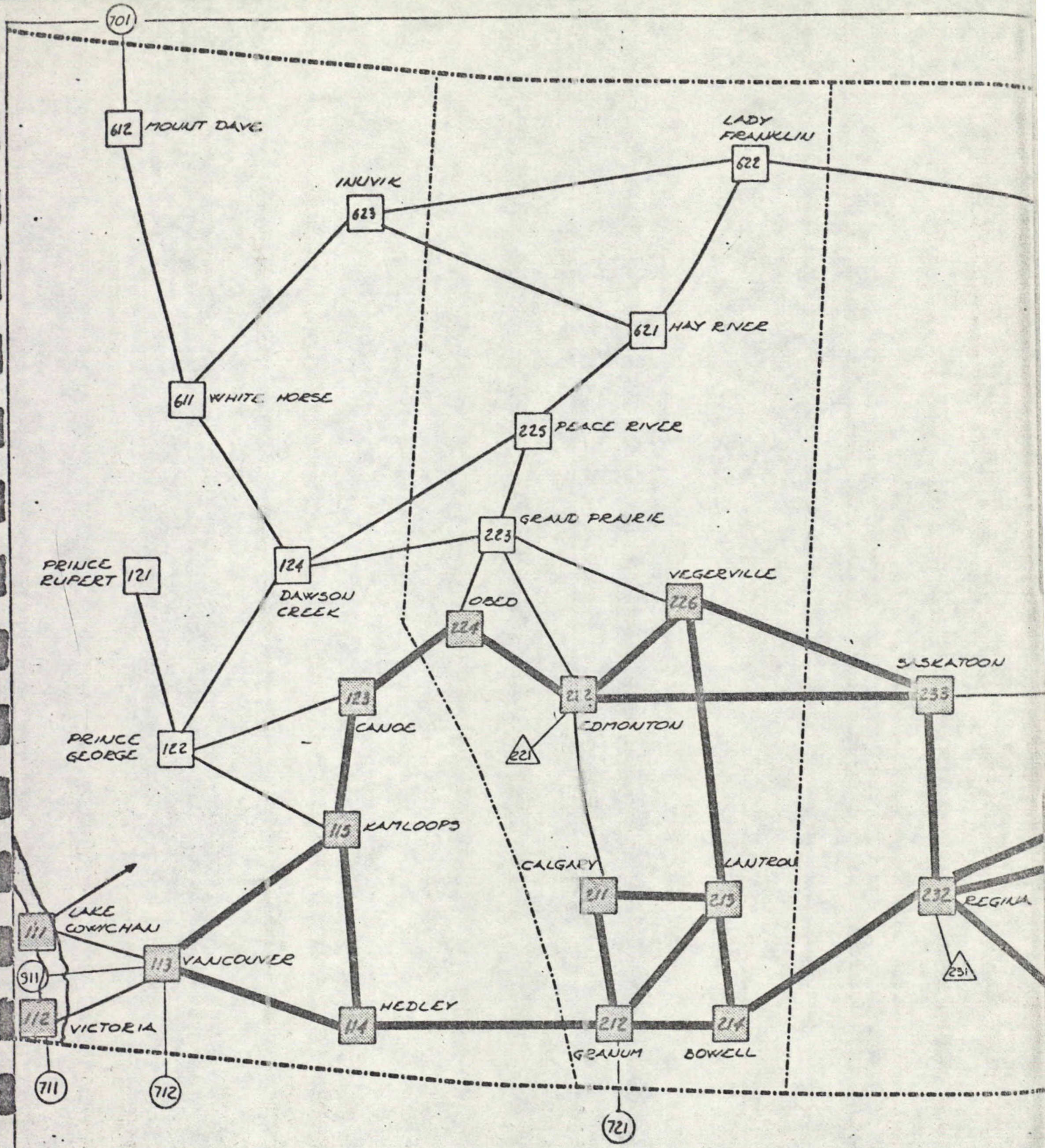


FIGURE 11
ADMISSIBLE CHAINS
TRIAL PROBLEM
MONTREAL - VANCOUVER
2. S. C.


622
RNCE
SCRVILLE
SASKATOON
233
FLINFLON
246
TYRELL
247
BRANDON
245
242
WINNIPEG
LANTRON
213
232
REGINA
231
244
BOISEVAIN
243
MORRIS
234
OKBOW
214
OWELL
332
331
THUNDER
BAY
722
723
731
732
733


701
612 MOUNT DAVE
LADY FRANKLIN
622
INUVIK
623
621 HAY RIVER
611 WHITE HORSE
225 PEACE RIVER
223 GRAND PRAIRIE
PRINCE RUPERT 121
124
DAWSON CREEK
VEGERVILLE
226
OBED
224
SASKATOON
233
123
CANOE
212
EDMONTON
221
PRINCE GEORGE
122
115 KAMLOOPS
CALGARY
211
LANTROU
213
232 REGINA
231
111
LAKE COWICHAN
911
113 VANCOUVER
112
VICTORIA
HEDLEY
114
212
GRANUM
214
BOWELL
711
712
721

Using the exclusion rules, a further 3 nodes and 7 links were eliminated. This is also illustrated in Figure 11.

A sample of the output of admissible chains is illustrated in Table 4-14. The first line of each set shows the node path using internal node numbers. The second line of the set shows the chains in the input format to TRANCHE, using the network node numbers. The first four entries on a line are the control information, giving: chain length, origin number, destination number, serial number of chains.

The final output is a count of the number of chains and a count of chains of given length. This is shown in Table 4-15.

4.4.3 Conclusion

The number of chains from Montreal to Vancouver has been reduced from approximately 30,000 to an eminently manageable 338. This illustrates the power and effectiveness of CADUCEE.

TABLE 4-14

Admissible Chains
(a sample)

for

Montreal - Vancouver

Chains are defined by internal node numbers (1st line) and by network node number (2nd line).

```
                   2    5    7  19  21  25  31  47  42  43  50
10  113  412     1 113  114 212 214 232 242 331 334 335 341 412
                   2    5    7  19  21  25  31  47  48  52  50
10  113  412     2 113  114 212 214 232 242 331 334 415 414 412
                   2    5    7  18  19  21  25  31  47  42  43  50
11  113  412     3 113  114 212 213 214 232 242 331 334 335 341 412
                   2    5    7  18  19  21  25  31  47  48  52  50
11  113  412     4 113  114 212 213 214 232 242 331 334 415 414 412
                   2    5    7  19  21  25  29  31  47  42  43  50
11  113  412     5 113  114 212 214 232 242 243 331 334 335 341 412
                   2    5    7  19  21  25  29  31  47  48  52  50
11  113  412     6 113  114 212 214 232 242 243 331 334 415 414 412
                   2    5    7  19  21  25  31  46  40  38  49  50
11  113  412     7 113  114 212 214 232 242 331 332 333 336 342 412
                   2    5    7  19  21  25  31  46  40  42  43  50
11  113  412     8 113  114 212 214 232 242 331 332 333 335 341 412
                   2    5    7  19  21  25  31  47  42  35  43  50
11  113  412     9 113  114 212 214 232 242 331 334 335 314 341 412
                   2    5    7  19  21  25  31  47  42  43  49  50
11  113  412    10 113  114 212 214 232 242 331 334 335 341 342 412
```

TABLE 4-15

Chain Statistics

for

Montreal-Toronto
2.s.c.

STATISTIQUES SUR LES CHAINES

****

NOMBRE TOTAL DE CHAINES = 338

NST(1) =	0	NST(10) =	2
NST(2) =	0	NST(11) =	12
NST(3) =	0	NST(12) =	40
NST(4) =	0	NST(13) =	74
NST(5) =	0	NST(14) =	97
NST(6) =	0	NST(15) =	79
NST(7) =	0	NST(16) =	30
NST(8) =	0	NST(17) =	4
NST(9) =	0	NST(18) =	0

5. TOWARDS A DEMAND MODEL

5.1 The problem of linking HERMES I with a demand model

The previous sections of this report have clearly demonstrated the great potential of the HERMES model. If this potential is to be fully utilised, it will not be sufficient to assume hypothetical increases of demand between given points expressed in number of channels, as it is done in the present version of the model; it should be possible to consider forecasts of different patterns of increases of demand for telecommunications facilities in Canada, provided by a demand model.

It is intended to work from the present concept of demand for facilities backward through the demand for telecommunication services to the demand for the transmission of messages and even beyond so as to begin to identify the real determinants of demand. In this way, progress towards a demand model will proceed step-by-step from a solid base of an operational, though manifestly narrow in scope, network capacity expansion model.

The important, and difficult, first stage of this development will be to introduce the considerations of the type of service (voice, TV, data transmission, etc.) of the quality of service and of the timing of demand.

The presence of joint costs and of decreasing costs makes it important to consider also the demand for local traffic although the model is primarily concerned with interregional traffic.

The demand for telecommunications is obviously generated by the social and economic activities and by the activities of public agencies. Public policies may have great direct effects (e.g. the policies of the state-owned TV networks, etc.) and indirect effects (e.g. encouragement of regional development, control of foreign investments, etc.). However, all these influences are certainly very complex and depend heavily on institutional arrangements.

There is also an interaction between the supply of telecommunications and the demand for them. Apart from the easy to name but difficult to evaluate,effect of prices on demand, there are also the effects on demand of the very existence of telecommunication facilities and of the quality of service.

Although HERMES I is a static model, its successors will be made progressively more dynamic. The importance of indivisibilities, and of the joint costs and of decreasing costs that go with them, makes it imperative to consider simultaneously several planning horizons to arrive at capacity expansion programs which are compatible with efficient sequencing in time of capacity expansion to face rapidly growing demand. The demand model to be aimed at is then a model capable of producing series of demand forecasts, for different horizons and, of course, corresponding to different hypotheses.

The results of any demand model, if they are to be used in the models of the HERMES series, should be compatible with the formulation of these models; thus, the results would have to be expressed in equivalent units (channels or smaller units envisaged) and would have to correspond to the present or envisaged configuration of the network. It is worthwhile noting that some research is being done on the demand problem inside the Department of Communications and while some results could be very interesting they could not, in their present form, be fed directly into models of the HERMES series. An important point also worth noting is the fact that the network actually used in the present version of the HERMES model, although very simplified, is largely supply oriented. Therefore, further extensions of this network aimed at bringing it closer to reality should take better account of the demand aspects of the problem.

Another specific problem is brought about by a characteristic of the telecommunications industry: the product being non stockable, the time of consumption is very important.

Still another problem is brought about by the growing variety of telecommunications services which can, and often do, use the same facilities but in substantially different amounts; also account has to be taken of the fact that the production of certain services can be delayed to escape the peak-hour problem and thus provide a better utilization of facilities; with the growing importance of data transmission this fact becomes more and more relevant to the problem.

As we proceed by successive steps from the demand for facilities all the way to analyzing the social and economic and other activities which ultimately generate the demand for telecommunications, we shall have to rely more and more on data, on results of analyses and on econometric models, originating in other agencies. This is bound to give rise to serious problems.

The potential inputs from other agencies are seldom in the formats directly usable for our purposes. And, of course, telecommunications, as other spatial aspects, are in most cases given a very summary treatment, if they are not entirely neglected in analyses and in econometric models originating in other agencies.

5.2 The elements available: methodology and data

Literature on telecommunications economics in general cannot be termed abundant; this is understandable though, in view of the fact that the field is fairly new with its technology evolving rapidly. Research in the field has been mainly focused on problems associated with regulation of the industry, such as pricing of services, rate of return of the firm, etc. Fairly numerous recent publications deal also with the broad problems of the impact of the telecommunications on the economy and the society as a whole.

In fact, there is no established and tested methodology directly relevant to the problem of demand forecasting for the purposes of the models of the HERMES series. There are however a certain number of studies that have tackled the problem of demand forecasting in the telecommunications industry. This section of the report deals with a few of these studies; no references will appear here though because we believe that the bibliography on the subject is not reasonably complete and that further exploration is needed. However, the studies referred to here, as well as a number of other studies, are available on file.

Studies of the Telecommission constitute the main source of readily available documentation on telecommunications in Canada. Once again, regulation caught a large part of the attention but a certain number of studies have been devoted to telecommunications economics.

One of these studies contains some highly aggregated regression analyses with an attempt to take account of the prices on the demand for telecommunications services. Another study contains forecasts of the volume of local and toll telephone messages up to 1980. The origin of the messages only is specified and the forecasting technique relies heavily on linear extrapolations.

A study has also been submitted by TCTS and CN-CP Telecommunications, titled "Telecommunications Carriers Market Projection and Analysis". It is too general to be useful in our study, representing more the thoughts of the carriers on the growth of their industry than a thorough analysis of the underlying factors.

In conclusion, we could say that studies of the Telecommission constitute a basic source of general reference on the telecommunications industry in Canada, but they cannot be expected to be useful for specific purposes such as building a demand model applying to an interregional network.

Other studies in the field of demand forecasting also include a recent US study, where traffic forecasts were made by assuming a specified annual rate of increase over a period of years and applying the resulting factors to the actual volume of traffic during a specified reference year. Results were also adjusted to take account of "impulse jumps" caused by a rapid rise in the number of telephones, noticeable improvement in the quality of service, etc.

This procedure was used to forecast telephone, telex and telegraph demand. To obtain circuit requirements the results were transformed using data concerning the average length of conversation, the average number of business days, the busy-hour to total-day relationship, and finally, the minutes charged to circuit-usage relationship.

Also treated in this study were the problems of routing and network configuration (radio-relay versus satellite, etc.), revenue projections and analysis of rates, and finally, economic feasibility of the proposed system.

Another study, though concerned only with estimating the price elasticity of demand for telegrams, proposes a methodology that merits a closer examination for the purposes of demand forecasting related to the HERMES series of models. This study was related mainly to issues in regulation but it can also provide valuable insights in the field of traffic forecasting. The object of the study was to estimate price elasticity for telegrams where price varies over distance between the point of origin of messages and their destination.

It was assumed that these telegrams were not only a good, having a price and providing utility, but also a method of social and economic interaction. It was expected thus that the volumes of messages were not only related to the basic variables of price and income, but also to spatial and gravitational characteristics such as distribution of population and income, physical distance between any two given points, levels of business activity at different points, etc.

Since all these elements influence the quantity of messages in a system, it appeared that an application of a gravity model in the investigation of demand for telegrams was appropriate. A functional relationship was sought linking the message volume to gravitational variables, demand variables, and indices of economic activity for each metropolitan area within the communications network.

Still another study dealing with forecasting the demand for international telecommunication contains an elaborate analysis of a large number of factors possibly affecting this demand. The results seem to show that some of these factors, including relative prices and quality of service have a significant effect on demand and on the choice of the type of service. There is no doubt, that work on actual demand forecasting will have to be preceded by analyses of this kind carried out with respect to Canadian data.

This is no place to attempt a comprehensive survey of the data available. Suffice it say that the development of demand forecasting procedures will involve a major effort in data integration. Fundamentally, data relevant to demand forecasting are available from two groups of sources: the carriers and the users. Any more than superficial analysis of the factors affecting the demand for telecommunications will have to give considerable importance to users' data, which ties in with the information on users characteristics.

The rapid growth and the rapid rate of change in the telecommunications industry mean that reliance on past trends may be very misleading. In any case, some of the phenomena involved have too short a history for any significant trends to emerge. The bulk of the effort will have to go into detailed cross-section analyses to uncover the mechanisms which determine the demand for telecommunications.

5.3 Possible approaches

There is no doubt that several approaches to the forecasting of the demand for telecommunications will have to be explored. The procedure arrived at will certainly contain features inspired by different approaches.

As pointed out above, there is no established methodology applicable to the problem in hand and a good deal of preliminary analysis and hypothesis testing will have to precede the stage of formulating forecasting procedures.

It is also evident that what is being aimed at are conditional forecasts, corresponding to alternative hypotheses and not "once for all" projections. And again, the forecasting procedures will serve to produce ranges of values rather than point forecasts. The models of the HERMES series into which these alternative forecasts will be fed will then have to be considered as being in fact simulation devices.

Broadly speaking one could consider three possible approaches to the questions of forecasting the demand for telecommunications. They are of course not mutually exclusive.

The extrapolations of past experience, with a gradual introduction of more explanatory variables and a gradual refinement of the level of detail. This is what might be termed the traditional approach.

The integration of detailed information collected by the telecommunication carriers in the course of their operations and for their internal purposes. In the first phase, there will have to be a coordination of the carriers own forecasts. Then, there will be the question of using carriers data in conjunction with information obtained from other sources. This will obviously be an essential component of the forecasting procedure. However, this approach is inevitably rather restrictive since the carriers data do not go very far concerning the characteristics of the users and their activities and cannot reveal some of the fundamental determinants of the demand for telecommunications.

The third approach is that of building detailed and elaborate models of the behaviour of telecommunications users, models whose structures will be basically those of activity analysis. This approach relies heavily on the collection and integration of the telecommunications users data: a major effort will have to be done in this area. For one, the users data are rather scanty and, in most cases, in formats which make them difficult to reconcile with each other and with the carriers data. Secondly, the purchases of telecommunications services represent in most cases relatively minor cost items and in addition, the accounting practices often make it difficult to identify them.

For a number of reasons, it may be expected that this third approach will play the main role in the procedure adopted, with the carriers data however being used to the fullest possible extent. This approach will involve the identifications of the fundamental determinants of the demand for telecommunications as well as of the successive stages through which social, economic and other activity gives rise to the need to communicate, to the demand for the transmission of messages, to the choices of the type of services, to the potential traffic flows all the way to the demand for telecommunication facilities and finally a definite capacity expansion program. By proceeding by clearly defined stages, the mechanisms of the successive transformations can be better formulated and tested. The model would include a large number of relatively simple relations based, as far as possible, directly on detailed data concerning observable phenomena. A model of such a structure is also more "transparent" and yields itself better to subsequent refinements and up-datings apart from being much more suitable for simulation purposes. A considerable experience has been acquired, by the Laboratoire d'économétrie and by Sorès Inc., in admittedly different contexts, in constructing and operating models of this kind.

6. EXTENSIONS AND FURTHER DEVELOPMENT

6.1 General remarks

The work undertaken in the preliminary phase of the HERMES project was done to a large extent in view of extensions and further developments, often at the expense of considerable additional effort. As it has been pointed out the HERMES I model is to be considered as the first of a series. The future steps fall naturally into four categories:

- improvements of the methodology already developped, including the software and its applications to more detailed and more elaborate networks,

- the introduction of additional factors affecting the planning of network expansions, including the considerations of the quality of service, the peak demands, etc.,

- the linking of the network capacity expansion models with demand models,

- the introduction of dynamic considerations: capacity expansion planning over time.

This chapter deals in some detail with the first of four categories. It covers what might be termed the natural extensions of the HERMES I model. Although the relevant methodology including the software is already available, their implementation will still call for a considerable effort, including once again a very heavy reliance on the expertise of the Department of Communications specialists.

The other three categories go a long way beyond the scope of the HERMES I model. The relevant operational formulations and implementations will have to be preceded by an intensive exploratory work, again calling for a close collaboration of all the participants to the projects.

Finally, it must be mentioned that, naturally, the efficiency and the usefulness of the model will increase considerably as the data base becomes generally more detailed and closer to reality. This concerns in particular the cost data.

6.2 Improvement of the operating characteristics of the HERMES I model

Although fully operational and, in fact quite efficient, the methodology of the HERMES I model, including the software, still requires a number of refinements especially concerning the format of the outputs of the model and a better utilization of its by-products which are of value in themselves. In addition, to handle problems where considerable demand increases are specified (e.g. more than 6 service channels) and the demand points concerned are far apart, a more powerful version of the CADUCEE program will have to be resorted to.

Software improvements will concern in the first place the output formats of the CADUCEE program. These outputs serve two purposes. The primary purpose is to constitute inputs into the TRANCHE program. In this case, the output is on punched cards according to the specification of TRANCHE, and no changes are contemplated at this stage (however, see below). The secondary, but important purpose of the CADUCEE outputs is to provide information concerning the analysis of the network. Output formats which are appropriate for this purpose are different from those specified for the inputs in the TRANCHE program and, in any case, many of these outputs are not needed in TRANCHE. Since these secondary outputs are destined to be analysed by various users, not necessarily familiar with the inner working of the model, they will have to be in a form which makes it easy to interpret them. More explicit identification of the elements of the network, including place names will have to be provided for, as well as better identification, with explanatory remarks, of the various derived concepts such as the minimum cost chains, at lower (upper) cost bounds, dominated nodes, etc.

In addition, clear indications will have to be given concerning the use of DØMINØ tables for the evaluation of non-existing, but contemplated links.

Finally, the CADUCEE II program will have to be developed. The conceptualization of this program has already been done. There still remains the actual programming and, undoubtedly, several successive improvements of the resulting software. CADUCEE II will be more powerful than CADUCEE I in eliminating dominated chains and thus in reducing the load on the program TRANCHE, at the expense, of course, of heavier computations at the CADUCEE stage. It will also require less core space than CADUCEE I, which is of considerable importance in some problems referred to above. The fundamental difference between CADUCEE II and CADUCEE I will be that the former will use a more restrictive definition of admissible chains, that is a wider definition of dominated chains. In CADUCEE I, a complete, or incomplete chain is dominated if any incomplete chain it contains is dominated. In CADUCEE II a complete, or incomplete chain will be dominated if any sub-chain it contains is dominated. Since, for any complete or incomplete chain, the incomplete chains it contains are a subset of the sub-chains it contains, the CADUCEE II definition of dominated chains is wider that the definition of CADUCEE I and therefore, will lead to the elimination of some chains which survive the elimination procedure of CADUCEE I. It goes without saying that CADUCEE II will not eliminate, as dominated, any chains which could appear in the optimal capacity expansion program. The program CADUCEE II requires the computation of the costs of minimum cost chains, at upper bound costs, for all the pairs of points of the network, instead of just for the pairs of points for which a demand increase is specified.

The use of the CADUCEE output for the evaluation of non-existing but contemplated links calls for no software development. However, the methodology has to be written up, examples worked out and the performance of this procedure examined with reference to real data.

Concerning the TRANCHE program, the situation is roughly analogous. The available software is operational and efficient. Output formats will have to be made substantially more elaborate and explicit to facilitate the utilizations of the program. Since large numbers of variables appear in the formulations of the problems and in the solutions, provisions will have to be made for optional summaries of results.

Finally, the CADUCEE and the TRANCHE programs will have to be integrated so that there is an option of running a given problem through both programs without human intervention. It may be expected that this option will not be very frequently used: in most cases, it will be thought advisable to examine the outputs of CADUCEE before proceeding further. However, when speed is essential, or when several problems have to be solved in rapid succession such an option of fully integrated solving may be found useful.

6.3 More detailed networks

The telecommunications network used so far by the HERMES I model is a highly simplified and aggregated network. With the existing methodology and with basically the same software (although the software improvements referred to in the preceding section will have to be done first) one can handle considerably more detailed networks. This may, however, necessitate in some cases a recourse to information stored on a disk, with an inevitable and substantial increase in computing cost. It is however too early to speak with any assurance of the computing costs involved in handling more detailed networks since, on the one hand we do not yet know what will be the increase in efficiency due to CADUCEE II and, on the other, the volume of computations and memory space requirements depend not only on the size but also, in a vital way, on the structure of the network concerned.

It is presently contemplated to construct a very detailed network containing up to 2,000 nodes to represent the Canadian telecommunications. This network would never by analysed as such but would serve more like a data base. A more aggregated version containing some 500 nodes would be the network actually analysed in the first stage of handling a given problem. The CADUCEE program identifies, as one of its by-products, the so-called non-dominated nodes, in any given problem. These non-dominated nodes define, in a non-arbitrary manner, the region which is relevant to the problem on hand. This definition is not arbitrary, since dominated nodes are the nodes which cannot, under any of the circumstances specified in the problem, appear in the optimal capacity expansion program aimed at. This relevant region being identified on the 500 node network, it can then be given a more detailed representation using the appropriate elements of the 2,000 node network, and the concluding stage of analysis carried out with reference to this new, partly blown-up network.

No experience has yet been acquired with this procedure and it is too early to say what its performance will be with real data and with the new improved software. Its objective is, basically, to reconcile the need for a more detailed analysis of certain regional problems, while avoiding the arbitrary defining of regions which might be particularly inappropriate in the case of telecommunications networks while, at the same time, keeping down to manageable proportions the dimensions of the problems.

The 500 and the 2,000 node networks referred to will originally be merely more detailed versions of the actual network. However, they could also be the enlarged networks described in the following section, although to handle such enlarged networks a certain amount of software development will have to be done.

6.4 Enlarged networks

The methodology already developed is applicable also to the so-called enlarged networks. This concept is introduced in order to be able to take into account such important characteristics of the telecommunications network as the existence of distinct carriers and of distinct facility systems and of certain joint costs relating to certain indivisible facilities capable of accommodating demand between more than one pair of points without one being able to allocate the cost of this indivisible facility between the pairs of points concerned. However, to retain the mathematical formulation, and hence the basic software of the original model, the enlarged network is constructed in such a way that its formal structure is strictly analogous to the network used in the HERMES I model.*

In the first place, it may be necessary in some cases to split a given node of the geographic network into two or more nodes. This will reflect the existence of more than one nodal facility at the same geographic location, and/or of more than one transmission facility between any given pair of nodes. Then either of the following two formulations may be adopted, depending on the purposes of the analysis. One can specify demand increases at the new nodes. In this case, the demand increases are assumed to arise already with an initial commitment to a given type of facility, that is, in fact, to a given system controlled by a given carrier. Alternatively, demand increases may be specified as arising in a given geographic location without any such original commitment. In these cases, a single dummy node will be introduced at the geographic location concerned and the demand increase specified as concerning this dummy node. This dummy node will then be connected by zero-cost links to the nodes representing the different nodal facilities available in that location. The analysis will then bear, among other things, on the most efficient choice of the initial nodal facility and hence also, to some extent, on the choice of the system and of the carrier.

To reflect the interconnection costs between distinct systems, the interconnection links will have cost functions associated with them, which will behave in the model as the cost functions of transmission facilities, although, in fact, the geographic distance involved may well be negligeable.

It may well be the case in various places of the enlarged network that facilities lying geographically close together are not in fact connected.

Another type of situation where dummy nodes will be introduced are the situations referred to already where we have a case of an indivisible transmission facility capable of accommodating the demand between more than one pair of points. In such cases, often only a small part of the cost of the facility can be allocated to the pairs of points concerned, the bulk of the cost being truly indivisible.

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* An outstanding example of such an indivisible facility is the satellite. In the HERMES I model, an arbitrary allocation of the satellite cost is imposed which clearly does not reflect the true economics of the situation. In an enlarged network, the joint costs will be treated as such, and only the identifiable additional costs will be allocated to the points served.

In this case the indivisible transmission facility is represented by a dummy node. The indivisible part of the cost is associated with this node. The costs which can be allocated to particular points, that is the costs of connecting these points to the indivisible facility and hence to other points so connected, are associated with the links connecting each of these points to the dummy node in question. Incidentally, we shall thus have a rather paradoxical situation of a transmission capacity expansion cost being (formally) associated with a (dummy) node.

It may thus be seen that the concept of the enlarged network makes for a substantial increase in flexibility and realism without altering the formal structure of networks submitted to the model.

6.5 Planning capacity expansion over time

As pointed out already in Chapter 5, a really efficient capacity expansion model will have to concern itself with the sequencing of capacity expansions over time. In the first place, this will require elaborate demand forecasting procedures working towards several different time horizons.

In the second place, the software will have to attain a very high level of efficiency since the volumes of computations involved in analyses covering several periods will inevitably be very large.

It is of course the presence of important indivisibilities, and hence of joint costs and of decreasing costs, coupled with the very rapid growth of demand for telecommunications which explain the interest in planning capacity expansion over time.

The dimensions and the complexity of the problems involved exclude, for the time being at any rate, the use of such rigorous procedures as those of dynamic programming or of the optimal control theory. The approach which appears most promising is that of repeated simulation runs, over various horizons and with different hypotheses, to calculate the alternative expansion programs and the orders of magnitude of the trade-offs between them.

APPENDIX

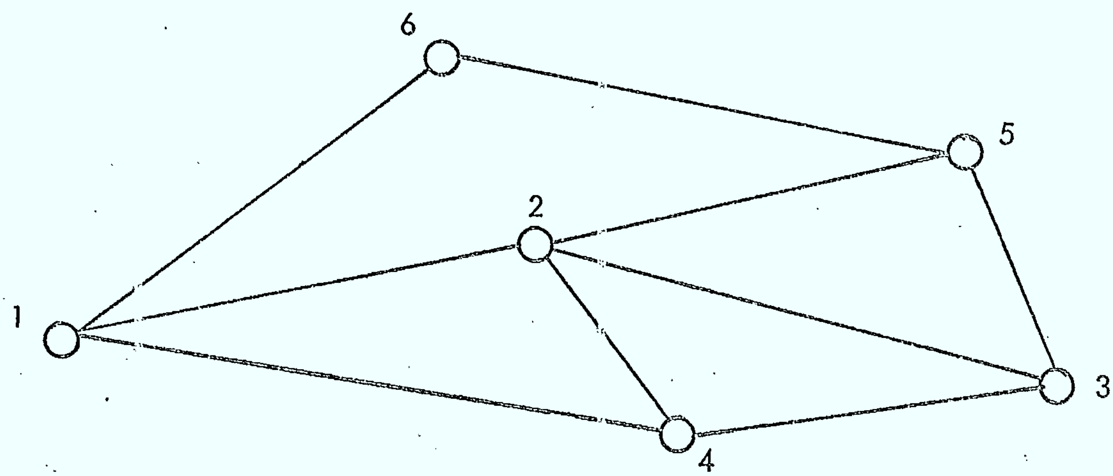
Reduced Example

1. INTRODUCTION

The example described in this appendix illustrates the functionning of the HERMES I model. Based on a very simplified network and fictitious cost data, it permits to follow every step of the calculations and underlines all the basic mechanisms of the software.

2. NETWORK AND COST DATA

The simplified network on which the example is based is shown in the following sketch.

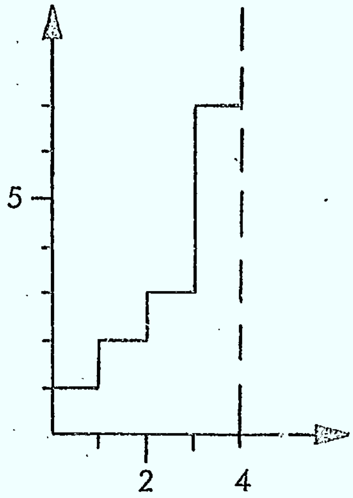


Mileage data is as follows:

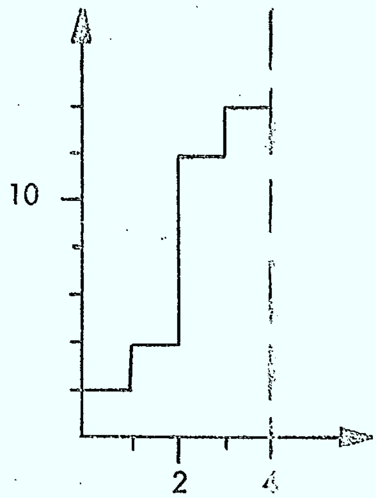
Link	Mileage
1-2	100
2-3	200
1-4	600
4-3	600
1-6	700
6-5	800
5-3	300
2-5	200
4-2	500

Capacity expansion cost functions for each link are as follows:

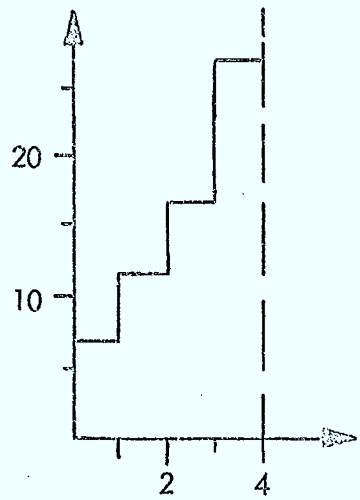
(The vertical axis represents cost per mile, the horizontal axis the increase in capacity measured in service channels.)



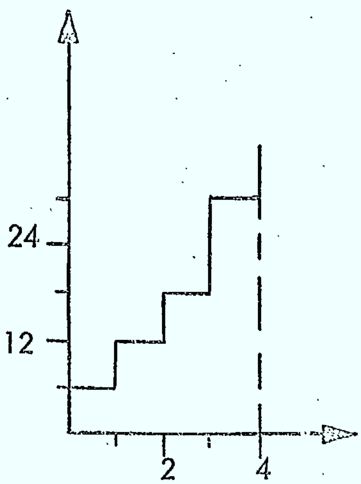
Link 1-2



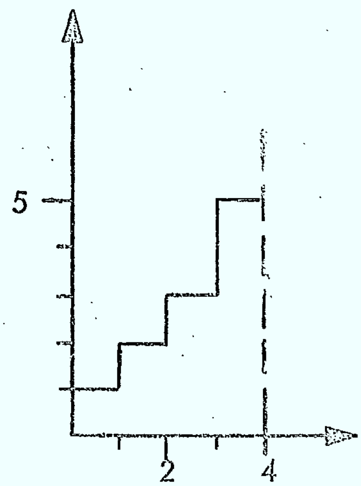
Link 2-3



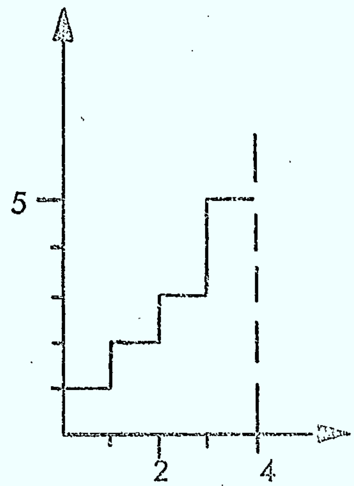
Link 1-4



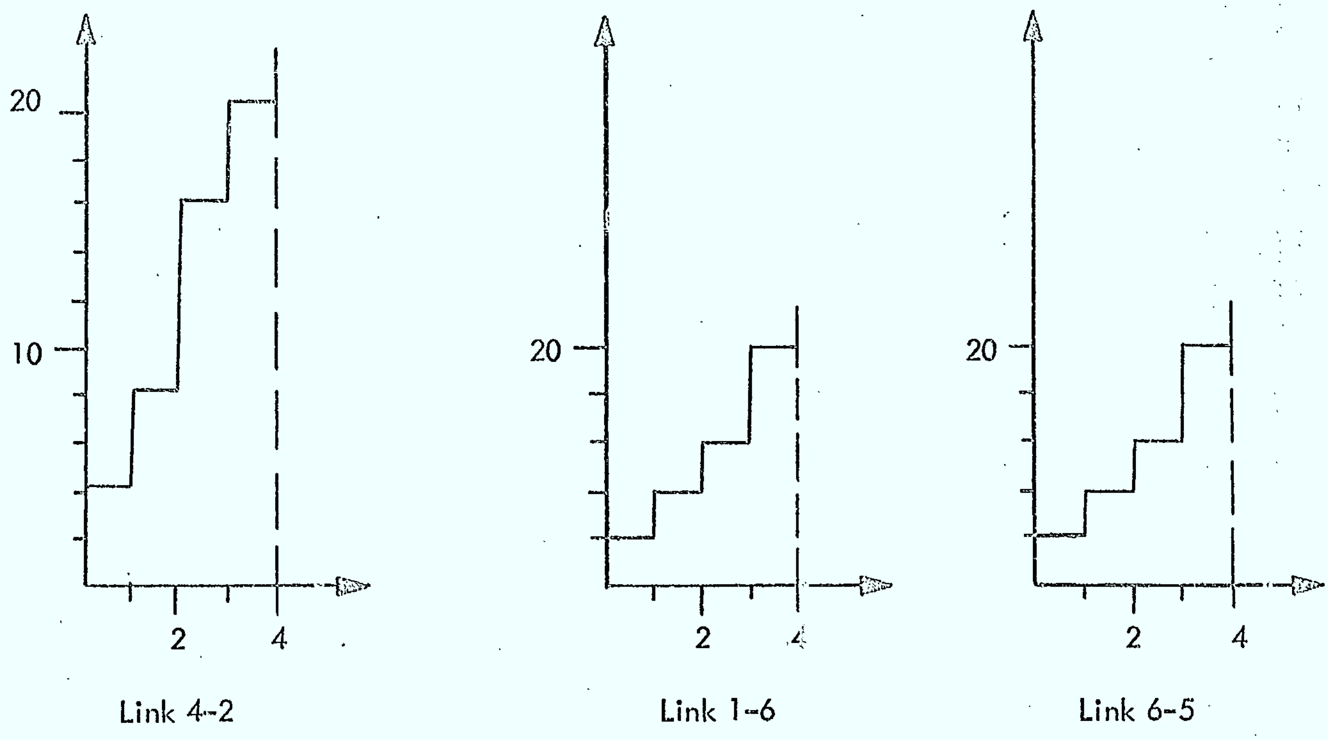
Link 4-3



Link 5-3



Link 2-5



3. THE PROBLEM

Determine the minimum capital cost capacity expansion program necessary to accommodate the following demand increases:

2 service channels between nodes 1 and 3
2 service channels between nodes 4 and 5

The maximum contemplated demand increase is thus 4 service channels.

4. SEARCHING THE ADMISSIBLE CHAINS (CADUCEE Program)

4.1 Demand points 1 and 3

Origin: NØRG = 1
Destination: NDEST = 3

a) Lower bound (LB) and upper bound (UB) on capacity expansion incremental costs for the maximum contemplated demand increase (4 service channels): sub-routine BØRNE.

These lower and upper bounds are calculated on each link as shown in the following table.

Link	Mileage	Lower bound on incremental cost per channel per mile.	LB (Incremental cost per channel)	Upper bound on incremental cost per channel per mile.	UB (Incremental cost per channel)
1-2	100	1	100	4	400
2-3	200	2	400	8	1 600
1-4	600	5	3 000	10	6 000
4-3	600	6	3 600	12	7 200
1-6	700	4	2 800	8	5 600
6-5	800	4	3 200	8	6 400
5-3	300	1	300	2	600
2-5	200	1	200	2	400
4-2	500	4	2 000	8	4 000

b) Search for dominated nodes.

The DØMINØ tables are first calculated. They contain the lower bounds (LB) and upper bounds (UB) of the minimum cost chains from every network node to the origin (NØRG) and from every network node to the destination (NDEST).

From NØRG to:	LB	UB	From NDEST to:	LB	UB
1	0	0	1	500	1 400
2	100	400	2	400	1 000
3	500	1 400	3	0	0
4	2 100	4 400	4	2 400	5 000
5	300	800	5	300	600
6	2 800	5 600	6	3 300	7 000

A node is dominated when the minimum cost chain through it has a LB higher than the UB of the minimum cost chain between the pair of demand points.

The two terms of the comparison, obtained from the DØMINØ table, are shown in the following table, for each node of the network except the origin and the destination. The dominated nodes are identified.

Node	LB on minimum cost chain through node	UB on minimum cost chain between 1 & 3	Dominated
2	100 + 400 = 500	1 400	NO
4	3 000 + 2 400 = 5 400	1 400	YES
5	300 + 300 = 600	1 400	NO
6	2 800 + 3 300 = 6 100	1 400	YES

c) Search for admissible chains.

Starting from NØRG , the chains having one link are identified. These are called "chains of length 1". The chains of length 1 are used in turn to identify chains of length 2, etc. Every time a node is arrived at the following two conditions for admissibility are tested:

i) An admissible chain (or incomplete chain) contains no dominated node. (Necessary but not sufficient).

ii) The LB on the cost of the incomplete chain concerned is less than or equal to the UB of the minimum cost chain connecting the node considered to NØRG . (Sufficient).

The successive testing for these two conditions is illustrated in the following table. If condition (i) is not fulfilled, the incomplete chain is immediately identified as non admissible. It is no longer used in further steps to construct chains of greater lengths.

If condition (i) is fulfilled, the node is tested for condition (ii). The first term of the comparison is calculated using the table of paragraph (a) above. The second term is found in the DØMINØ tables.

Length of chain	Chain	Dominated node in the chain	LB on cost	UB on minimum cost chain	Admissible
1	1-6	YES			NO
	1-2	NO	100	400	YES
	1-4	YES			NO
2	1-2-5	NO	300	800	YES
	1-2-3	NO	500	1 400	YES
	1-2-4	YES			NO
3	1-2-5-3	NO	600	1 400	YES

The following chains are admissible: 1-2-3
1-2-5-3

4.2 Demand points 4 and 5

NØRG = 4
NDEST = 5

The same reasoning as for the first pair of demand points is followed, starting from step (b). (The results previously obtained in step (a) are used again for the second pair of demand points).

b) Search for dominated nodes.

DØMINØ Tables

From NØRG to:	LB	UB	From NDEST to:	LB	UB
1	2 100	4 400	1	300	800
2	2 000	4 000	2	200	400
3	2 400	5 000	3	300	600
4	0	0	4	2 200	4 400
5	2 200	4 400	5	0	0
6	4 900	10 000	6	3 100	6 400

Node	LB on minimum cost chain through node	UB on minimum cost chain between 4 & 5	Dominated
1	2 100 + 300 = 2 400	4 400	NO
2	2 000 + 200 = 2 200	4 400	NO
3	2 400 + 300 = 2 700	4 400	NO
6	4 900 + 3 100 = 8 000	4 400	YES

c) Search for admissible chains.

Length of chain	Chain	Dominated node in the chain	LB on cost	UB on minimum cost chain	Admissible
1	4-1	NO	3 000	4 400	YES
	4-2	NO	2 000	4 000	YES
	4-3	NO	3 600	5 600	YES
2	4-1-6	YES			NO
	4-1-2	NO	4 000	4 000	YES
	4-2-1	"	2 100	4 400	"
	4-2-5	"	2 400	4 400	"
	4-2-3	"	2 400	5 600	"
	4-3-2	"	4 000	4 000	"
	4-3-5	"	3 900	4 400	"
3	4-1-2-5	NO	3 300	4 400	YES
	4-1-2-3	NO	3 500	5 600	YES
	4-2-1-6	YES			NO
	4-2-3-5	NO	2 700	4 400	NO
	4-3-2-5	NO	4 200	4 400	NO
	4-3-2-1	NO	4 100	4 400	NO
4	4-1-2-3-5	NO	3 800	4 400	NO
	4-3-2-1-6	YES			YES

The following chains are admissible: 4-2-5
4-3-5
4-1-2-5
4-2-3-5
4-3-2-5
4-1-2-3-5

4.3 Revision of upper and lower bounds

a) Maximum relevant demand increase.

The maximum relevant demand increases are calculated by considering in turn each link of the network and testing for the following conditions:

- If the link does not appear in any of the admissible chains listed for all the pairs of demand points considered in the problem, the maximum relevant demand increase on the link is zero. The link can effectively be considered as removed from the network.

- If the link appears in the lists of admissible chains for one or more demand point pairs, the maximum relevant demand increase for the link is equal to the sum of the demand increases for all the point pairs for which the link appears in the list of admissible chains.

The maximum relevant demand increases are calculated in the following table:

Link	MRD
1-2	4
1-6	0
1-4	2
6-5	0
2-5	4
4-2	2
4-3	2
5-3	4
2-3	4

Note that the maximum relevant demand increases on the links which constitute the admissible chains for the first pair of demand points (1 and 3), i.e. 1-2, 2-3 and 1-2, 2-5 and 5-3 is no different from the maximum demand increase as considered in the first part of the example. The admissible chains for this pair of points thus remain the same and only the second pair of demand points (4 and 5) has to be considered here.

b) LB and UB on capacity expansion incremental costs.

LB and UB on capacity expansion incremental costs for links 1-4, 4-2 and 4-3 have now to be revised as the maximum relevant demand increase on these links is now 2 service channels. The results are given in the following table.

Link	LB on incremental cost per channel	UB on incremental cost per channel
1-2	100	400
2-3	400	1 600
1-4	3 000	4 000
4-3	3 600	3 600
5-3	300	600
2-5	200	400
4-2	2 000	2 200

Note that links 1-6 and 6-5 are no longer considered.

c) Search for dominated nodes.

DØMINØ Tables

From NØRG to:	LB	UB	From NDEST to:	LB	UB
1	2 100	2 600	1	300	800
2	2 000	2 200	2	200	400
3	2 400	3 600	3	300	600
4	0	0	4	2 200	2 600
5	2 200	2 600	5	0	0

Node	LB on minimum cost chain through node	UB on minimum cost chain between 1 & 3	Dominated
1	2 100 + 300 = 2 400	2 600	NO
2	2 000 + 200 = 2 200	2 600	NO
3	2 400 + 300 = 2 700	2 600	YES

d) Search for admissible chains.

Length of chain	Chain	Dominated node in the chain	LB on cost	UB on minimum cost chain	Admissible
1	4-1	NO	3 000	2 600	NO
	4-2	NO	2 000	2 600	YES
	4-3	YES →			→ NO
2	4-2-5	NO	2 200	2 600	YES
	4-2-3	YES →			→ NO

Only one admissible chain remains: 4-2-5

4.4 Results of CADUCEE

Demand points 1 and 3:

Admissible chains: 1-2-3
1-2-5-3

Demand points 4 and 5:

Admissible chains: 4-2-5

5. THE CHOICE OF THE MINIMUM COST CAPACITY EXPANSION PROGRAM (TRANCHE Program)

5.1 General

For the two pairs of points that were considered at the outset of the problem, there remains a choice for only one: the pair 1 and 3, for which 2 chains are admissible. For the other pair, only one chain remains so that further processing with the TRANCHE program is not necessary.

For the demand between node 1 and node 3 it is evident that the problem can be solved by hand, and does not require an optimization model. Thus, the determination of the optimal solution by hand is shown. As a further example of formulating mixed integer linear programs in the manner chosen for TRANCHE the matrix for this problem is shown in Figure A-1 and discussed.

5.2 Hand Solution

Although there is only one chain for the demand from node 4-node 5, it must be considered since link 2-5 enters also in one of the chains from node 2 to node 3. Thus in selecting the least cost chain from node 2 to node 3 an existing 2 channels on link 2-5 must be assumed.

These 2 channels, for demand from 4-5, can be added at a cost of 200 units each, for a total of 400 units. The data on channel addition for each link is computed below:

Link	Milleage	Additions			
		1st channel		2nd channel	
		Cost per mile	Total cost	Cost per mile	Total cost
1-2	100	1	100	1	100
2-3	200	2	400	2	400
2-5	200	1	200	2	400
5-3	300	1	300	1	300

X_2 $^{1}0_{2}$	$^{D}X_2$ $^{1}0_{2}$		$^{D}X_1$ $^{1}0_{3}$	X_2 $^{1}0_{3}$	$^{D}X_2$ $^{1}0_{3}$		$^{D}X_1$ $^{1}0_{4}$	X_2 $^{1}0_{4}$		$^{D}X_1$ $^{1}0_{5}$	X_2 $^{1}0_{5}$		SIGN	R.H.S
													$\geq$	2
													$\geq$	2
													$\leq$	0
-1	-1												$\leq$	0
			-1	-1	-1								$\leq$	0
							-1	-1					$\leq$	0
										-1	-1		$\leq$	0
													$\geq$	0
-2													$\geq$	0
9	-1												$\geq$	0
			1	-2									$\geq$	0
				9	-1								$\geq$	0
							1	-3					$\geq$	0
										1	-3		$\geq$	0
1600	400		2000	4000	2000		200	400		300	600		N	
0 1	0 1		0 2	0 1	0 1		0 3	0 1		0 3	0 1			

: TRANCHE MATRIX FOR EXAMPLE PROBLEM

	R_1 13	R_2 13		R_1 45		DX_1 101	X_2 101		DX_1 102	X_2 102	DX_2 102
D 13	1	1									
D 45				1							
C 101	1	1				-1	-1				
C 102	1								-1	-1	-1
C 103				1							
C 104		1		1							
C 105		1									
X1 X2 101						1	-3				
X1 X2 102									1	-2	
X2 DX2 102										9	-1
X1 X2 103											
X2 DX2 103											
X1 X2 104											
X1 X2 105											
COST						100	400		400	1600	400
BOUNDS LO						0	0		0	0	0
BOUNDS UP						3	1		2	1	1

FIGURE A-1 : TRANCHE

The costs for adding one and two channels on each of the two chains are given below:

Chain	Cost	
	1 channel	2 channel
1-2-3	500	1 000
1-2-5-3	600	1 400

Thus the optimal expansion program is:

Link	Additional channels	Total cost
1-2	2	200
2-3	2	800
4-2	2	4 000
2-5	4	1 000

5.3 Matrix of Problem

There are a number of different approaches that could be taken in forming mixed-integer linear programs to solve the problems posed in Project HERMES.

The matrix in figure A-1 for the example problem uses the approach taken by the TRANCHE program but is not representative of the apparent form of the TRANCHE matrix. It is, however, representative of the effective form of the matrix. The difference lies in the generality of the TRANCHE program in regard to the status of channels on any link.

The links have been coded as follows:

Link no.	Nodes
101	1-2
102	2-3
103	4-2
104	2-5
105	5-3

The form of the problem is clearly seen, especially if the discussion on problem formulation in Chapter 3 is referred to.

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