## Capacity Analysis of Cellular CDMA

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## Summary

In this study, we have derived expressions for the capacity of a CDMA cellular system. We have analyzed the combined effects of fading, shadowing and mobile location on the service outage statistics. In our capacity calculations, we have assumed an outage probability threshold of $1 \%$ averaged over all cell locations. Outage statistics for a large number of positions were computed, allowing us to examine the dependence of the outage probability on the location within the cell in the presence of lognormal shadowing.

The effect of 3 cell-site diversity on the probability of outage was examined. A geometrical diversity region was assigned and mobiles within this region received 3 transmissions from the closest cell-sites. The cell-site diversity scheme presented reduced the worst case outage and the average outage statistics. As the outage probabilities were reduced the cell capacity could be increased. In the case of a $1 \%$ average probability of outage threshold the capacity ${ }^{*}$ doubled (from 25 to 53 users per cell) with the use of cell-site diversity. In the case of. a $10 \%$ threshold the number of users increased by 21 percent (from 268 to 326 users per cell).

The effect of a non-ideal power control method was investigated. Three models were considered: jitter, correlation, and a variance model. The variance model showed the most, promised and forms the main contribution of the analysis. By the introduction of an exponential accuracy parameter, $\kappa$, the accuracy of the transmitter's power control can be adjusted and the cell capacity can then be analysed with respect to this power control parameter. The results of the outage probability and capacity versus the parameter $\kappa$ were plotted and show that there exists a optimum value of $\kappa$ which delivers the maximum cell capacity. The improvement in capacity with the optimum $\kappa$ with non-ideal power control is slightly above three times the capacity when ideal power control is utilized.

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## Chapter 1

## Introduction

This document presents the results of a study of capacities for cellular Direct Sequence Code Division Multiple Access (DS-CDMA) systems.

There is considerable interest in commercial applications of spread spectrum (SS) communications for cellular mobile telephony such as QUALCOMM's proposal for the next generation of digital cellular [1]. The operational advantages of cellular CDMA such as frequency reuse, voice activation, and soft hand-off are apparent, but the capacity gain in terms of the number of users per cell per Hz remains debatable. A key issue is the translation of the link performance of DS-CDMA to capacities. The capacity calculations are complicated by the following aspects of DS-CDMA:
i) The combined effects of fading and location contribute to the service outage statistics. The mobile communications channel is typically prone to multipath fading and shadowing which affect the receiver's signal to noise ratio. The most typical fading model is Rayleigh fading, which is a fast varying process, with lognormal shadowing, which is a slower process. The combined process may cause service outages. In addition, in a CDMA cellular system different locations within the cell are subject to different levels of intercell interference. Thus different locations may have different outage probabilities. The actual outage experienced by a mobile user is the average of the outage probabilities at the locations along the path of travel. The combined effects of fading and location on outage probability will be analyzed.
ii) The effects of cell-site diversity. Mobiles at the cell edges can obtain improved coverage through the use of cell-site diversity which is in addition to the spread spectrum diversity inherent to DS-CDMA systems (ie. use of a RAKE receiver). However, as more transmissions per user are required when diversity is employed, the extra transmissions add to the the multiple access interference seen by other users. Thus, if the diversity is not applied properly, the system may actually experience a loss in capacity. The results are presented for 3 cell-site diversity with several different diversity regions.
iii) The effects of non-ideal power control. The effectiveness of power control will depend
both on its precision and on its performance in a cellular environment. In the uplink, power control is essential to alleviate the effects of the near-far problem. The objective of uplink power control is to ensure that the received signal at the receiving cell-site is constant. The mobile is required to vary its transmit power to achieve this objective, which in turn will increase the variance of the interference power at other cell-sites. Non-ideal power control will be examined as it is felt that the optimization of the such an uplink power control method may increase the overall capacity of the system. By using a non-ideal power control method, the intercell interference can be reduced at the cost of increased intracell interference. The effect of non-ideal uplink power control will be examined.

### 1.1 Background and Problem Formulation

To analyze the combined outage due to mobile location and lognormal fading, the mechanism behind location dependent outage must first be understood. In the downlink, a mobile closer to a cell edge is subject to more intercell interference. For any given location, an outage probability can be calculated for a given number of users per cell. If we assume a threshold of $1 \%$ outage probability for all locations we can then determine the maximum number of users per cell to satisfy a given BER performance.

In the uplink, the intercell interference affecting a given cell-site is due to mobiles in adjacent cells. The interference power caused by a mobile is a decreasing function of the distance between the mobile location and the cell-site in question. Since the location of the interfering mobile is random, the corresponding intercell interference power at the cell-site is also random. Since there are a large number of interfering mobiles, the central limit theorem can be applied, resulting in a Gaussian model for interference at the desired cell-site.

Cell-site diversity is used mainly to improve service coverage at the cell edge in the downlink. Obviously, not all positions within the cell will benefit from cell-site diversity. For example, if a mobile is close to a cell-site, a distant cell-site transmitting to this mobile will typically add to the interference seen by the users with little or no benefit to the mobile. A mobile near the cell edge is expected to benefit from cell-site diversity because it is subject to higher intercell interference and is almost equidistant between cell-sites.

The key technique which is required for DS-CDMA multiple access systems is uplink power control. Power control is necessary to combat the near-far problem at the cell-site. Ideal power control ensures that the signal power arriving at the cell-site is constant for all users regardless of the the user signals' propagation loss between the user and the cellsite. In the presence of lognormal power variations, non-ideal power control can only reduce the standard deviation of the power variation. On the other hand, given the fact that
intercell interference arrives through a different path than that between the cell-site and the mobile, the transmission power variation due to power control will increase the variation of the intercell interference. This may be modeled as lognormal shadowing with increased standard deviation. We examine a non-ideal power control strategy, to see if by reducing the standard deviation of the transmitted power from the mobile, the system capacity can be increased. This possible increase comes at the expense of increasing the overall intracell interference while reducing the overall intercell interference.

The cellular communications network considered in this document is a group of cell-sites organized in an idealized infinite hexagonal grid structure. Using the downlink, cell-sites transmit to mobile users, who are assumed to be uniformly distributed throughout the cells. The number $L_{c}$ of users per cell is assumed to be the same for each cell. There is no power control associated with the cell-site downlink. However, in the downlink, as well as the uplink, the system can take advantage of the fact that, in voice communications, users are not speaking all of the time. During speech pauses a transmitter will not output any signal, thus reducing the interference for other users. The voice activity factor $V_{o n}$ defines the percentage of time that the transmitter is active relative to the total transmission time. similar to [9], we will take it to be $3 / 8$. Each of the cell-sites is sectorized into $N_{\text {sect }}$ sectors to reduce mutual interference. The uplink and downlink are subject to lognormal shadowing which is a common assumption in mobile communications. Unlike other works, in this report we consider a practical non-ideal power control algorithm to explore potential capacity gain as will be discussed in later chapters.

### 1.2 Report plan

Chapter 2 develops expressions necessary to derive the system capacity in the presence of lognormal shadowing, in the presence of fast fading, without cell-site diversity. In the development both lognormal shadowing and location outage are taken into account. In Chapter 3, we define a diversity region where all users within the this region receive 3 diversity transmissions from the nearest cell-sites. Again the fading and location outage are taken into account. Non-ideal power control is examined in Chapter 4. A comparison between our capacity results for the CDMA and the capacities of FDMA and TDMA cellular systems is carried out in Chapter 5 . Concluding remarks are given in Chapter 6, as well as suggestions for future research.

## Chapter 2

## Capacity in Lognormal Shadowing with Fast Fading

### 2.1 Downlink Capacity

This section presents a derivation of the downlink capacity for a CDMA cellular system employing BPSK. Developments in this section are parallel to those presented in [13]. We assume that the paths are subject to lognormal shadowing and fast fading. Typically in mobile communications the fast fading is Rayleigh fading so we consider the Rayleigh fading case without shadowing first. The key performance parameter is equivalent SNR defined as $E_{s} / N_{0}$. Here, $E_{s}$ is the signal energy per symbol after despreading and $N_{0}$ is the one-sided spectral density of background interference. As is usually done in the analysis of CDMA systems [8], we assume that the system in limited by mutual interference and that thermal noise is negligible. Hence $N_{0}$ is due to multiple access interference alone. We assume that multiple access interference can be modeled as AWGN [2,3].

In DS/BPSK over a fading channel, the received signal is given by

$$
s(t)=\sqrt{2 P_{j}} A_{0} d_{j} c_{0}(t) \cos \left(2 \pi f_{c} t\right),(j-1) T_{s} \leq t<j T_{s}
$$

where $P_{j}$ is the transmitted signal power for the $j$-th symbol, $d_{j}= \pm 1$ represents the data bit transmitted during the $j$-th symbol interval, and $f_{c}$ is the carrier frequency. The DS spreading waveform, $c_{0}(t)$, for the reference user (ie. the 0 -th user), is given by

$$
c_{0}(t)=c_{l}= \pm 1,(j-1) T_{s}+l T_{c} \leq t<(j-1) T_{s}+(l+1) T_{c},
$$

for $l=0, \cdots, M-1$, where $T_{c}$ is the chip period. The received power at the receiver is the product of $\sqrt{2 P_{j}}$ and $A_{0}$, or $\sqrt{2 P_{j}} A_{0}$. The term $\sqrt{2 P_{j}}$ is a constant for all users and $A_{0}$ is the instantaneous fading amplitude experienced by the downlink signal and is common to all intracell users. We have assumed that there is no power control employed in the downlink,
hence $P_{j}$ is constant for all users. Although downlink power control was used in [9], it was stated that the capacity gain due to downlink power control was not significant and we did not examine it. There are $M$ chips per symbol period $T_{s}$. We assume that DS spreading sequences are random binary sequences. This assumption has been justified in several papers $[4,5,6,7]$. In order to perform symbol-by-symbol decision a matched filter is employed. This involves multiplying $s(t)$ plus the signals from other users by $\sqrt{\frac{2}{T_{s}}} c_{0}(t) \cos \left(2 \pi f_{c} t\right)$ and integrating the product over the time interval $\left[0, T_{s}\right]$. The signal component at the matched filter output is $d_{j} A_{0} \sqrt{E_{s j}}$, where $E_{s j}=P_{j} T_{s}$ is the $j$-th symbol energy. If, instead of interfering signals from other users, we have AWGN with one-sided spectral density $N_{0}$, the variance of the noisy component at the output of the matched filter is given by

$$
\begin{equation*}
\sigma_{n}^{2}=\frac{N_{0}}{2} \tag{2.1}
\end{equation*}
$$

Consider the case where only one other user occupies the cell occupied by the reference user. The interfering signal may then be written as

$$
\begin{equation*}
s_{1}(t)=\sqrt{2 P_{j}} A_{0} d_{j} c_{1}(t) \cos \left(2 \pi f_{c} t+\phi\right) \tag{2.2}
\end{equation*}
$$

where $c_{1}(t)$ is the spreading sequence for the interfering user. The chip boundaries of the interferer's spreading sequence, $c_{1}(t)$, are offset by $\tau$, which is uniformly distributed on the interval $\left[0, T_{c}\right.$ ), $f_{c}$ is the carrier frequency (which is common for all users), $\phi$ is the phase offset relative to the reference user. The phase offset, $\phi$, is assumed to be uniformly distributed on the interval $[0,2 \pi)$. Here we assume that the cell-site transmits the same power for each user and that interfering signals share the same propagation path. Based on the random sequence assumption, the variance of the interference component at the output of the match filter is then given by

$$
\begin{align*}
\sigma_{I}^{2} & =E\left[\left(\sqrt{\frac{P_{j}}{T_{s}}} A_{0} \cos (\phi) \int_{0}^{T_{s}} c_{0}(t) c_{1}(t) d t\right)^{2}\right] \\
& =\frac{P_{j}}{T_{s}} E\left[A_{0}^{2}\right] E\left[\cos ^{2}(\phi)\right] E\left[\left(\int_{0}^{T_{s}} c_{0}(t) c_{1}(t) d t\right)^{2}\right]  \tag{2.3}\\
& =\frac{P_{j}}{T_{s}} E\left[A_{0}^{2}\right] \int_{0}^{2 \pi} \frac{\cos ^{2}(\phi)}{2 \pi} d \phi \frac{1}{T_{c}} \int_{0}^{T_{c}} E\left[\left(\sum_{i=1}^{M}\left(a_{i} \tau+b_{i}\left(T_{c}-\tau\right)\right)\right)^{2}\right] d \tau \tag{2.4}
\end{align*}
$$

where the first expectation in Equation 2.3 is taken with respect to the fading parameter, the second with respect to $\phi$, and the third with respect to $\tau$ and independent random variables $a_{i}, b_{i}$ which take values $\pm 1$ with equal probability. In Equation 2.4, the first expectation is taken with respect to the fading parameter and the second is taken with respect to the random variables $a_{i}$ and $b_{i}$. This yields

$$
\sigma_{I}^{2}=\frac{P_{j}}{T_{s}} E\left[A_{0}^{2}\right] \frac{1}{2 T_{c}} \int_{0}^{T_{c}} E\left[\sum_{i=1}^{M} \sum_{l=1}^{M}\left(a_{i} a_{l} \tau^{2}+b_{i} b_{l}\left(T_{c}-\tau\right)^{2}+2 a_{i} b_{l}\left(T_{c}-\tau\right)\right)\right] d \tau
$$

$$
\begin{align*}
& =\frac{P_{j}}{T_{s}} E\left[A_{0}^{2}\right] \frac{M}{2 T_{c}} \int_{0}^{T_{c}}\left[\tau^{2}+\left(T_{c}-\tau\right)^{2}\right] d \tau \\
& =\frac{P_{j}}{T_{s}} E\left[A_{0}^{2}\right] \frac{M}{2} \frac{2 T_{c}^{2}}{3} \\
& =\frac{P_{j}}{T_{s}} E\left[A_{0}^{2}\right] \frac{M T_{c}^{2}}{3} \\
& =\frac{E_{s j}}{3 M} E\left[A_{0}^{2}\right] \tag{2.5}
\end{align*}
$$

The effect of interference on the pairwise error probability can be approximated by using the average variance over several symbols. The accuracy of this approximation improves as the number of symbols used in the decision process increases. From Equations 2.1 and 2.5, the respective average variances are given by

$$
\begin{equation*}
\bar{\sigma}_{n}^{2}=\frac{N_{0}}{2} \tag{2.6}
\end{equation*}
$$

and

$$
\begin{equation*}
\bar{\sigma}_{I}^{2}=\frac{1}{3 M} E_{s j} E\left[A_{k}^{2}\right] \tag{2.7}
\end{equation*}
$$

We assume that the transmitted symbol energy is constant from symbol to symbol thus we have $E_{s j}=E_{s}$ for all $j$. Comparing Equations 2.6 and 2.7, the contribution of one user to the equivalent $N_{0}$ is given by

$$
\begin{equation*}
N_{0}=\frac{2}{3 M} E_{s} E\left[A_{0}^{2}\right] . \tag{2.8}
\end{equation*}
$$

Suppose there are $L_{0}$ active interfering signals in the same cell as the reference user, where

$$
\begin{equation*}
L_{0}=\frac{V_{o n} L_{c}}{N_{s e c t}}-1 \tag{2.9}
\end{equation*}
$$

where $L_{c}$ is the number of users in the cell, $N_{\text {sect }}$ is the number of sectors per cell, and $V_{o n}$ is the voice activity factor, then the contribution by these users to the equivalent $N_{0}$ will be $L_{0}$ times that given in Equation 2.8.

We now consider the interference from an user which is outside the reference cell. The analysis is similar to the analysis for the intracell user. However, the received power is not the same for both users due to the independence of the propagation paths. Denoting the instantaneous fading amplitude for the $k$-th cell-site by $A_{k}$, the variance of the power at the matched filter output is given by

$$
\begin{align*}
\sigma_{I}^{2} & =E\left[\left(A_{k} \sqrt{\frac{P_{j}}{T_{s}}} \cos \phi \int_{0}^{T_{s}} c_{0}(t) c_{1}(t) d t\right)^{2}\right] \\
& =\frac{P_{j}}{2 T_{s}} E\left[A_{k}^{2}\right] E\left[\cos ^{2} \phi\right] E\left[\left(\int_{0}^{T_{s}}\left(c_{0}(t) c_{1}(t)\right) d t\right)^{2}\right] \tag{2.10}
\end{align*}
$$

where the first expectation in Equation 2.10 is taken with respect to the fading parameter, the second with respect to $\phi$ and the third with respect to $\tau$ and independent random variables $a_{i}, b_{i}$ which take values $\pm 1$ with equal probability. Carrying out the calculation yields

$$
\begin{align*}
\sigma_{I}^{2} & =\frac{P_{j}}{2 T_{s}} E\left[A_{k}^{2}\right] \frac{M}{T_{c}} \int_{0}^{T_{s}}\left[\tau^{2}+\left(T_{c}-\tau\right)^{2}\right] d \tau \\
& =\frac{P_{j} M T_{c}^{2}}{3 T_{s}} E\left[A_{k}^{2}\right] \\
& =\frac{1}{3 M} E_{s} E\left[A_{k}^{2}\right] \tag{2.11}
\end{align*}
$$

The respective average variances for AWGN and one intercell interferer are then given by

$$
\begin{align*}
\bar{\sigma}_{n}^{2} & =\frac{N_{0}}{2} \\
\bar{\sigma}_{I}^{2} & =\frac{1}{3 M} E_{s} E\left[A_{0}^{2}\right] . \tag{2.12}
\end{align*}
$$

The contribution to the equivalent $N_{0}$ by $K$ interfering cell-sites, each containing the same number of users as the reference cell, is given by

$$
\begin{equation*}
N_{0}=\left(L_{0}+1\right) \sum_{k=1}^{K} \frac{2}{3 M} E_{s} E\left[A_{k}^{2}\right] \tag{2.13}
\end{equation*}
$$

Here $A_{k}$ is the instantaneous fading amplitude for the path between the $k$-th interfering cellsite and the reference user. For Rayleigh fading, $A_{0}^{2}$ and $A_{k}^{2}$ are chi-squared distributed with 2 degrees of freedom. Hence, $E\left[A_{0}^{2}\right]=r_{i_{0}}^{-\gamma}$ and $E\left[A_{k}^{2}\right]=r_{i_{k}}^{-\gamma}$, where $r_{i_{k}}$ is the distance between the $i$-th mobile and the $k$-th cell-site and $\gamma$ is the propagation coefficient. Consequently, $\mathrm{E}\left[A_{0}^{2}\right]$ and $\mathrm{E}\left[A_{k}^{2}\right]$ are related by the following expression:

$$
\frac{E\left[A_{k}^{2}\right]}{E\left[A_{0}^{2}\right]}=\left(\frac{r_{i_{0}}}{r_{i_{k}}}\right)^{\gamma}
$$

Combining the signal, and the intracell and intercell interference terms, we can express the equivalent SNR as

$$
\begin{align*}
\frac{E_{s}}{N_{0}} & =\frac{E\left[A_{0}^{2}\right]}{\frac{2 L_{0}}{3 M} E\left[A_{0}^{2}\right]+\left(L_{0}+1\right) \sum_{k=1}^{K} \frac{2 E\left[A_{k}^{2}\right]}{3 M}}, \\
& =\frac{1}{\frac{2 L_{0}}{3 M}+\frac{2\left(L_{0}+1\right)}{3 M} \sum_{k=1}^{K}\left(\frac{r_{0}}{r_{i_{k}}}\right)^{\gamma}} . \tag{2.14}
\end{align*}
$$

This is the expression for equivalent SNR in the presence of Rayleigh fading alone. If we consider lognormal shadowing with fast fading, $E\left[A_{0}^{2}\right]$ is a lognormal variable. That
is, if $X_{0}$ is defined by $X_{0}=10 \log _{10} E\left[A_{0}^{2}\right]$, then $X_{0}$ is normally distributed with mean $m_{X_{0}}=10 \log _{10}\left[\left(\frac{1}{r_{i_{0}}^{(0)}}\right)^{\gamma}\right]$ and variance $\sigma_{x}^{2}$, where $r_{i_{0}}^{(0)}$ is the distance to the reference cell-site. The value of $\sigma_{x}$ is the standard deviation of the shadowing process between the user and the cell-site. Typical values are between 6 dB and $12 \mathrm{~dB}[10,13,9]$. Our numerical results are based on a $\sigma_{x}$ of 8 dB similar to [9].

The term $E\left[A_{k}^{2}\right]$ is the instantaneous received power of the interference signal from the $k$-th cell-site is also lognormally distributed. If we define $Y_{k}=10 \log _{10} E\left[A_{k}^{2}\right]$, then $Y_{k}$ is normally distributed with mean $m_{Y_{k}}=10 \log _{10}\left[\left(\frac{1}{r_{i_{k}}}\right)^{\gamma}\right]$ and variance $\sigma_{y_{k}}^{2}$. We assume that the variance of the shadowing is the same for the interfering signals as it is for the desired signal, or $\sigma_{y_{k}}^{2}=\sigma_{x}^{2}$. The equivalent SNR in the presence of lognormal shadowing and correlated Rayleigh fading is

$$
\begin{equation*}
\frac{E_{s}}{N_{0}}=\frac{10^{\frac{X_{0}}{10}}}{\frac{2 L_{0}}{3 M} 10^{\frac{X_{0}}{10}}+\frac{2\left(L_{0}+1\right)}{3 M} \sum_{k=1}^{K} 10^{\frac{Y_{k}}{10}}} \tag{2.15}
\end{equation*}
$$

The second term in the denominator involves a sum of independent lognormal random variables. This finite sum can be represented by another lognormal variable [11]. Let

$$
10^{\frac{y}{10}}=\sum_{k=1}^{K} 10^{\frac{\gamma_{k}}{10}}
$$

then $Y$ is approximately normally distributed with mean $m_{Y}$ and variance $\sigma_{Y}^{2}$ which can be calculated by a method presented in [11]. The equivalent SNR can then be expressed as

$$
\begin{equation*}
\frac{E_{s}}{N_{\mathbf{0}}}=\frac{10^{\frac{X_{0}}{10}}}{\frac{2 L_{0}}{3 M} 10^{\frac{X_{0}}{10}}+\frac{2\left(L_{0}+1\right)}{3 M} 10^{\frac{Y}{10}}} \tag{2.16}
\end{equation*}
$$

or equivalently as,

$$
\begin{equation*}
\frac{E_{s}}{N_{0}}=\frac{1}{\frac{2 L_{0}}{3 M}+\frac{2\left(L_{0}+1\right)}{3 M} 10^{\frac{Y-X_{0}}{10}}} . \tag{2.17}
\end{equation*}
$$

If we let $z=Y-X_{0}$, then $z$ is normally distributed with mean $m_{z}=m_{Y^{\prime}}-m_{x}$ and variance $\sigma_{z}^{2}=\sigma_{Y}^{2}+\sigma_{x}^{2}$. The equivalent $S N R, E_{s} / N_{0}$, is then a function of a Gaussian variable, allowing the density function to be found by standard techniques. The probability density function of $E_{s} / N_{0}$ can be expressed in terms of the mean and variance of $X_{0}$ and $Y$ as follows:

$$
\begin{equation*}
f_{\frac{E_{s}}{N_{0}}}(t)=\frac{10}{\ln 10 t\left(1-\frac{2 t L_{0}}{3 M}\right) \sqrt{2 \pi\left(\sigma_{x}^{2}+\sigma_{Y}^{2}\right)}} \exp \left(-\frac{\left(t^{\prime}-m_{Y}+m_{x}\right)^{2}}{2\left(\sigma_{x}^{2}+\sigma_{Y}^{2}\right)}\right) \tag{2.18}
\end{equation*}
$$

where

$$
\begin{equation*}
t^{\prime}=10 \log _{10}\left(\frac{3 M}{2 t\left(L_{0}+1\right)}-\frac{L_{0}}{L_{0}+1}\right) \tag{2.19}
\end{equation*}
$$

and $L_{0}$ is defined in Equation 2.9. The result presented in Equations 2.18 and 2.19 is well known. Its development is presented in Appendix A only to make the paper self-contained. The density function is parameterized by the location of the mobile, as a mobile's received signal and interference power are functions of the mobile position within the cell. Thus, the outage probability can be calculated for each cell location. We assume that an outage occurs whenever the SNR, $\left(E_{s} / N_{0}\right)$, is less than a certain level denoted by $b$. With this clefinition of outage, the probability of outage is given by

$$
\begin{equation*}
P_{o u t}\left(E_{s} / N_{0}<b\right)=\int_{0}^{b} f_{\frac{E_{0}}{N_{0}}}(t) d t \tag{2.20}
\end{equation*}
$$

where $b$ is the $\operatorname{SNR}\left(E_{s} / N_{0}\right)$ required for the given BER for reliable communications. In other words, we assume that if $E_{s} / N_{0}<b$ then the bit error rate is unacceptable and an outage occurs. The value of $b$ can be determined given the required BER. For example, in [9] a BER of $10^{-3}$ was used as the required BER with the corresponding $b$ equal to 5 d 13 . The average outage probability ( $\bar{P}_{\text {out }}$ ) over the desired cell can be obtained as follows; wo divide the cell into many small areas $d S$ and compute the outage probability for these small areas. We then sum the outage probabilities computed for each area and divide by the total amount of area in the cell. This can be expressed as

$$
\begin{equation*}
\bar{P}_{o u t}=\frac{1}{S} \int_{S} P_{o u t}\left(E_{s} / N_{0}<b\right) d S \tag{2.21}
\end{equation*}
$$

Numerical Results
In this section, numerical results are given for the downlink outage due to combined location and shadowing effects. No cell-site diversity is used. The variance of the the shadowing process is the same for all users and interference sources, that is

$$
\sigma_{x}=\sigma_{y_{k}}=8 d B \ldots \ldots
$$

The propagation coefficient $\gamma=4$ is assumed. We assume that there is no power control in the downlink, that is, the cell-site transmits the same power to all mobiles regardless of the mobiles' position within the cell. The number of chips per symbol $M$ is set to 512 .

In the case of an infinite cellular array, symmetry considerations allow us to restrict attention to a cell sector which is $1 / 12$ the area of the total cell area. A typical sector is shown in Figure 2.1. The mean and variance of $X_{0}$ and $Y$ are calculated for uniformly spaced points within the sector. The results are then used in Equation 2.18 to compute the outage probability for each location in the sector. The mobile within the sector shown in Figure 2.1 receives interference from all of the surrounding cell-sites. Figure 2.2 shows the first tier of interfering cell-sites' signal to a location within the sector. Three tiers of interfering cell-sites, or 36 cell-sites, are included in the interference calculations.

For the purposes of comparing our results with those in [9], we define an outage to occur when the $E_{b} / N_{0}$ is less than 5 dB . We also assume a rate $=1 / 2$ code a.s in [1] for the downlink. Thus, the minimum $S N R$ required for reliable communications is $E_{s} / N_{0}=2 d B$ That is to say that an outage occurs when the $E_{s} / N_{0}<2 \mathrm{~dB}$.

The results differ between this paper and those presented in [9] for several reasons. In [9], a. maximum of 18 cell-sites were included in the interference calculations while in this paper we have incorporated the effect of the first three tiers for a total of 36 interfering cell-sites. This is a minor difference as the third tier interference has a relatively small contribution to the interference. Another difference is we have not considered downlink power control which was included in [9]. We do not feel that the effect on the system capacity is affected much by this. A major difference in the approaches taken was that in [9] the method of assigning cell membership and thus how the signal-to-noise ratio was calculated. In [9], the received signal powers from the cell-sites were ordered and the cell-site with the maximum was assigned to transmit the signal to the mobile and the remaining signals were considered interference. In this section we assigned cell membership to the closest cell-site (ie. by distance) and considered the surrounding cell-sites as interference. In this section we do not; consider hand-off to the cell-site with the maximum power and it is this difference in the analysis which we expect has the greatest impact on the difference in the system capacities between this report and [9].

Table 2.1 shows various outage statistics for a number of different cell loads with the assumptions stated above. If we set a threshold of $1 \%$ for the average probability of outage, then the capacity per cell is 25 users. The table also shows that the worst case outage probability at any location in the cell is $6.1 \%$ and $33.52 \%$ of users experience a probability of outa.ge exceeding $1 \%$. If we assume that the average outage probability.should not exceed $10 \%$, then the number of users per cell increases to 286 . In this case, $74.72 \%$ and $41.76 \%$ of users, respectively, experience outage probabilities exceeding $1 \%$ and $10 \%$. The maximum outage probability at the worst case location is slightly over $36 \%$. Clearly, an average outage probability threshold of $10 \%$ results in unacceptable performance for a large percentage of users. This percentage will be used for comparison with other cellular systems in Chapter 5.

To illustrate the outage probability's dependence on the position within the cell, we present three contour maps for the cell sector shown in Figure 2.1. Figures 2.3-2.5 display contours for 4,8 , and 10 users per cell, respectively. The contour lines each denote an outage probability of 1 percent. The contour lines in the figures are labeled with the appropriate probability of outage. As a rectangular grid is used for calculations, the diagonal sector boundary appears as a jagged edge in the contours. The sector map is included for a point of reference. The relative position of the sector within the cell is presented in Figure 2.1. In


Figure 2.1: A Typical Sector (1/12 of cell) used to compute the outage probabilities


Figure 2.2: Illustration of the Interference from Cell-Sites within the First Tier on a Location within a Typical Sector

| $L_{c}$ | $P_{\text {out }}$ | $\max P_{\text {out }}$ | \% Area $>1 \%$ | \% Area $>10 \%$ |
| ---: | ---: | :---: | :---: | :---: |
| 24.0 | 0.0093 | 0.0588 | 0.3258 | 0 |
| 25.0 | 0.0098 | 0.0612 | 0.3315 | 0 |
| 25.2 | 0.0099 | 0.0617 | 0.3352 | 0 |
| 25.4 | 0.0099 | 0.0621 | 0.3371 | 0 |
| 25.6 | 0.0100 | 0.0626 | 0.3371 | 0 |
| 25.8 | 0.0101 | 0.0631 | 0.3390 | 0 |
| 26.0 | 0.0102 | 0.0635 | 0.3390 | 0 |
| 28.0 | 0.0111 | 0.0681 | 0.3596 | 0 |
| 30.0 | 0.0120 | 0.0726 | 0.3708 | 0 |
| 32.0 | 0.0129 | 0.0770 | 0.3858 | 0 |
| 34.0 | 0.0138 | 0.0813 | 0.4007 | 0 |
| 220 | 0.0812 | 0.3172 | 0.7135 | 0.3539 |
| 240 | 0.0870 | 0.3332 | 0.7266 | 0.3745 |
| 260 | 0.0927 | 0.3482 | 0.7360 | 0.3989 |
| 280 | 0.0982 | 0.3625 | 0.7453 | 0.4139 |
| 285 | 0.0996 | 0.3659 | 0.7472 | 0.4157 |
| 286 | 0.0999 | 0.3666 | 0.7472 | 0.4176 |
| 287 | 0.1001 | 0.3673 | 0.7491 | 0.4176 |
| 288 | 0.1004 | 0.3680 | 0.7491 | 0.4176 |
| 289 | 0.1007 | 0.3687 | 0.7491 | 0.4195 |
| 290 | 0.1009 | 0.3693 | 0.7491 | 0.4195 |
| 300 | 0.1036 | 0.3760 | 0.7509 | 0.4326 |

Table 2.1: Outage statistics for $L_{c}$ users per cell. $E_{s} / N_{0}=2 \mathrm{~dB}$ and standard deviation of lognormal shadowing is 8 dB .


Figure 2.3: Contour plot of outage probability for 20 users per cell with $1 \%$ probability contour lines
each of the figures, the probability of outage increases with the distance from the cell centre. The worst case outage probability occurs at a cell vertex (the lower right hand corner in the figures).

Additional results were obtained for the purposes of comparing our results with those presented in [8]. For this comparison, we were required to change a number of the parameters from the values stated earlier in this section. The $\mathrm{SNR}, E_{s} / N_{0}$ was set equal to -3.3 dB which corresponds to an $E_{b} / N_{0}=1.5 d B$ with rate $=1 / 3$ coding as used in [8]. For the comparison we followed the processing gain definition in [8], which is $P G=10 \log (W / R)$, where $W$ is the bandwidth of the signal and $R$ is the information bit rate. The expression for processing gain becomes $P G=2 M / r_{f e c}$ when the null to null signal bandwidth is used (ie. $W=2 R M / r_{f e c}$ ), where $M$ is the number of chips per symbol and $r_{f e c}$ is the coding rate. The number of chips per symbol was set to 210 to give the required processing gain of 31 dB as used in [8]. The average outage probabilities were calculated for the cell loads of 651 and 868 users per cell


Figure 2.4: Contour plot of Outage probability for 25 users per cell with $1 \%$ contour lines


Figure 2.5: Contour plot of Outage probability for 30 users per cell with $1 \%$ contour lines
which were $14.5 \%$ and $17.8 \%$, respectively. The corresponding results presented in [8] were $15.5 \%$ and $18.6 \%$, respectively. Given the parameters used in [8] our method showed good correspondence as the average outage probabilities for the given cell loads were within $1 \%$ of the presented results.

### 2.2 Uplink Capacity

We now study the uplink capacity in lognormal shadowing with fast fading. We adopt the terminology of the last section on the understanding that the receiver is now at a cell-site and that the interference is from other mobiles. Many of the results are taken from the previous section. The required SNR will be increased to take into account for the lack of a pilot signal for use in demodulation.

Power control is a necessity for DS-CDMA multiple access systems. Transmitters close to cell-sites cannot be allowed to saturate or capture the cell-site's receiver. Ideally, each mobile will compensate for lognormal shadowing by varying its transmitted power in such a way that the received signal strength at the cell-site is equal for all users. However, perfect power control is difficult to achieve. Moreover, transmitter power fluctuations may increase the variance of interference affecting other cell-sites. In the following development, we assume that mobiles do not have ideal power control. In the model presented in this section, we assume that the power control's estimate of the uplink attenuation and the actual uplink attenuation can differ. We assume that they are correlated, but not necessarily equal. We also assume that the variance of the transmit power can be constrained to be less than the variance of the shadowing process. Separate correlation and reduced variance power control methods are considered in detail in Chapter 4.

We first calculate the interference from a user in the same cell as the reference user under lognormal shadowing with fast fading with imperfect power control.

The interfering signal can be written as

$$
\begin{equation*}
s_{\mathbf{1}}(t)=\frac{A_{i_{k}}^{(o)}}{Q_{i_{k}}} \sqrt{2 P_{j}} d_{j} c_{\mathbf{l}}(t) \cos \left(2 \pi f_{c} t+\phi\right) \tag{2.22}
\end{equation*}
$$

where $c_{1}(t)$ is the spreading sequence for the interfering user. The chip boundaries of the interferer's spreading sequence, $c_{1}(t)$, are offset by $\tau$, which is assumed to be uniformly distributed on the interval $\left[0, T_{c}\right)$, where $T_{c}$ is the chip period. The carrier frequency is denoted as $f_{c}$ and is common for all users. The phase relative to the reference user, $\phi$, is assumed to be uniformly distributed on the interval $[0,2 \pi)$. The instantaneous fading amplitude affecting the signal from the $i$-th user in the $k$-th cell-site is denoted by $A_{i_{k}}^{(o)}$. The transmitter compensates for fading by applying a scale factor $Q_{i_{k}}$ to the transmitted signal. $Q_{i_{k}}$ is a function of the transmitter's estimate of the uplink attenuation affecting the signal. For ideal power control, if we assume the transmitter's estimate of the uplink attenuation then $Q_{i_{k}}$ is given by $Q_{i_{k}}=A_{i_{k}}$. The variance of the interference due to other users at the
cell-site receiver is calculated as follows:

$$
\begin{align*}
\sigma_{I}^{2} & =E\left[\left(\sqrt{\frac{P_{j}}{T_{s}}} \frac{A_{i_{k}}^{(o)}}{Q_{i_{k}}} \cos \phi \int_{0}^{T_{s}} c_{0}(t) c_{1}(t) d t\right)^{2}\right] \\
& =\frac{P_{j}}{2 T_{s}} E\left[\frac{A_{i_{k}}^{(o) 2}}{Q_{i_{k}}^{2}}\right] E\left[\cos ^{2} \phi\right] E\left[\left(\int_{0}^{T_{s}} c_{0}(t) c_{1}(t) d t\right)^{2}\right]  \tag{2.23}\\
& =\frac{P_{j}}{2 T_{s}} E\left[\frac{A_{i_{k}}^{(o) 2}}{Q_{i_{k}}^{2}}\right] \frac{M}{T_{c}} \int_{0}^{T_{c}}\left[\tau^{2}+\left(T_{c}-\tau\right)^{2}\right] d \tau \\
& =\frac{P_{j} M T_{c}^{2}}{3 T_{s}} E\left[\frac{A_{i_{k}}^{(o) 2}}{Q_{i_{k}}^{2}}\right] \\
& =\frac{1}{3 M} E_{s} E\left[\frac{A_{i_{k}}^{(o) 2}}{Q_{i_{k}}^{2}}\right] . \tag{2.24}
\end{align*}
$$

where the first expectation in Equation 2.23 is taken with respect to the power, the second with respect to $\phi$, and the third with respect to $\tau$ and independent random variables $a_{i}, b_{i}$ which take on values of $\pm 1$. Following the arguments presented in the downlink section, we clefine the average variances

$$
\begin{align*}
\bar{\sigma}_{n}^{2} & =\frac{N_{0}}{2}  \tag{2.25}\\
\bar{\sigma}_{I}^{2} & =\frac{1}{3 M} E_{s} E\left[\frac{A_{i_{k}}^{(o)}}{Q_{i_{k}}^{2}}\right] \tag{2.26}
\end{align*}
$$

Defining $\bar{\sigma}_{n}^{2}=\bar{\sigma}_{I}^{2}$, we have an equivalent noise contribution by one intracell user to the equivalent noise given by

$$
\begin{equation*}
N_{0}=\frac{2 E_{s}}{3 M} E\left[\frac{A_{i_{k}}^{(o) 2}}{Q_{i_{k}}^{2}}\right] \tag{2.27}
\end{equation*}
$$

If there are $L_{0}$ active interfering carriers within the same cell, then the equivalent $N_{0}$ is increased by $L_{0}$ times in the case of Rayleigh fading alone, where Rayleigh fading is typical fast fading process encountered in mobile applications. With lognormal shadowing, we assume that $E\left[A_{i_{0}}^{(o) 2} / Q_{i_{0}}^{2}\right]$ is lognormally distributed. If we define $Z_{i_{0}}$ for the $i$-th mobile in the reference cell-site as,

$$
\begin{equation*}
Z_{i_{0}}=10 \log _{10}\left(E\left[\frac{A_{i_{0}}^{(o) 2}}{Q_{i_{0}}^{2}}\right]\right), \tag{2.28}
\end{equation*}
$$

then $Z_{i_{0}}$ is normally distributed with $m_{Z_{i_{0}}}=0$ and variance

$$
\begin{equation*}
\sigma_{Z_{i 0}}^{2}=\sigma_{x}^{2}+\sigma_{X_{t}}^{2}-2 \rho \sigma_{x} \sigma_{X_{t}}, \tag{2.29}
\end{equation*}
$$

where $\sigma_{x}^{2}$ is the variance of the lognormal shadowing process, $\sigma_{X_{t}}^{2}$ is the variance of the transmitter power fluctuations, and $\rho$ is the correlation coefficient between the channel attenuation and the transmitter's compensation for the channel attenuation. Both $\sigma_{X_{1}}^{2}$ and $\rho$ are dependent on the choice of $Q$. All possible terms $Z_{i_{0}}$ have the same mean and variance since the intracell users are controlled by a common cell-site. Please note that the reference user will be denoted as $Z$ instead of $Z_{0_{0}}$ and it has the same mean and variance as $Z_{i_{0}}$. Thus, the intracell interference for a cell with $L_{0}$ active interference signals can be denoted as

$$
\begin{equation*}
I_{1}=\frac{2 E_{s}}{3 M} \sum_{i=1}^{L_{0}} 10^{\frac{z_{i 0}}{10}} \tag{2.30}
\end{equation*}
$$

For the intercell interference we consider that there are $K$ interfering cells which are equally loaded with $L_{c}$ users. The corresponding intercell contribution to $N_{0}$ is

$$
\begin{equation*}
I_{2}=\frac{2 V_{o n}}{3 M N_{s e c t}} \sum_{k=1}^{K} \sum_{i=1}^{L_{\mathrm{c}}} E\left[\frac{A_{i_{k}}^{(o) 2}}{Q_{i_{k}}^{2}}\right] \tag{2.31}
\end{equation*}
$$

where $A_{i_{k}}^{(o)}$ represents the fading amplitude affecting the path from the $i$-th interfering mobile in the reference cell-site ( 0 -th cell) and $Q_{i_{k}}$ is the transmitter compensation for the attenuation in the path between the mobile and its own cell-site (ie. the $k$-th cell-site): Note that the interference from a mobile in an adjacent cell arrives through a fading path independent of the signal path to its own cell-site. Thus, the interference from this user is dependent on its distance to its own cell-site and its distance to the reference cell-site. We define the distance of the $i$-th user to the $k$-th cell-site as $r_{i_{k}}$ and the distance to the reference cell-site as $r_{i_{k}}^{(o)}$ as in [8].

Similar to the intracell interference case with lognormal shadowing, we assume that $\mathrm{E}\left[A_{i_{k}}^{(o) 2} / Q_{i_{k}}^{2}\right]$ is lognormally distributed and we define $Z_{i_{k}}$ for the $i$-th mobile in the $k$-th cell-site,

$$
\begin{equation*}
Z_{i_{k}}=10 \log _{10}\left(E\left[\frac{A_{i_{k}}^{(0) 2}}{Q_{i_{k}}^{2}}\right]\right) \tag{2.32}
\end{equation*}
$$

then $Z_{i_{k}}$ is normally distributed with mean $m_{Z_{i_{k}}}=10 \log _{10}\left(\frac{r_{i_{k}}}{r_{i_{k}}^{(0)}}\right)^{\gamma}$ and variance

$$
\begin{equation*}
\sigma_{Z_{i_{0}}}^{2}=\sigma_{x}^{2}+\sigma_{X_{t}}^{2} \tag{2.33}
\end{equation*}
$$

It is assumed that the propagation paths between the $k$-th cell-site and the reference cellsite are independent. Thus, the correlation coefficient of a user's estimate of the uplink
attenuation to the $k$-th cell-site and the attenuation along the path to the reference cell-site is zero. Thus, the contribution to the equivalent noise is can be denoted as

$$
\begin{equation*}
I_{2}=\frac{2 V_{o n}}{3 M N_{s e c t}} \sum_{k=1}^{K} \sum_{i=1}^{L_{c}} 10^{\frac{z_{i i_{k}}}{10}} . \tag{2.34}
\end{equation*}
$$

The equivalent noise can be obtained by combining Equations 2.30 and 2.34. and is essentially summing the interference from all of the active interfering users:

$$
\begin{equation*}
N_{0}=\frac{2 E_{s}}{3 M}\left(\sum_{i=1}^{L_{0}} 10^{\frac{z_{i 0}}{10}}+\frac{V_{o n}}{N_{\text {sect }}} \sum_{k=1}^{K} \sum_{i=1}^{L_{c}} 10^{\frac{z_{i k}}{10}}\right) \tag{2.35}
\end{equation*}
$$

where $K$ is the number of cells included in the interference calculations, $N_{\text {sect }}$ in the number of sectors in reference cell-site, and $V_{o n}$ is the voice activity factor. The total contribution by interfering cells surronding the reference cell is computed, then this value is divided by $N_{\text {sect }}$ to obtain the contribution by $1 / N_{\text {sect }}$ to account for the sectors. Note that this is for symmetric antenna patterns. It would approximate the interference seen in one sector if the antenna patterns were non-symmetric.

The equivalent $S N R$ is given by

$$
\begin{equation*}
\frac{E_{s}}{N_{0}}=\frac{3 M 10 \frac{Z}{10}}{2 \sum_{i=1}^{L_{0}} 10^{\frac{z_{i j}}{10}}+2 \frac{V_{o n}}{N_{s e c z}} \sum_{k=1}^{K} \sum_{i=1}^{L_{c}} 10^{\frac{z_{i_{k}}}{10}}} \tag{2.36}
\end{equation*}
$$

Re-arranging yields

$$
\begin{equation*}
\frac{E_{s}}{N_{0}}=\frac{3 M}{2}\left(\sum_{i=1}^{L_{0}} 10^{\frac{w_{i o}}{10}}+\frac{V_{o n}}{N_{\text {sect }}} \sum_{k=1}^{K} \sum_{i=1}^{L_{c}} 10^{\frac{w_{i k}}{10}}\right)^{-1} \tag{2.37}
\end{equation*}
$$

where $w_{i_{0}}=Z_{i_{0}}-Z$ is a zero mean Gaussian variable with variance

$$
\begin{align*}
\sigma_{w_{i_{0}}}^{2} & =2 \sigma_{Z_{i_{0}}}^{2} \\
& =2 \sigma_{X_{\mathrm{a}}}^{2}+2 \sigma_{X_{\mathrm{t}}}^{2}-4 \rho \sigma_{X_{\mathrm{a}}} \sigma_{X_{\mathrm{t}}} \tag{2.38}
\end{align*}
$$

and $w_{i_{k}}=Z_{i_{k}}-Z$ is a Gaussian variable with mean

$$
m_{w_{i_{k}}}=10 \log _{10}\left(\frac{r_{i_{k}}}{r_{i_{k}}^{(o)}}\right)^{\gamma}
$$

and variance

$$
\begin{align*}
\sigma_{w_{i_{k}}}^{2} & =\sigma_{Z_{i_{k}}}^{2}+\sigma_{Z}^{2} \\
& =2\left(\sigma_{X_{a}}^{2}+\sigma_{X_{\mathbf{l}}}^{2}-\rho \sigma_{X_{a}} \sigma_{X_{\mathfrak{l}}}\right) \tag{2.39}
\end{align*}
$$

As there is a very large number of terms in the summations, we invoke the Central Limit Theorem and approximate the sums in the denominator of Equation 2.37 by normally distributed terms. To carry out the approximation we require the mean and variance of the lognormal terms.

The derivation of the density function for $\frac{E_{s}}{N_{0}}$ is given in Appendix B. We have

$$
\begin{equation*}
f_{\frac{E_{s}}{N_{0}}}(t)=\frac{3 M}{t^{2} \sqrt{2 \pi\left(\sigma_{u}^{2}+\sigma_{v}^{2}\right)}} \exp \left(-\frac{\left(t^{\prime}-\left(m_{u}+m_{v}\right)^{2}\right.}{2\left(\sigma_{u}^{2}+\sigma_{v}^{2}\right)}\right) \tag{2.40}
\end{equation*}
$$

where

$$
\begin{equation*}
t^{\prime}=\frac{3 M}{2 t} \tag{2.41}
\end{equation*}
$$

and where $m_{u}$ and $m_{v}$ are the respective means of the first and second terms in the denominator of Equation 2.37 with $\sigma_{u}^{2}$ and $\sigma_{v}^{2}$ being their respective variances. The uplink outage statistics can be calculated for a given number of users in each cell by computing

$$
\begin{equation*}
P_{o u t}\left(E_{s} / N_{0}<b\right)=\int_{0}^{b} f_{\frac{E_{s}}{N_{0}}}(t) d t \tag{2.42}
\end{equation*}
$$

where $b$ is the required SNR for the required bit error rate.
An alternative is to cast Equation 2.37 in the form

$$
\frac{E_{s}}{N_{0}}=\frac{3 M}{2}(U+V)^{-1}
$$

where $\mathrm{U}+\mathrm{V}$ is normally distributed with mean $m_{u}+m_{v}$ and variance $\sigma_{v}^{2}+\sigma_{u}^{2}$. Then

$$
\begin{align*}
P_{\text {out }} & =P\left(\frac{E_{s}}{N_{0}} \leq b\right) \\
& =P\left(U+V \geq \frac{3 M}{2 b}\right) \\
& =\frac{1}{2} e r f c\left(\frac{\frac{3 M}{2 b}-\left(m_{u}+m_{v}\right)}{\sqrt{2\left(\sigma_{u}^{2}+\sigma_{u}^{2}\right)}}\right) \tag{2.43}
\end{align*}
$$

where $\operatorname{erfc}(\cdot)$ is the complementary error function.
The uplink outage probability is evaluated using this equation. We assume that users are uniformly distributed within the cells and that mobiles are controlled by the nearest cell-site. Table 2.2 gives the maximum number of users per cell. The standard deviation of lognormal shadowing is 8 dB , the number of chips per symbol is $M=512$ and the mobiles are assumed to have ideal power control in this section. The number of sectors in the cell, $N_{\text {sect }}$, was set to 3 and the voice activity factor, $V_{o n}$, was again set to $3 / 8$. The interference for the entire cell was calculated by considering the interference seen by the $1 / 12$ sector then multiplying by 12 . Using this method there were 80 cells used in the interference calculations seen by the

| $E_{b} / N_{0} \mathrm{~dB}$ | M | $P_{\text {out }}$ | LC |
| :---: | :---: | :---: | :---: |
| 7 | 512 | .01 | 16 |
| 7 | 1024 | .01 | 59 |
| 7 | 512 | .10 | 43 |
| 7 | 1024 | .10 | 137 |

Table 2.2: Number of users per cell at $1 \%$ and $10 \%$ probability of outage on the Uplink. Standard deviation of lognormal shadowing $=8 \mathrm{~dB}$.
entire cell and this corresponds to 26.7 cells interfering with a sector when $N_{\text {sect }}=3$. The required SNR is 2 dB above that of BPSK for the downlink. In other words we assume that the required $E_{b} / N_{0}$ is 7 dB which was also used in [9] and rate $=1 / 3$ coding is used as in [1]. The results for $M=1024$ are also included. Doubling $M$ more than doubled the number of users per cell.

For the uplink we have assigned the users according to their location and not to the 'best' cell-site thus strong intercell interferers will exist along the border of the reference cell. This strong intercell interference will lower the capacity of the cell. In [9] cell membership was assigned not by location as in this report but by power. The reference cell-site would be assigned if the intercell interferer was above a given power threshold. This technique reduces the intercell interference as the strong intercell interferers become intracell interferers which create less interference for the reference cell-site. Comparing our results with those in [9] it is apparent that it is important to assign the mobiles according to the power received rather than by location. In this report we have assigned the mobiles to the cell-site according to their location, and thus we expect the results to be a lower bound on the number of users per cell.

## Chapter 3

## Capacity in Lognormal Shadowing with Fast Fading and Cell-Site Diversity

Cell-site diversity is employed to improve outage statistics for mobiles at the edge of a cell. Mobiles near the cell boundary are subject to interference from adjacent cell-sites. Cell-site diversity is also used to allow 'soft hand-off', where the mobile may choose which cell-site to receive while it is moving between cells. Both the downlink and the uplink make use of cell-site diversity. By this we mean that the mobile will select the best downlink signal from a set of $K_{\text {div }}$ cell-sites and we will assume that the mobile will make use of the corresponding uplink path to transmit to the best cell-site.

In the following analysis, all cell-site transmissions are assumed to be of equal power. For lognormal shadowing, the quantity $E\left\{A_{0, k}^{2}\right\}$ is a lognormal random variable. The value of $A_{0, k}$ is the instantaneous fading amplitude affecting the $k$-th cell-site's signal received at $\rightarrow$ the reference position. Since $E\left\{A_{0, k}^{2}\right\}$, is a lognormal random variable, $Y_{k}={ }^{\prime} 10 \log _{10} E\left\{A_{0, k}^{2}\right\}$ is a normally distributed random variable with mean $m_{Y_{k}}=10 \log _{10}\left\{\left(\frac{1}{r_{i_{k}}}\right)^{\gamma}\right\}$ and variance $\sigma_{Y_{k}}^{2}$, where $r_{i_{k}}$ is the distance between the $i$-th mobile and the $k$-th cell-site, and $\gamma$ is the propagation coefficient. The value of $i=0$ corresponds to the reference user ( $i=0$ for the 0 -th user). For diversity selection, we assume that the mobile receiver chooses to decode the received signal with the highest signal strength, or the maximum value of $E\left[A_{0, k}^{2}\right]$. If we define $X_{0}$ as

$$
\begin{equation*}
X_{0}=\max \left\{10 \log _{10} E\left\{A_{0,1}^{2}\right\}, 10 \log _{10} E\left\{A_{0,2}^{2}\right\}, \ldots, 10 \log _{10} E\left\{A_{0, K_{d i v}}^{2}\right\}\right\} \tag{3.1}
\end{equation*}
$$

then the expression for equivalent $S N R$ is similar to the expression in Section 2.1:

$$
\begin{equation*}
\frac{E_{s}}{N_{0}}=\frac{10^{\frac{X_{0}}{10}}}{\frac{2 L_{0}}{3 M} 10^{\frac{X_{0}}{10}}+\frac{2\left(L_{0}+1\right)}{3 M} \sum_{k=1}^{K} 10^{\frac{Y_{k}}{10}}} . \tag{3.2}
\end{equation*}
$$

Here $X_{0}$ is defined in Equation 3.1, $\sum_{k=1}^{K} 10^{\frac{Y_{k}}{10}}$ is the interference from adjacent cell-sites, and $L_{0}$ is the number of active interfering signals in the mobile's cell-site:

$$
\begin{equation*}
L_{0}=\frac{\left(1+\eta\left(K_{d i v}-1\right)\right) L_{c} V_{o n}}{N_{s e c t}}-1 \tag{3.3}
\end{equation*}
$$

Here $\eta$ is the percentage of users requiring diversity in a cell, $L_{c}$ is the number of users within a cell, $V_{o n}$ is the voice activity factor, $K_{d i v}$ is the number of diversity signals transmitted to the mobile, and $N_{\text {sect }}$ is the number of sectors in a cell. The number of active interfering signals from adjacent cell-sites is $L_{0}+1$.

We may replace the sum of lognormal random variables with a single lognormal random variable as follows:

$$
\begin{equation*}
10^{\frac{Y}{10}}=\sum_{k=1}^{K} 10^{\frac{Y_{k}}{10}} \tag{3.4}
\end{equation*}
$$

where $Y$ is approximately Gaussian with mean $m_{Y}$ and variance $\sigma_{Y}^{2}$ which can be computed using methods presented in [11]. Making this substitution and rearranging Equation 3.2, we obtain the following expression for the equivalent SNR:

$$
\begin{equation*}
\frac{E_{s}}{N_{0}}=\frac{1}{\frac{2 L_{0}}{3 M}+\frac{2\left(L_{0}+1\right)}{3 M} 10^{\frac{Y-X_{0}}{10}}} \tag{3.5}
\end{equation*}
$$

Following [ $8,9,13$ ], we assume that the intercell interference term ( $10 \frac{Y}{10}$ ) is independent of the cell-site with maximum power (ie. Y is independent of $X_{0}$ ). In other words, we assume that the mobile takes full advantage of the diversity transmissions from the $K_{\text {div }}$ cell-sites, but we assume that the transmission originates from the nearest cell-site to the user. (ie. the mobile uses the best signal but there is no 'hand-off' between the cells.) This slightly increases the interference as seen by the mobiles, but makes the analytical work much more tractable. As the interference is slightly increased we feel that the capacity results will be a lower bound to the actual capacity.

Using the independence of $Y$ and $X_{0}$, we can find the density function of the random variable

$$
\begin{equation*}
Z=Y-X_{0} \tag{3.6}
\end{equation*}
$$

The density function of $Z$ is given by

$$
\begin{equation*}
f_{Z}(z)=\int_{-\infty}^{\infty} f_{X_{0}}\left(x_{0}\right) f_{Y}\left(z+x_{0}\right) d x_{0} \tag{3.7}
\end{equation*}
$$

where the density function $f_{Y}(y)$ is given by

$$
\begin{equation*}
f_{Y}(y)=\frac{1}{\sqrt{2 \pi} \sigma_{Y}} \exp \left\{-\frac{\left(y-m_{Y}\right)^{2}}{2 \sigma_{Y}^{2}}\right\} \tag{3.8}
\end{equation*}
$$

The density function of $X_{0}$ is given in [8] as Equation 38, but is repeated here for completeness:

$$
\begin{array}{r}
f_{X_{0}}(x)=\sum_{i=1}^{K_{d i v}} \frac{1}{\sqrt{2 \pi} \sigma_{Y_{i}}} \exp \left(-\frac{\left(x-m_{Y_{i}}\right)^{2}}{2 \sigma_{Y_{i}}^{2}}\right) \times \\
\prod_{j=1, j \neq i}^{K_{d i v}}\left[1-Q\left(\frac{x-m_{Y_{j}}}{\sigma_{Y_{j}}}\right)\right] \tag{3.9}
\end{array}
$$

where

$$
\begin{equation*}
Q(x)=\frac{1}{\sqrt{2 \pi}} \int_{x}^{\infty} \exp \left(-\frac{t^{2}}{2}\right) d t \tag{3.10}
\end{equation*}
$$

Not all positions within a cell are expected to benefit from diversity transmissions. For example, if the mobile is near the centre of a cell-site, there is little benefit in having another cell-site transmit to the mobile. A cell then can be divided into two parts: a diversity region where mobiles receive $K_{\text {div }}$ copies of their signals, and a non-diversity region where mobiles receive only one signal from the closest cell-site. Within the diversity region, the outage probability experienced by a user at a given position is given by

$$
\begin{equation*}
P_{\text {out }}\left(E_{s} / N_{0}<b\right)=\int_{z_{\min }}^{z_{\max }} f_{Z}(z) d z \tag{3.11}
\end{equation*}
$$

where the density function $f_{Z}(z)$ is parameterized by the position within the cell and $z_{\max }=\infty$ and $z_{\min }$ depends on the $b$. Here, as in previous sections, $b$ is the SNR ( $E_{s} / N_{0}$ ) corresponding to required bit error rate. From Equation 3.5, $z_{\min }$ depends on the SNR as follows:

$$
\begin{equation*}
z_{\min }=10 \log _{10}\left(\frac{3 M}{2\left(L_{0}+1\right)}\left(\frac{1}{b}-\frac{2 L_{0}}{3 M}\right)\right) \tag{3.12}
\end{equation*}
$$

where $b$ is the required SNR for the required bit error probability on the downlink. Within the non-diversity region, the outage probability for any position is given by

$$
\begin{equation*}
P_{o u t}\left(E_{s} / N_{0}<b\right)=\int_{0}^{b} f_{\frac{E_{s}}{N_{0}}}(t) d t \tag{3.13}
\end{equation*}
$$

where the density function $f_{\frac{E_{0}}{N_{0}}}(t)$ is given by

$$
\begin{equation*}
f_{\frac{E_{0}}{N_{0}}}(t)=\frac{10}{\ln 10 t\left(1-\frac{2 t L_{0}}{3 M}\right) \sqrt{2 \pi\left(\sigma_{x}^{2}+\sigma_{Y}^{2}\right)}} \exp \left(-\frac{\left(t^{\prime}-m_{Y}+m_{X}\right)^{2}}{2\left(\sigma_{x}^{2}+\sigma_{Y}^{2}\right)}\right) \tag{3.14}
\end{equation*}
$$

where

$$
\begin{equation*}
t^{\prime}=10 \log _{10}\left(\frac{3 M}{2 t\left(L_{0}+1\right)}-\frac{L_{0}}{L_{0}+1}\right) \tag{3.15}
\end{equation*}
$$

and $L_{0}$ is defined in Equation 3.3. The derivation of Equation 3.14 is the same as Equation 2.18 in Section 2.1 and is given in Appendix A.

Equations 3.11 and 3.14 yield the outage probability for positions within the diversity and non-diversity regions, respectively, for a given value $L_{0}$ of active interfering signals where $L_{0}$ is defined in Equation 3.3 and is directly dependent on the cell load. Thus, the outage probability for a given diversity region and a given cell load can be calculated via the use of Equations 3.11 and 3.14. From Equation 3.3 it is apparent that the number of interfering signals is not equal to the number of users in the cell when cell-site diversity is employed. The number of users can be expressed in terms of the interfering signals by solving Equation 3.3 for $L_{c}$ :

$$
\begin{equation*}
L_{c}=\frac{N_{\text {sect }}}{\left(1+\eta\left(K_{d i v}-1\right)\right) V_{o n}}\left(L_{0}+1\right) \tag{3.16}
\end{equation*}
$$

where $\eta$ is the percentage of users requiring diversity. Assuming that the users are uniformly distributed throughout the cell, the percentage of users requiring diversity is the ratio of the area of the diversity region to the total area of the cell:

$$
\begin{equation*}
\eta=\frac{\text { Are } a_{\text {div }}}{\text { Are } a_{\text {cell }}} . \tag{3.17}
\end{equation*}
$$

There are several methods to specify the diversity region and thus the percentage $\eta$. The ideal method is to evaluate the probability of outage for each point and assign it to the diversity region if this probability exceeds a certain threshold. However, we choose to define a diversity region in the following straightforward manner. Draw a circle of radius $r_{t}$ inside the cell. The area within the circle is the non-diversity region while the region of the cell lying outside the circle is the diversity region. This approach appears to be sensible in view of the outage probability contour maps of Figures 2.3-2.5. In these figures, the contour lines appear to be approximately an arc of a circle with it centre at the cell-site. As the cell perpendicular is assigned to unit length, the area of the diversity region within the triangular cell sector containing $1 / 12$ the cell area is given by

$$
\begin{equation*}
\text { Area } a_{d i v}=\frac{\sqrt{2}}{2}-\frac{\pi r_{t}^{2}}{12} \tag{3.18}
\end{equation*}
$$

The diversity and non-diversity regions are illustrated in Figure 3.1. The ratio of the diversity region area to the total cell area is the same for the small sector as it is for the whole cell, thus using Equation 3.17 we obtain the percentage of users requiring diversity to be,

$$
\begin{equation*}
\eta=1-\frac{\pi r_{t}^{2}}{2 \sqrt{3}} \tag{3.19}
\end{equation*}
$$



Figure 3.1: A Typical Sector with Diversity Region.
In summary, a mobile at distance $r_{i_{o}}^{(o)}$ from the reference cell-site is in the diversity region if $r_{i_{o}}^{(o)}>r_{t}$. The percentage $\eta$ depends on the value of $r_{t}$ as indicated by Equation 3.19. Thus, by varying $r_{t}$, we obtain different values of $\eta$ and different outage statistics for positions within the cell. By determining the maximum of $L_{0}$ for a given $r_{t}$ (where $r_{t}$ defines the diversity region and the percentage of users which require diversity), the maximum number of users in the cell can be found using Equation 3.16. Note that the double integral obtained by combining Equations 3.7 and 3.11 is somewhat simplified if we first integrate $f_{Y}\left(z+x_{0}\right)$ with respect to $z$ and make use of the complimentary error function.

The average outage probability $\bar{P}_{\text {out }}$ in the cell can be obtained by using the method presented in Section 2.1. That is,

$$
\begin{equation*}
\bar{P}_{\text {out }}=\frac{1}{S} \int_{S} P_{\text {out }}\left(E_{s} / N_{0}<b\right) d S . \tag{3.20}
\end{equation*}
$$

In the following, we set $K_{d i v}=3$ and set the standard deviation of the lognormal shadowing to $\sigma_{x}=8 d B$. We make the same assumptions as Section 2.1: the number of chips per symbol $M=512$ and the required $\operatorname{SNR}\left(E_{s} / N_{0}\right)$ is 2 dB . We use this level as we have assumed $E_{b} / N_{0}=5 \mathrm{~dB}$ as presented in [9] with rate $=1 / 2$ coding [1] for the downlink. To make a fair comparison between the 3 cell-site diversity system and a system without diversity, we first find a good boundary radius $r_{t}$ for the non-diversity region. In Figure 3.2, the average outage probability for a fixed number of interfering signals is plotted vs. $r_{t}$. The upper curve corresponds to the case of $L_{0}=8$ and the lower curve corresponds to the $L_{0}=2$ case. $L_{0}$ is defined in Equation 3.3 and represents the number of active interfering signals from


Figure 3.2: Average Outage Probability vs. Threshold Distance of the Diversity Region.
the reference user's cell-site. The adjacent cell-sites contribute an additional $L_{0}+1$ active interfering signals each to the interference as seen by the user. A fixed number of interfering signals was chosen since different diversity regions have different cell load (users/cell) associated with the number of interfering signals. The relationship between active interference and number of users is given in Equation 3.16. From Figure 3.2 it is seen that the average probability of outage decreases with decreasing $r_{t}$, or in other words, the more mobiles in the diversity region the lower the probability of outage. However, the improvement to the average outage probability diminishes with increasing the diversity region (ie. decreasing $r_{t}$ ). For further evalulations of capacity, an $r_{t}=0.8$ was chosen. It was chosen since there was little improvement in the average outage probability when $r_{t}$ was decreased below 0.8 and there was a large increase in the average outage probability when $r_{t}$ was increased from 0.8. Thus, we felt $r_{t}=0.8$ was a good value to be examined. It must be pointed out that the capacity results obtained will change if different values of $r_{t}$ are used.

The data outage statistics for the cases where $L_{0}=2$ and $L_{0}=8$ are tabulated in

| $L_{c}$ | Fraction of users <br> employing diversity | $\bar{P}_{\text {out }}$ | max $P_{\text {out }}$ | Frac. Area $>1 \%$ <br> outage | $r_{t}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 9.4245 | 0.7733 | 0.0004 | 0.0015 | 0 | 0.5 |
| 10.2257 | 0.6735 | 0.0004 | 0.0015 | 0 | 0.6 |
| 11.3677 | 0.5556 | 0.0006 | 0.0023 | 0 | 0.7 |
| 13.0494 | 0.4196 | 0.0010 | 0.0067 | 0 | 0.8 |
| 15.6778 | 0.2654 | 0.0022 | 0.0132 | 0.0596 | 0.9 |
| 20.2327 | 0.0931 | 0.0054 | 0.0325 | 0.2185 | 1.0 |

Table 3.1: Outage statistics for $L_{c}$ users per cell. $E_{s} / N_{0}=2 \mathrm{~dB}, K_{\text {div }}=3, L_{0}=2$, and standard deviation of lognormal shadowing $=8 \mathrm{~dB}$.

| $L_{c}$ | Fraction of users <br> employing diversity | $P_{\text {out }}$ | $\max P_{\text {out }}$ | Frac. Area $>1 \%$ <br> outage | $r_{t}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 28.2735 | 0.7733 | 0.0035 | 0.0120 | 0.0927 | 0.5 |
| 30.6770 | 0.6735 | 0.0037 | 0.0120 | 0.0927 | 0.6 |
| 34.1032 | 0.5556 | 0.0044 | 0.0120 | 0.1060 | 0.7 |
| 39.1481 | 0.4196 | 0.0062 | 0.0267 | 0.2252 | 0.8 |
| 47.0335 | 0.2654 | 0.0103 | 0.0465 | 0.3576 | 0.9 |
| 60.6980 | 0.0931 | 0.0197 | 0.0940 | 0.5099 | 1.0 |

Table 3.2: Outage statistics for $L_{c}$ users per cell. $E_{s} / N_{0}=2 \mathrm{~dB}, K_{\text {div }}=3, L_{0}=8$, and standard deviation of lognormal shadowing $=8 \mathrm{~dB}$.

Tables 3.1 and 3.2. Comparing the results in Table 3.2 with Table 2.1, we see a significant reduction in the worst case outage and a reduction in the number of locations with outage probabilities exceeding $1 \%$. For example, for an $L_{c}$ of 30 , the worst case outage probabilities are $1.2 \%$ and $7.3 \%$ for the 3 cell-site diversity and no diversity cases, respectively.

In Table 3.3, outage statistics corresponding to various cell loads $L_{c}$ are tabulated with $r_{t}=0.8$ and $\sigma=8 \mathrm{~dB}$. For an one percent average outage probability, the capacity of the cell with 3 cell-site diversity is 52 users as compared with a capacity of 25 users with no cell-site diversity. The number of users per cell with cell-site diversity is approximately a factor of 2 or a $100 \%$ increase from the number of users per cell when no cell-site diversity is used.

With respect to Table 3.3 and Table 2.1 of Section 2.1 , it is interesting to compare the worst case outage probability and the percentage of the service area with outage probability exceeding one percent. For example, a cell load of $L_{c}=26$ has a worst case outage probability of $6.4 \%$ for the no diversity case, while the corresponding figure for $L_{c}=53$ with 3 cell-site
diversity is $3.8 \%$ (ie. almost double the number of users and the worse case outage probability is almost halved). Also $34 \%$ of the cell area has an outage probability exceeding $1 \%$ in the no diversity case, while the corresponding figure for $L_{c}=53$ with cell-site diversity case only slightly higher $37 \%$. This is to be expected as the cell-site diversity case involves more interference for a given cell load due to extra diversity transmissions. This is evident in Equations 2.9 and 3.3.

Table 3.4 the results for various cell loads but looking at $10 \%$ average outage probability. The capacity of the cell with the $10 \%$ threshold for average outage probability is 326 users. This is only $14 \%$ over the capacity from the no cell-site diversity case with the same threshold. Although this does not seem like a large increase, the number of interfering signals with diversity is much greater than without diversity. The extra interference introduced by the diversity transmissions increases other user's probability of outage. As we have chosen the performance threshold as the average outage probability over all locations in the cell, it is clear that we have reduced outage probability in some locations but we have also increased it in others (due to the extra interference). To summarize the above, the number of transmitted signals is greatly increased, but with diversity the number of users is not equal to the number of transmitted signals. Thus, the increase in the number of users is not as great as that the number of transmitted signals. Comparing Tables 3.4 and 2.1 it is evident that that the worst case probability of outage is reduced by about $15 \%$. However, a great number of users have high outage probabilities. For example, the number of users within the cell having greater than $1 \%$ and $10 \%$ outage probability is about $9 \%$ and $18 \%$ higher for the cell-site diversity case than the no cell-site diversity case, respectively.

To illustrate the dependence of the outage probability on the position within the cell, we present a contour map for the case $r_{t}=0.8$ in Figure 3.3. In the figure, $42 \%$ of the users are utilizing cell-site diversity. The positions on the boundary are assigned to the non-diversity region. For this contour map, the cell load is 39 users per cell. Comparing Figure 3.3 where cell-site diversity is employed and Figure 2.5 where no cell-site diversity is used, it obvious that cell-site diversity greatly improves the outage performance for the cell. The cell load for Figure 2.5 is 30 users per cell which is 9 less than for Figure 3.3 while the outage probability is much better in the second figure. As seen in Figure 3.3, the maximum outage probability is below $3 \%$ while in Figure 2.5 the maximum outage probability is below $7 \%$. The maximum outage probability occurs at the boundary of the diversity region rather than at the cell boundary when cell-site diversity is used. This is to be expected as the users within the diversity region are making use of the maximum signal while the users on the boundary (on the no diversity side) have only the reference cell-site's transmission to use and are subject to additional interference from the diversity transmissions.

| $L_{c}$ | $P_{\text {out }}$ | $\max \vec{P}_{\text {out }}$ | Fraction of Area $>1 \%$ <br> outage prob. |
| ---: | :---: | :---: | :---: |
| 47.8477 | 0.0087 | 0.0335 | 0.3664 |
| 50.0226 | 0.0093 | 0.0352 | 0.4427 |
| 51.1101 | 0.0096 | 0.0361 | 0.4580 |
| 52.1975 | 0.0099 | 0.0369 | 0.4656 |
| 52.6325 | 0.0100 | 0.0372 | 0.4656 |
| 52.8500 | 0.0101 | 0.0374 | 0.4656 |
| 53.2850 | 0.0102 | 0.0377 | 0.4656 |
| 54.3724 | 0.0105 | 0.0386 | 0.4733 |
| 56.5473 | 0.0111 | 0.0403 | 0.5038 |
| 65.2469 | 0.0136 | 0.0469 | 0.5954 |

Table 3.3: Outage statistics for $L_{c}$ users per cell near $1 \%$ average outage probability. $E_{s} / N_{0}=$ $2 \mathrm{~dB}, K_{\text {div }}=3, r_{t}=0.8$, code rate $=1 / 2$, standard deviation of lognormal shadowing $=8$ dB.

| $L_{c}$ | $\bar{P}_{\text {out }}$ | $\max P_{\text {out }}$ | \% Area > 1\% | \% Area > 10\% |
| ---: | ---: | :---: | :---: | :---: |
| 308.8352 | 0.0943 | 0.1961 | 0.8244 | 0.5344 |
| 317.5348 | 0.0972 | 0.2005 | 0.8244 | 0.5802 |
| 326.2344 | 0.1001 | 0.2049 | 0.8321 | 0.5878 |
| 334.9340 | 0.1030 | 0.2093 | 0.8321 | 0.6031 |
| 343.6336 | 0.1058 | 0.2136 | 0.8321 | 0.6107 |
| 374.0821 | 0.1158 | 0.2284 | 0.8550 | 0.6260 |
| 482.8269 | 0.1507 | 0.2871 | 0.8626 | 0.6794 |

Table 3.4: Outage statistics for $L_{c}$ users per cell near $10 \%$ average outage probability. $E_{s} / N_{0}=2 \mathrm{~dB}, K_{\text {div }}=3, r_{t}=0.8$, code rate $=1 / 2$, standard deviation of lognormal shadowing $=8 \mathrm{~dB}$.


Figure 3.3: Contour plot of Outage probability for 39 users per cell with $1 \%$ contour lines and 3 cell-site diversity.

The model implicitly assumes instantaneous switching between diversity signals. In a practical system, the switch-over to a diversity signal may not be instantaneous for various reasons (such as the use of the average signal level rather than the instantaneous signal level for a switching decision). The time statistics of the shadowing process have not been modelled here. Thus, an analysis of mean time between switch-over requires further study. Similarly, the effect of switch-over on the data stream also requires further study. Any such effect would depend on the actual receiver structure used.

The capacity of the uplink with cell-site diversity is difficult to achieve with analysis and we attempt to lower bound the actual capacity. Using uplink power control the mobile must adjust it transmitter power in an attempt to ensure that the received power at the cell-site is constant. Note that the standard deviation of the shadowing process and the transmitter power fluctuations are measured in dB , as in the previous section. However, the choice of the cell-site which the mobile uses may change according to its position within the cell when using cell-site cliversity. If the mobile is in the diversity region then we assume that the mobile is transmitting to the cell-site from which it is receiving the best downlink signal. If the mobile is in the non-diversity region, the mobile uses the reference cell-site at all times. It is felt that use of cell-site diversity reduces the interference as seen by the cell-site for the following reasons. The mobiles in the diversity region will have a reduced variance as they are using the best downlink and uplink out a possible $K_{\text {div }}$ choices. Thus, it is probable that mobiles which created strong intercell interference are now controlled by the cell-site and thus are intracell interferers (which create less interference than intercell interferers). Mobiles within the reference cell which are controlled by another cell-site can be considered as intercell interferers. However, when diveristy is utilized, the attenuation of the channel between the mobile and the reference cell-site and the mobile is higher than that between the mobile and the 'best' cell-site, otherwise the mobile would be using the reference cell-site instead. This implies that the mean of the received power from this type of mobile will then be lower than an intracell interferer due to the difference in the attenuations in the channels.

From these arguments it seems reasonable to assume that cell-site diversity reduces the interference as seen by the cell-sites. Thus, cell-site diversity will have a higher capacity for the uplink than the no cell-site diversity case. Thus the results in Section 2.2 will act as a lower bound on the capacity for uplink with cell-site diversity. More details of these arguments are presented in Appendix C. The actual capacity increase from the no cell-site cliversity case is difficult to achieve analytically and we feel that simulation of the system would be required achieve the capacity of the uplink with cell-site diversity.

To summarize the results of this chapter, 3 cell-site diversity increased the downlink capacity. For the downlink, the worst case outage probability is reduced by the utilization
of the 3 cell-site diversity. The increased interference due to the extra transmissions does increase the outage probability for users outside of the diversity region. This is evident as the percentage of users over the $1 \%$ and $10 \%$ outage probability threshold is increased when cell-site diversity is employed. In this report, we have assumed that there are 3 equal power signals transmitted from the 3 cell-sites which increases the interference to other users when diversity is in use. Thus, the results will act as a lower bound if the system can determine which signal will be the best signal and either reduce the power to the other transmissions or not transmit them. This increases the system's complexity but may reduce the interference due to the extra signals. For example, in [9] they considered the mobile received the best signal from one (and only one) of 18 cell-sites. Their results show 108 users/cell where we obtain 52 users/cell with 3 cell-site diversity. The main differences are they obtain the best signal from more cell-sites than we consider and only the best signal is transmitted. Therefore, there is no interference from diversity transmissions which are not used which we have considered in this report. Thus, we can conclude that it is essential that the best cell-site to transmit the signal to the mobile and that the reduction of the interference from the diversity transmissions can increase in the capacity.

## Chapter 4

## Effects of Non-ideal Power Control

In general, power control can be employed for both uplink and downlink. In a single cell system, if each user in a cell has the same downlink power level, each user is subject to the same signal to interference ratio. This is due to the fact that the interference and the signal follow a common propagation path (channel). Thus, any power-level variations (due to power control) will degrade the single cell system's performance. The situation changes slightly if we consider a multi-cell system. In this case, the intercell interference increases with the distance between the mobile user and the home cell-site. Fherefore, moderate downlink power control might improve the overall performance.

In the uplink, power control is crucial because of the near - far problem. We expect that the optimization of the uplink power control might contribute to overall system performance more significantly than the optimization of the downlink power control. For this reason we only address the issue of uplink power control.

### 4.1 Non-ideal Power Control

Ideally, uplink power control compensates for propagation loss, maintaining a constant power level which is equal for all users at the destination cell-site. Due to the difficulty of implementing ideal power control and to the possible advantages offered by non-ideal power control, we consider the non-ideal case. We present a general model of power control. Our objective is to investigate methods of introducing imperfections to the ideal power control technique which improve the system capacity. In this section, we consider three models of non-ideal power control. These models can be used with the results of Chapter 2.2 to determine the capacity of the uplink.

The first model is a "jitter" model. In this model, we introduce a jitter to the power control parameter, with the intention of modeling fast variations of the propagation path.

The second model is known as the correlation model. In this model, we address imperfec-
tions in estimating the propagation path attenuation by introducing a correlation coefficient between the estimate and the true propagation loss.

As shown in the following sections, the performance gains offered by these techniques are disappointing. System performance is actually improved by reducing the degree of imperfection in these forms of power control.

In the third model, we introduce a parameter $\kappa$ which reduces the variance of the transmitted signal with respect to the ideal power control strategy. System performance is then optimized relative to this parameter. We find this model to be promising, and concentrate our efforts on it. We derive expressions for the mean and variance of the received interfering power at the cell-site for both the intracell and intercell interference. In the analytical section, we present lower and upper bounds for the uplink signal-to-noise ratio. We present numerical results for outage probability and its dependence on coefficient $\kappa$.

### 4.1.1 Jitter model

Regardless of whether the system employs open or closed loop power control, there is always a discrepancy from the ideal case. This is due to various effects: uncorrelated uplinks and downlinks, power control processing delay. Qualcomm's proposal [1] shows that attempts to track Rayleigh fading with very fast power control can only provide a delayed finite-step approximation of the channel attenuation. Sophisticated prediction techniques would reduce the error. Our model may still be applied, since of its generality.

If we track the incoming signal level, and not just the mean, we obtain the following expressions by modifying Yung's results [8]. The received signal $s_{0_{i}}$ from the $i-t h$ user at the cell-site is given by

$$
\begin{equation*}
s_{0_{i}}=\frac{\sqrt{E_{s}} A_{i_{0}} d_{0_{i}}}{\left(1+x_{i_{0}}\right) \sqrt{A_{i_{0}}^{2}}}=\frac{\sqrt{E_{s}}}{\left(1+x_{i_{0}}\right)} d_{0_{i}} \tag{4.1}
\end{equation*}
$$

where $A_{i_{0}}$ is the signal "gain" between the $i-t h$ mobile and the $k$-th cell-site, $d_{0_{i}}$ represents the data bits and takes values $\pm 1$ with equal probability, $x_{i_{0}}$ is random variable representing the control jitter. The intracell interference $I_{1}$ and the intercell interference $I_{2}$ are given by

$$
\begin{align*}
& I_{1}=\frac{\sqrt{E_{s}}}{2} \sum_{i=1}^{L_{0_{a}}} \frac{1}{1+x_{i_{0}}} \sum_{n=1}^{8} f_{n, i}=\frac{\sqrt{E_{s}}}{2} \sum_{i=1}^{L_{0_{a}}} \sum_{n=1}^{8} \frac{f_{n}}{1+x_{i_{0}}}  \tag{4.2}\\
& I_{2}=\frac{\sqrt{E_{s}}}{2} \sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}} \frac{A_{i_{k}}^{(0)}}{1+x_{i_{0}}} \frac{f_{i_{k}}}{\sqrt{A_{i_{k}}^{2}}} \tag{4.3}
\end{align*}
$$

Here $E_{s}$ is the symbol energy and $A_{i_{k}}$ is the signal "gain" between the $i-t h$ mobile and the $k$-th cell-site. $A_{i_{k}}^{(0)}$ is the signal gain between the same mobile and $0-t h$ cell site, $L_{0_{a}}$

- number of active interferers in the 0 -th cell, $L_{k_{a}}$ - number of active interferers in the $k$-th cell, $K$ - number of interfering cells, $f_{n, i}, f_{i_{k}}$ are zero mean random variables depending on the detection procedure [8]. The dimensionless term $x_{i_{k}}$ is a random variable representing control jitter.

Suppose that for any $i_{k}$, the values of $x_{i_{k}}$ are mutually independent. As the control adjustment occurs quite frequently, we assume, that $\left(1+x_{i_{k}}\right) \approx 1$, or $\left|x_{i_{k}}\right| \ll 1$. In other words, we approximate the probability density function for $x_{i_{k}}$ by a function with a limited domain. However, the function is dependent on the actual system jitter performance, and is difficult to analyze. We approximate the function by assuming that the jitter is uniformly distributed over an interval $[-\delta,+\delta]$. This implies that the mean of the $x_{i_{k}}$ 's is zero and the variance is $\sigma_{J}^{2}=\delta^{2} / 3$. The value $\delta$ is assumed to be small, in conformance with the assumption $\left|x_{i_{k}}\right| \ll 1$.

Since both $f_{n}$ and $f_{i_{k}}$ have zero mean, both the intracell and intercell interferences also have zero mean. Thus, the variance of $I_{1}$ is equal to the mean of the square, given by

$$
\begin{equation*}
\operatorname{Var}\left(I_{1}\right)=E\left[I_{1}^{2}\right]=\frac{E_{s}}{4} E\left[\left(\sum_{i_{0}=1}^{L_{0 a}} \sum_{n=1}^{8} \frac{f_{n, i}}{1+x_{i_{0}}}\right)^{2}\right] \tag{4.4}
\end{equation*}
$$

where $L_{0_{a}}$ is number of active interfering users within the cell. If the $x_{i_{0}}$ are independent and equally distributed, then following Yung's derivations [8],

$$
\begin{equation*}
\operatorname{Var}\left(I_{1}\right)=L_{0}\left(\frac{\xi_{h}}{M}\right) E_{s} E\left[\frac{1}{(1+x)^{2}}\right] \tag{4.5}
\end{equation*}
$$

${ }^{-r}$ where $\xi_{h}$ is a factor depending on the chip shape (for rectangular chips, $\xi_{h}=\frac{1}{3}$ ) and M is number of chips per symbol. Evaluating the expectation in the last expression yields the following result:

$$
\begin{equation*}
\operatorname{Var}\left(I_{1}\right)=L_{0}\left(\frac{\xi_{h}}{M}\right) E_{s} \frac{1}{1-\delta^{2}} \tag{4.6}
\end{equation*}
$$

or equivalently

$$
\begin{equation*}
\operatorname{Var}\left(I_{1}\right)=L_{0}\left(\frac{\xi_{h}}{M}\right) E_{s} \frac{1}{1-3 \sigma_{J}^{2}} \tag{4.7}
\end{equation*}
$$

where $\sigma_{J}^{2}$ is the variance of the allowable power control jitter. Obviously, $0 \leq \sigma_{J}^{2} \leq \frac{1}{3}$, where the last equality drives the variance of the the received intracell interference to infinity.

Therefore $\sigma_{J}^{2}=\frac{1}{3}$ corresponds to the worst possible form of power control. We expect to have $\sigma_{J}^{2} \ll \frac{1}{3}$. The intercell variance can be expressed as:

$$
\begin{align*}
\operatorname{Var}\left(I_{2}\right)= & E\left[I_{2}^{2}\right] \\
= & \frac{E_{s}}{4} E\left[\left(\sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}} \frac{A_{i_{k}}^{(0)} f_{i_{k}}}{\left(1+x_{i_{k}}\right) \sqrt{A_{i_{k}}^{2}}}\right)^{2}\right] \\
= & \frac{E_{s}}{4} E\left[\sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}} \frac{\left(A_{i_{k}}^{(0)}\right)^{2} f_{i_{k}}^{2}}{\left(1+x_{i_{k}}\right)^{2} A_{i_{k}}^{2}}\right. \\
& \left.+\left.\sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}} \sum_{k^{\prime}=1}^{K} \sum_{i_{k}^{\prime}=1}^{L_{k_{a}}} \frac{A_{i_{k}}^{(0)} A_{i_{k}^{\prime}}^{(0)} f_{i_{k}} f_{i_{k}^{\prime}}}{\left(1+x_{i_{k}}\right)\left(1+x_{i_{k}^{\prime}}\right) A_{i_{k}} A_{i_{k}^{\prime}}}\right|_{\left(k, i_{k}\right) \neq\left(k^{\prime}, i_{k}^{\prime}\right)}\right] \\
= & E_{s} \frac{\xi_{h}}{M} \frac{1}{1-3 \sigma_{J}^{2}} \sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}} E\left[\left(\frac{A_{i_{k}}^{(0)}}{A_{i_{k}}}\right)^{2}\right] . \tag{4.8}
\end{align*}
$$

Here $K$ is the number of interfering cells and $L_{k_{a}}$ is the number of active users in the $k$-th cell. In terms of the voice activity factor $V_{o n}$, the number $N_{\text {sect }}$ of sectors per cell, and the cell load $L_{c}$, we can rewrite the previous results as follows:

$$
\begin{align*}
\operatorname{Var}\left(I_{1}\right) & =\left(\frac{\rho_{h} R_{b}}{W}\right) E_{b}\left(\frac{V_{o n} L_{c}}{N_{s e c t}}-1\right) \frac{1}{1-3 \sigma_{J}^{2}}  \tag{4.9}\\
\operatorname{Var}\left(I_{2}\right) & =\left(\frac{\rho_{h} R_{b}}{W}\right) E_{b} \sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{c}} \frac{V_{o n}}{N_{s e c t}} E\left[\left(\frac{A_{i_{k}}^{(0)}}{A_{i_{k}}}\right)^{2}\right] \frac{1}{1-3 \sigma_{J}^{2}} \tag{4.10}
\end{align*}
$$

The intracell and intercell interference can be represented by equivalent power spectral densities [13] of, respectively,

$$
\begin{align*}
& N_{1}=2 \operatorname{Var}\left(I_{1}\right)  \tag{4.11}\\
& N_{2}=2 \operatorname{Var}\left(I_{2}\right) . \tag{4.12}
\end{align*}
$$

For our model, these results can be rewritten as follows:

$$
\begin{align*}
N_{1} & =N_{01} \frac{1}{1-3 \sigma_{J}^{2}}  \tag{4.13}\\
N_{2} & =N_{02} \frac{1}{1-3 \sigma_{J}^{2}} \tag{4.14}
\end{align*}
$$

From these equations, we see that with increased jitter variance, the total interfering power increases. Therefore, the optimum value of $\sigma_{J}^{2}$ is $\sigma_{J}^{2}=0$. This trivial result causes us not to pursue this model further here. However, the model might be useful for analyzing the robustness of systems which try to follow the signal level.

### 4.1.2 Correlation Model

In this model we assume that the transmitted signal exhibits the same probability distribution as the "ideal" signal - but not fully correlated with the actual attenuation. We investigate correlation between the estimate of the uplink attenuation and the actual uplink attenuation, which may be due to the difference between the downlink and uplink. However, for a given correlation coefficient, the transmitted power variance may be smaller to find an optimum value, because it will affect both intracell and intercell interference in the similar way as mentioned in section 4.1.

It is obvious that for a decreasing correlation between links, the intracell interference increases, while the intercell interference remains at the same level. This model, like the previous model, provides trivial results for the optimization of system performance: performance improves with increasing correlation between the links.

This model might be of interest if one wishes to model "delayed" power control or similar applications. However, for the purpose of this study, we do not consider the model further.

### 4.1.3 Variance Model (the exponential form )

In the variance model, the transmitted signal tends to follow the channel's random changes. However, the tracking is not too 'tight' - the variance is compressed relative to that for ideal power control. It is shown that the reduced variance of the transmitted signall causes an increase in intracell interference and a decrease in intercell interference. The former is due to imperfect compensation of propagation losses, and the latter is due to the fact that the variance of the propagation path statistics remains the same but the additional variance due to power control is decreased. There are many ways to address this problem. The following method yields straightforward analytical results and indicates a possible implementation.

In this model, the output signal power is proportional to the inverse of the expectation of the incoming power. To introduce "variance compression", we raise the output signal level to a power $\kappa(0 \leq \kappa \leq 1)$. As shown in Equation 4.15, $\kappa=1$ corresponds to ideal power control and $\kappa=0$ corresponds to no power control. Again, we use results derived by Yung [8]. After detection, a user's signal arriving at the cell-site is given by:

$$
\begin{equation*}
s_{0_{i}}=\frac{\sqrt{E_{s}} A_{i_{0}}^{\prime} d_{0_{i}}}{\left(\sqrt{E\left[A_{i_{0}}^{2}\right]}\right)^{\kappa}} \tag{4.15}
\end{equation*}
$$

where $E_{\mathrm{y}}$ is the symbol energy, $A_{i_{0}}^{\prime}$ is the propagation "gain" of the reverse path and $A_{i_{0}}$ is the propagation "gain" of the forward path. In this model we assume that $A_{i_{0}}=A_{i_{0}}^{\prime} \cdot d_{0_{i}}$ represents the transmitted data bit, which takes values $\pm 1$ with equal probability. Intracell interference is given by

$$
\begin{equation*}
I_{1}=\frac{\sqrt{E_{s}}}{2} \sum_{i_{0_{a}}=1}^{L_{0_{a}}} \frac{A_{i_{0}}}{\left(\sqrt{E\left[A_{i_{0}}^{2}\right]}\right)^{\kappa}} \sum_{n=1}^{8} f_{n_{i_{0}}}, \tag{4.16}
\end{equation*}
$$

and the intercell interference by

$$
\begin{equation*}
I_{2}=\frac{\sqrt{E_{s}}}{2} \sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}} \frac{A_{i_{k}}^{(0)}}{\left(\sqrt{E\left[A_{i_{k}}^{2}\right]}\right)^{\kappa}} \sum_{n=1}^{8} f_{n_{i_{k}}}, \tag{4.17}
\end{equation*}
$$

where $A_{i_{k}}$ is the propagation "gain" of the path between the $i$-th user in the $k$-th cell and the $k$-th cell-site, $A_{i_{k}}^{(0)}$ is the propagation "gain" of the path between the $i$-th user in the $k$-th cell-site and the 0 -th cell-site, and $f_{n_{i_{k}}}$ are zero-mean functions depending on the detection method [8]. $L_{0_{a}}$ is the number of active interferers within the 0 -th cell, $L_{k_{a}}$ is the number of active users within the $k$-th cell, and $K$ is the number of interfering cells. Since $I_{1}, I_{2}$ are zero-mean, their variances are given by

$$
\begin{align*}
& \operatorname{Var}\left(I_{1}\right)=E\left[I_{1}^{2}\right]=E_{s} \frac{\xi_{h}}{M} \sum_{i_{0}=1}^{L_{0_{a}}}\left(E\left[A_{i_{0}}^{2}\right]\right)^{1-\kappa},  \tag{4.18}\\
& \operatorname{Var}\left(I_{2}\right)=E\left[I_{2}^{2}\right]=E_{s} \frac{\xi_{h}}{M} \sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}} \frac{E\left[\left(A_{i_{k}}^{(0)}\right)^{2}\right]}{\left(E\left[A_{i_{k}}^{2}\right]\right)^{1-\kappa} .} \tag{4.19}
\end{align*}
$$

If we introduce lognormal shadowing, $E\left[A_{i_{k}}^{2}\right]$ and $E\left[\left(A_{i_{k}}^{(0)}\right)^{2}\right]$ are lognormal variables with standard deviation $\sigma_{L N}$ (typically $\sigma_{L N} \approx 8$, see Gilhousen et al, [9]) and mean $10 \log \left(\frac{1}{r_{i k}}\right)^{\gamma}, 10 \log \left(\frac{1}{r_{i i_{k}}^{0}}\right)^{\gamma}$ respectively. We then calculate the average of the variances given by Equations 4.18 and 4.19 , obtaining

$$
\begin{align*}
& \overline{\operatorname{Var}\left(I_{1}\right)}=\frac{E_{s} \xi_{h}}{M}\left(\sum_{i_{0}=1}^{L_{0_{a}}}\left(\frac{1}{r_{i_{0}}}\right)^{\gamma(1-\kappa)}\right) \exp \left(\frac{1}{2} \sigma_{L N}^{2}\left(\frac{\ln 10}{10}\right)^{2}(1-\kappa)^{2}\right)  \tag{4.20}\\
& \overline{\operatorname{Var}\left(I_{2}\right)}=\frac{E_{s} \xi_{h}}{M}\left(\sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}}\left(\frac{\left(r_{i_{k}}\right)^{\kappa}}{r_{i_{k}}^{(0)}}\right)^{\gamma}\right) \exp \left(\frac{1}{2} \sigma_{L N}^{2}\left(\frac{\ln 10}{10}\right)^{2}\left(1+\kappa^{2}\right)\right) \tag{4.21}
\end{align*}
$$

where $r_{i_{k}}$ is distance of the $i$-th user in the $k$-th cell from the $k$-th cell-site, $r_{i_{k}}^{(0)}$ is the distance of the $i$-th user in the $k$-th cell from the 0 -th cell-site, $\gamma$ is the propagation coefficient (typically $\gamma \approx 4$ ), $M$ is number of chips per symbol, and $\xi_{h}$ is a coefficient depending on the chip shape (for a square chip, $\xi_{h}=\frac{1}{3}$ ).

If we apply equations 4.11 and 4.12 for 4.18 and 4.19 , we can compute the means and variances of the equivalent noise power density.

$$
\begin{align*}
& E\left[N_{1}\right]=2 \overline{\operatorname{Var}\left(I_{1}\right)}  \tag{4.22}\\
& E\left[N_{2}\right]=2 \overline{\operatorname{Var}\left(I_{2}\right)} \tag{4:23}
\end{align*}
$$

Combining these equations with the expressions for average variance, we see that with reduced power control "imperfection" (ie: increasing $\kappa$ ), the intracell interference increases and intercell interference decreases.

To compute the signal-to-noise ratio (SNR), the distribution of $N_{1}$ and $N_{2}$ must be determined. Since there are many interfering users, both intracell and intercell, we apply the Central Limit Theorem. Thus, we assume that the distributions for $N_{1}$ and $N_{2}$ both approach normal distributions. Since the normal distribution is determined by its mean and variance, we compute the variances of $N_{1}$ and $N_{2}$.

We have

$$
\begin{equation*}
E\left[N_{l}^{2}\right]=E\left[4 E_{s}^{2} \frac{\xi_{h}^{2}}{M^{2}}\left(\sum_{i_{0}=1}^{L_{0_{a}}}\left(E\left[A_{i_{0}}^{2}\right]\right)^{(1-\kappa)}\right)^{2}\right] \tag{4.24}
\end{equation*}
$$

which, after algebraic manipulation, can be expressed as:

$$
\begin{align*}
E\left[N_{1}^{2}\right]= & \left(E\left[N_{1}\right]\right)^{2} \\
& +4\left(\frac{E_{s} \xi_{h}}{M}\right)^{2} e^{\sigma_{L N}^{2}\left(\frac{\ln 10}{10}\right)^{2}(1-\kappa)^{2}}\left(e^{\sigma_{L N}^{2}\left(\frac{\ln 10}{10}\right)^{2}(1-\kappa)^{2}}-1\right) \sum_{i_{0}=1}^{L_{0 a}}\left(\frac{1}{r_{i_{0}}}\right)^{2 \gamma(1-\kappa)} \tag{4.25}
\end{align*}
$$

Therefore:

$$
\begin{equation*}
\operatorname{Var}\left(N_{1}\right)=4\left(\frac{E_{s} \xi_{h}}{M}\right)^{2} V_{1}\left(V_{1}-1\right) \sum_{i_{0}=1}^{L_{0_{a}}}\left(\frac{1}{r_{i_{0}}}\right)^{2 \gamma(1-\kappa)} \tag{4.26}
\end{equation*}
$$

where

$$
\begin{equation*}
V_{1}=e^{\sigma_{L N}^{2}\left(\frac{\ln 10}{10}\right)^{2}(1-\kappa)^{2}} \tag{4.27}
\end{equation*}
$$

Similarly, we have:

$$
\begin{equation*}
E\left[N_{2}^{2}\right]=E\left[4 E_{s}^{2} \frac{\xi_{h}^{2}}{M^{2}}\left(\sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}}\left(\frac{\left(r_{i_{k}}\right)^{\kappa}}{r_{i_{k}}^{(0)}}\right)^{\gamma}\right)^{2}\right] \tag{4.28}
\end{equation*}
$$

which, after some algebra, can be expressed as

$$
\begin{array}{r}
E\left[N_{2}^{2}\right]=\left(E\left[N_{2}\right]\right)^{2}-4\left(\frac{E_{s} \xi_{h}}{M}\right)^{2}\left(\sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}}\left(\frac{r_{i_{k}}^{\kappa}}{r_{i_{k}}^{(0)}}\right)^{2 \gamma}\right) e^{\sigma_{L N}^{2}\left(\frac{\ln 10}{10}\right)^{2}\left(1+\kappa^{2}\right)}+ \\
4\left(\frac{E_{s} \xi_{h}}{M}\right)^{2}\left(\sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k a}}\left(\frac{r_{i_{k}}^{\kappa}}{r_{i_{k}}^{(0)}}\right)^{2 \gamma}\right) e^{2 \sigma_{L N}^{2}\left(\frac{\ln 10}{10}\right)^{2}\left(1+\kappa^{2}\right)} \tag{4.29}
\end{array}
$$

We then obtain

$$
\begin{equation*}
\operatorname{Var}\left(N_{2}\right)=4\left(\frac{E_{s} \xi_{h}}{M}\right)^{2} V_{2}\left(V_{2}-1\right)\left(\sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}}\left(\frac{\left(r_{i_{k}}\right)^{\kappa}}{r_{i_{k}}^{(0)}}\right)^{2 \gamma}\right) \tag{4.30}
\end{equation*}
$$

where

$$
\begin{equation*}
V_{2}=e^{\sigma_{L N}^{2}\left(\frac{\ln 10}{10}\right)^{2}\left(1+\kappa^{2}\right)} \tag{4.31}
\end{equation*}
$$

Since

$$
\begin{equation*}
E\left[P_{0_{i}}\right]=E\left[s_{0_{i}}^{2}\right]=E_{s}\left(\frac{1}{r_{0_{i}}}\right)^{\gamma(1-\kappa)} e^{\frac{1}{2} \sigma_{L N}^{2}\left(\frac{\ln 10}{10}\right)^{2}(1-\kappa)^{2}} \tag{4.32}
\end{equation*}
$$

we can write the expectation of the SNR as follows:

$$
\begin{equation*}
\overline{S N R}=E\left[\frac{s_{0_{i}}^{2}}{N_{1}+N_{2}}\right]=E\left[s_{0_{i}}^{2}\right] E\left[\frac{1}{N_{1}+N_{2}}\right] \tag{4.33}
\end{equation*}
$$

Therefore

$$
\begin{equation*}
\overline{S N R_{0_{\mathrm{i}}}}=E_{s}\left(\frac{1}{r_{\mathrm{o}_{\mathrm{i}}}}\right)^{\gamma(1-\kappa)} e^{\frac{1}{2} \sigma_{L N}^{2}\left(\frac{\ln 10}{10}\right)^{2}(1-\kappa)^{2}} E\left[\frac{1}{N_{1}+N_{2}}\right] . \tag{4.34}
\end{equation*}
$$

We see that, in the worst case, $r_{0_{i}}$ is equal to maximum distance within the cell (i.e. the user is located at the cell boundary). Normalizing all distances to the worst case, we rewrite the above formula for the worst case:

Assume that $N_{1}$ and $N_{2}$ are independent Gaussian random variables with respective means $m_{1}, m_{2}$ given by Equations 4.22, 4.23 and variances $\sigma_{1}^{2}, \sigma_{2}^{2}$ given by 4.26, 4.30. Their sum is also Gaussian with mean $m_{1}+m_{2}$ and variance $\sigma_{1}^{2}+\sigma_{2}^{2}$. Under this assumption, we readily evaluate the expectation in Equation 4.35. Since this evaluation requires a numerical approach, at this stage we estimate it.

The Taylor series of $\cosh (\cdot)$ and $\sinh (\cdot)$ yield the following inequalities:

$$
\begin{align*}
\cosh \frac{x}{\sqrt{3}} & \leq \frac{\sinh x}{x}  \tag{4.36}\\
\frac{\cosh x}{3}+\frac{2}{3} & \geq \frac{\sinh x}{x} \tag{4.37}
\end{align*}
$$

These relations can be used to approximate the mean of the inverse of a Gaussian variable as follows (for details see Appendix D) :

$$
\begin{equation*}
\frac{m_{1}+m_{2}}{\sigma_{1}^{2}+\sigma_{2}^{2}} e^{-\frac{\left(m_{1}+m_{2}\right)^{2}}{3\left(\sigma_{1}^{2}+\sigma_{2}^{2}\right)}} \leq E\left[\frac{1}{N_{1}+N_{2}}\right] \leq \frac{m_{1}+m_{2}}{3\left(\sigma_{1}^{2}+\sigma_{2}^{2}\right)}\left(1+2 e^{-\frac{\left(m_{1}+m_{2}\right)^{2}}{2\left(\sigma_{1}^{2}+\sigma_{2}^{2}\right)}}\right) \tag{4.38}
\end{equation*}
$$

Using the formula 4.38, we can write an upper and a lower bound for the average signal-tonoise ratio:

$$
\begin{equation*}
E_{s} \sqrt{V_{1}} \frac{m_{1}+m_{2}}{\sigma_{1}^{2}+\sigma_{2}^{2}} e^{-\frac{\left(m_{1}+m_{2}\right)^{2}}{3\left(\sigma_{1}^{2}+\sigma_{2}^{2}\right)}} \leq \overline{S N R} \leq E_{s} \sqrt{V_{1}} \frac{m_{1}+m_{2}}{3\left(\sigma_{1}^{2}+\sigma_{2}^{2}\right)}\left(1+2 e^{-\frac{\left(m_{1}+m_{2}\right)^{2}}{2\left(\sigma_{1}^{2}+\sigma_{2}^{2}\right)}}\right) \tag{4.39}
\end{equation*}
$$

where $V_{1}$ is given by Equation $4.27, m_{1}$ by Equation $4.22, m_{2}$ by Equation $4.23, \sigma_{1}^{2}$ by Equation 4.26, and $\sigma_{2}^{2}$ by Equation 4.30 .
$E\left[\frac{1}{N_{1}+N_{2}}\right]$ can also be evaluated numerically, but this requires special attention because $\frac{1}{N_{1}+N_{2}}$ is a singular function.

Equation 4.35 gives some idea of how system performance depends on $\kappa$. However, to evaluate the outage probability, more comprehensive formulae showing the dependence of outage probability on the coefficient $\kappa$ must be obtained. Given an outage probability threshold, these results can then be used to compute the cell capacity as a function of $\kappa$. We choose an outage threshold of $1 \%$, i.e. the system must be outage free for at least $99 \%$ of the time.

The following equations are based on reasoning similar to our previous derivations. Equations $4.15,4.16$, and 4.17 can be used to modify Yung's results [8] as follows:

$$
\begin{equation*}
\frac{E_{s}}{N_{0}}=\frac{E_{s}\left(E\left[A_{0}^{2}\right]\right)^{1-\kappa}}{2\left(E_{s} \frac{\xi_{h}}{M} \sum_{i_{0}=1}^{L_{0}}\left(E\left[A_{i_{0}}^{2}\right]\right)^{1-\kappa}+E_{s} \frac{\xi_{h}}{M} \sum_{k=1}^{K} \sum_{i_{k}}^{L_{k_{a}}} \frac{E\left[\left(A_{i_{k}}^{(0)}\right)^{2}\right]}{\left(E\left[\left(A_{i_{k}}\right)^{2}\right]\right)^{\kappa}}\right)} . \tag{4.40}
\end{equation*}
$$

Introducing lognormal shadowing, we can rewrite the above formula as

$$
\begin{align*}
& E_{s} / N_{0}=  \tag{4.41}\\
& \frac{M / 2 \xi_{h}}{\sum_{i_{0}=1}^{L_{0_{a}}}\left(\frac{r_{0}}{r_{i_{0}}}\right)^{\gamma(1-\kappa)} 10^{\frac{x_{i_{0}}-x_{0}}{10}(1-\kappa)}+\sum_{k=1}^{K} \sum_{i_{k}}^{L_{k_{a}}} r_{0}^{\gamma(1-\kappa)}\left(\frac{\left(r_{i_{k}}\right)^{\kappa}}{r_{i_{k}}^{(0)}}\right)^{\gamma} 10^{\frac{x_{i_{k}}^{(0)}-x_{i_{k}} \kappa-x_{0}(1-\kappa)}{10}}}
\end{align*}
$$

where $r_{0}$ is the distance of the reference mobile from the 0 -th cell-site and $r_{i_{k}}$ is the distance of the $i$-th interfering mobile in the $k$-th cell from its home cell-site. The corresponding $x_{0}$ and $x_{i_{k}}$ are Gaussian random variables with zero mean and variance $\sigma_{L N}^{2} . r_{i_{k}}^{(0)}$ is the distance of the $i$-th mobile in the $k$-th cell to the 0 -th cell-site. For convenience, we denote the first term in the denominator as $N_{1}^{\prime}$ and the second term as $N_{2}^{\prime}$.

In the following we evaluate the first and second moments of $N_{1}^{\prime}$ and $N_{2}^{\prime}$. Since $x_{i_{0}}$ and $x_{0}$ are independent, identically distributed zero-mean Gaussian random variables with variance $\sigma_{L N}^{2}$, we can write the expectations of $N_{1}^{\prime}$ and $N_{2}^{\prime}$ as

$$
\begin{align*}
E\left[N_{1}^{\prime}\right] & =\sum_{i_{0}=1}^{L_{0_{a}}}\left(\frac{r_{0}}{r_{i_{0}}}\right)^{\gamma(1-\kappa)} E\left[10^{\frac{x_{i_{0}}-x_{0}}{10}(1-\kappa)}\right] \\
& =\sum_{i_{0}=1}^{L_{0_{a}}}\left(\frac{r_{0}}{r_{i_{0}}}\right)^{\gamma(1-\kappa)} e^{\sigma_{L N}^{2}\left(\frac{\ln (10)}{10}\right)^{2}(1-\kappa)^{2}} \tag{4.42}
\end{align*}
$$

and

$$
\begin{align*}
E\left[N_{2}^{\prime}\right]= & \sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}} r_{0}^{\gamma(1-\kappa)}\left(\frac{\left(r_{i_{k}}\right)^{\kappa}}{r_{i_{k}}^{(0)}}\right)^{\gamma} E\left[10^{\frac{i_{i_{k}}^{(0)}-x_{i_{k}} \kappa-x_{0}(1-\kappa)}{10}}\right] \\
& =\sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}} r_{0}^{\gamma(1-\kappa)}\left(\frac{\left(r_{i_{k}}\right)^{\kappa}}{r_{i_{k}}^{(0)}}\right)^{\gamma} e^{\sigma_{L N}^{2}\left(1-\kappa+\kappa^{2}\right)\left(\frac{\ln (10)}{10}\right)^{2}} . \tag{4.43}
\end{align*}
$$

We can also show that

$$
\begin{align*}
E\left[N_{1}^{\prime 2}\right]= & E\left[\sum_{i_{0}=1}^{L_{0}}\left(\frac{r_{0}}{r_{i_{0}}}\right)^{2 \gamma(1-\kappa)} 10^{\frac{x_{i_{0}}-x_{0}}{10} 2(1-\kappa)}+\left.\sum_{i_{0}=1}^{L_{0_{a}}} \sum_{i_{0}^{\prime}=1}^{L_{0}}\left(\frac{r_{0}^{2}}{r_{i_{0}} r_{i_{0}^{\prime}}}\right)^{\gamma(1-\kappa)} 10^{\frac{x_{i_{0} x_{i_{0}^{\prime}}-2 x_{0}(1-\kappa)}^{10}}{10}}\right|_{i_{0} \neq i_{0}^{\prime}}\right] \\
= & e^{4 \sigma_{L N}^{2}\left(\frac{\ln (10)}{10}\right)^{2}(1-\kappa)^{2}} \sum_{i_{0}=1}^{L_{0_{a}}}\left(\frac{r_{0}}{r_{i_{0}}}\right)^{2 \gamma(1-\kappa)} \\
& +\left.e^{3 \sigma_{L N}^{2}\left(\frac{\ln (10)}{10}\right)^{2}(1-\kappa)^{2}} \sum_{i_{0}=1}^{L_{0_{a}}} \sum_{i_{0}^{\prime}=1}^{L_{0_{a}}}\left(\frac{r_{0}^{2}}{r_{i_{0}} r_{i_{0}^{\prime}}}\right)^{\gamma(1-\kappa)}\right|_{i_{0} \neq i_{0}^{\prime} .} \tag{4.44}
\end{align*}
$$

Since

$$
\begin{equation*}
\left(E\left[N_{1}^{\prime}\right]\right)^{2}=e^{2 \sigma_{L N}^{2}\left(\frac{\ln (10)}{10}\right)^{2}(1-\kappa)^{2}}\left[\sum_{i_{0}=1}^{L_{0_{a}}}\left(\frac{r_{0}}{r_{i_{0}}}\right)^{2 \gamma(1-\kappa)}+\left.\sum_{i_{0}=1}^{L_{0 a}} \sum_{i_{0}^{\prime}=1}^{L_{0_{a}}}\left(\frac{r_{0}^{2}}{r_{i_{0}} r_{i_{0}^{\prime}}}\right)^{\gamma(1-\kappa)}\right|_{i_{0} \neq i_{0}^{\prime}}\right] \tag{4.45}
\end{equation*}
$$

we can rewrite Equation 4.44 as follows:

$$
\begin{align*}
& E\left[N_{1}^{\prime 2}\right]=\left(e^{4 \sigma_{L N}^{2}\left(\frac{\ln (10)}{10}\right)^{2}(1-\kappa)^{2}}-e^{3 \sigma_{L N}^{2}\left(\frac{\ln (10)}{10}\right)^{2}(1-\kappa)^{2}}\right) \sum_{i_{0}=1}^{L_{0_{a}}}\left(\frac{r_{0}}{r_{i_{0}}}\right)^{2 \gamma(1-\kappa)} \\
&+\left(E\left[N_{1}^{\prime}\right]\right)^{2}\left(e^{\sigma_{L N}^{2}\left(\frac{\ln (10)}{10}\right)^{2}(1-\kappa)^{2}}\right) \tag{4.46}
\end{align*}
$$

Thus

$$
\begin{align*}
& \operatorname{Var}\left[N_{1}^{\prime}\right]=\left(e^{4 \sigma_{L N}^{2}\left(\frac{\ln (10)}{10}\right)^{2}(1-\kappa)^{2}}-e^{3 \sigma_{L N}^{2}\left(\frac{\ln (10)}{10}\right)^{2}(1-\kappa)^{2}}\right) \sum_{i_{0}=1}^{L_{0 a}}\left(\frac{r_{0}}{r_{i_{0}}}\right)^{2 \gamma(1-\kappa)} \\
&+\left(E\left[N_{1}^{\prime}\right]\right)^{2}\left(e^{\sigma_{L N}^{2}\left(\frac{\ln (10)}{10}\right)^{2}(1-\kappa)^{2}}-1\right) \tag{4.47}
\end{align*}
$$

which is the second moment of $N_{1}^{\prime}$.
Following the same procedure for $N_{2}^{\prime}$, we obtain

$$
\begin{align*}
E\left[N_{2}^{\prime 2}\right]= & \sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}} r_{0}^{2 \gamma(1-\kappa)}\left(\frac{r_{i_{k}}^{\kappa}}{r_{i_{k}}^{(0)}}\right)^{2 \gamma} e^{4 \sigma_{L N}^{2}\left(1-\kappa+\kappa^{2}\right)\left(\frac{\ln (10)}{10}\right)^{2}}  \tag{4.48}\\
& +\left.\sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}} \sum_{k^{\prime}=1}^{K} \sum_{i_{k^{\prime}}^{\prime}=1}^{L_{k_{a}^{\prime}}^{\prime}} r_{0}^{2 \gamma(1-\kappa)}\left(\frac{r_{i_{k}}^{\kappa} r_{i_{k^{\prime}}^{\prime}}^{\kappa}}{r_{i_{k}}^{(0)} r_{i_{k^{\prime}}^{\prime}}^{(0)}}\right)^{\gamma} e^{\left(3 \sigma_{L N}^{2}\left(1+\kappa^{2}\right)-4 \kappa \sigma_{L N}^{2}\right)\left(\frac{\ln (10)}{10}\right)^{2}}\right|_{\left.\left(k, i_{k}\right) \neq\left(k^{\prime}, i_{k}^{\prime}\right)^{\prime}\right)} .
\end{align*}
$$

Utilizing the fact that

$$
\begin{align*}
E\left[N_{2}^{\prime}\right]^{2}= & e^{2 \sigma_{L N}^{2}\left(1-\kappa+\kappa^{2}\right)\left(\frac{\ln (10)}{10}\right)^{2}}  \tag{4.49}\\
& \times\left[\sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}} r_{0}^{2 \gamma(1-\kappa)}\left(\frac{r_{i_{k}}^{\kappa}}{r_{i_{k}}^{(0)}}\right)^{2 \gamma}\right. \\
& \left.+\left.\sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}} \sum_{k^{\prime}=1}^{K} \sum_{i_{k^{\prime}}^{\prime}=1}^{L_{k_{a}^{\prime}}} r_{0}^{2 \gamma(1-\kappa)}\left(\frac{r_{i_{k}}^{\kappa} r_{i_{k}}^{\prime}}{r_{i_{k}}^{(0)} r_{i_{k^{\prime}}^{\prime}}^{(0)}}\right)^{\gamma}\right|_{\left(k, i_{k}\right) \neq\left(k^{\prime}, i_{k^{\prime}}^{\prime}\right)}\right]
\end{align*}
$$

we obtain the variance

$$
\begin{align*}
\operatorname{Var}\left[N_{2}^{\prime}\right] & =\left[e^{4 \sigma_{L N}^{2}\left(1-\kappa+\kappa^{2}\right)\left(\frac{\ln (10)}{10}\right)^{2}}-e^{\left(3 \sigma_{L N}^{2}\left(1+\kappa^{2}\right)-4 \kappa \sigma_{L N}^{2}\right)\left(\frac{\ln (10)}{10}\right)^{2}}\right] \sum_{k=1}^{K} \sum_{i_{k}=1}^{L_{k_{a}}} r_{0}^{2 \gamma(1-\kappa)}\left(\frac{r_{i_{k}}^{\kappa}}{r_{i_{k}}^{(0)}}\right)^{2 \gamma} \\
& +\left(E\left[N_{2}^{\prime}\right]\right)^{2}\left(e^{\left(\sigma_{L N}^{2}\left(1+\kappa^{2}\right)-2 \kappa \sigma_{L N}^{2}\right)\left(\frac{\ln (10)}{10}\right)^{2}}-1\right) . \tag{4.50}
\end{align*}
$$

To evaluate the outage, we must set a BER threshold, above which an outage is declared. Wang [13] considers both $10^{-2}$ and $10^{-1}$ thresholds. Gilhousen [9] considers a threshold of $10^{-3}$. The corresponding $\frac{E_{0}}{N_{0}}$ level depends on the coding scheme employed. Yung [8] considers $1.5 d B$ while Gilhousen [9] considers $7 d B$ (for $10^{-3} B E R$ ). We select these levels for study. We avoid getting into details of coding schemes, since our objective here is to demonstrate the effect of non-ideal power control in the general case.

If we choose a $\frac{E_{b}}{N_{0}}$ threshold of $b^{\prime}$, the outage probability can be expressed as

$$
\begin{equation*}
P_{o u t}=P\left(\frac{E_{b}}{N_{0}}<b^{\prime}\right) \cdot=P\left(\frac{\frac{1}{\nu} E_{s}}{N_{0}}<b^{\prime}\right)=P^{*}\left(\frac{1}{\nu} \frac{\frac{M}{2 \xi_{h}}}{N_{1}^{\prime}+N_{2}^{\prime}}<b^{\prime}\right) \tag{4.51}
\end{equation*}
$$

where $N_{1}^{\prime}$ and $N_{2}^{\prime}$ are the terms appearing in the denominator of expression 4.42 and $\nu$ is the code rate. Since $N_{1}^{\prime}$ and $N_{2}^{\prime}$ represent power, they are non-negative. Thus the outage probability can be expressed as

$$
\begin{equation*}
P_{o u t}=P\left(N_{1}^{\prime}+N_{2}^{\prime}>\frac{M}{b^{\prime} \nu 2 \xi_{h}}\right) . \tag{4.52}
\end{equation*}
$$

Since both terms are formed from many addends, we invoke the Central Limit Theorem, and accept the normal approximation of these terms as satisfactory. The lognormal approximation of Schwartz [11] is impossible since the number of mobiles is larger than the validity limit of Schwartz's approximation. More accurate results can only be obtained numerically.

Taking this simplification into account, we see that both terms $N_{1}^{\prime}$ and $N_{2}^{\prime}$ are determined by their means and variances given in Equations 4.42, 4.43, 4.47, and 4.50. Hence the outage probability can be expressed as:

$$
\begin{equation*}
P_{o u t}=\frac{1}{2} \operatorname{erfc}\left(\frac{\frac{M}{b^{\prime} \nu 2 \xi_{h 1}}-E\left[N_{1}^{\prime}\right]-E\left[N_{2}^{\prime}\right]}{\sqrt{2\left(\operatorname{Var}\left[N_{1}^{\prime}\right]+\operatorname{Var}\left[N_{2}^{\prime}\right]\right)}}\right) \tag{4.53}
\end{equation*}
$$

where erfc is the complementary error function, $M$ is the number of chips per symbol, $\nu$ is the code rate, $\xi_{h}$ is the chip shape factor, and $b^{\prime}$ is the $\operatorname{SNR}\left(E_{b} / N_{0}\right)$ required for the minimum acceptable BER for reliable communication. The means and variances are given by Equations 4.42, 4.43, 4.47, and 4.50. The above formula does not account for diversity to emphasize a non-ideal power control. This relatively simple formula allows straightforward numerical evaluation.

## Numerical Results

In [9], the mobile is assigned to a cell-site according to the instantaneous signal power. The mobile chooses a cell-site with the largest instantaneous signal power. The performance results obtained by this method can be viewed as an upper bound on the capacity. We assigned the mobile according to its location and have not considered hand-off. Thus the performance results obtained can be considered as a lower bound on the actual capacity.

For a load of $L_{c}=30$ mobiles in the cell, a threshold of 5 dB , code rate $1 / 3$, and 512 chips per symbol, voice activity factor $V_{o n}=0.35$, the outage probability is plotted in Figure 4.1.

It is clear that the results are intuitively plausible. Initially the "stronger" the power control (larger $\kappa$ ), the lower the outage probability due to the decreased intracell interference. However, if the power control becomes too "strong", intercell interference becomes dominant and the outage probability begins to grow. To get a better view, the results are plotted logarithmically in Figure 4.2. It is evident that there is an optimum point for the power control strategy ( $\kappa \approx 0.75$ in our model).

We see that by proper application of power control, the outage probability drops by orders of magnitude.

Analogous results for a different threshold are plotted in Figure 4.3, 4.4 and 4.5.
By comparing these cases we observe that the optimum $\kappa$ (for minimal outage) does not move significantly with a change of the threshold.

However, to make general conclusions about the power control strategy, we must also investigate the outage probability for users in positions other than the worst case. We check whether an optimum for the worst case user yields optimal performance for any user in the cell. Our findings, plotted in Figure 4.6, show that the optimal $\kappa$ i has only a small


Figure 4.1: Outage probability for cell load 30 , threshold 5 dB , code rate $1 / 3, \mathrm{M}=512$.


Figure 4.2: Outage probability for cell load 30 , threshold 5 dB , code rate $1 / 3, \mathrm{M}=512$.


Figure 4.3: Outage probability for cell load 30 , threshold 1.5 dB , code rate $1 / 3, \mathrm{M}=512$.


Figure 4.4: Outage probability for cell load 30 , threshold 1.5 dB , code rate $1 / 3, \mathrm{M}=512$.


Figure 4.5: Outage probability for cell load 30 , threshold 7.0 dB , code rate $1 / 3, \mathrm{M}=512$.
dependence on distance. Therefore, use of the optimal $\kappa$ for the worst-case user provides near-optimal performance for any user. Figure 4.6 shows that the impact of non-ideal power. control increases with decreasing distance from the cell-site. Given that the performance for worst-case users can be improved by other means (e.g. cell-site diversity), power control optimization could provide further performance improvements.

It is evident that the decrease in outage probability obtained through power control allows an attendant increase in cell capacity. Cell capacity calculations for $1 \%$ outage for the worst-case user are given in Figures 4.7, 4.8 and 4.9. For the 7.0 dB threshold, the cell capacity can be increased by a factor of 2 , for the 5.0 dB threshold by a factor of 2.9 , for the 1.5 dB threshold by a factor of 3.6.

These results promise a significant increase in cell capacity if non-ideal power control is properly applied. For the exponential form of the variance model, the optimal $\kappa \approx 0.80-0.85$. Simulation will yield more exact results.


Figure 4.6: Outage probability for cell load 30 , threshold 7.0 dB , code rate $1 / 3, \mathrm{M}=512$ at $1,0.9,0.8$ and 0.7 of the maximal distance from the tower.


Figure 4.7: Cell capacity (user/cell) for threshold 1.5 dB , code rate $1 / 3, \mathrm{M}=512$ for the worst case user


Figure 4.8: Cell capacity (users/cell) for threshold 5.0 dB , code rate $1 / 3, \mathrm{M}=512$ for the worst case.user


Figure 4.9: Cell capacity (users/cell) for threshold 7.0 dB , code rate $1 / 3, \mathrm{M}=512$ for the worst case user

## Chapter 5

## Comparison with FDMA and TDMA

Present analog cellular mobile systems use 30 kHz FM channels with frequency division multiple access (FDMA). With 120 degree antenna sectors, the frequency reuse efficiency is limited to $14 \%$ due to the requirement of 17 dB carrier-to-interference ratio. This leads to the capacity of $0.14 /\left(30 \times 10^{3}\right)=4.67 \times 10^{-6}$ users per cell per Hz or 4.67 users per cell per MHz . In the currently proposed time division multiple access (TDMA) each 30 kHz channel is time shared by 3 users. Thus, the capacity is increased by a factor of 3 to $14 \times 10^{-6}$ users per cell per Hz or 14 users per cell per MHz . Note that this capacity is identical for both the uplink and the downlink.

In a CDMA system, assume the digital speech rate is $R$. FEC redundancy is extra corresponding to rate $r_{f e c}$ coding. Since each code symbol has $M$ chips, assuming binary PSK is used (either coherent or differentially coherent), the signal bandwidth (null to null) is

$$
\text { bandwidth }=2 R M / r_{f e c}
$$

Then the CDMA capacity is

$$
\text { capacity }=L_{c} / M \times r_{f e c} /(2 R) \text { users per cell per } H z
$$

Note that $L_{c} / M$ is inversely proportional to $E_{s} / N_{0}$ for large $L_{c}$. Thus, according to the above equation, the capacity is inversely proportional to $E_{b} / N_{0}$ where energy per bit is $E_{s} / r_{f e c}$.

Suppose the digital speech rate is $R=9.6 \mathrm{kbps}$ as in [1]. The error correction code rate is $r_{f e c}=1 / 2$. Then

$$
\text { capacity }=2.604 \times 10^{-5} \frac{L_{c}}{M} \text { users per cell per } H z
$$

Table 5.1 shows a comparison of results. The code rates for downlink and uplink are rate $=1 / 2$ and rate $=1 / 3$, respectively. All capacities are normalized with respect to that

| Type | Conditions | Number of Users <br> per Cell per MHz | Relative Capacity <br> to TDMA |
| :---: | :---: | :---: | :---: |
| FDMA |  | 4.67 | 0.33 |
| TDMA |  | 14 | 1 |
| CDMA | DL,ND, $1 \%$ | 1.3 | 0.09 |
| CDMA | DL,ND, $10 \%$ | 14.5 | 1.04 |
| CDMA | DL,3D $1 \% \dagger$ | 2.4 | .17 |
| CDMA | DL,3D 10\% $\dagger$ | 16.6 | 1.18 |
| CDMA | UL,IPC,ND,10\% $\dagger$ | 1.46 | .1 |
| CDMA | UL,IPC,ND,1\% $\dagger$ | 0.54 | 0.04 |
| CDMA | UL,ND,1\% $\ddagger$ | 2.31 | .165 |

DL - Downlink, UL - Uplink, ND - No Diversity
3D - 3 cell-site Diversity, IPC - Ideal Power Control
$\dagger$ The results presented are for the assumptions presented in the report.
They do not represent the maximum capacity achievable
$\ddagger, \kappa=0.78$, Non-ideal Power control, $E_{b} / N_{0}=7.0 d B$, worst case user

Table 5.1: Capacity Comparison.
of TDMA to obtain relative capacities. The standard deviation of the lognormal shadowing is 8 dB and the results for FDMA and TDMA outage probability is assumed to be $10 \%$ [13]. The results for $10 \%$ outage probability were calculated for comparison with TDMA and FDMA. As shown in the table the capacities in users/cell/MHz for the CDMA system for the cases evaluated are lower than that of TDMA. The Table 5.1 shows the results for the $1 \%$ average outage probability (except where noted). For the $1 \%$ cases, although they are not directly comparable with the other results; they are included to show the sensitivity of the capacity to performance threshold chosen. For the non-ideal power control capacities, the worst case user was used in the calculation of capacity.

With a $10 \%$ probability of outage, the results for CDMA are comparable with that of FDMA [13]. The capacity (in users/cell/ MHz ) of the downlink with no cell-site diversity is slightly better than that of TDMA. When 3 cell-site diversity is employed with the region as denoted in Chapter 3, the capacity of the downlink is increased to $18 \%$ above that of TDMA. we must emphasize the point that the 3 cell-site diversity scheme analyzed can be improved and thus these results are not the maximum achievable capacity of CDMA using cell-site diversity. The uplink capacity is higher than FDMA, but lower than TDMA. However, we consider the results for the uplink to be a lower bound to the achievable capacity since we did not consider 'hand-off' of the mobile which would improve the results.

## Chapter 6

## Concluding Remarks

In this study, we have derived expressions for the capacity of a CDMA cellular system. We have analyzed the combined effects of fading, shadowing and mobile location on the service outage statistics. In our capacity calculations, we have assumed an outage probability threshold of $1 \%$ averaged over all cell locations. Outage statistics for a large number of positions were computed, allowing us to examine the dependence of the outage probability on the location within the cell. As expected, locations near the cell boundary have the highest-outage probability when no cell-site diversity is used. Our approach allows several service criteria to be used in computing the capacity, such as worst case outage or an outage threshold which must be met by at least $10 \%$ of all locations.

We have examined the impact of 3 cell-site diversity on system capacity by varying the percentage of users employing diversity. We selected a geometrical diversity region. The diversity region was chosen to closely approximate a region having high outage statistics. It was shown that 3 cell-site diversity increases the system capacity and greatly reduces the maximum outage probability. We chose to examine a 3 cell-site diversity method which required 3 cell-sites to transmit a signal to the mobile if the mobile required diversity. It was shown that if diversity is not applied properly, one can increase the interference to other users without reducing the probability of outage. The use of cell-site diversity greatly reduced the the maximum outage probability over the case when no diversity is employed. Furthermore, for the chosen diversity region, there is a large reduction in the probability of outage for the diversity region, however there is an increase in the outage probability for location in the non-diversity region along the boundary of the non-diversity region and the diversity region. For practical implementations a received power threshold would be used instead of a distance measure. It is important to choose the average power level or received power threshold carefully, to ensure that the mobiles require it and that the number of mobiles requiring it does not exceed a certain percentage.

In Chapter 4, the effect of non-ideal power control was investigated. Of three power
control models considered there, a variance model showed the most promise. An exponential form of the variance model was chosen for ease of implementation and analysis. The purpose of this model is to reduce the intercell interference at the expense of increasing the intracell interference by reducing the transmitter variance relative to the shadowing process. The variance reduction is achieved by introducing an exponential parameter $\kappa$. Cell capacity was investigated as a function of $\kappa$. This model showed that a substantial increase in system capacity is possible. Our further analysis showed, that the optimal $\kappa$ is a function the mobile's position. However, choosing an optimal $\kappa$ for the worst-case user achieves better performance for any user in the cell. The cell capacity was evaluated using the worst-case user's performance, thus the presented results represent a lower bound on the capacity.

In conclusion, we have developed expressions necessary for the evaluation of CDMA capacity given the SNR for the required system BER performance. The combined effect of fading and location outage were taken into account in the development of the formulas. We used an average outage probability of $1 \%$ as the acceptable service performance, although the calculations can be applied to other service performance criteria. We considered a 3 cell-site diversity strategy, which was shown to reduce the worst case outage probability from the no cell-site diversity case. The chosen diversity scheme increased the the capacity of the cell and showed that if diversity is not applied properly, the benefits obtained are reduced. An investigation of non-ideal power control demonstrated that non-ideal control yields significant gains in the capacity of the CDMA system. With the lognormal shadowing model under study, a certain level of variance reduction relative to the variance of the shadowing resulted in an increase of the uplink capacity.

### 6.1 Suggestions for Further Work

We demonstrated a promising non-ideal power control strategy which increases system capacity dramatically. The strategy is easily implemented, but there may be better models in terms of higher capacity gains. As mentioned in Chapter 4, cell-site diversity improves the effect of non-ideal power control. We feel that the combined effect of cell-site diversity and non-ideal power control would result in higher system capacities and is worthy of further investigation.

We feel that numerical simulation would provide results which are difficult to obtain analytically. The simulations could be used to verify the analytical expressions given here and would provide useful new quantitative results. We are presently contemplating the development of a generic simulation tool for use in the evaluation of CDMA cellular systems.

In this study, we have assigned mobiles diversity transmissions on a geographical basis. However, in practice, the diversity assignments are made using different criteria. A combination of several diversity assignment strategies may provide optimal system performance.

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## Appendix A

## Density function of Equivalent SNR for Downlink

The expression for the equivalent SNR for the downlink without cell-site diversity is given by Equation 2.17 which is

$$
\begin{equation*}
\frac{E_{s}}{N_{0}}=\frac{1}{\frac{2 L_{0}}{3 M}+\frac{2\left(L_{0}+1\right)}{3 M} 10^{\frac{Y-x_{0}}{10}}} \tag{A.1}
\end{equation*}
$$

If we let $z=Y-X_{0}$, then $z$ is normally distributed with mean $m_{z}=m_{y}-m_{x}$ and variance $\sigma_{z}^{2}=\sigma_{x}^{2}+\sigma_{Y}^{2}$. The equivalent SNR given by $E_{s} / N_{0}$ is then a function of a Gaussian random variable. The density function of the SNR can be found by a transformation of a random variable. If we define a variable $t$, where $t=E_{s} / N_{0}$, then

$$
\begin{equation*}
t=\frac{1}{\frac{2 L_{0}}{3 M}+\frac{2\left(L_{0}+1\right)}{3 M} 10 \frac{2}{10}} . \tag{A.2}
\end{equation*}
$$

We then can solve for $z$ and compute the derivative, to obtain the following:

$$
\begin{align*}
z & =\frac{1}{\lambda} \ln \left(\frac{3 M^{*}}{2\left(L_{0}+1\right) t}-\frac{L_{0}}{L_{0}+1}\right)  \tag{A.3}\\
\frac{d z}{d t} & =-\frac{1}{\lambda t\left(1-\frac{2 L_{0} t}{3 M}\right)} \tag{A:4}
\end{align*}
$$

where $\lambda=\frac{\ln 10}{10}$. By standard transformation techniques for random variable [as an example see [12]] we can write $f_{t}(t)$ as

$$
\begin{equation*}
f_{t}(t)=\frac{1}{\lambda t\left(1-\frac{2 L_{0} t}{3 M}\right)} \exp \left(-\frac{\left(t^{\prime}-m_{z}\right)^{2}}{2 \sigma_{z}^{2}}\right) \tag{A.5}
\end{equation*}
$$

where

$$
t^{\prime}=\frac{1}{\lambda} \ln \left(\frac{3 M}{2\left(L_{0}+1\right) t}-\frac{L_{0}}{L_{0}+1}\right)
$$

This can then be put into the form as presented in Section 2.1.

## Appendix B

## Density Function for Equivalent SNR for Uplink

Contained in this appendix is the development of the density function of the equivalent SNR for the uplink in the presence of correlated Rayleigh fading and log-normal shadowing.

If X is normally distributed with mean $\mu_{x}$ and variance $\sigma^{2}$ then the first moment of $10 \frac{X}{10}$ is given by

$$
\begin{equation*}
E\left[10^{\frac{X}{10}}\right]=\exp \left\{\lambda\left(\mu_{x}+\frac{\sigma^{2} \lambda}{2}\right)\right\} \tag{B.1}
\end{equation*}
$$

where $\lambda=\frac{\ln 10}{10}$ and the second moment is given by

$$
\begin{equation*}
E\left[\left(10^{\frac{X}{10}}\right)^{2}\right]=\exp \left(2 \lambda\left(\mu_{x}+\sigma^{2} \lambda\right)\right) \tag{B.2}
\end{equation*}
$$

Given the first two moments the variance is can be computed by subtracting the square of the first moment from the second moment

$$
\begin{align*}
\operatorname{Var}\left(10^{\frac{x}{10}}\right) & =E\left[\left(10^{\frac{X}{10}}\right)\right]-\left(E\left[10^{\frac{X}{10}}\right]\right)^{2}  \tag{B.3}\\
& =\left(\exp \left\{\lambda^{2} \sigma^{2}\right\}-1\right) E\left[10^{\frac{x}{10}}\right]^{2} . \tag{B.4}
\end{align*}
$$

Now the expression for the equivalent SNR given by Equation 2.37 in Section 2.2 is

$$
\begin{equation*}
\frac{E_{s}}{N_{0}}=\frac{3 M}{2 \sum_{i=1}^{L_{0}} 10^{\frac{w_{i o}}{10}}+2 \frac{V_{o n}}{N_{s e c t}} \sum_{i=1}^{L_{c}} \sum_{k=1}^{K} 10^{\frac{w_{i}}{10}}} \tag{B.5}
\end{equation*}
$$

where $V_{o n}$ is the voice activity factor, and $N_{s e c t}$ is the number of sectors in a cell, $M$ is the number of chips per symbol, $L_{c}$ is the number of users in a cell, and $K$ is the number of cells which contain interfering users (assuming omni-directional antenna, since we then divide by the number of sectors). The term $w_{i_{0}}$ is a Gaussian variable with zero mean and a variance

$$
\begin{align*}
\sigma_{w_{i_{0}}}^{2} & =2 \sigma_{Z_{i_{0}}}^{2} \\
& =2 \sigma_{x}^{2}+2 \sigma_{X_{t}}^{2}-4 \rho \sigma_{x} \sigma_{X_{t}} \tag{B.6}
\end{align*}
$$

where $X_{t}$, and $\rho$ are defined in Section 2.2
and $w_{i_{k}}$ is a Gaussian variable with mean given by

$$
m_{w_{i_{k}}}=10 \log _{10}\left(\frac{r_{i_{k}}}{r_{i_{k}}^{(o)}}\right)^{\gamma}
$$

and a variance given by

$$
\begin{align*}
\sigma_{w_{i_{k}}}^{2} & =\sigma_{Z_{i_{k}}}^{2}+\sigma_{Z}^{2} \\
& =2\left(\sigma_{x}^{2}+\sigma_{X_{t}}^{2}-\rho \sigma_{x} \sigma_{X_{t}}\right) \tag{B.7}
\end{align*}
$$

We denote $U=\sum_{i=1}^{L_{0}} 10^{\frac{w_{i_{0}}}{10}}$ and $V=V_{o n} \sum_{i=1}^{L_{c}} \sum_{k=1}^{K} 10^{\frac{w_{i k}}{10}}$ then the expression for the equivalent SNR becomes

$$
\begin{equation*}
\frac{E_{s}}{N_{0}}=\frac{\frac{3 M}{2}}{U+V} \tag{B.8}
\end{equation*}
$$

As the number of terms in the summations for $U$ and $V$ are large we can approximate $U$ and $V$ as Gaussian random variables. Thus, $U$ is Gaussian distributed with mean $m_{u}$ and variance $\sigma_{u}^{2}$ and $V$ is Gaussian distributed with mean $m_{v}$ and variance $\sigma_{v}^{2}$. Let $T=U+V$ then $T$ is also Gaussian distributed with mean equal to $m_{u}+m_{v}$ and variance equal to $\sigma_{u}^{2}+\sigma_{v}^{2}$. Thus, $\frac{E_{s}}{N_{0}}$ is a function of a Gaussian variable so the density function can be written as

$$
\begin{equation*}
f_{\frac{E_{s}}{N_{0}}}(t)=\frac{3 M}{t^{2} \sqrt{2 \pi\left(\sigma_{u}^{2}+\sigma_{v}^{2}\right)}} \exp \left(-\frac{\left(t^{\prime}-\left(m_{u}+m_{v}\right)^{2}\right.}{2\left(\sigma_{u}^{2}+\sigma_{v .}^{2}\right)}\right) \tag{B.9}
\end{equation*}
$$

where

$$
\begin{equation*}
t^{\prime}=\frac{3 M}{2 t} \tag{B.10}
\end{equation*}
$$

where $m_{u}$ and $m_{v}$ are the means of the first and second term in the denominator in Equation 2.37, respectively and $\sigma_{u}^{2}$ and $\sigma_{v}^{2}$ are their respective variances. The means and variances of $U$ and $V$ can analytically computed from the means and variances of $w_{i_{o}}$ and $w_{i_{k}}$ using the mean and variance formulas shown in Equations B.1 and B.4.

## Appendix C

## Bound on Uplink Capacity with Cell-site Diversity

The mobile must adjust it transmitter power in an attempt to ensure that the received power at the cell-site is constant. However, the choice of the cell-site which the mobile uses may change according to its position within the cell. If the mobile is in the diversity region then we assume that the mobile is transmitting to the cell-site from which it is receiving the best downlink signal. If the mobile is in the non-diversity region, the mobile uses the reference cell-site at all times. Thus, the interference experienced by the cell-site from the mobiles in the non-diversity region is identical to the interference discussed in Section 2.2. For a transmitter in the diversity region, the mean and variance of the received signal at the cell-site are given by the following expressions:

$$
\begin{align*}
& m_{X_{0}}=\int_{0}^{\infty} X_{0} f_{X_{0}}\left(X_{0}\right) d_{X_{0}}  \tag{C.1}\\
& \operatorname{Var}\left(X_{0}\right)=\int_{0}^{\infty}\left(X_{0}\right)^{2} f_{X_{0}}(X) d_{X}-m_{X_{0}}^{2} \tag{C.2}
\end{align*}
$$

where $X_{0}$ is defined in Equation 3.1. The variance of a mobile's transmit power coincides with the variance of $X_{0}$ since $X_{0}$ is the basis for estimating the uplink attenuation. However, we assume that the mobile's transmit power $X_{t}^{2}$ can be represented as a lognormal random variable with reduced variance relative to a user without diversity. Thus, the variance of the received power will differ between users in the diversity region and those mobiles outside the diversity region.

The statistics of the transmit power variations are difficult to evaluate analytically. We take the following approximate approach. We assume that transmitter power control results in a lognormal power variation. The mean of the received power is bounded by

$$
0 \leq m_{i_{k}} \leq 10 \log _{10}\left(\frac{r_{i_{k}}}{r_{i_{k}}^{(o)}}\right)^{\gamma}
$$

and the variance is bounded by

$$
\begin{equation*}
\sigma_{Z_{i_{o}}}^{2}(\text { nodiv }) \leq \sigma_{Z_{i_{k}}}^{2}(\text { div }) \leq \sigma_{Z_{i_{k}}}^{2}(\text { nodiv }) \tag{C.3}
\end{equation*}
$$

where $\sigma_{Z_{i_{0}}}^{2}$ (nodiv), $\sigma_{Z_{i_{k}}}^{2}$ (nodiv) are the variances of the received power in the non-diversity region from intracell and intercell interferers, respectively. We assume as in [9] that the mobile is a member of the cell having the nearest cell-site to the mobile's position. Thus, for a. given position, the closest cell-site determines whether that position is included in intracell or intercell interference terms. This assumption slightly overestimates the interference seen at the cell-site. In other words, it yields an upper bound on the actual interference level and a. lower bound for the number of users in the cell.

## Appendix D

## Bounds on SNR

Let

$$
\begin{equation*}
S N R=\frac{1}{X_{1}+X_{2}} \tag{D.1}
\end{equation*}
$$

where $X_{1}$ and $X_{2}$ are independent gaussian random variable, each having mean $m_{1}, m_{2}$ and variance $\sigma_{1}^{2}$ respectively. Then we can write:

$$
\begin{equation*}
S N R=\frac{1}{X} \tag{D.2}
\end{equation*}
$$

where $X$ is normally distributed random variable with mean $m=m_{1}+m_{2}$ and variance $\sigma^{2}=\sigma_{1}^{2}+\sigma_{2}^{2}$. Then:

$$
\begin{align*}
E\left[\frac{1}{X}\right] & =\frac{1}{\sqrt{2 \pi \sigma^{2}}} \int_{-\infty}^{\infty} \frac{1}{X} e^{-\frac{(X-m)^{2}}{2 \sigma^{2}}} d X=\frac{e^{-\frac{m^{2}}{2 \sigma^{2}}}}{\sqrt{2 \pi \sigma^{2}}} \int_{\varepsilon \rightarrow 0}^{\infty} e^{-\frac{X^{2}}{2 \sigma^{2}}} \frac{e^{X \frac{m}{\sigma^{2}}}-e^{-X \frac{m}{\sigma^{2}}}}{X} d X= \\
& =\frac{e^{-\frac{m^{2}}{2 \sigma^{2}}}}{\sqrt{2 \pi \sigma^{2}}} \int_{\varepsilon \rightarrow 0}^{\infty} e^{-\frac{X^{2}}{2 \sigma^{2}} \frac{\sinh \left(X \frac{m}{\sigma^{2}}\right)}{X \frac{m}{\sigma^{2}}} d X} \tag{D.3}
\end{align*}
$$

using 4.37 we can write the following inequality:

$$
\begin{align*}
E\left[\frac{1}{X}\right] & \leq \frac{2 m}{\sigma^{2}} \frac{e^{-\frac{m^{2}}{2 \sigma^{2}}}}{\sqrt{2 \pi \sigma^{2}}} \int_{\varepsilon \rightarrow 0}^{\infty} e^{-\frac{X^{2}}{2 \sigma^{2}}}\left(\frac{\cosh \left(X \frac{m}{\sigma^{2}}\right)}{3}+\frac{2}{3}\right) d X \\
& =\frac{e^{-\frac{m^{2}}{2 \sigma^{2}}}}{\sqrt{2 \pi \sigma^{2}}}\left[\frac{2 m}{\sigma^{2}} \frac{2}{3} \int_{\varepsilon \rightarrow 0}^{\infty} e^{-\frac{X^{2}}{2 \sigma^{2}}} d X+\frac{m}{3 \sigma^{2}} \int_{\varepsilon \rightarrow 0}^{\infty} e^{-\frac{X^{2}}{2 \sigma^{2}}}\left(e^{\frac{X}{\sigma^{2}}}+e^{-\frac{X}{\sigma^{2}}}\right) d X\right] \\
& =\frac{2 m e^{-\frac{m^{2}}{2 \sigma^{2}}}}{3 \sigma^{2}}+\frac{m}{\sqrt{2 \pi \sigma^{2}} 3 \sigma^{2}}\left[\int_{\varepsilon \rightarrow 0}^{\infty} e^{-\frac{(X-m)^{2}}{2 \sigma^{2}}} d X+\int_{\varepsilon \rightarrow 0}^{\infty} e^{-\frac{(X+m)^{2}}{2 \sigma^{2}}} d X\right] \\
& =\frac{m}{3 \sigma^{2}}\left(1+2 e^{-\frac{m^{2}}{2 \sigma^{2}}}\right) \tag{D.4}
\end{align*}
$$

Thus:

$$
\begin{equation*}
E\left[\frac{1}{X}\right] \leq \frac{m}{3 \sigma^{2}}\left(1+2 e^{-\frac{m^{2}}{2 \sigma^{2}}}\right) \tag{D.5}
\end{equation*}
$$

which is the upper bound.
For lowerbound derivation we apply exactly the same procedure but instead of using 4.37 we use 4.36 and get:

$$
\begin{equation*}
E\left[\frac{1}{X}\right] \geq \frac{m}{\sigma^{2}} e^{-\frac{m^{2}}{2 \sigma^{2}}} \tag{D.6}
\end{equation*}
$$

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