Génie Electrique

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FINAL REPORT

D.O.C. Contract Serial Nr. OSU77-00181
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ANALYTICAL AND EXPERIMENTAL STUDY OF SELECTED PASSIVE MICROWAVE COMPONENTS IN PLANAR/FIN LINE GUIDE

By

Wolfgang J.R. Hoefer

March 1979

UNIVERSITÉ D'OTTAWA
UNIVERSITY OF OTTAWA

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Final Report to
Mr. Michel Cuhaci

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Ottawa, Ontario
Canada

FOREWORD

The work described in the present final report was sponsored by the Department of Communications, Communications Research Centre, Ottawa, under Contract Serial Number OSU77-00181, D.S.S. File Number 07SU 36100-7-9514.

The work was carried out in the Department of Electrical Engineering, University of Ottawa.

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1) INTRODUCTION

Fin line is an attractive transmission medium at millimeter wavelengths for its ease of fabrication and its compatibility with semiconductor devices. Its wide bandwidth, low dispersion and moderate attenuation (when compared with microstrip in that frequency range) would make it a favorite candidate for many designers, were it not for the cumbersome design procedures available to date.

Treatment of the fin line as a ridged waveguide of identical dimensions yields only poor accuracy, 15% at best for the effective permittivity and characteristic impedance.

Meier's [9] expressions for these parameters are certainly adequate for most applications, particularly at frequencies well above cutoff of the fundamental TE_{01}-mode, but unfortunately, the determination of the effective dielectric constant of the line requires a sample measurement, which is expensive and time consuming.

On the other hand, the exact solution of Maxwell's equations in fin line, as presented by Hofmann [2], tends to dissuade many designers by its involved mathematics, even though it is the ultimate approach.

The objective of the present study is to present an easier but nevertheless accurate method to theoretically predict the guided wavelength and characteristic impedance in fin lines.

To this end, the transverse resonance condition in a fin line resonator is solved using either a graphical method (very cheap but quite accurate) or a computer routine, either on a programmable pocket calculator or on a large computer. The equation for the transverse resonance condition contains an expression for the discontinuity susceptance of the fins and the substrate. It is this susceptance that will be presented in mathematically closed form, following the style of Marcuvitz' [6] Waveguide Handbook. This approach is familiar to all microwave circuit designers and combines the advantages of easy mathematical formalism and the accuracy of the exact solution from which it has been derived.

The exact solution presented in the present report is based on Cohn's [7] treatment of the unshielded slot line. Taking into account the transverse impedance of the fin line enclosure acting as a rectangular short-circuited waveguide below cutoff, the susceptance of the combined substrate-fin discontinuity is derived from Cohn's expressions which have already been conceived for fast convergence on a computer.

In conclusion, the fin line designer will benefit from the present approach because he will

a. save time by using the computer programs presented in this report
b. apply the results of an exact solution without having to solve the boundary value problem himself, and
c. have at his disposal a flexible method to evaluate geometries of arbitrary dimensions. He can therefore deviate from published data for very special geometries.

2) BASIC PROPERTIES OF FIN LINES

In a fin line structure, metal fins are printed on a dielectric substrate which bridges the broad side of a rectangular waveguide. Several different arrangements are possible, such as the examples shown in figure 2.1.

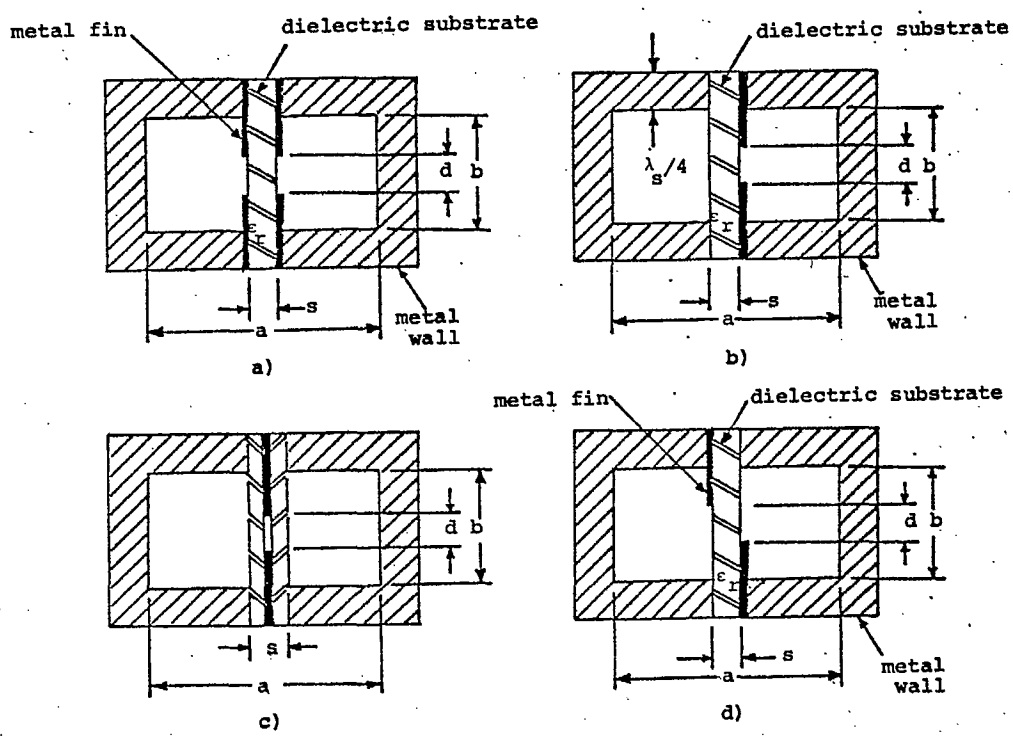

Figure 2.1 Cross-section of several fin lines

a) Bilateral or earthed fins; b) Unilateral fins
c) Central or insulated fins; d) Antipodal fins

In all cases, the thickness of the metal fins is negligeably small compared with the thickness of the dielectric substrate. The metal walls of the guide have a thickness of $\lambda_s/4$, where λ_s is the wavelength in the substrate, so that a RF short-circuit appears in the plane of the inner broad walls.

Even though the fin line is basically a loaded waveguide, its propagation properties are essentially determined by the relative fin spacing d/b and, to a lesser extent, by the dielectric constant of the substrate. Thus, the transmission line properties depend on the central planar structure rather than on the size of the metal enclosure.

An alternate way of describing a fin line is to call it a slotline (as analyzed by Cohn[1]), which has been placed into a rectangular waveguide.

A first understanding of the effect of the fins on the propagation in the guide can be gained by considering the fins and the dielectric as a capacitive loading which lowers the cut off frequency of the fundamental $TE_{1\bar{0}}$ mode and reduces its guided wavelength. The same applies to the higher TE_{m0}-modes with odd m, while all modes with even m are only slightly affected. This explains the larger bandwidth of the fundamental fin line mode, when compared with the $TE_{1\bar{0}}$ mode in rectangular waveguide.

By the same consideration, it can be seen that with decreasing d/b-ratio, the characteristic impedance of the fin line decreases. Characteristic impedances ranging from about

400 Ω to 100 Ω can be achieved with all types shown in figure 2.1. The antipodal line however can provide even lower impedances (down to 10Ω) by overlapping of the fins (d/b < 0) [2].

If active devices are to be added to the fin line, at least one of the fins must be insulated from the ground at dc to permit the application of bias, without disrupting the RF grounding of the fins. This can be done by inserting thin dielectric gaskets between the fins and the waveguide walls.

Figure 2.2 shows how the properties of fin line can be changed in longitudinal direction by varying the d/b-ratio, and how semiconductor devices can be added in series and in parallel.

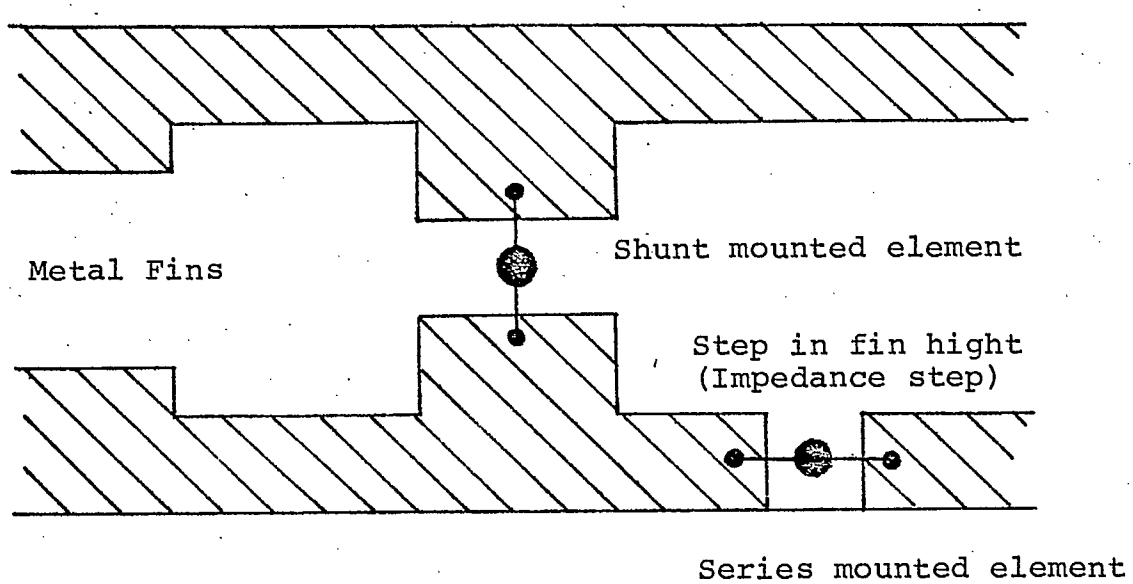

Fig. 2.2 Longitudinal view of fin line with impedance steps and mounted active elements.

In the following, expressions for the electromagnetic parameters of fin lines are derived. These parameters depend on the cross-sectional geometry and on the substrate permittivity.

3) DETERMINATION OF GUIDED WAVELENGTH AND CHARACTERISTIC IMPEDANCE OF FIN LINES

3.1) Guided Wavelength in Ridged Waveguide

A first glance at fin line geometries shows their similarities with ridged waveguide. In fact, if the permittivity of the substrate is low and the thickness of the substrate is small compared with the dimensions of the waveguide, the formalism for ridged waveguide may be applied to the fin line. Considerable design information for ridged waveguide is available in the literature [3][4][5][6], and detailed derivations for such structures will therefore not be given here. However, with the advent of small programmable calculators, it is easy to calculate the parameters of ridged waveguides of arbitrary geometry. The designer has thus more freedom in the choice of dimensions and can deviate from tabulated and charted data. For this reason, the method for evaluating the parameters of ridged waveguide will be outlined briefly.

As an example, the geometry shown in figure 2.1c will be chosen. When the influence of the dielectric is neglected, a waveguide with a central ridge of zero thickness results (see figure 3.1a).

If a wave travels in longitudinal direction, its guided wavelength being λ_g, the transverse electric field of the lowest

mode will be zero at both sidewalls as well as in transverse planes at distances $\lambda_g/2$ from each other. If electric walls are inserted in two such transverse planes, a standing wave is created and resonance occurs.

For further analysis, the resonator is now considered to be a rectangular waveguide of height b and width $\lambda_g/2$, containing

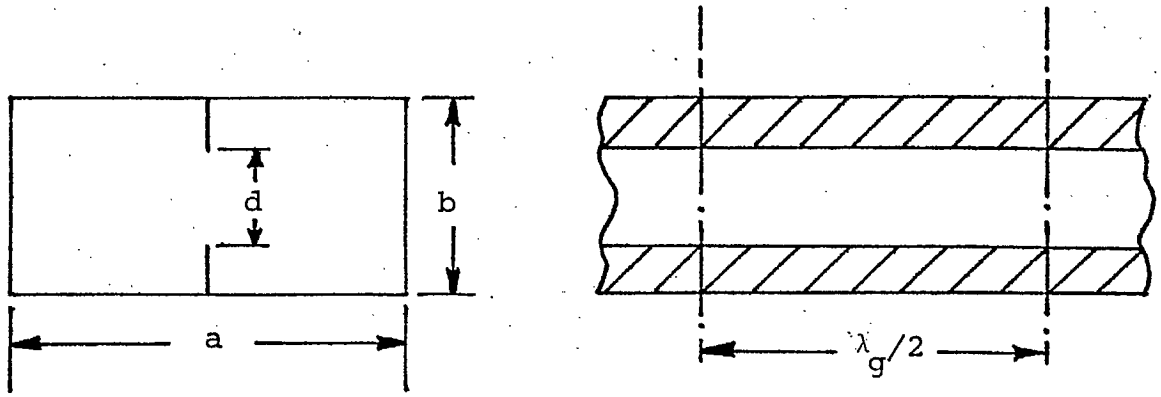

Fig. 3.1a Waveguide with central ridge of zero thickness

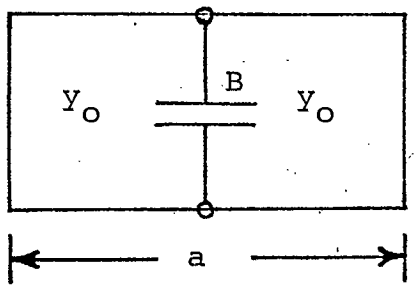

Fig. 3.1b Equivalent transverse resonant network of above waveguide

a transverse capacitive iris and being short-circuited at $-a/2$ and $+a/2$ from this discontinuity.

The transverse resonant condition is thus:

$$\frac{1}{2}\frac{B}{Y_o} - \cot\frac{\pi a}{\lambda_t} = 0 \qquad (3.1)$$

where λ_t is the transverse guided wavelength:

$$\lambda_t = \lambda[1 - (\frac{\lambda}{\lambda_g})^2]^{-\frac{1}{2}} \qquad (3.2)$$

λ is the free-space wavelength. The normalized discontinuity susceptance B/Y_o is given in the Waveguide Handbook [6], section 5.1, as:

$$\frac{B}{Y_o} = \frac{4b}{\lambda_t}\left[\ln(\csc\frac{\pi d}{2b}) + \frac{Q\cos^4\frac{\pi d}{2b}}{1+Q\sin^4\frac{\pi d}{2b}} + \right.$$

$$\left. + \frac{1}{16}(\frac{b}{\lambda_t})^2(1-3\sin^2\frac{\pi d}{2b})^2\cos^4\frac{\pi d}{2b}\right] \qquad (3.3)$$

with $Q = [1-(\frac{b}{\lambda_t})^2]^{-\frac{1}{2}} - 1$ (3.4)

For small values of d/b, and for d/b close to unity, somewhat simpler expressions for B/Y_o can be found in [6]. By fixing the b/a - ratio and choosing an arbitrary value for λ/λ_g, the corresponding b/λ is found as the root of eqn. (3.1). This root is obtained either graphically or perusing a standard routine readily available for programmable minicalculators.

The root of eqn. (3.1) can be found with considerably less computational effort if direct use is made of the graphical presentation of the transverse discontinuity susceptance in the Waveguide Handbook [6] (see Fig. 5.1-4 of this reference.) For this purpose, the transverse resonance condition in eqn. (3.1) is rewritten as follows:

$$\frac{1}{2}\left[\frac{B}{Y_o}\cdot\frac{\lambda_t}{b}\right]\frac{b}{\lambda_t} - \cot\pi\,\frac{a}{b}\,\frac{b}{\lambda_t}=0 \qquad (3.5)$$

where λ_t is the transverse guided wavelength.
The term in square brackets is the $[\frac{B}{Y_o}\,\frac{\lambda_g}{b}]$ presented in the Waveguide Handbook [6], Fig. 5.1-4, provided λ_g therein is replaced by λ_t of this report. Close inspection of Fig. 5.1-4 in [6] shows that the normalized susceptance is rather insensitive to changes in b/λ_t as long as b/λ_t is smaller than 0.4, which is practically always the case. Thus, a very good approximation for b/λ_t can be found by introducing into eqn. (3.5) the value of $[(B/Y_o)/(\lambda_t/b)]$ for $b/\lambda_t = 0$. The root obtained with this value is accurate within a few percent.

If a better approximation is desired, an improved value for $[(B/Y_o)/(\lambda_t/b)]$ corresponding to the just calculated b/λ_t is introduced into eqn. (3.5). The guided wavelength λ_g in the ridged waveguide is then found by writing

$$\lambda_g = \lambda\,[1-(\lambda/\lambda_t)^2]^{-1/2} \qquad (3.6)$$

where λ is the free space wavelength.

For the purpose of designing a waveguide, it is more desirable to write the transverse resonance condition (3.5) directly in terms of the free space wavelength λ as well as the guided wavelength in longitudinal direction, λ_g. This is done by writing

$$\lambda_t = \lambda[1-(\lambda/\lambda_g)^2]^{-1/2} \tag{3.7}$$

and introducing λ_t into eqn. (3.5). We use the following convenient abbreviations

$$\lambda/\lambda_g = p \; ; \quad b/\lambda = x$$

$$[1-(\lambda/\lambda_g)^2]^{1/2} = (1-p^2)^{1/2} = v \quad ; \quad b/a = z \; ; \; d/b = t$$

and obtain:

$$\frac{1}{2}\left[\frac{B}{Y_o}\cdot\frac{1}{vx}\right] vx - \cot(\pi vx/z) = 0 \tag{3.8}$$

The normalized susceptance of the transverse discontinuity (eqn. (3.3)) becomes with the same abbreviations:

$$\frac{B}{Y_o}\cdot\frac{1}{vx} = 4\left[\ln(\csc \pi t/2) + \frac{Q\cos^4\pi t/2}{1+Q\sin^4\pi t/2} + \right.$$

$$\left. + \frac{1}{16}(vx)^2(1-3\sin^2\pi t/2)^2\cos^4\pi t/2\right] \tag{3.9}$$

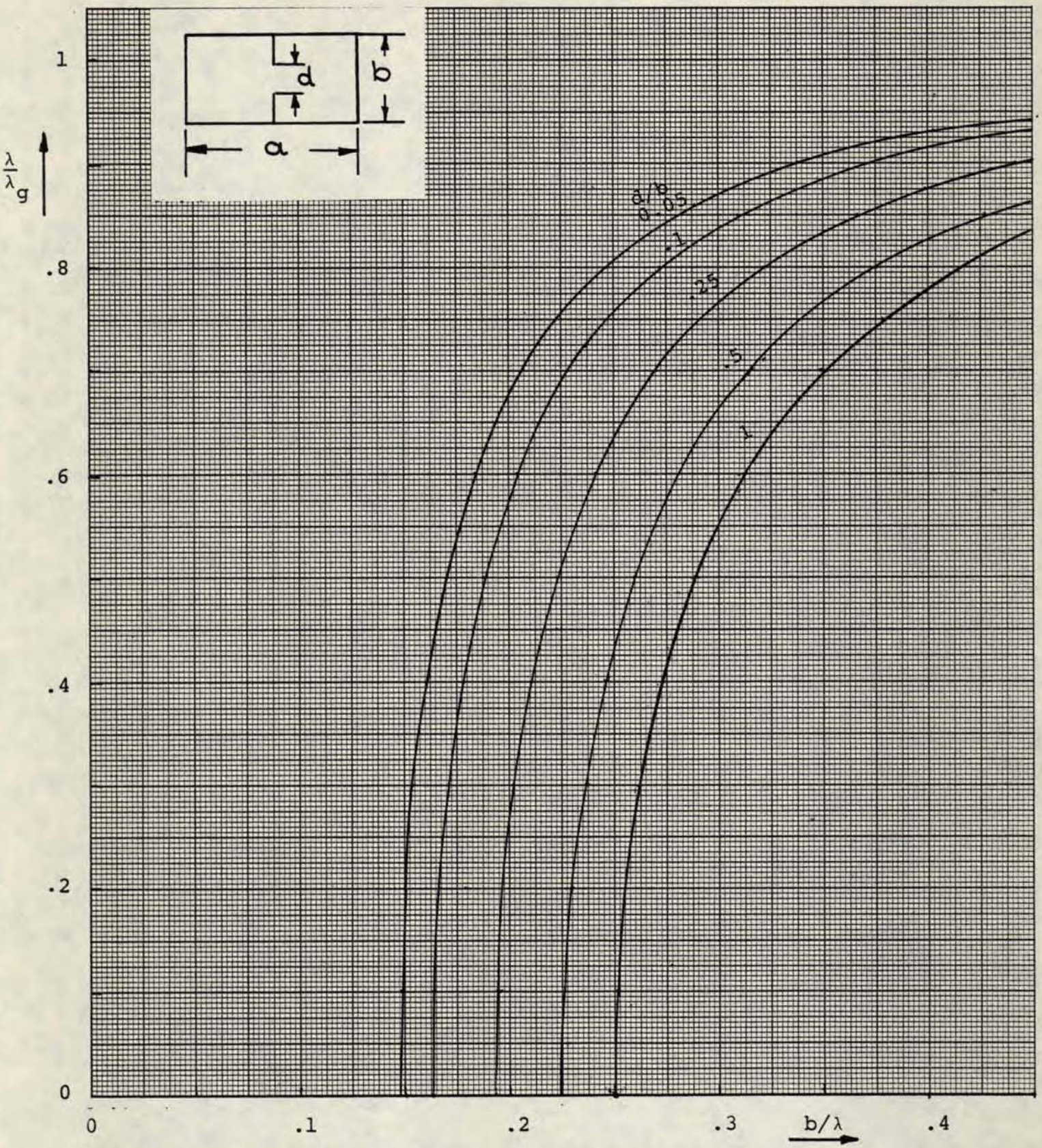

Fig. 3.2 Guided wavelength in ridged waveguide with centered ridge of zero thickness (thin fin) b/a=0.5

with

$$Q = [1-(vx)^2]^{-1/2} - 1 \qquad (3.10)$$

Eqn. (3.8) can be programmed in such a way that the value for any variable will be found if all others are given. (Program for calculating the roots of a function).

The value for the discontinuity susceptance may be taken from Fig. 5.1-4 of the Waveguide Handbook [6] or included explicitly in the program using eqn. (3.9).

Several such programs are given in Appendix 1. Typical results are presented in a diagram with $x = b/\lambda$ as abscissa and $p = \lambda/\lambda_g$ as ordinate, which has the advantage of being normalized (Fig. 3.2). Another way of presenting results would be to show $\varepsilon_{eff} = (\lambda/\lambda_g)^2$ as a function of frequency or angular frequency with t as parameter, or alternatively ω as a function of $\beta = 2\pi/\lambda$ or even ε_{eff} versus t with the frequency as a parameter. The choice finally depends on the characteristics of interest and on the habits of the designer.

In the case of bilateral or earthed fins (see Fig. 2.1a) the ridged waveguide approximation presents <u>a finite ridge width s</u> as shown in Fig. 3.3. In the equivalent transverse network of this structure, the thick capacitive obstacle must be represented by the network shown in section 5.9 of the Waveguide Handbook [6].

For transverse resonance to occur, the following condition must be satisfied: (TE_{no}-modes with odd n (n=1,3,5,7...)

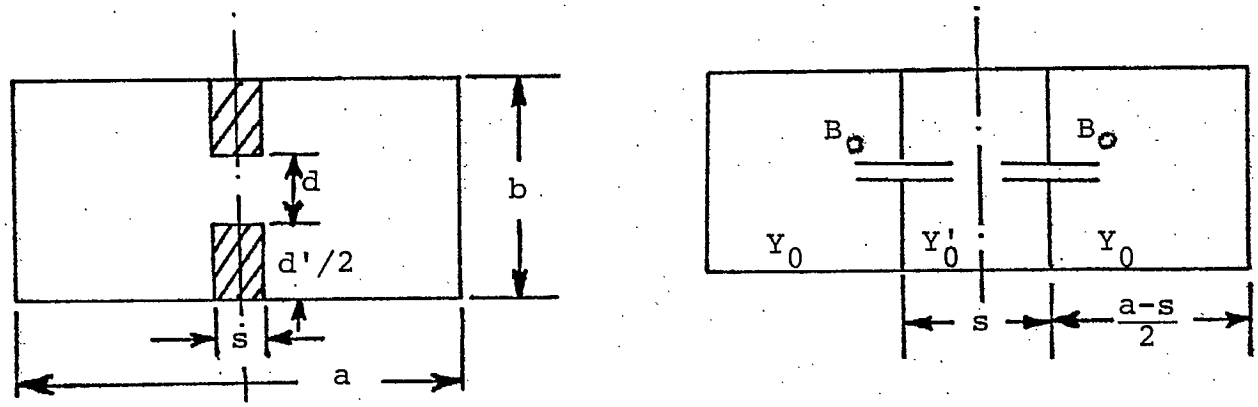

Fig. 3.3 Cross-section and equivalent transverse network of double ridged waveguide

$$\frac{1}{t}\tan\pi vxw + \left[\frac{B_o}{Y_o}\cdot\frac{1}{vx}\right] vx - \cot \pi vx\left(\frac{1}{z} - w\right) = 0 \qquad (3.11)$$

where $v = [1-p^2]^{1/2}$; $p = \lambda/\lambda_g$; $x = b/\lambda$;

$z = b/a$; $w = s/b$; $t = d/b$;

λ_g = is the guided wavelength in longitudinal direction

λ = is the free space wavelength

In this expression:

$$\frac{B_o}{Y_o} \cdot \frac{1}{vx} = \frac{1}{2} \left[\frac{B}{Y_o} \frac{1}{vx} \right] \tag{3.12}$$

where $\frac{B}{Y_o} \frac{1}{vx}$ is given by eqn. (3.9) within a few percent.

The transverse resonant condition (3.11) can be evaluated in the same manner as eqn. (3.8). Appropriate computer programs are given in Appendix A-2. For s = 0, the solutions are those for fins of zero thickness.

Results can be plotted in the same way as in Fig. 3.2 with the additional parameter s to be specified.

3.2. Characteristic Impedance of Ridged Waveguide

The characteristic impedance of any TE-mode in a uniform waveguide of arbitrary cross-section is given by

$$Z_{TE} = \sqrt{\frac{\mu}{\varepsilon}} \quad \frac{\lambda_g}{\lambda} = \sqrt{\frac{\mu}{\varepsilon}} \frac{1}{p} \tag{3.13}$$

If the waveguide is air-filled, this impedance becomes

$$Z_{TE} = [120\pi/p] \text{ Ohms} \tag{3.14}$$

where p can be evaluated using the methods described above.

In impedance matching problems, however, a characteristic impedance based on the voltage-to-current ratio is employed. In the formulation of this ratio, the transmission line current is separated into two components:

a) A longitudinal component on the top and bottom plates of the waveguide, which excites the principal fields.

b) A longitudinal component on the step wall which excites the local fields.

In order to evaluate these currents, let us consider the cross-section of a ridged waveguide as shown in Fig. 3.4

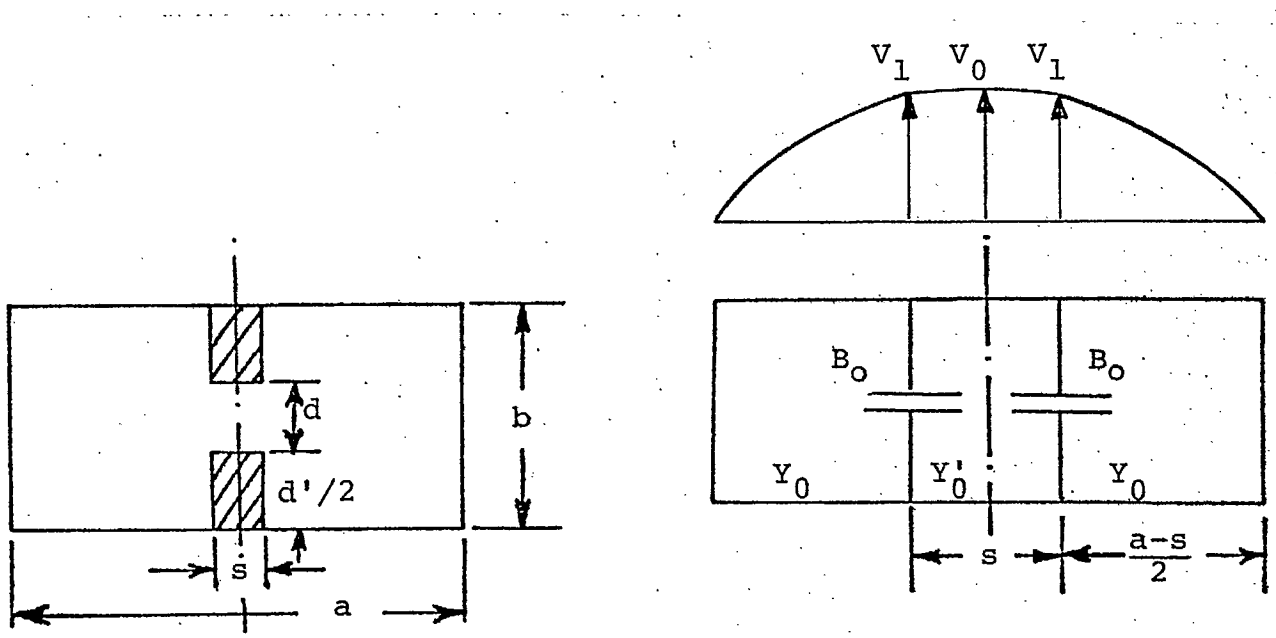

Fig. 3.4 Cross-section of double ridged waveguide with equivalent transverse network showing the voltage distribution

At the steps situated ± d/2 from the centre of the cross-section, the voltage must be continuous across the equivalent step discontinuity capacitance. As a next step, the normal electric field E_n along the top wall of the ridged guide is evaluated. The longitudinal linear current density J is then directly related to the normal electric field by the field impedance Z_{TE}:

$$J = \frac{E_n}{Z_{TE}} \tag{3.15}$$

Finally, the total longitudinal top wall current I_ℓ is obtained by integrating

$$I_\ell = \int_{-a/2}^{+a/2} J \, d\ell = 2 \int_0^{a/2} E_n/Z_{TE} \, d\ell \tag{3.16}$$

The characteristic impedance based on the Voltage-to-Current ratio is then

$$Z_o = \frac{V_o}{I_\ell} \tag{3.17}$$

a) <u>Evaluation of the longitudinal current in the central part of the cross-section</u>

In the TE_{10}-mode, the voltage decreases cosinusoïdally outwards from the centre:

$$V(\ell) = V_o \cos \frac{2\pi}{\lambda_t} \cdot \ell \tag{3.18}$$

The voltage V_1 at the step is thus

$$V_1 = V_o \cos \frac{\pi}{\lambda_t} \cdot s \tag{3.19}$$

The longit. current density at the top wall is, according to eqn. (3.15)

$$J(\ell) = \frac{1}{d} \frac{V_o}{Z_{TE}} \cos \frac{2\pi}{\lambda_t} \ell \tag{3.20}$$

and the longitudinal current in the central part becomes

$$\begin{aligned} I_{\ell_1} &= \frac{2}{d} \int_{\ell=0}^{s/2} \frac{V_o}{Z_{TE}} \cos \frac{2\pi}{\lambda_t} \ell \quad d\ell \\ &= \frac{1}{\pi} \frac{V_o}{Z_{TE}} \cdot \frac{\lambda_t}{d} \sin \frac{\pi s}{\lambda_t} \end{aligned} \tag{3.21}$$

b) <u>Evaluation of the longitudinal current in the lateral parts of the cross-section.</u>

In the lateral parts of the cross-section, the TE_{10}-voltage decreases in a cosinusoidal fashion, with a node situated at ± a/2. If ℓ' is the variable distance inward from the sidewalls, then the voltage varies in the transverse

direction as

$$V(\ell') = V_o \frac{\cos \frac{\pi s}{\lambda_t}}{\sin \frac{\pi(a-s)}{\lambda_t}} \sin \frac{2\pi}{\lambda_t} \ell' \tag{3.22}$$

The longitudinal current density in the top wall becomes thus

$$J(\ell') = \frac{1}{b} \frac{V_o}{Z_{TE}} \frac{\cos \frac{\pi s}{\lambda_t}}{\sin \frac{\pi(a-s)}{\lambda_t}} \sin \frac{2\pi}{\lambda_t} \ell' \tag{3.23}$$

The total longitudinal current in the lateral parts becomes:

$$I_{\ell_2} = 2 \int_{\ell'=0}^{\frac{a-s}{2}} \frac{1}{b} \frac{V_o}{Z_{TE}} \frac{\cos \frac{\pi s}{\lambda_t}}{\sin \frac{\pi(a-s)}{\lambda_t}} \sin \frac{2\pi}{\lambda_t} \ell' \, d\ell'$$

$$= -\frac{2}{b} \frac{V_o}{Z_{TE}} \frac{\cos \pi s/\lambda_t}{\sin \frac{\pi(a-s)}{\lambda_t}} \cdot \frac{\lambda_t}{2\pi} \cos \frac{2\pi}{\lambda_t} \ell' \Bigg|_0^{\frac{a-s}{2}}$$

$$= \frac{1}{\pi} \frac{V_o}{Z_{TE}} \frac{\lambda_t}{b} \frac{\cos \pi s/\lambda_t}{\sin \frac{\pi(a-s)}{\lambda_t}} \left[1 - \cos \frac{\pi(a-s)}{\lambda_t}\right] \tag{3.24}$$

Since

$$\frac{1-\cos\alpha}{\sin\alpha} = \tan \frac{\alpha}{2} \tag{3.25}$$

we can write eqn. (3.24) as follows:

$$I_{\ell_2} = \frac{1}{\pi} \frac{V_o}{Z_{TE}} \frac{\lambda_t}{b} \cos(\pi s/\lambda_t) \tan[\pi(a-s)/2\lambda_t] \tag{3.26}$$

c) Evaluation of the longitudinal current in the discontinuity region.

Assuming that the discontinuity region can be represented by a shunt capacitance C_s/unit length subject to the voltage $V_1 = V_o \cos \pi s/\lambda_t$, we can imagine it as a parallel plate capacitor of plate distance h and width ℓ in the transverse direction

$$C_s = \varepsilon_o \ell/h \tag{3.27}$$

The electric field strength in the capacitor is then

$$E_c = V_1/h = (V_o/h) \cos \pi s/\lambda_t \tag{3.28}$$

and the current in the top plate:

$$I_t = E_c \cdot \ell/Z_{TE} = \frac{V_o \ell}{hZ_{TE}} \cos \pi s/\lambda_t \tag{3.29}$$

Replacing ℓ/h by C_s/ε_o we obtain for I_t

$$I_t = \frac{V_o}{Z_{TE}} \frac{C_s}{\varepsilon_o} \cos \pi s/\lambda_t \tag{3.30}$$

The total discontinuity current, taking into account both halves of the cross-section, is then

$$I_{\ell_3} = \frac{2}{\varepsilon_o} \frac{V_o}{Z_{TE}} \frac{1}{\omega} \frac{\omega C_s}{Y_{ot}} \cdot Y_{ot} \cos \pi s/\lambda_t \tag{3.31}$$

where $Y_{ot} = \frac{\eta}{b}$; $\eta = \frac{\varepsilon_o}{\mu_o}$

after some further modification, this current becomes, for a finite real λ_g in longitudinal direction:

$$I_{\ell_3} = \frac{1}{\pi} \frac{V_o}{Z_{TE}} \frac{\lambda_t}{b} \left(\frac{B_o}{Y_o}\right) \cos \pi s/\lambda_t \tag{3.32}$$

The characteristic impedance, as defined in eqn. (3.17), is then for the ridged waveguide:

$$Z_o = Z_{TE}\, \pi \frac{b}{\lambda_t} / \left\{ \frac{b}{d} \sin \pi s/\lambda_t + \left[\left(\frac{B_o}{Y_o}\right) + \tan \frac{\pi(a-s)}{2\lambda_t}\right] \cos \pi s/\lambda_t \right\} \tag{3.33}$$

If we introduce the abbreviations already used in the evaluation of the guided wavelength, i.e.

$b/\lambda_t = vx$; $v = (1-p^2)^{1/2}$; $p = \lambda/\lambda_g$; $x = b/\lambda$

$b/a = z$; $d/b = t$; $s/b = w$

this expression for the impedance becomes:

$$Z_0 = \frac{\frac{120\pi}{p} \pi vx}{\frac{1}{t} \sin \pi wvx + \left[\left(\frac{B_o}{Y_o} \frac{1}{vx}\right) vx + \tan \frac{\pi vx}{2}\left(\frac{1}{z} - w\right)\right] \cos \pi wvx} \tag{3.34}$$

This expression can be evaluated if the transverse resonance condition (3.11) has been solved. Fig. 3.5 shows the characteristic impedance of ridged waveguide for several values of w and t with z = 0.5. The results were calculated using the program described in Appendix A 1.1.2. $(B_o/Y_0)/vx$ is the same as in eqn. (3.12).

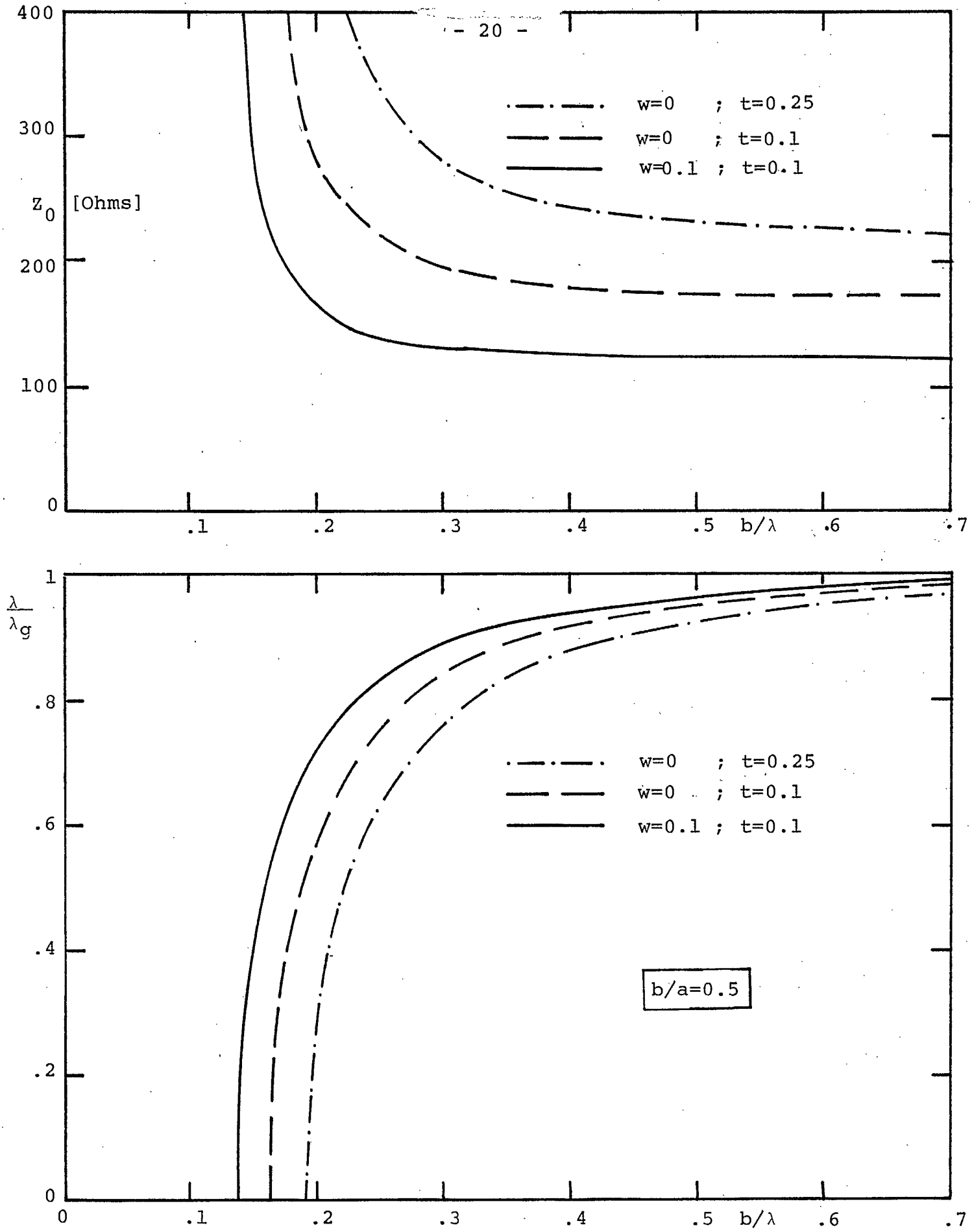

Fig.3.5 Characteristic impedance and guided wavelength in ridged waveguide

3.3 Guided Wavelength in Fin Lines

Fin lines can be treated in the same way as ridged waveguides with the additional complication that the presence of a dielectric sheet at the transverse discontinuity modifies its parameters.

In the present chapter, it is shown how the dielectric sheet changes the shunt susceptance of the fins.

3.3.1 Unilateral Fin Line

Let us study the unilateral fin line structure shown in Figure 2.1 b. Considering that this structure is a rectangular waveguide with a centered ridge of zero thickness to which a dielectric sheet of permittivity ε_r has been added, we can say that the susceptance of the covered half of the discontinuity is increased by a correction factor somewhere between 1 and ε_r, depending on the thickness s of the sheet.

It is convenient to normalize this correction factor in order to make it independent of the dielectric constant, and it becomes:

$$F = \frac{\text{Slot susceptance for finite sheet thickness}}{\text{Slot susceptance for infinite sheet thickness}}$$

An expression for F is derived in detail in section 3.3.4. We can now draw the transverse equivalent network shown in Figure 3.6.

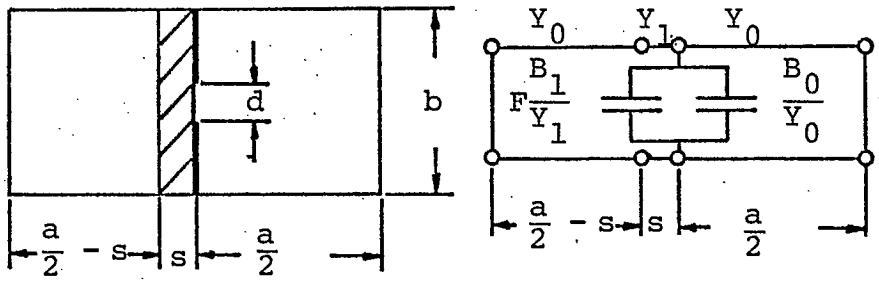

Fig. 3.6 Equivalent transverse network of unilateral fin line.

The transverse resonance condition in this circuit is found by setting the total susceptance in the plane of the fins equal to zero. The lowest root of this expression describes the guided wavelength of the fundamental mode in the fin line. Again, the following abbreviations will be used:

$$d/b = t \; ; \quad s/b = w \; ; \quad b/a = z \; ; \quad \lambda/\lambda_g = p$$

$$b/\lambda = x \; ; \quad [\varepsilon_r - p^2]^{1/2} = u \; ; \quad [1 - p^2]^{1/2} = v$$

$$\lambda_{to} = x/v \; ; \quad \lambda_{tl} = x/u \; ; \quad b/\lambda_{to} = vx$$

$$b/\lambda_{tl} = ux$$

where λ = free-space wavelength

λ_g = guided wavelength in the fin line

λ_{to} = transverse guided wavelength in air section

λ_{tl} = transverse guided wavelength in dielectric filled section of the fin line.

The transverse resonance condition is then for the unilateral fin line:

$$-(u/v)\ \cot\{2\pi wux + \tan^{-1}[(u/v)\ \tan 2\pi vx(\frac{1}{2z} - w)]\}$$

$$+ F\left[\frac{B_1}{Y_1}\frac{1}{ux}\right](\frac{u}{v})^2\ vx + \left[\frac{B_o}{Y_o}\frac{1}{vx}\right] vx - \cot\frac{\pi vx}{z} = 0 \qquad (3.35)$$

for $p < 1$.

In this expression, the term $(B_o/Y_o)/vx$ is one half the value of $(B/Y_o)/vx$ given by eqn. (3.9). $(B_1/Y_1)/ux$ is the slot susceptance for infinite s and is one half the value of $(B/Y_o)/vx$ provided that vx therein is replaced by ux.

It should be noted that due to the presence of the dielectric sheet, the guided wavelength λ_g in the fin line can become shorter than the free space wavelength λ. This leads to imaginary values for v, signifying that the air-filled part of the fin line is below cutoff. For the evaluation of the root, it is convenient to define v as follows:

$$v = |1 - p^2|^{1/2} \qquad (3.36)$$

and to replace, for $p > 1$, the term v by $-jv$ in eqn. (3.35) and also in the expression for $(B/Y_o)/vx$. This results in the following resonance condition:

$$-(u/v)\ \cot\{2\pi wux + \tan^{-1}[(u/v)\ \tanh 2\pi vx\ (\frac{1}{2z} - w)]\}$$

$$+ F\left[\frac{B_1}{Y_1}\frac{1}{ux}\right](\frac{u}{v})^2\ vx - \left[\frac{B_o}{Y_o}\frac{1}{vx}\right] vx - \coth\frac{\pi vx}{z} = 0 \qquad (3.37)$$

for $p > 1$ and $v = |1 - p^2|^{1/2}$

3.3.2 Insulated Fin Line

The insulated fin line structure is symmetrical about the fins, as can be seen from Fig. 2.1c. Only one half of the transverse equivalent circuit (Fig. 3.7) needs therefore to be considered.

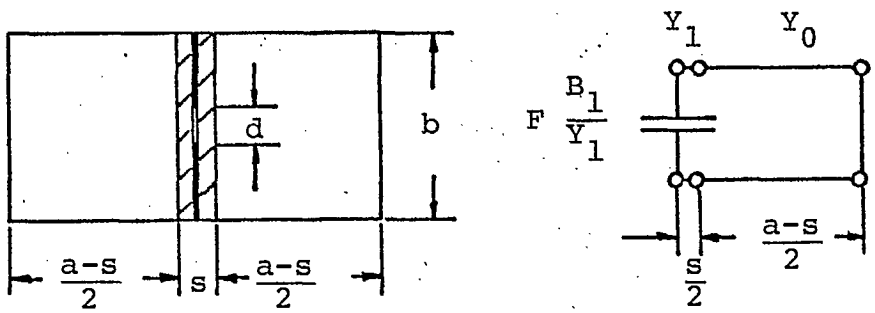

Fig. 3.7 Transverse equivalent circuit of insulated fin line. A magnetic wall is inserted in the plane of the fins.

The situation resembles that of the left hand side of unilateral fin line. The transverse resonant condition states that the total admittance at the plane of the magnetic wall must be zero:

$$- (u/v) \cot \{\pi wux + \tan^{-1} [(u/v) \tan \pi vx (\frac{1}{z} - w)]\} + F\left[\frac{B_1}{Y_1} \frac{1}{ux}\right] (\frac{u}{v})^2 vx = 0 \qquad (3.38)$$

for $p < 1$.

All expressions in eqn. (3.38) are the same as those in the resonance condition of the unilateral fin line.

Again, values for v can become imaginary, and in the above equation, v must be replaced by -jv for $\lambda > \lambda_g$. We obtain:

$$- (u/v) \cot \{\pi wux + \tan^{-1} [(u/v) \tanh \pi vx (\frac{1}{z} - w)]\}$$

$$+ F \left[\frac{B_1}{Y_1} \frac{1}{ux}\right] (\frac{u}{v})^2 vx = 0 \qquad (3.39)$$

for $p > 1$ and $v = |1 - p^2|^{1/2}$.

Appendix A.2 presents a Fortran IV program to evaluate these transverse resonance conditions.

3.3.3 Bilateral Fin Line

In the case of the bilateral fin line, the situation is still symmetrial about a central magnetic wall, but quite different from the two foregoing cases in that the dielectric sheet is now bounded by the fins on one side and by a magnetic wall rather than a section of empty waveguide on the other side. The correction factor for the slot susceptance on the dielectric side is then

$$G = \frac{\text{Slot Susceptance for finite sheet bounded by a magnetic wall}}{\text{Slot Susceptance for infinite sheet}}$$

A diagram for G is given in section 3.3.4.

The resulting transverse equivalent circuit is that of Fig. 3.8. Again, only one half of the circuit needs to be considered for symmetry reasons.

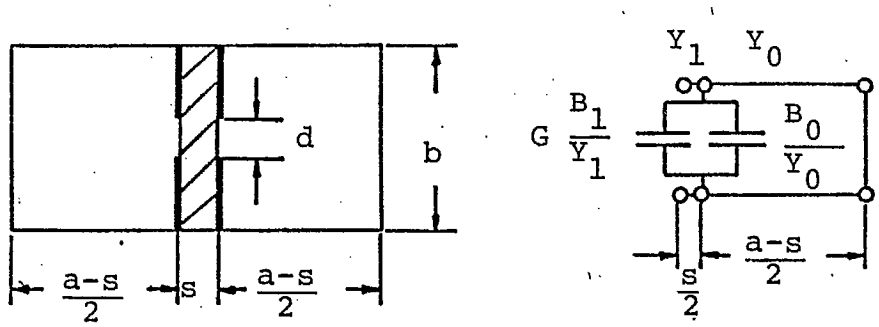

Fig. 3.8 Transverse equivalent circuit of bilateral fin line. A magnetic wall is inserted at the centre of the dielectric sheet.

The transverse resonant condition states that the total admittance in the plane of the fins is equal to zero. With our usual abbreviations, this reads:

$$(u/v)\ \tan \pi wux + G\left[\frac{B_1}{Y_1}\ \frac{1}{ux}\right]\left(\frac{u}{v}\right)^2 vx + \left[\frac{B_o}{Y_o}\ \frac{1}{vx}\right] vx - \cot \pi vx\left(\frac{1}{z} - w\right) = 0 \qquad (3.40)$$

for $p < 0$.

All expressions in the above equation are the same as those in the resonance condition of the unilateral fin line.

For $\lambda > \lambda_g$, v must be replaced by $-jv$ in eqn. (3.40) as well as in the expression for $(B_o/Y_o)/vx$. We obtain:

$$(u/v)\ \tan \pi wux + G\left[\frac{B_1}{Y_1}\ \frac{1}{ux}\right]\left(\frac{u}{v}\right)^2 vx - \left[\frac{B_o}{Y_o}\ \frac{1}{vx}\right] vx - \coth \pi vx\ \left(\frac{1}{z} - w\right) = 0 \qquad (3.41)$$

for $p > 1$ and $v = |1 - p^2|^{1/2}$.

Appendix A.2 presents a Fortran IV program for solving these resonance conditions for bilateral fin line.

3.3.4 Evaluation of the correction factors F and G

In his treatment of the slot line, Cohn [7] has developed expressions for the admittance of a slot backed by a dielectric sheet, which can be used directly in the evaluation of the correction factors F and G. Fig. 3.9 compares the structure characteristic of unilateral fin line with Cohn's model of the slot line. The only difference resides in the a - dimension which is infinite in the case of the slot line.

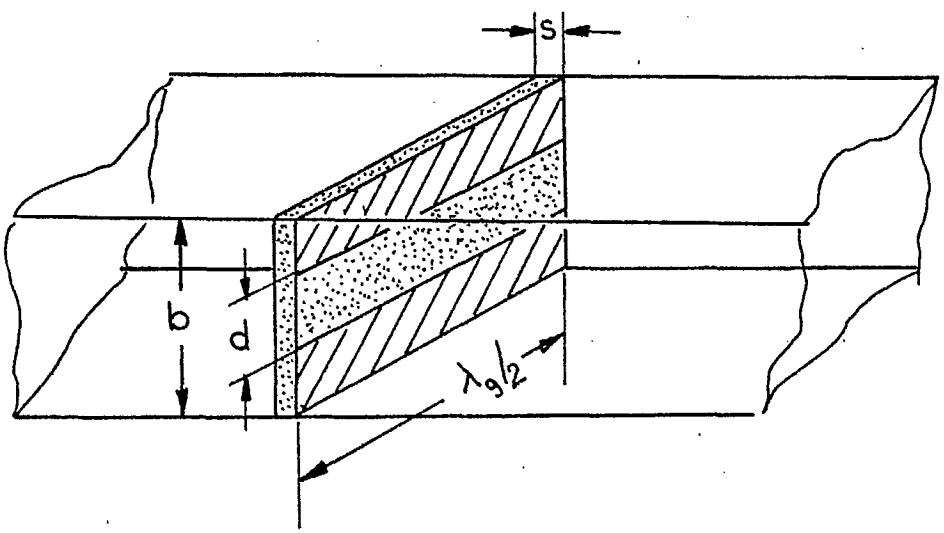

Fig. 3.9a Cohn's model of a slot line resonator. The guided wavelength is λ_g. Rectangular waveguide cut off at frequency of resonance.

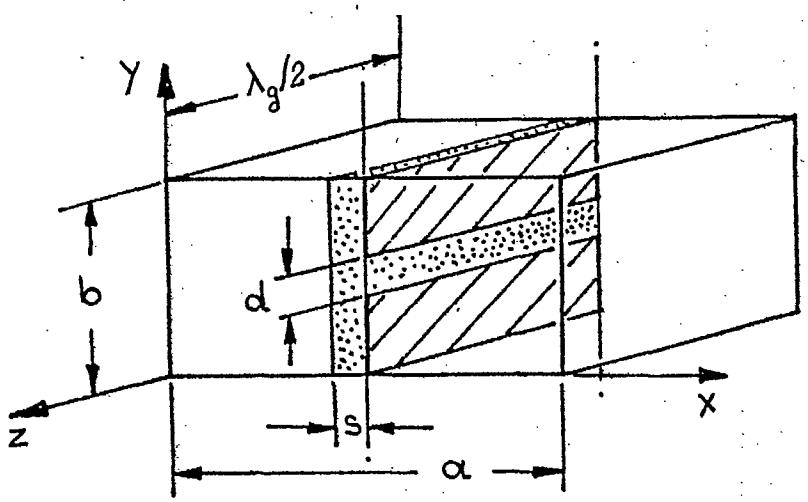

Fig. 3.9b Fin line resonator of identical fin structure as Cohns slot line, but rectangular waveguide short-circuited at + a/z.

In both cases shown in Fig. 3.9, the transverse susceptance of the dielectric-covered fins is the same, provided that a is large with respect to b ($a \geq 2b$). We can thus use the formula given by Cohn [7] for calculating the slot line susceptance. By letting w tend towards infinity, the slot susceptance for infinite sheet thickness is obtained. Finally, the correction factor F is just the ratio of the above two susceptances.

To evaluate the factor G used in the case of bilateral fin line, the expression for the slot susceptance must be slightly modified to take into account the presence of the magnetic wall at the other side of the dielectric sheet.

All these rather cumbersome expressions have been programmed and evaluated on a computer. It turns out that the values for F and G are related in the following way:

$$F = (v/u)^2 + G\,[1 - (v/u)^2] \tag{3.41}$$

if $p < 1$

If p increases beyond unity, v must be replaced by $-jv$, where $v = |1 - p^2|^{1/2}$, yielding

$$F = -(v/u)^2 + G\,[1 + (v/u)^2] \tag{3.42}$$

if $p > 1$

The correction factor G has been computed for several values of t and is presented as a function of the parameter w in Fig. 3.10. This diagram is used in the evaluation of fin line parameters through the solution of the transverse resonance conditions derived above.

3.4 Characteristic Impedance of Fin Lines

Once the guided wavelength in a fin line structure has been calculated, the characteristic impedance, based on a voltage-to-current ratio, can be found by dividing the characteristic impedance of a ridged waveguide of identical dimensions by the variable $p = \lambda/\lambda_g$. This method is sufficiently accurate for small values of ε_r and narrow dielectric sheets.

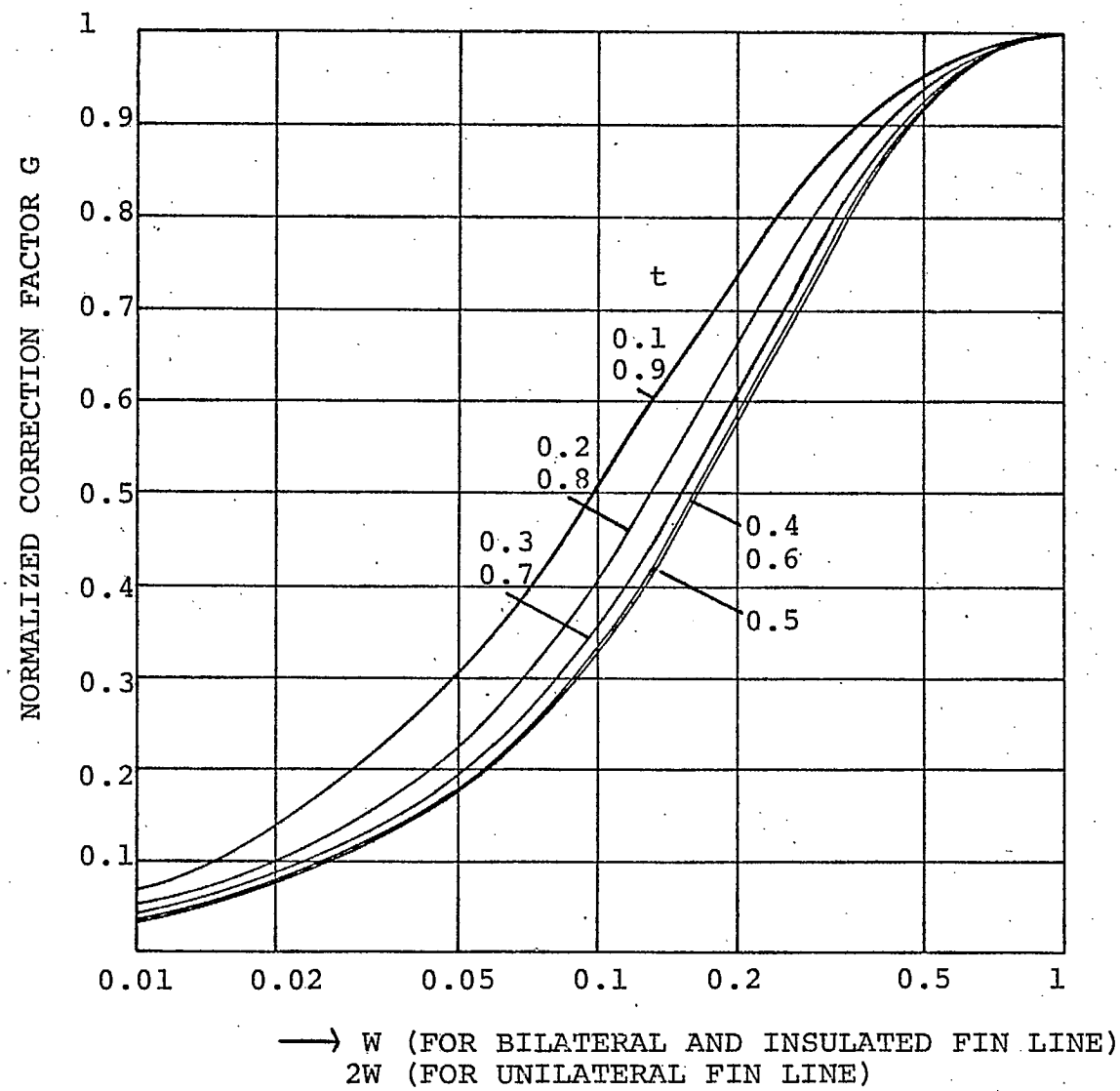

Fig. 3.10 Normalized correction factor G vs w for the evaluation of fin line parameters, with normalized fin spacing t as parameter.

In the case of unilateral and bilateral fin lines, the impedance chosen is that of a ridged waveguide with centered ridge of zero thickness. (Eqn. 3.34 with w = o).

In the case of bilateral fin line, the equivalent ridged waveguide has a ridge of thickness s. (Eqn. 3.34)

In order to evaluate the characteristic impedance of a fin line, the cut-off frequency of the commesurate ridged waveguide must be found first by solving eqn. (3.11) for p = 0.

We thus obtain the normalized cut-off frequency vx of the ridged waveguide.

This value for vx is then introduced into eqn. (3.34) to calculate

$$Z_{o\infty} = Z_o p = \frac{120\pi^2 \, vx}{\frac{1}{t}\sin \pi wvx + [(\frac{B_o}{Y_o}\frac{1}{vx})\, vx + \tan\frac{\pi vx}{2}(\frac{1}{z}-w)]\cos \pi wvx} \quad (3.43)$$

Finally, the fin line impedance is

$$Z_{Fin} = \frac{Z_{o\infty}}{p} \quad (3.44)$$

where p is now the value of λ/λ_g in the fin line, obtained from the solution of one of the transverse resonance conditions (3.35), (3.36), (3.38), (3.39), (3.40) or (3.41).

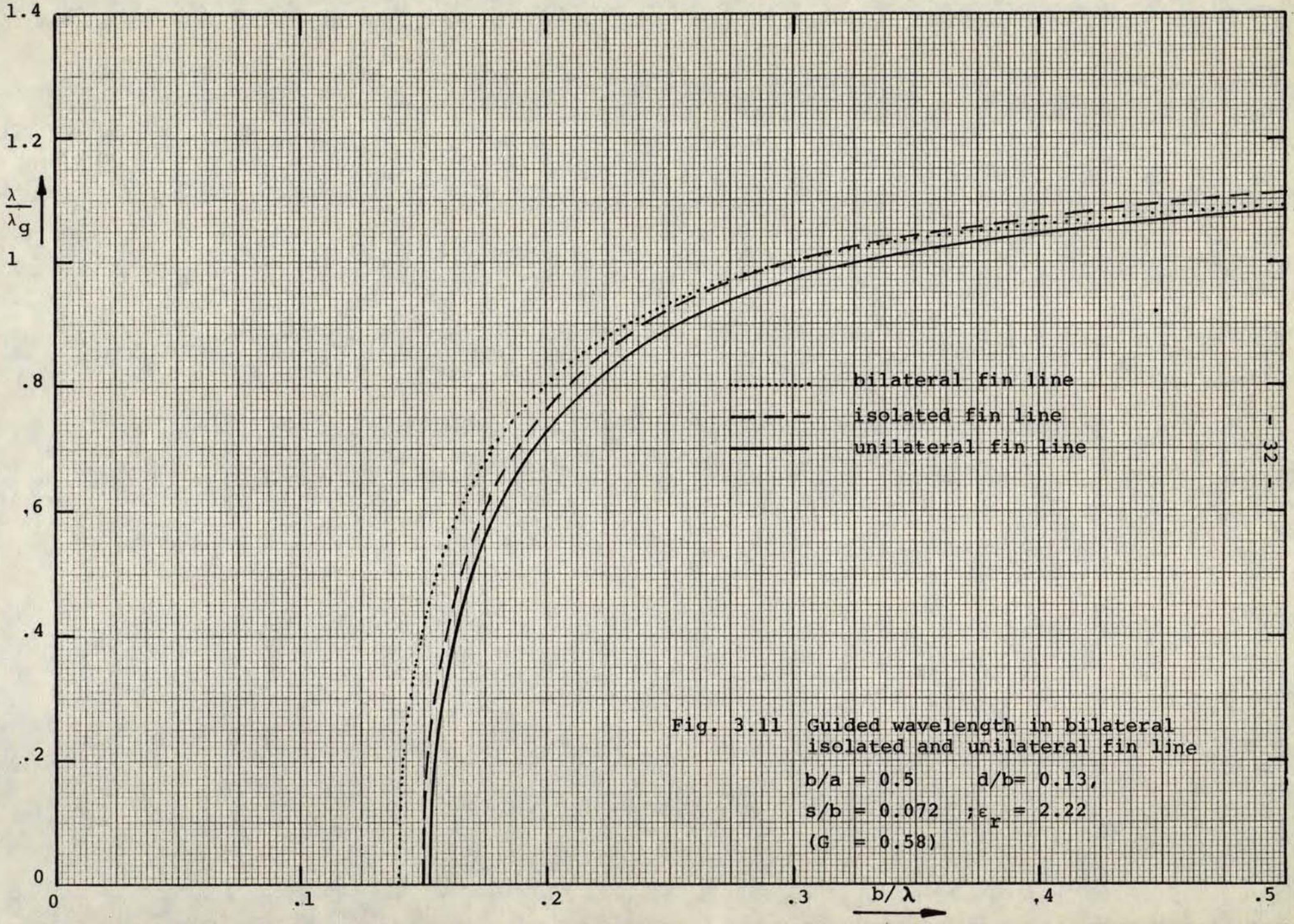

Fig. 3.11 Guided wavelength in bilateral isolated and unilateral fin line

b/a = 0.5 d/b= 0.13,

s/b = 0.072 ; ε_r = 2.22

(G = 0.58)

4) ATTENUATION IN FIN LINES

Saad and Begemann [7] have published expressions for the attenuation constant of fin lines, based on an approximate solution of the transverse resonance condition. It is considered that these expressions are sufficiently accurate for most applications, and no alternative formulae will be given here. From graphs presented by Saad and Begemann [7] it appears that the attenuation constant in fin lines is typically 0.05-0.1 Nep/m (0.43-0.87 db/m).

5) CONCLUSION

An original method for the evaluation of fin line parameters has been developed in the present report. This method combines accuracy with ease of application and versatility if use of the computer programs (given in the Appendix 2) is made.

The solutions obtained for the guided wavelength in unilateral fin line have been compared with measured data by Meier [9] and with numerical solutions published by Hofmann [2]. Excellent agreement is evident from Fig. 3 of Appendix 3 attached to the present report.

As far as the characteristic impedance of ridged waveguide is concerned, results agree very well with Chen's [5] results.

There is no experimental verification as yet of the values for the characteristic impedance for fin lines. However, it is suggested that the values calculated for Z_0 by Hofmann [2] are too high, and the expressions given in the present report appear more realistic. Further study is needed to explain these discrepancies.

A recent attempt has been made by the author to apply the TLM-method to the fin line problem. Some preliminary results of this study are presented in Appendix 4. This Appendix also contains data on an impedance step in ridged waveguide, calculated with the TLM-method. This approach seems quite promising and will be explored further with the aim of characterizing fin line discontinuities which cannot be treated with other methods because of the very difficult field problems they pose.

It is suggested that the results and methods presented here constitute a basis for accurate and versatile fin line design.

6) REFERENCES

[1] S.B. Cohn: "Slot Line on a Dielectric Substrate", IEEE-Transactions on Microwave Theory and Techniques, Vol. MTT-17, No. 10, Oct. 1969, pp. 768-778.

[2] H. Hofmann: "Dispension of Planar Waveguides for Millimeter-Wave Application", Archiv. F. Elektron. u. Ubertrag.-Techn., Vol. 31, No. 1, Jan. 1977, pp. 40-44.

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[4] S. Hopfer: "The Design of Ridged Waveguides" IRE-Trans. Microwave Theory and Techniques, Vol. MTT-3, pp. 20-29, Oct. 1955.

[5] T.-S. Chen: "Calculation of the Parameters of Ridge Waveguides", IRE-Trans.-Microwave Theory and Techniques, Vol. MTT-5, pp. 12-17, Jan. 1957.

[6] N. Marcuvitz: "Waveguide Handbook", MIT Radiation Laboratory Series, No. 10, Boston Technical Publishers, Inc., 1964.

[7] S.B. Cohn: "Slot Line on a Dielectric Substrate", IEEE-Trans. on Microwave Theory and Techniques, Vol. MTT-17, No. 10, Oct. 1969, pp. 768-778.

[8] A.M.K. Saad, G. Begemann: "Electrical Performance of Fin Lines of Various Configurations", Microwaves, Optics and Acoustics, Vol. 1, No. 2, Jan. 1977, pp. 81-88.

[9] P.J. Meier: "Integrated Fin Line Millimeter Components, IEEE-Trans. on Microwave Theory and Techniques, Vol. MTT-22, No. 12, Dec. 1974, pp. 1209-16.

[10] W.J.R. Hoefer: "Fin Line Design Made Easy", 1978 IEEE-MTT-S Intl. Microwave Symposium, June 27-29, Ottawa, Ont., Canada (See copy Appendix 3)

[11] W.J.R. Hoefer, A. Ros: "Fin Line Parameters Calculated With the TLM-Method", 1979 IEEE-MTT-S Intl. Microwave Symposium, April 20-May 2, Orlando, Florida, U.S.A. (see copy Appendix 4)

APPENDIX I

Appendix 1 presents several methods and programs to determine the guided wavelength and characteristic impedance in a rectangular waveguide with central longitudinal ridges of zero and finite thickness.

Fig. A1.1 shows the geometry of such a waveguide containing central fins of zero thickness.

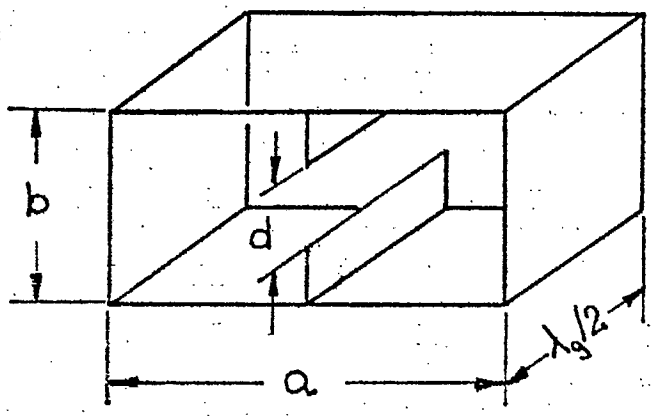


Fig. A1.1

The transverse resonance condition is given in eqn. (3.8) and can be solved either graphically or using a computer. This appendix shows how to solve this condition using a

A.1.1 Graphical Method

A.1.2 Program for HP 67/97 Calculator

a) Susceptance taken from Waveguide Handbook

b) Susceptance programmed directly

A.1.3 Fortran IV program with printing of diagram λ/λ_g vs b/λ.

A.1.1. Graphical Method

In the graphical solution, both terms of eqn. (3.8) are presented graphically and the point of intersection of the curves is obtained.

$$\frac{1}{2}\left[\frac{B}{Y_o}\ \frac{1}{vx}\right]\ vx = \cot\ (\pi vx/z) \qquad (3.8)$$

The value for $\left[\frac{B}{Y_o}\frac{1}{vx}\right]$ is taken from Waveguide Handbook [6]. In practically all cases, the curve for vx = 0 gives sufficient accuracy (see Fig. A.1.2), but $\frac{1}{2}\left[\frac{B}{Y_o}\ \frac{1}{vx}\right]$ can obviously be constructed with better accuracy to reflect the second order influence of the parameter vx.

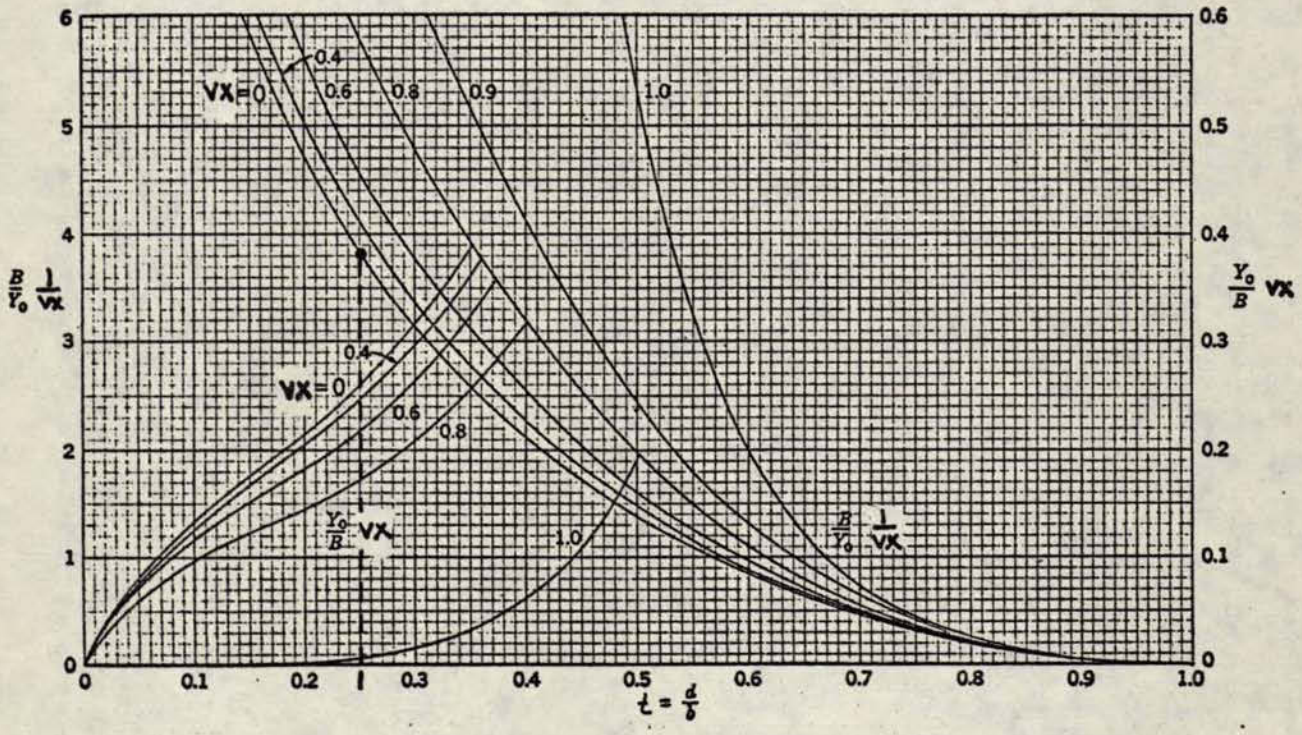


Fig. A.1.2 Susceptance of capacitive windows in rectangular guide. (From the "Waveguide Handbook" [6])

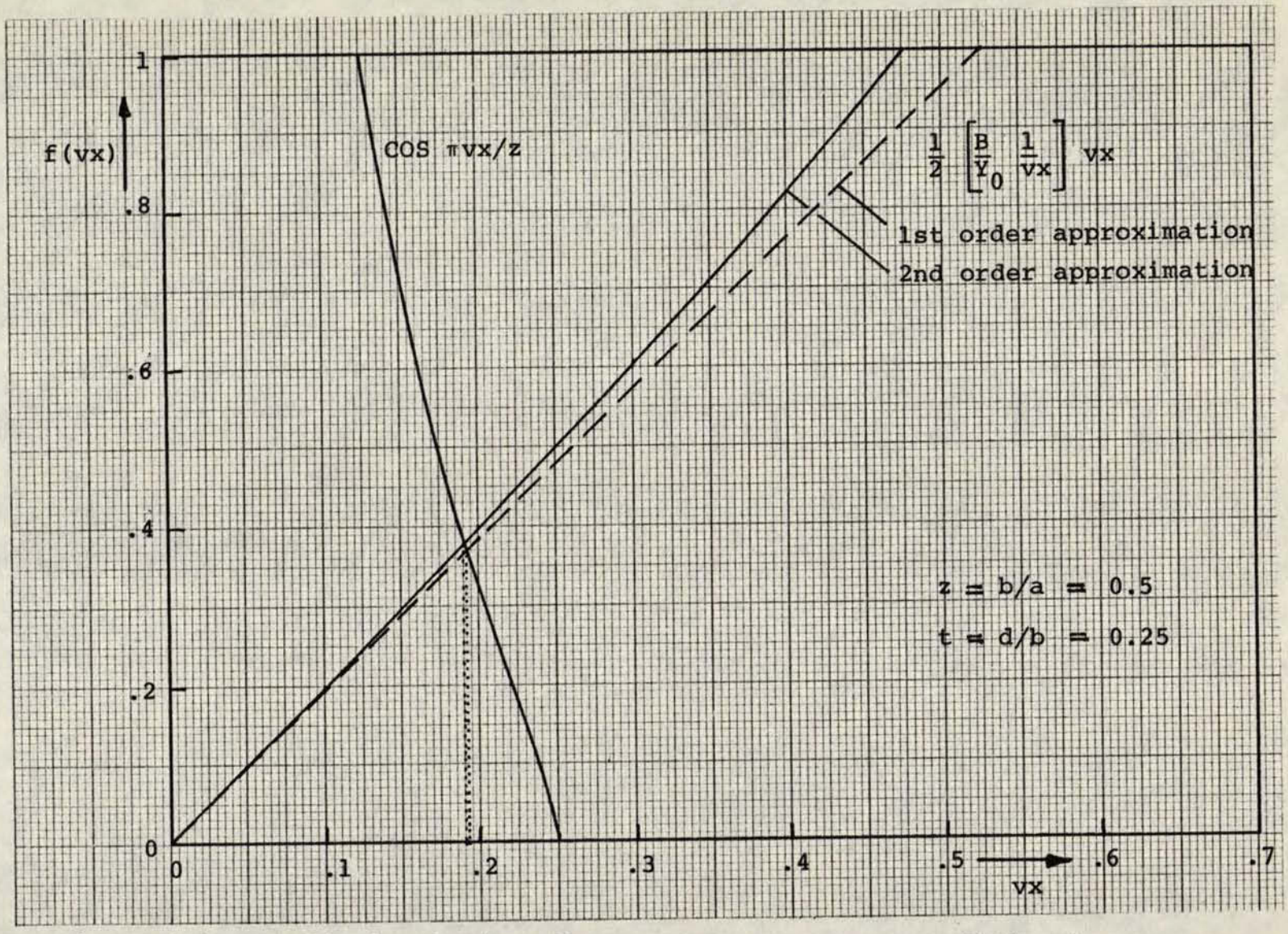


Fig. A 1.3 Graphical solution of transverse resonance condition in rectangular waveguide with longitudinal fins of zero thickness

If $z = b/a = 0.5$ and $t = d/b = 0.25$, Fig. A.1.3 shows the graphical solution of the transverse resonance condition, eqn. (3.8).

First, the value for $[\frac{B}{Y_o} \frac{1}{vx}]$ is found from Fig. A.1.2.

$$t = d/b = 0.25 \rightarrow [\frac{B}{Y_o} \frac{1}{vx}] = 3.8$$

This value gives an indication for the ordinate scale to be used in Fig. A.1.3.

If $z = 0.5$, a value of 1 for vx corresponds to one full period of the cos $(\pi vx/z)$-function. The graphical solution can now be drawn (Fig. A.1.3)

One obtains

$vx = .192$ based on the susceptance value for $vx = 0$ (dotted curve)

and $vx = .190$ based on a more accurate susceptance value (solid curve)

On the basis of this result, the function $p = f(x)$ can now be drawn since $v^2 = 1-p^2$.

Thus:

$$p = [1-(.190/x)^2]^{1/2}$$

This equation is represented graphically in Fig. 3.2 (see curve with the parameter $d/b = 0.25$)

There are also intersections of both functions for higher values of vx. These solutions are characteristic of higher modes

of the TE_{n0}-type with odd n, i.e. n = 3,5,7,9... It is obvious from Fig. A.1.3, that for the higher order solutions the exact value for the discontinuity susceptance must be used (solid line),

A.1.2 Programs for HP 67/97 Calculator

A.1.2.1. Rectangular Waveguide with Centered Fin of Zero Thickness

(susceptance value $[\frac{B}{Y_o} \frac{1}{vx}]$ taken from the Waveguide Handbook [6])

The standard pack of programs for programmable calculators HP 67/97 contains a routine called "Calculus and roots of f(x)". To calculate the root of a function f(x), this function is simply keyed in starting at program step 113. The variable to be determined (root) is placed into storage register R0.

The transverse resonance condition to be evaluated is again

$$\frac{1}{2}[\frac{B}{Y_o} \frac{1}{vx}]\ vx - \cot \pi vx/z = f(x) = 0 \qquad \text{(A1)}$$

Any variable in this expression could be placed in storage register R_I and solved for when all other parameters are fixed. In the following pages, the following three programs are described:

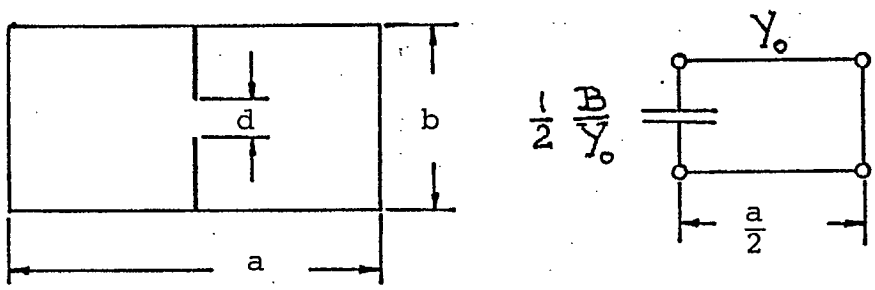
P-1: x is calculated when p, z and t are given
P-2: p is calculated when x, z and t are given
P-3: x is calculated when $px = b/\lambda_g$, z and t are given

PROPAGATION IN RECTANGULAR WAVEGUIDE WITH CENTERED RIDGE (FIN) OF ZERO THICKNESS

1 RW - Centr. Thin Fin Enter Susceptance

(HP 67/97)

This series of programs calculates the roots of the transverse resonant condition f(x) of the following structure:



Rectangular waveguide with centered ridge of zero thickness and equivalent transverse network.

$$f(x) = \frac{1}{2}\left[\frac{B}{Y_o}\ \frac{1}{vx}\right] vx - \cot\ \pi vx/z = 0 \qquad (1)$$

where $p = \lambda/\lambda_g$; $x = b/\lambda$;

$v = [1-p^2]^{1/2}$; $z = b/a$

λ = free space wavelength, λ_g = guided wavelength

The Standard Pack Routine "Calculus and Roots of f(x)" is used to find the root. Any of the above variables can be placed in the R_0-register. The function f(x) is programmed starting at step 113.

REMARKS:

The value for the discontinuity susceptance $\left[\frac{B}{Y_o} \quad \frac{1}{vx}\right]$ must be taken from the "Waveguide Handbook" by Marcuvitz, Fig. 5.1-4. In this figure, b/λ_g must be replaced by vx.

Since the first root of f(x) is in all practical cases situated between vx = 0 and vx = 0.4, a good approximation(within a few percent) is obtained by selecting the susceptance for vx=0.

If a better accuracy is desired, an improved value for $\left[\frac{B}{Y_o} \quad \frac{1}{vx}\right]$ corresponding to the just calculated vx is obtained from Fig. 5.1-4 of the "Waveguide Handbook", and introduced into Register R_3. The root obtained with this value should be sufficiently accurate for almost all design purposes.

The second and higher order roots are characteristic of higher order modes of the TE_{no}-type with odd n, i.e. n=3,5,7,9... For these higer modes, repeated refinement of the susceptance value must be made to obtain satisfactory accuracy.

The program can be used to evaluate other structures exhibiting a shunt susceptance in the centre of a waveguide.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 of HP-card "Calculus and roots of f(x)"			
2	Select one of programs P-1, P-2 or P-3 depending on choice of root			
3	Key in selected program or merge from magnetic card, starting at step 113 Select function nr. 1.	1	see HP instruc. A	1.00
4	Choose value for $(B/Y_o)/vx$ corresponding to vx=0 and for given value d/b in Waveguide Handbook and store in register R_3	$\frac{B}{Y_o}$ $\frac{1}{vx}$	STO 3	$\frac{B}{Y_o}$ $\frac{1}{vx}$
5	Store z = b/a in register R_2 and either p, x or px in register R_1, depending on program chosen. Note: p < 1	z p,x or px	STO 2 STO 1	z p,x or px
6	Key in guess and calculate root (x=0.2 is a good init. guess)	Guess	E	root
7	For higher accuracy, recall value obtained for vx		RCL 4	vx
8	Choose new value for $(B/Y_o)/vx$ corresponding to new vx and for given value d/b in Waveguide Handbook, and store in R 3	$\frac{B}{Y_o}$ $\frac{1}{vx}$	STO 3	$\frac{B}{Y_o}$ $\frac{1}{vx}$
9	Calculate new root starting with previous root		RCL 0 E	root
10	for higher accuracy, return to step 7			

PROPAGATION IN RECTANGULAR WAVEGUIDE WITH CENTERED RIDGE (FIN) OF ZERO THICKNESS

RW-Centr. Thin Fin P-1 Enter Susceptance

(HP 67/97)

This program calculates x if p,z and t are given. The susceptance value must be found from the "Waveguide Handbook" by Marcuvitz, Fig. 5.1-4.

PROGRAM:

Steps 000 to 112 see HP-routine "Calculus and Roots of f(x)"

113	*LBL1	21 01	Set to RAD-mode
114	RAD	16-22	
115	1	01	Compute v
116	RCL1	36 01	
117	X^2	53	$v = (1-p^2)^{1/2}$
118	-	-45	
119	√X	54	
120	RCL0	36 00	Compute vx and store in R_4
121	x	-35	
122	STO4	35 04	
123	RCL3	36 03	Compute $\frac{1}{2}\,[(B/Y_0)/vx]\,vx$
124	x	-35	
125	2	02	
126	÷	-24	
127	RCL4	36 04	Compute cot πvx/z and subtract from first term
128	PI	16-24	
129	x	-35	
130	RCL2	36 02	
131	÷	-24	
132	TAN	43	
133	1/X	52	
134	-	-45	
135	RTN	24	
136	R/S	51	

Registers

	R 0	$x = b/\lambda$
Store	R 1	$p = \lambda/\lambda_g$
	R 2	$z = b/a$
	R 3	$(B/Y_o)/vx$
	R 4	vx

PROPAGATION IN RECTANGULAR WAVEGUIDE WITH CENTERED RIDGE (FIN) OF ZERO THICKNESS

RW-Centr. Thin Fin P-2
Enter Susceptance

(HP 67/97)

This program calculates p if x, z and t are given. The susceptance value must be found from the "Waveguide Handbook" by Marcuvitz, Fig. 5.1-4.

PROGRAM:

Steps 000 to 112 see HP-routine "Calculus and Roots of f(x)"

113	*LBL1	21 01	Set to RAD-mode
114	RAD	16-22	
115	1	01	Compute v
116	RCL0	36 00	$v = (1-p^2)^{1/2}$
117	X²	53	
118	-	-45	
119	√X	54	
120	RCL1	36 01	Compute vx and store in R_4
121	×	-35	
122	STO4	35 04	
123	RCL3	36 03	Compute $\frac{1}{2}\,[(B/Y_0)/vx]\,vx$
124	×	-35	
125	2	02	
126	÷	-24	
127	RCL4	36 04	Compute cot πvx/z and subtract from first term
128	Pi	16-24	
129	×	-35	
130	RCL2	36 02	
131	÷	-24	
132	TAN	43	
133	1/X	52	
134	-	-45	
135	RTN	24	
136	R/S	51	

Registers

	R 0	$p=\lambda/\lambda_g$
Store	R 1	$x=b/\lambda$
	R 2	$z=b/a$
	R 3	$(B/Y_o)/vx$
	R 4	vx

PROPAGATION IN RECTANGULAR WAVEGUIDE WITH CENTERED RIDGE (FIN) OF ZERO THICKNESS

RW-Centr. Thin Fin P-3
Enter Susceptance

(HP 67/97)

This program calculates x if px,z and t are given. The susceptance value must be found from the "Waveguide Handbook" by Marcuvitz, Fig. 5.1-4.

PROGRAM:

Steps 000 to 112 see HP-routine "Calculus and Roots of f(x)"

113	*LBL1	21 01	
114	RAD	16-22	Set to RAD-mode
115	1	01	
116	RCL1	36 01	Compute v
117	RCL0	36 00	
118	÷	-24	$v = (1-p^2)^{1/2}$
119	X²	53	
120	-	-45	
121	√X	54	
122	RCL0	36 00	Compute vx and store in R_4
123	x	-35	
124	STO4	35 04	
125	RCL3	36 03	Compute $\frac{1}{2}$ [(B/Y_0)/vx] vx
126	x	-35	
127	.2	02	
128	÷	-24	
129	RCL4	36 04	Compute cot πvx/z and subtract from first term
130	Pi	16-24	
131	x	-35	
132	RCL2	36 02	
133	÷	-24	
134	TAN	43	
135	1/X	52	
136	-	-45	
137	RTN	24	
138	R/S	51	

Registers

	R 0	x
Store	R 1	$px=b/\lambda_g$
	R 2	z=b/a
	R 3	$(B/Y_o)/vx$
	R 4	vx

A.1.2.2. Rectangular Waveguide with Centered Ridge of Finite Thickness s.

The expression for $(B/Y_o)/vx$ given in eqn. 3.9 can be programmed directly, and thus it is not necessary to consult Fig. 5.1-4 of the Waveguide Handbook [6]. However, the computation time will be longer by a factor two.

For the sake of generality, a program for ridged waveguide with centered ridge of finite thickness will be given. The case of zero thickness is obviously included in this program.

Two variations of the program are given, namely

P-1: x is calculated when p, z, s and t are given

P-2: p is calculated when x, z, s and t are given

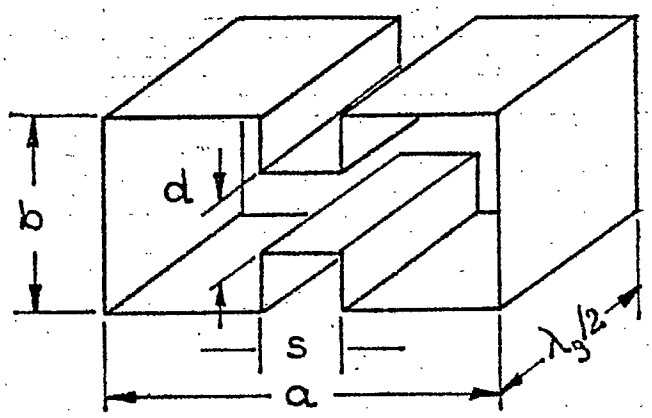
At the same time, the characteristic impedance of such a waveguide can be evaluated using the results of the calculation of the guided wavelength.

PROPAGATION IN RECTANGULAR WAVEGUIDE WITH CENTERED RIDGE OF THICKNESS s

RW - Double Ridged Guide λ_g and Z_0

(HP 67/97)

This series of programs calculates the roots of the transverse resonant condition f(x)=0 and the characteristic impedance of the following structure:



$$f(x) = \frac{1}{t} \tan \pi vxw + \left[\frac{B_o}{Y_o} \frac{1}{vx}\right] vx - \cot \pi vx\left(\frac{1}{z} - w\right) = 0 \qquad (1)$$

where $p = \lambda/\lambda_g$ $\quad x = b/\lambda$; $t = d/b$

$v = [1-p^2]^{1/2}$; $z = b/a$; $w = s/b$

λ = free space wavelength, λ_g = longit. guided wavelength

and
$$\frac{B_o}{Y_o} \frac{1}{vx} = 2 \left[\ln(\csc \pi t/2) + \frac{Q \cos^4 \pi t/2}{1+Q \sin^4 \pi t/2} + \right. \qquad (2)$$

$$\left. + \frac{1}{16}(vx)^2(1-3 \sin^2 \pi t/2)^2 \cos^4 \pi t/2\right]$$

$$\text{with } Q = [1-(vx)^2]^{-1/2} - 1 \qquad (3)$$

The Standard Pack Routine "Calculus and Roots of f(x)" is used to calculate the root. Any of the above variables can be placed in the R_0-register. The function f(x) is programmed starting at step 113.

The characteristic impedance based on a Voltage-to-Current ratio is given by the following expression:

$$Z_0 = Z_{0\infty}/p = \frac{\frac{120\pi}{p} \quad \pi vx}{\frac{1}{t}\sin \pi wvx + [(\frac{B_o}{Y_o}\frac{1}{vx})vx + \tan\frac{\pi vx}{2}(\frac{1}{z} - w)] \cos \pi wvx}$$

and is calculated following the solution of the transverse resonance condition.

Remarks:

The cutoff wavelength is obtained by setting $p = \lambda/\lambda_g$ equal to zero. However, since the characteristic impedance is infinity at cutoff, any attempt to evaluate Z_0 at this wavelength results in an "ERROR". However, the value $p \cdot Z_0 = Z_{0\infty}$ can be obtained by placing the value "one" (1) into the register containing p before evaluating Z_o.

Once the program has been keyed in, it can be stored on a blank magnetic card, including the routine "Calculus and Roots of f(x)". The "Rad" and "DSP" modes will be registered automatically on the card at storage and will not have to be set afterwards.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 of HP-card "Calculus and roots of f(x)			
2	Select one of programs (P-1 or P-2) depending on root to be evaluated			
3	Key in selected program or merge from magnetic card, starting at step 113		see HP instructions	
4	Select "RAD" mode and Display mode (ex:5 decimals)		f RAD DSP 5	
5	Select function no. 1	1	A	1.00000
6	Store value for p or x in R_1, depending on program selected	p or x	STO 1	p or x
7	Store values for parameters z, w and t	z w t	STO 2 STO 3 STO 4	z w t
8	Key in guess and calculate root (0.1 is a good initial guess in most cases)	Guess	E	root
9	Calculate characteristic impedance of lowest mode, Z_0		GSB 2	Z_0
10	To calculate root and Z_0 for different parameters, go to step 6			
11	Values for different terms can be recalled after root and Z_0 have been calculated		RCL 5 RCL 6 RCL 7 RCL 8	vx πvxw $(B_o/Y_o)/vx$ $\pi vx(\frac{1}{z} - w)$

PROPAGATION IN RECTANGULAR WAVEGUIDE WITH CENTERED RIDGE OF THICKNESS s (Double Ridged Waveguide)

RW - Double Ridged Guide λ_g and Z_0	P-1

(HP 67/97)

This program calculates b/λ and the characteristic impedance Z_o of double ridged waveguide if $p = \lambda/\lambda_g$, $z=b/a$, $w=s/b$ and $t=d/b$ are given

Program:

Steps 000 to 112 see HP-routine "Calculus and Roots of f(x)"

113	*LBL1	21 01	
114	1	01	
115	RCL1	36 46	Compute vx
116	RCL1	36 01	and store in R_5
117	X²	53	
118	-	-45	
119	√X	54	
120	RCL0	36 00	
121	×	-35	
122	STO5	35 05	
123	X²	53	
124	-	-45	Compute Q and
125	√X	54	store temporarily
126	1/X	52	in R_6
127	1	01	
128	-	-45	
129	STO6	35 06	
130	PI	16-24	
131	RCL4	36 04	Compute
132	×	-35	
133	2	02	
134	÷	-24	
135	STO7	35 07	
136	COS	42	$(B_o/Y_0)/vx$
137	X²	53	
138	STO8	35 08	
139	X²	53	
140	×	-35	R_7
141	RCL7	36 07	
142	SIN	41	
143	STO7	35 07	
144	X²	53	
145	X²	53	

Registers:

R_0	x	Guess
1	p	Store
2	z	
3	w	
4	t	
5	vx	
6	πwvx	
7	$(B_o/Y_0)/vx$	
8	$\pi vx(\frac{1}{z} - w)$	

146	RCL6	36 06	
147	×	-35	
148	1	01	
149	+	-55	
150	÷	-24	
151	RCL7	36 07	
152	1/X	52	
153	LN	32	
154	+	-55	
155	RCL7	36 07	
156	X²	53	
157	3	03	
158	×	-35	
159	1	01	
160	-	-45	
161	RCL5	36 05	
162	×	-35	
163	RCL8	36 08	
164	×	-35	
165	4	04	
166	÷	-24	
167	X²	53	
168	+	-55	
169	2	02	
170	×	-35	
171	STO7	35 07	
172	RCL5	36 05	Compute πvx and store temporarily in R_8
173	×	-35	
174	RCL5	36 05	
175	Pi	16-24	
176	×	-35	
177	STO8	35 06	
178	RCL3	36 03	
179	×	-35	πwvx in R_6
180	STO6	35 06	
181	TAN	43	
182	RCL4	36 04	Compute remainder of f(x)
183	÷	-24	
184	+	-55	
185	RCL2	36 02	
186	1/X	52	
187	RCL3	36 03	
188	-	-45	
189	RCL8	36 08	
190	×	-35	
191	STO8	35 08	
192	TAN	43	
193	1/X	52	
194	-	-45	
195	RTN	24	
196	*LBL2	21 02	Compute Z_0
197	RCL5	36 05	
198	RCL5	36 05	
199	RCL7	36 07	
200	×	-35	
201	RCL8	36 08	
202	2	02	
203	÷	-24	
204	TAN	43	
205	+	-55	
206	RCL6	36 06	
207	COS	42	
208	×	-35	
209	RCL6	36 06	
210	SIN	41	
211	RCL4	36 04	
212	÷	-24	
213	+	-55	
214	÷	-24	
215	RCL1	36 01	
216	÷	-24	
217	3	03	
218	7	07	
219	7	07	
220	×	-35	
221	Pi	16-24	
222	×	-35	Z_0
223	RTN	24	
224	R/S	51	

PROPAGATION IN RECTANGULAR WAVEGUIDE WITH CENTERED RIDGE OF THICKNESS s (Double Ridged Waveguide)

RW - Double Ridged Guide	
λ_g and Z_0	P-2

(HP 67/97)

This program calculates λ/λ_g and the characteristic impedance Z_o of double ridged waveguide if $x = b/\lambda$, $z=b/a$, $w=s/b$ and $t=d/b$ are given

Program:

Steps 000 to 112 see HP-routine "Calculus and Roots of f(x)"

113	*LBL1	21 01	
114	1	01	
115	RCL1	36 46	Compute vx
116	RCL0	36 00	and store in R_5
117	X²	53	
118	-	-45	
119	√X	54	
120	RCL1	36 01	
121	x	-35	
122	STO5	35 05	
123	X²	53	
124	-	-45	Compute Q and
125	√X	54	store temporarily
126	1/X	52	in R_6
127	1	01	
128	-	-45	
129	STO6	35 06	
130	Pi	16-24	
131	RCL4	36 04	Compute
132	x	-35	
133	2	02	
134	÷	-24	
135	STO7	35 07	
136	COS	42	$(B_o/Y_0)/vx$
137	X²	53	
138	STO8	35 08	
139	X²	53	
140	x	-35	
141	RCL7	36 07	R_7
142	SIN	41	
143	STO7	35 07	
144	X²	53	
145	X²	53	

Registers:

R_0	p	Guess
1	x	Store
2	z	Store
3	w	Store
4	t	Store
5	vx	
6	πwvx	
7	$(B_o/Y_0)/vx$	
8	$\pi vx(\frac{1}{z} - w)$	

146	RCL6	36 06	
147	×	-35	
148	1	01	
149	+	-55	
150	÷	-24	
151	RCL7	36 07	
152	1/X	52	
153	LN	32	
154	+	-55	
155	RCL7	36 07	
156	X²	53	
157	3	03	
158	×	-35	
159	1	01	
160	-	-45	
161	RCL5	36 05	
162	×	-35	
163	RCL8	36 08	
164	×	-35	
165	4	04	
166	÷	-24	
167	X²	53	
168	+	-55	
169	2	02	
170	×	-35	
171	STO7	35 07	Compute πvx and store temporarily in R_8
172	RCL5	36 05	
173	×	-35	
174	RCL5	36 05	
175	PI	16-24	
176	×	-35	
177	STO8	35 08	
178	RCL3	36 03	πwvx in R_6
179	×	-35	
180	STO6	35 06	
181	TAN	43	Compute remainder of f(x)
182	RCL4	36 04	
183	÷	-24	
184	+	-55	
185	RCL2	36 02	
186	1/X	52	
187	RCL3	36 03	
188	-	-45	
189	RCL8	36 08	
190	×	-35	
191	STO8	35 08	
192	TAN	43	
193	1/X	52	
194	-	-45	
195	RTN	24	
196	*LBL2	21 02	Compute z_0
197	RCL5	36 05	
198	RCL5	36 05	
199	RCL7	36 07	
200	×	-35	
201	RCL8	36 08	
202	2	02	
203	÷	-24	
204	TAN	43	
205	+	-55	
206	RCL6	36 06	
207	COS	42	
208	×	-35	
209	RCL6	36 06	
210	SIN	41	
211	RCL4	36 04	
212	÷	-24	
213	+	-55	
214	÷	-24	
215	RCL0	36 00	
216	÷	-24	
217	3	03	
218	7	07	
219	7	07	
220	×	-35	
221	PI	16-24	
222	×	-35	z_0
223	RTN	24	
224	R/S	51	

APPENDIX 2

Appendix 2 presents Fortran IV programs to evaluate the guided wavelength and the characteristic impedance of unilateral, bilateral and insulated fin line. They are based on the transverse resonance conditions and the expression for the characteristic impedance of ridged waveguide derived in the present report.

The program for unilateral fin line contains an additional routine that plots the normalized wavelength λ/λ_g vs the normalized frequency b/λ. The same routine may also be used for the other programs if desired.

In addition to the geometrical dimensions and the dielectric constant ε_r of the substrate material, the correction factor G must be input. G is obtained as a function of $w=s/b$ and $t=d/b$ from Fig. 3.10, p. 30 of the present report.

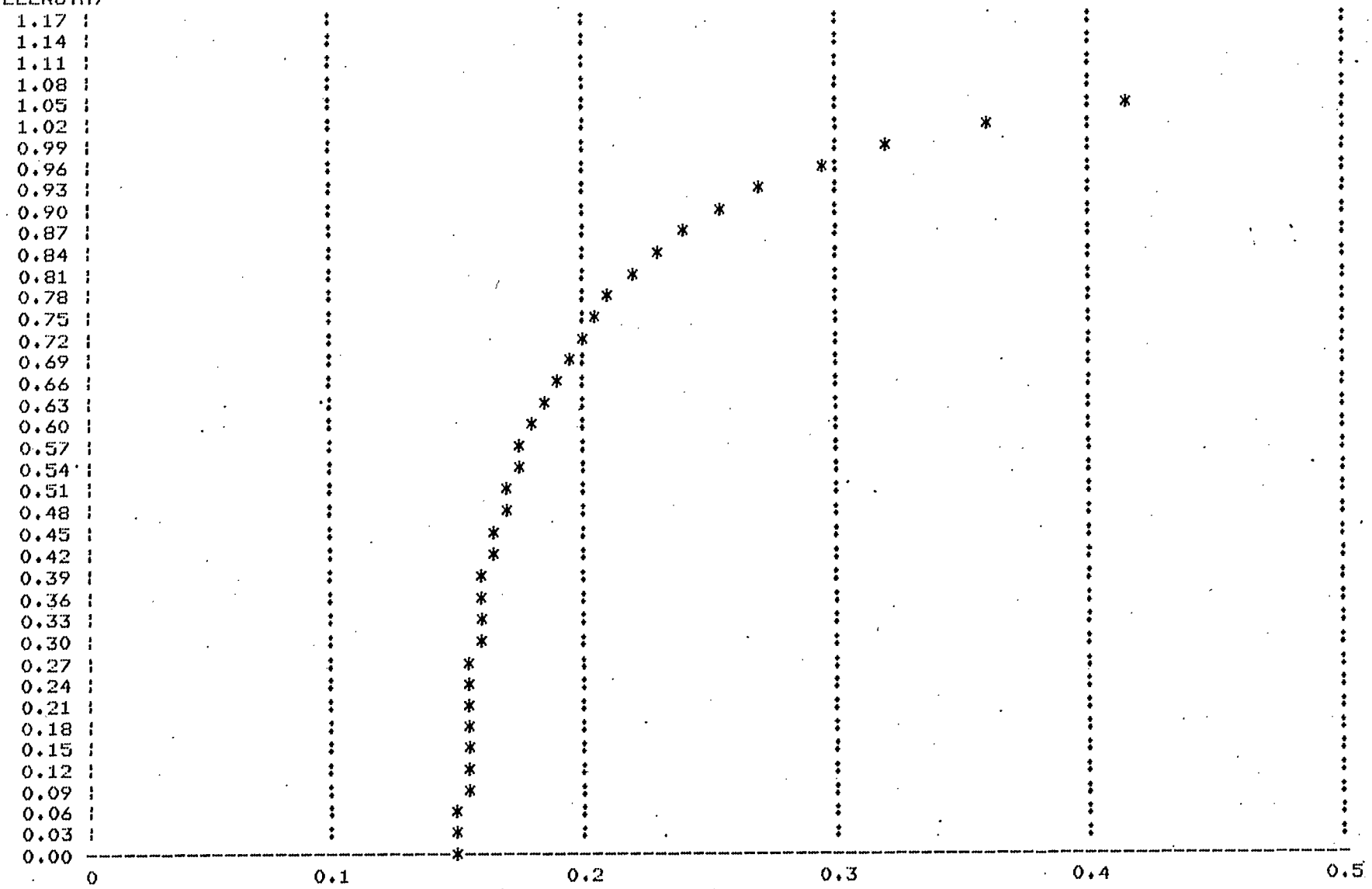
```
C
C     A.2.1
C
C     UNILATERAL FIN LINE
C          FORTRAN
C
C
C     FUNTION: THIS SUBROUTINE CALCULATES AND DRAWS THE GUIDED
C              WAVELENGTH IN UNILATERAL FIN LINE
C
C
C
C     INPUTS: Z(ASPECT RATIO,B/A); T(NORMALIZED GAP,D/B);W(NORMALIZED
C             DIELECTRIC THICKNESS,S/B); ER(DIELECTRIC CONSTANT);
C             G(NORMALIZED CORRECTION FACTOR)
C
C
C
      DIMENSION A(40,100),B(40)
      DIMENSION P(40),X(40),ZF(40)
      INTEGER DOT,HYPHEN,STAR,BLANK,DASH
      INTEGER A,IX,JX
      REAL XP,B,XI,ZO,ZF
      REAL AO,C,E,F,G,P,S,T,U,V,W,X,Z,DI,DO,ER,QI,QO,PI
      REAL A1,A2,A3,A4
      DATA DOT,HYPHEN,DASH/':','-','!'/
      DATA STAR,BLANK/'*',' '/
      TYPE *,'TNPUT DATA,Z,T,W,ER,G'
      ACCEPT *,Z,T,W,ER,G
      WRITE(6,1)
    1 FORMAT('1',5X,31HP(WAVELENGTH/GUIDED WAVELENGTH),
     1 5X,16H X(B/WAVELENGTH),5X,29HZF(CHARACT. IMP. OF FIN LINE),//)
      PI=3.14159
      S=SIN(T*PI/2.)
      C=COS(T*PI/2.)**4
      XI=0.5
      DO 5 I=1,30
      QI=1./((1.-XI**2)**0.5)-1.
      DI=2.*XI*(ALOG(1./S)+QI*C/(1.+QI*S**4)+XI**2/16.
     1 *(1.-3.*S**2)**2*C)
      E=DI-COS(PI*XI/Z)/SIN(PI*XI/Z)
      IF(ABS(E).LE.1.E-04)GO TO 10
      IF(E.LE.0.)GO TO 4
      XI=XI-0.5*0.5**I
      GO TO 5
4     XI=XI+0.5*0.5**I
5     CONTINUE
C
```

```
C
C     CHARACTERISTIC IMPEDANCE OF RIDGED WAVEGUIDE AT INFINITE
C     FREQUENCY IS CALCULATED.
C
10       ZO=120.*PI**2*XI/(DI+SIN(PI*XI/2./Z)/COS(PI*XI/2./Z))
         DO 100 I=1,40
         P(I)=0.03*I-0.03
2        U=(ABS(ER-P(I)**2))**0.5
         V=(ABS(1.-P(I)**2))**0.5
         S=SIN(T*PI/2.)
         C=COS(T*PI/2)**4
         X(I)=0.5
         DO 90 J=1,30
         QI=1./((1.-(U*X(I))**2)**0.5)-1.
         DI=2.*U*X(I)*(ALOG(1./S)+QI*C/(1.+QI*S**4)+(U*X(I))**2/16.
     1   *(1.-3.*S**2)**2*C)
         IF(P(I).GE.1.)GO TO 20
         QO=1./((1.-(V*X(I))**2)**0.5)-1.
         DO=2.*V*X(I)*(ALOG(1./S)+QO*C/(1.+QO*S**4)+(V*X(I))**2/16.
     1   *(1.-3.*S**2)**2*C)
         F=(V/U)**2+G*(1.-(V/U)**2)
         A1=2.*PI*X(I)*V*(0.5/Z-W)
         A2=2.*PI*W*X(I)*U+ATAN(U/V*SIN(A1)/COS(A1))
         A3=PI*X(I)*V/Z
         AO=U/V*COS(A2)/SIN(A2)+COS(A3)/SIN(A3)
         E=F*DI*U/V+DO-AO
         GO TO 30
20       F=-(V/U)**2+G*(1.+(V/U)**2)
         QO=1./((1.+(V*X(I))**2)**0.5)-1.
         DO=2.*V*X(I)*(ALOG(1./S)+QO*C/(1.+QO*S**4)-(V*X(I))**2/16.
     1   *(1.-3.*S**2)**2*C)
         A4=2.*PI*W*X(I)*U+ATAN(U/V*TANH(2.*PI*X(I)*V*(0.5/Z-W)))
         AO=U/V*COS(A4)/SIN(A4)+1./TANH(PI*X(I)*V/Z)
         E=F*DI*U/V-DO-AO
30       IF(ABS(E).LE.1.E-04)GO TO 92
         IF(E.LE.0.)GO TO 80
         X(I)=X(I)-0.5*0.5**J
         GO TO 90
80       X(I)=X(I)+0.5*0.5**J
90       CONTINUE
92       IF(P(I).EQ.0.)GO TO 93
         ZF(I)=ZO/P(I)
         GO TO 94
93       ZF(I)=1000.
94       WRITE(6,95)P(I),X(I),ZF(I)
95       FORMAT(' ',16X,F7.3,21X,F7.4,20X,F10.5)
100      CONTINUE
C
```

```
C ***************************************************************
C
C     THE FOLLOWING PART OF PROGRAM IS USED TO DRAW DIAGRAM
C
C ***************************************************************
C
      WRITE(6,110)
110   FORMAT('1',38HY-AXIS=P(WAVELENGTH/GUIDED WAVELENGTH),/,
     1 23H X-AXIS=X(B/WAVELENGTH))
      DO 200 I=1,40
      XP=X(I)*200
      IX=XP
      IF((XP-IX).GE.0.5)GO TO 150
      JX=IX
      GO TO 160
150   JX=IX+1
160   DO 180 J=1,100
      A(41-I,J)=BLANK
      IF(J.EQ.JX)GO TO 170
      IF(J.EQ.20.OR.J.EQ.40.OR.J.EQ.60.OR.J.EQ.80.OR.J.EQ.100)
     1 A(41-I,J)=DOT
      IF(J.EQ.1)A(41-I,J)=DASH
      IF(I.EQ.1)A(41-I,J)=HYPHEN
      GO TO 180
170   A(41-I,J)=STAR
180   CONTINUE
200   CONTINUE
      DO 300 I=1,40
      B(I)=(40-I)*0.03
      WRITE(6,280)B(I),(A(I,J),J=1,100)
280   FORMAT(' ',15X,F4.2,1X,100A1)
300   CONTINUE
      WRITE(6,350)
350   FORMAT(' ',20X,1H0,17X,3H0.1,17X,3H0.2,17X,3H0.3,17X,3H0.4,
     1 17X,3H0.5,//)
      WRITE(6,400)Z,T,W,ER,G
400   FORMAT(' ',15X,35HGUIDED WAVELENGTH IN UNILATERAL FIN,/,
     1 16X,4HB/A=,F4.2,6H; D/B=,F4.2,6H; S/B=,F5.3,5H; ER=,F5.2,
     2 4H; G=,F4.2)
      STOP
      END
```

P(WAVELENGTH/GUIDED WAVELENGTH)	X(B/WAVELENGTH)	ZF(CHARACT. IMP. OF FIN LINE)
0.000	0.1522	1000.00000 (∞)
0.030	0.1522	5891.69971
0.060	0.1524	2945.84985
0.090	0.1526	1963.90002
0.120	0.1530	1472.92505
0.150	0.1535	1178.33997
0.180	0.1542	981.95001
0.210	0.1549	841.67139
0.240	0.1558	736.46252
0.270	0.1568	654.63336
0.300	0.1579	589.16998
0.330	0.1592	535.60907
0.360	0.1607	490.97501
0.390	0.1623	453.20770
0.420	0.1641	420.83569
0.450	0.1661	392.78000
0.480	0.1684	368.23123
0.510	0.1709	346.57059
0.540	0.1737	327.31665
0.570	0.1767	310.08948
0.600	0.1802	294.58496
0.630	0.1840	280.55713
0.660	0.1883	267.80453
0.690	0.1931	256.16086
0.720	0.1985	245.48747
0.750	0.2047	235.66798
0.780	0.2118	226.60382
0.810	0.2200	218.21111
0.840	0.2296	210.41783
0.870	0.2410	203.16206
0.900	0.2547	196.38998
0.930	0.2716	190.05482
0.960	0.2930	184.11562
0.990	0.3210	178.53635
1.020	0.3594	173.28529
1.050	0.4155	168.33429
1.080	0.5031	163.65833
1.110	0.6407	159.23512
1.140	0.8160	155.04474
1.170	0.9692	151.06923

Y-AXIS=F(WAVELENGTH/GUIDED WAVELENGTH)
X-AXIS=X(B/WAVELENGTH)



GUIDED WAVELENGTH IN UNILATERAL FIN LINE
B/A=0.50; D/B=0.13; S/B=0.072; ER= 2.22; G=0.58

TTO -- STOP
>

A.2.2

```
C
C     INSULATED FINS
C        FORTRAN IV
C
C
C     FUNCTION: THIS SUBROUTINE CALCULATES THE GUIDED WAVELENGTH
C               AND CHARACTRISTIC IMPEDANCE IN INSULATED FINS
C
C
C     INPUTS: Z(ASPECT RATIO,B/A); T(NORMALIZED GAP,D/B); W(NORMA-
C             LIZED DIELECTRIC THICKNESS,S/B); ER(DIELECTRIC CONSTANT);
C             G(NORMALIZED CORRECTION FACTOR)
C
C
C ***********************************************************************
C
        DIMENSION X(40),P(40),ZF(40)
        REAL T,G,PI,Z,P,ER,W,V,U,X,S,C,QI,DI,E,F,A
        REAL XI,ZO,ZF,A1
        TYPE *,'INPUT DATA: Z,T,W,ER,G'
        ACCEPT *,Z,T,W,ER,G
        WRITE(6,1)Z,T,W,ER,G
1       FORMAT('1',5X,4HB/A=,F4.2,6H; D/B=,F4.2,6H; S/B=,F5.3,5H; ER=,
     1  F5.2,4H; G=,F4.2,//)
        PI=3.14159
        S=SIN(T*PI/2.)
        C=COS(T*PI/2.)**4
C
C ***********************************************************************
C
C    CHARACTRISTIC IMPEDANCE AND CUTOFF OF EQUIVALENT RIDGED
C    WAVEGUIDE IS CALCULATED BELOW
C
C ***********************************************************************
C
        XI=0.5
        DO 5 I=1,30
        QI=1./((1.-XI**2)**0.5)-1.
        DI=2.*XI*(ALOG(1./S)+QI*C/(1.+QI*S**4)+XI**2/16.
     1  *(1.-3.*S**2)**2*C)
        E=DI-COS(PI*XI/Z)/SIN(PI*XI/Z)
        IF(ABS(E).LE.1.E-04)GO TO 10
        IF(E.LE.0.)GO TO 4
        XI=XI-0.5*0.5**I
        GO TO 5
4       XI=XI+0.5*0.5**I
5       CONTINUE
10      ZO=120.*PI**2*XI/(DI+SIN(PI*XI/2./Z)/COS(PI*XI/2./Z))
        WRITE(6,15)XI,ZO
15      FORMAT(' ',5X,37HCUTOFF FOR EQUIV. RIDGED WAVEGUIDE IS,F6.4,
     1  //,5X,46HCHARACT. IMP. OF RIDGED WAVEGUIDE AT INFINITE ,
     2  8HFREQ. IS,F10.5,///,5X,31HP(WAVELENGTH/GUIDED WAVELENGTH),
     3  5X,16H X(B/WAVELENGTH),5X,25HZF(CHARACT. IMP. OF FINS),//)
C
```

```
C ***********************************************************************
C
C      THE GUIDED WAVELENGTH AND CHARACTRISTIC IMPEDANCE OF INSULATED
C      FINS ARE CALCULATED BELOW
C
C ***********************************************************************
C
        DO 100 I=1,40
        P(I)=0.03*I-0.03
        V=(ABS(1.-P(I)**2))**0.5
        U=(ABS(ER-P(I)**2))**0.5
        X(I)=0.45
        DO 90 J=1,30
        QI=1./((1.-(U*X(I))**2)**0.5)-1.
        DI=2.*U*X(I)*(ALOG(1./S)+QI*C/(1.+QI*S**4)+(U*X(I))**2/16.
     1  *(1.-3.*S**2)**2*C)
        IF(P(I).GE.1.)GO TO 20
        F=(V/U)**2+G*(1.-(V/U)**2)
        A1=PI*W*X(I)*U+ATAN(U/V*SIN(PI*X(I)*V*(1./Z-W))
     1  /COS(PI*X(I)*V*(1./Z-W)))
        A=U/V*COS(A1)/SIN(A1)
        E=F*DI*U/V-A
        GO TO 30
20      F=-(V/U)**2+G*(1.+(V/U)**2)
        A1=PI*X(I)*W*U+ATAN(U/V*TANH(PI*X(I)*V*(1./Z-W)))
        A=U/V*COS(A1)/SIN(A1)
        E=F*DI*U/V-A
30      IF(ABS(E).LE.1.E-04)GO TO 50
        IF(E.LE.0.)GO TO 40
        X(I)=X(I)-0.5*0.5**J
        GO TO 90
40      X(I)=X(I)+0.5*0.5**J
90      CONTINUE
50      IF(P(I).EQ.0.)GO TO 60
        ZF(I)=Z0/P(I)
        GO TO 70
60      ZF(I)=9000.
70      WRITE(6,80)P(I),X(I),ZF(I)
80      FORMAT(' ',16X,F7.3,21X,F7.4,20X,F10.5)
100     CONTINUE
        STOP
        END
```

B/A=0.50; D/B=0.13; S/B=0.072; ER= 2.22; G=0.37

CUTOFF FOR EQUIV. RIDGED WAVEGUIDE IS 0.1702 = XI

CHARACT. IMP. OF RIDGED WAVEGUIDE AT INFINITE FREQ. IS 176.75099 OHMS

P(WAVELENGTH/GUIDED WAVELENGTH)	X(B/WAVELENGTH)	ZF(CHARACT. IMP. OF FINS) OHMS
0.000	0.1487	9000.00000 (∞)
0.030	0.1487	5891.69971
0.060	0.1489	2945.84985
0.090	0.1491	1963.90002
0.120	0.1495	1472.92505
0.150	0.1499	1178.33997
0.180	0.1505	981.95001
0.210	0.1512	841.67139
0.240	0.1520	736.46252
0.270	0.1530	654.63336
0.300	0.1540	589.16998
0.330	0.1552	535.60907
0.360	0.1566	490.97501
0.390	0.1581	453.20770
0.420	0.1598	420.83569
0.450	0.1616	392.78000
0.480	0.1637	368.23123
0.510	0.1660	346.57059
0.540	0.1685	327.31665
0.570	0.1713	310.08948
0.600	0.1744	294.58496
0.630	0.1779	280.55713
0.660	0.1818	267.80453
0.690	0.1861	256.16086
0.720	0.1909	245.48747
0.750	0.1964	235.66798
0.780	0.2027	226.60382
0.810	0.2098	218.21111
0.840	0.2181	210.41783
0.870	0.2278	203.16206
0.900	0.2393	196.38998
0.930	0.2531	190.05482
0.960	0.2703	184.11562
0.990	0.2920	178.53635
1.020	0.3206	173.28529
1.050	0.3600	168.33429
1.080	0.4183	163.65833

```
C
C     A.2.3
C     BILATERAL FINS
C        FORTRAN IV
C
C
C     FUNCTION: THIS SUBROUTINE CALCULATES THE GUIDED WAVELENGTH
C               AND CHARACTRISTIC IMPEDANCE IN BILATERAL FINS
C
C
C     INPUTS: Z(ASPECT RATIO,B/A); T(NORMALIZED GAP,D/B); W(NORMA-
C             LIZED DIELECTRIC THICKNESS,S/B); ER(DIELECTRIC CONSTANT);
C             G(NORMALIZED CORRECTION FACTOR)
C
C
C ***********************************************************************
C
      DIMENSION X(40),F(40),ZF(40)
      REAL T,G,PI,Z,P,ER,W,V,U,X,S,C,QI,DI,E,F,A
      REAL XI,ZO,ZF,A1
      TYPE *,'INPUT DATA: Z,T,W,ER,G'
      ACCEPT *,Z,T,W,ER,G
      WRITE(6,1)Z,T,W,ER,G
1     FORMAT('1',5X,4HB/A=,F4.2,6H; D/B=,F4.2,6H; S/B=,F5.3,5H; ER=,
     1 F5.2,4H; G=,F4.2,//)
      PI=3.14159
      S=SIN(T*PI/2.)
      C=COS(T*PI/2.)**4
C
C ***********************************************************************
C
C    CHARACTRISTIC IMPEDANCE AND CUTOFF OF EQUIVALENT RIDGED
C    WAVEGUIDE IS CALCULATED BELOW
C
C ***********************************************************************
C
      XI=0.5
      DO 5 I=1,30
      QI=1./((1.-XI**2)**0.5)-1.
      DI=2.*XI*(ALOG(1./S)+QI*C/(1.+QI*S**4)+XI**2/16.
     1 *(1.-3.*S**2)**2*C)
      E=SIN(PI*W*XI)/COS(PI*W*XI)/T+DI-COS(PI*XI*(1./Z-W))/
     1 SIN(PI*XI*(1./Z-W))
      IF(ABS(E).LE.1.E-04)GO TO 10
      IF(E.LE.0.)GO TO 4
      XI=XI-0.5*0.5**I
      GO TO 5
4     XI=XI+0.5*0.5**I
5     CONTINUE
10    ZO=120.*PI**2*XI/(SIN(PI*W*XI)+DI+SIN(PI*XI/2.*(1./Z-W))
     1 *COS(PI*W*XI)/COS(PI*XI/2.*(1./Z-W)))
      WRITE(6,15)XI,ZO
15    FORMAT(' ',5X,37HCUTOFF FOR EQUIV. RIDGED WAVEGUIDE IS,F6.4,
     1 //,5X,46HCHARACT. IMP. OF RIDGED WAVEGUIDE AT INFINITE ,
     2 8HFREQ. IS,F10.5,///,5X,31HF(WAVELENGTH/GUIDED WAVELENGTH),
     3 5X,16H X(B/WAVELENGTH),5X,25HZF(CHARACT. IMP. OF FINS),//)
C
```

```
C ***********************************************************************
C
C      THE GUIDED WAVELENGTH AND CHARACTRISTIC IMPEDANCE OF BILATERAL
C      FINS ARE CALCULATED BELOW
C
C ***********************************************************************
C
        DO 100 I=1,40
        P(I)=0.03*I-0.03
        V=(ABS(1.-P(I)**2))**0.5
        U=(ABS(ER-P(I)**2))**0.5
        X(I)=0.45
        DO 90 J=1,30
        QI=1./((1.-(U*X(I))**2)**0.5)-1.
        DI=2.*U*X(I)*(ALOG(1./S)+QI*C/(1.+QI*S**4)+(U*X(I))**2/16.
     1  *(1.-3.*S**2)**2*C)
        IF(P(I).GE.1.)GO TO 20
        QO=1./((1.-(V*X(I))**2)**0.5)-1.
        DO=2.*V*X(I)*(ALOG(1./S)+QO*C/(1.+QO*S**4)+(V*X(I))**2/16.
     1  *(1.-3.*S**2)**2*C)
        A=COS(PI*X(I)*V*(1./Z-W))/SIN(PI*X(I)*V*(1./Z-W))
        E=U/V*SIN(PI*W*X(I)*U)/COS(PI*W*X(I)*U)+G*DI*U/V+DO-A
        GO TO 30
20      QO=1./((1.+(V*X(I))**2)**0.5)-1.
        DO=2.*V*X(I)*(ALOG(1./S)+QO*C/(1.+QO*S**4)-(V*X(I))**2/16.
     1  *(1.-3.*S**2)**2*C)
        A=U/V*SIN(PI*W*X(I)*U)/COS(PI*W*X(I)*U)
        E=A+G*DI*U/V-DO-1./TANH(PI*X(I)*V*(1./Z-W))
30      IF(ABS(E).LE.1.E-04)GO TO 50
        IF(E.LE.0.)GO TO 40
        X(I)=X(I)-0.5*0.5**J
        GO TO 90
40      X(I)=X(I)+0.5*0.5**J
90      CONTINUE
50      IF(P(I).EQ.0.)GO TO 60
        ZF(I)=ZO/P(I)
        GO TO 70
60      ZF(I)=9000.
70      WRITE(6,80)P(I),X(I),ZF(I)
80      FORMAT(' ',16X,F7.3,21X,F7.4,20X,F10.5)
100     CONTINUE
        STOP
        END
>
```

B/A=0.50; D/B=0.13; S/B=0.072; ER= 2.22; G=0.37

CUTOFF FOR EQUIV. RIDGED WAVEGUIDE IS 0.1525 = XI

CHARACT. IMP. OF RIDGED WAVEGUIDE AT INFINITE FREQ. IS 176.68607 OHMS

P(WAVELENGTH/GUIDED WAVELENGTH)	X(B/WAVELENGTH)	ZF(CHARACT. IMP. OF FINS) OHMS
0.000	0.1394	9000.00000 (∞)
0.030	0.1395	5889.53564
0.060	0.1396	2944.76782
0.090	0.1399	1963.17859
0.120	0.1402	1472.38403
0.150	0.1407	1177.90723
0.180	0.1413	981.58929
0.210	0.1419	841.36224
0.240	0.1427	736.19202
0.270	0.1437	654.39288
0.300	0.1447	588.95361
0.330	0.1459	535.41235
0.360	0.1472	490.79465
0.390	0.1487	453.04120
0.420	0.1504	420.68112
0.450	0.1522	392.63571
0.480	0.1543	368.09598
0.510	0.1566	346.44327
0.540	0.1591	327.19641
0.570	0.1619	309.97556
0.600	0.1651	294.47678
0.630	0.1686	280.45407
0.660	0.1725	267.70615
0.690	0.1769	256.06677
0.720	0.1819	245.39731
0.750	0.1876	235.58142
0.780	0.1941	226.52058
0.810	0.2016	218.13095
0.840	0.2104	210.34055
0.870	0.2209	203.08743
0.900	0.2335	196.31784
0.930	0.2490	189.98502
0.960	0.2687	184.04799
0.990	0.2946	178.47076
1.020	0.3303	173.22163
1.050	0.3831	168.27245
1.080	0.4471	163.59821

APPENDIX 3

"FIN LINE DESIGN MADE EASY"

FIN LINE DESIGN MADE EASY

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ABSTRACT

This paper presents a simple but accurate method to calculate the guided wavelength and characteristic impedance of various fin line configurations. The transverse resonance condition is solved using available data on capacitive irises in waveguides and a novel diagram for evaluating the influence of a dielectric sheet on the iris capacitance.

Introduction

The simplicity of Meier's[1,2] formulae for fin line parameters as well as the accuracy of Hofmann's[3,4] numerical solution of the fin line problem can only be fully exploited by the designer if he chooses the same geometries and dielectric constants as these authors in order to apply their results directly. Otherwise, either a test measurement or an involved computer solution is required to predict fin line performance. Saad and Begemann[5] have published approximate but still cumbersome expressions for the effective permittivity and characteristic impedance of fin lines, which are satisfactory for thin substrates only.

The purpose of this paper is to present an accurate, versatile and simple method to calculate the guided wavelength and impedance of various fin lines, using available data for waveguide discontinuities and a novel diagram for the evaluation of the susceptance of dielectrically supported irises in waveguides.

Theory

Calculation of the Guided Wavelength

The guided wavelength is found by evaluating the first root of the transverse resonance condition. This condition contains the susceptance of a capacitive iris modified by the presence of a dielectric sheet. Both the iris susceptance and the correction factor are available in graphical form, thus the computational effort is kept at a minimum.

The method is best explained by briefly revisiting the ridged waveguide (Fig. 1a). The transverse resonant condition can be written as follows:

$$\left[\frac{B_a}{Y_o}\frac{\lambda_t}{b}\right]\frac{b}{\lambda_t} - \cot \pi \frac{a}{b}\frac{b}{\lambda_t} = 0 \qquad (1)$$

where λ_t is the transverse guided wavelength.

The term in square brackets is $\frac{1}{2}$ the $[\frac{B}{Y_o}\ \frac{\lambda_g}{b}]$ presented in the Waveguide Handbook[6], Fig. 5.1-4, provided λ_g therein is replaced by λ_t of this paper. Close inspection of Fig. 5.1-4 in [6] shows that this term is rather insensitive to changes of the parameter b/λ_t as long as the latter is smaller than 0.4, which is practically always the case. Thus, a very good approximation for b/λ_t can be found by introducing into equ. (1) the value of $[(B_a/Y_o)(\lambda_t/b)]$ for $b/\lambda_t = 0$. The root is accurate within a few percent. If a better approximation is desired, an improved value for $[(B_a/Y_o)(\lambda_t/b)]$ corresponding to the just calculated b/λ_t is introduced into equ. (1). The guided wavelength λ_g is then found by writing:

$$\lambda_g = \lambda\ [1-(\lambda/\lambda_t)^2]^{-1/2} \qquad (2)$$

where λ is the free space wavelength.

Fin Lines

When a dielectric sheet of permittivity ε_r is added at one side of the ridge, as shown in Fig. 1b, the capacitance of the covered half of the discontinuity is increased by a factor somewhere between 1 and ε_r, depending on the thickness of the sheet. This correction factor has been calculated by numerically evaluating the ratio

$$F = \frac{\text{Slot susceptance for finite sheet thickness}}{\text{Slot susceptance for infinite sheet thickness}}$$

and normalizing it to make it independent of ε_r. The procedure for this evaluation was outlined by Cohn.[7] The results are presented graphically in Fig. 2. The guided wavelength in the fin line can now be determined in exactly the same way as for the ridged waveguide by introducing the graphically available values for the discontinuity susceptance[6] and the correction factor (Fig. 2) into the transverse resonance condition of the structure, and then finding the lowest root of this expression.

The resonance conditions for three fin line configurations will be given with the following abbreviations:

$d/b = t;\ s/b = w;\ b/a = z;\ \lambda/\lambda_g = p$

$b/\lambda = x;\ [|\varepsilon_r - p^2|]^{1/2} = u;\ [|1-p^2|]^{1/2} = v$

$\lambda_{to} = x/v;\ \lambda_{t1} = x/u;\ b/\lambda_{to} = vx,$

$b/\lambda_{t1} = ux.$

λ_g = fin line wavelength; λ_{to}, λ_{t1} = transverse guided wavelengths in air and dielectric respectively.

Resonant Condition for Unilateral Fin Line (Fig. 1.b)

for $p < 1$

$$-(u/v)\cot\{2\pi wxu + \tan^{-1}[(u/v)\tan 2\pi xv(\tfrac{1}{2z} - w)]\}$$
$$+ F\left[\frac{B_d}{Y_1}\frac{1}{ux}\right](\frac{u}{v})^2 vx + \left[\frac{B_a}{Y_o}\frac{1}{vx}\right] vx - \cot\frac{\pi xv}{z} = 0 \qquad (3a)$$

where $F = (v/u)^2 + G\,[1-(v/u)^2]$

for $p > 1$

$$-(u/v)\cot\{2\pi wxu + \tan^{-1}[(u/v)\tanh 2\pi xv(\tfrac{1}{2z} - w)]\}$$

$$+ F\left[\frac{B_d}{Y_1}\frac{1}{ux}\right](\frac{u}{v})^2 vx - \left[\frac{B_a}{Y_o}\frac{1}{vx}\right] vx - \coth\frac{\pi xv}{z} = 0 \qquad (3b)$$

where $F = -(v/u)^2 + G[1+(v/u)^2]$

Resonant Condition for Bilateral Fin Line (Fig. 1.c)

for $p < 1$

$$(u/v)\tan \pi wxu + G\left[\frac{B_d}{Y_1}\frac{1}{ux}\right](\frac{u}{v})^2 vx$$

$$+\left[\frac{B_a}{Y_o}\frac{1}{vx}\right] vx - \cot \pi xv(\frac{1}{z} - w) = 0 \qquad (4a)$$

for $p > 1$

$$(u/v)\tan \pi wxu + G\left[\frac{B_d}{Y_1}\frac{1}{ux}\right](\frac{u}{v})^2 vx$$

$$-\left[\frac{B_a}{Y_o}\frac{1}{vx}\right] vx - \coth \pi xv(\frac{1}{z} - w) = 0 \qquad (4b)$$

Resonant Condition for Insulated Fins (Fig. 1.d)

for $p < 1$

$$-(u/v)\cot\{\pi wxu + \tan^{-1}[(u/v)\tan\pi xv(\tfrac{1}{z} - w)]\}$$

$$+ F\left[\frac{B_d}{Y_1}\frac{1}{ux}\right](\frac{u}{v})^2 vx = 0 \qquad (5a)$$

where F as in (3a)

for $p > 1$

$$-(u/v)\cot\{\pi wxu + \tan^{-1}[(u/v)\tanh \pi xv(\tfrac{1}{z} - w)]\}$$

$$+ F\left[\frac{B_d}{Y_1}\frac{1}{ux}\right](\frac{u}{v})^2 vx = 0 \qquad (5b)$$

where F as in (3b)

As in the case of ridged waveguide, the values for $\left[\frac{B_d}{Y_1}\frac{1}{ux}\right]$ and $\left[\frac{B_a}{Y_o}\frac{1}{vx}\right]$ can, for a first approximation, be those for $1/ux$ and $1/vx = 0$. Once the root of the resonance condition is found, the choice of the susceptances can be improved accordingly.

For the solution of the resonant condition, either $p = \lambda/\lambda_g$ can be fixed while $x = b/\lambda$ is searched, or x is fixed and the corresponding p is found.

Characteristic Impedance

Satisfactory values for the characteristic impedance of fin lines can be obtained with the expression

$$Z_o = Z_{o\infty}\,\lambda_g/\lambda = Z_{o\infty}/p \qquad (6)$$

where $Z_{o\infty}$ is the characteristic impedance of a ridged waveguide of identical cross-sectional dimensions at infinite frequency, which can be obtained from Cohn's[8] paper on properties of ridged waveguides. The evaluation of p has just been described.

Results

The resonant conditions have been programmed on a HP-97 calculator and solved using the standard program "Calculus and Roots of f(x)". Fig. 3 shows $p = \lambda/\lambda_g$ as a function of $x = b/\lambda$ for a unilateral fin line and for the corresponding ridged waveguide ($\varepsilon_r = 1$). For comparison, measurements made by Meier[2] and calculations made by Hofmann[3] are shown for the same fin line. Agreement with both authors is excellent. The diagram also shows the cutoff wavelengths of the guides (at $\lambda/\lambda_g = 0$).

In order to find the lowest root of the resonant condition, an initial guess for x must be made such that the program converges to the desired value. A good initial guess is $x = b/\lambda_{co}$, where λ_{co} is the cutoff wavelength of the empty waveguide of identical dimensions.

Fig. 4 shows results for bilateral fin lines with values published by Saad and Bege mann[5]. Unfortunately, the authors do not specify the frequencies at which the dielectric constant has been measured. Discrepancies between results reach up to five percent for thick substrate ($c/b = 0.228$).

Conclusion

The availability in graphical form of values for capacitive irises in rectangular waveguides, and of a novel graph for evaluating the iris capacitance in the presence of a dielectric sheet, enable the fin line designer to accurately predict the guided wavelength in fin lines of any cross-sectional dimensions and substrate permittivity. The mathematical complexity of the solution does not go beyond finding the root of a transcendental equation, a task easily accomplished by small programmable computers. Nevertheless, the accuracy approaches that of a numerical solution and is limited only by the accuracy of the graphs, which is typically less than 4%.

Acknowledgement

This work was supported by the Communications Research Centre of Ottawa, and by the National Research Council of Canada. The author acknowledges the help of Mr. S. Kosmopoulos in the numerical evaluation of the expressions.

References

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[8] S.B. Cohn, "Properties of Ridge Wave Guide", Proc. of the I.R.E., Vol. 35, No. 8, August 1947, pp. 783-788.

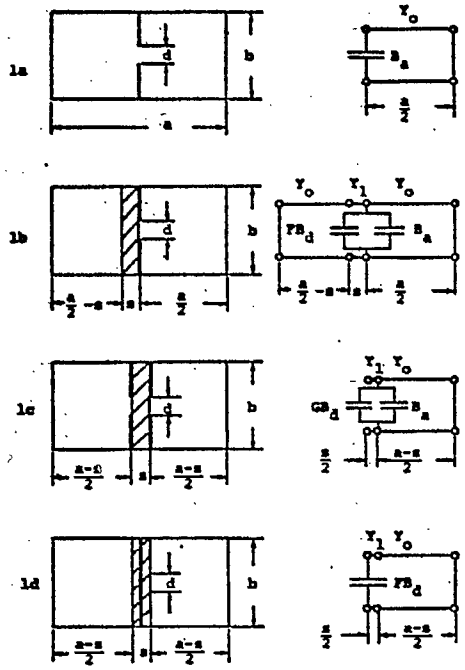

Fig. 1 Cross-section and equivalent transverse network of various fin line configurations.
a. Ridged waveguide b. unilateral fin line
c. bilateral fin line d. insulated fin line

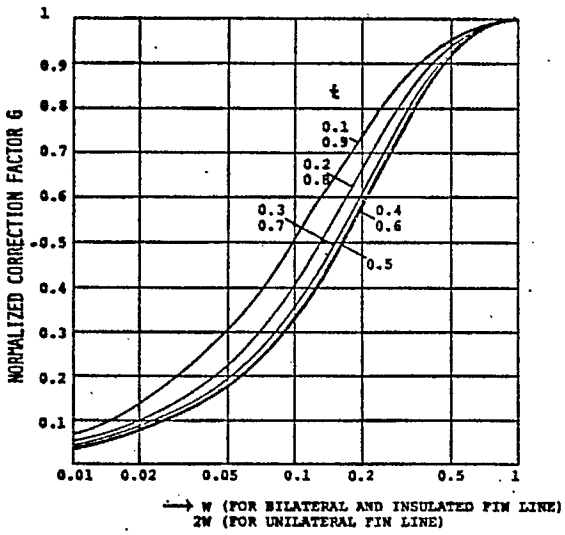

Fig. 2 Correction factor accounting for the presence of a dielectric substrate

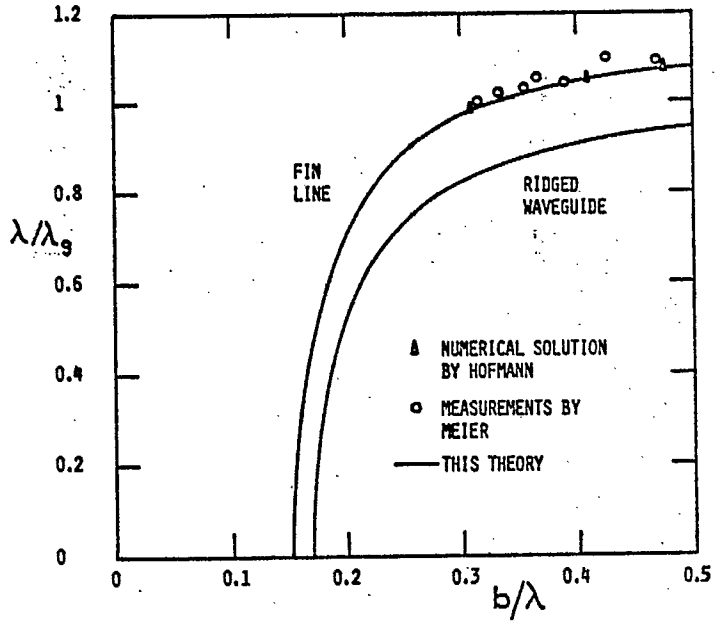

Fig. 3 Guided wavelength in unilateral fin line and in ridged waveguide.
$b/a = 0.5$; $d/b = 0.13$; $s/b = 0.072$; $\varepsilon_r = 2.22$

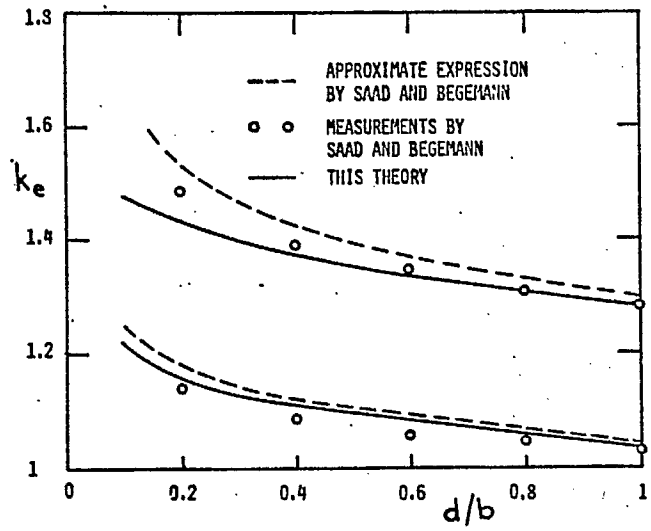

Fig. 4 Effective dielectric constant at cutoff in bilateral fin line.
$b/a = 0.5$; $\varepsilon_r = 2.22$
upper curves: $s/b = 0.228$
lower curves: $s/b = 0.05$

APPENDIX 4

"FIN LINE PARAMETERS CALCULATED WITH THE TLM-METHOD"

FIN LINE PARAMETERS CALCULATED WITH THE TLM - METHOD

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ABSTRACT

The guided wavelength in fin line is calculated using the Transmission Line Matrix (TLM) method. The resonant frequencies of fin line cavities are evaluated on a computer, yielding the dispersion characteristic of the fundamental and higher order modes of propagation.

Introduction

Various methods for the evaluation of fin-line parameters have been presented by Meier[1], Hofmann[2] Saad and Begemann[3] and Hoefer[4]. In addition, a method for analyzing the transition from fin-line to a below-cutoff waveguide has been reported by Saad and Schuenemann[5]. In the present paper, the application of the Transmission Line Matrix (TLM) technique to the fin line problem is demonstrated for the following reasons:

a) To verify and eventually to refine the various methods mentioned above,

b) To calculate the equivalent lumped element circuit of fin line discontinuities which have not been evaluated to date.

The number of results given in the present paper is rather limited because of the considerable size of memory and the large CPU-time required for the calculation of a given structure.

The TLM - Method

The transmission-line matrix (TLM) method was developed by Akhtarzad and Johns[6] and was applied by these authors to the analysis of three-dimensional resonating structures. The dispersion characteristics of waveguide structures and discontinuities can be obtained by calculating the resonance frequencies of cavities exhibiting the pertinent cross-sectional geometry and containing the discontinuities under investigation. To this end, field propagation in the structure is simulated by the propagation and scattering of impulses in a three-dimensional transmission line lattice characterised by the parameter $\Delta\ell$ (distance between adjacent nodes. Boundaries (electric and magnetic walls) and dielectric interfaces can be simulated by introducing stubs which modify in an appropriate way the impedance across nodes situated at the boundaries or inside the dielectric. Valid results are obtained if the distance between nodes is smaller than $0.1\,\lambda$, where λ is the free-space wavelength corresponding to the resonance frequency of interest. On the other hand, the minimum value for $\Delta\ell$ is limited by considerations of available computer memory.

To start the calculation, one or more nodes (depending on the mode to be investigated) are excited by an impulse. The propagation of the impulses across the three-dimensional network is calculated in real time. After a sufficient number of iterations (forth-and-back trips of impulses across the structure), the impulse response of the structure is picked up at strategic output points, chosen again according to the expected field distribution. Speaking in terms of measurements, the position of the input and output nodes is chosen in the same way as the position of field probes for excitation and detection of modes in a resonator. In the TLM-program, however, the "probes" do not interact with the field and thus are non-perturbing.

From the time domain output, the eigenvalues of the structure in the frequency domain are obtained via Fourier Transform. The number N of iterations must be sufficient to obtain satisfactory resolution in the frequency domain. The finite character of N limits the response in the time domain and thus determines the resolution of maxima and minima in the frequency spectrum.

Features of the Computer Program

The original program published by Akhtarzad[7] has been modified by A. Ros (co-author) and co-workers to gain a factor 5 in CPU-time and a factor 2 in memory size. This has been achieved by incorporating the subroutines into the main program. Still, for the structures calculated in the present paper, considerable memory is required, particularly because fin line structures with relatively thin dielectric and small fin-spacing require at least three nodes within the smallest dimension to yield satisfactory accuracy.

A value of $\Delta\ell = .4$ mm has been chosen for the distance between adjacent nodes. A typical program for a cavity of 20 x 10 x 4 mm requires a memory close to 1 M-octets, and an IBM 360 runs for about 240 CPU-minutes to execute 1000 iterations. These requirements are obviously the major drawback of the TLM-method, but on the other hand, any structure can be handled regardless of complexity of its geometry.

Computations and Results

A rectangular cavity containing a unilateral fin line structure was adopted for TLM computations. Fig.1 defines the parameters of the structure. Resonant modes are characterized, as in empty rectangular cavities, by indices ℓ, m, n representing the number of half-periods in x, y and z direction respectively.

Resonant frequencies were calculated with the TLM-method and compared with results obtained by solving transverse resonant conditions as shown by Hoefer[4]. Several special cases for which exact analytical solutions exist, were chosen to verify the accuracy of the TLM-program.

Since the TE_{10} fin line mode is of particular interest, the c-dimension should ideally be the longest dimension in order to separate it well from the other modes. However, this would require excessive computer memory, and shorter lengths $c < a,b$ had to be chosen.

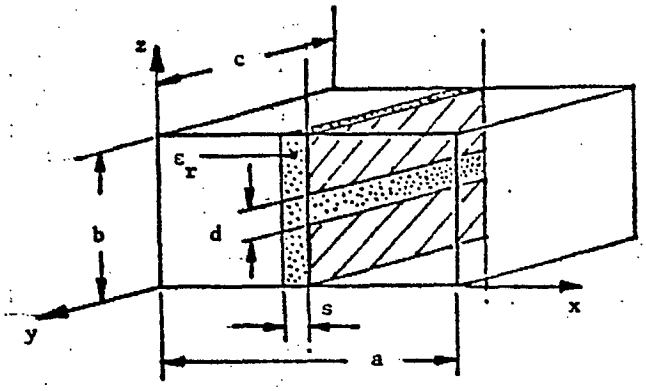

Fig. 1 Rectangular cavity containing unilateral fin line

CONFIGURATION 1: Empty Cavity

a = 20 mm d = b = 10.4 mm
b = 10.4 mm ε_r = 1
c = 7.2 mm

Mode	Resonant Frequency (GHz) TLM	Exact	Error
TE_{110}	16.22	16.257	-0.2%

Table 1 Resonant frequency of empty cavity. Comparison of TLM and exact solution.

CONFIGURATION 2: Dielectric-Filled Cavity

a = 20 mm d = b = 10.4 mm
b = 10.4 mm s = a = 20 mm
c = 7.2 mm ε_r = 2.22

Mode	Resonant Frequency (GHz) TLM	Exact	Error
TE_{110}	10.91	10.911	0%

Table 2 Resonant frequency of dielectric-filled cavity (ε_r = 2.22). Comparison of TLM and exact solution.

CONFIGURATION 3: Cavity with Dielectric Slab

a = 20 mm d = b = 10.4 mm
b = 10.4 mm s = 2 mm
c = 4 mm ε_r = 2.2

Mode	Resonant Frequencies (GHz) TLM	Exact	Error
TE_{101}	33.52	31.866	5.2%
TE_{301}	44.67	41.522	7.6%
TE_{501}	52.40	50.567	3.6%
TE_{102}	59.42	57.673	3 %
TE_{701}	63.39	62.026	2 %

Table 3 Resonant frequencies of cavity with dielectric slab (ε_r = 2.2). Comparison of TLM and exact solutions.

CONFIGURATION 4: Cavity Containing Centered Thin Fins

a = 20 mm d = 1.6 mm
b = 10.4 mm ε_r = 1
c = 7.2 mm

Mode	Resonant Frequencies TLM	Hoefer[4]	Effective Diel. Constant $\varepsilon_{eff} = (\lambda/\lambda_g)^2$ TLM	Hoefer[4]
TE_{110}	20.63	20.60	n/a	n/a
TE_{101}	23.25	21.473	0.80	0.94
TE_{301}	29.20	27.033	n/a	n/a

Table 4 Resonant frequencies of cavity with centered fins. Comparison of TLM solutions and solutions obtained using transverse resonance conditions. TE_{101} is the fundamental fin line mode.

CONFIGURATION 5: Cavity Containing Unilateral Fin Line

a = 20 mm d = 1.6 mm
b = 10.4 mm s = 1 mm
c = 7.2 mm ε_r = 2.22

Mode	Resonant Frequencies TLM	Hoefer[4]	Effective Diel. Constant $\varepsilon_{eff} = (\lambda/\lambda_g)^2$ TLM	Hoefer[4]
TE_{110}	14.49	-	n/a	n/a
TE_{101}	20.19	18.40	1.065	1.282

Table 5 Resonant frequencies of cavity containing unilateral fin line. Comparison of TLM solutions and solutions obtained with method described by Hoefer[4]. TE_{101} is the fundamental fin line mode.

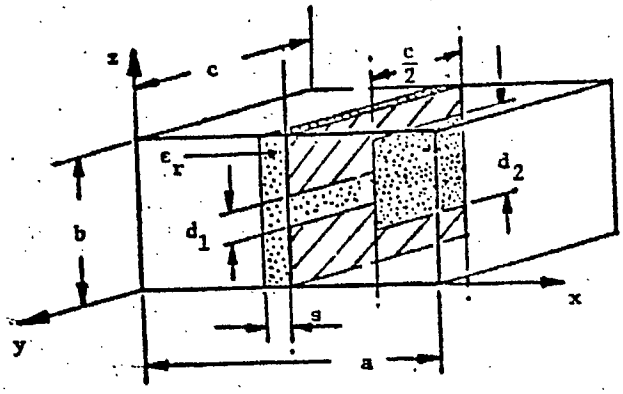

Fig. 2 Rectangular cavity containing a fin line step discontinuity at its centre.

CONFIGURATION 6: Cavity Containing a Fin Line Step Discontinuity at its Centre (See Fig. 2)

a = 20 mm		d_2 = 3.2 mm
b = 10.4 mm	d_1 = 1.6 mm	d_2 = 4 mm
c = 6.4 mm		d_2 = 5.6 mm
ε_r = 1		

Mode	Resonant Frequencies (GHz)			
	obtained with TLM-method			by extrapolation
	d_2=3.2 mm	d_2=4 mm	d_2=5.6 mm	d_2=d_1
TE_{101}	22.82	22.56	21.99	23.40
TE_{102}	45.91	46.13	46.82	45.7

Table 6 Resonant frequencies of cavity containing a fin line step discontinuity. In the TE_{101}-mode, a current node is situated at the discontinuity, while in the TE_{102}-mode, a voltage node occurs at this position. d_1= 1.6 mm.

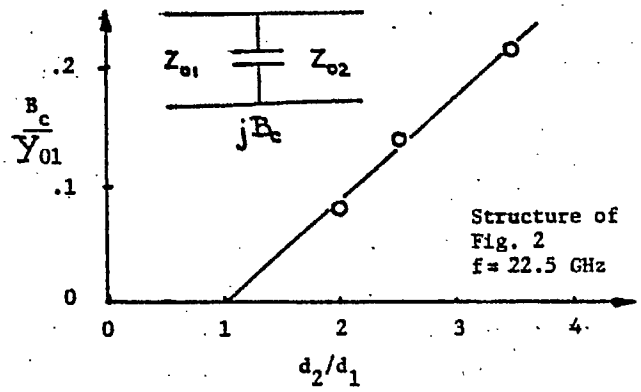

Fig. 3 Equivalent parameters for a step discontinuity of fin line, calculated with the TLM-method.

Fig. 3 shows the calculated equivalent parameters of the discontinuity for the resonant frequency of the fundamental fin line mode TE_{101}, as well as the equivalent circuit itself. The accuracy is estimated to be about ± 10%

Conclusion

The TLM-method yields resonant frequencies of rectangular cavities accurate within ± 0.5%, if the cavities are homogeneously filled with dielectric. In the presence of a centered dielectric slab (ε_r=2.2), the TLM-calculated frequencies are typically 5% too high. If fins are introduced into the cavity, TLM-frequencies are 8% higher than frequencies obtained with formulae for ridged waveguides. Similar discrepancies exist between TLM frequencies for unilateral fin line and frequencies obtained with Hoefer's[4] method. Consequently, the effective dielectric constant ε_{eff} for unilateral fin line is 17% smaller when calculated with the TLM-method. Further study is necessary to determine the reason for these differences.

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