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# Measurement of small forces on dielectric barrier discharge plasma actuators using parallel cable mechanism

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The scientific or technical validity of this contract report is entirely the responsibility of the contractor and the contents do not necessarily have the approval or endorsement of the Department of National Defence of Canada.

**Defence Research and Development Canada – Valcartier**

Contract Report  
DRDC Valcartier CR 2012-265  
March 2012

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## IMPORTANT INFORMATIVE STATEMENTS

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## **Abstract**

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It is desired to improve an existing measurement system that can measure very small forces (mgf) applied to a platform. The objective of this project is to propose new solutions in order to improve the measurement system and possibly measure additional force components. A survey of the existing force measuring systems is presented. Then, the preferred measurement systems are discussed. Cable mechanisms are introduced and their static analysis is presented. Several cable architectures are suggested and some examples of cable system analyses are presented.

## **Résumé**

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On veut améliorer un système existant qui peut mesurer de très petites forces (mgf) appliquées sur une plateforme. L'objectif de ce projet est de proposer de nouvelles solutions afin d'améliorer le système de mesure et possiblement de mesurer des composantes de force additionnelles. Une revue des appareils de mesure de force existants est présentée. Puis, les appareils de mesure préférés sont discutés. Les mécanismes à câbles sont introduits et leur analyse statique est présentée. Plusieurs architectures de câbles sont suggérées et quelques exemples de systèmes de câbles sont présentés.

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## Executive summary

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### Measurement of small forces on dielectric barrier discharge plasma actuators using parallel cable mechanism

Clément Gosselin; Thierry Laliberté ; DRDC Valcartier CR 2012-265; Defence Research and Development Canada – Valcartier; March 2012.

**Introduction or background:** It is desired to improve an existing measurement system that can measure very small forces applied to a platform. The objective of this project is to propose new solutions in order to improve the measurement system and possibly measure additional force components.

**Results:** It is shown that the measurement of several force/torque components is possible using a parallel cable mechanism and either load cells or balances. Geometric arrangements are discussed and proposed.

**Significance:** The ability to measure several force/torque components using a parallel cable mechanism opens the avenue to novel results for research in aerodynamics.

**Future plans:** It would be very interesting to implement some of the proposed measurement systems.

## Sommaire

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### **Measurement of small forces on dielectric barrier discharge plasma actuators using parallel cable mechanism**

**Clément Gosselin ; Thierry Laliberté ; DRDC Valcartier CR 2012-265 ;  
Recherche et développement pour la défense Canada – Valcartier ; mars 2012.**

**Introduction ou contexte :** On souhaite améliorer un système de mesure existant capable de mesurer de très petites forces (mgf) sur une plate-forme. L'objectif de ce projet est de proposer de nouvelles solutions capables d'améliorer le système de mesure et de mesurer des composantes de force additionnelles.

**Résultats :** Il est montré que la mesure de plusieurs composantes de forces/moments est possible à l'aide d'un mécanisme à câbles et des balances de précision ou des cellules de charge précises. Quelques arrangements géométriques sont analysés et proposés.

**Importance :** La capacité de mesurer plusieurs composantes de forces/moments à l'aide d'un mécanisme à câbles ouvre la porte à de nouveaux résultats pour la recherche en aérodynamique.

**Perspectives :** Il serait très intéressant de mettre en oeuvre certains des mécanismes proposés dans le rapport.



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# **1 Introduction**

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## **1.1 Overview**

It is desired to improve an existing measurement system that can measure very small forces applied on a platform. The objective of this project is to propose new solutions in order to improve the measurement system and possibly measure additional force components.

## **1.2 Existing measurement system**

The existing measurement system measures a vertical force only with the help of a Ohaus balance model number AV313C. The platform is rigidly attached to the balance. The platform weighs approximately 200 gf, with a maximum of 300 gf. The main force to be measured is around 10 mgf and the other forces are expected to be smaller. The balance AV313C has a capacity of 310 gf and a repeatability of 1 mgf. The size of the platform is approximately 200 mm x 100 mm x 6 mm. In order to avoid external perturbations, the measurement system is enclosed in an isolation chamber. The size of the chamber is approximately 600 mm x 600 mm x 600 mm. High voltages are applied to the platform. This must be taken into account for the selection of the force sensors.

## **1.3 Desired improvements**

While the existing measurement system measures only the force along the vertical axis, it is desired to measure force components along multiple axes. Moreover, the resolution is relatively coarse compared to the forces to be measured. Therefore, it is also desired to measure with a better resolution.

## **2 Review of force measuring systems**

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### **2.1 Challenge**

The main challenge regarding the measurement system is the large ratio between the needed capacity and the needed resolution. Moreover, the required resolution itself is extremely fine.

### **2.2 Existing force sensors**

A survey of the numerous force sensors commercially available has been performed. Since the required measurement range could be significantly smaller than the capacity of the force sensor, the accuracy of the sensor over a limited range may be better than over its full range. Indeed, the accuracy, characterized by linearity, repeatability and other indices, is generally specified for the full range. Therefore, the resolution of the sensor is also considered. The potential accuracy lies somewhere between the full scale accuracy and the resolution.

In Tables 1 and 2, the force sensors that are the closest to the requirements, regarding the sensing range (300 g) and/or the accuracy and resolution (as fine as possible), are presented with their main characteristics.

Table 1: Existing balances.

Sensor	Supplier	Capacity	Full range Accuracy	Resolution	Cost
Adventurer Pro	Ohaus	310 gf	2 mgf	1 mgf	\$1809
Adventurer Pro	Ohaus	260 gf	0.3 mgf	0.1 mgf	\$3245
Pioneer PA313	Ohaus	310 gf	2 mgf	1 mgf	\$1294
Pioneer PA214	Ohaus	210 gf	0.3 mgf	0.1 mgf	\$2472
Explorer EX423	Ohaus	420 gf	2 mgf	1 mgf	\$3301
Explorer EX324	Ohaus	320 gf	0.2 mgf	0.1 mgf	\$4991
Discovery DV314C	Ohaus	310 gf	0.5 mgf	0.1 mgf	\$4759
Discovery DV215CD	Ohaus	210 gf / 81 gf	0.2 mgf / 0.03 mgf	0.1 mgf / 0.01 mgf	\$5341
M-Power AZ214	Sartorius	210 gf	0.3 mgf	0.1 mgf	\$2455
CPA324S	Sartorius	310 gf	0.3 mgf	0.1 mgf	\$5750
Cubis MSE225S	Sartorius	220 gf	0.02 mgf	0.01 mgf	\$10480

Table 2: Existing sensors.

Sensor	Supplier	Capacity	Full range Accuracy	Resolution	Cost
<b>HANDHELD</b>					
MARK-10 M5-012	Wagner	50 gf	50 mgf	10 mgf	\$1145
MARK-10 M5-05	Wagner	250 gf	250 mgf	50 mgf	\$1145
DFG55-0.12	Omega	50 gf	50 mgf	10 mgf	\$1145
DFG41-250G	Omega	250 gf	250 mgf	50 mgf	\$1475
DPS-1	Imada (Jonar)	500 gf	1000 mgf	100 mgf	N/A
DPZ-1	Imada (Jonar)	500 gf	1000 mgf	100 mgf	N/A
<b>SIX AXIS LOAD</b>					
Nano17ti (SI-8-0.05)	ATI Industrial	1400 gf	N/A	150 mgf	N/A
50M31A	JR3	10 000 gf	N/A	4000 mgf	N/A
<b>ONE AXIS LOAD</b>					
LCL-454G/DP41-S	Omega	454 gf	1135 mgf	4 mgf	\$75+\$570
LCUA-500G/DP41	Omega	500 gf	1500 mgf	4 mgf	\$930+\$570
FT-S10000/FT-SC01	FemtoTools	1 gf	N/A	0.06 mgf (14 bits)	N/A
RSP2-0R6M-A*C01/ DI-1000U	Loadstar Sensors	600 gf	120 mgf	0.04 mgf (24 bit)	N/A
Model FD	Signer I & C	250 gf	125 mgf	N/A	N/A
Model FS	Signer I & C	30 gf	25 mgf	3 mgf	N/A



From Tables 1 and 2, only the Precision balances are able to certify an accuracy of one mgf or less and sustain a significant payload. It is noted that most of these balances include a hook, located under the balance, which allows measuring a tension force from beneath. This could be very useful for cable mechanisms.

The handheld sensors (Wagner, Omega, Imada) are clearly not able to provide the desired accuracy and are even not able to provide the desired resolution. The same comment applies to six-axis sensors.

The one-axis load cells, coupled with an appropriate acquisition system may have some potential. The sensor from FemtoTools is potentially very accurate, but the payload is very limited. Several experimental systems with force range and precision similar to FemtoTools exist [1][2]. The sensors from Loadstar and Signer have some potential, but the accuracy depends strongly on the acquisition system. This solution could require a significant amount of work. Indeed, many subsystems must be integrated and calibrated in order to obtain a functional system. However, because they are not integrated, it is possible to adjust the implementation to specific requirements.

In any case, the main challenge is to obtain a large payload to accuracy ratio. Generally, the cost of a force sensing device is dependent on this ratio and on the maximum resolution. The other challenge is to make sure that the measurement systems are immune to electromagnetic fields.

## **3 Proposed measurement systems**

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### **3.1 Commercial systems**

Among the commercial systems available, the most reliable force sensors are the commercial balances. Indeed, their behaviour is well documented and they have already been used in the existing setup. Their main drawback is the requirement to keep them horizontal. However, it is possible to alleviate this drawback. Firstly, it may be possible to use them even if they are slanted. Indeed, the horizontal requirement is related to the direction of gravity. Here, it is desired to measure the force along a cable. A test with a Scout Pro Ohaus balance demonstrated that it is still able to measure forces even if the balance is slanted at 45 degrees. Secondly, it is possible to include a cable arrangement that allows keeping the balances horizontal. This is discussed in the next section.

The other commercial systems with some potential are the one-axis load cells. They can be implemented with the help of data acquisition systems with sufficient resolution. In fact, because they are not fully integrated, it could be possible to use only a portion of their measurement range and to fit the required range of measurement with the acquisition system, which would allow to improve the accuracy. Indeed, it is not required to measure the weight of the platform but only the external forces applied on it, which is many orders of magnitude smaller. Unfortunately, there are many unknowns associated with such systems, such as the actual accuracy available, the quality of the calibration and the immunity to electromagnetic noise. Also, although the initial cost may seem small, the addition of an adequate acquisition system and the time needed to integrate the system could mean significant cost that cannot be neglected.

### **3.2 Photointerrupters with compliant mechanisms**

Another option is the use of photointerrupters combined with compliant mechanisms. Photointerrupters allow to measure very precisely small displacements. Since compliant mechanisms deform proportionally to the force applied on them, a properly located photointerrupter can indirectly measure the force applied. With a very soft compliant mechanism, very small forces can be measured. In the setup to be measured, it is not necessary to measure the weight of the platform, but only the external forces applied once the platform is suspended by the cables. Also, photointerrupters are non-contact measurement systems. Hence, compliant mechanisms can significantly deform without any measurement, then be measured in the range of interest. This strategy allows to maximize the accuracy. However, because of the significant deformations, the determination of the exact location of the components may be a challenge. Photointerrupters-based measurement systems have some potential and are relatively inexpensive, but similarly to the commercial one-axis load cells, there are many unknowns and the cost of the other components and integration must not be neglected.

## 4 Cable mechanisms

### 4.1 Analysis of cable mechanisms

The main advantage of using cable mechanisms is that they provide a link between the base and the platform that transmits a force along one axis and no moments, as long as the cable is sufficiently flexible in bending and sufficiently light. Since the forces to be transmitted are very small, it seems realistic to find such cables. Therefore, each sensor only needs to measure the force along one axis and does not support force along the other axes, as long as the sensor is properly aligned. A schematic illustration of cable mechanism is presented in Figure 4.1.

In order to correctly use cable mechanisms, the force applied on the platform must be properly transmitted to the sensors. This transmission is characterized by the Wrench Matrix  $\mathbf{W}$  of the mechanism. The kinematic modelling of the  $i$ th cable between the base and the platform is illustrated in Figure 4.2.

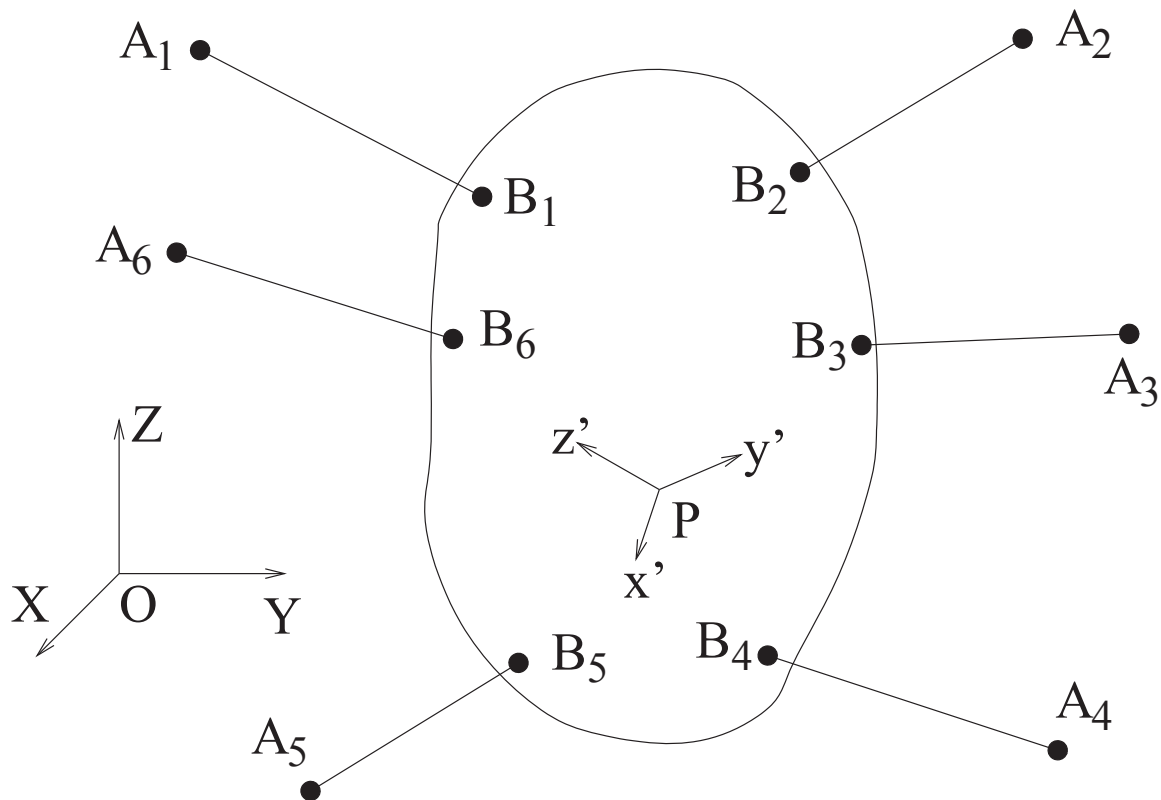


Figure 1: Schematic illustration of a cable mechanism.

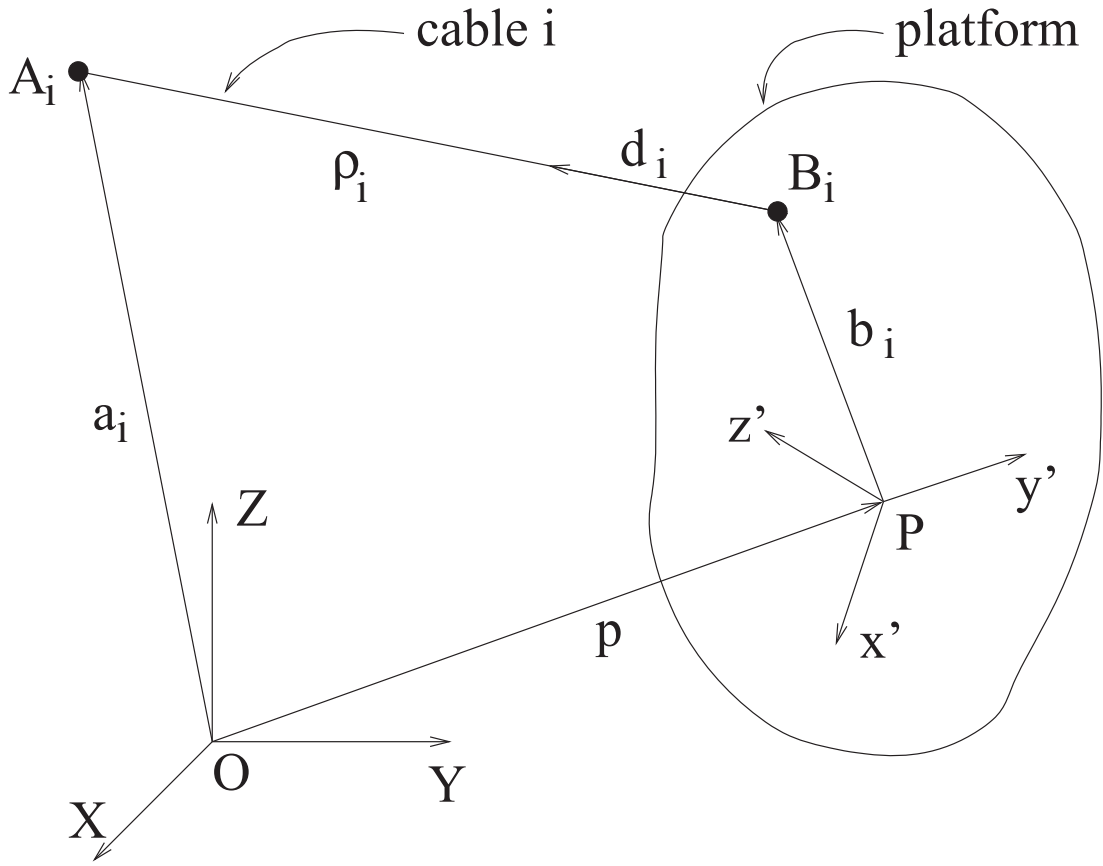


Figure 2: Kinematic model of the  $i$ th cable.

The vector from the base frame to the platform frame is noted  $\mathbf{p}$ . The attachment points of the  $n$  cables on the base are noted  $A_i$  and the attachment points on the platform are noted  $B_i$ . The vectors from the base frame to the attachment points on the base are noted  $\mathbf{a}_i$ . The vectors from the platform frame to the attachments points on the platform are noted  $\mathbf{b}_i$ . The length of the cables between their attachment points are noted  $\rho_i$ . The unit vector along the cable  $i$ , from the platform to the base, is obtained from:

$$\mathbf{d}_i = (\mathbf{a}_i - \mathbf{p} - \mathbf{b}_i) / \rho_i \quad (1)$$

If the attachment point on the platform is expressed in the platform frame as  $\mathbf{b}_i^{P_i}$ , then, it can be expressed in the base frame by:

$$\mathbf{b}_i = \mathbf{Q} \mathbf{b}_i^{P_i} \quad (2)$$

where  $\mathbf{Q}$  is the rotation matrix from the base to the platform. Since the platform is fixed in this application, we can assume that  $\mathbf{Q} = \mathbf{I}$ , where  $\mathbf{I}$  is the identity matrix.

The relationship between the tension in the cables  $\mathbf{t} = [t_1 \ t_2 \ \dots \ t_n]^T$  and the wrench applied on the platform  $\mathbf{w}_p = [f_x \ f_y \ f_z \ \tau_x \ \tau_y \ \tau_z]^T$  is given by:

$$\mathbf{W} \mathbf{t} = \mathbf{w}_p \quad (3)$$

where  $\mathbf{W} = [\mathbf{w}_1 \ \mathbf{w}_2 \ \dots \ \mathbf{w}_n]$  is the wrench transmission matrix and where

$$\mathbf{w}_i = [ \mathbf{d}_i^T (\mathbf{b}_i \times \mathbf{d}_i)^T ]^T \quad (4)$$

If it is desired to obtain the tensions in the cables for a given external force at the platform, then

$$\mathbf{W}^{-1} \mathbf{w}_p = \mathbf{t} \quad (5)$$

Then, the quality of transmission of the forces and moments between the base and the platform can be evaluated from the analysis of  $\mathbf{W}$ . Such an analysis can be performed using a mathematical tool such as Matlab. One of the indices that can be used to assess the quality of the force transmission is the condition number [3]. The condition number can be obtained as the ratio between the largest and the smallest singular values of  $\mathbf{W}$ , which are obtained from the Singular Value Decomposition (SVD) of  $\mathbf{W}$ . In fact, the study of the 6 singular values obtained is of interest. The SVD of a matrix is easily obtained using Matlab.

Since there are linear and rotational dimensions, the condition number is influenced by the units used. However, scaling factors can be used to compare forces and moments. In order to ensure consistency between the forces and the torques, the singular values associated with the torques are divided by the radius of the smallest sphere that envelopes the platform. This radius corresponds to the largest lever arm available from the centre of the platform in order to obtain the torques.

Another aspect that must be taken into account is that cables can only pull and not push. The mathematical analysis is much more complex to perform. Since the platform is fixed and since very well conditioned cable mechanisms are desired, it is generally possible to ensure that the cables will always be in tension from a visual inspection. If a mathematical analysis is required, the appropriate procedure can be found in [4].

#### 4.1.1 Special cable mechanism

In order to be able to use balances installed horizontally or to amplify the force measured, it is possible to include a special mechanism. The idea is to attach three cables together. A first cable is then attached to the platform, a second is attached to the force sensor and the third is attached

to the base. Once in tension, these three cables always lie in a plane. A cable mechanism that includes special mechanisms is illustrated in Figure 4.3. Also, a model of a special cable mechanism is illustrated in Figure 4.4. In practice, each of the cables, or some of the cables, of a cable mechanism could be replaced by a special cable mechanism.

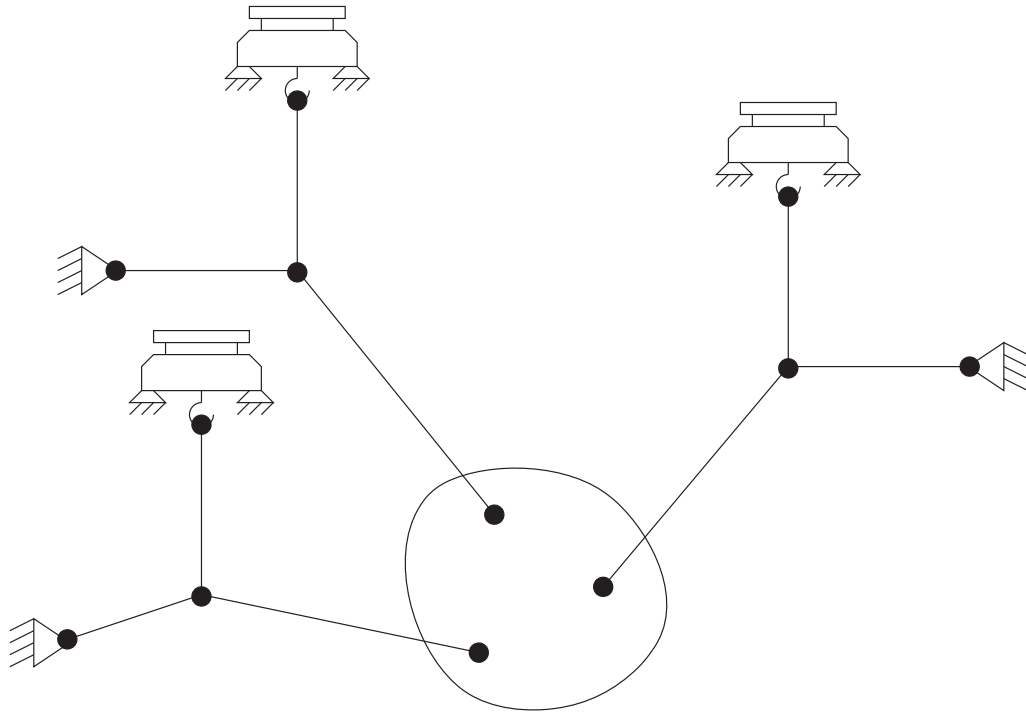
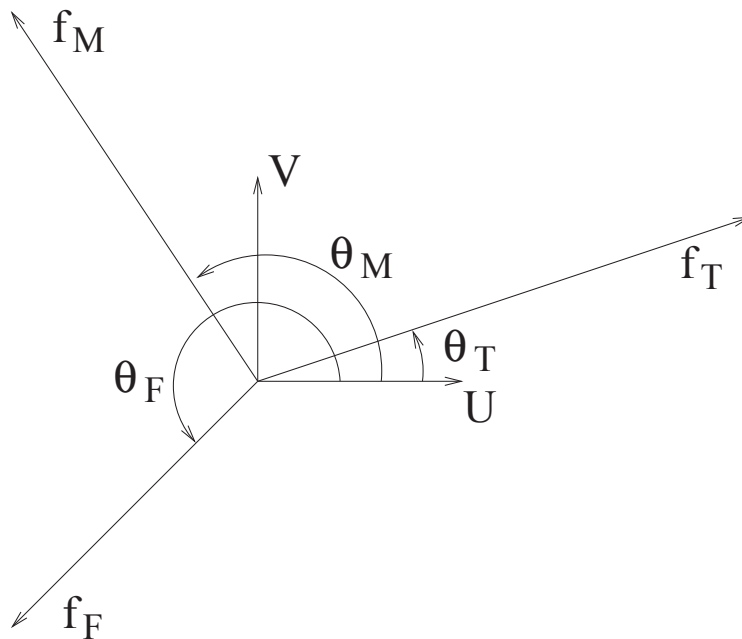


Figure 3: Schematic illustration of the use of special cable mechanisms.



*Figure 4: Kinematic model of the special cable mechanism.*

Knowing the geometry of the cable mechanism, the force in the cable attached to the measurement system  $\mathbf{f}_M$  can be related to the force in the cable attached to the platform  $\mathbf{f}_T$ . From the free body diagram of the special mechanism, the relationship between these forces can be obtained as:

$$\mathbf{f}_T = \mathbf{f}_M ( \sin(\theta_M) - \cos(\theta_M) \tan(\theta_F) ) / ( \cos(\theta_T) \tan(\theta_F) - \sin(\theta_T) ) \quad (6)$$

For the case for which the cable attached to the measurement system is vertical, then:

$$\mathbf{f}_T = \mathbf{f}_M / ( \cos(\theta_T) \tan(\theta_F) - \sin(\theta_T) ) \quad (7)$$

As a first example, if the cable attached to the measurement system is vertical, if the cable attached to the base is horizontal and if the cable attached to the platform is slanted at 45 degrees, then:

$$\mathbf{f}_T = 1.414 \mathbf{f}_M \quad (8)$$

As a second example, if the cable attached to the measurement system is vertical, if the cable attached to the base is almost vertical and slanted at 15 degrees and if the cable attached to the platform is horizontal, then:

$$\mathbf{f}_T = 0.268 \mathbf{f}_M \quad (9)$$

Interestingly, it is possible to adjust the ratio of force between what is applied to the platform and what is measured. Naturally, the position of the attachment points and the length of the cables must be well known in order to minimize the measurement errors. Also, large ratios should be used carefully because they can significantly amplify the effect of the errors on the location of the attachment points or the measurements.

#### **4.1.2 Use of a limited number of sensors**

The forces to be measured need the use of very precise force sensors, which are potentially very expensive. If budget is a limitation, it is possible to repeat an experiment many times and to attach the force sensors to different cables at each repetition. The cables that are not instrumented are fixed to the ground. In order to be successful, this technique requires a very good repeatability of the experiment.

## 4.2 Types of mechanisms

The geometry of the cable mechanism determines how the measurement of the forces is managed and how the platform is supported. These geometries can be classified in different types with common characteristics. Here these types are presented and illustrated with a typical example. Unless specified, the position of the platform is  $\mathbf{p} = [0, 0, 0]$ . Also, the scale of the mechanisms is representative of the expected sizes of the platform and workspace. For the illustration of the mechanisms, the circles represent the base and the polygon represents the platform.

### 4.2.1 Mechanisms constrained by gravity

This type of mechanism is constrained by gravity, which maintains the cables in tension. For a spatial system, 6 cables are needed. The advantage of this type of mechanism is its relative simplicity of implementation. Indeed, there is no need to very precisely position the attachment points or to preload the mechanism. If there is a sensor attached to each of the six cables, they allow to measure the three force components and three moment components applied on the platform. An example of such a mechanism with good properties is presented in Figure 4.5. The scaled singular values of the wrench matrix are  $[1.7303, 1.2296, 1.2268, 1.2249, 1.2233, 0.8701]$ . The resulting condition number is 0.503, which is very good.

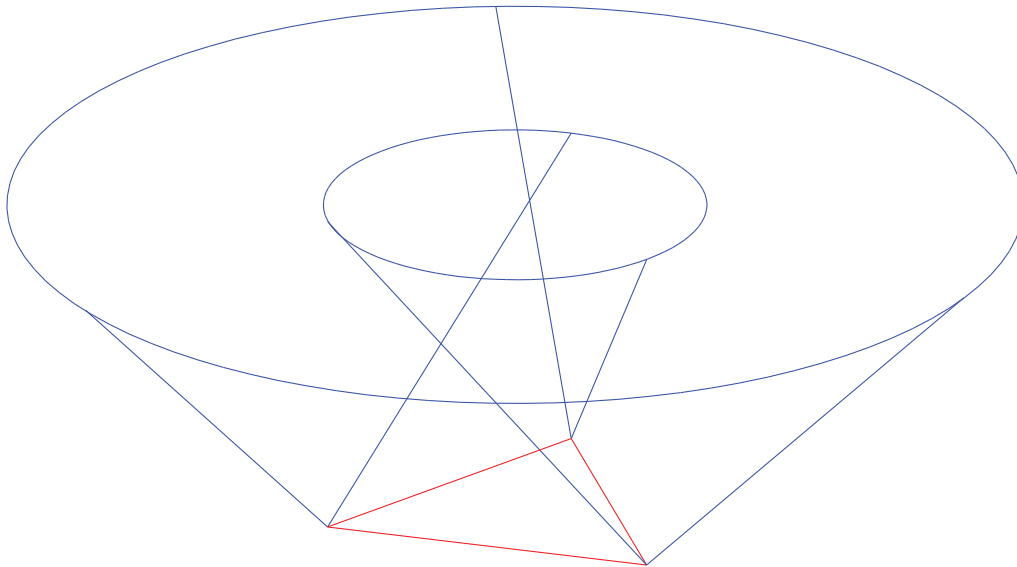


Figure 5: Gravity constrained cable mechanism that can measure the wrench components.



Table 3: Positions of the attachment points of the mechanism of Figure 4.5.

Cable number	$\mathbf{a}_i$	$\mathbf{b}_i^p$
1	(-0.087, 0.250, 0.173)	(0.000, 0.100, 0.000)
2	(0.087, -0.050, 0.173)	(0.000, 0.100, 0.000)
3	(0.260, -0.050, 0.173)	(0.087, -0.050, 0.000)
4	(-0.087, -0.050, 0.173)	(0.087, -0.050, 0.000)
5	(-0.173, -0.200, 0.173)	(-0.087, -0.050, 0.000)
6	(0.000, 0.100, 0.173)	(-0.087, -0.050, 0.000)

If the cables are arranged to be parallel by pairs, as illustrated in Figure 4.6, it is still possible to measure the six components. However, it is also possible to attach one sensor per pair of cables, oriented along their axis. Then three sensors can measure the forces applied. In order to maintain the orientation of the platform, the sensors must be able to sustain torques, at least along the axis orthogonal to the plane formed by two parallel cables. The advantage of such a mechanism is that fewer sensors are needed. An example of such a mechanism with good properties is presented in Figure 4.4. The scaled singular values of the wrench matrix are [1.7326, 1.5032, 1.5029, 1.5003, 0.8677, 0.8663]. The resulting condition number is 0.500, which is very good.

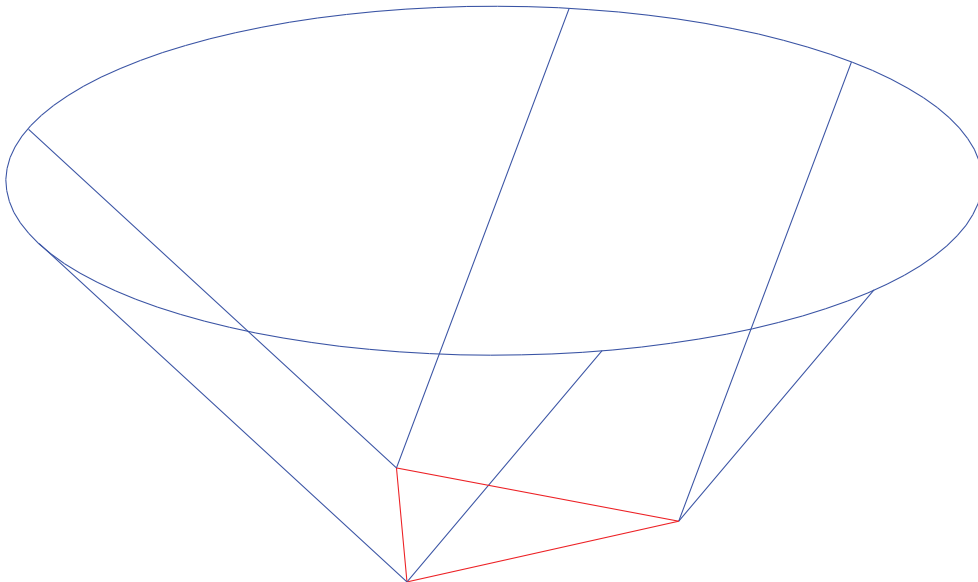


Figure 6: Gravity constrained cable mechanism that can include 6 sensors or only 3 sensors.

Table 4: Positions of the attachment points of the mechanism of Figure 4.6. The pairs of cables 2-3, 4-5 and 6-1 are parallel.

Cable number	$\mathbf{a}_i$	$\mathbf{b}_i^p$
1	(-0.173, 0.200, 0.200)	(0.000, 0.100, 0.000)
2	(0.173, 0.200, 0.200)	(0.000, 0.100, 0.000)
3	(0.260, 0.050, 0.200)	(0.087, -0.050, 0.000)
4	(0.087, -0.250, 0.200)	(0.087, -0.050, 0.000)
5	(-0.087, -0.250, 0.200)	(-0.087, -0.050, 0.000)
6	(-0.260, 0.050, 0.200)	(-0.087, -0.050, 0.000)

#### 4.2.2 General mechanisms with 7 or more cables

This type of mechanism is fully constrained (7 cables) or overconstrained (8 or more cables). For a spatial system, seven or more cables are included. The cable mechanism must be properly tensioned and internal forces are generated. The advantages of this type of mechanism are increased tolerance to external vibrations and redundant measurement of the forces (and moments) on the platform. The disadvantages are the needed overconstraint and the larger number of sensors. An example of such a mechanism with good properties is presented in Figure 4.7. The scaled singular values of the wrench matrix are [1.8411, 1.8411, 1.7498, 1.7498, 0.9686, 0.9686]. The resulting condition number is 0.526, which is very good.

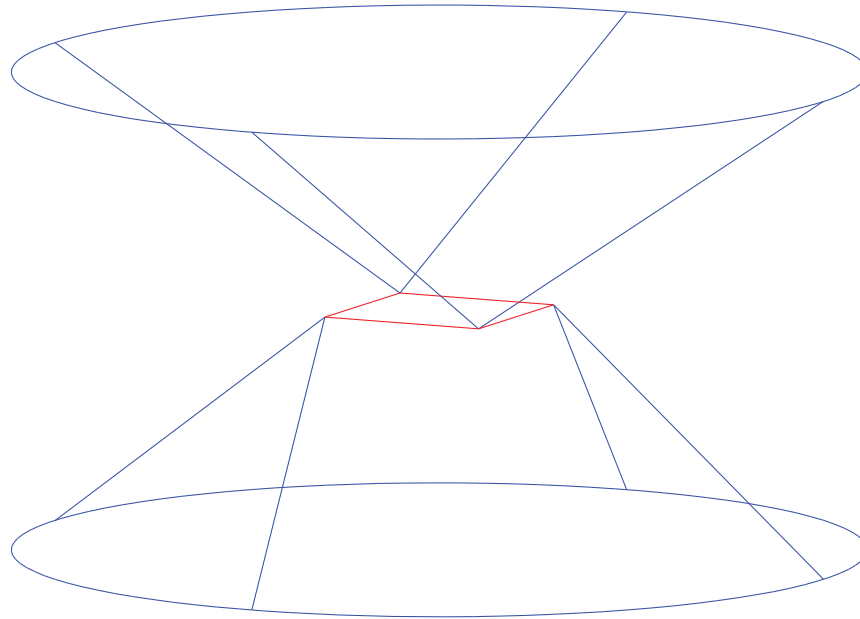


Figure 7: Overconstrained cable mechanism with 8 cables.

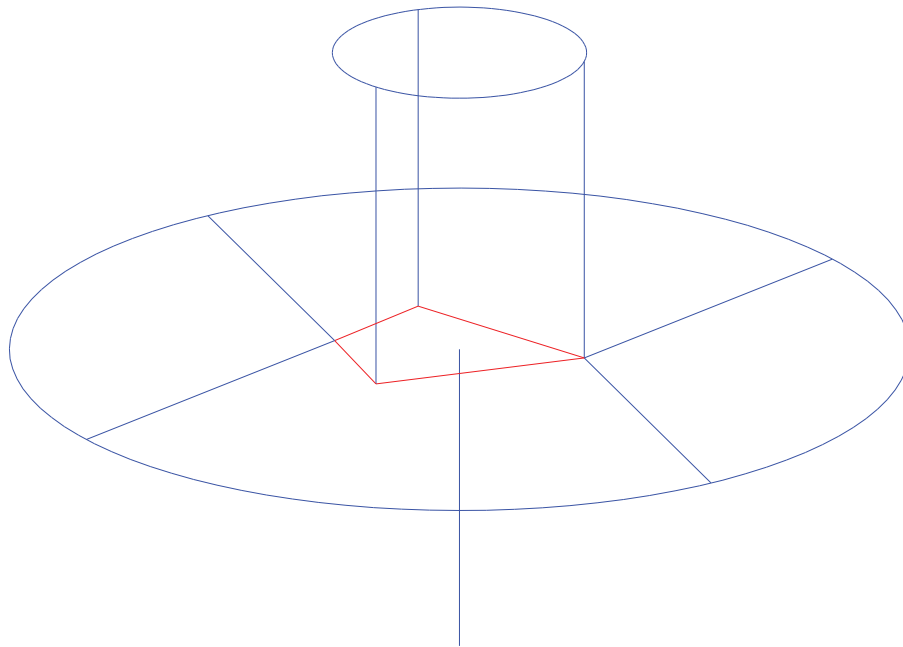
Table 5: Positions of the attachment points of the mechanism of Figure 4.7.

Cable number	$\mathbf{a}_i$	$\mathbf{b}_i^p$
1	(-0.250, -0.250, 0.250)	(-0.100, 0.000, 0.000)
2	(-0.250, 0.250, 0.250)	(-0.100, 0.000, 0.000)
3	(0.250, 0.250, 0.250)	(0.100, 0.000, 0.000)
4	(0.250, -0.250, 0.250)	(0.100, 0.000, 0.000)
5	(-0.250, -0.250, -0.250)	(0.000, -0.100, 0.000)
6	(-0.250, 0.250, -0.250)	(0.000, 0.100, 0.000)
7	(0.250, 0.250, -0.250)	(0.000, 0.100, 0.000)
8	(0.250, -0.250, -0.250)	(0.000, -0.100, 0.000)

### 4.2.3 Orthogonal architecture

Here, the idea is to use vertical cables in order to measure and sustain the platform along the vertical axis, then to measure the horizontal forces using a decoupled system. This allows to decouple the horizontal and vertical components and to use more sensitive sensors along the horizontal plane (compared to the sensors along the vertical axis).

Three vertical cables are needed to constrain the motions that are out of the horizontal plane, namely the vertical translation and the two horizontal rotations. Although it can be included, a fourth cable is not necessary since gravity keeps these cables in tension. However, along the horizontal plane, since there is no gravity, four cables are needed in order to fully constrain the two horizontal translations and the vertical rotation. An example of such a mechanism with good properties is presented in Figure 4.8. The scaled singular values of the wrench matrix are [2.0000, 1.7150, 1.7150, 1.2304, 1.2247, 1.0290]. The resulting condition number is 0.515, which is very good.



*Figure 8: Decoupled cable mechanism with 8 cables.*

Table 6: Positions of the attachment points of the mechanism of Figure 4.8.

Cable number	$\mathbf{a}_i$	$\mathbf{b}_i^p$
1	(0.000, 0.100, 0.250)	(0.000, 0.100, 0.000)
2	(0.087, -0.050, 0.250)	(0.087, -0.050, 0.000)
3	(-0.087, -0.050, 0.250)	(-0.087, -0.050, 0.000)
4	(0.000, 0.000, -0.250)	(0.000, 0.000, 0.000)
5	(-0.250, 0.250, 0.000)	(0.000, 0.100, 0.000)
6	(0.250, 0.250, 0.000)	(0.000, 0.100, 0.000)
7	(0.250, -0.250, 0.000)	(0.000, -0.100, 0.000)
8	(-0.250, -0.250, 0.000)	(0.000, -0.100, 0.000)

### 4.3 Examples of analysis

The following examples are included in order to illustrate how the wrench matrix can be used. The examples will also provide some hints for the design of the cable mechanism.

#### 4.3.1 Force at the sensors for a given weight of the platform

In order to know if the total force applied to the sensors is within their sensing range, the force applied by the weight of the platform must be converted into forces at the sensors. In other words, compute  $\mathbf{t}$  for a given value of  $\mathbf{w}_p$ . The mechanism of Figure 4.6 will be used here. From Table 4.2 and Equations (1), (2) and (4), the wrench matrix  $\mathbf{W}$  can be computed. Assume that the mass of the platform is 200 gf. Then,  $\mathbf{w}_p = [0 \ 0 \ 200 \ 0 \ 0 \ 0]$  gf in order to sustain this weight. Then, from equation (5), one obtains  $\mathbf{t} = [47.1195, 47.1195, 47.1195, 47.1405, 47.1405, 47.1195]$  gf. It can be observed that the weight of the platform is distributed among the sensors. Therefore, the capacity of the sensors could be reduced.

#### 4.3.2 Force at the platform for given measures at the sensors

In order to know what are the forces applied at the platform, the tensions measured by the sensors must be converted. In other words, compute  $\mathbf{w}_p$  for a given value of  $\mathbf{t}$ . In order to clarify the example, let's assume that the sensors have been tared and that the readings on the sensors are at

zero when the platform is suspended with no external forces. Assume that the measured variations of tension in the cables in response to the external forces are  $\mathbf{t} = [-5.8150, 10.5270, 2.3560, -5.7706, 10.4847, 2.3560]$  mgf. Since the variations of tension are measured, negative values are possible even if all the cables are in tension. Then Equation (3) can be used to compute the external forces. One obtains  $\mathbf{w}_p = [10 \ 0 \ 10 \ 0 \ 1 \ 0]$  mgf/mgf-m. It is observed that measured variations of tension can be smaller than the external forces. Therefore, this may require a better resolution of the sensors.

If only the external forces must be measured, it is possible to use only one sensor by pair of parallel cables. Knowing that the pairs of parallel cables are 2-3, 4-5 and 6-1, the forces read by the three sensors would be 12.883 mgf, 4.714 mgf and -3.459 mgf respectively. As a possible estimate of the values of  $\mathbf{t}$ , let's split equally the three measured values, which gives  $\mathbf{t} = [-1.7295 \ 6.4415 \ 6.4415 \ 2.357 \ 2.357 \ -1.7295]$  mgf. Using Equation (3), one obtains  $\mathbf{w}_p = [10 \ 0 \ 10 \ 0 \ -0.5 \ 0]$  mgf/mgf-m. The external forces applied are correctly obtained and valid. However, since the distribution of the tensions between the parallel cables could not be measured and have been assumed for the calculation, the external torques obtained are not correct and should not be used.

### 4.3.3 Example of a poorly conditioned wrench matrix

In order to study an example of a poorly conditioned wrench matrix, let's displace the platform of the mechanism of Figure 4.6 to  $\mathbf{p} = [0 \ 0 \ 0.18]$ . Since the platform is very close to the plane of attachment of the cables to the sensors, the cable mechanism is almost flat. The scaled singular values of the wrench matrix are  $[2.1156, 1.7261, 1.7251, 0.2439, 0.1498, 0.1492]$ . The resulting condition number is 0.071, which is very poor. Assume that only a vertical external force is applied, that is  $\mathbf{w}_p = [0 \ 0 \ 10 \ 0 \ 0 \ 0]$  mgf/mgf-m. Using equation (5), the resulting measured tensions are  $\mathbf{t} = [16.7351, 16.7351, 16.7351, 16.7498, 16.7498, 16.7351]$  mgf. Then, assume that only a horizontal external force is applied, that is  $\mathbf{w}_p = [10 \ 0 \ 0 \ 0 \ 0 \ 0]$  mgf/mgf-m. Using equation (5), the resulting measured tensions are  $\mathbf{t} = [-3.8750, 3.8750, 1.9291, -1.9308, 1.9308, -1.9291]$  mgf. It is clear that the transmission ratios are very different along the two directions. As a result, the errors measured along the vertical axis, that are significantly amplified, could degrade the quality of the measurement along the horizontal axis. Another problem related to poorly designed mechanisms will be discussed in the next section. As a final remark, the poorly designed mechanism presented here is only one example among a large range of possible poorly designed mechanisms, which could have very different behaviours.

### 4.3.4 Effects of position errors

In order to study the effect of position errors, let's estimate the error on the forces obtained for a given error of the estimation of the position of the end effector. Positioning the attachment points on the effector and on the base should be relatively precise. However, knowing the exact position of the platform may be a challenge. Consider that the theoretical position of the platform is perfectly centred but that its actual position is located a certain distance from the centre. In order

to visualize the effect, let's offset the platform 1 cm from the centred position, along the three main axes. Three different mechanisms will be studied: the mechanism of Figure 4.6, the example of a poorly conditioned mechanism and a mechanism with the same wrench matrix as the mechanism of figure 4.6, but with shorter cables.

For all three cases, let's assume that the real external wrench is  $\mathbf{w}_p = [10 \ 0 \ 10 \ 0 \ 1 \ 0]$  mgf/mgf-m. The exact measurements of  $\mathbf{t}$  with the platform in the offset positions are obtained from Equation (5). Then, using the centred, but incorrect, position of the platform for the calculation of the wrench matrix, an estimated  $\mathbf{w}_p$  is computed. The difference between the real and estimated  $\mathbf{w}_p$  will illustrate the effect of the position errors. The results for each of the cases are summarized in Tables 4.5 and 4.6.

Table 7: Measured tensions in the cables  $t$  (mgf).

Offset $\mathbf{p}$	Mechanism Fig 4.3	Poor design	Short cables
(0, 0, 0)	[-5.815 10.527 2.356 - 5.771 10.485 2.356]	[-6.431 39.901 -0.627 - 23.631 57.130 34.097]	[-5.781 10.503 2.351 - 5.771 10.485 2.351]
(0.01, 0, 0)	[-6.222 10.570 2.439 - 5.910 10.627 2.269]	[-8.730 40.058 0.323 - 24.627 58.168 34.564]	[-6.608 10.586 2.514 - 6.058 10.784 2.172]
(0, 0.01, 0)	[-5.880 10.535 2.426 - 6.019 10.369 2.695]	[-7.219 39.891 0.080 - 25.546 57.188 35.859]	[-5.913 10.528 2.491 - 6.275 10.239 3.022]
(0, 0, 0.01)	[-5.690 10.527 2.279 - 5.785 10.625 2.558]	[-8.954 75.644 -3.171 - 45.162 111.912 69.862]	[-5.536 10.528 2.197 - 5.812 10.794 2.771]

Table 8: Computed wrenches at the platform  $w_p$  (mgf/mgf-m).

Offset $p$	Mechanism Fig 4.3	Poor design	Short cables
(0, 0, 0)	[10 0 10 0 1 0]	[10 0 10 0 1 0]	[10 0 10 0 1 0]
(0.01, 0, 0)	[10.379 -0.132 9.742 - 0.026 1.007 0.000]	[12.532 -0.402 9.932 - 0.029 1.013 0.006]	[10.768 -0.277 9.472 - 0.052 1.015 0.001]
(0, 0.01, 0)	[9.880 0.382 9.992 - 0.006 1.025 -0.013]	[9.761 2.680 9.982 - 0.011 1.026 0.013]	[9.773 0.780 9.969 - 0.010 1.049 -0.026]
(0, 0, 0.01)	[9.753 -0.001 10.266 - 0.000 1.027 0.000]	[9.963 -0.001 19.925 - 0.000 1.993 0.000]	[9.515 0.000 10.569 0.000 1.057 -0.000]

Several observations can be made from Table 4.6:

For a fairly large error --- it should be easy to obtain an error of less than 1 cm --- the mechanism of Fig. 4.6 allows to obtain measurements with less than 4% of error in all the cases studied.

The errors obtained using the mechanism with short cables are approximately doubled compared to the reference mechanism. This shows that for a given absolute error, it is advantageous to use longer cables. However, a good design allows to keep the errors reasonable, even with short cables.

The errors obtained using the poorly designed cable mechanism can be critical, the largest errors being around 100%. Although the error on the position of the platform is large, this shows that huge potential errors can be obtained with a poorly designed mechanism.



## 5 Conclusion

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This report presented a preliminary study of a force measurement system. A survey of the existing force measurement systems is presented. The commercial precision balances are preferred because of their well-known behaviour. The one-axis load cells have some potential, but a significant amount of work is needed to implement them. Then, the static analysis of cable mechanisms is presented. Different types of cable mechanisms are suggested and described. Then, analysis examples are presented and guidelines are provided for the design of such mechanisms.

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It is desired to improve an existing measurement system that can measure very small forces (mgf) applied to a platform. The objective of this project is to propose new solutions in order to improve the measurement system and possibly measure additional force components. A survey of the existing force measuring systems is presented. Then, the preferred measurement systems are discussed. Cable mechanisms are introduced and their static analysis is presented. Several cable architectures are suggested and some examples of cable system analyses are presented.

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On veut améliorer un système existant qui peut mesurer de très petites forces (mgf) appliquées sur une plateforme. L'objectif de ce projet est de proposer de nouvelles solutions afin d'améliorer le système de mesure et possiblement de mesurer des composantes de force additionnelles. Une revue des appareils de mesure de force existants est présentée. Puis, les appareils de mesure préférés sont discutés. Les mécanismes à câbles sont introduits et leur analyse statique est présentée. Plusieurs architectures de câbles sont suggérées et quelques exemples de systèmes de câbles sont présentés.

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