## Imperfect Banking Competition and Macroeconomic Volatility: A DSGE Framework

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#### Abstract

This paper studies the impact of imperfect banking competition on aggregate fluctuations using a DSGE framework that features a Cournot banking sector. The paper highlights a new propagation mechanism of imperfect banking competition that operates via the dynamics of the expected marginal product of capital. Since capital is partly financed by bank loans, a higher expected return on capital implies that firms are more willing to borrow to invest in capital, making their capital and thus loan demand more inelastic. Market power enables banks to take advantage of the lower loan demand elasticity by charging a higher loan rate markup. Given that different shocks affect the dynamics of the expected return on capital differently, this paper finds that while the loan rate markup after a contractionary monetary policy shock increases and thus amplifies aggregate fluctuations, the impact of imperfect banking competition after a productivity shock is less clear and depends on the persistence of the shock.


Bank topics: Business fluctuations and cycles; Financial institutions; Interest rates
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## 1 Introduction

The banking sector tends to be dominated by a few large players. In most EU and OECD countries, the largest five banks account for more than $60 \%$ of the market. ${ }^{1}$ While there has been an increasing focus on the role of financial frictions in amplifying aggregate fluctuations since the global financial crisis, imperfect banking competition is often overlooked, as most of the existing literature tends to focus on agency problems between borrowers and lenders. Does imperfect banking competition matter for aggregate fluctuations? If so, via which channel?

With imperfect banking competition, banks tend to charge a loan rate markup above the marginal cost. If this loan rate markup endogenously changes over the business cycle, it can act as an internal propagation mechanism of macroeconomic shocks. ${ }^{2}$ A countercyclical loan rate markup can amplify the aggregate fluctuations by raising the cost of credit during bad times and thus reducing investment and output by more relative to the case of perfect banking competition; likewise, a procyclical loan rate markup would attenuate aggregate fluctuations. As a result, it is important to understand how banks endogenously adjust their loan rate markup in response to macroeconomic shocks.

This paper reveals a new mechanism behind the time-varying loan rate markup that operates through the general equilibrium dynamics in the expected marginal product of capital, after developing a dynamic stochastic general equilibrium (DSGE) model that features a Cournot banking sector. In the model, firms finance the purchase of capital using non-state-contingent bank loan contracts and their own net worth. When the expected marginal product of capital is higher, indicating better investment opportunities, firms are more willing to borrow to finance the purchase of capital. This tends to make their capital and thus loan demand less sensitive to the gross loan rate. Banks with market power respond to a more inelastic loan demand by raising their loan rate markup when the expected marginal product of capital is high.

Since different shocks affect the dynamics of the expected marginal product of capital differently, this paper finds that the impacts of imperfect banking competition on aggregate fluctuations are shock-specific. While the loan rate markup after a monetary policy shock

[^0]is countercyclical and thus amplifies aggregate fluctuations, it can be procyclical after a productivity shock, depending on the persistence of the shock.

More specifically, after a contractionary monetary policy shock, the real interest rate rises on impact and decreases over time. With perfect banking competition and no other frictions, the dynamics of the expected marginal product of capital mirror the dynamics of the real interest rate (equivalent to the real deposit or loan rate), as banks simply channel households' savings into financing firms' capital input for production. A higher expected marginal product of capital implies that firms are more willing to invest in capital, which makes their capital and thus loan demand more inelastic. Under imperfect banking competition, banks with market power will take advantage of the more inelastic loan demand by charging a higher loan rate markup. This countercyclical loan rate markup tends to raise the cost of credit and reduce firms' capital demand and hence output by more relative to the case of perfect banking competition. ${ }^{3}$ Although the static impact of a higher loan rate markup may be small, a persistently higher loan rate markup due to the dynamics of the expected marginal product of capital can greatly slow down the accumulation of capital and the output recovery.

By contrast, the impacts of imperfect banking competition after productivity shocks depend on the persistence of those shocks. After a persistent negative productivity shock, there is no clear amplification effect, as the loan rate markup is initially procyclical and later countercyclical. This is because there are two opposite forces that drive the expected marginal product of capital. On the one hand, a persistently low productivity tends to lower the expected marginal product of capital. On the other hand, there is upward pressure on the real interest rate to induce households to save for future consumption so that consumption can rise towards its steady state. A higher real interest rate tends to raise the expected marginal product of capital. As a result, the expected marginal product of capital falls during the early periods but then rises during later periods, making the capital and loan demand more elastic initially but more inelastic later on. If the negative productivity shock is fully transitory such that firms' productivity is only reduced in the current period, the downward pressure on the expected marginal product of capital due to the persistently low productivity disappears and the results are similar to a contractionary monetary policy shock.

This paper is closely related to the recent efforts in incorporating imperfect banking

[^1]competition into DSGE models. In the existing literature, imperfect banking competition is often modelled via monopolistic competition within the Dixit and Stiglitz (1977) framework (Airaudo and Olivero, 2019; Hafstead and Smith, 2012; Aliaga-Díaz and Olivero, 2010b; Dib, 2010; Gerali et al., 2010; Hülsewig, Mayer and Wollmershäuser, 2009). This monopolistic competition model implies a constant loan rate markup without further assumptions. ${ }^{4}$ There are a few papers that introduce an endogenously changing loan rate markup by using Salop's (1979) model of monopolistic competition (Andrés and Arce, 2012; Olivero, 2010), introducing large banks into the Dixit and Stiglitz (1977) framework (Cuciniello and Signoretti, 2015), or examining limit pricing strategy by banks to deter entry (Mandelman, 2011, 2010). This paper uses a Cournot banking sector to characterise oligopolistic competition among banks. The implication that the loan rate markup decreases in both the number of banks and the loan demand elasticity is similar to the former two approaches.

The main contribution of this paper is to reveal a new propagation mechanism of imperfect banking competition that operates via the dynamics of the expected marginal product of capital, which are embedded in a standard New Keynesian model with capital accumulation. While the existing frameworks study the role of imperfect banking competition in specific circumstances, i.e., when firms are financially constrained (Cuciniello and Signoretti, 2015; Andrés and Arce, 2012) or when banks practice limit pricing strategy to deter entry (Mandelman, 2011, 2010), they cannot explain how imperfect banking competition propagates macroeconomic shocks if borrowers are not financially constrained or if the competitive pressure from entry is minimal so that banks do not practice limit pricing to deter entry.

The propagation mechanism here differs from those papers largely because of the differences in modelling the loan demand and hence the interest rate elasticity of the loan demand. In this paper, the loan demand comes from firms' capital demand. Firms need to finance the purchase of new capital using non-state-contingent bank loan contracts and their own net worth. Due to a positive loan rate markup under imperfect banking competition, net worth becomes a cheaper source of financing than bank loans, and firms would invest all their net worth in capital and borrow the rest from banks. As a consequence, the loan demand elasticity is driven by the capital demand elasticity, which decreases in the expected marginal product of capital. Since different shocks affect the dynamics of the expected marginal product of capital differently, this paper also finds that the cyclicality of the loan rate markup is shock-specific, which differs from the existing literature (Cuciniello

[^2]and Signoretti, 2015; Andrés and Arce, 2012; Mandelman, 2011, 2010; Olivero, 2010). While the loan rate markup is countercyclical after a monetary policy shock, it can be procyclical after productivity shocks, depending on the persistence of those shocks.

This paper is related to a large literature that incorporates financial frictions into DSGE models. Most papers incorporate an agency problem between borrowers and lenders, which is often modelled by costly debt enforcement (e.g., Gertler, Kiyotaki and Queralto, 2012; Gertler and Karadi, 2011; Gertler and Kiyotaki, 2010; Iacoviello, 2005; Kiyotaki and Moore, 1997) or costly state verification (e.g., Christiano, Motto and Rostagno, 2014; Gilchrist, Ortiz and Zakrajsek, 2009; Bernanke, Gertler and Gilchrist, 1999; Carlstrom and Fuerst, 1997; Bernanke and Gertler, 1989). ${ }^{5}$ As borrowers' balance sheet conditions worsen during bad times, agency problems become more severe, and the resulting increased difficulty in obtaining external finance tends to amplify any shocks that adversely affect the balance sheet conditions (Bernanke, Gertler and Gilchrist, 1996).

While this paper focuses on the impact of imperfect banking competition in the absence of agency problems, I obtain similar qualitative results via the role of firms' net worth. ${ }^{6}$ This paper finds that an adverse shock that reduces firms' net worth can increase firms' reliance on bank loans, which tends to make the loan demand more inelastic, leading to a countercyclical loan rate markup that amplifies the effect of the adverse shock. In addition, while the literature often links the borrowers' or financial intermediaries' balance sheet conditions to the credit spread (e.g., Gertler and Karadi, 2011; Bernanke, Gertler and Gilchrist, 1999), ${ }^{7}$ this paper models the credit spread as a function of the degree of banking competition and the loan demand elasticity, where the latter depends on the capital demand elasticity as well as the borrowers' leverage ratio.

The remainder of the paper is structured as follows. Section 2 introduces the DSGE framework with a perfectly competitive banking sector, which is used as a benchmark in the dynamic analysis. Cournot banking competition is then introduced to replace the perfectly competitive banking sector, while the rest of the model set-up remains the same. Section 3 explains the calibration of model parameters. Section 4 shows the impulse responses of

[^3]some key variables after a monetary shock, a persistent productivity shock, and a transitory productivity shock. Section 5 discusses the robustness checks, and Section 6 concludes.

## 2 The Model

The model aims to show the effect of imperfect banking competition relative to perfect banking competition on aggregate fluctuations in a New Keynesian DSGE framework. Section 2.1 shows the model set-up for perfect banking competition, and Section 2.2 replaces the perfectly competitive banking sector with a Cournot banking sector.

### 2.1 Perfect Banking Competition Benchmark

There are six types of agents: households, firms, capital producers, retailers, banks, and a central bank. Households consume, supply labor to the firms, and decide how much to save via one-period non-state-contingent nominal bank deposit contracts or one-period risk-free nominal bonds. Perfectly competitive firms start with some net worth in the initial period, which is insufficient to finance the purchase of capital. In each period, they purchase new capital from capital producers for production in the following period, where capital is financed by net worth and one-period non-state-contingent nominal bank loan contracts. The wholesale good produced by firms cannot be consumed directly and is sold to monopolistically competitive retailers who then differentiate the wholesale good costlessly into different varieties. Each retailer uses the wholesale good as the only input to produce a different variety. The final consumption good is a composite CES (constant elasticity of substitution) bundle of all the varieties. Perfectly competitive capital producers buy the undepreciated capital from firms and consumption goods from retailers to produce new capital, which is sold back to the firms.

Banks offer two types of one-period contracts: deposit contracts and loan contracts. The contracts are denominated in nominal terms, which means they are not inflation-indexed and the borrowing or saving decisions are made on the basis of a preset contractual nominal loan or deposit rate. Assuming nominal bank deposits and one-period riskless nominal bonds are perfect substitutes to households under full deposit insurance, the gross nominal deposit rate must equal the gross nominal interest rate $R_{t}$ earned on the riskless nominal bond invested in period $t$. Following Andrés and Arce (2012), this paper abstracts away from the deposit insurance premium in the banking sector's optimization problem. Since banks are perfectly competitive, each of them takes the nominal loan rate as given and maximizes its profit with respect to the loan (or deposit) quantity. Assuming costless financial intermediation and no
expected default on loans, ${ }^{8}$ the gross nominal loan rate $R_{b, t}$ equals the gross nominal deposit rate $R_{t}$, which is controlled by the central bank.

### 2.1.1 Households

There is a continuum of identical infinitely-lived households of unit mass. The representative household maximizes the following expected utility:

$$
\begin{equation*}
E_{t} \sum_{s=0}^{\infty} \beta^{s}\left[\ln \left(c_{t+s}\right)+\phi \ln \left(1-l_{t+s}\right)\right] \tag{1}
\end{equation*}
$$

which depends on consumption $c$ and labor supply $l$, with $E_{t}$ being the expectation operator conditional on information in period $t$, and $\beta \in(0,1)$ being the subjective discount factor of the household. The total time endowment is normalised to 1 , so $\left(1-l_{t}\right)$ denotes the amount of period- $t$ leisure time, and $\phi>0$ is the relative utility weight on leisure.

In each period $t$, the household consumes $c_{t}$, saves $d_{t}$ in real (final consumption) terms, and supplies labor hours $l_{t}$. Assume there is zero net supply of risk-free nominal bonds, so in equilibrium, households hold only nominal bank deposits. The nominal deposits $d_{t-1}$ saved in period $t-1$ earn a gross nominal interest rate $R_{t-1}$ at the beginning of period $t$. Let $p_{t}$ denote the unit price of the final consumption good, then the gross inflation rate is $\pi_{t} \equiv \frac{p_{t}}{p_{t-1}}$. Assume households own the firms, retailers, and banks. Given the gross real interest earnings on deposits $\frac{R_{t-1} d_{t-1}}{\pi_{t}}$ at the beginning of period $t$, real labor income $w_{t} l_{t}$, and real dividends $\Pi_{t}^{F}, \Pi_{t}^{R}, \Pi_{t}^{C P}$, and $\Pi_{t}^{B}$ from firms, retailers, capital producers, and the banking sector, respectively, households decide how much to consume and save in period $t$. Hence, the representative household faces the following budget constraint:

$$
\begin{equation*}
c_{t}+d_{t}=\frac{R_{t-1} d_{t-1}}{\pi_{t}}+w_{t} l_{t}+\Pi_{t}^{F}+\Pi_{t}^{R}+\Pi_{t}^{C P}+\Pi_{t}^{B} \tag{2}
\end{equation*}
$$

Let $\lambda_{t}$ denote the Lagrange multiplier associated with the budget constraint or, equivalently, the marginal utility of consumption. The first order conditions with respect to consumption $c_{t}(3)$, labor supply $l_{t}(4)$, and bank deposits $d_{t}(5)$ are as follows:

$$
\begin{equation*}
\lambda_{t}=\frac{1}{c_{t}} \tag{3}
\end{equation*}
$$

[^4]\[

$$
\begin{gather*}
\frac{\phi}{1-l_{t}}=\lambda_{t} w_{t}  \tag{4}\\
\lambda_{t}=\beta E_{t}\left[\lambda_{t+1} \frac{R_{t}}{\pi_{t+1}}\right] \tag{5}
\end{gather*}
$$
\]

Equation (5) is the standard intertemporal Euler equation, which can also be written as:

$$
\begin{equation*}
1=E_{t}\left[\Lambda_{t, t+1} \frac{R_{t}}{\pi_{t+1}}\right] \tag{6}
\end{equation*}
$$

where $\Lambda_{t, t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_{t}}=\beta \frac{u^{\prime}\left(c_{t+1}\right)}{u^{\prime}\left(c_{t}\right)}$ is the stochastic discount factor in period $t$ for real payoffs in period $t+1$, with $u(c)=\ln (c)$.

### 2.1.2 Firms

A continuum of perfectly competitive firms of unit mass purchase new capital $k_{t-1}$ from capital producers at a real price $q_{t-1}$ in period $t-1$ for production in period $t$. Capital $k_{t-1}$ and labor $l_{t}$ hired from households are used to produce the wholesale good $y_{w, t}$ via a constant-returns-to-scale Cobb-Douglas production technology:

$$
\begin{equation*}
y_{w, t}=z_{t} k_{t-1}^{\alpha_{k}} l_{t}^{\alpha_{l}} \tag{7}
\end{equation*}
$$

where $\alpha_{k} \in(0,1)$ and $\alpha_{l} \in(0,1)$ are the output elasticities of physical capital and labor, respectively. The wholesale good produced in period $t$ is sold to retailers at a nominal price $p_{w, t}$, who then produce the final consumption good sold at a nominal price $p_{t}$. Productivity $z_{t}$ follows an $\mathrm{AR}(1)$ process in logs:

$$
\begin{equation*}
\ln z_{t}=\psi \ln z_{t-1}+e_{z, t} \tag{8}
\end{equation*}
$$

with $\psi \in(0,1)$ indicating the persistence of the process, and $e_{z, t}$ normally distributed with mean zero and variance $\sigma_{z}^{2}$.

Assume firms start with net worth $n_{0}$ in the initial period, which is insufficient to finance the capital $k_{0}$. Hence, firms borrow $b_{0}$ from banks to purchase the new capital $k_{0}$ at a real capital price $q_{0}$ from capital producers for production in the following period. Let $R_{b, t-1}$ denote the gross nominal loan rate in period $t-1$, then at the beginning of period $t$, the gross real loan interest payment is $\frac{R_{b, t-1} b_{t-1}}{\pi_{t}}$. In each period $t$, the net worth of a firm $j$ equals the sum of the realized output in terms of the final consumption units $\frac{y_{w, t}(j)}{x_{t}}$ and the revenue from selling the undepreciated capital stock to capital producers $q_{t}(1-\delta) k_{t-1}(j)$,
net of the real wage cost $w_{t} l_{t}(j)$ and the gross real loan interest payment $\frac{R_{b, t-1} b_{t-1}(j)}{\pi_{t}}$ :

$$
\begin{equation*}
n_{t}(j)=\frac{y_{w, t}(j)}{x_{t}}-w_{t} l_{t}(j)+q_{t}(1-\delta) k_{t-1}(j)-\frac{R_{b, t-1} b_{t-1}(j)}{\pi_{t}} \tag{9}
\end{equation*}
$$

where $x_{t} \equiv \frac{p_{t}}{p_{w, t}}$ denotes the markup of the price of the final consumption good over the price of the wholesale good. After the net worth $n_{t}(j)$ is realized and before choosing the capital $k_{t}(j)$ for production in period $t+1$, assume there is an exogenous death shock such that the firm exits with a probability $\varphi \in(0,1)$, in which case the firm transfers its net worth to households as dividend payments. This assumption ensures that firms cannot quickly accumulate enough net worth to self-finance the purchase of capital. A surviving firm $j$ chooses the amount of capital $k_{t}(j)$ to purchase at a real price $q_{t}$. Given its net worth $n_{t}(j)$, the firm needs to borrow $b_{t}(j)$ from banks:

$$
\begin{equation*}
b_{t}(j)=q_{t} k_{t}(j)-n_{t}(j) \tag{10}
\end{equation*}
$$

Since bank loans are assumed to be non-state-contingent, firms' net worth is introduced as a buffer to absorb any ex post losses. If firms had no net worth, a negative productivity shock that lowers the realized output could cause firms to go bankrupt. To determine firms' loan demand and thus examine the interest rate elasticity of the loan demand, which is a key component for the loan rate markup under imperfect banking competition, I assume that capital is financed by the firm's net worth and bank loans. ${ }^{9}$ This assumption is innocuous when the lending rate and the saving rate are identical, in which case it does not matter whether firms use net worth or debt to finance the capital. By contrast, when the lending rate is higher than the saving rate due to imperfect banking competition, net worth becomes a cheaper source of financing compared to bank loans. In this case, firms prefer to invest all their net worth into capital and have incentives to delay consumption until exit to accumulate net worth over time.

Let $\Lambda_{t, t+s} \equiv \beta^{s} \frac{u^{\prime}\left(c_{t+s}\right)}{u^{\prime}\left(c_{t}\right)}$ denote the stochastic discount factor, since households own the firms. Each surviving firm $j$ in period $t$ chooses its capital $k_{t}(j)$ and labor $l_{t}(j)$ to maximize the expected discounted terminal net worth:

$$
\begin{equation*}
E_{t} \sum_{\tau=0}^{\infty} \varphi(1-\varphi)^{\tau} \Lambda_{t, t+1+\tau} n_{t+1+\tau}(j) \tag{11}
\end{equation*}
$$

[^5]Since firms are financially unconstrained, their net worth does not affect their optimal choices of capital and labor. Hence, I neglect the subscript $j$ in the first order conditions with respect to capital (12) and labor (13):

$$
\begin{gather*}
E_{t} \Lambda_{t, t+1}\left[\frac{z_{t+1} \alpha_{k} k_{t}^{\alpha_{k}-1} l_{t+1}^{\alpha_{l}}}{x_{t+1}}+q_{t+1}(1-\delta)-\frac{R_{b, t} q_{t}}{\pi_{t+1}}\right]=0  \tag{12}\\
\frac{z_{t} \alpha_{l} k_{t-1}^{\alpha_{k}} l_{t}^{\alpha_{l}-1}}{x_{t}}=w_{t} \tag{13}
\end{gather*}
$$

Rearranging (12), the total demand for capital $k_{t}=\int k_{t}(j) d j$ for a given level of labor hours is:

$$
\begin{equation*}
k_{t}=\left(\frac{E_{t} \Lambda_{t, t+1}\left[\frac{R_{b, t} q_{t}}{\pi_{t+1}}-q_{t+1}(1-\delta)\right]}{E_{t} \Lambda_{t, t+1}\left[\frac{z_{t+1} \alpha_{k} l_{t+1}^{\alpha}}{x_{t+1}}\right]}\right)^{-\frac{1}{1-\alpha_{k}}} \tag{14}
\end{equation*}
$$

which decreases in the gross loan rate $R_{b, t}$ set by the banking sector. Firms choose the optimal capital after knowing their net worth $n_{t}(j)$ in period $t$ and borrow the difference between the two, $b_{t}(j)$. As a result, the market loan demand $b_{t}=\int b_{t}(j) d j$ is the difference between the value of optimal capital and the aggregate net worth $n_{t}=\int n_{t}(j) d j$ :

$$
\begin{equation*}
b_{t}=q_{t} k_{t}-n_{t} \tag{15}
\end{equation*}
$$

Assume in each period $t$ the fraction $\varphi$ of exiting firms are replaced by new firms, each with an initial net worth of $\frac{\omega}{\varphi} q_{t} k_{t-1}$ transferred from households, where $\omega \in(0,1)$. The aggregate net worth is the sum of the net worth of the surviving firms $(1-\varphi) \int n_{t}(j) d j$ and the net worth of the new entering firms $\omega q_{t} k_{t-1}$ :

$$
\begin{equation*}
n_{t}=(1-\varphi)\left[\frac{y_{w, t}}{x_{t}}-w_{t} l_{t}+q_{t}(1-\delta) k_{t-1}-\frac{R_{b, t-1} b_{t-1}}{\pi_{t}}\right]+\omega q_{t} k_{t-1} \tag{16}
\end{equation*}
$$

where $\omega$ helps determine the steady state firm leverage ratio $\frac{k}{n}$. The net dividend received by households $\Pi_{t}^{F}$ is the total net worth of the exiting firms net of the transfer to the new entering firms. When the production function exhibits constant returns to scale, the firm's steady state net worth is $n=(1-\varphi) \frac{R_{b} n}{\pi}+\omega q k$ by substituting the first order conditions (12) and (13) into (16). Positive values of $\varphi$ and $\omega$ ensure a positive steady state net worth $n=\frac{\omega q k}{1-(1-\varphi) \frac{R_{b}}{\pi}}$ that can absorb ex post losses.

### 2.1.3 Capital Producers

Perfectly competitive capital producers purchase undepreciated capital $(1-\delta) k_{t-1}$ at the real price $q_{t}$ from firms and $i_{t}$ units of final consumption goods from retailers to produce new capital $k_{t}$ at the end of period $t$ :

$$
\begin{equation*}
k_{t}=i_{t}+(1-\delta) k_{t-1} \tag{17}
\end{equation*}
$$

where $i_{t}$ is also gross investment. The new capital produced will be sold back to the entrepreneur at the real price $q_{t}$, which will be used to produce the wholesale good in period $t+1$. Following Christiano, Eichenbaum and Evans (2005), assume capital producers face investment adjustment costs that depend on the gross growth rate of investment $\frac{i_{t}}{i_{t-1}}$. Assume old capital can be converted one-to-one into new capital and a quadratic unit investment adjustment cost $f\left(\frac{i_{t}}{i_{t-1}}\right)=\frac{\chi}{2}\left(\frac{i_{t}}{i_{t-1}}-1\right)^{2}$ is only incurred in the production of new capital when using the final consumption good as the input, where $f(1)=f^{\prime}(1)=0, f^{\prime \prime}(1)>0$ and $\chi>0$. This specification of the adjustment cost implies that fewer units of new capital would be produced from one unit of investment whenever $\frac{i_{t}}{i_{t-1}}$ deviates from its steady state value of one and the parameter $\chi$ reflects the magnitude of the cost.

Hence, the representative capital producer chooses the gross investment level $i_{t}$ to maximize the sum of the expected discounted future profits made from the sales revenue of new capital $q_{t} k_{t}$ net of the input cost $\left[q_{t}(1-\delta) k_{t-1}+i_{t}\right]$ and the investment adjustment cost $f\left(\frac{i_{t}}{i_{t-1}}\right) i_{t}$ :

$$
\begin{equation*}
E_{t} \sum_{s=0}^{\infty} \Lambda_{t, t+s}\left[q_{t} k_{t}-q_{t}(1-\delta) k_{t-1}-i_{t}-\frac{\chi}{2}\left(\frac{i_{t}}{i_{t-1}}-1\right)^{2} i_{t}\right] \tag{18}
\end{equation*}
$$

where $\Lambda_{t, t+s} \equiv \beta^{s} \frac{u^{\prime}\left(c_{t+s}\right)}{u^{\prime}\left(c_{t}\right)}$ is the stochastic discount factor, since households own the capital producers. Using (17), the objective function (18) can be simplified to:

$$
\begin{equation*}
E_{t} \sum_{s=0}^{\infty} \Lambda_{t, t+s}\left[\left(q_{t}-1\right) i_{t}-\frac{\chi}{2}\left(\frac{i_{t}}{i_{t-1}}-1\right)^{2} i_{t}\right] \tag{19}
\end{equation*}
$$

Taking the first order condition with respect to investment $i_{t}$ gives the following expression for the real price of capital:

$$
\begin{equation*}
q_{t}=1+\frac{\chi}{2}\left(\frac{i_{t}}{i_{t-1}}-1\right)^{2}+\chi \frac{i_{t}}{i_{t-1}}\left(\frac{i_{t}}{i_{t-1}}-1\right)-\chi E_{t}\left[\Lambda_{t, t+1}\left(\frac{i_{t+1}}{i_{t}}\right)^{2}\left(\frac{i_{t+1}}{i_{t}}-1\right)\right] \tag{20}
\end{equation*}
$$

In the steady state, the real price of capital $q$ is one, since $i_{t+1}=i_{t}=i_{t-1}$. Any real
profits $\Pi_{t}^{C P}$ (which only arise outside the steady state) are rebated to the households, where $\Pi_{t}^{C P}=\left(q_{t}-1\right) i_{t}-\frac{\chi}{2}\left(\frac{i_{t}}{i_{t-1}}-1\right)^{2} i_{t}$. To focus on the role of imperfect banking competition, the investment adjustment cost parameter $\chi$ is set to zero in the baseline analysis, so that the real capital price $q_{t}$ remains one throughout.

### 2.1.4 Retailers

To analyse monetary policy shocks, it is essential to introduce nominal price rigidity, which makes monetary policy have real effects. Nominal rigidity is introduced by assuming the retailers are monopolistically competitive and set prices à la Calvo (1983).

A continuum of retailers of unit mass, indexed by $j$, buy the wholesale good at a nominal price $p_{w, t}$ from entrepreneurs and use it as the only input to produce differentiated retail goods costlessly. Assume that one unit of the wholesale good can produce one unit of the differentiated product, so the marginal cost of production is the real price of the wholesale good $\frac{p_{w, t}}{p_{t}}$. Each retailer $j$ produces a different variety $y_{t}(j)$ and charges a nominal price $p_{t}(j)$ for the differentiated product. The output of the final consumption good $y_{t}$ is a constant elasticity of substitution (CES) composite of all the different varieties produced by the retailers, using the Dixit and Stiglitz (1977) framework:

$$
\begin{equation*}
y_{t}=\left[\int_{0}^{1} y_{t}(j)^{\frac{\epsilon-1}{\epsilon}} d j\right]^{\frac{\epsilon}{\epsilon-1}} \tag{21}
\end{equation*}
$$

where $\epsilon>1$ is the elasticity of intratemporal substitution between different varieties. Given the aggregate output index $y_{t}$, it can be calculated from the cost minimization problem of the buyers of the final consumption good that each retailer $j$ faces a downward-sloping demand curve:

$$
\begin{equation*}
y_{t}(j)=\left[\frac{p_{t}(j)}{p_{t}}\right]^{-\epsilon} y_{t} \tag{22}
\end{equation*}
$$

It can be shown that the aggregate consumption-based price index is:

$$
\begin{equation*}
p_{t}=\left[\int_{0}^{1} p_{t}(j)^{1-\epsilon} d j\right]^{\frac{1}{1-\epsilon}} \tag{23}
\end{equation*}
$$

which is defined as the minimum expenditure to obtain one unit of consumption $y_{t}$ in the cost minimization problem for the final output users.

Each retailer $j$ sets its own price $p_{t}(j)$ taking the aggregate price $p_{t}$ and the demand curve (22) as given. Under Calvo pricing, each retailer $j$ is only allowed to change its price $p_{t}(j)$ in period $t$ with probability $(1-\theta)$. The probability of price adjustment is independent
of the time since the last adjustment, so in each period, a fraction $(1-\theta)$ of retailers reset their prices whereas a fraction $\theta$ of retailers keep their prices fixed. Hence, $\theta \in(0,1)$ reflects the degree of price stickiness. Let $p_{t}^{*}(j)$ denote the optimal reset price in period $t$; then the corresponding demand facing retailer $j$ who adjusted its price in period $t$ but cannot adjust its price in period $t+s$ is:

$$
\begin{equation*}
y_{t+s}^{*}(j)=\left[\frac{p_{t}^{*}(j)}{p_{t+s}}\right]^{-\epsilon} y_{t+s} \tag{24}
\end{equation*}
$$

Retailer $j$ chooses $p_{t}^{*}(j)$ to maximize the expected discounted value of real profits while its price is kept fixed at $p_{t}^{*}(j)$ :

$$
\begin{equation*}
\sum_{s=0}^{\infty} \theta^{s} E_{t}\left[\Lambda_{t, t+s}\left\{\frac{p_{t}^{*}(j)}{p_{t+s}} y_{t+s}^{*}(j)-\frac{1}{x_{t+s}} y_{t+s}^{*}(j)\right\}\right] \tag{25}
\end{equation*}
$$

subject to the demand function (24), where $\Lambda_{t, t+s} \equiv \beta^{s} \frac{u^{\prime}\left(c_{t+s}\right)}{u^{\prime}\left(c_{t}\right)}$ is the stochastic discount factor, since households own the retailers, $\theta^{s}$ is the probability that $p_{t}^{*}(j)$ would remain fixed for $s$ periods, and $\frac{1}{x_{t+s}}=\frac{p_{w, t+s}}{p_{t+s}}$ is the price of the wholesale good in terms of the consumption units or the real marginal cost of production in period $t+s$. Taking the first order condition to solve for $p_{t}^{*}(j)$ gives the following optimal pricing equation:

$$
\begin{equation*}
p_{t}^{*}(j)=\frac{\epsilon}{\epsilon-1} \frac{\sum_{s=0}^{\infty}(\beta \theta)^{s} E_{t}\left[u^{\prime}\left(c_{t+s}\right) x_{t+s}^{-1} p_{t+s}^{\epsilon} y_{t+s}\right]}{\sum_{s=0}^{\infty}(\beta \theta)^{s} E_{t}\left[u^{\prime}\left(c_{t+s}\right) p_{t+s}^{\epsilon-1} y_{t+s}\right]} \tag{26}
\end{equation*}
$$

The derivation is shown in Appendix B.1. In a symmetric equilibrium, all the retailers that adjust their prices in period $t$ will set the same optimal price, such that $p_{t}^{*}(j)=p_{t}^{*}$. It is proved in Appendix B. 2 that the aggregate price level evolves as follows:

$$
\begin{equation*}
p_{t}^{1-\epsilon}=\theta p_{t-1}^{1-\epsilon}+(1-\theta)\left(p_{t}^{*}\right)^{1-\epsilon} \tag{27}
\end{equation*}
$$

which is independent of the heterogeneity of the retailers due to the convenience of the Calvo assumption. With randomly chosen price-adjusting retailers and the large number of retailers, there is no need to keep track of each retailer's price evolution.

Since there is a one-to-one conversion rate from the wholesale good to the differentiated retail good, in equilibrium the supply of wholesale good output $y_{w, t}$ is equal to the demand $y_{t}(j)$ over the entire unit interval of retailers $j$. Using retailer $j$ 's individual demand function (22), the wholesale good output can be expressed as:

$$
\begin{equation*}
y_{w, t}=\int_{0}^{1} y_{t}(j) d j=y_{t} \int_{0}^{1}\left[\frac{p_{t}(j)}{p_{t}}\right]^{-\epsilon} d j \tag{28}
\end{equation*}
$$

This shows that the final consumption good output $y_{t}$ differs from the wholesale good output $y_{w, t}$ by a factor of the price dispersion $\int_{0}^{1}\left[\frac{p_{t}(j)}{p_{t}}\right]^{-\epsilon} d j$. In a zero-inflation steady state, the price dispersion is one and the final output $y_{t}$ would equal the wholesale good output $y_{w, t}$. Using (28) and letting $f_{3, t} \equiv \int_{0}^{1}\left[\frac{p_{t}(j)}{p_{t}}\right]^{-\epsilon} d j$ denote the price dispersion, the real profit $\Pi_{t}^{R}$ made by the retailers is:

$$
\begin{equation*}
\Pi_{t}^{R}=y_{t}-\frac{y_{w, t}}{x_{t}}=\left(\frac{1}{f_{3, t}}-\frac{1}{x_{t}}\right) y_{w, t} \tag{29}
\end{equation*}
$$

which will be rebated back to the households. The recursive formulation of the price dispersion used for numerical computation and the derivation for $\Pi_{t}^{R}$ are shown in Appendix B.3.

### 2.1.5 Banking Sector

Assume there is a continuum of banks of mass one, indexed by $j$, which are perfectly competitive with no price-setting power. The gross nominal interest rate $R_{t}$ is controlled by the central bank and is thus taken as given. Following Andrés and Arce (2012) and Cuciniello and Signoretti (2015), assume all bank profits $\Pi_{t}^{B}(j)$ are distributed as dividends to households each period, so $\Pi_{t}^{B}=\sum_{j} \Pi_{t}^{B}(j)$. In addition, assume there is zero bank capital, so bank loans (assets) equal the deposits (liabilities):

$$
\begin{equation*}
b_{t}(j)=d_{t}(j) \tag{30}
\end{equation*}
$$

In each period $t$, the total outflow of funds, consisting of the dividend payment to households $\Pi_{t}^{B}(j)$, loans granted to firms $b_{t}(j)$, and the gross real deposit interest payments to households $\frac{R_{t-1} d_{t-1}(j)}{\pi_{t}}$, equals the total inflow of funds from the deposits saved by households $d_{t}(j)$ and the gross real loan interest payments received from firms $\frac{R_{b, t-1} b_{t-1}(j)}{\pi_{t}}$. Assuming costless financial intermediation and no default on loans, ${ }^{10}$ each bank $j$ faces the following budget constraint:

$$
\begin{equation*}
\Pi_{t}^{B}(j)+b_{t}(j)+\frac{R_{t-1} d_{t-1}(j)}{\pi_{t}}=d_{t}(j)+\frac{R_{b, t-1} b_{t-1}(j)}{\pi_{t}} \tag{31}
\end{equation*}
$$

Each bank $j$ chooses the units of loans $b_{t}(j)$ and the units of deposits $d_{t}(j)$ to maximize the sum of the expected discounted value of real profits:

$$
\begin{equation*}
E_{t} \sum_{s=0}^{\infty} \Lambda_{t, t+s} \Pi_{t}^{B}(j) \tag{32}
\end{equation*}
$$

[^6]subject to the balance sheet identity (30) and the budget constraint in real terms (31). Substituting (30) into (31) simplifies the bank's real profit $\Pi_{t}^{B}(j)$ to:
\[

$$
\begin{equation*}
\Pi_{t}^{B}(j)=\frac{1}{\pi_{t}}\left(R_{b, t-1}-R_{t-1}\right) b_{t-1}(j) \tag{33}
\end{equation*}
$$

\]

Taking the first order condition of (32) with respect to $b_{t}(j)$ gives:

$$
\begin{equation*}
E_{t}\left[\Lambda_{t, t+1} \frac{1}{\pi_{t+1}}\left(R_{b, t}-R_{t}\right)\right]=0 \tag{34}
\end{equation*}
$$

Since $\Lambda_{t, t+1}>0$ and $\pi_{t+1} \equiv \frac{p_{t+1}}{p_{t}}>0$, the nominal loan interest margin $\left(R_{b, t}-R_{t}\right)$ is zero. With perfect banking competition and no expected default on loans, the market-determined gross nominal loan rate $R_{b, t}$ equals $R_{t}$.

### 2.1.6 Central Bank

Suppose monetary policy is implemented by a Taylor rule with interest rate smoothing, which responds to both the deviation of the gross inflation rate from the inflation target $\pi$ and the deviation of output from its steady state $y$. The central bank controls the gross nominal interest rate $R_{t}$ on risk-free bonds and bank deposits, following the Taylor rule specification below:

$$
\begin{equation*}
R_{t}=\rho_{r} R_{t-1}+\left(1-\rho_{r}\right)\left[R+\kappa_{\pi}\left(\pi_{t}-\pi\right)+\kappa_{y}\left(y_{t}-y\right)\right]+e_{r, t} \tag{35}
\end{equation*}
$$

where variables without time subscript represent steady state values, and $e_{r, t}$ is a monetary policy shock, which is a white noise process with zero mean and variance $\sigma_{r}^{2}$. The coefficient $\rho_{r} \in[0,1]$ is the interest rate smoothing parameter, and $\kappa_{\pi}$ and $\kappa_{y}$ are non-negative feedback parameters that reflect the sensitivity of the interest rate to output and inflation deviations. Due to interest rate smoothing, this policy rule implies a partial adjustment of $R_{t}$. The policy rate $R_{t}$ is a weighted average of the lagged nominal interest rate $R_{t-1}$ and the current target rate, which depends positively on the deviation of inflation from its target and the deviation of output from its steady state value.

To focus on the model mechanism, I assume the Taylor rule takes the simplest possible form by setting $\rho_{r}$ and $\kappa_{y}$ to zero in the baseline analysis. ${ }^{11}$ Letting $R_{r, t}$ denote the gross real interest rate, the relation between the nominal and real interest rates is given by the Fisher equation:

$$
\begin{equation*}
R_{r, t}=E_{t}\left[\frac{R_{t}}{\pi_{t+1}}\right] \tag{36}
\end{equation*}
$$

[^7]
### 2.2 Imperfect Banking Competition

This section replaces the perfectly competitive banking sector with an imperfectly competitive banking sector in the model set-up described in Section 2.1. As the banking sector tends to be dominated by a few large players, a Cournot banking sector is used to characterise oligopolistic competition and capture the market power possessed by banks. In a Cournot equilibrium, banks' quantity-setting decisions affect the market loan rate. Assume now there are $N$ banks in the economy, indexed by $j$, which operate under Cournot competition. Each individual bank takes the quantities of loans chosen by the other banks $m \neq j$ as given. However, it takes into account the effect of its choice $b_{t}(j)$ on the (partial) equilibrium in the loan market, through the total loan quantity $b_{t}$ and the loan rate $R_{b, t}$, but it ignores general equilibrium effects and takes other prices and aggregate quantities as given. Each bank $j$ sets the quantity of loans $b_{t}(j)$ to maximize the sum of the present discounted value of future profits:

$$
\begin{equation*}
E_{t} \sum_{s=0}^{\infty} \Lambda_{t, t+s} \Pi_{t}^{B}(j) \tag{37}
\end{equation*}
$$

where

$$
\begin{equation*}
\Pi_{t}^{B}(j)=\frac{1}{\pi_{t}}\left[R_{b, t-1}\left(b_{t-1}(j)+\sum_{m \neq j} b_{t-1}(m)\right)-R_{t-1}\right] b_{t-1}(j) \tag{38}
\end{equation*}
$$

The real profit $\Pi_{t}^{B}(j)$ is positive due to imperfect competition and will be rebated back to the households. A key difference from Section 2.1.5 is that $R_{b, t}($.$) now represents the inverse$ loan demand function, which depends on $b_{t}$ and thereby $b_{t}(j)$. This is crucial for introducing imperfect banking competition. The dependence of $R_{b, t}$ on $b_{t}(j)$ means that each bank $j$ has some control over the equilibrium gross loan interest rate by altering its own quantity of loans given the other banks' loan quantities, and this is taken into consideration by bank $j$ under Cournot competition when choosing $b_{t}(j)$. Solving the profit maximization problem with respect to $b_{t}(j)$ gives the following first order condition:

$$
\begin{equation*}
E_{t}\left[\Lambda_{t, t+1} \frac{1}{\pi_{t+1}}\left\{\frac{\partial R_{b, t}}{\partial b_{t}(j)} b_{t}(j)+R_{b, t}-R_{t}\right\}\right]=0 \tag{39}
\end{equation*}
$$

In a Cournot equilibrium, the total optimal loan quantity is $b_{t}=b_{t}(j)+\sum_{m \neq j} b_{t}(m)$ and each bank produces a share of the total quantity. Assuming banks are identical, then $b_{t}(j)=\frac{b_{t}}{N}$ in equilibrium. Since $\frac{\partial R_{b, t}}{\partial b_{t}(j)}=\frac{\partial R_{b, t}}{\partial b_{t}} \frac{\partial b_{t}}{\partial b_{t}(j)}=\frac{\partial R_{b, t}}{\partial b_{t}}$ in Cournot equilibrium, the first order condition (39) can be rewritten as:

$$
\begin{equation*}
E_{t}\left[\Lambda_{t, t+1} \frac{1}{\pi_{t+1}}\left\{\frac{\partial R_{b, t}}{\partial b_{t}} \frac{b_{t}}{N}+R_{b, t}-R_{t}\right\}\right]=0 \tag{40}
\end{equation*}
$$

where the market loan demand $b_{t}$ is given by $b_{t}=q_{t} k_{t}-n_{t}$ (15). Since the firms' net worth $n_{t}(16)$ is independent of the current period loan rate $R_{b, t}$, the effect of $R_{b, t}$ on $b_{t}$ works through firms' demand for capital, taking the real price of capital $q_{t}$ as given. When bank $j$ chooses $b_{t}(j)$, which affects the equilibrium gross loan rate $R_{b, t}$ under Cournot competition, it needs to consider how firms would respond by changing their demand for physical capital $\frac{\partial k_{t}}{\partial R_{b, t}}$.

It is shown in Appendix A that firms' demand for capital decreases in the gross loan rate $\frac{\partial k_{t}}{\partial R_{b, t}}<0$ due to diminishing returns to capital, and the interest rate elasticity of capital demand $\mathrm{PEK}_{t} \equiv-\frac{\partial k_{t}}{\partial R_{b, t}} \frac{R_{b, t}}{k_{t}}$ monotonically decreases in the expected marginal product of capital:

$$
\begin{equation*}
\mathrm{PEK}_{t}=\frac{1}{1-\alpha_{k}}\left(1+\frac{E_{t} \Lambda_{t, t+1}\left[q_{t+1}(1-\delta)\right]}{E_{t} \Lambda_{t, t+1}\left[\mathrm{MPK}_{t+1}\right]}\right) \tag{41}
\end{equation*}
$$

where $\mathrm{MPK}_{t+1} \equiv \frac{z_{t+1} \alpha_{k} \alpha_{t}^{\alpha_{k}-1} l_{t+1}^{\alpha_{l}}}{x_{t+1}}$ is the marginal product of capital in real (final consumption) terms. Intuitively, a higher expected marginal product of capital implies better investment opportunities for firms. As firms are more willing to invest in capital, their capital demand becomes more inelastic. In the baseline analysis, I assume old capital can be costlessly converted into new capital by perfectly competitive capital producers so that the real price of capital $q_{t}$ is one throughout.

As capital is financed by loans and net worth $n_{t}$, the market loan demand elasticity $\mathrm{PED}_{t}$ is driven by the capital demand elasticity $\mathrm{PEK}_{t}$ as well as the leverage ratio that captures firms' reliance on bank loans. Using (15), the market loan demand is more inelastic (i.e., lower $\mathrm{PED}_{t}$ ) when capital demand is more inelastic and the leverage $\frac{b_{t}}{q_{t} k_{t}}$ is higher:

$$
\begin{equation*}
\mathrm{PED}_{t} \equiv-\frac{\partial b_{t}}{\partial R_{b, t}} \frac{R_{b, t}}{b_{t}}=-\frac{\partial k_{t}}{\partial R_{b, t}} \frac{R_{b, t}}{k_{t}} \frac{k_{t}}{b_{t}}=\mathrm{PEK}_{t} \frac{q_{t} k_{t}}{b_{t}} \tag{42}
\end{equation*}
$$

Under perfect banking competition, these elasticities are not internalized by banks and hence do not influence the model dynamics or the steady state. By contrast, under imperfect banking competition, banks will respond to the changes in the loan demand elasticity.

Since $\mathrm{PED}_{t}$ (42) only depends on period $t$ variables and expectations, together with $\Lambda_{t, t+1}>0$ and $\pi_{t+1} \equiv \frac{p_{t+1}}{p_{t}}>0$, the first order condition (40) implies that $\left(\frac{\partial R_{b, t}}{\partial b_{t}} \frac{b_{t}}{R_{b, t}} \frac{1}{N}+1\right) R_{b, t}$ equals $R_{t}$. It follows that the equilibrium loan rate depends on the policy rate $R_{t}$, the number of banks $N$, and the elasticity of loan demand $\mathrm{PED}_{t}$ :

$$
\begin{equation*}
R_{b, t}=\frac{1}{1-\frac{1}{N} \mathrm{PED}_{t}^{-1}} R_{t} \equiv \mu_{t} R_{t} \tag{43}
\end{equation*}
$$

where $\mu_{t} \equiv \frac{1}{1-\frac{1}{N} \mathrm{PED}_{t}^{-1}}$ is the loan rate markup. ${ }^{12}$ With perfect banking competition, each bank faces a perfectly elastic loan demand, so $N \mathrm{PED}_{t} \rightarrow \infty$ and $R_{b, t}=R_{t}$, although the market loan demand is downward-sloping. With Cournot competition, banks with market power can affect the equilibrium loan rate by taking advantage of the endogenously changing loan demand elasticity. From (43), the loan rate markup $\mu_{t}$ decreases in both the number of banks $N$ (implying more intense banking competition) and the loan demand elasticity $\mathrm{PED}_{t}$. For a given level of imperfect banking competition implied by $N$, changes in the loan demand elasticity over the business cycle will cause the loan rate markup to change, which then affects aggregate fluctuations.

### 2.3 Equilibrium Conditions

In equilibrium, the aggregate resource constraint is:

$$
\begin{equation*}
c_{t}+i_{t}+\frac{\chi}{2}\left(\frac{i_{t}}{i_{t-1}}-1\right)^{2} i_{t}=y_{t} \tag{44}
\end{equation*}
$$

which is also the goods market clearing condition. In equilibrium, households' labor supply equals firms' labor demand and the new capital supplied by capital producers equals firms' capital demand. Let $b_{t}^{B}$ and $d_{t}^{B}$ denote the total units of loans given out and deposits taken in by the banking sector, respectively. Under perfect banking competition with a continuum of banks of unit mass, $b_{t}^{B}=\int_{0}^{1} b_{t}(j) d j$ and $d_{t}^{B}=\int_{0}^{1} d_{t}(j) d j$, while under Cournot banking competition, $b_{t}^{B}=\sum_{j=1}^{N} b_{t}(j)$ and $d_{t}^{B}=\sum_{j=1}^{N} d_{t}(j)$. In equilibrium, the supply of loans from the banking sector $b_{t}^{B}$ equals the market loan demand $b_{t}$, and the demand for deposits from the banking sector $d_{t}^{B}$ equals the supply of deposits from households $d_{t}$. Based on banks' balance sheet identity, the total loan supply equals the total deposit holding $b_{t}^{B}=d_{t}^{B}$.

## 3 Calibration

The two models with different types of banking competition are solved numerically using Dynare after calibrating the parameters to a quarterly frequency. Using the ECB's harmonized monetary financial institutions' (MFI) interest rates from 2000 to 2018, the average annualised household deposit rate is around $2.16 \%$. Hence, the household subjective discount factor $\beta$ is set at 0.995 , giving an annualised net real deposit rate of $\left(\frac{1}{0.995}-1\right) * 4 \approx 2 \%$.

[^8]In order to focus on the role of imperfect banking competition, the gross inflation target $\pi$ is set to one and the investment adjustment cost parameter $\chi$ is set to zero in the baseline analysis so that the price dispersion $f_{3, t}(28)$ and the real capital price $q_{t}(20)$ remain a constant one. The elasticity of substitution among differentiated retail goods $\epsilon$ is chosen to be 6 , to generate a final good price markup $x$ over the wholesale good of $20 \%\left(x=\frac{\epsilon}{\epsilon-1}\right)$ in this zero-inflation steady state. The probability $\theta$ of retailers keeping prices fixed in each period is set at 0.75 to give a price rigidity of $\frac{1}{1-0.75}=4$ quarters on average. The calibration for $\epsilon$ and $\theta$ is in line with the literature (e.g., Andrés and Arce, 2012; Gerali et al., 2010). Using the first order condition with respect to capital (14), the steady state capital-to-output ratio is:

$$
\begin{equation*}
\frac{k}{y}=\frac{\alpha_{k}}{x\left(\frac{R_{b}}{\pi}-1+\delta\right)} \tag{45}
\end{equation*}
$$

where final good output is equal to the wholesale good output $y=y_{w}$ in a zero-inflation steady state, and the real loan rate $\frac{R_{b}}{\pi}$ is equal to the real deposit rate $\frac{R}{\pi}=\frac{1}{\beta}$ under perfect banking competition. Given the calibration for $\beta$ and $x$, the capital share $\alpha_{k}$ and depreciation rate $\delta$ are calibrated to match the capital-to-output ratio of 4.9 and the labor share of 0.56 , which are mean values for EU countries over 2000-2017 from the Penn World Table (PWT). Assuming a constant-returns-to-scale production function, the average capital share $\alpha_{k}=1-\alpha_{l}$ is thus 0.44 . If $\frac{k}{y}$ is equal to 4.9 , setting $\alpha_{k}$ to 0.44 implies a depreciation rate of 0.07 .

Given the calibration for $\beta, \epsilon, \alpha_{k}, \alpha_{l}$, and $\delta$, the relative utility weight on leisure time $\phi$ is set to 1.8 to yield a steady state labor $l$ of around 0.28 , which achieves the target of the average annual hours worked by the employed people (i.e., 1756 hours on average across EU countries over 2000-2017 from PWT). Assuming people work five days a week, 1756 working hours implies people work 6.8 hours a day on average, and hence the labor time normalized by 24 hours is around 0.28 .

The death rate of firms $\varphi$ governs the amount of dividend paid to households by the exiting firms. Households' transfer of $\omega$ fraction of physical capital to new entering firms ensures that these new firms have some initial net worth. From the evolution of aggregate net worth (16), the parameters $\varphi$ and $\omega$ pin down the steady state asset-to-equity ratio $\frac{k}{n}$ :

$$
\begin{equation*}
\frac{k}{n}=\frac{1-(1-\varphi) \frac{R_{b}}{\pi}}{\omega} \tag{46}
\end{equation*}
$$

where $\varphi$ is calibrated to match the average annual enterprise death rate of around $10 \%$ for EU countries over 2008-2017 from the OECD database. After setting $\varphi$ to 0.025 , meaning that $2.5 \%$ of firms exit each quarter, $\omega$ is calibrated to match the average asset-to-equity
ratio across non-financial firms in Europe of around 4.7 over 2000-2008, as documented in Kalemli-Ozcan, Sorensen and Yesiltas (2012). Hence, $\omega$ is set to 0.0042 to give an asset-toequity ratio $\frac{k}{n}$ of around 4.8 .

To focus on the model mechanism, the Taylor rule is kept to its simplest form in the baseline analysis. There is no interest rate smoothing ( $\rho_{r}=0$ ), and the feedback coefficient on output $\kappa_{y}$ is set to zero. Monetary policy only responds to the deviation of the gross inflation rate from the inflation target $\pi$, where the feedback coefficient on inflation $\kappa_{\pi}$ is set to 1.5 . I also use different calibrations for $\rho_{r}, \kappa_{\pi}$, and $\kappa_{y}$ as robustness checks in Section 5. The standard deviation for the monetary policy shock $\sigma_{r}$ is 0.0025 and for the productivity shock $\sigma_{z}$ is 0.01 . I look at two types of productivity shocks. When the productivity shock has a persistent effect, I set the parameter $\psi$ in the $\operatorname{AR}(1)$ process to 0.95 . When looking at a fully transitory productivity shock, $\psi$ is set to 0 . The parameter $\kappa_{\pi}$ and the shock-related parameters are in line with the literature (e.g., Andrés and Arce, 2012; Gerali et al., 2010).

Given the calibration for $\beta, \epsilon, \alpha_{k}, \delta, \varphi$, and $\omega$, the number of banks $N$ is set to 4 to get a steady state gross loan rate $R_{b}$ of 1.01, implying an annualised net real loan rate of $(1.01-1) * 4 \approx 4 \%$ and a real loan margin of around 200 basis points. This matches the average annualised corporate loan rate of around $4.14 \%$ and the loan interest margin of around 198 basis points across EU countries over the past 19 years, using the ECB's harmonized monetary financial institutions' (MFI) interest rates from 2000 to 2018. Table 1 summarizes the calibrated parameters discussed above, which are used for the baseline analysis. Under this calibration, the steady state values of the key variables can be found in Table 2 in Appendix C.

With imperfect banking competition, a higher loan rate lowers the capital-to-output ratio (45) and the asset-to-equity ratio (46), among other variables, as shown in Table 2 in Appendix C. When comparing the dynamics of the two models with different types of banking competition in Section 4, I assume that there is a lump sum tax that redistributes the positive profit from the banking sector to households such that the steady state under imperfect banking competition is identical to that under perfect banking competition.

Figure 1 plots the steady state values for some variables against the number of banks $N$ that ranges from 1 to 100 . A higher $N$ implies more intense competition. When there is a monopoly bank, the annualised loan margin is around $150 * 4=600$ basis points. As $N$ increases, the loan margin $\left(R_{b}-R\right)$ approaches zero and output increases.

From (43), the equilibrium loan rate decreases in both the number of banks and the interest rate elasticity of loan demand. One key component that determines the loan demand elasticity is the capital demand elasticity. From (41), capital demand is more elastic when the marginal product of capital is lower. Under the assumption of a constant-returns-to-scale

Table 1: Calibration of Parameters in Baseline Analysis

| Parameter | Value | Description |
| :--- | :---: | :--- |
| Households |  |  |
| $\beta$ | 0.995 | Subjective discount factor |
| $\phi$ | 1.8 | Relative utility weight on leisure time |
| Firms |  |  |
| $\alpha_{k}$ | 0.44 | Physical capital share |
| $\delta$ | 0.07 | Depreciation rate for physical capital |
| $\varphi$ | 0.025 | Death rate of firms |
| $\omega$ | 0.0042 | Transfer from households to new firms |
| Capital producers |  |  |
| $\chi$ | 0 | Investment adjustment cost |
| Retailers | 6 | Elasticity of substitution between retail goods |
| $\epsilon$ | 0.75 | Probability of not adjusting price |
| $\theta$ | 4 | Number of banks |
| Banking sector |  |  |
| $N$ | 0 | Interest rate smoothing |
| Central bank | 1.5 | Feedback coefficient on inflation |
| $\rho_{r}$ | 0 | Feedback coefficient on output |
| $\kappa_{\pi}$ | 0.0025 | Standard deviation of monetary policy shock |
| $\kappa_{y}$ | 0.01 | Standard deviation of productivity shock |
| Shocks |  |  |

production function, the marginal product of capital $\alpha_{k} z\left(\frac{k}{l}\right)^{\alpha_{k}-1}$ decreases in the capital-to-labor ratio. As the number of banks $N$ increases, a lower loan rate makes capital cheaper relative to labor, raising the capital-to-labor ratio and thus reducing the marginal product of capital. Hence, Figure 1 shows that as $N$ increases, capital demand elasticity also increases.

If capital were financed by bank loans only, then the loan demand elasticity would be identical to the capital demand elasticity. However, capital is financed by both bank loans and net worth, so the loan demand elasticity also depends on firms' leverage ratio $\frac{b}{q k}=\frac{q k-n}{q k}$ (42). Figure 1 shows that despite the capital demand becoming more elastic, the loan demand is more inelastic due to a higher leverage ratio as $N$ increases. A higher leverage ratio implies greater reliance on bank loans, which tends to make the loan demand more inelastic. As $N$ increases, the firm's net worth $n$ falls as a lower loan rate reduces the benefit of using internal financing (net worth) relative to external financing. As a result, the leverage ratio rises as $N$ increases, which tends to make the loan demand more inelastic.

Figure 1 shows that in the long-run equilibrium, the loan rate is mainly driven by the number of banks $N$ instead of the market loan demand elasticity. A higher $N$ directly reduces the equilibrium loan rate and thus the loan interest margin, although it is associated with

Figure 1: Steady State Values for Different Number of Banks


Note: The figure shows the steady state values of variables against the number of banks $N$ ranging from 1 to 100 . The loan margin $\left(R_{b}-R\right)$ is expressed in percentage points. The marginal product of capital is computed as $\alpha_{k} z k^{\alpha_{k}-1} l^{\alpha_{l}}$, and the leverage ratio refers to the loan-to-asset ratio $\frac{b}{q k}$. Capital and loan demand elasticities are calculated based on (41) and (42), respectively.
a more inelastic loan demand. The next section shows that, conditional on the number of banks, endogenous changes in the loan demand elasticity in response to shocks can drive the changes in the loan interest margin over the business cycle.

## 4 Dynamic Analysis

In this section, I investigate how aggregate output responds to a monetary policy shock, a persistent productivity shock, and a transitory productivity shock under imperfect and perfect banking competition. To compare models with different types of banking competition, I assume that the entire steady steady profit made by the banking sector is taxed and transferred to households, so the two models have the same steady state. ${ }^{13}$ This nonlinear model is solved using a first order Taylor approximation around the steady state in Dynare.

### 4.1 Monetary Policy Shock

Figure 2 shows the impulse responses to an unexpected one-time monetary policy shock, where the white noise term $e_{r, t}$ in the Taylor rule is raised by 25 basis points at the beginning of period 1. The nominal deposit rate (policy rate) $R_{t}$ increases as a consequence.

Under perfect banking competition, output decreases by around $1.2 \%$ immediately after a contractionary monetary policy shock, but quickly rises back to the steady state. As a result, the output accumulated flattens and the total percentage deviation of output from its steady state is only around $1.8 \%$ in period 40 . However, under imperfect banking competition, the accumulated output is $3.2 \%$ lower than its steady state in period 40 , which is around $78 \%$ higher relative to perfect banking competition. This amplification effect of imperfect banking competition can be explained by a rise in the real loan margin.

With perfect banking competition, households and firms face the same real interest rate, and thus the loan interest margin $\left(R_{b, t}-R_{t}\right)$ is zero. An increase in the real interest rate after the contractionary monetary policy shock reduces households' consumption and firms' capital investment, resulting in a drop in output. The dynamics of the real interest rate reflect the intertemporal substitution of consumption. As consumption rises towards the steady state (i.e., $c_{t+1}>c_{t}$ ), the real interest rate gradually decreases but remains above the steady state during this transition to induce households to save for future consumption. The real interest rate dynamics also govern the dynamics of the expected marginal product of capital as firms choose their optimal capital to equate the expected marginal return on capital to the real interest rate (12). Combining the Euler equation (6) and the firm's first

[^9]Figure 2: Impulse Responses to a Contractionary Monetary Policy Shock


Note: The horizontal axis shows quarters after a contractionary monetary policy shock of 25 basis points at the beginning of period 1. The vertical axis shows the percentage deviation from the steady state for variables other than the interest rates and the loan margin, which are expressed in deviations from the steady state in percentage points. The leverage ratio refers to the debt-to-asset ratio $\frac{b_{t}}{q_{t} k_{t}}$, and the marginal product of capital is $\frac{\alpha_{k} y_{t}}{k_{t-1}}$.
order condition with respect to capital (12) gives:

$$
\begin{equation*}
1=\beta E_{t}\left[\frac{c_{t}}{c_{t+1}} \frac{R_{t}}{\pi_{t+1}}\right]=\beta E_{t}\left[\frac{c_{t}}{c_{t+1}} \frac{\left(\mathrm{MPK}_{t+1}+q_{t+1}(1-\delta)\right)}{q_{t}}\right] \tag{47}
\end{equation*}
$$

where the loan rate $\frac{R_{b t}}{\pi_{t+1}}$ in (12) equals the deposit rate $\frac{R_{t}}{\pi_{t+1}}$ under perfect banking competition and $\mathrm{MPK}_{t+1}=\frac{\alpha_{k} z_{t+1} k_{t}^{\alpha_{k}-1} l_{t+1}^{\alpha_{l}}}{x_{t+1}}$ is the marginal product of capital in real (final consumption) terms. This section assumes there is no investment adjustment cost so that $q_{t}=q_{t+1}=1$. From (47), when the expected return to capital is higher, households want to consume less and save more today.

Figure 2 shows a higher marginal product of capital from period 2 onwards. Since capital is predetermined, the initial drop in marginal product of capital in Figure 2 is caused by the drop in labor hours. A higher expected marginal product of capital implies that firms are more willing to borrow to purchase the new capital, making their capital and thus loan demand less sensitive to the loan rate (41). In addition, the rise in firms' leverage ratio $\frac{b_{t}}{q_{t} k_{t}}$ implies that firms rely more on bank loans, which also tends to make their loan demand more inelastic (42). Figure 2 shows that the loan demand elasticity $\mathrm{PED}_{t}$ falls by $0.55 \%$ immediately after the shock, which is due to a higher expected marginal product of capital and a higher leverage ratio.

Under perfect banking competition, despite the market loan demand becoming more inelastic, each bank faces a perfectly elastic loan demand and takes the equilibrium loan rate as given. By contrast, when banks have market power, they can take advantage of the lower loan demand elasticity by reducing their loan quantities to achieve a higher equilibrium loan rate. The higher real loan rate then reduces the firms' demand for capital and therefore output by more. However, Figure 2 shows that the real loan rate under imperfect banking competition is initially lower than that under perfect banking competition. This suggests that the slight amplification effect during the early periods is likely driven by other forces.

Apart from amplifying the reduction in capital through a higher loan rate, imperfect banking competition also distorts households' consumption-saving decisions through a lower deposit rate. Figure 2 shows that the real deposit rate under imperfect banking competition is much lower relative to the perfect banking competition benchmark during the early periods. Under imperfect banking competition, the relative price of consumption today is distorted by the loan rate markup $\mu_{t}=\frac{R_{b, t}}{R_{t}}$ :

$$
\begin{equation*}
1=\beta E_{t}\left[\frac{c_{t}}{c_{t+1}} \frac{R_{b, t}}{\pi_{t+1} \mu_{t}}\right]=\beta E_{t}\left[\frac{c_{t}}{c_{t+1}} \frac{\left(\mathrm{MPK}_{t+1}+q_{t+1}(1-\delta)\right)}{\mu_{t} q_{t}}\right] \tag{48}
\end{equation*}
$$

where the loan rate markup $\mu_{t}=\frac{1}{1-\frac{1}{N} \mathrm{PED}_{t}^{-1}}$ (43) increases if the loan demand becomes more inelastic (i.e., lower $\mathrm{PED}_{t}$ ). As the number of banks $N$ approaches infinity and the banking sector becomes perfectly competitive, this loan rate markup is one. With a small number of banks, any change in the loan demand elasticity induces banks to optimally adjust the markup. When the expected return on capital is higher, the loan demand becomes more inelastic and the loan rate markup $\mu_{t}$ is larger. Instead of getting the expected return on capital by saving more today, households only get a fraction $\frac{1}{\mu_{t}}$ of that expected return. Figure 2 shows that the fall in consumption in period 1 is smaller under imperfect banking competition, indicating that households want to save less due to the distorted expected return from saving. As a result, investment and capital drop by more, given the initial fall in output is similar between the two types of banking competition.

Due to the capital accumulation process, a persistently higher loan rate under imperfect banking competition is able to have a persistent effect on capital stock and output over time. Since capital stock depends on the accumulated past investment, a lower investment under imperfect banking competition at each point in time can slow down the accumulation of capital, leading to a more persistent reduction in output.

### 4.2 Productivity Shocks

Figure 3 shows the impulse responses after a persistent negative productivity shock. Under imperfect banking competition, output is initially attenuated but amplified later on, so the output accumulated is slightly larger in later periods. The differential responses of output under the two types of banking competition can be explained by changes in the real loan margin, which are driven by the dynamics of the expected marginal product of capital.

Unlike the monetary policy shock, there are now two opposite forces that act upon the expected marginal product of capital. On the one hand, a persistently low productivity directly reduces the expected marginal product of capital. On the other hand, there is an upward pressure on the real interest rate to induce households to save for future consumption so that consumption can rise towards its steady state. More savings ensure that capital stock can be gradually built up, and output eventually recovers. Initially, the effect of the lower productivity dominates, so the expected marginal product of capital falls. Later on, as productivity rises towards its steady state, the upward pressure on the real interest rate and the expected return on capital dominates. Consequently, the expected marginal product of capital is lower during the early periods and is higher later on, making the capital and thus loan demand initially more elastic but later on inelastic. With imperfect banking competition, banks respond to these changes in the loan demand elasticity by adjusting their

Figure 3: Impulse Responses to a Persistent Negative Productivity Shock


Note: The horizontal axis shows quarters after a one-standard-deviation negative productivity shock at the beginning of period 1 . The shock has a persistent effect because productivity follows an $\operatorname{AR}(1)$ process with a persistence parameter $\psi=0.95$. The vertical axis shows the percentage deviation from the steady state for variables other than the interest rates and the loan margin, which are expressed in deviations from the steady state in percentage points. The leverage ratio refers to the debt-to-asset ratio $\frac{b_{t}}{q_{t} k_{t}}$, and the marginal product of capital is $\frac{\alpha_{k} y_{t}}{k_{t-1}}$.

Figure 4: Impulse Responses to a Transitory Negative Productivity Shock


Note: The horizontal axis shows quarters after a one-standard-deviation negative productivity shock at the beginning of period 1 . The persistence parameter $\psi$ in the $\mathrm{AR}(1)$ process for productivity is set to zero, so the shock is fully transitory. The vertical axis shows the percentage deviation from the steady state for variables other than the interest rates and the loan margin, which are expressed in deviations from the steady state in percentage points. The leverage ratio refers to the debt-to-asset ratio $\frac{b_{t}}{q_{t} k_{t}}$, and the marginal product of capital is $\frac{\alpha_{k} y_{t}}{k_{t-1}}$.
loan margin, so the real loan margin is initially procyclical and turns countercyclical during later periods.

In contrast to the persistent productivity shock, Figure 4 shows that the output accumulated is around $17 \%$ lower under imperfect banking competition after a transitory negative productivity shock. I assume the productivity no longer follows an $\operatorname{AR}(1)$ process in this case. Productivity falls at the beginning of period 1 and returns to its steady state the next period. As a result, the marginal product of capital falls in the first period due to lower productivity $z_{t}$ and labor hours $l_{t}$, but rises immediately afterwards due to a higher real interest rate, unlike after a persistent productivity shock where continuously low productivity tends to drive down the expected marginal product of capital. A higher expected marginal product of capital and a higher leverage ratio make the loan demand more inelastic, leading to a fall in the loan demand elasticity $\mathrm{PED}_{t}$ in Figure 4. Under imperfect banking competition, banks respond to the more inelastic loan demand by reducing their loan quantities to achieve a higher loan rate, leading to a rise in the real loan margin. A higher loan rate relative to the perfect banking competition benchmark reduces the firms' capital demand by more and thus amplifies the drop in output.

Despite a rise in the real loan margin, the real loan rate under imperfect banking competition is initially lower. The intertemporal substitution channel may explain why capital is still amplified during early periods. When the expected return on capital is higher, households want to postpone their consumption and save more under perfect banking competition. However, a higher expected return on capital leads to a higher loan rate markup $\mu_{t}$ under imperfect banking competition, which tends to reduce the expected return from saving (48). Therefore, the drop in households' consumption is smaller in the initial period, and given a similar initial fall in output, capital investment needs to drop by more under imperfect banking competition.

## 5 Sensitivity Analysis

I check the robustness of the baseline results in Section 4 by changing the investment adjustment cost parameter $\chi$, the number of banks $N$, the output elasticities of capital $\alpha_{k}$ and labor $\alpha_{l}$, the depreciation rate $\delta$, the initial transfer from households $\omega$, and the parameters in the Taylor rule ( $\rho_{r}, \kappa_{\pi}$, and $\kappa_{y}$ ), while all the other parameters are calibrated as in the baseline analysis. Overall, the baseline results in Section 4 are robust to these parameter changes except for two cases: when the investment adjustment cost parameter $\chi$ or the feedback coefficient on output $\kappa_{y}$ in the Taylor rule is positive, the loan rate markup after a transitory productivity shock can be procyclical and thus attenuates the aggregate fluctuations. This
section discusses the sensitivity of the baseline results to these parameters in turn.
In the baseline analysis in Section 4, I assume there is no investment adjustment cost (i.e., $\chi=0$ ) so that the real price of capital is always one. In the presence of an investment adjustment cost, the dynamics of consumption, investment, and capital are quite different. Comparing Figure 5 with Figure 2, due to the adjustment cost, capital no longer adjusts instantly in response to the contractionary monetary policy shock. In fact, the initial drop in capital is very small. The response of investment is smoothed, and changes in investment become much smaller, which reduces households' ability to smooth consumption at an aggregate level. As a result, consumption is more volatile and moves more closely with the output. The fall in investment and capital in Figure 5 is smaller, which leads to a smaller drop in output. Figure 5 shows that the initial drop in output is only around $0.2 \%$, whereas output drops by $1.2 \%$ if capital could adjust immediately in Figure 2.

In the presence of the investment adjustment cost, imperfect banking competition greatly slows down capital accumulation compared to the perfect banking competition benchmark. As a result, the drop in output is much more persistent, and even after 40 quarters output is still far from reaching the steady state. Figure 5 shows that the accumulated drop in output under imperfect banking competition is almost 4 times the accumulated drop under perfect banking competition in period 40. This is because the higher loan rate under imperfect banking competition now has a more persistent effect on output via the smoothed investment process in addition to capital accumulation.

After a persistent productivity shock, the results are similar to Figure 3 and there is not much difference between the two types of banking competition. When the productivity shock is fully transitory, there is very little change in capital, and thus changes in the expected marginal product of capital are minimal. Figure 7 in Appendix D shows that in the presence of investment adjustment cost, firms' leverage ratio can decrease in this case, ${ }^{14}$ making the loan demand more elastic and leading to a procyclical loan rate markup that attenuates the output.

Figure 6 shows the impulse responses of output, marginal product of capital, loan demand elasticity $\mathrm{PED}_{t}$, and the real loan margin after three types of shocks when the number of banks $N$ is two, eight, and infinity (i.e., perfect competition). When there are only two banks, the amplification effect is much larger after the contractionary monetary policy shock and the transitory productivity shock. After a persistent productivity shock, it is still difficult to see the differences clearly since output is initially attenuated but later is amplified. When

[^10]Figure 5: Impulse Responses to a Contractionary Monetary Policy Shock when $\chi=2$


Note: The horizontal axis shows quarters after a contractionary monetary policy shock of 25 basis points at the beginning of period 1. The vertical axis shows the percentage deviation from the steady state for variables other than the interest rates and the loan margin, which are expressed in deviations from the steady state in percentage points. The leverage ratio refers to the debt-to-asset ratio $\frac{b_{t}}{q_{t} k_{t}}$, and the marginal product of capital is $\frac{\alpha_{k} y_{t}}{k_{t-1}}$.

Figure 6: Impulse Responses for Different Shocks and Number of Banks $N$


$$
\text { -o- Perfect Banking Competition ——Imperfect Banking Competition }(\mathrm{N}=2) \cdots \text { Imperfect Banking Competition }(\mathrm{N}=8)
$$

Note: The horizontal axis shows quarters after the shock that occurs at the beginning of period 1. The vertical axis shows the percentage deviation from the steady state for variables other than the loan margin, which is expressed in deviations from the steady state in percentage points. Each row shows the impulse responses of four variables after a given type of shock: a 25 basis-point contractionary monetary policy shock, a one-standard-deviation persistent negative productivity shock $(\psi=0.95)$, and a one-standard-deviation transitory negative productivity shock $(\psi=0)$.
$N$ increases to eight, banks' market power is greatly reduced and the outcome is closer to the perfect banking competition benchmark.

Reducing the output elasticity of capital $\alpha_{k}$ from 0.44 to 0.25 gives a slightly stronger amplification effect of imperfect banking competition. This is because a lower $\alpha_{k}$ implies that capital is used less intensively in the production, so the reduction in capital is larger after the negative shocks, which is associated with a higher expected marginal product of capital. The latter leads to a more inelastic loan demand and a higher loan interest margin that amplifies output. Figure 8 in Appendix D shows that the magnitude of the rise in loan interest margin is larger compared to the baseline results in Section 4.

Changing the depreciation rate $\delta$ will change the steady state value of the loan demand elasticity PED. Using (41) and (42),

$$
\begin{equation*}
\mathrm{PED}=\frac{1}{1-\alpha_{k}}\left(1+\frac{1-\delta}{\mathrm{MPK}}\right) \frac{q k}{b} \tag{49}
\end{equation*}
$$

where $q=1$ and MPK $=\frac{z \alpha_{k} k^{\alpha_{k}-1} l^{\alpha}}{x}=\frac{R_{b}}{\pi}-1+\delta$ (12). A higher depreciation rate $\delta$ lowers PED directly but also indirectly through raising the steady state marginal product of capital in final consumption terms MPK. Similarly, changing the parameter $\omega$ that governs the transfer from households to new entering firms also affects PED, but through the leverage ratio $\frac{b}{q k}$ instead. From (46), a lower $\omega$ leads to a lower total net worth $n$ of firms and a higher asset-to-equity ratio $\frac{q k}{n}$. Since $b=q k-n$, the leverage ratio $\frac{b}{q k}=\left(1-\frac{n}{q k}\right)$ is also higher, which leads to a more inelastic loan demand in the steady state (49).

A lower steady state value of loan demand elasticity leads to a larger percentage deviation of loan demand elasticity from its steady state PED and thus a larger change in the loan margin. Figure 9 and Figure 10 in Appendix D show that when the depreciation rate $\delta$ and the initial transfer from households $\omega$ are lower relative to the baseline calibration, the rise in the loan margin is smaller, and hence the amplification effect of imperfect banking competition is weaker in the former case, whereas the opposite happens in the latter case.

The baseline results are robust to changing the interest rate smoothing parameter $\rho_{r}$. In the baseline analysis, the Taylor rule takes the simplest possible form in the baseline analysis where both $\rho_{r}$ and the feedback coefficient on output $\kappa_{y}$ are set to zero. When changing $\rho_{r}$ to 0.7 , the contractionary monetary policy shock leads to a persistent increase in the nominal interest rate due to interest rate smoothing, thus increasing the effective size of the shock. Figure 11 in Appendix D shows that the responses of output under both types of banking competition are much larger when $\rho_{r}=0.7$.

Increasing $\kappa_{\pi}$ leads to a greater response to the deviation of inflation from its target. Figure 12 in Appendix D shows that the fall in output is smaller under both types of banking
competition after the contractionary monetary policy shock that is deflationary, but larger after the negative transitory productivity shock that is inflationary. In the latter case, the real loan rate rises more to bring down inflation, causing a larger fall in output.

However, the amplification effect after a transitory productivity shock in the baseline analysis is not robust to changes in the sensitivity $\kappa_{y}$ of the policy rate to the output gap. This is because after the negative productivity shock, the fall in output is large. With $\kappa_{y}>0$, the central bank's response to the output gap leads to a lower policy rate, and thereby a lower real loan rate. This leads to a smaller reduction in the firms' net worth and a lower leverage ratio that tends to make the loan demand more elastic. Figure 13 in Appendix D shows that when $\kappa_{y}=0.125$, the loan demand becomes more elastic and the loan margin decreases after a negative transitory productivity shock.

## 6 Conclusions

This paper studies how imperfect banking competition affects aggregate fluctuations via a time-varying loan rate markup. The paper highlights a new mechanism behind the timevarying loan rate markup that works through the general equilibrium dynamics in the expected marginal product of capital. Intuitively, firms would be more willing to borrow to invest in capital when the expected marginal product of capital is high. This tends to make their capital and thus loan demand less sensitive to the loan rate. Market power enables banks to charge a higher loan rate markup in response to a more inelastic loan demand.

After a contractionary monetary policy shock, the rise in the real interest rate leads to a higher expected marginal product of capital, which makes firms' loan demand more inelastic and hence raises the loan rate markup. A higher loan rate markup tends to raise the cost of credit and reduce firms' capital demand and hence output by more relative to the case of perfect banking competition. By contrast, the impacts of imperfect banking competition on aggregate fluctuations after productivity shocks are less clear, because they depend on the persistence of those shocks.

The results in this paper suggest that imperfect banking competition can have important amplification effects of monetary policy shocks. With imperfect banking competition, even a transitory contractionary monetary policy shock can lead to a persistently lower output relative to the case of perfect banking competition. Although the initial impact of a higher loan rate markup on output may be small, a persistently higher loan rate markup can greatly slow down the accumulation of capital and output recovery.

## Appendices

## A Elasticities of Capital and Loan Demand

Differentiate the optimal capital demand (14) for a given level of labor:

$$
\begin{equation*}
k_{t}=\left(\frac{E_{t} \Lambda_{t, t+1}\left[\frac{R_{b, t} q_{t}}{\pi_{t+1}}-q_{t+1}(1-\delta)\right]}{E_{t} \Lambda_{t, t+1}\left[\frac{z_{t+1} \alpha_{k} l_{t+1}^{\alpha}}{x_{t+1}}\right]}\right)^{-\frac{1}{1-\alpha_{k}}} \tag{50}
\end{equation*}
$$

with respect to the gross loan rate $R_{b, t}$ :

$$
\begin{align*}
\frac{\partial k_{t}}{\partial R_{b, t}} & =-\frac{1}{1-\alpha_{k}}\left(\frac{E_{t} \Lambda_{t, t+1}\left[\frac{R_{b, t} q_{t}}{\pi_{t+1}}-q_{t+1}(1-\delta)\right]}{E_{t} \Lambda_{t, t+1}\left[\frac{z_{t+1} \alpha_{k} l_{t+1}^{\alpha_{l}}}{x_{t+1}}\right]}\right)^{-\frac{2-\alpha_{k}}{1-\alpha_{k}}} \frac{E_{t} \Lambda_{t, t+1}\left[\frac{q_{t}}{\pi_{t+1}}\right]}{E_{t} \Lambda_{t, t+1}\left[\frac{z_{t+1} \alpha_{k} l_{t+1}^{\alpha_{l}}}{x_{t+1}}\right]}  \tag{51}\\
& =-\frac{1}{1-\alpha_{k}} k_{t}^{2-\alpha_{k}} \frac{E_{t} \Lambda_{t, t+1}\left[\frac{q_{t}}{\pi_{t+1}}\right]}{E_{t} \Lambda_{t, t+1}\left[\frac{z_{t+1} \alpha_{k} l_{t+1}^{\alpha_{l}}}{x_{t+1}}\right]}<0
\end{align*}
$$

where the second step uses the capital demand (14).
Using (12), (14), and (51), the interest rate elasticity of capital demand $\mathrm{PEK}_{t}$ is:

$$
\begin{align*}
\operatorname{PEK}_{t} \equiv-\frac{\partial k_{t}}{\partial R_{b, t}} \frac{R_{b, t}}{k_{t}} & =\frac{1}{1-\alpha_{k}} k_{t}^{1-\alpha_{k}} \frac{E_{t} \Lambda_{t, t+1}\left[\frac{R_{b, t} q_{t}}{\pi_{t+1}}\right]}{E_{t} \Lambda_{t, t+1}\left[\frac{z_{t+1} \alpha_{k} l_{t+1}^{\alpha_{l}}}{x_{t+1}}\right]} \\
& =\frac{1}{1-\alpha_{k}} \frac{E_{t} \Lambda_{t, t+1}\left[\frac{R_{b, t} q_{t}}{\pi_{t+1}}\right]}{E_{t} \Lambda_{t, t+1}\left[\frac{R_{b, t} q_{t}}{\pi_{t+1}}-q_{t+1}(1-\delta)\right]}  \tag{52}\\
& =\frac{1}{1-\alpha_{k}} \frac{E_{t} \Lambda_{t, t+1}\left[\mathrm{MPK}_{t+1}+q_{t+1}(1-\delta)\right]}{E_{t} \Lambda_{t, t+1}\left[\mathrm{MPK}_{t+1}\right]} \\
& =\frac{1}{1-\alpha_{k}}\left(1+\frac{E_{t} \Lambda_{t, t+1}\left[q_{t+1}(1-\delta)\right]}{E_{t} \Lambda_{t, t+1}\left[\mathrm{MPK}_{t+1}\right]}\right)>0
\end{align*}
$$

where MPK $_{t+1} \equiv \frac{{ }_{z_{t+1} \alpha_{k} k_{t}^{\alpha_{k}-1} l_{t+1}^{\alpha}}^{x_{t+1}}}{x_{t+1}}$ denotes the marginal product of capital in real (final consumption) terms and $E_{t} \Lambda_{t, t+1}\left[\frac{R_{b, t} q_{t}}{\pi_{t+1}}\right]=E_{t} \Lambda_{t, t+1}\left[\mathrm{MPK}_{t+1}+q_{t+1}(1-\delta)\right]$ comes from the first order condition (12).

Since net worth is unaffected by the current period loan rate, differentiate the market
loan demand $b_{t}(15)$ with respect to the loan rate $R_{b, t}$ to get:

$$
\begin{equation*}
\frac{\partial b_{t}}{\partial R_{b, t}}=q_{t} \frac{\partial k_{t}}{\partial R_{b, t}} \tag{53}
\end{equation*}
$$

Hence, the elasticity $\mathrm{PED}_{t}$ of the market loan demand to the loan rate is:

$$
\begin{equation*}
\mathrm{PED}_{t} \equiv-\frac{\partial b_{t}}{\partial R_{b, t}} \frac{R_{b, t}}{b_{t}}=-\frac{\partial k_{t}}{\partial R_{b, t}} \frac{R_{b, t}}{k_{t}} \frac{q_{t} k_{t}}{b_{t}}=\mathrm{PEK}_{t} \frac{q_{t} k_{t}}{b_{t}}>0 \tag{54}
\end{equation*}
$$

which increases in the capital demand elasticity $\mathrm{PEK}_{t}$ and the inverse leverage ratio $\frac{q_{t} k_{t}}{b_{t}}$.

## B Calvo Pricing

## B. 1 Optimal Pricing Equation

Substitute in $y_{t+s}^{*}(j)$ and rearrange:

$$
\begin{equation*}
\operatorname{Max}_{p_{t}^{*}(j)} \quad \sum_{s=0}^{\infty} \theta^{s} E_{t}\left[\Lambda_{t, t+s}\left(\frac{p_{t}^{*}(j)}{p_{t+s}}-\frac{1}{x_{t+s}}\right)\left(\frac{p_{t}^{*}(j)}{p_{t+s}}\right)^{-\epsilon} y_{t+s}\right] \tag{55}
\end{equation*}
$$

Take the first order condition:

$$
\begin{equation*}
\sum_{s=0}^{\infty} \theta^{s} E_{t} \Lambda_{t, t+s}\left[\left(\frac{1}{p_{t+s}}\right)\left(\frac{p_{t}^{*}(j)}{p_{t+s}}\right)^{-\epsilon} y_{t+s}+\left(\frac{p_{t}^{*}(j)}{p_{t+s}}-\frac{1}{x_{t+s}}\right)(-\epsilon)\left(\frac{p_{t}^{*}(j)}{p_{t+s}}\right)^{-\epsilon-1} \frac{y_{t+s}}{p_{t+s}}\right]=0 \tag{56}
\end{equation*}
$$

Simplify the above equation:

$$
\begin{equation*}
\sum_{s=0}^{\infty} \theta^{s} E_{t} \Lambda_{t, t+s}\left[(1-\epsilon)\left(\frac{y_{t+s}}{p_{t+s}}\right)\left(\frac{p_{t}^{*}(j)}{p_{t+s}}\right)^{-\epsilon}+\epsilon \frac{1}{x_{t+s}} p_{t}^{*}(j)^{-\epsilon-1}\left(\frac{1}{p_{t+s}}\right)^{-\epsilon} y_{t+s}\right]=0 \tag{57}
\end{equation*}
$$

Multiply by $\frac{p_{t}^{*}(j)^{\epsilon+1}}{1-\epsilon}$ :

$$
\begin{equation*}
\sum_{s=0}^{\infty} \theta^{s} E_{t} \Lambda_{t, t+s}\left[p_{t}^{*}(j)\left(\frac{1}{p_{t+s}}\right)^{1-\epsilon} y_{t+s}+\frac{\epsilon}{1-\epsilon} \frac{1}{x_{t+s}}\left(\frac{1}{p_{t+s}}\right)^{-\epsilon} y_{t+s}\right]=0 \tag{58}
\end{equation*}
$$

Rearrange to solve for $p_{t}^{*}(j)$ and get the optimal pricing equation (26):

$$
\begin{equation*}
p_{t}^{*}(j)=\frac{\epsilon}{\epsilon-1} \frac{\sum_{s=0}^{\infty} \theta^{s} E_{t}\left[\Lambda_{t, t+s} x_{t+s}^{-1} p_{t+s}^{\epsilon} y_{t+s}\right]}{\sum_{s=0}^{\infty} \theta^{s} E_{t}\left[\Lambda_{t, t+s} p_{t+s}^{\epsilon-1} y_{t+s}\right]}=\frac{\epsilon}{\epsilon-1} \frac{\sum_{s=0}^{\infty}(\beta \theta)^{s} E_{t}\left[u^{\prime}\left(c_{t+s}\right) x_{t+s}^{-1} p_{t+s}^{\epsilon} y_{t+s}\right]}{\sum_{s=0}^{\infty}(\beta \theta)^{s} E_{t}\left[u^{\prime}\left(c_{t+s}\right) p_{t+s}^{\epsilon-1} y_{t+s}\right]} \tag{59}
\end{equation*}
$$

To numerically implement the optimal pricing equation in Dynare, summarize the equation above with two recursive formulations such that:

$$
\begin{equation*}
p_{t}^{*}=\frac{\epsilon}{\epsilon-1} \frac{g_{1, t}}{g_{2, t}} \tag{60}
\end{equation*}
$$

where

$$
\begin{align*}
& g_{1, t} \equiv u^{\prime}\left(c_{t}\right) p_{t}^{\epsilon} y_{t} x_{t}^{-1}+\beta \theta E_{t}\left[g_{1, t+1}\right]=\frac{1}{c_{t}} p_{t}^{\epsilon} y_{t} x_{t}^{-1}+\beta \theta E_{t}\left[g_{1, t+1}\right]  \tag{61}\\
& g_{2, t} \equiv u^{\prime}\left(c_{t}\right) p_{t}^{\epsilon-1} y_{t}+\beta \theta E_{t}\left[g_{2, t+1}\right]=\frac{1}{c_{t}} p_{t}^{\epsilon-1} y_{t}+\beta \theta E_{t}\left[g_{2, t+1}\right] \tag{62}
\end{align*}
$$

Let $f_{1, t} \equiv p_{t}^{-\epsilon} g_{1, t}$, then

$$
\begin{equation*}
f_{1, t} \equiv p_{t}^{-\epsilon} g_{1, t}=\frac{1}{c_{t}} y_{t} x_{t}^{-1}+\beta \theta E_{t}\left[\pi_{t+1}^{\epsilon} f_{1, t+1}\right] \tag{63}
\end{equation*}
$$

Let $f_{2, t} \equiv p_{t}^{1-\epsilon} g_{2, t}$, then

$$
\begin{equation*}
f_{2, t} \equiv p_{t}^{1-\epsilon} g_{2, t}=\frac{1}{c_{t}} y_{t}+\beta \theta E_{t}\left[\pi_{t+1}^{\epsilon-1} f_{2, t+1}\right] \tag{64}
\end{equation*}
$$

The optimal pricing equation $p_{t}^{*}=\frac{\epsilon}{\epsilon-1} \frac{g_{1, t}}{g_{2, t}}$ becomes:

$$
\begin{equation*}
p_{t}^{*}=\frac{\epsilon}{\epsilon-1} \frac{f_{1, t} p_{t}^{\epsilon}}{f_{2, t} p_{t}^{\epsilon-1}}=\frac{\epsilon}{\epsilon-1} \frac{f_{1, t}}{f_{2, t}} p_{t} \tag{65}
\end{equation*}
$$

Divide both sides by $p_{t-1}$ and let $\pi_{t}^{*}=\frac{p_{t}^{*}}{p_{t-1}}$ denote the gross reset price inflation rate to eliminate the price levels:

$$
\begin{equation*}
\pi_{t}^{*}=\frac{p_{t}^{*}}{p_{t-1}}=\frac{\epsilon}{\epsilon-1} \frac{f_{1, t}}{f_{2, t}} \pi_{t} \tag{66}
\end{equation*}
$$

## B. 2 Aggregate Price Evolution

Rearrange the aggregate price index (23):

$$
\begin{equation*}
p_{t}^{1-\epsilon}=\int_{0}^{1} p_{t}(j)^{1-\epsilon} d j \tag{67}
\end{equation*}
$$

Following Sims (2014), the above integral can be broken up into two parts by ordering the retailers along the unit interval:

$$
\begin{equation*}
p_{t}^{1-\epsilon}=\int_{0}^{1-\theta}\left(p_{t}^{*}\right)^{1-\epsilon} d j+\int_{1-\theta}^{1} p_{t-1}(j)^{1-\epsilon} d j=(1-\theta)\left(p_{t}^{*}\right)^{1-\epsilon}+\int_{1-\theta}^{1} p_{t-1}(j)^{1-\epsilon} d j \tag{68}
\end{equation*}
$$

Given the assumptions that the price-adjusting retailers in each period are randomly chosen and the number of retailers is large, the integral of individual prices over $[1-\theta, 1]$ of the unit interval is equal to a proportion $\theta$ of the integral over the entire unit interval, where $\theta$ is the length of the subset $[1-\theta, 1]$. That is,

$$
\begin{equation*}
\int_{1-\theta}^{1} p_{t-1}(j)^{1-\epsilon} d j=\theta \int_{0}^{1} p_{t-1}(j)^{1-\epsilon} d j=\theta p_{t-1}^{1-\epsilon} \tag{69}
\end{equation*}
$$

Hence, the aggregate price level evolves according to (27):

$$
\begin{equation*}
p_{t}^{1-\epsilon}=(1-\theta)\left(p_{t}^{*}\right)^{1-\epsilon}+\theta p_{t-1}^{1-\epsilon} \tag{27}
\end{equation*}
$$

To compute the model numerically, it is necessary to rewrite the price evolution in terms of the inflation rates because the price level may not be stationary. Eliminating the price levels in the equation above by dividing both sides by $p_{t-1}^{1-\epsilon}$ :

$$
\begin{equation*}
\left(\frac{p_{t}}{p_{t-1}}\right)^{1-\epsilon}=\theta+(1-\theta)\left(\frac{p_{t}^{*}}{p_{t-1}}\right)^{1-\epsilon} \tag{70}
\end{equation*}
$$

Letting $\pi_{t} \equiv \frac{p_{t}}{p_{t-1}}$ and $\pi_{t}^{*} \equiv \frac{p_{t}^{*}}{p_{t-1}}$ denote the gross inflation rate and the gross reset price inflation rate, respectively, the equation above can be rewritten as:

$$
\begin{equation*}
\pi_{t}^{1-\epsilon}=\theta+(1-\theta)\left(\pi_{t}^{*}\right)^{1-\epsilon} \tag{71}
\end{equation*}
$$

## B. 3 Price Dispersion

Use the Calvo assumption to break up the integral into two parts by ordering the retailers along the unit interval:

$$
\begin{equation*}
f_{3, t} \equiv \int_{0}^{1}\left[\frac{p_{t}(j)}{p_{t}}\right]^{-\epsilon} d j=\int_{0}^{1-\theta}\left(\frac{p_{t}^{*}}{p_{t}}\right)^{-\epsilon} d j+\int_{1-\theta}^{1}\left[\frac{p_{t-1}(j)}{p_{t}}\right]^{-\epsilon} d j \tag{72}
\end{equation*}
$$

Rearrange and simplify by using the definitions for $\pi_{t}$ and $\pi_{t}^{*}$ :

$$
\begin{equation*}
f_{3, t}=\int_{0}^{1-\theta}\left(\frac{p_{t}^{*}}{p_{t-1}} \frac{p_{t-1}}{p_{t}}\right)^{-\epsilon} d j+\int_{1-\theta}^{1}\left[\frac{p_{t-1}(j)}{p_{t-1}} \frac{p_{t-1}}{p_{t}}\right]^{-\epsilon} d j=(1-\theta)\left(\pi_{t}^{*}\right)^{-\epsilon} \pi_{t}^{\epsilon}+\pi_{t}^{\epsilon} \int_{1-\theta}^{1}\left[\frac{p_{t-1}(j)}{p_{t-1}}\right]_{(-\infty)}^{-\epsilon} d j \tag{73}
\end{equation*}
$$

Use the same method as in Appendix B. 2 to simplify the last term in the equation above:

$$
\begin{equation*}
\int_{1-\theta}^{1}\left[\frac{p_{t-1}(j)}{p_{t-1}}\right]^{-\epsilon} d j=\theta \int_{0}^{1}\left[\frac{p_{t-1}(j)}{p_{t-1}}\right]^{-\epsilon} d j=\theta f_{3, t-1} \tag{74}
\end{equation*}
$$

Hence, the price dispersion $f_{3, t}$ can be written recursively:

$$
\begin{equation*}
f_{3, t}=(1-\theta)\left(\pi_{t}^{*}\right)^{-\epsilon} \pi_{t}^{\epsilon}+\pi_{t}^{\epsilon} \theta f_{3, t-1} \tag{75}
\end{equation*}
$$

The index $j$ has been eliminated in the above expression, so there is no need to keep track of the individual prices. Using (28), (72), and (75), the final consumption good output $y_{t}$ is:

$$
\begin{equation*}
y_{t}=\frac{y_{w, t}}{f_{3, t}}=\frac{y_{w, t}}{(1-\theta)\left(\pi_{t}^{*}\right)^{-\epsilon} \pi_{t}^{\epsilon}+\pi_{t}^{\epsilon} \theta f_{3, t-1}} \tag{76}
\end{equation*}
$$

The real profit $\Pi_{t}^{R}$ made by the continuum of unit mass retailers is:

$$
\begin{equation*}
\Pi_{t}^{R}=\int_{0}^{1}\left[\frac{p_{t}(j)}{p_{t}} y_{t}(j)-\frac{1}{x_{t}} y_{t}(j)\right] d j=\int_{0}^{1} \frac{p_{t}(j)}{p_{t}} y_{t}(j) d j-\frac{1}{x_{t}} \int_{0}^{1} y_{t}(j) d j \tag{77}
\end{equation*}
$$

Use retailer $j$ 's individual demand function $y_{t}(j)=\left[\frac{p_{t}(j)}{p_{t}}\right]^{-\epsilon} y_{t}(22)$, the wholesale good output expression $y_{w, t}=\int_{0}^{1} y_{t}(j) d j(28)$, the aggregate price index $p_{t}=\left[\int_{0}^{1} p_{t}(j)^{1-\epsilon} d j\right]^{\frac{1}{1-\epsilon}}$ (23), and (76) to get (29):
$\Pi_{t}^{R}=\int_{0}^{1} \frac{p_{t}(j)}{p_{t}}\left[\frac{p_{t}(j)}{p_{t}}\right]^{-\epsilon} y_{t} d j-\frac{y_{w, t}}{x_{t}}=y_{t} p_{t}^{\epsilon-1} \int_{0}^{1} p_{t}(j)^{1-\epsilon} d j-\frac{y_{w, t}}{x_{t}}=y_{t}-\frac{y_{w, t}}{x_{t}}=\left(\frac{1}{f_{3, t}}-\frac{1}{x_{t}}\right) y_{w, t}$

## C Steady State Values

Table 2: Steady State Values under Baseline Calibration

|  | Perfect Competition | Imperfect Competition |
| :--- | :---: | :---: |
| Gross Inflation Rate $\pi$ | 1 | 1 |
| Productivity $z$ | 1 | 1 |
| Output $y$ | 0.983 | 0.880 |
| Consumption $c$ | 0.647 | 0.607 |
| Investment $i$ | 0.336 | 0.273 |
| Physical Capital $k$ | 4.806 | 3.899 |
| Real Price of Capital $q$ | 1 | 1 |
| Bank Loan $b$ | 3.802 | 2.597 |
| Labor $l$ | 0.283 | 0.273 |
| Real Wage $w$ | 1.623 | 1.503 |
| Gross Real Deposit Rate $R_{r}$ | 1.005 | 1.005 |
| Gross Real Loan Rate $R_{r b}$ | 1.005 | 1.013 |
| Firms' Total Net Worth $n$ | 1.004 | 1.302 |
| Leverage Ratio $\frac{b}{q k}$ | 0.791 | 0.666 |
| Marginal Product of Capital $\frac{\alpha_{k} y}{k}$ | 0.090 | 0.099 |
| Capital Demand Elasticity PEK | 23.921 | 21.857 |
| Loan Demand Elasticity PED | 30.240 | 32.815 |

Note: The table shows the steady state values of selected variables from two models with perfect banking competition and Cournot banking competition, respectively. The steady state values for gross inflation rate and productivity are exogenously set to one. The steady state values of all other variables are determined in equilibrium, based on the parameter values in Table 1.

## D Robustness Checks

Figure 7: Impulse Responses to a Negative Transitory Productivity Shock when $\chi=2$


Note: The horizontal axis shows quarters after a one-standard-deviation negative productivity shock at the beginning of period 1 . The persistence parameter $\psi$ in the $\operatorname{AR}(1)$ process for productivity is set to zero, so the shock is fully transitory. The vertical axis shows the percentage deviation from the steady state for variables other than the interest rates and the loan margin, which are expressed in deviations from the steady state in percentage points. The leverage ratio refers to the debt-to-asset ratio $\frac{b_{t}}{q_{t} k_{t}}$, and the marginal product of capital is $\frac{\alpha_{k} y_{t}}{k_{t-1}}$.

Figure 8: Impulse Responses for Different Shocks when $\alpha_{k}=0.25$ and $\alpha_{l}=0.75$


Note: The horizontal axis shows quarters after the shock that occurs at the beginning of period 1. The vertical axis shows the percentage deviation from the steady state for variables other than the loan margin, which is expressed in deviations from the steady state in percentage points. Each row shows the impulse responses of four variables after a given type of shock: a 25 basis-point contractionary monetary policy shock, a one-standard-deviation persistent negative productivity shock ( $\psi=0.95$ ), and a one-standard-deviation transitory negative productivity shock $(\psi=0)$.

Figure 9: Impulse Responses for Different Shocks when $\delta=0.025$


Note: The horizontal axis shows quarters after the shock that occurs at the beginning of period 1. The vertical axis shows the percentage deviation from the steady state for variables other than the loan margin, which is expressed in deviations from the steady state in percentage points. Each row shows the impulse responses of four variables after a given type of shock: a 25 basis-point contractionary monetary policy shock, a one-standard-deviation persistent negative productivity shock $(\psi=0.95)$, and a one-standard-deviation transitory negative productivity shock $(\psi=0)$.

Figure 10: Impulse Responses for Different Shocks when $\omega=0.0021$


Note: The horizontal axis shows quarters after the shock that occurs at the beginning of period 1. The vertical axis shows the percentage deviation from the steady state for variables other than the loan margin, which is expressed in deviations from the steady state in percentage points. Each row shows the impulse responses of four variables after a given type of shock: a 25 basis-point contractionary monetary policy shock, a one-standard-deviation persistent negative productivity shock $(\psi=0.95)$, and a one-standard-deviation transitory negative productivity shock $(\psi=0)$.

Figure 11: Impulse Responses for Different Shocks when $\rho_{r}=0.7$


Note: The horizontal axis shows quarters after the shock that occurs at the beginning of period 1. The vertical axis shows the percentage deviation from the steady state for variables other than the loan margin, which is expressed in deviations from the steady state in percentage points. Each row shows the impulse responses of four variables after a given type of shock: a 25 basis-point contractionary monetary policy shock, a one-standard-deviation persistent negative productivity shock ( $\psi=0.95$ ), and a one-standard-deviation transitory negative productivity shock $(\psi=0)$.

Figure 12: Impulse Responses for Different Shocks when $\kappa_{\pi}=3$


Note: The horizontal axis shows quarters after the shock that occurs at the beginning of period 1. The vertical axis shows the percentage deviation from the steady state for variables other than the loan margin, which is expressed in deviations from the steady state in percentage points. Each row shows the impulse responses of four variables after a given type of shock: a 25 basis-point contractionary monetary policy shock, a one-standard-deviation persistent negative productivity shock $(\psi=0.95)$, and a one-standard-deviation transitory negative productivity shock ( $\psi=0$ ).

Figure 13: Impulse Responses for Different Shocks when $\kappa_{y}=0.125$


Note: The horizontal axis shows quarters after the shock that occurs at the beginning of period 1. The vertical axis shows the percentage deviation from the steady state for variables other than the loan margin, which is expressed in deviations from the steady state in percentage points. Each row shows the impulse responses of four variables after a given type of shock: a 25 basis-point contractionary monetary policy shock, a one-standard-deviation persistent negative productivity shock ( $\psi=0.95$ ), and a one-standard-deviation transitory negative productivity shock $(\psi=0)$.

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[^0]:    ${ }^{1}$ Author's calculation based on ECB and Bankscope data in 2007 and 2014. For empirical evidence on banks' market power using different measures of bank competition, see Corbae and D'Erasmo (2013), Bikker and Haaf (2002), Ehrmann et al. (2001), De Bandt and Davis (2000), Oxenstierna (1999), Berg and Kim (1998), Molyneux, Lloyd-Williams and Thornton (1994), etc.
    ${ }^{2}$ Olivero (2010) documents that banks' price-cost margin is countercyclical in $58 \%$ (using Bankscope data for 1996-2007) to $79 \%$ (IMF International Financial Statistics for 1970-2008) of the selected OECD countries. Aliaga-Díaz and Olivero $(2010 a, 2011)$ provide evidence for the countercyclical loan margin in the US.

[^1]:    ${ }^{3}$ Drechsler, Savov and Schnabl (2017) focus on how deposit market competition affects the monetary transmission. They find that as the policy rate rises, banks tend to raise their deposit spread and households reduce their deposit holdings as a result. The contraction in deposit funding induces banks to cut lending. This paper focuses on the loan market competition instead, and a higher loan rate markup after a rise in the policy rate directly reduces firms' demand for loans.

[^2]:    ${ }^{4}$ In all these papers, changes in the loan rate markup over the business cycle are generated by introducing exogenous shocks to the elasticity of substitution between different loan or deposit products (Gerali et al., 2010), bank's marginal cost of producing loans (Hafstead and Smith, 2012), deep habits in banking (Airaudo and Olivero, 2019; Aliaga-Díaz and Olivero, 2010b), or interest rate stickiness à la Calvo (1983) or Rotemberg (1982) (Dib, 2010; Gerali et al., 2010; Hülsewig, Mayer and Wollmershäuser, 2009).

[^3]:    ${ }^{5}$ With a costly debt enforcement problem, borrowers cannot be forced to repay unsecured debt (Beck, Colciago and Pfajfar, 2014), so creditors would not lend an amount that exceeds the value of collateralized assets and borrowers would face a collateral constraint. The costly state verification of Townsend (1979) leads to an endogenous external finance premium, which then raises the cost of borrowing and amplifies business cycle fluctuations.
    ${ }^{6}$ In this paper, the loan contract is non-state-contingent, so an adverse shock can cause firms to go bankrupt ex post. This paper models firms' net worth accumulation to abstract away from firms' default probability, as net worth can absorb the potential losses.
    ${ }^{7}$ Recent asset pricing papers also focus on the role of risk premium and relate the financial intermediaries' balance sheet conditions to the credit spread (e.g., Drechsler, Savov and Schnabl, 2018; Muir, 2017; Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2013).

[^4]:    ${ }^{8}$ Under reasonable calibration, the steady state net worth of the firm is high enough that even after a large and persistent negative productivity shock, the net worth is still far from being negative. Hence, this paper neglects the possibility of default on loans.

[^5]:    ${ }^{9}$ In Andrés and Arce (2012) and Cuciniello and Signoretti (2015), firms are always financially constrained, so the effective market loan demand is given by the binding borrowing constraint that is tied to the collateral value as well as the loan rate. Due to the binding constraint, a higher loan rate directly reduces firms' borrowing capacity and hence the loan demand.

[^6]:    ${ }^{10}$ Recall that this paper neglects the possibility of loan default since it is extremely unlikely under reasonable calibration, as discussed at the beginning of Section 2.1.

[^7]:    ${ }^{11}$ When $\kappa_{y}=0$, the Taylor principle implies that $\kappa_{\pi}>1$ will ensure the nominal interest rate $R_{t}$ is raised sufficiently in response to an increase in the gross inflation rate $\pi_{t}$ so that the real interest rate rises.

[^8]:    ${ }^{12}$ Due to the differences in modelling the loan demand and banks' strategic considerations, the loan demand elasticity and hence the loan rate markup determinants in this paper differ from the existing literature (e.g., Cuciniello and Signoretti, 2015; Andrés and Arce, 2012; Mandelman, 2011, 2010; Olivero, 2010).

[^9]:    ${ }^{13}$ More specifically, the tax rate $\tau \in(0,1)$ is set to $\frac{\mu-1}{\mu}$, where $\mu=\frac{R_{b}}{R}$ is the steady state loan rate markup (43). In this way, the steady state profit made by the banking sector after tax $\left[R_{b}(1-\tau)-R\right] b$ becomes zero. In the dynamic analysis, the loan rate set by the banking sector is $R_{b, t}=\frac{\mu_{t} R_{t}}{\mu}$ under Cournot competition, where the denominator $\mu$ ensures that the steady state loan margin ( $R_{b}-R$ ) is zero, and hence the model steady state will be identical to that under perfect banking competition. The results from the dynamic analysis are robust to the case without the tax, i.e., where the two models with different banking competition have different steady states.

[^10]:    ${ }^{14}$ This is because firms' net worth increases due to a higher capital price and hence a higher value of undepreciated capital. So borrowing $\left(b_{t}=q_{t} k_{t}-n_{t}\right)$ falls more than the value of capital, leading to a drop in the leverage ratio $\frac{b_{t}}{q_{t} k_{t}}$.

