6 - INFLATION (FINANCE)

PRICES AND INCOMES COMMISSION

J. F. McCollum

Inflation and Interest Rates in Canada

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A study prepared for the Prices and Incomes Commission

by

J. F. McCollum

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PREFACE

This study was undertaken to test the various theories relating real rates of interest, nominal rates of interest and price expectations. While the study is primarily empirical, several theoretical issues are examined. In contrast to earlier work in this area, the investigation concentrates on how price expectations become built into nominal interest rates and how the behavior of final lenders and borrowers may be affected. Canadian data are used.

Fred Nold made a number of useful comments on the statistical work in this paper. Valuable research assistance was provided by Barbara Young. Typing assistance was obtained from Miss Elizabeth Galazka.

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chapter one

INTRODUCTION AND STATEMENT OF THE ISSUES TO BE CONSIDERED

This study is an investigation into the influence of inflation in Canada on Canadian rates of interest. At the present time no consensus exists among economists concerning the impact of inflation on interest rates and capital market decisions. This lack of concern reflects a disagreement over theory and over the interpretation of empirical evidence. In addition, there is considerable controversy surrounding the implications of price expectational influences on interest rates for contemporary monetary policy.

Recent statements by public authorities in Canada would appear to take the influence of price expectations upon interest rates as accepted knowledge. "The best way to reduce high interest (in Canada) is to reduce inflation (Canadian)."[1] Thus stated Edgar Benson, the Minister of Finance, before the House of Commons Committee on Interest Rates, on October 30, 1969. Other testimonies given before that committee suggested that recent inflation in Canada was primarily responsible for the high levels reached by interest rates at that time.[2]

Other statements by the Canadian authorities would seem to suggest that while nominal rates of interest are influenced by price expectations, these rates are somewhat rigid, and hence, real rates fall when there is inflation. Consider the following statement by Louis Rasminsky, the Governor of the Bank of Canada:

> "I would like now to raise the question: Why haven't the high interest rates that we have seen been more effective in restraining the use of credit and the volume of

[1] Edgar Benson, in *House of Commons Standing Committee on Finance, Trade and Economic Affairs: Respecting Interest Rates*, October 1969, No. 6.

[2] See in particular the statement made by Professor E. Neufeld.

spending?—I believe that the willingness of many borrowers to increase obligations to pay high interest rates, often for long periods is in large part due to the strong inflationary psychology that has developed."[3]

The argument made above is that inflation complicates the use of counter-cyclical monetary policy, requiring wider swings in nominal rates of interest than might otherwise be the case. It also implies a belief that nominal interest rates did not rise sufficiently during 1968 and 1969 to account for inflationary expectations and did not deter expenditures.

Despite the wide recognition by official policy-makers in Canada of the influence of price expectations on interest rates, no empirical documentation of its influence exists. Since policy is being formulated on the belief that inflation in Canada has a substantial effect on Canadian nominal rates of interest, it behooves us to attempt to quantify and assess this effect.

The next chapter contains a discussion of the Fisherian theory of real and nominal rates of interest and price expectations. Following this discussion, a number of empirical results using Canadian data over the period 1952–70 are presented. An excursion through the empirical literature on price expectations and interest rates is also contained in chapter two.

A number of investigators have suggested that price expectations have a different impact on different parts of the term structure of interest rates.[4] In chapter three, the distinction between the nominal and real term structures is explored. Empirical tests of the "Differential Effects According to Maturity" hypothesis are presented.

In reviewing the first edition of Irving Fisher's book, *The Purchasing Power of Money*,[5] John Maynard Keynes[6] argued that while Fisher showed changes in the quantity of money as affecting the price level he did not show how it was affected.[7] The literature on interest rates and inflation suffers from the same difficulties. Although it can be demonstrated that inflation and nominal rates of interest are positively correlated, the mechanism whereby inflation influences the nominal rate of interest has generally received little attention.

In chapter four, a plausible explanation of how inflationary premiums become embedded in nominal rates of interest is explored. Chapter five serves as a preliminary attempt to verify this approach. In chapter six some supple-

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[3] Louis Rasminsky, "Interest Rates and Inflation", *A Statement before the House of Commons Standing Committee on Finance, Trade, and Economic Affairs*, July 3, 1969, pp. 8–9.

[4] See for example, Milton Friedman, "Factors Affecting the Level of Interest Rates", *Saving and Residential Financing*, Chicago, Saving and Loan League, 1968. Also relevant is the discussion contained in William E. Gibson, *Effects of Money on Interest Rates*, Washington, D.C.: Board of Governors of the Federal Reserve System, 1968.

[5] Irving Fisher, *The Purchasing Power of Money*, New York, Augustus M. Kelly, 1963.

[6] J. M. Keynes, "Review of Fisher's 'The Purchasing Power of Money' ", *The Economic Journal*, 21: 393–98, 1911.

[7] Fisher's answer is illuminating. He referred Keynes to his (Fisher's) *Elementary Principles of Economics*, New York, Macmillan, 1911, pp. 242–47. The transmission mechanism specified there is precisely that outlined by Milton Friedman and David Meiselman in "The Relative Stability of Monetary Velocity and the Investment Multiplier in the United States, 1897–1958", *Commission on Money and Credit: Stabilization Policies*, Englewood Cliffs, N.J., Prentice-Hall, 1963. Also see M. Friedman, "The Demand for Money", *Proceedings of the American Philosophical Society*, June 1961.

mentary results on the impact of price expectations on investment behavior are presented. In chapter seven, the main theoretical and empirical results of the previous chapters are drawn together and their importance from the point of view of policy is assessed.

chapter two

THE FISHERIAN THEORY OF THE RELATIONSHIP BETWEEN REAL AND NOMINAL INTEREST RATES

In order to appreciate the Fisherian theory of real and nominal interest rates it is necessary to discuss first Fisher's theory of real interest rate determination.

The Fisherian theory of interest rate determination is non-monetary. That is to day, money does not enter as one of the basis determinants of interest rates. As Fisher pointed out:

> "In other words interest changes with absolutely no relation to the quantity of money in circulation."[1]

In Fisher's analysis the rate of interest is determined by time preference and the intertemporal rate of transformation. If we make the usual assumptions concerning convexity of utility and transformation functions, an aggregate excess demand curve for loans can be derived.[2] The rate of interest is determined by the market clearing condition that "desired lending is equal to desired borrowing". The rate determined is a real rate, in the sense that it gives the rate at which present goods can be traded for future goods. This rate is shown as "r" in Figure 1.

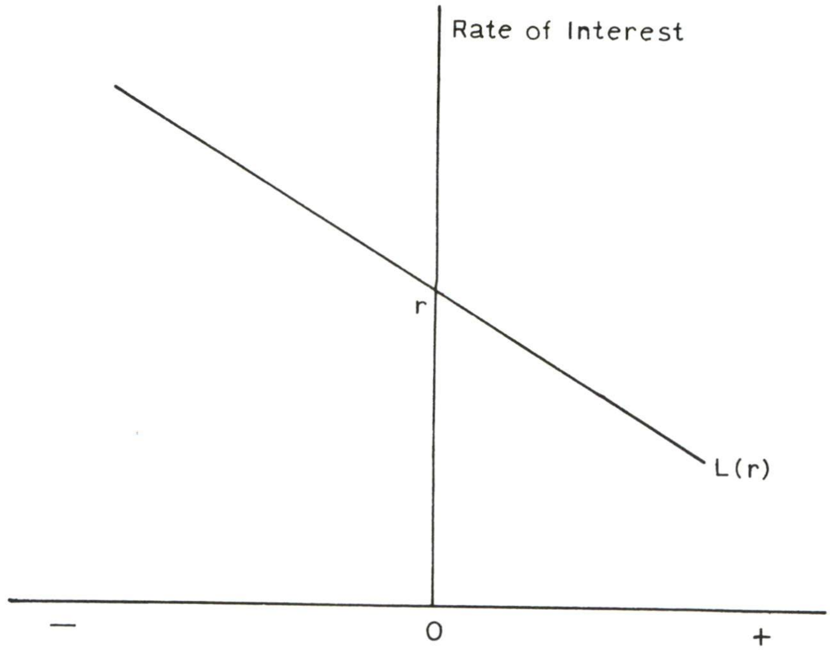
Loans, however, are made and repaid not in terms of commodities but in terms of nominal dollars. So long as the price level is not changing, this fact does

[1] I. Fisher, *Elementary Principles of Economics*, p. 359. Also relevant are Fisher's *The Theory of Interest*, p. 47, Fisher's *The Rate of Interest*, p. 78, and Joseph Schumpeter's discussion of Fisher's monetary theory in *The Theory of Economic Development*, Cambridge, Harvard University Press, 1934.

[2] For exposition purposes it is assumed that we are dealing with a closed economy at this point.

Figure 1

Aggregate Excess Demand Curve for Loans



not alter the analysis. Purchasing power over "x" of real output is given up for x(1 + r) worth of real output one year hence.

> "It is perfectly true, as is often pointed out, that when a man lends $100 of this year in order to obtain $105 next year, he is really sacrificing not $100 in literal money, but one hundred dollar's worth of other goods such as food, clothing, shelter or pleasure trips, in order to obtain next year, not $105 in literal money, but one hundred and five dollar's worth of other goods."[3]

Thus in Fisher's analysis the real rate of interest takes on the nature of a relative price—between goods today and goods tomorrow. The question then arises: should inflation occur, what changes will occur in the nature of financial contracts?

Fisher postulated that under condition of perfect foresight of the future course of prices, the nominal rate of interest would rise by the rate of inflation. This situation is illustrated in Figure 2, where "i" is the nominal rate, "r" is the real rate and "π" is the rate of inflation (actual and expected).

> "When prices are rising or falling, money is depreciating or appreciating relatively to commodities. Our theory would therefore require high or low interest accordingly as prices are rising or falling, provided we assume that the rate of interest in the commodity standard should not vary."[4]

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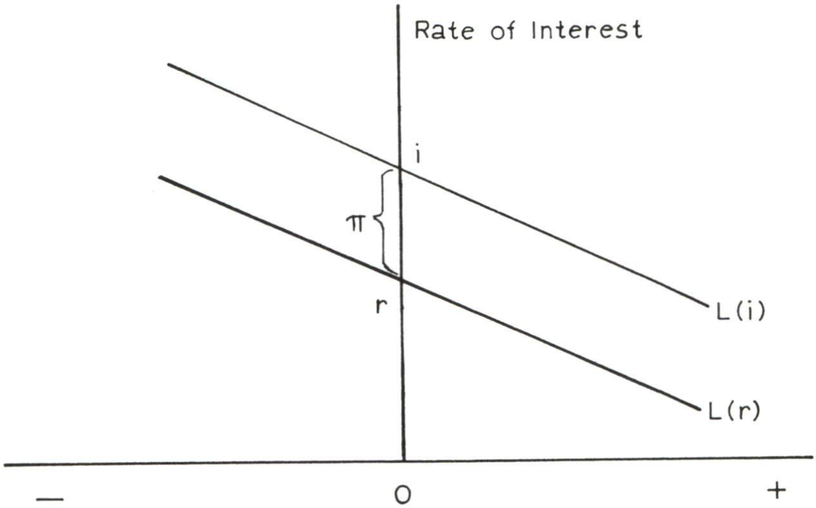
[3] Irving Fisher, *The Theory of Interest*, 1930, p. 36.

[4] I. Fisher, *Appreciation and Interest*, New York, Macmillan, 1898, p. 58. See also *Ibid.*, p. 1.

Figure 2

Aggregate Excess Demand Curve for Loans Under Inflation



This result is not surprising, since Fisher viewed the rate of interest as a special relative price, and most economists viewed relative price determination to be divorced from the determination of the absolute price level. Consequently should the absolute price level begin to change, Fisher argued that the nominal interest rate would alter in such a manner that the old intertemporal terms of trade (old real rate) were preserved.

The relation between the nominal rate of interest, the real rate of interest and the rate of inflation is given by the formula:

(1) $(1 + i) = (1 + r)(1 + \pi)$

or

(2) $i = r + \pi$

Thus, in order for the rate of interest in terms of real purchasing power over commodities to remain unchanged, the nominal rate of interest needs to rise by $\pi + \pi r$.[5]

The real rates of interest "r" in Figures 1 and 2 are identical in Fisher's analysis. The rationale given by Fisher was that lenders and borrowers are concerned with intertemporal trading in real terms, and that a change in the price level would have no effect on the terms of trading things over time.

It is not enough to note that Irving Fisher distinguished between real and nominal rates of interest without defining precisely how the component parts of the "Fisher equation" (i.e. equation (1) or (2)) are derived. A number of econo-

[5] It is customary to disregard the term πr which is usually small. Thus in Figure 2 we show the nominal rate of interest rising by π rather than $\pi + \pi r$.

mists have used the Fisher equation in indiscriminate and contradictory ways. Equation (2) can be variously interpreted as:

(a) An identity relating the nominal rate of interest, the actual rate of inflation, and the ex-post real rate of interest.

(b) A definition of the expected real rate of interest.

(c) A definition of the expected rate of inflation.

(d) An equilibrium condition.

(e) A hypothesis which suggests that the level of nominal interest rates adjusts to incorporate the expected rate of inflation.

As our previous argument showed, interpretation (e) is the Fisherian hypothesis relating nominal interest rates, real interest rates and the expected rate of inflation. A careful reading of Fisher's writing on real and nominal rates of interest reveal, however, the Fisher did not always distinguish clearly among the various interpretations listed above.

In Fisher's analysis the real rate of interest is not affected by the rate of inflation, in equilibrium. In our earlier discussion this was noted by pointing out that the "real rates" were identical in Figures 1 and 2. Should inflation affect the real rate directly, then of course it is still possible that the real rate is exceeded by the nominal rate by the expected rate of inflation in equilibrium. In most of our work, however, we follow Fisher in assuming that the direct effect of inflation upon the real rate is zero in equilibrium.

Fisher observed that the nominal interest rate did not appear to move a sufficient amount to compensate for contemporaneous movements in the rate of inflation. He suggested that the nominal rate adjusted not to the actual rate of inflation but to the expected rate of inflation. He suggested, further, that expectations were formed on the basis of a weighted average of past rates of inflation. This concept may be represented by the formula:

$$(3) \quad \pi_t^e = \sum_{i=0}^{N} \alpha_i \, \pi_{t-i}^a ,$$

where π_t^e is the expected rate of inflation in period t, and π_t^a is the actual rate of inflation in period t, and α_i is the weight given to the rate of inflation in period $(t - i)$ in forming expectations in period t. Consequently, Equation (2) appears as:

$$(4) \quad i_t = r_t + b \sum_{i=0}^{N} \alpha_i \, \pi_{t-i}^a$$

The Fisher Hypothesis is the hypothesis that "b", the coefficient on the expected inflation term, is equal to unity. The less specific hypothesis that price expectations exert a positive influence on the nominal rate of interest is represented by the hypothesis $b > 0$. The test of the Fisher Hypothesis is conditional upon two things, a maintained hypothesis on the formation of price expectations and a maintained hypothesis on the determination of the real rate. As will be pointed out later in this chapter, failure to recognize this fact has led some authors to make groundless assertions on the basis of their empirical work.

In actual estimation procedures, the weights in equation (4) are usually estimated rather than imposed. This means that the coefficient "b" is entangled with the estimated weights. Assuming there is no bias in the formation of price expectations (i.e., on the average, people neither over nor under-predict the future course of prices) a sum of weights which is equal to one is consistent with the Fisher Hypothesis, providing we can separately account for the real rate.[6]

EMPIRICAL TESTS OF THE FISHERIAN MODEL

This section consists of empirical tests of the Fisherian Hypothesis that the level of nominal interest rates adjusts to incorporate the expected rate of inflation. The central equation that is used is:

$$(5) \quad i_t = g(x)_t + b\pi^e_t + u_t$$

where i is the nominal rate of interest, π^e is the expected rate of inflation, and g(x) is a function which determines the real rate of interest. u_t is an error or disturbance term. The Fisher Hypothesis, in the context of equation (5), above, is represented by the hypothesis that $b = 1$. In the following sections of this chapter the Fisher Hypothesis is tested under different assumptions about the function g(x), and different assumptions about the construction of the price expectations variable. Since there are no reasonable behavioral grounds for preferring one method of representing the expectational variable over another method, simple methods will be employed.

Our test of the Fisher Hypothesis is conditional upon the maintained hypotheses on the real rate of interest and the formation of expectations. The variety of combinations of assumptions on the real rate of interest and price expectation formation is infinite, consequently no general test of the Fisher Hypothesis can be made using equation (5).[7]

The time period of observation in this chapter is 1952–1970 for monthly data, and 1955–1970 for quarterly data. The price index used is the Canadian Consumer Price Index. Six interest rates are used: the Treasury Bill rate (3 months), the 1 to 3, the 3 to 5, 5 to 10, and the over 10 years Government of Canada bond rates, and the McLeod, Young, Weir industrial bond index.

THE NAIVE FISHERIAN MODEL

In this section, the real rate is assumed to be constant over time. This is the same assumption that Irving Fisher used in his empirical work.[8]

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[6] If the inflation is consistently accelerating and if $\Sigma w_i = 1$, equation (3) always under predicts the future course of price changes. It is assumed that expectations are not formed on time derivations of a higher order than the first.

[7] The strongest statistical result one can obtain using equation (3) is a rejection of a conditional version of the Fisher hypothesis.

[8] See Irving Fisher, *The Theory of Interest*, New York, MacMillan, 1930, pp. 399–492.

Expectations of the future course of price changes are expectations about the average rate of inflation over a period. Thus, for a loan contract made in period t and maturing in period t + k, we have:

$$(6) \quad P_t . e^{k({}_{t+k}\pi^e_t)} = {}_{t+k}P^e_t$$

where P_t is the price level in period t and ${}_{t+k}P^e_t$ is the level at which the price level is expected to be k periods hence. The expected rate of inflation over this period (of length k) formed in the period t is represented by the symbol ${}_{t+k}\pi^e_t$.[9]

From equation (6) we have: $e^{{}_{t+k}\pi^e_t} = \left[\frac{{}_{t+k}P^e_t}{P_t}\right]^{1/k}$
thus

$$(7) \quad {}_{t+k}\pi^e_t = \frac{1}{k}(\ln{}_{t+k}P^e_t - \ln P_t).$$

It is postulated that expectations about the rate of inflation depend solely upon the past history of price changes.[10] That is to say other variables such as government policy and unemployment, which might influence the formation of price expectations are ignored. It is also assumed that expectations are single-valued. Alternatively, we could assume that individuals form a subjective probability distribution about the future course of inflation, but react only to the mean of that function.[11] Under these assumptions price expectations can be assumed to be generated by an extrapolative or adaptive generating mechanism. This permits the representation of the expected rate of inflation by a weighted average of past rates of change of prices.[12] Consequently, we have:

$$(8) \quad \pi^e_t = \sum_{i=0}^{n} \alpha_i \, \pi^a_{t-i}$$

Substituting equation (8) into equation (5), and assuming that the real rate is constant over time we have:

$$(9) \quad i_t = a + b \sum_{i=0}^{n} \alpha_i \, \pi^a_{t-i} + u_t .$$

However, the actual estimating equation used is:

$$(10) \quad i_t = a + \Sigma \, w_i \, \pi^a_{t-i} + u_t .$$

If it is assumed that there is no bias in the formation of price expectations (i.e. $\Sigma\alpha_i = 1$), then a sum of weights ($\Sigma \, w_i = 1$) in the above equation is consistent with the Fisher Hypothesis.

There are several reasons why our empirical results may go astray. If b does not equal unity one cannot be sure if the trouble lies with the hypothesis that price expectations influence nominal rates of interest, or the maintained hypoth-

[9] To simplify the presentation, the preceding subscript (t + k) will be generally omitted.

[10] This assumption is made despite the warning by Samuel Taylor Coleridge that: "To most men, experience is like the stern lights of a ship, which illumine only the track it has passed."

[11] In other words, other parameters of the subjective probability distribution such as the variance are assumed not to matter.

[12] There are several ways of looking at this problem. We can view individuals, in their capital market decisions, as reacting to the "permanent" rate of inflation. Alternatively, we can look upon the weighted sum of past rates of inflation as giving a forecast of inflation to come.

esis on the formation of price expectations. Furthermore, a sum of coefficients which is equal to unity is consistent with both the Fisher Hypothesis and the assumed price expectation hypothesis being violated, though in offsetting ways. These reservations must be kept in mind while interpreting the empirical results of this chapter.

It must be stressed that the naive Fisherian model is constructed on the assumption of a constant real rate. If it is found that $b \neq 1$, it is incorrect to conclude that the real rate must have changed.[13] This conclusion obviously violates the assumptions under which the model was estimated, and if true equation (10) would not provide a test of the Fisher Hypothesis.

It is also the case that a real rate series cannot be constructed using the estimated coefficients of equation (10). That is to say, one cannot construct a real rate series of the form:

$$(11) \quad \hat{r}_t = i_t - \sum_{i=0}^{n} \hat{w}_i \, \pi^a_{t-i}$$

since the computed $\hat{r}$ violates the assumptions under which we measured the effect of price expectations.[14]

It should also be noted that a series of positive coefficients in a regression of the form of equation (10) can be generated by a variety of underlying structural models, one of which is the Fisherian model. In a period of rising prices, the monetary authorities may react by restricting the growth or reducing the level of the money supply. This may have the effect of raising the nominal rate of interest. If the authorities react with a lag, or if there is a lag in the effect of their actions upon interest rates or both, a positive relationship between nominal rates of interest and past rates of inflation would be observed. Consequently, an observed series of positive coefficients in equation (10) does not yield sufficient information to conclude that price expectations have been at work.

Estimates of equation (10) using monthly and quarterly data appear in Tables I and II respectively. The rate of price change is compounded and expressed at an annual percentage rate.

The t value for each variable appears in parentheses below the estimated regression coefficient. The summary statistics appearing below each regression are: D.W.—the Durbin Watson Statistic, S.E.E.—the standard error of the estimate, $\bar{R}^2$—the adjusted coefficient of determination, A.L.—the average lag, S—the sum of the coefficients on the price terms, S.E.S.—the standard error of S and St_1—the t statistic for S under the hypothesis that $S = 1$.

The average lag is not reported if negative coefficients appear in the regression. Under this circumstance there is no generally meaningful formula for computing

13 For example, this inference is incorrectly drawn by W. E. Gibson. See W. E. Gibson, "Price-Expectations Effect on Interest Rates", *Journal of Finance*, March 1970, p. 33.

14 The Federal Reserve Bank of St. Louis has published series based on computations using an equation such as (11). Despite the obvious contradictions implicit in the construction of such a series, it may also be remarked that such a series is of no interest to the authorities since they cannot affect the real rate of interest as defined by equation (11). Movements in the real rate, defined by equation (11), consist entirely of movements in the estimated residual term of equation (10).

TABLE I–(a)

Rates of Price Change and Interest Rates

January 1952—December 1970, Monthly Observations (228) O.L.S. 12 Unconstrained Lags

	Intercept	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
T.B.	2.30 (16.23)	0.1113 (4.53)	0.0931 (3.63)	0.0749 (2.92)	0.0858 (3.34)	0.0577 (2.27)	0.0589 (2.31)	0.0689 (2.69)	0.0315 (1.26)	0.0493 (1.99)	0.0482 (1.92)	0.0096 (0.40)	−0.0176 (−0.77)
		D.W. = 0.180			S.E.E. = 1.328 S = 0.6717 St_1 = 6.219 Average Lag = *N.A.*			$\bar{R}^2$ = 0.42 S.E.S. = 0.0528					
1–3	3.17 (26.44)	0.0858 (4.13)	0.0774 (3.57)	0.0634 (2.93)	0.0711 (3.28)	0.0469 (2.18)	0.0490 (2.28)	0.0545 (2.52)	0.0293 (1.39)	0.0429 (2.05)	0.0446 (2.11)	0.0126 (0.62)	−0.0018 (−0.09)
		D.W. = 0.158			S.E.E. = 1.121 S = 0.5757 St_1 = 4.520 Average Lag = *N.A.*			$\bar{R}^2$ = 0.42 S.E.S. = 0.0446					
3–5	3.55 (30.93)	0.0774 (3.89)	0.0703 (3.38)	0.0566 (2.72)	0.0641 (3.08)	0.0486 (2.35)	0.0499 (2.42)	0.0540 (2.60)	0.0275 (1.36)	0.0408 (2.03)	0.0419 (2.06)	0.0162 (0.83)	−0.0005 (−0.03)
		D.W. = 0.134			S.E.E. = 1.075 S = 0.5467 St_1 = −10.606 Average Lag = *N.A.*			$\bar{R}^2$ = 0.41 S.E.S. = 0.0427					
5–10	3.76 (31.22)	0.0732 (3.51)	0.0596 (2.73)	0.0522 (2.39)	0.0675 (3.09)	0.0379 (1.75)	0.0499 (2.31)	0.0523 (2.40)	0.0265 (1.25)	0.0386 (1.83)	0.0432 (2.02)	0.0182 (0.88)	0.0096 (0.50)
		D.W. = 0.176			S.E.E. = 1.128			$\bar{R}^2$ = 0.36					

					S = 0.5288 $St_1 = -10.506$ Average Lag = 2.473 Months			S.E.S. = 0.0449					
10+	4.01 (33.72)	0.0640 (3.11)	0.0608 (2.83)	0.0444 (2.07)	0.0544 (2.53)	0.0438 (2.06)	0.0459 (2.15)	0.0455 (2.12)	0.0253 (1.21)	0.0361 (1.73)	0.0353 (1.68)	0.0174 (0.86)	0.0075 (0.39)
		D.W. = 0.071			S.E.E. = 1.112 S = 0.4804 $St_1 = -11.752$ Average Lag = 2.418 Months			$\bar{R}^2 = 0.32$ S.E.S. = 0.0442					
M.Y.W.	4.61 (37.77)	0.0738 (3.49)	0.0694 (3.14)	0.0530 (2.40)	0.0622 (2.82)	0.0537 (2.45)	0.0559 (2.55)	0.0521 (2.37)	0.0288 (1.34)	0.0439 (2.06)	0.0414 (1.91)	0.0207 (1.00)	0.0121 (0.62)
		D.W. = 0.077			S.E.E. = 1.142 S = 0.5671 $St_1 = -9.534$ Average Lag = 2.475 Months			$\bar{R}^2 = 0.39$ S.E.S. = 0.0454					

the average lag.[15] S.E.S., the standard error of the sum of the coefficients, is obtained by pre and post-multiplying the variance-covariance matrix by a vector of zeros and ones.

Table I-(a) contains results for 12 unconstrained lags, using monthly data. Under the assumptions made, we would expect to find $a > 0$, and $b_i > 0$ for $i = 1$ to k. This result is in fact found. Most of the coefficients are significantly different from zero at the 0.05 level.

The coefficients tend to decline over time, but not uniformly so. Insofar as negative or insignificant coefficients are obtained, they tend to occur near the tail of the lag.

The sum of the coefficients range from a high of 0.67 for the Treasury Bill rate to a low of 0.48 for the long-term Government bond rate. At first sight, these results suggest that if the rate of inflation were to increase by one per cent and remain at that new higher level, the Treasury Bill rate would (on these calculations) rise by 67 basis points, and the long-term Government bond rate would rise by 48 basis points, by the time 12 months had expired.

The Fisher Hypothesis, interpreted as suggesting that the sum of coefficients is equal to unity, can be rejected for all maturities at the 0.01 level of significance. The weaker hypothesis, that price expectations have a positive effect on nominal interest rates is not rejected.

Mean lags are not calculable for the three shortest-term rates. There is no apparent difference among the mean lags of the remaining rates. The adjusted coefficients of determination decline with term of maturity on Government securities, the highest being 0.42 for Treasury Bills, and the lowest being 0.32 for long-term issues.

TABLE I–(b)

Rates of Price Change and Interest Rates

January 1952–December 1970, Monthly Observations

O.L.S. 18 Unconstrained Lags

Dependent Variable	Intercept	Sum of Weights	D.W.	S.E.E.	$\bar{R}^2$
T.B.	2.32	0.667	0.191	1.334	0.41
	(15.46)	(−5.431)			
1–3	3.12	0.597	0.193	1.126	0.41
	(24.59)	(−8.530)			
3–5	3.50	0.569	0.166	1.081	0.40
	(28.78)	(−9.497)			
5–10	3.70	0.555	0.205	1.134	0.36
	(29.98)	(−9.344)			
10+	3.95	0.504	0.095	1.120	0.31
	(31.34)	(−10.551)			
M.Y.W.	4.54	0.596	0.110	1.147	0.39
	(35.14)	(−8.182)			

[15] Zvi Griliches, "Distributed Lags: A Survey", *Econometrica*, January 1967, pp. 16–47.

The pattern of the intercept terms suggests that the real rate of interest increases with term to maturity. The relatively high real rate (4.61) for the McLeod, Young, Weir industrial bond index suggests that there is a risk premium attached to industrial securities in comparison with Government securities. The pattern of real rates is in accord with standard liquidity preference notions.

In summary, while these results suggest that price expectations may affect the level of nominal interest rates, the evidence is not clear cut. The Fisher Hypothesis is uniformly rejected by the data. A number of other statistical difficulties are inherent in these regression results; they will be discussed shortly.

In Table I-(b), the results using 18 unconstrained lags on monthly data over the period January 1952 to December 1970 are found. Only the sum of the coefficients is presented. The "t" statistic appearing under the sum of the weights has been calculated for the null hypothesis that the sum is equal to unity. None of the six additional lag terms were significant. Most were negative. The summary statistics remained essentially unchanged. Similar results are obtained if the length of the lag is extended to 24 months.

In Table II the results for quarterly data are presented using 12 unconstrained lags. Lags after the second or third quarter are not significant. Furthermore, most of the response occurs within the first three quarters. For example, in the case of Treasury Bills, the sum of the coefficients is 0.90, while after three quarters it is 0.80. A similar statement can be made for the other maturities. The adjusted coefficient of determination remains in the same range as was the case using monthly data (0.48 to 0.38). The basic difference between these results and those previously obtained using monthly data is that the sum of the coefficients is closer to unity. In three cases out of six we are unable to reject the Fisher Hypo-

TABLE II

Rates of Price Change and Interest Rates
1955–1970, Quarterly Observations
O.L.S. 12 Unconstrained Lags

Dependent Variable	Intercept	Sum of Weights	D.W.	S.E.E.	$\bar{R}^2$
T.B.	1.83 (5.92)	0.898 (−0.820)	0.551	1.273	0.46
1–3	2.50 (10.39)	0.880 (−1.245)	0.562	0.992	0.54
3–5	2.95 (12.35)	0.821 (−1.867)	0.457	0.984	0.50
5–10	3.13 (12.51)	0.816 (−1.838)	0.390	1.030	0.46
10+	3.45 (13.30)	0.735 (−2.556)	0.247	1.067	0.38
M.Y.W.	3.93 (15.30)	0.875 (−1.215)	0.334	1.056	0.48

thesis (using a one-tailed test).[16] The hypothesis that the sum of the weights is equal to zero can also be rejected for all maturities at the 0.01 level of significance.

The body of regression results taken as a whole, although interesting, reveal a number of statistical problems. First, there is a relatively serious multicollinearity problem. In many cases, the correlation matrix of regressors is close to being singular. This, in itself, would tend to raise the standard errors of the coefficients. The degree of serial correlation is also quite severe. This means that our estimators are not efficient. At the same time the variances of the obtained estimators will be biased downward, tending to raise the t statistics. This combination of severe serial correlation combined with multicollinearity means that it is difficult to place much meaning on the tests of significance.

Autocorrelation by itself still leaves us with unbiased estimators. There are reasonable grounds for suspecting that the coefficients on the lagged inflation terms are biased upwards. The serial correlation suggests that important variables have been omitted. If the omitted variables are collinear or at least depart substantially from orthogonality with the included regressors, the estimated coefficients will be biased. Thus, it appears that the equations as they stand suffer from specification error. The low proportion of the variance explained in the monthly and quarterly cases is also suggestive of omitted variables.

In the following section, a variety of efforts are made to reduce the serial correlation problem in the context of equation (10). This is followed by some discussion of the difficulties produced by multicollinearity and some attempted remedies. In the latter sections of this chapter, the assumption of an unchanging real rate is no longer retained.

Representative results of simplified attempts to reduce the serial correlation problem are contained in Table III. Two interest rates were used, the 90-day

TABLE III

Hildreth-Lu and First Difference Regressions
January 1952–December 1970, Monthly Observations
12 Unconstrained Lags

Dependent Variable	Intercept	Sum of Weights	D.W.	S.E.E.	$\overline{R}^2$	ρ	Transformation
T.B.	2.98 (1.84)	0.076 (−15.584)	1.449	0.33	0.00	0.99	Hildreth-Lu
T.B.	0.02 (0.94)	0.0005 (−162.5270)	1.545	0.34	0.00	1.00	First Difference
10+	5.02 (7.77)	0.028 (−41.589)	1.337	0.13	0.01	0.99	Hildreth-Lu
10+	0.03 (1.58)	−0.007 (−214.257)	1.242	0.26	0.04	1.00	First Difference

[16] The reason for the one-tailed test was that the most reasonable alternative *a priori* was $S = 0$. The possibility of $S > 1$ was thought most unlikely. A one-tailed test provides a more powerful test under these conditions.

Treasury Bill (T.B.) rate and the long-term Government of Canada bond yield (10+).[17] Using first differences, the serial correlation is considerably reduced. The regressions, however, do not explain any of the variation in the rate of interest. Using an F test the regressions as a whole would be rejected. The sum of the coefficients is 0.0005 for the short-term rate and 0.0070 for the long-term rates. Similar results are obtained using the Hildreth-Lu technique. The estimated autoregressive parameter is 0.99 (i.e., it is at the extremity of the range) suggesting that the serial correlation may be of a higher order than the first. The adjusted coefficients of determination are effectively zero.

None of the individual coefficients in any of the regressions is significantly different from zero. At the same time, several coefficients are negative in sign. At face value, these results suggest that our earlier regressions of interest rates on lagged rates of price change were picking up bogus lags induced by the serial correlation.

Tables IV-A and IV-B contain results using the Durbin-Two-Step estimation procedure.[18] This procedure allows us to obtain estimates which are asymtotically efficient.

Suppose that we have an equation of the form:

$$y_t = \beta_1 x_{1t} + \ldots + \beta_1 x_{kt} + u_t$$
$$u_t = \alpha_1 u_{t-1} + \ldots + \alpha_m u_{t-m} \quad (t = 1 \ldots N)$$

A preliminary regression of the form:

$$y_t = a_1 y_{t-1} + \ldots + a_m y_{t-m} + \sum_j^m \sum_i^k w_{ij} x_{ijt} + \varepsilon_t$$

is estimated. The variables

$$\hat{y} = y_t - \hat{a}_1 y_{t-1} - \ldots - \hat{a}_m y_{t-m}$$
$$\hat{x}_{it} = x_{it} - \hat{a}_1 x_{it-1} - \ldots - \hat{a}_m x_{it-m}$$

can be created. The Durbin-Two-Step estimates of the β's are obtained by regressing $\hat{y}_t$ on the $\hat{x}$'s using ordinary least squares.

In the empirical work reported below it was assumed that the residuals followed a first order autoregressive scheme. The estimated autoregressive parameter appears as ø among the summary statistics reported for each regression result in Tables IV-(a) and IV-(b).

The Durbin-Two-Step procedure yielded estimates of the autoregressive parameter which were lower than those obtained with the Hildreth-Lu technique. Autoregressive parameters in the neighbourhood of 0.9 were usually obtained.

The results obtained using the Durbin-Two-Step procedure are in accord with our earlier efforts using first differences and the Hildreth-Lu technique. The sum of the coefficients drops substantially. The Fisher Hypothesis is rejected across

17 The results are not sensitive to the maturity of the bond chosen. Since broadly similar results were obtained, only results for a representative short and long-term rate of interest are presented.

18 J. Durbin, "The Fitting of Time-Series Models", *Revue de L'Institut International de Statistique*, The Hague, 1960, pp. 233–243.

Also, J. Durbin, "Estimation of Parameters in Time-Series Regression Models", *Journal of the Royal Statistical Society*, Series B, vol. 22(1): 139–153, 1960.

TABLE IV–(a)

Interest Rates and Rates of Price Change
Durbin Two-Step Procedure
January 1952–December 1970
12 Unconstrained Lags, Monthly Observations

Dependent Variable	Intercept	Sum of Weights	Auto Regressive Coefficient	D.W.	S.E.E.	$\bar{R}^2$
T.B.	0.22	0.173	0.940	1.471	0.33	0.02
	(9.84)	(−13.193)				
1–3	0.42	0.254	0.898	1.301	0.33	0.05
	(17.22)	(−12.046)				
3–5	0.50	0.282	0.886	1.184	0.31	0.08
	(21.10)	(−12.488)				
5–10	0.28	0.087	0.943	2.437	0.35	0.00
	(11.97)	(−13.745)				
10+	0.47	0.143	0.908	1.013	0.26	0.05
	(24.77)	(−17.449)				
M.Y.W.	0.62	0.233	0.890	0.903	0.30	0.09
	(27.61)	(−13.729)				

TABLE IV–(b)

Interest Rates and Rates of Price Change
Durbin Two-Step Procedure
1955–1970, Quarterly Observations
12 Unconstrained Lags

Dependent Variable	Intercept	Sum of Weights	Auto Regressive Coefficient	D.W.	S.E.E.	$\bar{R}^2$
T.B.	0.68	0.981	0.665	1.108	0.60	0.28
	(4.27)	(−0.099)				
1–3	0.99	0.961	0.622	1.347	0.47	0.46
	(8.01)	(−0.303)				
3–5	0.99	0.895	0.705	1.389	0.40	0.39
	(8.54)	(−0.747)				
5–10	0.86	0.872	0.741	1.421	0.38	0.35
	(8.70)	(−0.873)				
10+	0.54	0.622	0.870	1.107	0.23	0.19
	(8.70)	(−2.629)				
M.Y.W.	0.44	0.561	0.919	0.883	0.21	0.17
	(9.48)	(−2.710)				

the board for all maturities. The adjusted coefficients of determination are uniformly either zero or a negligible distance from zero.

In contrast to the monthly results, the quarterly results using the Durbin-Two-Step procedure are broadly consistent with the naive version of the Fisherian theory. The Fisher Hypothesis can only be rejected for the industrial bond yield and the long-term Government bond yield. On the other hand the null hypothesis that the sum of the weights is equal to zero can be rejected for all maturities.

It was previously pointed out that estimation of equation (10) by ordinary least squares was complicated by multicollinearity difficulties. One way of evading the multi-collinearity problem in the context of a distributed lag regression such as equation (10) is through the use of the Almon lag technique.[20] Tables V-(a) and V-(b) present the results for monthly data, while the results for quarterly data are contained in Tables V-(c) and V-(d). A third degree polynomial was used throughout, and 12, 24 and 36 lags were tried.

In the case of monthly data, the results are substantially unchanged from those using unconstrained lags. The adjusted coefficients of determination hover about 0.40. The Fisher Hypothesis is uniformly rejected. The results for quarterly data were not appreciably different from those obtained previously. The only

TABLE V–(a)

Rates of Interest and Rates of Price Change
January 1952–December 1970, Monthly Observations
3rd Degree Polynomial, 12 Lags

Dependent Variable	Intercept	Sum of Weights	D.W.	S.E.E.	$\bar{R}^2$	Average Lag
T.B.	2.40 (16.88)	0.618 (−7.351)	0.10	1.352	0.40	4.018
1–3	3.24 (27.15)	0.534 (−10.669)	0.10	1.135	0.40	4.310
3–5	3.62 (31.79)	0.510 (−11.761)	0.09	1.082	0.40	4.418
5–10	3.82 (32.22)	0.496 (−11.608)	0.13	1.128	0.36	4.644
10+	4.06 (34.86)	0.451 (−12.880)	0.04	1.107	0.33	4.599
M.Y.W.	4.67 (38.91)	0.535 (−10.609)	0.04	1.141	0.39	4.646

TABLE V–(b)

Rate of Interest and Rates of Price Change
January 1952–December 1970, Monthly Observations
3rd Degree Polynomial, Representative Results

Dependent Variable	Intercept	Sum of Weights	Number of Lags	D.W.	S.E.E.	$\bar{R}^2$
T.B.	2.39 (14.65)	0.616 (−6.344)	24	0.123	1.370	0.38
10+	3.66 (28.74)	0.625 (−7.652)	24	0.067	1.030	0.42
T.B.	2.54 (12.74)	0.523 (−6.417)	36	0.067	1.520	0.23
10+	3.29 (25.11)	0.804 (−3.520)	36	0.035	0.96	0.50

[20] Shirley Almon, "The Distributed Lag Between Capital Appropriations and Expenditures", *Econometrica*, January, 1965.

Table V–(c)

Rates of Interest and Rates of Price Change
1955–1970, Quarterly Observations
3rd Degree Polynomial, 12 Lags

Dependent Variable	Intercept	Sum of Weights	D.W.	S.E.E.	$\bar{R}^2$	Average Lag
T.B.	1.92 (8.64)	1.034 (0.6029)	0.757	0.853	0.695	5.73
1–3	2.58 (15.12)	0.993 (−0.096)	0.853	0.655	0.773	5.48
3–5	2.99 (18.45)	0.953 (−0.722)	0.723	0.623	0.775	5.66
5–10	3.14 (19.01)	0.976 (−0.357)	0.640	0.634	0.778	6.14
10+	3.42 (21.24)	0.923 (−1.186)	0.347	0.619	0.767	6.71
M.Y.W.	3.78 (24.56)	1.092 (1.482)	0.306	0.592	0.830	5.75

Table V–(d)

Rates of Interest and Rates of Price Change
1955–1970, Quarterly Observations
3rd Degree Polynomial, 24 Lags

Dependent Variable	Intercept	Sum of Weights	D.W.	S.E.E.	$\bar{R}^2$	Average Lag
T.B.	1.38 (4.47)	1.329 (2.316)	0.615	0.84	0.69	8.215
1–3	2.16 (9.52)	1.213 (2.070)	0.713	0.61	0.79	n.a.
3–5	2.44 (11.78)	1.246 (2.595)	0.600	0.56	0.81	8.475
5–10	2.56 (12.31)	1.275 (2.896)	0.486	0.56	0.82	8.453
10+	2.88 (14.90)	1.198 (2.253)	0.163	0.52	0.83	8.491
M.Y.W.	3.01 (21.51)	1.505 (7.850)	0.340	0.38	0.93	8.760

substantive difference appears to be that the t statistics on individual coefficients tend to be higher than those previously obtained. Serial correlation remains a distinct difficulty.[21]

The results of our experimentation with the Naive Fisherian model and monthly data may be summarized as follows: Using unconstrained lags the Fisherian hypothesis was rejected by the data. The weaker hypothesis that price expectations exert a positive influence on nominal interest rates was not imme-

[21] Estimating the lag employing a Koyck transformation on equation (10) produces unsatisfactory results. The estimated decay parameter was uniformly greater than one, which implies expectations are explosive. In the presence of serial correlation, however, that coefficient is biased upwards.

diately rejected. Two statistical problems were evident, multicollinearity and serial correlation. An attempt was made to circumvent the multicollinearity problem by re-estimating the equations using the Almon lag technique. This had little impact on the estimated coefficients. Attempts to treat serial correlation, however, resulted in the effective disappearance of the relationship. This suggested that our earlier estimates of the impact of inflation may have been bogus lags which were induced by the serial correlation. The severe degree of serial correlation and relatively low coefficients of determination suggested that other factors in addition to price expectations may have a systematic effect on interest rates. This in turn suggests that the Naive Fisherian model may be an inappropriate device for studying the impact of price expectations on interest rates.

In contrast to the results obtained using monthly data, the results using quarterly data stand up relatively well once the variables have been adjusted for serial correlation. In most cases we were unable to reject the hypothesis that the sum of the weights is equal to unity. At the same time, we were able to reject the hypothesis that the sum of the weights is equal to zero. The ambiguity of the monthly and quarterly results suggest that there may be some advantage in going beyond the Naive Fisherian model and looking at behavioral relationships.

A TEST OF THE RADCLIFFE POSITION

It is often argued that the reason that nominal rates of interest rise during an inflationary episode is that individuals have the alternative of holding equities. Consider the following statement from the Radcliffe report:

> "Insofar as people foresee a steady rise of prices of two per cent per annum, they will look for a five per cent rate of interest instead of three per cent. Since there are alternative securities, especially equities, which can be expected to increase in money value if inflation continues, bondholders will tend to switch from bonds to equities unless a rise in bond yield offers satisfactory compensation for the expected fall in the real value of the fixed money interest earned on the bonds. There is no doubt at all that this has been a real force in the course of security markets in the post war years: as a realization of continuing inflation gained hold."[22]

The position taken by the Radcliffe Commission is that the upward pressure on interest rates comes from the lender's side of the market. Nothing is said about the behavior of borrowers. Attempts to put the Radcliffe position on a more rigorous basis have continued this neglect.[23]

Three fairly important assumptions are implicit in the Radcliffe statement. It is assumed that the rate of return on equities in real terms is not affected by inflation, and that bonds and equities are perfect substitutes in investors' portfolios. Furthermore, it is assumed that the supply of bonds (demand for bond finance) is completely inelastic with respect to the rate of interest.

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22 *Report of the Committee on the Working of the Monetary System*, London, 1959, p. 279.

23 Charles Kennedy, "Inflation and the Bond Rate", *Oxford Economic Papers*, October 1960. R. J. Ball, "Inflation and the Bond Rate", *Oxford Economic Papers*, October 1962.

The substitutability assumption on the lender's side of the market can be stated as "equivalent assets should earn equivalent rates of return whether in real or nominal terms". However, this is an assertion about an equilibrium position and does not imply that inflation pushes up the nominal rate of interest.

In a similar fashion, Milton Friedman writes:

"Let us designate the nominal interest rate by R_B (the B for bonds) and the real rate by R_E (the E for Equity). Now

$$\frac{1}{p}\frac{dP}{dt}$$

is the percentage rate at which prices are changing at time t. Let an asterisk attached to it stand for an anticipated rate, so

$$\left(\frac{1}{p}\frac{dP}{dt}\right)^*$$

is the anticipated rate of change in prices. Then the relation Fisher developed is

$$R_B = R_E + \left[\frac{1}{p}\frac{dP}{dt}\right]^*.$$

In other words the nominal rate of interest on the market will be equal to the real rate plus the anticipated rate of price change. Therefore if R_E stays the same but the anticipated rate of price change goes up, the nominal rate of interest will also go up."[24]

There are a number of reasons to suggest that the rate of return on equities may be affected by inflation (i.e., $\frac{dR}{d\pi}E \neq 0$). One fairly direct link between inflation and equity returns is through depreciation charges. Since depreciation charges are pegged in nominal terms, the real value of non-taxable corporate cash flow falls. Consequently, firms with large amounts of depreciable assets may experience reductions in the market value of their common stocks with the onset of inflation.

Another problem centres around the initial conditions at the time inflation begins. If a firm is a net creditor, initially inflation will have an adverse effect on the value of its common stocks. These considerations are neglected in what follows, however.

Assuming that the auxiliary premises implicit in the Radcliffe position are met, a simplified version of the Radcliffe position can be tested using the following equation:

$$(11) \quad i_t = a_0 + a_1 rE_t + b\pi^e_t + u_t$$

[24] Milton Friedman, "Factors Affecting the Level of Interest Rates", *op. cit.*, p. 20. See also P. Cagan, *Determinants of the Money Supply in the United States*, New York, N.B.E.R., 1965, Chapter 6.

An argument similar to that presented by Friedman can be found in R. F. Harrod, *Money*, London, MacMillan, 1970, p. 180. ". . . in periods when inflation is expected, the rate of interest on bonds should be higher than the yield on equities of comparable standing." This is a statement of an equilibrium condition. It simply says that the own rates of return expressed in terms of the same standard should be the same on bonds and equities. Later Harrod argues "And a decline in the yield of equities relatively to bonds owing to fears of inflation is precisely what has been happening in recent years when people have been gloomily settling down to the prediction that inflation is likely to continue." This turns the Fisher Hypothesis on its head; it implies that the effect of inflation is to push down the real rate rather than raise the nominal rate.

where rE is the rate of return on equities and the other symbols have been previously defined.[25] The coefficient a_0 can be assumed to pick up any risk differential between bonds and equities. *A priori* we might expect $a_0 < 0$. A test of the Radcliffe version of the Fisherian Hypothesis involves the joint hypothesis $(a_1, b) = (1, 1)$.

There are a number of possible candidates to play the role of rE in equation (11).[26] There are a number of difficulties in constructing an index number which purports to measure the rate of return on equities. Part of the difficulty lies in the fact that a considerable portion of the return on equities accrues in the form of capital gains. The procedure adopted here is to obtain a measure of the rate of return on shareholders' equity in Canadian Industrial Corporations. This avoids the problem of having to construct an index number of the rate of return on equities. The proxy that is used for rE is obtained by dividing profits of Canadian Industrial Corporations by shareholders' equity.[27]

Results for Canadian quarterly data over the period 1950–I to 1970–II are contained in Table VI. Two interest rates were used: the Government of Canada long-term rate (10+) and the McLeod, Young, Weir Industrial Bond rate. Twelve lags were used, with the lag structure estimated in both unconstrained form and constrained by a third degree polynomial.

The F statistic reported among the summary statistics represents a test of the joint hypothesis that $(a_1, b) = (1, 1)$. The critical F value at the 0.01 level of significance is 5.72. Thus if $F > 5.72$ we reject the hypothesis that $(a_1, b) = (1, 1)$. It should be pointed out that this test is conditioned on the premise that our method of representing price expectations is the correct one.

In testing the hypothesis that the sum of the weights is equal to unity, using a one-tailed test the critical t value is 2.508, at the 0.01 level of significance. This test is conditioned on an adequate representation of the real rate and the hypothesis on price expectation formation.

A priori we would expect the intercept term (a_0) to be less than zero, since one may expect a portfolio of stocks to carry a risk premium over a bond portfolio. As it turned out, however, a_0 was positive and significant in all of the equations fitted. Contrary to *a priori* expectations, the coefficient of our measure of the real rate was always negative. In the case of the M.Y.W. industrial bond rate, the coefficient of rE was negative and close to being significant at the 0.01 level. (It was significant at the 0.05 level.) The joint hypothesis that the coefficient of rE and the sum of the weights are each equal to unity was strongly rejected in all cases.

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[25] Equation (11) above simply involves the substitution of $g(x)_t$ in Equation (10) with $a_1 rE_t$.

[26] Another possible candidate for the real rate is the marginal product of capital. Some preliminary explorations along these lines were tried, but it did not appear to be a fruitful avenue for research. One problem is that most estimates of f_k are far in excess of the nominal rate. Another problem is that the estimate of f_k is extremely sensitive to the form of the production function that is fitted. Furthermore, there is a substantial theoretical controversy of the exact relationship between the rate of interest and the marginal product of capital.

[27] The data are available for the period 1962–70 on a quarterly basis in DBS 61–003, *Industrial Corporations—Financial Statistics*. The method adopted was to divide base profit by total shareholders' equity.

TABLE VI

Radcliffe Regressions, Quarterly Data, 1962–I to 1970–IV, 36 Observations

$$i_t = a_0 + a_1 rE_t + \sum_{i=0} b_i\, \pi^a_{t-i}$$

Rate	Lag	a_0	a_1	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}
10+	UNC*	5.586	−0.280	−0.058	0.075	0.059	0.061	0.038	0.062	0.026	0.120	0.159	0.089	0.096	0.112
		(5.85)	(−1.685)	(−1.140)	(1.206)	(1.060)	(1.046)	(0.754)	(1.242)	(0.504)	(2.066)	(3.089)	(1.587)	(1.888)	(2.1403)
		D.W. = 0.804			S.E.E. = 0.418 S = 0.841 St_1 = −5.778			$\bar{R}^2$ = 0.849 S.E.S. = 0.027			$\tilde{F}$ = 34.405 A.L. = N.A.				
M.Y.W.	UNC	5.877	−0.346	−0.049	0.100	0.065	0.096	0.082	0.113	0.087	0.153	0.196	0.105	0.106	0.111
		(7.324)	(−2.477)	(−1.130)	(1.903)	(1.404)	(1.955)	(1.896)	(2.680)	(1.986)	(3.150)	(4.521)	(2.206)	(2.474)	(2.522)
		D.W. = 1.020			S.E.E. = 0.351 S = 1.167 St_1 = 8.610			$\bar{R}^2$ = 0.940 S.E.S. = 0.019			$\tilde{F}$ = 48.943 A.L. = N.A.				
10±	Almon	5.297	−0.237	0.011	0.023	0.037	0.051	0.064	0.077	0.088	0.096	0.101	0.104	0.101	0.094
		(6.492)	(−1.682)	(0.651)	(0.961)	(1.476)	(2.426)	(4.283)	(6.215)	(5.403)	(4.458)	(4.159)	(4.329)	(4.163)	(2.472)
		D.W. = 0.689			S.E.E. = 0.389 S = 0.848 St_1 = −6.876			$\bar{R}^2$ = 0.870 S.E.S. = 0.022			$\tilde{F}$ = 43.88 A.L. = 6.94 Quarters				
M.Y.W.	Almon	5.520	−0.292	0.019	0.040	0.062	0.083	0.103	0.119	0.132	0.139	0.139	0.132	0.117	0.091
		(7.863)	(−2.409)	(1.340)	(1.925)	(2.883)	(4.623)	(7.952)	(11.213)	(9.430)	(7.474)	(6.621)	(6.424)	(5.565)	(2.769)
		D.W. = 0.844			S.E.E. = 0.334 S = 1.177 St_1 = 10.874			$\bar{R}^2$ = 0.946 S.E.S. = 0.016			$\tilde{F}$ = 60.75 A.L. = 6.50 Quarters				

* UNC refers to unconstrained. F is not the F statistic for the regression as a whole, but for the hypothesis.

$$\begin{bmatrix} a_1 \\ \sum_{i=0}^{N} b_i \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

In the case of the M.Y.W. industrial bond index there appears to be some support for the Fisher Hypothesis. As mentioned previously, however, this provides a test of the Fisher Hypothesis on the premise of an adequate representation of the real rate. The F test and the t test on the coefficient of rE suggests, however, that this criterion has not been met.

A MODIFIED FORM OF THE FISHERIAN MODEL

The empirical results of the first section of this chapter suggest that the assumption of a constant real rate is a poor one. In this section, an additional representation of the determinants of the real rate is considered. That is to say, an alternative way of specifying the function g (x) in the equation $i_t = g(x)_t + b\pi_t^e$ is considered.

A simple liquidity preference theory is utilized. In such a setting the interest rate is determined by the demand and supply of money. If it is assumed that the money supply is exogenous and that the money market always clears, then it is reasonable to regress interest rates against the money supply rather than vice versa. This formulation allows us to test a conjecture put forward by Sir Roy Harrod:

> "When we raise the question in Britain why we now have much higher rates of interest, the answer often is that the prospect of continuing inflation makes this inevitable ... I would suggest, on the contrary, that the reason why interest rates are so high is that the money supply is so low relatively to PT."[28]

The disadvantage of such a formulation is that in the context of a liquidity preference theory of interest, it is not obvious how inflationary premiums become embedded in nominal rates of interest. Indeed, the demand and supply of money should determine the nominal rate rather than the real rate.

Given the arguments presented above, a function representative of the liquidity preference theory of interest can be written as:

$$(12) \quad r_t = a + b\left(\frac{m}{y}\right)_t$$

where $\frac{m}{y}$ is the observed ratio of money to income, and y is defined as GNP.

Substituting equation (12) into equation (5) we obtain:

$$(13) \quad i_t = a + b_1\left(\frac{m}{y}\right)_t + b_2\pi_t^e + u_t\,.$$

Table VII contains results of confronting several variants of the liquidity preference model with the data. Two representative interest rates were chosen. These are the 90-day Treasury Bill rate and the average rate on an index of long-term federal government bonds (10+). Only the results for the narrow definition of money[29] (M1) are reported. Use of other monetary aggregates such as the monetary base or M2 produced substantially poorer results.

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[28] Harrod, *op. cit.* p. 180.

[29] Currency in the hands of the public plus demand deposits at chartered banks.

The first two regressions reported in Table VII are the results of simply regressing the nominal rate of interest against the ratio of M1 to y(GNP). The signs of the regression coefficients on the money-income ratio are in accord with standard liquidity preference theory. The estimated equations are plagued by positive autocorrelation, however, and in addition leave over 50 per cent of the variation in interest rates unexplained. The results of adding lags of 12 and 24 quarters on the rate of inflation are reported in equations (3) and (4), and (5) and (6) respectively. In general, these results do not conflict with the Fisher Hypothesis. The strict Fisher Hypothesis is not supported by equation (4), however. The equations continued to be troubled by serial correlation.

TABLE VII

Liquidity Preference Model Almon Lag 3rd Degree Polynomial
1955–1970 Quarterly Data S.A.

Rate	Lag Length	Intercept	$\left[\frac{M1}{y}\right]$	S	S.E.S.	$\bar{R}^2$	S.E.E.	D.W.	A.L.
1. T.B.		21.845 (8.598)	−37.058 (−6.990)			0.43	1.165	0.279	N/A
2. 10+		20.642 (10.163)	−32.073 (−7.567)			0.47	0.931	0.102	N/A
3. T.B.	12	6.768 (2.659)	−9.447 (−1.895)	0.927	0.108	0.75	0.777	0.611	6.41
4. 10+	12	10.253 (5.913)	−13.361 (−3.935)	0.733	0.074	0.83	0.530	0.220	7.68
5. T.B.	24	9.866 (3.957)	−17.197 (−3.434)	1.213	0.134	0.75	0.758	0.634	9.23
6. 10+	24	12.373 (10.455)	−19.183 (−8.070)	1.054	0.064	0.92	0.360	0.333	9.76

PREVIOUS EMPIRICAL EVIDENCE ON REAL AND NOMINAL RATES OF INTEREST

All previous empirical work on the relationship between interest rates and inflation has involved estimating the relationship using the reduced form technique. No attempt has ever been made to specify or empirically investigate how inflationary premiums become embedded in nominal rates of interest. It follows, therefore, that these studies do not deal with the issues raised by Friedman[30] concerning the ability of the monetary authorities to lower the real market rate of interest by inflation in the short run.

The basic procedure adopted by most investigators has been to regress nominal interest rates upon a distributed lag of past rates of price change. While these studies have produced widely different estimates of the impact of inflation

[30] M. Friedman, "The Role of Monetary Policy", *American Economic Review*, March 1968.

on nominal rates of interest, they are basically similar in approach and subject to similar criticisms. For this reason, we shall consider only one recent study in this section.[31]

In a recent article in *The Federal Reserve Bank of St. Louis Review*, William P. Yohe and Denis S. Karnosky[32] attempted to update the work of Irving Fisher on the relationship between inflation and interest rates. Their work appears to have received widespread approval.[33]

The purpose of this section is to demonstrate why some of the procedures used by the authors are questionable. In addition, attention is drawn to some of the doubtful assertions made by the authors on the basis of their empirical work.[34]

In what follows, the notation utilized by Yohe and Karnosky is employed. The symbols have the following meanings: rn is the nominal rate of interest, $\dot{P}^e$ is the expected rate of price change, $\dot{P}$ is the actual rate of price change, rr is the "real" rate of interest, and rm is the "real market" rate of interest.

The authors set out to: "test the hypotheses about the effect of price expectations on the level of nominal interest rates."[35]

The following two relationships were used:

$$(16)\quad rn_t = \dot{P}^e_t + rr_t$$

$$(17)\quad \dot{P}^e_t = \sum_{i=0}^{n} w_i P_{t-i}$$

Substituting (17) into (16) they obtained "the form of the equation that is usually estimated".[36]

$$(18)\quad rn_t = \sum_{i=0}^{n} w_i P_{t-i} + rr_t$$

In their discussion of the term rr_t in equations (16) and (18), Yohe and Karnosky assert that: "Fisher used 'real' rate in the sense of 'virtual' or 'true' rate."[37] The appendix to Fisher's *The Theory of Interest* suggests that Fisher's virtual rate of

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31 A detailed commentary on past empirical studies in this area is contained in J. F. McCollum, *Inflation and Capital Markets* (unpublished, Prices and Incomes Commission, Ottawa, 1971).

32 W. P. Yohe and D. S. Karnosky, "Interest Rates and Price Level Changes, 1952–69", *The Federal Reserve Bank of St. Louis Review*, December 1969, pp. 18–38.

33 The following comments by various authors are suggestive. "W. P. Yohe and D. S. Karnosky in a comprehensive article . . . provide a succinct statement of the Fisher theory and a careful discussion of the theoretical aspect of the Gibson Paradox. They derive alternative estimates of the real rate, and relate this analysis to explain interest rate movements in recent years." D. I. Fand, "Monetarism and Fiscalism", *Banca Nazionale del Lavoro*, September 1970, pp. 283–84. "More recently, the Federal Reserve Bank of St. Louis has been estimating the 'real rate' and their estimates are remarkably stable despite very large changes in nominal rates." M. Friedman, "A Monetary Theory of Nominal Incomes", *Journal of Political Economy*, March-April 1971. "For a detailed study of interest rates and the Fisher effect, see W. P. Yohe and D. S. Karnosky . . ." L. C. Anderson and K. M. Carlson, "A Monetarist Model for Economic Stabilization, *The Federal Reserve Bank of St. Louis Review*, April 1970, p. 10.

34 In other words, we are criticizing the article on its own terms. A variety of other questions concerning inflation and interest rates are not considered in this section.

35 Yohe and Karnosky, *op. cit.*, p. 22.

36 *Ibid.*, p. 20.

37 *Ibid.*, p. 19.

interest is an ex-post real rate. Using the notation above, Fisher's "virtual" rate of interest would be:

Virtual rate in period t $= rn_t - \dot{P}_t$.[38]

Using equation (16) this implies $\dot{P}^e_t = \dot{P}_t$ for all t. That is to say, expectations are always fulfilled, and equation (17) is irrelevant. One may conclude, therefore, that rr_t in equations (16) and (18) is not the "virtual" rate of interest as defined by Irving Fisher.

Employing monthly U.S. data for the period January 1952 to September 1969, Yohe and Karnosky estimated equations of the following form:

$$(19) \quad rn_t = A_0 + A_1\dot{P}_t + A_2\dot{P}_{t-1} + \ldots + A_{n+1}\dot{P}_{t-n} + U_t .^{39}$$

Equation (19) was estimated by using a variety of techniques for representing distributed lags, although the procedure most heavily relied upon in the study was the Almon lag technique.

Regressions of the above form, using monthly data, yield extremely low Durbin-Watson statistics which are in the neighbourhood of 0.05 to 0.40, depending on the time span and interest rate series chosen. There is no mention of difficulties raised by serial correlations in the Yohe and Karnosky paper.

It was not possible to test whether the mean lags for short-term rates of interest were different from long-term rates since the mean-lag calculation is meaningful only if all the coefficients are positive. Consequently, it is difficult to design a useful test for the hypothesis that the time horizon, in forming price expectations, increases with longer-termed securities. The evidence as presented by Yohe and Karnosky appears to be consistent with the notion that there is no appreciable difference for securities of different lengths.

In order to test the hypothesis that price expectations influence nominal rates of interest (using equation (19)) it is implicitly assumed that the real rate of interest is constant over time. Thus, the "real" rate would be represented by a constant term in equation (19). In constructing a "real" rate series,[40] Yohe and Karnosky make the following calculation using the estimated weights from equation (19):

$$(20) \quad rm_t = rn_t - \sum_{i=1}^{n} \hat{A}_i P_{t-i}$$

where rm_t is the "real" rate.[41]

---

[38] This is interpretation (a) on page 8 of this chapter. Under interpretation (a), equation (16) does not constitute a theory.

[39] Two interest rates were employed, a short rate which was the yield on four to six monthly commercial papers, and a long rate which was the yield to maturity on corporate Aaa bonds. Usually the rate of change of prices was calculated using the Consumer Price Index, although the implicit GNP deflator was occasionally used. For some of their regressions a seasonally-adjusted interest rate series was used. It should be noted that, to obtain similar treatment for the rate of change of prices, one should first calculate the rate of change of the price index and then seasonally adjust the result. Taking the rate of change of the seasonally-adjusted price index does not result in the same series.

[40] This is the Real-Rate 2 on page 38 of Yohe and Karnosky *op. cit.*

[41] *Ibid.*, p. 38. The authors never demonstrate the relationship between rr_t and rm_t.

This is not an acceptable procedure since equation (20) in conjunction with equation (19) implies $rm_t = \hat{A}_0 + \hat{U}_t$. That is to say, the real rate is a random variable. The time series for the "real" rate derived by Yohe and Karnosky is obtained by using a price expectational variable constructed on the basis of a constant real rate. It follows that the "real" rate series constructed by Yohe and Karnosky is without interest.

Yohe and Karnosky appear to be inconsistent in their use of notation. Different symbols are used to represent the same concept or variable, and the same symbol is used to represent different concepts or variables.

We have already pointed out the confusing switch of notation from rr_t in equations (16) and (18) to rm_t in equation (20). On page 24 of their article the authors attach an entirely different meaning to "rm", the real market rate, than the one found in equation (20):

> "Assuming an equilibrium position with expected price changes equal to zero, then $rn_t = rm_t$. If price expectations increase by one per cent per year, after four years the nominal interest rate will rise by 69 basis points, thus:
>
> (1) $rn_{t+48} - rm_{t+48} = 1.00$
>
> (2) $rn_{t+48} - rn_t = 0.69$
>
> Since $rn_t = rm_t$ equations (1) and (2) reduce to
>
> (3) $rm_{t+48} - rm_t = -0.31$
>
> Thus, the market rate falls by 31 basis points following the increase in price expectations."[42]

It is clear that rm_t above is not the same rm_t obtained from equation (20). Equation (11) is the key in the above equation. In equation (11), it is assumed that the Fisher equation always holds true. Consequently, price expectations can depress the real rate of interest, and the nominal rate of interest need not rise by the full amount of the expected rate of change of prices. If this is the case, however, we could never reject the Fisher Hypothesis using equation (19).

The real market rate of the above equation will equal the real market rate of equation (20) only if the sum of the coefficients on the lagged inflation terms equals unity. In this interpretation the purpose of estimating equation (19) is not to test the Fisher Hypothesis, but simply to estimate the depressive effects of inflation on the real rate of interest, assuming that the Fisher equation holds true. Since the real market rate obtained in this fashion is inconsistent with the approach used in obtaining the price expectation variable, it may be concluded that the calculations in the quotation are without meaning.

The contrast between these various rates is shown in Figure 3. The intercept and assumed real rate are shown as a straight line. Assume that the rate of price change rises from zero to one per cent per year. The estimated intercept plus the computed residual for any point $(\hat{A}_0 + \hat{U}_t)$ is the real rate of interest presented by Yohe and Karnosky. The curve labelled rm is Yohe and Karnosky's second estimate of the real market rate. In general, $rm_t \neq \hat{A} + \hat{U}_t$. This real market rate appears to have nothing to do with Wicksell, contrary to the assertions by

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[42] *Ibid.*, p. 24.

the authors, and is constructed on the basis that the Fisher equation always holds true.[43]

In summary, the work of Yohe and Karnosky is subject to criticism on the following grounds:

(1) Construction of a misleading real rate of interest series.

(2) Construction of a defective "real market" rate of interest (Yohe and Karnosky, *op cit*, p. 24).

(3) Using notation which suggests that the above two series are equal while, in fact, they are not.

(4) Ambiguous approach to the use of the Fisherian relationship between the expected rate of price change, nominal interest rates, and real interest rates. They originally set out to test Fisher's Hypothesis that price expectations influence nominal rates of interest, and later assert that the Fisher relation always holds (Yohe and Karnosky, *op cit*, p. 24).

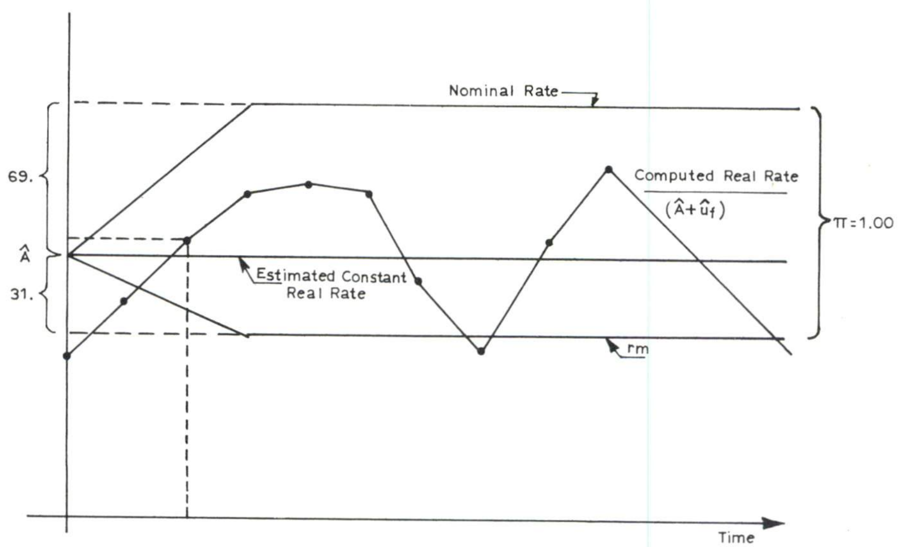
(5) An apparent misinterpretation of Fisher's "virtual" rate of interest.

(6) The high degree of serial correlation present in their regressions suggests that their model is mis-specified. This possibility, however, is never alluded to in the article.

(7) Incorrect use of seasonal adjustment procedures.

Figure 3

St. Louis Real Rates



[43] Real rate 1 on page 34 of the article (*Ibid*) is the original "real" rate series constructed at the St. Louis Federal Reserve Bank. This real rate series was constructed on the basis that price expectations are known. A similar series for Canada may be found in T. J. Courchene, "Recent Canadian Monetary Policy", *Journal of Money, Credit and, Banking*, February 1971. Calculations of this sort have nothing to say about whether price expectations affect nominal rate of interest; that is assumed to be true in the construction of the series.

SUMMARY OF SINGLE EQUATION RESULTS

The results for the naive Fisherian model were as much in accord with Samuel Coleridge as Irving Fisher. That is to say, evidence that price expectations based solely on the past history of rates of change of prices become incorporated in nominal rates of interest is ambiguous. This does not mean, of course, that price expectations do not affect the nominal rate of interest. It does suggest, however, that efforts to measure the effects of price expectations using an adaptive expectations model and the naive Fisherian model (constant real rate) are subject to question.

TABLE VIII

Sensitivity to the Measure of the Impact of Price Expectations to Specification and Method of Estimation

Form of Equation	Treasury Bill Rate	10+
1. Constant Real Rate, 12 unconstrained lags, monthly data, 1952–1970	0.672	0.480
2. Constant Real Rate, 12 lags Almon 3rd Degree Polynomial	0.618	0.451
3. Constant Real Rate, 12 unconstrained lags, quarterly data	0.898	0.735
4. Constant Real Rate, 12 lags, monthly data, Durbin-Two-Step	0.173	0.143
5. Constant Real Rate, Stepwise Orthogonal Regression, monthly data (see Chapter 3)	0.505	0.352
6. Constant Real Rate, Stepwise Orthogonal Regression, quarterly data, 12 lags	0.329	0.339
7. Simple Liquidity Preference Model, quarterly data, 12 lags, Almon 3rd Degree Polynomial	0.927	0.733
8. U.S. Rate and expected change in exchange rate, 12 lags, Almon 3rd Degree Polynomial, monthly data (see Chapter 3)	0.042	−0.0005

The strict version of the Fisherian Hypothesis using monthly data was generally rejected by the data. What relationship there was between past rates of price change and interest rates disappeared if any attempt was made to treat the estimated equations for serial correlation. On the other hand, results using quarterly data tended not to conflict with the naive version of the Fisherian model. The data did not support the Radcliffe version of the Fisherian Hypothesis.

In summary, it would appear that an equation such as equation (5), incorporating a linear price expectations generating model is an inappropriate device for measuring the impact of price expectations. In addition, equations such as equation (5) do not permit testing asymmetric reactions to inflation by lenders or borrowers. If price expectations do affect nominal rates of interest, the behavior of capital market participants must be altered in the presence of inflation. This issue is examined in chapters four and five.

chapter three

EXTENSIONS OF THE FISHERIAN MODEL

"George Horwich:	David, are those real or nominal rates that you are talking about?
"David Meiselman:	That is a very good question."[1]

"Further, it is such 'real' rate series that should be employed in studies of the term structure of interest rates and of the effects of international interest rate differentials on short and long-term capital flows."[2]

The main purpose of this chapter is to explore the relationship between the real term structure, the nominal term structure, and price expectations.

THE FISHERIAN MODEL OF THE TERM STRUCTURE OF INTEREST RATES

A theory of the term structure of interest rates is contained within the Fisherian theory of the determination of interest rates.[3] This term structure theory follows almost directly from the Fisherian real interest rate theory briefly discussed in chapter two, and is obtained by extending the Fisherian theory to cover more than two time periods.

In the n period case, this permits the derivation of a "... separate rate of interest for each separate period."[4]

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1 David Meiselman, "The Policy Implications of Current Research in the Term Structure of Interest Rates", *Savings and Residential Financing*, Chicago, Conference of the Savings and Loans League, 1968.

2 William P. Yohe and Denis S. Karnosky, "Interest Rates and Price Level Changes, 1952–1969", *The Federal Reserve Bank of St. Louis Review*, December 1969, p. 36.

3 Irving Fisher, *The Theory of Interest*, New York, Macmillan, 1930, pp. 293–301, 302–310, 313–314, 508–510, 512–513, and 515–516.

Fisher, *Appreciation and Interest*, Publications of the American Economic Association, 1898, pp. 26–29.

4 Fisher, *The Theory of Interest, op cit*, p. 313.

These single period rates of interest can be represented by the symbols: $r_1, r_2, ..., r_n$.[5] Fisher further points out that:

> "Since the element of risk is supposed to be absent, it does not matter whether we consider these second-year rates of interest (r_2) and time preference as the ones which are expected, or those which will actually obtain, for under our assumed conditions of no risk, there is no discrepancy between expectations and realizations."[6]

Consequently, the single period rates of interest can be represented by expected rates: $r_1, \underset{\sim}{r_2}, \underset{\sim}{r_3}, ..., \underset{\sim}{r_n}$.

Under optimum conditions there will be no incentive for intertemporal arbitrage. Thus, for an interval greater than one period in length, say two periods long, the same rate of return must be obtained by engaging in a two-period contract as will be made in two successive one-period contracts. This provides a theory of the term structure of interest rates. Thus, Fisher notes:

> "Since, in practice, no loan contracts are made in advance so that there are no market quotations for a rate of interest connecting, for example, one year in the future with two years in the future, we never encounter such separate year to year rates. We do, however, have such rates implicitly in long term loans. The rate of interest on a long term loan is virtually an average of the separate rates for the separate years constituting that long term."[7]

Thus, the rate of interest on a two-year loan appears as:

$$(1) \quad (1 + {}_2R_1)^2 = (1 + r_1)\,(1 + r_2)$$

or

$$(2) \quad {}_2R_1 = \sqrt{(1 + r_1)\,(1 + r_2)} - 1$$

where the preceding subscript refers to the length of the loan and the following subscript refers to the period in which the contract begins.[8]

For the n period case we have:

$$(4) \quad {}_nR_1 = \sqrt[n]{(1 + r_1)\,(1 + r_2) \ldots (1 + r_n)} - 1$$

If the expected real rates for future one-year contracts are substituted into equations (1) and (4)[9] the "Expectations Theory" of the term structure of interest rates emerges:

$$(5) \quad {}_2R_1 = \sqrt{(1 + r_1)\,(1 + \underset{\sim}{r_2})} - 1$$

---

[5] The Fisherian model for the determination of multi-period interest rates is a general equilibrium model. It is mathematically equivalent to the general equilibrium models of price theory and international trade, which did not arise until the 1930s and 1940s with John Hicks' *Value and Capital*, Abba Lerner's *Economics of Control*. The first presentation of Fisher's multi-period equilibrium model is found in *The Rate of Interest* (1908).

[6] *Ibid.*, p. 295.

[7] *Ibid.*, pp. 313–314.

[8] It is assumed that all interest accrues at the end of the period of the loan. Alternately, if interest is paid during the life of the contract, this formulation assumes that it can be reinvested at the rate of ${}_2R_1$. The formula otherwise alters to:

$$(3) \quad {}_2R_1 = \frac{(1 + r_1)\,(1 + r_2)}{(2 + r_2)}.$$

[9] See p. 295 of Fisher's *The Theory of Interest, op cit.*

$$
\begin{aligned}
{}_3R_1 &= \sqrt[3]{(1 + r_1)(1 + \underset{\sim}{r_2})(1 + \underset{\sim}{r_2})} - 1 \\
&\vdots \\
{}_nR_1 &= \sqrt{(1 + r_1)(1 + \underset{\sim}{r_2}) \ldots (1 + \underset{\sim}{r_n})} - 1
\end{aligned}
$$

The term structure of interest rates so derived is a term structure of real rates of interest, since individuals are interested in trading in real terms. It gives the rates at which real purchasing power over current goods can be traded for purchasing power over future goods at various times in the future.

If the Fisher price expectations hypothesis holds for the individual rates of interest in the term structure of interest rates (i.e., the rate of change of the price level has no effect on the terms of exchanging things over time) the nominal rate of interest for each maturity will be raised by the average rate of inflation expected over that period.

Let i_j represent the nominal one-year rate of interest in period j, i_j the one-year rate of interest expected to prevail in period j, and ${}_jI_1$ the nominal long-term rate of interest on a loan contract of j years. The term structure of interest rate relationships can be expressed as:

$$
\begin{aligned}
(6)\quad {}_2I_1 &= \sqrt{(1 + i_1)(1 + i_2)} - 1 \\
&\vdots \\
{}_nI_1 &= \sqrt[n]{(1 + i_1)(1 + i_2) \ldots (1 + i_n)} - 1
\end{aligned}
$$

for the certainty formulation. The "Expectations" version appears as:

$$
\begin{aligned}
(7)\quad {}_2I_1 &= \sqrt{(1 + i_1)(1 + \underset{\sim}{i_2})} = 1 \\
&\vdots \\
{}_nI_1 &= \sqrt[n]{(1 + i_1)(1 + \underset{\sim}{i_2}) \ldots (1 + \underset{\sim}{i_n})} - 1
\end{aligned}
$$

If the Fisher price expectations hypothesis is correct for each period, then:[10]

$$
\begin{aligned}
(8)\quad I_1 &= r_1 + b_1\, {}_1\pi^e_t \\
{}_2I_1 &= {}_2R_1 + b_2\, {}_2\pi^e_t \\
&\vdots \\
{}_nI_1 &= {}_nR_1 + b_n\, {}_n\pi^e_t \\
\text{and } b_1 &= b_2 = \ldots = b_n = 1
\end{aligned}
$$

Assuming that the Fisherian theory of the real term structure is correct and that the constituent real rates are not affected by the rate of inflation, what conditions must be placed on the "b" coefficients above if the nominal term structure is to conform to the "Expectations Hypothesis" of the term structure?

[10] The interaction term has been ignored here. It should be noted that the real rates are not assumed to be constant; all that is assumed is that inflation does not affect these real rates. ${}_n\pi^e$ is the average rate of inflation expected to prevail over the next n periods.

With a steady state rate of inflation where $\pi_t^e = \pi^e$ for all t, the condition sought is that $b_i = b$ for all i. Suppose that in our empirical work we find that $b_i \neq b_j$ for $i \neq j$, then inflation alters the nominal term structure of interest rates. The observed nominal term structure cannot be consistent with the "Expectations Hypothesis" when it is assumed that this hypothesis is obeyed by the underlying Fisherian real term structure. This proposition can readily be demonstrated. According to the Fisherian theory of term structure, we have:

$$(1 + {}_2R_1)^2 = (1 + r_1)(1 + r_2) .$$

Assume that a steady state rate of inflation is experienced so that $\pi^e = \pi^a$. The relationship between price expectations, real rates, and nominal rates appears as:

$$i_1 = r_1 + b_1 \pi_1^e$$
$$i_2 = r_2 + b_2 \pi_2^e$$
$${}_2I_1 = {}_2R_1 + B_2 \, {}_2\pi_1^e .$$

Since we assumed a steady state rate of inflation, we have $\pi_1^e = \pi_2^e = {}_2\pi_1^e$. Furthermore $b_1 = b_2$, if inflation is to have the same effect continuously on the one-year rate. If $B_2 \neq b_1 = b_2$, then

$$(1 + {}_2I_1)^2 \neq (1 + i_1)(1 + i_2),$$

which is what we wanted to show. Furthermore, if $B_2 = b_1 = b_2$, then $(1 + {}_2I_1)^2 = (1 + i_1)(1 + i_2)$.

Assume that the underlying Fisherian real term structure holds. What occurs to the nominal term structure if there are long lags in the formation of expectations? In general, there is no way to tell unless we indicate the interval over which we are interested and know how expectations are formed. Even if $b_1 = b_j$ for all i, j, the nominal term structure will only reflect the real term structure in equilibrium. If there are long lags in the process of adjustment from one rate of inflation to another and if the lengths of these lags vary for different rates of interest, the nominal term structure will generally not obey the "Expectations Theory" of term structure over the interval of adjustment. Further, if the rate of inflation is variable then the nominal term structure will, in general, not obey the "Expectations Theory". Consequently, tests of the "Expectations Theory" of the term structure of interest rates using nominal interest rates involve a mixture of effects, including lags in the formation of price expectations.

An attempt to construct a term structure of interest rates employing the real rate series of Yohe and Karnosky[11] yields misleading results. As mentioned previously, each of the real rate series constructed by Yohe and Karnosky is a random variable and appears as:

$$r_t^s = \hat{A}_s + \hat{u}_t$$
$$r_t^L = \hat{A}_L + \hat{\varepsilon}_t$$

where r^s and r^L are the real short-term and real long-term rates as constructed

[11] Yohe and Karnosky, *op. cit.* p. 36.

by Yohe and Karnosky. As a result, any relationship between r^s and r^L consists of a relation between two random variables. If u and ε are truly random one would expect changes in r^s to be unrelated to changes in r^L.

THE DIFFERENTIAL EFFECTS ACCORDING TO TERM TO MATURITY HYPOTHESIS

Milton Friedman has hypothesized that there are differential effects of price expectations on interest rates according to the term to maturity. In particular, it is suggested that the time horizon relevant to the formation of price expectations varies directly with the security's term to maturity. This conjecture may be termed "The Differential Effects Hypothesis".

> "As a purely theoretical matter, one would expect that it would take longer for long rates than for short rates. When you are buying a security with a short life, you are really interested in extrapolating price movements over a shorter future period of time than when you are buying very long-term security. It seems not unreasonable that if you are extrapolating for a short period, you will look back for a shorter period than when you are extrapolating for a longer period."[12]

It should be noted that this is a hypothesis about the formation of price expectations rather than about the effect of price expectations on interest rates. Assuming that the above hypothesis is true, in steady-state equilibrium there would be no differential effects, according to maturity, of price expectations. There is a tendency in the literature to confuse this notion with the notion that the estimated coefficients of the price expectational variables should be different.[13]

If the differential effects hypothesis is correct, the mean lag in the formation of price expectations for short-term rates of interest should be significantly less than the mean lag for long-term rates of interest.

Friedman notes that:

> "I regard it as very strong empirical confirmation of this interpretation of the evidence that it does turn out that the period it takes to get to full adjustment tends to be much longer for long rates than it does for short rates.... The mean period of price anticipation turns out to be something like 10 years for short rates and 20 years for long rates. Since these are average periods, they imply that people take an even longer period of past history into account. The results are wholly consistent with Fisher's."[14]

The last sentence in this quotation is, of course, incorrect. The relative effects found by Fisher on short and long-term rates of interest were just the opposite of those effects mentioned by Friedman.[15]

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[12] Milton Friedman, "Factors Affecting the Level of Interest Rates", *Saving and Residential Financing: 1968 Conference Proceedings*, Chicago, Savings and Loan League, 1968, p. 21.

[13] For example, see Thomas Sargent, "Commodity Price Expectations and the Interest Rate", *Quarterly Journal of Economics*, February 1969, pp. 135–36.

[14] Milton Friedman, *op. cit.*, p. 21.

[15] Irving Fisher, *The Theory of Interest*, New York, Macmillan, 1930, Chapter XIX.

EMPIRICAL TESTS OF THE DIFFERENTIAL EFFECTS HYPOTHESIS

The differential effects according to maturity hypothesis relates to the formation of expectations *per se*. Let ${}_L\pi^e$ and ${}_S\pi^e$ refer to expectations relevant to the long-term rate of interest and short-term rate of interest respectively. Suppose that:

$${}_L\pi^e = \sum_{i=0}^{N} w_i \, \pi^a_{t-i}, \text{ and}$$

$${}_S\pi^e = \sum_{i=0}^{M} w_i \, \pi^a_{t-i};$$

then $N > M$. Note that this version of the Differential Effects Hypothesis need not imply that short-term rates respond faster than long-term rates, since the speed of response is also conditioned by the shape of the lag distribution.

The regression results reported in chapter two offer no support for the hypothesis that $N > M$. Further evidence on the Differential Effects Hypothesis is contained in Tables IX-(a) and (b).

The procedure used to obtain the results reported in Table IX was stepwise orthogonal regression. The lagged price terms were introduced one at a time in stepwise fashion. If the Differential Effects Hypothesis in the above version is correct, one would expect to find more significant lagged price terms entering the regression for long-term rates than for short-term rates. In the case of monthly data, there is no appreciable difference of the length of the lag according to term to maturity. The contribution of additional lagged values of the rate of price change tends to disappear after seven or eight lags.

The Differential Effects Hypothesis is not supported by the quarterly results either. After 12 or 13 lags, additional lags do not make a significant contribution to reducing the unexplained variation in the rate of interest. The clear exception to this is the McLeod, Young, and Weir Industrial Bond Index where as many as 20 lags are found to be significant. With respect to the Differential Effects Hypothesis this result does not have a clear-cut interpretation.

The results reported in Tables IX-(a) and (b) are interesting for another reason. In chapter two, it was pointed out that one difficulty in using the unconstrained lag technique is the multicollinearity of the regressors. Consequently, the coefficients obtained are suspect. Using orthogonal regression, multicollinearity is absent since the regressors are perpendicular to each other. The result of removing the multicollinearity in this fashion is to reduce the sum of the coefficients. The summary statistics S and St_1 reported beneath each regression suggest that the Fisherian price expectations hypothesis can be rejected for monthly and quarterly data.

Another version of the differential effects hypothesis can be specified in terms of the average lag. In other words, if the short-term rate of interest responds to inflation at a faster rate than does the long-term rate, then the average lag for

Table IX—(a)

Differential Effects According to Maturity
Stepwise Orthogonal Regression
January 1952—December 1970, Monthly Observations (228)

	Intercept a	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}
T.B.	55.65	0.0844	0.0649	0.0627	0.0617	0.0531	0.0524	0.0518	0.0329	0.0366	
	(42.05)	(6.38)	(5.24)	(4.74)	(4.66)	(4.02)	(3.96)	(3.92)	(2.49)	(2.76)	
		D.W. = 0.173			S.E.E. = 1.32			$\bar{R}^2 = 0.42$			
		S = 0.505			$St_1 = -12.382$						
					Average Lag = 3.342 months						
1–3	65.83	0.0671	0.0579	0.0526	0.0519	0.0438	0.0441	0.0434	0.0316	0.0338	0.0272
	(58.73)	(5.98)	(5.17)	(4.69)	(4.63)	(3.91)	(3.94)	(3.87)	(2.82)	(3.01)	(2.43)
		D.W. = 0.158			S.E.E. = −1.12			$\bar{R}_2 = 0.42$			
		S = 0.454			$St_1 = -15.472$						
					Average Lag = 3.774 months						
3–5	70.81	0.0605	0.0524	0.0482	0.0485	0.0446	0.0443	0.0427	0.0302	0.0327	0.0266
	(65.85)	(5.63)	(4.87)	(4.49)	(4.51)	(4.15)	(4.12)	(3.97)	(2.81)	(3.04)	(2.47)
		D.W. = 0.134			S.E.E. = 1.07			$\bar{R}^2 = 0.41$			
		S = 0.431			$St_1 = -16.788$						
					Average Lag = 3.859 months						
5–10	73.51	0.0561	0.0465	0.0543	0.0494	0.0387	0.0441	0.0420	0.0305	0.0326	0.0293
	(65.26)	(4.98)	(4.13)	(4.02)	(4.38)	(3.44)	(3.91)	(3.73)	(2.71)	(2.90)	(2.60)
		D.W. = 0.182			S.W.E. = 1.13			$\bar{R}^2 = 0.37$			
		S = 0.414			$St_1 = -16.430$						
					Average Lag = 3.982 months						
10+	75.66	0.0506	0.0445	0.0394	0.0426	0.0401	0.0400	0.0372	0.0279	0.0295	
	(68.13)	(4.56)	(4.01)	(3.55)	(3.84)	(3.62)	(3.60)	(3.35)	(2.52)	(2.66)	
		D.W. = 0.087			S.E.E. = 1.11			$\bar{R}^2 = 0.33$			
		S = 0.352			$St_1 = -19.283$						
					Average Lag = 3.949 months						
M.Y.W.	87.50	0.0585	0.0512	0.0467	0.0499	0.0487	0.0478	0.0433	0.0329	0.0358	0.0291
	(76.71)	(5.13)	(4.49)	(4.09)	(4.38)	(4.27)	(4.19)	(3.79)	(2.88)	(3.14)	(2.55)
		D.W. = 0.084			S.E.E. = 1.14			$\bar{R}^2 = 0.37$			
		S = 0.444			$St_1 = -15.394$						
					Average Lag = 3.978 months						

TABLE IX—(b)

Orthogonal Regression
Quarterly Observations (64)
1955–1970

	Intercept a	b_1 b_{13}	b_2 b_{14}	b_3 b_{15}	b_4 b_{16}	b_5 b_{17}	b_6 b_{18}	b_7 b_{19}	b_8 b_{20}	b_9 b_{21}	b_{10} b_{22}	b_{11} b_{23}	b_{12} b_{24}
T.B.	32.91 (40.27)	0.0559 (6.84)	0.0482 (5.90)	0.0448 (5.49)	0.0290 (3.55)	0.0145 (1.77)	0.0117 (1.44)	0.093 (1.13)	0.0186 (2.27)	0.0197 (2.41)	0.0241 (2.95)	0.0261 (3.19)	0.0280 (3.43)
		D.W. = 0.659 S = 0.329			S.E.E. = 0.82 $St_1 = -15.004$ Average Lag = 4.455 quarters			$\bar{R}_2 = 0.72$					
1–3	37.32 (62.79)	0.0518 (8.72) 0.0170 (2.86)	0.0449 (7.59)	0.0431 (7.26)	0.0305 (5.13)	0.0142 (2.40)	0.0117 (1.97)	0.076 (1.27)	0.0175 (2.95)	0.0180 (3.03)	0.0203 (3.42)	0.0207 (3.48)	0.0238 (4.01)
		D.W. = 0.727 S = 0.321			S.E.E. = 0.59 $St_1 = -21.908$ Average Lag = 4.690 quarters			$\bar{R}^2 = 0.81$					
3–5	39.87 (75.13)	0.0456 (8.59) 0.0183 (3.45)	0.0437 (8.24)	0.0405 (7.63)	0.0296 (5.57)	0.0163 (3.08)	0.0132 (2.48)	0.086 (1.61)	0.0186 (3.51)	0.0156 (2.94)	0.0180 (3.40)	0.0219 (4.12)	0.0223 (4.20)
		D.W. = 0.708 S = 0.312			S.E.E. = 0.53 $St_1 = -23.991$ Average Lag = 4.796 quarters			$\bar{R}^2 = 0.84$					
5–10	41.20 (78.64)	0.0417 (7.95) 0.0214	0.0421 (8.04) 0.0137	0.0401 (7.65)	0.0330 (6.30)	0.0205 (3.92)	0.0158 (3.02)	0.089 (1.69)	0.0193 (3.68)	0.0195 (3.72)	0.0227 (4.34)	0.0220 (4.20)	0.0243 (4.64)

		(4.09)	(2.62)										
		D.W. = 0.546				S.E.E. = 0.52			$\bar{R}_2 = 0.85$				
		S = 0.344				$St_1 = -23.551$							
						Average Lag = 5.373 quarters							
10+	42.38	0.0342	0.0371	0.0355	0.0305	0.0241	0.0205	0.0156	0.0193	0.0212	0.0212	0.0205	0.0231
	(78.12)	(6.30)	(6.84)	(6.54)	(5.63)	(4.45)	(3.78)	(2.87)	(3.57)	(3.90)	(3.92)	(3.79)	(4.25)
		0.0206	0.0160										
		(3.80)	(2.95)										
		D.W. = 0.214				S.E.E. = 0.54			$\bar{R}^2 = 0.82$				
		S = 0.339				$St_1 = -23.221$							
						Average Lag = 5.602 quarters							
M.Y.W.	48.61	0.0436	0.0474	0.0445	0.0375	0.0289	0.0234	0.0186	0.0186	0.0201	0.0175	0.0165	0.0197
	(123.80)	(11.10)	(12.07)	(11.34)	(9.55)	(7.37)	(5.95)	(4.73)	(4.74)	(5.12)	(4.47)	(4.20)	(5.02)
		0.0176	0.0176	0.095	0.071	0.098	0.0106	0.0124	0.0103				
		(4.48)	(4.50)	(2.42)	(1.82)	(2.49)	(2.71)	(3.15)	(2.64)				
		D.W. = 0.441				S.E.E. = 0.39			$\bar{R}^2 = 0.92$				
		S = 0.430				$St_1 = -28.531$							

the short-term rate will be shorter. Using the linear unconstrained model, the average lag can be computed using the formula:

$$AL = \frac{\sum_{i=0}^{N} iw_i}{\sum w_i}$$

where w_i are the estimated coefficients. Average lags cannot be computed for most of the regression results reported in chapter two due to the presence of negative terms. The regression results reported in Tables IX-(a) and (b), however, are amiable to average lag calculations. The computed average lags for monthly and quarterly data are reported in Table X.

TABLE X

Differential Effects According to Term to Maturity Average Lags

	Based on Table V-A monthly data	Based on Table V-D quarterly data	Based on Table IX-(a) monthly data	Based on Table IX-(b) quarterly data
T.B.	4.0 months	8.2 quarters	3.3 months	4.5 quarters
1–3	4.3	N.A.*	3.8	4.7
3–5	4.4	8.5	3.9	4.8
5–10	4.6	8.4	4.0	5.4
10+	4.6	8.5	4.0	5.6

*N.A. means that the average lag calculation is not applicable.

Another way to approach the problem is by examining the percentage of the total response that occurs over a specific interval. Or alternatively, the period over which a given percentage of the response (say 50 per cent) occur. Tables XI-(a), (b) and (c) contain computations based on the results reported in Table I-(a) of chapter two and Tables IX-(a) and (b) of this chapter. Of these computations, those based on Table I-(a) of chapter two should be viewed with the most suspicion.[16] The support for the Differential Effects Hypothesis implicit in Tables XI-(a), (b) and (c) is extremely marginal. The percentage of total response that occurs in the most recent past periods tends to be greater for shorter-termed securities.

On balance it seems reasonable to conclude that, using a variety of techniques, the hypothesis of "differential effects according to term maturity" is not supported by the data.

[16] The first three regressions reported in Table I contain a small and insignificant negative term at the tail of the lag. For these regressions, this particular technique is not entirely appropriate.

Table XI–(a)

Percentage of Total Response Monthly, January 1952—December 1970

Based on Results Reported in Table I–(a)

	0	1	2	3	4	5	6	7	8	9	10	11
T.B.	16.57	30.43	41.59	54.36	62.95	71.72	81.98	86.67	94.01	101.19	102.62	100
1–3	14.90	28.35	39.36	51.71	59.86	68.37	77.84	82.92	90.38	98.12	100.31	100
3–5	14.15	27.01	37.36	49.09	57.97	67.10	76.97	82.00	89.47	97.13	100.09	100
5–10	13.84	25.12	34.99	47.76	54.93	64.36	74.26	79.27	86.57	94.74	98.18	100
10+	13.32	25.98	35.22	46.54	55.66	65.22	74.69	79.95	87.47	94.82	98.44	100
M.Y.M.	13.02	25.26	34.60	45.57	55.04	64.90	74.09	79.17	86.91	94.21	97.87	100

TABLE XI–(b)

Percentage of Total Response Monthly, January 1952–December 1970
Based on Stepwise Orthogonal Regressions of Interest Rates
on Rates of Price Change

	0	1	2	3	4	5	6	7	8	9
T.B.	16.72	30.47	42.88	55.10	65.62	75.99	86.25	92.77		
1–3	14.78	27.56	39.17	50.62	60.28	70.01	79.59	86.56	94.00	
3–5	14.06	26.21	37.42	48.69	59.05	69.32	79.25	86.26	93.84	
5–10	13.55	24.76	35.68	47.60	56.95	67.58	77.70	85.07	92.94	
10+	13.44	25.25	35.74	47.07	57.74	68.36	78.26	85.68	93.53	
M.Y.W.	13.20	24.72	35.24	46.49	57.45	68.24	14.07	85.38	93.46	

CAPITAL MOVEMENTS, EXCHANGE RATES AND REAL AND NOMINAL INTEREST RATES

The purpose of this section is to explore the meaning of the distinction between real and nominal interest rates in an open economy. This is done by utilizing the Fisherian theory of real and nominal interest rates, the purchasing power parity theory of exchange rates, and the theory of interest arbitrage. The following notation is used:

c_i = the Canadian nominal interest rate

u_i = the nominal interest rate in the United States

${}^c\pi^e$ = the expected rate of change of prices in Canada

${}^u\pi^e$ = the expected rate of change of prices in the United States

${}^cP$ = the Canadian price level

${}^uP$ = the price level in the United States

${}^cr$ = the real rate of interest in Canada

${}^ur$ = the real rate of interest in the United States

E_s = the spot rate of exchange—the price of the Canadian dollar in terms of U.S. dollars

E_f = the forward exchange rate

E_s^e = the spot rate expected to prevail at some future date

$\lambda = \frac{\dot{E}_s}{E_s}$ = the rate of appreciation or depreciation of the Canadian dollar in terms of U.S. dollars

λ^e = the expected rate of change of the price of the Canadian dollar in terms of U.S. dollars.

Table XI–(c)

Percentage of Total Response Quarterly 1955–1970

Based on Stepwise Orthogonal Regression

	0 12	1 13	2 14	3 15	4 16	5 17	6 18	7 19	8 20	9 21	10 22	11 23
T.B.	16.94	31.55	45.13	53.92	58.32	61.87	64.69	70.23	76.29	83.60	91.51	
1–3	16.13	30.11	43.54	53.04	57.46	61.10	63.47	68.92	74.52	80.85	87.29	94.71
3–5	14.61	28.60	41.58	51.06	56.28	60.51	63.26	69.22	74.21	79.98	86.99	94.14
5–10	12.09 96.03	24.29	35.91	45.48	51.42	56.00	58.58	64.17	69.83	76.41	82.78	89.83
10+	10.08 95.29	21.01	31.47	40.45	47.55	53.59	58.09	63.88	70.12	76.37	82.41	89.22
M.Y.M.	10.11 82.07	21.10 86.15	31.42 88.36	40.12 90.00	46.82 92.28	52.25 94.74	56.56 97.61	60.88	65.54	69.60	73.42	77.99

Abstracting initially from capital flows the purchasing power parity theory suggests that the exchange rate is determined by the price level in the United States relative to that in Canada. Thus, we have:

$$(9) \quad E_s = \frac{{}^{u}P}{{}^{c}P}$$

According to equation (9), if the price level in the United States doubles, *ceteris paribus*, the price of the Canadian dollar in terms of U.S. dollars doubles.

Taking logs of both sides of equation (9), means:

$$(10) \quad \ln E_s = \ln {}^{u}P - \ln {}^{c}P.$$

Differentiating the above expression yields:

$$\frac{\dot{E}_s}{E_s} = \frac{{}^{u}\dot{P}}{{}^{u}P} - \frac{{}^{c}\dot{P}}{P}$$

or

$$(11) \quad \lambda = {}^{u}\pi - {}^{c}\pi$$

where λ is the rate of appreciation or depreciation of the Canadian dollar in terms of U.S. dollars. Equation (11) says that the difference between the rates of inflation in Canada and the United States is reflected in the rate of change of the exchange rate. Similarly, the expected rate of change of the exchange rate (λ^e) is a reflection of the anticipated rates of inflation in the two countries. Therefore:

$$(12) \quad \lambda_t^e = {}^{u}\pi_t^e - {}^{c}\pi_t^e.$$

Under conditions of perfect foresight of exchange rate movements, and abstracting from transactions costs, the forward rate of exchange will be equal to the spot rate of exchange expected to prevail at the date of consummation of the forward contract. The spot rate expected to prevail at any time k in the future appears as:

$$(13) \quad {}_kE_s^e = E_s \cdot e^{\lambda^e k}$$

The forward rate of exchange reflects the expected change in the exchange rate, and hence the expected rate of inflation in Canada relative to that in the United States.

$$(14) \quad {}_kE_f = E_s \cdot e^{({}^{u}\pi^e - {}^{c}\pi^e)\,k}$$

where ${}_kE_f$ is the currently quoted forward rate of length k.

What relationship must exist between the equilibrium rates of inflation in the two countries, the rate of appreciation of the exchange rate, and the rates of interest? Suppose that a U.S. investor is deciding whether to purchase a Canadian or U.S. security which will be held to maturity. Assume that no forward market exists. If the investor purchases a one dollar U.S. dollar denominated security, by time t it will have grown to $e^{u_{it}}$ in nominal U.S. dollars. In real terms the value of the security will be $e^{u_{it}}. e^{u - \pi_t} = e^{(u_i - u\pi)_t} = e^{u_{rt}}$.

If he simply holds Canadian currency, he will obtain $e^{\lambda t}$ in U.S. nominal funds at time t. If he purchases a comparable Canadian security he will receive:

$$e^{\lambda t} \cdot e^{c_{it}} = e^{(\lambda + c_i)t}$$

in nominal U.S. funds. In terms of real purchasing power over U.S. goods and services, he obtains $e^{(\lambda + {}^{c}i - {}^{u}\pi)t}$. If these two returns are to be equal, we have:

$$(15) \quad e^{(\lambda + {}^{c}i - {}^{u}\pi)t} = e^{({}^{u}i - {}^{u}\pi)t}$$

taking antilogs:

$$(16) \quad \lambda + {}^{c}i - {}^{u}\pi = {}^{u}i - {}^{u}\pi$$

or

$$(17) \quad {}^{c}i = {}^{u}i - \lambda$$

Equation (17) does not imply causality; it is simply a modified statement of the interest arbitrage equilibrium condition. The U.S. investor need not care about the real rate of interest in Canada (${}^{c}r = {}^{c}i - {}^{c}\pi$); his only concern with Canadian inflation lies in its effect on λ.

Equation (11) can be made into a theory of Canadian interest rate determination if we are willing to assert something about the determination of λ and ${}^{u}i$.

Assume that the U.S. nominal rate fully adjusts to take account of inflation and that the U.S. real rate is not affected by U.S. inflation. For present purposes, let the U.S. real rate be constant. Thus,

$$(18) \quad {}^{u}i = {}^{u}r + {}^{u}\pi^{e}.$$

Assume also that the purchasing power parity theory holds:

$$(19) \quad \lambda^{e} = {}^{u}\pi^{e} - {}^{c}\pi^{e}$$

The forward market will then reflect the relative rates of inflation in the two countries. Thus, if the forward price of U.S. dollars is at a discount, the Canadian dollar is expected to appreciate *vis-à-vis* the U.S. dollar. Therefore:

$$(20) \quad \xi = \lambda^{e},$$

where ξ is the percentage premium on forward Canadian dollars over the spot rate. Combining equations (20) and (17), we have:

$$(21) \quad {}^{c}i = {}^{u}i - \xi$$

In this equation, changes in the rate of inflation in Canada affect the Canadian nominal rate by influencing the forward premium or discount on U.S. dollars.

PRICE EXPECTATIONS, THE FORWARD MARKET, AND INTEREST DIFFERENTIALS

In this section, we test whether Canadian inflation plays a significant role in determining Canadian interest rates once U.S. interest rates as well as the percentage premium on forward over spot exchange are taken into account.

Results for regressions of selected Canadian nominal interest rates on U.S. nominal rates with similar characteristics and the percentage premium of forward over spot exchange (expressed as an annual percentage) are contained in Table XII-(a). The percentage premium of forward over spot is used as a proxy for expected movements in the spot exchange rate. Simply regressing Canadian interest rates on their U.S. counterparts produces a reasonably close relationship. This relationship is closer for longer-term securities than for shorter-term securities. The percentage premium on forward exchange improves the relationship somewhat, and is always significant.

To test whether Canadian inflation makes any independent contribution to the explanation of the level of Canadian interest rates once U.S. interest rates are accounted for, a 12-month distributed lag on the rate of change of the price level was added to the regressions. Table XII-(b) contains results for a third degree polynomial Almon constrained lag structure. The sum of the weights is close to zero in all cases. The individual lags are not reported. The shape of the lag distribution is quite unstable whether or not the Almon lag procedure is used. Several of the lag distributions contain a number of negative terms. In the unconstrained lag case, most of the coefficients are insignificantly different from zero and a large portion of them are negative. The sum of the weights is about 0.04. The results using the Almon lag technique are similar. In one case (long-term government bonds) the sum of the weights is negative.

The results of splitting the period into fixed and flexible exchange regimes are reported in Tables XII-(c) and XII-(d). The relationship between rates is closer for the fixed rate period. A number of considerations suggest this result, including the increasing degree of financial integration of the North American capital market. Only the results using an Almon constrained lag are reported. The sum of the coefficient is in the neighbourhood of 0.02 for the flexible rate period. While higher for the fixed rate period, the sum of the coefficient is significantly different from unity and insignificantly different from zero.

These results are in accord with the theoretical developments found in this chapter. They suggest that Canadian inflation has had little direct effect on the height of nominal interest rates in Canada, except insofar as this may have affected expectations of movements in the exchange rate.

TABLE XII–(a)

Relationship Between Selected Canadian Interest Rates and American Counterparts
Monthly Observations (228) OLS
January 1952—December 1970

$i^c = a_0 + a_1{}^u$
$i^c = a_0 + a_1\, i^u + a_2\xi$

Canadian Rate	U.S. Rate	Intercept	a_1	a_2	$\bar{R}^2$	S.E.E.	D.W.
Treasury Bill	Treasury Bill	0.3087 (3.1286)	1.0107 (37.9444)	—0—	0.86	0.6439	0.25
1–3	1	1.3500 (18.9046)	0.8236 (46.4552)	—0—	0.90	0.4534	0.36
3–5	3–5	1.0738 (14.0491)	0.9015 (50.4465)	—0—	0.92	0.4007	0.30
10+	10+	0.0351 (0.6192)	1.2316 (90.9291)	—0—	0.97	0.2211	0.29
M.Y.W.	U.S. CORP. B.	0.9299 (16.7200)	1.0423 (91.2467)	—0—	0.97	0.2388	0.36
Treasury Bill	Treasury Bill	−0.1664 (−1.5771)	1.0871 (42.7851)	42.0929 (8.0100)	0.89	0.5692	0.34
1–3	1	0.9615 (13.3414)	0.8811 (54.9655)	34.1641 (9.7391)	0.93	0.3812	0.51
3–5	3–5	0.7042 (8.6466)	0.9587 (55.4860)	26.8230 (8.0907)	0.94	0.3535	0.38
10+	10+	−0.1198 (−1.8075)	1.2590 (85.8879)	8.4815 (4.1434)	0.97	0.2136	0.30
M.Y.W.	U.S. CORP. B.	0.7055 (11.6018)	1.0748 (93.3647)	13.8579 (6.7293)	0.98	0.2184	0.46

TABLE XII–(b)

Relationship Between Selected Canadian Interest Rates and American Counterparts
Monthly Observations (228) OLS Almon 3rd Degree Polynomial 12 Lags
January 1952—December 1970

$$i^c = a_0 + a_1 i^u + a_2 \xi + b\pi_t^e$$

Canadian Rate	U.S. Rate	Intercept	a_1	a_2	Sum of Weights	S.E.E.	$\bar{R}^2$	D.W.
Treasury Bill	Treasury Bill	−0.1070 (−1.0021)	1.0487 (32.4181)	0.3998 (7.6003)	0.0415 (−34.2357)	0.5632	0.90	0.36
1–3	1	0.9813 (13.4512)	0.8485 (42.0964)	0.3346 (9.5308)	0.0491 (−51.2805)	0.3777	0.93	0.52
3–5	3–5	0.7605 (9.3147)	0.9103 (43.7677)	0.2582 (7.9434)	0.0674 (−56.6745)	0.3423	0.94	0.39
10+	10+	−0.1380 (−1.9590)	1.2627 (70.0892)	0.0891 (4.2863)	−0.0005 (−98.3202)	0.2137	0.97	0.31
M.Y.W.	U.S. CORP. B.	0.8060 (13.9191)	1.0246 (79.8808)	0.1279 (6.7491)	0.0660 (−98.7361)	0.1990	0.98	0.52

Table XII–(c)

Relationship Between Selected Canadian Interest Rates and American Counterparts
Monthly Observations (124) Almon, OLS
January 1952—April 1962

$$i^c = a_0 + a_1 i^u + a_2 \xi + b\pi_t^e$$

Canadian Rate	U.S. Rate	Intercept	a_1	a_2	Sum of Weights	S.E.E.	$\bar{R}^2$	D.W.
Treasury Bill	Treasury Bill	−0.2677 (−1.6581)	1.1639 (17.9356)	0.2331 (3.2176)	0.0283 (−30.8427)	0.5777	0.77	0.39
1–3	1	1.0049 (10.9433)	0.8082 (25.9125)	0.3764 (8.8045)	0.0296 (−52.3199)	0.3416	0.87	0.67
3–5	3–5	0.7970 (6.8632)	0.8864 (25.6605)	0.2785 (7.1897)	0.0268 (−58.2814)	0.3081	0.86	0.55
10+	10+	−0.4003 (−2.9020)	1.3395 (34.1515)	0.0710 (2.3880)	−0.0091 (−80.4254)	0.2315	0.91	0.26
M.Y.W.	U.S. CORP. B.	0.9212 (8.9132)	0.9963 (38.3275)	0.1635 (7.5027)	0.0279 (−104.0442)	0.1707	0.93	0.35

TABLE XII–(d)

Canadian Rates Against U.S. Rates
Split Period—May 1962—May 1972
Almon Observations (97)

$$i_t^c = a + a_1 i_t^u + a_2 \xi_t + \sum_{i=0} w_i \pi_{t-i}^a$$

	Intercept a	a_1	a_2	Sum of Weights	S.E.E.	$\bar{R}^2$	D.W.
T.B.	0.1881 (0.8595)	1.0178 (10.6848)	0.5562 (6.7483)	0.0130 (−9.9717)	0.4789	0.8753	0.4081
1–3	1.3570 (1.8201)	0.8142 (10.7130)	0.3184 (4.7933)	0.0170 (−11.2463)	0.3983	0.8893	0.4654
3–5	1.0346 (5.9006)	0.8802 (13.7166)	0.2524 (4.8034)	0.0666 (−14.4890)	0.3242	0.9229	0.3798
10+	0.1198 (0.9885)	1.2363 (33.1075)	0.1194 (5.0873)	−0.0269 (−40.4826)	0.1485	0.9778	0.6747
M.Y.W.	0.6986 (6.7591)	1.0136 (32.7519)	0.0613 (2.3761)	0.1388 (−28.7226)	0.1675	0.9842	1.1848

chapter four

PRICE EXPECTATIONS AND THE CREDIT MARKET

There are two recognized bodies of economic theory which are concerned with the relationship between inflation and interest rates. These are the Fisherian price expectations theory[1] and the Wicksellian market-natural rate theory.[2] In Fisher's analysis, inflation leads to higher nominal interests via price expectations, while in Wicksell's analysis, a fall in the market rate below the natural rate leads to inflation. Whereas Fisher tended to disregard the initial impact of money supply changes on interest rates, Wicksell tended to disregard the impact of price expectations on interest rates.

Milton Friedman has recently suggested a framework whereby changes in the money supply result in a temporary depressive effect on nominal interest rates which is ultimately reversed as inflationary expectations take hold.[3] Friedman's suggestion incorporates elements of Wicksell's and Fisher's analyses of the determinants of interest rates. In this chapter we attempt to formalize Friedman's system of Wicksell's and Fisher's views concerning the relationship among changes in the money supply, changes in the price level and interest rates. The results obtained from our analysis appear to be at variance with Friedman's in several regards. Within the framework developed in this chapter

[1] Irving Fisher, *The Theory of Interest*, New York, Macmillan, 1930.

[2] Knut Wicksell, "The Influence of the Rate of Interest on Prices", *The Economic Journal*, XVII: 213–19, 1907. Wicksell, *Interest and Prices*, London, Allen and Unwin, 1936.

[3] Milton Friedman, "The Role of Monetary Policy", *The American Economic Review* (March 1968), reprinted in M. Friedman, *The Optimum Quantity of Money and Other Essays*, Chicago, Aldine, 1969.

it is demonstrated that the monetary authorities can have a permanent effect on the real rate of interest. The condition required in the model to produce a steady rate of inflation also results in a permanent reduction in the real rate of interest.

In addition to the permanent effect of an increased rate of expansion of the money supply on the real interest rate, a temporary effect may also occur. It is this temporary effect which is focussed on in Friedman's synthesis.[4] The temporary effect occurs not because of inflation *per se,* nor even because inflation is not fully anticipated, but rather is due to asymmetric expectations on the part of capital market participants. If participants on both sides of the capital market form expectations in the same manner, then the temporary effect disappears. It is commonly argued that the reason why interest rates rise during an inflation is that:

> "Borrowers will then be willing to pay, and lenders will then demand higher interest rates. . .as Irving Fisher pointed out long ago."[5]

One of the purposes of this chapter is to provide a theoretical model which gives a plausible description of how nominal rates of interest are bid up during an inflationary process. This model in turn provides a guideline for subsequent empirical tests.

In order to highlight the mechanisms at work, a highly simplified model is used. For present illustrative purposes, it will be assumed that there is a single homogeneous market for credit, in which a single rate of interest is determined. Furthermore, it is assumed that all finance is direct, and is not indirectly routed from primary savers through financial intermediaries to ultimate borrowers.[6] It is assumed that credit contracts are made for one period, or, if not, the terms are recontracted each period. This assumption permits us to evade a number of stock-flow difficulties. Changes in the price level are assumed to arise from disequilibrium in the commodity market, while changes in interest rates are assumed to arise from disequilibrium in the credit market. It is assumed that resources are fully employed.

THE NO INFLATION CASE

Assuming an absence of inflation, the rate of interest (in both real and nominal terms) is determined along with the volume of credit by the interaction of supply and demand in the credit market. The following simplified model of the credit market is utilized.

(1) $d_t = a + bi_t$

(2) $s_t = h + ki_t$

(3) $d_t = s_t$

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[4] *Ibid,.* p. 104.

[5] *Ibid.*, p. 100.

[6] The terminology employed here is that of Gurley and Shaw. J. Gurley and E. S. Shaw, *Money in a Theory of Finance*, Washington, Brookings, 1960.

where equations (1), (2), and (3) are the demand-for-credit equation, the supply-of-credit equation, and the market-clearing assumption, respectively, and i is the nominal (and real) rate of interest. Instantaneous clearing of the market is assumed.

The equilibrium rate of interest is:

$$(4) \quad i_t^* = \frac{h - a}{b - k}, \; t = 1 \ldots \infty ,$$

while the equilibrium real quantity of credit extended is:

$$(5) \quad l_t^* = \frac{bh - ak}{b - k}, \; t = 1 \ldots \infty .$$ [7]

PERFECTLY ANTICIPATED INFLATION

Let us assume that the economy moves from one steady state rate of inflation (say zero) to a new permanent higher level (say $\bar{\pi}$). Assume that capital market participants are concerned with the real cost of borrowing and the real return on lending, and that they correctly perceive the new rate of inflation. It is also assumed that the rate of inflation has no effect on any other of the determinants of the demand and supply functions of credit. The aggregate demand and supply function for credit are functions of the perceived real cost (return) on credit transactions. The model, thus, alters to:

$$(1)' \quad d_t = a + b(i - \bar{\pi})$$

$$(2)' \quad s_t = h + k(i - \bar{\pi})$$

$$(3)' \quad d_t = s_t .$$

In equilibrium we have:

$$(4)' \quad i_t^* = \frac{h - a}{b - k} + \bar{\pi} .$$

Equation (4)′ demonstrates that under the assumed conditions of our model, the nominal rate of interest has risen by the rate of inflation.

From equation (4)′ it is also evident that the real rate has remained unchanged. That is:

$$(5) \quad i_t^* - \bar{\pi} = \frac{h - a}{b - k}$$

which is identical to the result we obtained under conditions of no inflation.

The volume of real credit in equilibrium is also unaltered at:

$$(6) \quad l_t^* = \frac{bh - ak}{b - k}$$

which is the same as the result obtained in the no inflation case.

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[7] Symbols with an asterisk represent steady state values of the corresponding variable.

IMPERFECTLY FORESEEN INFLATION WITH SYMMETRIC EXPECTATIONS

Let us assume that both lenders and borrowers do not foresee perfectly the rate of inflation, but adjust their expectations of inflation with equal speed. Our model becomes:

$$(1)'' \quad d_t = a + b(i_t - \pi^e_t)$$

$$(2)'' \quad s_t = h + k(i_t - \pi^e_t)$$

$$(3)'' \quad d_t = s_t$$

$$(4)'' \quad \pi^e_t = \pi^e_{t-1} + \lambda(\pi^a_t - \pi^e_{t-1})$$

$$\text{or } \pi^e_t = \sum_{i=0}^{\infty} (1 - \lambda)^i \lambda \, \pi^a_{t-1}$$

$$(5)'' \quad r^e_t = i_t - \pi^e_t$$

$$(6)'' \quad r^a_t = i_t - \pi^a_t$$

$$(7)'' \quad \pi^e_t = \pi^a_t \text{ (in steady state equilibrium)}$$

where r^e_t is the real ex-ante rate of interest at time t, r^a_t is the ex-post real rate of interest, and π^e_t and π^a_t are the expected and actual rates of inflation. Equation (4)″ describes the adaptive expectations mechanism. If the coefficient of adaptive expectations "λ" is equal to unity then this period's rate of inflation is perfectly anticipated. On the other hand, if $\lambda = 0$, then the actual rate of inflation has no effect on this period's expected rate of inflation. The closer λ is to zero the longer the time horizon of market participants. Assume that the rate of inflation increases from zero to a new steady state rate $\bar{\pi}$. ($\pi^a_t = \bar{\pi}$ for $t = 1...N$). The full equilibrium values for the nominal rate of interest, the volume of borrowing, and the real rates ex-ante and ex-post can be derived by substituting $\bar{\pi}$ for π^e_t in equations (1)″ and (2)″. The steady state results are identical to the results under perfectly foreseen inflation.[8] The real rate of interest ex-post in equilibrium appears as:

$$(8)'' \quad i^*_t - \bar{\pi} = \frac{h - a}{b - k},$$

while the real rate ex-ante is:

$$(9)'' \quad i^*_t - \pi^e = i^*_t - \bar{\pi} = \frac{h - a}{b - k}.$$

In full equilibrium, the real rates ex-ante and ex-post, are unchanged from the no inflation case. In this model, as it stands, the authorities cannot change either the real rate of interest or the real volume of credit by inflation in the long run. We now consider the process of adjustment on the way to equilibrium when $\pi^e \neq \bar{\pi}$, since it is commonly alleged that it is unanticipated inflation rather than inflation *per se* which has real effects.

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[8] This assumes that the distribution effects generated on the path to equilibrium did not in fact alter the final equilibrium position.

At any time t > 0, the expected rate of inflation is:

$$(10)'' \quad \pi_t^e = \sum_{i=0}^{N} (1 - \lambda)^i \lambda \bar{\pi}_{t-i}^a .$$

Substituting this result into equations (1)″ and (2)″ and solving for the temporary equilibrium value for the nominal rate, we have:

$$(11)'' \quad i_t' = \left(\frac{h - a}{b - k}\right) + \pi_t^e = \left(\frac{h - a}{b - k}\right) + \sum_{i=0}^{N} (1 - \lambda)^i \lambda \bar{\pi}_{t-i}^a$$

where i_t' is the market clearing nominal rate of interest at time t, and $\lim_{t \to \infty} i_t' = i^*$

Thus, at any time, t, the nominal rate of interest exceeds the no-inflation real rate by the expected rate of inflation. It follows, therefore, that the real ex-ante rate has not altered, since:

$$(12)'' \quad r_t^e = i_t' - \pi_t^e = \frac{h - a}{b - k} .$$

Similarly, the real volume of credit extended remains at:

$$(13)'' \quad l_t' = \frac{bh - ak}{b - k} .$$

Equations (12)″ and (13)″ demonstrate that the perceived market-clearing results are not altered by unanticipated inflation. That is, there is neither an increase in the amount of real credit extended for a temporary lowering in the real ex-ante rate of interest.

The real rate of interest ex-post, however, does alter. The ex-post real rate at any time t is:

$$(14)'' \quad r_t^a = \left(\frac{h - a}{b - k}\right) - \pi_t^a + \sum_{i=0}^{N} \lambda(1 - \lambda)^i \pi_{t-1}^a$$

or alternately:

$$r_t^a = \left(\frac{h - a}{b - k}\right) - \left(\pi_t^a - \pi_t^e\right) .$$

Figure 4 illustrates the impact on the real ex-ante and real ex-post rates of interest over time of a rise in the rate of inflation.

Short of full equilibrium,[9] however, with an increase in the rate of inflation, borrowers gain at lenders' expense.[10] Consider the temporary equilibrium that occurs immediately after the rise to a new higher rate of price increase.

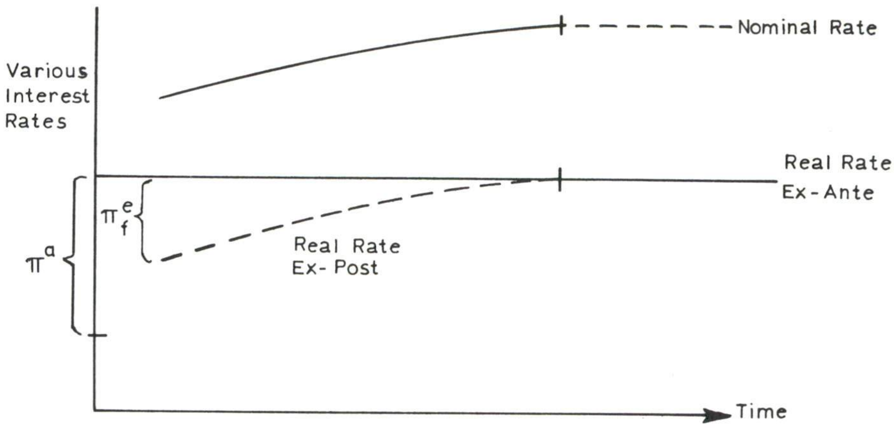
Ultimate savers extended l^* of loans in terms of current purchasing power, expecting to receive $l^* (1 + r^e)$ in return. In fact, they received $l^* (1 + r^a)$ or $l^* (\Pi^a - \Pi^e)$ less than they anticipated. $(\Pi^a - \Pi^e)$ is a measure of the unan-

[9] The word "equilibrium" is being used in two senses here. Full equilibrium refers to the notion that there is no inherent reason for values of variables to change. Temporary equilibrium refers to the notion that the market clears.

[10] With a fall in the rate of inflation, a transfer in the reverse direction occurs.

Figure 4

Impact of Inflation on Selected Interest Rates



ticipated inflation. Borrowers anticipate paying a real rate of interest of $r_t^e = (i_t - \lambda\Pi_t^a) = (i_t - \Pi_t^e)$. They actually pay an actual ex-post rate of $r_t^a = (i_t - \Pi_t^a)$.

Retrospectively, the system was in disequilibrium. If lenders could have foreseen correctly the extent of the inflation, at the real rate of interest, r^a, they would have been willing to extend a smaller amount of credit. In a similar fashion, had borrowers foreseen correctly the real rate of interest they would have acquired more loans.

There is a lump-sum transfer of $l^* (\Pi_t^a - \Pi_t^e)$ from primary lenders to primary borrowers. This transfer is a distribution effect and is assumed to cancel out for the economy as a whole. It is assumed in our analysis that wealth transfers of this type have no effect on the behavior of lenders or borrowers.[11] The net transfer from lenders to borrowers in any particular period is:

$$(15)'' \quad T_t = \left(\frac{bh - ak}{b - k}\right) \_ \left(\pi_t^a - \pi_t^e\right)$$

where $\lim_{t \to \infty} T_t = 0$. That is to say, in steady state equilibrium, the transfer effect is zero.

If it is possible to identify lenders and borrowers with specific groups (e.g., once a lender, always a lender), then the transfer effect is cumulative and is at any particular time, "t",

$$(16)'' \quad CT_t = \int_0^t T(N)dN$$

[11] Should ultimate lenders attempt to recoup their transferred savings, or the behavior of borrowers be affected by their gain, the demand and supply curves would shift. This phenomenon is generally known as Historisis. (The path to equilibrium affects the ultimate equilibrium achieved.) Historisis is generally ignored in economics. And for good reason, for should trading occur at what is ultimately a non-equilibrium price on the way to equilibrium, the whole analysis of demand and supply appears to break down. See John R. Hicks, *Value and Capital*, 2nd Edition, London, Oxford University Press, 1946, p. 128,

If, as is likely the case, lenders are not forever lenders, and borrowers are not forever borrowers, CT, the cumulative transfer, has no precise meaning.

Equations (14)″ and (15)″ show that the effect of unanticipated inflation, where both sides of the market are equally poor forecasters, causes a temporary fall in the real ex-post rate of interest, and a transfer from lenders to borrowers. Ex-ante magnitudes are not affected. Insofar as it is legitimate to neglect distribution effects, no real effects occur. It should be noted that if either borrowers or lenders fail to fully adjust to inflation, permanent effects do occur.

The important point here is that not only does perfectly anticipated inflation not have any real effects in the model, but unanticipated inflation does not affect behavior either, so long as it is assumed that distributional effects cancel out.

In the analysis, there is neither a temporary nor a permanent trade-off between inflation and real effects. No temporary trade-off results from unanticipated inflation. Capital market decisions, the real ex-ante rate of interest and the real value of loan contracts remain unchanged throughout. The most that occurs is a distribution effect which results from the transfer of wealth from lenders to borrowers (if $\lambda < 1$). This transfer effect occurs because loan contracts are written in nominal money terms. At the same time, this distributional effect is customarily neglected in economics. It is not possible for the authorities to depress the real, ex-ante rate of interest by inflation in the context of this model.[12] In order to have a real impact (aside from wealth transfer) something else must be true other than the fact that inflation is imperfectly foreseen.

INFLATION WITH ASYMMETRIC EXPECTATIONS

In this section, the assumption that borrowers and lenders form expectations in an identical manner is modified. Our model now alters to:

(1)+ $d_t = a + b(i_t - {}^d\pi^e_t)$

(2)+ $s_t = h + k(i_t - {}^s\pi^e_t)$

(3)+ $d_t = s_t$

(4)+ ${}^d\pi^e_t = {}^d\pi^e_{t-1} + \gamma\{\pi^a_t - {}^d\pi^e_{t-1}\}$

or

$${}^d\pi^e_t = \sum_{i=0}^{\infty} (1-\gamma)^i\, \gamma\pi^a_{t-i}$$ (formation of expectations, borrowers)

(5)+ ${}^s\pi^e_t = {}^s\pi^e_{t-i} + \delta\{\pi^a_t - {}^s\pi^e_{t-1}\}$

$${}^s\pi^e_t = \sum_{i=0}^{\infty} (1-\delta)^i\, \delta\pi^a_{t-i}$$ (formation of expectations, lenders)

(6)+ ${}^d r^e_t = i'_t - {}^d\pi^e_t$ (real ex-ante rate for borrowers)

(7)+ ${}^s r^e_t = i'_t - {}^s\pi^e_t$ (real ex-ante rate for lenders)

(8)+ $r^a_t = i'_t - \pi^a_t$

(9)+ ${}^d\pi^e_t = {}^s\pi^e_t = \pi^e = \pi^a$, in steady state equilibrium.

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12 "The monetary authorities can make the market rate less than the natural rate only by inflation. It can make the market rate higher than the natural rate only by deflation." Friedman, *op. cit.*, p. 101. Note that market rate in this context is not the augmented loanable funds induced market rate of Wicksell. Friedman appears to have misrepresented Wicksell. Wicksell would not have argued that the "monetary authority can make the market rate less than the natural rate only by inflation."

In the previous model, it was assumed that $\delta = \gamma$, otherwise nothing has been altered. The steady state equilibrium results are as before. That is to say, the steady equilibrium nominal rate is given by:

$$(10)+ \quad i_t^* = \frac{h - a}{b - k} + \pi_t .$$

The real volume of credit contracts remains at:

$$(11)+ \quad l_t^* = \frac{bh - ak}{b - k} .$$

The real rate ex-ante to borrowers appears as:

$$(12)+ \quad {}^d r_t^{*a} = i_t^* - {}^d\pi_t^e = i_t^* - \bar{\pi} = \left(\frac{h - a}{b - k}\right),$$

while for lenders we have:

$$(13)+ \quad {}^s r_t^{*e} = i_t^* - {}^s\pi_t^e = \left(\frac{h - a}{b - k}\right)$$

In steady state equilibrium the real ex-post rate of interest is unchanged at:

$$(14)+ \quad r_t^{*a} = \frac{h - a}{b - k}$$

The path to equilibrium, however, is altered considerably. It is no longer true that the real volume of loan contracts is unaffected by unanticipated inflation, nor is it the case that the real ex-ante rate of interest remains unchanged.

The market-clearing nominal rate i_t' in any period t, can be found by solving equations (1) + and (2) + for i_t.

The following expression for i_t' is obtained:

$$(15)+ \quad i_t' = \frac{h - a}{b - k} + \frac{b\,{}^d\pi_t^e - k\,{}^s\pi_t^e}{b - k} .$$

Alternatively, we have:

$$(16)+ \quad i_t' = \frac{h - a}{b - k} + (b - k)^{-1} \sum_{i=0}^{\infty} \{b(1 - \gamma)^i \gamma - k(1 - \delta)^i \delta\} \bullet \pi_{t-i}^a$$

$$\text{where } \lim_{t \to \infty} i_t' = \frac{h - a}{b - k} .$$

Suppose that the real factors in determining the rate of interest remain unchanged. Equation (16) + shows that the nominal rate at any time t is not a simple addition of the real rate and the expected rate of inflation. Equation (16)+ also demonstrates one of the reasons why regressions of nominal rates of interest as past rates of inflation can yield highly misleading results. If equation (16) + represents the true reduced form expression for the nominal rate of interest, then the estimated coefficients of lagged inflation terms represent a complex mixture of the slopes of the two demand and supply functions and expectations on either side of the market.

At any time, t, the real ex-ante rate of interest to borrowers is lower than it initially was, until full equilibrium is reached. (See Figure 5.) The path of the real ex-ante rate of interest to borrowers is given by:

$$(17)+ \quad {}^{d}r^{e}_{t} = i'_{t} - {}^{d}\pi^{e}_{t} .$$

Alternatively:

$$(18)+ \quad {}^{d}r^{e}_{t} = \frac{h - a}{b - k} + (b - k)^{-1} \{\sum_{i=0}^{\infty} (b(1 - \gamma)^{i} \gamma - k(1 - \delta)^{i} \delta) \pi^{a}_{t-i}\} - \sum_{i=0}^{\infty} (1 - \gamma)^{i} \gamma\pi^{a}_{t-i} .$$

We also have:

$$(19)+ \quad \lim_{t \to \infty} {}^{d}r^{e}_{t} = \frac{h - a}{b - k}$$

Under the hypothesis[13] that $\gamma > \delta$, equation (18)+ demonstrates that there is a temporary trade-off between inflation and the real ex-ante rate of interest to borrowers. If the authorities are interested in the real cost of credit to borrowers, then ${}^{d}r^{e}_{t}$ is the relevant rate for consideration.

The ex-ante rate of interest to lenders rises and is represented by:

$$(20)+ \quad {}^{s}r^{e}_{t} = i'_{t} - {}^{s}\pi^{e}_{t} , \text{ or}$$

$$(21)+ \quad {}^{s}r^{e}_{t} = \frac{h - a}{b - k} + (b - k)^{-1} \{\Sigma b(1 - \gamma)^{i} \gamma - k(1 - \delta) \delta\pi^{a}_{t-i} - \sum_{i=0}^{\infty} (1 - \delta) \delta\pi^{a}_{t-i} ,$$

where,

$$(22)+ \quad \lim_{t \to \infty} {}^{s}r^{e}_{t} = \frac{b - a}{h - k} .$$

Short of steady state equilibrium, the real ex-ante rate of interest to lenders is above its equilibrium value. (See Figure 6.) From equations (18) + and (21) +, we can see that in the short run the real ex-ante rate of interest perceived by lenders rises, and that perceived by borrowers falls. Consequently, the real volume of loan contracts in the short run is higher than its equilibrium value. It is also clear that the term "real rate of interest" does not have an unambiguous meaning short of steady state equilibrium.

The real volume of loan contracts rises, in the short run, above its long-run equilibrium value. (See Figure 8.) This short-run effect is due to asymmetric expectations and may be termed "bilkage". It is not the same thing as the transfer effect discussed previously. The transfer effect occurs because of disequilibrium

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[13] This is the Fisher-Friedman Asymmetric Expectations hypothesis. A rationalization for this asymmetry in the formation of price expectations may be that borrowers are frequently business firms which have fewer prices to predict than do ultimate savers. Fisher argued as follows:

> "... in general, borrowers foresee better than lenders. The great borrowers of today are not, as is often supposed, the ignorant poor, but the alert and well informed and rich. It is the function of these people to look ahead, and the consequence is that they foresee a rise or fall of prices more quickly than the lenders or bondholders, who are only silent partners in the business .... The consequence, therefore, is an inflation of loans stimulated from both sides of the market." Irving Fisher, *Elementary Principles of Economics*, New York, Macmillan, 1911, pp. 362–63.

Figure 5

Time Path of Real Market Rate Facing Borrowers

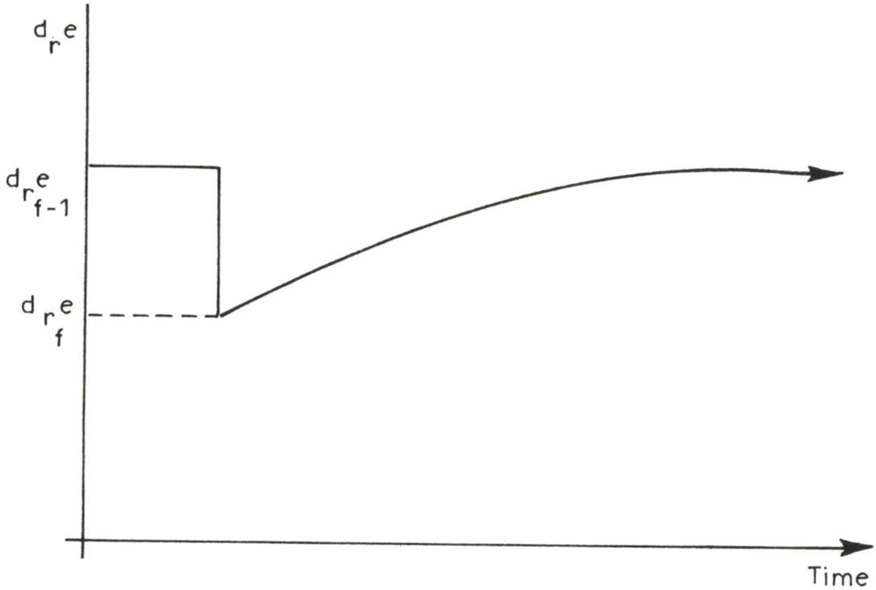


Figure 6

Time Path of Real Market Rate Facing Lenders

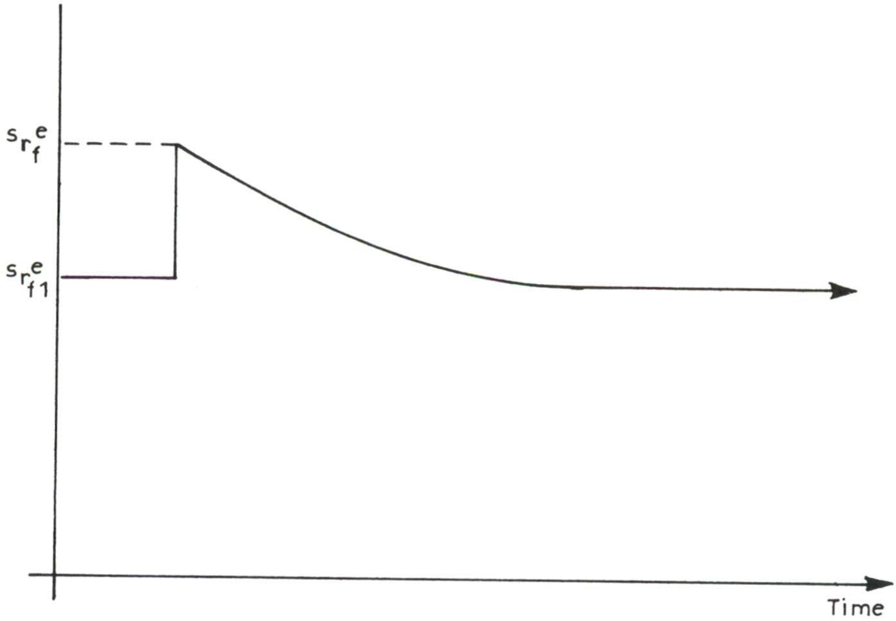


Figure 7

Short and Long-run Trade-off Between Reductions in the Real Rate to Borrowers and Inflation

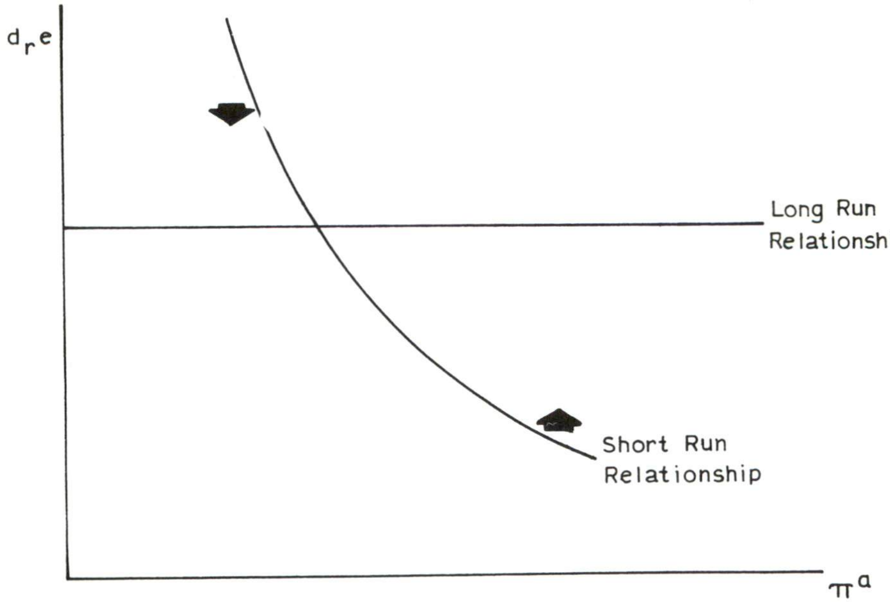
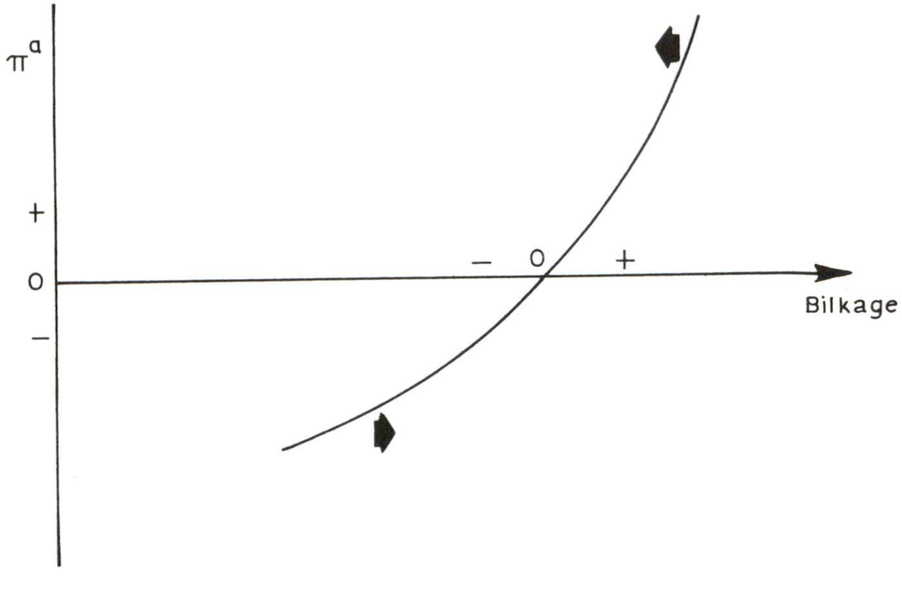


Figure 8

Short-Run Trade-off Between Bilked Funds and Inflation



in retrospect, while bilkage is a change in the temporary equilibrium position induced by asymmetric expectations. Bilkage leads to a change in the real ex-ante equilibrium position, and so has real effects (aside from distribution effects).

Suppose that after the period is over, borrowers compensated lenders for the fall in the real value of the principal and interest payments. Under these conditions inflation would still have a temporary real effect. While compensation removes the transfer effect, it does not remove the bilkage effect. Should ex-post compensation become commonplace, the model would need to be modified.

The real volume of credit extended in any period can be obtained by substituting the value for i'_t into either equation (1)+ and (2)+ to obtain:

$$(23)+ \quad l'_t = (b - k)^{-1} \{bh - ak + bk \cdot ({}^{d}\pi^{e} - {}^{s}\pi^{e})\}.$$

In the long run, the real volume of credit extended is unaffected, since:

$$(24)+ \quad \lim_{t \to \infty} l'_t = \frac{bh - ak}{b - k}.$$

The amount of bilkage occurring each period is given by:

$$(25)+ \quad \beta_t = l'_t - e^*_t \text{, or}$$

$$(26)+ \quad \beta_t = (b - k)^{-1} \Sigma\{(1 - \gamma)^i \gamma - (1 - \delta)^i \delta\} \pi^a_{t-i}.$$

From (26)+ it is seen that in the long run, bilkage is zero for:

$$(27)+ \quad \lim_{t \to \infty} \beta_t = 0.$$

From equation (26)+, it is also evident that the amount of bilkage occurring each period is affected not only by the coefficients of expectations but also by the elasticity of demand for and supply of credit with respect to the rate of interest. The cumulative total increase in the amount of bilked credit over a given period of length (a, b) is given by the expression:

$$(28)+ \quad CB_t = \int_a^b B(z)\, dz.$$

Whether inflation causes a temporary increase or a temporary decrease in the volume of real credit extended depends upon whether $\gamma \lesseqgtr \delta$. If $\delta > \gamma$, the effect of inflation in the short run is to withdraw credit from the market and raise the real rate ex-ante to borrowers.

The ex-post real rate of interest is readily derived using equation (8)+. The path of the ex-post real rate of interest is influenced by the asymmetry in expectations, since the market-clearing nominal rate is affected. In addition, the amount and nature of the ex-post transfer effect is altered.

There are a number of reasons to regard the application of the above model to the real world with suspicion. One of the more important reasons is that we have ignored bilkage effects on the real determinants of the system. If the amount of bilkage is sizeable, then one would expect that the size of the capital stock would be altered from what it otherwise might be. This, in turn, will have an effect on the real rate of interest. Should this occur, inflation would produce real effects even in the long run. Nevertheless, we disregard effects of this type.

BILKAGE AND THE PHILLIPS CURVE

In this section, the direct impact of money on the credit market, and transfer effects of imperfectly foreseen inflation, will be ignored. The analysis concentrates solely on the concept of bilkage.

The bilkage model is formally analagous to the Friedmanian explanation of the Phillips Curve[14]. The conditions required to generate a short-run Phillips Curve are precisely the same as those required to generate bilkage. The analysis is heavily dependent upon the view that factor suppliers are less perceptive about the rate of inflation or adapt less quickly than factor purchasers. In neither case is there a long-run trade-off between inflation and real effects.

Both the bilkage model and the Friedmanian analysis of the short-run Phillips Curve have the interesting property that, should factor suppliers adapt to inflation more quickly than the factor purchasers, the real effects are opposite in direction. Under this set of conditions the Phillips Curve is positively inclined and bilkage is negative.

In Figure 9, it is assumed that factor suppliers are less perceptive than factor purchasers. This produces the Friedmanian results. The abscissa in Figure 9 measures either the inverse of the unemployment rate or the real rate (ex-ante) of interest charged to borrowers. In the short run, both of these variables can be affected by the rate of inflation. When full adjustment occurs there is no trade-off. If factor suppliers and purchasers adjust to the new rate of inflation at the same speed, there is neither a short-run nor a long-run trade-off. Figure 10 illustrates this. Should factor suppliers adjust to inflation at a quicker pace than factor purchasers, the effect of inflation in the short run is to raise rather than to reduce the level of unemployment, and to increase the real ex-ante rate of interest to borrowers. This situation is presented graphically in Figure 11.

THE FISHERIAN MODEL OF MONEY AND CREDIT

As previously mentioned, Fisher did not consider the direct linkage between the money supply or its rate of increase and the capital market. The role of money in Fisher's analysis is to determine the price level or its rate of change. Introducing money, in the Fisherian manner, into the previously discussed model does not alter the conclusions. The Fisherian system is bloc recursive. The rate of change of the money supply determines the rate of increase of the price level. The rate of increase in the price level generates inflationary expectations, which in turn affect the nominal rate of interest through their impact on the demand and supply of credit.

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[14] M. Friedman, *op. cit.*

Figure 9

Short and Long-run Trade-off Between Inflation and Real Effects

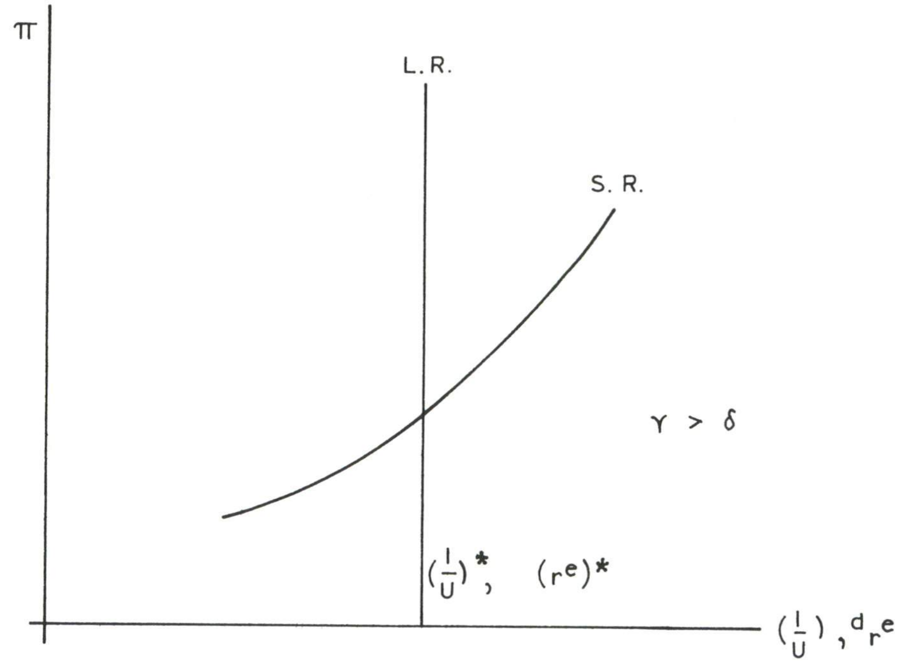


Figure 10

Short and Long-run Trade-off Between Inflation and Real Effects

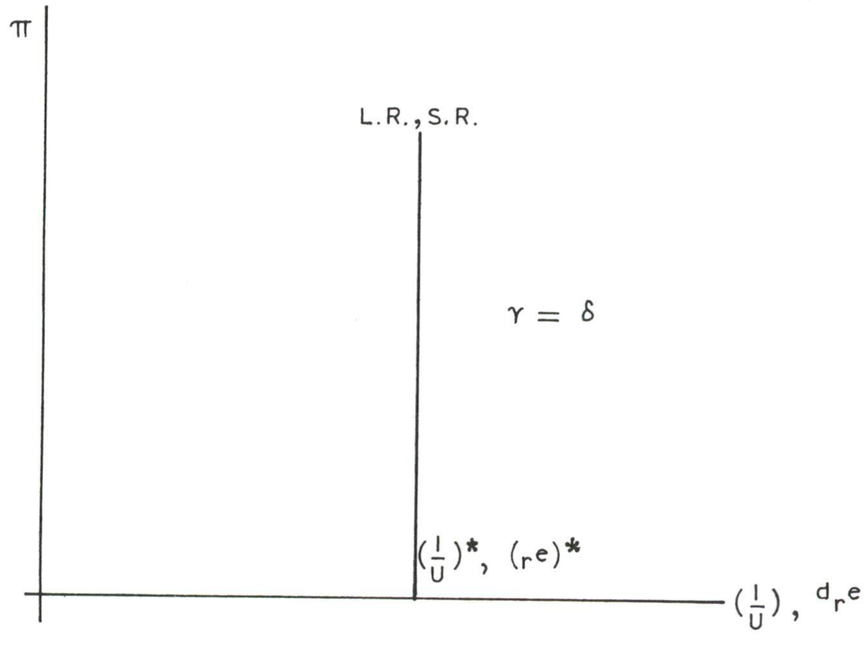
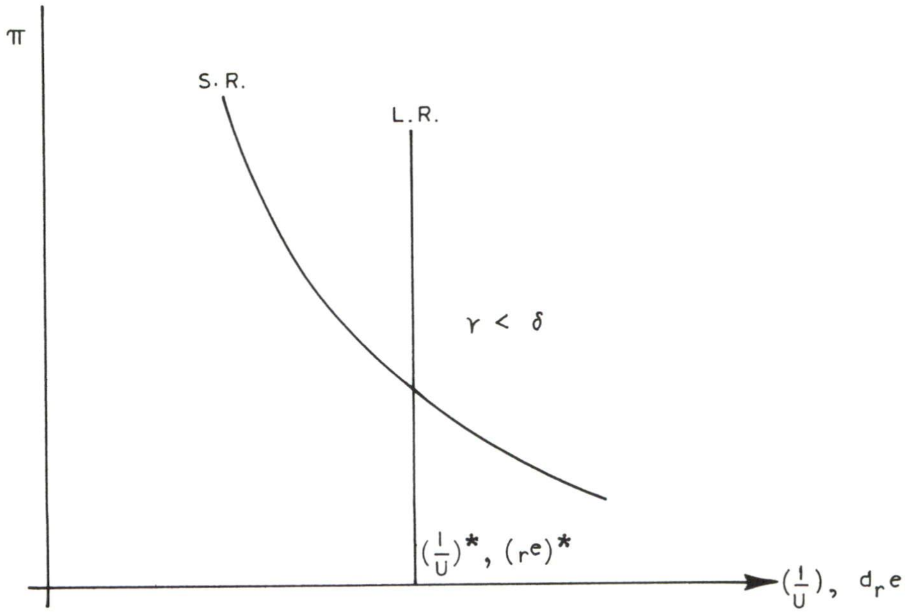


Figure 11

Short and Long-run Trade-off Between Inflation and Real Effects



THE WICKSELL-OHLIN MODEL

The main difference between the Fisherian model and the Wicksell-Ohlin model is that the system is no longer bloc recursive. Changes in the money supply directly affect the volume of real credit extended. Changes in the aggregate price level are a consequence of disequilibrium in the commodity market while changes in the interest rate arise from disequilibrium in the capital market.

In Wicksell's analysis, however, disequilibrium in the commodity market (excess aggregate demand or supply) arises from the effects of money on the credit market. In particular, increasing the supply of money has the effect of augmenting the supply of loanable funds and so permits excess aggregate demand to occur. Wicksell was still a quantity theorist, however, and his system can be regarded as an attempt to describe how changes in the money supply affect the price level.[15]

A simple version of the Wicksell-Ohlin model can be formulated as follows:

$$(1){+}{+} \quad d_t = a + b(i_t - \pi_t^e)$$

$$(2){+}{+} \quad s_t = h + k(i_t - \pi_t^e) + \left[\frac{\Delta M}{p}\right]_t$$

$$(3){+}{+} \quad d_t = s_t + \left[\frac{\Delta M}{p}\right]_t$$

[15] See chapter one.

$$(4)++ \quad \pi_t^e = \Sigma(1-\lambda)^i \lambda \pi_{t-i}^a$$

$$(5)++ \quad r_t^e = i_t' - \pi_t^e$$

$$(6)++ \quad r_t^a = i_t' - \pi_t^a$$

$$(7)++ \quad \pi_t^e = \pi_t^a$$

$$(8)++ \quad \pi_t^a = f\left[\frac{\Delta M}{p}\right]_t.$$ [16]

Only the case of symmetric expectations is considered. The introduction of asymmetric expectations would change the results, but in a straightforward way. The essential modifications made in this model are the endogenous nature of inflation, and the direct effect of additions to the money supply on the credit market.[17]

Consider the result of increasing the money supply by the same nominal amount period after period. The money supply function which is added to the system of eight equations is:

$$(9)++ \quad \Delta M_t^s = \overline{\Delta M} \qquad t = 1 \ldots \infty.$$

As the price level rises, the net addition to the supply of credit, each period, falls. This, in turn, puts less pressure on the price level. We have:

$$(10)++ \quad \lim_{t \to \infty} \left[\frac{\Delta M}{p}\right]_t = 0, \text{ and}$$

$$(11)++ \quad \lim_{t \to \infty} \pi_t^a = 0.$$

In the long run, the nominal rate of interest returns to its original level. The nominal rate of interest in any period, t, is given by:

$$(12)++ \quad i_t' = (b-k)^{-1} \{(h-a) - \left[\frac{\Delta M}{p}\right]_t\}$$

where

$$(13)++ \quad \lim_{t \to \infty} i_t' = \frac{h-a}{b-k}.$$

The increase in the supply of money leads to forced savings of the amount $\left[\frac{\Delta M}{p}\right]_t$ each period. The amount of forced savings shrinks with time, and approaches zero in the limit. The notion of forced savings is conceptually distinct from the transfer effect and bilkage. Forced savings arise through the ability of the authorities to increase the supply of credit by printing money. Bilkage occurs because of different speeds of reactions on the part of capital market participants to inflation. Transfer effects arise because of difference in ex-ante and ex-post equilibrium positions.

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[16] Suppose that v and y are constant, then $M_1v = p_1y$, $M_2v = p_2y$ and

$$\frac{M_2 - M_1}{p_1} = v\frac{p_2 - p_1}{p_1}y. \text{ Thus } \pi_t^a = f\left[\frac{\Delta M}{p}\right]_t.$$

[17] We have only considered inflationary pressure coming from increases in the money supply. Removing this restriction complicates the analysis considerably.

The short-run dynamics of this model can be easily investigated, but are not pursued here. It should be noted, however, that the nominal rate of interest generally will not exceed the equilibrium real rate by the rate of inflation on the path to long-run equilibrium. The occurrence of this result would be an accident.

Consider, now, the situation where the monetary authorities increase the money supply at a constant percentage rate, so that $\left[\frac{\Delta M}{p}\right]_t$ is constant for all t. The money supply function is altered to:

$$(14)++ \quad \left[\frac{\dot{M}}{M}\right]_t^s = c.$$

By equation (8)++, the rate of inflation will remain constant at $\bar{\pi}^a$. We will consider only the steady state solution. By equation (4)++, the expected rate of inflation is also constant and equal to $\bar{\pi}^a$. ($\pi_t^e = \bar{\pi}_t^a$ for all t), once full adjustment has taken place.

The market-clearing nominal rate of interest, i_t^*, is given by:

$$(15)++ \quad i_t^* = \frac{h - c - a}{b - k} + \pi.$$

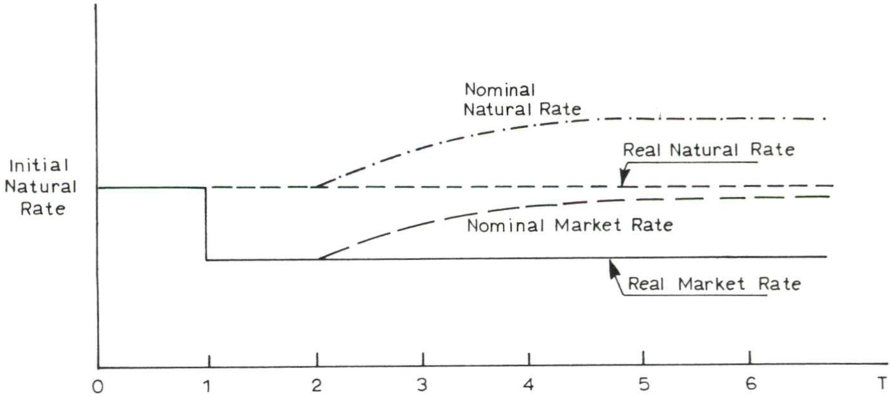
In steady state equilibrium, the nominal rate of interest does not simply exceed the real natural rate by the rate of inflation. (See Figure 12.) The real ex-ante rate of interest in equilibrium is altered to:

$$(16)++ \quad r_t^e = \frac{h - c - a}{b - k} .$$

Similar results can be obtained for the ex-post real rate. On the path to equili-

Figure 12

Effect of a Continuous Increment of Real Balances in the Wicksellian Model Embodying Symmetric Price Expectations



brium, the nominal rate will exceed the old real rate by the expected rate of inflation only by accident.

The interesting aspect of this model is that it demonstrates that adding "Fisherian wrinkles" to Wicksell, contrary to Friedman's assertions, does not suggest that the authorities can only temporarily affect the real rate of interest.[18]

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[18] Friedman, *op. cit.*, p. 101.

chapter five

EMPIRICAL TESTS OF THE REACTIONS OF BORROWERS AND LENDERS TO INFLATION

Why do interest rates rise during an inflationary period? It is commonly alleged that lenders and borrowers react to inflation in such a way as to force up interest rates.

> "The basic reason why interest rates have risen so high and bond prices have fallen so low is the belief of investors and borrowers that the value of money will continue to decline significantly."[1]

Despite this allegation, and the basing of policy decisions upon it, the merit of this proposal has received little attention in the empirical literature. The empirical work in this chapter is designed to provide some preliminary evidence on the behavior of lenders and borrowers under inflationary conditions.

In chapter four, it was noted that the Fisher-Friedman asymmetric expectations hypothesis suggests that borrowers adapt to inflation at a faster rate than do lenders. As a consequence, the real ex-ante rate to borrowers falls, and the volume of real credit rises in the short run. A second purpose of the work in this chapter is to provide some preliminary empirical evidence on the Fisher-Friedman asymmetric expectations hypothesis. The problems of transfer effects, and forced frugality are ignored.

Interest rates in our model are assumed to be determined by the demand and supply of debt, rather than by liquidity preference. Inflation is assumed to be exogenous to the market for claims. Typical real sector variables such as income, savings, investment, and wealth are assumed to be exogenous to the financial

[1] *Bank of Canada: Annual Report 1968*, Ottawa, 1968, p. 7.

sector. This assumption appears reasonable since most economic models, to date, suggest that to the extent the financial sector has an impact on the real sector, these impacts occur with substantial lags. The use of quarterly data makes this assumption more viable. Due to various data limitations, quarterly data over the period 1962-70 (inclusive) were employed.

The purpose of the model to be presented must be kept in mind—that is, to test the effect of inflation on borrower's and lender's behavior. That means that we are interested primarily in such relationships as the demand for loans, the demand for various financial assets and so forth. We do not attempt to specify and estimate an entire financial sub-model. The behavior of financial institutions is not considered. Instead, we focus entirely on ultimate borrowers and ultimate lenders. In the present context this appears reasonable, since the upward pressure on interest rates due to inflation, is hypothesized to result from the reactions of primary borrowers and lenders. In the case of financial institutions, both sides of the balance sheet are in nominal terms and thus, at least to a first order of approximation, we can assume that their behavior is passive.

Nominal yields of financial claims are determined so as to clear the markets for financial claims. It is assumed that holdings of claims can be adjusted relatively easily and quickly, with most of the adjustment occurring within one quarter. Thus we do not utilize the stock adjustment model in describing portfolio behavior.

GENERALIZED PRIMARY BORROWER AND LENDER EQUATIONS

Attention is now turned to the derivation of the demand and supply equations which are used in the model. The case of ultimate lenders is considered first. This involves the specification of demand functions for various financial assets. Assume that there exist N financial assets among which individuals distribute their wealth. The demand for any financial asset S^j is asserted to depend upon a vector of rates of interest available to ultimate lenders on various financial assets, the expected rate of inflation (${}^s\pi^e$), and income or wealth, or both.

The demand function for the j^{th} security can be written as:

$$(1) \quad S^j = S^j(i_1 \ldots i_j \ldots i_N, {}^s\pi^e, W)\ j = 1 \ldots N\,,$$

where N of the above equations exist. *A priori* the following characteristics on the signs of partial derivatives are anticipated.

$$(2) \quad \frac{\partial S^j}{\partial i_j} > 0$$

$$(3) \quad \frac{\partial S^j}{\partial i_k} \leq 0 \quad \text{for } k \neq j$$

$$(4) \quad \frac{\partial S^j}{\partial {}^s\pi^e} \leq 0$$

$$(5) \quad \frac{\partial S^j}{\partial W} \geq 0.$$

Equation (2) says that a rise in the own rate on a security, *ceteris paribus*, increases the demand for that security. Condition (3) indicates that a rise in the rate on an alternative security reduces the demand for the particular security in question. A *ceteris paribus* rise in the rate of expected inflation decreases the demand for any given financial security. Condition (5) implies that there are no inferior securities.

Primary borrower behavior may be described by a similar set of equations. The demand function for the j^{th} form of loan finance can be described by the following demand function:

$$(6) \quad d^j = d^j(\rho_1 \ldots \rho_j \ldots \rho_N, {}^d\pi^e, x), j = 1 \ldots N$$

where $<\rho>$ represents a vector of borrowing rates, ${}^d\pi^e$ is the rate of inflation expected by borrowers, and, x represents a vector of other variables (such as total indebtedness) which may affect the demand for funds.

A priori we anticipate the following signs on the partial derivatives:

$$(7) \quad \frac{\partial d^j}{\partial \rho_j} > 0$$

$$(8) \quad \frac{\partial d^j}{\partial \rho_k} \geq 0 \qquad k \neq j$$

$$(9) \quad \frac{\partial d^j}{\partial\, {}^d\pi^e} \geq 0$$

If x represents total indebtedness, we expect that:

$$(10) \quad \frac{\partial d^j}{\partial x} \geq 0.$$

If all finance is direct or if pure competition reigns in the intermediary business, we have:

$$(11) \quad i_j \approx \rho_j \qquad \text{for } j = 1 \ldots N.$$

Thus, the model would determine N quantities, and N nominal rates of interest.

The nominal rate of interest in any sub-market is assumed to adjust (even in the short run) until that market is cleared. At any moment in time, however, the real rate expected in that market may differ for lenders and borrowers. The conditions under which this may be so were dealt with at length in chapter four.

If financial institutions are not characterized by perfect competition, the rates on the two sides of its balance sheet (the IN rate "i_j" and the OUT rate "ρ_j") may be linked by a simple condition such as:

$$(12) \quad \rho_j = ki_j \quad \text{where } k = 1 + \theta \text{ and } \theta \text{ is a markup factor.}$$

A variety of other possibilities could be used to link rates paid by borrowers to rates received by lenders.

There is no agreed-upon wealth constraint for ultimate borrowers, as opposed to final lenders. It seems reasonable, however, to use total debt as the analogue to the wealth constraint. In addition, a number of other real sector variables may enter the demand for credit functions. These are subsumed in the vector x, and may include such variables as investment, value of the capital stock, and so forth.

We do not require the estimated equations to obey portfolio consistency constraints. Data limitations do not allow us to estimate demand equations for all items in the portfolio. It is not clear that imposing portfolio consistency constraints, when we estimate a reduced number of equations, is more desirable than not imposing them. Thus, we take the pragmatic position that those financial assets for which we do not estimate demand equations are the buffer items in the portfolio.

EMPIRICAL REPRESENTATION OF PRICE EXPECTATIONS

A central difficulty in examining the reactions of lenders and borrowers to inflationary expectations is that expectations are not directly observable. Consequently, indirect methods of measuring price expectations are adopted. The procedure usually adopted is to assume that expectations of inflation are based on past inflationary experience. This procedure has a number of shortcomings which were discussed in chapter two. In this chapter, two different techniques of representing the dependence of inflationary expectations on past behavior are employed. These techniques and the implications of using them in an effort to test the Fisherian hypothesis and the Fisher-Friedman asymmetric expectations hypothesis are discussed below.

For demonstration purposes let us consider the following asset demand equation:

(13) $S_t = a + br^e_t + f(x)$

S_t represents the real amount of a particular asset demanded (supply of credit), r^e is the real rate of interest portfolio holders expect to receive if they hold asset S, and f(x) summarizes a variety of other influences on the demand for S. A simplified form of equation (13) appears as:

(14) $S_t = a + br^e_t$.

The real rate of interest which holders of financial assets expect to be paid is defined as the nominal rate minus the rate of inflation expected by primary lenders. The demand function for S can then be written as:

(15) $S_t = a + b(i_t - c\pi^e_t)$, or

(16) $S_t = a + bi_t - d\pi^e_t$.

This allows a test of the Fisher Hypothesis since an estimate of "c" can be obtained by dividing the coefficient of the expected rate of inflation by the coefficient of the interest rate. We do not perform hypothesis tests on "c" directly since the required distribution theory is complicated. Instead, we test the Fisher Hypothesis by comparing the coefficient of the interest rate variable to the coefficient of the expected rate of price change variable.[3]

We now consider the effect of using different lag procedures, in terms of testing the Fisher Hypothesis, and in terms of testing the Fisher-Friedman asymmetric expectations hypothesis.

Assuming that inflationary expectations are based on past values of the rate of inflation, and using equation (16), we have:

$$(17)\quad S_t = a + bi_t - bc \sum_{i=0}^{n} \alpha_i \pi^a_{t-i} ,$$

where π^a is the actual rate of inflation. The actual equation estimated, however, is of the form:

$$(18)\quad S_t = a + bi_t - \sum_{i=o}^{n} w_i \pi^a_{t-i} .$$

If the sum of the weights ($\Sigma\, w_i$) is not equal to zero, an estimate of "c" may be obtained by dividing the sum of the weights by the coefficient of the interest rate variable (i.e., $c = \frac{\Sigma\, w_i}{b}$, assuming $\Sigma\alpha : I\ 1$). This allows us to test the Fisher Hypothesis (i.e., $c = 1$). The distributed lag may be estimated using the Almon lag technique. An explicit theory of price expectations need not be adopted using this model, other than the notion that forecast values of inflation depend upon what has happened in the past. As it turned out, this procedure produced poor results and was not used beyond some initial exploratory work.

Alternatively, a geometric lag structure can be imposed. If it is assumed that price expectations are formed using the adaptive expectations formula, we have,

$$(19)\quad \pi^e_t = \pi^e_{t-1} + \lambda(\pi^a_t - \pi^e_{t-1}),\ 0 < \lambda < 1,$$

where the coefficient of adaptive expectations is λ. Equation (19) can be alternatively expressed as:

$$(20)\quad \pi^e_t = \sum_{i=0}^{\infty} (1 - \lambda)^i\, \lambda\pi^a_{t-1} .$$

Substituting (20) into (15) gives:

$$(21)\quad S_t = a + bi_t - bc\sum_{i=0}^{\infty} (1 - \lambda)^i\, \lambda\pi^a_{t-1} .$$

The procedure adopted is to iterate for λ over the interval (0,1). A value of unity for c would be consistent with the Fisher hypothesis. A value for the coefficient of adaptive expectations (λ) which is larger for borrowers than for lenders is consistent with the Fisher-Friedman Asymmetric expectations hypothesis.

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[3] Thus, if $c = 1$, we have $(d - b) = 0$. If b and d have signs consistent with *a priori* theory, then, $(d - b) > 0$, if $c > 1$, while $(d - b) < 0$ if $1 > c > 0$.

We may force the sum of the weights to unity by dividing each weight by the sum. Using this procedure, we obtain:

$$(22) \quad \pi_t^e = \sum_{i=0}^{n} \left[\frac{(1-\lambda)^i \lambda}{\Sigma(1-\lambda)^i \lambda} \right] \pi_{t-i}^a .$$

In practice, this latter approach was adopted.[4]

A third possibility is to use a Fisher-Arithmetic lag. The expected rate of inflation may be written as:

$$(23) \quad \pi_t^e = \sum_{i=0}^{n} \frac{n-i}{\frac{n(n+1)}{2}} \pi_{t-i}^a .$$

Using equation (23), the sum of the weights is constrained to sum to unity. In the case of the geometric lag, the end point of the lag was fixed at (t−48) and the common ratio of the geometric progression was varied. Using the arithmetic lag, however, the common difference of the arithmetic progression was set at unity while the end point of the lag was varied to obtain different lag structures. With the arithmetic lag procedure, the finding of longer average lags for lenders rather than borrowers would be consistent with the asymmetric expectations hypothesis.

THE DATA

The existing state of financial data in Canada does not permit the estimation of a detailed portfolio balance model for primary savers and borrowers. There are several difficulties. One very obvious point is that financial data are not collected with the behavior of ultimate lenders and borrowers in view. Data on the holdings of financial assets by ultimate lenders are far from complete. No adequate measure of financial wealth is available to use as a constraint in the primary lender equations. Consequently, real GNP was utilized.[5] Another problem is that interest rate data for assets held by ultimate lenders lacks completeness. An obvious omission is the absence of a yield-to-maturity series for Canadian Savings Bonds.

A possible way out of the data difficulties on financial quantities for ultimate lenders is to recast the model in first difference form and utilize data from the Financial Flow Accounts. This procedure was not adopted for three reasons.

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[4] An alternative procedure consistent with the use of the geometric lag structure is to estimate λ directly using a Koyck transformation. Suppose our asset demand equation appears as:

(a) $S_t = a + br_t^e + dW_t$, where the expected real rate is defined as:

(b) $r_t^e = (i_t - c\pi_t^e)$. Equation (c) describes the formation of price expectations:

(c) $\pi_t^e = \pi_{t-1}^e + \lambda(\pi_t^a - \pi_{t-1}^e)$. Employing the Koyck transformation, we obtain:

(d) $S_t = a\lambda + (1-\lambda)S_{t-1} + bi_t - b(1-\lambda)i_{t-1} + dW_t + d(1-\lambda)W_{t-1} - bc\lambda\pi_t$.

Using equation (d), we obtain seven coefficient estimates for five separate parameters. Consistent estimates of the parameters may be obtained using a non-linear regression program. The advantage of this approach is it allows an explicit test of the asymmetric expectations hypothesis. This procedure was not followed, however, for the compelling reason that the available non-linear regression program did not appear to be working properly.

[5] Permanent income was tried as a proxy for wealth. Since poorer fits were obtained, GNP was retained.

First, the problem of a lack of information on relevant rates of return remained. Second, the Financial Flow Accounts are subject to constant and sizable revisions, consequently, an "errors in variables" problem of unknown magnitude would exist.[6] Thirdly, exploratory econometric work using the Financial Flow Accounts proved unsatisfactory.

Data problems of a similar nature arise on the borrower side. This is particularly true with regard to rates of interest paid by primary borrowers in the household sector. In addition, there is some difficulty in obtaining satisfactory measures of total financial indebtedness for various categories of final borrowers. Total financial liabilities of Industrial Corporations in Canada were used as measure of business sector financial debt.[7] In the case of the household sector, total consumer credit outstanding was used as a measure of household financial indebtedness. This differs from total household indebtedness by household sector mortgage debt outstanding.

The primary saver equations presented in this chapter include total personal saving deposits at Chartered Banks, non-chequable deposits at Trust and Mortgage Loan Companies, and investment certificates and term deposits at Trust and Mortgage Loan Companies. A dearth of data did not permit a finer breakdown, nor did it permit the estimation of demand functions for some financial assets which are obviously held by ultimate leaders such as Canada Savings Bonds. A number of other primary saver equations, based on Financial Flow Accounts data were estimated, but the results were of dubious quality and are not reproduced here.

A special interest rate series was constructed for personal saving deposits at Chartered Banks. With the change in the Bank Act in May of 1967, the Chartered Bank offered three categories of personal saving deposits. The procedure adopted was to weight the interest rate payable in each category of deposit by the portion of saving deposits in that category. In the case of personal certificates of deposit, no interest rate series is available, either in published or unpublished form. In this case, the rate of interest payable on Non-personal Term and Notice Deposits (Corporate C.D.s) was used.

The equations presented include only the own rate of interest on the financial claim in question. Substitution effects among financial assets or among different financial liabilities are ignored.

The primary borrower equations estimated include Corporate Bonds Outstanding Consumer Loans at Sales Finance and Loan Companies; personal loans at Chartered Banks and business loans at Chartered Banks. Often the appropriate interest rate series did not exist, so a proxy rate had to be used.

Quarterly data on Corporate Bonds Outstanding are not published. There are, however, published data on year-end stocks of bonds outstanding and quarterly data on net new issues. Combining these two sources of information, it is possible to construct a quarterly series for stocks.

[6] The main problem here is that the household sector accounts are obtained as a residual and are probably the poorest quality data in the accounts.

[7] D.B.S. Cat. No. 61-003.

A list of definitions of the variables used follows.

List of Symbols Used

S_1 = Personal saving deposits at Chartered Banks.
S_2 = Non-chequable deposits in Trust and Mortgage Loan Companies.
S_3 = Term deposits and investment Certificates in Trust and Mortgage Loan Companies.
i_{s1} = Weighted interest rate on personal saving deposits.
i_{s2} = Rate on chequable deposits in Trust Companies.
i_{s3} = Rate on five-year investment Certificates in Trust Companies.
y = GNP in constant dollars.
π^e = Weighted average of past rates of inflation.
d_1 = Total Corporate Bonds Outstanding (Canadian $ pay plus U.S. pay).
d_2 = Consumer loans at Sales Finance and Consumer Loan Companies.
d_3 = Business loans at Chartered Banks.
d_4 = Personal loans at Chartered Banks.
i_{d1} = McLeod-Young-Weir Industrial Bond Yield.
i_{d2} = 90-day Finance Company paper rate.
$i_{d'2}$ = Conventional Mortgage Rate.
i_{d3} = Prime rate at Chartered Banks.
D_1 = Financial Liabilities of Industrial Corporations in Canada.
D_2 = Total Consumer Credit Outstanding.

The primary saver equations estimated are of the form,

$$(24) \quad S_j = a + bi_{sj} - d\pi^e + ey + u,$$

where S_j is the financial asset in question expressed in real terms, i_{sj} is the corresponding nominal rate of interest, π^e is the rate of change of prices expected (scaled by -100) and y is Gross National Product in constant dollars. *A priori* the following signs are expected for the coefficients $b > 0, d > 0, e > 0, c = 1$.

The primary borrower equations are of the form,

$$(25) \quad d_j = a + bi_{dj} - d\pi^e - eD_k + u,$$

where d_j is a financial liability expressed in real terms, i_{dj} is the relevant nominal rate of interest, π^e is the expected rate of price change (scaled by -100), and D_k is the relevant total indebtedness variable. *A priori* we expect the following pattern of signs, $b < 0, d < 0, e > 0, c = 1$.

The bracketed term under the coefficients are t statistics. The parameter c is obtained by dividing the coefficient of the inflation term by the coefficient of the interest rate term. The t statistic appearing under the computed value of c represents a test of the hypothesis $c = 1$.[8]

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[8] The hypothesis tested is $(b - d) = 0$. The value for t is obtained from

$$t = \frac{(b - d)}{\hat{S}\sqrt{Z^T(X^TX)^{-1}Z}}$$

with $n - k$ degrees of freedom, where $Z = (0, 1, -1, 0)$. At the 0.05 level of significance using a one-tailed test, the critical value for t is 1.694.

The primary lender and borrower equations were initially estimated using Ordinary Least Squares (OLS), and employing both arithmetic and geometric progressions to represent the lag structure. Since interest rates are not exogenous, estimation using Ordinary Least Squares is not appropriate. The equations were then re-estimated using Two-Stage Least Squares (TSLS).[9]

EMPIRICAL RESULTS: PRIMARY LENDERS

Results for primary lenders, using Ordinary Least Squares and Fisher-Arithmetic lags are contained in Table XIII-(a) through(c). The column entitled "length of lag" indicates how many lagged inflation terms are included in the regression. For example, a lag length of zero includes only the contemporaneous value of the rate of price change, while a lag length of five indicates that a total of six values of the rate of price change was included.

The results for personal saving deposits are recorded in Table XIII-(a). The overall fit of this equation is quite good. There appears to be a slight problem of positive autocorrelation, however. The interest rate coefficients are of the correct sign and are uniformly significant.[10] The coefficient of the expected rate of inflation has the correct sign for all lags except the longest lag run (48 quarters). Except for the 48 lag case, the expected rate of inflation produced a coefficient which was significantly different from zero or nearly so. In all cases the Fisher Hypothesis that $c = 1$ was rejected by the data.

Unfortunately, the results do not provide any strong reason to choose one lag length over another. Changing the lag length had little bearing on the fit of the equation. The inflation term reached maximum significance with a total length of five quarters. On the other hand, the Fisher Hypothesis came closest to not being rejected with a lag length of 23 quarters.

The non-chequable saving deposit equation produces a good overall fit. The degree of autocorrelation appears to be severe, however. All of the coefficients have signs which are consistent with the postulated theory. In general, however, neither the coefficients for the interest rate nor the coefficients for the expected inflation term are significant. In the case of 48 lags, a number of the coefficients reversed sign, and the result is not recorded in Table XIII-(b). The point estimate of c was low, but increased as the length of the lag was increased.

The data do not permit a rejection of the hypothesis that $c = 1$. It should be pointed out, however, that on the one hand, the values of c are closer to zero than to unity, and on the other hand, the standard error of $(b-d)$ is sufficiently large that the data would not reject the hypothesis $c = 0$, either. Again, the existence of strong information on the length of the lag is absent. If we chose the

[9] The additional instrumental variables used in the first stage are variables which can reasonably be taken to be exogenous to the financial sector. The variables used were autonomous expenditures (Federal Government expenditures, exports, investment in plant and equipment), consumer expenditures on automobiles and other durable goods, Bank Rate, the change in the monetary base, and total housing starts. All variables were expressed in real terms.

[10] Critical t for a two-tailed test at the 0.05 level of significance is $\pm$ 2.038.

lag length on the basis of the equation which produces a value of c closest to unity, the lag length would be 23 quarters.[11]

The results for term deposits and investment certificates in Trust and Mortgage Loan Companies are reported in Table XIII-(c). These equations had a good overall fit, but positive autocorrelation was indicated. The coefficient of the expected rate of inflation was constantly significantly different from zero. The coefficient of the interest rate variable was significantly different from zero in the case of a 48 quarter lag. Part of the difficulty lies in the fact that term deposits and investment certificates in Trust and Mortgage Loan Companies consist of several different types of financial claims,[12] and consequently the interest rate variable is not entirely appropriate. A number of other alternatives were tried, but did not alter the results in any substantive way. The estimated values for c tend to be excessively large, usually exceeding unity. If we chose the lag length on the basis of the value of c closest to unity a lag of one-quarter would be chosen.

Tables XV-(a) to (c) contain results for primary lenders using ordinary least squares and geometric lags.

The results for personal saving deposits contained in Table XV-(a) are broadly similar to those found in Table XIII-(a). All the coefficients have correct signs. The coefficient for the expected rate of inflation is never significantly different from zero. The Fisher Hypothesis is uniformly rejected. A decay parameter of 0.3 produced a value for c which was closest to unity. The point estimates of c tended to be somewhat less than those obtained using arithmetic lags.

The results produced in Table XV-(b) again are similar to those found in Table XIII-(b). The coefficients generally have the correct signs. The coefficient of the interest rate variable was never significantly different from zero. The point estimates of c ranged from 0.1120 with $\lambda = 0.9$ to 0.5013 with $\lambda = 0.2$. Inadmissable results were obtained by setting $\lambda = 0.1$. The confidence interval for c was extremely wide.

The results in Table XIII-(c) and XV-(c) are generally similar. This equation tends to produce excessive values for c. The standard error for (b—d) is large, however. Values for c ranged from 0.65 with $\lambda = 0.9$, to 2.61 with $\lambda = 0.2$.

Table XVIII contains sample results for primary lenders using two-stage-least-squares and arithmetic lags. Only the values for zero, five and 23 lags are included.

In the case of personal saving deposits at Chartered Banks (equations 1, 2, 3, in Table XVIII) the primary effect of using two-stage-least-squares was to reduce the significance of the expected rate of inflation term, and to produce a lower point estimate for c.

In the case of S_2, the two-stage regression procedure raised the t values for both the interest rate variable and the inflation variable. The point estimates of c were raised, but the standard error was lowered. Of the three results presented in Table XVIII, the Fisher Hypothesis was rejected in one case, and came close

[11] Constraining the parameter c to equal unity (assuming that borrowers and lenders eventually fully adapt to the rate of inflation) still left the summary statistics relatively insensitive to the lag structure used. These results are not reported.

[12] A more detailed breakdown of deposits in Trust and Mortgage Loan Companies exists only for the period following 1966 IV.

to rejection, in another case. The two-stage results for S_3 were generally unsatisfactory. The coefficient of the interest rate variable had the wrong sign in two cases out of three, and was insignificant in all three regressions.

Two-stage regression results for primary savers using geometric lags are recorded in Table XX. These results parallel those found in Table XVIII.

EMPIRICAL RESULTS: PRIMARY BORROWERS

Primary borrower equations employing Fisher-Arithmetic lags and ordinary least squares are contained in Tables XIV-(a) to (c). Only results for consumer loans at Sales Finance and Loan Companies, and Corporate Bond Level equations are presented. Neither the business loan nor personal loan equations at Chartered Banks are reported.[13]

The estimates for the corporate level equations produced coefficients which were all in accord with the postulated theory. The coefficient of the expected inflation term was never significantly different from zero, however. The Fisher Hypothesis was rejected by the data for all but the 23-quarter lag. There are no reasonable grounds for picking one lag length as opposed to any other on the basis of the statistical results. Using the criterion of the equation producing a value of c closest to unity, one would choose the 23-quarter lag.[14]

There is no quarterly interest rate series, of any kind, for consumer loans. In Table XIV-(b), an interest rate series from the other side of the balance sheet of sales finance companies is used. This choice can be rationalized using the markup hypothesis discussed at an earlier point in this chapter.

The results for consumer loans at Sales Finance and Loan Companies are recorded in Table XIV-(b). All of the coefficients exhibited signs which were consistent with *a priori* theory. The values of c tended to increase as the lag lengthened. For lags in excess of 3 quarters, we were unable to reject the Fisher Hypothesis. A lag length of 11 quarters produced the value of c which was closest to unity.

The only difference between the equations presented in Table XIV-(b) and XIV-(c) is that the conventional mortgage rate was used in place of the 90-day finance paper rate. This alteration has the effect of raising the coefficient of determination and the Durbin-Watson statistic. With lags exceeding 11 quarters in length, the $\overline{R}^2$ and the Durbin-Watson statistic decline markedly. Only in one case are we unable to reject the Fisher Hypothesis. Using the criterion of a value of c closest to unity, a lag with a length of 23 quarters would be chosen.

Tables XVI-(a) through (c) contain primary borrower equations using geometric lags. The Corporate Bond equations in Table XVI-(a) are similar to those found in Table XIV-(a). The Fisher Hypothesis was rejected for all values of λ except $\lambda = 0.1$. The highest point estimate for c was 0.1764 and was obtained with $\lambda = 0.1$.

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[13] The reason for this ommission will be discussed below.

[14] For lag lengths longer than 23 quarters, the value of c declined.

In Table XVI-(b), consumer loans using geometric lags are produced. Generally speaking, the coefficients were significant and had signs which were in accord with *a priori* theory. For values of $\lambda > 0.4$, the Fisher Hypothesis of $c = 1$ was rejected. The maximum value of c was obtained with $\lambda = 0.2$. Table XVI-(c) corresponds to Table XIV-(c) except geometric lags were used. Only representative results are reproduced in Table XVI-(c). The Fisher Hypothesis was rejected for all regressions run. The value of c was closest to unity when $\lambda = 0.1$. This equation, however, was inferior to all of the other equations in terms of the values for the coefficient of determination and the Durbin-Watson statistic.

Representative results for primary borrowers using two-stage-least-squares and arithmetic lags are found in Table XIX.

In the case of corporate bonds, we were unable to reject the Fisher equation for lags in excess of one quarter. A maximum value for c was obtained when the length of the lag was set at 23 quarters. In the case of Consumer Loans, the value of c was lowered by using two-stage-least-squares. We were unable to reject the Fisher Hypothesis for those equations in which the length of the lag was 11 quarters or greater. The value of c closest to unity was 0.7835 and was obtained using a 23-quarter lag. The Consumer Loan equations, using the conventional mortgage rate, are not recorded in Table XIX. In this case, however, the Fisher Hypothesis was rejected for all lags.

Table XXI parallels Table XIX, with the exception that geometric lags were used. The Fisher Hypothesis is rejected in all but one case.

In Table XVII, two-stage regression results using geometric lags for personal and business loans at Chartered Banks are recorded. Only the results for $\lambda = 0.5$ are reported. In all of the regressions run, the coefficients on the interest rate were of the wrong sign. This result was invariate to whether ordinary or two-stage-least-squares was used. It was also invariate to the type of length of the lag used. The uses of alternative interest rates produced coefficients with the same signs. The difficulty centres in part around the absence of suitable interest rate series. No series for the rate on personal loans at chartered banks exists. In addition, until recently, the only business loan rate available was prime rate. Until May 1967, prime rate could not exceed six per cent under the Bank Act. Through a variety of devices (such as compensating balances) the banks were able to charge effective rates in excess of six per cent.

On balance, these results more often conflict with the Fisher Hypothesis than support it. In the majority of cases, the Fisher Hypothesis was rejected. This result was not particularly sensitive to the type of estimation technique or to the type of lag structure used. There was, however, a tendency for the long-run impact of price expectations on borrower and lender behavior to increase with the average lag of the price expectational variable.

The results did not permit us to say anything concrete about the Asymmetric Expectations hypothesis on statistical grounds. If, however, we chose the length of the lag on the basis of the value of c closest to unity, there is no apparent support of the Asymmetric Expectations hypothesis.

TABLE XIII—(a)

Personal Saving Deposits in Chartered Banks

Constrained Arithmetic Progression (OLS).

$S_1 = a + bi_{s1} - d\pi^e + ey + u$

a	b	−d	e	c	Length of Lag Quarters	Average Lag	S.E.E.	D.W.	$\bar{R}^2$
0.1026 (2.9243)	0.0709 (9.3646)	0.0041 (1.8374)	0.1125×10^{-4} (10.4482)	0.0578 (8.0482)	0	0.00	0.0238	1.06	0.973
0.0924 (2.5578)	0.0700 (9.1966)	0.0060 (1.9641)	0.1161×10^{-4} (10.2147)	0.0853 (7.1910)	1	0.33	0.0236	1.13	0.974
0.0761 (2.0872)	0.0688 (9.3343)	0.0090 (2.5410)	0.1217×10^{-4} (10.5188)	0.1303 (6.6556)	2	0.67	0.0228	1.12	0.975
0.0589 (1.5992)	0.0679 (9.4341)	0.0117 (2.9304)	0.1271×10^{-4} (10.6445)	0.1723 (6.1086)	3	1.00	0.0222	1.16	0.977
0.0405 (1.0387)	0.0672 (9.5453)	0.0143 (3.2496)	0.1325×10^{-4} (10.7104)	0.2132 (5.6882)	4	1.33	0.0217	1.15	0.978
0.0261 (0.6236)	0.0669 (9.4652)	0.0160 (3.2535)	0.1363×10^{-4} (7.5678)	0.2390 (5.2041)	5	1.67	0.0217	1.19	0.978
−0.1350 (−1.0818)	0.0719 (9.7771)	0.0345 (2.0928)	0.1694×10^{-4} (5.2797)	0.4798 (1.969)	23	5.67	0.0235	1.09	0.974
0.1649 (0.7408)	0.0724 (6.9593)	−0.0089 (−0.2209)	0.0948×10^{-4} (1.9091)	−0.1229 (2.1339)	47	11.67	0.0250	1.03	0.970

TABLE XIII—(b)

Saving Deposits in Trust and Mortgage Loan Companies

Constrained Arithmetic Lag: (OLS)

$S_2 = a + bi_{s2} - d\pi^e + ey + u$

a	b	−d	e	c	Length of Lag	Average Lag	S.E.E.	D.W.	$\bar{R}^2$
−10.0334 (−2.2314)	1.0212 (0.6537)	0.0733 (1.0296)	0.2435×10^{-3} (7.1721)	0.0718 (0.6190)	0	0.00	0.6994	0.29	0.877
−11.7924 (−2.4853)	1.5651 (0.9638)	0.1478 (1.4613)	0.2410×10^{-3} (7.2494)	0.0944 (0.9054)	1	0.33	0.6883	0.28	0.880
−12.5629 (−2.5824)	1.7506 (1.0693)	0.1960 (1.6048)	0.2447×10^{-3} (7.5304)	0.1120 (0.9886)	2	0.67	0.6839	0.27	0.882
−13.0758 (−2.6209)	1.8414 (1.1170)	0.2364 (1.6583)	0.2499×10^{-3} (7.7848)	0.1284 (0.0210)	3	1.00	0.6822	0.27	0.883
−13.9842 (−2.7201)	1.9540 (1.1943)	0.3184 (1.7955)	0.2632×10^{-3} (8.1110)	0.1629 (0.0603)	5	1.67	0.6776	0.26	0.885
−14.6637 (−2.5019)	1.7130 (1.0420)	0.4550 (1.5512)	0.2996×10^{-3} (6.7402)	0.2656 (0.8386)	11	3.67	0.6856	0.24	0.882
−22.3414 (−3.6941)	1.8616 (1.3665)	1.4126 (2.9680)	0.4714×10^{-3} (5.9232)	0.7388 (0.3580)	23	5.67	0.6296	0.41	0.900

TABLE XIII—(c)

Term Deposits and Investment Certificates in Trust and Mortgage Loan Companies

Lag Form: Constrained Arithmetic Progression (OLS)

$S_3 = a + bi_{s3} - d\pi^e + ey + u.$

a	b	−d	e	Implied C	Average Lag (quarters)	S.E.E.	D.W.	$\bar{R}^2$
−39.662 (−29.056)	0.395 (1.077)	0.237 (2.807)	0.0013 (20.987)	0.600 (0.405)	0.00	0.917	1.02	0.992
−40.346 (29.814)	0.317 (0.892)	0.375 (3.361)	0.0014 (21.586)	1.183 (−0.147)	0.33	0.880	1.05	0.992
−40.999 (−29.776)	0.283 (0.815)	0.480 (3.663)	0.0014 (21.800)	1.696 (−0.495)	0.67	0.859	1.05	0.993
−41.690 (−29.271)	0.258 (0.753)	0.580 (3.880)	0.0014 (21.803)	2.248 (−0.795)	1.00	0.843	1.04	0.993
−42.394 (−28.161)	0.230 (0.674)	0.667 (3.963)	0.0014 (21.430)	2.900 (−1.047)	1.33	0.838	1.03	0.993
−43.054 (−26.564)	0.205 (0.595)	0.738 (3.912)	0.0014 (20.700)	3.600 (−1.222)	1.67	0.842	1.04	0.993
−47.482 (−18.373)	0.082 (0.227)	1.225 (3.777)	0.0016 (16.629)	14.939 (−2.024)	3.67	0.851	1.02	0.993
−55.345 (−12.334)	0.440 (1.308)	2.264 (3.808)	0.0017 (13.651)	5.146 (−2.558)	5.67	0.849	1.120	0.993
−57.553 (−6.572)	1.358 (2.591)	3.431 (2.155)	0.0016 (9.201)	2.527 (−1.614)	15.67	0.956	0.878	0.990

TABLE XIV–(a)

Primary Borrowers: Corporate Bond Borrowing

Constrained Arithmetic Lag (OLS)

$$d_1 = a + bi_{d1} - d\pi^e + eD_1 + u$$

a	b	d	e	c	Length of lag	Average lag	S.E.E.	D.W.	$\bar{R}^2$
−18.3363 (−5.3064)	−2.2289 (−3.3816)	−0.0102 (−0.0669)	0.4355 (16.8834)	0.0046 (−3.9457)	0	0.00	1.6545	1.13	0.985
−18.0405 (−5.1080)	−2.2029 (−3.3371)	−0.0680 (0.3252)	0.4333 (16.4788)	0.0387 (−2.9651)	1	0.33	1.6519	1.11	0.985
−17.7772 (−4.6799)	−2.1942 (−3.3081)	−0.1007 (−0.3433)	0.4319 (15.6341)	0.0458 (−2.7150)	3	1.00	1.6516	1.12	0.985
−17.6471 (−4.1706)	−2.1924 (−3.2743)	−0.1061 (−0.2850)	0.4314 (14.5573)	0.0484 (−2.4998)	5	1.67	1.6525	1.12	0.985
−17.5194 (−2.8230)	−2.2004 (−3.2086)	−0.1038 (−0.1638)	0.4312 (11.3862)	0.0472 (−1.9687)	11	3.67	1.6539	1.12	0.985
−10.4643 (−0.9878)	−2.2129 (−3.4137)	−0.9915 (−0.7869)	0.4009 (7.8645)	0.4480 (−0.8476)	23	5.67	1.6389	1.04	0.985

TABLE XIV–(b)

Primary Borrowers: Consumer Loans at Consumer Loan and Sales Finance Companies

Constrained Arithmetic Lag (OLS)

$$d_2 = a + bi_{d2} - d\pi^e + eD_2 + u$$

a	b	d	e	c	Length of lag	Average lag	S.E.E.	D.W.	$\bar{R}^2$
7.4388 (12.7856)	−0.9208 (−4.1072)	−0.1749 (−2.9329)	0.1176 (4.5146)	0.1899 (−3.1354)	0	0.00	0.6505	0.76	0.503
7.6376 (13.8222)	−0.8797 (−4.1525)	−0.2863 (−3.7233)	0.1043 (4.1318)	0.3254 (−2.5229)	1	0.33	0.6121	0.76	0.560
8.1753 (15.1248)	−0.8480 (−4.2934)	−0.4548 (−4.5865)	0.0829 (3.3271)	0.5363 (−1.6775)	3	1.00	0.5692	0.73	0.619
8.7473 (15.1299)	−0.8281 (−4.2680)	−0.5809 (−4.8227)	0.0646 (2.4662)	0.7015 (−1.0077)	5	1.67	0.5576	0.69	0.635
10.5136 (12.6498)	−0.8378 (−4.3136)	−0.9072 (−4.7923)	0.0199 (0.6140)	1.0828 (0.2373)	11	3.67	0.5591	0.73	0.633
12.0225 (7.0231)	−1.0312 (−4.6096)	−1.1220 (−2.9154)	0.0093 (0.1851)	1.0880 (0.2093)	23	5.67	0.6510	0.66	0.502

TABLE XIV–(c)

Consumer Loans at Sales Finance and Consumer Loans Companies

$d_2 = a + bi'_{d2} - d\pi^e + e\,D_2 + u$

Arithmetic Progression Lag (OLS)

a	b	d	e	c	Length of lag	Average lag	S.E.E.	D.W.	$\bar{R}^2$
10.6097 (19.2234)	−1.3050 (−7.3536)	−0.0901 (−1.9113)	0.1696 (7.7552)	0.0690 (−6.1483)	0	0.00	0.4901	0.96	0.718
10.8883 (23.5190)	−1.1980 (−7.8732)	−0.3171 (−4.1563)	0.1371 (6.6101)	0.2647 (−4.5880)	3	1.00	0.4169	1.05	0.796
11.3026 (25.0939)	−1.1856 (−8.2613)	−0.4267 (−4.7956)	0.1224 (5.8511)	0.3599 (−3.9486)	5	1.67	0.3946	1.03	0.817
12.4375 (20.3944)	−1.1801 (−7.6682)	−0.6206 (−4.1434)	0.0933 (3.3023)	0.5259 (−2.2230)	11	3.67	0.4174	0.92	0.795
14.0206 (11.4034)	−1.3701 (−8.6236)	−0.7879 (−2.8869)	0.0906 (2.3721)	0.5751 (−1.7728)	23	5.67	0.4608	0.82	0.750
13.2789 (4.6436)	−1.5303 (−6.9645)	−0.6263 (−0.8871)	0.1420 (2.6004)	0.4093 (−4.3441)	47	11.67	0.5111	0.71	0.693

TABLE XV–(a)

Personal Saving Deposits in Chartered Banks

Constrained Geometric Progression (OLS)

$$S_1 = a + bi_{s1} - d\pi^e + ey + u$$

a	b	−d	e	c	λ	S.E.E.	D.W.	$\bar{R}^2$
0.1068 (2.8832)	0.0725 (9.2653)	0.0025 (0.9179)	0.1098×10^{-4} (9.5930)	0.0345 (8.7500)	0.9	0.0247	1.14	0.971
0.1041 (2.784)	0.0723 (9.2483)	0.0032 (1.0265)	0.1109×10^{-4} (9.5300)	0.0443 (7.6778)	0.8	0.0246	1.15	0.971
0.1001 (2.6392)	0.0720 (9.2380)	0.0041 (1.1760)	0.1123×10^{-4} (9.4648)	0.0569 (7.3804)	0.7	0.0245	1.17	0.972
0.0939 (2.4265)	0.0716 (9.2393)	0.0053 (1.3646)	0.1145×10^{-4} (9.3856)	0.0740 (7.0532)	0.6	0.0243	1.19	0.972
0.0848 (2.1162)	0.0712 (9.2576)	0.0071 (1.5795)	0.1175×10^{-4} (9.2590)	0.0997 (6.6082)	0.5	0.02411	1.21	0.973
0.0714 (1.6775)	0.0708 (9.3017)	0.0095 (1.7872)	0.1215×10^{-4} (9.0091)	0.1342 (5.9515)	0.4	0.0239	1.23	0.973
0.0531 (1.1156)	0.0709 (9.4024)	0.0127 (1.9066)	0.1266×10^{-4} (8.4811)	0.1791 (6.3874)	0.3	0.0237	1.23	0.973
0.0358 (0.6167)	0.0724 (9.6649)	0.0163 (1.7205)	0.1305×10^{-4} (7.3817)	0.1754 (4.3828)	0.2	0.0239	1.18	0.973
0.0796 (1.0448)	0.0753 (9.2671)	0.0100 (0.5468)	0.1163×10^{-4} (5.2225)	0.1328 (23.3214)	0.1	0.0249	1.07	0.971

TABLE XV–(b)

Saving Deposits in Trust and Loan Companies

Constrained Geometric Progression (OLS)

$S_2 = a + bi_{s2} - d\pi^e + ey + u$

a	b	d	e	c	λ	S.E.E.	D.W.	$\bar{R}^2$
−10.2436 (−2.3933)	1.0408 (0.7052)	0.1157 (1.3914)	0.2485×10^{-3} (7.6299)	0.1120 (0.6390)	0.9	0.6903	0.29	0.880
−10.6345 (−2.4766)	1.1480 (0.7786)	0.1415 (1.5400)	0.2495×10^{-3} (7.7237)	0.1233 (0.6979)	0.8	0.6860	0.30	0.882
−11.1204 (−2.5780)	1.2748 (0.8659)	0.1749 (1.7067)	0.2513×10^{-3} (7.8502)	0.1372 (0.7662)	0.7	0.6806	0.31	0.884
−11.7316 (−2.7041)	1.4237 (0.9698)	0.2198 (1.8992)	0.2544×10^{-3} (8.0231)	0.1544 (0.8449)	0.6	0.6739	0.32	0.886
−12.5093 (−2.8651)	1.5935 (1.0923)	0.2829 (2.1293)	0.2602×10^{-3} (8.2570)	0.1776 (0.9307)	0.5	0.6653	0.33	0.889
−13.5100 (−3.0803)	1.7732 (1.2334)	0.3789 (2.4218)	0.2712×10^{-3} (8.5495)	0.2137 (1.0116)	0.4	0.6535	0.36	0.893
−14.8162 (−3.3980)	1.9190 (1.3827)	0.5446 (2.8441)	0.2939×10^{-3} (8.8321)	0.2838 (1.0399)	0.3	0.6351	0.39	0.898
−16.3451 (−3.9567)	1.8024 (1.4275)	0.9035 (3.6229)	0.3490×10^{-3} (8.9889)	0.5013 (0.7468)	0.2	0.5987	0.49	0.910
−14.7023 (−4.5453)	−0.0081 (−0.0078)	1.9102 (5.2580)	0.4935×10^{-3} (9.4862)	−1.9183 (−1.7149)	0.1	0.5207	0.75	0.932

TABLE XV–(c)

Term Deposits and Investment Certificate in Trust and Loan Companies

$S_3 = a + bi_{s3} - d\pi^e + ey + u$

Constrained Geometric Lag

a	b	−d	e	c	λ	S.E.E.	D.W.	$\bar{R}^2$
−39.8763 (−28.3571)	0.4221 (1.1380)	0.2751 (2.6071)	0.1331×10^{-2} (20.5391)	0.6517 (0.3664)	0.9	0.9293	0.99	0.991
−40.0783 (−28.6059)	0.4129 (1.1288)	0.3232 (2.8074)	0.1339×10^{-2} (20.7807)	0.7828 (0.2241)	0.8	0.9166	1.02	0.992
−40.3517 (−28.8252)	0.4036 (1.1212)	0.3833 (3.0291)	0.1349×10^{-2} (21.0427)	0.9497 (0.0505)	0.7	0.9021	1.06	0.992
−40.7316 (−28.9307)	0.3942 (1.1149)	0.4598 (3.2678)	0.1361×10^{-2} (21.2957)	1.1664 (−0.1639)	0.6	0.8860	1.09	0.992
−41.2748 (−28.7623)	0.3852 (1.1111)	0.5609 (3.5167)	0.1378×10^{-2} (21.4825)	1.4561 (−0.4355)	0.5	0.8690	1.12	0.992
−42.0862 (−28.0508)	0.3804 (1.1206)	0.7036 (3.7708)	0.1140×10^{-2} (21.5118)	1.8496 (−0.7862)	0.4	0.8514	1.16	0.993
−43.3679 (26.4178)	0.3987 (1.2036)	0.9311 (4.0346)	0.1436×10^{-2} (21.2719)	2.3353 (−1.2448)	0.3	0.8331	1.20	0.993
−45.5109 (−23.5299)	0.5300 (1.6521)	1.3814 (4.3167)	0.1482×10^{-2} (20.7220)	2.6064 (−1.6545)	0.2	0.8135	1.26	0.993
−48.3678 (−19.1474)	1.2022 (3.3857)	2.6632 (4.2442)	0.1508×10^{-2} (19.7091)	2.2153 (−2.5286)	0.1	0.8185	1.30	0.993

TABLE XVI–(a)

Primary Borrowers: Corporate Bonds Borrowing

Constrained Geometric Lag (OLS)

$$d_1 = a + bi_{d1} - d\pi^e + eD_1 + u$$

a	b	d	e	c	λ	S.E.E.	D.W.	$\bar{R}^2$
−17.9400 (−5.2592)	−2.2226 (−3.4254)	−0.1351 (−0.7318)	0.4328 (17.1007)	0.0609 (−3.0753)	0.9	1.6409	1.07	0.985
−17.8671 (5.2000)	−2.2204 (−3.4197)	−0.1450 (−0.7089)	0.4324 (16.9910)	0.0653 (−3.0227)	0.8	1.6418	1.07	0.985
−17.7794 (−5.1187)	−2.2186 (−3.4144)	−0.1565 (−0.6828)	0.4319 (16.8447)	0.0705 (−2.9590)	0.7	1.6427	1.08	0.985
−17.6707 (−5.0017)	−2.2171 (−3.4092)	−0.1698 (−0.6521)	0.4314 (16.6387)	0.0766 (−2.8819)	0.6	1.6437	1.08	0.985
−17.5381 (−4.8265)	−2.2163 (−3.4046)	−0.1848 (−0.6112)	0.4308 (16.3336)	0.0834 (−2.4456)	0.5	1.6450	1.09	0.985
−17.3911 (−4.5580)	−2.2178 (−3.4029)	−0.1999 (−0.5490)	0.4302 (15.8651)	0.0901 (−2.6507)	0.4	1.6469	1.09	0.985
−17.2705 (−4.1452)	−2.2259 (−3.4124)	−0.2112 (−0.4511)	0.4299 (15.1403)	0.0949 (−2.4760)	0.3	1.6494	1.10	0.985
−17.2618 (−3.5242)	−2.2499 (−3.4352)	−0.2181 (−0.3132)	0.4304 (14.0784)	0.0969 (−2.2106)	0.2	1.6521	1.10	0.985
−16.9104 (−2.5425)	−2.3544 (−2.9238)	−0.4223 (−0.2551)	0.4301 (12.7590)	0.1794 (−1.4300)	0.1	1.6529	1.10	0.985

TABLE XVI–(b)

Primary Borrowers Consumer Loans at Sales Finance and Loan Companies

Constrained Geometric Lag (OLS)

$d_2 = a + bi_{d2} - d\pi^e + eD_2 + u$

a	b	d	e	c	λ	S.E.E.	D.W.	$\bar{R}^2$
7.4920 (13.0196)	−0.9636 (−4.3757)	−0.2231 (−3.1114)	0.1179 (4.6006)	0.2315 (−3.1645)	0.9	0.6420	0.72	0.516
7.5641 (13.2422)	−0.9506 (−4.3622)	−0.2559 (−3.2605)	0.1136 (4.4378)	0.2692 (−2.9486)	0.8	0.6348	0.71	0.526
7.6610 (13.4917)	−0.9392 (−4.3611)	−0.2966 (−3.4246)	0.1086 (4.2402)	0.3158 (−2.7068)	0.7	0.6268	0.72	0.538
7.7946 (13.7456)	−0.9302 (−4.3749)	−0.3475 (−3.5936)	0.1027 (3.9917)	0.3736 (−2.4259)	0.6	0.6185	0.72	0.551
7.9828 (13.9345)	−0.9254 (−4.4027)	−0.4119 (−3.7407)	0.0955 (3.6661)	0.4451 (−2.0951)	0.5	0.6112	0.71	0.561
8.2508 (13.8834)	−0.9282 (−4.4413)	−0.4947 (−3.8058)	0.0868 (3.2312)	0.5330 (−1.7361)	0.4	0.6080	0.69	0.566
8.6217 (13.1888)	−0.9479 (−4.4859	−0.6006 (−3.6518)	0.0770 (2.6753)	0.6336 (−1.2647)	0.3	0.6156	0.67	0.555
8.9850 (11.0172)	−1.0043 (−4.4924)	−0.7054 (−2.9889)	0.0718 (2.1421)	0.7024 (−0.9152)	0.2	0.6521	0.60	0.500
7.6699 (6.6432)	−1.0148 (−3.8941)	−0.1793 (−0.3587)	0.1233 (2.8407)	0.1767 (−1.6828)	0.1	0.7313	0.45	0.372

TABLE XVI–(c)

Primary Borrower Consumer Loans at Sales Finance and Loan Companies

Constrained Geometric Lag

$$d_2 = a + bi'_{d2} - d\pi^e + eD_2 + u$$

a	b	d	e	c	λ	S.E.E.	D.W.	$\bar{R}^2$
10.6916 (20.4238)	−1.3173 (−7.9467)	−0.1391 (−2.5802)	0.1672 (8.1218)	0.1056 (−6.3549)	0.9	0.4707	0.94	0.740
11.2257 (23.7749)	−1.2914 (−8.8749)	−0.3794 (−4.1983)	0.1413 (7.0416)	0.2938 (−4.9032)	0.4	0.4155	1.01	0.797
11.6142 (23.5594)	−1.3168 (−9.2338)	−0.4812 (−4.3414)	0.1332 (6.4122)	0.3654 (−4.3139)	0.3	0.4104	0.98	0.802
12.2615 (20.7684)	−1.3961 (−9.6703)	−0.6364 (−4.0567)	0.1264 (5.5749)	0.4558 (−3.5139)	0.2	0.4204	0.91	0.792
13.1033 (13.5923)	−1.6208 (−9.2646)	−0.9488 (−2.8315)	0.1333 (4.9979)	0.5854 (−2.1954)	0.1	0.4626	0.80	0.748

Table XVII

Primary Borrowers. Constrained Geometric Lag*

$d_j = a + bi_{dj} - d\pi^e + eD_j + u$

TSLS
$\lambda = .5$

Dependent Variable	a	b**	−d	e***	c	S.E.E.	D.W.	$\bar{R}^2$	Method of Estimation
Personal loans at Chartered Banks d_3	−18.6475 (−3.9338)	3.4754 (2.1076)	−0.9173 (−1.3456)	0.3711 (2.3292)	−0.2694 (6.2272)	1.3714	2.06	0.821	TSLS
Business loans at Chartered Banks d_4	−18.4519 (−9.0735)	2.3708 (4.7512)	−0.6662 (−2.7086)	0.1848 (12.8025)	−0.2810 (5.0794)	1.3244	0.9672	0.981	TSLS

*Changing the lag form produced broadly similar results.

**Interest rate used was Chartered Bank Prime Rate. Using an interest from the other side of the bank's balance sheet such as the certificate of deposit rate produced similar results.

***D_j is equal to total Consumer credit outstanding for the first regression, and total Industrial financial liabilities for the second regression.

TABLE XVIII

Primary Savers: Constrained Arithmetic Lags (TSLS)

$$S_j = a + bi_{sj} - d\pi^e + ey + u$$

	Dependent Variable	a	b	−d	e	c	Length of Lag	Average Lag	S.E.E.	D.W.	$\bar{R}^2$
1	S_1	0.1345 (2.3802)	0.0839 (5.3960)	0.0032 (1.0170)	0.0970×10^{-4} (4.7809)	0.0381 (4.8036)	0	0.00	0.0333	0.92	0.948
2	S_1	0.0616 (0.8659)	0.0776 (5.1008)	0.0137 (1.829)	0.1209×10^{-4} (4.8301)	0.1765 (3.2602)	5	1.67	0.0314	0.87	0.953
3	S_1	−0.0778 (−0.4299)	0.0845 (5.7484)	0.0307 (1.3164)	0.1482×10^{-4} (3.0696)	0.3633 (1.7993)	23	5.67	0.0329	0.91	0.949
4	S_2	−15.6213 (−1.5845)	3.0003 (0.8626)	0.1141 (1.1936)	0.2064×10^{-3} (3.0616)	0.0380 (0.8472)	0	0.00	0.6960	0.35	0.878
5	S_2	−38.5914 (−2.5652)	9.9625 (2.0396)	0.8097 (2.4464)	0.1685×10^{-3} (2.6766)	0.0813 (1.9952)	5	1.67	0.6515	0.51	0.893
6	S_2	−36.1660 (−3.1979)	5.5375 (1.9170)	1.9127 (3.2948)	0.4850×10^{-3} (6.2077)	0.3454 (1.4317)	23	5.67	0.6134	0.63	0.906
7	S_3	−40.8880 (−27.1800)	−0.2086 (−0.4427)	0.2608 (3.0189)	0.1425×10^{-2} (17.9713)	−1.2502 (0.1050)	0	0.00	0.9302	1.08	0.991
8	S_3	−44.4897 (−25.2547)	−0.3358 (−0.7736)	0.8192 (4.2624)	0.1539×10^{-2} (18.2998)	−2.4395 (0.9109)	5	1.67	0.8385	1.11	0.993
9	S_3	−56.5697 (−12.0949)	0.0661 (0.1547)	2.3323 (3.8139)	0.1775×10^{-2} (12.9790)	35.2844 (−2.8690)	23	5.67	0.8709	1.14	0.992

TABLE XIX

Primary Borrowers—Constrained Arithmetic Lags (TSLS)

$$d_j = a + bi_{dj} - d\pi^e eD_j + u$$

	Dependent Variable	a	b	d	e	c	Length of Lag	Average Lag	S.E.E.	D.W.	$\bar{R}^2$
1	d_1	−16.1063 (−3.9876)	−1.6788 (−1.9876)	−0.0261 (−0.1556)	0.4148 (12.7432)	0.0156 (−1.8686)	0	0.00	1.8186	1.03	0.982
2	d_1	−8.1452 (−0.6918)	−1.7064 (−2.0701)	−1.0330 (−0.7443)	0.3808 (6.5338)	0.6054 (−0.4097)	23	5.67	1.7974	0.97	0.982
3	d_2	7.1917 (12.8090)	−1.3111 (−4.7572)	−0.1640 (−2.8985)	0.1591 (5.1421)	0.1251 (−3.9719)	0	0.00	0.6153	1.34	0.555
4	d_2	8.4040 (15.4373)	−1.2249 (−5.2899)	−0.5385 (−4.8359)	0.1100 (3.7431)	0.4396 (−2.4664)	5	1.67	0.5102	1.28	0.694
5	d_2	10.0439 (12.9422)	−1.2418 (−5.3885)	−0.8431 (−4.8467)	0.0692 (1.9997)	0.6789 (−1.2593)	11	3.67	0.5091	1.30	0.695
6	d_2	11.9492 (7.6568)	−1.4979 (−5.6645)	−1.1736 (−3.3409)	0.0521 (1.0814)	0.7835 (−0.7098)	23	5.67	0.5937	1.18	0.585

TABLE XX

Primary Savers: Constrained Geometric Lag (TSLS)

$S_j = a + bi_{sj} - d\pi^e + ey + u$

	Dependent Variable	a	b	−d	e	c	λ	S.E.E.	D.W.	$\bar{R}^2$
1	S_1	0.1395 (2.4294)	0.0854 (5.6211)	0.0017 (0.4228)	0.0942×10^{-4} (4.6196)	0.0199 (5.0120)	0.9	0.0336	0.89	0.947
2	S_1	0.1184 (1.8880)	0.0830 (5.5336)	0.0055 (0.8701)	0.1020×10^{-4} (4.5756)	0.0663 (4.2818)	0.5	0.0330	0.95	0.948
3	S_1	0.0732 (0.8519)	0.0845 (5.8433)	0.0144 (1.0976)	0.1140×10^{-4} (4.0770)	0.1704 (3.3067)	0.2	0.0330	0.96	0.949
4	S_2	−15.0207 (1.7762)	2.7237 (0.9191)	0.1503 (1.5306)	0.2174×10^{-3} (3.7830)	0.0552 (0.8863)	0.9	0.6876	0.38	0.881
5	S_2	−21.8131 (−2.3648)	4.7748 (1.5213)	0.4044 (2.3981)	0.2098×10^{-3} (3.8866)	0.0847 (1.4466)	0.5	0.6544	0.54	0.892
6	S_2	−25.8518 (−3.2598)	4.8718 (1.9286)	1.1037 (3.9034)	0.3144×10^{-3} (6.9324)	0.2265 (2.3725)	0.2	0.5844	0.79	0.914
7	S_3	−40.8485 (−26.4085)	−0.0473 (−0.1005)	0.2950 (2.7318)	0.1407×10^{-2} (17.5516)	−6.2579 (0.4947)	0.9	0.9478	1.01	0.991
8	S_3	−42.2219 (−26.8503)	−0.0422 (−0.0959)	0.5912 (3.6143)	0.1449×10^{-2} (18.4636)	−14.0095 (1.1046)	0.5	0.8855	1.15	0.992
9	S_3	−46.1277 (−22.2996)	0.2379 (0.5786)	1.3904 (4.1915)	0.1529×10^{-2} (18.2376)	5.8445 (−2.1426)	0.2	0.8430	1.25	0.993

TABLE XXI

Primary Borrower Constrained Geometric Lag (TSLS)

$$d_j = a + b i_{dj} - d\pi^e + e\,D_j + u$$

	Dependent Variable	a	b	d	e	c	λ	S.E.E.	D.W.	$\bar{R}^2$
1	d_1	−15.9081 (−3.8858)	−1.7087 (−2.0685)	−0.1387 (−0.6843)	0.4138 (13.0217)	0.0812 (−1.8339)	0.9	1.8017	0.99	0.982
2	d_1	−15.4616 (−3.5714)	−1.6998 (−2.0519)	−0.1957 (−0.5897)	0.4115 (12.5121)	0.1151 (−1.6557)	0.5	1.8049	1.00	0.982
3	d_1	−15.3710 (−2.7490)	−1.7099 (−2.0436)	−0.1747 (−0.2277)	0.4118 (11.2193)	0.1022 (−1.2972)	0.2	1.8178	1.02	0.982
4	d_2	7.1927 (13.7576)	−1.4383 (−5.6866)	−0.2173 (−3.3975)	0.1679 (5.9207)	0.1511 (−4.6303)	0.9	0.5724	1.24	0.615
5	d_2	7.6562 (14.9681)	−1.4016 (−5.9266)	−0.3917 (−4.0564)	0.1470 (5.2699)	0.2795 (−3.8181)	0.5	0.5347	1.28	0.664
6	d_2	8.6979 (12.1375)	−1.5147 (−6.0519)	−0.7183 (−3.3858)	0.1240 (3.7189)	0.4742 (−2.4258)	0.2	0.5686	1.18	0.620

chapter six

SURVEY STUDY RESULTS

This chapter is concerned with the effect of inflationary expectations on business-spending behavior. A considerable amount of research time has been devoted to examining the effect of price expectations on consumer spending behavior using direct survey methods.[1] Investigations of the impact of inflationary expectations on investment spending are, on the other hand, basically nonexistent. It has been argued, however, that a principal reason for the small responses to tight money in 1969 was inflationary expectations on the part of entrepreneurs.[2]

One of the research projects considered at the Prices and Incomes Commission during the winter of 1970–71 was a survey of the impact of monetary policy on business spending decisions. The project was designed along the lines of the Young-Helliwell survey which was carried out for the Royal Commission on Banking and Finance.[3] One of the ways in which this study departed from the Young-Helliwell study was that it inquired into the response of business-spending decisions to inflation. For various reasons it was decided not to carry out the study in its entirety. Nevertheless, a small pilot survey was carried out, which does provide answers which are suggestive.[4]

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[1] E. Mueller, "Consumer Reactions to Inflation", *Quarterly Journal of Economics*, May 1959, p. 246–62.

G. Katona and E. Mueller, *Consumer Attitudes and Demand 1950–52*, Michigan, Survey Research Centre, 1953.

[2] See chapter one.

[3] John H. Young and John F. Helliwell, "The Effect of Monetary Policy on Corporations", *Royal Commission on Banking and Finance*, Appendix Volume, Ottawa, Queen's Printer, 1964.

[4] The other members of the pilot survey project were Fred C. Nold and Wayne Thirsk.

The procedure used was to mail copies of the survey questionnaire to the company involved and then after an elapsed time of one to two weeks, to meet with the president and financial officers of the company. The companies concerned ranged from small-scale manufacturing companies to large-scale multinational corporations and utilities.

The portion of the questionnaire that is relevant to the current discussion reads as follows:

"12. During the period 1966 to 1970 prices have risen at a faster rate than has been common in the post-War II period. Over the period 1960 to 1965, for example, the rate of change of the Canadian consumer price index was 1.7%, while over the period 1966 to 1970 it increased at the average rate of 4%. Has this rise in prices led you to:

	YES	NO
(a) Speed up your capital expenditure program? If yes, could you please indicate the amounts, the nature of the projects, the number of months brought forward, and the dates at which the decisions were made.	___	___
(b) Acquire more inventories of inputs in anticipation of further price rises. If yes, please indicate amounts and dates.	___	___

	YES	NO
(c) Make any alterations in your calculations of the profitability of capital expenditures. If yes, please specify the sort of changes you made.	___	___
(d) Change the manner in which you calculate the cost of funds? If yes, please specify the sort of changes you made.	___	___
(e) Make any changes in the method in which you finance capital expenditures as a direct consequence of the recent inflation? If yes, please specify.	___	___

"

A total of 14 firms were interviewed.

Questions 12(a) and 12(b) were designed to elicit information on anticipatory spending on the part of firms in the face of prospective inflation. All 14 firms gave a negative answer to question 12(b). Only one firm gave an affirmative answer to question 12(a). The firm in question stated that beginning in 1968 there was some tendency to undertake construction earlier than might otherwise have been the case in order to avoid higher const uction costs in the future.

Question 12(c) and 12(d) are overlapping questions, the purpose of which was to find out if and/or how firms altered their investment decision criteria during the current inflation. Most of the larger firms used a range of investment criteria in evaluating projects. Only one firm of the 14 interviewed made any modification to any of its investment criteria. The firm was the same firm which gave an affirmative reply to question 12(a). The alteration was to use construction cost inflators (the company was not a construction company) in its DCF (discounted cash flow) calculations. Construction wage costs and other construction material costs were projected over a two to three year period. There was no attempt to alter the projected cash inflow for inflation; and current prices were used. The firm in question also pointed out that this alteration affected the timing but not the total size of their investment program.

TABLE XXII

Impact of Inflation on Business Expenditure Decisions:
Results of the Pilot Survey

	Number	Per Cent
Anticipatory Expenditures on Plant and Equipment	1	7.14
Anticipatory Expenditures on Inventories	0	0
Alterations in Investment Criteria	1	7.14
Alterations in Calculating Cost of Funds	0	0
Changes in Financing Procedures	0	0

Question 12(e) received a uniform negative reply. This was somewhat surprising in view of the variety of qualitative changes in capital markets over the past few years.

A summary of the survey results is contained in Table XXII.

Because of the restricted size of the survey and the limitations of the survey technique itself, these results are extremely tentative. They do, however, suggest three things. The limited inflationary experience of recent years has not led to wholesale alterations in investment criteria.

It is commonly argued that anticipatory business expenditures on plant and equipment and inventories will accompany inflation in its early stages. The survey results give only a small amount of support to the notion that current investment on plant may be affected. No support was found for anticipatory stocking up of inventories.

There was no visible evidence of modifications of investment criteria that would have the effect of reducing the impact of tight monetary policy.

chapter seven

PRINCIPAL CONCLUSIONS AND SUMMARY

It is often alleged that the reason interest rates are poor indicators of monetary policy is that they are susceptible to the impact of price expectations.[1] Empirical evidence that price expectations play an important role in determining the level of nominal interest rates is, however, far from being strong or unambiguous.[2] In addition, more persuasive arguments exist against the use of interest rates as the sole indicator of the stance of monetary policy. These arguments centre around the notion that the Central Bank cannot, in general, control either the cost or the availability of credit. Consequently, it is possible for credit conditions to become tighter while the Central Bank is undertaking operations that would, *ceteris paribus*, tend to ease credit conditions and reduce interest rates, and vice versa. Thus, it would appear that interest rates used alone are poor indicators of monetary policy for more fundamental reasons than the possibility that the level of interest rates may be influenced by price expectations.

Nevertheless, since the authorities in Canada appeared to believe (during the 1950s and 1960s) that they actually controlled the cost and availability of credit,[3]

[1] Typical are M. Friedman, "The Role of Monetary Policy", *American Economic Review*, March, 1968, p. 101; D. I. Fand, "Keynesian Monetary Theories, Stabilization Policy and the Recent Inflation", *Journal of Money Credit and Banking*, August, 1969, pp. 556–87; and D. I. Fand, "A Monetarist Model of the Monetary Process, *Journal of Finance*, Proceedings, 1970, pp. 275–325.

[2] A discussion of the various subtleties involved in interpreting the empirical results is contained in chapters two, three and four. On the other hand, W. H. Gibson claims that his results "have some very clear policy implications". W. H. Gibson, "Price–Expectations Effects on Interest Rates", *Journal of Finance*, March, 1970, p. 34. A commentary on the Gibson article is contained in J. F. McCollum, "Price–Expectations Effects on Interest Rates?", *Journal of Finance* (forthcoming).

[3] *Appendix Volume Royal Commission on Banking and Finance*, Ottawa, Queen's Printer, 1964.

they may have been led into some poor policy decisions in the mid-1960s. Some of the upward pressure on interest rates during the 1960s is likely attributable to price expectations (even though the statistical evidence is weak). The authorities, however, appear to have interpreted this as a tightening of credit markets, and consequently did not follow the tighter monetary policy that may have been warranted.

From the discussions in chapters two and four, it is clear that a question of the form, "What is the real rate of interest in Canada?", does not have an unambiguous answer. It is possible to construct a number of variables, all of which have some claim to being called the real rate of interest. Which definition of the real rate one chooses depends on the use to which one wishes to put it. Insofar as the authorities are interested in attempting to affect spending by manipulating the rate of interest, the real rate of interest anticipated by borrowers is the appropriate choice. This means that we require some measure of price expectations on the part of borrowers. The limited survey results reported in chapter six, however, suggest that corporate borrowers did not, in general, take inflation into account in any systematic way in making expenditure decisions.

In the course of this study a number of conclusions have emerged. Both theoretical and empirical results were obtained, of which the principal ones are summarized here.

1. The "Fisher Equation" stating that the nominal rate of interest is equal to the real rate of interest plus an inflationary premium does not constitute an adequate basis for the investigation of the influence of inflation on nominal rates of interest. The "Fisher Equation" can be variously interpreted as:

(i) a definition of the ex-post real rate of interest,
(ii) a definition of the ex-ante real rate of interest,
(iii) a definition of the expected rate of inflation,
(iv) an equilibrium condition,
(v) an hypothesis suggesting price expectations affect nominal interest rates in a particular manner.

2. Within the confines of the Fisherian model of interest rate determination, it is not unanticipated inflation (or lags in the adjustment to inflation) which grant the authorities the ability to reduce the real rate of interest in the short run by inflation, but asymmetric expectations on the part of lenders and borrowers.

3. Within the confines of the Loanable Funds model of interest rate determination, the authorities can affect the level of the real rate of interest. Introducing price expectations into this framework modifies the analysis, but leaves the principal conclusions unaltered. Introducing the asymmetric expectations hypothesis into the Loanable Funds model permits the authorities to affect the real rate of interest through two routes. They can have a permanent effect through their ability to augment the supply of loanable funds, and a short-run effect through asymmetric expectations.

4. It was argued that a series of positive coefficients in a regression of interest rates on a distributed lag of rates of price change can be explained by hypotheses in which price expectations play no role.

5. Using monthly data, we were able to reject the conditional hypothesis that the Fisher Hypothesis is true given that the maintained hypotheses on the formation of the real rate and price expectations are correct. Using quarterly data, however, we were generally unable to reject the naive Fisherian model. These results, however, were subject to a number of reservations due to the presence of several econometric problems.

6. A naive version of the "Radcliffe Hypothesis" was rejected by the data.

7. There was no visible support for the "differential effects according to term to maturity" hypothesis.

8. We were unable to find any direct impact of price expectations on Canadian interest rates once account had been taken of U.S. interest rates.

9. The statistical evidence on the effect of inflationary expectations on the behavior of ultimate borrowers and lenders is not completely clear. It would appear possible, however, to make the following generalizations:

(i) The long-run impact of price expectations on borrowers and lenders behavior was less than that implied by the Fisher Hypothesis. In several cases, however, we were unable to reject the Fisher Hypothesis.

(ii) We were unable to provide clear cut evidence on the asymmetric expectations hypothesis. However, it may be the case that with modification of the specification of the questions contained in chapter five and with the use of alternative estimation techniques, unproved estimates could be obtained.

10. The results of a small scale corporate study were consistent with the notion that firms have not made systematic revisions in their investment critetia in order to take account of price expectations.

APPENDIX

DATA SOURCES

Price Indexes:

Time series on the Canadian Consumer Price Index (CPI), the Wholesale Price Index (WPI), and the Implicit Price Index (IPI), were all obtained from *Prices and Price Indexes* D.B.S. Cat. No. 62–002. U.S. price indexes were obtained from the *Survey of Current Business*, U.S. Department of Commerce. Monthly and quarterly rates of change of prices were compounded and expressed as annual rates. Unless otherwise mentioned in the body of the paper, the data was not seasonally adjusted.

Interest Rates:

Interest rates on financial assets are in the form of yields to maturity. Most of the Canadian interest rates used are published in *The Bank of Canada Statistical Summary* and *Supplement*. A number of unpublished interest rate time series were obtained from the Research Department of the Bank of Canada. U.S. interest rates were obtained from *The Federal Reserve Bulletin*.

Exchange Rates:

Data in spot and forward exchange rates were obtained from *The Bank of Canada Statistical Summary*.

Financial Assets and Liabilities:

This data was obtained from three principal sources which include: *The Bank of Canada Statistical Summary* and *Supplement*, *The Financial Flow Accounts*, D.B.S. Cat. No. 13–002, and *Industrial Corporations Financial Statistics*, D.B.S. Cat. No. 61–003. In several cases, stock data was available only on an annual basis. A variety of financial data in flow form is available on a quarterly basis beginning in 1962. Taken in conjunction with the annual data on stocks, the flow data permits the calculation of end of quarter stocks. In many cases, however, the sum of the net flows over four quarters did not equal the difference between the two corresponding annual observations. In these cases, the annual stock observations were assumed to be correct, and the discrepancy was spread equally over the four quarters.

Other Data:

Data on various macroeconomic aggregates (such as GNP, etc.) were obtained from *The National Income and Expenditure Accounts*, D.B.S. Cat. No. –531. The current rate of profit on shareholders equity for Canadian industries was calculated on the basis of information contained in Tables I and II in *Industrial Corporations Financial Statistics*, D.B.S. Cat. No. 61–003.

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G-INFLATION (FINANCE)

INFLATION AND INTEREST RATES IN CANADA