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CURRENT METERS, THE EFFECTS OF UNSYMMETRICAL RESPONSES WHEN OPERATING IN WAVES

BY

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ES-526

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Engineering Services Section
Hydraulics Division
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MANAGEMENT OVERVIEW

Chosing a current meter for a research project can be risky because of unexpected errors inherent in the meter and its application.

For this reason, a study was carried out to designate the best current meter to purchase and test for the Littoral Zone which is characterized by high wave orbital speeds and a wide dynamic range for the net current or drift.

One aspect of this study involved analyses of the effects of not having ideal responses at all angles of incidence on the current meters. The results show that this is an important factor for designers as well as users of current meters for wave zones.

CURRENT METERS, THE EFFECTS OF UNSYMMETRICAL RESPONSES WHEN OPERATING IN WAVES

ABSTRACT

Mathematical analysis shows that some current meters can become totally unreliable in wave zones with low currents. This can be caused by polar responses that are unsymmetrical and non-compensating between the right and left half-planes (forward and backward current components).

In some cases compensation can be realized such as the Plessey 9021 and in cases non-symmetry between the lower and upper half planes (up and down current components).

Greater errors appear when the orbitals are flattened and the response is non-ideal in the horizontal plane. That is, a two axes current meter with 10% distortion from a circular response becomes useless when the drift speed falls to less than 9.5% of the orbital speed.

INTRODUCTION

Current meters are often used in zones where wave motion is superimposed on the mean (drift) current. Because the mean current is usually the one to be measured, the current meters are sometimes designed to minimize the effects of waves on the readings.

A brief study of the errors expected from wave orbitals was done using a model developed by McCullough, (1979). His model analysed the errors caused by the non-cosinusoidal responses of current meters such as $\cos^{0.5}\theta$ or $\cos^{2.0}\theta$ in the vertical plane. These models showed worst case errors of about 15% of reading as the mean speed drops to zero for a given orbital speed.

This note reports on the theoretical effects of having an unsymmetrical response in the meter such as $\cos^{0.5}\theta$ in the upper half plane and $\cos^{2}\theta$ in the lower half plane. As well, non-symmetry was analysed for the left-half and right-half planes. See figure 1.

The results show that non-symmetry between the upper and lower half-planes, Fig. 1C, could lead to a compensating effects and worst case errors were no worse than those predicted by McCullough, Fig. 1B. However, the left to right symmetry, Fig. 1D, was more important because errors in circular symmetry of 10% (worst case) Fig. 4 could lead to errors of $\pm$417% of the ideal reading as the mean speed approaches 1% of the orbital speed. The error can be great enough to reverse the apparent direction of the mean speed when it falls below 4% of the orbital speed.

In the Littoral Zone applications where the drift to orbital speed ratio could be low, the support structure and current meter itself could cause non-symmetry of 10% from and ideal response. It is most important to attend to the array's design and followup response tests to ensure that measurements are representative of the current in the zone.

MATHEMATICAL MODEL

For solid-state models, the current meter's physical axis of symmetry is usually normal to the horizontal plane (the earth's surface). The orbital and drift speeds are assumed to be co-planar and in line with one of the current meter's sensing axes. The problem then reduces to a two-dimensional analysis.

Using Vector analysis, see Fig., 2, McCullogh (1979), appears to have used the equations.

$$B' = \left[\left(V + C \cos A \right)^2 + \left(C \sin A \right)^2 \right]^{\frac{1}{2}}$$

$$T = \arctan \left(\frac{C \sin A}{V + C \cos A} \right)$$

$$B = B' \left(\cos T \right)^D$$

ere B' is the amplitude of the resultant current vector, m/s

V is the mean current (drift), m/s

C is the amplitude speed of the orbital velocity, m/s

A is the angular difference between the drift and the orbital velocity, degrees

T is the angle B' makes relative to the horizontal plane, degrees

B is the horizontal component of the speed and read by the current meter, m/s

D is the distortion factor in the cosinusoidal response of the current meter.

Using numerical methods, the mean speed is computed by summing all values of B as A is incremented from 1 to 360 degrees. The sum is normalized as in:

$$S = \frac{1}{360} \sum_{A=1}^{360} B_A$$

Where B_A is the component of B at angle A.

Finally, S is compared to V to predict how much error will accumulate in the current meter for the waves' orbital speed.

In the case of a flattened orbitals, the model is changed to

$$S = \left(\frac{\varepsilon_1 - \varepsilon_2}{360}\right) \sum_{A=1}^{360} \cos A$$

Where ε_1 is the fractional error from an ideal response on a given bearing where ε_2 is the fractional error from an ideal response on the opposite bearing. The cos A term models the speed of the wave motion having an amplitude of unity.

In this case, the model is one dimensional and does not operate in the vertical plane but along one line in the horizontal plane. This line lies in any bearing relative to the sensitive axis of the current meter. In fig. 4, $\varepsilon_1 = 1.05$ and $\varepsilon_2 = 0.90$ along the 135°/315° bearings respectively.

ERRORS CAUSED BY NON-SYMMETRY BETWEEN THE UPPER AND LOWER HALF-PLANES

In the first set of equations, a calculator is programmed to change the distortion factor, D, when T becomes 180°. A non-symmetry results between the upper and lower half-planes of the current meter's response to circular orbitals. See the program in the appendix and figure 1C.

The results are shown in Table I

TABLE I

D		C m/s	V m/s	S m/s	% Error (of reading)
Upper	Lower				
0.5	2.0	1.0	0.01	0.0098	-2%
1.0	2.0	1.0	0.01	0.0092	-8%
2.0	2.0	1.0	0.01	0.0085	-15%
0.5	2.0	1.0	0.5	0.4904	-2%
1.0	2.0	1.0	0.5	0.4622	-8%
2.0	2.0	1.0	0.5	0.4244	-16%

The table illustrates that the degree of non-symmetry between the upper and lower semicircle can compensate errors in the current meter and the worst case errors occur when there is good symmetry for a given distortion figure.

ERRORS CAUSED BY NON-SYMMETRY BETWEEN THE LEFT AND RIGHT HALF-PLANES

In this case, the value of D is changed as T goes beyond 90° and is changed back as T goes beyond 270°. This causes a left half-plane and right-half plane distortion difference, as shown in figure 1D.

When the response is modelled in this manner, the results are quite different as shown in table II.

TABLE II

D		C m/s	V m/s	S m/s	% Errors of reading
Left Half Plane	Right Half Plane				
2.0	0.5	1.0	0.001	0.1324	13,240
2.0	0.5	1.0	0.01	0.1412	1,412
2.0	0.5	1.0	0.1	0.2294	229
2.0	0.5	1.0	0.2	0.3274	63
2.0	0.5	1.0	0.5	0.6208	24
2.0	0.5	1.0	0.00	1.1024	11
0.5	0.5	1.0	2.0	2.0619	3
0.5	2.0	1.0	0.99	0.8396	-15
0.5	2.0	1.0	0.5	0.3600	-28
0.5	2.0	1.0	0.2	0.0649	-28
0.5	2.0	1.0	0.1	-0.0333	-133
0.5	2.0	1.0	0.01	-0.1216	-1320
0.5	2.0	1.0	0.001	-0.1303	-13130

Two adverse effects appear. The error, expressed as percent of reading, approaches infinity as the drift speed goes to zero. As well, the apparent drift speed can reverse direction from the original drift.

Table II is not representative of the kind of errors seen in good current meters. The distortion from the ideal cosine response is around +91% and -238% for D=2 and 0.5 respectively.

By changing the value of D to something closer to 1.0, the distortion can be reduced to about ±50% from ideal for the worst case. The values of D=1.3 and D- 0.85 were used to simulate typical current meters with fair if not good polar responses.

To illustrate the effects of such distortions, Figure 4 shows the polar plot of a two-axis current meter having distortion figures (Dx, Dy) in the horizontal plane as shown in each quadrant. This illustrates that such distortions are typical of current meter responses in the horizontal plane. It is conjectured that the vertical plane can be no better than the horizontal plane, because the blockage is often worse. Therefore, the D = 1.3 and 0.85 values are likely conservative estimates of the distortions in the vertical plane.

When these values are used, the results are shown in Table III and in Figure 5. The errors are considerably smaller but are still serious for some applications such as littoral zone and limnology studies.

TABLE III

D		C	V	S	% error
Left Half Plane (Distortion)	Right Half Plane (Distortion)	m/s (Orbital)	m/s (Drift)	m/s (Simulated)	of reading
0.85	1.3	1.0	0.001	-0.0406	-4160
0.85	1.3	1.0	0.01	-0.0317	-417
0.85	1.3	1.0	0.1	0.0573	-42
0.85	1.3	1.0	0.2	0.1561	-22
0.85	1.3	1.0	0.5	0.4529	-9.4
0.85	1.3	1.0	0.9	0.8485	-5.7
1.3	1.3	1.0	1.5	1.4546	-3.0
1.3	1.3	1.0	2.0	1.9643	-1.8
0.85	0.85	1.0	2.0	2.0182	0.9
0.85	0.85	1.0	1.5	1.5237	1.6
1.3	0.85	1.0	.9	0.9307	3.4
1.3	0.85	1.0	.5	0.5355	9.3
1.3	0.85	1.0	.2	0.2392	20
1.3	0.85	1.0	.1	0.1404	40
1.3	0.85	1.0	.01	0.0515	415
1.3	0.85	1.0	.001	0.0426	4160

AN EXAMPLE USING THE PLESSEY 9021 RESPONSE CURVES

In Exposure Vol. 9., No. 5, D. Crump gives the polar response curves for a Plessey (Grundy) 9021 with and without his modifications. The cosine curves are quite distorted and would appear to introduce severe errors in wave motion but this is not the case for circular orbitals.

To model the response of the 9021, a polar equation was developed of the form $L \cos^{D}(M-A) + PA$. The coefficients L,M,D and P are all controlled by the value of the angle A. In this way, a variety of shapes can be generated including the double, rear-lobes is shown in Fig. 3A and 3B.

By setting the drift speed to 0.001 metres per second and the orbital speed to 1 metre per second and comparing the results of the sum of speed components as an orbital passes, an indication of the current meter's performance is obtained. If the performance is non ideal, the sum is not 0.001 m/s.

The results were S=0.087 for the unaltered 9021 and S=0.002 for the altered 9021. Another way of expressing this error is that the apparent current caused by a given orbital of speed R is 8.7% and 0.2% of R respectively. The model indicates that the Plessey with its response to currents approaching from the rear has remarkably good compensation for circular orbitals. Furthermore, the addition of Crump's modification improves the response. Flattened orbitals would be a problem.

THE LITTORAL ZONE AS A SPECIAL CASE

The foregoing applies to open ocean work where the orbital motions are assumed to be circular and on a fixed bearing. Therefore, all the errors are accumulated in the vertical plane.

In the Littoral Zone, it is assumed that the orbitals are suppressed to a back and forth motion only and the waves can approach at different bearings.

In fig. 4 waves which impinge at some angle relative to the most sensitive axis, will generate erroneous measurements because the back stroke will not cancel the forestroke. For example, with the polar distortion figures, $(\cos A)^{D}$, of D=0.85 and 1.3 a wave impinging on the worst case bearings of 315° and 135° will generate significant errors. For a given orbital, the meter will read 9.5% of the orbital speed, even though the mean current is zero. Another way of expressing the problem is that the signal to distortion ratio falls to 1 when the actual current speed equals 9.5% of the orbital speed. At that point, the apparent current could read zero or twice the actual current depending upon the direction of that current.

CONCLUSION

A brief look at some of the calibrations done on current meters in NWRI, for example C. Der and B. White 1976, will give the reader the impression that most current meters do not have sufficiently good polar response characteristics to be better than the model shown in Fig. 3. If that is the case, then in the Littoral Zone such meters will cause totally unreliable readings at current speeds of at least 9.5% of the orbital speeds and will only reach 90 to 95% accuracies when the drift (current) speed reaches at least 95% of the orbital speed.

In the vertical plane, it is likely that the cosine response is worse because of the shapes of the current meter and its supports. In this case, (open ocean) the meter becomes totally unreliable for mean currents below 4% of the orbital speeds and does not achieve 90 to 95% accurate reading until the mean current reads at last 40% of the orbital speeds. An exception can occur if there is a compensation effect in the current meter's polar response.

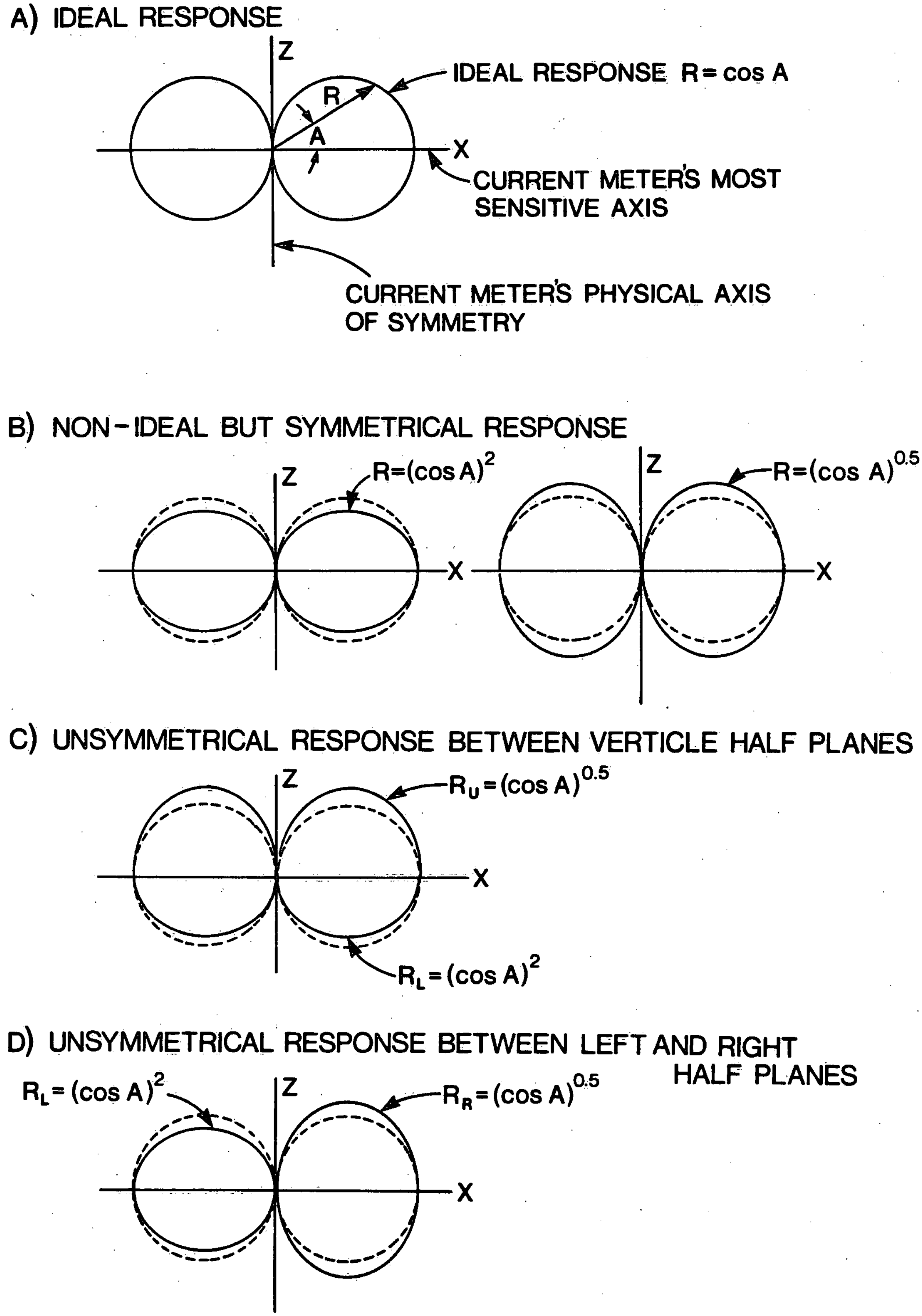
In view of this, extreme care will have to be taken to minimize distortion of the polar response in both the vertical and horizontal planes, if the current meters are to be used to measure low currents in moderate waves.

REFERENCES

J. R. McCullough, "Near-Surface Ocean Current Sensors: Problems and Performance", Woods Hole Oceanographic Institute Technical Report, WHOI - 79-90, Dec. 1979

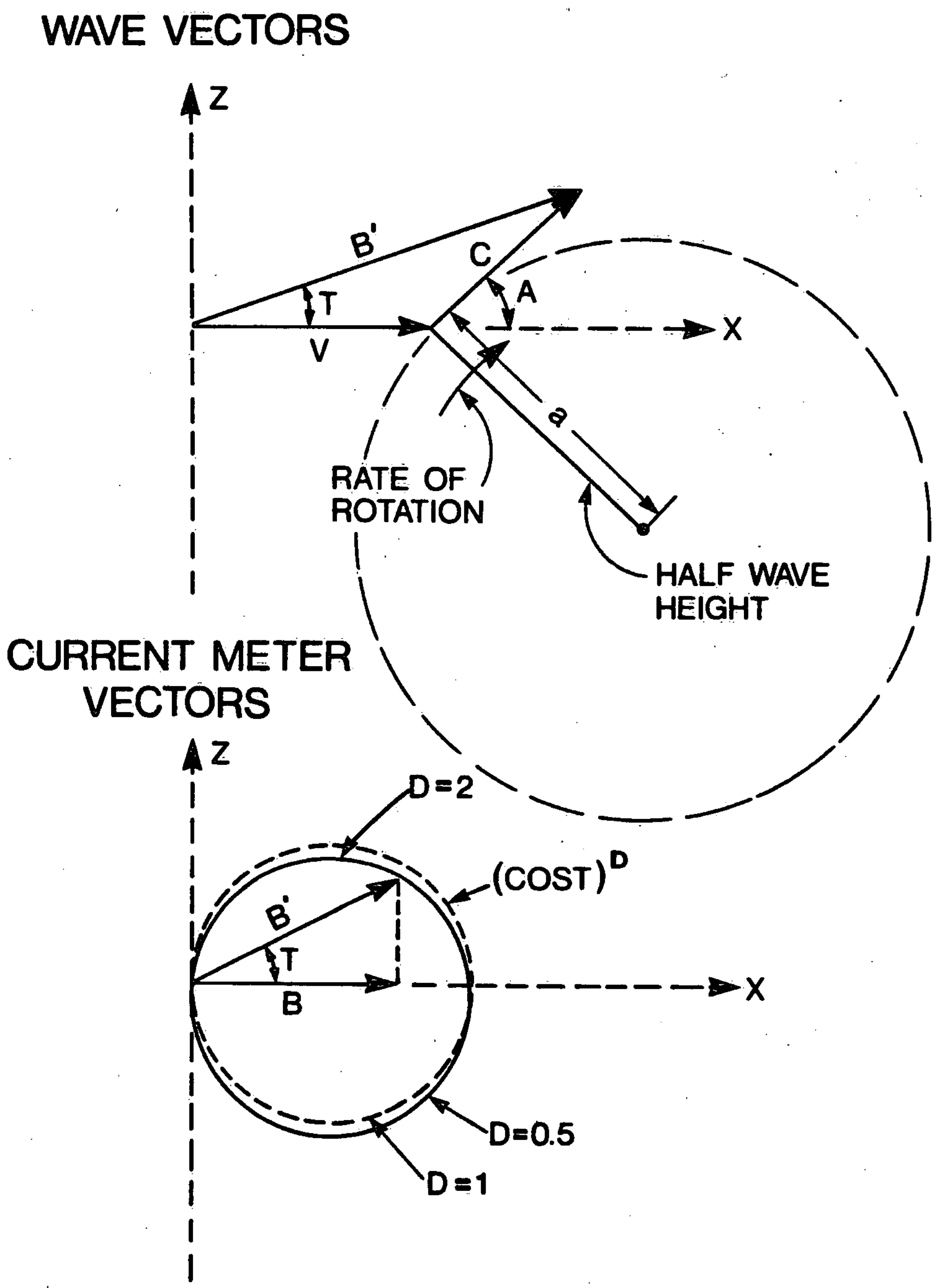
D. Crump, "Test Results of a Flotsam Shedder for the Grundy Model 9021 Current Meter" Exposure Vol. 9. No. 5., Nov. 1981.

C. Y. Der, B. F. White "Sensor Engineering Project: Evaluation Data on Solid-State, 2-Axis Water Velocity Sensors", National Water REsearch Institute Unpublished Report, ES-511, July 1976.



A) IDEAL RESPONSE
Z
R
A
X
IDEAL RESPONSE R = cos A
CURRENT METER'S MOST SENSITIVE AXIS
CURRENT METER'S PHYSICAL AXIS OF SYMMETRY
B) NON-IDEAL BUT SYMMETRICAL RESPONSE
Z
R=(cos A)^2
X
Z
R=(cos A)^0.5
X
C) UNSYMMETRICAL RESPONSE BETWEEN VERTICLE HALF PLANES
Z
R_U = (cos A)^0.5
X
R_L = (cos A)^2
D) UNSYMMETRICAL RESPONSE BETWEEN LEFT AND RIGHT HALF PLANES
R_L = (cos A)^2
Z
R_R = (cos A)^0.5
X

FIGURE 1.



WAVE VECTORS
Z
B'
C
T
A
X
V
a
RATE OF ROTATION
HALF WAVE HEIGHT
CURRENT METER VECTORS
Z
D=2
(COST)^D
B'
T
B
X
D=0.5
D=1

FIGURE 2.

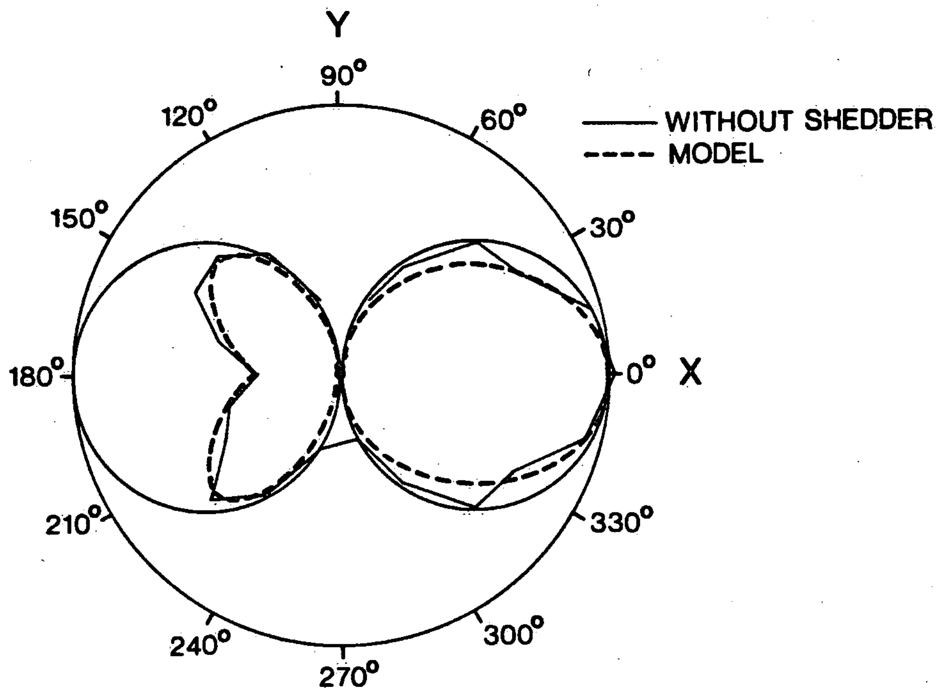


FIGURE 3a. PLESSEY 9021 SN#52 ROTOR
HORIZONTAL DIRECTIVITY RESPONSE

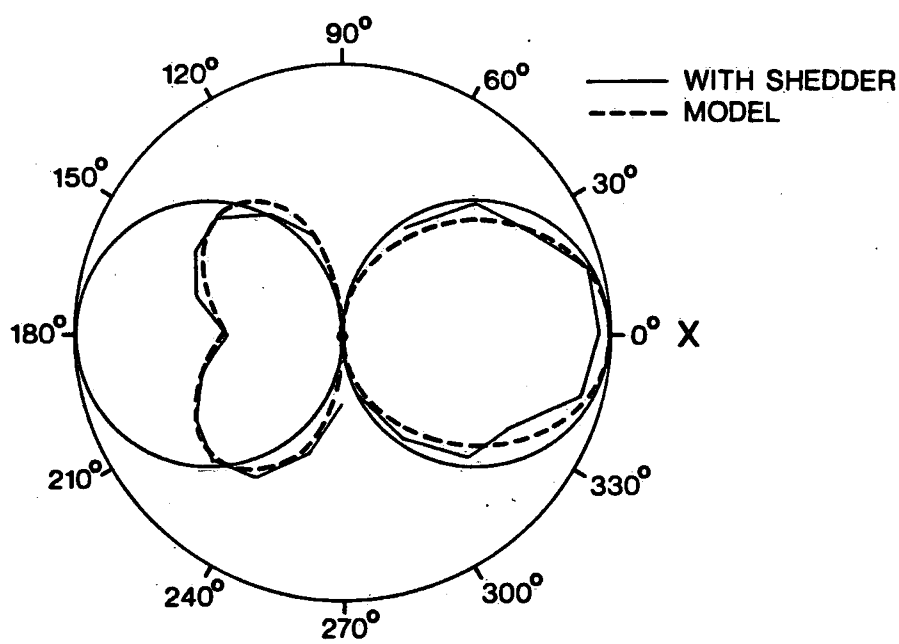


FIGURE 3b. PLESSEY 9021 SN#52 ROTOR
HORIZONTAL DIRECTIVITY RESPONSE

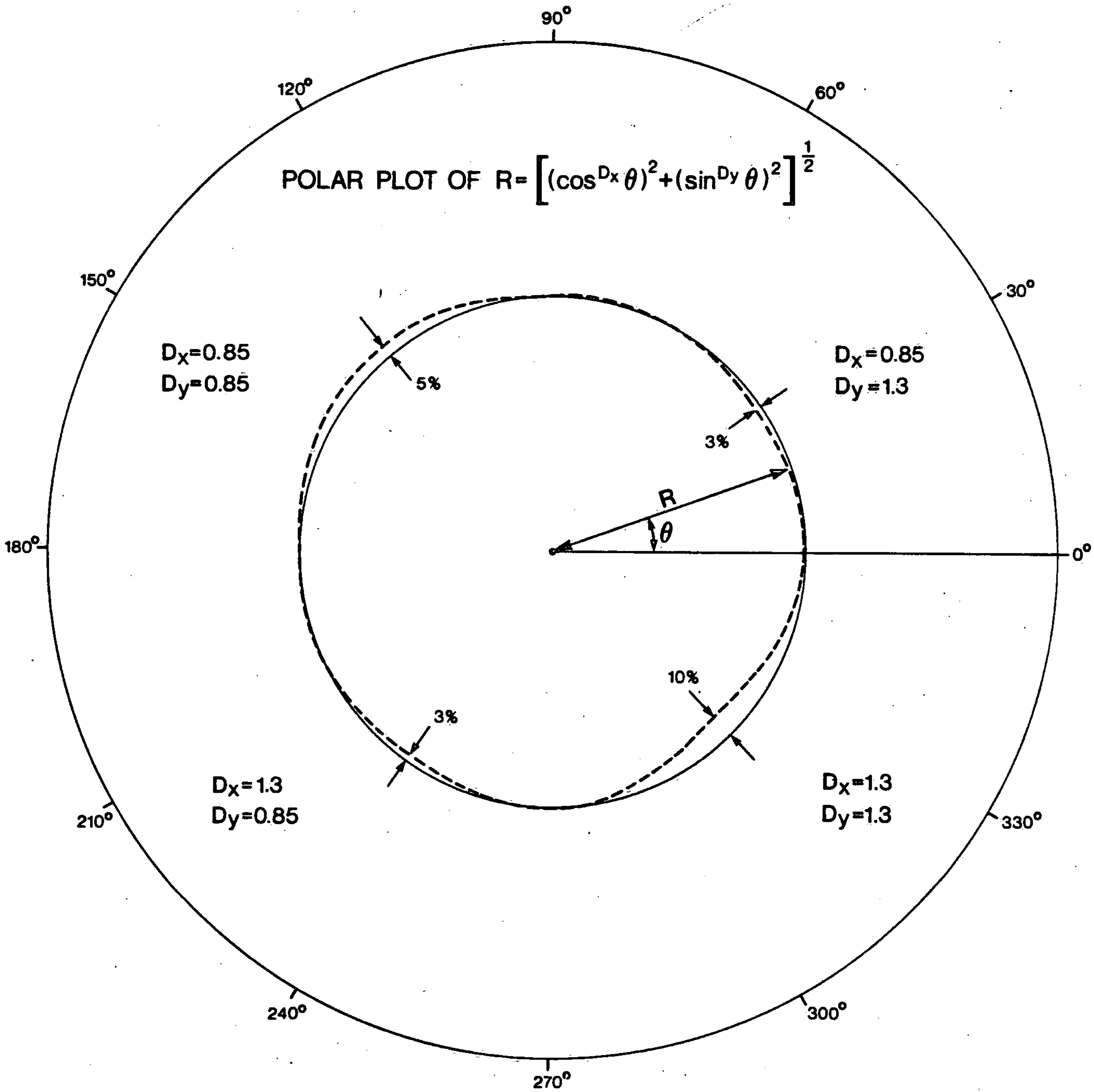

90°
120°
60°
POLAR PLOT OF $R=\left[(\cos^{D_x}\theta)^2+(\sin^{D_y}\theta)^2\right]^{\frac{1}{2}}$
150°
30°
D_x=0.85
D_y=0.85
5%
D_x=0.85
D_y=1.3
3%
R
θ
180°
0°
10%
3%
D_x=1.3
D_y=0.85
D_x=1.3
D_y=1.3
210°
330°
240°
300°
270°

FIGURE 4.

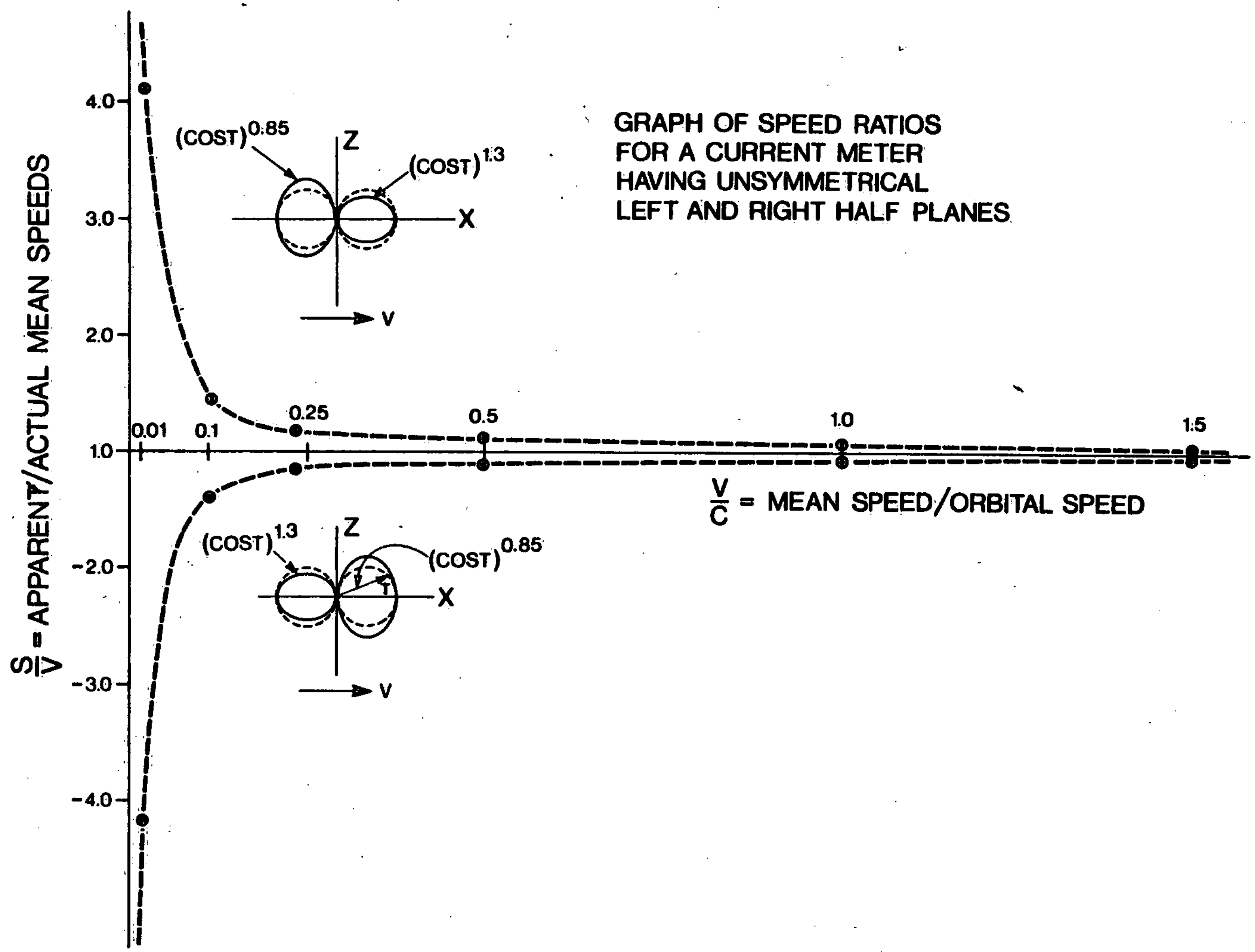

GRAPH OF SPEED RATIOS
FOR A CURRENT METER
HAVING UNSYMMETRICAL
LEFT AND RIGHT HALF PLANES
$\frac{S}{V}$ = APPARENT/ACTUAL MEAN SPEEDS
$\frac{V}{C}$ = MEAN SPEED/ORBITAL SPEED
4.0
3.0
2.0
1.0
-2.0
-3.0
-4.0
0.01
0.1
0.25
0.5
1.0
1.5
(COST)$^{0.85}$
(COST)$^{1.3}$
Z
X
V
(COST)$^{1.3}$
(COST)$^{0.85}$
Z
X
V

FIGURE 5.

APPENDIX: HP 9825 PROGRAM TO MODEL A CURRENT METER'S ERROR DUE TO WAVES WHEN THE RESPONSE IS UNSYMMETRICAL

```
0 :     dim B[360], T [360]; 1→C; 0.5→D; 0.5→V
        Set up constants and memory locations.

1 :     for A=1 to 360
        Set up a loop for the phase angle A.

2 :     V + C cos (A)→E
        Calculate the horizontal components.

3 :     atn (C sin (A)/E)→T[A]; if T[A]*sgn(E)<0;2→D
        Calculate the resultant current angle for current plus orbital.
        Check for T> 180° and if so change D to 2.
4 :     sgn(E)*sgn(cos(T[A])*abs(cos(T[A]))↑D*
                (E↑2 + (C*sin(A))↑0.5→B[A]
        Calculate the distorted component of the meters reading.

5 :     dsp B[A], D; wait 10
        Display the intermediate results

6 :     next A
        Return to step 1

7 :     for N = 1 to 180; B[2*N]+B [2N-1]+S→S
        Calculate the sum of the distorted components.

8 :     dsp S; wait 20; next N; prt "C",C, "D", D, "V", V,
                "S", S/360

        Display and print the results.

9 :     end
```