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A CORRECTION TECHNIQUE FOR SENSORS HAVING EXPONENTIAL TIME RESPONSES

by

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ABSTRACT

Many sensors have an exponential time-response to a sharp change in the medium being sensed. In some cases it is desirable to correct for this response characteristic and estimate the sharpness of the actual change. A simple method is explained and applied to an oxygen sensor that was used to profile the water column in a lake.

RÉSUMÉ

De nombreux détecteurs ont un temps de réponse exponentiel à une variation brusque dans le milieu étudié. Dans certains cas, il est souhaitable de corriger la réponse en fonction de cette caractérestique et d'évaluer l'acuité de la variation réelle. Une méthode simple est présentée; elle est appliquée à un détecteur d'oxygène utilisé pour l'établissement du profil d'une colonne d'eau dans un lac.

MANAGEMENT PERSPECTIVE

In water research, the layers of distinctly different water that form and change in the lakes with the seasons, are most important to the scientific understanding of the lake's behaviour. The common method of measuring these layers is to take a profile of the water from the surface to the bottom using an array of sensors. Some sensors may exhibit a slow or delayed response to a stimulus which makes accurate profiling tedious and time-consuming. Regardless of the time and patience taken, the profiles are not as accurate as they should be.

This report demonstrates a method of correcting the profile data to reconstruct a profile as if the sensor were much quicker in its responses. Within reason, this method will allow the profile to be taken more rapidly thereby saving valuable ship-time.

The work was done to assist in the National Water Research Institute's Study 427 to do with the oxygen profiling of lakes.

T. Milne Dick
Chief
Hydraulics Division

June 1982

PERSPECTIVE DE GESTION

Dans le domaine de la recherche sur les eaux, les couches d'eaux nettement différentes qui se forment dans les lacs et qui changent avec les saisons ont une importance considérable pour la compréhension scientifique du comportement du lac. La méthode usuelle pour étudier ces couches consiste à établir un profil de l'eau depuis la surface jusqu'au fond en utilisant un réseau de détecteurs. Certains détecteurs peuvent répondre lentement ou à retardement à un stimulus, ce qui rend l'établissement d'un profil précis, long et fastidieux. Quels que soient le temps et la patience mis en jeu, les profils ne sont jamais aussi précis qu'ils devraient l'être.

Le présent rapport fait état d'une méthode de correction des données permettant de reconstruire un profil comme si les réponses du détecteur étaient plus rapides. Normalement, cette méthode permettra d'établir plus rapidement le profil et d'écourter les sorties en bateau.

Le travail a été effectué pour aider l'Institut national de recherche sur les eaux dans son étude 427 relative à l'établissement des profils de l'oxygène dans des lacs.

T. Milne Dick
Chef
Division de l'hydraulique
June 1982

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INTRODUCTION

The errors in time-series records, generated by sensors having delay and exponential time-response characteristics were analysed using exponential equations. A method has been derived to produce corrections on the time-series data to minimize the effects of time responses of the sensor.

The method is tested using a non-linear waveform generated with Laplace transforms. The waveform is corrected and shown to be coincident with the original to within 1% error, full scale.

The step-response data for typical membrane-type oxygen sensors were analysed. Using the resulting constants for the time-response characteristics of the sensors, a set of time-series data from a pair of profiles, taken on a stratified lake were corrected to demonstrate the method. In turn, these data can be used to reconstruct a profile which will be more indicative of the boundaries and gradients of the layers that the sonde sensed as it passed through the water column.

An apparent discrepancy between laboratory test data and field data, as it applies to the sensor characteristics, requires further investigation.

THEORY

Figure 1A and B show the classical responses of a sensor that has a limited bandwidth and is responding to in a step stimulus. The response is an exponential function according to the solution of the differential equations that describe the sensor's behaviour. For example, the response of a single-pole low-pass filter to a step function (stimulus) is (VanValkenburg, 1955):

$$y = y_o \, (1 - e^{-t/\tau}) \tag{1}$$

Similarily the response to a step down is

$$y = y_o \, e^{-t/\tau} \tag{2}$$

where y = the response at time t (volts)

t = the time since the event (step) occurred (sec)

y_o = the original values before the step (volts)

τ = the characteristics time constant of the filter (sec)

The difference between the time series plot of the step function and the exponential response of the sensor is the error, d, that the sensor generates. The error itself is a function of the form:

$$d = k(-\frac{y_o}{\tau} \; e^{-t/\tau}) = k \frac{dy}{dt} \tag{3}$$

which is a function of the slope of the sensor's response with time.

If the error is estimated and added to the original response, the ideal response can be obtained, as shown in Figure 1C. That is:

$$y + d = y_o \tag{4}$$

The function of the error term can be derived by substitution of equations (2) and (3) into (4)

$$y_o(1 - e^{-t/\tau}) + k\left(-\frac{y_o}{\tau} e^{-t/\tau}\right) = y_o \tag{5}$$

The result is that k equals τ.

Therefore $d = \tau\frac{dy}{dt}$ (6)

From calculus, (Thomas 1953), random waveforms can be produced using a series of discrete step functions. If the steps are arbitrarily small, the error, from the ideal waveform and the approximate waveform, can be made arbitrarily small. In a time series waveform, this infers that, if the stepping period Δt is small enough, the error in estimating d can be kept small enough for practical purposes.

If each discrete step after Δt is treated as a step function for the sensor, the correction for that step function can be applied to approximate the ideal step response. Therefore, the ideal response to the random waveform can be accurately approximated.

Electrical analogues for various physical and chemical functions are common practice for representing and analysing systems (Everett; Anner, 1956), therefore the correction process is quite general. The next section will make use of this fact to demonstrate the correction process for a purely electrical circuit.

TEST CASE

Using exactly generated and differentiable functions, the theory can be tested using Laplace transforms (Nixon, 1960).

The waveform generation and testing is simulated by a low pass filter driven by a voltage generator. See Figure 2.

The equations of state are:

$$v_i(t) = i(t)R + \frac{1}{C} \int i(t)\, dt \tag{7}$$

$$v_o(t) = \frac{1}{C} \int i(t)\, dt = v_i(t) - i(t)R \tag{8}$$

The Laplace transform for equation (7) is

$$V_i(s) = I(s) + \frac{I(s)}{Cs} + q\, \frac{0+}{Cs} \tag{9}$$

If $v_i(t)$ is made a ramp function

$$v_i(t) = t, \text{ and } q(0+) \text{ is set to zero,} \tag{10}$$

then

$$V_i(s) = 1/s^2 \text{ and} \tag{11}$$

$$I(s) = \frac{1}{s(sR + 1/C)} \tag{12}$$

The inverse Laplace for this equation is:

$$i(t) = C(1 - e^{-t/RC}) \tag{13}$$

from equation (8)

$$v_o(t) = t - RC\,(1 - e^{-t/RC}) \tag{14}$$

Using the same technique, an input ramp function that rises then reverses direction can be generated. Therefore, the output and the slope of the low pass filter's response can be known exactly. This allows the corrections to be applied exactly and thereby test the theory. The output voltage at the capacitor is:

$$v_o(t) = \left[t - RC(1-e^{-t/RC})\right] - 2u(t-20)\left[(t-20) - RC(1-e^{-\frac{(t-20)}{RC}})\right] \tag{15}$$

where u(t-20) is zero for t<20 and 1 for t $\geq$ 20. The slope of the output waveform is:

$$\frac{dv_o}{dt} = [1-e^{-t/RC}] - 2u(t-20)\,[1-e^{-(\frac{t-20}{RC})}] \tag{16}$$

The corrected time function is from equations (4) and (6)

$$v_c(t) = V_o(t) + RC\frac{dv_o}{dt} \tag{17}$$

If the correction is valid

$$v_c(t) = V_i(t) \tag{18}$$

which is the input triangular wave. Table I gives the values of $v_i(t)$, $v_o(t)$, $v_c(t)$ and some intermediate results. The main values are shown in Figure 2.

By comparing $v_c(t)$ with $v_i(t)$ in Table I, the correction process is seen to be accurate to within 1% of full-scale for the triangular wave.

CHARACTERISTICS OF A MEMBRANED OXYGEN SENSOR

The theory can be applied to slowly responding sensors such as membraned oxygen probes that exhibit exponential time-responses to a reasonable degree. Hitchman (1978) discusses the nature of the time response for membraned oxygen sensors beginning on page 93. The response is expressed as a series of exponential functions with respect to time. It also exhibits a delay function.

TABLE I. Values of the input, $v_i(t)$, output, $v_o(t)$ and intermediate voltage values for a low pass filter driven by a triangular waveform. In this case the filter's time constant, RC = τ = 10 seconds.

t(sec)	v_i(t)(volts)	A(volts)	B(volts)	v_o(t)(volts)	dv_o/dt(v/s)	v_c(t)(volts)
0	0	0	0	0	0	0
2	2	0.19	0	0.19	0.18	1.99
4	4	0.7	0	0.7	0.33	4.0
6	6	1.5	0	1.5	0.45	6.0
8	8	2.5	0	2.5	0.55	8.0
10	10	3.7	0	3.7	0.63	10.0
12	12	5.0	0	5.0	0.70	12.0
14	14	6.5	0	6.5	0.75	14.0
16	16	8.1	0	8.1	0.80	16.1
18	18	9.6	0	9.6	0.84	18.0
20	20	11.4	0	11.4	0.86	20.0
22	18	13.1	0.4	12.7	0.53	18.0
24	16	14.9	1.4	13.5	0.25	16.0
26	14	16.7	3.0	13.7	0.03	14.0
28	12	18.6	5.0	13.6	-0.16	12.0
30	10	20.5	7.4	13.1	-0.31	10.0
32	8	22.4	10.0	12.4	-0.44	8.0
34	6	24.3	13.0	11.3	-0.53	6.0
36	4	26.3	16.2	10.1	-0.63	3.8
38	2	28.2	19.2	9.0	-0.70	2.0
40	0	30.2	22.8	7.4	-0.74	0
42	- 2	32.2	26.2	6.0	-0.80	- 2.0
44	- 4	34.1	29.8	4.3	-0.83	- 4.0
46	- 6	36.1	33.4	2.7	-0.87	- 6.0
48	- 8	38.1	37.2	0.9	-0.89	- 8.0
50	-10	40.1	41.2	- 0.9	-0.91	-10.0
52	-12	42.1	44.8	- 2.7	-0.93	-12.0
54	-14	44.1	48.6	- 4.5	-0.95	-14.0
56	-16	46.1	52.6	- 6.5	-0.95	-16.0
58	-18	48.1	56.4	- 8.3	-0.97	-18.0
60	-20	50.1	60.2	-10.1	-0.97	-19.8

If the dominant exponential response is used at the onset of the response (after the delay period), the response can be corrected to approximate zero delay and zero time constant using the methods described here.

Two constants are needed to make the corrections: the delay time, in seconds, before the sensor begins to respond significantly and the exponential time constant, in seconds, once it begins to respond.

These are obtained by laboratory tests on the sensor. The tests involve plunging the sensor from one oxygen concentration to another as quickly as possible and measuring its response with time. Figures 3 and 4 give the rise and fall characteristics respectively.

Using regression analysis, the following time constants were found for two sensors labelled green and blue. R^2 is the coefficient of determination.

	rising	R^2	falling	R^2
green	10.53	0.9989	12.11	0.9979
blue	12.05	0.9959	12.2	0.9944

For simplicity, the grand mean was used for further corrections. The grand mean is 11.72 seconds.

The delay time was estimated to be about seven seconds. A better estimate is not available because it is not clear in Figures 3 and 4 where the dominant exponential response begins.

With these two prime pieces of information about the sensors, the following methodology was used to improve the time-series data.

METHODOLOGY FOR THE CORRECTIONS

Figure 5A shows a truly exponential step-up, step-down, response pair. The vertical lines designate the sampled values of the response values, V, coming from the sensor. The sampling procedure is used because the data are usually recorded on digital, magnetic tape.

Figure 5B shows a truly exponential response pair but with a delay of two sample periods.

Figure 5C shows a typical response pair from the oxygen probes (Yellow Springs) tested at the National Water Research Institute. Some system noise is present that must be smoothed by averaging.

In each case, the equations that are appropriate for making the corrections are shown below. See also Figure 1C.

Case A: (Figure 5A)

$$dv/dt \approx \Delta V_N/\Delta t = \frac{V_{N+1} - V_N}{\Delta t} \qquad (19)$$

$$V_{CN} = \tau(\Delta V_N/\Delta T) + V_N \qquad (20)$$

where: dv/dt = the exact slope of the oxygen signal (volts/second)

ΔV_N = the difference in the samples of the oxygen signal (volts)

Δt = the time interval between samples (seconds)

V_N = the present sample of the oxygen signal (volts)

V_{N+1} = the following sample of the oxygen signal (volts)

V_{CN} = the present corrected oxygen signal (volts)

τ = the time constant of the oxygen sensor (seconds)

Case B: (Figure 4B)

$$\frac{V_{N+2}}{\Delta t} = \frac{V_{N+3} - V_{N+2}}{\Delta t} \qquad (21)$$

$$V_{CN2} = \frac{V_{N+2}}{\Delta t} + V_{N+2} \qquad (22)$$

where: V_{N+2} is the sample of the oxygen signal taken two sample periods later than the present (volts).

V_{CN2} is the corrected oxygen signal taking into account the delay of two sample periods caused by the oxygen sensor (volts).

Case C: (Figure 5C)

$$\frac{\overline{V_{N+2}}}{\Delta t} = \frac{V_{N+3} - V_{N+1}}{2\Delta t} \tag{23}$$

$$V_{CN2M} = \frac{\overline{V_{N+2}}}{2\Delta t} + V_{N+2} - f(\tau_1) \tag{24}$$

where: $\overline{V_{N+2}}$ = the mean difference in the samples of the oxygen signal from the present to two sample periods hence (volts).

V_{CN2M} = the corrected signal in the present taking into account account the delay and the initial rise in response that is characteristic of membraned oxygen sensors (volts).

$f(\tau_1)$ = the initial response of the sensor after the stimulus but before the main response begins (volts).

In case C; the correction for the initial time constant, τ_1, was found to be negligible so it was neglected for the balance of the work. That is, $f(\tau_1)$ was set to zero.

Equation (23) includes a method of deriving a mean slope from a noisy signal. This is important because the correction factor $\tau(dV/dt)$ has a strong effect if dV/dt is noisy.

Equation (24) provides for a time shift of two sampling periods which would be 10 seconds if the sample period, D, is 5 seconds.

Because the sensor had a seven second delay (about one sample period) and because the smoothing of dV/dt was done graphically, equation (24) was simplified to:

$$V_{oN1} = \frac{\tau}{\Delta t}(V_{N+2} - V_{N+1}) + V_N \tag{25}$$

Table II gives the values of the raw data from a graph provided by E. Harrison. The graph was a time series of an oxygen profile that operated from the top to the bottom of the lake and back. The raw data were sampled from the time series graph of the data. Using the program listed in Appendix I, the raw data were corrected. Figure 6 shows the results.

The unacceptable overshoot seen in Figure 6 was investigated for the effects of the following:

1. The fineness of the sample interval.
2. The accuracy of the time constant.
3. The application of the correction sooner or later in the time series.

The fineness of the sample interval did not affect the results significantly.

The applicability of the time constant from the laboratory tests is in serious question when the following logic and data are used.

TABLE II. Raw Data and Corrected Data for the Time Series of an Oxygen Profile.

Time (seconds)	Raw Data (mg/L)	Corrected Data (mg/L)	Time (seconds)	Raw Data (mg/L)	Corrected Data (mg/L)
5	8.70	8.70	205	2.55	2.55
10	8.70	8.70	210	2.55	2.55
15	8.70	8.70	215	2.55	2.55
20	8.70	8.70	220	2.55	2.58
25	8.70	8.70	225	2.56	2.56
30	8.70	8.70	230	2.56	2.56
35	8.70	6.71	235	2.56	2.56
40	8.70	5.02	240	2.56	2.56
45	7.50	4.01	245	2.56	2.59
50	6.00	3.48	250	2.57	2.57
55	4.80	2.84	255	2.57	2.60
60	4.00	2.14	260	2.58	2.58
65	3.30	2.24	265	2.58	2.61
70	2.90	2.37	270	2.59	2.59
75	2.70	2.65	275	2.59	2.62
80	2.68	2.60	280	2.60	2.73
85	2.65	2.62	285	2.65	3.58
90	2.64	2.59	290	3.00	5.12
95	2.62	2.59	295	3.80	8.32
100	2.61	2.58	300	5.50	8.16
105	2.60	2.57	305	6.50	9.42
110	2.59	2.56	310	7.60	8.80
115	2.58	2.55	315	8.05	8.45
120	2.57	2.57	320	8.20	8.33
125	2.57	2.57	325	8.25	8.38
130	2.57	2.57	330	8.30	8.43
135	2.57	2.57	335	8.35	5.67
140	2.57	2.57	340	8.47	8.60
145	2.57	2.57	345	8.52	8.76
150	2.57	2.52	350	8.61	8.72
155	2.55	2.28	355	8.65	8.76
160	2.45	1.79	360	8.69	8.77
165	2.20	1.54	365	8.72	8.80
170	1.95	1.82	370	8.75	8.83
175	1.90	2.70	375	8.78	8.81
180	2.20	2.73	380	8.79	8.82
185	2.40	2.53	385	8.80	8.85
190	2.45	2.64			
195	2.52	2.57			
200	2.54	2.57			

The Harrison profile passes through the interface of two layers of water and returns to the surface through the interface. If the interface is sharp, a step function in oxygen and temperature is experienced. The oxygen sensor would respond exponentially. Regresson analysis on the actual profile responses yielded time constants of 8.56 seconds for the step down and 8 seconds for the step up giving a mean of 8.28 seconds. These regressions had R^2 coefficients of 0.962 and 0.849 respectively. Figure 7 and Table II show the result of using the 8.28 second time constant in equation (25) for τ.

The effect of advancing the correction function by one step (sample interval) can be seen by comparing Figure 7 with Figure 8. The delay time is shortened by five seconds and the overshoot increases. This effect will be discussed later.

Once a corrected time series is generated, a corrected profile of oxygen versus depth is simple to generate. The advantage of this methodology is that delay and exponential response have been corrected so that the oxygen sensor's response now matches the pressure sensor's more closely and the distortion in the profile is minimized.

DISCUSSION AND CONCLUSIONS

Further laboratory and field tests will have to be carried out to resolve the discrepancy in the apparent time constants between the laboratory and field experience. The discrepancy might be explained by a pressure effect on the membrane and electrolyte, or the stretch applied to the membrane from one time to the next.

The importance of using the correct delay period in the technique is shown by the differences in Figure 7 and Figure 8. A better estimation of the response slope and a finer delay discrimination would be possible when the readings are sampled more rapidly, say, once a second. In this case the corrected funcion can be generated with:

$$V_{CN3M} = V_{N+7} + \frac{\tau}{3\Delta t}(V_{N+9} - V_{N+6}) \tag{26}$$

There is a correct averaging and delaying relationship for a given system. Otherwise the corrected response becomes either "spikey" or "sluggish". If the samples were taken once per second and equation (26) used, the corrected time series should fall between Figure 7 and Figure 8.

The importance of using this correction technique can be seen in the way it removes the sensor's delaying and lagging response time and sharpens the details of the response. A more subtle but equally important aspect of the correction technique is seen when the oxygen concentration is a gradient with depth. An example is visible in Figures 7 and 8 from the 330th second onward.

ACKNOWLEDGEMENTS

Acknowledgement is given to E. Harrison for his graphs and his cooperation in providing additional information about the graphs and how they were derived. N. M. Charlton's comments were beneficial in the review process.

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APPENDIX I

Program for Correcting Dissolved Oxygen Profiles

The following program for the Hewlett-Packard 9825 was generated to correct the time series response of a dissolved oxygen sensor having a time constant r_1 = 8.28 seconds while the profile is being sampled at D = 5 second intervals.

0: dim A[100]

Set up memory space for the data

1: for N = 1 to 80; ent "D.O.", A[N]

Start a looping process to hand enter the oxygen data values into a list.

2: dsp N, A[N]; wait 500

Display the sample number and the entered value

3: next N

Complete the looping process

4: 8.28 → r_1; 5 → D

Enter the time constant and the sampling intervals

5: for N = 1 to 80; prt "TIME", (N-1)*D,A[N]

Produce a check list of the oxygen data versus time

6: next N

Complete the list

7: For N = 2 to 77

Set up a new looping process

8: A[N+1] + (A[N+2] - A[N+1])*r1/D → V

Calculate the corrected value V for the oxygen reading.

9: `prt "T",D*N,"RAW",A[N],"COR",V;spc`

Print the time, corrected value and the original values

10: `next N`

Complete the looping

11: `end`

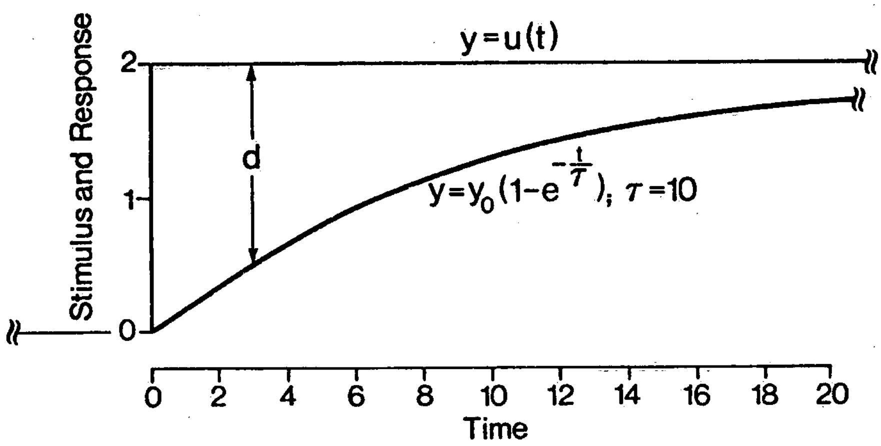

Figure 1a A step up stimulus and the exponential response

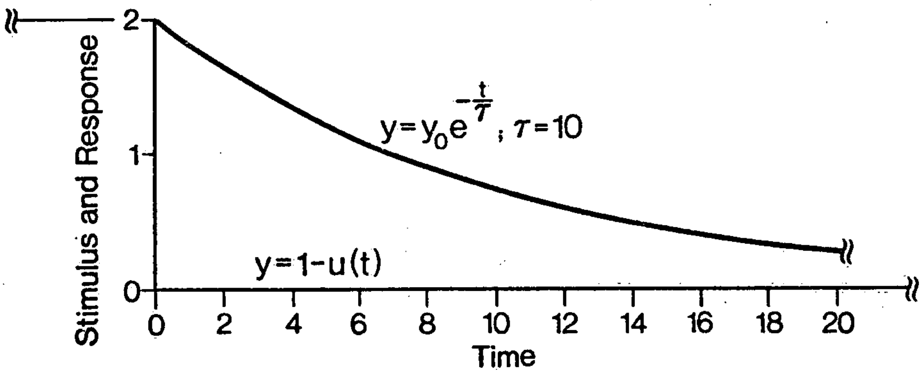

Figure 1b A step down stimulus and the exponential response

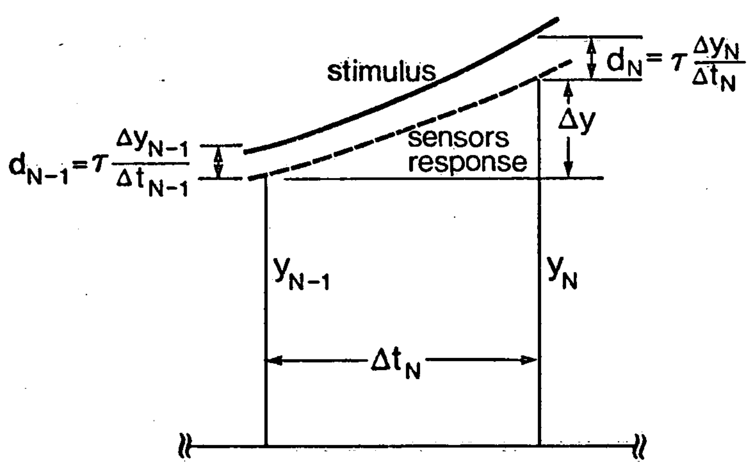

Figure 1c Time and response differences

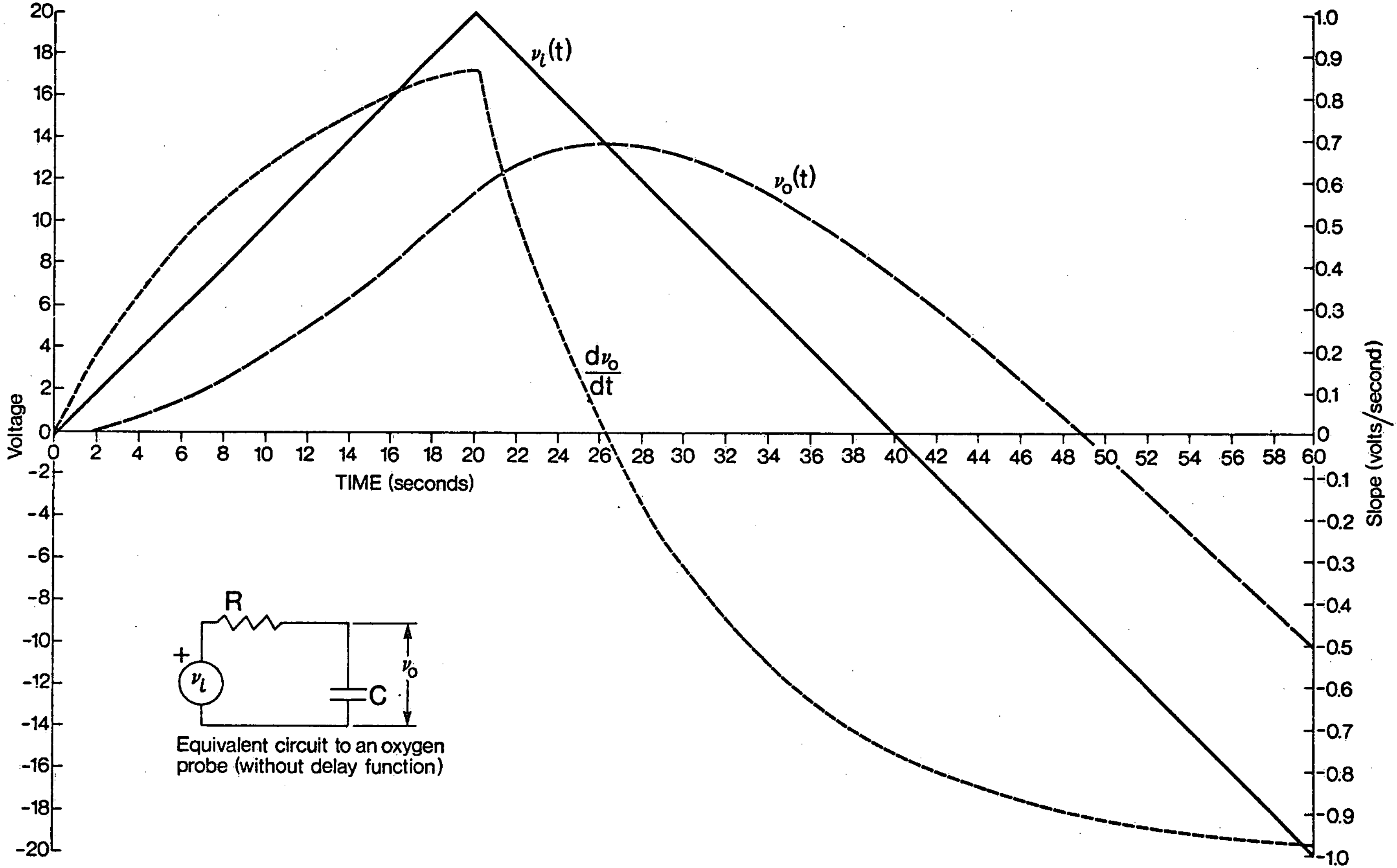

Figure 2 Response of lowpass filter to triangular input voltage time series

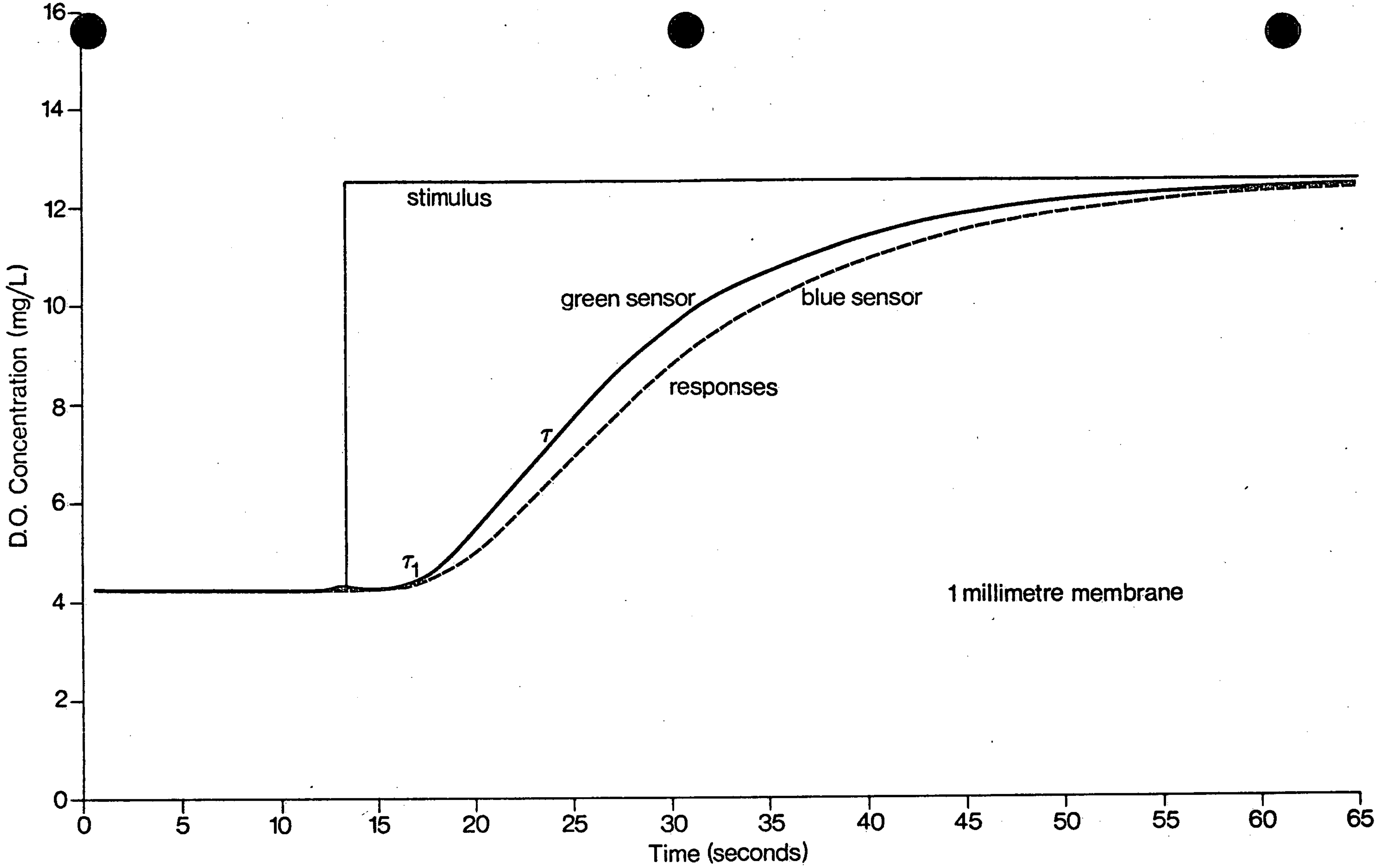

Figure 3 Laboratory Calibration Response to a Step Increase in Oxygen Concentration

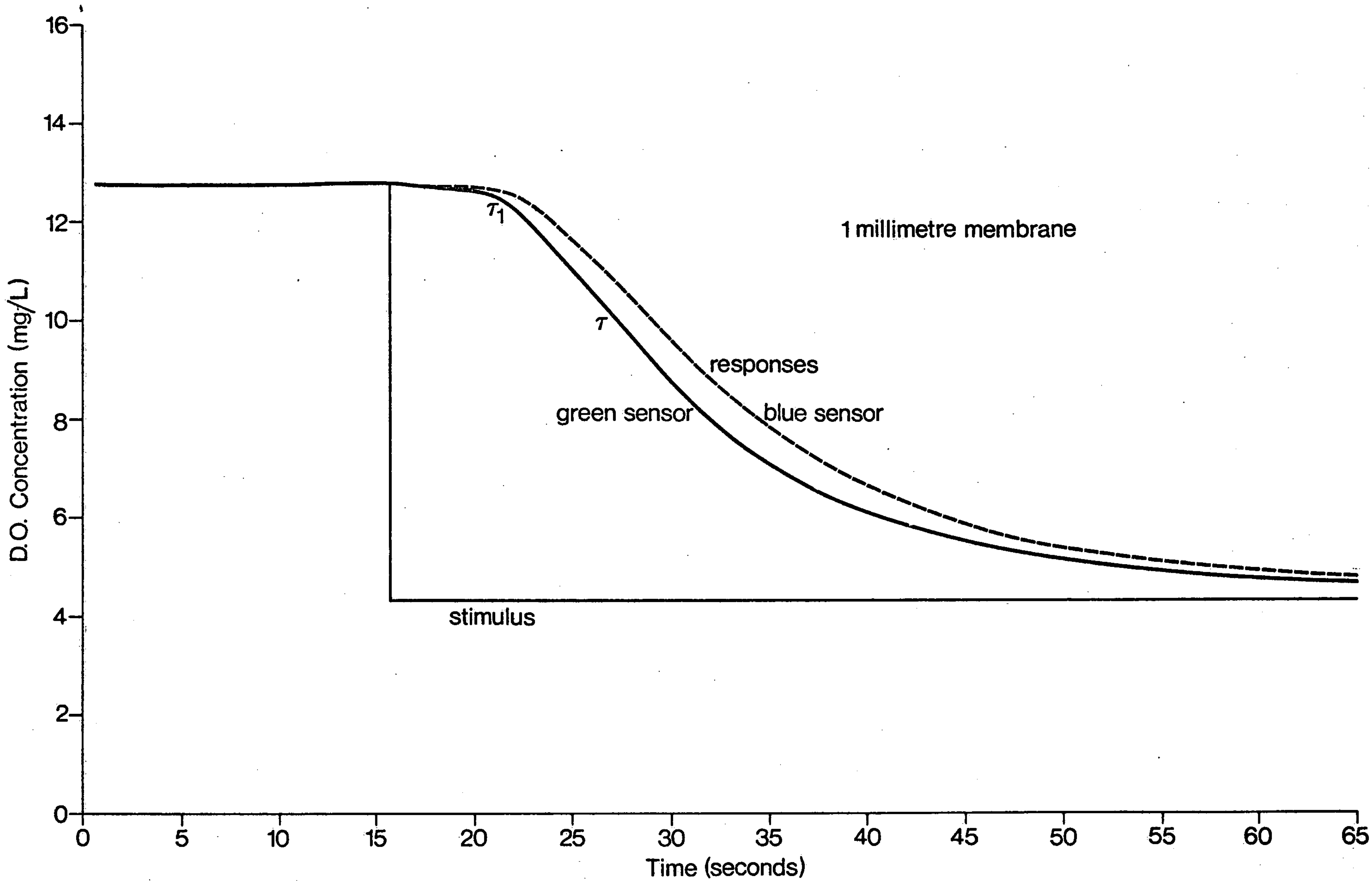

Figure 4 Laboratory Calibration Response to a Step Decrease in Oxygen Concentration

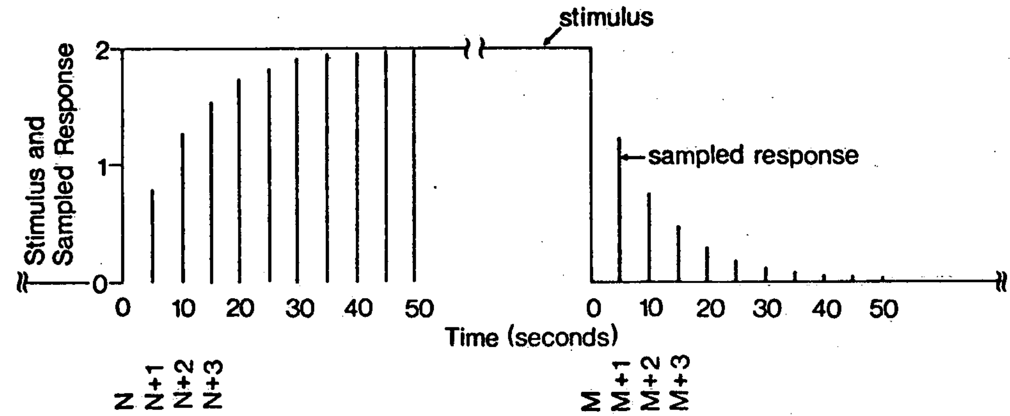

Figure 5a Stimulus and Sampled Values for exact Exponential Response

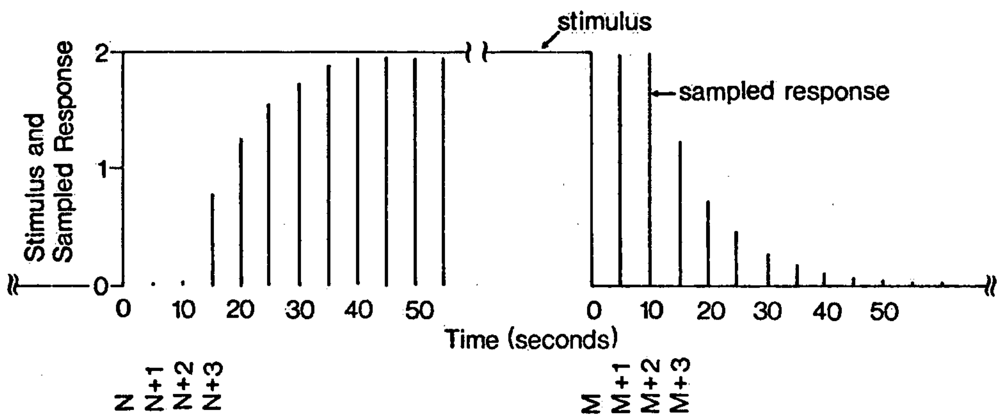

Figure 5b Stimulus and Sampled Values for a delayed Exponential Response

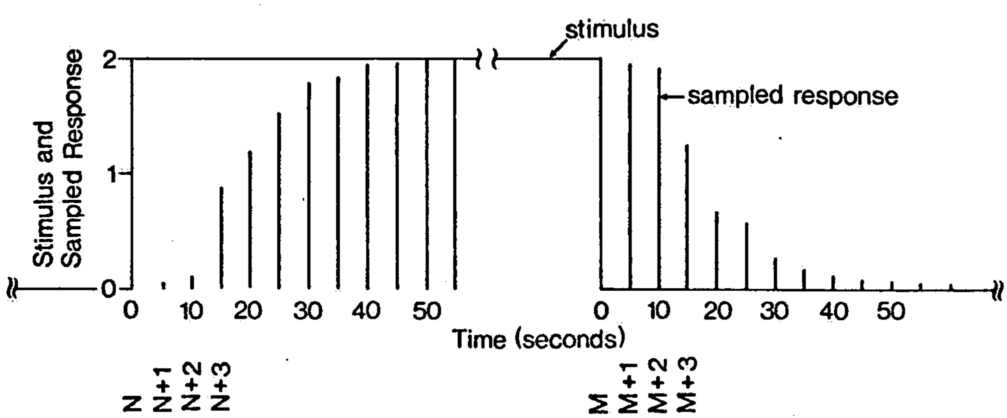

Figure 5c Stimulus and Sampled Values for a Simplified Oxygen Sensor Response (including noise)

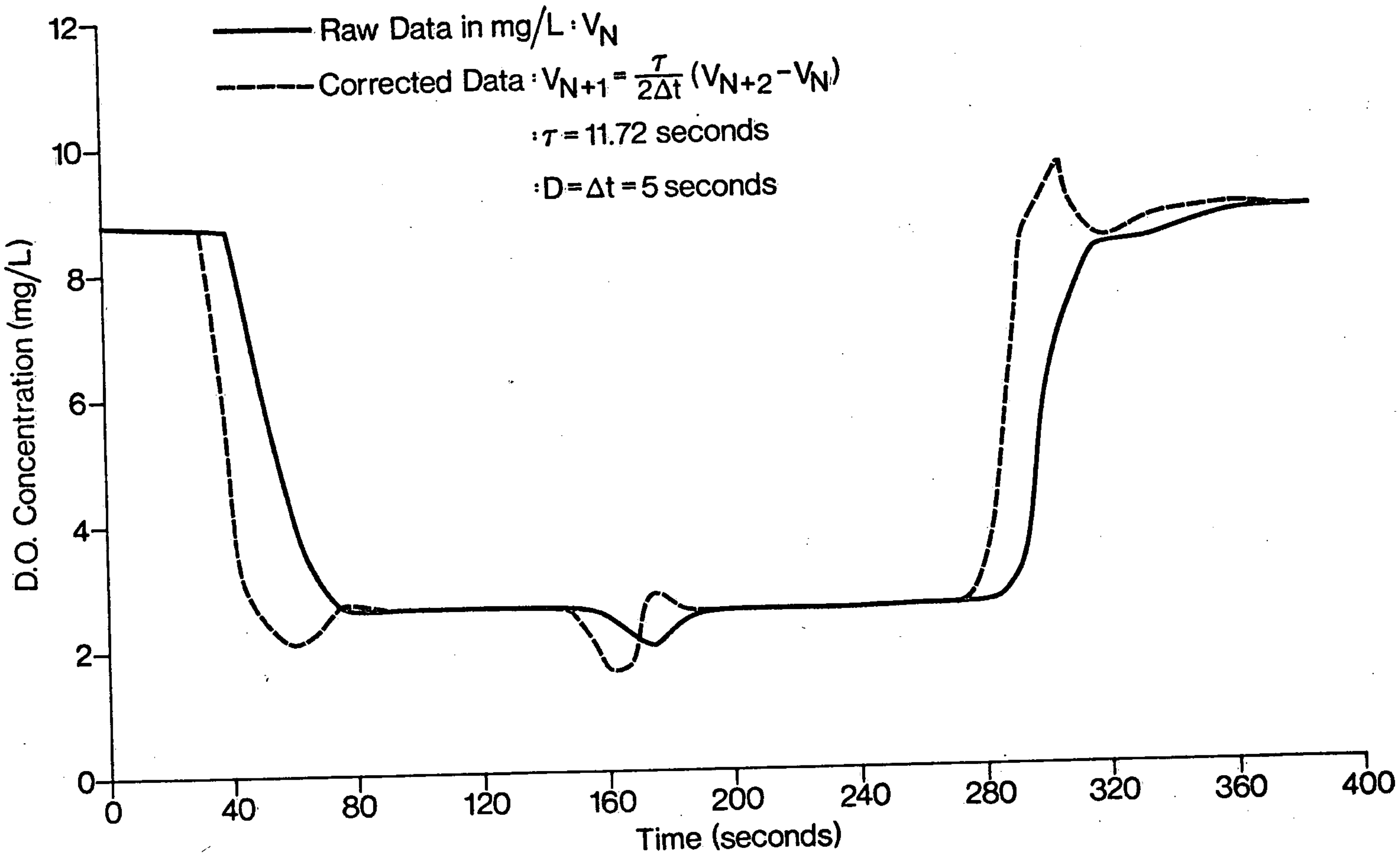

Figure 6 Time Series Plot of Oxygen Profile

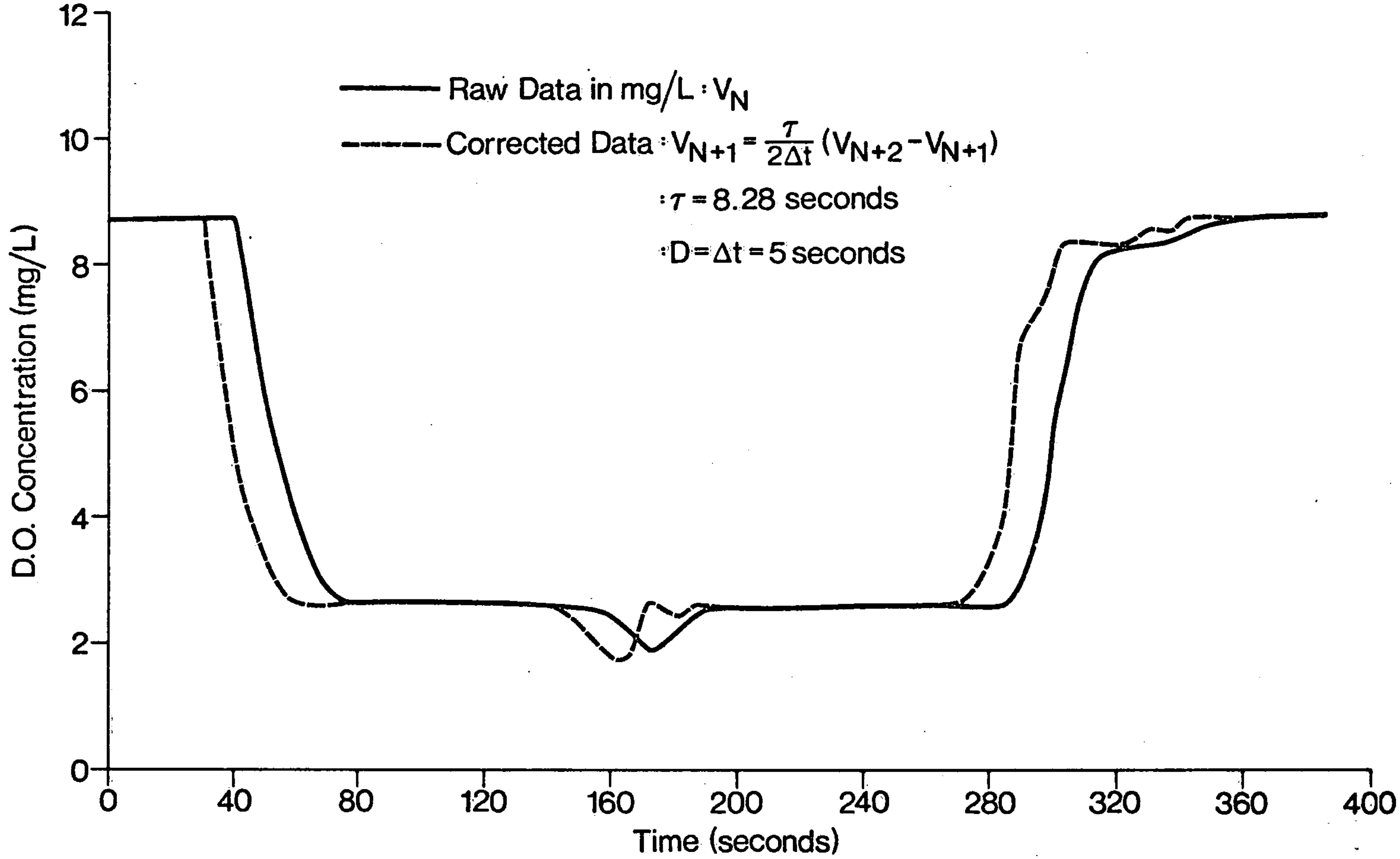

Figure 7 Time Series Plot of Oxygen Profile

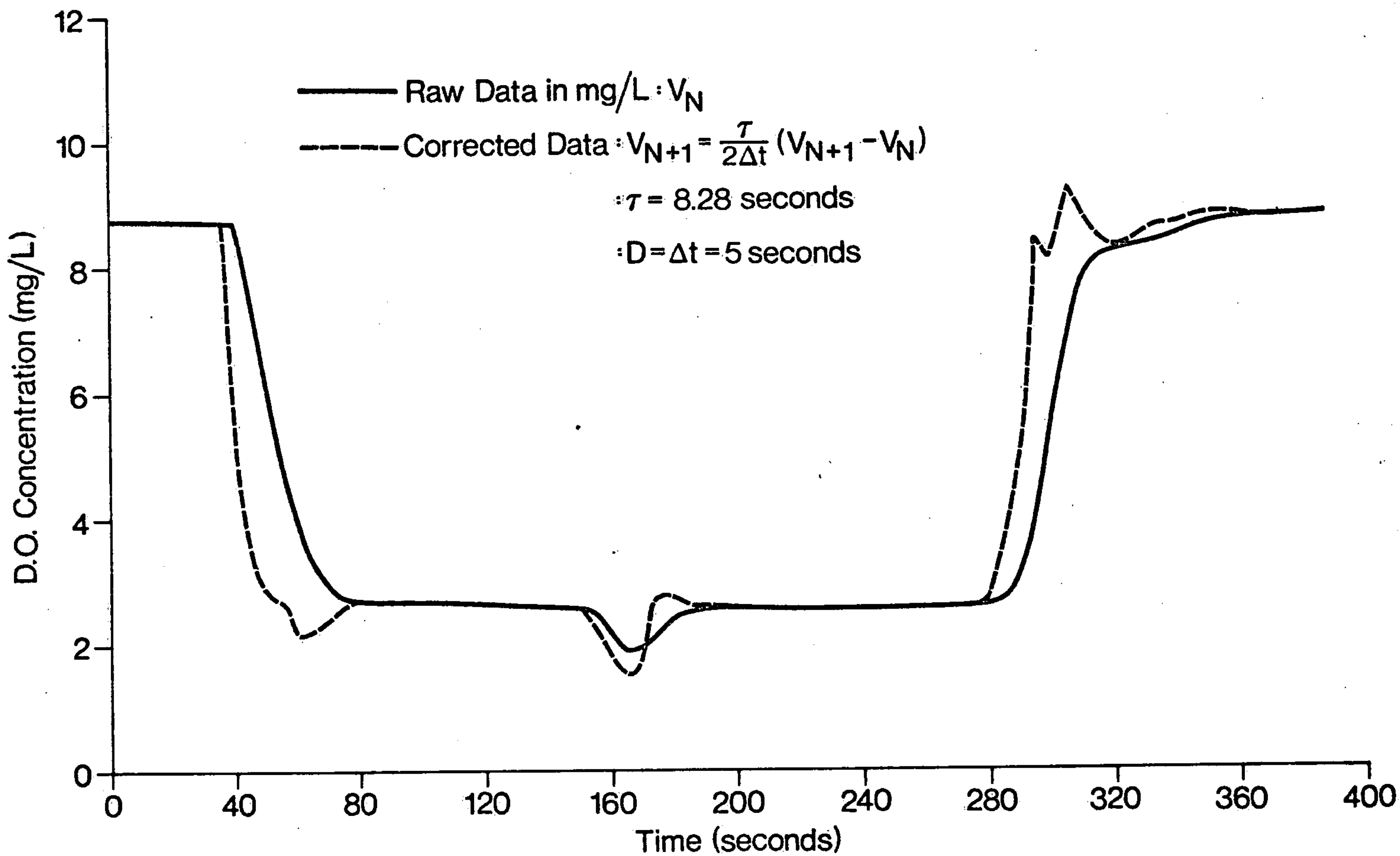

Figure 8 Time Series Plot of Oxygen Profile