BELTAOS

Environment Canada

Environnement Canada

National Water Research Institute

Institut National de Recherche sur les Eaux

DISPERSION IN TUMBLING FLOW

by

S. Beltaos

This manuscript has been submitted to
ASCE, J. of Hydraulics Division for publication
and the contents are subject to change.

This copy is to provide information
prior to publication.

DISPERSION IN TUMBLING FLOW

by

S. Beltaos

Environmental Hydraulics Section
Hydraulics Research Division
National Water Research Institute
Canada Centre for Inland Waters
April 1980

ABSTRACT

Steep mountain streams are often in the "tumbling" flow regime, that is alternating sub- and super-critical flow with energy dissipation caused mainly by sudden changes in slope and cross section. Though longitudinal dispersion characteristics of tumbling flows are important in time-of-travel, stream gauging, and contaminant spread considerations, very little pertinent information is available. The results of a laboratory study intended to provide preliminary data on tumbling flow dispersion are discussed herein. To simulate tumbling flow, a 1 m wide flume was divided into 11 pools by placing identical triangular weirs at 3 m intervals. Slugs of fluorescent dye were injected at the first weir and the resulting concentration-time variations measured downstream at every second weir. Test results are analyzed according to existing theoretical models and corresponding parameters evaluated. It is concluded that conventional river dispersion models can be successfully adapted to describe tumbling flow dispersion but it is unclear at this time whether storage dispersion models are realistic.

RÉSUMÉ

Les cours d'eau torrentiels de montagne sont souvent en régime d'écoulement "turbulent" où alternent des écoulement sous-critiques et surcritiques accompagnés d'une dissipation d'énergie due principalement à de brusques variations de pente et de profil. Malgré l'importance des caractéristiques de dispersion longitudinales des écoulements turbulents en ce qui touche le temps de parcours, le jaugeage des cours d'eau et la dispersion des contaminants, on dispose de très peu de données sur le sujet. Les résultats d'une étude de laboratoire visant à recueillir des données préliminaires sur la dispersion dans les écoulements turbulents sont analysés. Pour simuler les écoulements turbulents, on a divisé un canal de 1 m de largeur en 11 mouilles, en échelonnant des déversoirs triangulaires identiques, à des intervalles de 3 m. On a injecté des quantités de colorant fluorescent à la sortie du premier déversoir et mesuré les variations résultantes de concentration en fonction du temps à tous les deux déversoirs. Les résultats des essais sont analysés à l'aide de modèles théoretiques existants, et les paramètres correspondants, évalués. On conclut qu'il est possible d'adapter les modèles classiques de dispersion dans les cours d'eau pour décrire la dispersion dans les écoulement turbulents, mais il n'est pas encore établi si les modèles de stockage par dispersion son réalistes.

MANAGEMENT PERSPECTIVE

Most of our knowledge of dispersion in rivers has been obtained for relatively tranquil river or channel flow. Dispersion in rivers with attenuating pools and rapids is much less well known. This study indicates that there is a lot yet to be learned but nevertheless, for applied problems of dispersion in steep rivers, calculations could be made from existing knowledge provided some field data was first obtained.

The study adds to the base of expertise in river dispersion.

T. M. Dick
Chief
Hydraulics Division
June 9, 1980

PERSPECTIVE - GESTION

Une grande partie de ce que nous savons sur la dispersion fluviale porte sur des écoulements calmes dans des cours d'eau ou des canaux. La dispersion dans des cours d'eau où alternent mouilles et rapides est beaucoup moins connue. La présente étude indique qu'il reste beaucoup à appendre, mais, dans le cas de problèmes pratiques de dispersion dans les cours d'eau torrentiels, il serait possible d'effectuer des calculs à partir de nos connaissances actuelles à la condition de recueillir au préable des données sur le terrain.

L'étude complète les notions acquises sur la dispersion dans les cours d'eau.

T. M. Dick
Chef
Division d'hydraulique
9 juin 1980

DISPERSION IN TUMBLING FLOW

S. Beltaos[1]

INTRODUCTION

The study of dispersion processes in natural streams finds application in many areas of hydraulic engineering, such as environmental assessments, time-of-travel studies, and stream gauging. Longitudinal dispersion is commonly recognized as the final stage of the mixing process where cross-sectional concentration distributions become nearly uniform so that knowledge of the cross-sectional average concentration suffices for engineering purposes.

To date, much research has been carried out on longitudinal dispersion in prismatic laboratory channels and in many rivers (18, 6, 7, 8, 13, 4). However, little is known about dispersion in steep mountain streams that are characterized by alternating super- and sub-critical flow and by energy dissipation due to rapid changes in cross section and slope. The descriptive term "tumbling" has been applied to this type of flow (15, 10) and will be retained herein. Undoubtedly, the tumbling flow configuration has an effect on dispersion and it is the object of the present study to help elucidate this effect. To simulate the tumbling flow configuration, a rectangular flume was divided into a series of pools by placing triangular weirs at equal distances along the flume. The resulting flow pattern is considered a first approximation to the tumbling flow regime. In addition, the present experiments would be relevant to dispersion through a series of closely spaced dams in a river.

---

[1]Research Scientist, Hydraulics Research Division, National Water Research Institute, Canada Centre for Inland Waters, Burlington, Ontario, Canada; Formerly: Research Officer, Alberta Research Council, Edmonton, Alberta, Canada.

This paper presents the results of several dispersion tests in the aforementioned flume along with an analysis based on two different theoretical models.

THEORETICAL BACKGROUND

River Dispersion - Longitudinal dispersion in straight prismatic channels is well understood through previous work, both experimental and theoretical (18, 1, 5, 6, 17). The well known theory of Taylor (18) gives the cross-sectional average concentration, C, as

$$C = \frac{M}{2A\sqrt{\pi Dt}} \exp\left[-\frac{(x - Vt)^2}{4Dt}\right] \qquad (1)$$

in which M is the mass of tracer injected instantaneously at x (longitudinal distance)=0 and t (time)=0; A is the cross-sectional area of the flow; V is the average flow velocity; and D is the dispersion coefficient. Equation 1 applies only downstream of a certain stream length, herein termed the Taylor length, L_T. Because of the similarities between Taylor's theory of dispersion and Fick's laws of mass transfer by diffusion, this process has often been termed "Fickian dispersion". This description is retained herein.

Taylor's analysis for a circular pipe of radius a resulted in $D=10.1\ aV_*$ and Elder (5) calculated D as $5.9\ dV_*$ for two-dimensional open-channel flow of depth d, with V_* being the shear velocity. As a first approximation to natural stream processes, Fischer (6, 7, 8) calculated D for prismatic channels of large width-to-depth ratios; this resulted in a rather complex expression, which, with plausible assumptions, can be simplified to (11, 4):

$$\frac{D}{RV_*} = B\left(\frac{W}{R}\right)^2 \qquad (2)$$

in which R=hydraulic radius, W=channel width; and B= a dimensionless coefficient that depends on the cross-sectional geometry of the stream [see also Sooky (17)].

Theory has not been able to predict the length required for the dispersive process to become of the Fickian type. Fischer (7) used dimensional analysis and experimental results in prismatic channels to formulate the following equation:

$$L_T = 1.8\, W'^2\, V/RV_* \qquad (3)$$

in which W' is the horizontal distance between the maximum velocity location and the farthest bank; for symmetrical flow cross sections W' is equal to W/2.

Certain complications arose when the above understanding was applied to rivers, presumably caused by the rivers' characteristic lack of prismaticity and rectilinearity. To explain such complications, several investigators produced so-called dead-zone models (2, 9, 19, 14, 21, 22, 20). In these models, a river is assumed to consist of a core of essentially prismatic flow; at the boundaries of the core are attached series of stagnant fluid pockets. Dead-zone models have been able to predict certain observed effects, such as increased rates of spread and Taylor lengths. However, the geometrical and diffusive characteristics of the dead zones cannot be directly perceived (and thence measured) in rivers; rather, they have to be deduced from field test data.

To circumvent this difficulty and minimize the number of experimental parameters involved, Beltaos (4) assumed that the major effect of river non-uniformities is an increase in L_T. The resulting dispersion model assumes a prismatic channel (for ease of mathematics) in which, however L_T is allowed to exceed the value given by Eq. 3. Comparison of this model with numerous sets of field data was favourable and it was possible to explain frequently encountered non-Fickian dispersion. This model is briefly outlined below.

The model indicates the existence of three distinct dispersion ranges, delineated in terms of a characteristic channel length, L.

(i) **$x \lesssim L$:** In this range the concentration is

$$C = (M/\sqrt{2\pi\beta}\,Ax)\left\{(Vt/x)\exp\left[1-(Vt/x)\right]\right\}^{1/\beta} \qquad (4)$$

in which β is a dimensionless coefficient. From Eq. 4, it can be shown that the time to peak concentration, t_p, is simply

$$t_p = x/V \qquad (5)$$

As explained in (4), it is not known whether Eq. 5 remains valid beyond the range $x \lesssim L$ but it may be considered approximately correct for all x. Rearrangement of Eq. 4 results in

$$\frac{C}{C_p} = f(\tau) = \frac{1}{2}\left[\exp(-\tau\sqrt{\alpha\beta})\left\{1+\left[\exp(\sqrt{\alpha\beta})-1\right]\tau\right\}\right]^{1/\beta} \qquad (6)$$

in which C_p=peak concentration; $\tau=(t-t_1)/\Delta T$; $\Delta T=t_2-t_1$; t_1, t_2=times when $C=0.5\,C_p$ (see also Fig. 1 for a definition of various characteristics of the concentration-time curve); and α is a dimensionless coefficient such that $\alpha=\Delta T^2/\bar{\sigma}_t^2$ with $\bar{\sigma}_t^2$ being the variance of a C-t curve. For values of β less than about 0.2, α is very nearly equal to 5.55. For larger β's, α increases slightly with β as explained in Appendix III. Equation 6 shows that, in the range x L, C-t curves are similar, that is they define a single curve when C/C_p is plotted versus the modified time variable τ. The parameters C_p, ΔT and t_1 vary with x as indicated in the following equations.

$$\Delta T \simeq \sqrt{\alpha\beta}\, x/V \tag{7}$$

$$C_p\, \Delta T / \int_o^\infty Cdt \doteq \sqrt{\alpha/2\pi} = 0.94 \tag{8}$$

(note that, for a conservative tracer, $\int_o^\infty Cdt = M/Q$),

$$t_1 = t_p - \left\{ \frac{1}{\sqrt{\alpha\beta}} - \frac{1}{\exp(\sqrt{\alpha\beta}) - 1} \right\} \Delta T \tag{9}$$

(ii) **$x \gtrsim 3L$ ($=L_T$): -** In this range, the square of the temporal spread of C-t curves increases linearly with x; this behaviour is indicative of Fickian dispersion and Taylor's theory (Eq. 1) applies. The dispersion coefficient is related to β and L by

$$D = \beta L V \tag{10}$$

(iii) **$L \lesssim x \lesssim 3L$: -** This is an intermediate range for which an analytical solution for C(x,t) has not been found. Only an approxiamte method to predict C has been suggested (4), as outlined next.

- Compute ΔT from Eq. 11 below.

$$\Delta T^2 = 2\alpha\beta \left(\frac{L}{V}\right)^2 \left[\frac{x}{L} + e^{-x/L} - 1\right] \tag{11}$$

- Compute C_p from Eq. 8
- Compute t_p from Eq. 5
- Compute t_1 from Eq. 9
- Finally compute C/C_p and C using Eq. 6 and the values of C_p, ΔT, t_1 computed as above.

This method is based on the expectation that Eqs. 6, 8 and 9 remain approximately valid for x values well beyond L see also (3) . It should be noted here that Eq. 11 applies to all three dispersion ranges; for small x/L it simplifies to Eq. 7, while for large x/L it suggests Fickian behaviour ($d\Delta T^2/dx$=const.) and Taylor's theory (Eq. 1) applies with $D=\beta LV$ (Eq. 10). Considering the parameters β and L, Beltaos (4) argued that

$$\beta \propto \overline{u'^2}/V^2 \tag{12}$$

$$L \propto W^2 V/\varepsilon_Z \tag{13}$$

in which u'=local velocity deviation from the average flow velocity V; W=channel width; ε_Z= transverse mixing coefficient; and the overbar denotes an ensemble average. In most applications, the contribution of turbulence to this average can be neglected so that spatial averaging of time-average values is usually sufficient (8).

The model described above is a somewhat generalized version of Taylor's model; generalization consists of providing methods to predict C(x, t) prior to the Taylor length L_T. This range ($x \leq L_T$) has considerable practical significance as analysis of river data has shown (3, 4). Field values of L_T were between three and thirty times those that would have been predicted for corresponding prismatic channels (that is, channels with dimensions equal to the average dimensions of the rivers in question).

The models described so far have been tested in prismatic channels and in "normal" natural streams; the latter being channels with essentially uni-directional flow and where there is some, but not extreme, downstream fluctuation in geometry and velocity field. Clearly, steep mountain streams consisting of alternating pools and falls, represent an extreme case where such fluctuations predominate; a corresponding prismatic channel could perhaps be defined mathematically but would have little physical meaning.

Storage Dispersion - A theory that may be relevant to mountain stream dispersion has been advanced by McMullin and Weber (12) and independently by Kellerhals (10). This theory assumes a series of identical pools, each of volume v, joined by falls (or tubes) that carry a discharge Q (see also Fig. 2). Tracer injection occurs at t=0 so that the 0th pool is "loaded" with a mass M of tracer at a uniform concentration $C_o=M/v$. It is assumed that tracer entering any one pool is instantaneously mixed so that at any one time the concentration in any one pool is uniformly distributed.

Applying the principle of conservation of tracer mass for the ith pool gives

$$\frac{\partial C_i}{\partial t} = \frac{C_{i-1} - C_i}{T_R} \tag{14}$$

in which T_R is a time characteristic defined as

$$T_R = v/Q \tag{15}$$

Clearly, T_R is the residence time of the fluid in any one pool and could also be viewed as the time required for a fluid particle to travel the length of the pool (if A is the average cross-sectional area of each pool and ℓ its length, then $v=A\ell$ and $T_R=\ell/V$, with $V=Q/A$=average flow velocity).

For each value of i, an equation analogous to Eq. 14 can be written; with the initial condition described earlier, the resulting system of equations can be solved to give (10):

$$C_n = \frac{M}{n!v} \left(\frac{t}{T_R} \right)^n \exp \left(- \frac{t}{T_R} \right) \tag{16}$$

where C_n=concentration in the nth pool. From Eq. 16, the time t_p can be found as

$$t_p = n\,T_R \tag{17}$$

Considering that $T_R=\ell/V$ and taking for x the distance from the center of the nth pool to the center of the 0th pool, $(x=n\,\ell)$ gives

$$t_p = x/V \tag{18}$$

which coincides with Eq. 5, derived from Beltaos' dispersion model. The peak concentration in the nth pool is [after application of Stirling's approximation (16)].

$$C_{n,p} \simeq \frac{M}{v\sqrt{2\pi n}} \tag{19}$$

which shows $C_{n,p}$ to vary as $x^{-1/2}$. Dividing Eq. 16 with Eq. 19 and using Eq. 17 gives

$$C_n/C_{n,p} = (\theta e^{1-\theta})^n \tag{20}$$

in which $\theta=t/t_p$. Considering the "one-half" time spread of C_n-t curves, it can be shown from Eq. 20 that

$$\frac{\Delta T}{T_R} \simeq \sqrt{\alpha \frac{t_p}{T_R}} \tag{21}$$

in which $\alpha=5.55$ as found before. Clearly, storage dispersion is of the Fickian type (but for a different reason than does conventional shear flow dispersion). The corresponding dispersion coefficient can be shown to be:

$$D = 0.5\, T_R V^2 \tag{22}$$

It is noteworthy, that if the storage dispersion theory is applied to common stream dispersion using vanishingly small pool volumes, Eq. 22 shows that D would also vanish, that is, there would be no dispersion; the same can be derived from Eq. 15: If $v\rightarrow 0$, then $\ell\rightarrow 0$ and $C_{i-1}-C_i=-\ell(\partial C/\partial x)$; putting $T_R=\ell/V$, Eq. 15 would give $(\partial C/\partial t)+V(\partial C/\partial x)=0$, which is the convection equation with no dispersion. It may thus be concluded that storage dispersion occurs essentially because of the finite size of the pools and the assumed instantaneous mixing in these pools. In practice, this condition will be approached when the diffusivity of the fluid is very large so that the time required for full tracer mixing in any one pool is much less than the corresponding travel time. In this sense, the storage dispersion model may also be viewed as a series of dead zones connected by strong jets that produce very fast mixing.

The storage dispersion model was developed by Kellerhals (10) in conjunction with a field study of runoff concentration in steep channel networks. Tracer tests were performed as a means of stream gauging as well as defining the relationships between discharge, velocity and channel area. As a rule, only one or two concentration-time curves were obtained for each test while only three tests involved three sampling locations. Though this information is hardly adequate for concrete conclusions regarding dispersion characteristics, Kellerhals' preliminary application of the storage dispersion model indicated poor parameter consistency. When n and T_R were determined so as to optimize agreement between theory and data, it was found that n did not necessarily increase with x (see Eqs. 17 and 18) neither did T_R decrease with discharge (see Eq. 15). More satisfactory results were obtained from applications of Taylor's model.

EXPERIMENTS

To simulate the pool-fall sequence of steep mountain streams, 11 identical weirs were placed at 3.05 m intervals in a 0.90 m wide by 36.5 m long flume, as depicted in Fig. 3.

Before commencing tracer tests, the pool volume, ΔV, was measured carefully to define the volume-discharge relationship. It was found that, for a given discharge, all pool volumes were nearly identical and the average pool volume increased with discharge as indicated below:

$$\Delta V = 0.252 + 2.296\, Q^{0.42} \qquad (23)$$

in which Q is flow discharge in m^3/s and ΔV is pool volume in m^3; the volume corresponding to zero discharge is 0.252 m^3.

Tracer tests consisted of injecting known amounts of Rhodamine W.T. fluorescent dye at the 0th weir and sampling at the 2nd, 4th, 6th, 8th and 10th weirs to define the corresponding C-t curves. Dye concentration was recorded by means of a flow-through Turner fluorometer. To optimize accuracy, the injected dye mass was varied with flow discharge and downstream sampling location. Thus, measured concentrations are not directly comparable, but can be made such if they are divided by the corresponding values of $\int_0^\infty Cdt$.

Four different values of discharge were used, Q=0.0028 m^3/s, 0.0085 m^3/s, 0.017 m^3/s and 0.026 m^3/s. In all, 20 injections were performed, one for each C-t curve.

Table 1 summarizes hydraulic parameters for each set of tests and Fig. 4 shows the observed concentration-time curves (for consistency, concentrations have been divided by $\int_0^\infty Cdt$). Table 2 summarizes pertinent characteristics of concentration-time curves.

ANALYSIS OF RESULTS

Time of Travel - It was shown earlier that both the river and storage dispersion models indicate that the time to peak concentration, t_p, grows linearly with x at a rate dictated by the average flow velocity, V. Figure 5 reinforces this prediction for the present experiments. It is noted that individual plots of t_p versus x showed a virtual origin effect, that is, there was an initially rapid rate of growth of t_p that shortly afterwards became equal to 1/V. The virtual origin coordinates, x_o, are indicated in Fig. 5.

River Dispersion Model - To determine the parameters β and L required for application of the river dispersion model, Eq. 11 was used as explained in Appendix III; and to account for the virtual origin effect, x was replaced by x' ($=x-x_o$). From individual plots of ΔT versus x', it was noticed that data from different runs could be made to collapse on a single curve, if values of ΔT for each run were multiplied by a constant factor. This implies a common value of L for all runs but different values of β; after multiplication of the experimental values of ΔT with the appropriate factors, ΔT^2 was plotted versus x' for all runs and showed a linear trend beyong x'=9 m. This permitted direct evaluation of β and L as described in Appendix III. Table 3 summarizes the parameters found for the river dispersion model. Using values of β and L from Table 3 and hydraulic data from Table 1, the quantity $\Delta T/(L/V)/\sqrt{2\alpha\beta}$ was calculated and plotted versus x'/L in Fig. 6. The data points appear to collapse on a single curve, as suggested by Eq. 11.

To test whether the river dispersion model provides adequate predictions of C_p, experimental values of $\Delta T\ C_p/\int_o^\infty Cdt$ are listed in column (10) of Table 2; by Eq. 8, this quantity should be a constant, equal to 0.94. The observed values of $\Delta T\ C_p/\int_o^\infty Cdt$ are seen to range between 0.85 and 0.95 without evidence of a consistent increase or decrease with x. Prediction is thus accurate to within 12 percent.

Figure 6 shows that x'/L exceeds 1.0 for all sampling sites. It is thus of interest to examine whether the similarity property of C-t curves, implied by Eq. 6, persists beyond x'=L, as has been suggested in (4). Figure 7 shows the data for each run plotted in the form suggested by Eq. 6. With the exception of the first sampling site (2nd weir), Fig. 7 indicates that the various curves are approximately similar and adequately described by Eq. 6.

Finally, Eq. 9 is tested by plotting predicted values of t_1 versus observed ones in Fig. 8, which shows satisfactory agreement. It is noted that, to apply Eq. 9, ΔT and t_p were calculated from Eqs. 7 and 5 respectively.

Storage Dispersion Model - To evaluate the parameter T_R, Eq. 19 is rearranged by putting $M=Q \int_0^\infty Cdt$ and $v=T_R Q$. This gives

$$C_p / \int_0^\infty Cdt = (2\pi T_R t_p)^{-1/2} \tag{24}$$

in which the suffix n has been omitted since the RHS of the equation depends on the continuously varying quantity t_p. Figure 9 shows log $(C_p / \int_0^\infty Cdt)$ plotted versus log t_p. If Eq. 24 were exact, the data points would define straight lines of slope minus 1:2, that is, they would be parallel to the solid line drawn in Fig. 9. It is evident that Eq. 24 is only an approximation. Nevertheless, "best-fit" values of T_R were obtained from these data, and they are summarized in Table 4. It is seen that T_R decreases with increasing dicharge, as might have been expected, but is not equal to $\Delta V/Q$, the pool travel time. In turn, this implies that the pool volume required for the model to fit the data is not equal to the actual pool volume.

The model prediction for the "one-half" spread of C-t curves is tested in Fig. 10 where $\Delta T/T_R$ is plotted versus t_p/T_R, along with Eq. 21. It is seen that Eq. 21 provides fair predictions of ΔT.

Finally, it is noted that the quantity $T_R V^2/2$, which by Eq. 22 should be equal to D, is shown in column (5) of Table 4. Comparison with column (5) of Table 3 indicates fair agreement.

DISCUSSION

We have seen so far that, with proper choices of the parameters β, L and

T_R, both the river- and storage-dispersion models provide fair predictions of the observed dispersion process. However, this does not necessarily imply that the models are realistic; it should be possible to show, in addition, that the experimental values of β, L and T_R can be correlated with flow and channel characteristics in a plausible manner. This question is considered next.

Coefficient β - It has been suggested (4) that β should vary in proportion to the spatial velocity variance $\overline{u'^2}/V^2$ of the channel. In ordinary river flows, this parameter is primarily a function of V_*/V, being roughly proportional to $(V_*/V)^2$. It was thus possible to correlate β with V_*/V using published river data (3, 4); the value of β was generally less than 0.04, there being only one test with β as high as 0.07.

Inspection of Tables 1 and 3 indicates two differences between the present data and those obtained in ordinary rivers: Firstly, β has much larger values in the present case than the highest ordinary river value and, secondly, β appears to decrease when V_*/V increases. To explain these differences, consider the flow pattern in any one pool. The jet issuing from the upstream weir plunges into the pool and then grows along the flume bed, not unlike a wall jet. Because the water surface is relatively close to the flume bed, a vertical eddy forms above the wall jet. At the same time, the jet expands laterally since its initial width is less than the width of the flume; this causes formation of two horizontal eddies downstream of the weir. Moreover, additional eddies form upstream of the downstream weir as the flow separates from the boundaries to form the next jet. Thus, a significant portion of the pool volume is occupied by eddying motions which causes an extreme variation of the velocity field in all directions. This variation is not reflected in V_* which is calculated according to a formula borrowed from uniform flow theory. At the same time, we would expect that the velocity variance in any one pool exceeds by far corresponding values that occur in ordinary rivers.

An approximate calculation has shown that the velocity variance is given by (see Appendix III):

$$\frac{\overline{u'^2}}{V^2} \approx (1+\lambda)\frac{1-(1-\mu^2)r_R}{\left[1-(1+\mu)\,r_R\right]^2} - 1 \qquad (25)$$

in which r_R=fraction of pool volume occupied by reversed flow; λ =velocity variance in reversed flow region, assumed equal to that of the forward flow region for simplicity; and μ=ratio of average reverse speed to average forward speed ($\mu > 0$). Note that Eq. 25 is given here to illustrate trends rather than quantitative relationships. With plausible assumptions regarding the rates of vertical and horizontal jet expansions, r_R was found roughly to be 0.36, 0.39, 0.43 and 0.46 for Runs 1, 2, 3 and 4 respectively, that is r_R increases with Q. Examination of Eq. 25 indicated that the RHS increases with increasing r_R if μ and λ remain constant. In turn, this explains (tentatively) the tendency of β to increase with Q. Moreover, if μ and λ are "guessed" at a common value of 1/3 and r_R is taken as noted above, Eq. 25 may be used to compute the ratio $\beta/(\overline{u'^2}/V^2)$. This operation gave ratios of 0.07, 0.08, 0.08 and 0.06 for Runs 1, 2, 3 and 4 respectively. These values are close to each other, which is in fair agreement with the prediction of Eq. 12 and lie within the range found for river data [0.025 to 0.1 (3, 4)].

Length L - It has been argued (4) that $L \propto W^2V/\varepsilon_Z$, as indicated earlier. The transverse mixing coefficient ε_Z can be assumed equal to the product $u_M \ell_M$ where u_M is the r.m.s. value of the lateral velocities responsible for lateral mixing, and ℓ_M is a "mixing" length associated with such velocities. Assuming that u_M scales on the average flow velocity, V, gives $L \propto W^2/\ell_M$. It is not clear at present how ℓ_M relates to hydraulic parameters. The following is an attempt to elucidate this question.

Lateral mixing is caused primarily by two mechanisms: turbulence, for which ℓ_T would be proportional to H; and lateral convection by eddying motions, for which $\ell_M \propto W$. If the former effect predominates, then $L \propto W^2/H$, that is, L would decrease slightly with Q. On the other hand, if, as is more likely, lateral mixing is governed by the horizontal eddies, then $\ell_M \propto W$, that is, L would be a constant. In general, it could be stated that L should either be a constant or decrease slightly with increasing discharge. Essentially, this agrees with the present finding of a constant L. The experimental value of L/W is 2.9/0.9=3.2; this is very low relative to ordinary river data (3, 4), as might have been expected since eddies are not as prominent a feature in rivers as they are in the present tests.

Time Characteristic T_R - Ordinarily we would expect that T_R should be equal to $\Delta V/Q$, the actual pool residence time. Since this condition is not satisfied (see Table 4), it is difficult to decide what stream parameters T_R correlates with. The only conclusion that can be drawn at this time is that the ratio $T_R/\frac{\Delta V}{Q}$ increases with discharge, tending to assume a constant value of about 0.45.

Assessment of Dispersion Models - From the preceding discussion, it appears that the river dispersion model provides consistent results when applied to the present test data. It is thus concluded that, despite the extreme flow variations in the longitudinal direction, this model can realistically be used to predict dispersion in steep mountain streams; however, practical application requires field data to correlate field values of β and L with stream characteristics.

The storage dispersion model has the advantage of introducing only one experimental parameter; however, it is not clear at present whether this model is physically realistic. The agreement with the present data when T_R is suitably chosen may have been coincidental and only reflected the fact that the observed dispersion process was mostly Fickian.

SUMMARY AND CONCLUSIONS

The results of a laboratory study intended to provide preliminary understanding of dispersion in tumbling flow have been presented and discussed. The data were analyzed in terms of two dispersion models, a conventional river dispersion model and a storage dispersion model.

The following conclusions were drawn:

1. The time of travel in a series of pools and falls can be calculated in a straight-forward manner; it is equal to the time of travel for one pool times the number of pools.
2. With suitable choice of the dispersion parameters β and L, the river dispersion model provides satisfactory predictions of observed dispersion characteristics. Moreover, the dispersion parameters appear to correlate plausibly with hydraulic stream characteristics. Thus, the river dispersion model is deemed realistic for mountain stream dispersion; however, field data are needed to enable practical application.
3. With suitable choice of the pool residence time T_R, the storage dispersion model gives satisfactory predictions of dispersion characteristics. However, the experimental values of T_R are not physically plausible. Thus, it is unclear whether this model is realistic and its partial agreement with the data may have been fortuitous.

ACKNOWLEDGMENTS

A part of the work reported herein was performed under a continuing research program to study mixing characteristics of Alberta rivers; this program is carried out by the Transportation and Surface Water Engineering Division of Alberta Research Council in cooperation with Alberta Environment. Experiments were performed in the "T. Blench" hydraulics laboratory of the University of Alberta; the experimental set up had originally been prepared by R. Kellerhals (formerly Associate Professor, Civil Engineering Department) who kindly permitted the writer to use this set up for the dispersion tests. The tests were carried out with the assistance of G. Fraser. Review comments by T. M. Dick and Y. L. Lau are appreciated.

APPENDIX I. - REFERENCES

1. Aris, R. "On the Dispersion of a Solute in a Fluid Flowing Through a Tube," Proceedings of the Royal Society of London, London, England, Series A, Vol. 235, 1956, pp. 67-77.
2. Aris, R., "The Longitudinal Diffusion Coefficient in Flow Through a Tube with Stagnant Pockets," Chemical Engineering Science, Vol. 11, 1959, pp. 194-198.
3. Beltaos, S., "An Interpretation of Longitudinal Dispersion Data in Rivers," Internal Report No. SWE 78-3, Transportation and Surface Water Engineering Division, Alberta Research Council, Edmonton, Canada, 1978.
4. Beltaos, S., "Longitudinal Dispersion in Rivers," Journal of the Hydraulics Division, ASCE, Vol. 106, No. HY1, Proc. Paper 15118, January, 1980, pp. 151-172.

5. Elder, J. W., "The Dispersion of Marked Fluid in Turbulent Shear Flow," Journal of Fluid Mechanics, Vol. 5, Part 4, 1959, pp. 544-560.

6. Fischer, H. B., "Longitudinal Dispersion in Laboratory and Natural Streams," Thesis presented to the California Institute of Technology, at Pasadena, Calif., in 1966, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

7. Fischer, H. B., "The Mechanics of Dispersion in Natural Streams," Journal of the Hydraulics Division, ASCE, Vol. 93, No. HY6, Proc. Paper 5592, Nov. , 1967, pp. 187-216.

8. Fischer, H. B., "Longitudinal Dispersion and Turbulent Mixing in Open Channel Flow," Annual Review of Fluid Mechanics, Vol. 5, 1973, pp. 59-77.

9. Hays, J. R., "Mass Transport Mechanisms in Open Channel Flow," Thesis presented to Vanderbilt University, at Nashville, Tenn., in 1966, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

10. Kellerhals, R., "Runoff Concentration in Steep Channel Networks," Thesis presented to the University of British Columbia, at Vancouver, B. C., Canada, in 1969, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

11. Liu, H., "Predicting Dispersion Coefficients in Streams," Journal of the Environmental Engineering Division, ASCE, Vol. 103, No. EE1, Proc. Paper 12724, Feb., 1977, pp. 59-69; see also closure by H. Liu, Journal of the Environmental Engineering Division, ASCE, Vol. 104, No. EE4, Proc. Paper 13908, Aug., 1978, pp. 825-828.

12. MacMullin, R. B. and Weber, M., Jr., "The Theory of Short-Circuiting in Continuous Flow Mixing Vessels in Series and the Kinetics of Chemical Reactions in Such Systems," Transactions of American Institute of Chemical Engineers, Vol. 31, 1935, pp. 409-458.

13. Nordin, C. F. and Sabol, G. V., "Empirical Data on Longitudinal Dispersion in Rivers," USGS Water Resources Investigations 20-74, United States Geological Survey, Washington, D. C., 1974.

14. Okubo, A., "Effects of Shoreline Irregularities on Streamwise Dispersion in Estuaries and Other Embayments," Netherlands Journal of Sea Research, 6 (1-2), 1972, pp. 213-224.

15. Peterson, D. F. and Mohanty, P. K., "Flume Studies of Flow in Steep, Rough Channels," Journal of the Hydraulics Division, ASCE, Vol. 86, No. HY9, Proc. Paper 2653, Sept., 1960, pp. 55-76.

16. Selby, S. M., "Standard Mathematical Tables," The Chemical Rubber Company, 18th edition, 1970, p. 49.

17. Sooky, A., "Longitudinal Dispersion in Open Channels," Journal of the Hydraulics Division, ASCE, Vol. 95, No. HY4, Proc. Paper 6697, July, 1969, pp. 1327-1345.

18. Taylor, G. I., "The Dispersion of Matter in Turbulent Flow Through a Pipe," Proceedings of the Royal Society of London, London, England, Series A, Vol. 223, 1954, pp.446-468.

19. Thackston, E. L. and Schnelle, K. B., Jr., "Predicting Effects of Dead Zones on Stream Mixing," Journal of the Sanitary Engineering Division, ASCE, Vol. 96, No. SA2, Proc. Paper 7218, Apr., 1970, pp. 319-331.

20. Tsai, Y. H. and Holley, E. R., "Temporal and Spatial Moments for Longitudinal Mixing in Prismatic Channels with Storage in Separation Zones," Hydraulic Engineering Series, Report No. 35, Department of Civil Engineering, University of Illinois at Urbana - Champaign, Urban, Ill., Aug., 1979.

21. Valentine, E. M. and Wood, I. R., "Longitudinal Dispersion with Dead Zones," Journal of the Hydraulics Division, ASCE, Vol. 103, Proc. Paper 13208, Sept., 1977, pp. 975-990.

22. Valentine, E.M. and Wood, I. R., "Dispersion in Rough Rectangular Channels," Journal of the Hydraulics Division, ASCE, Vol. 105, No.HY12, Proc. Paper 15088, Dec., 1979., pp. 1537-1553.

APPENDIX II. - NOTATION

A	=	cross-sectional area
a	=	pipe radius
B	=	dimensionless coefficient
C	=	tracer concentration
D	=	dispersion coefficient
d	=	two-dimensional flow depth
f	=	a function
g	=	acceleration of gravity
H	=	average flow depth
i	=	suffix indicating pool number
k	=	a constant
L	=	a channel length
L_T	=	stream length required for onset of Fickian dispersion
ℓ	=	pool length
ℓ_M	=	lateral mixing length
M	=	injected tracer mass
n	=	suffix indicating pool number
Q	=	flow discharge
R	=	hydraulic radius
r_R	=	fraction of pool volume occupied by reversed flow
S	=	flume slope
T_R	=	pool residence time
t	=	time from injection of tracer
u_M	=	root-mean-square of velocities responsible for lateral mixing

u'	=	deviation of longitudinal velocity from average flow velocity
V	=	average flow velocity
V_*	=	shear velocity
v	=	pool volume in storage dispersion model
W	=	channel width
x	=	channel length downstream of injection site
α	=	a constant
β	=	dimensionless coefficient
ΔT	=	duration of concentrations exceeding one-half of the peak concentration
ε_Z	=	transverse mixing coefficient
θ	=	dimensionless time, t/tp
λ	=	velocity variance in the pool region of reversed flow; assumed equal to that of the forward flow region
μ	=	ratio of reversed flow average speed to forward flow average speed
σ_t^2	=	variance of a concentration-time curve
τ	=	modified time variable

APPENDIX III. - DERIVATIONS

(i) Calculation of α: This coefficient may be determined using Eq. 6 and stipulating that at $t=t_p$, $C=C_p$ and $df/d\tau=0$. This operation leads to a trial-and-error solution for α as a function of β. When $\beta \to 0$, $\alpha \to 8 \ln 2 = 5.545$ while for usual values of β, α remains very close to 5.55, increasing to 5.633 at $\beta=0.20$ and 5.986 at $\beta=1.00$.

(ii) Evaluation of β and L: This may be accomplished by optimizing agreement between Eq. 11 and experimental values of ΔT. In general,a trial-and-error process is required in which the value of L is guessed and the corresponding value of β is determined to fit the data best with the particular choice of L; this is repeated with other values of L and that pair (β, L) is selected which gives best overall agreement with the data. However, the evaluation of β and L can be direct in two special, but frequent, cases:

1. If a plot of ΔT versus x is linear, the data are in the range $x<L$ and the value of $d\Delta T/dx$ can be used to calculate β, taking into account Eq. 7. All that can be said about L is that it should exceed the length of the study reach, since the dispersion process does not depend on L in the range $x \lesssim L$ (see Eq. 4).

2. If a plot of ΔT^2 versus x becomes linear beyond a certain value of x, the data extend into the range $x \gtrsim 3L$ where $\Delta T^2 \simeq 2\alpha\beta(L/V)^2 \left[(x/L) - 1\right]$ (see also Eq. 11). This shows that extrapolation of the linear portion of a ΔT^2-x graph intersects the x-axis at x=L. This property may be used to find L and the slope of the linear portion of the graph may then be used to calculate β.

(iii) Calculation of velocity variance $\overline{u'^2}/V^2$: Let $\Delta\forall_1$ and $\Delta\forall_2$ be the volumes of the forward and reversed flow regions respectively in any one pool. Then $\Delta\forall_1 + \Delta\forall_2 = \Delta\forall$ and $r_R = \Delta\forall_2 / \Delta\forall$. Further, let u_1 and u_2 denote forward and reversed speeds respectively ($u_1 \geq 0$, $u_2 \geq 0$). Since $\overline{u'^2} = (1/\Delta\forall) \int_{\Delta\forall} u'^2 d\forall$ we may write:

$$\overline{u'^2} = \overline{(u_1 - V)^2}\,(1 - r_R) + \overline{(-u_2 - V)^2}\; r_R \tag{26}$$

Moreover, the average flow speed, V, is

$$V = \overline{u_1}\,(1 - r_R) - \overline{u_2}\; r_R \tag{27}$$

Using Eq. 27, Eq. 26 may be rearranged as

$$\overline{u'^2} = \overline{u_1^2} - V^2 + r_R\,(\overline{u_2^2} - \overline{u_1^2}) \tag{28}$$

If λ_1 and λ_2 are the velocity variances for the forward and reversed flow regions respectively, Eq. 28 may be rearranged, taking into account Eq. 27, to read:

$$\frac{\overline{u'^2}}{V^2} = \frac{(1 + \lambda_1) - r_R\left[(1 + \lambda_1) - \mu^2\,(1 + \lambda_2)\right]}{\left[1 - r_R\,(1 + \mu)\right]^2} - 1 \tag{29}$$

in which $\mu = \overline{u_2}/\overline{u_1}$. For $\lambda_1 = \lambda_2 = \lambda$, Eq. 29 simplifies to Eq. 25.

TABLE 1. - Hydraulic Parameters of Test Runs

Run No. (1)	Discharge (m^3/s) (2)	Pool volume, $\Delta\forall$ (m^3) (3)	$\frac{\Delta\forall}{Q}$ (min) (4)	V (m/s) ($=\ell Q/\Delta\forall$) (5)	Average Cross-Sectional Area[1] (m^2) (6)	Average Depth[2] (m) (7)	$\frac{V_*}{V}$[3] (8)
1	0.0028	0.45	2.63	0.019	0.15	0.16	6.56
2	0.0085	0.56	1.10	0.046	0.18	0.20	3.07
3	0.017	0.67	0.65	0.078	0.22	0.24	1.99
4	0.026	0.75	0.49	0.11	0.25	0.27	1.56

[1]Defined as: pool volume/pool length

[2]Defined as: pool volume/(pool width x pool length)

[3]Shear velocity defined as $\sqrt{gHS}$; g=acceleration of gravity, H=average depth, S=flume slope =0.01

TABLE 2. - Summary of Observed Dispersion Characteristics

Discharge (m^3/s) (1)	Weir No. (2)	x (m) (3)	t_p (min) (4)	t_1 (min) (5)	ΔT (min) (6)	C_p (μg/ℓ) (7)	$\int_0^\infty Cdt$ (μg min/ℓ) (8)	$\frac{C_p}{\int_0^\infty Cdt}$ (min^{-1}) (9)	$\frac{C_p \Delta T}{\int_0^\infty Cdt}$ (10)
0.0028	2	6.10	4.00	2.89	3.19	3.33	12.49	0.267	0.85
	4	12.19	9.09	6.40	5.58	3.10	19.15	0.162	0.90
	6	18.29	14.15	10.80	7.50	3.21	25.87	0.124	0.93
	8	24.38	19.97	15.50	9.50	3.89	39.81	0.098	0.93
	10	30.48	23.92	19.62	10.10	3.48	37.84	0.092	0.93
0.0085	2	6.10	1.28	0.89	1.65	3.79	6.97	0.544	0.90
	4	12.19	3.63	2.66	2.68	3.30	9.97	0.331	0.89
	6	18.29	6.44	4.86	3.53	4.37	16.99	0.257	0.91
	8	24.38	8.36	6.42	4.58	4.06	21.35	0.190	0.87
	10	30.48	9.12	7.32	4.45	3.30	16.75	0.197	0.88
0.017	2	6.10	0.86	0.43	0.96	3.28	3.68	0.894	0.86
	4	12.19	2.09	1.18	1.84	3.10	6.36	0.487	0.90
	6	18.29	3.36	2.35	2.33	2.56	6.49	0.394	0.92
	8	24.38	4.52	3.52	2.67	2.90	8.29	0.351	0.94
	10	30.48	5.77	4.43	2.93	2.68	8.43	0.317	0.93
0.026	2	6.10	0.48	0.31	0.61	3.27	2.33	1.402	0.86
	4	12.19	1.56	1.08	1.09	2.88	3.29	0.875	0.95
	6	18.29	2.65	1.89	1.68	3.45	6.46	0.535	0.90
	8	24.38	3.35	2.57	2.08	3.18	7.25	0.439	0.91
	10	30.48	4.46	3.50	2.18	3.27	8.13	0.400	0.87

TABLE 3. - Parameters of River Dispersion Model

Q (m^3/s) (1)	V (m/s) (2)	β (3)	L (m) (4)	D (m^2/s) (5)
0.0028	0.019	0.170	2.90	0.010
0.0085	0.046	0.224	2.90	0.030
0.017	0.078	0.275	2.90	0.062
0.026	0.11	0.269	2.90	0.082

TABLE 4. - Parameters of Storage Dispersion Model

Q (m^3/s) (1)	T_R (min) (2)	$\frac{\Delta \forall}{Q}$ (min) (3)	$\frac{T_R Q}{\Delta \forall}$ (4)	$T_R V^2/2$ (m^2/s) (5)
0.0028	0.76	2.63	0.29	0.0084
0.0085	0.45	1.10	0.41	0.029
0.017	0.29	0.65	0.45	0.053
0.026	0.21	0.49	0.44	0.070

LIST OF FIGURES

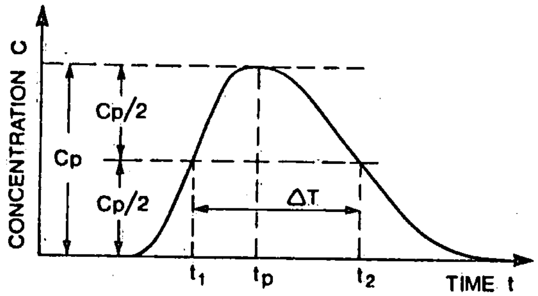


Fig. 1 Definition sketch; concentration time curve at a fixed downstream location

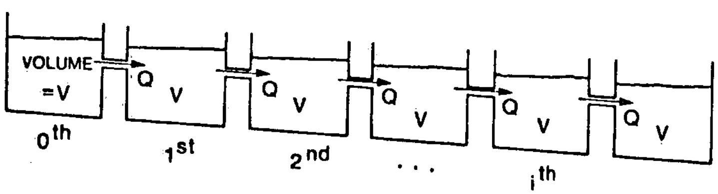
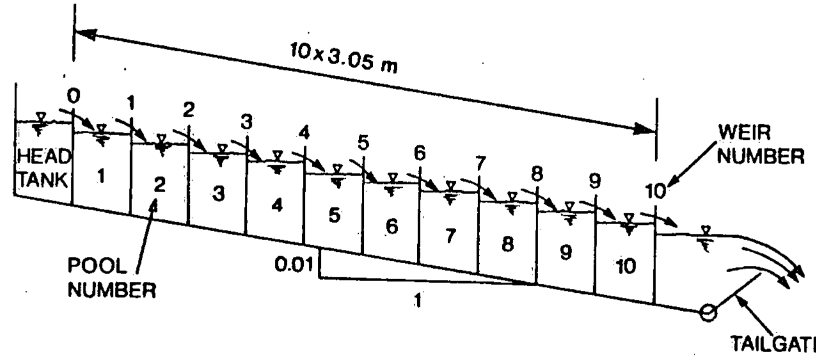


Fig. 2 Schematic illustration of storage dispersion concept

(a) Longitudinal Profile



(b) Flume cross-section and weir details

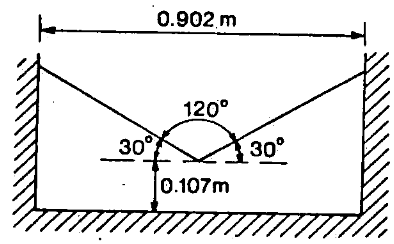


Fig. 3 Schematic illustration of experimental set up

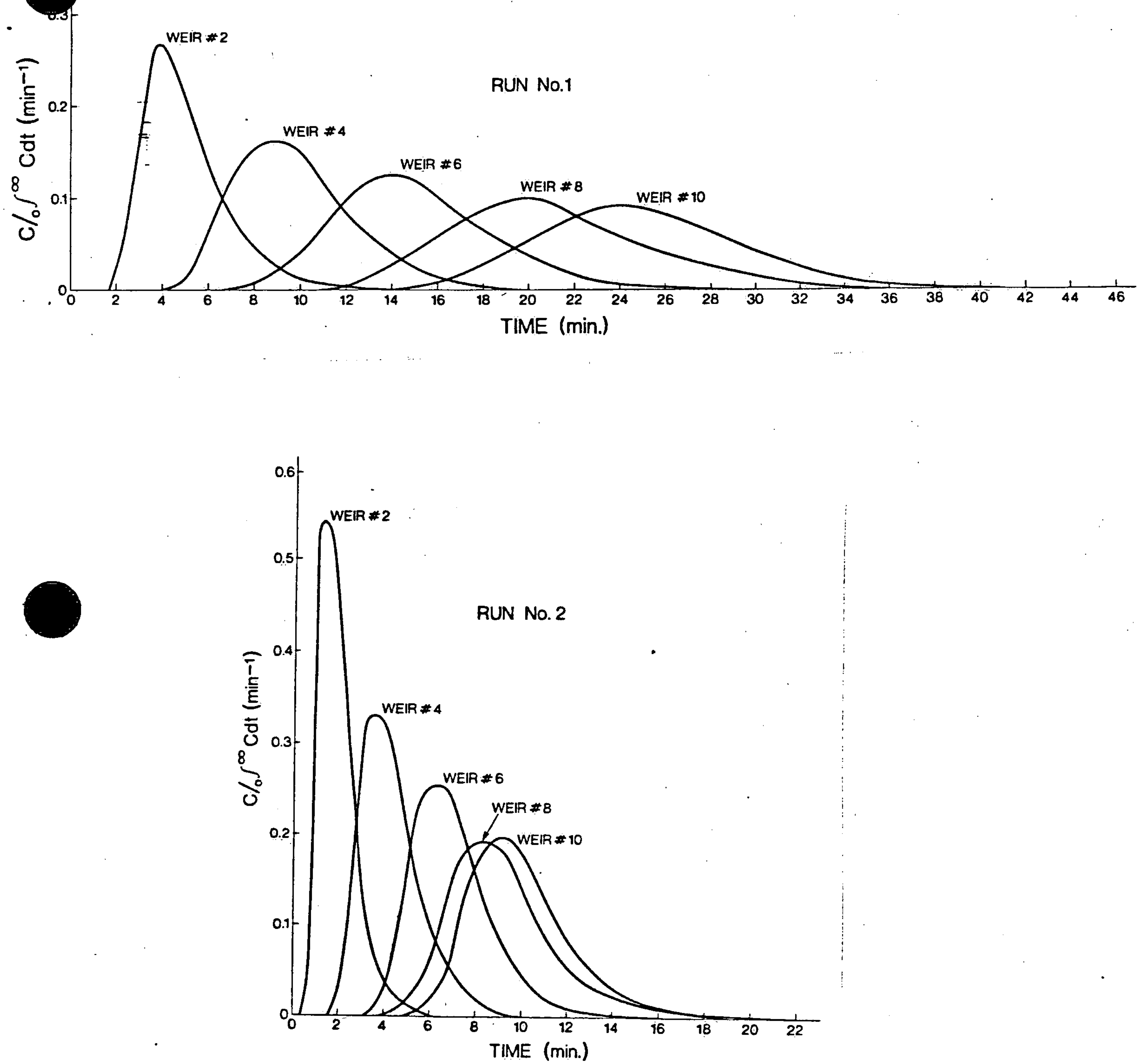
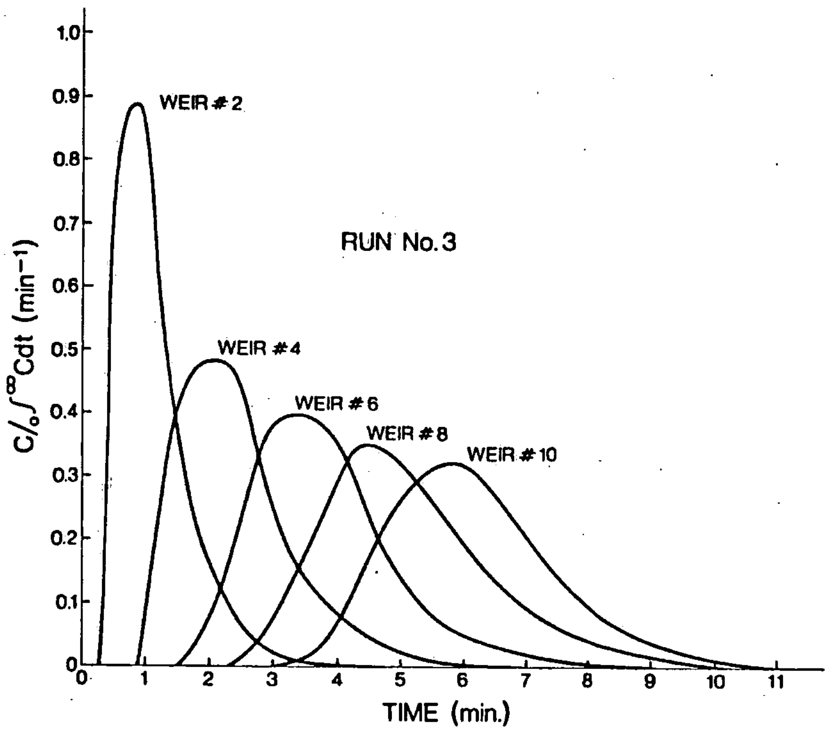


Fig. 4 Observed concentration-time curves



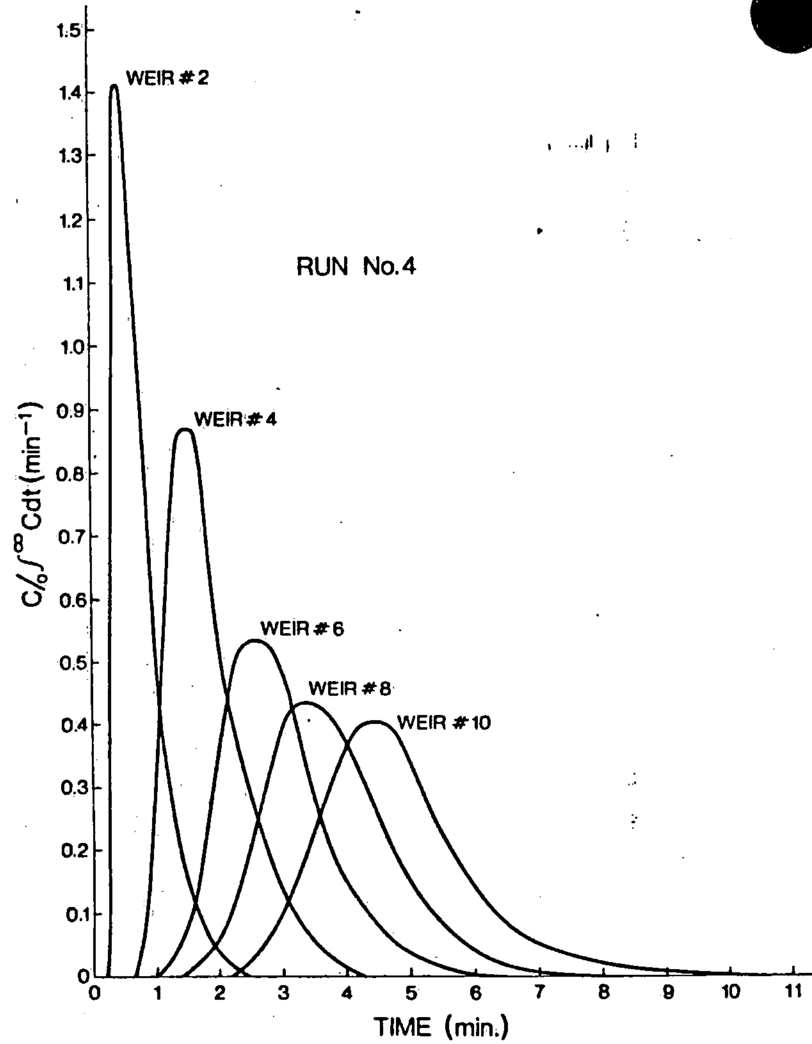


Fig. 4 cont. Observed concentration-time curves

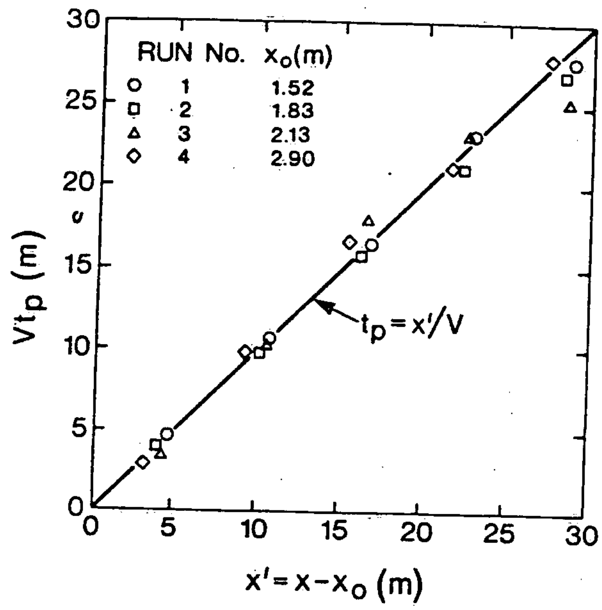


Fig. 5 Downstream growth of t_p

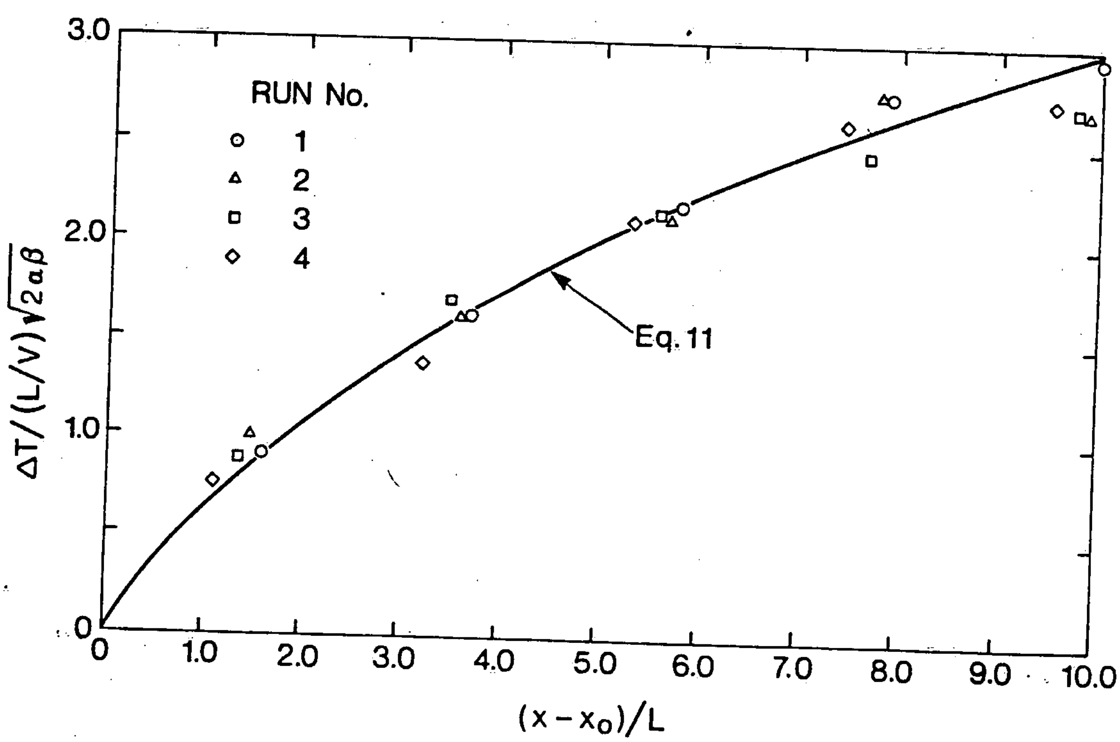


Fig. 6 Test of Eq. 11; river dispersion model

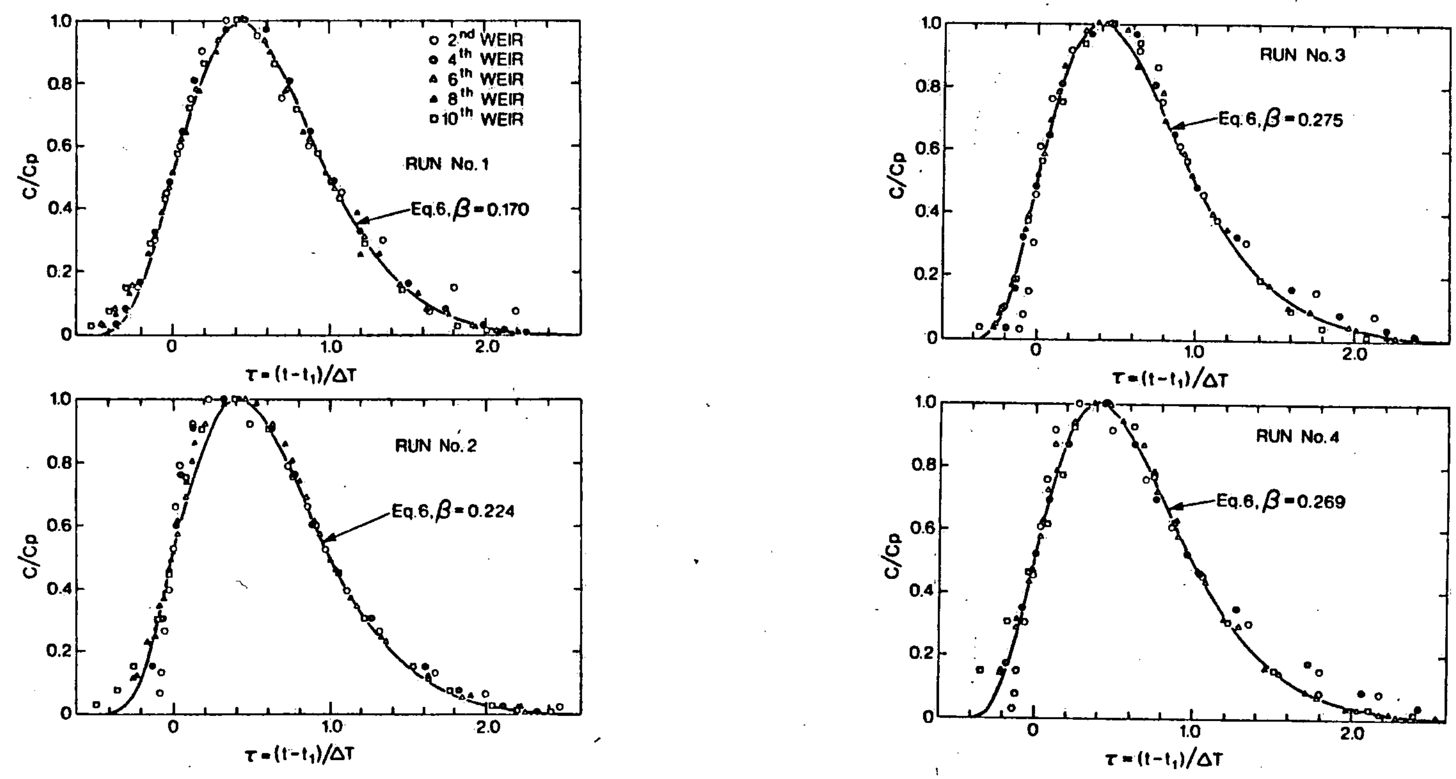


Fig. 7 Similarity of concentration-time curves; river dispersion model

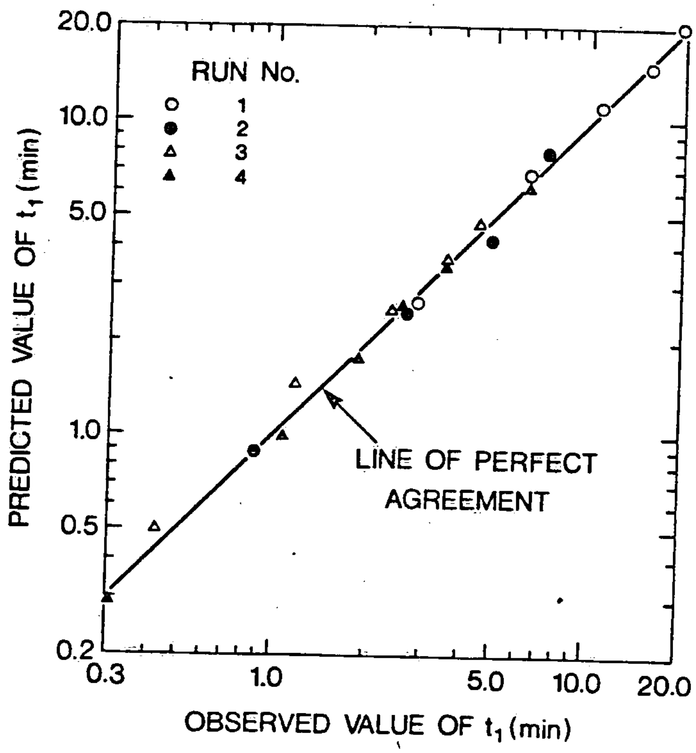


Fig. 8 Test of Eq. 9; river dispersion model

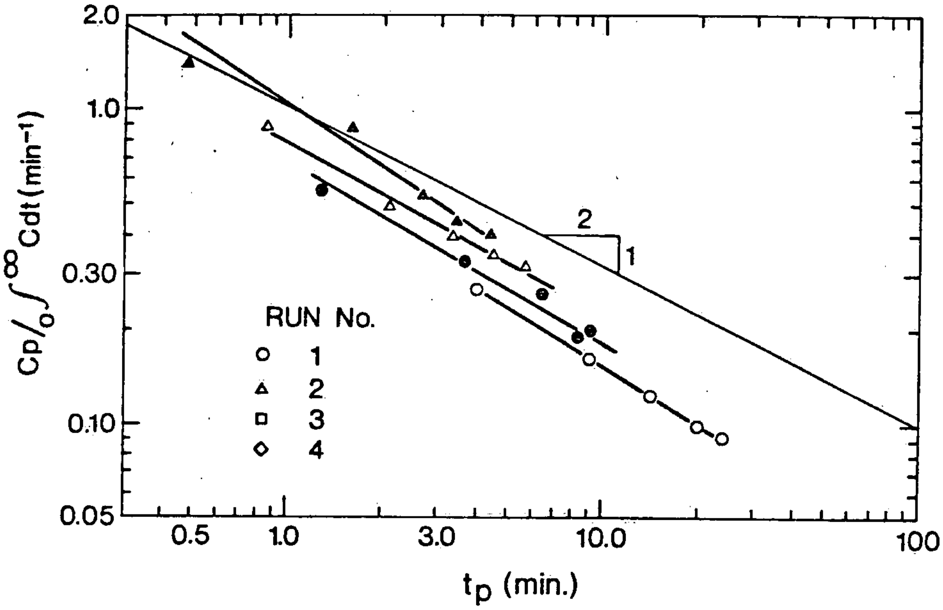


Fig. 9   Test of Eq. 24; storage dispersion model

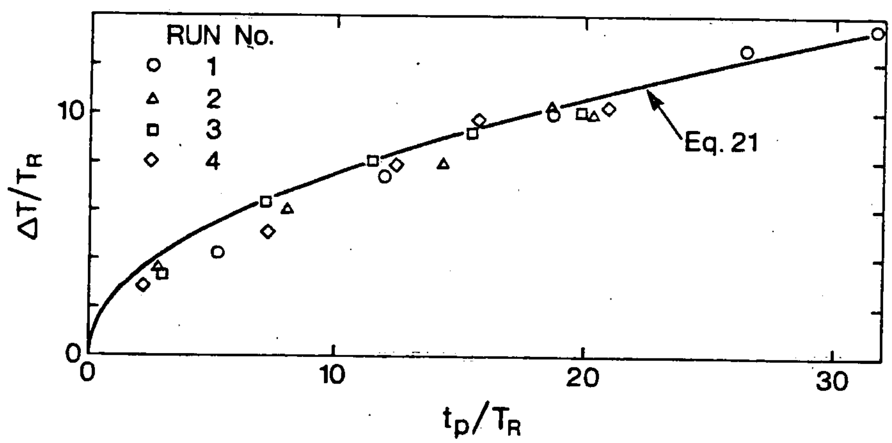


Fig. 10   Test of Eq. 21; storage dispersion model

Date Due
