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DISPERSION OF DREDGED SPOIL WHEN DUMPED AS A SLUG IN DEEP WATER: THE KRISHNAPPAN MODEL

by

Krishnappan (38)
Study

This manuscript has been submitted for the workshop on Drilling Fluid Dispersion Models to be held at Santa Barbara, California during Feb. 7-10, 1983 for publication and the contents are subject to change.

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January 1983

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INTRODUCTION

In some of the existing methods for studying the dispersion of dredged spoil when dumped as a slug in deep water such as the Koh-Chang method (1) and the Edge-Dysart method (2), it is assumed that the dredged spoil behaves in the same manner as a denser liquid of equivalent density. Krishnappan (3, 4) had demonstrated using laboratory experiments that the behaviour of the clouds formed by the solid particles is different from those formed by the denser liquids. He formulated the motion of clouds formed by solid particles of uniform size using the theory of dimensions and proposed a model to predict the behaviour of the dredged spoil which consists of sand particles of varying sizes using a "superposition" principle. The details of this model together with the formulation of the motion of uniform-size particle clouds and some illustrative examples are described in this paper.

BEHAVIOUR OF CLOUDS OF UNIFORM-SIZE PARTICLES

Laboratory experiments of Krishnappan indicated that the motion of the clouds of uniform-size particles resulting from the release of a slug of solid particles without any initial downward momentum in a body of stagnant water can be considered in two distinct phases, namely, the initial "entrainment" phase when the size of the cloud increased mainly due to the incorporation of the surrounding water into the cloud and the final "settling" phase when the downward velocity of the cloud coincided with the terminal fall velocity of the individual solid particles constituting the cloud. The theoretical formulation of the entrainment phase was made using the theory of dimensions similar to the approach of Batchelor (5) who considered the motion of the "liquid-clouds". Accordingly, the vertical downward velocity of the cloud, W, and the horizontal dimension (radius) of the cloud, R, are expressed as:

$$\left\{\begin{array}{l} W = \dfrac{\beta F^{1/2}}{Z} \\ \text{and} \\ R = \alpha Z \end{array}\right. \qquad (1)$$

where F is the total negative buoyancy of the solid particles forming the cloud, given in terms of the density of the solid particles, ρ_s, the density of the receiving fluid medium, ρ, the acceleration due to gravity, g, and the volume of the solid particles, V_s, as:

$$F = \frac{\rho_s - \rho}{\rho} g V_s \qquad (2)$$

Z is the position of the cloud measured from the virtual origin as shown in Fig. 1 and α and β are dimensionless parameters and were treated as functions of dimensionless variable $(\gamma_s \rho D^3/\mu^2)$ where γ_s is the submerged specific weight of the solid particles, i.e.

$$\gamma_s = (\rho_s - \rho) g \qquad (3)$$

D is the size of the solid particles and μ is the absolute viscosity of the fluid medium. In other words, α and β were expressed as:

$$\left.\begin{aligned} \alpha &= \psi_\alpha \left(\frac{\gamma_s \, \rho D^3}{\mu^2} \right) \\ \\ \beta &= \psi_\beta \left(\frac{\gamma_s \, \rho D^3}{\mu^2} \right) \end{aligned}\right\} \qquad (4)$$

The form of the functions ψ_α and ψ_β were determined experimentally by Krishnappan and are shown here in Figs. 2 and 3 respectively.

The "settling" phase of the solid particle-clouds was considered to start when the vertical downward velocity W of the cloud reached the fall velocity (terminal velocity ω) of the individual particles forming the cloud. Therefore, the distance from the virtual origin (Z_f) at which the settling phase begins can be evaluated as:

$$Z_f = \frac{\beta F^{1/2}}{\omega} \qquad (5)$$

and the size (R_f) of the cloud at the beginning of the settling phase is:

$$R_f = \alpha Z_f \qquad (6)$$

Krishnappan adopted the method of Koh (6) for predicting the spread of solid particles during the settling phase. According to this method which considers only the horizontal turbulent diffusion (effects of vertical diffusion and hindered settling are neglected), the distribution of the concentration of the solid particles is assumed to be Gaussian and the horizontal turbulent diffusion coefficient, K, is assumed to follow the 4/3 power law commonly used in ocean turbulence studies.

Accordingly, knowing the standard deviation at the beginning of the settling phase, σ_o, which can be related to the size of the cloud at the beginning of the settling phase, R_f, as:

$$\sigma_o = R_f/4 \qquad (7)^1$$

the standard deviation at any other time from the beginning of the settling phase, σ_t is given by:

$$\sigma_t = \sigma_o \left[1 + 4^{4/3} \frac{2}{3} \frac{At}{\sigma_o^{2/3}} \right]^{3/2} \qquad (8)$$

where A is the dissipation parameter that appears in the 4/3 power law for the diffusion coefficient, i.e.

$$K = A\,(4\,\sigma_t)^{4/3} \qquad (9)$$

and the time, t, is measured from the start of the settling phase. The height of the mound produced at the bed of the deep water as a result of the settling of the solid particles is also considered to be distributed according to the Gaussian distribution with the standard deviation σ_t. In other words the height of the mount, h, is given by:

$$h = h_{max} \exp\left[- \frac{r^2}{2\,\sigma_t^2} \right] ; \quad - 4\sigma_t \leq r \leq 4\sigma_t \qquad (10)$$

---

[1]The Gaussian distribution with σ_o as given by Equation 7 encompasses 99.994% of the solid particles within the radius of R_f.

The value of h_{max} can be computed knowing the volume of the solid particles V_s and the porosity, n, of the mound formed as:

$$h_{max} = \frac{V_s}{(1-n)\ 2\pi\ \sigma_t^2} \tag{11}$$

To solve the motion of the cloud of solid particles during the settling phase, the fall velocity ω of the solid particles has to be known. This can be obtained from the measurements made for the spherical particles as given in Fig. 4. The effect of the particle shape on its fall velocity is not very well established as yet. Until a better method is devised which would correctly include the effects of the shape of particles, Fig. 4 can be used to predict the fall velocity of the particles constituting the dredged spoil. Note that the parameter $(\gamma_s\ \rho D^3/\mu^2)$ governing the parameters of the entrainment phase also governs the fall velocity of the solid particles.

BEHAVIOUR OF THE DREDGED SPOIL

In the model formulated by Krishnappan to predict the behaviour of the dredged spoil, it is assumed that the turbulence of the receiving body of water has negligible effect during entrainment phase and it becomes important only during the settling phase. The effect of density gradient of the receiving body of water is considered to be small since a majority of the dredged spoil is quartz and the density differential between the particles and the water is large compared to the variation of density of water over the depth. The receiving body of water is considered to have a uniform current of magnitude U. The total volume of the dredged material dumped is V_s and the depth of water is d. The dredged spoil is considered to consist of particles of different specific weights and grain sizes. V_{sij} is the volume of a fraction whose submerged specific weight is γ_{si} and the grain size is D_j.

Entrainment Phase

The position of the dump is taken as the origin and the vertical distance is measured from the level of dump and is represented by ζ (see Fig. 5). Therefore, the relationship between ζ and Z becomes

$$Z = \zeta + \frac{R_o}{\alpha_m} \qquad (12)$$

where R_o is the size of the dump and it is equated to the initial size of the cloud and α_m is the entrainment coefficient corresponding to the mixture of particles. When the mixture of different specific weight and grain-size materials are moving together as a cloud, it is hypothesized that each fraction exerts influence on the total behaviour of the cloud in the same proportion as its negative buoyancy. In other words, if F_{ij} is the buoyancy of the fraction (ij) and F is the total buoyancy, then the influence of the fraction (ij) on the total behaviour of the cloud (or on the coefficients describing the total behaviour) is proportional to F_{ij}/F. The ratios F_{ij}/F can be termed "weighting coefficients" which determine the behaviour of the whole cloud from the behaviour of the individual fractions. For example, if α_m and β_m are the dimensionless coefficients governing the motion of the total cloud, then they can be evaluated using the above "weighting coefficients" as follows:

$$\alpha_m = \sum_i \sum_j \frac{F_{ij}}{F} \alpha_{ij} \tag{12}$$

where

$$\left.\begin{aligned} F_{ij} &= \frac{\gamma_{si} V_{sij}}{\rho} \\ &\text{and} \\ F &= \sum_i \sum_j F_{ij} \end{aligned}\right\} \tag{13}$$

a_{ij} and β_{ij} are the dimensionless coefficients that can be determined from Figs. 2 and 3 corresponding to parameter $(\gamma_{si}\ \rho\ D_j^3)$.

The behaviour of the cloud of mixture of particles can, therefore, be expressed as:

$$\left\{\begin{aligned} R &= R_o + \alpha_m \zeta \\ &\text{and} \\ W &= \frac{\beta_m F^{1/2}}{(\zeta + R_o/\alpha_m)} \end{aligned}\right. \tag{14}$$

As the cloud of the dredged spoil moves down, the downward velocity decreases and it might become less than the fall velocity of one of its constituents. In such a case, the fraction having the fall velocity greater than the cloud velocity is assumed to separate out of the main cloud and to undergo settling phase while the main cloud still undergoes entrainment phase: but, now, the total buoyancy has been reduced by the amount of the buoyancy of the separated fraction, say, $F_{k\ell}$. Therefore, the total buoyancy of the main cloud F' is

$$F' = F - F_{k\ell} \tag{15}$$

and hence the weighting coefficients would also be altered as:

$$w'_{ij} = \frac{F_{ij}}{F'} \quad \text{for } i \neq k \text{ and } j \neq \ell \tag{16}$$

and consequently the parameters defining the motion of the new cloud become:

$$\left.\begin{aligned} \alpha'_m &= \sum_{i \neq k} \sum_{j \neq \ell} w'_{ij} \, \alpha_{ij} \\ \beta'_m &= \sum_{i \neq k} \sum_{j \neq \ell} w_{ij} \, \beta_{ij} \end{aligned}\right\} \tag{17}$$

The level at which such a separation would occur can be calculated knowing the fall velocity of the separated fraction, say, $\omega_{k,\ell}$. Denoting this level by $\zeta_{fk\ell}$, it can be evaluated as:

$$\zeta_{fk\ell} = \left(\frac{\beta_m \, F^{1/2}}{\omega k\ell} - \frac{R_o}{\alpha_m} \right) \tag{18}$$

The size of the main cloud, which is also the size of the separated cloud is:

$$R_{fk\ell} = R_o + \alpha_m \, \zeta_{fk\ell} \tag{19}$$

The time elapsed from the instant the material is dumped and the instant the fraction (k,ℓ) separated, denoted by $t_{fk\ell}$ can be calcualted using Equation 14 as follows:

Since $W = d\zeta/df$, Equation 14 can be rearranged as:

$$\left(\zeta + \frac{R_o}{\alpha_m} \right) d\zeta = \beta_m \, F^{1/2} \, dt \tag{20}$$

Integrating the above equation and using the condition that at $t=0$, $\zeta = \zeta_{fk\ell}$, $t_{fk\ell}$ can be evaluated as

$$t_{fk\ell} = \frac{\left(\frac{\zeta_{fk\ell}}{2} + \frac{R_o}{\alpha_m}\right)}{\beta_m \; F^{1/2}} \; \zeta_{fk\ell} \tag{21}$$

The lateral distance travelled by the cloud ($L_{k\ell}$) at the time of separation due to the ambient current can be calculated as:

$$L_{k\ell} = U \; t_{fk\ell} \tag{22}$$

After the separation of the fraction ($k\ell$) the behaviour of the main cloud is described by:

$$\left.\begin{aligned} R &= R_{fk\ell} + \alpha'_m \; (\zeta - \zeta_{fk\ell}) \\ W &= \frac{\beta'_m \; F'^{1/2}}{\left(\zeta + \frac{R_o}{\alpha_m}\right)} \end{aligned}\right\} \text{ for } \zeta > \zeta_{fk\ell} \qquad (23$$

Again, as can be seen from Equation 23, the downward velocity of the cloud decreases as the cloud moves down and it could reach a value equal to the fall velocity of the next heavier fraction, say, (pq). In this case, the fraction (pq) will settle out of the cloud and would undergo settling while the main cloud undergoes entrainment. The new buoyancy F'' is given by:

$$F'' = F - (F_{k\ell} + F_{pq}) \tag{24}$$

The weighting coefficients are:

$$w''_{ij} = \frac{F_{ij}}{F''} \quad \text{for } i \neq k, p \text{ and } j \neq \ell, q \tag{25}$$

α''_m and β''_m are given by:

$$\alpha''_m = \sum_{i \neq k,p} \sum_{j \neq \ell,q} w''_{ij} \; \alpha_{ij}$$

$$\beta''_m = \sum_{i \neq k,p} \sum_{j \neq \ell q} w''_{ij} \; \beta_{ij} \tag{26}$$

The level at which the second separation occurs is given by:

$$\zeta_{fpq} = \left(\frac{\beta'_m \; F'^{1/2}}{\omega pq} - \frac{R_o}{\alpha_m} \right) \tag{27}$$

The size of the cloud at the time of second separation is

$$R_{fpq} = R_{fk\ell} + \alpha'_m \; (\zeta_{fpq} - \zeta_{fk\ell})$$

The time at which the second separation occurs can be calculated from:

$$(t_{fpq} - t_{fk\ell}) = \frac{\left(\frac{\zeta^2_{fpq} - \zeta^2_{fk\ell}}{2}\right) - \frac{R_o}{\alpha_m} (\zeta_{fpq} - \zeta_{fk\ell})}{\beta'_m \; F'^{1/2}} \tag{28}$$

The lateral distance travelled by the main cloud:

$$L_{pq} = U\, t_{fpq} \tag{29}$$

The behaviour of the main cloud after ζ_{fpq} is given by:

$$\left.\begin{aligned} R &= R_{fpq} + \alpha''_m (\zeta - \zeta_{fpq}) \\ W &= \frac{\beta''_m \; F''^{1/2}}{(\zeta + \frac{R_o}{\alpha_m})} \end{aligned}\right\} \quad \zeta > \zeta_{fpq} \tag{30}$$

and the process continues until all the fractions are settled out of the cloud and/or the bottom of the deep water is reached.

Settling Phase

The fractions that separate out of the main cloud undergo settling phase. The motion of the fraction (k,ℓ) is considered here as an example. The level at which the separation from the main cloud occurred is $\zeta_{fk\ell}$ and the size of the fraction at time of separation is $R_{fk\ell}$. The time at which the separation occurred is $t_{fk\ell}$. Adopting the method described earlier for the settling phase, the standard deviation $\sigma_{ok\ell}$ of the distribution of the particles at the time of separation is given by:

$$\sigma_{ok\ell} = R_{fk\ell}/4.0 \tag{31}$$

The standard deviation of the distribution of the particles at the time of deposition at the bed of deep water is given by:

$$\sigma_{fk\ell} = \sigma_{ok\ell} \left[1 + 4^{4/3} \; \frac{2}{3} \; \frac{A}{\sigma_{ok\ell}^{2/3}} \; \left(\frac{d - \zeta_{fk\ell}}{\omega k\ell}\right) \right]^{3/2} \tag{32}$$

The height distribution of the mound formed at the bed due to the settlement of the fraction ($k\ell$) is:

$$h_{k,\ell} = h_{maxk,\ell} \exp \left[- \frac{\{r - U\,(t_{fk\ell} + \frac{(d-\zeta_{fk\ell})}{\omega k\ell})\}^2}{2\,\sigma^2_{fk\ell}} \right] \qquad (33)$$

where $h_{maxk\ell}$ is given by:

$$hmax_{k\ell} = \frac{V_{sk\ell}}{(1-n)\ 2\pi\ \sigma^2_{fk\ell}} \qquad (34)$$

Similar expressions can be derived for all the fractions that settle out of the main cloud and hence the total height of mound formed by all the fractions can be obtained by simply superimposing them as:

$$h = \sum_k \sum_\ell h_{k\ell} \qquad (35)$$

When the cloud reaches the bottom before the separation of any of the fractions, which is possible for shallow waters, the mound formed at the bottom can be calculated by assuming that the Gaussian distribution for the concentration of solid particles is valid even during the entrainment phase. However, when the cloud undergoing entrainment phase hits the bottom, further spreading will occur which has not been considered in the model of Krishnappan.

EXAMPLES TO ILLUSTRATE THE APPLICATION OF THE MODEL

The following three examples are selected to illustrate the application of the Krishnappan model.

Example 1: 8 m^3 of dredged material with the following size and specific weight distributions were dumped as a slug in a deep water where the depth is 150 m.

Fraction	Grain Size	Specific Weight in Water	% by Volume
1	0.700	1650 kg/m^3	10
2	0.253	1650 kg/m^3	20
3	0.180	1650 kg/m^3	30
4	0.044	1650 kg/m^3	40

Assuming that the radius of the cloud at the dump level is 2 m, determine the size of the mound formed due to this dump. Assume also the numerical value of the dissipation parameter A=.000068 $m^{2/3}/sec$ and the porosity of the mound formed is 0.333.

Solution: Since the specific weight of all the constituents is the same, the subscript i will be dropped and j varies from 1 to 4.

i) Evaluation of the Weighting Coefficients: $w_j = F_j/F$

$$F = \frac{\gamma_s V_s}{\rho} = \frac{\gamma_s}{\gamma} g V_s = 1.65 \times 9.81 \times 8 = 129.49 \ m^4/sec^2$$

$$F_1 = 1.65 \times 9.81 \times 0.8 = 12.949 \ m^4/sec^2$$

$$F_2 = 1.65 \times 9.81 \times 1.6 = 25.898 \ m^4/sec^2$$

$$F_3 = 1.65 \times 9.81 \times 2.4 = 38.847 \ m^4/sec^2$$

$$F_4 = 1.65 \times 9.81 \times 3.2 = 51.796 \ m^4/sec^2$$

$$w_1 = F_1/F = 0.1$$

$$w_2 = F_2/F = 0.2$$

$$w_3 = F_3/F = 0.3$$

$$w^4 = F^4/F = 0.4$$

ii) Determination of α_j, β_j and ω_j

The parameter $(\gamma_s \rho D^3{}_j/\mu^2)$ for each fraction and the values of α_j, β_j and ω_j obtained from Figs. 2, 3 and 4 respectively are as follows:

Fraction	Grain Size (mm)	$(\gamma_s \rho D^3{}_j/\mu^2)$	α_j	β_j	ω_j cm/s
1	0.700	5552.0	0.232	3.54	10.0
2	0.253	262.0	0.272	1.52	2.8
3	0.100	94.4	0.295	1.30	1.9
4	0.044	1.38	0.312	1.10	0.16

$$\alpha_m = \sum_{j=1}^{4} w_j \alpha_j = (0.1x0.232)+(0.2x0.272)+(0.3x0.295)+(0.4x0.312)$$

$$= \underline{0.291}$$

$$\beta_m = \sum_{j=1}^{4} \omega_j \beta_j = (0.1x3.54)+(0.2x1.52)+0.3x1.30)+(0.4x1.10)$$

$$= \underline{1.488}$$

The equations describing the behaviour of the dumped material are:

$$R = 2 + 0.291\ \zeta \text{ (taking } R_0 \text{ as 2m)}$$

$$W = \frac{1.488x129.49^{1/2}}{\zeta + \frac{2}{0.291}} = \frac{16.93}{\zeta + 6.87} \qquad (36)$$

To check whether Fraction 1 would separate out of the cloud before the cloud reaches the bottom:

Equating W and ω_1 and solving for ζ in Equation 36, we get:

$$\zeta_{f_1} = \frac{16.93}{0.1} - 6.87 = 169.3 - 6.87 = 162.46 \text{ metres.}$$

Since the depth is only 150 m, the separation will not occur and the whole cloud undergoes entrainment phase until it hits the bottom.

The radius of the cloud when it hits the bottom is:

$$R_f = 2 + 0.291 \times 150 = 45.65 \text{ m}$$

The standard deviation of the distribution of the material is:

$$\sigma_f = R_f/4.0 = 45.65/4 = 11.41 \text{ m}$$

The maximum height of the mound formed at the bottom:

$$h_{max} = \frac{V_s}{(1-n)\ 2\pi\sigma_f^2} = \frac{8}{0.667 \times 2\pi \times 11.41^2} = \underline{.015} \text{ m}$$

<u>Example 2</u>: The size of the dump is reduced to 1 m^3 while everything else remains the same as in Example 1. (Assume the initial radius of the cloud is 1 m.)

<u>Solution</u>: The weighting coefficients and α_m and β_m take values that are the same as in Example 1.

The equation describing the motion of the cloud is:

$$R = 1.0 + 0.291\ \zeta$$

$$W = \frac{1.488 \times (1.65 \times 9.81 \times 1)^{1/2}}{\zeta + (\frac{1.0}{0.291})} = \frac{5.987}{\zeta + 3.44} \qquad (37)$$

To check whether separation of Fraction 1 will occur:

The depth required for separation is:

$$\zeta_{f_1} = \frac{5.987}{0.1} - 3.44 = 56.43$$

Since the total depth is 150 m, the separation of 0.70 mm fraction will occur at the depth of 56.43 m and it would undergo settling phase while the main cloud undergoes entrainment phase.

The radius of the cloud at the instant of separation is:

$$R_{f_1} = 1 + 0.291 \times 56.43 = 17.42 \text{ m}$$

The time required for the first separation is:

$$t_{f_1} = \frac{(\frac{\zeta_{f_1}}{2} + \frac{R_o}{\alpha_m})\, \zeta f_1}{\beta_m F^{1/2}} = \frac{(\frac{56.43}{2} + 3.44)\, 56.43}{5.986} = \underline{4.97} \text{ min}$$

<u>Motion of the Main Cloud after the Separation of Fraction 1</u>

$$F^1 = F - F_1 = 1.65 \times 9.81 \times 1 - 1.65 \times 9.81 \times 0.1$$
$$= 14.567 \text{ m}^4/\text{sec}^2$$
$$F_2 = 1.65 \times 9.81 \times 0.2 = 3.237 \text{ m}^4/\text{sec}^2$$
$$F_3 = 1.65 \times 9.81 \times 0.3 = 4.856 \text{ m}^4/\text{sec}^2$$
$$F_4 = 1.65 \times 9.81 \times 0.4 = 6.475 \text{ m}^4/\text{sec}^2$$

$$w_2 = \frac{F_2}{F^1} = \frac{3.237}{14.567} = 0.222$$

$$w_3 = \frac{F_3}{F^1} = \frac{4.856}{14.567} = 0.333$$

$$w_4 = \frac{F_4}{F^1} = \frac{6,475}{14.567} = 0.444$$

$$\alpha_m' = \sum_{j=2}^{4} w_j \alpha_j = (0.222x0.272) + (0.333x0.295) + (0.444x0.312)$$

$$= \underline{0.297}$$

$$\beta_m' = \sum_{j=2}^{4} \omega_j \beta_j = (0.222x1.52) + (0.333x1.30) + (0.444x1.10)$$

$$= \underline{1.260}$$

The equations describing the behaviour of the cloud become:

$$R = 17.42 + 0.297 (\zeta - 56.43) \text{ for } \zeta > 56.43 \text{ m}$$

(38)

$$W = \frac{1.260 \text{ x } (14.567)^{1/2}}{\zeta + 3.44} = \frac{4.809}{\zeta + 3.44}$$

To check whether the separation of Fraction 2 will occur:

The depth required for the second separation is:

$$\zeta_{f_2} = \frac{4.809}{.028} - 3.44 = \underline{170.36} \text{ m}$$

Since the depth is only 150 m, the separation of Fraction 2 will not occur and the cloud reaches the bottom while undergoing entrainment phase.

The radius of the cloud when it hits the bottom is:

$$R_f = 17.42 + 0.297\ (150 - 56.43) = 45.21\ \text{m}$$

The standard deviation of the particle distribution is:

$$\sigma_f = R_f/4 = 45.21/4 = \underline{11.30}\ \text{m}$$

The maximum height of the mound formed at the bottom owing to the main cloud is:

$$h_{max} = \frac{V_s - V_{s_1}}{(1-n)\ 2\pi\sigma_f^{\,2}} = \frac{0.9}{0.667 \times 2\pi \times (11.30)^2} = \underline{.00168}\ \text{m}$$

<u>Motion of Fraction 1 Undergoing Settling Phase</u>

The standard deviation of the distribution of the particles at the instant of separation (initial distribution) is:

$$\sigma_o = \frac{R_f}{4} = \frac{17.42}{4} = 4.36\ \text{m}$$

The time required for the settling particles to reach the bottom t_1 is:

$$t_1 = \frac{d - \zeta_{f_1}}{\omega_1} = \frac{150 - 56.43}{0.1} = 935.7\ \text{secs}$$

The standard deviation of the distribution of the particles settling at the bottom is:

$$\sigma_f = 4.36 \left[1 + 4^{4/3} \frac{2}{3} \cdot \frac{0.000068}{(4.36)^{2/3}} \times 935.7 \right] = \underline{5.04} \text{ m}$$

The radius of the cloud when hitting the bottom is:

$$R_{f_1} = 4\sigma_f = 4 \times 5.04 = 20.16 \text{ m}$$

The maximum height of the mound formed owing to the settling fraction is:

$$h_{max} = \frac{V_{s_1}}{(1-n)\ 2\pi\sigma_f^2} = \frac{0.1}{0.667 \times 2\pi \times (5.04)^2} = \underline{0.00094} \text{ m}$$

Therefore, the total height of the mound formed because of the dumping of 1 m^3 of the dredged material is:

$.00168 + .00094 = .00262$ m

If eight such dumps are made then the maximum height of the mound will be

$.00262 \times 8 = .021$ m

Note that if all 8 m^3 of the dredged material were dumped as one slug, the maximum height of the mound formed is only .015 m. The radius of the cloud in both cases is more or less the same.

Example 3: An ambient current of 0.10 m/sec is assumed for Example 2.

Solution: For this case, when Fraction 1 separates from the main cloud, the cloud would have been displaced in the horizontal direction an amount, L, equal to:

$$L_1 = U\, t_{f_1} = \frac{U\left[\frac{\zeta_{f_1}}{2} + \frac{R_o}{\alpha_m}\right]\zeta_{f_1}}{\beta_m\, F^{1/2}} = 0.1 \times 298 = \underline{29.8}\ \text{m}$$

After the first separation, the main cloud would have moved horizontally a distance of L_2 before hitting the ground. The distance L_2 is given by:

$$L_2 = U\,(t_f - t_{f_1})$$

The value of t_f can be determined from the equation governing the motion of the main cloud, i.e.

$$W = \frac{4.809}{\zeta + 3.44}$$

or

$$\frac{d\zeta}{dt} = \frac{4.809}{\zeta + 3.44}$$

Rearranging and integrating we get:

$$\frac{\zeta^2}{2} + 3.44\,\zeta = 4.809\,t + C \quad \text{(C is a constant of integration)}$$

when $t = t_{f_1}$, $\zeta = \zeta_{f_1} = 56.43$

$$t_f = 2127 \text{ secs}$$

Therefore, L_2 can be calculated as $\underline{182.9}$ m.

The total horizontal distance moved by the cloud undergoing entrainment is:

$L = 182.9 + 29.8 = \underline{212.7}$ m

The height distribution of the mound formed at the bottom due to this cloud is:

$$h = h_{max} \exp \left[- \frac{(r - 212.7)^2}{2 \times (11.30)^2} \right]$$

where $h_{max} = .00168$ m

Consider the Fraction Undergoing Settling Phase

The lateral distance travelled during settling phase is

$L_s = Ut_1 = 0.1 \times 935.7 = \underline{93.57}$ m

The total horizontal displacement $= 93.58 + 29.8 = 123.38$ m.

The distribution of the height of mound formed by the settling fraction is:

$$h_1 = h_{max_1} \exp \left[- \frac{(r - 123.38)^2}{2 \times (5.04)^2} \right]$$

where $h_{max_1} = .00094$ m

The net weight distribution of both mounds is:

$$h = h + h_1 = .00168 \exp\left[- \frac{(r-212.7)^2}{255.4}\right] + .00094 \exp\left[- \frac{(r-123.38)^2}{50.8}\right]$$

$$h_{212.7} = .00168 + .00094 \exp [- 157.05] = .00168 \text{ m}$$

$$h_{123.38} = .00168 \exp [-31.23] + .00094 = .00094 \text{ m}$$

The maximum height formed is .0168 m. Note that the cloud in this case has been broken up into two pieces and they are deposited on the bottom, far apart from each other. If there were eight such dumps, then the maximum height formed would only be .013 m and it would occur at a lateral distance of 213 m from the location of the dump.

SUMMARY

The Krishnappan model for predicting the dispersion of the dredged spoil when dumped as a slug in deep water is described in detail along with three illustrative examples showing the application of the model. In this model, the deredged spoil is considered in various fractions of uniform size particles and it is assumed that each fraction exerts influence on the total behaviour of the dredged spoil in proportion to its negative buoyancy. The behaviour of the uniform size particles has been formulated using the theory of dimensions and laboratory experiments. The method can be used to predict the vertical height and the horizontal size distribution of the "mound" formed due to the deposition of the dredged spoil at the bed of deep water. The model indicates how the characteristics of the mound depend on the volume of dump, the size distribution of the dredged spoil, the water depth and the ambient current and the turbulence characteristics, thereby providing guidance for the selection of optimum dump size and location for the disposal of the dredged spoil.

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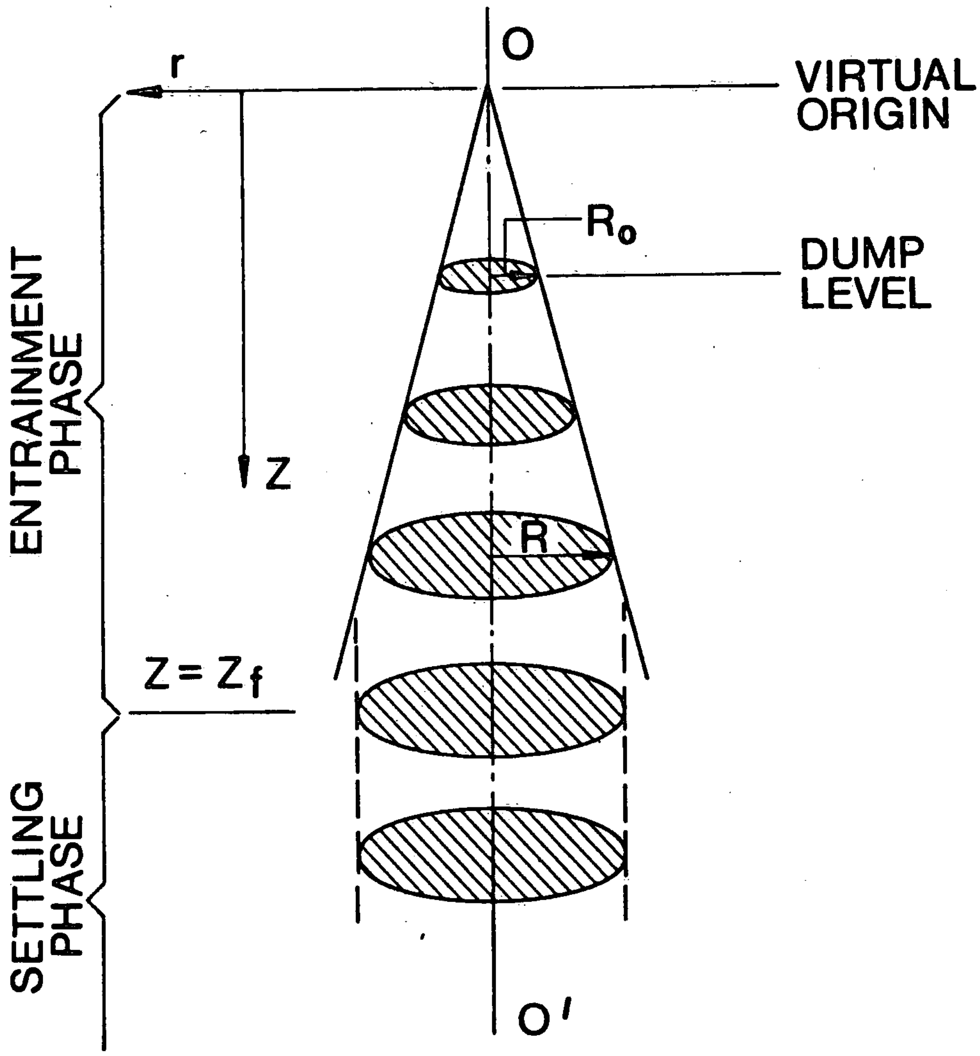


Fig. 1 Co-ordinate system for the motion of uniform size particle clouds in stagnant water

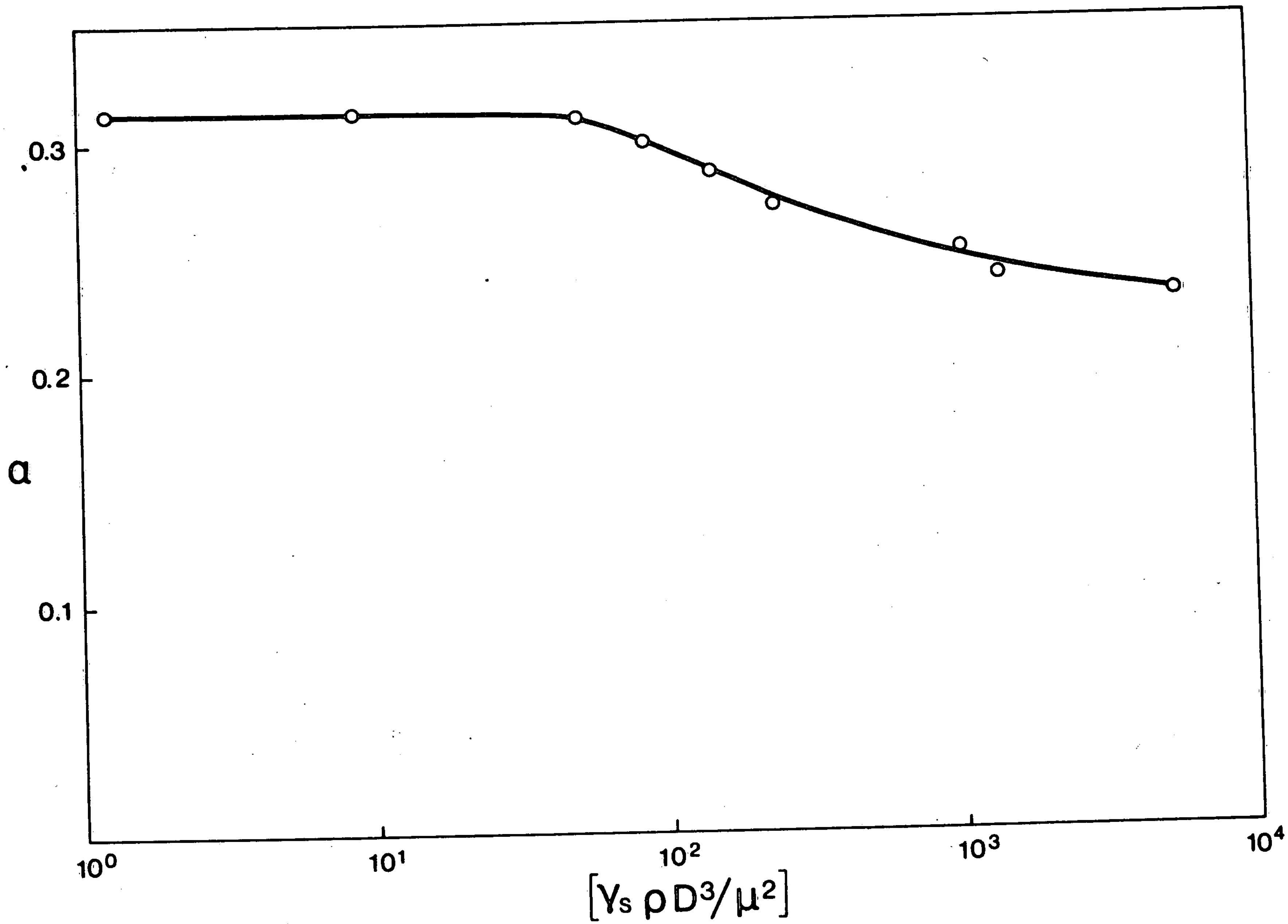


Fig. 2 Graph representing ψ_a

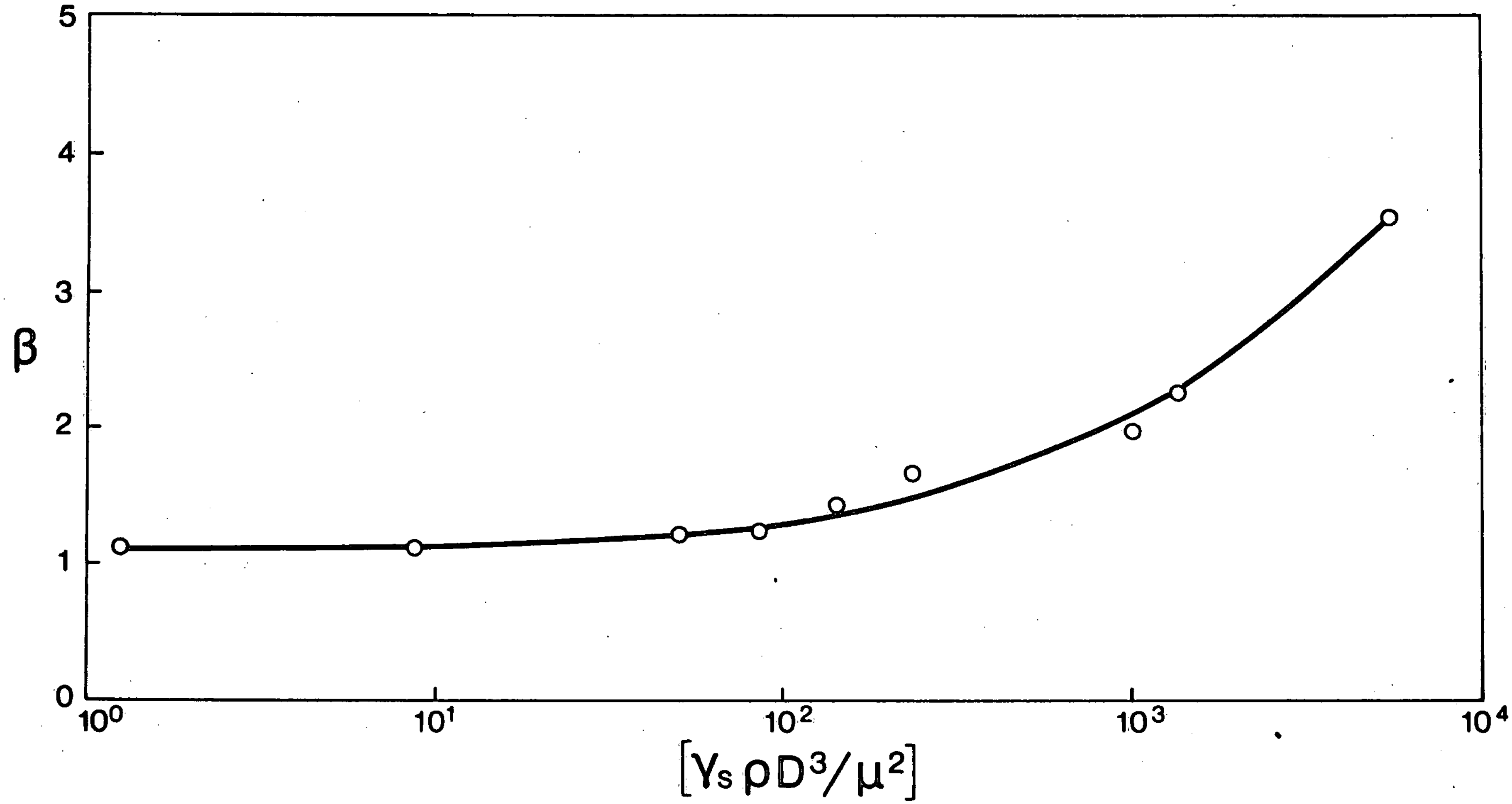
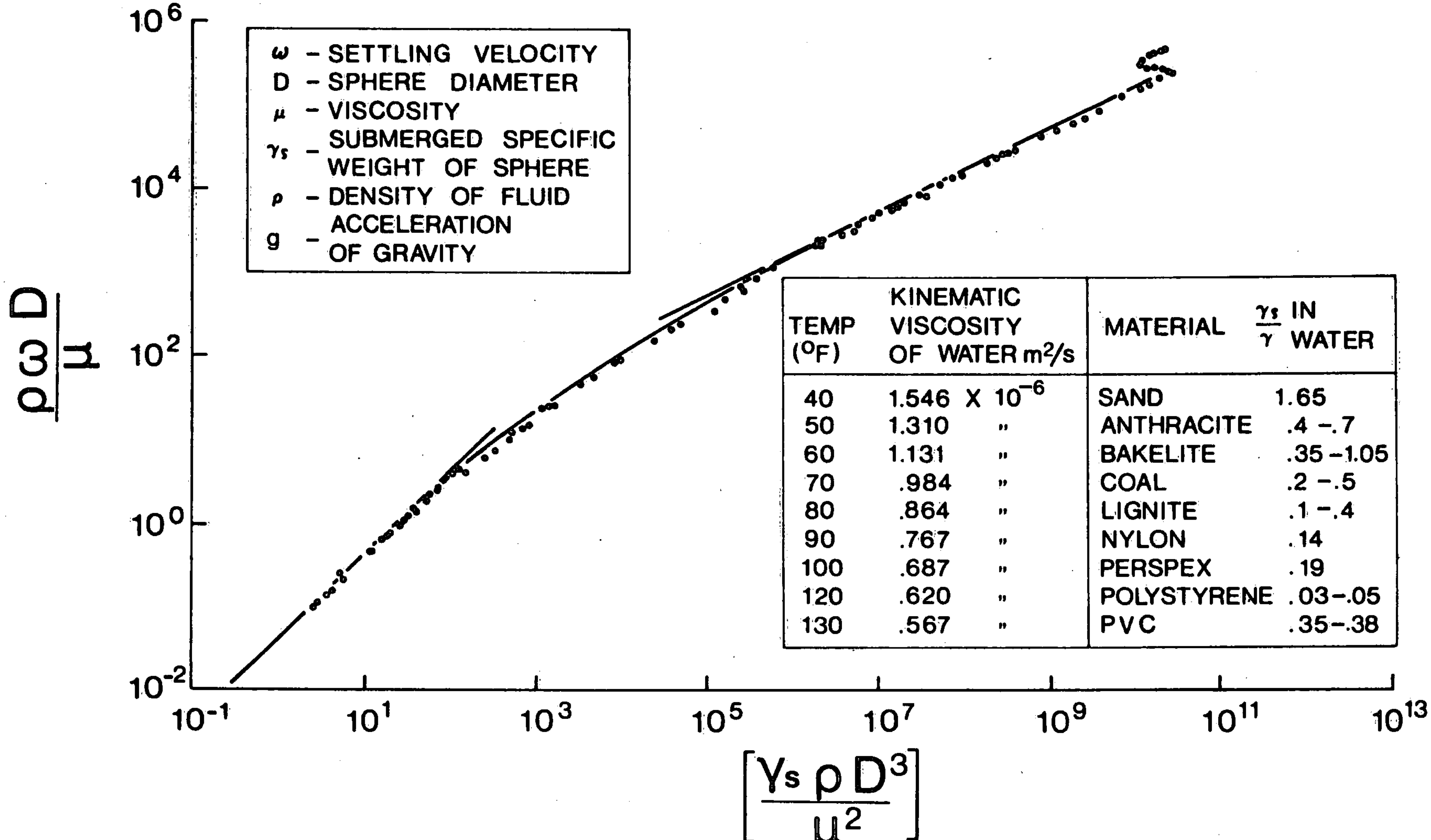


Fig. 3 Graph representing ψ_β



TEMP (^{o}F)	KINEMATIC VISCOSITY OF WATER m²/s	MATERIAL	$\frac{\gamma_s}{\gamma}$ IN WATER
40	1.546 X 10^{-6}	SAND	1.65
50	1.310 "	ANTHRACITE	.4 -.7
60	1.131 "	BAKELITE	.35-1.05
70	.984 "	COAL	.2 -.5
80	.864 "	LIGNITE	.1 -.4
90	.767 "	NYLON	.14
100	.687 "	PERSPEX	.19
120	.620 "	POLYSTYRENE	.03-.05
130	.567 "	PVC	.35-.38

Fig. 4 Fall velocity of spherical particles

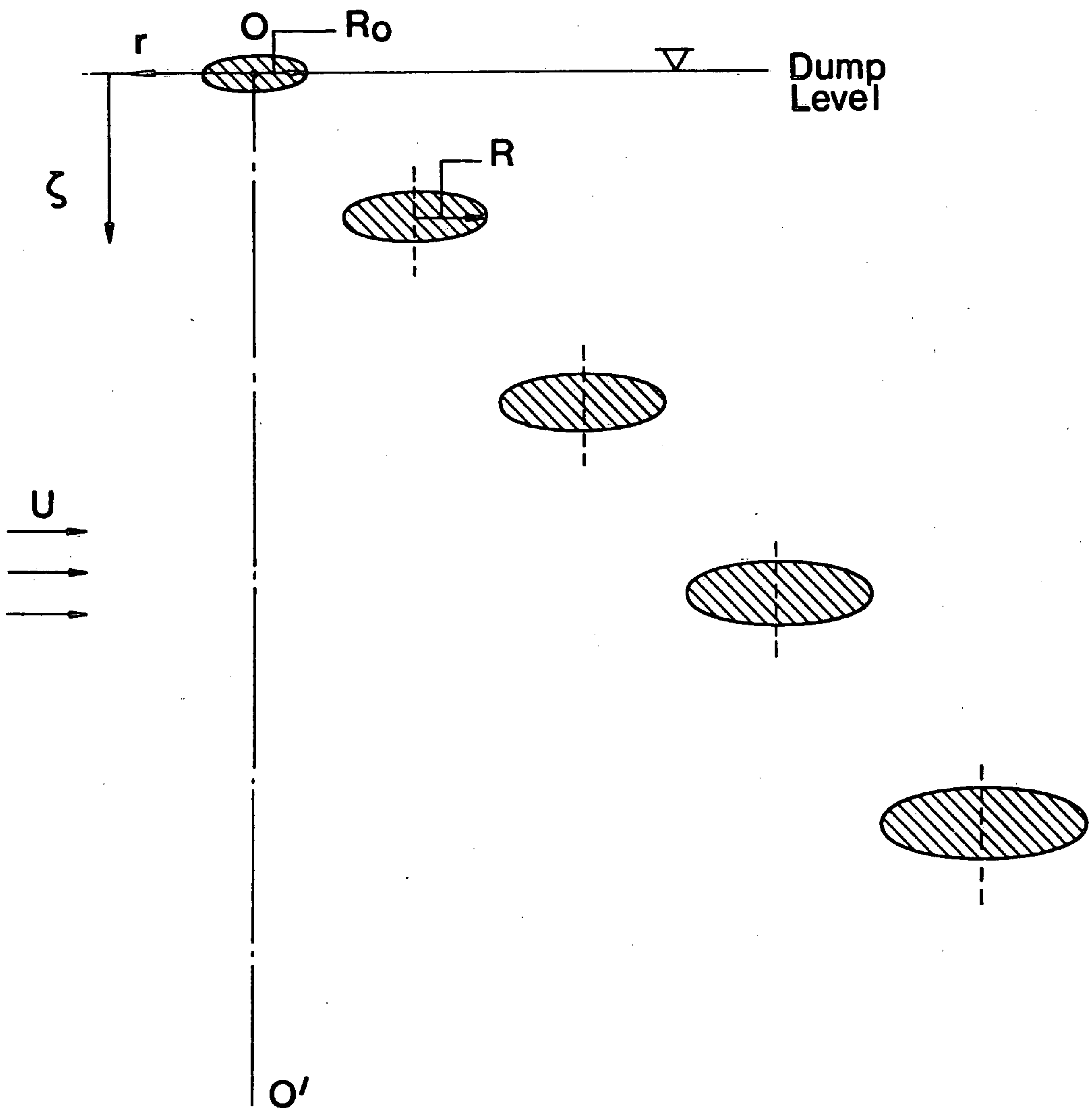


Fig. 5 Co-ordinate system used for the motion of the dredged spoil