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DISTRIBUTION OF SUSPENDED SEDIMENT IN ICE-COVERED FLOWS

by

B. G. Krishnappan

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October 1981

MANAGEMENT PERSPECTIVE

Up until now, the concentration of sediment under an ice cover as a function of depth was not available without taking several samples in a vertical plane.

This paper provides a method to accurately estimate the distribution of sediment concentration provided certain hydraulic variables and sediment characteristics are measured.

The transport of sediment under ice is of significance for reservoir design and could be an important factor in the economic design of year round water transfer canals or aqueducts.

T. Milne Dick

Chief, Hydraulics Division

October 2, 1981

PERSPECTIVE D'AMÉNAGEMENT

Jusqu'à maintenant, il n'était pas possible d'obtenir la concentration des sédiments sous une couverture de glace en fonction de la profondeur sans prélever plusieurs échantillons dans un plan vertical.

Le présent article porte sur une méthode précise d'estimation de la distribution des concentrations des sédiments à partir de la mesure de certaines variables hydrauliques et caractéristiques des sédiments.

La connaissance des phénomènes de transport des sédiments est importante pour la conception des réservoirs; ce pourrait être aussi un facteur important lors de la conception de réseaux économiques de canaux de transport de l'eau ou d'aqueducs, utilisables toute l'année.

T. Milne Dick

Chef, Division de l'hydraulique

Le 2 octobre 1981

KEY WORDS: ice-covered, fluid flow, vertical distribution, suspended-sediments, fully developed turbulence, bed, roughness, concentration.

ABSTRACT: The vertical distribution of the suspended sediment in two-dimensional, uniform and fully developed turbulent flows under ice cover is formulated using the results of the k-ε turbulence model of Launder and Spalding. Three sets of distribution curves each corresponding to a particular value of the ratio between the ice cover roughness and bed roughness are presented in a form similar to the distribution curves of H. Rouse for free-surface flows. For the same value of the ratio between the shear velocity and the fall velocity of sediment particles, the relative concentration is less in ice-covered flows than in free-surface flows. Under similar conditions, the relative concentration of suspended sediment decreases as the ratio of the ice cover roughness to bed roughness increases. The agreement between the present formulation and the measurements found in the literature is favourable.

RÉSUMÉ

La distribution verticale des sédiments en suspension dans un écoulement turbulent bidimensionnel en régime permanent sous une couverture de glace est décrite à l'aide des résultats du modèle de turbulence k- de Launder et Spalding. Le présent article traite de courbes de distribution réparties en trois ensembles dont chacun correspond à une valeur particulière du rapport des rugosités de la couverture de glace et du lit; elles sont analogues aux courbes de distribution de H. Rouse pour les écoulements à surface libre. À valeur égale du rapport de la vitesse de cisaillement à la vitesse limite de chute, la concentration relative des sédiments dans un écoulement couvert de glace est inférieure à celle dans un écoulement à surface libre et elle diminue avec l'augmentation du rapport des rugosités de la couverture de glace et du lit. Cette formulation donne des résultats qui concordent avec les valeurs expérimentales rapportées dans la documentation.

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DISTRIBUTION OF SUSPENDED SEDIMENT IN ICE-COVERED FLOWS

By Bommanna G. Krishnappan[1]

INTRODUCTION

Knowledge of the concentration distribution of suspended sediment in stream flows is useful for the solution of a number of engineering and environmental problems. Problems, such as positioning of water intakes, planning of reservoirs, prediction of pollutant transport attached to the suspended sediments, are some of the examples that can be solved knowing the distribution of suspended sediment. For two-dimensional free-surface flows, the suspended sediment distributions had been studied extensively starting from the work of M. P. O'Brien (7) as early as 1933. In 1937, H. Rouse (8) published the results of A. Ippen, who derived a family of distribution curves which are still widely used in practice. For ice covered flows, the number of studies is rather limited and, as yet, there are no such accepted distribution curves that can be readily applied to practical problems. In this paper, a set of distribution curves that can be used for flows in ice-covered streams is derived. The derivation of the distribution curves is based on the prediction of momentum diffusivity distributions in ice-covered streams using the k-ε turbulence model (5).

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LITERATURE REVIEW

H. T. Shen and T. O. Harden (10) studied the effect of ice cover on vertical transfer in stream channels by dividing the flow into two layers of equal thickness each of which is assumed to have logarithmic velocity profile and linear shear stress distribution. They assumed that the mass transfer coefficient, Γ_y, is the same as the momentum diffusivity ν_t, and evaluated ν_t using the Boussinesq's hypothesis:

$$\nu_t \frac{\partial u}{\partial y} = \frac{\tau}{\rho} \qquad (1)$$

where u is the time-averaged local velocity along the channel, y is the coordinate in the vertical direction, τ is the shear stress and ρ is the fluid density. The resulting distribution for the mass transfer coefficient is a double parabola with a zero value at the middle as shown in Fig. 1 with a partly dotted line. In Fig. 1, both the mass transfer coefficient and the vertical ordinate are made dimensionless using the shear velocity at the bed, v_{*b} and the total depth h. Such a distribution for the mass transfer coefficient is unrealistic as it implies that no mass transfer can take place across the mid plane. This obviously is not true. To avoid this shortcoming, Shen and Harden arbitrarily modified the distribution by assuming a constant value at the centre portion of the flow as shown by the solid line in Fig. 1. The constant value used by Shen and Harden was obtained from measurements made by Ismail (2) of sediment suspension in a closed rectangular channel. Shen and Harden then solved the mass balance equation for a steady, uniform, two-dimensional flow numerically and simulated the concentration distributions downstream from a given boundary source distribution. They performed simulations for flows with and without ice cover and evaluated the effect of ice cover. They found a substantial reduction in the mixing capacity due to the presence of ice cover.

W. W. Sayre and G. B. Song (9) adopted a similar approach to that of Shen and Harden for predicting the effects of ice cover on alluvial channel flow and sediment transport processes. In their approach, Sayre and Song considered the ice-covered flow in two layers, separated by the line of maximum velocity. In each layer, they also assumed logarithmic velocity distribution and linear shear stress distribution to prevail. Treating Γ_y as proportional to ν_t and adopting Boussinesq's hypothesis, Sayre and Song obtained double parabola distribution for the mass transfer coefficient as shown in Fig. 2 with zero value at the line of maximum velocity. Unlike Shen and Harden, Sayre and Song modified the distribution shown in Fig. 2 by joining the vertices of the two parabolas by a straight line. Using this modified distribution for Γ_y, Sayre and Song solved the Schmidt-O'Brien equation:

$$\Gamma_y \frac{dC}{dy} + wC = 0 \qquad (2)$$

(where w is the fall velocity of sediment particles and C is the volumetric sediment concentration) to obtain a steady-state, fully-developed concentraion profile in the vertical in an ice-covered flow.

In both of the above studies, the distribution for Γ_y was not deduced rigorously and the distribution of Γ_y in the central portion had to be modified arbitrarily. In Sayre and Song's approach, it also becomes necessary to know the velocity profile a priori in order to establish the line of maximum velocity for the division of flow into two layers. Measurements of velocity profiles by Larsen (3, 4) under ice-covered flows suggest that the profiles have near zero velocity gradients over substantial portions of the central region of the flow which would make the division of the flow very subjective.

In a recent study by Y.L. Lau and B. G. Krishnappan (5), an altogether different approach has been presented. They adopted the k-ϵ turbulence model proposed by Launder and Spalding (6) to investigate the effects of ice cover on the stream flows and vertical mixing. According to their approach, the momentum diffusivity, ν_t, is expressed in terms of the kinetic energy of turbulent motion, k, and the rate of dissipation of kinetic energy, ϵ, as:

$$\nu_t = c_\mu \frac{k^2}{\epsilon} \tag{3}$$

where c_μ is an empirical constant.

The distributions for k and ϵ in turn were obtained by solving a set of semi-empirical transport equations for k and ϵ developed at the Imperial College, London, by Launder and Spalding (6).

The resulting distribution of ν_t from Lau and Krishnappan (5) is shown in Figs. 3 and 4 for two different flow conditions. Figure 3 shows the ν_t distribution for a flow with equivalent sand grain roughness for bed, k_{sb}, of 5 mm and a hydraulically smooth ice cover. The flow depth is 30 cm and the overall slope of the uniform flow is .0012. Figure 4 shows the distribution of ν_t for a flow with equal bed and ice cover roughness of 5 mm, a flow depth of 30 cm and a slope of .0019. Figures 3 and 4 also contain the double parabola distributions that would result if the flows were divided into two regions at the lines of maximum velocity and if logarithmic velocity distribution and linear shear stress distributions are assumed to exist in each layer as has been done by Sayre and Song. The work of Lau and Krishnappan permits the precise evaluation of the line of maximum velocity since the velocity distribution is also predicted by solving the x-momentum equation with ν_t as given by Eq. 3.

From Figs. 3 and 4, it can be seen that the ν_t distributions of Lau and Krishnappan are closer to Sayre and Song's distribution than Shen and Harden's. The symmetric distribution of Shen and Harden is valid only for the case where the roughnesses of the bed and ice cover are identical.

Lau and Krishnappan used the ν_t distributions resulting from the k-ε turbulence model to predict the concentration distributions along the length of the ice-covered flows for different boundary distributions of a neutrally-buoyant tracer. They performed similar calculations in free-surface flows as well to establish the effect of ice cover on the vertical mixing.

PRESENT CONTRIBUTION

In this study, the method of Lau and Krishnappan is used to predict the fully-developed vertical distribution of suspended sediment in ice-covered flows by solving the Schmidt-O'Brien equation, i.e. Eq. 2. The solution of Eq. 2 can be written as:

$$\frac{C}{C_a} = \exp \left[- w \int_a^y \frac{1}{\Gamma_y} dy \right] \qquad (4)$$

where C_a is the concentration at a level y=a which can be selected following the convention as:

$$a = .05\,h \qquad (5)$$

The mass transfer coefficient Γ_y is assumed to be proportional to the momentum diffusivity ν_t as has been done by Sayre and Song, i.e.

$$\Gamma_y = \beta \nu_t \tag{6}$$

β is the proportionality constant and is equal to the reciprocal of the turbulent Schmidt number.

For a two-dimensional, uniform, turbulent flow under ice cover with bottom roughness k_{sb}, ice cover roughness k_{st}, flow depth h and slope S, the momentum diffusivity ν_t can be expressed as:

$$\nu_t = f\ (k_{sb};\ k_{st};\ h;\ v_*;\ \rho;\ \mu;\ y) \tag{7}$$

ρ and μ are the density and absolute viscosity of fluid respectively and v_* is the net shear velocity defined as:

$$v_* = \sqrt{gsh} \tag{8}$$

g is the acceleration due to gravity.

The dimensional relation (7) can be expressed in dimensionless form by selecting ρ, v_* and h as repeating variables

$$\frac{\nu_t}{v_* h} = \phi_1 \left[\frac{k_{sb}}{h}, \frac{k_{st}}{h}, \frac{\rho\, v_* h}{\mu}, \frac{y}{h} \right] \tag{9}$$

If fully rough turbulent flows are considered, then the Reynolds number $(\rho v_* h/\mu)$ ceases to be a parameter and the relation can be rewritten as:

$$\frac{\nu_t}{v_* h} = \phi_2 \left[\frac{k_{st}}{k_{sb}}; \frac{k_{sb}}{h}; \frac{y}{h} \right] \tag{10}$$

The method of Lau and Krishnappan was applied to a large number of flow conditions for different values of the dimensionless parameters appearing on the right hand side of Eq. 10. The parameter k_{sb}/h varied between .001 to .1 for four different values of the ratio k_{st}/k_{sb}, namely, 1.00, 0.50, 0.10 and 0.00. Table 1 shows the distribution of the dimensionless momentum diffusivity $(\nu_t/v_* h)$ for three sets of two flows. Each set corresponds to a different value of the ratio (k_{st}/k_{sb}). In each set, flow number 1 corresponds to the lowest value of the parameter (k_{sb}/h) whereas flow number 2 represents the flow with largest value.

As can be seen from Table 1, for ratio k_{st}/k_{sb}=1.00, i.e. when the ice cover roughness equals the channel bed roughness, the dimensionless momentum diffusivity is independent of the parameter (k_{sb}/h) for the whole range of (k_{sb}/h) tested. The difference in $(\nu_t/v_* h)$ values for flow number 1 with (k_{sb}/h)=.001 and for flow number 2 with (k_{sb}/h)=.10 is noticeable only at the fifth decimal place. (The values given in Table 1 are rounded off to the third decimal place.) A similar result is evident when the ratio (k_{st}/k_{sb})=0.50, i.e. when the ice cover roughness is half of the channel bed roughness. For the ratio (k_{st}/k_{sb})=0.10, the lower limit of the parameter (k_{sb}/h) has to be greater than or equal to .02 so that the flow is fully rough turbulent at both the boundaries. Therefore, the flow number 1 listed in Table 1 has a value of .02 for the parameter (k_{sb}/h). The flow number 2 corresponds to (k_{sb}/h) value of .10 as in the other two cases. For this case, the difference in $(\nu_t/v_* h)$ values between flow number 1 and flow number 2 is slightly larger but still less than 5 percent. For practical purposes, the $(\nu_t/v_* h)$ values for this case too can be treated as independent of the parameter (k_{sb}/h).

For (k_{st}/k_{sb})=0, i.e. when the ice cover behaves as an hydraulically smooth surface, the values of $(\nu_t/v_* h)$ are not listed in Table 1. For this case, since the

TABLE 1. - The Distribution of the Dimensionless Momentum Diffusivity (ν_t/v_*h) for Ice-Covered Flows

$\eta=y/h$	$(k_{st}/k_{sb})=1.0$		$(k_{st}/k_{sb})=0.50$		$(k_{st}/k_{sb})=0.10$	
	$(k_{sb}/h)=.001$ Flow No. 1	$(k_{sb}/h)=0.10$ Flow No. 2	$(k_{sb}/h)=.001$ Flow No. 1	$(k_{sb}/h)=.10$ Flow No. 2	$(k_{sb}/h)=.02$ Flow No. 1	$(k_{sb}/h)=.10$ Flow No. 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.00	.007	.007	.007	.007	.008	.008
0.05	.017	.017	.018	.018	.019	.020
0.10	.023	.023	.025	.025	.028	.028
0.15	.028	.028	.030	.031	.034	.035
0.20	.031	.031	.034	.034	.039	.040
0.25	.033	.033	.036	.037	.042	.044
0.30	.035	.035	.037	.038	.044	.046
0.35	.036	.036	.038	.039	.045	.047
0.40	.037	.037	.038	.039	.045	.047
0.45	.038	.038	.039	.039	.044	.046
0.50	.039	.039	.039	.039	.044	.045
0.55	.038	.038	.038	.038	.042	.044
0.60	.037	.037	.036	.036	.040	.041
0.65	.036	.036	.034	.034	.036	.037
0.70	.034	.034	.032	.032	.031	.032
0.75	.033	.033	.031	.030	.027	.027
0.80	.031	.031	.029	.028	.024	.024
0.85	.028	.028	.026	.026	.022	.021
0.90	.023	.023	.022	.022	.019	.018
0.95	.017	.017	.016	.016	.014	.014
1.00	.007	.007	.007	.007	.006	.006

flow near the upper boundary is dependent on the fluid viscosity, the effects of the Reynolds number (v_*h/ν) cannot be ignored. Hence, it is not possible to consider the distribution of the dimensionless momentum diffusivity as a function of the ratio (k_{st}/k_{sb}) alone as has been done for the other three cases.

Taking advantage of the fact that, for the ice-covered flows where the effects of Reynolds number are negligible, the distribution of the dimensionless momentum diffusivity is a function of the (k_{st}/k_{sb}) alone, a set of families of curves for the distribution of the suspended sediment in ice-covered flows is evaluated in the present work. Figure 5 shows the dimensionless momentum diffusivity distributions for three different ratios of the ice cover and bed roughnesses. The solid line is for the case when (k_{st}/k_{sb})=1.0. The dotted line is for (k_{st}/k_{sb})=0.5 and the dash-dot line is for (k_{st}/k_{sb})=0.1. The dash-dot line was plotted using an average value of the last two columns in Table 1.

The distribution curves shown in Fig. 5 were used to evaluate the integral on the right hand side of Eq. 4. For this purpose, Eq. 4 is rewritten as:

$$\frac{C}{C_a} = \exp\left[-\frac{w}{\beta v_*} A\right] \qquad (11)$$

where A stands for the integral:

$$\int_{\eta_a}^{\eta} \frac{1}{(\nu_t/v_*h)}\, d\eta \qquad (12)$$

The integral A is evaluated numerically using the trapezoidal rule and is plotted as a function of η in Fig. 6. The values used in Fig. 6 are also listed in Table 2. For a given value of the parameter ($w/\beta v_*$), the relative concentration distribution (C/C_a) in an ice-covered flow can be evaluated using Eq. 11 and the values of A.

TABLE 2. - Values of $A = \int_{\eta_a}^{\eta} (\nu_t/v_*h)^{-1} d\eta$ for Various Values of η and (k_{st}/k_{sb})

$\eta=y/h$	$\frac{\eta-\eta_a}{1-\eta_a}$	$A = \int_{\eta_a}^{\eta} (\nu_t/v_*h)^{-1} d\eta$		
		$(k_{st}/k_{sb})=0.10$	$(k_{st}/k_{sb})=0.50$	$(k_{st}/k_{sb})=1.00$
(1)	(2)	(3)	(4)	(5)
$0.05=\eta_a$	0	0	0	0
0.10	0.053	2.143	2.389	2.558
0.15	0.105	3.750	4.196	4.537
0.20	0.158	5.089	5.737	6.237
0.25	0.211	6.296	7.148	7.801
0.30	0.263	7.433	8.482	9.272
0.35	0.316	8.532	9.781	10.662
0.40	0.368	9.619	11.063	12.014
0.45	0.421	10.718	12.345	13.348
0.50	0.474	11.829	13.627	14.647
0.55	0.526	12.966	14.926	15.946
0.60	0.579	14.157	16.279	17.279
0.65	0.632	15.442	17.708	18.631
0.70	0.684	16.899	19.225	20.021
0.75	0.737	18.607	20.839	21.493
0.80	0.790	20.574	22.565	23.057
0.85	0.842	22.752	24.420	24.756
0.90	0.895	25.204	26.517	26.736
0.95	0.947	28.306	29.216	29.293
1.0	1.000	34.258	34.350	34.335

For each value of the ratio (k_{st}/k_{sb}), a family of curves for the variation of the relative concentration C/C_a as a function of the dimensionless height η with $(\beta v_*/w)$ as a parameter is produced and is given in Figs. 7, 8 and 9. In these figures, the vertical ordinate is plotted as $((\eta-\eta_a)/(1-\eta_a))$ instead of just η to conform with the concentration distribution curves produced by H. Rouse (8) for free-surface flows. The distribution curves of Rouse are reproduced in Fig. 10 for comparison.

DISCUSSION OF THE RESULTS

The dimensionless momentum diffusivity distribution curves shown in Fig. 5 indicate that when the ratio (k_{st}/k_{sb}) takes a value of one, the distribution is symmetrical and as the ratio decreases the distributions become more and more skewed towards the rougher boundary. The integrals of the distributions (see Fig. 6) tend to collapse into single lines near both the boundaries and deviate only at the central regions of the flow. For a given elevation from the channel bed, the value of the integral A, is large for larger values of the ratio (k_{st}/k_{sb}). Consequently the ratio C/C_a is less at the central regions of the flow for higher values of (k_{st}/k_{sb}) (see Figs. 7, 8 and 9). In other words, for a given value of $(\beta v_*/w)$ as the ice cover roughness increases in relation to the channel bed roughness, the relative concentration (C/C_a) decreases.

Comparing the concentration distribution curves of the ice-covered flows with those of the free-surface flows shown in Fig. 10, it can be seen that for the same value of the parameter $(\beta v_*/w)$ the relative concentration in ice-covered flows is less than that in the free-surface flows. This applies to all ratios of (k_{st}/k_{sb}).

For the same value of $(\beta v_*/w)$, the absolute concentration C in an ice-covered flow will be much less than that in an "equivalent" free-surface flow because the reference concentration C_a in the former is less than that of the latter. Note that an "equivalent" free-surface flow is defined as the flow with same energy slope and same discharge per unit width as that of an ice-covered flow and herefore for a given value of the total shear velocity v_*, the shear velocity at the channel bed (responsible for sediment transport) of the ice-covered flow will be less than that of the free-surface flow.

There is no reliable method, as yet, to predict the reference concentration C_a, even in a free-surface flow. H. A. Einstein (1) proposed a method wherein the reference concentration is considered at a distance of two particle diameters above the bed is related to the theoretical bed-load transport of sediment and the velocity at the outer edge of the viscous sublayer. In order to apply a similar approach to the ice-covered flows, the shear stress at the channel bottom has to be specified. The method of Lau and Krishnappan permits the evaluation of the shear stress acting at the channel bed knowing the total shear stress and the momentum diffusivity at the boundaries as follows.

Assuming that near the wall region the production and dissipation of turbulent kinetic energy are approximately in balance and the turbulent shear stress is constant and is equal to the wall shear stress, the values of the kinetic energy (k_{wt} and k_{wb}) and the dissipation rate (ε_{wt} and ε_{wb}) near the boundaries can be related to the boundary shear velocities as:

$$k_{wt} = \frac{v_{*t}^2}{\sqrt{c_\mu}} \quad \text{and} \quad \varepsilon_{wt} = \frac{v_{*t}^3}{\kappa y_w}$$

$$k_{wb} = \frac{v_{*b}^2}{\sqrt{c_\mu}} \quad \text{and} \quad \varepsilon_{wb} = \frac{v_{*b}^3}{\kappa y_w} \tag{13}$$

Where κ is the Von Karman constant and y_w is the thickness of the wall region. Using Eq. 3 and considering the same value for y_w at the top and bottom boundaries, the momentum diffusivities near the boundaries can be expressed in terms of the shear velocities at the boundary as:

$$\frac{\nu_{t_{y=h}}}{\nu_{t_{y=0}}} = \frac{v_{*t}}{v_{*b}} \tag{14}$$

The total shear velocity v_* can be expressed as:

$$v_*^2 = v_{*t}^2 + v_{*b}^2 \tag{15}$$

Using Eqs. 14 and 15 and knowing the L.H.S. of both equations, the shear velocities at the top and bottom boundaries can be evaluated.

The application of the distribution curves given in Figs. 7, 8 and 9 to practical problems is rather simple. The variables that need to be known for the application of these distribution curves are a) the total shear velocity v_* which can be evaluated using Eq. 8 knowing the net slope of the ice-covered stream S and the total flow depth h; b) the fall velocity of the sediment particles; c) the value of β which is close to 1.0; and d) an indication of the roughness of the ice cover roughness in relation to the channel bed roughness. The last item may be the most difficult one to establish. A visual examination of the bottom side of the ice cover might give a clue. If a measurement of the velocity profile is available, then it is possible to establish the precise ratio of two roughnesses using the method of Lau and Krishnappan (5). If the values of the parameters (k_{st}/k_{sb}) and $(\beta v_*/w)$ do not coincide with the ones for which the distribution curves are given, then the distribution can be obtained by interpolation. If the parameter (k_{st}/k_{sb}) coincides with one of the three cases considered and the parameter $(\beta v_*/w)$ does not coincide with the one for which the distribution curve is drawn in Figs. 7, 8 and 9, the distribution can be computed using the values of A listed in Table 2 and Eq. 11.

Comparison with Experimental Data. - The only measurement the author could find in the literature on suspended sediment concentration in ice-covered flow is the one reported by Sayre and Song (9). Sayre and Song measured suspended sediment concentrations in six different flows with simulated ice covers. Three flows had smooth top cover while the other three had rough cover. Data were presented in the form of distribution curves only for four flows, two with smooth cover and the other two with rough cover. The smooth cover flows were denoted by AS and BS while the rough cover runs were denoted by AR and BR. A comparison of the present curves with the experimental data is shown in Figs. 11 and 12. Fig 11 shows the comparison for runs AS and AR while Fig. 12 shows the same for runs BS and BR. The smooth top cover runs were compared with the distribution curves resulting from $(k_{st}/k_{sb})=0.10$ and the rough cover runs were compared with the distribution curves of $(k_{st}/k_{sb})=1.0$. Each set of experimental data follows one particular distribution curve reasonably well. However, the resulting values of β are much too large. Table 3 lists the values of β together with the other hydraulic parameters of Sayre and Song's experiments. Sayre and Song evaluated β using their model of the flow and also found unreasonably large values for β. They obtained different β values for the two layers. Their values ranged between 1.9 to 25.0. Sayre and Song attributed the inaccuracies in the measurement of small sediment concentrations and the possibility that the sediment particles actually in suspension might have been much finer than the sediment size used to compute the fall velocity w, for such a result.

TABLE 3. - Summary of Hydraulic Parameters for Sayre and Song's Experiments

Run (1)	Mean Depth h in Metres (2)	Slope S (3)	Shear Velocity v_* in Metres/Second (4)	Fall velocity w in Metres/Second (5)	$\beta v_*/w$ (6)	β (7)
AS	0.118	0.00180	0.046	0.033	20.0	14.3
AR	0.121	0.00180	0.046	0.033	12.0	8.6
BS	0.122	0.00204	0.049	0.033	8.0	5.4
BR	0.184	0.00211	0.055	0.033	5.0	3.0

SUMMARY AND CONCLUSIONS

Three sets of suspended sediment concentration distribution curves are presented in this paper for two-dimensional and uniform turbulent flows under ice cover analogous to the distribution curves of H. Rouse for the free-surface flows. The distribution curves were derived using the flow predictions based on the k-ε turbulence model. For fully rough turbulent flows, the vertical distribution of the dimensionless mass transfer coefficient depends only on the ratio of the ice cover roughness to the channel bed roughness. For the same value of the parameter ($\beta v_*/w$), the relative concentration is less in the ice-covered flow than in the free-surface flow. For the same value of the parameter ($\beta v_*/w$), the relative concentration vdecreases as the ratio of ice cover roughness to bed roughness increases. The agreement between the present distribution curves and the measurement found in the literature is favourable.

ACKNOWLEDGMENTS

The author wishes to acknowledge the fruitful discussions he had with Drs. Y. L. Lau and S. Beltaos of the National Water Research Institute. The author also wishes to thank Dr. Y. L. Lau and Dr. T. M. Dick for their critical review of the manuscript.

APPENDIX I. - REFERENCES

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APPENDIX II. - NOTATION

The following symbols are used in this paper:

a = reference level;

C = volumetric sediment concentration;

C_a = C at a

c_μ = empirical constant;

g = acceleration due to gravity;

h = flow depth;

k = kinetic energy of turbulent motion;

k_{sb} = equivalent sand roughness of channel bed;

k_{st} = equivalent sand roughness of ice cover;

S = slope of the channel bottom;

u = time-averaged local velocity along the channel;

v_* = shear velocity;

v_{*b} = shear velocity at channel bed;

v_{*t} = shear velocity at ice cover;

y = coordinate in the vertical direction;

y_w = thickness of wall region;

β = proportionality constant;

Γ_y = mass transfer coefficient in the vertical direction;

ϵ = rate of dissipation of kinetic energy

η = y/h

κ = Von Karman constant;

ν_t = momentum diffusivity;

ρ = fluid density;

τ = shear stress;

ϕ_1, ϕ_2 = dimensionless functions.

Civil Engineering Abstract: Concentration distribution of suspended sediments in ice-covered flows is formualted for different ratios of ice cover and bed roughnesses. The agreement between the present formulation and the measurements found in the literature is favourable.

LIST OF FIGURES

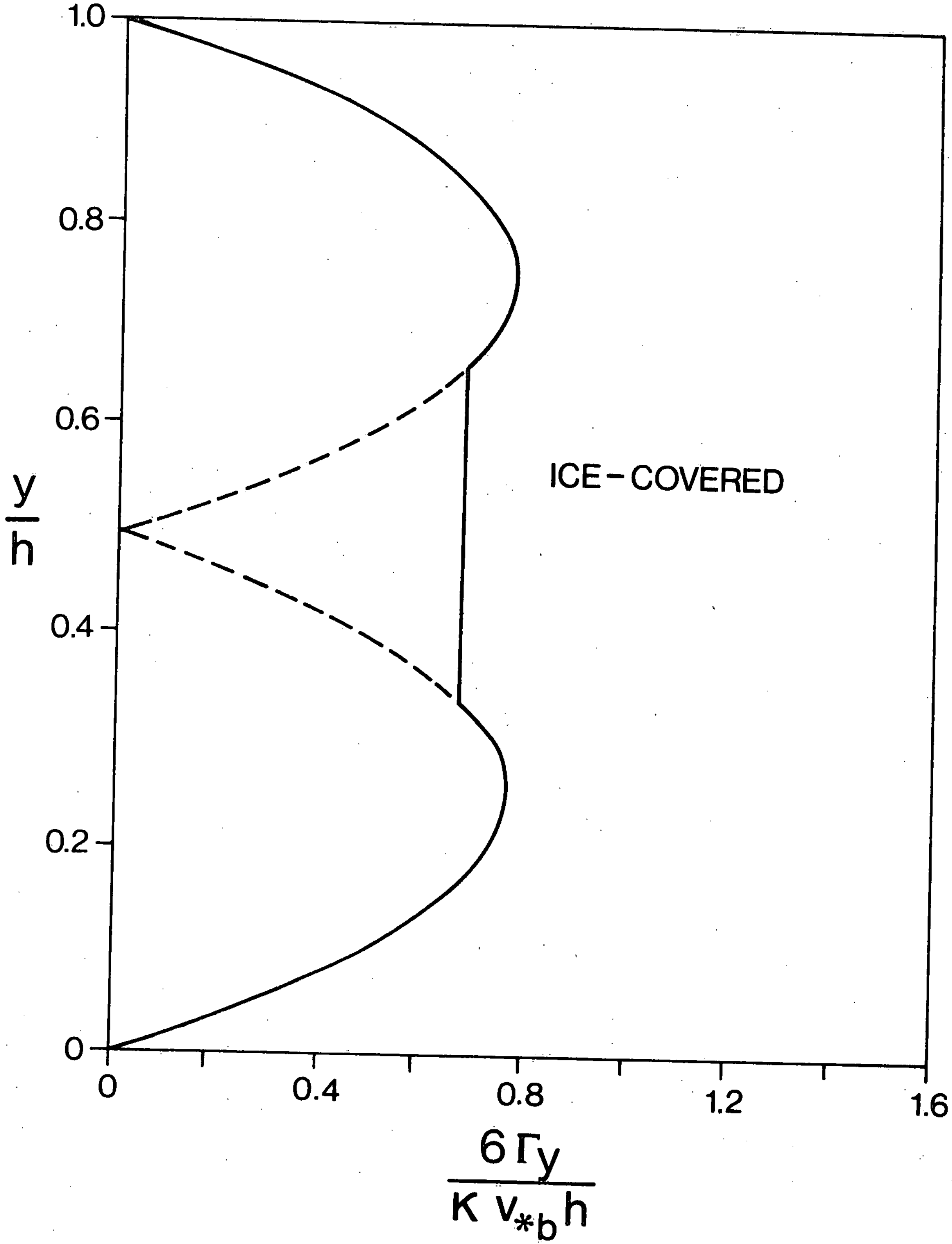

Figure 1. Distribution of mass transfer coefficient used by Shen and Harden (10).

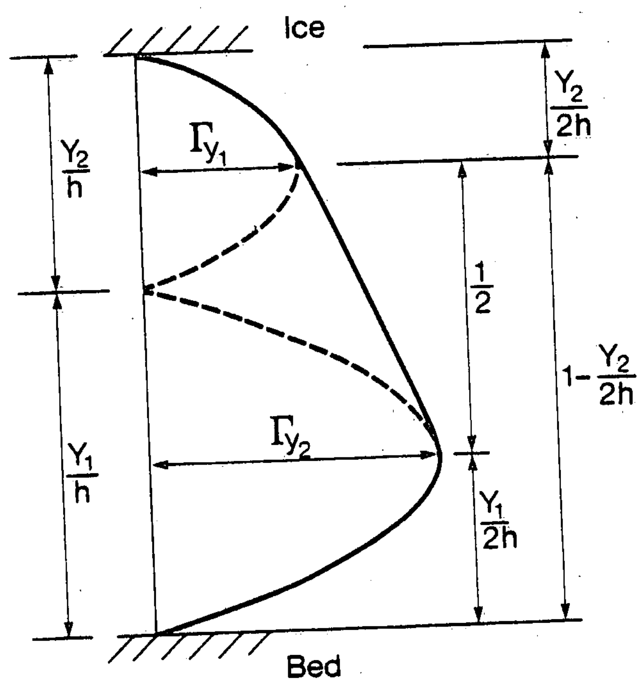

Figure 2. Schematic representation of the distribution of mass transfer coefficient used by Sayre and Song (9).

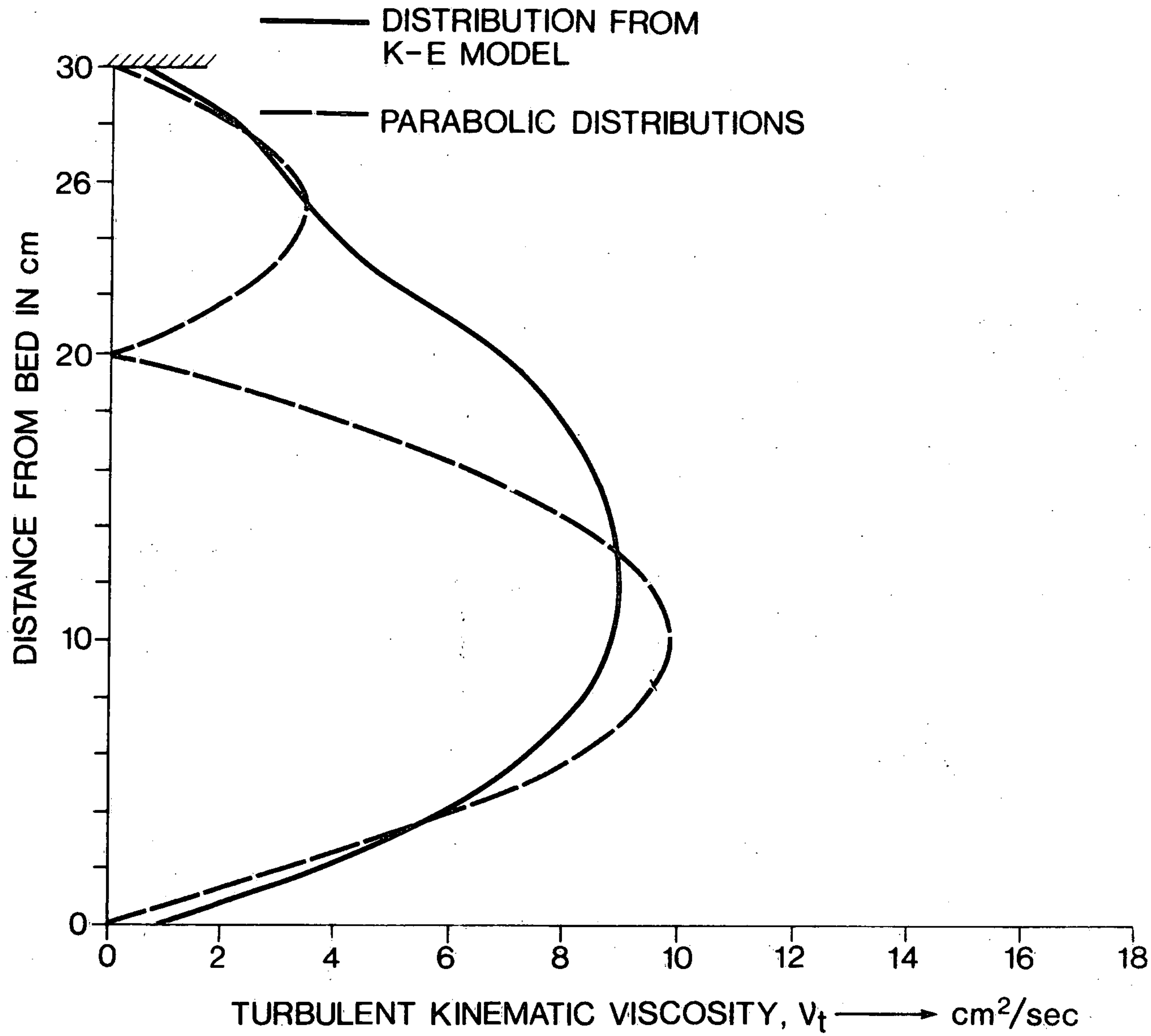

FIGURE 3 ν_t-DISTRIBUTION FOR RUN # 2 (REF(5))

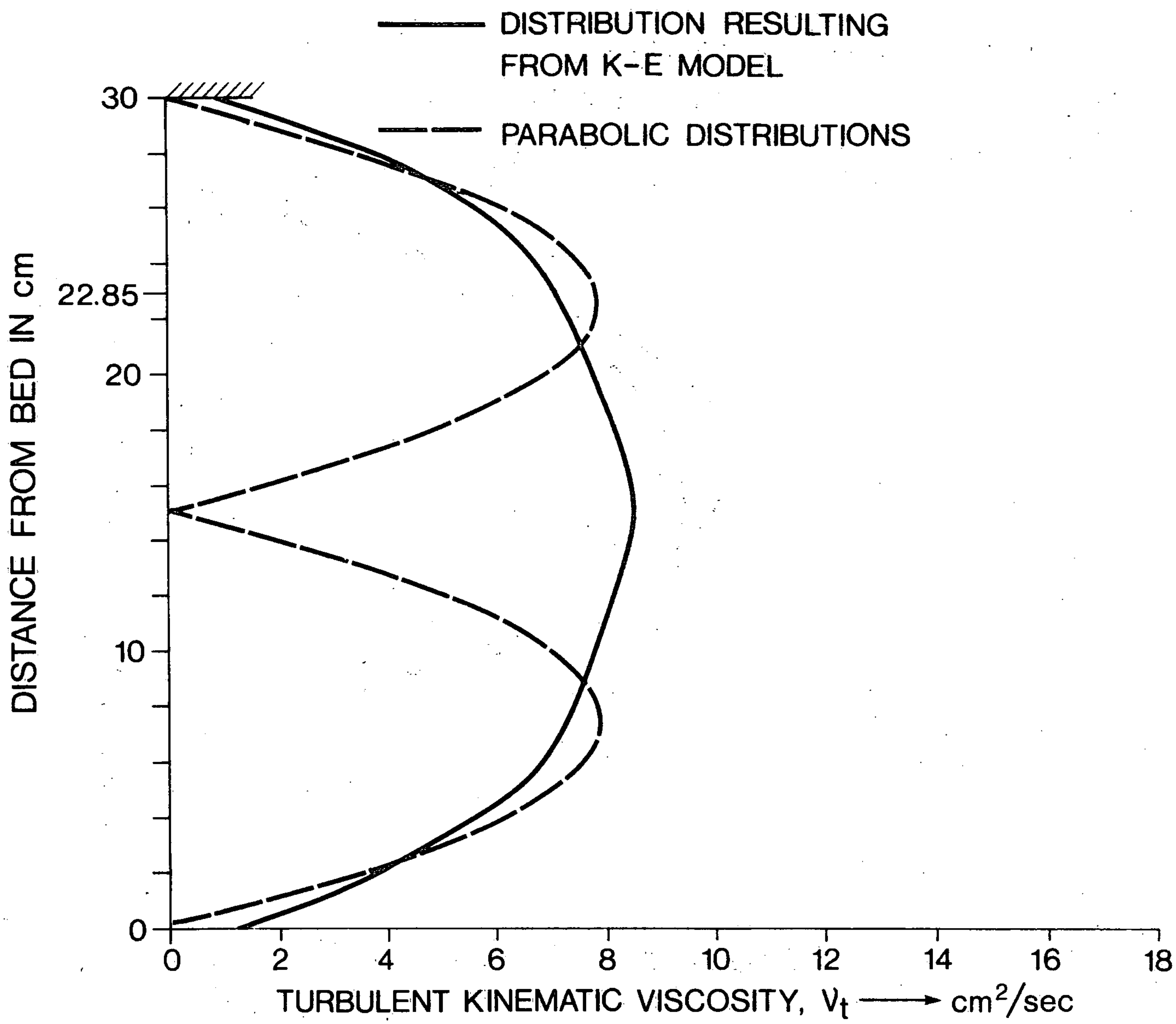

FIGURE 4 ν_t - DISTRIBUTION FOR RUN # 3 (REF(5))

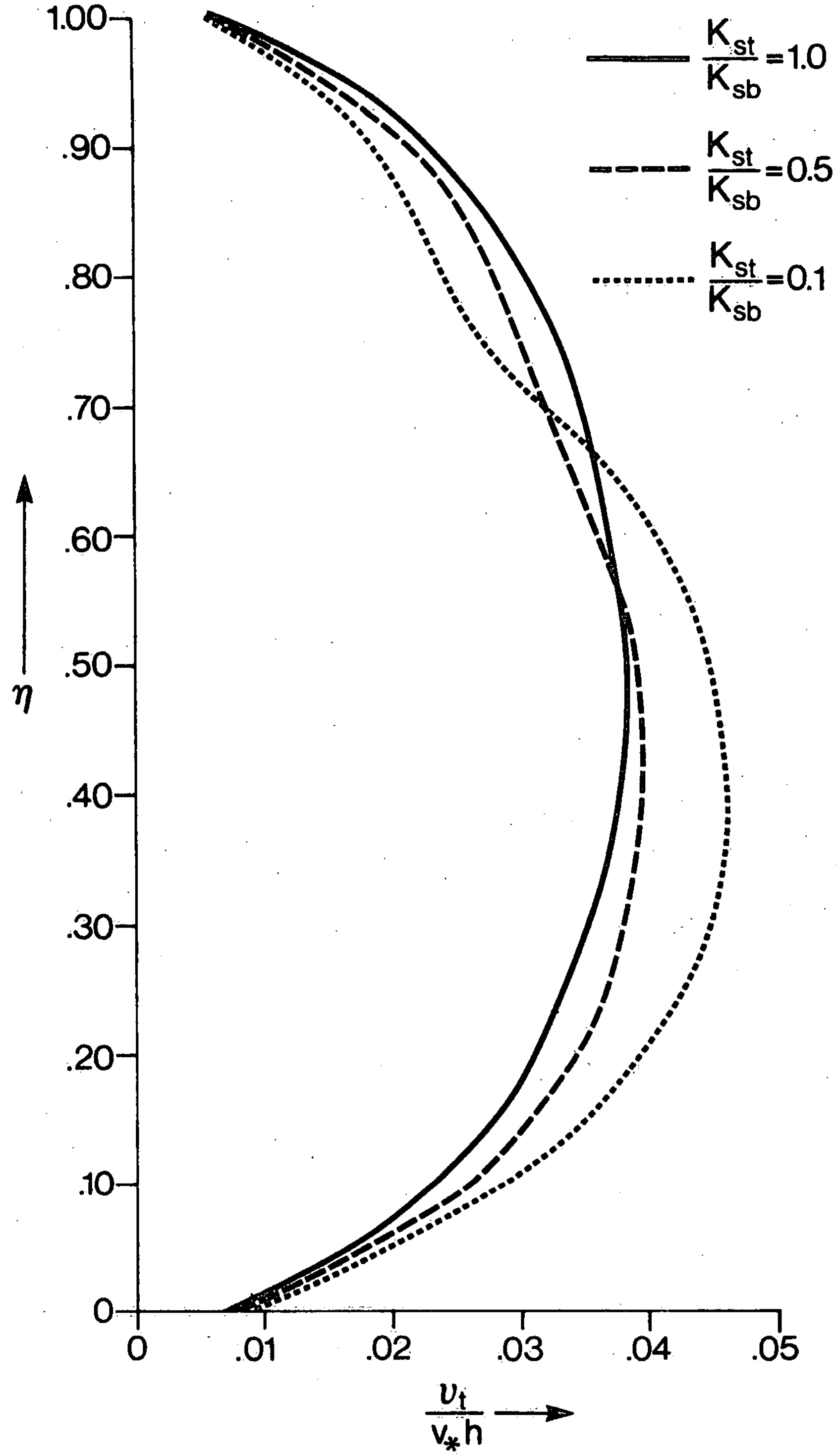

Figure 5 Distribution of the dimensionless momentum diffusivity for ice cover flows resulting from K-E turbulence model.

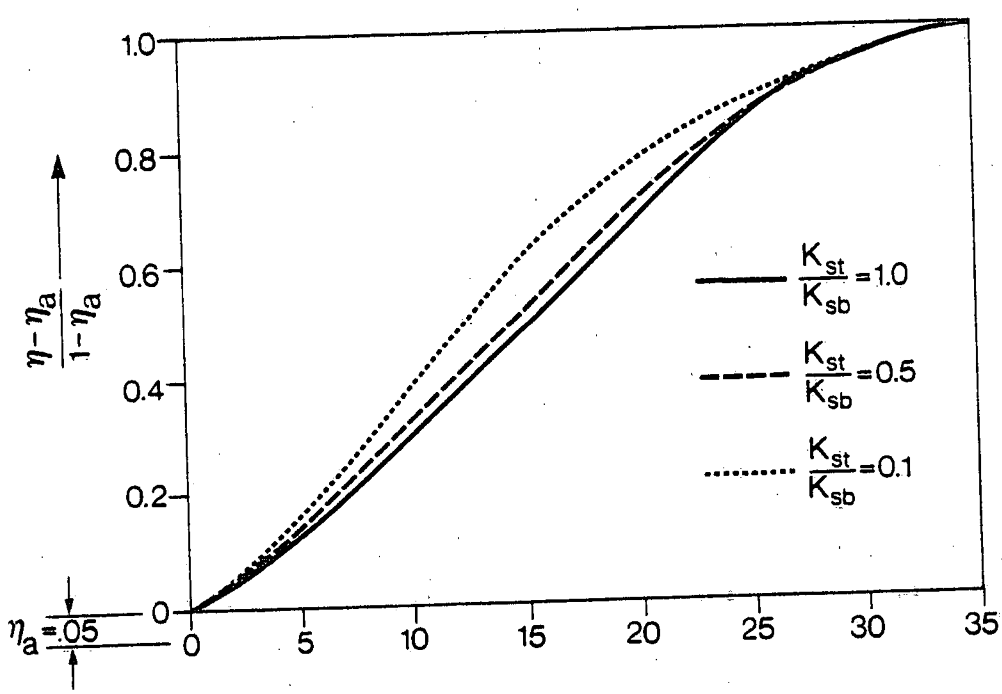

Figure 6 Variation of A with η

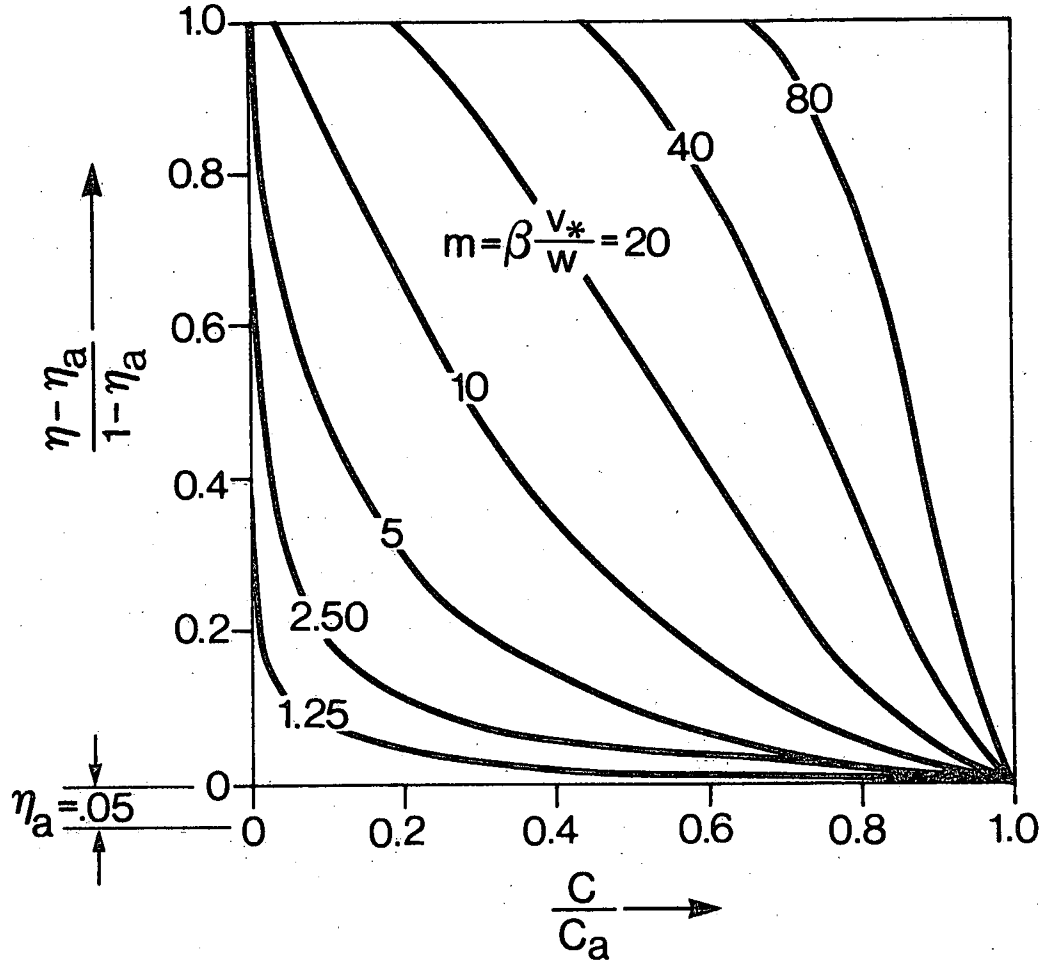

Figure 7 Concentration distribution curves for ice covered flows. The ratio between ice cover roughness to bed roughness is one tenth.

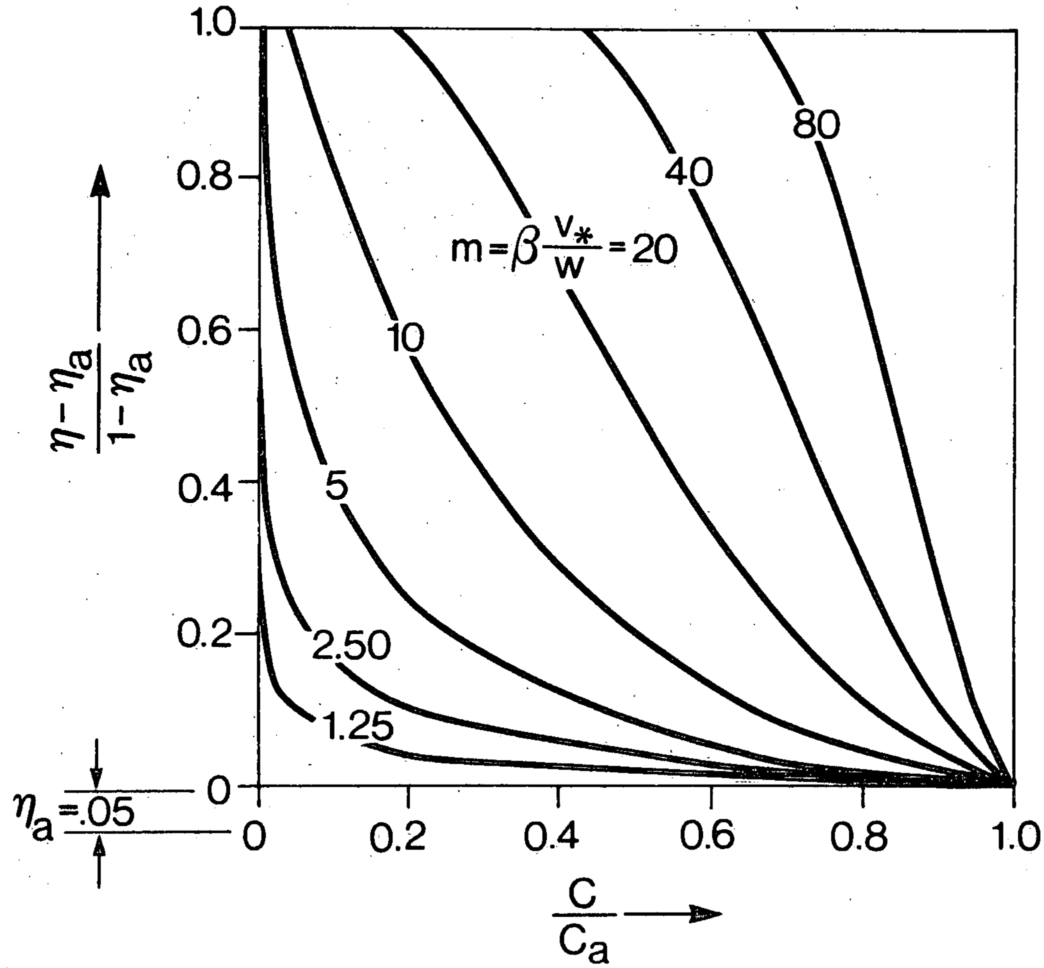

Figure 8 Concentration distribution curves for ice covered flows. The ratio between ice cover roughness to bed roughness is one half.

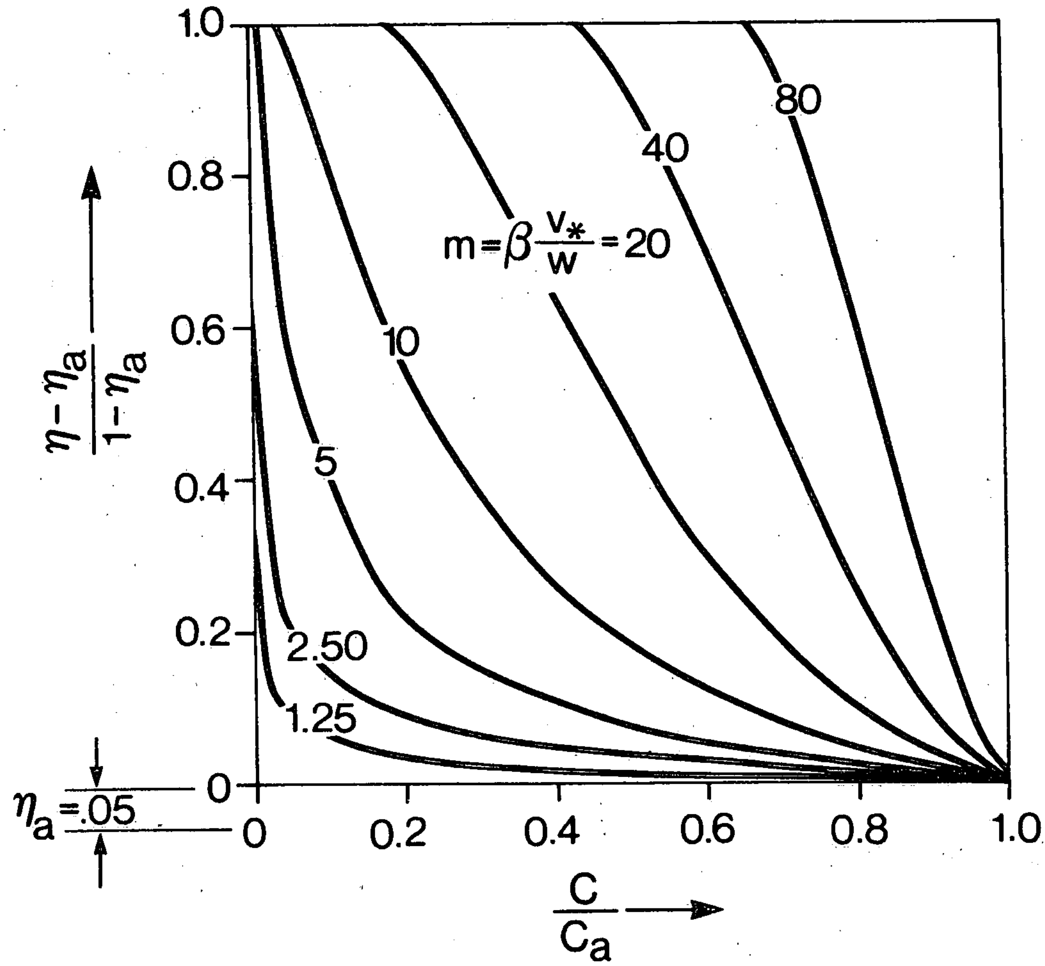

Figure 9 Concentration distribution curves for ice covered flows. The underside of ice cover is as rough as the channel bed.

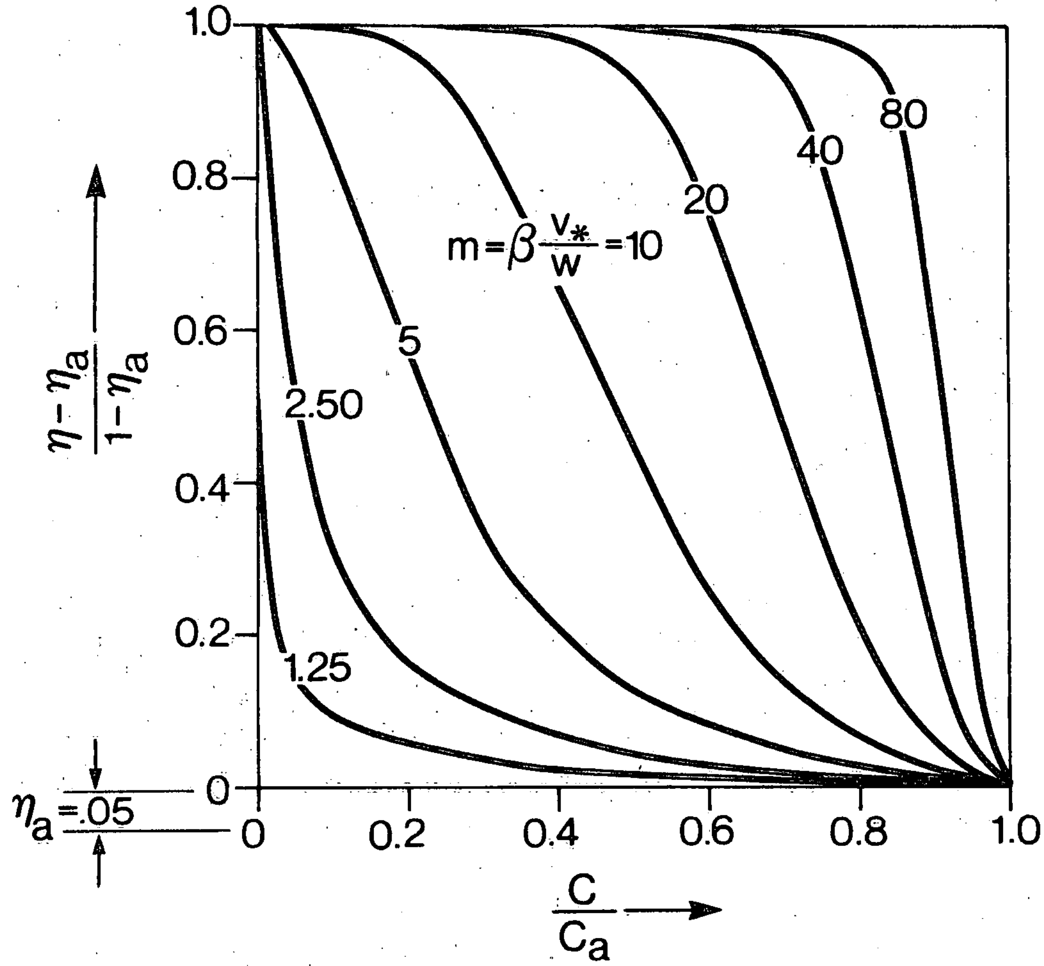

Figure 10 Concentration distribution curves of H. Rouse (8) for free surface flows.

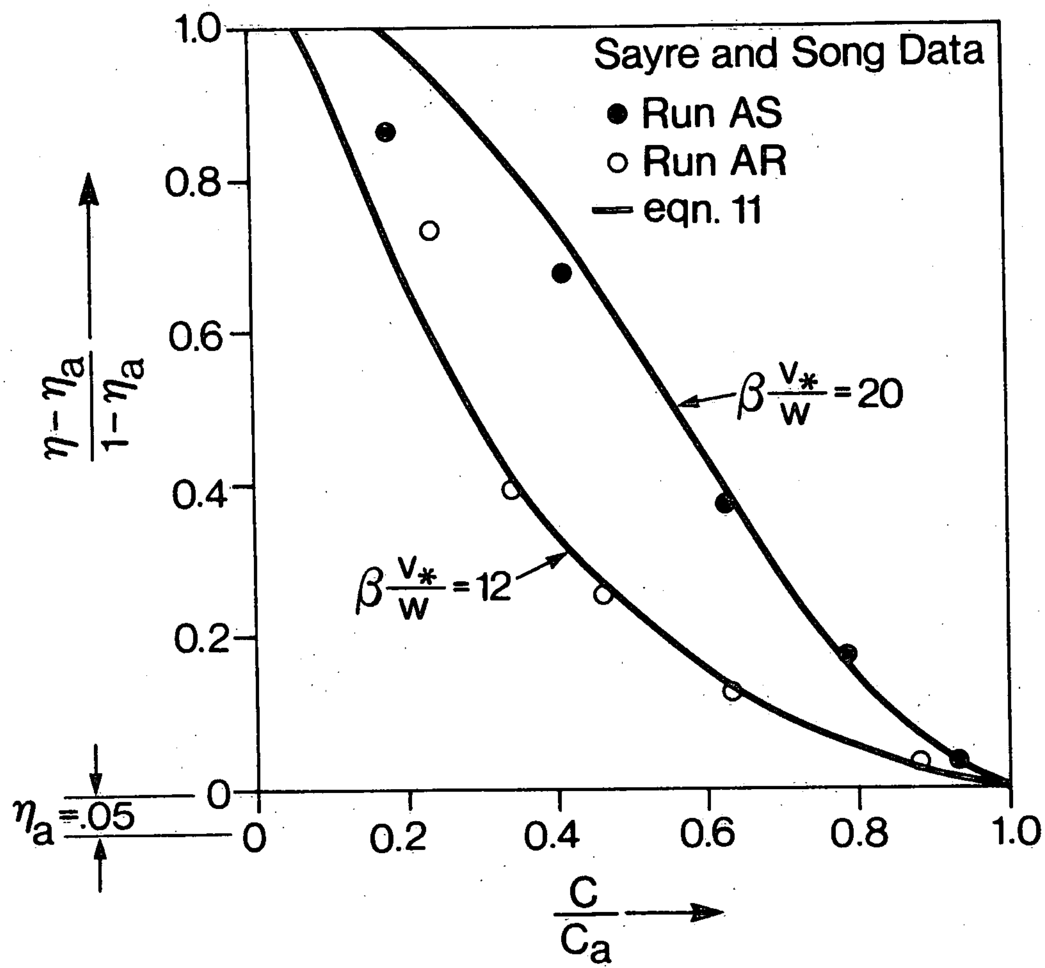

Figure 11 Comparison with experimental data

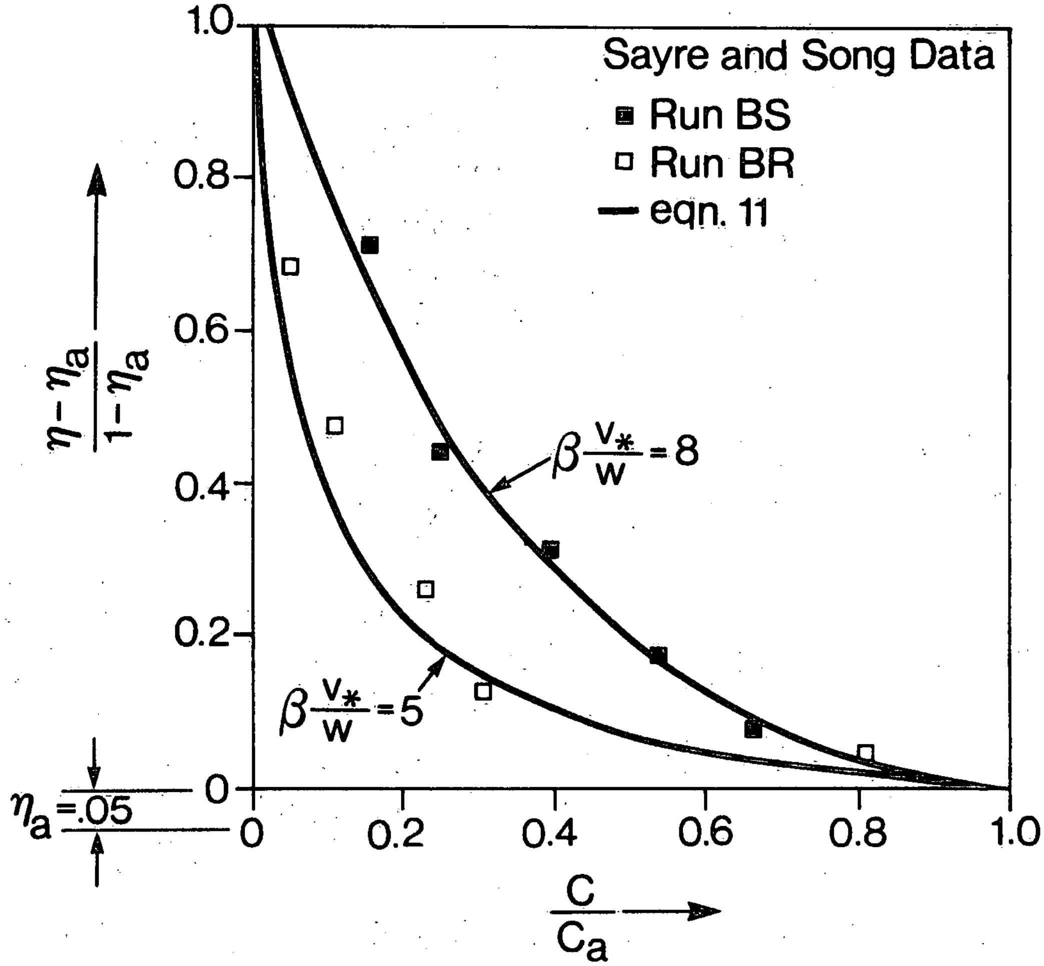

Figure 12 Comparison with experimental data