



This manuscript summarizes lecture notes presented partly at the IWD Training Seminar on "Hydraulics of Ice Covered Rivers and Ice Jam Analysis", June 14 and 15, 1982, Hull, Quebec. The seminar was organized by Water Planning and Management Branch Environment Canada, Ottawa, Ontario

NOTES ON ICE HYDRAULICS

By  
S.Beltaos

Environmental Hydraulics Section  
Hydraulics Division  
National Water Research Institute  
Canada Centre for Inland Waters  
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## FOREWORD

A training seminar for the Inland Waters Directorate was held on June 14 and 15, 1982, at Place Vincent Massey, Hull, Quebec. This seminar had the theme "Hydraulics of Ice Covered Rivers and Ice Jam Analysis" and was organized by the Water Planning and Management Branch of Environment Canada, Ottawa, Ontario. The seminar speakers were D. J. Calkins, U. S. Army Cold Regions Research and Engineering Laboratory; R. Gerard, Department of Civil Engineering, University of Alberta; and S. Beltaos, Environmental Hydraulics Section, Hydraulics Division, National Water Research Institute.

This report summarizes lecture notes by S. Beltaos and has been prepared in accordance with Divisional requirements and procedures, including reviews by T. M. Dick, Chief, Hydraulics Division and Y. L. Lau, Head, Environmental Hydraulics Section.

## AVANT-PROPOS

Un séminaire ayant pour thème l'hydraulique des cours d'eau couverts de glace et analyse des embâcles a eu lieu les 14 et 15 juin 1982 à la Place Vincent-Massey, Hull (Québec). Ce séminaire avait été organisé à l'intention de la Direction générale des eaux intérieures par la Direction de la planification et de la gestion des eaux d'Environnement Canada, Ottawa (Ontario). Les conférenciers étaient D. J. Calkins, U. S. Army Cold Regions Research and Engineering Laboratory, R. Gerard, Département du Génie civil de l'Université d'Alberta et S. Beltaos, Section de l'hydraulique environnementale, Division de l'hydraulique, Institut national de recherche sur les eaux.

Le présent rapport est un résumé des notes de conférence de S. Beltaos et a été préparé conformément aux exigences et procédures de la Division qui comprennent une révision par T. M. Dick, chef, Division de l'hydraulique et par Y. L. Lau, chef, Section de l'hydraulique environnementale.

## MANAGEMENT PERSPECTIVE

These lecture notes provide up-to-date information on the hydraulic phenomena related to ice jams in rivers.

Theories and procedures for analysis are explained and illustrated with examples.

This report should provide a valuable reference to all engineers faced with flooding and ice jam problems.

T. Milne Dick, Chief  
Hydraulics Division  
December 17, 1982

## PERSPECTIVE DE GESTION

Les présentes notes de conférence fournissent des renseignements mis à jour sur les phénomènes hydrauliques se rapportant aux embâcles des cours d'eau.

Théories et procédés d'analyse sont expliqués et illustrés à l'aide d'exemples.

J'estime que le présent rapport constitue un document de référence précieux pour tous les ingénieurs qui ont à résoudre des problèmes causés par les inondations et les embâcles.

T. Milne Dick  
Chef, Division de l'hydraulique  
Le 17 décembre 1982

## ABSTRACT

Part I of this report deals with the "Hydraulics of Ice Covered Rivers". First, a brief review of basic concepts of open channel flow is given, leading to the various hydraulic resistance relationships and definitions of the associated coefficients. Extension of these concepts to flow under an ice cover is presented next, leading to composite hydraulic resistance relationships and coefficients; values of the latter for different ice cover types are reviewed. A brief discussion of the effects of a moving cover on flow depth is included. Part II of the report is entitled "Breakup and Ice Jams". In this part, a qualitative description of ice breakup, jamming and means of alleviation of their effects, is followed by a review of possible procedures to predict potential stages caused by ice jams.

## RÉSUMÉ

La première partie de ce rapport traite de l'hydraulique des cours d'eau couverts de glace. Elle comporte, d'abord, un bref aperçu des concepts de base de l'écoulement à surface libre, puis passe aux divers rapports de résistance hydraulique et à la définition des coefficients associés. Ensuite, on étend ces concepts à l'écoulement sous une couverture de glace, ce qui amène aux rapports et aux coefficients composés de résistance hydraulique; on examine ce que représentent ces derniers pour les différents genres de couverture de glace. De plus, on parle brièvement des effets d'une couverture mouvante sur la hauteur d'écoulement. La 2<sup>e</sup> partie du rapport traite des débâcles et embâcles et des moyens d'atténuer leurs effets; vient ensuite une étude des moyens possibles de prévoir les niveaux éventuels créés par les embâcles.

## PART I

## HYDRAULICS OF ICE COVERED RIVERS

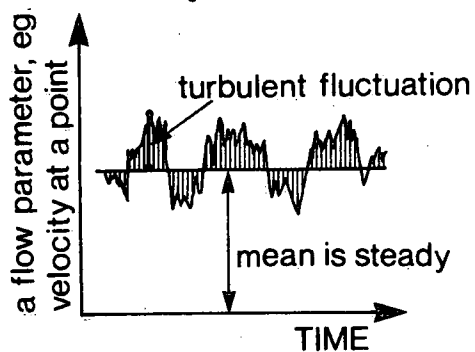
### 1.0 BASICS - OPEN CHANNEL FLOW

#### 1.1 Definitions

Unsteady flow:

Flow parameters, e. g. velocity at a given point, change with time.

Steady flow:



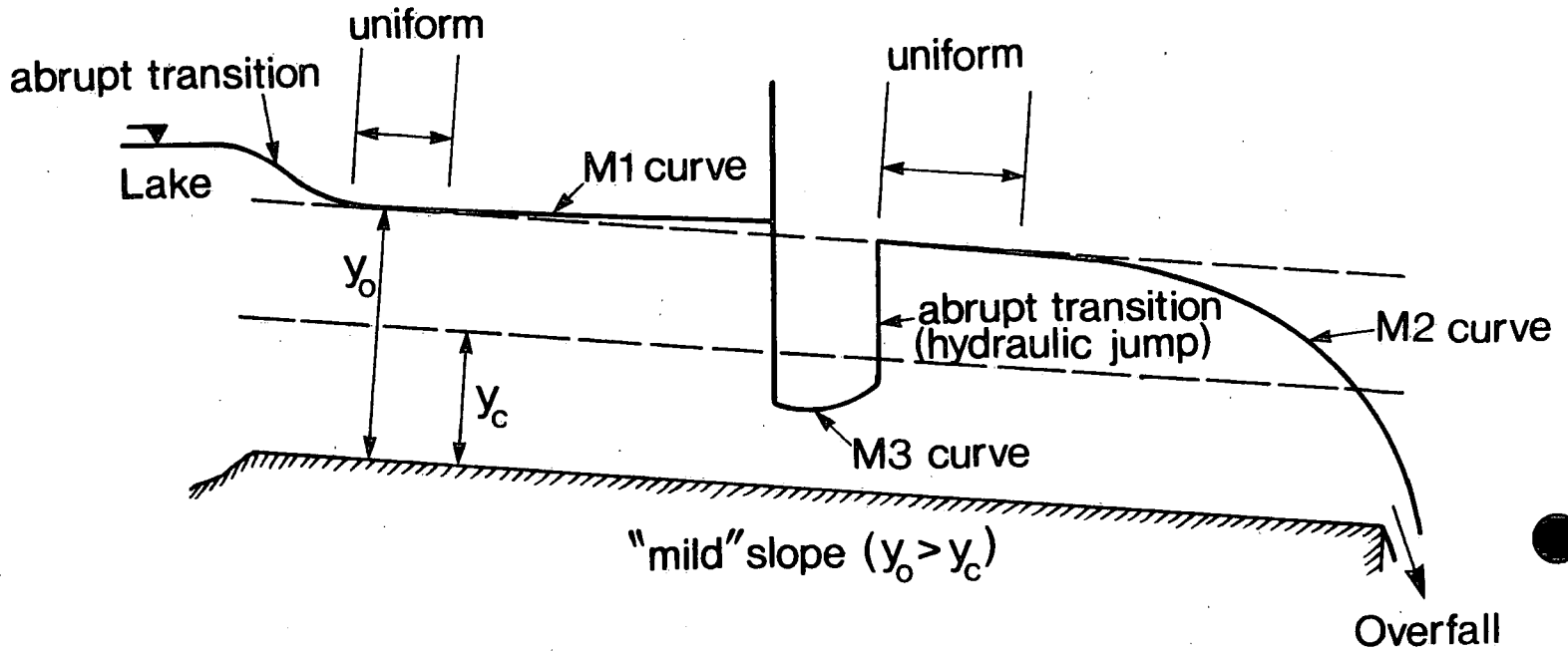
Flow parameters do not change with time. Turbulent flow is, strictly speaking, unsteady. However, there is a class of turbulent flows where flow parameters fluctuate about stable mean values. Such flows we may consider "steady" insofar as the mean values are concerned. Note that in most applications in hydraulics we have to deal with turbulent flow.

- Steady flow may be subdivided into two categories: Non-uniform and uniform.
- Uniform flow in a channel or conduit occurs when flow parameters do not change with downstream distance. According to Henderson<sup>1</sup>, uniform flow "is the state which the flow tends to assume in a long uniform channel when no other controls" (i.e. other than bed slope and resistance) "are present. If there are other controls, they tend to pull the flow away from the uniform condition, and there will be a transition-gradual or abrupt-between the two states of flow".

<sup>1</sup>Open Channel flow, the Macmillan Co., 1966.



Example: Illustration of the three basic-gradually varied flow-curves in a channel of mild slope.  $y_0$  = uniform flow depth,  $y_c$  = critical flow depth.



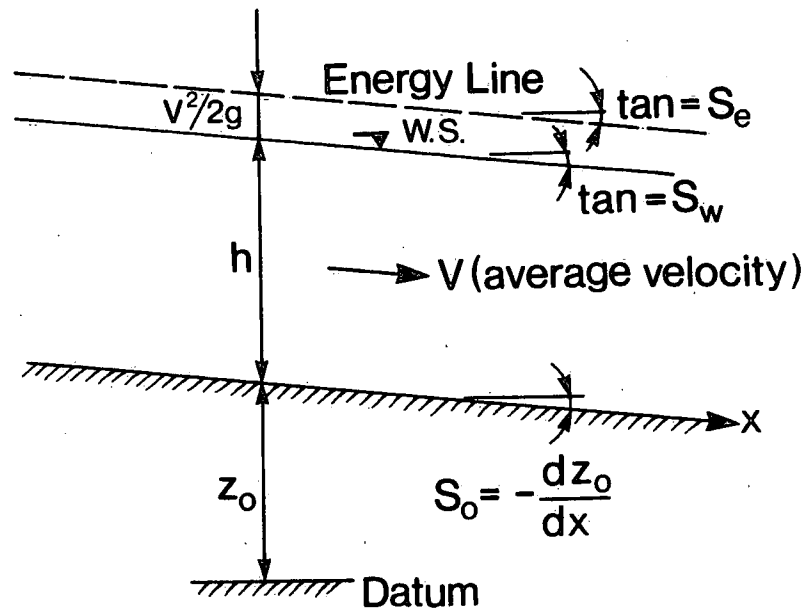
### 1.2 Uniform Flow

Assume two-dimensional flow for simplicity. See sketch on top of p. 3.

Note:  $h, V$  do not change along  $x$ , i.e.

$W, S$  and energy line are parallel to channel bed,

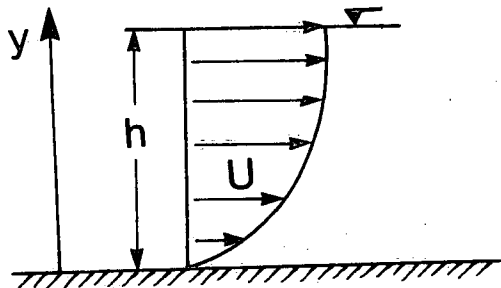
$$S_e = S_0 = S_w$$



note:  $g$  = acceleration of gravity

If the discharge changes,  $h$  and  $V$  will change. But so long as we do get uniform flow,  $S_w$  and  $S_e$  will remain equal to  $S_o$ , i.e. the W.S. and energy lines will shift positions but remain parallel to the bed.

The velocity distribution is given by:



$$\frac{U}{U_*} = \frac{1}{\kappa} \ln \left( \frac{y}{K_s} \right) + B \quad (1)$$

(Note:  $\ln x = 2.3 \log_{10} x$ )

- $U$  = velocity at a distance  $y$  from the bed
- $\kappa$  = Von Karman constant = 0.40

- $U_*$  = shear velocity  $\equiv \sqrt{\tau_0/\rho}$ ;  $\tau_0$  = bed shear stress;  
 $\rho$  = density of water  
 $K_S$  = "equivalent sand roughness" height = measure of average roughness height,  $e$ , of the bed, i.e.  $K_S \propto e$ .  
 $B$  = 8.5 for "fully rough" flow (independent of viscosity).

Fully rough flow occurs for:  $U_* K_S / \nu > 70$  (2)

( $\nu$  = kinematic viscosity). For  $U_* = (1/15) V$ , and  $V = 0.6$  m/s, the criterion of Equation 2 becomes  $K_S > 2$  mm which should be satisfied in most practical instances [see also Appendix].

Equation 1 may be transformed to a simpler version, i.e.

$$U = 2.5 U_* \ln (30 y / K_S) \quad (3)$$

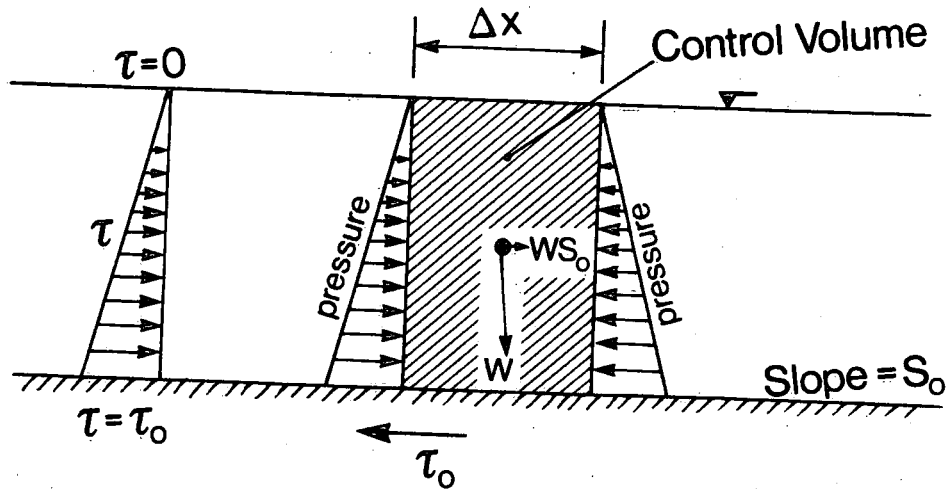
The average velocity  $V$  works out to be:

$$V = \frac{1}{h} \int_0^h 2.5 U_* \ln \left( \frac{30y}{K_S} \right) dy = 2.5 U_* \ln \left( \frac{11h}{K_S} \right) \quad (4)$$

If we measure the velocity at a depth of  $0.6 h$  from the surface, we will be at  $y = 0.4 h$ . This velocity will be (Equation 3):  $U_{0.4} = 2.5 U_* \ln (12h/K_S)$  which is very close to  $V$  (Equation 4). Similarly, we can check that  $(U_{0.2} + U_{0.8})/2 = V$ .

The surface velocity ( $U_S$ ) is (Equation 3):  $2.5 U_* \ln (30h/K_S)$ . And (Equation 4):  $V/U_S = \ln(11h/K_S)/\ln(30h/K_S)$ . For  $h/K_S$  ranging from 20 to 1,000,  $V/U_S$  varies only from 0.843 to 0.903, i.e.  $V/U_S = \text{const} \approx 0.87$ .

To find the shear stress,  $\tau_0$ , and thence the shear velocity,  $U_*$ , we can apply the momentum equation, which (for steady flow) states that the net momentum flux in and out of a control volume equals the sum of the forces acting on the control volume.



Because the flow is uniform the momentum flux entering the control volume is the same as that exiting, i.e. the net momentum flux is zero. The net pressure force is also zero, because (again) the flow is uniform. Moreover, in uniform flow the shear stress  $\tau$  is linearly distributed, being zero at the free surface and  $\tau_0$  at the bed. Therefore, the momentum equation in the streamwise direction reads:

$$0 = 0 + WS_0 - \tau_0 \Delta x B$$

in which  $W$  = weight of control volume =  $\rho gh \Delta x B$  ( $\rho$  = density of water;  $g$  = acceleration of gravity; and  $B$  = channel width). Simplifying gives

$$\tau_0 = \rho gh S_0 \text{ and } U_* \equiv \sqrt{\tau_0 / \rho} = \sqrt{gh S_0} \quad (5)$$

For uniform flow of arbitrary cross section, Equation 5 still holds, provided  $\tau_0$  is replaced by  $\bar{\tau}_0$  (= average over wetted perimeter); and  $h$  is replaced by  $R$  (= hydraulic radius).

### 1.3 Resistance Relationships

In practice, the usual problem in uniform flow computations is: Given channel width, slope and roughness, determine the depth required to pass any given discharge, i.e. we are given  $B$ ,  $S_0$  and  $K_S$  and want to determine  $h$  as a function of  $Q$ .

Now,  $Q = B \cdot h \cdot V$ . So, if we could derive a general relationship between  $V$  and  $h$ , then we would have  $h$  as a function of  $Q$ . Relationships between  $V$  and  $h$  are the "resistance relationships".

#### 1.3.1 Friction factor, $f$

$$f \equiv 8 (U_* / V)^2 \text{ (definition)} \quad (6)$$

Note that  $U_* = \sqrt{ghS_0}$  (Equation 5), i.e. if we knew  $f$ , Equation 6 would give the desired relationship between  $h$  and  $V$ . But we found earlier (Equation 4) that

$$V = 2.5 U_* \ln (11h/K_S) \quad (4)$$

From this equation we can solve for  $U_* / V$ , take the square, and multiply by 8 to find  $f$ ; after some algebra and using the more common  $\log_{10}$  in place of  $\ln$ , we get

$$f = [ 2.12 + 2.04 \log_{10} (h/K_S) ]^{-2} \quad (7)$$

which, according to earlier assumptions, applies only to fully rough flow. Note that when the "relative roughness",  $K_S/h$ , increases,  $f$  increases. It has been found that, to fit the data, the coefficients in Equation 7 have to be adjusted slightly. For example, Henderson gives:

$$f = [ 2.16 + 2 \log_{10} (h/K_S) ]^{-2} \quad (8)$$

### 1.3.2 Chezy coefficient, C

$$V = C \sqrt{hS_0} \quad (9)$$

Equation 9 gives  $V = (C/\sqrt{g}) \underbrace{\sqrt{ghS_0}}_{U_*}$ , from which  $8(U_*/V)^2 = f = 8(\sqrt{g}/C)^2$ , i.e.

$$C = \sqrt{8g/f} \quad (10)$$

### 1.3.3 Manning coefficient, n

$$V = \frac{1}{n} h^{2/3} S_0^{1/2} \text{ (metric units)} \quad (11)$$

Comparing with the friction factor formula, we can show that

$$n = \sqrt{f/8g} h^{1/6}, \quad \text{or putting } g = 9.8 \text{ m/s}^2,$$

$$n = 0.113 \sqrt{f} h^{1/6} \text{ (metric units)} \quad (12)$$

For a channel of arbitrary cross section, Equation 12 is still valid, provided we replace  $h$  by the hydraulic radius,  $R$ .

Figure 1 shows  $f$  plotted vs.  $h/K_S$ , according to Equation 8. In the range  $h/K_S = 5$  to 700,  $f$  can be approximated by a simpler relation of the form

$$f = \text{const} \times (K_S/h)^{1/3} \quad (13)$$

Substituting Equation 13 in Equation 12 and simplifying gives

$$n = \underbrace{(\text{const})}_{=0.04} \cdot K_S^{1/6} \left[ 5 < \frac{h}{K_S} < 700 \right] \quad (14)$$

$=0.04$ ;  $K_S$  in metres

which explains why  $n$  tends to be more stable than  $f$  or  $C$ : in a very common range of  $h/K_S$ ,  $n$  depends only on  $K_S$ , i.e. only on bed features. In addition, Equation 14 shows that channels of different  $K_S$ 's might have similar  $n$ 's because  $n$  varies as a small power of  $K_S$ .

In general, however,  $n$  may depend on  $h$  as well as  $K_S$ .

#### 1.3.4 Extension to natural streams

There are two obvious objections to applying the uniform flow theory to natural streams:

- (i)  $Q$  changes with time, i.e. flow is unsteady. However, except for periods associated with intense runoff generation, the rate of change of hydraulic parameters such as  $Q$ ,  $W$ ,  $h$ , is very small and can be neglected.

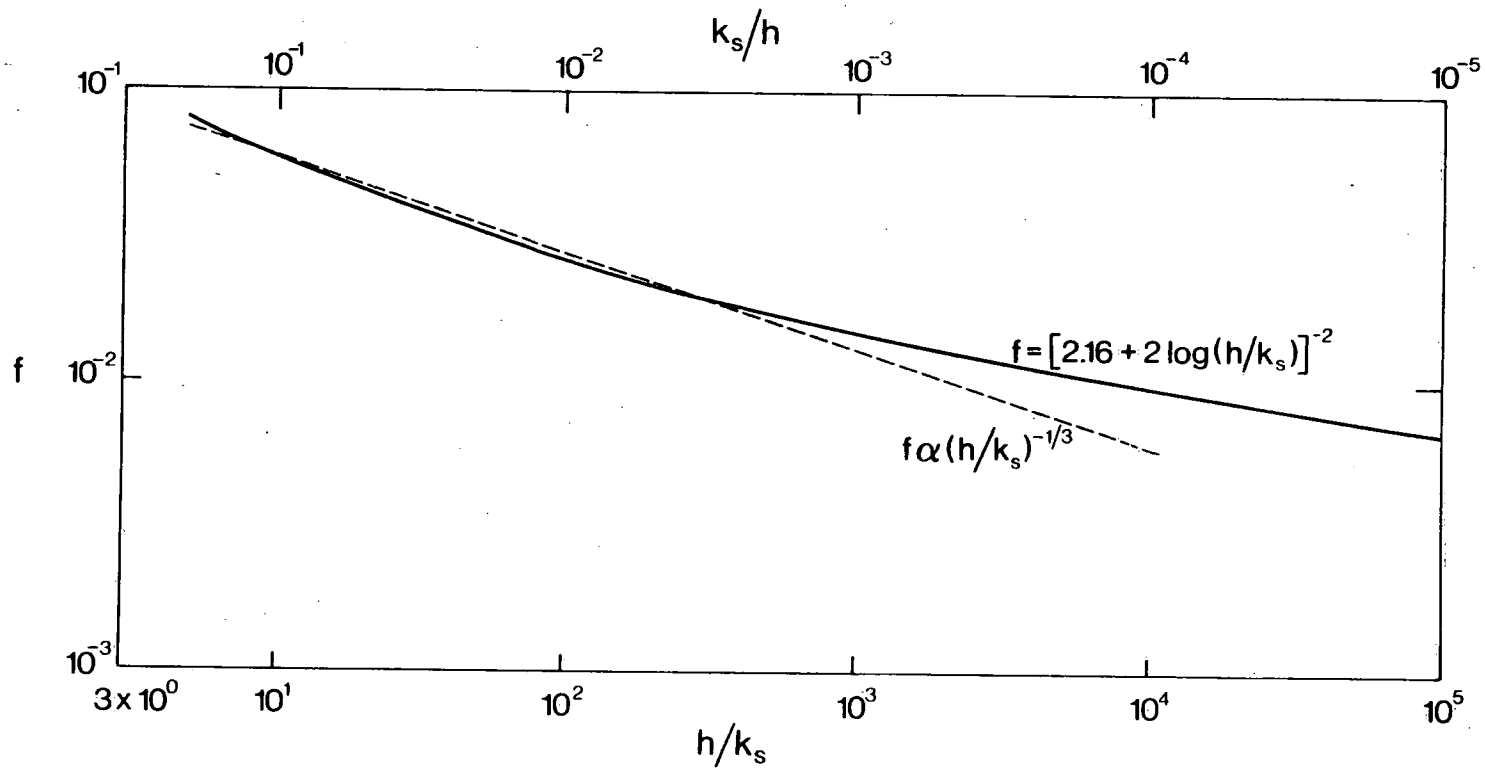


Figure 1 VARIATION OF FRICTION FACTOR  $f$  WITH  $h/k_s$  FOR FULLY ROUGH TURBULENT FLOW



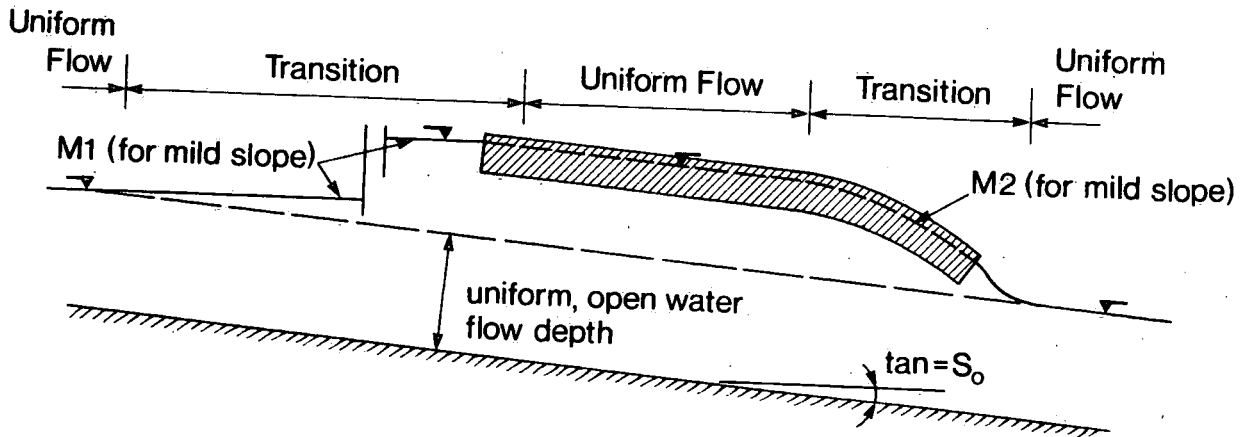
- (ii) The cross-sectional geometry and even the slope change in the downstream direction, i.e. flow is non-uniform. However, in most rivers the changes within reaches of approximately constant  $Q$  show no trend but, rather, represent fluctuations about stable reach-average values of  $B$ ,  $h$ ,  $S$  and  $V$ . In a "reach-average" sense, the uniform flow concept can thus be applied to a river. The reach must be long enough to enable meaningful averaging of the various irregularities. Of course, the uniform flow concept will not apply to extremely irregular streams (e.g. tumbling flow - mountain streams) or where there are significant control influences.

It should also be kept in mind that, in natural streams, the various irregularities become more prominent as the flow (or stage) decreases.

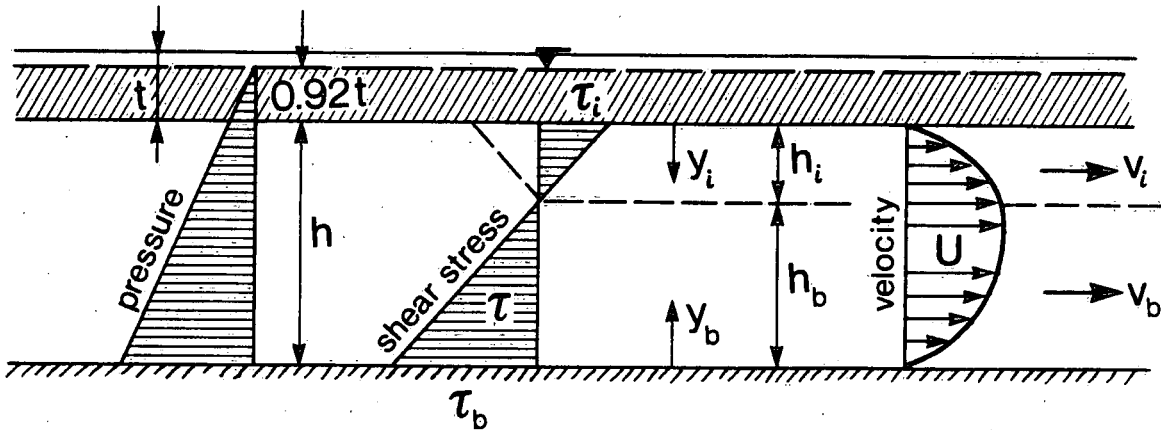
2.0 FLOW UNDER AN ICE COVER

2.1 Complete, Stationary Cover-Basics

- Assumptions:
- ice cover has uniform thickness  $t$
  - ice cover is free to float, i.e. W.S. is  $0.92 t$  above the bottom of the cover
  - our channel is two-dimensional; and there are no significant control effects
  - ice cover is long enough to permit development of uniform flow underneath.



In the uniform flow section under the cover we have the following distributions:



Note that  $\tau$  is still linearly distributed. If we choose our  $x$  and  $y$  axes so that  $\tau_b$  is +ive, then  $\tau_i$  is -ive. So as to avoid dealing with -ive  $\tau$ 's,  $\tau_i$  will imply the absolute value in the following. There is a point where  $\tau = 0$  and by the similarity of triangles we have

$$h_i / h_b = \tau_i / \tau_b \quad (15)$$

We now assume that the maximum velocity occurs at the point where  $\tau = 0$ ; and that the velocity distributions in the two layers adjacent to the river bed and ice bottom respectively, are the same as for open channel flow (the upper layer is inverted). These two assumptions are not quite true but are fair approximations for the present purposes. Then we have:

$$U_i = 2.5 U_{*i} \ln (30 y_i / K_{si}) \quad (16)$$

$$U_b = 2.5 U_{*b} \ln (30 y_b / K_{sb}) \quad (17)$$

Also, separate application of the momentum equation to the two layers will give

$$\tau_i = \rho g h_i S_0; \quad U_{*i} = \sqrt{g h_i S_0} \quad (18)$$

$$\tau_b = \rho g h_b S_0; \quad U_{*b} = \sqrt{g h_b S_0} \quad (19)$$

If  $K_{Si} = K_{Sb}$ , then  $\tau_i = \tau_b$ ,  $V_i = V_b$  and  $h_i = h_b$ , i.e. the maximum velocity occurs at mid-depth. In this case, one could measure the mid-depth velocity and multiply by 0.87 (average to maximum velocity ratio found for open water flow) to get a good estimate of the average velocity in the vertical. This approach is not justified, however, if  $K_{Si} \neq K_{Sb}$ .

Now, using Equations 16 and 17, we will find that over wide ranges of  $K_{Si}/K_{Sb}$ , the average velocities  $V_i$  and  $V_b$  are about equal. This has been confirmed by many measurements. Lately, a turbulence-model analysis by Lau (1982) showed that  $V_i$  and  $V_b$  were within 11% of each other, even outside the fully rough flow regime [ $K_{Si}/h$  and  $K_{Sb}/h$  were between 0 and 0.10;  $K_{Si}/K_{Sb}$  was between 0 and 1.00]. Another interesting finding by Lau was that  $0.5 (U_{0.2} + U_{0.8})$  was always within 3% of  $V$  (= average in vertical).

## 2.2 Complete, Stationary Cover-Resistance Relationships

Here, again, the objective is to find a relationship between  $V$  and  $h$  so that we can calculate  $h$  when we are given  $B$ ,  $S_0$ ,  $K_{Si}$ ,  $K_{Sb}$ , and  $\theta$ . First of all, we can define "composite" resistance coefficients,  $f_0$ ,  $C_0$  and  $n_0$  such that our earlier equations - modified to account for the doubling of the wetter perimeter - are still valid. Then:

$$f_0 \equiv 8 (U_{*0}/V)^2 = 8 (\sqrt{g(h/2)S_0}/V)^2 \quad (20)$$

$$V = C_0 \sqrt{(h/2) S_0} \quad (21)$$

$$V = (1/n_0) (h/2)^{2/3} S_0^{1/2} \quad (22)$$

The problem becomes how to relate the composite coefficients to those of the channel bed and ice underside. A multitude of formulae have been proposed in the past. See Pratte (1979 - Canadian Hydrology Symposium) for a comprehensive review. Only those equations deemed to optimize the balance between rigour and practicality will be given here. These equations are also generally preferred by engineers.

The basic assumption is  $V_i = V_b (=V \text{ also})$ .

### 2.2.1 Friction factor, $f_0$

Since  $V_i = V_b = V$ , we have:

$$f_i = 8 (U_{*i}/V)^2; \quad f_b = 8 (U_{*b}/V)^2 \text{ and}$$

$$\begin{aligned} f_i + f_b &= 8 \frac{U_{*i}^2 + U_{*b}^2}{V^2} = 8 \frac{gh_i S_0 + gh_b S_0}{V^2} = \\ &= 8 (2) \cdot \frac{g(h/2)S_0}{V^2} = 2 (8) \left( \frac{U_{*0}}{V} \right)^2 = 2 f_0 \quad \text{i.e.} \end{aligned}$$

$$\underline{f_0 = (f_i + f_b)/2} \quad (23)$$

$$\text{Note also that } f_i/f_b = U_{*i}^2/U_{*b}^2 = \tau_i/\tau_b = h_i/h_b \quad (24)$$

### 2.2.2 Chezy coefficient, $C_0$

We found earlier (Equation 10) that  $C = \sqrt{8g/f}$ , i.e.  $f = 8g/C^2$ . Substituting in Equation 23 gives:

$$C_o = \sqrt{\frac{2}{(1/C_i^2) + (1/C_b^2)}} \quad (25)$$

### 2.2.3 Manning coefficient, $n_o$

We have:

$$V_i = V_b = V = \frac{1}{n_i} h_i^{2/3} S_o^{1/2} = \frac{1}{n_b} h_b^{2/3} S_o^{1/2} = \frac{1}{n_o} \left(\frac{h}{2}\right)^{2/3} S_o^{1/2} \quad (26)$$

Adding (after solving for  $h_i/h$  and  $h_b/h$ ) and noting that  $h_i+h_b = h$ , gives

$$2 = (n_i/n_o)^{3/2} + (n_b/n_o)^{3/2} \quad \text{or,}$$

$$n_o = \frac{(n_i^{3/2} + n_b^{3/2})}{2}^{2/3} \quad (27)$$

This is the so-called "Savaneev" equation; Larsen's derivation which has better theoretical foundation reduces to this form when  $V_i \approx V_b$  (see Pratte, 1979).

### 2.3 Complete Stationary Cover - Effect on Stage

Suppose we are given a wide channel of width  $B$ , slope  $S_o$ , and discharge  $Q$ . We want to determine the effect of an ice cover with uniform thickness  $t$  under free flotation and uniform flow conditions.

$$\text{For open-water:} \quad V_{\text{open}} = (1/n_{b,\text{open}}) h_{\text{open}}^{2/3} S_o^{1/2}$$

Multiply by  $B \cdot h_{\text{open}}$  to find  $Q$  and solve for  $h_{\text{open}}$ :

$$h_{\text{open}} = \left( \frac{n_{b,\text{open}}^0}{B S_0^{1/2}} \right)^{3/5} \quad (28)$$

Similarly,  $V_{\text{cov}} = (1/n_0) (h_{\text{cov}}/2)^{2/3} S_0^{1/2}$  and

$$h_{\text{cov}} = \left( \frac{2^{2/3} n_0^0}{B S_0^{1/2}} \right)^{3/5} \quad (29)$$

Dividing Equations 28 and 29:

$$\frac{h_{\text{cov}}}{h_{\text{open}}} = \left( 2^{2/3} \frac{n_0}{n_{b,\text{open}}} \right)^{3/5} \quad (30)$$

Though not necessarily true, let's assume for simplicity that  $n_{b,\text{open}} \approx n_{b,\text{cov}} = n_b$ . Then use of Equation 27 will give

$$h_{\text{cov}}/h_{\text{open}} = [1 + (n_i/n_b)^{3/2}]^{2/5} \quad (31)$$

which shows that  $h_{\text{cov}}$  is larger than  $h_{\text{op}}$ . Figure 2 shows  $h_{\text{cov}}/h_{\text{open}}$  plotted vs.  $(n_i/n_b)$  according to Equation 31. For example, when  $n_i = n_b$ ,  $h_{\text{cov}} = 1.3 h_{\text{open}}$ .

To find the total depth  $H_{\text{cov}}$ , we must add  $0.92 t$  to  $h_{\text{cov}}$ , i.e.

$$\frac{H_{\text{cov}}}{h_{\text{open}}} = \left( \frac{h_{\text{cov}}}{h_{\text{open}}} \right) + 0.92 \left( \frac{t}{h_{\text{open}}} \right) \quad (32)$$

This equation illustrates the very large effect of an ice jam on stage because an ice jam is both:

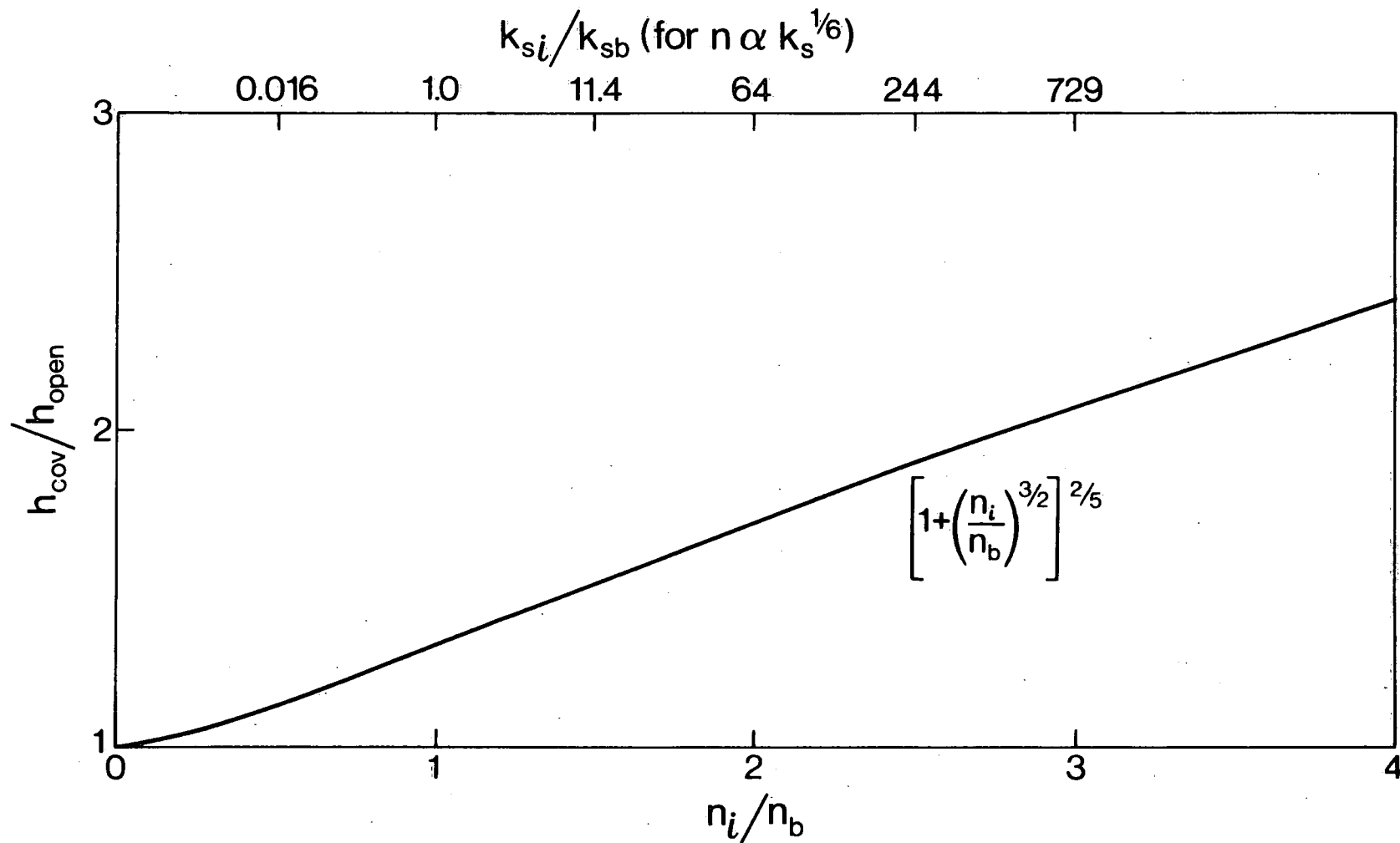


Figure 2 EFFECT OF AN ICE COVER ON FLOW DEPTH, ASSUMING UNIFORM FLOW AND  $n_{b,open} = n_{b,cov} = n_b$ .



- very rough, i.e.  $n_i$  could be several times  $n_b$ , and
- very thick, i.e.  $t$  could be several times the thickness of the individual fragments it comprises.

## 2.4 Values of Resistance Coefficients of Ice Covers

The hydraulic resistance of an ice cover depends on the configuration of its underside which in turn may depend on its mode of formation, state and heat transfer from the water.

### 2.4.1 Beginning of freeze up

The resistance reaches very high values at this time because the cover is made of unconsolidated, surface or thickened accumulations of frazil slush and, sometimes, solid ice floes. A large set of

field data are those reported by Nezhikhovsky (see Pratte 1979) - Table 1. Note These data are crude and have yet to be duplicated. A few values reported recently by Gerard (1982; Edmonton Workshop on River Ice Hydraulics), also showed high resistance but did not confirm the trends evident in Table 1.

### 2.4.2 Early and mid-winter

Once freeze up is complete, the resistance drops quickly because the accumulations freeze into solid masses while bottom irregularities are "smoothed" out by heat transfer effects. Typical early and mid-winter values of the Manning coefficient are between 0.008-0.012 and they characterize covers with smooth and planar undersides.

However, hanging dams (large deposits of frazil slush) which stay in place for most of the winter have larger resistance as one might suspect. Table 2 gives a few values for a large hanging dam that forms annually in the lower Smoky River (Alberta).

TABLE 1. VALUES OF  $n_i$  FOR SLUSH-ICE ACCUMULATIONS AT FREEZE-UP  
(Nezhikhovskiy; quoted from Pratte 1979 with changes)

Initial Thickness of Slush-Ice Cover feet/metres		Slush Ice Cover Formed From:		
		Loose Slush, Frazil Ice Flocks	Dense Slush, Frazil Ice Flocks With Ice Plates	Ice Floes
0.3	0.1	--	--	0.015
1.0	0.3	0.01	0.013	0.04
1.6	0.5	0.01	0.02	0.05
2.2	0.7	0.02	0.03	0.06
3.2	1.0	0.03	0.04	0.07
4.8	1.5	0.04	0.06	0.08
6.4	2.0	0.04	0.07	0.09
9.7	3.0	0.05	0.08	0.10
16.0	5.0	0.06	0.09	--

TABLE 2. HYDRAULIC RESISTANCE OF SMOKY R. HANGING DAM  
(Beltaos & Dean)

Date of Measurement	Thickness of Frazil Accumulation (m)	$h_i$ (m)	$K_{si}$ (m)	$f_i$	$n_i$	Remarks
Apr. 5, 1977	9.0	1.45	2.15	0.30	0.066	Poor fit of log-law*; h=3.5 m; V ≈ 0.40 m/s
Apr. 6, 1977	7.9	1.60	0.04	0.034	0.023	Good fit; h=2.6 m; V ≈ 0.6 m/s
Apr. 6, 1977	3.8	1.41	0.52	0.11	0.040	Good fit; h = 3.0 m; V ≈ 1.3 m/s
Mar. 15, 1978	13.2	2.0	2.30	0.24	0.062	Fair fit; h=2.8 m; V ≈ 0.07 m/s

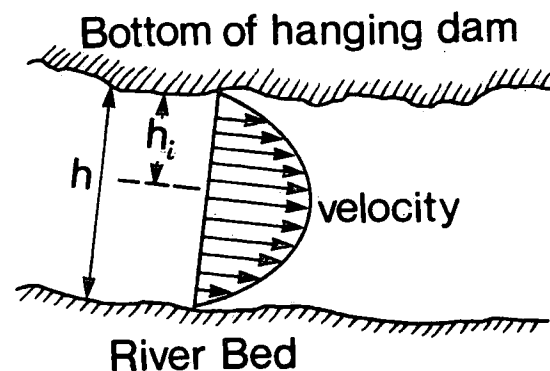
$K_{si}$  = "equivalent sand roughness" of ice bottom

$f_i$  = friction factor of ice bottom

$n_i$  = Manning coefficient of ice bottom

V = Average velocity in vertical

\* $U_{*i}$  and  $K_{si}$  were determined by fitting straight lines to plots of  $U_i$  vs.  $\log y_i$  ("log-law", Eq. 16).



### 2.4.3 Late winter

With the onset of mild weather, the water temperature rises above 0°C and the boundary heat transfer causes ripple-like features to form on the underside of the cover. Then, the hydraulic resistance increases (e.g. see Ashton: "River Ice", Ann. Rev. Fluid Mech., 1978, pp. 369-392). Measurements by Carey (quoted by Pratte, 1979) gave  $n_i$  values of up to 0.028; the amplitude and wavelength of the forms were between 1 and 4 cm and 15 and 30 cm respectively.

With continued heat transfer, these regular wavy forms give way to irregular, random-like roughness. One measurement by Carey gave a value of  $n_i=0.025$ ; the roughness height was between 3 and 6 cm.

### 2.4.4 Breakup

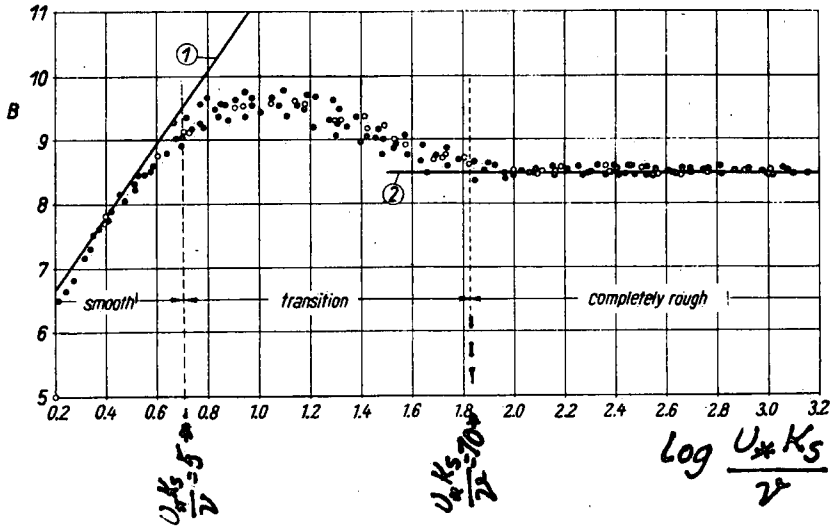
Once the cover has broken up and ice jams have formed, it is very difficult to measure flow parameters. No data are available (to the writer's knowledge) for the resistance of breakup ice jams. Nezhikhovsky's data for ice floe accumulations at freeze up could be used as a rough indication.

## 2.5 MOVING COVER

If a stationary cover loses its restraint at the river banks, it will be set in motion under the influence of two forces: the hydrodynamic force and the streamwise component of its own weight. Initially, the cover should accelerate but eventually should assume a constant velocity. In the latter case, assuming uniform flow under the cover, we could again calculate the flow depth. However, because the cover is now moving, the ice effect is much less than with a stationary cover. A rough calculation showed that  $H_{\text{moving cov}}/h_{\text{open}}$  depends mainly on  $t/h_{\text{open}}$ , being 1.00 when  $t/h_{\text{open}} = 0$  and increasing to  $\approx 1.2$  when  $t/h_{\text{open}} = 0.3$ .

Velocity Distribution in General

$U/U_* = 2.5 \ln (y/K_s) + B$  ; B is generally a function of  $U_* K_s / \nu$



From Schlichting,  
"Boundary Layer Theory",  
McGraw Hill, 1968.

Roughness function B in terms of  $U_* K_s / \nu$ ,  
for Nikuradse's sand roughness. Curve (1):  
hydraulically smooth; curve (2):  $B = 8.5$ ;  
completely rough.

$$\frac{U_* K_s}{\nu} < 5 \rightarrow B = 5.5 + 2.5 \ln \frac{U_* K_s}{\nu} \rightarrow$$

$$\frac{U}{U_*} = 2.5 \ln \left( \frac{U_* y}{\nu} \right) + 5.5:$$

hydraulically smooth flow

$$5 < \frac{U_* K_s}{\nu} < 70 :$$

transition

$$\frac{U_* K_s}{\nu} > 70 :$$

fully rough  $B = 8.5$ .

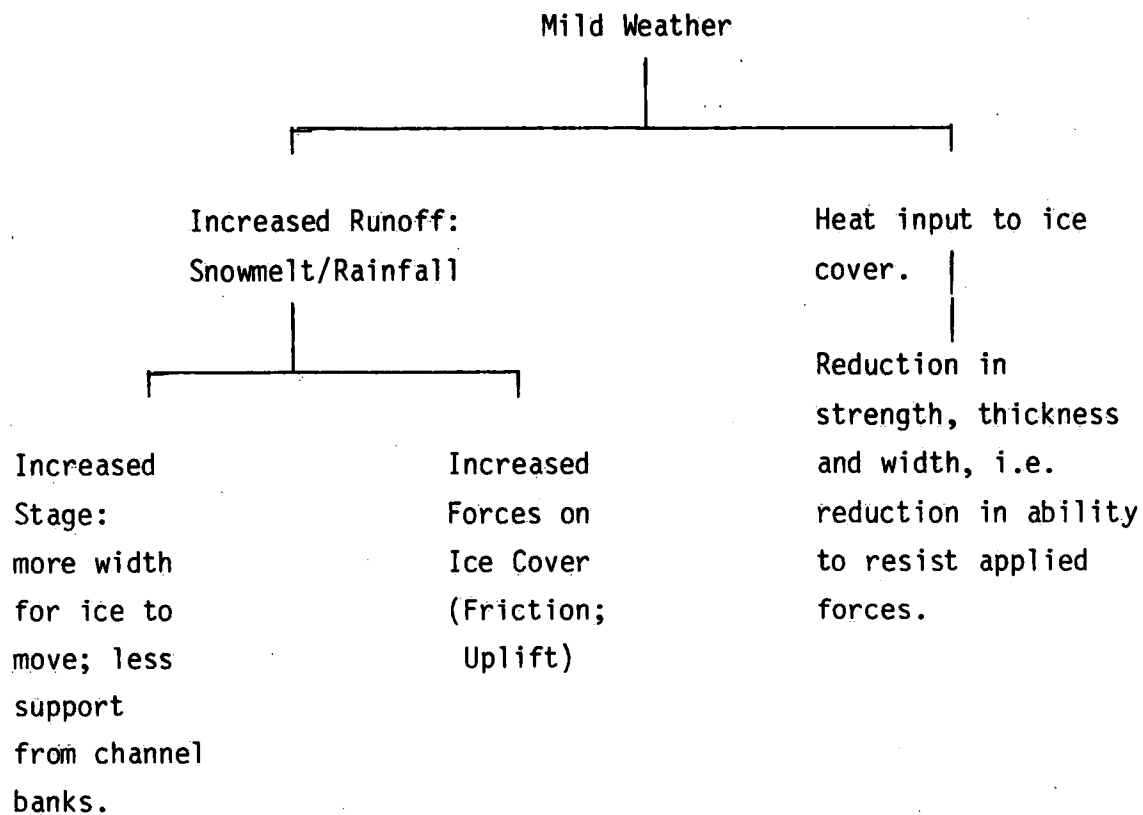
Part II

BREAKUP AND ICE JAMS

Ice breakup is a brief but very important part of the ice season, because it is accompanied by ice jams which have considerable potential for flooding, erosion and violent ice action against river structures.

1.0 INITIATION OF BREAKUP

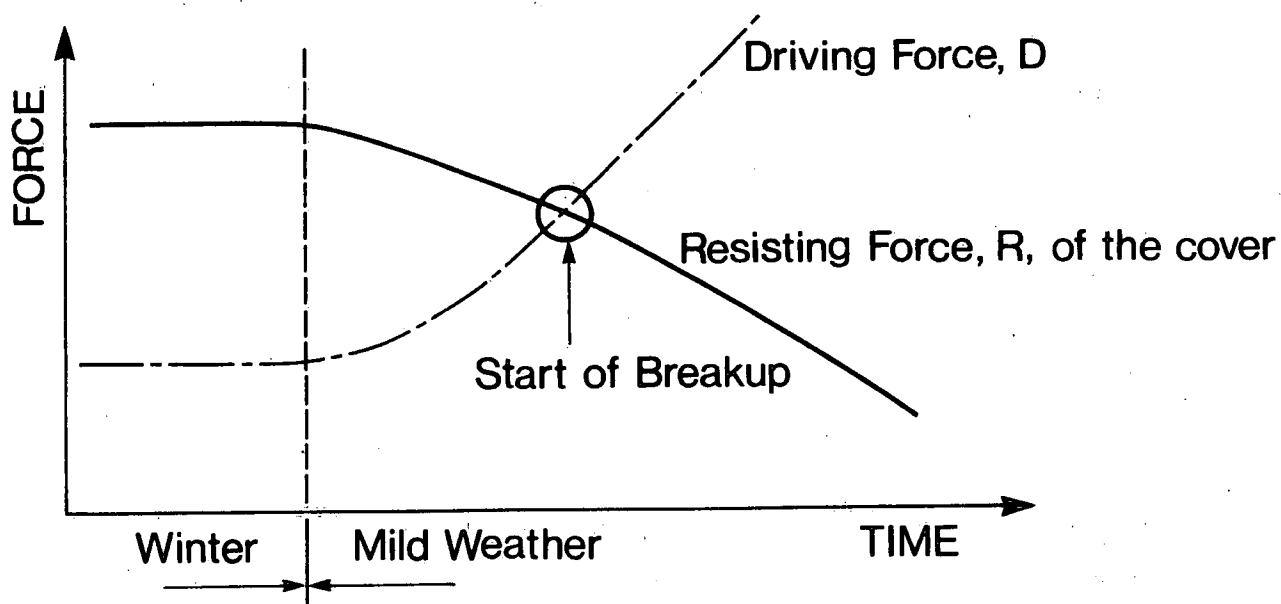
When mild (temperature  $> 0^{\circ}\text{C}$ )<sup>2</sup> weather sets in, a number of things happen as illustrated below:



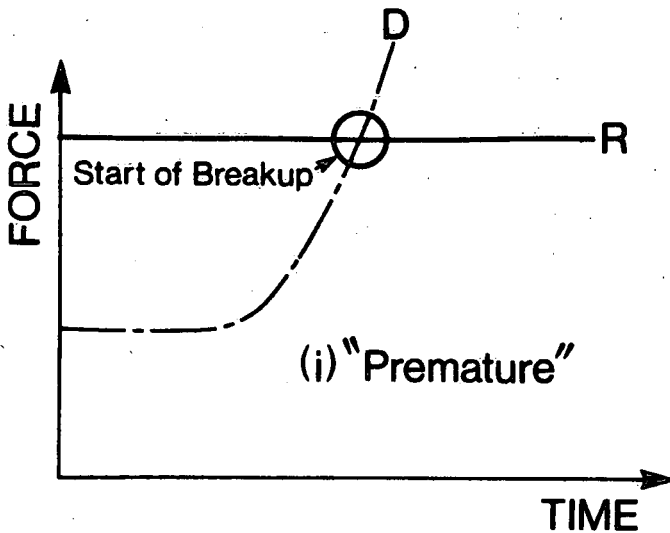
<sup>2</sup>By and large, this is a fair criterion for "mild" weather in the present context. It is possible, however, that snowmelt and ice deterioration start even at air temperatures less than  $0^{\circ}\text{C}$ , if there is intense solar radiation.

Eventually, the driving forces become capable of breaking the cover at various locations. Mostly large, separate ice sheets form. The increased water surface width permits some of these sheets to move past obstructions and pick up speed. On impact with other sheets or with river boundaries, there is more breaking and fragmentation. Small jams begin to form, causing additional stage increases, more impacts and breaking and so on. Eventually, a few large ice jams remain in a reach which is cleared when these jams release. All this time, heat transfer continues to reduce the strength and volume of the ice. In a given reach, the onset of breakup may be accelerated by the presence of areas that never froze over during the winter; by earlier breakup of tributaries; and by "holes" in the cover that may develop due to a variety of reasons.

A qualitative illustration of the initiation of breakup is given in the following sketch:

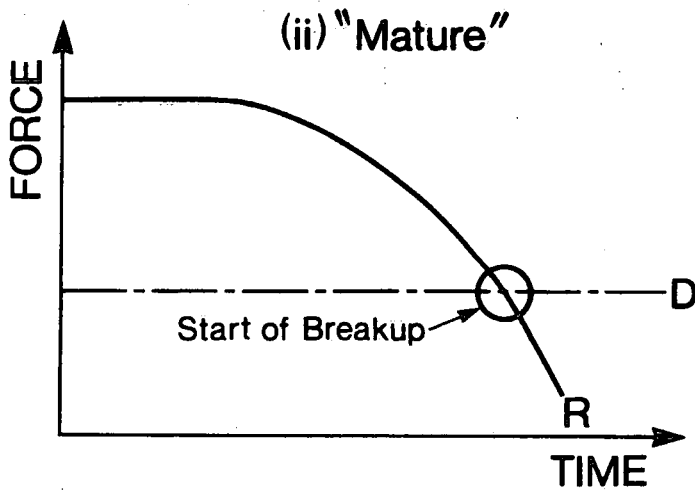


In terms of this qualitative sketch, we could have two extremes:



(i) "Premature" breakup

Fast rise of driving force with little decline of resisting force. Usually, this happens during a brief mid-winter thaw with intense rainfall and little heat transfer to the cover. Such events are the worst.



(ii) "Mature" breakup

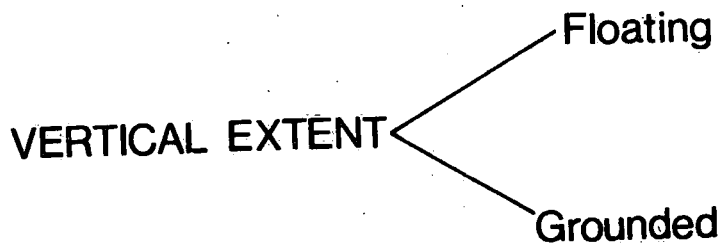
Decline of resisting force with little or no rise of driving force. Usually, this happens when there is little runoff, i.e. no rain and very little snow on the ground. Such events are usually of no concern with regard to damage.



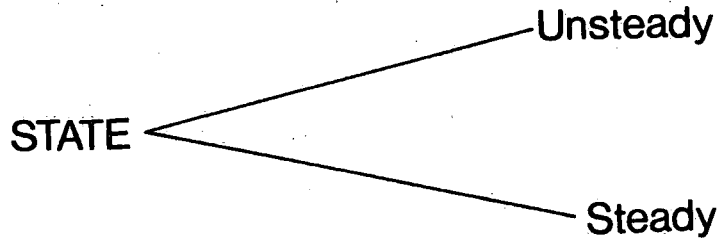
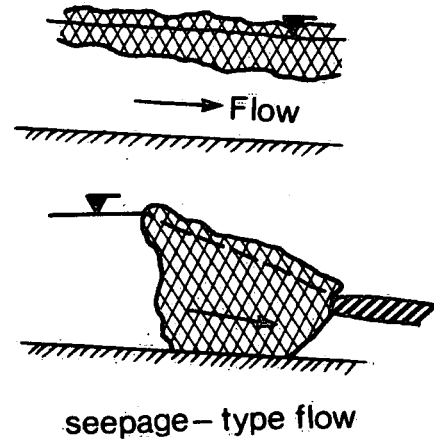
## 2.0 ICE JAMS

Ice jamming is a central event during breakup. Anyone who has seen a major ice jam knows that its thickness and underside roughness should be very large relative to those of sheet ice, judging by the appearance of the jam's visible part. These two factors combine to produce very high stages relative to the open-water stage for the same discharge. Table 3 gives a few illustrations.

### 2.1 Types of Ice Jams



(sometimes, there are both grounded and floating sections within the same jam)



### 2.2 Initiation - Causes of Jams

Any feature that obstructs the downstream flux of the ice fragments is a potential jam instigator. E.g:

TABLE 3. SELECTED CASE HISTORIES

Ice Jam Location (Alberta)	Year of Occurrence	Source	Return Period of Equal-Maximum Stage Summer Flood (yrs)	Remarks
South Saskatchewan River above Sandy Point	1974	Gerard (1975)	>3	Floating surface jam, initiated by "arching" of ice floes.
Peace River near town of Peace River	1974	Beltaos (Unpublished data)	>100	Floating thickened jam, possibly grounded at toe. Second highest breakup stage during 1974-79.
Athabasca River near Fort McMurray	1974 1977 1978 1979*	Gerard (1975) Doyle (1977) Doyle (1978) Doyle (1979)	>100 >100 >100 >100	Grounded jam. Grounded toe. Grounded toe. Grounded toe.
Wapiti River near Grande Prairie	1976	Beltaos (Unpublished data)	70	Floating, thickened jam; likely floating toe.
Smoky River near the mouth	1976	Beltaos (Unpublished data)	≈400	Likely grounded toe; the 1976 stage was exceeded by at least 2 m in 1979 (1000 yr?)

\* During this breakup, a relative stage excess of 4.5 m was recorded at a location some 12 km upstream of Fort McMurray.

Note

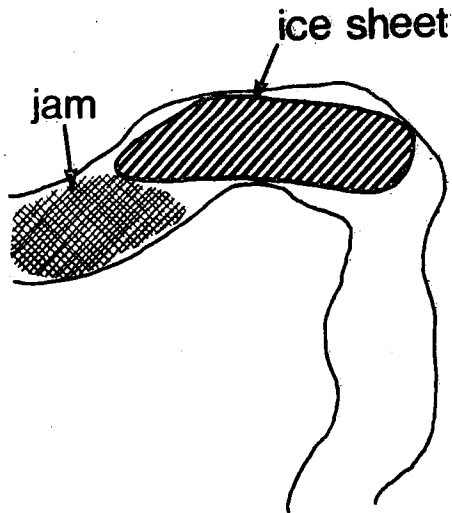
Some of the quoted return periods are based on extrapolations of frequency curves derived from relatively short periods of record which may or may not be an accurate reflection of reality. However, these return periods do show that very rare open-water flood stages can be attained when ice jams are present.

- **Undisturbed Ice Cover:** When an ice run arrives at the upstream edge of an undisturbed and still fairly competent ice cover, there is some breakage by the impact and local piling up. But the rate of breaking may be too slow (due to ice strength and thickness) to permit continued advance of the run. Then a jam is initiated.
- **Morphological Features:** Constrictions, bends, shallows, islands, abrupt reductions in slope and velocity.
- **Man-Made Obstacles:** e.g. bridge piers or other obstructions.

Often, though not always, jams form because of the presence of two or more features such as the above. A very common combination is an undisturbed cover plus something else. E.g.:

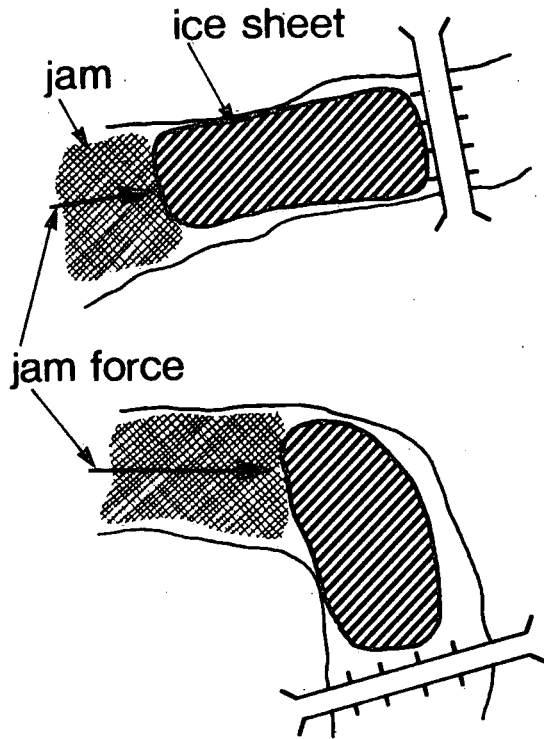
- Reaches at and upstream of river mouths at lakes.

Usually, the flow in such reaches is very tranquil owing to lake control. The cover is relatively thick and stays in place well after the cover further upstream has broken up. This strong cover combines with the reduced velocity to cause ice jams.



Bends. Often, a large ice sheet cannot move past a sharp bend. A jam may form upstream and remain in place until the large sheet can move either due to increased water level and width or due to reduction in dimensions by breaking and melting, or both.

Bridge Piers. An ice sheet may be lodged against bridge piers. The situation is greatly aggravated when the bridge is located just downstream of a bend (the force driving the sheet against the piers is reduced).



### 2.3 Evolution and Equilibrium of Jams

When an ice jam is initiated, there is a local reduction of flow, i.e. less flow is going past the toe of the jam (downstream end) than is coming in. An unsteady condition then prevails. During this time, the jam stage generally increases. Eventually, a steady-state condition is established. Assuming that there are no controls in the vicinity and the jam is long enough, a part of the jam's length will be in a condition of "equilibrium", i.e., it will have relatively uniform thickness with relatively uniform flow underneath. Barring the occurrence of severe grounded jams or other unusual circumstances, the equilibrium stage of a floating ice jam can be considered the maximum possible at a given site for a given discharge (see sketch on next page).

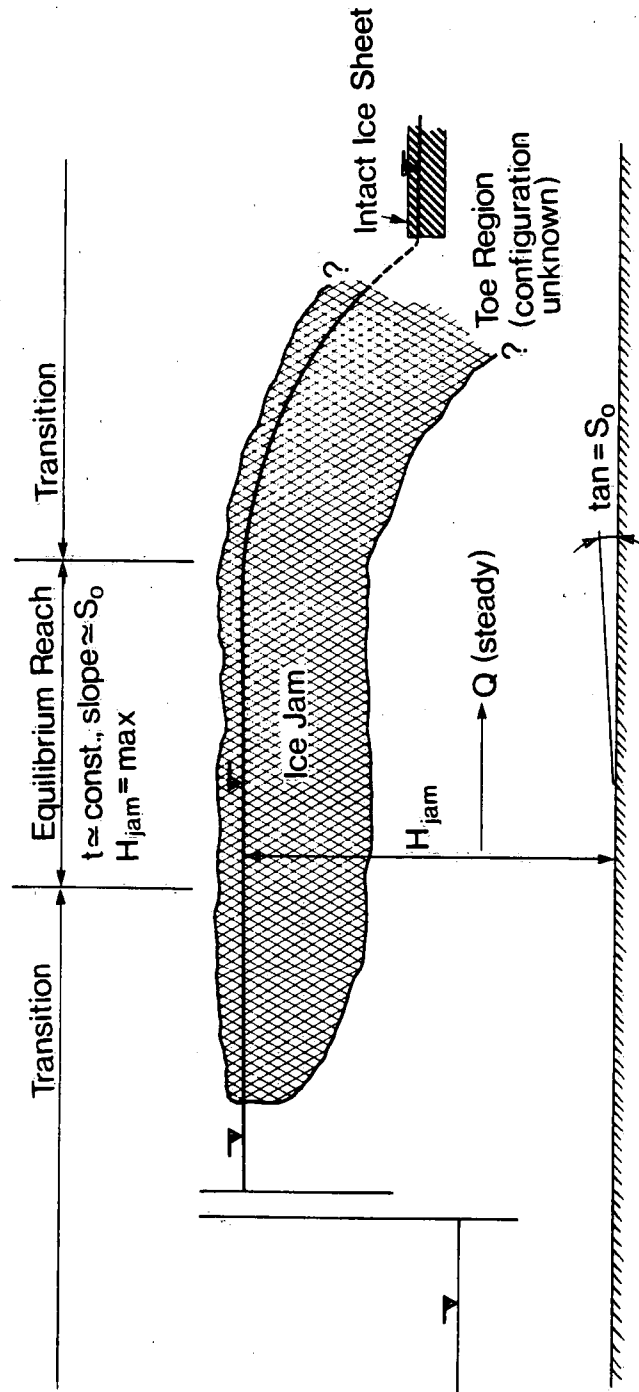
Of course, an ice jam may never attain an equilibrium state: it may release while still in evolution, or it may be too short to include an equilibrium reach. The "Life" of an ice jam can vary from a few minutes to several days.

### 2.4 Release of Jams

Ice jams may release gradually or suddenly. The mechanics of ice jam release is unknown at present and it is possible that there are several release mechanisms, depending on mode of formation and conditions at the toe. Usually, release is preceded by intense activity at the toe (downstream end of a jam).

Witness accounts often mention violent ice runs with extreme velocities and rates of stage rise. Velocities of 5-6 m/s and such rates of rise as 17 m in less than 1 hour or 1.5 m in 30 seconds have been reported occasionally. Recent work (Gerard & Henderson 1981 - IAHR Ice Symposium, Quebec City) suggests that such events are caused by sudden releases of ice jams. Quantitative prediction of stage and velocity caused by a released jam is a matter for future research. Rough calculations can be made either by an approximate theory (Gerard and Henderson, 1981) or by adapting unsteady, open-water flow models

(Mercer and Cooper, 1977, CSCE Hydrotechnical Conference, Quebec City; Beltaos and Krishnappan, 1982, Can. J. of Civ. Eng., Vol. 9, No. 2).



PROFILE OF A FLOATING JAM WITH AN EQUILIBRIUM REACH

### 3.0 FORECASTING

#### 3.1 Short-Term Forecasting

- onset of breakup
- severity of breakup

No general method is available. Existing methods are empirical and site-specific; they are based on historical data (see Shulyakovsky, 1963). The various factors determining the onset and severity of breakup are known in qualitative terms but the available data are relatively few to permit derivation of general relationships.

#### 3.2 Long-Term Forecasting

The objective here is to come up with a peak breakup stage-frequency relationship which is necessary for flood risk assessments or design of bridge piers. The state of the art regarding this problem is discussed in another instructor's notes (R. Gerard).

#### 4.0 ALLEVIATION OF BREAKUP EFFECTS

Various methods to alleviate breakup effects are employed at sites prone to jamming. However, because our understanding of jam formation and release mechanisms is limited, ice jam control depends largely on knowledge of local conditions and experience.

In general, control methods can be subdivided into ice modification and river modification (see Bolsenga 1968 for an extensive literature review on this subject).

##### 4.1 Ice Modification

- Ice Breaking. Ice breakers or other vessels are used to break the ice downstream of ice jams or at critical areas prior to breakup. Sometimes, different equipment is used to cut open leads in the cover prior to breakup. This method is usually satisfactory but costly.
- Explosives: Blasting and bombing. Always better to apply before ice jams form. Removal of ice jams once they have formed is uncertain.
- Dusting: Solid material of low albedo is spread from aircraft on the ice surface to promote heat transfer to the cover. Effectiveness depends on weather conditions.
- Thermal regime modification: water temperature is altered to prevent formation of or weaken ice cover at critical areas.

##### 4.2 River Modification

These are more permanent measures resulting in alterations of the normal flow pattern of the river. This can be done by



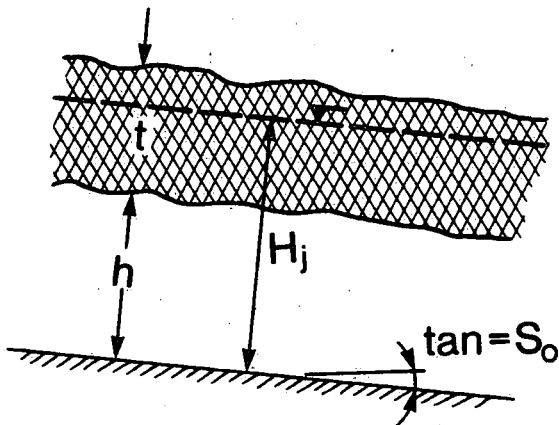
- channelization, i.e. eliminating morphological features conducive to jamming.
  
- building ice retention or diversion structures, i.e. ice booms, dykes, dams.

5.0 CALCULATION OF JAM STAGE

Assumptions:

- Floating, equilibrium jam of the "wide-channel" type. Flow through the voids of the jam is neglected.
- River can be replaced by a very wide rectangular channel of equivalent average dimensions.
- Special constraints such as low flood plains, by-pass channels, etc., are ignored.

Problem: Given  $B$ ,  $S_0$ ,  $Q$  and channel roughness, predict  $H_j$  = overall water depth.



Profile in equilibrium reach of a jam

Assuming that the porosity of the jam is the same both above and below the water surface, it can be shown that the submerged portion of the jam thickness is =  $0.92 t$ . Hence,

$$H_j = h + 0.92 t \quad (1)$$

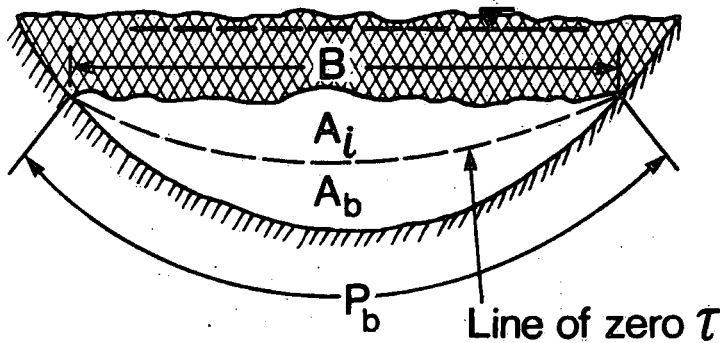
To calculate  $t$ , we start from the equilibrium relation (derived in Dr. Gerard's notes).

$$\mu \rho_i (1 - s_i) g t^2 = (\tau_i + \rho_i g S_0 t) \quad (2)$$

We have neglected cohesion - should be a fair assumption where the air temperature is not less than  $0^\circ\text{C}$  which is usually the case for breakup jams.

- $\mu$  = dimensionless coefficient related to the internal friction of the jam
- $\rho_i$  = ice density
- $s_i$  = specific gravity of ice =  $\rho_i/\rho = 0.92$

Our earlier relationships, derived for two-dimensional flow under an ice cover, may now be applied to a river cross section by subdividing the flow area using the line of zero shear stress. Then, in place of  $h_i$  and  $h_b$  we may put the hydraulic radii  $R_i$  and  $R_b$ .



Note  $R_i = A_i/B$ ;  $R_b = A_b/P_b = A_b/B$ ;  $h =$  average flow depth =  $(A_i + A_b)/B = R_i + R_b$ .

From earlier results, we have:  $\tau_i = \rho g S_0 R_i$  and  $R_i/R_b = f_i/f_b$ .

$$\frac{R_i}{R_i + R_b} = \frac{f_i}{f_i + f_b}$$

Hence,  $R_i = (f_i/2f_0) h$  and

$$\tau_i = \frac{1}{2} (f_i/f_0) \rho g h S_0 \quad (3)$$

Substitute Equation 3 in Equation 2 and solve for  $t$  ( $s_i = 0.92$ )

$$\frac{t}{S_0 B} = \frac{6.25}{\mu} \left[ 1 + \sqrt{1 + 0.174 \left( \frac{f_i}{f_0} \right) \frac{h}{S_0 B}} \right] \quad (4)$$

Now  $V = \sqrt{\frac{8}{f_0}} \sqrt{g \left(\frac{h}{2}\right) S_0}$  and thus

$$Q = B.h.V = Bh^{2/3} \sqrt{\frac{4gS_0}{f_0}} \dots \text{Solving for } h \text{ gives}$$

$$h = \left( \frac{q \sqrt{f_0}}{2\sqrt{gS_0}} \right)^{2/3} = 0.63 f_0^{1/3} (q^2/gS_0)^{1/3} \quad (5)$$

in which  $q = Q/B =$  discharge intensity.

It is now convenient to define  $\xi$ , a dimensionless variable, as

$$\xi = (q^2/gS_0)^{1/3} / S_0 B \quad (6)$$

Then Equation 4 becomes

$$\frac{t}{S_0 B} = \frac{6.25}{\mu} \left[ 1 + \sqrt{1 + 0.11 f_0^{1/3} (f_i/f_0) \xi} \right] \quad (7)$$

and letting  $\eta = H_j/S_0 \cdot B$ , we can substitute Equations 5, 6 and 7 in Equation 1 to get

$$\eta = \frac{H_j}{S_0 B} = 0.63 f_0^{1/3} \xi + \frac{5.75}{\mu} \left[ 1 + \sqrt{1 + 0.11 f_0^{1/3} (f_i/f_0) \xi} \right] \quad (8)$$

which gives  $H_j$  in terms of  $B$ ,  $S_0$ ,  $Q$  and the coefficients  $f_i$ ,  $f_0$  and  $\mu$ . The main variable in Equation 8 is the dimensionless discharge  $\xi$ . The coefficient  $\mu$  is determined by the internal friction of the jam and, if the theory is valid, should be a constant. The

parameter  $f_i/f_0$  is always between 0 and 2 (note  $f_i/f_0 = 2f_i/(f_i + f_b) = 2/[1 + (f_b/f_i)]$ ) while the RHS of Equation 8 is fairly insensitive to  $f_i/f_0$ .

Using data of several case studies performed in recent years, a graph of  $n$  vs.  $\xi$  has been prepared (Beltaos 1982 - NWRI Hydraulics Division Unpublished Report). The data points indicated that Equation 8 gives a fair description with  $\mu = 1.2$ ;  $f_i/f_0 = 1.25$ ; and  $f_0$  between 0.1 and 0.6. There seems to be a trend for  $f_0$  to decrease when  $\xi$  increases. For practical purposes, it is convenient to draw an average line through the data points and use it for calculations (Figure 3).

Example:

We are given that, at a given site,  $B = 100$  m;  $S_0 = 0.5 \times 10^{-3}$  and the breakup discharges range from 200 to 1000  $m^3/s$ . Estimate on upper envelope for the peak breakup stages.

We have  $B.S_0 = 0.05$  m. Tabulate:

- Find  $\xi$  from Equation 6;
- Find  $n$  from Figure 3;
- Find  $H_j = n \times (S_0 \times B)$

Q ( $m^3/s$ )	$\xi$	n	$H_j$ (m)
200	187	85	4.3
400	297	113	5.7
600	389	139	7.0
800	471	162	8.1
1000	547	183	9.2

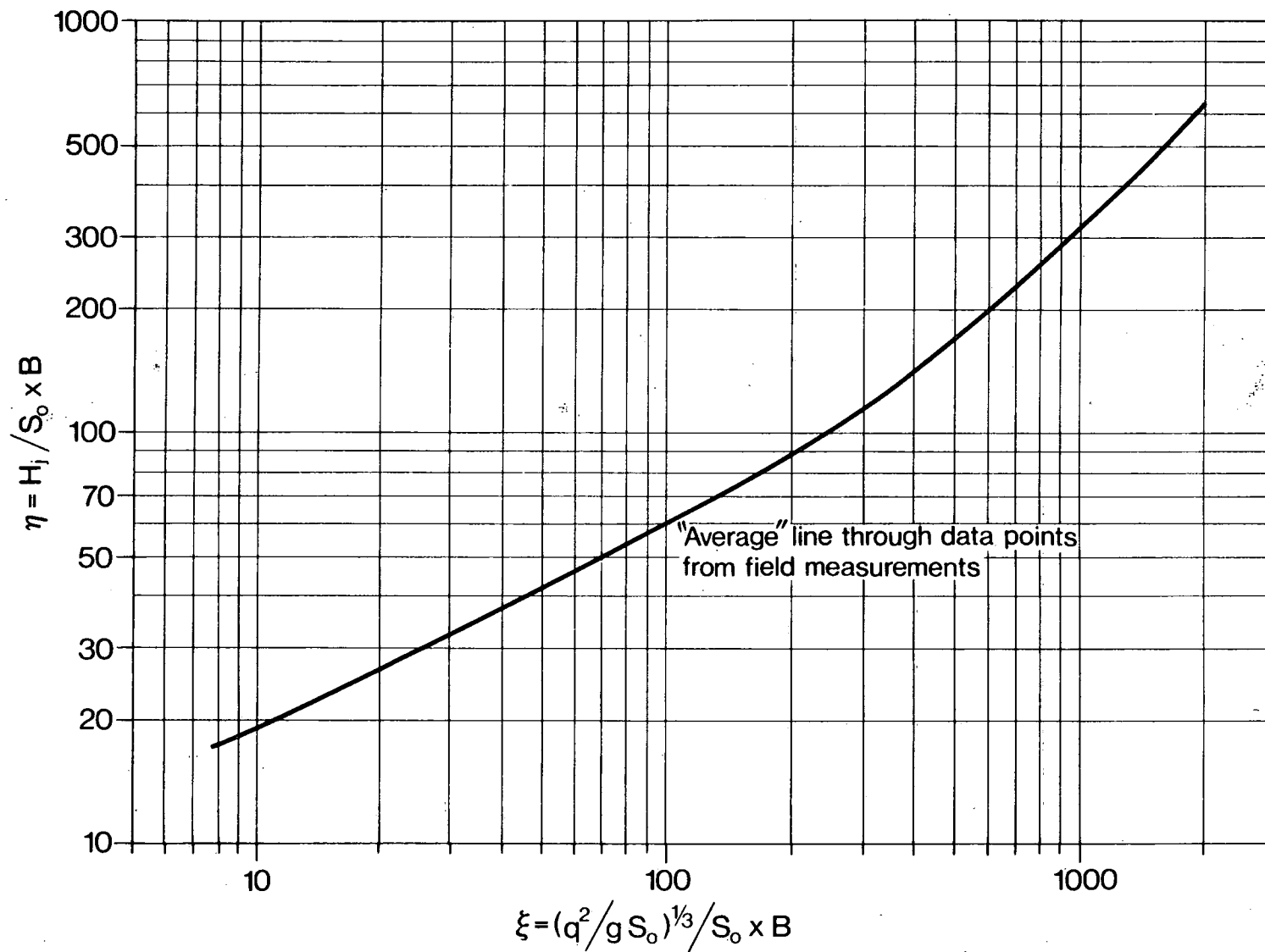


Figure 3 DIMENSIONLESS WATER DEPTH DUE TO AN EQUILIBRIUM, FLOATING, WIDE CHANNEL JAM vs. DIMENSIONLESS DISCHARGE

These values of  $H_j$  should be considered an upper envelope because an ice jam may or may not form in any given year; or it may not fully affect our site. In addition, the elevation of the flood plain is an important consideration because, once the W.L. exceeds it, ice jams will not be as severe as calculated. Much of the flow will probably escape to the flood plain.

There is another more detailed method to calculate  $H_j$ , as described by Beltaos (1982 - NWRI, Hydraulics Division Unpublished Report). This method takes into account changes in width with stage and makes use of channel bed resistance characteristics if such information is available from hydrometric surveys. Resistance characteristics of the jam are evaluated based on Nezhikhovsky's data for the beginning of freeze up.

Both methods involve considerable uncertainties and should always be supplemented with careful site inspections and historical evidence where possible. A few examples of the performance of these methods in actual situations are given in the above mentioned report by Beltaos.

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