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ABSTRACT

Field tests to determine the transverse mixing coefficients of three river reaches, under both open water and ice covered conditions, are described. To interpret the test results, the streamtube method for mixing calculations, developed earlier by others, is used. For evaluating transverse mixing coefficients, the widely used moments method is modified so as to be consistent with the streamtube approach. Application of the modified method to test data gave consistent results, thus reinforcing the streamtube approach. For the tests described here, the transverse mixing coefficient ranges between 0.01 m^2/s and 0.09 m^2/s; when this coefficient is non-dimensionalized with the hydraulic radius and shear velocity, it lies between 0.4 and 2.5. Open water values of the dimensionless transverse mixing coefficient are less than the corresponding values under an ice cover by as much as two and one-half times.

RÉSUMÉ

L'auteur décrit des tests effectués sur le terrain pour établir les coefficients de mélange transversal de trois tronçons de cours d'eau tant dans le cas des eaux libres qu'en temps de glace. Pour interpréter les résultats des tests, il se sert de la méthode du tube de courant mise au point par d'autres en vue des calculs relatifs aux mélanges. Pour évaluer les coefficients de mélange transversal, la méthode des moments, d'usage répandu, a été modifiée de sorte qu'elle soit conforme à celle du tube de courant. Le traitement des données des tests, suivant la méthode modifiée, a abouti à des résultats conséquents, ce qui confirme la validité de la méthode du tube de courant. En ce qui concerne les tests décrits dans cette étude, le coefficient de mélange transversal établi varie de 0.01 m^2/sec à 0.09 m^2/sec; mais lorsqu'il est exprimé, au moyen du rayon hydraulique et de la vélocité de cisaillement, sous une forme sans dimensions, il est de 0.4 à 2.5. Les valeurs du coefficient de mélange transversal sans dimensions en cas d'eaux libres sont jusqu'à deux fois et demie moins élevées que les valeurs correspondantes en temps de glace.

MANAGEMENT PERSPECTIVE

The mixing coefficient of a river has to be known in order to calculate the concentration of pollutants downstream of a source. At present, this coefficient cannot be predicted with any confidence for natural streams and only a limited number of field determinations have been reported. The mixing coefficient under ice covered flow is virtually unknown and has not been reported.

This report provides some additional field data which will be useful to those who have to assess the effects of contaminant releases. Especially valuable are the data for the mixing coefficient under ice covered flows.

This report also describes a streamtube method of analysis which will be useful to those who plan to carry out field determinations of mixing coefficients.

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PERSPECTIVE - GESTION

Il faut connaître le coefficient de mélange d'un cours d'eau pour calculer la concentration des polluants en aval de leur source. À l'heure actuelle, on ne peut prévoir ce coefficient avec certitude dans le cas des cours d'eau naturels et on ne dispose que d'un nombre limité de données établies sur le terrain à cette fin. Le coefficient de mélange des eaux recouvertes de glace reste quasiment inconnu et n'a fait l'objet d'aucun rapport.

Le présent rapport fournit d'autres données obtenues sur le terrain, lesquelles seront utiles à ceux qui doivent évaluer les effets des déversements de contaminants. Les données touchant le coefficient de mélange en temps de glace sont particulièrement précieuses.

Ce rapport décrit également une méthode d'analyse au moyen d'un tube de courant, laquelle sera utile à ceux qui comptent faire des tests sur le terrain pour établir les coefficients de mélange.

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By S. Beltaos[1]

INTRODUCTION

The study of mixing processes in natural streams is important in regulating pollution sources as well as evaluating the risks involved in accidental contaminant releases. Transverse mixing of neutral substances is one of the fundamental processes that require investigation. Suitable methods for pertinent engineering calculation are available but require knowledge of the transverse mixing coefficient of a stream. At present, this coefficient cannot be predicted in terms of observable stream characteristics and only a few field determinations have been carried out to date (6, 8, 12, 13, 15, 16, 17). Additional field data, under both open water and ice covered conditions, are furnished herein and test descriptions are given to illustrate field procedures. A convenient method for analyzing field measurements is developed and shown to give consistent results.

BACKGROUND INFORMATION

When a tracer is injected into a stream, its concentration C, will generally vary with respect to both space and time. The basic tool available for predicting the concentration is the principle of conservation of tracer mass, expressed in differential form.

In its most general form, the resulting partial differential equation involves formidable difficulties; even a numerical solution would be so laborious as to be impractical, at best. To reduce the problem to a more tractable form, two assumptions are utilized, deriving from physical understanding of the processes involved [see for example, Fischer (6); Holley, Siemons and Abraham (9)]:

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1. Since natural streams are much wider than they are deep, it is reasonable to suppose that an acceptable degree of vertical mixing is established relatively soon, that is, within a distance of 100 to 1000 river depths below the injection site. If this is so, acceptable engineering calculation can be made in terms of the depth average concentration, C_d, where the suffix d denotes a depth average value.

2. Since it is known that dispersion is much more effective in spreading the tracer in the longitudinal direction than longitudinal diffusion, the latter can be neglected as a first approximation.

Assuming further that there is no tracer transport across the flow boundaries, and using the notation explained in Fig. 1, it can be shown that, for prismatic channels [see also (9)]:

$$h \frac{\partial C_d}{\partial t} + hu_d \frac{\partial C_d}{\partial x} = \frac{\partial}{\partial z} (h\varepsilon_{zd} \frac{\partial C_d}{\partial z}) \qquad (1)$$

in which ε_z is the transverse diffusivity, t is time from injection and C_d=C_d(x, z, t). Though Eq. 1 is not amenable to analytical solution, numerical algorithms can be developed for efficient computation.

For steady state concentration distributions, which for example result from prolonged tracer injections at constant rate, the term $\partial C_d/\partial t$ vanishes and Eq. 1 simplifies to

$$hu_d \frac{\partial C_d}{\partial x} = \frac{\partial}{\partial z} (h\varepsilon_{zd} \frac{\partial C_d}{\partial z}) \qquad (2)$$

Because both h and u_d vary with z, an analytical solution of Eq. 2 has not been found so far. However, a simple and effective method for solving Eq. 2 indirectly was presented by Yotsukura and Cobb (16), as follows.

Letting q denote the flow discharge between, say, the left bank (z=0) and any one vertical located at z,

$$q = \int_0^z hu_d\, dz = q\,(z) \tag{3}$$

Since q is a function of z, the latter can be replaced by q in Eq. 2 which is then transformed to:

$$\frac{\partial C_d}{\partial x} = \frac{\partial}{\partial q}\left(h^2 u_d\, \varepsilon_{zd} \frac{\partial C_d}{\partial q}\right) \tag{4}$$

Using numerical examples, Yotsukura and Cobb showed that, although the quantity $h^2 u_d \varepsilon_{zd}$ varies with z (or q), the solution of Eq. 4 is insensitive to such variation and, for practical purposes, it can be assumed that

$$h^2 u_d\, \varepsilon_{zd} \approx \text{const.} = (h^2 u_d\, \varepsilon_{zd})_q = D_z \tag{5}$$

where the suffix q denotes an average value with respect to q, that is

$$D_z = \frac{1}{Q} \int_0^Q h^2 u_d\, \varepsilon_{zd} dq = e_z\,(h^2 u_d)_q \tag{6}$$

in which Q is the total river discharge and e_z is an overall average transverse diffusion coefficient. The "diffusion factor", D_z, can be written further as

$$D_z = \psi\, e_z\, VH^2 \tag{7}$$

with V and H being the cross-sectional average values of velocity and depth respectively; and ψ being a dimensionless "shape-velocity" factor defined by

$$\psi = \frac{(h^2u_d)_q}{H^2V} = \frac{1}{Q} \int_o^Q h^2u_d dq \tag{8}$$

With the assumption implied in Eq. 5, Eq. 4 reduces to

$$\frac{\partial C_d}{\partial x} = D_z \frac{\partial^2 C_d}{\partial q^2} \tag{9}$$

which can be solved analytically (16). This approach, herein termed the "streamtube" approach (surfaces of equal q along the stream define a system of time- and depth-average streamsurfaces and streamtubes), eliminates the need for numerical solution of the mixing problem when $\partial C/\partial t=0$.

Recently, Yotsukura and Sayre (17) extended the theoretical development to natural streams, so as to include flow expansion and contraction as well as curvature effects; they showed that Eq. 9 still provides a realistic approximation, if the definition of D_z is generalized as

$$D_z = (m_x h^2 u_d E_{zd})_q \tag{10}$$

in which m_x is a metric coefficient accounting for divergence or convergence of streamsurfaces; and E_{zd} is now a transverse mixing coefficient such that

$$E_{zd} = \varepsilon_{zd} + \varepsilon_{zD} \tag{11}$$

with ε_{zD} being a transverse dispersion coefficient, introduced to account for transverse dispersion effects that arise from the helical motions at river beds. [It has been pointed out (11) that dispersive effects are present even in straight channels due to secondary flow. Strictly speaking then, ε_{zd} should be the straight-channel component of the mixing coefficient and the diffusion/dispersion terminology used herein should be regarded as conventional.] Clearly, in the case of natural streams, E_{zd}, ψ and D_z will vary along the channel but all three should have well defined average values. Moreover, if a reach contains several bends, the average value of m_x should be close to unity. In a "reach average" context it would then be permissible to write

$$D_z = \psi \; E_z \; VH^2 \tag{12}$$

where D_z, ψ, E_z, V and H are taken as reach average values.

The advantages of the streamtube approach are several, from both the analytical and practical points of view, and have already been illustrated in (16) and (17). As will be shown later, the writer has also found this approach to provide a "rugged" and practical means of analysis for natural stream applications.

The preceding discussion has shown that convenient means are available for engineering solutions to mixing problems; the procedures involved require knowledge of stream hydraulics and the transverse mixing coefficient. The former can be measured relatively simply; the latter can only be obtained by means of tracer tests.

To date, relatively few field studies have been carried out to determine E_z, especially under ice covered conditions. One of the aims of this paper is to provide additional field data and furnish further testing of the streamtube approach; the second aim is to illustrate field procedures and techniques by giving detailed descriptions of a few representative tests. Before proceeding with the data presentation, however, the methods used for data interpretation will be described.

METHODS OF ANALYSIS

Method of Moments. Analysis of the test results has been based on the streamtube approach. The method of moments which has been used extensively in the past was applied to the basic differential equation describing the depth average concentration C_d, in terms of the coordinate system x and q (Eq. 9). Normalizing C_d and q via the definitions

$$\eta \equiv q/Q \; ; \quad C' \equiv C_d/C_\infty \tag{13}$$

gives (note that $C_\infty = \int_0^1 C_d d\eta$=final, fully mixed concentration):

$$\frac{\partial C'}{\partial x} = \frac{D_z}{Q^2} \frac{\partial^2 C'}{\partial \eta^2} \tag{14}$$

The centroid of the C'-η distribution is given by $\eta_o = \int_0^1 \eta \, C' d\eta$ and can be determined by taking the first η-moment of both sides of Eq. 14:

$$\frac{d\eta_o}{dx} = \frac{D_z}{Q^2} \int_0^1 \eta \frac{\partial^2 C'}{\partial \eta^2} d\eta \tag{15}$$

Integrating by parts and assuming no tracer transport across the channel banks gives

$$\frac{d\eta_o}{dx} = \frac{D_z}{Q^2} \left(C'_o - C'_1 \right) \tag{16}$$

where the suffixes o and 1 denote the left (η=0) and right (η=1) banks respectively. Multiplying Eq. 15 by $2\eta_o$ and integrating by parts gives

$$\frac{d\eta_o^2}{dx} = - \frac{2D_z}{Q^2} \int_0^1 \eta_o \frac{\partial C'}{\partial \eta} d\eta \tag{17}$$

Next, consider the second moment of Eq. 14. After some manipulation, this operation gives

$$\frac{d}{dx} \int_0^1 \eta^2 C' d\eta = - \frac{2D_z}{Q^2} \int_0^1 \eta \frac{\partial C'}{\partial \eta} d\eta \tag{18}$$

Subtracting Eq. 17 from Eq. 18 and manipulating gives

$$\frac{d\sigma_\eta^2}{dx} = \frac{2D_z}{Q^2} \quad [\,1 - (1 - \eta_o)\, C'_1 - \eta_o\, C'_o\,] \tag{19}$$

where σ_η^2 is the variance of the C'-η distribution.

For such distributions that the diffusing substance has not reached either bank, $C'_o = C'_1 = 0$; then Eq. 19 reads simply:

$$\frac{d\sigma_\eta^2}{dx} = \frac{2D_z}{Q^2} \tag{20}$$

which is a generalized version of the well known moments equation that has been used extensively in the past:

$$\frac{d\sigma_z^2}{dx} = \frac{2E_z}{V} \tag{21}$$

where σ_z^2 is the variance of the C-z distribution. However, validity of Eq. 21 is subject to the restrictions:

(i) Constant value of u_d (=V) across the channel, at least within the region defined by the outer limits of the diffusing plume.

(ii) Constant value of depth within the same region.

These conditions are rarely fulfilled in natural streams and use of Eq. 21 should thus be avoided. On the other hand, Eq. 20 is based on the streamtube approach, which takes into account lateral variations of depth and velocity.

When C'_o or C'_1 are not zero, one may proceed by setting

$$f(x) \equiv 1 - (1 - \eta_o)\, C'_1 - \eta_o C'_o \tag{22}$$

where f (x) can be easily determined from a set of measured lateral concentration distributions. Substituting Eq. 22 in Eq. 19 and integrating gives

$$\sigma_\eta^2 = \frac{2D_z}{Q^2} \int_o^x f(x)\,dx \tag{23}$$

This suggests that plotting σ_η^2 versus corresponding values of $\int_o^x f(x)\,dx$ should result in a straight line of slope $2D_z/Q^2$ which can be used to compute the corresponding value of E_z.

The above analysis and Eq. 23 apply only when concentration is independent of time, that is, for steady-state experiments. Beltaos (1) showed that this method can also be applied to the results of transient tests, if C is replaced by the dosage θ, defined by

$$\theta(x, q) \equiv \int_o^\infty C(x, q, t)\,dt \tag{24}$$

For injections at either river bank and prior to the tracer reaching the opposite bank, a simple Gaussian type equation applies for the lateral distributions of C' (2, 4). The maximum value of C' ($=C'_m$) decreases as $x^{-1/2}$ whereas the spread, $\Delta\eta$, of the plume increases as $x^{1/2}$. If this is the case, one need not compute variances but use these properties of the plume to determine E_z by simply plotting C'_m and $\Delta\eta$ versus x on logarithmic paper.

Flow Distribution Curves. To determine transverse profiles of concentration in terms of q or η, flow distribution curves (q versus z) are needed for the various sampling sites. If a cross section is current metered in detail, transverse profiles of h and u_d can be produced readily and q(z) can be determined by graphical integration of the profile of hu_d versus z.

Occasionally, it may be impractical to current meter all of the intended sampling sites, even though depth profiles can be obtained readily; moreover, in applications of the streamtube method, the situation where depth profiles are available but u_d profiles are not, occurs often. This difficulty can be overcome by assuming a local resistance type of law:

$$u_d/V = (h/H)^a \tag{25}$$

where a takes the values 1/2 and 2/3 for the Chezy and Manning formulae respectively. When velocity data are missing, Eq. 25 provides a means for estimating $u_d(z)$ and thence q(z) [see also Sayre and Yeh (13)]. Clearly, this approach will be invalid in dead zone and eddy areas that occur occasionally near the stream boundaries. If such areas are detected at a sampling site, it is advisable to current meter this site.

FIELD TESTS

Three sets of field tests will be described in this section. Each set consists of evaluations of the transverse mixing coefficient under both open water and ice covered conditions. The river reaches tested are located in Alberta, Canada (2).

For illustration purposes, the first two tests will be described in some detail but only brief descriptions of the remaining tests will be presented. More detailed descriptions of these tests may be found in (2).

Test 1 - Athabasca River below Fort McMurray, Ice Covered Condition, February 1-2, 1974. A slug of 63.5 kg of 20 percent Rhodamine WT fluorescent dye was injected 34 m off the left bank at Mildred Lake Dock (some 37 river kilometres downstream of the town of Fort McMurray) at 1530 hours on February 1, 1974 (see Fig. 2). This test was intended to provide preliminary data necessary for a comprehensive test that had been planned for later. For this reason, sampling was carried out at only two sites, located 6.3 and 11.8 km downstream of the injection site.

Prior to injection, several holes were cut in the ice across each sampling site to permit current metering, as well as to provide access for taking water samples during the test. Each site was provided with a tent equipped with a fluorometer and a heater for in situ analysis of the samples. Sampling consisted of taking water samples across the stream as often as possible, after dye was detected and until dye concentrations dropped to negligibly low values. The extremely low temperature prevailing during the test caused a substantial reduction in the intended sampling frequency. At the first sampling site, where the temporal spread of the dye cloud was relatively small, this proved to be detrimental. Typically, only two or three measurements of non-negligible concentrations were obtained at each hole.

About three weeks after the test, the study reach was documented in more detail by measuring the cross-sectional geometry of several more intermediate sites, and current metering the injection site. Water surface slopes near the original sites were also measured. Figure 2 shows a plan of the study reach and the locations of the various sites. Note that these sites are designated with numbers whereas capital letters are used to denote sites associated with the open water test (Test 2).

Using Water Survey of Canada records for the gauging station at Fort McMurray (14), the discharge during the test was estimated as 240 m^3/s. Field data were adjusted to this discharge (2) and reach average hydraulic parameters were computed as listed in Table 1.

Because adequate concentration data were available for only the second sampling site, determination of E_z was based on the transverse dosage distribution at this site. Dosage was calculated by planimetering the observed concentration-time curves (Fig. 3) and was plotted versus $\eta(=q/Q)$ as shown in Fig. 4. To find E_z, several values of $2D_z/Q^2$ were tried in conjunction with the equation describing steady-state concentrations (dosages in this case) for point source injections (16). A value of 6.56 (10^{-6}) m^{-1} for $2D_z/Q^2$ resulted in optimum agreement between calculated and observed profiles, as shown in Fig. 4. This value corresponds to ψE_z=0.105 m^2/s. Using an average ψ of 2.56 (Table 1) gives E_z=0.041 m^2/s and $K_z(\equiv E_z/RV_*)$=1.16. It should be recognized that this value represents an average for the 12 km long reach between sections 0 and 7.

It is of interest to point out here that using the above value of E_z and a numerical algorithm described in (2), C-t curves were simulated for the sampling points of section 7 and compared favourably with the observed curves (2). Moreover, a comprehensive test, carried out in February 1978 in a 27 km long reach beginning some 30 km below Mildred Lake Dock (see Fig. 2), gave K_z=1.44 (3).

Test 2 - Athabasca River below Fort McMurray, Open Water Condition, September 26, 1974. A slug of 1.4 kg of 20 percent Rhodamine WT dye was injected at 1204 hrs, 85 m off the left bank at a site somewhat downstream of the Mildred Lake Dock, as shown in Fig. 2 (section 0 for open water test). The injection point was chosen so as to be at the centroid of flow, η=0.5. Sampling was carried out at seven sites (sections A, B, C, E, F, G and H) located as shown in Fig. 2. Several sampling points were established across the river at each of these sections, using floats attached to heavy concrete blocks. All sections were sounded to define the cross-sectional geometry. In addition, sections 0, B, C, D and F were current metered to determine the transverse velocity distribution. Velocity distributions for the remaining sections were generated as outlined previously and average hydraulic data are summarized in Table 1.

Analysis of the samples showed that the frequency of sampling was not adequate to permit good definition of the corresponding time-concentration curves at all sampling points. However, a sufficient number of reasonably well defined curves were obtained. These were planimetered to determine corresponding values of the dosage, θ, and the latter are shown plotted in terms of η (=q/Q) in Fig. 5. At site A, the dosage was nil at η=.42 and .82, and 59 μg min/ℓ at η=0.61. As these observations are not sufficient to define the dosage profile, site A is not included in Fig. 5. The remaining profiles were used to compute corresponding variances and determine E_z. Since at the last three sites the dye had reached the banks, evaluation of E_z requires use of Eq. 23 with f(x) defined in Eq. 22. The results of this calculation are summarized in Table 2. Note that section G has been omitted due to poor definition of the profile.

Figure 6 shows σ_η^2 plotted versus $\int_0^x f dx$. The data points define a straight line of slope $4.18 \times 10^{-6} m^{-1}$ which, by Eq. 23, equals $2D_z/Q^2$. Therefore, D_z=1.26 m^5/s^2 and, with ψ=2.96, E_z=0.093 m^2/s and K_z=0.75.

Test 3 - Beaver River near Cold Lake, Open Water Condition, October 9, 1974.

For this test, a 1.45 % solution of Rhodamine WT was injected at a constant rate of 4.25 cm^3/s. The source was located near the flow centroid at the injection site and sampling was carried out at several sections within a river length of some 1.5 km. Cross-sectional geometry and velocity distributions were measured at several sections and the reach average hydraulic data for October 9, 1974 are listed in Table 1 [see (2) for a river plan and section location] .

Figure 7 shows C_d-η profiles and Fig. 8 is a plot of σ_η^2 versus $\int_0^x f dx$. The latter is linear for the major portion of the test reach. The last five data points correspond to sections where the concentration profiles were spread considerably across the stream and accurate detection of change in variance is difficult. Using the slope of the straight line shown in Fig. 8 and hydraulic data from Table 1, E_z is found as 0.043 m^2/s which corresponds to K_z=1.01.

Figure 7 shows that the maximum concentration occurs generally near the flow centroid, as would have been expected from the fact that the source was located near η=0.5. However, section 3 does not fit this trend; the maximum concentration seems to be located too far towards the right bank. Figures 9a and 9b show respectively the depth and concentration profiles for section 3. It is seen that, in the range $0 \leq z \leq 8.4$ m, $\partial C_d/\partial z = 0$. Recalling Eq. 2 shows that either u_d or $\partial C_d/\partial x$ or both should vanish in this range. Since the longitudinal concentration gradient is controlled by mixing in the live stream where $\partial C_d/\partial x \neq 0$, it is more likely that u_d=0. In turn, this suggests that a dead zone existed near the left bank at section 3. Assuming this to be the case, the η-z profile was resynthesized and is shown together with the one used originally in Fig. 9c. Figure 9d compares the C_d-η

profiles that result from the flow distribution curves of Fig. 9c. Though the above reasoning is somewhat simplistic, Fig. 9d illustrates that the C_d-η profile is improved when the dead zone is taken into consideration.

Test 4 - Beaver River near Cold Lake, Ice Covered Condition, February 11-12, 1975. This test involved two steady-state injections and subsequent sampling, one on February 11, 1975, near the centroid of the flow, and the other on February 12, 1975, very near the right bank. The two injections were carried out to investigate possible effects of source location [the layout of this test is shown in (2)].

Figure 10 shows σ_η^2 plotted versus $_o\int^x$ fdx for both injections. For the midstream injection, the data points define a straight line with little scatter. The scatter for the side injection is larger but a straight line, parallel to the previous one, seems to fit the results satisfactorily, with the exception of the first 100 m. In this early phase, mixing seems to have been more pronounced for the side injection than for the central injection. This is reasonable since the injection site was located at the entrance of a bend (2) where intense transverse dispersion is expected to occur. This effect would be pronounced for the side injection where the main portion of the diffusing plume was located at the outside (right bank) of the bend. However, for the central injection, this transverse dispersion effect would be experienced only by the outer-right portion of the plume and the overall effect is likely to have been small.

The common slope of the straight lines drawn in Fig. 10 is 0.218 x 10^{-3} m^{-1} which, with the data listed in Table 1, gives E_z=0.02 m^2/s and K_z=2.54.

Test 5 - Athabasca River below Athabasca, Open Water Condition, September 16, 1974. Injection was carried out as a slug from the bridge at the town of Athabasca (2) near the flow centroid at 1030 hrs on September 16, 1974. Sampling was carried out at seven sites within a 17 km long reach shown in (2) ; however, passage of the dye cloud at site 1 was so rapid that it was not possible to define corresponding time-concentration curves. For the remaining sites, profiles of the dosage, θ, versus η, showed that dye did not generally reach the banks, except at the farthest downstream site where the side concentrations were only 10% of the maximum. For this reason, E_z was determined by plotting σ_η^2 directly versus x, as shown in Fig. 11a. The discharge on the date of the test, September 16, 1974, was estimated as 566 m^3/s (15). Cross-sectional geometry and occasional velocity distributions were measured at the injection site, the seven sampling sites and two additional sections (2). From this information, reach average hydraulics were found as shown in Table 1. Using Fig. 11a and these data, E_z is computed as 0.067 m^2/s and K_z=0.41. The testing procedure was identical to that described earlier regarding test 2.

Test 6 - Athabasca River below Athabasca, Ice Covered Condition, February 27, 1975. A slug of dye was injected at 0830 hrs near the centroid of the flow at a site located 30 m upstream of the bridge at the town of Athabasca (2) and sampling was carried out at five sites (2). From the hydrometric survey carried out in the study reach, the average hydraulic parameters were found as listed in Table 1.

Analysis of the test results showed again that only insignificant amounts of dye reached the banks within the study reach and thus Fig. 11b shows the variance of dosage profiles plotted directly versus distance. It is seen that, even though the data points are described well by a straight line, the first 2 km of the test reach were characterized by a higher mixing capacity than the straight line indicates. It is not known why this is so, especially in view of the uniformity of mixing intensity observed in the same reach during the open water test (Fig. 11a). A possible explanation may be the fact that the longitudinal gradient of the variance ($d\sigma_\eta^2/dx$) is proportional to $D_z(=\psi E_z VH^2)$ which is influenced not only by E_z but also by the factor ψ. The latter has a reach average value of 3.64; however, its value at 2 km was computed as 5.1 which may be indicative of more pronounced initial mixing (note that for the open water test, the 2 km value of ψ was 1.65 which is very close to the corresponding average value of 1.72). Using the slope of the straight line shown in Fig. 11b and average hydraulic parameters gives E_z=0.010 m^2/s and K_z10.56.

Beltaos (1) suggested that the dosage method could be employed profitably to combine transverse mixing and longitudinal dispersion tests, which in the past have always been carried out separately, in a single slug-injection test. This could be done efficiently by measuring transverse dosage profiles in the early portion of the test reach using automatic sampling techniques and observing time-concentration variations far downstream where a one-dimensional condition is established. Automatic measurement of the dosage at a desired location would require a device drawing water at a constant rate throughout the anticipated time of passage of the tracer cloud at this location. If C_f is the final concentration in a sample obtained in this way, it can be shown that the corresponding value of the dosage θ is equal to $C_f \Delta t_s$ where Δt_s is the duration of sampling.

The feasibility of this technique was investigated during this test using a number of pails fitted in holes cut in the ice cover and equipped with small heat sources to prevent the samples from freezing. This set-up is shown schematically in Fig. 12a. Constant head was provided by the free water surface and sampling was started after adjusting the rate with clamps. To counteract buoyancy, each pail was secured to the ice cover using ice blocks for weight, as shown in Fig. 12a.

Pails were installed at the first four sampling sites where manual sampling was also carried out. Use of the pails enabled more frequent manual sampling which improved definition of the time-concentration curves. Figure 12b shows transverse dosage profiles obtained by a combination of automatic sampling and planimetering of observed time-concentration curves. It is seen that data points obtained by these different methods define single curves with satisfactory consistency.

DISCUSSION

The results of several transverse mixing tests, together with methods of evaluating field data were outlined in the previous sections.

The streamtube approach was adopted as an efficient means for engineering calculations of transverse mixing in rivers. To analyze field data, the commonly used method of moments was modified so as to be consistent with the streamtube concept and a generalized equation, accounting for non-zero bank concentrations, was derived. This equation shows E_z to be related to the slope of variance versus modified distance graphs; when E_z does not vary appreciably in the study reach, such plots should be linear. This was found to be generally the case when the test results were processed, as illustrated in Figs. 6, 8, 10 and 11. Readers familiar with previous pertinent literature would agree that the streamtube method of moments gives much more consistent variance plots than plots based on the variance of concentration-lateral distance profiles.

The possible effects of dead zones were discussed in conjunction with the "misfit" C_d-η profile obtained at section 3 of test 3. A dead zone is a region of stagnant fluid, that is, u_d=0. If this is the case, then Eq. 2 shows that $h\varepsilon_{zd}$ $(\partial C_d/\partial z)$ should be independent of z within the dead zone. Since dead zones are adjacent to the stream boundaries where $\partial C_d/\partial z=0$, C_d should be independent of z within the dead zone. However, this argument neglects the effect of longitudinal diffusion, which may be significant in the absence of velocity gradients.

It has been suggested (1) that the transverse mixing coefficient can be conveniently determined from unsteady concentration data, based on the spread of dosage rather than concentration. The present test data seem to support this expectation since consistent results were obtained when the dosage method was used. A preliminary evaluation of the feasibility of using automatic sampling for dosage measurement showed some promise.

Considering the effects of an ice cover, the only previously reported test known to the writer is that by Engmann and Kellerhals (5). These investigators suggested that an ice cover reduces the mixing capacity of a stream in a way that the dimensionless coefficient $K_z(=E_z/RV_*)$ is about the same for both ice covered and open water conditions. However, inspection of Table 1 shows that values of K_z are larger under an ice cover than with open water flow by as much as two and one-half times. It is probably premature to attempt an explanation of this finding, however, it is believed that much may be learned by studying the helical bend flow under an ice cover [see also Fischer (7)].

In a recent paper (4), the writer summarized and discussed the results of previous tests and the tests described herein. No quantitative correlation between E_z and stream hydraulic characteristics could be found and it was suggested that this could be partly due to a lack of standardized methods for measuring E_z. Some investigators disregard velocity and depth gradients that are always present in rivers, while others have assumed that transverse advection at bends behaves in the same manner as transverse dispersion and included transverse advection into the E_z term (5). The streamtube approach which implicitly accounts for these effects seems to be the best non-numerical method for analyzing test results.

SUMMARY AND CONCLUSIONS

Transverse mixing characteristics of three river reaches in Alberta were documented by means of tracer tests under both open water and ice covered flow conditions.

The streamtube approach was adopted as an efficient means for engineering calculations in natural stream applications; to analyze field data, the method of moments was modified in accordance with the streamtube concept and a generalized equation that accounts for non-zero bank concentrations was derived. The consistency of the results presented herein lends support to the streamtube method of analysis.

The observed transverse mixing coefficients vary between 0.01 m^2/s and 0.09 m^2/s. When these coefficients are non-dimensionalized with the hydraulic radius and the shear velocity, they range between 0.4 and 2.5. Under ice covered conditions, the dimensionless coefficient is larger than with open water flow by as much as 2.5 times. It is suggested that a study of the helical bend flow under an ice cover would help explain this finding.

Use of dosage in place of concentration for time-dependent situations, such as with slug injection tests, gave consistent results; this supports earlier theoretical work suggesting that, other things being the same, the dosage for a slug test varies in the same manner as the concentration of a steady state test.

ACKNOWLEDGMENTS

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Throughout the course of some three and one-half years, R. Gerard provided valuable criticism and encouragement and participated in some of the tests. Occasional discussion with R. Kellerhals is appreciated. Thanks are due to B. G. Krishnappan, and N. Yotsukura for comments on an internal report on which this paper is based. Review comments by T. M. Dick and Y. L. Lau are appreciated. The tests described herein were carried out with the help of M. Anderson, G. Childs, E. Karpuk, G. Putz and H. Schultz.

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APPENDIX II. - NOTATION

a	an exponent
C	concentration
d	suffix denoting depth average value
D_z	diffusion factor
e_z	average diffusion coefficient
E_z	transverse mixing coefficient
f	a function
h	local flow depth
H	average flow depth
K_z	dimensionless mixing coefficient
q	cumulative stream discharge; suffix denoting discharge weighted average value
Q	total stream discharge
R	hydraulic radius
t	time
u	streamwise component of velocity
V	average flow velocity
V_*	average shear velocity
x	longitudinal space coordinate
z	transverse space coordinate

ε	diffusivity
η	normalized cumulative discharge (q/Q)
θ	dosage
σ^2	variance
ψ	a dimensionless factor

TABLE 1. - Reach Average Hydraulic Parameters of Study Reaches

Test No. (1)	River Site and Channel Description (2)	Flow Condition (3)	Discharge (m^3/s) (4)	Width (m) (5)	Depth (m) (6)	Velocity (m/s) (7)	Slope ($x10^4$) (8)	Factor (9)	Transverse Mixing Coefficient (m^2/s) (10)	K_z (11)
1	Athabasca R. below Fort McMurray; straight with occasional bars, islands.	Ice Covered	240	252	1.9	0.49	1.44	2.56	0.041	1.16
2		Open Water	776	373	2.2	0.95	- 11 -	2.96	0.093	0.75
3	Beaver R. near Cold Lake; regular meanders, point bars and large dunes.	Open Water	20.5	42.7	0.96	0.50	2.1 [a]	2.15	0.043	1.01
4		Ice Covered	6.5	38.7	0.61	0.28	- 11 -	2.33	0.020	2.54
5	Athabasca R. below Athabasca; irregular meanders with occasional bars, islands.	Open Water	566	320	2.05	0.86	3.1 [a]	1.72	0.067	0.41
6		Ice Covered	105	276	0.96	0.40	- 11 -	3.64	0.010	0.56

[a] From Kellerhals et al (10). Note: 1 m = 3.28 ft.

TABLE 2. - Analysis of Lateral Dosage Distributions - Test 2

Section (1)	x (km) (2)	Centroid η_o (3)	Variance σ_η^2 (4)	$f(x)$ (5)	$\int_0^x f dx$ (6)
B	2.6	0.50	0.0115	1.00	2.6
C	5.8	0.54	0.0353	1.00	5.8
E	8.7	0.53	0.0345	1.00	8.7
F	11.6	0.51	0.0502	0.67	11.1
H	17.6	0.50	0.0568	0.45	14.5

Note: 1 km = 3281 ft.

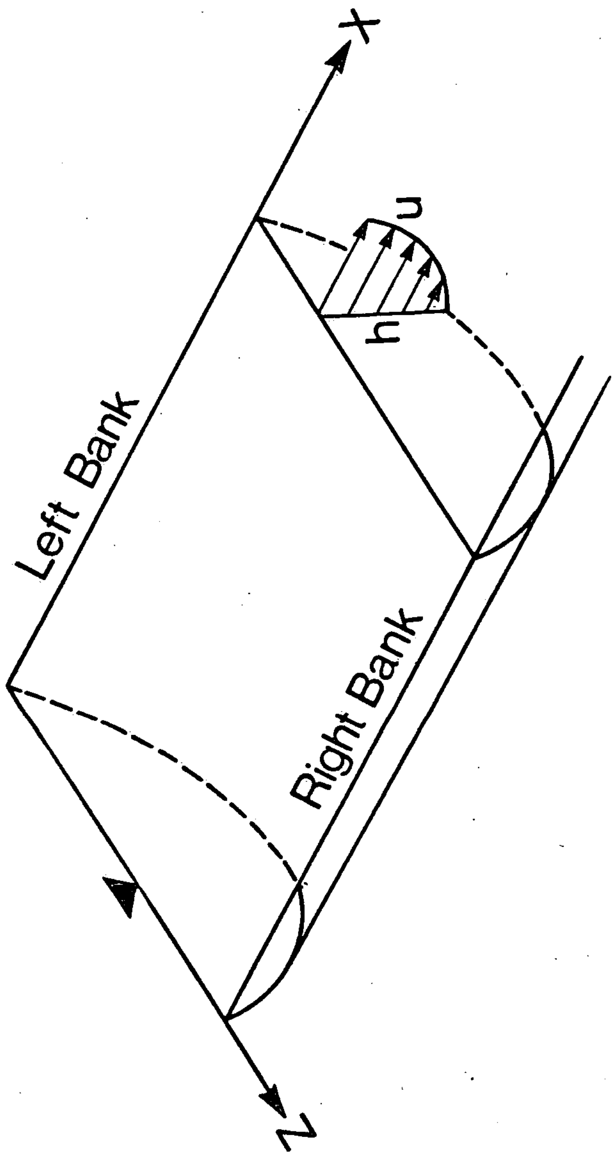


Fig. 1 Prismatic channel; definition sketch

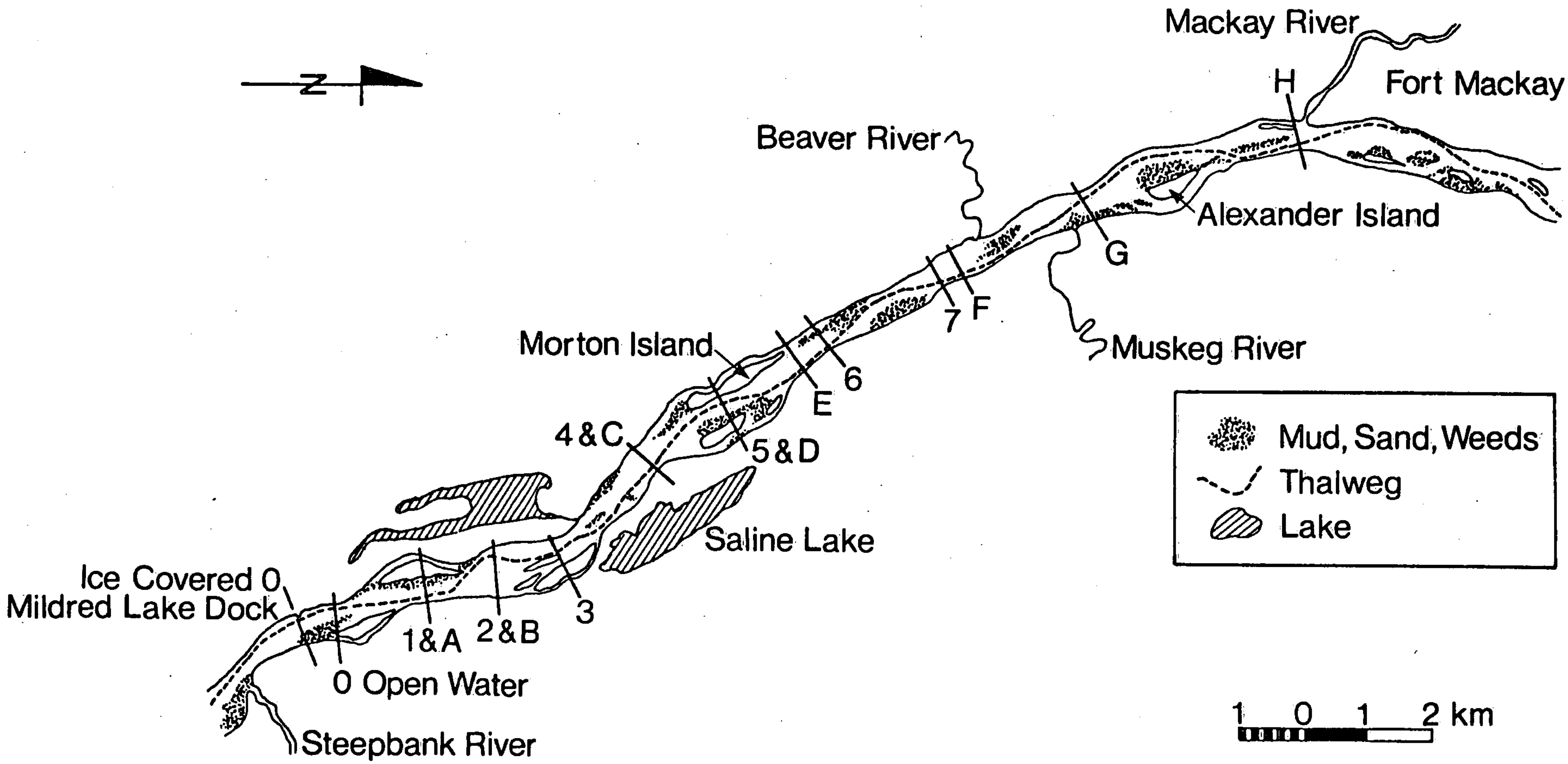


Fig. 2 Test reach, Athabasca River below Fort McMurray

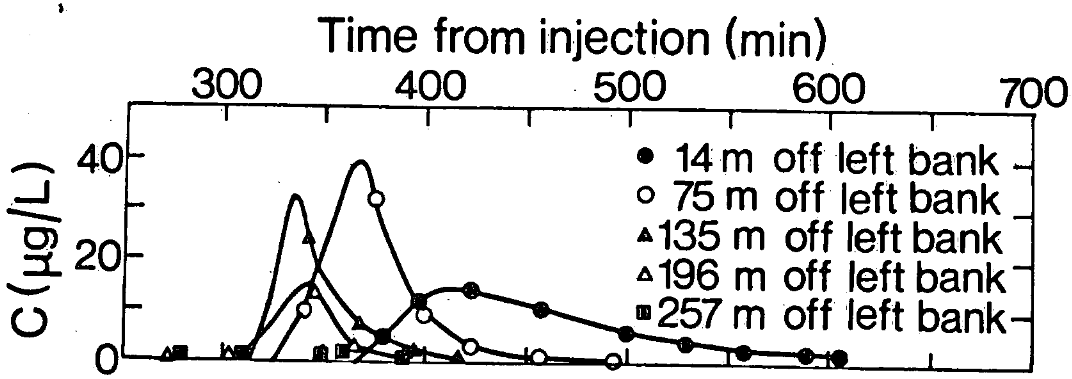


Fig. 3 Concentration-time variations observed across site 7 (x=11.8 km, width=292 m); Test 1

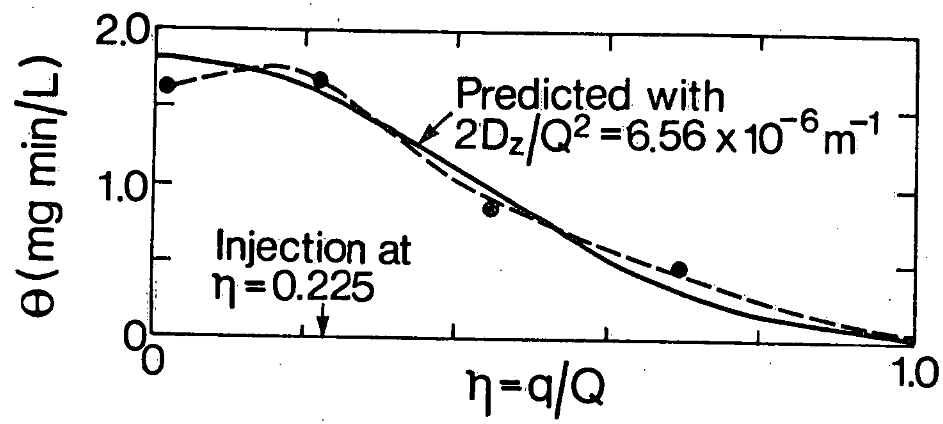


Fig. 4 Observed dosage profile and "optimum" prediction for site 7 (x=11.8 km); Test 1

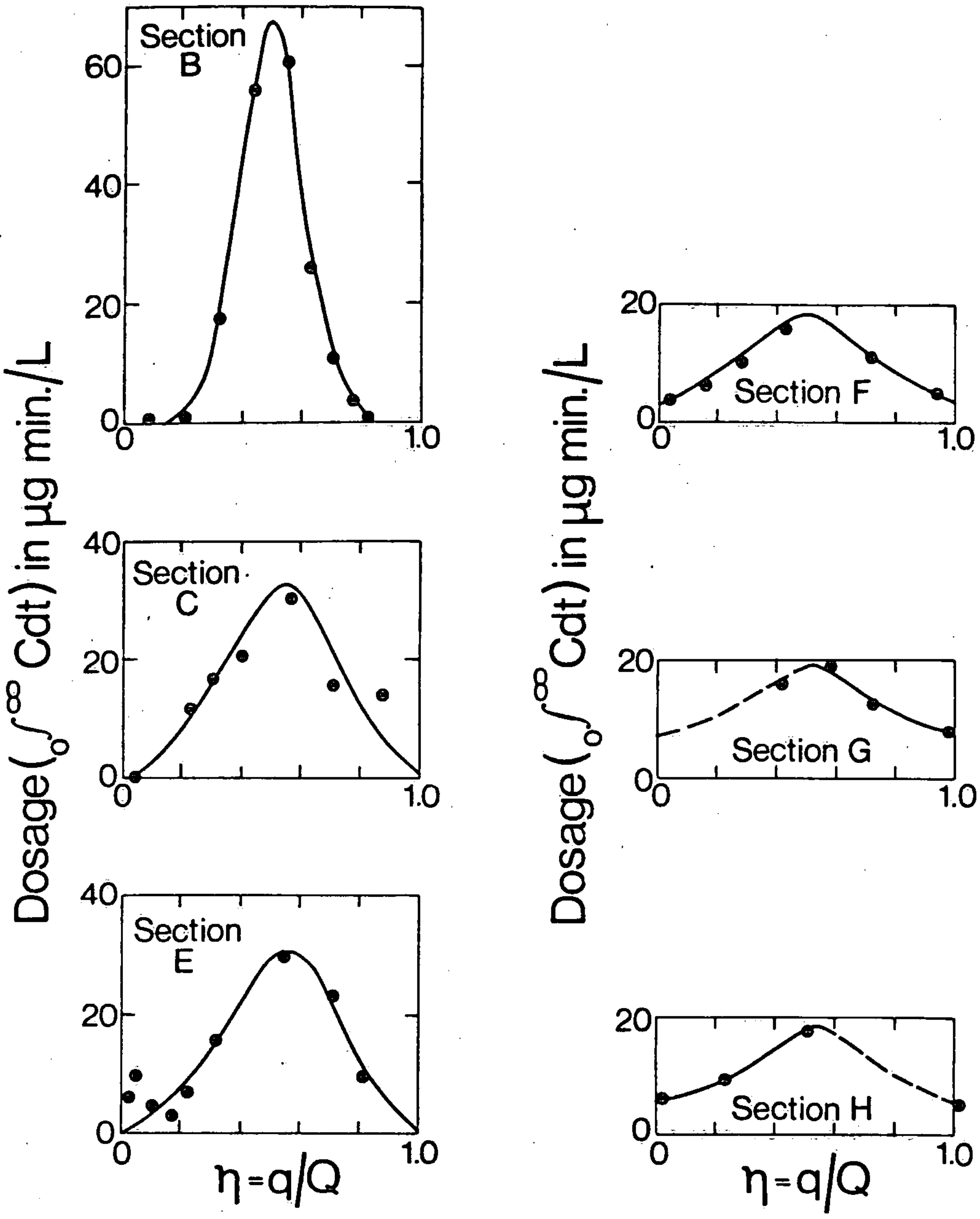


Fig. 5 Dosage profiles in terms of cumulative discharge; Test 2

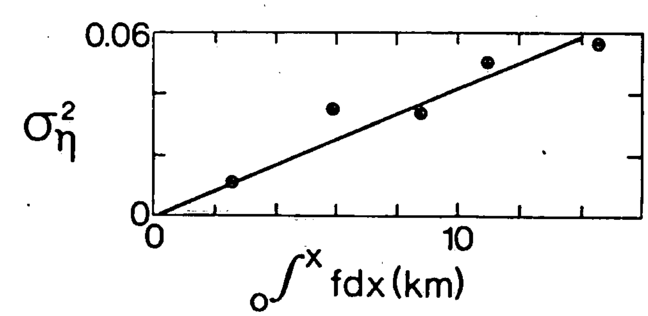


Fig. 6 Plot of variance versus modified distance to determine the transverse mixing coefficient; Test 2 (1 km=3281 ft.)

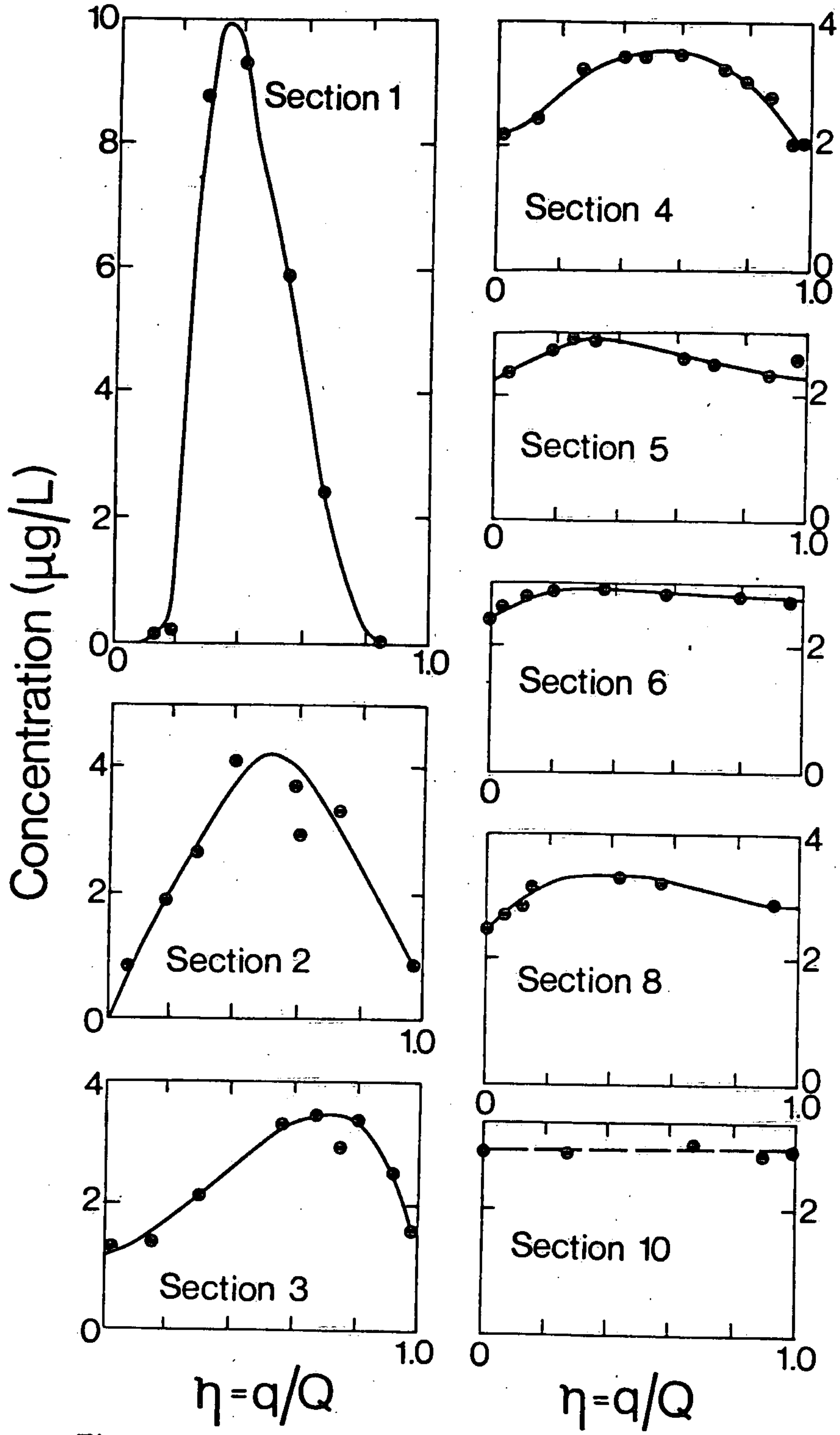
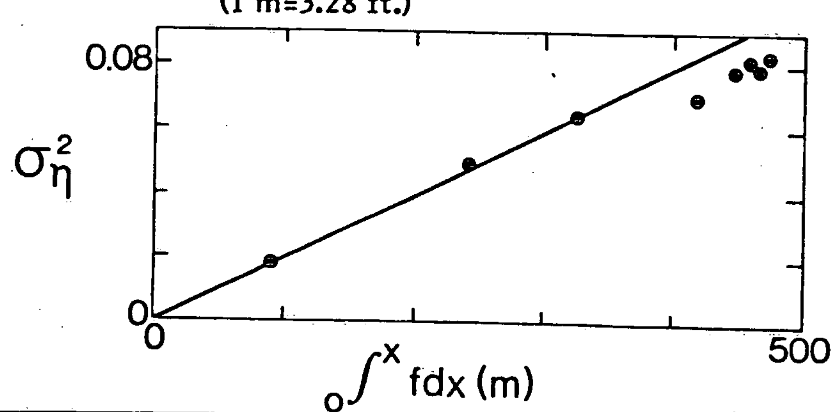


Fig. 7 Concentration profiles in terms of cumulative discharge; Test 3

Fig. 8 Plot of variance versus modified distance; Test 3 (1 m=3.28 ft.)



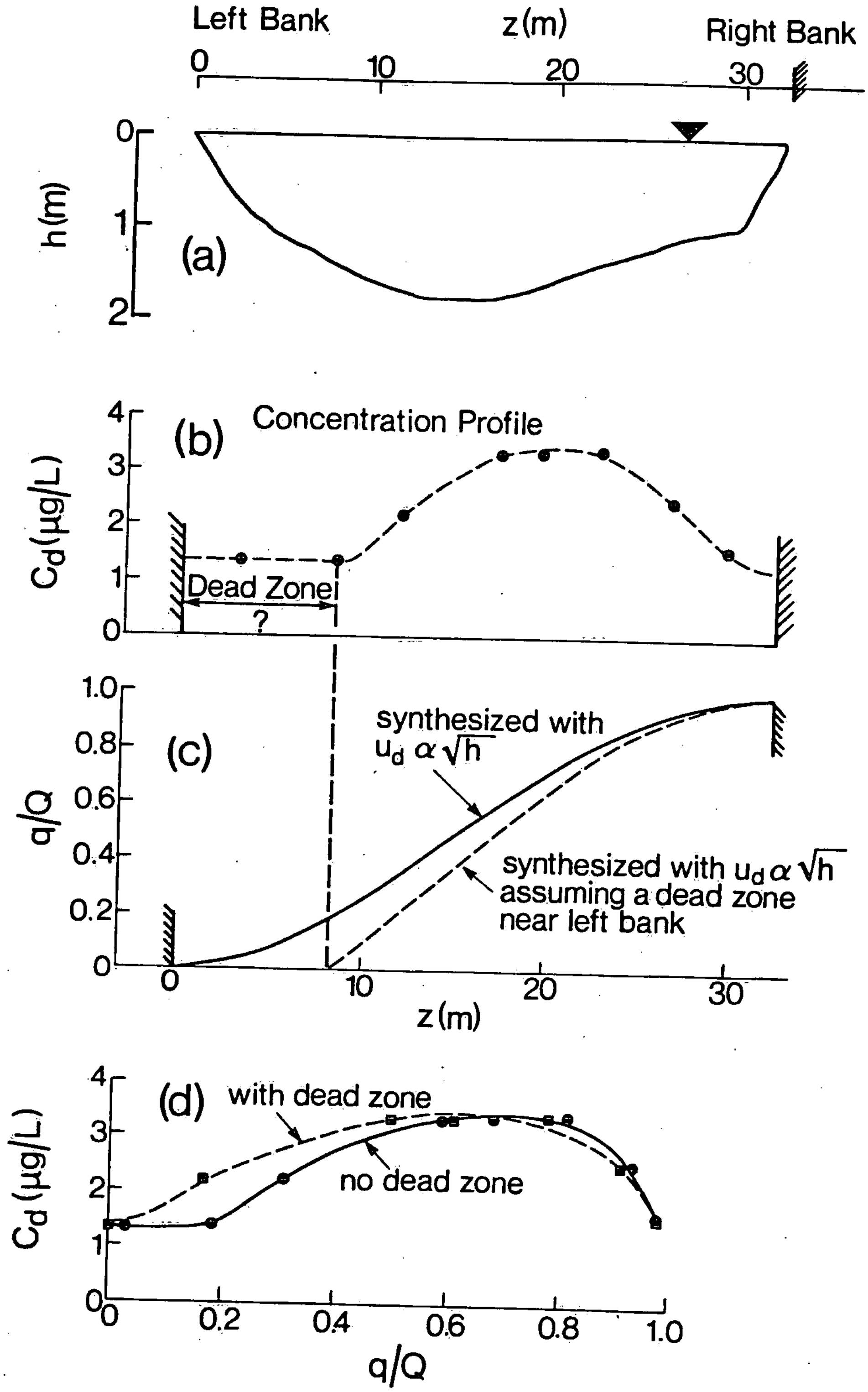


Fig. 9 Illustration of possible dead zone effect; section 3, Test 3 (1 m=3.28 ft.)

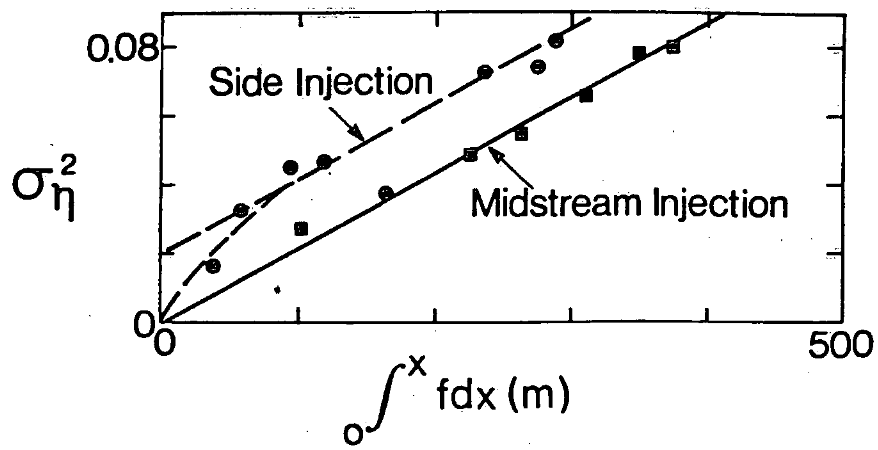


Fig. 10 Plots of variance versus modified distance; Test 4, midstream and side injections (1 m=3.28 ft.)

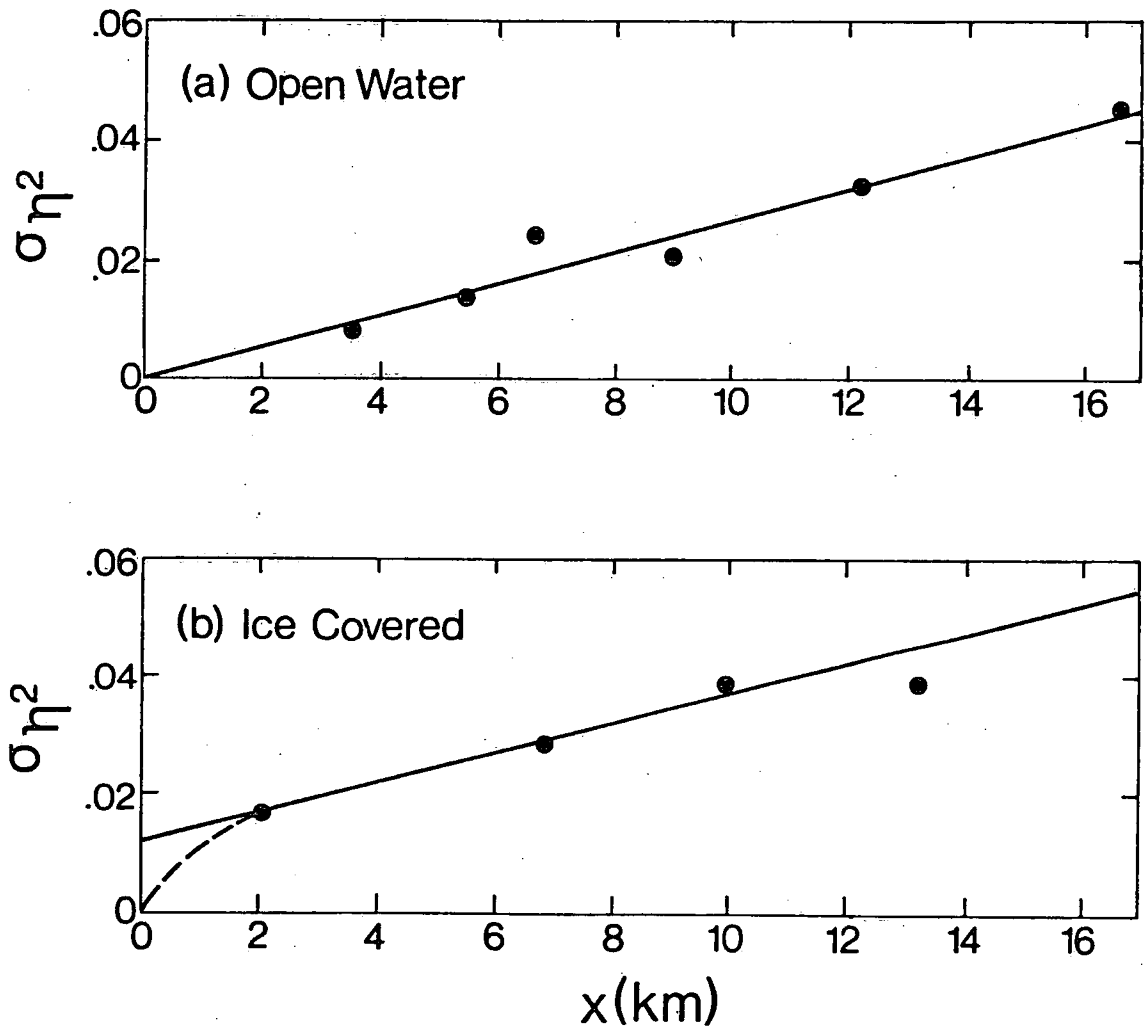


Fig. 11 Plots of variance versus distance; Tests 5 and 6 (1 km=3281 ft.)

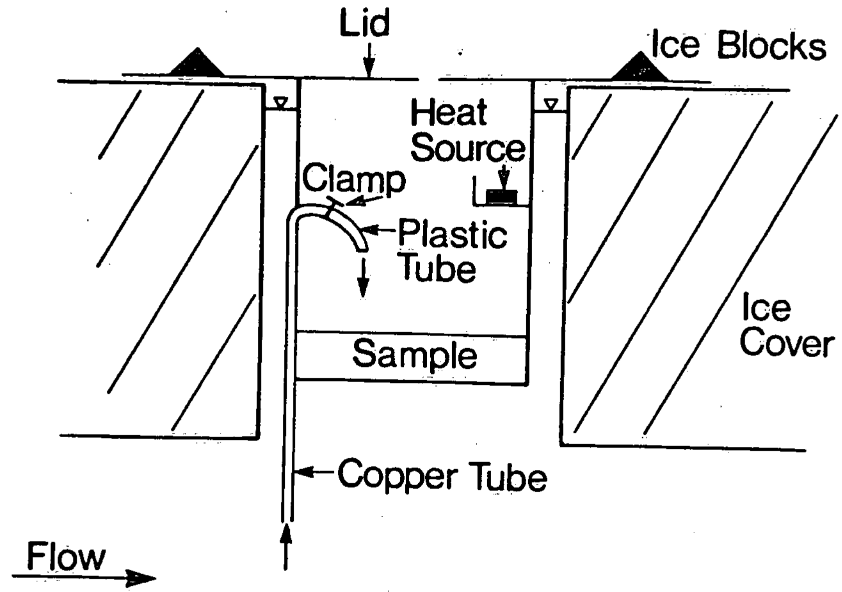


Fig. 12a Schematic illustration of automatic sampling device for measuring dosage; Test 6

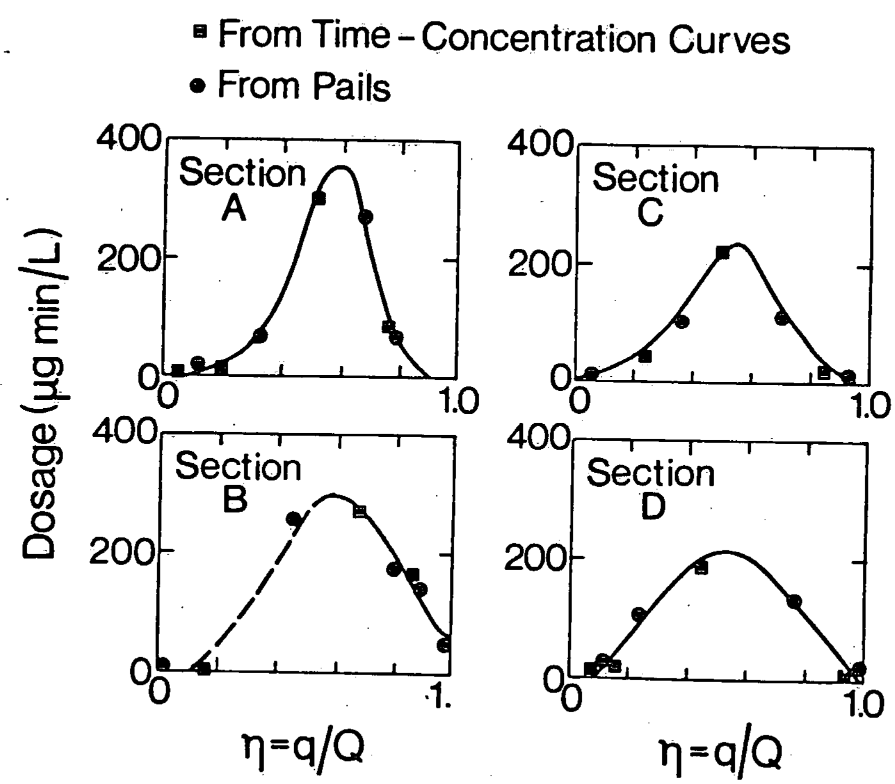


Fig. 12b Comparison between dosage observations obtained by different sampling methods; Test 6