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DISPERSION COEFFICIENT FOR NATURAL STREAMS

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DISPERSION COEFFICIENT FOR NATURAL STREAMS

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ABSTRACT

Results of several field experiments to determine the transverse mixing coefficient are presented. The mixing coefficients are evaluated by comparing observed data with simulated data from a numerical model. Inspection of all available data on the dimensionless mixing coefficient confirms that sinuosity is an important variable.

RÉSUMÉ

Les résultats de plusieurs expériences conduites sur le terrain, pour déterminer le coefficient de mélange transversal, sont présentés. Le coefficient de mélange est évalué par comparaison d'observations avec les résultats d'une simulation numérique. Un examen de toutes les données disponibles, relatives au coefficient de mélange sand dimensions, confirme que la sinuosité est une variable importante.

MANAGEMENT PERSPECTIVE

Water quality in rivers is not uniform. Dissolved substances disperse across a river or channel, reducing the concentrations, as the distance increases downstream from the source. The computation of the concentration depends on knowing the lateral dispersion coefficient. Unfortunately that coefficient varies with the river flow, the river dimensions and its sinuosity. Previously there was no systematic method to discriminate between the influences of the significant variables on the dispersion coefficient. This paper provides a method to select the lateral dispersion coefficient so that water quality concentrations may be accurately estimated for a large range of flow conditions without recourse to an expensive series of field experiments.

T. Milne Dick
Chief, Hydraulics Division
November 4, 1981

PERSPECTIVE DE GESTION

La qualité de l'eau des cours d'eau n'est pas uniforme. Lorsqu'on s'éloigne de la source, vers l'aval, les substances dissoutes se dispersent dans le cours d'eau ou le chenal, ce qui entraîne une diminution de leur concentration. Pour calculer la concentration, on doit connaître le coefficient de dispersion latérale. Malheureusement, ce coefficient dépend du débit, des dimensions et de la sinuosité du cours d'eau. Auparavant, il n'existait pas de méthode systématique pour distinguer les incidences respectives des différentes variables sur le coefficient de dispersion. Ce rapport offre une méthode de sélection du coefficient de dispersion latérale qui permet une estimation précise de la qualité de l'eau, pour une gamme étendue de débits, sans qu'il soit nécessaire de recourir à une série d'expériences coûteuses sur le terrain.

T. Milne Dick
Chef de la Division d'hydraulique
4 novembre 1981

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DISPERSION COEFFICIENT FOR NATURAL STREAMS

By Y. Lam Lau[1]

INTRODUCTION

Recent studies have shown that the transverse mixing of solutes in natural streams can be modelled by using the cumulative discharge approach and writing the governing equations in a general orthogonal coordinate system. This approach takes into account the effects of changes in width and depth as well as channel curvature and has been proven successful using natural stream data (2, 4).

In order to calculate the dispersion of a substance along a reach of a river, one needs information on flow variables such as width and depth, which are relatively easy to obtain, as well as the value of the transverse dispersion coefficient e_z, which can be determined only from field tests using tracers. As tracers tests are not always feasible one often has to choose a value of e_z based on knowledge of the bulk stream variables.

Published data on the transverse dispersion coefficient for natural streams have been summarized by Lau and Krishnappan (2). It was found that the dimensionless dispersion coefficient e_z/U_*H (U_*=shear velocity; H=average depth) was relatively constant for nearly all the streams, independent of width-to-depth ratio and friction factor variations. However, two of the streams had much larger values of e_z/U_*H than the rest. Lau and Krishnappan reasoned that the difference in secondary circulation caused by channel curvature could have been responsible for the increase in e_z/U_*H. They suggested that the

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dimensionless dispersion coefficient should depend on some variable characterizing the stream curvature and used the sinuosity S_n as that variable. It was found that the two reaches concerned indeed have the largest values of sinuosity. However more field data are required to assess whether S_n is the proper variable to use and to allow practicing engineers to select e_z with more confidence.

This article gives the results of measurements of the dispersion coefficient in several river reaches and the comparison with previous data.

DESCRIPTION OF FIELD TESTS

Five stretches of river around southern Ontario were chosen as test sites. The test reaches varied in length from 2 km to 155 m, depending on how rapidly transverse mixing took place. The bulk hydraulic variables are given in Table 1.

Rhodamine B dye was used as the tracer at four of the five sites. A 2 percent solution of Rhodamine B, made neutrally buoyant by the addition of methanol, was injected near one bank at about 100 ml/min using a constant rate diaphram pump. Sampling was done using a Turner model 111 fluorometer. Measurements of concentration, velocity and depth were made at each transect, in the same manner as described in Ref. 2.

For the Grand River site, there is a small tributary with very high concentrations of sulphates, about five times the river background concentration. This tributary discharge was used as a convenient source of tracer and bottle samples were collected at each transect where fluorometer readings would have been taken in the case of a dye test. Sulphate and chloride concentrations as well as conductivity were used in the data analysis.

ANALYSIS OF RESULTS

The dispersion coefficients were obtained by comparing measured and simulated concentration profiles, as described in detail by Lau and Krishnappan (2). Therefore only a brief description is given here.

Simulated concentrations were obtained from numerical solutions of the equation

$$\frac{\partial c}{\partial x} = \frac{1}{Q^2} \frac{\partial}{\partial \eta} \left[uh^2 m_x e_z \frac{\partial c}{\partial \eta} \right] \qquad (1)$$

in which x=longitudinal distance coordinate in an orthogonal, curvilinear system; η=dimensionless cumulative discharge, ratio between the cumulative discharge and the total discharge; c=depth-averaged concentration; Q-total discharge; u=depth-averaged velocity in the x direction; h=local flow depth; m_x=metric coefficient; and e_z=transverse dispersion coefficient.

The measured concentration profile at the first transect was used as input boundary condition and Eq. 1 was solved using a constant value for e_z. The simulated concentration profiles at the various downstream transects were compared with the measured profiles. Different e_z values were tried and the one giving the closest fit was selected as the value for that river reach.

In general the agreement between simulated and measured concentrations is quite satisfactory. Comparisons of two of the reaches are shown in Figs. 1 and 2. For the Grand River site, the sulphate, chloride and conductivity data all gave the same value for e_z.

DIMENSIONLESS DISPERSION COEFFICIENT

The dimensionless dispersion coefficient, e_z/U_*, was computed for each site. These values are listed in Table 2 together with values for the width-to-depth ratio, friction factor and sinuosity. The sinuosity, defined as the ratio between the thalweg length and the down valley distance, was obtained from aerial maps of the test sites. In Fig. 3, e_z/U_*H is plotted against W/H. All the previously published data given in Ref. 2 are also included. The values of S_n are written beside each data point.

Figure 3 confirms that the sinuosity S_n does indeed influence the value of e_z/U_*H more than any other variable. The Grand River site has $S_n=1.07$ and its value of e_z/U_*H is very close to those found for other relatively straight reaches. The other four sites have S_n values varying from 1.20 to 1.37 and their values of e_z/U_*H are all higher. From an inspection of all the data in Fig. 3, it seems that for straight uniform reaches with S_n close to unity, e_z/U_*H should be approximately equal to 0.25. As the sinuosity increases to about 1.4, e_z/U_*H has increased to about 1.0.

None of the tested reaches has sinuosity large enough to compare with the result of Sayre and Yeh (3) which has $S_n=2.1$ and $e_z/U_*H=3.30$. Their test reach consisted mainly of one large, severe bend. In the present tests it was observed that, at the sections with bends, most of the mixing took place at the downstream portion of the bends. If the value of e_z was allowed to vary from transect to transect in the simulations, its value for the downstream end of the bend was sometimes more than twice the value of the reach average. This kind of variation was also observed by Cheng (1) and Sayre and Yeh (3). Therefore, with large sinuosities, values of e_z/U_*H of 3.0 or higher are quite possible.

SUMMARY

Some new data from field measurements of transverse mixing have been reported. These results reinforce the suggestion that stream curvature plays an important role in determining the value of the dimensionless transverse dispersion coefficient. For constant values of sinuosity, e_z/U_*H can be considered constant, irrespective of width to depth ratio or friction factor. Some field data from reaches with sinuosity larger than 1.5 are still required to better define the value of e_z/U_*H at the very large sinuosities.

ACKNOWLEDGMENTS

The author wishes to thank B. G. Krishnappan and F. Dunnett who took part in all the field tests. F. Dunnett also carried out all the reduction of the raw data.

APPENDIX I. - REFERENCES

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3. Sayre, W. W., and Yeh, T. P., "Transverse Mixing Characteristics of the Missouri River Downstream from the Cooper Nuclear Station," IIHR Report 145, Iowa Institute of Hydraulics Research, University of Iowa, Iowa City, Iowa, 1973.

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APPENDIX II. - NOTATION

The following symbols are used in this paper;

c = depth-averaged concentration;
e_z = transverse dispersion coefficient;
f = friction factor;
H = average flow depth;
h = local flow depth;
m_x = metric coefficient;
Q = total discharge;
S_n = sinuosity;
U_* = mean shear velocity;
u = depth-averaged local velocity;
W = average width;
x = longitudinal coordinate;
η = dimensionless cumulative discharge.

TABLE 1. - Summary of Hydraulic Data

Test Site (1)	Reach length in meters (2)	Average width in meters (3)	Average depth, H, in meters (4)	Average velocity, U, in meters per second (5)	Channel Slope S (6)	Shear velocity, U_*, in meters per second (7)
Grand River below West Montrose	2000	48.0	0.41	0.40	12.0×10^{-4}	0.069
Big Creek above Walsingham	150	11.5	0.69	0.36	4.6×10^{-4}	0.056
Nith River above Canning	650	36.0	0.54	0.16	9.6×10^{-4}	0.071
Nith River near Philipsburg	250	15.0	0.48	0.04	0.2×10^{-4}	0.010
Nith River near Plattsville	155	18.0	0.55	0.08	2.5×10^{-4}	0.037

TABLE 2. - Data for Transverse Dispersion Coefficient

Test Site (1)	Dispersion Coefficient, e_z, in meters squared per second (2)	$\frac{W}{H}$ (3)	Friction Factor, f (4)	Sinuosity S_n (5)	$\frac{e_z}{U_* H}$ (6)
Grand River below West Montrose	0.007	117.0	0.24	1.07	0.25
Big Creek above Walsingham	0.030	16.7	0.19	1.25	0.78
Nith River near Canning	0.028	66.7	1.59	1.20	0.73
Nith River near Philipsburg	0.004	31.3	0.47	1.30	0.86
Nith River near Plattsville	0.020	32.7	1.68	1.37	0.98

LIST OF FIGURE CAPTIONS

Fig. 1 Comparison of measured and simulated sulphate concentration profiles - Grand River below West Montrose

Fig. 2 Comparison of measured and simulated dye concentration profiles - Nith River near Philipsburg

Fig. 3 Natural stream data of e_z/U_*H against W/H. The value of sinuosity is written beside each data point

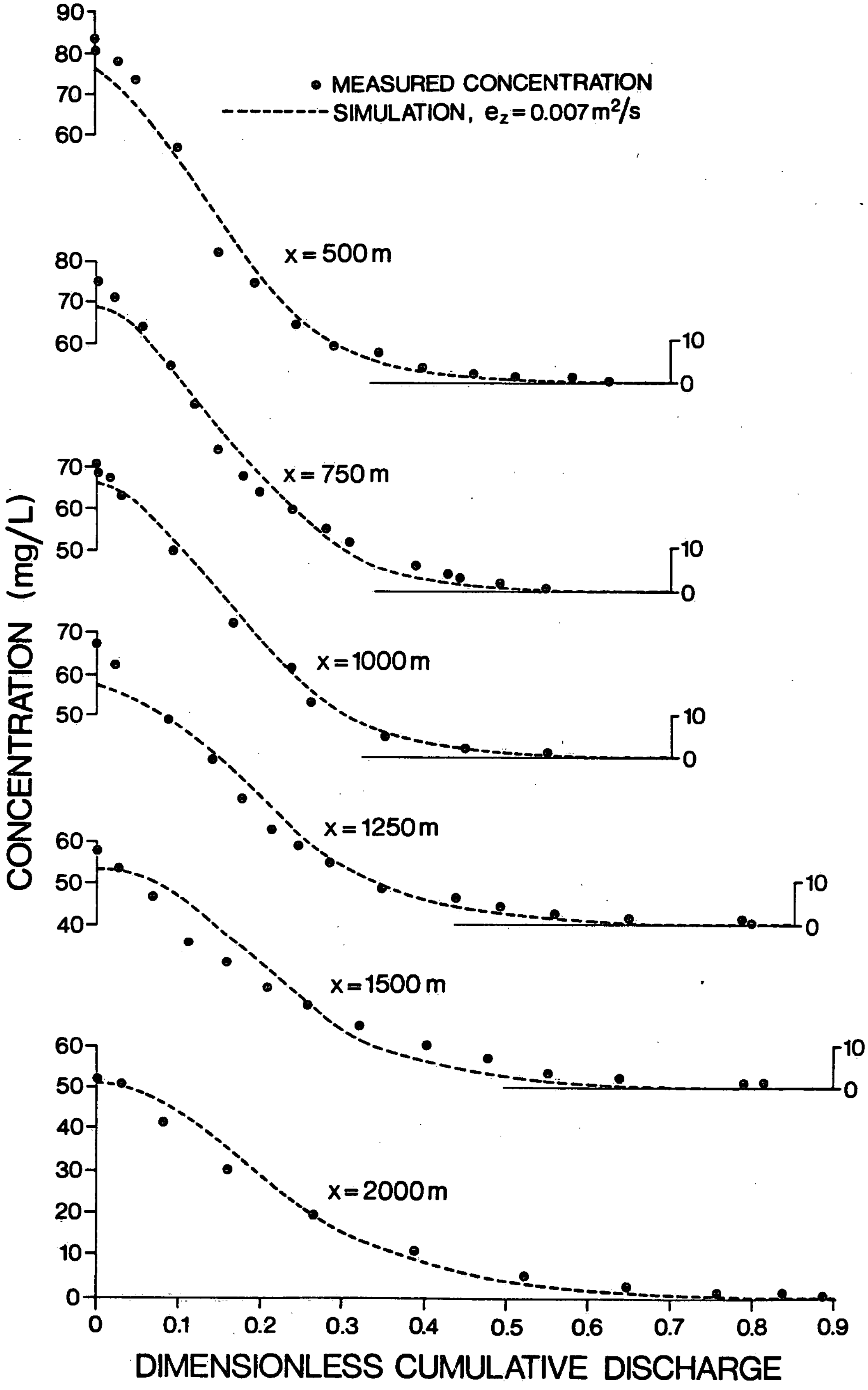

Fig. 1 Comparison of measured and simulated sulphate concentration profiles - Grand River below West Montrose

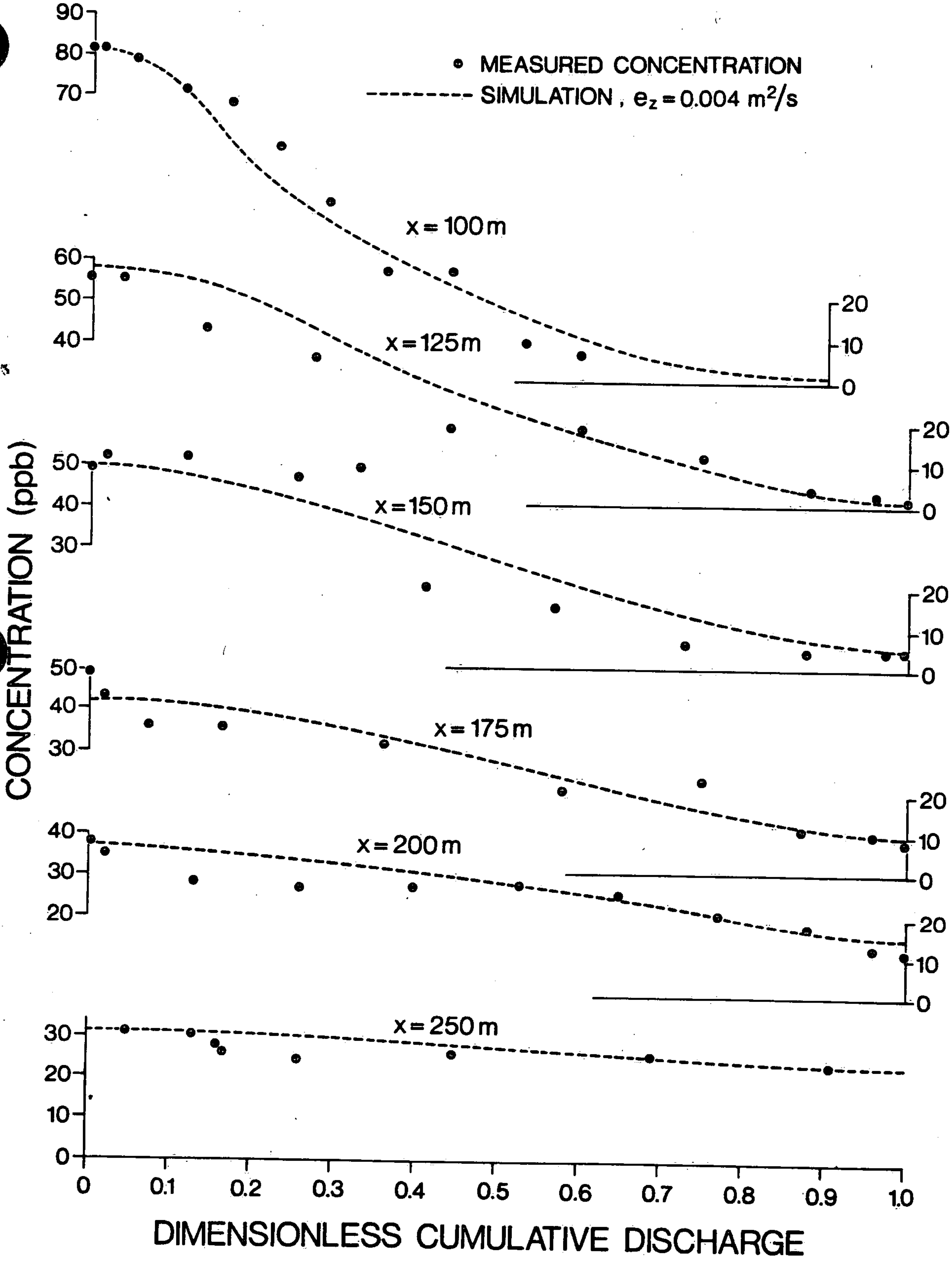

Fig. 2 Comparison of measured and simulated dye concentration profiles - Nith River near Philipsburg

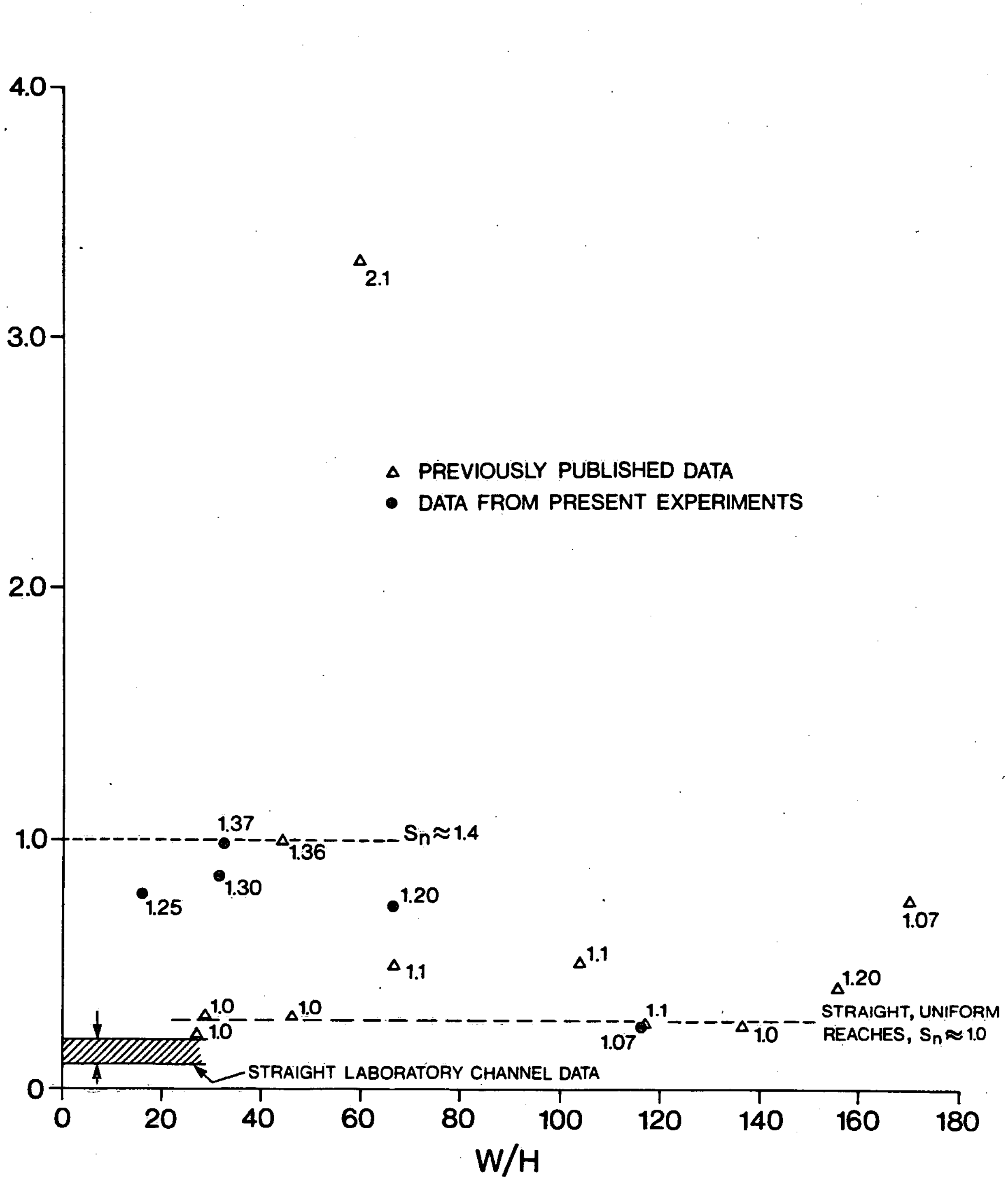

Fig. 3 Natural stream data of e_z/U_*H against W/H. The value of sinuosity is written beside each data point.