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THEORETICAL INVESTIGATION ON MODEL SIMULATION CRITERIA OF BUBBLE PLUMES

By

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TABLE OF CONTENTS

ABSTRACT: A theoretical investigation is made to derive the parameters that govern the dynamic similarity between prototype and model bubble plumes. It is shown in the study that, for a complete similarity between the prototype and the model, the buoyancy flux from the source, the size of the bubbles and the expansion of the gas bubbles all have to be simulated. This is translated into the observation of the Froude law, the Weber law and the expansion law in the model study. The expansion law requires that, for a reduced model, the study be conducted under partial vacuum and, for an enlarged model, the study be conducted under pressure.

The physical properties of common fluids impose severe constraints on the model study. If the Weber law is to be satisfied, the smallest model permissible will only be half the prototype size. To conduct experiments under partial vacuum, the boiling of the model liquid has to be contended with. For practical reasons, a certain degree of violation of the similarity laws seems to be unavoidable. The theoretical investigation calls for systematic experiments to examine the quantitative effect of the different similiarity parameters and the degree of violation of the similarity laws permissible in solving practical problems.

RÉSUMÉ: L'étude théorique a pour but de calculer les paramètres qui régissent la similitude dynamique entre les panaches de bulles et leurs modèles. Elle révèle que, pour obtenir une similitude complète entre un panache et son modèle, il faut simuler le flux hydrostatique émis par la source, la taille des bulles et l'expansion des bulles de gaz. Pour ce faire, il faut respecter dans le modèle, la loi de Froude, la loi de Weber et la loi d'expansion d'un gaz. Cette dernière stipule que l'étude sur modèle réduit doit être menée sous vide partiel, et celle du panache réel, sous pression.

Les propriétés physiques des fluides courants imposent de lourdes contraintes dans l'étude sur modèle. La loi de Weber requiert que le modèle ait des dimensions d'au moins la moitié de celles d'un vrai panache. Pour faire des expériences sous vide partiel, il faut limiter l'ébullition du liquide dans le modèle. Dans la pratique, il semble qu'il faille violer les lois de similitude jusqu'à un certain point. L'étude théorique doit reposer sur des expériences systématiques visant à déterminer l'effet quantitatif des différents paramètres de similitude et le degré admissible de violation des lois de similitude pour résoudre des problèmes pratiques.

MANAGEMENT PERSPECTIVE

Gases emitted at the bottom of a body of water, cause a vertical current to rise. This may also occur as a release of natural gas and oil in a blowout. Theories for currents and mixing caused by gas plumes are mostly based on model studies.

This report indicates that many model studies may not have taken into account all the significant fluid dynamic scale effects and, consequently, theories used to predict the behaviour of full-sized plumes may be incorrect.

Additional studies are required.

T. Milne Dick

Chief, Hydraulics Division

January 6, 1983

PERSPECTIVE DE GESTION

Les gaz émis au fond d'un plan d'eau engendrent un courant vertical ascendant. Cela peut aussi se produire lors d'une éruption de gaz naturel et de pétrole. La plupart des théories des courants et des mélanges causés par des panaches gazeux sont issues d'études sur modèle.

Le présent rapport montre qu'il est possible que de nombreuses études sur modèle n'aient pas tenu compte de tous les grand effets dynamiques d'échelle des fluides et, partant, que les théories utilisées pour prévoir le comportement des panaches soient erronées.

D'autres études s'imposent.

T. Milne Dick

Chef, Division de l'hydraulique

Le 6 janvier 1983

THEORETICAL INVESTIGATION ON MODEL SIMULATION CRITERIA OF BUBBLE PLUMES

By Gee Tsang[1]

INTRODUCTION AND PRESENT STATE OF THE KNOWLEDGE

When one fluid is released into another fluid of different density, a plume is formed. Depending on whether the released fluid is lighter or heavier than the ambient fluid, the plume can be either a rising plume or a sinking plume. If the released fluid is miscible with the ambient fluid, the plume is called a simple plume, otherwise, the plume is called a bubble plume.

Simple plumes have been experimentally and theoretically studied by Rouse et al (6) and Morton et al (5), among others. It was found that simple plumes are self-preserving, or that the velocity profiles and buoyancy profiles at different distances from the source are similar. Today, scientific knowledge of simple plumes has been sufficiently advanced to solve most practical problems.

For bubble plumes, depending on whether the buoyancy flux is small or large, the plume is known either as a weak bubble plume or a strong bubble plume. Weak bubble plumes, in the form of air bubbles released

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in water, have been studied by Kobus (3). In his treatment of the problem, Kobus adopted the same theoretical approach as that used by previous researchers in studying simple plumes, but took into account also the relative velocity of the air bubbles in the plume and the expansion of the air bubbles. Wilkinson (9) compared the experimental results by Kobus (3) and Rouse et al (6) and concluded that, for weak bubble plumes, although the simple plume theory may be approximately applied, the vital proportionality constant specifying the distributions of density, velocity and momentum, which for simple plumes are equal, are now different from each other and from experiment to experiment. Thus, for weak bubble plumes of different source parameters, the vital proportionality constants have to be individually obtained, presumably experimentally.

Strong bubble plumes, also in the form of air bubbles in water, have been studied by Bulson (1). A comparison of his experimental findings with those by Rouse et al by Wilkinson showed that there is little agreement between simple plume prediction and experimental results of strong bubble plumes. As of today, a working theoretical model for strong bubble plumes has not yet been developed.

The classification of bubble plumes into strong ones and weak ones was proposed by Wilkinson. In his study of two-dimensional plumes, Wilkinson found that the behaviour of air bubble plumes in water is greatly affected by the inertial force to surface tension force ratio, or the Weber number of the plume, which can be expressed as

$$W = \frac{\rho\beta_0^{4/3}}{\sigma g} \tag{1}$$

where ρ is the density of the ambient fluid, σ is the interfacial tension between the released fluid and the ambient fluid, g is the gravitational acceleration and $\rho\beta_0$ gives the buoyancy flux of the plume per unit length of the line source. Wilkinson called the plumes with large Weber numbers strong bubble plumes and plumes with small Weber numbers weak bubble plumes. After comparing the experimental results of 1, 3 and 6, Wilkinson concluded that the larger the Weber number, the farther away will a bubble plume depart from a simple plume. He reasoned that, at low Weber numbers, the surface tension will cause the air stream to collapse into small bubbles. The small bubbles are then dispersed by turbulence within the plume in a manner similar to that which occurred in a simple plume of miscible buoyancy. At high Weber numbers, the surface tension effects are dominated by buoyancy and the larger bubbles do not disperse but rather rise as an air-water core imparting relatively little momentum to the surrounding water.

Wilkinson's reasoning, however, was not supported by experimental findings by Topham (8). In his work on air bubbles released from a submerged orifice, Topham found that, at low flow rates (i.e., at low Weber numbers), single bubbles were formed and released by the orifice when the buoyant force became equal to the holding force of the surface tension at the orifice. The ratio of the bubble radius r_b to the radius of the orifice r_0 at the moment of bubble release is given by

$$\frac{r_b}{r_o} = \left[\frac{3\sigma}{2r_o^2 \rho g} \right]^{1/3} \qquad (2)$$

At high flow rates (i.e., high Weber numbers), the bubble shape was modified. The emergant bubbles had a fluted appearance and bursted into small bubbles within a short distance. The higher the flow rate, the more violent was the bursting.

One may also question the validity of Wilkinson's Weber number criterion by looking at the case of an oil plume released in water. If dispersant is slowly added to the oil, then the interfacial tension between the oil and water will be gradually reduced and, according to Eq. 1, the plume will depart further and further away from a simple plume. However, this is impossible because, when the interfacial tension is reduced to being negligible, the oil will become miscible with water and the plume will become a simple plume.

To sum up, one sees that, until today, research results from past works on bubble plumes are still inconclusive and relationships governing the behaviour of bubble plumes have still not been established. On the other hand, problems involving bubble plumes are many and happen in different fields. The rising of an air bubble plume from a de-icing air bubbler, the blowout of a gas or oil well under water and the passing of compressed oxygen through molten steel for carbon and impurity control are but a few of the practical examples. Whether the governing relationships have been found or not, one still has to know the amount of heat brought up by the air bubble plume, the size of the gas or oil plume when it reaches the water surface so effective containment and combustion can be prepared, and the interaction between oxygen bubbles and the surrounding molten steel. In the absence of established relationships, to study the plumes in-situ or to study their simulated models in the laboratory offers a practical alternative.

Because a model study enables one to study the behaviour of a plume under systematically controlled conditions in addition to its economical advantage, it should be a natural choice in studying plume problems. However, as of today, guidance in properly conducting bubble plume model studies has not yet been developed. There were enough examples showing that bubble plumes were simulated by simply releasing the buoyant fluid under ordinary conditions into another fluid. In this paper, a theoretical investigation will be made to identify the important parameters that govern the proper simulation of bubble plumes. Based on the theoretical work shown here and expected future experimental work, a well laid groundwork can be expected to guide future model study of bubble plumes.

Because of the compressibility of gas, in the investigation here, attention will be paid to gas in liquid plumes. The conclusions drawn, however, are applicable to liquid in liquid plumes also if, for the latter, the plume fluid is considered as having a negligible compressibility. Although the present study will be limited to a two-fluid system, a natural extension of the present work is to a three-fluid system, as exemplified by an oil/gas plume following a subsea blowout.

PARAMETERS GOVERNING MODEL SIMULATION

Pressure Simulation . - For a gas in liquid plume, as the gas bubbles rise, they will expand as a result of reducing ambient pressure. In the case of a gas in water plume, if the source is at a depth of D (metres), the gas bubbles will expand to approximately D/10 times its initial volume when reaching the surface. Therefore, in model study of bubble plumes, unless the water depth involved is small, the expansion of the gas bubbles has to be simulated. However, in a recent workshop on subsea oil well blowouts, none of the reported model studies (2, 4, 7) had taken the expansion of the gas bubbles into consideration.

For a gas bubble, as it rises, it obeys the gas law

$$pv = RT \tag{3}$$

where p is the pressure, v is the specific volume, T is the absolute temperature and R is the universal gas constant. As a bubble plume rises, it entrains the ambient fluid and sets up a mixing motion. As a consequence of this mixing motion, after a certain initial period, the ambient fluid should come to a rather uniform temperature. The expansion of the gas bubbles, therefore, may be approximately considered to be isothermal. However, to relax the condition to a more general situation, it is assumed that the expansion of the gas bubbles follows the polytropic law

$$pv^n = \text{Const.} \tag{4}$$

where n is the expansion index and can assume different values.

The pressure on a gas bubble at a depth d below the free surface is the sum of the hydrostatic pressure at that depth plus the pressure produced by the surface tension of the bubble. In mathematical terms, one writes

$$p = p_{at} + \rho gd + p_{st} \tag{5}$$

where p_{at} is the pressure at the free surface and p_{st} is the pressure produced by the surface tension. From equating the surface tension of a gas bubble and the internal pressure caused by it, one obtains

$$\sigma 2\pi R_g = p_{st}\, \pi R_g^2 \tag{6}$$

where σ is the surface tension and R_g is the radius of the gas bubble. From Eqs. 5 and 6, one has

$$p = p_{at} + \rho gd + \frac{2\sigma}{R_g} \tag{7}$$

From the above equation, one sees that if

$$R_g = \frac{2\sigma}{\rho g} \cdot \frac{1}{d} \tag{8}$$

then the pressure produced by surface tension will be comparable to that produced by the hydrostatic head. However, for most common liquids, the parameter $2\sigma/\rho g$ is quite small. For instance, for water, $2\sigma/\rho g$ is approximately equal to $15 \times 10^{-6} m^2$. Therefore, even for a depth of only 1 m, the pressure produced by surface tension will be insignificant compared to the hydrostatic pressure unless the diameter of the gas bubble is of the order of 0.03 mm or less. However, for gas bubbles this small, their motion is subject to the Stokes' law with a high drag coefficient. So they will more drift with the flow than produce the upward current. Thus, when dealing with the buoyant motion of a bubble plume, the pressure on the gas plumes may be simply considered to be hydrostatic.

It should be mentioned that strictly speaking the ambient fluid density ρ in the above equation should be a function of temperature and pressure. However, it is well known that, for most common liquids, their density is not sensitive to temperature and pressure changes. Therefore, for evaluating the ambient pressure using Eq. 5, the density of the ambient fluid may be considered to be constant.

The substitution of Eq. 7, after neglecting the surface tension force term, into Eq. 4 leads to

$$(p_{at} + \rho g d)\, v_d^n = \text{Const.} \tag{9}$$

where v_d is the specific volume of the gas at depth d. At the depth of the plume source, Eq. 9 becomes

$$(P_{at} + \rho g D)\, v_D^n = \text{Const.} \tag{10}$$

where v_D is the specific volume of the gas at the source's depth D. From Eqs. 9 and 10, one has

$$\frac{v_d}{V_D} = \left[\frac{p_{at} + \rho g D}{p_{at} + \rho g d} \right]^{1/n} \tag{11}$$

The geometric similarity between the model and prototype plumes requires that:

If

$$\left(\frac{d}{D} \right)_m = \left(\frac{d}{D} \right)_p \tag{12}$$

then

$$\left[\frac{v_d}{v_D} \right]_m = \left[\frac{v_d}{v_D} \right]_p \tag{13}$$

where the subscripts m and p indicate the model and the prototype respectively. From Eqs. 11 and 13 and assuming the same expansion index for both the prototype and the model, one obtains

$$\left[\frac{p_{at} + \rho g D}{p_{at} + \rho g d} \right]_m = \left[\frac{p_{at} + \rho g D}{p_{at} + \rho g d} \right]_p \tag{14}$$

By dividing the numerators and the denominators by ρgD, the above equation becomes

$$\left[\frac{p_{at}/\rho g D + 1}{p_{at}/\rho g D + d/D} \right]_m = \left[\frac{p_{at}/\rho g D + 1}{p_{at}/\rho g D + d/D} \right]_p \tag{15}$$

Because the d/D ratios between the model and the prototype are equal by virtue of Eq. 12,, the satisfaction of the above equation requires that

$$\left[\frac{p_{at}}{\rho g D} \right]_m = \left[\frac{p_{at}}{\rho g D} \right]_p \tag{16}$$

$(p_{at}/\rho gD)$ may be called the expansion parameter and denoted as E. From the above equation, one sees that for a model study of bubble plumes, if the expansion of the gas bubbles is to be simulated, the expansion parameter for both the model and the prototype should be the same.

Equation 16 may be rewritten as

$$\frac{p_{at_m}}{p_{at_p}} = \frac{\rho_m}{\rho_p} \frac{D_m}{D_p} \tag{17}$$

The above equation states that for geometric similarity between the model and prototype plumes, the surface pressure scale should be the product of the density scale and the length scale. If the prototype is exposed to the atmosphere, the model and prototype ambient fluids are the same and the model is smaller than the prototype, the above equation states that the model surface pressure should be smaller than the atmospheric pressure, or that the model study should be conducted under vacuum. Conversely, if the model is larger than the prototype, the model study should be conducted under pressure.

Simulation of Bubble Sizes and Buoyancy Flux . - The parameter controlling the behaviour of a bubble plume may also be studied by dimensional analysis. For a gas in liquid plume as shown in Fig. 1, the characteristic radius of the plume at a depth d and the local velocity at a point r from the axis of the plume and at the same depth are functions of the following form:

$$R_d = f\,(Q_o, D, \rho, d, \mu, g, \sigma, p_{at}) \qquad (18)$$

and

$$w = F\,(Q_o, D, \rho, d, \mu, g, \sigma, p_{at}, r) \qquad (19)$$

where Q_o is the rate of discharge of the bubble gas at the source, μ is the viscosity of the ambient fluid, and the meaning of the other symbols have been defined earlier. In the above equations, the temperature of the ambient fluid, as a function of depth, is not included because, as it has been said earlier, the mixing action of the plume will lead to a uniform distribution of the ambient temperature. The orifice size of the plume source is also not treated as a parameter because both Bulson and Kobus have shown from their experiments on two-dimensional plumes that the spacing and size of the air nozzles had little effect on the structure of the plume.

From Eqs. 18 and 19, by dimensional analysis, one obtains

$$\frac{R_d}{D} = f_1\left(\frac{d}{D}, \frac{\rho Q_o}{D\mu}, \frac{Q_o^2}{gD^5}, \frac{\rho Q_o{}^2}{D^3\sigma}, \frac{p_{at}}{\rho gD}\right) \tag{20}$$

and

$$\frac{w^2}{Dg} = F_1\left(\frac{d}{D}, \frac{\rho Q_o}{D\mu}, \frac{Q_o^2}{gD^5}, \frac{\rho Q_o^2}{D^3\sigma}, \frac{r}{D}, \frac{p_{at}}{\rho gD}\right) \tag{21}$$

respectively.

The parameter $p_{at}/\rho gD$ in the above equation, which governs the expansion of the gas bubbles, has been derived from physical principles in the last section. Here, it is reaffirmed from dimensional considerations.

For parameter $\rho Q_o/D\mu$, it is a form of the Reynolds number for the plume. The Reynolds number of a flow describes the turbulence level of the flow and, in the case of a rising plume, the entrainment of the ambient fluid by turbulence. $\rho Q_0/D\mu$, therefore affects the turbulent entrainment of ambient fluid into the plume. However, it is learnt from fluid mechanics that, if a flow is sufficiently turbulent, then a change of the Reynolds number will cause little change to the turbulent condition of the flow. Therefore, if one limits the present discussion to turbulent bubble plumes, the actual value of the Reynolds number will have little effect on the turbulence and turbulent entrainment of the plume and may be dropped from the above equations.

Q^2/gD^5 in Eqs 20 and 21 is a form of the Froude number for the plume and may be denoted as F. This parameter may also be written as $((\rho - \rho_g)/\rho)^{-1}.(Q^2{}_0/gD^5)$, ρ_g being the density of the gas and is negligible. When in such a form, the Froude number is known as the densimetric Froude number. In the study of plume motions, the densimetric Froude number describes the buoyant force that sets up the rising motion of the plume.

$\rho Q^2{}_0/D^3\sigma$ is a form of the Weber number and may be denoted as W. It indicates the ratio between the inertial force and the interfacial tension between the gas and the ambient fluid. Because the interfacial tension between the gas and the ambient fluid controls the size of the gas bubble and it is common knowldge that smaller bubbles rise slower than large bubbles, the Weber number therefore contributes to the control of the upward motion of the plume through its control on the bubble sizes. According to Wilkinson, the bubble sizes also affect the entrainment of the ambient fluid, the larger the bubble, the less will the ambient fluid be entrained. The Weber number, therefore, also controls the upward motion and the growth of the plume through its effect on ambient entrainment.

Following the above discussion and Eqs. 20 and 21, one sees that, for model simulation of a bubble plume, in addition to Eq. 16, one also need to have

$$\left[\frac{Q_o^2}{gD^5} \right]_m = \left[\frac{Q_o^2}{gD^5} \right]_p \tag{22}$$

and

$$\left[\frac{\rho Q_o^2}{\sigma D^3} \right]_m = \left[\frac{\rho Q_o^2}{\sigma D^3} \right]_p \tag{23}$$

While Eq. 22 guarantees that the rate of buoyancy inflow is similar, Eq. 23 guarantees the similarity of the bubble sizes.

Equations 16, 22 and 23, respectively, may be written as:

$$R_p = \frac{p_{atm}}{p_{atp}} = \frac{\rho_m}{\rho_p} \frac{D_m}{D_p} = R_\rho R_L \tag{24}$$

$$R_{Q_o} = \frac{Q_{om}}{Q_{op}} = R_L^{5/2} \tag{25}$$

and

$$R_{\sigma} = \frac{\sigma_m}{\sigma_p} = R_L^2 \; R_{\rho} \qquad (26)$$

where R_p, R_{ρ}, R_L, R_{Q_o} and R_{σ} are the surface pressure scale, the density scale, the length scale, the discharge scale and the interfacial tension scale respectively. For a simulated study of bubble plumes, the above scales relationship have to be observed.

PHYSICAL CONSTRAINTS IN BUBBLE PLUME MODEL STUDY

According to the last section, one sees that, in order to obtain a complete dynamic similarity between the model and the prototype, the expansion law, the Froude law and the Weber law, or Eqs. 24, 25 and 26 have to be simultaneously observed. While the observation of the Froude law, or Eq. 25, is relatively easy, the observation of Eqs. 24 and 26 are subject to physical constraints.

The difficulty in observing the expansion law, or Eq. 24 comes from the boiling of the model ambient fluid. It has been mentioned earlier that, for a reduced scale model, the atmospheric pressure in the model has to be less than the prototype atmospheric pressure, or that the model study has to be conducted under vacuum, unless the model fluid is much heavier than the prototype ambient fluid. For most practical cases, water will be the ambient fluid for both the prototype and the model, so the conduction of the model experiment under vacuum is unavoidable. It is known that the boiling point of water will decrease if the atmospheric pressure is reduced. At a low temperature of 15°C, say, water will boil at an atmospheric pressure of 0.017 at. press. To ensure that water will not boil at this temperature, according to Eq. 24, the smallest model scale permissible will be about 1/60. For a high summer temperature of 30°C, it can be shown that the smallest scale can only be 1/18.

The above limitation on the scale of the model, while undesirable, appears to be bearable. The greater constraint on the length scale of the model is imposed by the Weber law. It is known that, for most common fluids, their interfacial tension with air, or their surface tension, falls within the narrow range of 10-100 dynes/cm. Although the addition of surfactants to the fluids may reduce the surface tension somehow, it will not reduce it to less than the lower limit of the above range. Thus, from the above, one sees that the smallest surface tension scale R_σ obtainable can only be about 1/10. It follows from Eq. 26, that, unless the model ambient fluid is much heavier than the prototype ambient fluid, the smallest length scale of the model obtainable can only be about 1/3.5. For most practical model studies, water will be the ambient fluid for both the model and the prototype. Assuming that a surfactant can be found which can reduce the surface tension of water from its ordinary value of 72-73 dynes/cm to a realistic value of 20 dynes/cm, then the smallest model scale obtainable will only be 1/2. In other words, the model size can only be half that of the prototype.

CONCLUSIONS AND DISCUSSIONS

It is seen from the preceding sections that, to have a complete dynamic similarity between prototype and model bubble plumes, the expansion of the gas bubbles, the flux of buoyancy from the source and the size of the bubbles all have to be simulated. In the past, to the author's knowledge, the observation of the expansion law and the observation of the Weber law have not been reported in the published literature. It will be imperative, therefore, to study the quantitative effect of these two parameters on the behaviour of a bubble plume, especially a strong bubble plume, because of its practical importance and because, intuitively, it should be more sensitive to the influences of bubble size and bubble expansion.

Because of the physical properties of common fluids, the observation of the expansion law and the Weber law, especially the latter, are subject to severe restrictions. To satisfy the Weber law, the smallest model permissible, for all practical reasons, is simply too large. It appears that from a practical point of view, the Weber law, and possibly the expansion law may have to be violated to have an economical and manageable model. The question then becomes what is the degree of violation permissible, or whether the parameter can be violated at all. If it cannot, then the whole idea of model study of bubble plumes will be futile. Only systematic laboratory experiments can answer the above questions.

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APPENDIX II. - NOTATION

The following symbols are used in this paper:

d = depth;

D = depth of plume source below free surface;

g = gravitational acceleration;

n = polytropic gas expansion index;

p = pressure;

p_{at} = pressure at free surface of ambient fluid;

p_{st} = pressure produced by surface tension;

Q_o = rate of discharge of plume fluid at source;

r = distance from axis of plume;

r_b = radius of bubble at moment of release from orifice;

r_o = radius of orifice;

R = universal gas constant;

R_d = characteristic radius of plume at depth d;

R_g = radius of gas bubble;

R_L = length scale

R_p = surface pressure scale;

R_{Q_o} = discharge scale;

R_ρ = density scale;

R_σ = interfacial tension scale

T = absolute temperature;

v = specific volume;

V_d = specifc volume at depth d;

r = distance from axis of plume;

r_b = radius of bubble at moment of release from orifice;

r_o = radius of orifice;

R = universal gas constant;

R_d = characteristic radius of plume at depth d;

R_g = radius of gas bubble;

R_L = length scale

R_p = surface pressure scale;

R_{Q_o} = discharge scale;

R_ρ = density scale;

R_σ = interfacial tension scale

T = absolute temperature;

v = specific volume;

V_d = specifc volume at depth d;

V_D = specific volume at depth D;

w = local velocity of plume at r from axis at depth d;

W = Weber number;

μ = viscosity of ambient fluid;

ρ_g = density of plume gas;

σ = interfacial tension between plume and ambient fluids;

$\sigma\beta_o$ = buoyancy flux of plume per unit length of line source;

Subscripts m and p - denote the model and the prototype respectively.

FIGURE

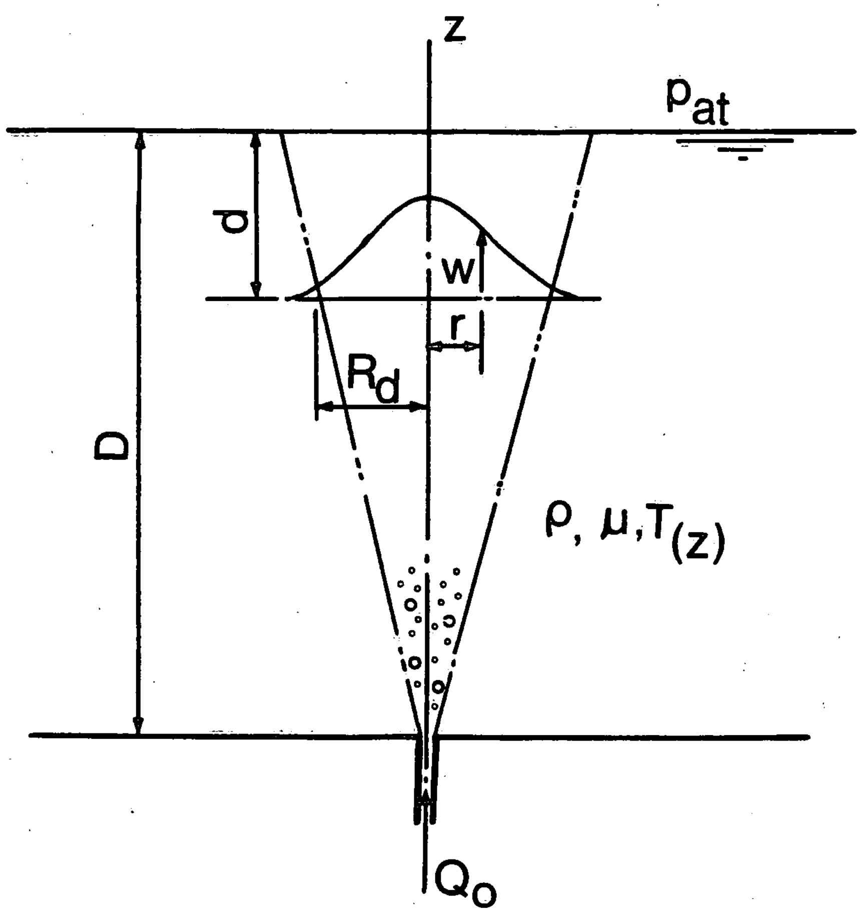

Fig.1 Definition Diagram

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