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Stochastic Modelling of Hydrometeorologic Time Series From the Arctic

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STOCHASTIC MODELLING OF HYDROMETEOROLOGIC TIME SERIES FROM THE ARCTIC

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Summary

In response to the increasing pace of development in the Canadian North, there is an intensified need for flexible hydrologic models to be used for engineering design, water resources planning, environmental impact assessment, and other applications. Consideration must be given to a number of special problems such as missing data points, shortness of hydrologic records, and the physical characteristics of the unique hydrologic regimes found in the Arctic. This paper is concerned with the examination of a broad family of stochastic models to be used for application to monthly hydrometeorologic time series from the Northwest Territories of Canada. Univariate models are fitted to hydrometric data and transfer function-noise models are developed to link hydrometric and meteorologic time series. Thus, the relatively long meteorologic record can be used to "extend" the shorter hydrometric record, and more precise estimates may be obtained for missing data points. Values for missing data points are estimated using an intervention analysis approach. Various other applications of the models are developed, and the limitations of the models and the available data are pointed out.

Conclusions

Box-Jenkins models and other closely related models can be employed to successfully model monthly hydrometeorologic data from the Canadian Arctic. These models have been shown to be useful in the following areas:

- summarizing and describing data sets
- filling in missing data points
- "extension" of hydrometric records
- intervention analysis and environmental impact assessment
- simulation and engineering design

The application of intervention models to the assessment of the impact of both human and natural interventions on the environment should be of particular interest to Environment Canada as man's activities in the Arctic increasingly alter the natural environment. Because intervention analysis allows the calculation of confidence limits for the change in the mean level of a time series relevant conclusions can be drawn from the analysis.

Of the hydrometric stations in the North West Territories, only about sixteen stations have a length of record that is adequate for stochastic modelling purposes. Data collection at these stations should be continued in order to improve the reliability of the models for such purposes as engineering design and intervention analysis. In particular, the minimum change in level that can be detected by an intervention model will decrease as more data is available.

Another forty or more stations will have adequate lengths of record if data collection continues for the next decade. At that time a regional analysis taking into account various physiographic regions and basin characteristics should also be viable.

Notation

Abbreviations

ACF - Autocorrelation Function

AR - Autoregressive

AIC - Akaike Information Criterion

ARIMA - Autoregressive Integrated Moving Average

ARMA - Autoregressive Moving Average

df - Degrees of Freedom (for a Chi-squared test)

MAICE - Minimum Akaike Information Criterion Estimation

ML - Maximum Likelihood

PACF - Partial Autocorrelation Function

RACF - Residual Autocorrelation Function

S.E. - Standard Error of Estimation

S.L. - Significance Level

W.S.C. - Water Survey of Canada

Symbols

a_t - white noise innovation at time t

m - month

N - length of time series

N_t - ARMA noise component at time t

y - year

x_t - input series observation at time t

z_t - value of time series at time t

Greak Symbols

$\delta(B)$	- denominator of transfer function
θ_i	- the ith moving average parameter
$\theta(B)$	- moving average operator
λ	- box-cox transformation parameter
μ	- mean of a time series
ξ_i	- the ith intervention series
ϕ_i	- the ith autoregressive parameter
$\phi(B)$	- autoregressive operator
σ	- standard deviation of a time series
$\omega(B)$	- numerator of transfer function

1 INTRODUCTION

The purpose of this study is to examine stochastic modelling techniques for application to water related problems in the Canadian Arctic. A number of linear stochastic time series models are scrutinized and sample applications are considered.

The study is divided into four sections. The introductory section presents some of the background and philosophical considerations required for the later sections. The second section describes the various models that are used. The mathematics of the models as well as the techniques for model identification, estimation, and diagnostic checking are discussed briefly. In the third section, the models are fitted to monthly hydrometric and meteorologic time series from the Northwest Territories. Modelling procedures and results are indicated for each class of models. Finally, the fourth section deals with sample applications of the models. Where possible, "real life" problems from the Arctic are considered. In other cases, potential applications are discussed.

1.1 Available Data

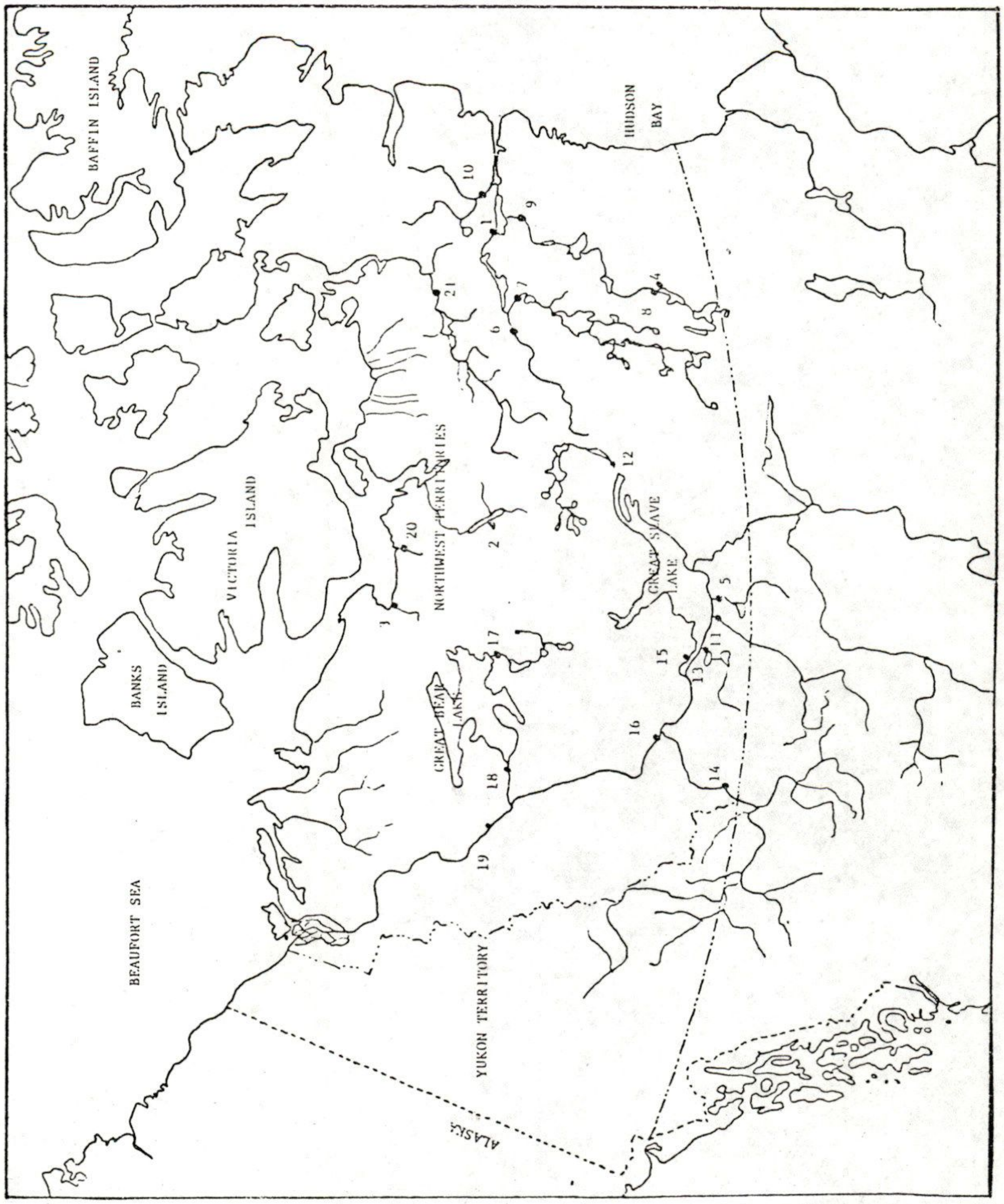
In response to the increasing need for hydrometric data in the North, the Water Survey of Canada (W.S.C.) has set up and is currently expanding a network of hydrometric gauging stations in the Northwest Territories. Of these stations, sixteen locations have between seven and fifteen years of reasonably continuous monthly data, a bare minimum required for time series analysis. Also in place is a network of meteorologic stations run by the Atmospheric Environment Service of Canada and having up to forty-six years of continuous data. Of these stations, five are considered to be near enough to a gauged watershed to be of hydrometeorologic interest. The hydrometric and meteorologic stations from which data was used for this study are listed in Table 1 and their locations are marked on a map of the Northwest Territories in Figure 1.

Table 1

Available Data

Sta.No.	Name (map code)	Position lat.	long.
	Meteorologic Stations		
2300500	Baker Lake (1)	64 18	96 00
2200850	Contwoyto Lake (2)	65 29	110 22
2200900	Coppermine (3)	67 50	115 07
230110	Ennadai Lake (4)	61 08	100 55
220240	Hay River A (5)	60 51	115 46
	Hydrometric Stations		
06jc002	Thelon R at out. of Beverly L (6)	64 32	101 24
06kc003	Dubawnt R bl Marjorie L (7)	64 16	99 35
06la001	Kazan R at Ennadai L (8)	61 15	100 28
06lc001	Kazan R at Kazan Falls (9)	63 40	95 45
06mb001	Quoich R ab St. Clair Falls (10)	64 27	94 07
07ob001	Hay R nr Hay River (11)	60 44	115 51
07rd001	Lockhart R at Artillery L (12)	62 53	108 28
07uc001	Kakisa R at out. Kakisa L (13)	60 56	117 25
10ed001	Liard R at Fort Liard (14)	60 14	123 28
10fb001	Mackenzie R nr F. Providence (15)	61 15	117 30
10gc001	Mackenzie R nr F. Simpson (16)	61 52	121 20
10ja002	Camsell R at out. Clut L (17)	65 35	117 45
10jc002	Great Bear R at Great Bear L (18)	65 08	123 30
10ka001	Mackenzie R at Norman Wells (19)	65 16	126 51
10qa001	Tree R nr the mouth (20)	67 38	111 52
10rc001	Back R bl Deep Rose L (21)	66 05	96 30

Figure 1
Locations of Hydrometric and Meteorologic Stations



1.2 Time Series

A time series is defined as a set of chronologically ordered observations. The order of observations is crucial to time series analysis. If the chronological ordering of, say, a river flow series were ignored much of the information in the data would be lost and a hydrologist would have a difficult time in forecasting future flows.

Some types of observation such as temperature or water level can be recorded continuously, and hence form a continuous time series. Other types of observation, such as suspended solids concentration or rainfall may be taken at discrete intervals, and constitute a discrete time series. In the case of discrete time series with regular measurement intervals the mathematics of time series modelling is greatly simplified. Fortunately, many types of geophysical time series are discrete with regular measurement intervals. Often, data that is not in the correct form, such as continuous temperature or runoff records, can be converted to discrete equispaced data by taking averages. The time series used in this study are produced by the monthly averaging of daily data. This is done by the various governmental agencies involved in data collection and is a routine procedure.

1.3 Stochastic Processes

A stochastic process may be defined for the purposes of this study as a process consisting of the sum of two components, one deterministic and the other random. The deterministic component is usually considered as a signal or overall trend, while the random component is interpreted as white noise. Thus, a river flow may be thought of as being stochastic in that it is to some extent predictable but there will always be some error in the prediction. In this report it is not necessary to distinguish between randomness that is due to some specific physical phenomenon and randomness due to the analyst's imperfect knowledge of the physics of the process. (For an interesting discussion of the scientific and philosophical aspects of this question see Klemes (1978).) From an engineering standpoint one need only develop a method for dealing with the analysis and modelling of a stochastic process. A stochastic model is one which describes mathematically both the deterministic and the random components of a process. Stochastic models are usually of the form:

process = deterministic component + random component
(signal) (white noise)

The deterministic component may be represented by a polynomial, a generalized transfer function, or any other suitable mathematical function. The mathematics of the model are greatly simplified if a linear function is used, and this study deals exlusively with linear stochastic time series models. The random component is usually described in terms of a probability distribution such as the normal, log-normal or log-Pearson distributions.

1.4 Stationarity

Stationarity of a stochastic process can be defined as a form of statistical equilibrium. This means that the underlying statistical properties of the process, such as mean, variance and serial correlation, do not change with time. For example, if a natural river basin is not subjected to changes such as urbanization, cultivation, forest fire, or climatic change, it would be reasonable to assume that the statistical properties of the streamflow time series would not change significantly over the design period of an engineering project. When modelling a geophysical time series for use in engineering design one would normally use stochastic models designed in such a way that the mean and variance of the model are independant of time, unless there is some reason to believe that some change or intervention has taken place in the underlying process. As explained by Hipel et al. (1979a) the use of nonstationary models in simulation for engineering design is usually not appropriate. In addition, the method used in this study to estimate missing data points requires the use of a stationary model. This study therefore concerns itself only with classes of stationary models. These models will be desribed in more detail in Section 2.

1.5 Akaike Information Criterion

A question that frequently arises in stochastic modelling is the question of which of two or more competing models should be used to model a process. A useful criterion for choosing between models should consider two general modelling principles: simplicity of the model (parsimony) and good statistical fit. The principle of model parsimony is of great statistical and practical importance. In intervention analysis, for example, the detectability of environmental changes is greatly impaired by the use of an overly complex model (Lettenmaier et al. 1978). From a practical viewpoint, more complex models are more difficult to work with, and frequently more expensive in terms of computation and data collection. Model parsimony can be quantified in terms of the number of estimated parameters in the fitted model, while goodness of fit can be quantified in terms of the maximum likelihood of the fitted model. A model with slightly higher maximum likelihood, and hence better statistical goodness of fit, but having many parameters may be less acceptable than one that has slightly lower maximum likelihood but fewer parameters. The Akaike Information Criterion (AIC) (Akaike 1974) is based on information theory and considers both the aforesaid principles. The AIC for a fitted statistical model is defined by:

$$AIC = -2\ (\ln ML) + 2k \qquad (1.5\text{-}1)$$

where ML = the maximum likelihood

k = the number of model parameters

The 2k term reflects model parsimony while goodness of fit is incorporated in the -2 (ln ML) term. The model with the lowest value of the AIC is considered to be the best model. Thus the model may be chosen according to the minimum AIC. This is termed minimum AIC estimation or MAICE.

It is also possible to calculate the relative plausibilility of two competing models if the AIC values of the models are known. The relative plausibility is given by:

$$\text{relative plausibility} = \exp\left(\frac{AIC2 - AIC1}{2}\right) \qquad (1.5\text{-}2)$$

where AIC1 = the AIC of the 1st model

and AIC2 = the AIC of the 2nd model

As a general modelling philosophy, when modelling complex stochastic situations it is better to start with simple models, and following this, perhaps examine more complex models. If, for example, one has a river flow series and a precipitation series, and one is interested in a model for the river flows, one might first fit a univariate model to the flow series. Then, if desired, one could go on to fit a transfer function model to link the flows to precipitation. The information gained in constructing the univariate model may be useful in identifying the most appropriate form for the more complex transfer function model.

2 DESCRIPTION OF THE MODELS

A class of linear stochastic time series models commonly known as Box-Jenkins (Box and Jenkins 1970) or ARIMA (Autoregressive Integrated Moving Average) models have been gaining increasing acceptance for use in the field of hydrology (Hipel and McLeod 1979a) and these models constitute the basis for the procedures used in this study. In recent years numerous theoretical and technical advances have taken place in ARIMA modelling. The utility of the models has been increased through advances in the identification, estimation and diagnostic checking stages of the modelling process, as well as through the extension of the models for use in intervention analysis and transfer function-noise modelling. For a detailed description of the mathematics and theory underlying the models the reader is referred to Box and Jenkins (1970), and for an account of some of the more contemporary advances the reader is referred to the relevant statistical and engineering literature (see for example Hipel et al. 1977a, Box and Tiao 1975, and McLeod and Hipel 1978b). A brief outline of the models and model building techniques used in this paper is given in this section. Sections 2.1 to 2.5 deal with models for nonseasonal time series, while models for seasonal time series are considered in sections 2.6 to 2.11.

2.1 Nonseasonal Autoregressive Models

An Autoregressive (AR) model is a special type of Box-Jenkins model which describes a time series in terms of previous observations, and in terms of a series of white noise innovations. The most simple example of an AR model is the Markov model, defined by:

$$z_t = \phi_1 z_{t-1} + a_t \tag{2.1-1}$$

where

z_t = the value of the process at time t (usually after the subtracting the mean level of the series)

ϕ_1 = the lag 1 AR parameter

a_t = the random or white noise component, assumed to be identically and independently distributed with mean zero and variance σ_a^2. (ie $a_t \sim IID(0, \sigma_a^2)$)

Thus the value of the series at time t depends on the value of the observation at time t-1, and on the random value of the white noise series at time t. This is equivalent to regressing the series at time t on the same series at time t-1. An alternative name for the Markov model is therefore the AR model of order one or AR(1) model. For the purpose of algebraic manipulation, the model may be written in a more convenient notation using the backshift operator, B, defined by:

$$B z_t = z_{t-1} \quad \text{and} \quad B^k z_t = z_{t-k} \qquad (2.1\text{-}2)$$

B is a linear operator and can therefore be manipulated as if it were a variable by using the normal rules of algebra. The model can now be written in several alternate forms:

$$z_t = \phi_1 z_{t-1} + a_t \qquad (2.1\text{-}3)$$

$$z_t = \phi_1 B z_t + a_t \qquad (2.1\text{-}4)$$

$$z_t - \phi_1 B z_t = a_t \qquad (2.1\text{-}5)$$

$$(1 - \phi_1 B) z_t = a_t \qquad (2.1\text{-}6)$$

$$\text{or} \quad \emptyset(B) z_t = a_t \qquad (2.1\text{-}7)$$

where $\emptyset(B) = 1 - \phi_1 B$ = the AR operator of order 1.

General Autoregressive Model

The AR(1) model can be extended to general case where there are p AR parameters. The AR model of order p is denoted by AR(p) and is written as:

$$z_t - \phi_1 z_{t-1} - \phi_2 z_{t-2} - \dots - \phi_p z_{t-p} = a_t \qquad (2.1\text{-}8)$$

where ϕ_i = the ith AR parameter.

Thus the value of the series at time t depends on the value of the series at the p preceeding times, plus a white noise term.

The model can also be written using the B notation:

$$z_t - \phi_1 B^1 z_t - \phi_2 B^2 z_t - \dots - \phi_p B^p z_t = a_t \qquad (2.1\text{-}9)$$

$$(1 - \phi_1 B^1 - \phi_2 B^2 - \dots - \phi_p B^p) z_t = a_t \qquad (2.1\text{-}10)$$

$$\text{or} \quad \Phi(B) z_t = a_t \qquad (2.1\text{-}11)$$

where $\Phi(B) = (1 - \phi_1 B^1 - \phi_2 B^2 - \dots - \phi_p B^p)$ = the AR operator of order p.

2.2 Nonseasonal Moving Average Models

The Moving Average (MA) model is one which describes a time series in terms of a white noise time series. An MA model of order q, denoted by MA(q), defines the value of a time series at time t in terms of the q most recent white noise innovations at times t, t-1, t-2,...,t-q+1. The MA(q) model is written as:

$$z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \dots - \theta_q a_{t-q} \tag{2.2-1}$$

where θ_i = the ith MA parameter.

In B notation the model is written:

$$z_t = a_t - \theta_1 B^1 a_t - \theta_2 B^2 a_t - \dots - \theta_q B^q a_t \tag{2.2-2}$$

$$z_t = (1 - \theta_1 B^1 - \theta_2 B^2 - \dots - \theta_q B^q) a_t \tag{2.2-3}$$

$$\text{or } z_t = \theta(B) a_t \tag{2.2-4}$$

where $\theta(B) = (1 - \theta_1 B^1 - \theta_2 B^2 - \dots - \theta_q B^q)$ = the MA operator of order q.

It should be noted that the MA(q) model may also be equivalently written as an AR model of infinite order by rewriting equation 2.2-4 as:

$$\frac{1}{\theta(B)} z_t = a_t \tag{2.2-5}$$

The $1 / \theta(B)$ term may then be expanded by polynomial expansion. Similarily the AR(p) model may be written as an MA model of infinite order by writing equation 2.1-11 as:

$$z_t = \frac{1}{\phi(B)} a_t \qquad (2.1\text{-}6)$$

2.3 Nonseasonal Autoregressive Moving Average Models

An autoregressive moving average or ARMA model is one having both AR and MA terms, defined by:

$$\phi(B)\, z_t = \theta(B)\, a_t \qquad (2.3\text{-}1)$$

Thus the value of the series at time t depends on the noise term at time t, the q preceeding noise terms, and the p preceeding values of the series.

An ARMA model having p AR parameters and q MA parameters is denoted by ARMA(p,q). As an example the ARMA(1,1) model is given by:

$$z_t - \phi_1 z_{t-1} = a_t - \theta_1 a_{t-1} \qquad (2.3\text{-}2)$$

This can be equivalently written as:

$$z_t - \phi_1 B z_t = a_t - \theta_1 B a_t \qquad (2.3\text{-}3)$$

$$\text{or} \quad (1 - \phi_1 B)\, z_t = (1 - \theta_1 B)\, a_t \quad \text{or} \qquad (2.3\text{-}4)$$

It may be noted that the notation ARMA(1,0) is synonymous with AR(1) and likewise ARMA(0,1) is the same as MA(1).

Some properties of ARMA models are:

1) ARMA models are parsimonious. This means that in most practical applications only a few AR and MA parameters are required to provide an adequate description of a time series.

2) ARMA models have been found to provide good statistical fits to hydrometric, meteorologic and other geophysical time series (Hipel et al. 1977a, McLeod et al. 1977, Hipel and McLeod 1978).

2.4 Box-Cox Power Transformation

A power transformation frequently used in Box-Jenkins modelling (Box and Cox 1974, Hipel et al. 1977a) is the Box-Cox transformation given by:

$$z^{(\lambda)} = \frac{z + \text{const}}{\lambda} - 1 \qquad \text{for } \lambda \neq 0 \qquad (2.4\text{-}1)$$

$$= \ln(z + \text{const}) \qquad \text{for } \lambda = 0 \qquad (2.4\text{-}2)$$

The Box-Cox transformation can often be used to correct situations where an examination of the model residuals indicates that some of the underlying modelling assumptions have been violated. The model residuals constitute an estimate of the white noise series, and can therefore be statistically tested for violations of the assumption of whiteness, as well as the less important assumptions of constant variance and normality of the white noise series. For a more detailed description of the use of the transformation please see Section 2.7.

2.5 Nonseasonal Autoregressive Integrated Moving Average Models

Autoregressive Integrated Moving Average or ARIMA models are used to model nonstationary time series. The procedure is to first transform the data by differencing the series to remove the nonstationarity and to then fit an ARMA model to the stationary transformed series. The differencing transformation is defined by:

$$w_t = (1-B)^d z_t \qquad (2.5-1)$$

where w_t = the value of the transformed series at time t

d = the degree of differencing

To illustrate the differencing transformation, for d = 1:

$$w_t = (1-B)^1 z_t = z_t - B z_t = z_t - z_{t-1} \qquad (2.5-2)$$

Therefore differencing of order one simply subtracts the value at time t-1 from the value at time t.

ARIMA models are denoted by ARIMA(p,d,q)

where p = number of AR parameters

d = degree of differencing

q = number of MA parameters

ARIMA models have been found to be particularly useful for applications such as forecasting, but, as has been explained in Section 1.4, they are not appropriate for the applications considered in this study.

2.6 Deseasonalized Models

Many types of data, such as weekly or monthly hydrometric and mateorologic data show seasonal or cyclic trends. In the case of stationary seasonal series, they can be modelled by the following procedure:

1) The application of a deseasonalizing transformation.
2) Using an ARMA model to represent the transformed series (which is now nonseasonal).

Two alternate deseasonalizing transformations are available and are described below for the case of monthly data. Similar transformatiors are available for weekly data.

1) $$z'_{y,m} = z_{y,m} - \mu_m \qquad (2.6\text{-}1)$$

2) $$z'_{y,m} = (z_{y,m} - \mu_m) / \sigma_m \qquad (2.6\text{-}2)$$

where $z_{y,m}$ = the value of the series for the mth month of the yth year

$z'_{y,m}$ = the deseasonalized value

μ_m = the mean of the mth month

σ_m = the standard deviation of the mth month

In general it is preferable to use the first transformation in order to reduce the total number parameters. However, in some cases the seasonality is not entirely removed by simply subtracting out the monthly means, because the standard deviation also changes from month to month, so it is also necessary to divide by the monthly standard deviations.

In those instances where the monthly means and standard deviations follow a roughly sinusoidal pattern, it is possible to reduce the number of model parameters by using a Fourier series representation of the monthly means and standard deviations. In the case of monthly data perhaps 4 or 5 Fourier components can be used to represent 12 monthly values, for a modest reduction in the number of parameters. For weekly data the reduction in the number of parameters can be dramatic, with 52 weekly values being represented by only a few Fourier components.

If Fm and Fs are the number of Fourier components used for the monthly means and standard deviations respectively, then $\hat{\mu}$ and $\hat{\sigma}$ are estimated by the following equations:

$$\hat{\mu}_m = A_0 + \sum_{k=1}^{Fm} A_k \cos \frac{2\pi km}{s} + B_k \sin \frac{2\pi km}{s} \qquad (2.6\text{-}3)$$

$$\hat{\sigma}_m = C_0 + \sum_{h=1}^{Fs} C_h \cos \frac{2\pi km}{s} + D_h \sin \frac{2\pi km}{s} \qquad (2.6\text{-}4)$$

where $m = 1,2,\ldots,s$

$$A_0 = \frac{1}{s}\sum_{m=1}^{s}\mu_m$$

$$C_0 = \frac{1}{s}\sum_{m=1}^{s}\sigma_m$$

$$A_k = \frac{2}{s}\sum_{m=1}^{s}\mu_m \cos\frac{2\pi km}{s}$$

$$B_k = \frac{2}{s}\sum_{m=1}^{s}\mu_m \sin\frac{2\pi km}{s}$$

$$C_h = \frac{2}{s}\sum_{m=1}^{s}\sigma_m \cos\frac{2\pi km}{s}$$

$$D_h = \frac{2}{s}\sum_{m=1}^{s}\sigma_m \sin\frac{2\pi km}{s}$$

$k = 1,2,\ldots,Fm$; $h = 1,2,\ldots,Fs$

s = the number of seasons (ie. 12 for monthly data)

Thus for each Fourier component there are two estimated parameters, one coefficient for the sine term and one for the cosine term. Note also that if $Fm = 0$ there is no seasonal adjustment for the means and therefore the mean of the entire series is used for $\hat{\mu}$. If $Fs = 0$ there is no adjustment for the standard deviations and $\hat{\sigma}$ is set to unity in equations 2.6-2 and 2.6-4.

2.7 Modelling Techniques

The recommended procedure for the construction of both seasonal and nonseasonal models is a process that may be divided into three distinct stages, each consisting of a number of operations (Box and Jenkins 1970, Hipel et al. 1977a). The first stage is to tentatively identify from the data the most appropriate model. In the second stage the model parameters are estimated, and hence the model is fitted to the data. In the third stage the model is tested for adequate fit and to insure that the underlying assumptions have not been violated. If evidence of poor fit or violations of the model assumptions is found the process is repeated iteratively until an adequate model is found. The three stages of model construction are outlined in Table 2.

Identification

The very first step in model construction is to examine a plot of the data. A data plot will show immediately if the series is seasonal or nonseasonal, although for geophysical data this is usually known in advance. Figure 2 is a plot of the monthly mean flows of the Liard River at Fort Liard. Figure 3 shows the same series after the application of the deseasonalizing transformation. It can be readily seen that the cyclic or seasonal component has been removed.

Table 2

The Three Stages of Model Construction

1) Identification

a) Plot of the data

b) Plot of autocorrelation function (ACF)

c) Plot of partial autocorralation function (PACF)

2) Estimation

a) Maximum likelihood estimate of parameters

b) Box-Cox power transformation

c) Akaike Information Criterion (AIC)

3) Diagnostic Checks

is a_t $\left\{\begin{array}{l}\text{independent ?}\\ \text{homoscedastic ?}\\ \text{normal ?}\end{array}\right.$

homoscedastic ? / normal ? } can often be corrected by Box-Cox transformation

Figure 2
Monthly Flows of the Liard River from 1960 to 1976

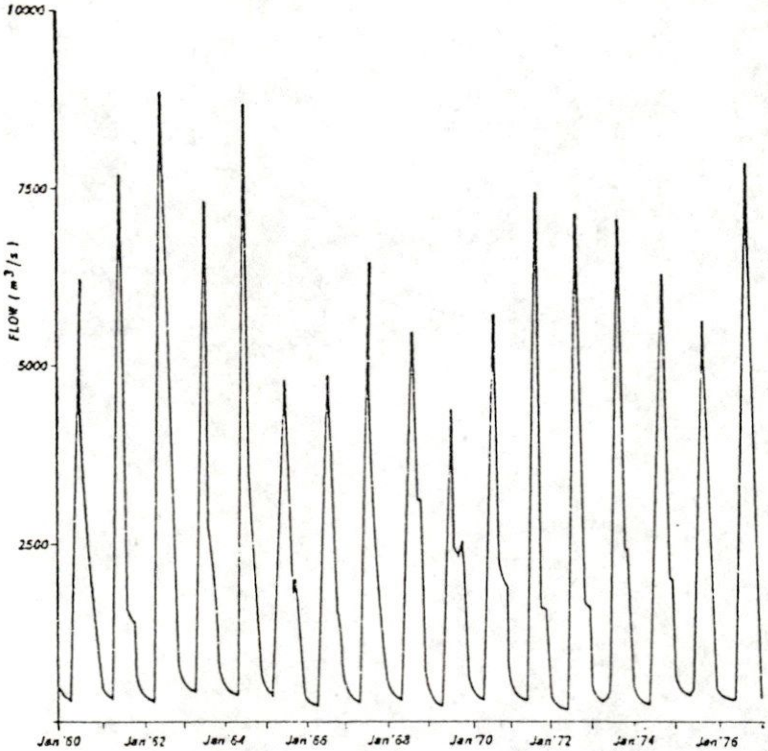
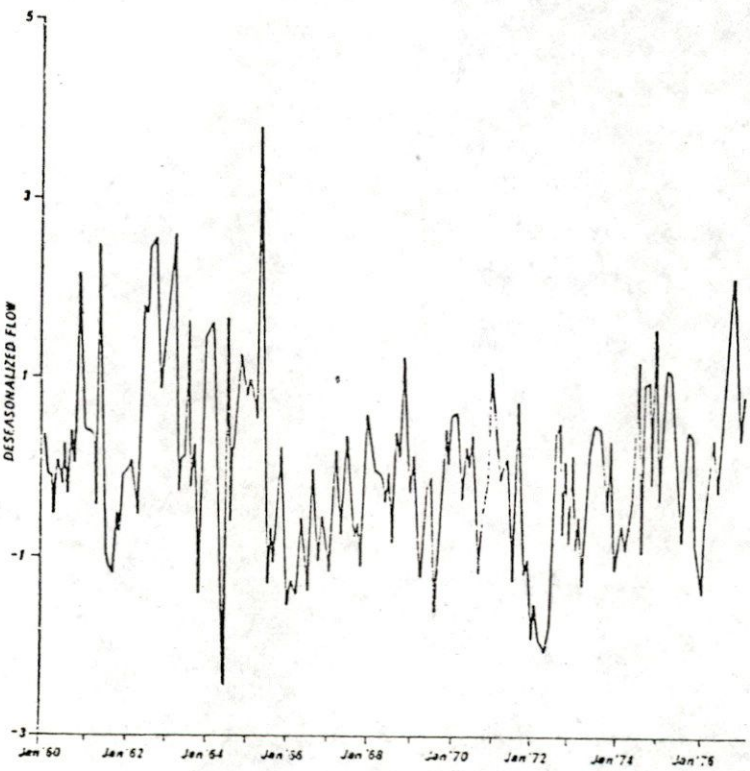


Figure 3
Deseasonalized Liard River Series



Two very useful tools in identifying Box-Jenkins models are the Auto Correlation Function (ACF) and the Partial Auto Correlation Function (PACF) (Strictly speaking, the ACF and PACF calculated from measured data are known as the sample ACF and sample PACF, but this distinction is dropped here for reasons of convenience). The ACF at lag k gives the linear dependence or correlation of values of the time series separated in time by k lags. The PACF at lag k gives the value of the kth AR parameter of an AR(k) model fitted to the series (Pagano 1972). Plots of the ACF and the PACF are used in determining how many MA and AR parameters will probably be necessary to model a given series. The ACF and PACF are are calculated from nonseasonal or deseasonalized data for use as an identification tool for nonseasonal ARMA models and ARMA models fit to deseasonalized data.

The use of an AR(p) model is indicated if the ACF dies off slowly towards zero and the PACF is not significantly different from zero (truncated) after lag p. Conversely, the use of an MA(q) model is indicated if the PACF dies off slowly and the ACF is truncated after lag q. If both the ACF and the PACF die off slowly the use of an ARMA(p,q) model is indicated. If the ACF and PACF show cyclic fluctuations, this may indicate that the series has not been adequately deseasonalized, and a transformation with more Fourier components may be necessary.

Combining the information from the ACF and PACF plots gives a good idea of the number of AR and MA parameters a model will need. This information greatly simplifies the problem of choosing the most appropriate model, although some experience may be necessary in dealing with ambiguous plots.

The ACF and PACF for the deseasonalized Liard River flow series are shown as sample plots in Figures 4 and 5. A further description of the identification process for this series may be found in Section 3.1.

Figure 4
ACF for the Deseasonalized Liard River Series

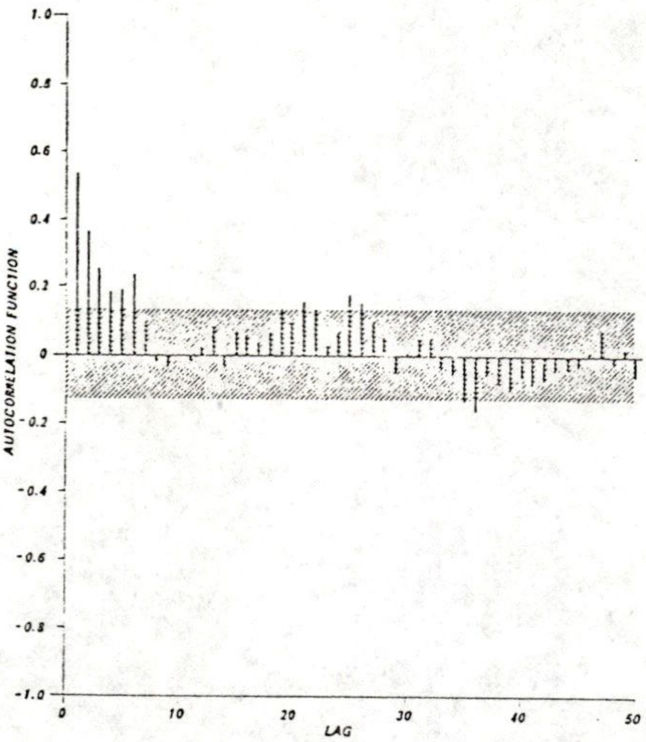
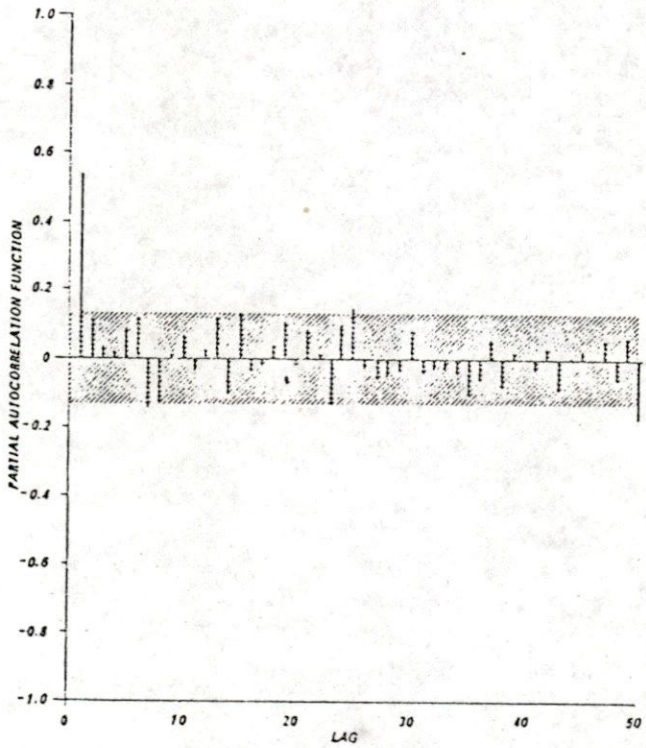


Figure 5
PACF for the Deseasonalized Liard River Series



Estimation

The values of the AR and MA parameters of the ARMA model are not known, and must be estimated from the data. This is commonly known as fitting the model. In addition, the standard errors of estimation can be calculated. An estimated parameter can be compared to its standard error of estimation to check if it is significantly different from zero. If not it should be omitted from the model. The residuals of the fitted model constitute an estimate for the white noise series so a_t and σ_a^2 can be readily calculated.

Parameters are estimated by maximizing the log likelihood function for the model in question (maximum likelihood estimation). For large sample sizes (100 or more) the least squares estimates formed by minimizing the residual sum of squares is almost identical to the maximum likelihood estimates (Box and Jenkins 1970). For smaller sample sizes the modified sum of squares method proposed by Mcleod (1977) has been shown to give a closer approximation to the true maximum likelihood estimates, especially for the MA parameters. The programs used to estimate ARMA parameters in this study use the modified sum of squares method.

Diagnostic Checks

After a time series has been fitted, it is important to check to insure that the assumptions made in the model are not violated. The primary assumption of Box-Jenkins models is that the a series is independently distributed. If the noise is correlated, the model will not give valid results for simulation and forecasting applications. Furthermore, the model parameters may be very poorly estimated. The assumption of an independent noise series is checked by testing the model residuals for whiteness. If significant residual autocorrelations are found, particularly at low lags, the model cannot be accepted (McLeod 1979a). The model may then be corrected by iteratively repeating the the three stages of model construction.

Another frequent assumption is that the variance of noise term is constant. This is referred to as homoscedasticity. Non-constant variance is termed heteroscedasticity. Statistical tests are available to test for changes in variance depending on the time and also on the current level of the series (Hipel et al. 1977a). Also, it is frequently assumed that the white noise component follows a particular probability distribution, usually the normal distribution. The skewness of the estimeated residuals provides a test of the normality assumption (D'Agostino 1970). The above tests are included in the estimation programs used in this study.

2.8 Monthly Autoregressive Models

Monthly river flow data frequently exhibits an autocorrelation structure that depends not only on the time lag between observations, but also on the month or season of the year. Seasonal variation in the autocorrelation structure may be due to such physical conditions as the presence or absence of ice cover, whether the precipitation is in the form of snow or rain, etc.. Autoregressive models with monthly varying parameters were first suggested by Thomas and Fiering (1962) and later by Jones and Brelsford (1967), Yevjevich (1972), Croley and Rao (1973), Clark (1973), and Tao and Delleur (1976). Recent advances in the identification, estimation, and diagnostic checking stages (McLeod and Hipel 1978b) have greatly increased the utility of monthly varying autoregressive (MAR) models.

Model Description

Let $z_{y,m}^{(\lambda)}$ be the value of a transformed time series on the mth month of the yth year. It may be noted that $z_{7,12}^{(\lambda)}$, $z_{6,24}^{(\lambda)}$, and $z_{8,0}^{(\lambda)}$ all refer to the same observation. The monthly autoregressive model of order $(p_1, p_2, \ldots p_{12})$ for the month m is defined by:

$$z_{y,m}^{(\lambda)} - \mu_m^{(\lambda)} = \sum_{i=1}^{p_i} \phi_{i,m} \left(z_{y,m-i}^{(\lambda)} - \mu_{m-i}^{(\lambda)} \right) + a_{y,m} \qquad (2.8\text{-}1)$$

where $\mu_m^{(\lambda)}$ = the mean of $z_{y,m}^{(\lambda)}$ for the mth month

$\phi_{i,m}$ = the ith AR coefficient for the mth month

$a_{y,m}$ = the white noise term for the mth month of the yth year

Model Identification and Estimation

Because the parameter estimates for the 12 months are independent it is possible to estimate the MAR parameters using multiple linear regression (McLeod and Hipel 1978b). The algorithm of Morgan and Tatar (1972) can therefore be used to calculate the residual sum of squares and hence the AIC for all possible MAR models. For instance, if all models up to order 12 are considered, the AIC value for 12×2^{12} possible regressions may be looked at in about 50 seconds of Honeywell Series 66 computer time. If only the 1st, 2nd, 3rd, and 12th parameters for each month are considered, 12×2^{4} possible regressions are needed and computation time drops to about 1.8 seconds. Thus the model may be identified and estimated automatically using a MAICE procedure. Typically MAICE will choose a model with two to four AR parameters for each month, usually at lags 1, 2, 3, or 12. It may be noted that MAICE will break down if one attempts to fit a model where the number of parameters approaches the number of data points, (ie. if one tries to fit an MAR(12) model to less than 13 years of data). For short data sets this problem can be overcome by constraining some of the parameters to zero, thus reducing the number of parameters to be estimated. In this study the parameters of order 4-11 are set to zero for this reason. As before, the model chosen by MAICE need not contain all the possible parameters, and often only one or two will be chosen for each month.

Diagnostic Checks

As with the other models already described, MAR models are rejected if there is evidence of significant residual autocorrelation. Tests based on the residual autocorrelation and a Chi-squared portmanteau statistic, Q, due to McLeod and Hipel (1978b) are included in the programs used in this study. A model may be rejected if the residual autocorrelations are larger than twice their standard error, or equivalently if the Q statistic is too large.

2.9 Transfer Function-Noise Models

If a process is affected by some external inputs, and time series data is available for the inputs, a transfer function-noise model can be used to link the process with the inputs. In this study meteorologic series such as precipitation and temperature are considered as external inputs to the river flow process. The model can be interpreted as a black box in which inputs of precipitation are transferred into an output of streamflow. Temperature enters into the system in that it is the controlling factor in determining at what lag precipitation in the form of snow will melt to form runoff.

The transfer function-noise model is made up of the sum of two components, the transfer functions for each of I_1 input series x_{ti}, $i = 1,2,...,I_1$, and the noise component which is represented by an ARMA model. Transfer function models may be applied to either seasonal, nonseasonal, or deseasonalized data depending on the situation. In Section 3.3 it is explained why deseasonalized series are the most appropriate for the Arctic data modelled in this study.

Model Description

A transfer function-noise model is defined by:

response = inputs + noise (2.9-1)

If there are I_1 inputs the modal is written:

$$z'_t = \sum_{i=1}^{I_1} V_i(B)\,(x'_{ti}) + N_t \qquad (2.9\text{-}2)$$

where

z_t = the value of the output series at time t

z'_t = the (deseasonalized) Box-Cox transform of z_t

x_{ti} = the value of the ith input series at time t

x'_{ti} = the (deseasonalized Box-Cox transform of x_{ti}

and

$$V_i(B) = \frac{\omega^{(i)}(B)}{\delta^{(i)}(B)} = \frac{\omega_0^{(i)} - \omega_1^{(i)} B - \dots - \omega_v^{(i)} B^{v_i}}{1 - \delta_1^{(i)} B - \dots - \delta_u^{(i)} B^{u_i}} \qquad (2.9\text{-}3)$$

= the ith transfer function

b_i = the ith delay parameter

$$N_t = \frac{\theta(B)\, a_t}{\phi(B)}$$, an ARMA noise term

It may be noted that the deseasonalization and Box-Cox transformation of z and x are optional. The transformations are applied only where necessary to remove seasonality or to correct for violations of model assumptions.

Model Identification

The method of choosing an appropriate transfer function-noise model may be broken down into the steps shown below:

1) ARMA models are fitted separately to the response series and the input series and the residuals are calculated.
2) The cross-correlation structure of the residuals is examined as described in Box and Jenkins (1970) and Haugh and Box (1977). (This procedure is similar to that used in the identification of ARMA models, except the cross-correlation function is used instead of the autocorrelation function.) The form of the transfer function should be chosen in the light of a physical understanding of the process in order to choose a reasonable model.
3) When the transfer functions have been identified a model is estimated using an ARMA(0,0) noise component (ie. no noise term) and the residuals are calculated.
4) An ARMA model is identified for the time series of residuals of the model in step (3) using the techniques previously described in Section 2.7. This ARMA model becomes the noise component. Linking this noise model to the transfer functions identified in step (2) gives

the complete transfer function-noise model.

Parameter Estimation and Diagnostic Checks

The dynamic and noise components are estimated simultaneously by numerically maximizing the log likelihood function. The method of doing this is described by McLeod (1979b).

The diagnostic checks used for transfer function-noise models are very similar to those for ARMA models. Because the noise term of the transfer function-noise model is in fact an ARMA model, the identical diagnostics, described in Sections 2.7 and 3.1, are used for this component. In addition, several tests are available for the transfer functions. Each estimated parameter can be compared to its standard error to check whether the parameter is significantly different from zero. If not, it should not be included in the model. Equally important, the transfer functions must be reasonable in the light of a physical understanding of the process. For instance, a negative relationship between precipitation and runoff may be significant from the point of view of the various statistical tests, but it must be rejected because it does not make sense physically. Finally, alternate models may be estimated to choose the one with the lowest AIC.

2.10 Intervention Models

A special form of the transfer function model is the intervention model which considers external interventions on a process as a special type of transfer function (Box and Tiao 1975, Hipel et al. 1975, 1977b). An intervention on a river flow process might be a man-made change such as the construction of a dam, the removal of forest cover, or the construction of irrigation or drainage works. A forest fire is an example of a natural intervention. In the intervention model the intervention is considered as an input time series denoted by ξ_t. ξ_t is a binary variable whose value is zero when the intervention is not occurring and one when the intervention is occurring. The effect of a dam construction, for example, is represented by $\xi_t = 0$ before the construction and $\xi_t = 1$ after the dam is in operation.

Model Description

The form of the intervention model is given by:

$$\text{response} = \text{interventions} + \text{noise} \qquad (2.10\text{-}1)$$

For the general case of I_2 interventions the model is defined by:

$$z'_t = \sum_{i=1}^{I_2} V_i(B)\, \xi_{ti} + N_t \qquad (2.10\text{-}2)$$

Model Construction

In general the modeller will know the cause of an intervention and will therefore be able to choose the intervention model in the light of a physical understanding of the process and the intervention and also an understanding of the mathematical behaviour of the transfer functions used to model the intervention. Descriptions of this procedure, as well as some other useful aids to identification can be found in Box and Tiao (1975) and Hipel et al. (1977b).

The estimation and diagnostic stages are identical to those for the transfer function-noise model of which the intervention model is a special case.

2.11 The General Intervention Model

Model Description

The general intervention model is a combination of the transfer function-noise model descibed in Section 2.9 and the the intervention model described in Section 2.10. The general intervention model can therefore include I_1 inputs and I_2 external intervention in a model of the form:

$$\text{response} = \text{inputs} + \text{interventions} + \text{noise} \qquad (2.11\text{-}1)$$

or

$$z'_t = \sum_{i=1}^{I_1} V_i(B)\,(x'_{ti}) + \sum_{i=I_1+1}^{I_2} V_i(B)\,\xi_{ti} + N_t \qquad (2.11\text{-}2)$$

The three stages of model construction are the same as described for transfer function-noise models and intervention models.

3 MODELLING ARCTIC RIVERS

In this section of the report the deseasonalized ARMA models, monthly autoregressive models, and transfer function-noise models which were described in Section 2 are fitted to monthly hydrometric time series from the Northwest Territories. In the case of transfer function-noise models, monthly rainfall, snowfall, and temperature data are included as input series. The use of the identification, estimation, and diagnostic checking stages in the modelling procedure is explained for each type of model, and the results are presented in tabular form.

3.1 Deseasonalized ARMA Models

In keeping with the modelling philosophy of starting with simple models and examining more complex models only if the simple models are not adequate, the first models fitted to the monthly hydrometric data from the Arctic are deseasonalized ARMA models.

Procedures

The recommended stochastic modelling procedure consists of three stages; identification of a reasonable model, estimation of the model parameters, and diagnostic checking of the fitted model (Box and Jenkins 1970, Hipel et al. 1979a). To illustrate the use of the three stage procedure, the modelling of the flow series of the Liard River at Fort Liard is given as an example. The Liard River drains an area of 230,000 km^2 extending into a variety of physiographic regions, including the Rocky Mountains and their foothills, the Fort Nelson Lowlands, the Liard Plateau, the Liard Plain, and the Hyland Plateau. Monthly flow data is available from 1960 onwards.

1) Identification

The first step in the identification stage is to examine a plot of the monthly flow data. Figure 2 (page 25), shows that the Liard River data is highly seasonal, with monthly means ranging from about 325 m^3/s in February to about 6,300 m^3/s in June. The series must therefore be deseasonalized by one of the two methods defined by equations 2.6-1 and 2.6-2. The preferred method is to subtract out the monthly means and divide by the monthly standard deviations as defined in equation 2.6-2. As was shown in Section 2.6, when a Fourier representation is used for the monthly means and standard deviations, equation 2.6-1 becomes a special case of equation 2.6-2. Figure 3 (page 25), a plot of the deseasonalized series, shows that the seasonal component has been removed.

The next step is the inspection of the ACF and PACF of the deseasonalized series. The ACF and PACF for the deseasonalized Liard River series are shown in figures 4 and 5 (page 28). It can be seen that the PACF is truncated after lag 1, indicating that a ϕ_1 AR term should be included in the model. This is further supported by the ACF which dies away from lag 1 to lag 4. A final spike in the ACF at lag 6 indicates that a θ_6 MA term may also be necessary. Therefore the model is tentatively identified as an AR(1) or possibly an ARMA(1,6) model with θ_1 to θ_5 constrained to zero.

2) Estimation

The model parameters are estimated from the data by using the method of maximum likelihood. This step is done numerically using appropriate computer programs. The estimation programs used for this study are from A.I. McLeod's T.S. package of interactive time series analysis programs on the University of Waterloo Mathematics Faculty Honeywell Series 66 system. The diagnostics needed for the next stage are also calculated by the same programs. Sample outputs are shown in Appendix 1.

3) Diagnostic Checks

Models are rejected if the assumption of an independently distributed noise term is violated. Evidence against the assumption is given by a value of the residual autocorrelation function (RACF) greater than twice its standard error of estimation, especially at low lags. It is also desirable to have a model with a normally distributed noise term with constant variance. The hypothesis of normality can be rejected if significant evidence of skewness can be found and the assumption of constant variance can be rejected if there is evidence of heteroscedasticity. In this study various statistical tests are used for this purpose. Skewness is indicated by a skewness statistic significance level of less than 0.05 (D'Agostino 1970). Heteroscedasticity is indicated by Chi statistics in the tests for changes in the variance depending on the current level and changes in the variance over time being greater than twice their respective standard errors of estimation (Hipel et al. 1977a). Finally, of the models which pass the above tests, the one with the lowest AIC is chosen as the best model. For the two models identified in the preceding section, the diagnostics are listed in Table 3.

Table 3

Diagnostic Results for the Liard River Models

Model	AIC	RACF at	Skewness (S.L.)	Heteroscdasticity with level (S.E.)	Heteroscdasticity over time (S.E.)
AR(1)	2340.5	lag 6	0.61	0.00019	-0.0042
			(0.00090)	(0.000047)	(0.0016)
ARMA(1,6)	2312.4	none	0.37	0.00018	-0.0033
			(0.028)	(0.000047)	(0.0016)

(significance level (S.L.) and standard error (S.E.) in brackets)

The AR(1) model is rejected because it has a significant residual autocorrelation at lag 6. The ARMA(1,6) model residuals are badly skewed, with a skewness test significance level of only 0.028.

Residual skewness can often be corrected in practice by using a Box-Cox transformation. Incorporating a Box-Cox parameter $\lambda = 0$ transformation into the model, yields the results listed in Table 4.

Table 4

Diagnostic Results for the Revised Liard River Models

Model	AIC	RACF at	Skewness (S.L.)	Heteroscedasticity with level (S.E.)	Heteroscedasticity over time (S.E.)
AR(1)	2315.9	lag 6	0.13	0.13	-0.0032
			(0.43)	(0.08)	(0.0017)
ARMA(1,6)	2312.4	none	-0.021	0.17	-0.0027
			(0.90)	(0.08)	(0.0016)

(significance level (S.L.) and standard error (S.E.) in brackets)

Again the AR(1) model is unacceptable due to a significant residual autocorrelation at lag 6. The ARMA(1,6) model has no significant residual autocorrelations and appears to satisfy the normality assumptions reasonably well except for a slightly significant heteroscedasticity Chi statistic. The ARMA(1,6) model also has a lower AIC.

Fourier Representation

After the best deseasonalized ARMA model has been fitted, model parsimony can be improved by using a Fourier representation of the monthly means and standard deviations. Using six Fourier components to represent the means and six for the standard deviations is equivalent to an exact calculation of each. The model is re-estimated with smaller numbers of Fourier components until the model with the lowest AIC value is found.

For the Liard River, the improvement in the AIC with the Fourier representation is illustrated in Table 5.

Table 5

AIC of Deseasonalized Models

Number of Fourier Components		AIC
for means	for s.d.'s	
6	6	2312.4
6	5	2308.9 - lowest AIC
6	4	2312.4
5	5	2310.6
6	0	2334.5

The model that is finally chosen to represent the monthly flows of the Liard River at Fort Liard is an ARMA(1,6) deseasonalized using six Fourier components for the monthly means and five Fourier components for the monthly standard deviations, with a Box-Cox transformation parameter of zero. For some data sets a model having zero Fourier components for the standard deviations is found to give the lowest AIC. In such cases perhaps 4 or 5 Fourier components would be used to represent the monthly means but none would be used for the monthly standard deviations.

Because the Fourier representation requires considerably more use of the computer, it has not been applied to the remaining models of Arctic river flows in this study. It is felt that one example is sufficient to illustrate the technique and that the computing resources should be conserved for other applications.

Results

Using the procedures outlined in the previous section, models are fitted to each of 16 hydrometric series from the Northwest Territories. Table 6 lists the model specifications, constrained parameters, and AIC values for these models. Further information is contained in Appendix 1.

Table 6

Deseasonalized ARMA Models

Sta.No.	Name	Model (AR,MA) constrained parameters	Lambda	Type of Deseasonalization	AIC
06jc002	Thelon	(1,1)	0	1	444.4
06jc003	Dubawnt	(1,1)	1	2	794.0
06la001	Kazan	(1,1)	0	1	674.1
06lc001	Kazan	(1,0)	0	2	506.0
06mb001	Quoich	(0,1)	0	1	367.0
07ob001	Hay	(1,10) θ_2 to θ_{11}	0	2	830.5
07rd001	Lockhart	(1,12) θ_2 to θ_{11}	0	1	641.5
07uc001	Kakisa	(1,3) θ_2	0	1	584.2
10ed001	Liard	(1,6) θ_1 to θ_5	0	2	2312.4
10fb001	Mackenzie	(6,0) ϕ_2 to ϕ_5	1	2	1495.6
10gc001	Mackenzie	(1,0)	1	2	1912.2
10ja002	Camsell	(1,1)	0	1	563.1
10jc002	Gr. Bear	(1,1)	1	1	514.2
10ka001	Mackenzie	(1,1)	0	2	1817.1
10qa001	Tree	(1,1)	0	1	367.8
10rc001	Back	(1,2) θ_1	0	2	1235.5

Types of deseasonalization:

1 - as defined by eq. 2.6-1 (subtract monthly means)

2 - as defined by eq 2.6-2 (subtract monthly means, divide by monthly standard deviations)

A number of points of interest regarding these models are worthy of mention:

1) In most cases it is necessary only to subtract monthly means to deseasonalize the data. Nonetheless, in some situations it is also necessary to divide by the standard deviations in order to remove seasonality in the residual ACF. This also has the effect of lowering the AIC of the fitted model, in spite of the 12 extra parameters needed for this operation. The more complex form of deseasonalization is only used where the final model can be justified by a lower AIC.

2) Model uncertainty can take on two different forms: model uncertainty and parameter uncertainty. In the first case the modeller is not certain if the correct model has been identified, and in the second he is not sure if the parameters have been correctly estimated. In both instances, the more data available, the less uncertainty in the identification and estimation of the stochastic model. The estimation programs used in this study calculate the standard errors of all estimated parameters so it is possible to have some idea of the magnitude of the parameter uncertainty. Similarly, at the identification stage, approximate confidence limits are calculated for the ACF and the PACF. Because standard errors and confidence intervals decrease as the amount of data increases, the need for continued collection of data in the Arctic is emphasized. The

length of record currently available is the bare minimum required for stochastic modelling. In many cases the estimated parameters have large standard errors, and the identification of the correct model is made difficult by the large uncertainty of the ACF and PACF estimates that are used in the model identification stage.

3) In all cases the assumption of an independent noise term is well satisfied, and none of the models show significant residual autocorrelation. However, many of the models showed highly skewed or heteroscedastic residuals, and thus do not satisfy the less important assumptions of a normally distributed noise series having constant variance. When using these models for simulation it would be important not to try to use a normally distributed random generator for the noise term. Rather, as suggested by McLeod and Hipel(1978a), the model residuals themselves may be placed in a table and chosen at random to form the noise term. As explained by McLeod (1979b), non-normal residuals are frequently a sign that an important covariate series has been left out of the model, and that a transfer function-noise model might be more appropriate. With this in mind , the next step in the modelling process is to fit transfer function-noise models that include the meteorologic data as covariate series (see section 3.3).

3.2 Monthly Autoregressive Models

Procedures

The MAICE technique developed by McLeod and Hipel (1978b) is used to automatically fit the best monthly autoregressive model of order 12 according to the lowest AIC. Because of the short time series available from the Arctic it is necessary to constrain some of the model parameters to zero to avoid the overfitting of the model. The parameters of order 4 to order 11 are constrained to zero for the automatic identification and estimation of MAR models for 12 Arctic river flow series.

Results

The resulting models, fitted with Box-Cox parameters of zero and unity, are shown in Table 7 and sample outputs listing the complete estimated models are included in Appendix 2. A portemanteau statistic, Q, is calculated for each model. Each value of Q has associated with it a number of degrees of freedom, df, that depends on the number of estimated parameters in the model and on the quantity of data. The Q statistic is distributed as Chi-squared with df degrees of freedom, so any standard mathematical tables of the Chi-squared distribution can be used to test whether the value of Q is significant. A large value of Q indicates model inadequacy.

Table 7

Monthly Autoregressive Models

Sta. #	Name	MAR(12)λ = 0		MAR(12)λ = 1	
		AIC	Q/df	AIC	Q/df
06jc003	Dubawnt	805.9	172/161	807.5	158/161
07ob001	Hay	792.9	204/165	875.5	199/154
07rd001	Lockhart	589.1	199/159	599.2	184/158
07uc001	kakisa	540.6	194/164	562.6	204/161
10ed001	Liard	2335.8	168/167	2359.8	162/166
10fb001	Mackenzie	1535.1	225/159	1539.9	217/161
10gc001	Mackenzie	1979.7	195/164	1984.8	206/164
10ja002	Camsell	495.8	170/159	489.6	169/158
10jc002	Gr. Bear	551.2	200/160	551.3	200/159
10ka001	Mackenzie	1942.2	192/161	1916.4	199/162
10qa001	Tree	354.3	190/159	385.5	197/157
10rc001	Back	1262.2	194/164	1262.0	183/165

For many of the models the Q statistic indicates a rather poor fit. This could be due to the necessity of constraining parameters. In spite of the high values of Q, five of the MAR models show a much lower AIC than do the deseasonalized ARMA models for the same series. The improved AIC would indicate that there does indeed exist important variation in the autocorrelation structure from month to month. For those series where the MAR models are significantly better, it would also make sense to try fitting a transfer function-noise model incorporating a monthly autoregressive noise term. However, the programming of transfer function MAR models remains a subject for future study.

3.3 Transfer Function-Noise Modelling

Four river flow time series are chosen on the basis of proximity of a meteorologic station for analysis using transfer function-noise models. River flows are modelled with temperature, rainfall, and snowfall as input series, and using an ARMA noise term. In all four cases the transfer function-noise model is superior in terms of the AIC and the other diagnostics than were the previously fitted deseasonalized ARMA models.

Procedures

1) Transformation and Deseasonalization

The models are fitted using transformed series. The flow series are transformed using a Box-Cox transformation with $\lambda = 0$, and then deseasonalized by subtracting out monthly means. The rainfall and temperature series are deseasonalized in the same way but without the Box-Cox transformation.

Where the seasonal variation of a time series constitutes a significant part of the total variation of the series, transfer function-noise models are significantly improved if the series is first deseasonalized by subtracting out the monthly means. This improvement is due to a much less severe assumption of linearity for the deseasonalized model. In transfer function modelling, the transfer function is assumed to operate linearly over the entire range of the series, from the smallest to the largest value. When the monthly means have been subtracted out, the range of the deseasonalized series is much smaller than the range of the original series. This is indicated for the Tree River in Table 8.

Table 8

Range of Tree River Series

Series	Min. Value	Max. Value	Range
	m^3/s	m^3/s	m^3/s
Tree R. (original)	1.9	145.6	143.7
Tree R. (deseas.)	- 4.6	78.7	93.3

The range of the deseasonalized series is equal to 0.65 times the range of the original data. The decrease in the variance is even more dramatic. The variance of the transformed series is 1.0863. After deseasonalization the residual variance is only 0.1425, a drop to 0.131 times the original variance. Thus, a deseasonalized transfer function needs only assume linearity over a much smaller range. The non-deseasonalized model assumes linearity over the entire range of values that the time series can take on, while the deseasonalized transfer function-noise model assumes only that the deviation of the from the monthly means are linear. This is analogous to the assumption of linear small signal gain in an amplifier that is nonlinear for large signals. In practice, deseasonalized transfer function-noise models fit monthly hydrologic series from the Arctic much better than do non-deseasonalized models. As an example, deseasonalized and non-deseasonalized transfer function-noise models are fitted to the Back River data, linking the series to precipitation and temperature series from the Baker Lake and Contwoyto Lake weather stations. The resulting AIC values are shown in Table 9.

Table 9

AIC values for Back River Models

Model	AIC
deseasonalized	1196
non-deseasonalized	1316

The relative plausibility of the models is given by:

$$\text{relative plausibility} = \exp\left(\frac{AIC2 - AIC1}{2}\right)$$

$$= \exp\left(\frac{1319 - 1196}{2}\right) = 1.14 \text{ E } 26$$

Thus the deseasonalized model is much more plausible than the non-deseasonalized model. In fact, a relative plausibility greater than, say, 1.5, would normally be quite sufficient for discriminating between two competing models.

2) Snowmelt

The monthly snowfalls are summed over each winter, and then the total snowfall for the winter is introduced as a pulse input to the model during the first month that the mean temperature rises above zero Celsius, for each year. The snowmelt pulse input is "shaped" by the transfer function to more closely resemble the shape of the actual hydrograph. As explained by Box and Jenkins (1970) and Hipel et al. (1977) the transfer function can be used to model a wide variety of impulse responses. For example, a transfer function with $\omega_0 = 0.5$, $\omega_1 = 2.0$, $\delta_1 = 0.5$ would produce the impulse response shown in Figure 6.

Figure 6

Snowmelt Transfer Function Impulse Response

```
          INPUT                        OUTPUT

        !                         !   x
        !                         !   x
      2 +                       2 +   x
x       !                z        !   x
  t     !                  t      !   x
        !                         !   x
      1 +  x                    1 +   xx
        !  x                      !   xxx
        !  x                      !  xxxxx
        !  x                      !  xxxxxxx
      0 +----+----+----+        0 +----+----+----+
        0    5    10   15         0    5    10   15

          TIME                      TIME
```

As can be seen by the graph of the output, this impulse response constitutes a plausible discrete representation of the peak and recession limb of a snowmelt hydrograph.

Snowfalls that occur during months when the mean temperature was above zero Celsius are assumed to have melted immediately, and are added to the rainfall series rather than summed in with the winter's snow accumulation.

3) Model Identification

As suggested by Box and Jenkins (1970), Haugh and Box (1977), and Hipel et al. (1977c), the transfer function-noise models are identified by first fitting univariate models to each of the covariate series and then examining the cross-correlation function of the residuals for the two series. In this way spurious correlations due to the autocorrelation or seasonality of the covariate series can hopefully be rejected. When temperature is used as an input series, the cross-correlation function is calculated for each month of the year. The temperatures for a given month are included in the model only if a significant residual cross-correlation with the flow series is found for that month. In some cases the temperature may have a positive residual cross-correlation with the flow for one month, but a negative residual cross-correlation with the flow for the following month. A physical explanation can be given for this phenomenon. Consider a river where peak runoff usually occurs in May or June due to snowmelt. If the May temperatures were higher than usual, more snow would melt in May and May runoff would be higher. Because the peak flows would have already occurred in May, and most of the snow would be already melted, the flows in June would be lower than usual. Conversely, if May temperatures were lower than normal, May runoff would also be lower because less snowmelt would occur and June runoff would be higher because more snow would be left over from May. Thus for the example cited there would be a positive correlation between May temperatures and May flows, but a negative correlation between May temperatures and June flows. One could therefore include the May temperature twice in the model, with a positive coefficient for a non-delayed term, and a negative coefficient for a term with a delay of 1.

An input series is included in the model only if a significant and physically reasonable residual cross-correlation is found between the input series and the flow series. In some cases a statistically significant cross-correlation is found, but the relationship is rejected because it does not make sense in the light of a physical understanding of the process. For instance, if the statistical tests indicate a negative correlation between rainfall and runoff, it must none the less be rejected on physical grounds. Therefore the models used in this study do not necessarily have three input series. Only the series that were statistically and reasonably acceptable are included.

4) Weighting of Meteorologic Data

Where there are more than one meteorologic stations in or near a watershed, the weighted average of data from the various stations is frequently used in hydrologic studies. Methods commonly in use for determining the weighting factors are the Thiessen polygon method and the Isohyetal method (Bruce and Clark 1966).

In this study it is only necessary to calculate weighting factors in one instance; for the analysis of the Back River below Deep Rose Lake. The Back River drains an area of 98200 km^2 about midway between the Baker Lake and Contwoyto Lake meteorologic stations. Because only two weather stations are involved, a weighting factor may be calculated by including the two sets of meteorologic data separately in the transfer function-noise model, and comparing the maximum likelihood estimates of the model parameters for each input series. The weighting factors calculated in this manner may be considered to be optimal in that they minimize the modified sum of squares of the final model. In addition, the cumbersome application of the more complicated methods are avoided. For the Back River the weighting turns out to be a 53:47 weighting ratio for data from Baker Lake and Contwoyto Lake.

Results

The transfer function-noise models fitted to four Arctic River flow series are shown in Table 10 and computer listings of the estimated models are presented in Appendix 3.

Table 10

Transfer Function-Noise Models

River	Input Series	AIC
Back	(1) rainfall - Baker Lake -Contwoyto Lake (2) temperature - Baker L - Contwoyto L	328.1
Kakisa	(1) rainfall - Hay River (2) temperature- Hay R	568.1
Kazan	(1) snowfall - Ennadai Lake (2) rainfall - Ennadai L (3) temperature - Ennadai L	635.7
Tree	(1) rainfall - Coppermine (2) temperature - Coppermine	356.5

In all cases the transfer function-noise model is an improvement over the deseasonalized ARMA model in terms of the residual variance, the AIC, and the various diagnostic tests. The reduction in the skewness of the residuals of the transfer function-noise model verifies the earlier presumption that skewed residuals indicate that an important covariate input series is missing.

To illustrate the reduction in residual variance as the sophistication of the model is increased, the residual variances of various models fitted to the Tree River flow series are shown in Table 11.

Table 11

Residual Variance of Tree River Models

	Variance	%	AIC
transformed series	1.0863	100.0	595.2
residuals - deseasonalized ARMA(0,0)	0.1425	13.1	422.2
residuals - deseasonalized ARMA(1,1)	0.0770	7.1	367.8
residuals - transfer function-ARMA(1,0)	0.0632	5.8	356.5

The variance of the series after the appropriate Box-Cox transformation is performed ($\lambda = 0$) is 1.0863. This is reduced by 86.7% to 0.1425 when the series is modelled as a sinusoid (deseasonalized) using six Fourier components for the monthly means. The residual variance of this model is reduced by 45.8% by the introduction of an ARMA noise term. A further reduction of 18.3% in the residual variance is due to the transfer function term. It may be noted however that this final reduction is equal to only 1.3% of the original variance. The same relative importance of the different terms in the model is shown in the AIC. There is a large drop in the AIC when the Fourier and the ARMA terms are added to the model, and a somewhat smaller drop when the transfer function terms are included. Thus, while the transfer function term makes a statistically significant improvement to the final model, the practical importance may be limited. None the less, when the input series data is available, it makes sense to use the stochastic model that gives the best statistical goodness of fit.

As with the deseasonalized ARMA and the MAR models there may be more parameter uncertainty than is indicated by the calculated standard errors of estimation. In the future, as more data becomes available, this problem will become less severe.

4 APPLICATIONS

4.1 Uses of Box-Jenkins Models

Because of the flexibility of Box-Jenkins models and their extensions, they have been utilized in many fields of study. A number of the potential applications of Box-Jenkins modelling that are relevant to water resource problems are listed below:

1) The "extension" of hydrometric records using the dynamic relationships established between hydrometric and meteorologic time series.
2) The analysis of interventions and environmental impacts due to man-made or natural causes.
3) The estimation of missing data points.
4) The simulation of possible occurrences of a process for use in the design of engineering projects.
5) The presentation of an efficient summary of hydrometric and meteorologic data using only a few model parameters.
6) The forecasting of future events.

Examples of some of these applications are presented and discussed in this section of the report. Please note that use (5), data summary, is implicit in the modelling process and requires no further discussion. Application (6), forecasting, is left for a further study.

4.2 Extension of Hydrometric Record

Weather records have been kept in the Arctic for a much longer period of time than have hydrometric records. Based on a knowledge of the dynamic relationship between hydrometric series and meteorologic series, it is possible to give an estimate of the values the hydrometric series is likely to have taken during the period when weather data is available, but before flow records were kept. This may be thought of as an artificial extension of the hydrometric record. The true values of the unmeasured flows can of course never be obtained by this method, but likely values, given the covariate meteorologic input series, can be calculated. These estimates are simply the output of the transfer function-noise model with the noise term set to zero (the expected value of the noise term).

The value of this type of extension of record is due to the possibility of a persistence effect in the meteorologic series. A sequence of above-average or below-average precipitations will often last for a number of years. If the flow data were collected during those years, analysis of that data would give a misleading idea of the long term mean flow. That is, flow data collected during a period of atypical weather will itself be atypical. If the flow model takes into account the extra information provided by studying the long term weather data, a better understanding can be gained of the long term behaviour of the river flows.

As an illustration, the extension of record technique is applied to the Tree River flow series. The Tree River flow series used to fit the transfer function-noise model is eight years long, from 1969 to 1976. The covariate input series, rainfall and temperatures from the Coppermine weather station are 44 years long, from 1933 to 1977. The output series, predicted flow, is plotted in Figure 10. The predicted and actual measured flows are plotted on one graph for comparison purposes, in Figure 11. It can be seen that the the predicted flows follow the actual measured flows fairly well and that the largest errors are in the prediction of peak flows.

Figure 7
Predicted Flows for the Tree River from 1933 to 1977

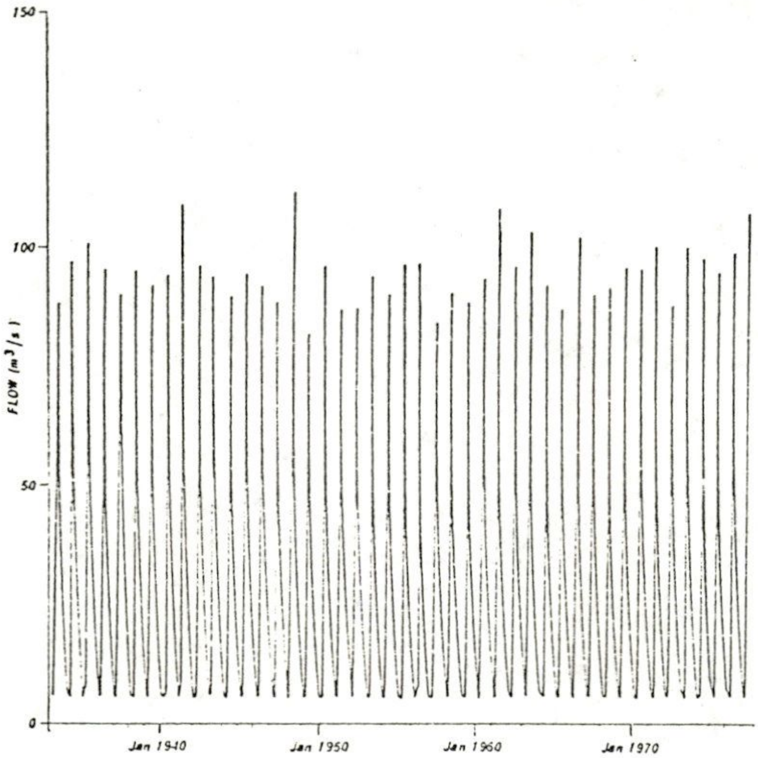
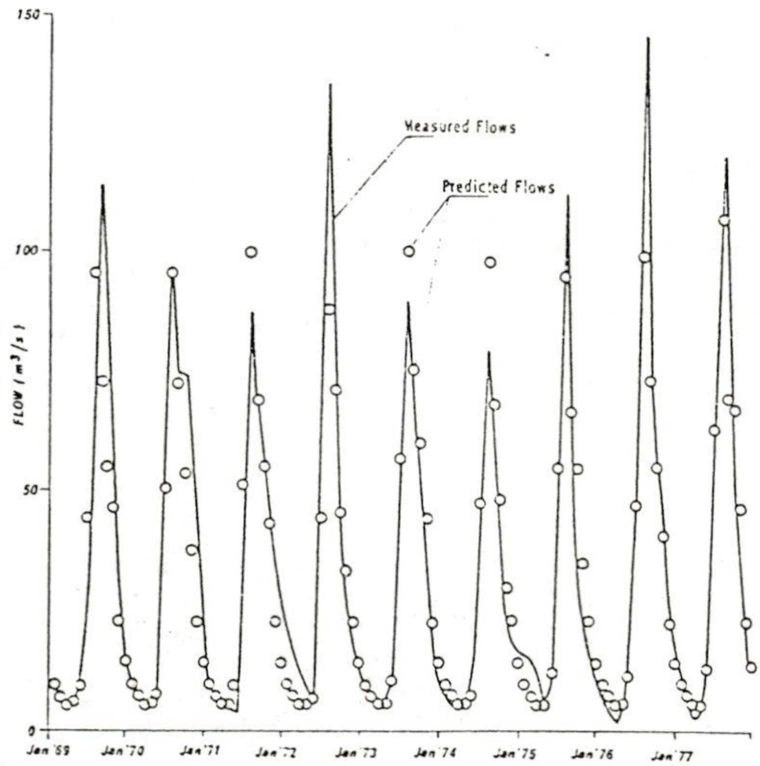


Figure 8
Predicted vs Measured Flows for the Tree River



The means of the artificially extended and measured flow series are shown in Table 12.

Table 12

Means of Tree River Series

series	mean m^3/s	S.E.	length of record
artificially extended	31.98404	1.22	44yrs
measured series	34.01510	3.35	8yrs

The mean of the artificially extended series is more than 2 m^3/s lower than the mean of the measured series. The difference is, however, less than twice the standard error and could plausibly be accounted for by random variation. Therefore there is no evidence that the mean of the measured series is not representative of the long term mean of the process.

4.3 Environmental Impact Assessment

As a result of the increasing rate of development in the North, and the heightened public awareness of environmental issues, an effort is currently being made to collect data and develop analytic tools in order to quantify the impact of human activities on the environment. Environmental impact assessment techniques can be classified as either before-the-fact or after-the-fact assessment. To date most of the effort has been in before-the-fact assessment of large projects. An example is the multi-million dollar Mackenzie Valley Pipeline study which attempted to predict the possible effects of the construction of a natural gas pipeline. Typically, the impact of a proposed project may be predicted by using physically based simulation models and/or simply relying on expert opinion.

On the other hand, after-the-fact assessment has received relatively little attention. Even in the case of large projects such as the Nelson River development in Northern Manitoba, where a considerable effort at before-the-fact assessment took place, comparitively little follow-up work has been done to verify the actual impact of the hydro project on the environment.

It is in the area of after-the-fact assessment that intervention analysis promises to be particularly useful. Although the technique is still new, intervention analysis has already been used several times to successfully model the effect of interventions on hydrologic systems. Hipel et al. (1975) used intervention analysis to determine the effects of the Aswan dam on the average annual flows of the Nile River. Hipel et al. (1977b) used an intervention model to describe the effects of the Gardiner dam on the monthly flows of the South Saskatchewan River. Hipel et al. (1977c) also model the effect of a devastating forest fire on the monthly hydrologic characteristics of the Pipers Hole River basin in Newfoundland. In particular, the sudden change in the runoff regime when the vegetation cover was destroyed by the fire, and the gradual return to normal as the basin was revegetated, was modelled parsimoniously using a minimum of estimated parameters. D'Astous and Hipel (1979) used an intervention model in the analysis of the effect of the introduction of new sewage treatment facilities on the monthly mean phosphorous levels in the Speed River at Guelph, Ontario, and in the Grand River at Cambridge, Ontario.

Thus intervention analysis can be used to model both natural and man-made environmental impacts. In fact, changes in a time series that are due to modifications to the data collection procedure, rather than to some specific physical intervention in the underlying process may also be modelled. The utility of the models is greatly increased because confidence levels can be calculated for the effect of the intervention. Furthermore, the models may also be used for forecasting and simulation.

Data Collection for Intervention Analysis

Intervention analysis is a statistical tool for determining changes in the mean level of a stochastic process. Intervention analysis can be thought of as a test for distinguishing between the effects of an external intervention on a physical process and the variation due to randomness and measurement errors. The ability of the model to detect changes can be improved by the use of an appropriate data collection program. Intervention analysis is especially useful in those cases where the standard t test cannot be used because the data is serially dependent (autocorrelated). Because the effects of an intervention may be masked by the noise term of a series, or the random occurrence of the noise may purely by chance appear to be the effect of an intervention, it is impossible to be absolutely sure that any statistical analysis will correctly model the intervention. The probability that an intervention that really does exist will be detected by the analysis is called the power of the test. Conversely, the probability that analysis will indicate an intervention where none actually exists is called the significance level. The significance level and the power of a test are inversely related, and depend also on other considerations such as sample size, residual variance of the model, and the complexity of the model. The power of the model to correctly detect an intervention increases with the sample size, and decreases with the residual variance and model complexity. Clearly, in order to correctly detect the effects of an environmental intervention, it is necessary to collect the appropriate type and amount of data.

In planning a data collection program for use in intervention analysis the environmental manager is concerned with four main questions (Lettenmaier et al 1978). These are:

1) What relative lengths of pre-intervention and post-intervention data records should be collected?

2) What sampling frequency should be used?

3) For existing data collection programmes, how long should collection continue after the intervention has occurred?

4) How does the minimum detectable change vary with (a) monitoring system design, and (b) monitoring system cost?

Questions 1 to 4a are dealt with extensively by Lettenmaier et al (1978) where, among other observations, it is noted that:

1) Data must be collected at even intervals in time.

2) Contrary to what might be intuitively expected, it is not necessarily better for the pre-intervention and post-intervention records to be of the same length. In three of four examples the power of the model was improved when the post-intervention record was substantially longer than the pre-intervention record.

3) Unless the sample size is at least 50, and preferably 100, the power of the model to detect changes will be quite low.

The question concerning program costs (4a) is more appropriately handled by experts in the particular areas of data collection. It is recommended, however, that data collection systems intended to supply data for after-the-fact environmental impact assessment should be planned with consideration of the guidelines set forth by Lettenmaier et al (1978).

Sample Application

No suitable intervention due to project construction is currently available in the Arctic, so an example is shown where the effects of an intervention due to a change in data collection procedure are tested.

In the early 1960's, new snow gauges of the Nipher type were installed at most meteorologic stations in the Northwest Territories. Some concern has been expressed (Wedel 1975) that the measured quantity of snowfall may have been affected by this change. Intervention analysis is used to test for a significant change in the mean annual snowfall measurement at the Coppermine weather station as a result of the installation of the new equipment in October 1963. The period of record is from 1933 to 1977.

Because the intervention occurs during the thirty-first year of the series, and because the change in equipment could be expected to have an immediate and permanent effect on the amount of measured snowfall, the intervention is modelled by a dynamic step response of the form:

$$z_t = \mu + \omega_0 \xi_t \tag{4.3-1}$$

where $\xi_t = 0$ for $t < 31$ and $\xi_t = 1$ for $t \geq 31$

The noise term N is identified as ARMA(1,0) by examining the ACF and PACF of the series. The complete model, including both the intervention and noise terms given in equation 4.3-2 and maximum likelihood estimates for the parameters are shown in Table 13.

$$z_t = \mu + \omega_0 \xi_t + \frac{1}{1-\phi_1 B} a_t \tag{4.3-2}$$

Table 13

Intervention Analysis Parameter Estimates

Estimated parameter	Estimate	Standard Error
ω_0	263	183
δ_1	0.571	0.124

Diagnostic checks indicate that the model does indeed give an adequate fit to the data. Because the estimate of ω_0 is smaller than twice it's standard error of estimation it is concluded (at a 5% significance level) that the evidence does not indicate that there is a change in the measured snowfall due to the installation of the new gauges. It may be noted that this does not rule out the possibility that such a change may actually exist; it is simply not detectable from the data currently available. The power of the model with the present length of record is such that the minimum detectable change, at a 95% confidence level, is about 360 mm, or approximately a 34% change in the mean level.

4.4 Estimating Missing Data Points

The problem of missing data points occurs in time series modelling because of the requirement for measurements at equally spaced intervals in time. Due to the difficulties of access to Arctic hydrometric stations, and because of problems due to low temperatures in the winter and ice conditions in the spring, sections of hydrometric record are sometimes lost. In order to carry on with time series analysis of the data it is necessary to have a method for filling in the missing data points with reasonable estimates. Some of the traditional methods currently in use include graphical methods and polynomial interpolation. No matter how powerful the analytic tools used for the estimation of missing data points, the true value of the unmeasured point can never be precisely known. The problem becomes one of recognizing, from the incomplete data, certain patterns or relationships between points, and using these to deduce what value the missing point may most probably have taken. An experienced analyst may develop a good deal of skill at this task, but the experience needed to perform competently is gained from years of working in the field of hydrometric analysis.

A newly developed method for filling of missing data points, based on the intervention model, is particularly suitable for use in time series analysis. Some advantages of this method are;

1) the method does not depend on the experience of the user, so it can be applied by novice users;

2) only part of the series is needed to fit the model;

3) confidence limits can be calculated for the estimated points;

4) the method can be used to fill more than one missing point at a time;

5) the method can be used to fill points anywhere in the series, including the initial and final points;

6) diagnostic checks are available to confirm the applicability of the model.

The model used for the filling of missing data points is a special case of the transfer function-noise intervention model described in Section 2.11. The model used to fill one missing data point at time T may be written as:

$$z'_t = \omega_0 \xi_t + N_t \tag{4.5-1}$$

with z'_T set to zero

At time T the model reduces to:

$$-\omega_0 = N_T \tag{4.5-2}$$

The maximum likelihood estimate of $-\omega$ constitutes an estimate of the missing observation z_T. Because this estimate depends only on the ARMA noise term N the autocorrelation structure of the series is preserved.

If more than one observation must be filled, the model is extended by simply adding more intervention terms. For I missing points the model is:

$$z'_t = \sum_{i=1}^{I} \omega_0^{(i)} \xi_{ti} + N_t \qquad (4.5\text{-}3)$$

The model can also be extended to the general case of multiple missing points, multiple interventions due to known external causes, and multiple input series. The performance of the intervention model data filling technique is assessed by D'Astous and Hipel (1979) by estimating observations where the actual historical values are known. The estimated values were in all cases within one standard error of measured values. This result is consistent with the stochastic nature of the processes involved.

In this study the intervention model method is used to provide estimates for unobserved flows for several rivers in the Northwest Territories. Two estimates are obtained for each point, one using only the flow series itself in the intervention model, and the other including meteorologic input series in a general intervention model. These estimates are compared to estimates supplied by the Water Survey of Canada. The results are listed in Table 14. As a further reference for the performance of the technique, the values of known historical measurements on the Tree River are estimated. These values are within one standard error of the true values. For the estimates of actual missing values, the values obtained by the intervention model are in good agreement with the estimates obtained by the Water Survey of Canada, with the Water Survey estimates lying well within the 95% confidence interval of the intervention model estimates. As would be expected, the general intervention models which include the meteorologic input series yield estimates with tighter confidence bounds than do the more simple intervention models. This is due to the extra information included in the general intervention models.

Table 14

Estimates of Missing Data Points

(95% confidence limits in brackets)

River	Date	W.S.C.	ARMA	Transfer Function-Noise
Back	Oct 67	558	545	606
			(270-1100)	(363-1010)
Back	Jun 71	273	384	238
			(66.5-2220)	(121-474)
Back	Jul 71	1660	1740	1560
			(1040-2890)	(1070-2290)
Kakisa	Jul 71	18.4	25.8	26.3
			(18.9-35.2)	(19.4-35.6)
Kakisa	Aug 71	17.0	17.2	18.1
			(12.6-23.5)	(13.5-24.3)
Kazan	Dec 76	84.1	82.9	82.1
			(63.4-108)	(65.7-103)
Quoich	Jul 72	869	310	537
			(141-636)	(186-1550)
Tree	Jul 70	93.5*	125	112
			(84.7-184)	(79.9-158)
Tree	Apr 71	4.05*	3.64	3.59
			(2.47-5.36)	(2.59-4.97)

Flows in m^3/s

* measured value - not an estimate

4.5 Simulation and Engineering Design

In the design of a water resources project such as a reservoir the design engineer would ideally like to know what flows into the reservoir will occur during the design life of the project. Because it is impossible to know what flow sequences will occur in the future, the design must be based on a knowledge of the past flows. However it is certain that the historic sequence of flows will not occur again in the future. A common approach to this problem is to use simulation. Simulated flow sequences are generated with the same statistical properties as the historical sequence, and used to test the proposed design on the computer. In this way, a variety of alternative designs can be compared at relatively low cost. The designs are compared based on their performances under simulated flow conditions, conditions that could have occurred in the past, and are just as likely to occur in the future.

Recent advances in Box-Jenkins model simulation techiques include exact simulation methods that eliminate bias in the initial values, as well as a procedure for incorporating parameter uncertainty into the simulation (McLeod and Hipel 1978a). Because of the short period of hydrometric record in the Arctic, and the resulting high degree of parameter uncertainty, it is strongly suggested that these techniques be used in the simulation of Arctic river flow series. It is also stressed that in the interest of reliable engineering design, data collection should be continued and extended in order to reduce uncertainty in the models.

4.6 Regional Analysis

A regional hydrologic analysis is one that would, for instance, link the type of model that best fits a particular river flow series to such physical factors as basin area, physiographic region, and latitude. However, with data from only 16 stations covering an immense area that includes some 22 major physiographic regions (GSC-Map 1254A, 1967, "Physiographic Regions of Canada") such a general analysis is not possible; the spatial distribution of the data is simply not adequate. The only observations that can be made at this time are rather basic. For instance, basin storage, which is reflected in the autocorrelation structure of the flow series, increases with the size of the river basin and the number of lakes. Another example is that spring runoff occurs later at higher latitudes. More data will be available during the next 10 to 20 years from the many stations currently having only 1-3 years of data. At that time a regional analysis may yield more interesting results.

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APPENDIX I

ARMA Models for 16 Arctic River Series

In Section 3.1 of the accompanying paper deseasonalized ARMA models are fitted to 16 Arctic river series. These models are presented here in the form of computer listings of A.I. McLeod's USES program for Box-Jenkins models. The computer listings are more or less self-explanatory, but a brief description of some of the special notations is included in this appendix. For further details please see McLeod, A.I., Box-Jenkins Computer Program Manual, University of Waterloo, 1979.

Special Notation

SARIMA (p, d, q) (P, D, Q)s - denotes the general seasonal ARIMA model of order (p, d, q) (P, D, Q) with s seasons, as defined in Box and Jenkins (1970) where

p is the order of the nonseasonal AR operator $\phi(B)$

d is the order of nonseasonal differencing

q is the order of the nonseasonal MA operator $\theta(B)$

P is the order of the seasonal AR operator $\Phi(B)$

D is the order of the seasonal differencing

Q is the order of the seasonal MA operator $\Theta(B)$

The nonseasonal ARIMA model is described in Section 2.5. The seasonal ARIMA model contains the nonseasonal ARIMA model as a special case, and also includes the seasonal terms of orders P, D and Q. The deseasonalized ARMA (p, q) models used in the modelling of monthly Arctic river data are the special case of the SARIMA model denoted by:

SARIMA (p, 0, q) (0, 0, 0) 12

The seasonal operators are defined below:

The seasonal AR operator is defined by :

$$\Phi(B) = 1 - \Phi_1 B^s - \Phi_2 B^{2s} - \ldots - \Phi_P B^{Ps}$$

where Φ_i = the ith seasonal AR parameter.

The seasonal AR parameter may be written alternatively as a nonseasonal AR parameter as shown below, for $s = 12$ (i.e. 12 months in a year): $\Phi_1 = \phi_{12}$, $\Phi_2 = \phi_{12}$, etc.

Seasonal differencing is defined by:

$$Z_t \, B^D = Z_{t-sD}$$

Seasonal differencing subtracts values of a time series that are separated by s lags.

The seasonal MA operator is defined by:

$$\Theta(B) = 1 - \Theta_1 \, B^s - \Theta_2 \, B^{2s} - \ldots - \Theta_Q \, B^{Qs}$$

where Θ_i = the ith seasonal AR parameter.

As with the seasonal AR parameters, the seasonal MA parameters can be alternatively written as nonseasonal MA parameters, that is:

$$\Theta_1 = \theta_{12}, \quad \Theta_2 = \theta_{24}, \quad \text{etc.}$$

BETA parameters - denotes the vector of estimated AR and MA parameters which are listed in the order $(\phi_1, \phi_2, \ldots, \phi_p, \theta_1, \theta_2, \ldots, \theta_q, \Phi_1, \Phi_2, \ldots, \Phi_P, \Theta_1, \Theta_2, \ldots, \Theta_Q)$. For an ARMA (1,1) model BETA would be (ϕ_1, θ_1).

Diagnostic Checks

Statistical tests of the model assumption are calculated under the heading of RESIDUAL ANALYSIS. Model inadequacy is indicated by evidence against the assumptions.

1) Test for SKEWNESS of residuals
   - evidence against assumption of normally distributed noise term if the significance level, SL, of the G1 statistic is less than 0.05.
2) Tests for HETEROSCEDASTICITY and TRENDS in the variance of the residuals
   - evidence against assumption of a homoscedastic noise term if the CHI statistic is greater than twice its standard error, SE(CHI).
3) Test for RESIDUAL AUTOCORRELATIONS
   - evidence against assumption of a white noise series if the residual autocorrelation at lag L, RA(L), is greater than twice its standard error, SE(L). It is especially important that there be no large residual autocorrelations in the low lags (i.e. lags 1 - 6) as this is an indication of gross model inadequacy.

06JC002 THELON R AT BEVERLY L 72-76 FLOW

SARIMA(1, 0, 1)(0, 0, 0)12

LENGTH OF THE INPUT TIME SERIES = 60

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
5.80525616D 04	1.66507942D-01	4.44485573D 02

BOX-COX TRANSFORMATION PARAMETERS

LAMDA	CONS
0.	0.

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS
(TRANSFORMED SERIES)

SEASON	MEAN	S.D.
1	3.1773410 00	1.0000000D 00
2	2.871136D 00	1.0000000D 00
3	2.549443D 00	1.0000000D 00
4	2.650087D 00	1.0000000D 00
5	4.256726D 00	1.0000000D 00
6	6.439538D 00	1.0000000D 00
7	6.043775D 00	1.0000000D 00
8	5.674070D 00	1.0000000D 00
9	5.702863D 00	1.0000000D 00
10	4.891961D 00	1.0000000D 00
11	4.229948D 00	1.0000000D 00
12	3.475812D 00	1.0000000D 00

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 0

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.2798	C.2115
-0.3664	C.2050

CORRELATION MATRIX OF BETA

1.000	
0.810	1.000

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
0.1743	0.546478

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
0.033437	0.120853

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
-0.025328	0.010542

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	0.02197	0.01323	0.03043
2	-0.02600	0.01808	0.07378
3	-0.14037	0.12796	1.35962
4	0.09547	0.12833	1.96507
5	0.13591	0.12907	3.21447
6	0.19107	0.12909	5.72939
7	0.08414	0.12910	6.22628
8	0.02948	0.12910	6.28846
9	-0.04245	0.12910	6.41993
10	-0.06547	0.12910	6.73880
11	-0.06993	0.12910	7.11011
12	-0.13575	0.12910	8.53829
13	0.03359	0.12910	8.62758
14	0.16705	0.12910	10.88432
15	-0.08256	0.12910	11.44780
16	-0.19223	0.12910	14.57202
17	-0.29844	0.12910	22.27731
18	0.02948	0.12910	22.35429
19	0.09012	0.12910	23.09116
20	0.04748	0.12910	23.30084
21	-0.09613	0.12910	24.18231
22	0.06926	0.12910	24.65191
23	-0.03685	0.12910	24.78841
24	-0.27988	0.12910	32.88273

06KC003 DUBAWNT R BL MARG. L 69-76 FLOW

SARIMA(1, 0, 1)(0, 0, 0)12

LENGTH OF THE INPUT TIME SERIES = 96

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
2.13754080D 05	4.55194414D-01	7.93990395D 02

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS

SEASON	MEAN	S.D.
1	2.473112D 02	9.199913D 01
2	2.164112D 02	8.677019D 01
3	1.948200D 02	7.629474D 01
4	2.029962D 02	4.632050D 01
5	2.369412D 02	3.882725D 01
6	4.536375D 02	1.296234D 02
7	5.121737D 02	9.509811D 01
8	4.541325D 02	7.731338D 01
9	4.279400D 02	5.498897D 01
10	3.833050D 02	6.358763D 01
11	3.225237D 02	5.360097D 01
12	2.553475D 02	6.585235D 01

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 6

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.4660	0.1142
-0.5137	0.1108

CORRELATION MATRIX OF BETA

1.000	
0.612	1.000

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
0.8997	0.001025

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
0.002222	0.001078

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
-0.009215	0.005209

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	-0.02815	0.02443	0.14410
2	-0.04658	0.02517	0.36121
3	0.03048	0.09917	0.45517
4	0.06965	0.09922	0.95121
5	0.02150	0.10188	0.99900
6	0.02694	0.10189	1.07485
7	0.09863	0.10205	2.10312
8	-0.01770	0.10205	2.13660
9	0.02195	0.10206	2.18869
10	-0.02437	0.10206	2.25365
11	0.09798	0.10206	3.31627
12	0.00999	0.10206	3.32745
13	0.08133	0.10206	4.07809
14	0.08851	0.10206	4.97684
15	-0.07848	0.10206	5.69222
16	0.10318	0.10206	6.94426
17	-0.10826	0.10206	8.34003
18	0.01142	0.10206	8.35575
19	0.06628	0.10206	8.89245
20	-0.00580	0.10206	8.89661
21	-0.00363	0.10206	8.89826
22	-0.06696	0.10206	9.46822
23	-0.02270	0.10206	9.53465
24	0.05290	0.10206	9.90033

06LA001 KAZAN R AT ENNADAI L 67-76 FLOW

SARIMA(1, 0, 1)(0, 0, 0)12

LENGTH OF THE INPUT TIME SERIES = 120

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
2.52944970D 04	1.92082303D-02	6.74102048D 02

BOX-COX TRANSFORMATION PARAMETERS

LAMDA	CONS
0.	0.

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS
(TRANSFORMED SERIES)

SEASON	MEAN	S.D.
1	4.419319D 00	1.0000000D 00
2	4.331659D 00	1.0000000D 00
3	4.269649D 00	1.0000000D 00
4	4.277634D 00	1.0000000D 00
5	4.484332D 00	1.0000000D 00
6	5.105759D 00	1.0000000D 00
7	5.114013D 00	1.0000000D 00
8	4.971702D 00	1.0000000D 00
9	4.838622D 00	1.0000000D 00
10	4.765935D 00	1.0000000D 00
11	4.636050D 00	1.0000000D 00
12	4.503418D 00	1.0000000D 00

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 0

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.8710	0.0481
-0.3244	0.0926

CORRELATION MATRIX OF BETA

1.000	
0.362	1.000

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
1.2979	0.000014

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
0.651194	0.129010

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
0.000713	0.003727

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	-0.01219	0.02579	0.01827
2	-0.05540	0.06900	0.39906
3	0.06845	0.08464	0.98541
4	0.10344	0.08509	2.33582
5	-0.04884	0.08710	2.63947
6	0.12784	0.08801	4.73813
7	0.03235	0.08885	4.87371
8	-0.00356	0.08943	4.87537
9	0.01080	0.08989	4.89076
10	-0.12625	0.09023	7.01216
11	-0.00113	0.09048	7.01233
12	-0.05209	0.09068	7.38020
13	0.02615	0.09083	7.47376
14	-0.11274	0.09094	9.22934
15	-0.00060	0.09102	9.22939
16	-0.01342	0.09109	9.25474
17	-0.04967	0.09113	9.60538
18	0.01418	0.09117	9.63425
19	-0.09750	0.09120	11.01219
20	-0.05853	0.09122	11.51377
21	0.02084	0.09124	11.57801
22	0.03213	0.09125	11.73225
23	-0.16343	0.09126	15.76579
24	0.18015	0.09127	20.71488

06LC001 KAZAN R AT KAZAN FALLS 72-76 FLOW

SARIMA(1, 0, 0)(0, 0, 0)12

LENGTH OF THE INPUT TIME SERIES = 60

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
1.673847170 05	5.932439850-02	5.060223340 02

BOX-COX TRANSFORMATION PARAMETERS

LAMDA	CONS
0.	0.

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS
(TRANSFORMED SERIES)

SEASON	MEAN	S.D.
1	4.3417100 00	1.0000000 00
2	4.0036310 00	1.0000000 00
3	3.8008760 00	1.0000000 00
4	4.1667260 00	1.0000000 00
5	5.6230080 00	1.0000000 00
6	6.9711980 00	1.0000000 00
7	6.9187390 00	1.0000000 00
8	6.5295130 00	1.0000000 00
9	6.2439340 00	1.0000000 00
10	5.8054950 00	1.0000000 00
11	5.2725510 00	1.0000000 00
12	4.7143140 00	1.0000000 00

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 0

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.8922	0.0583

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
0.0568	0.843957

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
0.006171	0.128433

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
0.013676	0.010542

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	0.06409	0.11518	0.25901
2	0.12318	0.11815	1.23216
3	-0.00170	0.12047	1.23234
4	-0.01005	0.12228	1.23905
5	-0.15872	0.12370	2.94289
6	-0.09922	0.12482	3.62112
7	0.18809	0.12571	6.10417
8	-0.03051	0.12641	6.17076
9	-0.07959	0.12696	6.63284
10	-0.09694	0.12740	7.33199
11	-0.05847	0.12775	7.59155
12	-0.05671	0.12802	7.84081
13	0.17542	0.12824	10.27653
14	0.00059	0.12842	10.27655
15	0.14487	0.12856	12.01158
16	-0.12613	0.12867	13.35650
17	-0.12275	0.12876	14.66001
18	-0.11058	0.12883	15.74305
19	0.01334	0.12888	15.75920
20	0.01116	0.12893	15.77079
21	0.01449	0.12896	15.79082
22	0.19936	0.12899	19.68175
23	-0.01896	0.12901	19.71788
24	-0.26399	0.12903	26.91936

06MB001 QUOICH RIVER 72-76 FLOW

SARIMA(0, 0, 1)(0, 0, 0)12

LENGTH OF THE INPUT TIME SERIES = 60

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
1.650135860 04	3.046457040-01	3.670112060 02

BOX-COX TRANSFORMATION PARAMETERS

LAMDA	CONS
0.	0.

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS
(TRANSFORMED SERIES)

SEASON	MEAN	S.D.
1	1.377184D 00	1.000000D 00
2	9.738483D-01	1.000000D 00
3	1.135750D 00	1.000000D 00
4	1.446611D 00	1.000000D 00
5	2.959159D 00	1.000000D 00
6	5.972809D 00	1.000000D 00
7	6.047031D 00	1.000000D 00
8	5.200575D 00	1.000000D 00
9	5.476098D 00	1.000000D 00
10	4.543771D 00	1.000000D 00
11	3.515768D 00	1.000000D 00
12	2.122093D 00	1.000000D 00

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 0

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
-0.6800	0.0947

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
-0.2338	0.420922

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
-0.033924	0.091839

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
0.002541	0.010542

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	0.09994	0.08779	0.62973
2	0.06650	0.11191	0.91333
3	0.04682	0.12145	1.05637
4	0.10113	0.12562	1.73575
5	-0.06707	0.12750	2.04004
6	0.06606	0.12836	2.34069
7	0.05477	0.12876	2.55121
8	0.09295	0.12894	3.16925
9	0.06076	0.12903	3.43856
10	0.10866	0.12907	4.31696
11	-0.20298	0.12908	7.44497
12	-0.03474	0.12909	7.53849
13	0.07628	0.12910	7.99905
14	0.00659	0.12910	8.00257
15	0.03859	0.12910	8.12564
16	-0.03008	0.12910	8.20216
17	-0.06030	0.12910	8.51672
18	0.04559	0.12910	8.70080
19	0.06587	0.12910	9.09452
20	-0.04615	0.12910	9.29256
21	-0.14521	0.12910	11.30396
22	-0.00148	0.12910	11.30417
23	-0.11275	0.12910	12.58233
24	-0.23091	0.12910	18.09189

07OB001 HAY RIVER NR HAY RIVER 64-76 FLOW

SARIMA(1, 0,10)(0, 0, 0)12

LENGTH OF THE INPUT TIME SERIES = 156

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
2.20683599D 04	5.55518070D-01	8.30518891D 02

BOX-COX TRANSFORMATION PARAMETERS

LAMDA	CONS
0.	0.

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS
(TRANSFORMED SERIES)

SEASON	MEAN	S.D.
1	1.1453100 00	3.5357140-01
2	7.3718150-01	5.1061130-01
3	4.0603350-01	5.5519230-01
4	3.4302250 00	1.6020470 00
5	6.0407630 00	3.1018260-01
6	5.3232070 00	3.3615170-01
7	4.9311280 00	4.9482440-01
8	4.4201340 00	8.5569410-01
9	3.8844400 00	1.0300650 00
10	3.6649180 00	8.5003180-01
11	2.9363280 00	7.5243590-01
12	1.9898280 00	7.0159750-01

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 6

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.8591	0.0567
0.4100	0.0966
0.	0.
0.	0.
0.	0.
0.	0.
0.	0.
0.	0.
0.	0.
0.	0.
-0.2042	0.0744

CORRELATION MATRIX OF BETA

1.000		
0.680	1.000	
0.311	0.286	1.000

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
-0.2778	0.145845

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
0.080011	0.060443

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
0.003441	0.002514

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	0.00691	0.03635	0.00760
2	-0.12432	0.07137	2.48131
3	0.07407	0.07294	3.36516
4	-0.00234	0.07348	3.36604
5	0.07466	0.07459	4.27591
6	0.04553	0.07580	4.61658
7	-0.07345	0.07684	5.50908
8	0.00179	0.07766	5.50961
9	0.01054	0.07829	5.52824
10	0.02937	0.03642	5.67390
11	-0.10321	0.07199	7.48475
12	-0.09841	0.07644	9.14238
13	0.16481	0.07833	13.82435
14	0.15867	0.07920	18.19421
15	0.02232	0.07960	18.28131
16	-0.13802	0.07978	21.63510
17	-0.01514	0.07988	21.67573
18	-0.00510	0.07994	21.68037
19	-0.09997	0.07998	23.47829
20	-0.02746	0.07835	23.61498
21	0.04460	0.07908	23.97816
22	-0.07251	0.07967	24.94528
23	0.13134	0.07992	28.14206
24	-0.01834	0.08002	28.20489

07RD001 LOCKHART R AT ARTIL. L 63-76 FLOW

SARIMA(1, 0, 1)(0, 0, 1)12

LENGTH OF THE INPUT TIME SERIES = 168

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
6.24786215D 03	2.53751310D-03	6.41493149D 02

BOX-COX TRANSFORMATION PARAMETERS

LAMDA	CONS
0.	0.

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS
(TRANSFORMED SERIES)

SEASON	MEAN	S.D.
1	4.809444D 00	1.0000000D 00
2	4.694966D 00	1.0000000D 00
3	4.572160D 00	1.0000000D 00
4	4.474510D 00	1.0000000D 00
5	4.462068D 00	1.0000000D 00
6	4.676423D 00	1.0000000D 00
7	4.866900D 00	1.0000000D 00
8	4.998442D 00	1.0000000D 00
9	5.045535D 00	1.0000000D 00
10	5.014313D 00	1.0000000D 00
11	4.953758D 00	1.0000000D 00
12	4.883296D 00	1.0000000D 00

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 0

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.9135	0.0330
-0.3964	0.0737
-0.1371	0.0773

CORRELATION MATRIX OF BETA

1.000		
0.277	1.000	
0.148	0.041	1.000

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
0.4232	0.024583

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
0.153302	0.108476

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
-0.002820	0.002250

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	0.04062	0.02787	0.28212
2	0.08055	0.06084	1.39853
3	-0.00264	0.07236	1.39973
4	0.00562	0.07235	1.40524
5	-0.03380	0.07378	1.60542
6	0.03691	0.07418	1.84564
7	0.03852	0.07475	2.10882
8	-0.03703	0.07513	2.35361
9	0.02322	0.07548	2.45050
10	0.03358	0.07575	2.65435
11	-0.12066	0.07599	5.30260
12	0.00374	0.01055	5.30516
13	-0.01196	0.07634	5.33151
14	-0.04939	0.07648	5.78386
15	0.00221	0.07659	5.78477
16	-0.10557	0.07668	7.87871
17	-0.02141	0.07676	7.96540
18	-0.07131	0.07683	8.93372
19	-0.02434	0.07688	9.04729
20	-0.02134	0.07692	9.13517
21	-0.01792	0.07696	9.19753
22	0.01355	0.07699	9.23343
23	-0.07564	0.07702	10.36034
24	0.05670	0.07622	10.99799

07UC001 KAKISA R AT KAKISA L 64-76 FLOW

SARIMA(1, 0, 3)(0, 0, 0)12

LENGTH OF THE INPUT TIME SERIES = 156

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
5.30631493D 03	4.22915291D-02	5.84180313D 02

BOX-COX TRANSFORMATION PARAMETERS

LAMDA	CONS
0.	0.

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS
(TRANSFORMED SERIES)

SEASON	MEAN	S.D.
1	2.8032620 00	1.0000000 00
2	2.545564D 00	1.000000D 00
3	2.279656D 00	1.000000D 00
4	2.319986D 00	1.000000D 00
5	3.886125D 00	1.000000D 00
6	4.586437D 00	1.000000D 00
7	4.055811D 00	1.000000D 00
8	3.724303D 00	1.000000D 00
9	3.682545D 00	1.000000D 00
10	3.655167D 00	1.000000D 00
11	3.407193D 00	1.000000D 00
12	3.115734D 00	1.000000D 00

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 0

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.9083	0.0386
-0 .2257	0.0829
0.	0.
0.2275	0.0807

CORRELATION MATRIX OF BETA

1.000		
0.404	1.000	
0.342	0.139	1.000

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
0.6332	0.002192

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
0.175848	0.096705

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
0.004560	0.002514

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	-0.02255	0.02515	0.08090
2	-0.08339	0.06559	1.19395
3	-0.00741	0.02524	1.20281
4	-0.02510	0.07120	1.30496
5	-0.00055	0.07444	1.30501
6	0.01895	0.07652	1.36401
7	0.06035	0.07623	1.96648
8	0.01750	0.07749	2.01746
9	-0.02953	0.07833	2.16367
10	-0.02119	0.07822	2.23949
11	-0.04422	0.07871	2.57181
12	-0.06931	0.07899	3.39409
13	-0.01564	0.07910	3.43623
14	0.12134	0.07931	5.99195
15	0.12859	0.07944	8.88265
16	0.01164	0.07954	8.90649
17	0.11747	0.07964	11.35325
18	-0.08824	0.07971	12.74400
19	-0.02811	0.07977	12.88613
20	0.04717	0.07982	13.28931
21	0.04275	0.07987	13.62292
22	0.02203	0.07990	13.71220
23	-0.01734	0.07993	13.76791
24	0.04534	0.07995	14.15181

10ED001 LIARD R AT FORT LIARD 60-76 FLOW

SARIMA(1, 0, 6)(0, 0, 0)12

LENGTH OF THE INPUT TIME SERIES = 204

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
1.29801408D 07	6.85218643D-01	2.31240547D 03

BOX-COX TRANSFORMATION PARAMETERS

LAMDA	CONS
0.	0.

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS
(TRANSFORMED SERIES)

SEASON	MEAN	S.D.
1	5.965423D 00	2.277805D-01
2	5.785219D 00	2.367709D-01
3	5.694918D 00	2.184731D-01
4	6.074430D 00	4.855309D-01
5	8.046683D 00	3.369769D-01
6	8.752478D 00	2.090166D-01
7	8.451651D 00	3.031838D-01
8	7.939480D 00	3.536124D-01
9	7.677683D 00	2.628292D-01
10	7.352265D 00	1.785471D-01
11	6.556445D 00	2.729607D-01
12	6.166901D 00	2.373222D-01

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 6

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.5331	0.0593
0.	0.
0.	0.
0.	0.
0.	0.
0.	0.
-0.1654	0.0691

CORRELATION MATRIX OF BETA

1.000	
0.036	1.000

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
-0.0213	0.898208

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
0.174288	0.080153

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
-0.002699	0.001681

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	-0.04779	0.03726	0.47274
2	0.06205	0.06248	1.27376
3	0.05138	0.06796	1.82571
4	0.01583	0.06944	1.87836
5	0.02149	0.06985	1.97585
6	0.00511	0.01158	1.98138
7	0.02592	0.07000	2.12468
8	-0.06555	0.07001	3.04605
9	-0.04452	0.07001	3.47315
10	-0.00588	0.07001	3.48063
11	-0.05290	0.07001	4.09004
12	0.00699	0.06907	4.10074
13	0.13071	0.07001	7.86001
14	-0.17325	0.07001	14.49888
15	0.09746	0.07001	16.61102
16	-0.00476	0.07001	16.61609
17	-0.04058	0.07001	16.98621
18	0.00450	0.06999	16.99078
19	0.06816	0.07001	18.04601
20	-0.03184	0.07001	18.27759
21	0.05708	0.07001	19.02588
22	0.10543	0.07001	21.59260
23	-0.07117	0.07001	22.76847
24	-0.04577	0.07001	23.25757
25	0.13493	0.07001	27.53497
26	0.05544	0.07001	28.26054
27	0.00257	0.07001	28.26211
28	0.03355	0.07001	28.53083
29	-0.09956	0.07001	30.91105
30	0.05207	0.07001	31.56594
31	-0.00222	0.07001	31.56713
32	0.05669	0.07001	32.35246
33	-0.05515	0.07001	33.09990
34	0.04163	0.07001	33.52837
35	-0.06673	0.07001	34.63565
36	-0.14011	0.07001	39.54596
37	0.04706	0.07001	40.10329
38	-0.02757	0.07001	40.29568
39	-0.07620	0.07001	41.77433
40	0.03927	0.07001	42.16958
41	-0.06395	0.07001	43.22405
42	-0.01625	0.07001	43.29258
43	-0.01274	0.07001	43.33496
44	[illegible]	0.07001	43.33497

10FB001 MACKENZIE R NR F PROV. 64-73 FLOW

SARIMA(6, 0, 0)(0, 0, 1)12

LENGTH OF THE INPUT TIME SERIES = 120

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
1.94589905D 07	3.06502207D-01	1.49555936D 03

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS

SEASON	MEAN	S.D.
1	2.160009D 03	5.350472D 02
2	1.974816D 03	5.010850D 02
3	1.820772D 03	4.439752D 02
4	1.860133D 03	4.300061D 02
5	4.825190D 03	7.908666D 02
6	6.722420D 03	7.589968D 02
7	7.039568D 03	1.096209D 03
8	6.651627D 03	1.167084D 03
9	6.243863D 03	1.165695D 03
10	5.739826D 03	1.133044D 03
11	3.768971D 03	9.031658D 02
12	2.417126D 03	4.276578D 02

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 6

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.6740	0.0634
0.	0.
0.	0.
0.	0.
0.	0.
0.2063	0.0640
-0.1354	0.0918

CORRELATION MATRIX OF BETA

1.000		
-0.478	1.000	
0.023	0.138	1.000

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
-0.3381	0.118746

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
-0.000013	0.000058

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
-0.001887	0.003727

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	-0.05869	0.06569	0.42377
2	-0.01257	0.08067	0.44337
3	0.01358	0.08663	0.46645
4	0.04577	0.08920	0.73088
5	-0.11423	0.09035	2.39209
6	-0.03906	0.06857	2.58803
7	-0.02676	0.08304	2.68079
8	-0.00870	0.08730	2.69068
9	0.02637	0.08895	2.78242
10	0.03916	0.08981	2.98655
11	0.11967	0.09035	4.91008
12	-0.00810	0.01228	4.91897
13	0.00890	0.08924	4.92981
14	-0.07822	0.08942	5.77485
15	-0.01262	0.08982	5.79706
16	0.09930	0.09021	7.18513
17	-0.01037	0.09052	7.20041
18	-0.05052	0.09058	7.56666
19	0.09500	0.09056	8.87473
20	0.02566	0.09056	8.97114
21	-0.01055	0.09061	8.98760
22	0.03408	0.09070	9.16106
23	-0.14685	0.09080	12.41588
24	-0.02652	0.08982	12.52312

10GC001 MACKENZIE R AT F SIMP. 65-76 FLOW

SARIMA(1, 0, 0)(0, 0, 0)12

LENGTH OF THE INPUT TIME SERIES = 144

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
5.871052730 07	5.442487650-01	1.912237550 03

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS

SEASON	MEAN	S.D.
1	2.734699D 03	5.669348D 02
2	2.443507D 03	5.288892D 02
3	2.290361D 03	4.877128D 02
4	2.770567D 03	5.919110D 02
5	9.809427D 03	2.030682D 03
6	1.418438D 04	1.319113D 03
7	1.290776D 04	1.936923D 03
8	1.027665D 04	1.508630D 03
9	8.688552D 03	1.052168D 03
10	7.341142D 03	8.424859D 02
11	4.354659D 03	9.063236D 02
12	2.889970D 03	3.166064D 02

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 6

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.6737	0.0616

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
0.0115	0.953310

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
0.000054	0.000027

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
0.002684	0.002835

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	-0.04522	0.05614	0.30067
2	0.01642	0.07227	0.34058
3	-0.00024	0.07851	0.34059
4	0.07832	0.08118	1.26176
5	-0.03399	0.08236	1.43651
6	0.10889	0.08289	3.24300
7	0.01629	0.08313	3.28373
8	-0.02176	0.08324	3.35692
9	0.03634	0.08329	3.56261
10	0.01090	0.08331	3.58127
11	0.08678	0.08332	4.77171
12	0.06297	0.08333	5.40324
13	0.11766	0.08333	7.62519
14	0.01460	0.08333	7.65968
15	0.17278	0.08333	12.52490
16	0.00413	0.08333	12.52771
17	-0.05201	0.08333	12.97548
18	0.07228	0.08333	13.84725
19	0.07494	0.08333	14.79175
20	0.04623	0.08333	15.15405
21	0.11285	0.08333	17.33097
22	0.06189	0.08333	17.99106
23	-0.01125	0.08333	18.01306
24	-0.04464	0.08333	18.36214

10JA002 CAMSELL R AT CLUT L 65-76 FLOW

SARIMA(1, 0, 1)(0, 0, 0)12

LENGTH OF THE INPUT TIME SERIES = 144

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
5.75520338D 03	5.26092051D-03	5.63078676D 02

BOX-COX TRANSFORMATION PARAMETERS

LAMDA	CONS
0.	0.

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS
(TRANSFORMED SERIES)

SEASON	MEAN	S.D.
1	4.458441D 00	1.0000000D 00
2	4.375898D 00	1.0000000D 00
3	4.284617D 00	1.0000000D 00
4	4.190382D 00	1.0000000D 00
5	4.241290D 00	1.0000000D 00
6	4.552829D 00	1.0000000D 00
7	4.679392D 00	1.0000000D 00
8	4.648349D 00	1.0000000D 00
9	4.597222D 00	1.0000000D 00
10	4.533636D 00	1.0000000D 00
11	4.493153D 00	1.0000000D 00
12	4.450419D 00	1.0000000D 00

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 0

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.9088	0.0362
-0.4147	0.0789

CORRELATION MATRIX OF BETA

1.000	
0.276	1.000

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
0.9632	0.000083

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
-0.095270	0.119089

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
-0.000187	0.002835

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	0.01885	0.03141	0.05224
2	0.04543	0.06481	0.35782
3	0.03694	0.07805	0.56128
4	-0.00930	0.07805	0.57427
5	0.00366	0.07976	0.57629
6	0.07109	0.08018	1.34616
7	0.05574	0.08083	1.82298
8	-0.00669	0.08124	1.82990
9	-0.07143	0.08162	2.62441
10	-0.02210	0.08192	2.70105
11	-0.07479	0.08217	3.58515
12	0.05119	0.08237	4.00246
13	-0.04961	0.08254	4.39752
14	0.03926	0.08268	4.64676
15	-0.08874	0.08279	5.93010
16	-0.11357	0.08289	8.04868
17	0.00492	0.08296	8.05268
18	-0.03259	0.08303	8.22939
19	0.00240	0.08308	8.23086
20	0.09401	0.08313	9.72932
21	0.04891	0.08316	10.13823
22	0.01686	0.08319	10.18721
23	-0.16925	0.08322	15.16457
24	0.08724	0.08324	16.49810

10JC003 GR BEAR R AT GR BEAR L 69-76 FLOW

SARIMA(1, 0, 1)(0, 0, 0)12

LENGTH OF THE INPUT TIME SERIES = 96

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
1.48925289D 04	1.52297573D 02	5.14249609D 02

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS

SEASON	MEAN	S.D.
1	5.245687D 02	1.0000000D 00
2	5.203225D 02	1.0000000D 00
3	5.118262D 02	1.0000000D 00
4	5.061625D 02	1.0000000D 00
5	5.167812D 02	1.0000000D 00
6	5.567812D 02	1.0000000D 00
7	5.819125D 02	1.0000000D 00
8	5.914712D 02	1.0000000D 00
9	5.886362D 02	1.0000000D 00
10	5.677562D 02	1.0000000D 00
11	5.369587D 02	1.0000000D 00
12	5.245700D 02	1.0000000D 00

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 0

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.7993	0.0673
-0.4100	0.1022

CORRELATION MATRIX OF BETA

1.000	
0.413	1.000

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
0.3446	0.150542

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
0.004449	0.004466

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
0.001357	0.005209

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	-0.04770	0.03345	0.22529
2	-0.09728	0.06246	1.17243
3	0.11698	0.09424	2.55669
4	0.07692	0.09479	3.16174
5	-0.06002	0.09858	3.53417
6	-0.06088	0.09956	3.92167
7	0.07981	0.10057	4.59493
8	0.07361	0.10108	5.17423
9	-0.17982	0.10145	8.67094
10	0.21822	0.10167	13.88021
11	0.05175	0.10181	14.17663
12	-0.05859	0.10190	14.56115
13	0.01944	0.10196	14.60397
14	0.03987	0.10200	14.78639
15	-0.01706	0.10202	14.82019
16	-0.11145	0.10204	16.28096
17	0.19503	0.10204	20.81058
18	0.01563	0.10205	20.84006
19	-0.15775	0.10206	23.88064
20	0.00470	0.10206	23.88337
21	0.09566	0.10206	25.03118
22	0.00235	0.10206	25.03189
23	0.11232	0.10206	26.65791
24	-0.07113	0.10206	27.31909

10KA001 MACKENZIE R AT N.W. 66-76 FLOW

SARIMA(1, 0, 1)(0, 0, 0)12

LENGTH OF THE INPUT TIME SERIES = 132

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
8.21773569D 07	6.19510305D-01	1.81708967D 03

BOX-COX TRANSFORMATION PARAMETERS

LAMDA	CONS
0.	0.

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS
(TRANSFORMED SERIES)

SEASON	MEAN	S.D.
1	8.119974D 00	2.923979D-01
2	8.006352D 00	1.993945D-01
3	7.943718D 00	1.722347D-01
4	8.011462D 00	1.443483D-01
5	9.342719D 00	2.611963D-01
6	9.829354D 00	9.454074D-02
7	9.737921D 00	1.171897D-01
8	9.501217D 00	1.404400D-01
9	9.295991D 00	1.083740D-01
10	9.092230D 00	9.351650D-02
11	8.557970D 00	1.448463D-01
12	8.141105D 00	1.620277D-01

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 6

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.9093	0.0515
0.5716	0.1016

CORRELATION MATRIX OF BETA

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1.000
0.711  1.000
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------RESIDUAL ANALYSIS-------

SKEWNESS

G1	SL
-0.2458	0.232058

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
0.190474	0.127500

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
-0.002617	0.003230

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	0.02352	0.04524	0.07467
2	-0.02781	0.07667	0.17993
3	-0.09512	0.08167	1.42059
4	-0.06075	0.08222	1.93056
5	-0.03454	0.08223	2.09673
6	0.14600	0.08243	5.08920
7	0.01222	0.08284	5.11032
8	-0.01625	0.08335	5.14801
9	-0.03687	0.08387	5.34348
10	-0.07373	0.08436	6.13169
11	-0.05969	0.08479	6.65251
12	-0.05048	0.08517	7.02811
13	0.13996	0.08549	9.93985
14	0.01781	0.08575	9.98738
15	0.03754	0.08597	10.20043
16	-0.02086	0.08616	10.26678
17	0.00481	0.08631	10.27033
18	0.10417	0.08644	11.95395
19	0.22161	0.08654	19.64108
20	-0.04734	0.08663	19.99503
21	0.05644	0.08670	20.50257
22	-0.04449	0.08676	20.82082
23	-0.04468	0.08681	21.14480
24	0.01031	0.08685	21.16221

100A001 TREE RIVER 69-76 FLOW

SARIMA(1, 0, 1)(0, 0, 0)12

LENGTH OF THE INPUT TIME SERIES = 96

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
3.17297178D 03	7.70108422D-02	3.67815268D 02

BOX-COX TRANSFORMATION PARAMETERS

LAMDA	CONS
0.	0.

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS
(TRANSFORMED SERIES)

SEASON	MEAN	S.D.
1	2.2921839D 00	1.0000000D 00
2	2.0344510D 00	1.0000000D 00
3	1.7928110D 00	1.0000000D 00
4	1.7987100D 00	1.0000000D 00
5	2.2314540D 00	1.0000000D 00
6	3.9075450D 00	1.0000000D 00
7	4.5638340D 00	1.0000000D 00
8	4.2668740D 00	1.0000000D 00
9	3.9797140D 00	1.0000000D 00
10	3.6541490D 00	1.0000000D 00
11	3.1361090D 00	1.0000000D 00
12	2.6638410D 00	1.0000000D 00

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 0

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.3936	0.1314
-0.4453	0.1279

CORRELATION MATRIX OF BETA

1.000	
0.700	1.000

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
-0.2840	0.233335

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
-0.005533	0.113694

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
-0.004551	0.005209

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	-0.00348	0.01791	0.00120
2	0.01159	0.01897	0.01463
3	0.06713	0.10040	0.47053
4	-0.03450	0.10048	0.59226
5	-0.01306	0.10200	0.60988
6	-0.10329	0.10201	1.72511
7	-0.09870	0.10206	2.75493
8	-0.19453	0.10206	6.80036
9	-0.12265	0.10206	8.42718
10	-0.00634	0.10206	8.43229
11	-0.01343	0.10206	8.45226
12	-0.06333	0.10206	8.90149
13	0.03934	0.10206	9.07689
14	0.02429	0.10206	9.14460
15	0.03567	0.10206	9.29237
16	0.11658	0.10206	10.89059
17	0.00840	0.10206	10.89899
18	0.00543	0.10206	10.90260
19	-0.00691	0.10206	10.90844
20	-0.01990	0.10206	10.95746
21	0.03160	0.10206	11.03269
22	0.00335	0.10206	11.03412
23	-0.02731	0.10206	11.18379
24	-0.04352	0.10206	11.43128

10RC001   BACK R BL DEEP ROSE L   65-76   FLOW

SARIMA(1, 0, 2)(0, 0, 0)12

LENGTH OF THE INPUT TIME SERIES = 144

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
5.19673326D 05	5.82739279D-01	1.23552450D 03

BOX-COX TRANSFORMATION PARAMETERS

LAMDA	CONS
0.	0.

FITTED SEASONAL MEANS AND STANDARD DEVIATIONS
(TRANSFORMED SERIES)

SEASON	MEAN	S.D.
1	4.192235D 00	4.044443D-01
2	3.916754D 00	3.916921D-01
3	3.725521D 00	3.624297D-01
4	3.580370D 00	2.125766D-01
5	3.563176D 00	4.879933D-01
6	6.395430D 00	1.176627D 00
7	7.541956D 00	3.423634D-01
8	6.735737D 00	2.624991D-01
9	6.527119D 00	5.494011D-01
10	6.026533D 00	6.090438D-01
11	5.301305D 00	5.176331D-01
12	4.656621D 00	5.060843D-01

NO. OF FOURIER COMPONENTS FOR MEAN 6
NO. OF FOURIER COMPONENTS FOR SD 6

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.7524	0.0670
0.	0.
0.3219	0.0963

CORRELATION MATRIX OF BETA

1.000	
0.574	1.000

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
0.4082	0.042593

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
0.064316	0.080542

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
0.007036	0.002835

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	0.03222	0.04953	0.15264
2	-0.03313	0.02639	0.31514
3	-0.04695	0.07419	0.64384
4	0.08765	0.07864	1.79760
5	-0.09955	0.08052	3.29665
6	0.01378	0.08227	3.32553
7	-0.00957	0.08244	3.33962
8	-0.01417	0.08297	3.37066
9	0.08869	0.08305	4.59578
10	-0.01707	0.08320	4.64148
11	-0.01691	0.08324	4.68668
12	-0.07248	0.08329	5.52339
13	0.17205	0.08330	10.27404
14	0.10251	0.08332	11.97332
15	0.07329	0.08332	12.84886
16	-0.06867	0.08333	13.62341
17	-0.00046	0.08333	13.62345
18	0.06582	0.08333	14.34627
19	-0.03738	0.08333	14.52260
20	-0.00377	0.08333	14.52502
21	-0.04902	0.08333	14.93569
22	-0.04535	0.08333	15.29790
23	0.02411	0.08333	15.39893
24	0.19118	0.08333	21.80274

APPENDIX II

MAR Models

In Section 3.2 of the accompanying paper MAR models are fitted to 12 streamflow series from the Arctic. Two sample listings of the MONAR subroutine output are presented here. Because of space limitations the other 22 models are not shown.

Special Notation

phi(j,i) - the ith AR parameter for the jth month, eg. phi(3,7) is the 7th AR parameter for March, the 3rd month

se(phi) - standard error of phi

ra - residual autocorrelation

se - standard error

q - portemanteau statistic

df - degrees of freedom

```
0ed001    liard r at fort liard  60-76  flow

     number of years of data =  17
     total length of series  =  204
     box-cox parameter        =    0.

   means and standard deviations of transformed data
    month        mean              s.d.
      1      5.965423d 00    2.277805d-01
      2      5.785219d 00    2.367709d-01
      3      5.694918d 00    2.184731d-01
      4      6.074480d 00    4.855309d-01
      5      8.046683d 00    3.369769d-01
      6      8.752478d 00    2.050166d-01
      7      8.451651d 00    3.031838d-01
      8      7.939480d 00    3.536124d-01
      9      7.677683d 00    2.628292d-01
     10      7.352265d 00    1.785471d-01
     11      6.556445d 00    2.729607d-01
     12      6.166901d 00    2.373222d-01

   series corrected by monthly means
          (transformed series)
   monthly-varying ar(12) fitted
   monthly residual standard deviations estimated

        aic =     2.334813d 03

                   month  1

     residual standard deviation =     1.143170d-01
   phi( 1,i)          0.8486    0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.
     se(phi)          0.1191    0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.

                   month  2

     residual standard deviation =     9.067862d-02
   phi( 2,i)          0.9705    0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.
     se(phi)          0.0971    0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.

                   month  3

     residual standard deviation =     8.388628d-02
   phi( 3,i)          0.8564    0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.
     se(phi)          0.0859    0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.

                   month  4

     residual standard deviation =     4.918436d-01
   phi( 4,i)          0.        0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.
     se(phi)          0.        0.      0.      0.      0.      0.      0.      0.      0.      0.      0.      0.

                   month  5
```

residual standard deviation =	3.469382d-01											
phi(5,l)	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
se(phi)	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

month 6

residual standard deviation =	1.786300d-01											
phi(6,l)	0.	-0.2336	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.4395
se(phi)	0.	0.0965	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.2274

month 7

residual standard deviation =	3.110003d-01											
phi(7,l)	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
se(phi)	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

month 8

residual standard deviation =	2.391584d-01											
phi(8,l)	0.3849	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.2916
se(phi)	0.1951	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.1872

month 9

residual standard deviation =	1.899774d-01											
phi(9,l)	0.5215	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
se(phi)	0.1304	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

month 10

residual standard deviation =	1.273522d-01											
phi(10,l)	0.2632	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.5366
se(phi)	0.1213	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.1773

month 11

residual standard deviation =	1.824179d-01											
phi(11,l)	0.6120	0.3310	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
se(phi)	0.2859	0.1557	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

month 12

residual standard deviation =	1.573768d-01											
phi(12,l)	0.6575	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
se(phi)	0.1558	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

residual autocorrelations

	month 1			month 2		
lag	ra	se	q	ra	se	q
1	0.1213	0.2425	0.3	0.1522	0.2425	0.4
2	-0.1320	0.2425	0.5	0.1297	0.2425	0.7
3	-0.0175	0.2425	0.6	-0.0461	0.2425	0.7
4	-0.3100	0.2425	2.2	0.0500	0.2425	0.9
5	0.0448	0.2425	2.2	-0.6926	0.2425	8.9
6	0.3348	0.2425	4.9	0.0886	0.2425	9.0
7	-0.2058	0.2425	5.6	0.1757	0.2425	9.5

lag	ra	se	q	ra	se	q
8	-0.0795	0.2425	5.7	0.1396	0.2425	9.9
9	0.1225	0.2425	6.0	0.0508	0.2425	9.9
10	-0.1753	0.2425	6.5	-0.2084	0.2425	10.7
11	0.0553	0.2425	6.5	0.0931	0.2425	10.8
12	0.3906	0.2425	9.1	0.1044	0.2425	11.0
13	0.2296	0.2425	10.0	0.0748	0.2425	11.1
14	-0.2562	0.2425	11.1	-0.2068	0.2425	11.8
15	0.2646	0.2425	12.3	-0.3499	0.2425	13.9

	month 3			month 4		
lag	ra	se	q	ra	se	q
1	-0.0641	0.2425	0.1	0.2134	0.2425	0.8
2	-0.4715	0.2425	3.8	0.0885	0.2425	0.9
3	0.2740	0.2425	5.1	0.0039	0.2425	0.9
4	-0.0026	0.2425	5.1	0.2731	0.2425	2.2
5	0.0225	0.2425	5.1	-0.0550	0.2425	2.2
6	0.3768	0.2425	7.5	-0.2482	0.2425	3.3
7	0.4666	0.2425	11.2	0.0676	0.2425	3.4
8	-0.1048	0.2425	11.4	0.5724	0.2425	8.9
9	0.3644	0.2425	13.7	-0.0461	0.2425	9.0
10	0.0753	0.2425	13.8	0.3038	0.2425	10.5
11	-0.1317	0.2425	14.1	-0.5756	0.2425	16.2
12	0.1007	0.2425	14.3	-0.1063	0.2425	16.3
13	-0.1171	0.2425	14.5	0.1761	0.2425	16.9
14	-0.2955	0.2425	16.0	0.0595	0.2425	17.0
15	0.3000	0.2425	17.5	-0.0806	0.2425	17.2

	month 5			month 6		
lag	ra	se	q	ra	se	q
1	0.1326	0.2425	0.3	-0.0571	0.2425	0.1
2	0.0658	0.2425	0.4	0.1490	0.2425	0.4
3	0.0550	0.2425	0.4	0.2790	0.2425	1.8
4	-0.2337	0.2425	1.8	0.5284	0.2425	6.5
5	-0.1626	0.2425	2.2	-0.2340	0.2425	7.4
6	-0.1531	0.2425	2.6	0.4767	0.2425	11.3
7	-0.0389	0.2425	2.7	0.3710	0.2425	13.0
8	-0.0391	0.2425	2.7	-0.1733	0.2425	13.6
9	0.2187	0.2425	3.5	-0.1987	0.2425	14.2
10	-0.2713	0.2425	4.8	0.0587	0.2425	14.3
11	0.0336	0.2425	4.8	-0.0115	0.2425	14.3
12	-0.1262	0.2425	5.0	0.2771	0.2425	15.6
13	-0.1563	0.2425	5.5	-0.0228	0.2425	15.6
14	-0.3776	0.2425	8.1	-0.4798	0.2425	19.5
15	0.0971	0.2425	8.3	0.3871	0.2425	22.1

	month 7			month 8		
lag	ra	se	q	ra	se	q
1	0.1950	0.2425	0.6	-0.0525	0.2425	0.0
2	0.0573	0.2425	0.7	-0.0562	0.2425	0.1
3	-0.0734	0.2425	0.8	-0.1280	0.2425	0.4
4	0.0223	0.2425	0.9	-0.2031	0.2425	1.1
5	0.5396	0.2425	5.9	-0.1279	0.2425	1.4
6	0.3776	0.2425	8.3	-0.3566	0.2425	3.5
7	0.2795	0.2425	9.6	0.0055	0.2425	3.5
8	-0.2270	0.2425	10.5	0.2746	0.2425	4.8
9	0.2112	0.2425	11.3	-0.3319	0.2425	6.7
10	-0.4136	0.2425	14.2	-0.2005	0.2425	7.4
11	0.0146	0.2425	14.2	-0.0275	0.2425	7.4
12	0.2167	0.2425	15.0	-0.4608	0.2425	11.0
13	0.4659	0.2425	18.7	0.0524	0.2425	11.0
14	0.3173	0.2425	20.7	-0.1498	0.2425	11.4
15	0.1353	0.2425	21.0	0.2227	0.2425	12.3

	month 9			month 10		
lag	ra	se	q	ra	se	q
1	0.0158	0.2425	0.0	-0.0868	0.2425	0.1

2	-0.0198	0.2425	0.0	0.2885	0.2425	1.5
3	0.2291	0.2425	0.9	-0.0814	0.2425	1.7
4	-0.0276	0.2425	0.9	-0.1126	0.2425	1.9
5	0.1828	0.2425	1.5	0.7681	0.2425	11.9
6	0.3000	0.2425	3.0	0.1582	0.2425	12.3
7	-0.0511	0.2425	3.1	-0.0966	0.2425	12.5
8	-0.2277	0.2425	3.9	-0.2020	0.2425	13.2
9	0.3600	0.2425	6.1	-0.1847	0.2425	13.8
10	0.2873	0.2425	7.5	-0.1564	0.2425	14.2
11	0.1394	0.2425	7.9	-0.0038	0.2425	14.2
12	0.0553	0.2425	7.9	-0.3595	0.2425	16.4
13	0.3075	0.2425	9.5	0.1265	0.2425	16.6
14	-0.1098	0.2425	9.7	-0.1555	0.2425	17.1
15	-0.0234	0.2425	9.8	-0.3038	0.2425	18.6

	month 11			month 12		
lag	ra	se	q	ra	se	q
1	-0.0147	0.2425	0.0	-0.0458	0.2425	0.0
2	0.1914	0.2425	0.6	0.0778	0.2425	0.1
3	-0.0568	0.2425	0.7	0.0125	0.2425	0.1
4	-0.1411	0.2425	1.0	0.0954	0.2425	0.3
5	-0.0932	0.2425	1.2	0.1463	0.2425	0.7
6	-0.0145	0.2425	1.2	0.0083	0.2425	0.7
7	-0.2200	0.2425	2.0	0.1289	0.2425	0.9
8	-0.0940	0.2425	2.1	-0.2545	0.2425	2.0
9	-0.1072	0.2425	2.3	0.2542	0.2425	3.1
10	0.3929	0.2425	5.0	-0.0054	0.2425	3.1
11	0.1209	0.2425	5.2	-0.2526	0.2425	4.2
12	0.0217	0.2425	5.2	0.2496	0.2425	5.3
13	-0.1054	0.2425	5.4	0.0078	0.2425	5.3
14	0.2315	0.2425	6.3	0.3584	0.2425	8.0
15	0.1901	0.2425	6.5	0.1966	0.2425	8.6

overall q-statistic = 168.0 on 162 df

0ed001 liard r at fort liard 60-76 flow

number of years of data = 17
total length of series = 204

means and standard deviations of raw data

month	mean	s.d.
1	3.595165d 02	8.600890d 01
2	3.342182d 02	7.395015d 01
3	3.044306d 02	6.606258d 01
4	5.109341d 02	4.128746d 02
5	3.278091d 03	9.140211d 02
6	6.464568d 03	1.333733d 03
7	4.509646d 03	1.476828d 03
8	2.996423d 03	1.162091d 03
9	2.237696d 03	6.182510d 02
10	1.584078d 03	2.718530d 02
11	7.309641d 02	1.958759d 02
12	4.898647d 02	1.111411d 02

series corrected by monthly means
monthly-varying ar(12) fitted
monthly residual standard deviations estimated

aic = 2.359850d 03

month 1

residual standard deviation = 4.169974d 01

phi(1,i)	0.9047	-0.1497	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
se(phi)	0.1749	0.0980	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

month 2

residual standard deviation = 2.678114d 01

phi(2,i)	0.8079	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
se(phi)	0.0758	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

month 3

residual standard deviation = 2.673134d 01

phi(3,i)	0.8214	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
se(phi)	0.0877	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

month 4

residual standard deviation = 4.223066d 02

phi(4,i)	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
se(phi)	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

month 5

residual standard deviation = 9.420185d 02

phi(5,i)	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

se(phi)	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

month 6

residual standard deviation = 1.133882d 03

phi(6,i)	0.	-1.8866	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.4572
se(phi)	0.	0.7474	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.2372

month 7

residual standard deviation = 1.519080d 03

phi(7,i)	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
se(phi)	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

month 8

residual standard deviation = 6.360863d 02

phi(8,i)	0.6939	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.4744
se(phi)	0.1098	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.1425

month 9

residual standard deviation = 4.157635d 02

phi(9,i)	0.4009	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
se(phi)	0.0870	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

month 10

residual standard deviation = 1.917917d 02

phi(10,i)	0.1660	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.5412
se(phi)	0.0779	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.1770

month 11

residual standard deviation = 1.289913d 02

phi(11,i)	0.2486	0.0959	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
se(phi)	0.1335	0.0590	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

month 12

residual standard deviation = 7.308466d 01

phi(12,i)	0.4385	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
se(phi)	0.1064	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

residual autocorrelations

	month 1			month 2		
lag	ra	se	q	ra	se	q
1	0.0000	0.2425	0.0	0.1428	0.2425	0.3
2	-0.0798	0.2425	0.1	0.0797	0.2425	0.5
3	-0.0622	0.2425	0.2	-0.0696	0.2425	0.5
4	-0.2817	0.2425	1.5	0.0189	0.2425	0.5
5	0.0950	0.2425	1.7	-0.6787	0.2425	8.4
6	0.3121	0.2425	3.3	0.0146	0.2425	8.4
7	-0.0042	0.2425	3.3	0.2095	0.2425	9.1
8	-0.1538	0.2425	3.7	0.1624	0.2425	9.6
9	0.1538	0.2425	4.1	0.0413	0.2425	9.6

10	-0.2297	0.2425	5.0	-0.1627	0.2425	10.0
11	0.0040	0.2425	5.0	0.0705	0.2425	10.1
12	0.3748	0.2425	7.4	0.1480	0.2425	10.5
13	0.2417	0.2425	8.4	0.1053	0.2425	10.7
14	-0.2031	0.2425	9.1	-0.1361	0.2425	11.0
15	0.2380	0.2425	10.1	-0.2640	0.2425	12.2

	month 3			month 4		
lag	ra	se	q	ra	se	q
1	-0.1225	0.2425	0.3	0.0156	0.2425	0.0
2	-0.4026	0.2425	3.0	-0.0272	0.2425	0.0
3	0.2207	0.2425	3.8	0.1263	0.2425	0.3
4	-0.0309	0.2425	3.9	0.0764	0.2425	0.4
5	-0.0054	0.2425	3.9	0.0511	0.2425	0.4
6	0.4032	0.2425	6.6	-0.3418	0.2425	2.4
7	0.4793	0.2425	10.5	0.0803	0.2425	2.5
8	-0.0162	0.2425	10.5	0.3884	0.2425	5.1
9	0.3138	0.2425	12.2	-0.1141	0.2425	5.3
10	0.0078	0.2425	12.2	0.3757	0.2425	7.7
11	-0.1010	0.2425	12.4	-0.5474	0.2425	12.8
12	0.0223	0.2425	12.4	-0.1286	0.2425	13.1
13	-0.1191	0.2425	12.6	0.0362	0.2425	13.1
14	-0.3446	0.2425	14.6	0.1072	0.2425	13.3
15	0.1911	0.2425	15.3	-0.0751	0.2425	13.4

	month 5			month 6		
lag	ra	se	q	ra	se	q
1	0.0723	0.2425	0.1	-0.0468	0.2425	0.0
2	0.1117	0.2425	0.3	0.2162	0.2425	0.8
3	0.0625	0.2425	0.4	-0.0075	0.2425	0.8
4	-0.3611	0.2425	2.6	0.4880	0.2425	4.9
5	-0.1882	0.2425	3.2	-0.0456	0.2425	4.9
6	0.1197	0.2425	3.4	0.4052	0.2425	7.7
7	-0.1323	0.2425	3.7	0.3488	0.2425	9.8
8	0.0743	0.2425	3.8	-0.1087	0.2425	10.0
9	0.2205	0.2425	4.6	-0.2389	0.2425	10.9
10	-0.2211	0.2425	5.5	-0.1074	0.2425	11.1
11	0.0071	0.2425	5.5	0.0871	0.2425	11.3
12	-0.1066	0.2425	5.7	0.2380	0.2425	12.2
13	-0.1603	0.2425	6.1	-0.0201	0.2425	12.2
14	-0.3209	0.2425	7.9	-0.4666	0.2425	15.9
15	0.1377	0.2425	8.2	0.3214	0.2425	17.7

	month 7			month 8		
lag	ra	se	q	ra	se	q
1	0.2373	0.2425	1.0	-0.1666	0.2425	0.5
2	0.0425	0.2425	1.0	0.0741	0.2425	0.6
3	-0.1337	0.2425	1.3	-0.0951	0.2425	0.7
4	0.1198	0.2425	1.5	-0.1787	0.2425	1.3
5	0.4960	0.2425	5.7	-0.3741	0.2425	3.6
6	0.2677	0.2425	6.9	-0.3374	0.2425	5.6
7	0.3076	0.2425	8.6	-0.2313	0.2425	6.5
8	-0.2911	0.2425	10.0	0.1406	0.2425	6.8
9	0.3164	0.2425	11.7	-0.1808	0.2425	7.4
10	-0.3854	0.2425	14.2	-0.0722	0.2425	7.5
11	0.1366	0.2425	14.6	-0.0593	0.2425	7.6
12	0.2302	0.2425	15.5	-0.5662	0.2425	13.0
13	0.4214	0.2425	18.5	-0.1428	0.2425	13.4
14	0.4151	0.2425	21.4	-0.3815	0.2425	15.8
15	0.0533	0.2425	21.4	0.1575	0.2425	16.3

	month 9			month 10		
lag	ra	se	q	ra	se	q
1	-0.0466	0.2425	0.0	-0.0957	0.2425	0.2
2	0.0058	0.2425	0.0	0.1765	0.2425	0.7
3	0.1605	0.2425	0.4	[illegible]	0.2425	[illegible]

4	-0.0729	0.2425	0.5	-0.0537	0.2425	0.7
5	0.0071	0.2425	0.6	0.9095	0.2425	14.8
6	0.2754	0.2425	1.9	0.1191	0.2425	15.1
7	-0.1002	0.2425	2.1	0.0144	0.2425	15.1
8	-0.2104	0.2425	2.8	-0.1125	0.2425	15.3
9	0.2568	0.2425	4.0	-0.3317	0.2425	17.1
10	0.2475	0.2425	5.0	-0.1606	0.2425	17.6
11	0.1760	0.2425	5.5	0.1669	0.2425	18.1
12	0.1251	0.2425	5.8	-0.2555	0.2425	19.2
13	0.2500	0.2425	6.9	0.1812	0.2425	19.7
14	-0.0354	0.2425	7.0	-0.0147	0.2425	19.7
15	0.0200	0.2425	7.0	-0.2598	0.2425	20.9

	month 11			month 12		
lag	ra	se	q	ra	se	q
1	-0.0310	0.2425	0.0	-0.0827	0.2425	0.1
2	0.2149	0.2425	0.8	0.1422	0.2425	0.5
3	-0.1079	0.2425	1.0	0.1339	0.2425	0.8
4	-0.0880	0.2425	1.1	-0.1322	0.2425	1.1
5	-0.0043	0.2425	1.1	0.2041	0.2425	1.8
6	-0.0298	0.2425	1.1	-0.1104	0.2425	2.0
7	-0.1068	0.2425	1.3	0.1726	0.2425	2.5
8	-0.0142	0.2425	1.8	-0.3085	0.2425	4.1
9	-0.0926	0.2425	2.0	0.2991	0.2425	5.6
10	0.3209	0.2425	3.7	-0.0004	0.2425	5.6
11	0.2274	0.2425	4.6	-0.2680	0.2425	6.8
12	-0.0140	0.2425	4.6	0.2970	0.2425	8.3
13	-0.1808	0.2425	5.1	-0.0055	0.2425	8.3
14	0.2676	0.2425	6.4	0.4812	0.2425	12.3
15	0.0042	0.2425	6.4	0.2595	0.2425	13.4

overall q-statistic = 162.2 on 166 df

APPENDIX III

Transfer Function-Noise Models for 4 Arctic River Series

In Section 3.3 of the accompanying paper, transfer function-noise models are used to dynamically link hydrometric and meteorologic time series. The models are presented in the form of computer listings of the output of A,I, McLeod's TEST program. The outputs are identical to those for the USES program (see Appendix I) except that the transfer functions $V_i(B)$ from equations 2.9-2 and 2.9-3 are also specified.

Special Notation

IV - order of the numerator of $V_i(B)$, (i.e. one plus the number of ω parameters)

IU - order of the denominator of $V_i(B)$, (i.e. the number of δ parameters)

IDELAY - the delay term in transfer function

TAU - the vector of estimated parameters, always listed in the order

$$(\omega_0,\omega_1,\ldots,\omega_{v_i},\ _1,\delta_2,\ldots,\delta_{u_i})$$

KAKISA R 64-76 - HAY R DSRAIN,APRILTEMP

SARIMA(1, 0, 3)(0, 0, 0) 0

LENGTH OF THE INPUT TIME SERIES = 156

LENGTH OF OUTPUT SERIES OF TRANSFER FUNCTION = 154

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
5.86888553D 00	3.76072884D-02	568.1

ESTIMATED MEAN OF SERIES

MEAN	S.E.(MEAN)
4.624272D-02	1.010588D-03

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.9010	0.0396
-0.2637	0.0821
0.	0.
0.2051	0.0798

CORRELATION MATRIX OF BETA

1.000		
0.390	1.000	
0.324	0.113	1.000

TRANSFER FUNCTION NO. 1, IU=0, IV=0, IDELAY= 1
MEAN CORRECTION = 3.230769D-06

TAU	S.D.(TAU)
1.729754D-03	6.657821D-04

TRANSFER FUNCTION NO. 2, IU=0, IV=0, IDELAY= 2
MEAN CORRECTION =-1.987179D-07

TAU	S.D.(TAU)
-3.144248D-02	9.060987D-03

CORRELATION MATRIX OF TAU

1.000	
-0.025	1.000

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
0.4721	0.016396

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
-0.785529	0.253698

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
0.C02950	0.002514

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	-0.01970	0.02542	0.06172
2	-0.07603	0.06375	0.98700
3	0.00302	0.02591	0.98847
4	0.00081	0.07012	0.98858
5	-0.03938	0.07455	1.24171
6	0.-02970	0.07682	1.38669
7	0.10974	0.07626	3.37882
8	0.05042	0.07770	3.80216
9	-0.02637	0.07842	3.91879
10	-0.07317	0.07838	4.82273
11	0.04903	0.07886	5.23144
12	-0.12760	0.07910	8.01843
13	0.02602	0.07923	8.13514
14	0.08283	0.07942	9.32599
15	0.13284	0.07954	12.41093
16	0.03584	0.07963	12.63708
17	0.10883	0.07972	14.73942
18	-0.04828	0.07978	15.15577
19	-0.06451	0.07983	15.90442
20	0.05655	0.07988	16.48401
21	0.04728	0.07991	16.89216
22	0.04644	0.07994	17.28883
23	-0.00537	0.07996	17.29418
24	0.08221	0.07998	18.55622

Response: Kakisa R. Flow 64-76 m**3/s lambda=0 Fm=6 Fs=0

0.869483D 00	0.594701D 00	0.269789D 00	0.137035D 00
0.416723D 00	0.403724D 00	0.129288D 00	-0.882972D-01
0.149569D 00	0.596608D 00	0.918395D 00	0.804257D 00
0.679823D 00	0.627058D 00	0.701470D 00	0.517922D 00
0.155522D 00	0.143338D 00	0.219743D 00	0.412143D 00
0.265231D 00	0.259253D 00	0.307841D 00	0.323080D 00
0.403136D 00	0.397295D 00	0.392422D 00	0.274522D 00
0.277124D 00	0.350337D 00	0.457025D 00	0.218055D 00
-0.243823D-01	-0.104683D 00	0.395426D-02	-0.283348D-01
-0.133260D 00	-0.145852D 00	-0.571967D-01	-0.104049D 00
-0.319413D 00	0.530299D 00	0.598721D 00	0.456372D 00
0.118323D 00	-0.570333D-01	-0.678715D-01	-0.361205D-01
-0.114055D 00	-0.367781D-01	-0.117483D 00	-0.338985D 00
-0.420389D 00	-0.326719D 00	-0.187531D 00	0.430038D 00
0.467392D 00	0.406138D 00	0.328331D 00	0.295413D 00
0.246959D 00	0.199782D 00	0.293720D 00	0.257955D 00
0.263812D 00	0.435236D-01	-0.108035D 00	-0.370896D 00
-0.472508D 00	-0.282026D 00	-0.204041D 00	-0.320284D 00
-0.310883D 00	-0.226122D 00	-0.177964D 00	-0.214634D 00
0.114824D 00	-0.340346D-01	-0.141391D 00	-0.276540D 00
-0.441125D 00	-0.485902D 00	-0.581064D 00	-0.666455D 00
-0.383783D 00	-0.120761D 00	-0.421426D-01	-0.315807D 00
-0.192009D 00	-0.730138D 00	-0.114346D 01	-0.891678D 00
-0.878579D 00	-0.104363D 01	-0.120002D 01	-0.982752D 00
-0.118586D 01	-0.128144D 01	-0.130133D 01	-0.680990D 00
-0.549644D 00	0.946754D-01	0.195965D 00	-0.515531D-01
-0.388191D 00	-0.560042D 00	-0.506871D 00	-0.422459D 00
-0.264814D 00	-0.172520D 00	-0.757866D-01	-0.210986D 00
-0.684192D 00	-0.112070D 01	-0.894988D 00	-0.424769D 00
-0.115833D 00	-0.427133D 00	-0.739271D 00	-0.535517D 00
-0.393618D 00	-0.389161D 00	-0.311146D 00	-0.124138D-01
-0.335646D 00	-0.310873D-01	0.133541D 00	0.201425D 00
0.155616D 00	0.100436D 00	0.167397D 00	0.227834D 00
0.110718D 00	0.134037D 00	-0.341460D-02	0.323407D-01
0.307461D 00	0.760585D-01	-0.394279D-01	-0.149713D 00
0.265231D 00	0.435002D 00	0.518535D 00	0.571394D 00
0.476144D 00	0.419709D 00	0.429061D 00	0.658091D 00
0.965827D 00	0.600726D 00	0.780550D 00	0.535415D 00
0.899254D 00	0.116302D 01	0.105468D 01	0.769945D 00

Transfer Function No. 1: Hay River Rainfall mm
lambda=1 Fm=6 Fs=0

0.	0.	-0.2307690 00	-0.3461540 01
0.6923080 01	-0.2592310 02	-0.2415380 02	-0.1107690 02
0.7700000 02	0.2169230 02	-0.3076920 00	0.
0.	0.	-0.2307690 00	-0.2461540 01
-0.1407690 02	0.8076920 01	-0.1153850 01	-0.1307690 02
0.1200000 02	0.3692310 01	-0.3076920 00	0.
0.	0.	-0.2307690 00	-0.3461540 01
0.1923080 01	0.2007690 02	0.7846150 01	-0.3107690 02
0.1900000 02	-0.2307690 01	0.1692310 01	0.
0.	0.	-0.2307690 00	-0.3461540 01
-0.1076920 01	-0.9230770 00	0.2084620 02	-0.3407690 02
-0.2800000 02	-0.2307690 01	-0.3076920 00	0.
0.	0.	-0.2307690 00	0.4538460 01
-0.1076920 01	-0.6923080 01	0.3384620 02	-0.1507690 02
0.1100000 02	-0.4307690 01	-0.3076920 00	0.
0.	0.	-0.2307690 00	0.1053850 02
-0.1307690 02	-0.1192310 02	-0.2315380 02	0.1792310 02
-0.2700000 02	-0.1530770 02	-0.3076920 00	0.
0.	0.	0.2769230 01	-0.3461540 01
-0.7076920 01	-0.1492310 02	-0.1915380 02	-0.1107690 02
-0.2100000 02	-0.9307690 01	-0.3076920 00	0.
0.	0.	-0.2307690 00	-0.3461540 01
0.1492310 02	-0.2692310 02	0.8461540 00	-0.1607690 02
-0.2800000 02	-0.9307690 01	-0.3076920 00	0.
0.	0.	-0.2307690 00	-0.2461540 01
0.2923080 01	0.9076920 01	-0.2515380 02	0.1492310 02
0.1700000 02	-0.1030770 02	-0.3076920 00	0.
0.	0.	-0.2307690 00	-0.2461540 01
-0.2076920 01	0.1807690 02	0.2984620 02	0.5792310 02
-0.4100000 02	-0.1430770 02	-0.3076920 00	0.
0.	0.	-0.2307690 00	-0.2461540 01
0.9230770 00	0.1407690 02	0.7846150 01	0.8923080 01
0.2100000 02	-0.6307690 01	-0.3076920 00	0.
0.	0.	-0.2307690 00	0.7538460 01
0.6923080 01	-0.2292310 02	-0.1415380 02	0.3292310 02
-0.1100000 02	0.4169230 02	-0.3076920 00	0.
0.	0.	-0.2307690 00	0.4538460 01
0.3923080 01	0.4107690 02	0.5846150 01	-0.1076920 01
-0.1000000 01	0.6692310 01	0.1692310 01	0.

Transfer Function No. 2: Hay River Temperature(April only) C
mean of series has been subtracted

```
0  0  0  -3.72308  0  0  0
0  0  0  0  0
0  0  0  3.07692  0  0  0
0  0  0  0  0
0  0  0  -6.42308  0  0  0
0  0  0  0  0
0  0  0  -3.32308  0  0  0
0  0  0  0  0
0  0  0  -1.42308  0  0  0
0  0  0  0  0
0  0  0  2.07692  0  0  0
0  0  0  0  0
0  0  0  .376923  0  0  0
0  0  0  0  0
0  0  0  3.37692  0  0  0
0  0  0  0  0
0  0  0  -4.92308  0  0  0
0  0  0  0  0
0  0  0  .776923  0  0  0
0  0  0  0  0
0  0  0  -.123077  0  0  0
0  0  0  0  0
0  0  0  3.37692  0  0  0
0  0  0  0  0
0  0  0  6.87692  0  0  0
0  0  0  0  0
```

KAZAN R 67-76 - ENNADAI SNOW,DSRAIN,JUNE TEMP

SARIMA(1, 0, 1)(0, 0, 0) 0

LENGTH OF THE INPUT TIME SERIES = 120

LENGTH OF OUTPUT SERIES OF TRANSFER FUNCTION = 119

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
1.514657740 00	1.258535620-02	635.7

ESTIMATED MEAN OF SERIES

MEAN	S.E.(MEAN)
6.2454240-03	2.8221120-03

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.7898	0.0651
-0.2176	0.1036

CORRELATION MATRIX OF BETA

1.000	
0.511	1.000

TRANSFER FUNCTION NO. 1, IU=1, IV=1, IDELAY= 0
MEAN CORRECTION = 9.9858330 01

TAU	S.D.(TAU)
9.7959530-01	3.4413700-02
1.1408420-04	2.9356760-05
-6.6090760-05	3.0406640-05

TRANSFER FUNCTION NO. 2, IU=1, IV=1, IDELAY= 0
MEAN CORRECTION =-2.9490300-18

TAU	S.D.(TAU)
8.8114090-01	3.8640010-02
2.1382120-03	4.9288730-04
-2.4900170-03	5.0673340-04

TRANSFER FUNCTION NO. 3, IU=0, IV=0, IDELAY= 1
MEAN CORRECTION =-1.8070040-20

TAU	S.D.(TAU)
-2.1363780-02	1.0381150-02

CORRELATION MATRIX OF TAU

1.000						
0.020	1.000					
0.072	-0.202	1.000				
-0.152	-0.014	-0.015	1.000			
-0.045	-0.036	0.054	-0.041	1.000		
0.021	-0.010	-0.014	0.249	0.121	1.000	
-0.014	-0.003	-0.165	0.033	0.069	0.052	1.000

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
0.1783	0.403613

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
1.080076	0.363518

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
-0.001226	0.003727

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	0.01619	0.01569	0.03224
2	0.03665	0.06318	0.19885
3	-0.03114	0.08284	0.32022
4	-0.00738	0.08525	0.32709
5	0.00230	0.08773	0.32777
6	0.10190	0.08906	1.66113
7	0.01853	0.08991	1.70560
8	-0.00915	0.09043	1.71655
9	-0.15798	0.09075	5.00842
10	-0.11677	0.09095	6.82324
11	-0.05269	0.09108	7.19612
12	-0.02881	0.09116	7.30866
13	0.17212	0.09121	11.36206
14	-0.00538	0.09124	11.36605
15	-0.13174	0.09126	13.78585
16	-0.02148	0.09127	13.85079
17	-0.06923	0.09127	14.53204
18	0.05053	0.09128	14.89851
19	0.00760	0.09128	14.90690
20	-0.01909	0.09128	14.96024
21	-0.02888	0.09129	15.08356
22	-0.10655	0.09129	16.77951
23	-0.11730	0.09129	18.85600
24	0.20924	0.09129	25.53272

```
Response: Kazan R. at Ennadai Flow 67-76  m**3/s lambda=0 Fm=6 Fs=0
  -0.716254D-01  -0.133856D 00  -0.213179D 00  -0.118751D 00
  -0.224614D 00  -0.113627D 00   0.182452D 00   0.180028D 00
   0.196771D 00   0.267635D 00   0.261342D 00   0.291628D 00
   0.344818D 00   0.413012D 00   0.472497D 00   0.449577D 00
   0.140052D 00   0.589084D 00   0.763134D 00   0.732614D 00
   0.801404D 00   0.116527D 01   0.105879D 01   0.848251D 00
   0.758298D 00   0.754022D 00   0.712793D 00   0.594045D 00
   0.277415D 00   0.260904D-01   0.178185D 00   0.146711D 00
   0.502975D-01  -0.122314D 00  -0.184031D 00  -0.185397D 00
  -0.147666D 00  -0.121163D 00  -0.174805D 00  -0.256219D 00
  -0.473007D 00  -0.388600D 00  -0.314264D 00  -0.183710D 00
  -0.106379D 00  -0.106088D 00  -0.836477D-01  -0.680876D-01
  -0.426822D-01   0.876380D-02  -0.138945D-01  -0.110039D 00
  -0.620003D-02  -0.322443D 00  -0.374312D 00  -0.493570D 00
  -0.645036D 00  -0.675766D 00  -0.604468D 00  -0.534070D 00
  -0.522004D 00  -0.505848D 00  -0.431488D 00  -0.197051D 00
  -0.248922D 00  -0.927266D-01  -0.112352D 00   0.143294D 00
   0.456440D 00   0.521978D 00   0.453150D 00   0.374752D 00
   0.251170D 00   0.181176D 00   0.195454D 00   0.286610D 00
   0.413060D 00   0.231058D 00   0.442944D-01  -0.764023D-01
  -0.211495D 00  -0.307295D 00  -0.392424D 00  -0.376122D 00
  -0.287358D 00  -0.315276D 00  -0.360831D 00  -0.415432D 00
  -0.418215D 00  -0.574451D 00  -0.598112D 00  -0.344574D 00
  -0.231055D 00  -0.263796D 00  -0.220951D 00  -0.170450D 00
  -0.200400D 00  -0.208889D 00  -0.975721D-01  -0.546034D-01
   0.799120D-01  -0.431007D-01  -0.318640D-01  -0.128617D 00
  -0.274378D 00  -0.310658D 00  -0.167616D 00  -0.109092D 00
  -0.825516D-01  -0.719415D-01  -0.889739D-01  -0.178136D 00
   0.460520D 00   0.688717D 00   0.262838D 00   0.242261D-01
  -0.365702D-01  -0.163970D 00  -0.120148D 00  -0.714115D-01
Transfer Function No. 1: Ennadai Accumulated Snowfall  mm
0 0 0 0 0 1729 0 0 0 0 0 0
0 0 0 0 0  689 0 0 0 0 0 0
0 0 0 0 0 1126 0 0 0 0 0 0
0 0 0 0 0 1291 0 0 0 0 0 0
0 0 0 0 0  976 0 0 0 0 0 0
0 0 0 0 0  951 0 0 0 0 0 0
0 0 0 0 1398 0 0 0 0 0 0 0
0 0 0 0 0  473 0 0 0 0 0 0
0 0 0 0 0 1101 0 0 0 0 0 0
0 0 0 0 2249 0 0 0 0 0 0 0
```

Transfer Function No. 2:Ennadai Rainfall mm
lambda=1 Fm=6 Fs=0

0.	0.	0.	-0.7000000 00
-0.6100000 01	0.4000000 01	0.2910000 02	-0.1620000 02
-0.3400000 01	-0.1220000 02	0.	0.
0.	0.	0.	-0.7000000 00
-0.9100000 01	0.9000000 01	0.2910000 02	0.2800000 01
0.1446000 03	0.7280000 02	0.	0.
0.	0.	0.	-0.7000000 00
-0.7100000 01	-0.2500000 02	-0.2290000 02	-0.1520000 02
-0.4640000 02	-0.1520000 02	0.	0.
0.	0.	0.	-0.7000000 00
-0.9100000 01	0.	0.1410000 02	0.7800000 01
-0.2540000 02	-0.9200000 01	0.	0.
0.	0.	0.	-0.7000000 00
-0.4100000 01	-0.4000000 01	-0.3490000 02	-0.1220000 02
-0.4400000 01	-0.6200000 01	0.	0.
0.	0.	0.	-0.7000000 00
0.1190000 02	0.1100000 02	0.2210000 02	0.1980000 02
0.4860000 02	-0.1520000 02	0.	0.
0.	0.	0.	-0.7000000 00
0.9000000 00	0.2300000 02	-0.2290000 02	0.4800000 01
-0.3440000 02	-0.1520000 02	0.	0.
0.	0.	0.	0.3000000 00
-0.1010000 02	-0.1900000 02	0.5510000 02	0.1480000 02
-0.5240000 02	-0.6200000 01	0.	0.
0.	0.	0.	-0.7000000 00
0.1590000 02	0.1200000 02	-0.2690000 02	0.1480000 02
-0.4540000 02	0.1680000 02	0.	0.
0.	0.	0.	0.5300000 01
0.1690000 02	-0.1100000 02	-0.4190000 02	-0.2120000 02
0.1860000 02	-0.1020000 02	0.	0.

Transfer Function No. 3: Ennadai Temperature (June only) C
mean has been subtracted

0	0	0	0	0	-1.81	0
0	0	0	0	0		
0	0	0	0	0	-1.91	0
0	0	0	0	0		
0	0	0	0	0	-5.51	0
0	0	0	0	0		
0	0	0	0	0	2.49	0
0	0	0	0	0		
0	0	0	0	0	-.51	0
0	0	0	0	0		
0	0	0	0	0	-.11	0
0	0	0	0	0		
0	0	0	0	0	1.69	0
0	0	0	0	0		
0	0	0	0	0	2.49	0
0	0	0	0	0		
0	0	0	0	0	1.29	0
0	0	0	0	0		
0	0	0	0	0	1.89	0
0	0	0	0	0		

back r 65-76 - RMEAN,TM,TCJUNE

SARIMA(1, 0, 4)(1, 0, 0)12

LENGTH OF THE INPUT TIME SERIES = 144

LENGTH OF OUTPUT SERIES OF TRANSFER FUNCTION = 143

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
5.817753250 01	4.013358350-01	328.1

ESTIMATED MEAN OF SERIES

MEAN	S.E.(MEAN)
7.286240D-04	2.449283D-01

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.4748	0.0966
-0.4123	0.0973
0.	0.
0.	0.
-0.1947	0.0742
-0.2755	0.0801

CORRELATION MATRIX OF BETA

1.000			
0.649	1.000		
0.072	0.030	1.000	
-0.017	-0.026	0.008	1.000

TRANSFER FUNCTION NO. 1, IU=1, IV=0, IDELAY= 0
MEAN CORRECTION = 1.8611110-06

TAU	S.D.(TAU)
7.8340670-01	1.2557510-01
1.3484150-02	6.4095960-03

TRANSFER FUNCTION NO. 2, IU=0, IV=1, IDELAY= 0
MEAN CORRECTION = 1.3784720-07

TAU	S.D.(TAU)
1.6666850-01	3.0598060-02
-1.2644700-01	2.9714470-02

TRANSFER FUNCTION NO. 3, IU=0, IV=0, IDELAY= 1
MEAN CORRECTION = 1.9444440-07

TAU	S.D.(TAU)
-2.0451710-01	5.7313030-02

CORRELATION MATRIX OF TAU

1.000				
-0.133	1.000			
0.001	-0.200	1.000		
0.029	0.139	-0.453	1.000	
-0.089	0.379	0.007	0.159	1.000

------RESIDUAL ANALYSIS------

	SKEWNESS	
	G1	SL
	0.1815	0.355785

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
0.131942	0.150335

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
0.006981	0.002835

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	0.00181	0.01846	0.00048
2	0.02093	0.02911	0.06536
3	-0.01911	0.08163	0.11982
4	-0.00470	0.03651	0.12314
5	-0.05132	0.07463	0.52145
6	0.04768	0.08129	0.86778
7	0.09346	0.08255	2.20830
8	0.09232	0.08230	3.52588
9	0.06728	0.08259	4.23089
10	-0.05118	0.08296	4.64179
11	0.01855	0.08318	4.69620
12	-0.00585	0.02296	4.70166
13	0.29050	0.08329	18.24545
14	0.07092	0.08330	19.05884
15	0.10636	0.08331	20.90236
16	-0.01989	0.08333	20.96734
17	0.01699	0.08333	21.01511
18	0.02087	0.08333	21.08777
19	-0.06994	0.08333	21.91059
20	0.05579	0.08333	22.43840
21	0.00349	0.08333	22.44047
22	0.09845	0.08333	24.11081
23	0.02171	0.08333	24.19270
24	0.04251	0.08035	24.50931

Response: Back R. Flow 65-76 m**3/s lambda=0 Fm=6 Fs=6

-0.890410D 00	-0.893667D 00	-0.644453D 00	-0.539259D 00
-0.108441D 01	-0.190944D 00	0.132838D 01	0.207732D 00
-0.692203D 00	-0.637937D 00	-0.538747D 00	-0.119328D 00
0.166852D 00	0.560879D 00	0.816420D 00	0.958449D-02
0.716362D-01	0.616489D 00	0.223899D 00	-0.325307D 00
0.131112D 01	0.134056D 01	0.101820D 01	0.323865D 00
0.557690D-01	-0.586241D 00	-0.551138D 00	-0.623808D 00
-0.109231D 01	-0.773019D 00	0.907797D 00	0.128846D 01
0.257323D 00	0.489010D 00	0.114046D 01	0.115618D 01
0.894168D 00	0.824966D 00	0.124273D 01	0.182538D 01
0.718955D 00	-0.281861D 00	0.970571D 00	0.592907D-01
0.395961D 00	0.135537D 01	0.132977D 01	0.151818D 01
0.153083D 01	0.167557D 01	0.125584D 01	0.331872D 00
-0.943424D 00	-0.219978D 01	0.110325D 01	0.215858D 01
0.519099D 00	-0.260269D-01	-0.480571D-01	-0.136348D 00
-0.149029D 00	0.107129D 00	0.154970D 00	-0.642495D-01
-0.579190D 00	0.604740D 00	-0.960707D 00	-0.496550D 00
0.174928D 01	0.142280D 01	0.814653D 00	0.733862D 00
0.159811D 01	0.141663D 01	0.774051D 00	0.761201D 00
-0.199675D 00	-0.667976D 00	-0.371556D 00	-0.706558D 00
-0.124174D 01	-0.823443D 00	-0.875556D-01	0.459012D 00
0.235059D 00	-0.263539D 00	0.294107D 00	0.916924D 00
-0.350454D 00	-0.105141D 01	-0.128226D 01	0.327176D 00
-0.648630D 00	-0.673030D 00	-0.760078D 00	-0.463900D 00
-0.112923D 01	-0.853596D 00	-0.164241D 01	-0.183994D 01
0.843524D 00	0.919230D 00	-0.184270D 01	-0.119985D 01
0.115807D 01	0.839855D 00	0.850142D 00	0.844379D 00
0.402541D 00	0.717526D 00	0.788579D 00	0.916924D 00
0.224540D 00	0.701544D 00	-0.546999D 00	0.588404D 00
-0.503744D 00	-0.102846D 01	-0.105645D 01	-0.112856D 01
-0.173147D 01	-0.117470D 01	-0.815981D 00	-0.218233D 00
0.262732D 01	0.140104D 01	-0.352834D 00	-0.168608D 01
-0.101747D 01	-0.775974D 00	-0.634773D 00	-0.128548D 01
-0.983188D 00	-0.153096D 01	-0.167272D 01	-0.147640D 01
-0.236505D 00	0.921948D 00	0.823162D 00	-0.215293D 00
-0.128708D 01	-0.148272D 01	-0.202756D 01	-0.190187D 01

Transfer Function No. 1: averaged rainfall mm
Baker L. and Contwoyto L. lambda=1 Fm=6 Fs=0

0 0 0 -.1666665
-4.708335 -3.3749985 -10.125 -18.41665
-31.29165 -2.25 0 0
0 0 0 -.1666665
8.291665 -4.875 1.875 -5.91665
-4.29165 -2.25 0 0
0 0 0 -.1666665
-5.208335 23.125 13.875 -5.416665
8.708335 5.25 0 0
0 0 0 -.1666665
-1.708335 -15.375 -12.625 -13.416665
25.20835 .75 0 0
0 0 0 -.1666665
-6.208335 -4.375015 23.875 -11.41665
-24.79165 -1.75 0 0
0 0 0 -.1666665
-5.208335 12.124985 13.375 17.583335
18.70835 5.25 0 0
0 0 0 -.1666665
-1.708335 -9.374985 1.375 2.08335
16.208335 6.75 0 0
0 0 0 -.1666665
-2.2083335 -13.375 -9.125 3.083335
-17.791665 -3.25 0 0
0 0 0 -.1666665
.2916665 -.875 -28.125 33.58335
20.20835 -2.75 0 0
0 0 0 -.1666665
-.208335 8.125015 7.875 -8.91665
-22.29165 -3.25 0 0
0 0 0 1.833335
-.208335 -7.874985 7.375 26.58335
.208335 .75 0 0
0 0 0 -.1666665
18.79165 16.625015 -9.625 -19.41665
11.208335 -3.25 0 0

Transfer Function No. 2: Averaged Temperatures C
Baker L. and Contwoyto L. lambda=1 Fm=6 Fs=0
months 1 4 7 8 11 12 constrained

```
0  -5.47083  1.28125  0  -1.13125  -1.970835  0
0  -2.68125  -2.2375  0  0
0  1.02916925  1.35625  0  .21875  .82916725  0
0  2.01875  -2.4125  0  0
0  -2.69583  -1.14375  0  -1.78125  -2.220835  0
0  .31875  2.1125  0  0
0  2.27917  3.88125  0  -2.25625  -.1958335  0
0  1.94375  3.4125  0  0
0  5.75417  -.36875  0  -3.43125  -3.845835  0
0  -.25625  1.8125  0  0
0  -1.17083075  1.75625  0  -3.48125  .9291665  0
0  .01875  1.0625  0  0
0  3.50417  .75625  0  1.76875  1.154165  0
0  1.56875  2.8625  0  0
0  -4.39583  .28125  0  -1.58125  -2.79583575  0
0  -3.03125  -5.5375  0  0
0  -.72083075  1.00625  0  4.01875  2.404165  0
0  2.34375  3.0375  0  0
0  -.195833075  -3.26875  0  1.71875  3.029165  0
0  -3.18125  -5.0875  0  0
0  2.87917  -2.36375  0  3.76875  3.029165  0
0  1.01875  .7375  0  0
0  -.795830825  -3.16875  0  2.16875  -.3458335  0
0  -.08125  .2375  0  0
```

Transfer Function No. 3: Contwoyto Lake Temperature C
June only

```
0  0  0  0  0  -2.43333  0
0  0  0  0  0
0  0  0  0  0  2.76667  0
0  0  0  0  0
0  0  0  0  0  -3.73333  0
0  0  0  0  0
0  0  0  0  0  -.433333  0
0  0  0  0  0
0  0  0  0  0  -4.53333  0
0  0  0  0  0
0  0  0  0  0  .766667  0
0  0  0  0  0
0  0  0  0  0  1.06667  0
0  0  0  0  0
0  0  0  0  0  -.933333  0
0  0  0  0  0
0  0  0  0  0  3.66667  0
0  0  0  0  0
0  0  0  0  0  1.66667  0
0  0  0  0  0
0  0  0  0  0  2.26667  0
0  0  0  0  0
0  0  0  0  0  -.133333  0
0  0  0  0  0
```

TREE RIVER [illegible] COPPERMINE - DSRAIN,t,APRILTEMP

SARIMA(8, 0, 1)(0, 0, 0) 0

LENGTH OF THE INPUT TIME SERIES = 96

LENGTH OF OUTPUT SERIES OF TRANSFER FUNCTION = 95

SUM OF SQUARES	RESIDUAL VARIANCE	AIC
6.09264692D 00	6.31712618D-02	356.5

ESTIMATED MEAN OF SERIES

MEAN	S.E.(MEAN)
-1.598157D-03	2.236219D-01

ESTIMATED BETA PARAMETERS

BETA	SE(BETA)
0.3165	C.1195
0.	0.
0.	0.
0.	0.
0.	0.
0.	0.
0.	0.
-0.2508	0.0931
-0.5732	0.1076

CORRELATION MATRIX OF BETA

1.000		
0.112	1.000	
0.629	0.061	1.000

TRANSFER FUNCTION NO. 1, IU=0, IV=0, IDELAY= 0

MEAN CORRECTION =-3.817295D-19

TAU	S.D.(TAU)
1.807221D-03	1.655544D-03

TRANSFER FUNCTION NO. 2, IU=0, IV=0, IDELAY= 0

MEAN CORRECTION =-3.388132D-20

TAU	S.D.(TAU)
3.983774D-02	1.662417D-02

TRANSFER FUNCTION NO. 3, IU=0, IV=0, IDELAY= 1

MEAN CORRECTION =-9.035018D-21

TAU	S.D.(TAU)
3.095679D-02	1.749545D-02

CORRELATION MATRIX OF TAU

1.000		
0.130	1.000	
-0.241	-0.349	1.000

------RESIDUAL ANALYSIS------

SKEWNESS

G1	SL
-0.3037	0.203322

TEST FOR HETEROSCEDASTICITY DEPENDING ON THE CURRENT LEVEL

CHI	SE(CHI)
-0.546156	0.510702

TEST FOR TRENDS IN THE VARIANCE OVER TIME

CHI	SE(CHI)
-0.003757	0.005209

RESIDUAL AUTOCORRELATIONS

L	RA(L)	SE(L)	Q(L)
1	-0.00208	0.02699	0.00043
2	0.03533	0.04741	0.12538
3	0.07335	0.09776	0.66970
4	-0.04083	0.09947	0.84016
5	-0.00714	0.10148	0.84543
6	-0.05341	0.10183	1.14362
7	-0.07235	0.10199	1.69691
8	0.01542	0.04142	1.72232
9	-0.00311	0.09383	1.72337
10	0.03769	0.10012	1.87879
11	-0.03388	0.10163	2.04610
12	-0.08549	0.10200	2.86474
13	0.02760	0.10205	2.95106
14	-0.05394	0.10206	3.28486
15	0.03762	0.10206	3.44927
16	-0.01292	0.09936	3.46890
17	-0.00697	0.10083	3.47469
18	-0.00773	0.10163	3.48190
19	0.02729	0.10193	3.57288
20	-0.02801	0.10203	3.66999
21	0.02509	0.10206	3.74896
22	-0.04225	0.10206	3.97588
23	0.07760	0.10206	4.75203
24	-0.11389	0.10189	6.44699

Response: Tree R. Flow 69-76 m**3/s lambda=0 Fm=6 Fs=0

-0.162768D 00	-0.182852D 00	-0.169470D 00	-0.127237D 00
0.232399D 00	-0.827931D 00	-0.522330D 00	0.472827D 00
0.666694D 00	0.440695D 00	0.284237D 00	0.294343D-01
-0.123136D 00	-0.135333D 00	-0.593871D-01	-0.305604D-01
-0.465267D-01	0.406738D 00	-0.314572D-01	0.511474D-01
0.323134D 00	0.482297D 00	0.592712D 00	0.240872D 00
-0.241958D-01	-0.761971D-02	-0.136490D 00	-0.399993D 00
-0.860273D 00	-0.563977D 00	-0.940500D-01	-0.479546D-01
0.316110D-01	0.152958D 00	0.438482D 00	0.633477D 00
0.716953D 00	0.698617D 00	0.715975D 00	0.360005D 00
-0.444707D 00	0.544474D 00	0.341121D 00	-0.192362D-01
-0.204428D 00	-0.381164D 00	-0.337609D 00	-0.290797D 00
-0.190497D 00	-0.687385D-01	0.148804D 00	0.271943D 00
0.348763D 00	0.201360D 00	-0.685186D-01	-0.125964D 00
-0.123415D 00	-0.795587D-01	-0.435419D 00	-0.551206D 00
-0.372330D 00	-0.147382D 00	0.897028D-01	0.150053D 00
0.310935D 00	-0.185714D 00	-0.192247D 00	-0.195793D 00
-0.351116D 00	-0.462850D 00	-0.212947D 00	0.129776D 00
0.458920D 00	0.645199D 00	0.567099D 00	0.883596D-01
0.187849D-01	0.286041D 00	0.155846D 00	-0.265925D 00
-0.358578D 00	-0.418219D 00	-0.502064D 00	-0.370296D 00
-0.302946D 00	-0.801891D 00	-0.115623D 01	-0.312570D 00
0.440624D 00	0.139009D 00	0.411636D 00	0.130903D 00
0.160983D-01	0.265842D 00	0.172608D 00	0.178741D 00

Transfer Function No. 1: Coppermine Rainfall mm

lambda=1 Fm=6 Fs=0

0.	0.	0.	0.
0.225000D 01	-0.113750D 02	-0.837500D 01	0.127500D 02
-0.612500D 01	0.237500D 01	0.	0.
0.	0.	0.	0.
-0.775000D 01	-0.113750D 02	0.362500D 01	0.875000D 01
0.308750D 02	-0.162500D 01	0.	0.
0.	0.	0.	0.
-0.375000D 01	0.862500D 01	0.126250D 02	-0.142500D 02
0.587500D 01	0.337500D 01	0.	0.
0.	0.	0.	0.
-0.675000D 01	-0.153750D 02	-0.173750D 02	-0.250000D 00
-0.712500D 01	-0.162500D 01	0.	0.
0.	0.	0.	0.
0.425000D 01	0.436250D 02	-0.113750D 02	0.337500D 02
-0.125000D 00	0.237500D 01	0.	0.
0.	0.	0.	0.
-0.775000D 01	-0.375000D 00	0.156250D 02	-0.232500D 02
-0.812500D 01	-0.162500D 01	0.	0.
0.	0.	0.	0.
0.250000D 00	-0.103750D 02	-0.337500D 01	-0.322500D 02
-0.112500D 01	-0.162500D 01	0.	0.
0.	0.	0.	0.
0.192500D 02	-0.337500D 01	0.136250D 02	0.147500D 02
-0.141250D 02	-0.162500D 01	0.	0.

Transfer Function No. 2: Coppermine Temperature C
lambda=1 Fm=6 Fs=0 months 1 2 3 4 8 11 12 constrained

0 0 0 0 -1.5125 -2.225 .25
0 1.1375 4.5875 0 0
0 0 0 0 -2.7125 .975 -.35
0 -1.2625 -.5125 0 0
0 0 0 0 1.6875 .575 .45
0 .6375 2.6875 0 0
0 0 0 0 -3.2125 -1.925 -1.55
0 -3.4625 -3.2125 0 0
0 0 0 0 2.6875 1.475 1.55
0 3.0375 3.4875 0 0
0 0 0 0 -.8125 -.925 -.35
0 -2.1625 -6.5125 0 0
0 0 0 0 3.0875 3.075 -.05
0 .6375 -2.0125 0 0
0 0 0 0 .7875 -1.025 .05
0 1.4375 1.4875 0 0

Transfer Function No. 3: Coppermine Temperature C
April only mean has been subtracted

0 0 0 2.125 0 0 0
0 0 0 0 0
0 0 0 -1.475 0 0 0
0 0 0 0 0
0 0 0 -.675 0 0 0
0 0 0 0 0
0 0 0 -4.775 0 0 0
0 0 0 0 0
0 0 0 -.175 0 0 0
0 0 0 0 0
0 0 0 -5.075 0 0 0
0 0 0 0 0
0 0 0 5.725 0 0 0
0 0 0 0 0
0 0 0 4.325 0 0 0
0 0 0 0 0