ADVANCES IN DEFINING CRITICAL LOADING LEVELS FOR PHOSPHORUS IN LAKE EUTROPHICATION

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1. Introduction

It is now well accepted that eutrophication of lakes depends on excessive discharges of phosphorus and nitrogen to inland waters. This led to the development of what is now called the nutrient loading concept (cf. Vollenweider and Dillon 1974; Vollenweider 1975). This concept implies that a quantifiable relationship exists between the amount of nutrients reaching a lake and its trophic degree measurable with some kind of trophic scale index. The call for a trophic scale index evolved from a need generally felt to give better meaning and significance to the classical limnological categories of oligo-, meso- and eutrophy. This question is not entirely solved as yet, but advancement toward the development of a universal scale of this sort is underway by a number of researchers.

Meanwhile, further progress has been made toward improved criteria for estimating critical levels for phosphorus loading. This notion applies to lakes in which the production level is controlled by phosphorus, and implies that the trophic

*) This paper is part of the OECD Cooperative Programme on Eutrophication. However, the views expressed here are those of the author alone, and are not a policy recommendation of OECD.

nature of a lake may change if discharge of phosphorus exceeds a certain but quantifiable level, which in turn, depends on the limnological characteristics of the body of water in question.

The establishment of such critical levels has undergone several stages in the past. Because of the scanty information available at the time, the first possible estimation of the transition range for critical loading of phosphorus accounted for the effect of mean depth as the sole reference parameter, giving the following approximation

$$L_c \, (mg/m^2.y) \approx (25 \text{ to } 50) \, \bar{z}^{0.6} \qquad (1)$$

(Vollenweider 1968). Further improvement on this criterion was possible using a simplified mixed reactor model which - in addition to mean depth - included terms for the hydraulic residence time and a sedimentation function (Vollenweider, 1969, 1975; cf. also Dillon and Rigler 1974, Lorenzen 1974). At this basis, then, the next level of approximation may be written

$$L_c \, (mg/m^2.y) \approx [P]_c^{sp} \bar{z} \, (\rho_w + \sigma_p) \qquad (2)$$

$$\approx [P]_c^{sp} (\bar{z}/\tau_w + \bar{z}\sigma_p) \qquad (2a)$$

where $[P]_c^{sp}$ is a critical concentration of total phosphorus (mg PT/m^3), for simplicity taken at spring overturn, $\bar{z}$ is the mean depth (m), $\rho_w = 1/\tau_w = Q_{y/V}$ defining the flushing rate per year, and σ_p relates to the sedimentation rate of phosphorus.

Common limnological experience suggests that the lower limit of $[P]_c^{sp}$ may safely be assumed to be 10 mgP/m^3 with an upper limit not in excess of 20 mg/m^3. σ_p, the only unknown in (2), however, could not be estimated independently, but had to be derived from available data indirectly (see below).

Eq. (2a) offers two interpretations:

a) The critical loading of comparable lakes is directly proportional to their mean depth, and to some extent indirectly proportional to the hydraulic residence time of water *); in addition, the loading tolerance depends also on the apparent velocity of sedimentation of phosphorus (second term in parenthesis);

b) Considering the meaning of $\bar{z}/\tau_w$ which equals the hydraulic load q_s in m/y per unit surface, however, it appears that mean depth, as an independent parameter, in part is lost. The extent to which this may be true depends on the relationship between $\bar{z}$ and σ_p.

2. Numerical Solutions for Eq. 2

According to what has been said above, the development of (2) into an equation useful for estimating critical loadings depends on how σ_p is dealt with. In principle, several ways are

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*) The proportionality to $\bar{z}$ relates to the dilution of the incoming phosphorus load and the reciprocal proportionality to τ_w to its likely time of residence in the lake.

open. From the steady state equation underlying (2), one can derive that

$$\sigma_p = \frac{F_p\ (\bar{z})}{\bar{z}.[P]_\lambda}$$

where $F(\bar{z})$ represents the flux of P through the horizon at $\bar{z}$ in mg P/m^2.y, and $[P]_\lambda$ represents an average total phosphorus concentration in mg P/m^3 over the column from 0 to $\bar{z}$. Accordingly (2a) becomes

$$L_c\ (\text{mg/m}^2.\text{y}) \approx [P]_c^{sp}\,(\bar{z}/\tau_w + \frac{F_p\ (\bar{z})}{[P]_\lambda}) \qquad (3)$$

The shortcoming of this equation lies in the scarcity of data available for evaluating $F(\bar{z})/[P]$. Table 1 lists some values calculated for a few Swiss lakes using estimated deep point sedimentation rates and spring overturn concentrations. The few values are in the same order of magnitude; however, it would be premature to generalize these data over a large spectrum of lakes.

For a second attempt, σ_p may also be estimated from

$$\sigma_p = \frac{L_P}{\bar{z}.[P]_\lambda} - \rho_w$$

which follows directly from the steady state equation pertaining to the mixed reactor lake model (cf. Vollenweider l.c.). In this procedure, it was possible to use a much larger data bank including lakes of various physiographic and trophic characteristics. Results for some 25 lakes are plotted in Figure 1 against mean depth.

The trend over this large spectrum indicates that σ_p depends inversely on $\bar{z}$ although the scattering of the individual lakes is but little related to either trophic conditions or other limnological characteristics except that lakes of considerable surface extension (e.g. the Laurentian Great Lakes) consistently show higher σ_p values than smaller lakes of comparable mean depth. This peculiarity cannot be explained as yet.

As a general rule, however, σ_p may be approximated by

$$\sigma_p \doteq 10/\bar{z}$$

Introducing this relation into (2a), then L_c would be given as

$$L_c \ (mg/m^2.y) = [P]_c^{sp} \, (\bar{z}/\tau_w + 10) \qquad (4)$$

or, assuming for $[P]_c^{sp} = 10 \ mg/m^3$

$$L_c \ (mg/m^2.y) = 100 + 10 \ \bar{z}/\tau_w \qquad (4a)$$

Lakes receiving a phosphorus load essentially below the corresponding estimates from (4a) should be oligotrophic. Conversely, it is reasonable to predict that lakes receiving at least twice the load as calculated from (4a) would be on the eutrophic side.

Eq. 4 has been tested in a workshop exercise on a large variety of lakes recently studied in connection with the North American Project of the OECD Cooperative Programme on Eutrophication *). In order to account for the statistical

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*) OECD Workshop 1974 at CCIW, Burlington. Participants have been requested to plot their P loading values against $\bar{z}/\tau_w$ of a blank diagram indicating at the same time the trophic character (oligo-, meso-, eutrophic) of the lake in question. Eq. 4 has been drawn after completion of the exercise. Names of the lakes in Figure 2 are omitted to preserve the anonymity of participants.

uncertainties regarding σ_p in (3), allowance has been made for some marginal variability of the constants in (4).

The result of this workshop exercise is shown in Figure 2. From the more than 60 lakes plotted, only a small fraction (three or four lakes defined as "eutrophic" and "mesotrophic", respectively) do not fit the expected pattern. Conversely, a corresponding exercise plotting L_p against $\bar{z}$ only resulted in a much larger fraction of misplaced lakes.

In view of this experience, it can be concluded that (4) represents a considerable improvement over (1). It is further remarkable that the two procedures discussed thus far give practically the same order of magnitude for the apparent settling velocity, i.e. about 10 m/y *). However, this value lies considerably below experimentally determined sinking velocities (cf. e.g. Burns and Pashley 1974); accordingly, the apparent settling velocity, as used in this paper, is not a physical entity per se, but a balance value integrated in time over all positive and negative settling velocities which in addition includes the effects due to remineralization over that period of time. Therefore, it would be misleading to introduce real settling velocities into

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*) In an unpublished comment of S. C. Chapra, using data of Kirchner and Dillon (1975) and their empirical relationship of phosphorus retention against hydraulic load derived on 15 Ontario lakes, the author found by least square techniques 16 m/y for the apparent settling velocity.

Eq. (4). If done, this would result in an increased apparent loading tolerance, particularly for lakes of low hydraulic load, and hence, lead to erroneous predictions.

3. Critical Assessment

Although Eq. (4) appears to be by far more satisfactory for predicting loading tolerances than the original criteria based on mean depth only, there is legitimate ground to argue that, using Eq. (4) as reference criterion, one ignores, or at least underestimates, the effect of mean depth, i.e. the dilution function. Indeed, $\bar{z}/\tau_w = q_s$ represents the hydraulic load which is independent of mean depth.

At the basis of his two-layer model - which is more elaborate than our mixed reactor model - Imboden (1974) has demonstrated that the effects of mean depth and hydraulic load on permissible loading levels can be separated in principle. At a first glance, it would appear that his criteria are more adequate than ours. However, a critical analysis of his diagram reveals an inconsistency not easily noted, i.e. the strong negative bending of derived loading tolerance curves for lakes having a mean depth of less than 20 to 30 m. From his theory, it would follow e.g. that for lakes having a mean depth of some 20 m, the loading tolerance would range from 100 $mg/m^2.y$ for lakes receiving a hydraulic load of about 9 m/y (0.025 m/day), to only 250 $mg/m^2.y$

for lakes receiving a hydraulic load of 73 m/y (0.2 m/day). This is in contradiction to experience on highly flushed lakes. Indeed, for the above example, the average inflow concentration in the first case would be 11 mg/m^3 (100/9), in the second, however, only some 3.5 mg/m^3 (250/73). Studies conducted by Dillon and Rigler (l.c.)and Kerekes (1975) on Canadian lakes, show that highly flushed lakes with average inflow concentration as high as 10 mg P/m^3 can receive a total annual load of several hundred or thousands of mg/m^2.y without noticeable alteration of their oligotrophic character. Accordingly, one would expect the loading tolerance curves in Imboden's diagram to bend in the opposite sense, i.e. flatten out with decreasing mean depth. The usefulness of the notion of "average inflow concentration" shall be further explored later on.

The two arguments have given rise to further studies on the critical loading concept.

4. Implications of phosphorus Residence Time as a Reference Paramater

In light of what has been stated above, Eq. (4) needs to be modified to account for mean depth in addition to hydraulic load. As has been discussed elsewhere (cf. Vollenweider l.c.) the basic attempt in pursuing a mixed reactor model was to conserve, as far as possible, the dimensional consistency. However, this cannot be done without forcing the system by over-simplifying the connected relationships. In order to overcome the problem, one has at least two ways open, either to attempt to model lakes from the point of view of an interactive dynamic more-layer system in the sense as discussed by Imboden (l.c.), or to try to circumvent the non-trivial difficulties connected with this concept by giving up the dimensional consistencies, using instead statistically given parameter connections. This, indeed, can be defended not only because of our inability to sufficiently cope with the complex interactions between the different systems components in real situations, but also that at least some of the interactions, in addition to being non-linear, are to a considerable degree stochastic in nature. Accordingly, whatever the way of choice might be, we are forced into adopting certain more or less defendable shortcuts.

The possibility of a shortcut results from consideration of the residence time of phosphorus. An adequate definition of this notion encounters the same difficulty as the corresponding definition of the water residence time (cf. Piontelli and Tonolli 1964; Boyce 1975). However, for a variety of practical purposes, theoretical filling-time of a lake, $\tau_W = V/Q\gamma$ appears to be as appropriate as any other more elaborate definition. In principle, the same concept can be expanded to any substance entering a lake and requires the sole knowledge of the total load and the average, or an appropriately selected value for the total content of that substance in the lake, i.e.

$$\overline{\tau_M} = \frac{\mathcal{M}\lambda}{\mathcal{L}_M} ;$$

or, for phosphorus

$$\overline{\tau_P} = \frac{\mathcal{P}\lambda}{\mathcal{L}_P} = \frac{\mathcal{P}\lambda/V}{\mathcal{L}_{P/V}} = \frac{[P]\lambda}{\ell_{p,v}} \quad (5)$$

Eq. (5) defines the filling time, i.e. the hypothetical time necessary to bring the phosphorus concentration of a lake to its present level starting from concentration equal 0. This version has been used by Sonzogni et. al. 1973; Vollenweider 1975, a.o.

However, this definition contains some unrealistic connotations. Because of the fact that the phosphorus loading - with the exception of direct inlets of high concentration which account only marginally for the total water discharge - is not independent from the water balance, $\ell_{p,v}$ (the volumnar loading) has primarily computational meaning. Therefore, it would probably be more meaningful for comparative purposes to consider the residence time of phosphorus relative to that of water, i.e.

$$\pi_r = \overline{\tau_P}/\tau_w = \frac{\mathcal{P}\lambda}{\mathcal{L}_P} \cdot \frac{1}{V/Q} = \frac{[P]\lambda}{[P]_j} \quad (6)$$

which defines the ratio between average lake concentration, $[P]\lambda$, and the average inflow concentration $[P]_j$. The question then arises of what relation exists between π_r and the residence time τ_w, considering a large enough sample of lakes of different limnological characteristics.

In Figure 3, numerical values of $\overline{\tau_p}/\tau_w$ from some 21 lates are plotted against τ_w. Lakes having $\tau_w < 1$ have been excluded. As expected, the statistical relationship between $\overline{\tau_p}/\tau_w$ is neither independent nor inversely proportional to τ_w but somewhere in between, i.e. $\overline{\tau_p}/\tau_w$ tends to decrease with increasing τ_w. Regression calculations have been made in two steps, originally including only 14 lakes, supplemented in a later stage by another 7 lakes. Both steps are shown in order to demonstrate that inclusion of further data would hardly alter the picture. The two regression lines are not significantly different. Therefore, it can be concluded that the relative residence time of phosphorus depends on the residence time of water by a statistical relationship of the form

$$\overline{\tau_p}/\tau_w = \kappa \cdot (\tau_w)^{-\alpha} \quad ; \quad 0 < \alpha < 1 \tag{7}$$

From the mixed reactor theory follows further that

$$\overline{\tau_p}/\tau_w = \frac{\rho_w}{\rho_w + \sigma_p} = \pi_r \tag{8}$$

i.e. σ_p can be estimated from π_r whatever the specific meaning of σ_p may be. Combining (7) and (8), and introducing this in (2), one therefore can postulate that

$$L_c = \frac{[P]_c}{\kappa} \cdot (\bar{z}/\tau_w) \cdot \tau_w{}^{\alpha} = \frac{[P]_c}{\kappa} \cdot (\bar{z}/\tau_w)^{(1-\alpha)} \cdot \bar{z}^{\alpha} \tag{9}$$

The first expression of (9) gives the critical loading as a function of the hydraulic load and water residence time, the second as a function of the hydraulic load and mean depth; both expressions are equivalent. As has been discussed in the introduction to this section, Eq. (9) is no longer correct in the dimensions unless κ is redefined.

5. Improved loading Criteria.

As a first step one may introduce in (9) values for α and κ which result directly from the correlation analysis of Figure 3. Accordingly,

$$L_c \ (mg/m^2.y) = 15.4 \ (\bar{z}/\tau_w)^{0.585}. \ (\bar{z})^{0.415} \qquad (9a)$$

which may be simplified to

$$L_c \ (mg/m^2.y) = 17 \ (\bar{z}/\tau_w)^{0.6}. \ (\bar{z})^{0.4} \qquad (9b)$$

With Eq (9a) and (9b) it is therefore possible to define the relative contribution of the hydraulic load and mean depth separately, and accordingly can be developed in a simple monogramme (cf. Fig. 4). Prior to discussing its shortcomings, we shall discuss its relation to (1). Essentially, Eq. (9b) and (1) differ in the magnitude of the exponent in $\bar{z}$ (0.4 versus 0.6); further, the factor which takes care of the hydraulic load in (9b) can be thought of to be included in the proportionality factor of (1). Indeed, a review of all lakes which originally had been included in the $L_c - \bar{z}$ diagramme revealed that the average $(\bar{z}/\tau_w)$ for all lakes was about 8.5. From this the following loading tolerances for lakes of 10 and 100 m respectively, using Eq. (1) (lower limit) and Eq. (9b), are computed.

	Eq. (1) $25.^{0.6}$	$17.(8.5)^{0.6}\ \bar{z}^{0.4}$
$\bar{z}$		
10 m	99	154
100 m	396	387

Accordingly, for lakes of an average hydraulic load of some 10 m/y, the estimated loading tolerance results in the same order of magnitude, yet, Eq. (1) would fail for lakes of very low and very high flushing rates respectively. It is, however, comforting how close the estimates are for average conditions.

In regard to the shortcomings of (9), one can see from Figure 4 that the loading tolerance curves - although accounting for the hydraulic load - do not flatten out for lakes of lower mean depth, as expected, yet, the pattern generally speaking, is considerably different from that of Imboden's diagramme.

The inadequacy of (9) for the mean depth range of less that 20 m and lakes of increased hydraulic load is due to the fact that the statistical relation given in Figure 3 regarding the dependency of $\overline{\tau p}/\tau_W$ on τ_W cannot be linearly extrapolated below $\tau_W < 1$. Indeed, one would have to expect that

$$\overline{\tau p}/\tau_W \rightarrow 1, \text{ for } \tau_W \rightarrow 0.$$

An approximation which takes care of this would be

$$\overline{\tau p}/\tau_W = \frac{1}{1 + \tau_W^{\kappa}} ,$$

which, for our purposes can be simplified to

$$\overline{\tau p}/\tau_W = \frac{1}{1 + \sqrt{\tau_W}} = \frac{1}{1 + \sqrt{\bar{z}/q_s}} \qquad (10)$$

Numerically, (10) gives practically the same values for $\overline{\tau p}/\tau_W$ at values of $\tau_W > 5$, with an intersect of 0.5 for $\tau_W = 1$, i.e. slightly less than the statistical intersection. Therefore, it might be that (10) underestimates the average trend slightly for τ_W values < 1. Unless larger inconsistencies are encountered, however, there is no reason for essentially modifying Eq. (10).

Combining (10) with (8) and (2), then the more generalized relationship

$$L_c \text{ (mg P/m}^2\text{.y)} = [P]_c^{sp} \, . \, \bar{z} \left(\frac{1 + \sqrt{\tau_W}}{\tau_W} \right) \qquad (11)$$

gives a criterion which holds over the total spectrum of combinations of mean depth and hydraulic load, considering that $\bar{z}/\tau_W = q_s$, and $\tau_W = \bar{z}/q_s$; (11) may also be written as

$$\boxed{L_c \text{ (mg P/m}^2\text{y)} = 10 \, . \, q_s \left(1 + \sqrt{\bar{z}/q_s}\right)} \qquad (11a)$$

(11a) expresses the loading tolerance in terms of the sole characteristics of morphometric properties of the lake curvette - condensed in the term of mean depth $\bar{z}$ - , and the hydraulic load $q_s = \bar{z}/\tau_w$ which - in essence - expresses the relationship between the hydrologic properties of the basin and the lake. This means that, in principle, the loading tolerance for phosphorus - or more generalized, the production capacity - of any lake can be understood as governed to a large extent by two independent functional properties.

(11a) has been developed in the form of two equivalent diagrammes. In version 1 (Figure 5) L_c is plotted against mean depth, and parameterized as a function of q_s; in version 2 (Figure 6), instead, L_c is plotted against the hydraulic load and split up in terms of mean depth. Although identical in content, each version has its proper merit.

Version (1) shows the characteristic flattening toward the left; this means that the loading tolerance of lakes of moderate mean depth is almost entirely governed by flushing; vice versa, version 2 shows that the loading tolerance of lakes, receiving a modest hydraulic load, is governed more by mean depth.

With (11) and (11a), therefore, a loading tolerance definition is obtained essentially different than that of Imboden. Remarkably, the same pattern - though not in absolute terms - has been derived by Snodgrass (1974) using a two-layer simulation model which - in contrast to that of Imboden - puts particular emphasis on the vertical exchange processes between the layers. Our method, and the method of Snodgrass, therefore, seem to be complementary.

On the other hand, it is also worthwhile to note that our new criteria, and those of Imboden, are practically identical for large, deep lakes as shown from Table 2; cf. column (6) versus (3) and (4). This would mean that the model assumptions made by Imboden are probably applicable for such lakes but would fail for shallower lakes.

Figure 6 supports further a version which I proposed earlier, and which was derived from simple inspection of lakes plotted in an L_c-$(\bar{z}/\tau_w)$ diagramme (cf. Vollenweider 1975). In this diagramme, the loading criterion for separating oligotrophic from eutrophic lakes was assumed to be

$$L_c\ (mg/m^2.y) \approx (100\ \text{to}\ 200)\ (z/\tau_w)^{0.5} \qquad (12)$$

which indeed is a shortcut well in agreement with (11a). Its usefulness has already been demonstrated by Michalski et. al. (1973), Stockner et. al.(1974) and several reports in preparation by G. Fred Lee and his collaborators. Its main weaknesses, of course, are the same as those of Eq. (4a).

6. Verification

Loading criteria cannot be verified, of course, in the proper sense of the term. However, any of the former equations implicitly contain the hypothesis that a relation exists between the actual phosphorus loading and production level attained by the lake(s) in question. Up until recently, it was impossible to discuss this aspect in quantitative terms because of lack of comparable data. With the initial results available from the OECD programme, however, a preliminary exploration is now feasible.

For this, Eq. (11) may be rewritten, not in the terms of critical loading and critical concentration, but simply relating phosphorus concentration to loading, i.e.

$$\overline{[P]}_\lambda = \left(L_p/q_s\right) \cdot \left(\frac{1}{1 + \sqrt{\bar{z}/q_s}}\right) \qquad (13)$$

In this (L_p/q_s) represents the average inflow concentration as discussed earlier. Its further implication shall be considered further below.

Several authors have shown that a relation exists between the spring overturn phosphorus concentration and the average chlorophyll build-up during the following season (Sakamoto 1966; Dillon 1974). According to this, we may postulate that (13) could be read in the sense of a production equation in which the LHS is measured by a biological entity such as chlorophyll.

In order to test this hypothesis, average epilimnetic chlorophyll concentrations for some 60 lakes have been plotted against

$$(L_p/q_s) \cdot \left(\frac{1}{1+\sqrt{\bar{z}/q_s}}\right);$$

(cf. Figure 7). Although there are a few uncertain values*), the relation is unquestionable, having a correlation coefficient of 0.868. Also, the exponent derived from the least square fit, though slightly less than 1, deviates only marginally from unity.

In addition to this, each lake plotted has been characterized as oligo-, meso-, or eutrophic, according to the subjective judgement provided by the author. The above discussion on critical phosphorus loadings implies that oligotrophic lakes would be found to the left of the 10 mg/m^3 mark, and eutrophic to the right of the 20 mg mark. This is indeed the case for the majority of lakes considered with a few noticeable exceptions. The data available do not allow, however, analysis of the nature of such uncertain lakes.

As a whole, however, Figure 7 can be considered as valid proof for the realism of the new phosphorus loading criteria

*) The values plotted refer in a large part to summer averages; however, included also are a few yearly averages and averages for which it was not possible to clearly understand their time limits.

presented here. The spectrum of lakes covered includes shallow to deep, and little to highly flushed basin, and hence, represents a valid sample of lakes of the temperate zone. Lakes outside this zone, however, will probably have to be excluded from the application of this criteria.

7. Prediction of Chlorophyll from Loading Characteristics

It is self-evident that the relationship in Figure 7 can be used to predict biomass, in terms of chlorophyll, in relation to the specific phosphorus loading characteristics. In absolute terms, the prediction is probably quite good for low and medium productive lakes, yet less certain for highly productive lakes. According to the experience gained thus far in applying the prediction to lakes not included in the correlation analysis, one can expect that at least the peak values of chlorophyll measured will fall within the 99% confidence limits given in Figure 8. For lakes receiving a very high phosphorus load one has reason to believe that their production level is not solely controlled by phosphorus, and hence, one would expect the relationship to break down.

A specific application has been made to Lake Washington over the period of its recovery, cf. Figure 9. During the period from 1957 to 1964 during which the load still increases, the summer chlorophyll also increases within the confidence limits of the prediction; after sewage diversion has been implemented, however, the measured chlorophyll values remain slightly above prediction until 1971 which may be interpreted as lag phenomenon. From 1972 to 1974, the summer chlorophyll follows the variations of the loading characteristics practically on the line, including the 1957 pre-diversion situation. It has to be mentioned that the loading characteristics over all years have been derived from variations in the phosphorus load and the variations of the hydraulic discharge each year; mean depth has been treated constant.

8. The π_r Ratio

Eq. (6) defines the π_r ratio as the quotient between the average lake concentration and the average inflow concentration. In principle, this definition extends to any substance flowing into a lake. For highly flushed lakes with but little sedimentation, this ratio is expected to approach unity, yet, with increasing involvement of the substance in question into the lake metabolism, this ratio can deviate more or less from 1. Values < 1 signify that the substance has a positive net flow to sediments; conversely, values > 1 signify that the net exchange with sediments is negative, i.e. the substance accumulates in the water phase of the lake system.

In the case of phosphorus, one would expect that. statistically,

$$\pi_r = [P]_\lambda \Big/ [P]_j \sim \frac{1}{1 + \sqrt{\bar{z}/q_s}} = \frac{1}{1 + \sqrt{\tau_w}} \qquad (14)$$

Indeed, this relationship has proved to be extremely valuable. First of all, it serves as a guideline to judge the validity of the basic data. Strong deviations of π_r from the expected value make the basic data suspect. If, for example, in a highly flushed lake, π_r is significantly above the value from (14), one is entitled to believe that the loading has been underestimated; conversely, if the calculated π_r is lower than the value from (14), one has to suspect that the total loading has been overestimated. These considerations are particularly useful in cases where the loading has been obtained from indirect estimates.

If, however, the basic data are such that any uncertainty would have little effect on π_r, then this ratio gives additional information about the metabolic properties of a lake relative to phosphorus. In the case of π_r significantly smaller than $1/(1 + \sqrt{\tau_w})$ the accumulation of phosphorus in the sediments of the lake in question must occur at an above rate; this situation may be caused by high settling rates of inorganic suspensions, or by phytoplankton having high specific density such as highly silicated diatoms. Strong positive deviations of π_r from (14), on the other hand, may signify sedimentation rates below average as caused by light tiny phytoplankton or mineral turbidities, or reduced capacity of sediments to take up phosphorus permanently as can be expected in highly eutrophic lakes. Indeed, in a few cases of highly eutrophied lakes, the π_r ratio has been found to be in excess of 1.

In this light, then, it is also possible to define loading criteria for phosphorus as average inflow concentrations. This is done by dividing both sides of Eq. (9b) and (11a) by $q_s = \bar{z}/\tau_w$; the resulting relationship is shown in Figure 10 which also includes a few specific examples. These, in addition to demonstrating the transition range from oligo- to eutrophy, corroborates also the correction which was deemed necessary in (9) to properly describe the character of highly flushed lakes. It is evident that the loading tolerance of lakes to the left of the diagramme which are highly flushed oligotrophic lakes, would erroneously be judged with Eq. (9), yet, they are reasonably comprehended within the boundaries of (11).*)

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*) Because of the washout effect which can be expected to occur at water residence times comparable with the mean life span of phytoplankton populations, it could be argued that the loading tolerance in terms of average inflow concentrations may increase somewhat at very high flushing rates, i.e. at residence times < 1 ÷ 2 weeks. Yet, at the present state of knowledge, no precise quantitative statements as to the magnitudes involved can be made but the problem could be explored at the basis of appropriate dynamic growth models.

Used in this sense, the average inflow concentration gives a reference figure valid for the lake as a whole. However, the reference figure for the critical loading does not necessarily imply that local areas may not be affected adversely. Indeed, judging from the experience made on a number of lakes listed in Figure 10, one would have to conclude that local problems, such as increased growth of periphyton, noticeably Cladophora, would have to be expected for average inflow concentrations exceeding 30 to 40 mg P/m^3. The implications for a number of cases have been discussed elsewhere (Vollenweider 1975).

9. Discussion

The improvement on available loading criteria for phosphorus was essentially possible only by including terms for water residence time and relative residence times of phosphorus in the lake. The importance of water residence time was recognized by Vollenweider (1968, 1969) and has been stressed further by Dillon (1974), Imboden (1974), Snodgrass (1974), Kerekes (1975), whereas the phosphorus residence time has been considered by Lerman (1974), Sonzogni and Lee (1974).

There are a number of aspects which are worth discussing separately. First, the agreement in principle of our results, with those derived from Snodgrass' theory appears to be more than accidental, although the underlying concepts used are quite different. Snodgrass follows the way designed by O'Melia

(1972) and Imboden (l.c.)., considering mass balances as depending on the internal thermal cycles and the mass exchanges taking place between vertically connected layers, whereas in our treatment, this dynamic aspect has been largely neglected and substituted for by a global treatment of the phosphorus residence time.

Although not supporting fully Imboden's qualitative conclusions, the partial agreement between his and our criteria for deeper not too highly flushed lakes could, in no sense, be expected a priori. Indeed, Imboden's mensural criterion refers to oxygen depletion in the hypolimnion during summer stagnation - arbitrarily set at $\Delta O_2 = 1$ g/m^3, which is quite stringent - whereas the mensural criterion used here refers to an average phosphorus concentration of 10 mg/m^3 arbitrarily selected at spring overturn time. Therefore, it appears that these criteria for lakes of the kind in question are limnologically compatible.

On the other hand, it is also clear why Imboden's criterion fails for shallower, highly flushed lakes. First, an oxygen depletion of 1 g/m^3, for lakes having a hypolimnion of 5 m has quite a different meaning in terms of the epilimnetic productivity than the same rate would have for lakes having a hypolimnion of 100 m. In addition - and this seems to be the more stringent reason - the seasonal cycle and the vertical thermal structure, and their consequence for the intensity and rate of water and material exchange between superposed layers in shallow highly flushed lakes, is hardly comparable to that of deep modestly flushed lakes. The shear forces acting between the layers of a highly flushed shallow lake undoubtedly favour a higher rate of vertical oxygen transport to the hypolimnion over the period of stagnation (if such a stagnation ever occurs at all) as compared to deep lake situations. Accordingly, in the former case, the apparent hypolimnetic oxygen depletion rate represents a balance value between supply and respiratory consumption, whereas in the latter case the same oxygen depletion rate will be closer to respiratory consumption alone. Therefore the hypolimnetic oxygen depletion rate, for a large spectrum of lakes, cannot be taken as a reference without due consideration of the connected vertical oxygen transport functions.

Conversely, the nutrient availability in the production carrying epilimnetic layers, seems to be more closely connected to productivity, at least in terms of average standing crop, generally. Difficulties will arise, however, with the variability of food transfer from phyto- to zooplankton in different lakes, and this may account for part of the statistical variability of the measured average phytoplankton standing crops (in terms of chlorophyll) in lakes of different limnological characteristics. The phytoplankton - zooplankton interrelationship in highly eutrophic lakes appears to be particularly dependent on the kind of species composition of the biota; hence, if the phytoplankton is composed primarily of species edible for zooplankton, one may find a relatively low phytoplankton standing crop vis-a-vis the standing crop of zooplankton and *vice versa*. As shown by Steel (1975), Shapiro et. al. (1975) a.o., this area needs careful consideration for improving on our understanding of the trophic nature of lakes as depending on nutrient loadings.

Secondly, the relation of the relative phosphorus residence time, or π_r, as defined here, to the residence time of water, seems to integrate a number of basic limnological processes in balance. Implicitly, the same relationship has been found recently by Larsen and Mercier (1975) using data from a different set of lakes, concluding that the retention coefficient, statistically can be approximated by

$$R_c = \frac{1}{1+\sqrt{\rho w}}.$$

This, indeed, is identical to what one wouldderive from Eq. (6), defining

$$R_c = \frac{[P]_i\,Q - [P]_\lambda\,Q}{[P]_i\,Q} \equiv 1 - \pi_r \qquad (15)$$

Substituting $\pi_r = \overline{\tau p}/\tau_w$ from Eq. (10, then

$$R_c = \frac{\sqrt{\tau_w}}{1+\sqrt{\tau_w}} = \frac{\sqrt{\bar{z}/q_s}}{1+\sqrt{\bar{z}/q_s}} = \frac{1}{1+\sqrt{\rho w}} \qquad (16)$$

or

$$R_c = \frac{\sqrt{\bar{z}}}{\sqrt{q_s}+\sqrt{\bar{z}}} \qquad (16a)$$

Accordingly, R_c would be an increasing function of mean depth with an upper limit of 1 for very deep lakes ($\bar{z} >> q_s$), but inversely related to q_s for lakes of comparable mean depth. The sense of this is obviously that the likelihood of settling of particles, formed in, or reaching the lake from outside, decreases with the rate of flushing, whereas the dependency on mean depth relates to the probability of "trapping" of particles in the hypolimnion as well as the extent of mineralization of organic particles along their way of sedimentation.

Although (16) is in agreement with our previous deductions, one might suspect that R_c would be slightly underestimated for $q_s < 10$. As has been pointed out earlier, the statistical relationship between $\overline{\tau_p}/\tau_w$ versus τ_w gives an intersection of 0.6 - 0.7 for $\tau_w = 1$, yet $1/(1 + \sqrt{\tau_w})$ would give only 0.5. Indeed, Kirchner and Dillon's statistical analysis for Ontario lakes of some 10 m mean depth would give higher R_c values for $q_s << 10$, yet values for $q_s \geq 10$ are in fair agreement with ours. Contrary to this, Snodgrass' theory would give lower retention coefficients for $q_s > 10$, $\bar{z} > 100$ m. The only known case to the author, apt to provide a judgement whether or not (16a) or Snodgrass' estimates agree more closely with reality, is Lake Maggiore ($\bar{z}$ = 177 m, q_s = 177 m, q_s = 44 m; τ_w = 4 y). The two values would be 0.67 and 0.32 respectively. From available data on loading and outflow (Calderoni in Barbanti et. al. 1974) R_c has been estimated to 0.6. Therefore, (16a) seems to give the better estimate of R_c for such lakes. However, prior to giving a final judgement, much more data from "extreme" lakes should be available.

The discussion of these two aspects indicates the limit of simple mass balance models. If further progress should be possible, then more complex models are needed. It seems to be particularly important to obtain a better hold on parameters which also exert an influence on loading tolerance, such as length of stratification, mixing cycles, depth of thermocline, hypolimnetic entrainment, water discharge and loading cycles, etc. Also, the trophic-dynamic interrelationships in the sense of Lindeman (1942) requires much more sophisticated analyses.

Attemps of this nature are underway in several places; however, there are a number of pitfalls to be avoided in order to significantly exceed what Riley et. al. (1949) have already prospected. In spite of the large amount of limnological literature on the subject the tropho-dynamic interrelations are still insufficiently understood; also, careful mass balance studies, broken down into monthly, or even timely closer episodes, are scant. In addition, much of the data used for "verification" of limnological models have been drawn from non-reliable or at least inappropriate data banks, and hence, this has hardly been to the advantage of model development.

Finally, it has to be stated that - whilst much effort has been put into the analysis of the phosphorus situation, the dynamics of other substances has been largely neglected. If we should arrive at more global criteria in the future relative to eutrophication, then, over the next period of time, more substantial studies are required for other nutrients, such as carbon, nitrogen, macro- and micro-elements, and their metabolic interactions.

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Table 1. Evaluation of $F_P(\bar{z})/[\bar{P}]_\lambda^{sp}$

	$F_p(\bar{z})$ Sedimentation*) mg/m²·y	$[\bar{P}]_\lambda^{sp}$ Spring Overturn Concentration*) mg/m³	$F_P(\bar{z})/[\bar{P}]_\lambda^{sp}$ Apparent Sedimentation Velocity m/y
Aegerisee	130	7.6	17
Zürichsee	310	32	9.7
Hallwilersee	460	40	11.5
Greifensee	950	118	8.1

Average 11.6 ± 3.9

*) For original data cf. Vollenweider 1969

Table 2. Phosphorus Loading Tolerance of Selected Deep Lakes.

Lake	Mean Depth ($\bar{z}$)	Hydraulic Residence Time (τ_W) Years	Present Phosphorus Loading(*) g $P/m^2.y$	Trophic Conditions	Critical Loading Estimates L_c (g $PT/m^2.y$)					
					(1)	(2)	(3)	(4)	(5)	(6)
Lake Superior 1)	148	185	0.03(0.03)	0	0.50	0.11	0.11	0.12	0.09	0.10
Lake Michigan 2)	84	113	0.14(0.29)	0-m	0.36	0.11	0.08	0.09	0.09	0.10
Lake Huron 1)	61	21	0.13(0.15)	0-m	0.30	0.13	0.17	0.16	0.17	0.12
Lake Erie 1)	18	2.6	1.06(0.98)	m-e	0.14	0.17	0.17	0.18	0.26	0.09
Lake Ontario 1)	84	7.9	0.65(0.86)	m	0.36	0.21	0.41	0.40	0.33	0.32
Lake Tahoe 3)	300	700	0.04	0	0.75	0.10	0.10	0.12	0.07	0.16
Lake Maggiore 4)	177	4	~3	→ m	0.55	0.54	1.31	1.33	0.67	1.5
Lake Léman 5)	154	12	0.7→ 1.9	→ m	0.51	0.23	0.59	0.57	0.36	0.47

(*) 1) Sources: IJC Reports (1969; 1973)
Patalas (1972 figures in parenthesis)
2) Lee 1974
3) Goldman (1974, unpublished)
4) Calderoni (personal communication)
5) Jaquet (personal communication)

(1) from Mean Depth (cf. Vollenweider (1968)
(2) from $0.1 + 0.01\ (\bar{z}/\tau_W)$ (cf. Vollenweider 1975)
(3) from $0.007\ (\bar{z}/\tau_W)^{0.6}\ (\bar{z})^{0.6}$ (present paper)
(4) from $0.01\ (\bar{z}/\tau_W)\ (1 + \sqrt{\bar{z}/q_s})$ (present paper)
(5) from $0.1\ (\bar{z}/\tau_W)^{0.5}$ (present paper)
(6) from Imboden (1974)

APPENDIX 1

USE OF FIGURES 5 AND 6, RESPECTIVELY FOR PREDICTING TOLERANCE LEVELS FOR PHOSPHORUS LOADINGS

Enter diagrammes either with the appropriate figures for mean depth $\bar{z}$ (in metres), and hydraulic load q_s (in metres per year), or, alternatively, follow the isolines for the residence time $\tau_w = \bar{z}/q_s$ (in years) instead of separately calculating q_s. Both entries are equivalent.

Example: Lake Ontario ($\bar{z}$ = 84 m, τ_w = 7.9 m,

$q_s = \bar{z}/\tau_w$ = 10.6 m/y).

The loading tolerance, from either figure 5 or 6 , would approximatively be 350 and 400 mg P/m^2.y. This value specifies the lower limit not to be exceeded to keep the lake in acceptable oligotrophic conditions. Excess loading over about twice the above values, i.e. 700 - 800 mg/m^2.y would cause the lake to become eutrophic.

The loading of this lake, prior to the initiation of the phosphorus reduction programme which is now in full progress according to the US-Canada Agreement, was estimated to 680 mg/m^2.y (cf. Anon. IJC Report 1969). Indeed, the condition of the lake has been judged mesotrophic exhibiting pronounced problems of eutrophication inshore (extended growth of Cladophora). This is in agreement with the statement made in the paper (cf. p.23) that

Appendix 1 (continued)

local problems are to be expected, if the average inflow concentration exceeds 30 to 40 mg $\underline{P}/m^3$. From the data given here, the average inflow concentration would be

$$680/10.6 = 64 \text{ mg } \underline{P}/m^3.$$

SYMBOLS

	Meaning	Dimensions CGS-System	Dimensions Practical
V	Lake Volume	L^3	m^3 ; km^3
A_o	Lake Surface	L^2	m^2 ; ha ; km^2
z	Depth	L	m
$\bar{z}$	Mean Depth	L	m
Q_y	Total Yearly Water Discharge	L^3T^{-1}	m^3/y ; km^3/y
q_s	Hydraulic Load $(Q_y/A_o = \bar{z}/\tau_w)$	LT^{-1}	m/y
τ_w	Water Filling Time (Water "Residence Time") $(V/Q_y = \bar{z}/q_s)$	T	y
ρ_w	Flushing Coefficient $(1/\tau_w)$	T^{-1}	1/y
$\mathscr{P}_\lambda$	Total Amount of Phosphorus in Lake	M	t ; kg ; g
$[P]_\lambda$	Phosphorus Concentration in Lake	ML^{-3}	g/m^3 ; mg/m^3
$[P]_\lambda^{sp}$	Lake Concentration at Spring Overturn $(\mathscr{P}_\lambda^{sp}/V)$	ML^{-3}	g/m^3 ; mg/m^3
$\mathscr{L}_P$	Total Phosphorus Loading	MT^{-1}	t/y ; kg/y g/y
L_P	Specific Surface Loading $(\mathscr{L}_P/A_o)$	$ML^{-2}T^{-1}$	$g/m^2.y$; $mg/m^2.y$
ℓ_V	Specific Volumnar Loading $(\mathscr{L}_P/V)$	$ML^{-3}T^{-1}$	$g/m^3.y$; $mg/m^3.y$
$\overline{[P]}_j$	Average Inflow Concentration $(\mathscr{L}_P/Q_Y = L_P/q_s)$	$ML^{-3}T^{-1}$	$g/m^3.y$; $mg/m^3.y$
L_c	Critical Specific Loading	$ML^{-2}T^{-1}$	$g/m^2.y$; $mg/m^2.y$
$[P]_c^{sp}$	Critical Lake Concentration at Spring Overturn	$ML^{-3}T^{-1}$	g/m^3 ; mg/m^3
σ_P	Sedimentation Coefficient of Phosphorus	T^{-1}	1/y
$F_P(z)$	Flux of Phosphorus through the z - horizon	$ML^{-2}T^{-1}$	$g/m^2.y$; $mg/m^2.y$

...2/

Symbols (continued)

Appendix 2 (cont'd)

	Meaning	Dimensions	
		GS - System	Practical
$\overline{\tau_p}/\tau_w$	"Residence Time" of phosphorus relative to "Residence Time" of water	0	
r_π	Ratio between Average Lake Concentration and Average Inflow Concentration (e.g. $[P]_\lambda/[P]_j$)	0	
R_c	Retention Coefficient	0	

CAPTIONS TO FIGURES

Figure 1: $\sigma_p = \dfrac{L_p}{\bar{z}\,[P]_\lambda} - \rho_w$.

Figure 2: Test of loading tolerance for phosphorus according to Eq. (4). Result from an OECD Workshop.

Figure 3: Relative phosphorus residence time as function of water residence time (filling time).

Figure 4: Critical Loading (lower limit) for phosphorus according to Eq. (9b).

Figure 5 : Critical Loading (lower limit) for phosphorus according to Eq. (11a) plotted as function of mean depth.

Figure 6: Critical Loading (lower limit) for phosphorus according to Eq. (11a) plotted as function of hydraulic load.

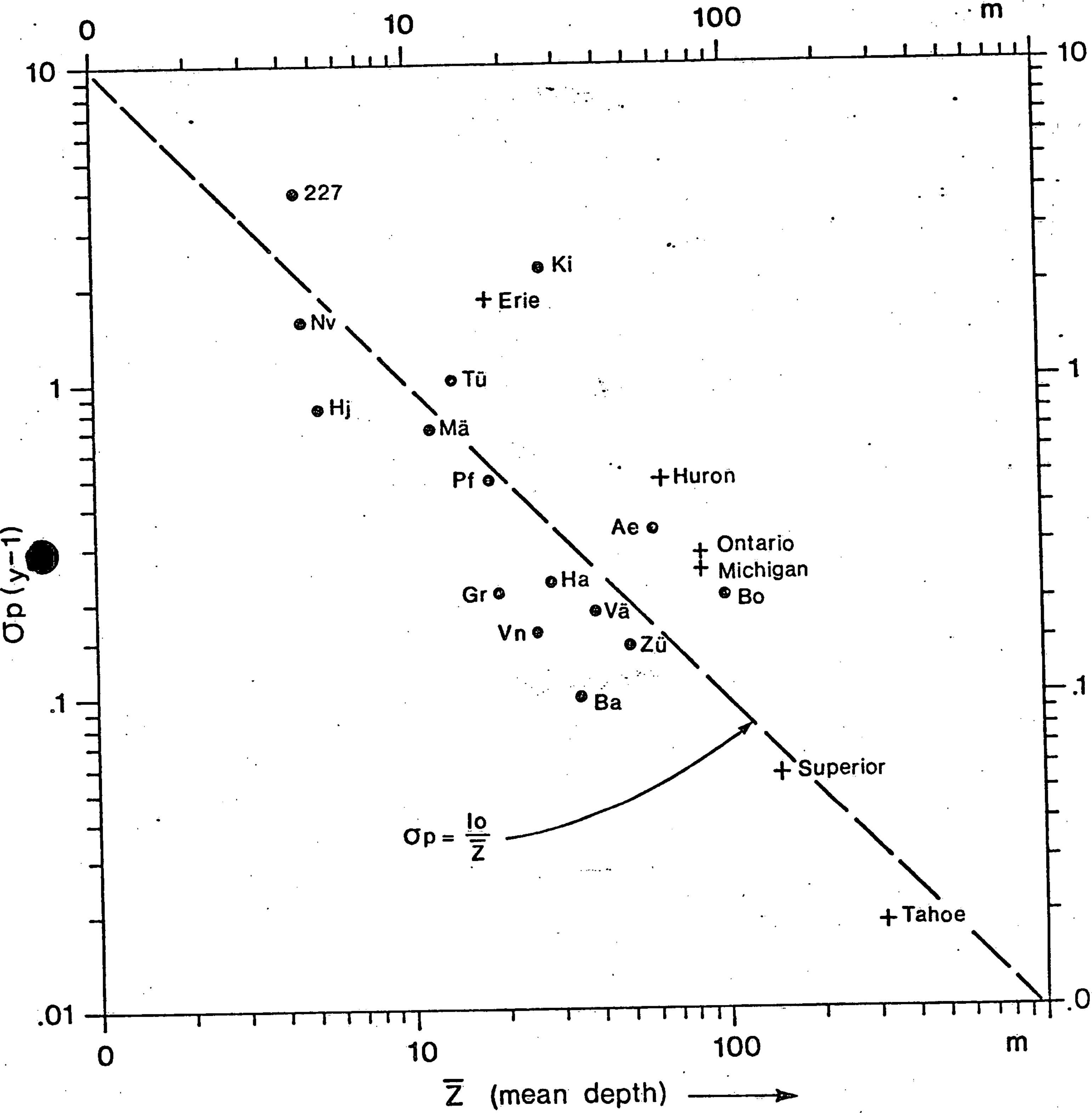
Figure 7: Statistical relationship between average epilimnetic chlorophyll concentration and phosphorus loading characteristics according to Eq. (13). Original data by courtesy of authors collaborating in the OECD Cooperative Programme on Eutrophication, North American Project: P. Brezonik, G.J. Brunskill, R. Carlson, N. Clesceri, D. Cook, R. Daley M. Dickman, P.J. Dillon, W.T. Edmondson, C.R. Goldman, U.T. Hammer, L. Hetling, N.A. Jaworski, J. Kalff, J. Kerekes, G.F. Lee, K. Malueg, R.O. Megard, J.K. Neel, T.G. Northcote, R. Oglesby, C.F. Powers, J. Robinson, C. Schelske, D. Schindler, J. Shapiro, J. Stockner, S.J. Tarapchak, R.R. Weiler, C. Weiss, E. Welch, a.o. sources.

Figure 8: Prediction of average chlorophyll and trophic character of lakes relative to phosphorus loading characteristics.

...2/

Figure 9: Application of the phosphorus loading concept to the evolution of Lake Washington prior and after sewage diversion. Original data by courtesy of Professor W. T. Edmondson.

Figure 10: Generalization of the phosphorus loading concept in terms of average inflow concentration (L_p/q_s) plotted as function of water residence (filling) time. Lower and upper limits are given. Points plotted referring to a number of known examples; unidentified points referring to oligotrophic highly flushed Ontario lakes (original data from Dillon 1974).



0
10
100
m
10
10
227
Ki
Erie
Nv
1
1
Tü
Hj
Mä
Pf
Huron
Ae
Ontario
Michigan
σp (y−1)
Gr
Ha
Bo
Vä
Vn
Zü
.1
.1
Ba
Superior
σp = Io / Z̄
Tahoe
.01
.01
0
10
100
m
Z̄ (mean depth)

Figure 1

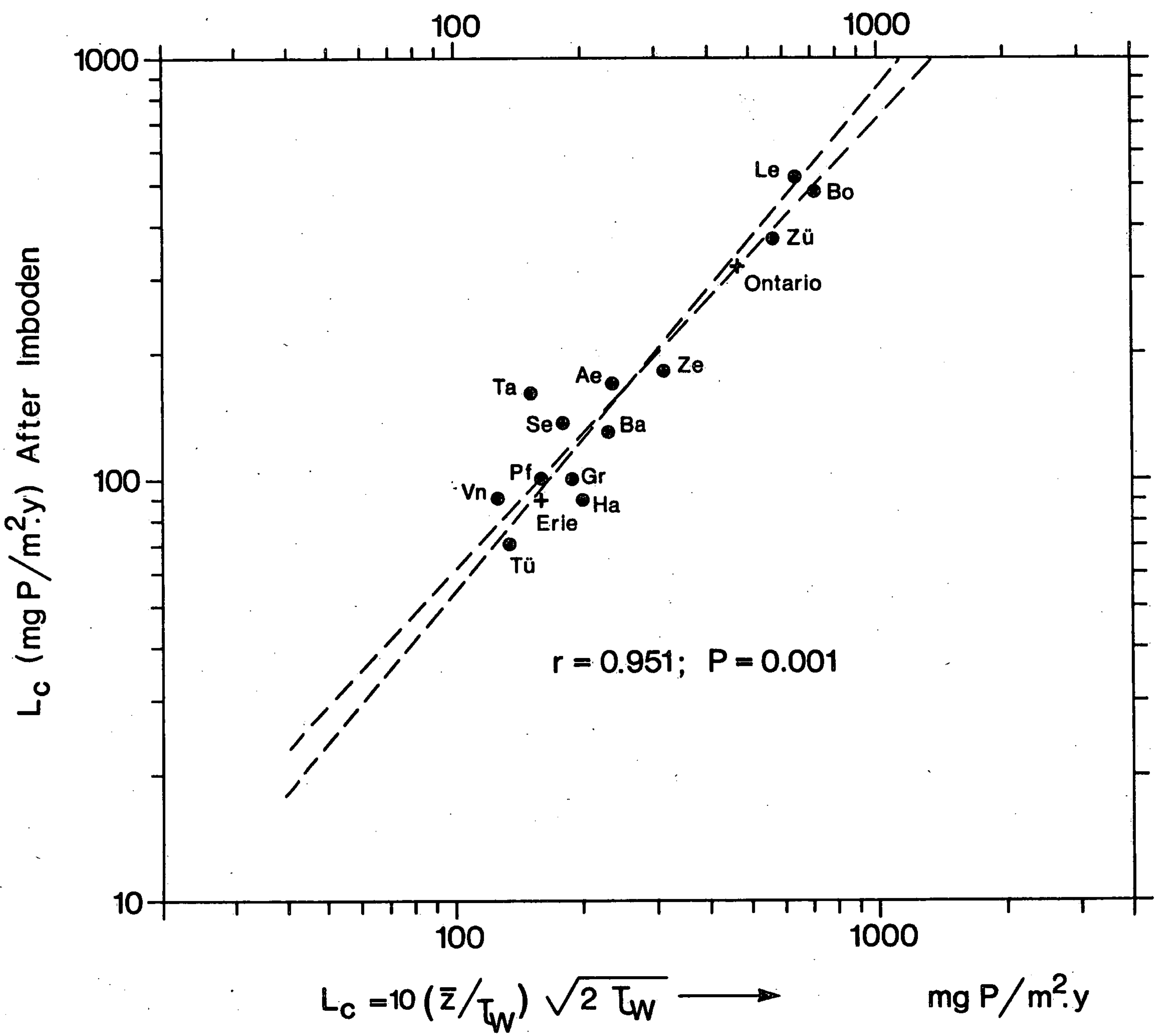
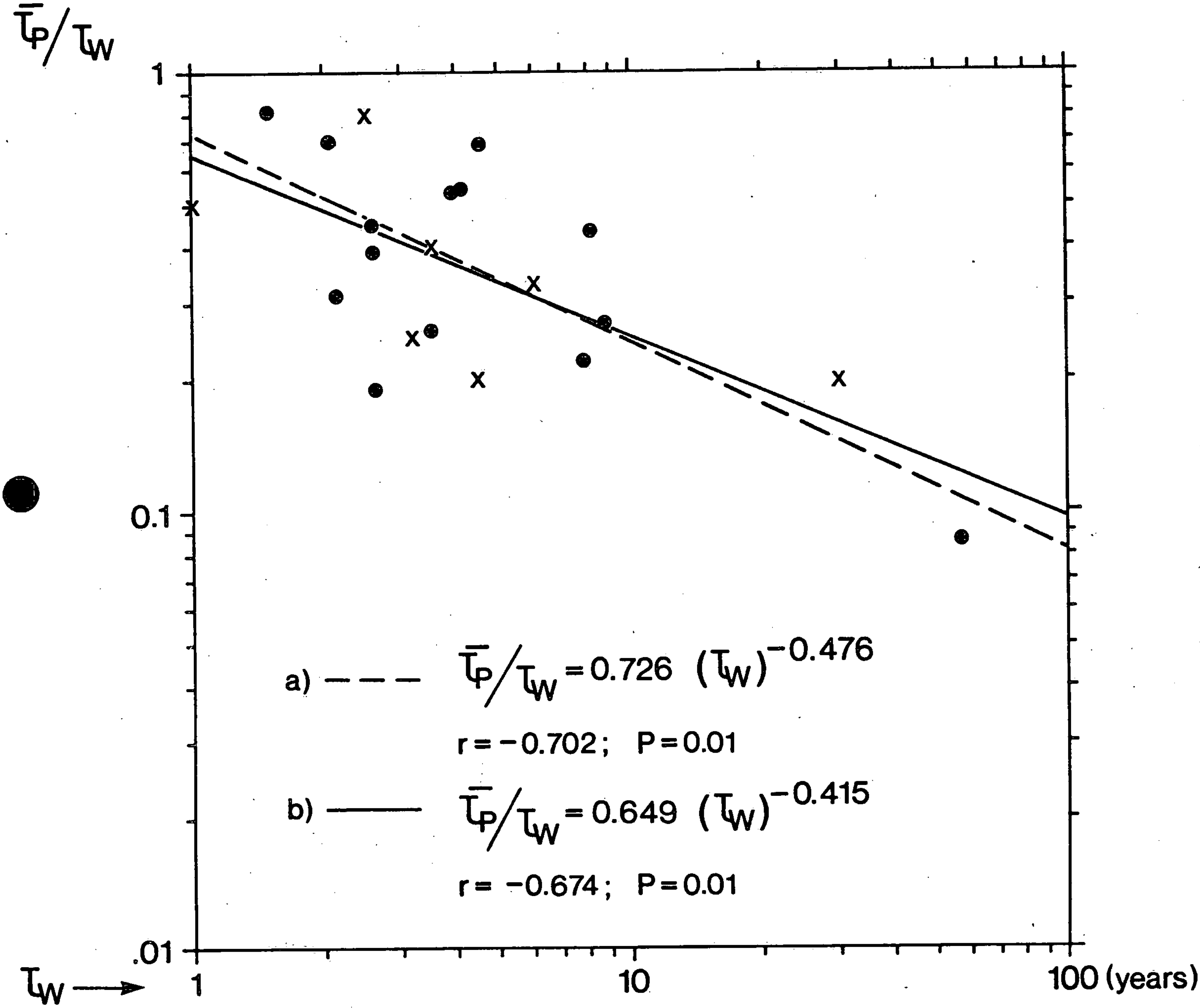

100
1000
1000
100
10
Le
Bo
Zü
Ontario
Ze
Ta
Ae
Se
Ba
Pf
Gr
Vn
Ha
Erie
Tü
r = 0.951; P = 0.001
L_c (mg P/m^2·y) After Imboden
$L_c = 10(\bar{z}/\tau_W)\sqrt{2\tau_W}$ ⟶
mg P/m^2·y

Fig 2



$\bar{\tau}_P/\tau_W$
1
0.1
.01
τ_W →
1
10
100 (years)
a) $\bar{\tau}_P/\tau_W = 0.726\ (\tau_W)^{-0.476}$
r = −0.702; P = 0.01
b) $\bar{\tau}_P/\tau_W = 0.649\ (\tau_W)^{-0.415}$
r = −0.674; P = 0.01

Figure 3

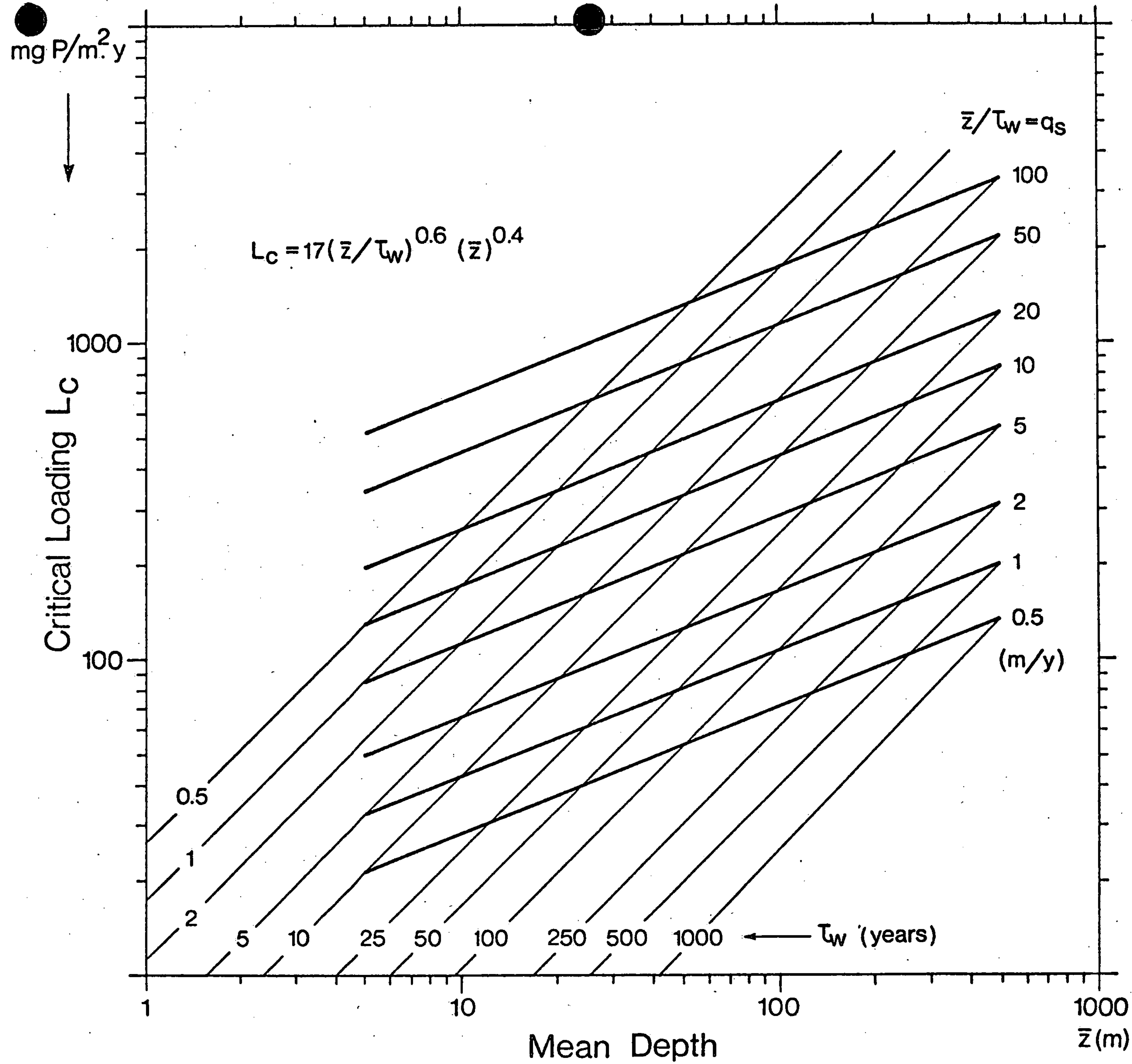
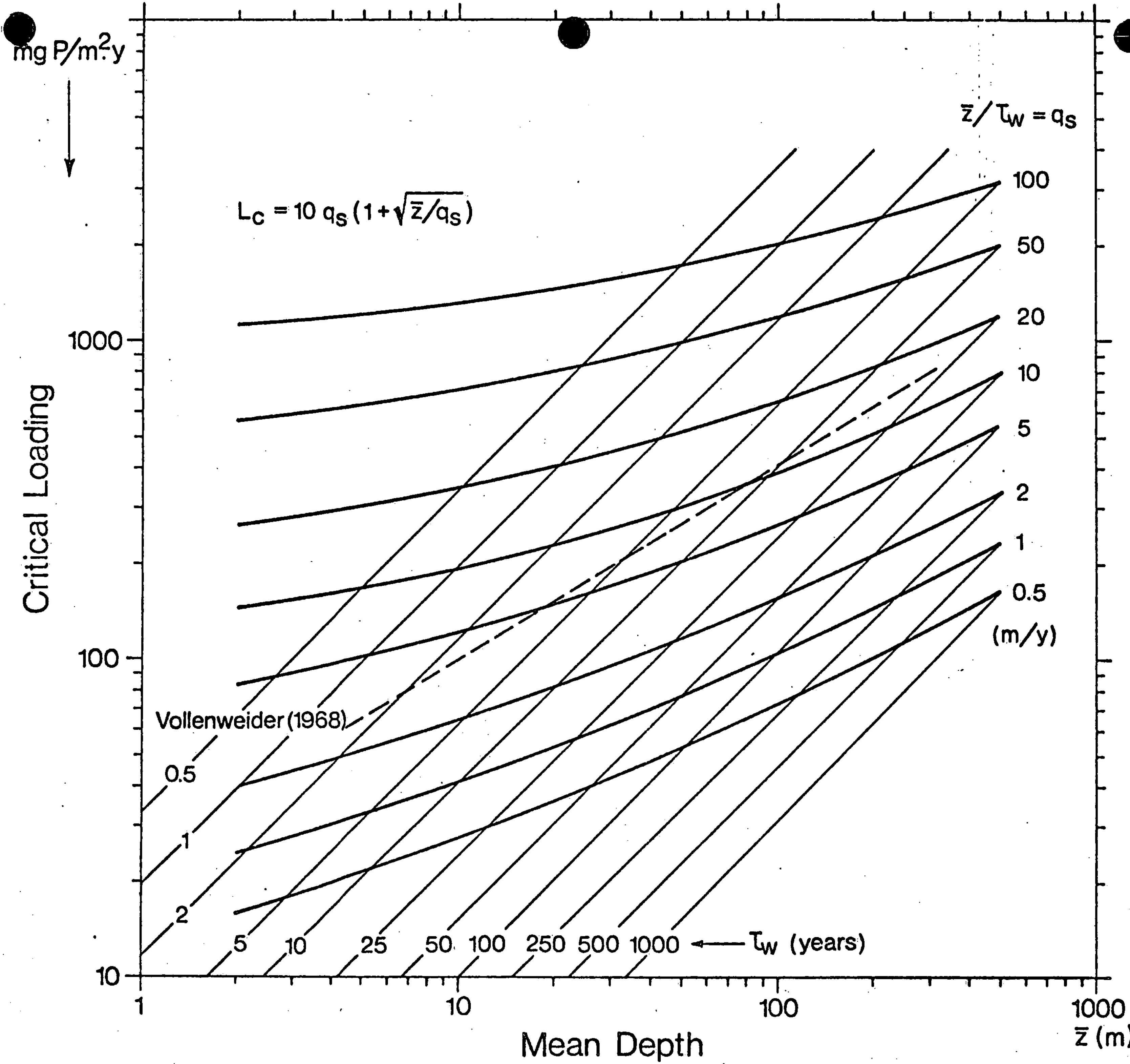

mg P/m².y
Critical Loading L_c
1000
100
$L_c = 17(\bar{z}/\tau_w)^{0.6}(\bar{z})^{0.4}$
$\bar{z}/\tau_w = q_s$
100
50
20
10
5
2
1
0.5
(m/y)
0.5
1
2
5
10
25
50
100
250
500
1000
τ_w (years)
1
10
100
1000
$\bar{z}$ (m)
Mean Depth

Figure 4



mg P/m²·y
Critical Loading
$L_c = 10\, q_s (1 + \sqrt{\bar{z}/q_s})$
$\bar{z}/\tau_w = q_s$
100
50
20
10
5
2
1
0.5
(m/y)
Vollenweider (1968)
0.5
1
2
5
10
25
50
100
250
500
1000
τ_w (years)
1000
100
10
1
10
100
1000
Mean Depth
$\bar{z}$ (m)

Figure 5

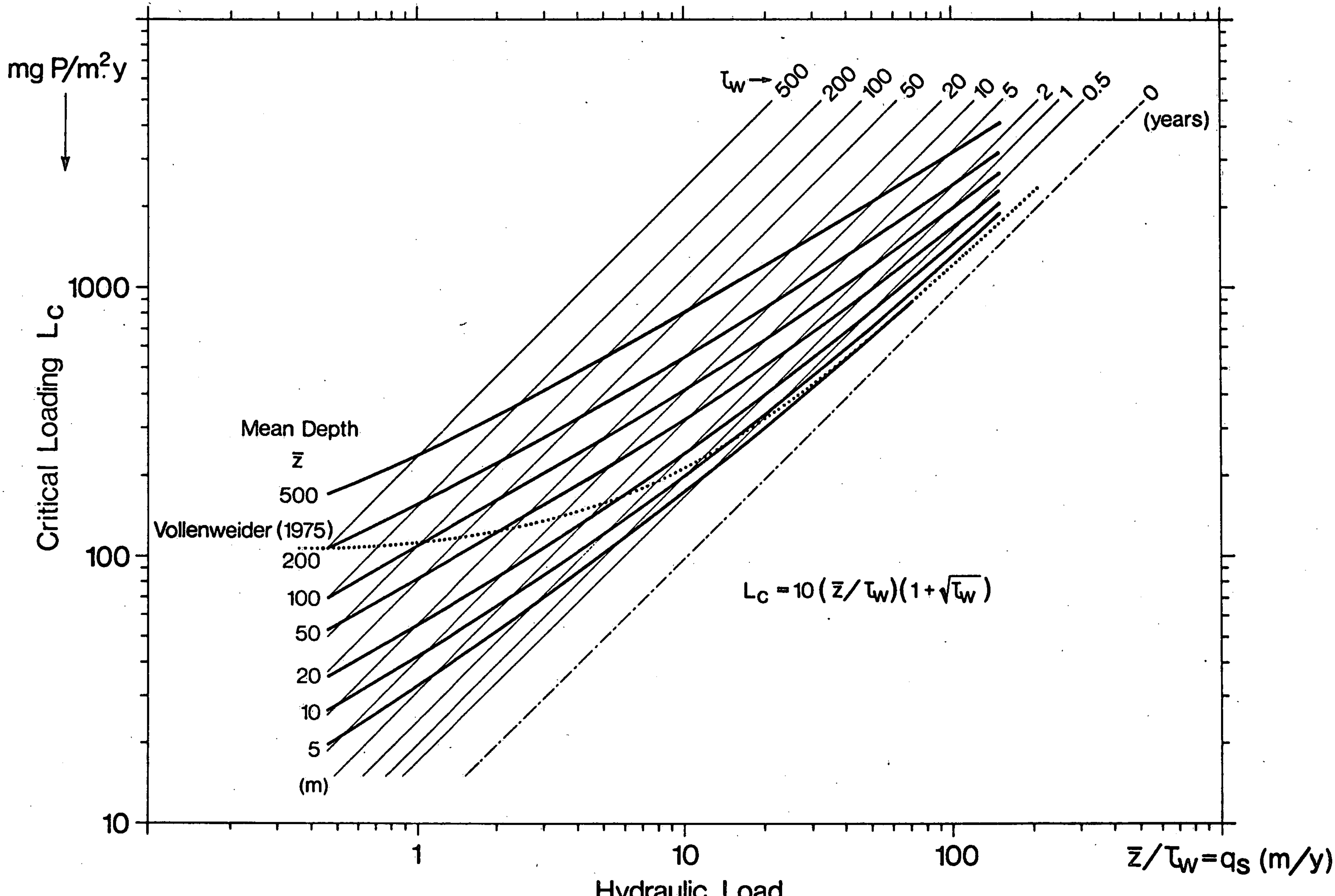

mg P/m²·y
Critical Loading L_c
1000
100
10
Mean Depth
$\bar{z}$
500
Vollenweider (1975)
200
100
50
20
10
5
(m)
$\tau_w \rightarrow$ 500 200 100 50 20 10 5 2 1 0.5 0
(years)
$L_c = 10(\bar{z}/\tau_w)(1+\sqrt{\tau_w})$
1
10
100
Hydraulic Load
$\bar{z}/\tau_w = q_s$ (m/y)

Figure 6

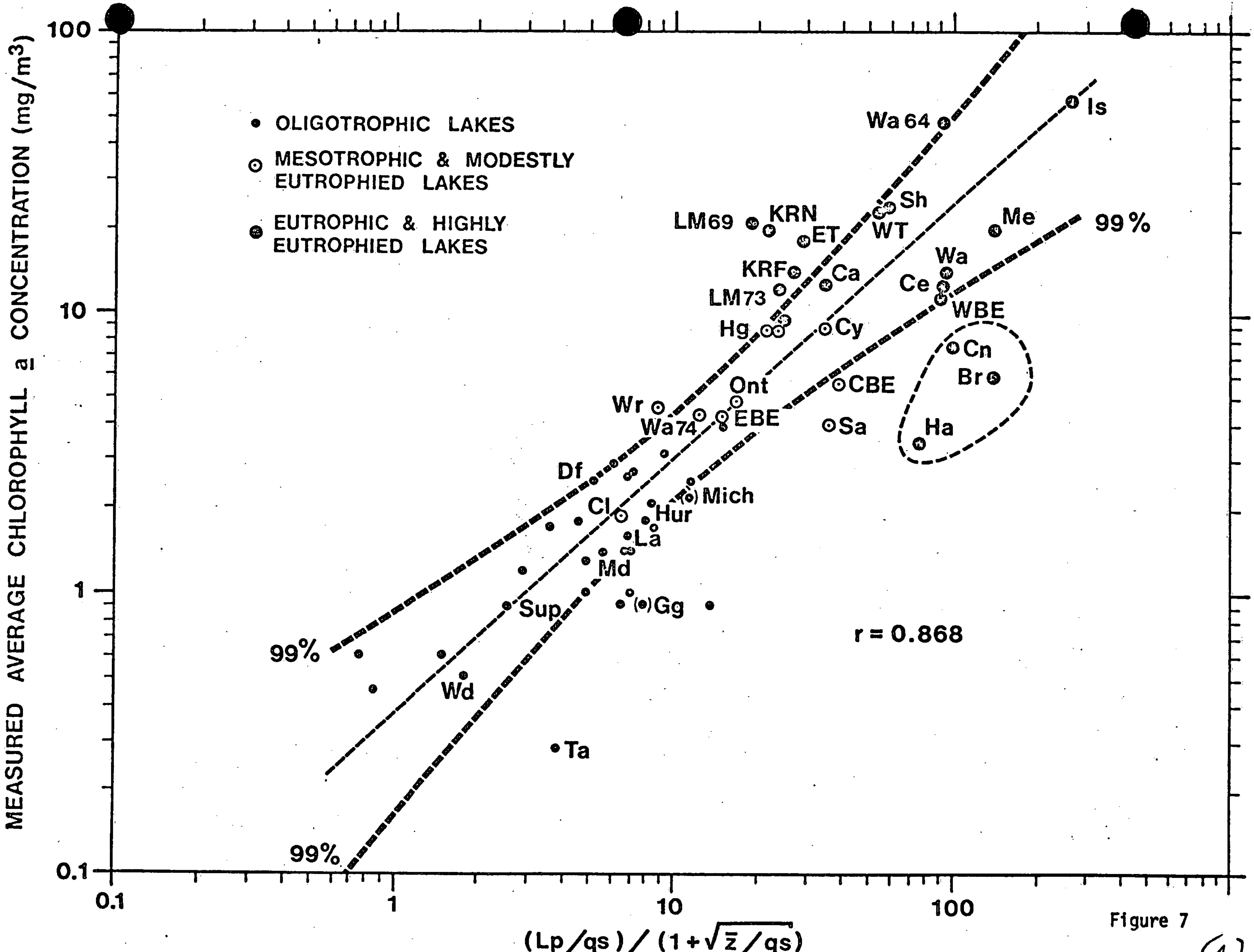
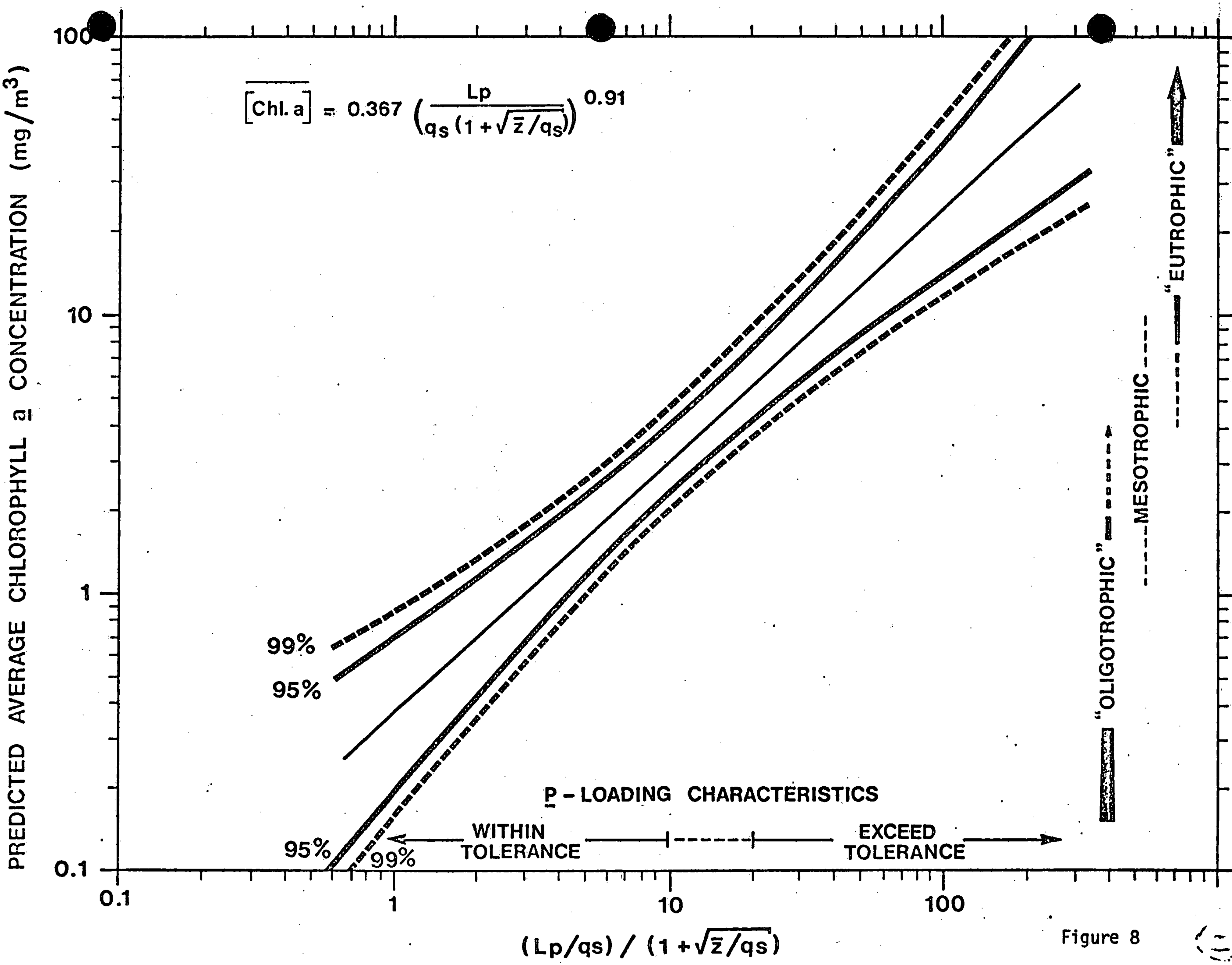

OLIGOTROPHIC LAKES
MESOTROPHIC & MODESTLY EUTROPHIED LAKES
EUTROPHIC & HIGHLY EUTROPHIED LAKES
MEASURED AVERAGE CHLOROPHYLL a CONCENTRATION (mg/m³)
(Lp/qs) / (1 + √z̄/qs)
100
10
1
0.1
0.1
1
10
100
r = 0.868
99%
99%
99%
99%
Wa 64
Is
Sh
WT
LM69
KRN
ET
Me
Wa
Ce
WBE
KRF
Ca
LM73
Hg
Cy
Cn
Br
Ont
CBE
Wr
Wa74
EBE
Sa
Ha
Df
Mich
Cl
Hur
La
Md
Sup
Gg
Wd
Ta

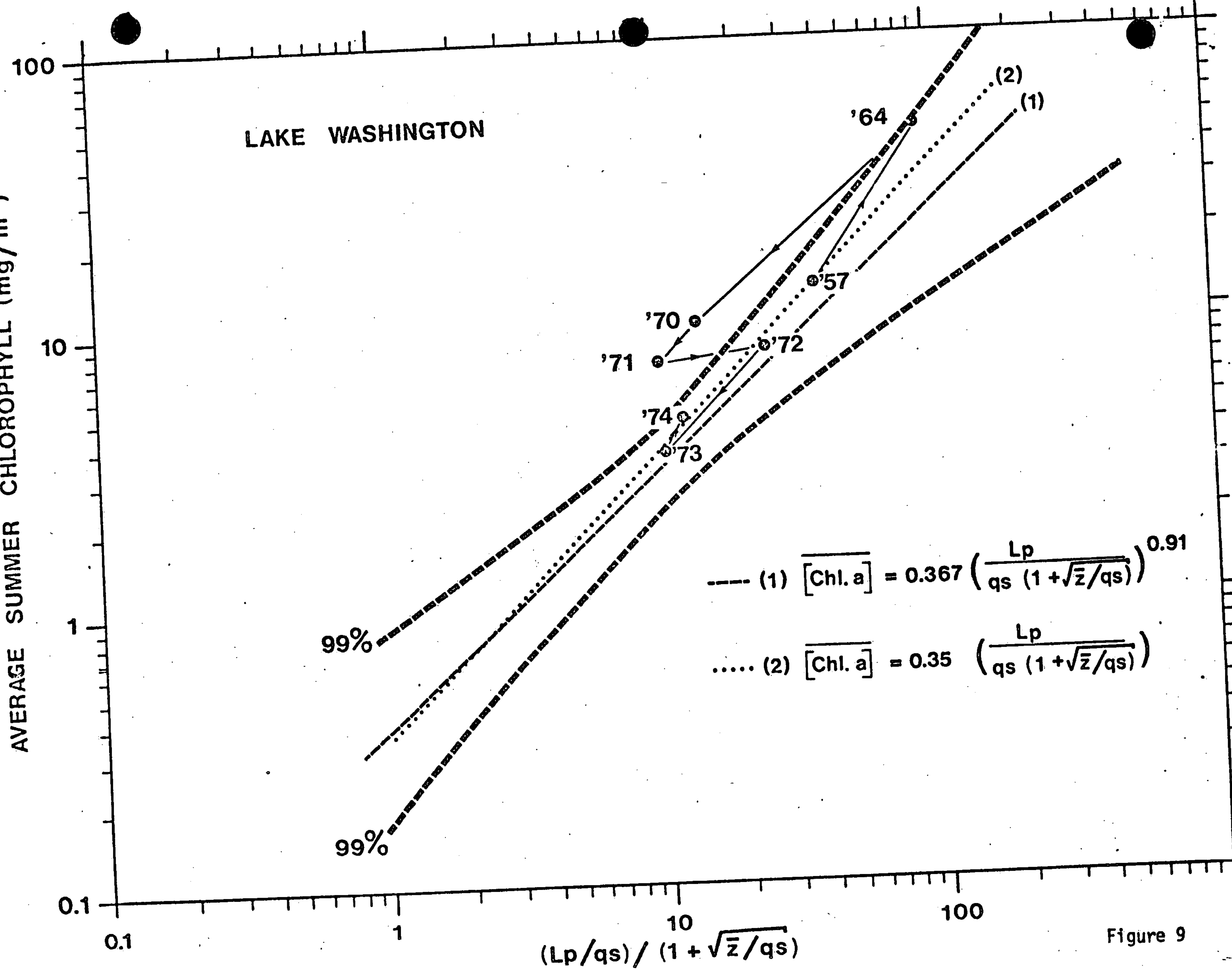
Figure 7

(1)



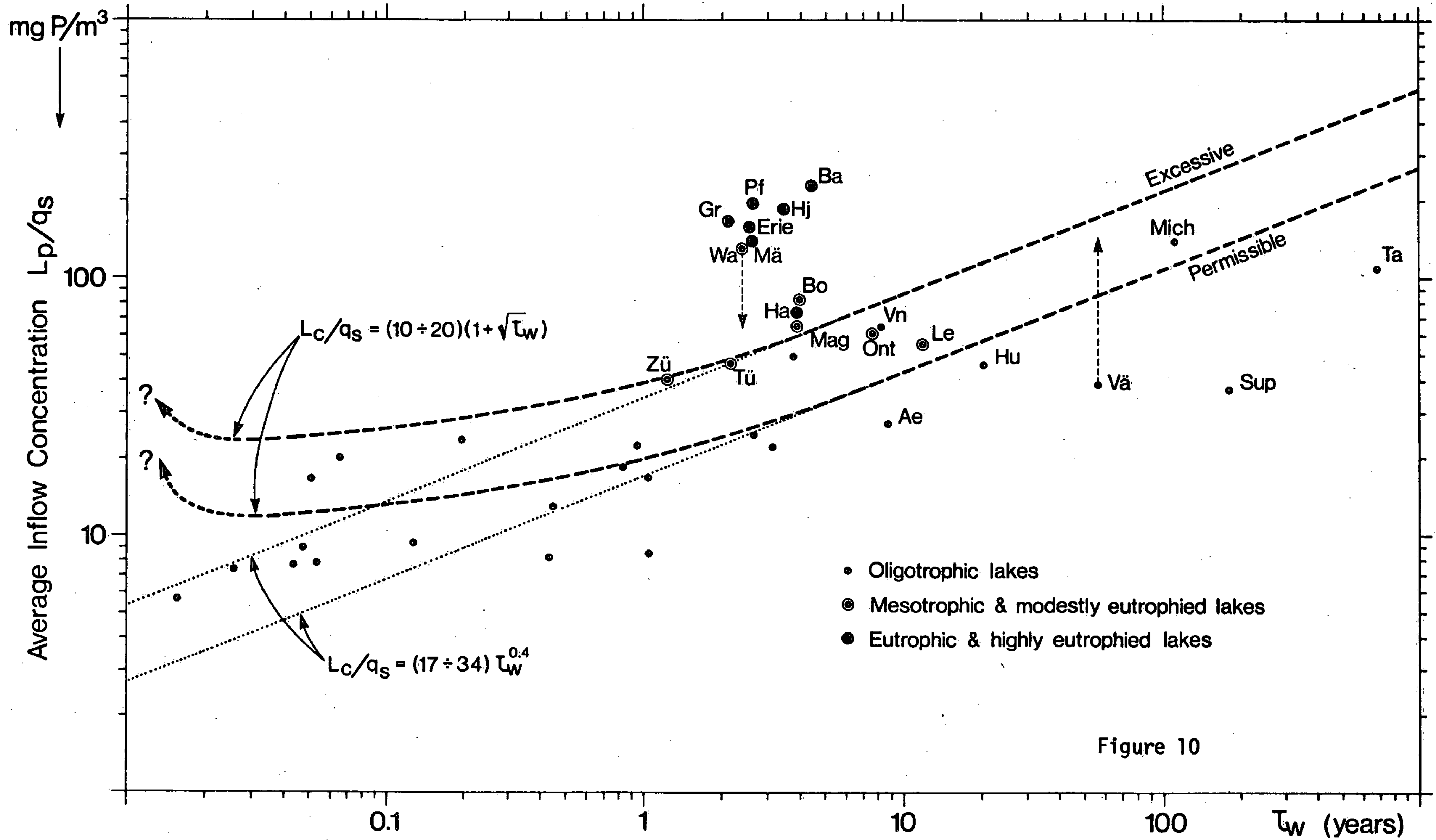
[Chl. a] = 0.367 (Lp / qs(1 + √(z̄/qs)))^0.91
PREDICTED AVERAGE CHLOROPHYLL a CONCENTRATION (mg/m3)
(Lp/qs) / (1 + √(z̄/qs))
100
10
1
0.1
0.1
1
10
100
99%
95%
95%
99%
P - LOADING CHARACTERISTICS
WITHIN TOLERANCE
EXCEED TOLERANCE
"OLIGOTROPHIC"
MESOTROPHIC
"EUTROPHIC"

Figure 8



LAKE WASHINGTON
AVERAGE SUMMER CHLOROPHYLL (mg/m3)
100
10
1
0.1
0.1
1
10
100
(Lp/qs) / (1 + √z̄/qs)
'64
'57
'70
'71
'72
'74
'73
(1)
(2)
99%
99%
---- (1) [Chl. a] = 0.367 (Lp / qs (1 + √z̄/qs))^0.91
..... (2) [Chl. a] = 0.35 (Lp / qs (1 + √z̄/qs))

Figure 9



mg P/m³
Average Inflow Concentration L_p/q_s
100
10
0.1
1
10
100
τ_w (years)
Excessive
Permissible
Ba
Pf
Hj
Gr
Erie
Wa
Mä
Bo
Ha
Mag
Vn
Ont
Le
Zü
Tü
Hu
Vä
Mich
Sup
Ta
Ae
?
?
$L_c/q_s = (10 \div 20)(1+\sqrt{\tau_w})$
$L_c/q_s = (17 \div 34)\,\tau_w^{0.4}$
Oligotrophic lakes
Mesotrophic & modestly eutrophied lakes
Eutrophic & highly eutrophied lakes

Figure 10