Communications Research Centre

AUTOMATIC TRACKING TECHNIQUES FOR SURVEILLANCE RADARS

by

A.W. Bridgewater

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ABSTRACT

This report is a survey of target tracking techniques for automated radar surveillance. It does not go into great technical detail, but attempts to highlight the important issues and to indicate how they are related. These issues are:

1) the choice of coordinate system in which to make target measurements, to initiate or associate tracks, and to update and display these tracks;

2) the choice of filter for prediction and smoothing of track data;

3) the techniques to be employed for conflict resolution, when ambiguities arise in the association of new target observations with existing tracks;

4) the use of manoeuvre-detection procedures, when the target-in-track deviates from its modelled mode of behavior; and

5) the process of initiating a new track automatically, by combining past measurements to form a likely trajectory of a target of interest.

An extensive bibliography pertaining to these topics is provided.

1. INTRODUCTION

The great advances in digital technology in recent years are making possible the development of automatic surveillance and tracking radars, with the potential for increasing the number and accuracy of target tracks and decreasing the response time while simultaneously reducing the cost of manning the radar. An automatic tracking system will consist of digital computing hardware interfaced to one or more radar sensors, and will be programmed to adapt automatically to the environment in order to conserve resources and to apply them most effectively. It must cope with natural and man-made interference. It must track manoeuvring targets, and targets flying in formation or in crossing trajectories. If it is part of a surveillance network, it must be able to correlate tracks and radar returns from more than one sensor.

This report provides an overview of the main elements of automatic target tracking with surveillance radars, without going into technical detail. It attempts only to highlight the important issues and to indicate how they are related. Further detailed information is available from the sources listed in the reference.

Five main sub-topics are identified under Tracking Techniques. These are Coordinate Systems, Tracking Filters, Conflict Resolution, Manoeuvre Detection and Automatic Initiation. They are examined in the following sections. Single-radar tracking is generally assumed; the topic of multi-radar surveillance and tracking is discussed briefly in a separate section. The effect of the type of radar sensor (regular scanning or agile beam) on tracking methods is mentioned, together with some comments on the impact of recent advances in computing technology.

1.1 RADAR DATA PROCESSING[1]

In radar surveillance, the data processing problem is to form and maintain tracks on air targets of interest, from a sequence of detections or plots. Multiple targets in the region, manoeuvring targets, and false plots generated by natural or man-made interference must be accommodated. Traditionally, this task has been carried out by human operators working with radar displays. In recent years there have been increasing efforts to automate the various processes involved, with the provision for manual intervention if necessary.

The techniques and approaches mentioned in the subsequent sections have been advocated, simulated, tested or implemented by researchers and systems developers, for reasons ranging from (some definition of) mathematical optimality to practical expediency. The increasing speed and power of data-processing hardware are leading to the reconsideration of procedures and algorithms previously dismissed as impractical, yet it is not clear whether the greater sophistication and complexity of such procedures can be justified by their results. Moreover, there are differences of opinion amongst researchers as to what are the "best" procedures for particular applications.

2. COORDINATE SYSTEMS

The tracking process requires a frame of reference in which to make measurements on the target(s) of interest, to initiate or associate tracks, and to update and display these tracks. There are often conflicting requirements in the choice of a suitable coordinate system, a choice which is essentially between polar (range plus angle measurements) and cartesian (orthogonal distance measurements).[2]

2.1 POLAR COORDINATES

Conventional scanning surveillance radars provide target position measurements in range ρ and azimuth θ. They may also provide a target doppler measurement $\dot{\rho}$ as an indication of radial velocity or range rate. Elevation ϕ is sometimes provided, by means of either electronically-scanned elevation beams or an ancillary height-finder sensor. The system of coordinates is orthogonal and the measurements in (ρ, θ, ϕ) are independent (Figure 1).

Electronically-agile or phased-array radars provide range-angle measurements in a system which differs from the above, because the target of interest is not necessarily on the boresight of the antenna. For the 3-D sensor, the measurements are in range ρ and two direction cosines u, v (or direction angles A,B) relative to the antenna plane perpendicular to the boresight[3,4]. Range rate $\dot{\rho}$ may also be provided. The measurements in (ρ, u, v) are independent, but the coordinate system is now non-orthogonal.[5] (Figure 1).

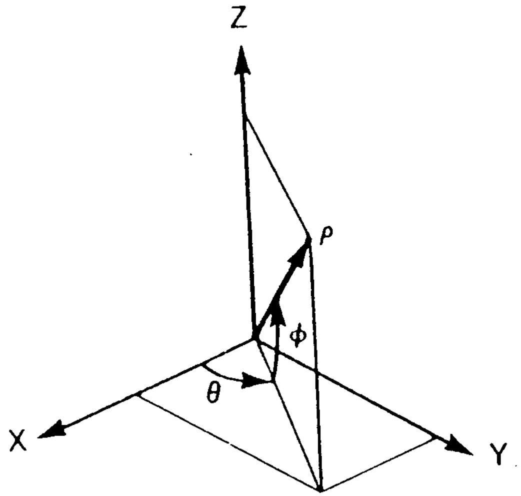
Since polar coordinates of either form represent the system in which radar measurements are taken, they are also the natural system for carrying out plot-to-track association, track initiation and track filtering. Since the measurements are independent, the target-track dynamics may be modelled separately in each coordinate, and the track filters correctly represented by 2 or 3 uncoupled operations. This is a useful computational saving. However, the equations of target motion in these coordinates are nonlinear, leading to the necessity of propagating second and higher-order derivatives in the tracking filters, even for constant-velocity targets, or accepting large dynamic errors and the generation of artificial acceleration components. This has the effect of re-introducing computational complexities.

2.2 CARTESIAN COORDINATES

It is often considered advantageous, overall, to carry out track filtering in cartesian coordinates, at some computational cost in transforming the radar measurement coordinates.[6,7]

There are three possibilities: fixed or earth-referenced, line-of-sight or sensor-oriented, and track-referenced or velocity-oriented cartesian coordinates.

Fixed coordinates are centred at the radar site, or at some predetermined regional centre in the case of a multi-site distributed network of sensors. All tracks are maintained with reference to the fixed centre

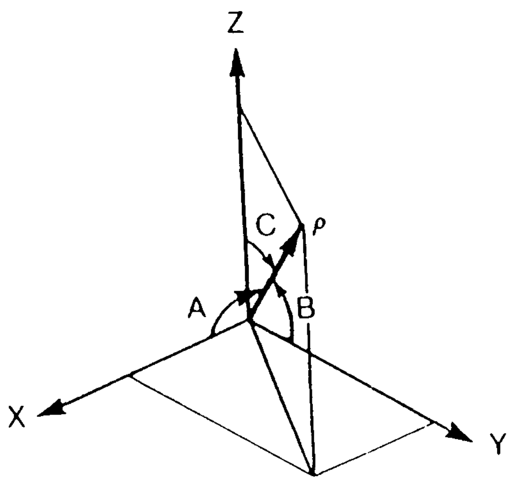


(A) SPHERICAL COORDINATES (ρ, θ, ϕ)

$X = \rho \cos\theta \cos\phi$

$Y = \rho \sin\theta \cos\phi$

$Z = \rho \sin\phi$



(B) SINE-SPACE COORDINATES (ρ,A,B) OR (ρ,u,v)

$X = \rho \cos A = \rho u$

$Y = \rho \cos B = \rho v$

$Z = \rho \cos C = \rho\sqrt{1-u^2-v^2}$

Figure 1. Polar Coordinate Systems for Radar Target Measurements

and a common ground plane tangent to the curved earth at this centre. This is the most appealing and conceptually straightforward system to use. Target dynamics are easily modelled as linear processes. In these coordinates the problem of track registration between distributed sites within a region and between regions is most easily handled. However, radar measurements expressed in fixed cartesian coordinates are not independent, and the filter equations may not be strictly decoupled. Fully-coupled filters are computationally expensive. In some applications, the requirement for coupling is ignored at the expense of larger tracking errors.

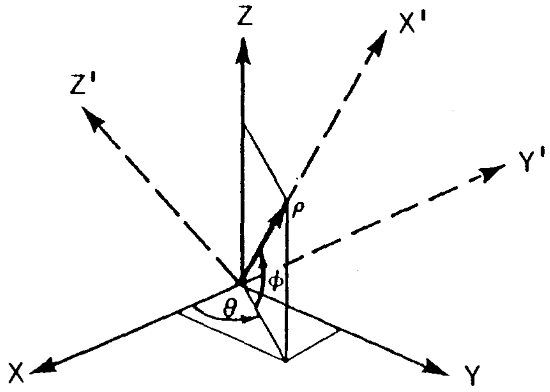
Line-of-sight coordinates, centred at the (single) radar, rotate in accordance with the current position of the target-in-track[6]. A set of coordinates is maintained for each target. One cartesian coordinate is continually realigned along the range vector to the target with each update. Because of this alignment, measurements expressed in these coordinates are independent and decoupled filters may be correctly employed. The cost is the large number of coordinate rotations to be carried out. (Figure 2). However, because they represent only incremental realignments, the rotations can be implemented using a small-angle approximation, with a very modest cost in computing load.

Track-referenced coordinates are sometimes employed as an added refinement in the filtering (smoothing) process[7,8]. Since it is more likely that target manoeuvres take the form of turns than of changes in velocity along the current line of travel, an empirical weighting may be assigned to "across-track" and "along-track" filter gains. The technique requires coordinate transformation operations on each target for every update. It has been used with decoupled filters, tracking high-velocity targets in fixed cartesian coordinates. (Figure 2).

2.3 DISCUSSION

From a survey of the literature on tracking techniques, it appears that both polar (ρ,θ) coordinates and fixed cartesian (X,Y) coordinates are used for 2-D surveillance. Track association and filtering are done in polar coordinates for typical radar geometries in which the nonlinearities introduced by possible target manoeuvres are much greater than the apparent nonlinearities of the equations of motion[9,10]. This condition applies to single-sensor or co-located multi-sensor systems at medium to long ranges. For target headings at high crossing angles (roughly between 75° and 105°) to the range vector[11], and for distributed multi-sensor networks where integrated tracking, track registration and handover are required[7], and generally for short ranges (less than 20 miles), fixed cartesian coordinates are preferred for track filtering. With moving platforms, a fixed cartesian system of coordinates is preferred for a network of distributed sensors; with co-located (or single) sensors, relatively fixed coordinates, centred on the sensor(s), grid-oriented and moving with the platform, can be used. Sensor-oriented (rotating) cartesian coordinates have been proposed and tested in several systems[6,12,13]. Track-oriented smoothing techniques have long been employed in 2-D air surveillance[7].

In 3-D tracking and surveillance, the literature surveyed has concerned the use of electronically-agile or phased-array radars, generally for the

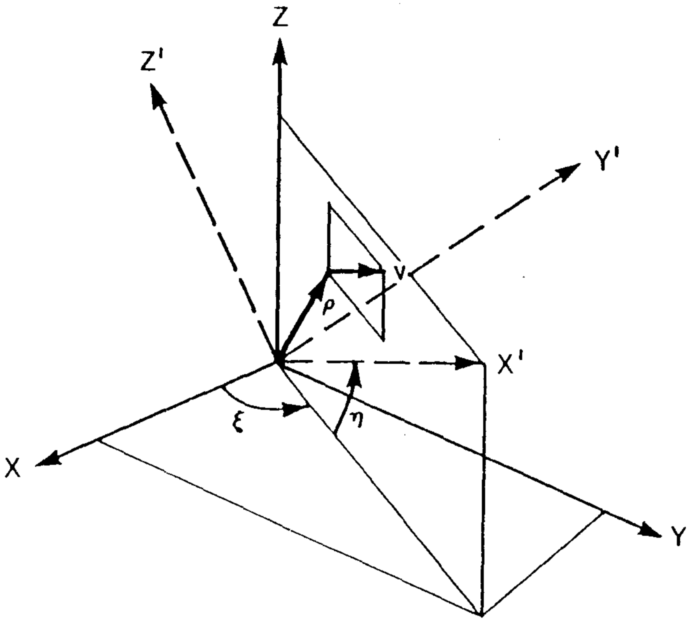


(A) LINE-OF-SIGHT COORDINATES (X' Y' Z')

X' ALIGNED ALONG LINE-OF-SIGHT FROM RADAR TO TARGET

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix}$$

CW ROTATION ABOUT 3rd AXIS — CCW ROTATION ABOUT 2nd AXIS



(B) TRACK-ORIENTED COORDINATES (X' Y' Z')

X' ALIGNED ALONG VELOCITY VECTOR V OF TRACK

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\xi & -\sin\xi & 0 \\ \sin\xi & \cos\xi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\eta & 0 & -\sin\eta \\ 0 & 1 & 0 \\ \sin\eta & 0 & \cos\eta \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix}$$

Figure 2. Cartesian Coordinate Systems for Radar Target Tracking

tracking of re-entry vehicles (ballistic or manoeuvrable). Here, both polar coordinates (ρ,u,v) and sensor-oriented cartesian have been used for track association and filtering[4,5,11,14].

In the past, design compromises were often made in meeting the conflicting requirements in the choice of coordinates for the various tracking functions. With current advances in computer speeds, such compromises should no longer be necessary. All the necessary coordinate conversion and registration operations should be easily handled so that each function may be carried out in the coordinates most appropriate.

3. TRACKING FILTERS

There are two basic types of tracking filter[15]. One involves a *dynamic* algorithm, which uses a postulated model for the target's motion and attempts to combine new measurements with the track history in accordance with that model. This is by far the most common method employed in radar surveillance applications. The other involves a *kinematic* algorithm, which attempts to fit mathematical curves to the measurement data, by means of n^{th}-order polynomials. Its advantage is that it works equally well (or badly) in any coordinate system. A kinematic estimation algorithm is used with polynomial extrapolation for track prediction. The polynomial mathematics, uncoupled in the component coordinates, are easy to handle. The drawback is that the method does not permit extrapolation as far into the future as does the dynamic algorithm. Its use is probably limited to electronically-agile radars with variable update rates, handling a relatively small number of targets at a time, or for high-data-rate tracking radars on a single target[16].

The remainder of this section will look at the dynamic algorithm in more detail.

3.1 FILTER CATEGORIES

Tracking filters can be classified under the following headings:

a) recursion

b) adaptivity

c) coordinate coupling

d) order

e) dimensionality.

They are most easily discussed with reference to the Kalman filter, which is the general solution to the recursive, linear, mean-square estimation problem[17,18,19]. Kalman filtering combines a track forecast, which is derived from the previous best estimate of the track state in accordance with the equations of motion, with the most recent physical measurement, to produce a weighted mean. The set of weighting factors is chosen to minimize the variance. (Figure 3). For strict optimality, the noise statistics assumed in the formulation must be gaussian, and the noise terms uncoupled from one sampling instant to the next. Because it provides for the inclusion of all possible couplings of covariance terms in its general matrix formulation, the Kalman filter is independent of the coordinate system in which the state variables and measurement variables are expressed, provided that the assumed model for target motion is linear. When it is not strictly linear, as is the case with polar coordinate systems, it can be made approximately so by means of the piece-wise linear approximations of the "extended" Kalman filter[19].

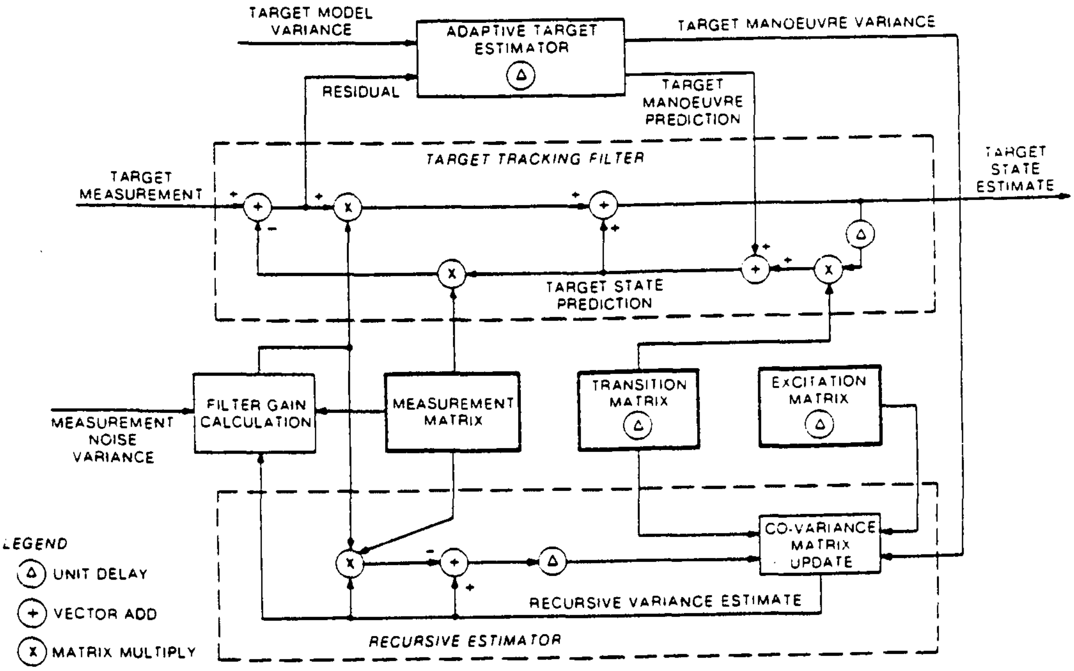
3.1.1 Recursion

The drawback of the full Kalman filter is its computational cost. The recursive procedures require, at each sampling instant, and for each target, the multiplication of matrices of order nxn and the inversion of a matrix of order mxm (n is the length of the state vector and m is the length of the measurement vector, $m \leq n$). By dispensing with various components of the full apparatus of the Kalman filter, simpler approximations to the solution of the estimation problem can be obtained, with corresponding reductions in computer loading. This must be accomplished, of course, without degrading the overall tracking performance of the system to an unacceptable degree.

One approach to simplification is to adopt a constant gain or set of weighting factors in place of the recursively computed set. This steady-state or limiting value of gain is dependent only on the measurement noise covariance, the target manoeuvre covariance and the update interval, all of which must be fixed over the period for which that gain is applied[20,21]. The optimal form of solution is sometimes called the Wiener filter.

3.1.2 Adaptivity

Whatever method is adopted for track filtering, it is usually necessary to combine it with some form of adaptation[22,23,24]. An adaptive system is one which continually adjusts its own parameters in the course of time to meet a certain performance criterion. By this definition the recursive filter is not by itself adaptive. The sequence of values for the gain could be computed off-line and stored prior to being applied to a sequence of target measurements. On-line adaptation is required when significant changes occur in the target motion (manoeuvres), radar measurement accuracy, or sampling rate. Changes in the tracking environment are reflected in the appropriate adjustment of these three parameters during the recursive estimation process or in the recomputation of the steady-state gain. Changes in measurement covariance (brought about by a significant change in the position of the target-in-track with respect to the sensor) and update interval (brought about in track-while-scan systems by missed plots on individual scans, or by asynchronous combining of reports on one target from several radars, and in agile-radar systems by the nature of their operation) will occur as a matter of course during the life of the track and are easily identified. To determine whether a change in manoeuvre covariance is necessary, some form of manoeuvre detection is required. This is discussed in a subsequent section.



TARGET MODEL: $X_k = \Phi_{k-1} X_{k-1} + \Gamma_{k-1} U_{k-1}$

MEASUREMENT MODEL: $Y_k = M_k X_k + V_k$

TRACKING FILTER: $\tilde{X}_k = \Phi_{k-1} \hat{X}_{k-1} + \tilde{X}_{m,k}$

$\hat{X}_k = K_k Y_k + (I - K_k M_k) \tilde{X}_k$

RECURSIVE ESTIMATOR: $\tilde{P}_k = \Phi_{k-1} \hat{P}_{k-1} \Phi^t_{k-1} + \Gamma_{k-1} Q_{k-1} \Gamma^t_{k-1} + \tilde{P}_{m,k}$

$\hat{P}_k = (I - K_k M_k) \tilde{P}_k$

FILTER GAIN: $K_k = \tilde{P}_k M_k^t (M_k \tilde{P}_k M_k^t + R_k)^{-1}$

ADAPTIVE ESTIMATOR: $\tilde{X}_{m,k} = fn_1(Q_{k-1}, (Y_{k-1} - M_{k-1} \tilde{X}_{k-1}))$

$\tilde{P}_{m,k} = fn_2(Q_{k-1}, (Y_{k-1} - M_{k-1} \tilde{X}_{k-1}))$

(~) – forecast

(∧) – estimate

superscript (t) – matrix transpose

superscript (-1) – matrix inverse

X – state vector; P – Covariance matrix (state uncertainty)

U – noise vector; Q – Covariance matrix (model uncertainty)

V – noise vector; R – Covariance matrix (measurement uncertainty)

Γ – model excitation matrix

Φ – state transition matrix

Y – measurement vector

M – measurement (selection) matrix

K – (optimum) gain matrix

Figure 3. The Adaptive Recursive Kalman Tracking Filter

3.1.3 Coordinate Coupling

Another simplifying procedure which may be applied to the general matrix formulation of the Kalman filter is the elimination of coordinate interaction terms in the covariance expressions. This reduces the filter algorithm to a set of algebraic equations where each coordinate is treated separately[25]. For measurement-noise covariance in sensor coordinates (i.e., polar or line-of-sight Cartesian), the assumption of decoupled coordinates is correct. In fixed cartesians it is not, and gives rise to some error which, depending on the application, may or may not be ignored. The errors are angle-dependent unless the gains are the same in all dimensions. Manoeuvre-noise covariance in track-oriented coordinates is decoupled for randomly manoeuvring air targets. It may be assumed to be so in other coordinate systems as well without apparent degradation in tracking performance.

The decoupled, or reduced, Kalman filter may be simplified further when covariances and update interval are constant, and when radar observations do not include doppler measurements. It is then often termed the α-β(-γ) filter. The scalar weights, α,β(and γ) replace the matrix of gain coefficients of the Kalman weighting factor in the position, velocity (and acceleration) estimation procedure for each coordinate of the target motion. The reduced filter does not propagate all possible covariance terms, as does the Kalman filter. If manoeuvre covariance is assumed to be zero, the iterative values of the weights can be specified as explicit functions of the iteration number of the track. Non-recursive or fixed gain α-β filters are commonly employed[25,26].

It is generally desirable to use fixed cartesian coordinates for surveillance tracking since linear models of target motion may then be most effectively employed. However, as stated earlier, the use of decoupled filters on each coordinate, as required by the reduced forms of Kalman filter, is strictly valid only for a line-of-sight or sensor-oriented coordinate system, for which the component measurement errors are independent. The fully-coupled but computationally expensive Kalman filter, operating in fixed cartesian coordinates, absorbs the resulting cross-terms in the measurement covariance matrix directly. A decoupled filter ignores them from the outset. This deficiency can be overcome, if desired, by the application of a suitable coordinate transformation to the gain coefficients of the decoupled filter[27,28]. The technique is an alternative to the computation of the gain matrix directly in fixed coordinates using the fully-coupled formulation, and allows the decoupled formulation to be used without loss of generality and with some reduction in computational load. It may be applied to both recursive and steady-state versions of the tracking filter. Alternatively, a less general but computationally less expensive technique is to use a form of "hybrid" filter: carry out filtering in polar coordinates and prediction in cartesian coordinates, to eliminate pseudo-accelerations which give rise to tracking biases for high-speed targets at close range.

Although current computer speeds would, in principle, allow full adaptive Kalman filters for track estimation to be implemented where accuracy requirements seem to demand it, there is still a question whether such implementations would provide improvements in performance in proportion to the processing power expended. Sub-optimal approximations to the filtering process would probably work nearly as well as full-blown optimal approaches, and in addition would be more "robust" in performance when dealing with faulty or incomplete data.

Doppler measurements are more easily incorporated in a tracking filter using line-of-sight or polar coordinates, and work is being carried out on how best to use doppler information to enhance the tracking process[29,30].

3.1.4 Order

For linear tracking filters the choice is generally between second-order filters, for which target position and velocity are modelled and estimated, and third-order filters, for which acceleration is also included as an explicit term in the state vector. There is some debate amongst system designers and researchers over the use of third-order filters instead of second-order filters, particularly when radar measurements are on target position only (no doppler information). Current opinion suggests that acceleration smoothing is not appropriate unless data rates are sufficiently high to monitor an aircraft's turn over a reasonably large number of sampling intervals, and that velocity smoothing alone is better for straight-line trajectories. Certainly, a third-order filter is no replacement for a second-order filter plus a manoeuvre detector[7]. It is possible, however, that a properly designed third-order filter in conjunction with a manoeuvre detector would be superior in overall performance to a second-order filter with an equivalent manoeuvre detector.

The use of "cascaded" filters has been suggested to control the estimation error in the higher-order elements of the state vector: that is, the application of secondary, low-order smoothing to the velocity or acceleration terms following the primary estimation filter[13].

In (ρ, u, v) coordinates, where the target is tracked by an electronically-agile radar with variable data rates and using separate reduced-Kalman filters on each coordinate, it is common to use a higher-order filter in ρ and lower-order filters in u and v[11]. This sort of arrangement incorporates doppler ($\dot{\rho}$) measurements directly in the filter.

3.1.5 Dimensionality

The dimensionality of the tracking filter is governed by that of the sensor itself. If the filter is designed to accommodate interaction terms between coordinates, 3-D filtering will impose considerably greater computational loading, in both memory requirements and processing time, than 2-D filtering. This is a further reason for the current preference for decoupled filters in (ρ, u, v) coordinates for the modern phased-array radars.

3.2 DISCUSSION

Two-dimensional track-while-scan systems have favored various forms of α-β filter with regular update interval and position-only information. Where fixed Cartesians have been employed, the coordinate coupling has generally been ignored. Often the same α-β coefficients have applied in both dimensions using combined or collapsed coordinates. The full range of these forms of filter has been used, from recursive to fixed-gain, and including various empirical adaptive mechanisms. The tendency has been to use the simplest version of filter consistent with satisfactory performance for the particular application.

Three-dimensional electronically-agile surveillance and tracking systems have preferred the more general and complex Kalman formulations, in order to accommodate variable update rates and direct velocity measurements. The reduced (i.e., decoupled) recursive third-order Kalman filter has been the standard version considered. Since the applications have included missile defence and reentry vehicle tracking, high accuracy was vital and computational cost was of less concern in the design trade-off.

4. CONFLICT RESOLUTION

The role of an automatic tracking system is to provide a sequence of best estimates of the target's position and velocity, based on the available measurements and knowledge of the limitations of the sensor performance and the target's speed and manoeuvrability. The tracking filter carries out the track-estimation operation and supports the critical function of plot-to-track association by providing a prediction of the position where the next plot or radar return is expected and a measure of the allowable error or deviation of that measurement from the prediction. The aim is to ensure an unambiguous measurement which may reliably be used to update the track estimate.

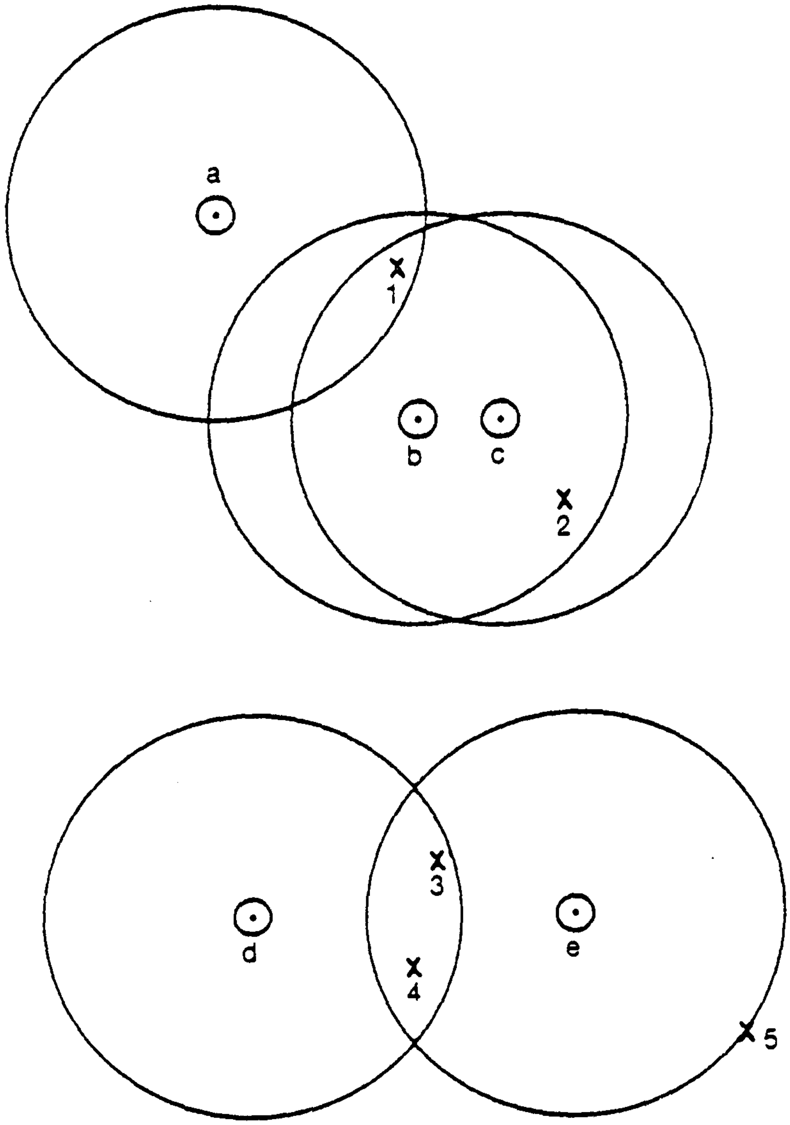
Difficulties arise when more than one observation or measurement occurs within the association window determined by the allowable error, or when one observation can be associated with more than one track in case of overlapping association windows on nearby tracks. Radar signal processing, plot detection and stationary plot filtering contribute to removing false plots, but inevitably some ambiguities will occur. These conflicts must be resolved as accurately as possible since incorrect associations lead rapidly to track seduction and track loss[12]. (We distinguish the term "conflict resolution" as used here from the problem of averting potential collisions in the air traffic control environment.)

The techniques for conflict resolution described below apply principally to track-while-scan systems for which the association logic must accept and sort the incoming plots according to existing tracks as best it can. Conflicting plots are assumed to have occurred at roughly the same time. An electronically-agile system can "time-share" its attention to individual tracks and can resolve ambiguities more directly be repeated interrogations in the local neighborhood of the track(s) in question.

The techniques discussed below involve radar information only. Other data may also be used to resolve association conflicts, such as ESM information, known target performance limits (velocity versus turning rate constraints), probable target scenarios (expected goals or trajectories), and so forth.

4.1 TYPICAL PLOT-TO-TRACK ASSOCIATION RULES

Three types of association procedures are given. Their function in each case is to resolve a "confusion matrix" of multiple plot-to-track assignments into a single consistent set of associations. All entries in the initial matrix satisfy some maximum-distance criterion. As the example in Figures 4 and 5 shows, these procedures can produce different resolutions for the same data set.



LEGEND

⊙ PREDICTED POSITIONS OF 5 TRACKS LABELLED a-e
CENTRED ON NORMALIZED ASSOCIATION GATES (LARGE CIRCLES)

x MEASURED POSITIONS OF 5 PLOTS NUMBERED 1-5

Figure 4. Example of Conflict Resolution: Diagram of Track Positions and Target Measurements.

TRACKS

PLOTS	a	b	c	d	e
1	(9)	7	9		
2		8	(6)		
3				9	8
4				(8)	7
5					(10)

COMMON PLOT ELIMINATION

- Drop 1b, 1c by Rule (a)
- Drop 3e, 4e by Rule (b)
- Choose 1a and 5e
- Drop 3d by Rule (c)
- Drop 2b by Rule (d)
- Choose 2c and 4d

PLOTS	a	b	c	d	e
1	9	(7)	9		
2		8	(6)		
3				(9)	8
4				8	(7)
5					10

MINIMUM DISTANCE ELIMINATION

- Choose 2c as absolute minimum entry in table
- Eliminate Row 2 Column c from further consideration
- Find next minimum entry and proceed as above, etc.
- Choose 1b, 4e and 3d as a result.

PLOTS	a	b	c	d	e
1	9	(7)	9		
2		8	(6)		
3				9	(8)
4				(8)	7
5					10

GLOBAL MINIMIZATION

- Choose overall selection which minimizes global mean square distance
- Result is 1b, 2c, 3e, 4d.

Numerical entries in tables are normalized distances from plots to predicted track positions.

Figure 5. Example of Conflict Resolution: Three Different Resolution Procedures Applied to Confusion Matrix

4.1.1 Elimination by Common Plots[31]

This procedure applies the following two rules repeatedly until no further eliminations can be made:

(a) For a track which correlates with several observations, reject any observations held in common with another track, if the common observation is the only observation correlating with the second track.

(b) For a track correlating with several observations, some of which are *not* held in common with other tracks, reject those observations which *are* held in common with other tracks.

If, after repeated application of these rules there are still left multiple assignments, the following two rules are applied in sequence:

(c) When several observations correlate with one track, the closest observation (nearest neighbor) is associated with that track.

(d) When several tracks correlate with one observation, the observation is associated with the closest track.

4.1.2 Elimination by Minimum Distances

(a) Search the confusion matrix for the nearest-neighbor plot-to-track correlation (minimum distance), and make the indicated assignment.

(b) Cancel all other correlations connected with that plot and track. Repeat rule (a) using the resulting reduced confusion matrix, and continue the application of (a) and (b) until all the assignments have been made.

4.1.3 Resolution by Global Averaging

Using a linear-programming optimization procedure find the set of mutually exclusive plot-to-track assignments within the confusion matrix which minimized the *overall* mean-square distance. That is, instead of searching for the best individual fits of plot-to-track data, look for the best global fit of the data. This is a time-consuming operation which is not as practical as the other two previous methods.

4.2 OPTIMAL ASSOCIATION PROCEDURES

4.2.1 Track Branching

It is generally agreed that the best approach to association in a multi-target cluttered environment is to use a multi-hypothesis branching algorithm. That is, provisional tracks are generated in accordance with all possible assignments of plots-to-tracks[32,33,34]. Some form of likelihood function for each track is computed and further measurements are obtained. New branchings are generated if necessary, all likelihood functions are updated, and those tracks are dropped whose functions fall below a certain threshold. Use of this sequential decision-theoretic approach avoids making irrevocable decisions and needlessly throwing away information at each scan, by retaining all relevant data until a more confident decision can be made. Of course, this procedure makes very heavy computational and memory storage demands on the data processor. Various techniques have been suggested for reducing the processing load and rendering the procedure more practical. These techniques include:

(a) arbitrary truncation of the branching tree and depth-of-scan search to control the growing memory[36];

(b) a recursive branching algorithm designed to limit the number of possible provisional tracks held in memory at any time[37]; and

(c) an integer-programming approach intended to alleviate combinatorial computing difficulties[38].

4.2.2 Probabilistic Association

Theoretical studies have examined the possibility of weighting the contribution of a new plot toward updating an existing track in accordance with its probability of representing the correct radar return[35]. The probability is computed on the basis of an exponential function of the distance of the new measurement from the predicted position of the track. These probabilistic weightings are incorporated in a modified Kalman filter for track smoothing and estimation. The different elements of this class of association-cum-filtering procedures include:

(a) nearest-neighbor assignment versus all-neighbors weighted-average assignment;

(b) a priori estimation versus a posteriori estimation of the probability of an incorrect measurement;

(c) single-scan fixed-memory operation versus N-scan growing-memory operation (i.e., track branching with probabilistic association).

4.2.3 Discussion

These optimal approaches to track association show promise of leading to significant improvements in automatic tracking performance. However, they are still in an early stage of development and have a number of unresolved problems:

(a) The formalisms used to date provide no means of treating manoeuvring targets in a multi-target tracking environment, and it is not at all clear how manoeuvre detection can be incorporated in the general scheme.

(b) There is as yet no means of including adaptive techniques (modification of the filter parameters) in accordance with changing conditions in the tracking environment.

(c) The techniques are, of course, computationally very expensive.

These approaches could become important for automatic tracking techniques if further research is able to resolve the above difficulties, particularly points (a) and (b). The recent advances in computing hardware, which have in the past few years far outstripped developments in software, make these optimal but hitherto impractical methods worthy of consideration. The designer of an automatic tracking system is no longer constrained to adopt the simplest or cheapest solutions or to employ the most elegant sub-optimal approximations. He may now consider "brute-force" procedures to enhance performance because of the excess of computing power that will be available at relatively low cost. The advances in hardware will allow the use of standardized modular routines, resulting in greater ease of programming, more portability of software and lower development costs. These comments also apply to the implementation of track filtering algorithms (Section 3.2).

5. MANOEUVRE DETECTION

In discussing the problem of manoeuvre detection two things should be kept in mind:

(a) Manoeuvre detection can only take place under tracking conditions relatively free of clutter. If the probability of false alarm is high, track seduction is likely[39].

(b) Manoeuvre detection is closely associated with tracking filter adaptation. The filter parameters should be capable of changing appropriately upon initiation and termination of a target manoeuvre. The filter must in turn provide appropriate measures of track accuracy to assist in the reliable declaration of the target's "manoeuvre status"[24].

5.1 ASSOCIATION GATES

The automatic tracker searches a prescribed area centred on the predicted track state. The size of this area, termed an association gate or window, is governed by the current estimation accuracy of the filter. This gate may be of dimension greater than 2, depending on the type of the sensor, and should perhaps be viewed in general as an N-dimensional "volume". A 3-σ contour or hyper-surface establishes the normal limits of this gate.

If, after all conflicts have been resolved, no new plots are found within the standard or non-manoeuvre gate, the tracker must search a larger gate enclosing the first. The size of this gate is governed by certain assumed limitations on the dynamics of the target-in-track. For instance, a target flying at a certain altitude with a certain velocity is constrained in the duration of the manoeuvre and in the amount of acceleration. If an unambiguous plot is established in this gate, then a manoeuvre may be declared.

If no plots are found, in either the manoeuvre or non-manoeuvre gate, the track must be coasted. The longer the condition of missed plots persists, the greater the uncertainty of track position. The size of the association gate grows as a power function of the time interval from the last update of the track.

5.2 DETECTION PROCEDURES

From the above general schemes, it is possible to devise a number of empirical algorithms for manoeuvre detection[10,40]. These usually involve some form of provisional declaration, that is, a delay pending the arrival of corroborating data. For instance, the tracker may apply a deviation test on subsequent data to determine whether a short sequence of measurements is consistently biased in one direction, thereby indicating a target manoeuvre. Alternatively, at the first occurrence of a plot in the manoeuvre gate, the tracker may split the track, generating a provisional track on the potential manoeuvre and coasting the original track. The bifurcation is then resolved upon the arrival of a new measurement at the next interval, when one or other of the tracks is dropped. One might also consider "polyfurcation" of tracks in a more general scheme for track splitting under manoeuvre. These algorithms have relevance to track initiation procedures as well.

5.3 FILTER ADAPTATION

During manoeuvre, the tracking filter should provide light smoothing. The target is deviating from its modelled mode of behavior and greater weight should be placed on incoming measurements. With no manoeuvre, the target motion is presumed to be accurately modelled, and heavy smoothing of the data should be applied. A satisfactory compromise must always be sought in the filter's responsiveness to a potential manoeuvre[24]. If it is too quick to open up, then erratic behavior and poor estimation will result. If it is too sluggish, the likelihood of track loss increases rapidly. The best solution is to provide two parallel filters on the same track (or one each of the parent and provisional tracks) one with heavy smoothing, one with light, during the transitional period preceding a firm declaration of a manoeuvre[23].

5.4 ELECTRONICALLY-AGILE SENSORS

The discussions in this section have applied in general terms to regularly-scanned sensors with predetermined update intervals. For electronically-agile radars the possibility of a higher-order adaptation exists. The update interval on a specific target can be varied and the association areas surrounding the target can be rapidly interrogated for confirmation of manoeuvre. The procedures for manoeuvre detection become at once more sophisticated and more controlled. Potential target manoeuvre suggests greater uncertainty in the track estimates. The remedy is a higher information rate, and this can be provided by the radar under feedback control from the automatic tracker[41]. The central problem is then one of beam scheduling - that is, an optimal sharing of the available radar energy amongst all existing tracks according to their potential uncertainty, while carrying out surveillance and search for new targets in the region[42].

For electronically-agile radars, fully automatic data handling is essential. Considerable effort will be necessary in the design of adaptive algorithms and the development of reliable software for the automatic control of the surveillance and tracking functions.

6. TRACK INITIATION

The process of forming a new track consists of combining a sequence of plots which indicate the trajectory of a likely target of interest. The human operator, working with a radar display, can be very quick and accurate at this task when the number of targets he must handle is small. He is able to bring considerable intuition and experience to bear in coping with noisy and cluttered environments. But for modern surveillance systems, with many targets to be acquired and rapid response times to be achieved, automatic initiation is to be preferred, with the provision for a manual override function.

6.1 AUTOMATIC INITIATION

The general procedures for automatic initiation with scanning radars and agile-beam radars are similar. The latter type of sensor has the capability of rapidly updating a tentative track to establish a firm one, or further interrogating an unassociated plot or potential track to declare a tentative track[43].

A single plot, unassociated with existing tracks, is paired with all possible similar plots occurring some interval of time T_o later and found at a distance d, where $V_{min}T_o < d < V_{max}T_o$, and V_{min} and V_{max} are a priori velocity limits for targets of interest. These tentative tracks are projected ahead for an additional time interval T_1. For each prediction, associations are attempted on new plots occurring at the indicated time within an association window, centred on the prediction, whose size is governed by the criterion, $aT_1^2/2$, where "a" is an a priori acceleration limit for the targets of interest. This procedure should eliminate most of the false pairings, and the remaining tentative tracks will be declared firm once the promotion logic has been satisfied. Doppler clues, if available, may also be used to establish a track.

To minimize errors in establishing an azimuth rate at long range, an a priori rate-initialization technique has been suggested which constrains the variance of the estimated velocity to lie within prescribed limits[13]. In most instances, this will result in a more robust initialization.

6.2 TRACK PROMOTION

6.2.1 Fixed Logic

An initiation process which has a set of rules, such as 3 consecutive "hits", or 4 out of 6, etc., is defined as fixed. Such schemes can be optimized only with difficulty and much empirical testing, and only under specific operating conditions. A design for high false-alarm conditions would be unsatisfactory in clear conditions, and vice versa. Furthermore, a design intended for a given clutter density would not be effective at all ranges of the surveillance space, since clutter density varies with range. Range partitioning with multiple designs would be required.

6.2.2 Adaptive Initiation Logic

Three types of adaptive logic can be identified:

(a) the identification of clutter areas from environment maps and the inhibition of automatic initiation in areas of dense clutter;

(b) as suggested in Section 6.2.1, the establishment of a set of different fixed-logic designs to be applied in accordance with range and the identified clutter condition (from the map); and

(c) the formation of cumulative probabilities that the tentative track is genuine on the basis of the observed local environment.

The third type has been tried in both track-while-scan systems and agile-beam systems[12,43]. The technique involves a sequential decision procedure. Information which can contribute to the estimation of probabilities includes: clutter densities, forecasting accuracy, target-detection probabilities and assumed probability of track validity. The threshold for track confirmation is a specified cumulative probability of track likelihood and is set according to the acceptable false-alarm rate. If the probability falls below a lower threshold the tentative track is declared false and dropped. This lower threshold may be adapted to suit the current system load.

6.3 DISCUSSION

Automatic track initiation is certainly the most difficult of any sub-topic of automatic tracking. Empirical approaches have been tried and attempts have been made to base an adaptive logic on a mathematical foundation of probability and sequential decision theory. The problem may also be treated as part of the plot-to-association problem, when provision is included for the formation and deletion of tracks[44].

This problem may be classified as one of unsupervised pattern recognition, for which a large body of theoretical and experimental work exists. Perhaps an appeal to these references and the techniques developed there may suggest useful approaches which could be applied. One such approach is to carry out a "retrospective" search through past plot data, looking for probable track trajectories in space and time[45].

7. MULTI-RADAR TRACKING

The integration of plot data from several radars with overlapping coverage into a single system track file is an important feature of automatic tracking in modern surveillance networks. The distinction is made between colocated radar systems (for example on board a naval vessel) and distributed multi-site network (as in land-based or airborne air-defence systems).

7.1 TRACK INTEGRATION

The type of radar integration used depends on the radar's performance, the environment and whether the system is colocated or not. Several integration methods are[46]

(a) track selection - generate a track with each radar and choose one of the tracks as the system track;

(b) average track - generate a track with each radar and weight the individual tracks to form a system track;

(c) augmented track - generate a track with each radar, choose one as the system track, and use selected plots from other radars to update the system track;

(d) average plot - for a given time interval, average all radar plots on a single target and use this average to update the system track;

(e) plot-to-track - use all radar plots to update the system track; tracks may or may not be initiated using plots from all radars.

Theoretically, the plot-to-track method yields the best tracks because no information is lost through averaging or selection. However, the plots must be weighted according to their estimated reliability and accuracy and care must be taken that bad data do not corrupt good data. Figure 6 shows three ways in which the system track file might be maintained in multi-radar tracker[47].

7.2 COLOCATED RADARS[48]

In track initiation, a plot from one radar can be correlated with a saved plot from another radar resulting in a track declaration sooner than is possible from any individual radar. The tracking algorithms for a multiple radar system are quite similar to those for a single radar system. However, the filter must update the target with plots whose accuracies vary with the radar and which arrive irregularly in time (see Section 3.1.2 on adaptive filters). The system must also provide range and angle alignment between the radars. This can be done by feedback techniques. With colocated radars there is no problem associated with communication links between sites; all the information is in one place.

7.3 MULTI-SITE RADAR INTEGRATION[47,49]

When exchanging plot and track information between distributed sites, the methods used depend on whether the sites are fixed or mobile and what communication links are available. They also depend on the extent of overlapping coverage provided by the radars in the network. The major problems to be solved concern the maintenance of gridlock over the network using targets of opportunity, the determination of the type of data to be transmitted over limited bandwidth communication links, and the way in which plot

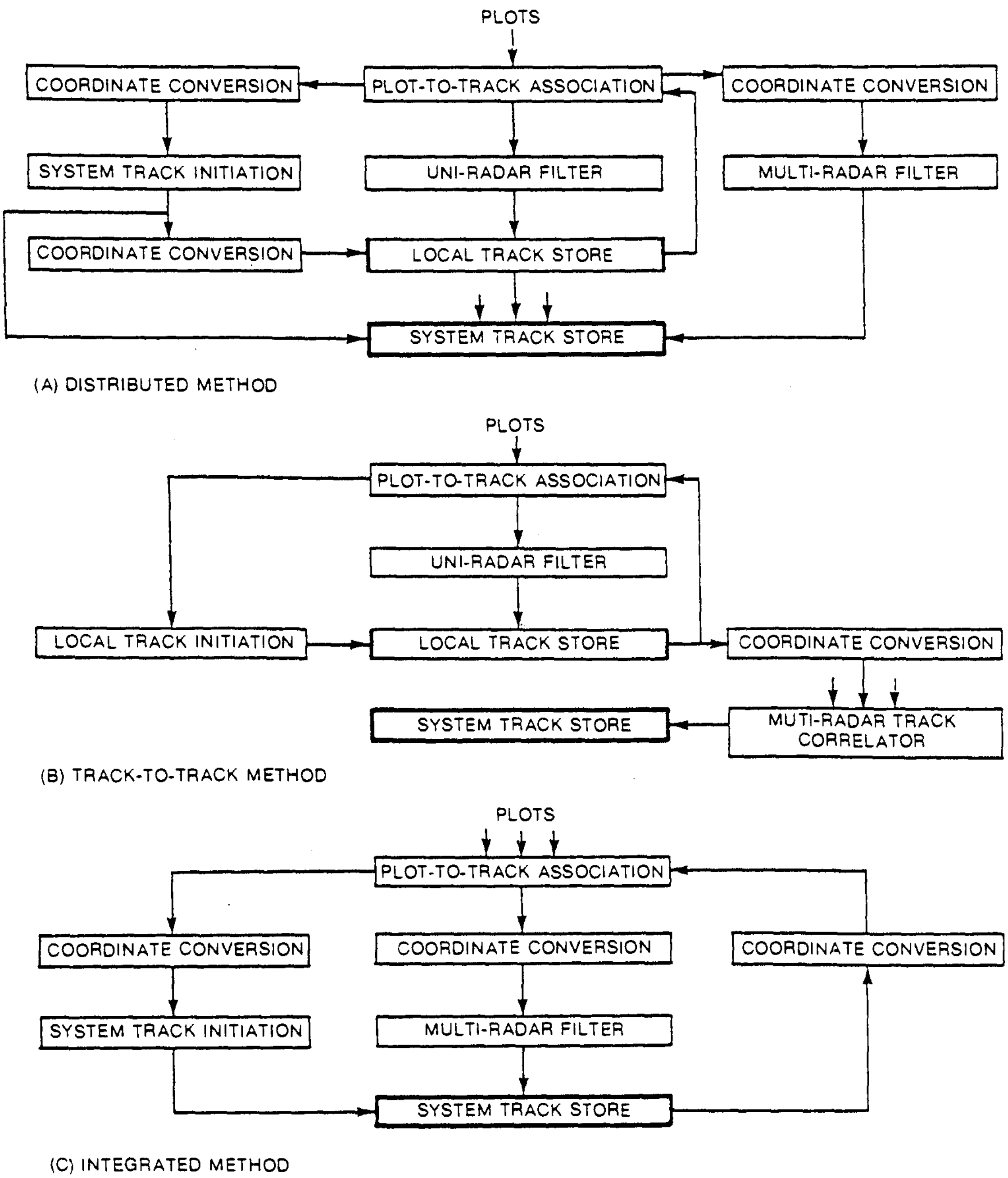


Figure 6. Three Methods for Multi-Radar Tracking

and track information is to be processed and distributed throughout the network for reasons of redundancy and survivability.

7.4 SUMMARY

The problems associated with automatic surveillance and tracking systems generally, such as false track generation, track initiation and plot-to-track association in a dense plot environment, manoeuvre handling[50], etc., become still more difficult when attempting multi-radar integration.

8. CONCLUDING REMARKS

In the field of automatic tracking, the most important topics to be addressed are Track Association (including Conflict Resolution and Manoeuvre Detection) and Track Initiation. Both these topics may be considered under the general heading of False-Track Control. A current problem of particular interest concerns the dense target environment, that is, the automatic formation and maintenance of "raid" targets. Although certain questions remain in the implementation of track-filtering and coordinate-conversion procedures, these are not critical for the further development of automatic tracking systems.

The most important factor in the current development of automatic tracking techniques is the rapid progress made in digital storage and processing technology. Computer hardware costs are diminishing in relation to software and overall system costs. Processor speeds and memory sizes are continually increasing. Software development and reliability continue to be problems, however.

As a result of hardware advances, the more computationally intensive techniques considered for automatic tracking will be increasingly used in present and future systems - e.g., (i) coordinate transformations, as required, on each track and at every update interval; (ii) full Kalman-filter formulations, recursive and adaptive; (iii) parallel filters on each track; (iv) multi-hypothesis track-branching algorithms for plot-to-track association and manoeuvre detection; and (v) sequential, adaptive, decision-theoretic procedures for track initiation and promotion. However, there will be competition for computing resources amongst these different processes and some compromises may still have to be made.

With computing hardware no longer seen as a fundamental limitation there will be increasing effort made in the development of radar sensors and front-end signal processors to provide better information to the data processors. In addition, the development of software and man/machine interactive techniques will provide more effective control of the complete system.

Constraints imposed by out-dated hardware will remain. Actual radar installations are tied to design freezes and long-lead-time procurement cycles; often it is necessary to make the best of obsolescent equipment during the life-cycle of the installation.

One topic which has not been dealt with in this report is Tracking in an ECM Environment. This involves an examination of possible countermeasures to tracking processes. It is assumed that jamming and ECM is a "normal" environment for military radar systems, and that weaknesses in the tracking processes will be attacked. Efforts must therefore be made to remove or reduce these weaknesses as far as possible.

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