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A THEORY OF DISPLACED PHASE CENTER ANTENNA FOR SPACE BASED RADAR APPLICATIONS

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ABSTRACT

A theory of the displaced phase center antenna system for space based radar applications is presented. The matching condition required to compensate for the motion of the satellite platform so that clutter cancellation can be achieved is first derived. Analytical expressions for the signal and clutter covariance matrices are given. With the aid of a simplified model, numerical values of an improvement factor are obtained. These results illustrate the dependence of the level of clutter rejection on radar parameters such as: grazing angle, pulse train duration, pulse repetition rate and antenna aperture size.

1.0 INTRODUCTION

With the rapid progress in satellite technology an increasing number of sensors are launched into space for various observation purposes. For microwave sensors, the synthetic aperture radar SEASAT, for observation of the oceans, was launched in 1978 and this was followed by the shuttle imaging radars SIR-A in 1981 and SIR-B in 1984 (Refs 1-3).

The defence of the Canadian territorial integrity requires a surveillance system which can cover the vast land masses and its surrounding oceans. A space based radar (SBR) could be a key element in a system designed to fulfill the needs of Canada in safeguarding its sovereignty by ensuring that intrusion into even the remotest corner of the nation could be detected, thereby allowing rapid tactical decisions to be made.

Before space based radars can be deployed as part of an integrated surveillance system a number of outstanding technical and technological issues must first be resolved. One key problem is the ability of the SBR to extract target signals from the interference of unwanted electromagnetic energy reflected from the earth's surface.

In Refs. 4 and 5, the target detection capability of a base line real aperture, pulse-doppler, space based radar was examined. These and a recent detailed study of the spectral characteristics of clutter from different types of terrain as seen by a spaced-based radar, [6] demonstrate quantitatively that space based radar clutter return can be several orders of magnitude stronger than that of a typical target signal of interest. Consequently, an SBR system must possess an efficient clutter rejection scheme which can adapt to the high degree of variability of the clutter spectrum as a function of radar geometry and terrain.

Although the SBR clutter spectrum is dependent on many factors, its variability is mainly due to the motion of the radar platform relative to that of the earth's surface illuminated by the radar. For airborne radars a clutter rejection method known as the displaced phase center antenna technique has been shown to be effective in a monopulse implementation [7]. In principle, this technique seeks to compensate for the platform motion, thereby reducing the target masking effect produced by the background clutter. This is done by controlling the timing of the emitted pulses so that the clutter returns to different receiving phase centers at different sampling times emulate the clutter returns seen by a virtual stationary receiver. A proposal to apply such a technique to space based radars has been outlined by workers of the Lincoln Laboratory [8,9].

In this report we present a theory of the displaced phase center antenna technique as applied to a space based radar. Our attention will be focussed on a two phase center system, although some discussion on higher order phase center systems will also be given. The objective of this theoretical investigation is to derive an optimum processor which maximizes the signal to clutter ratio. In parallel, analytical results required for quantitative evaluation of the performance of the displaced phase center antenna system will also be obtained. A theory of application of the displaced phase center antenna technique for an airborne moving target indicator has been discussed by Hofstetter et al [10]. Although the general approach adopted in our analysis is similar to that of Ref. [10] the constraints imposed by the radar operation on a space platform require a different system configuration which leads to significantly different results.

To make the analysis mathematically tractable a number of simplifications have been made. The effects of some of these simplifications are secondary and therefore can be ignored. Others are subjects for investigation to be carried out in our on-going studies.

Section 2 describes the propagation of electromagnetic pulses emitted by a space based radar in orbital motion and backscattered by a target on the earth's surface. The matching condition is derived from the time delays between the transmission and reception of the individual pulses by the different phase centers. Section 3 is devoted to the analytical description of the signal and clutter return vector. The clutter covariance matrix which characterizes the radar background interference is also derived. In section 4, we describe an optimum processor for the displaced phase center antenna system and define an improvement factor to evaluate the performance of the overall scheme. Key parameters such as the radar beam grazing angle, pulse repetition rate and number of pulses in a burst are varied and numerical results are presented. The conclusions of this report, which is the first of several on the subject of displaced phase center antenna for space based radars, are found in section 5.

2.0 GEOMETRICAL CONFIGURATION AND SIGNALLING FOR A DISPLACED PHASE CENTER ANTENNA SYSTEM

The spectrum of the clutter return of a space based radar is highly sensitive to the orbital motion of the radar platform and the radar pointing direction [4-6]. In this section we establish the viewing geometry of the radar and the signalling sequence of the displaced phase center antenna. To simplify our analysis we will restrict ourselves to the consideration of circular orbits. For the surveillance of an extensive area with a minimum number of satellites, circular orbits are preferred [11]. In addition, we will neglect the rotation of the earth. In Ref. 6 it has been shown that the earth's rotation has only a small effect on the clutter spectrum. The basic displaced phase center system being considered is assumed to have two phase centers. In Appendix I higher order displaced phase center antenna system will be discussed.

2.1 A Basic Displaced Phase Center Antenna System

The basic displaced phase center antenna system we consider is assumed to utilize a phased array antenna controlled electronically to form a single transmitter with phase center at T and two separate identical receivers with phase centers at R_1 and R_2 respectively (Fig. 1). It may be recalled that the phase center of a radiating source is the center of curvature of the spherical wavefront in the far field zone of the emitting source. Choosing a geocentered coordinate system Oxyz such

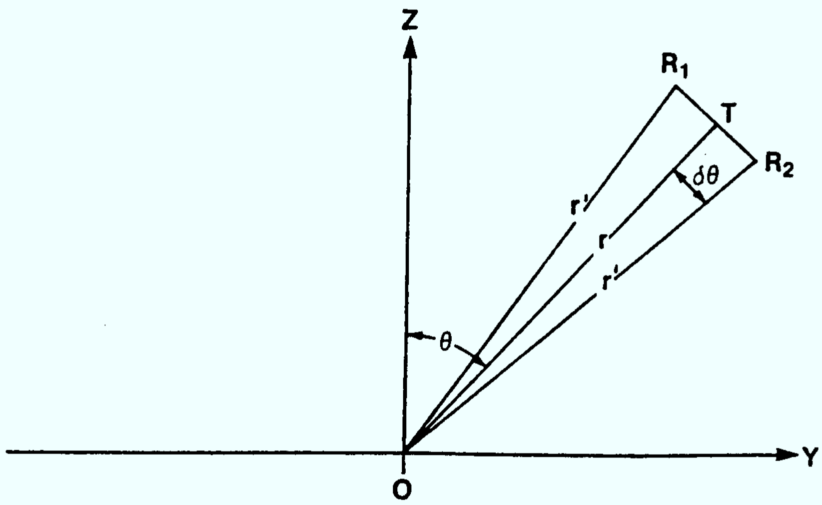

Fig. 1 - Positions of the transmission and reception phase centers T, R_1 and R_2 in a geocentric coordinate system.

that the circular orbit is in the yOz plane, we can express the coordinates of the phase centers as in Table 1. The symbols used in Table 1 have the following meanings:

r_e : radius of the earth

r : radius of the satellite orbit

θ	:	angular position of T from Oz
$\delta\theta$	:	angular separation of R_1 and R_2 from T
S	:	an arbitrary point on the earth's surface
θ_s	:	polar angle of S (Fig. 2)
ϕ_s	:	azimuth angle of S (Fig. 2)
r'	:	distance of the phase centers R_1, R_2 to the center of the earth.

Table 1 - Coordinates of Phase Centers and an Arbitrary Point on the Earth's Surface

	Spherical Coordinates	Cartesian Coordinates
T	$(r,\ \theta,\ 0)$	$(x = 0,\ y = r\sin\theta,\ z = r\cos\theta)$
R_1	$\left(\frac{r}{\cos\delta\theta},\ \theta-\delta\theta,\ 0\right)$ $= (r',\ \theta-\delta\theta,\ 0)$	$(x = 0,\ y = r'\sin(\theta-\delta\theta),\ z = r'\cos(\theta-\delta\theta))$
R_2	$\left(\frac{r}{\cos\delta\theta},\ \theta+\delta\theta,\ 0\right)$ $= (r',\ \theta+\delta\theta,\ 0)$	$(x = 0,\ y = r'\sin(\theta+\delta\theta),\ z = r'\cos(\theta+\delta\theta))$
S	$(r_e,\ \theta_s,\ \phi_s)$	$(x = r_e\sin\theta_s\cos\phi_s,\ \ y = r_e\sin\theta_s\sin\phi_s$ $z = r_e\cos\theta_s)$

Let the angular velocity of the satellite be denoted by ω and we choose the time t=0 to be the instant that T is on the z axis. Assume that a pulse is transmitted from T at $t=t_1$. At $t=t_2$ the pulse is supposed to have reached S and at $t=t_3$ the pulse reflected from S arrives at R_1. With the assumption that the earth is stationary, $ST(t_1)$, the distance between S and T at $t=t_1$ is given by

$$[ST(t_1)]^2 = (r_e\sin\theta_s\cos\phi_s)^2 + (r_e\sin\theta_s\sin\phi_s - r\sin\theta_1)^2 + (r_e\cos\theta_s - r\cos\theta_1)^2 \qquad (1)$$

where $\theta_1 = \omega t_1$.

The distance between S and R_1 at $t=t_3$ is given by

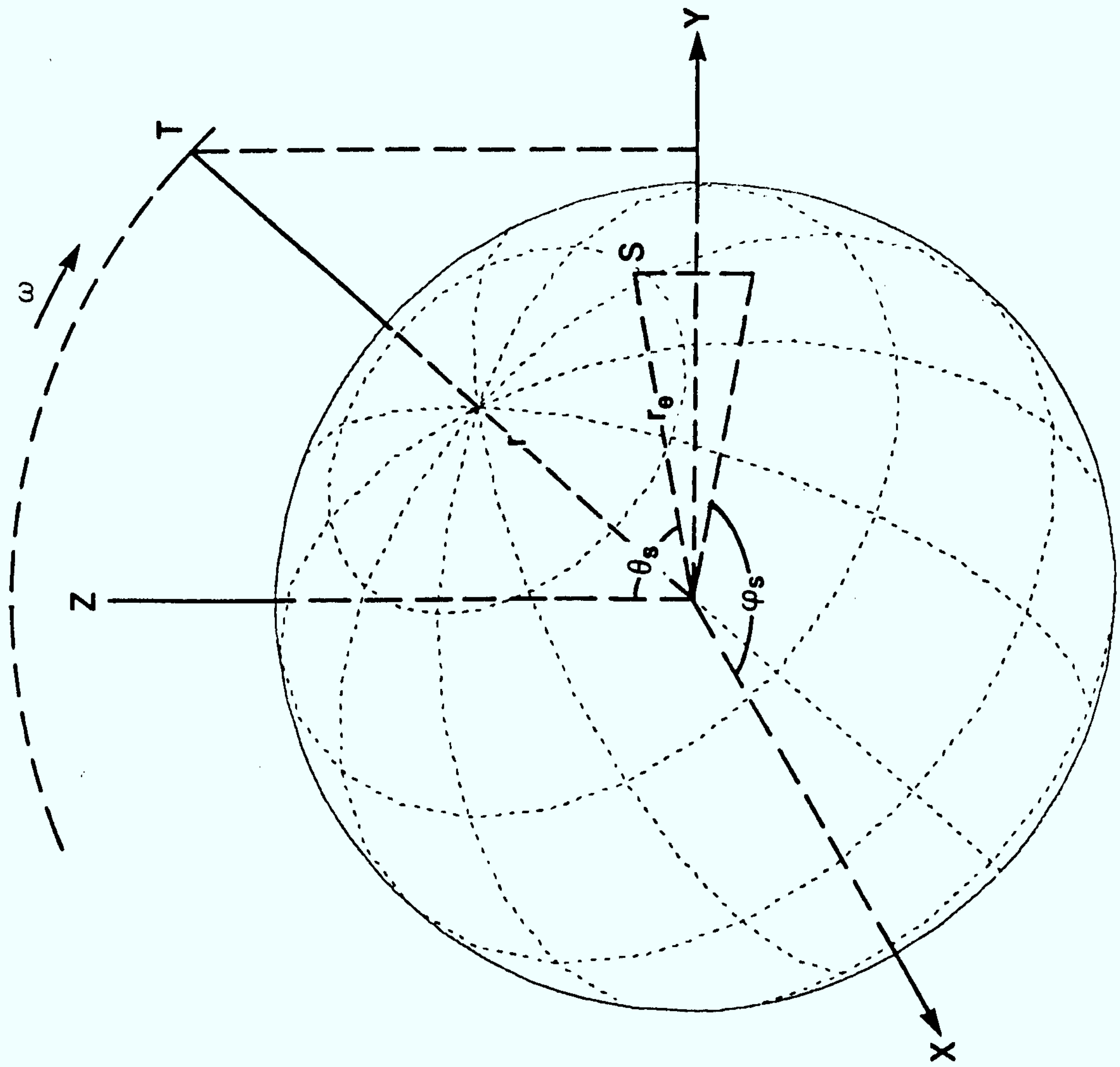

Fig. 2 - Geometrical configuration of the relative positions of the satellite orbit and a point scatterer on the surface of the earth.

$$[SR_\imath(t_3)] = (r_e\sin\theta_s\cos\phi_s)^2 + (r_e\sin\theta_s\sin\phi_s - r'\sin\theta_-)^2$$
$$+ (r_e\cos\theta_s - r'\cos\theta_-)^2 \qquad (2)$$

where

$$\theta_- = \theta_3 - \delta\theta = \theta_\imath + \omega(t_3 - t_\imath) - \delta\theta$$
$$= \theta_\imath + \omega\Delta_\imath - \delta\theta$$

$\Delta_\imath$ is the time required for the pulse transmitted at $t_\imath$ to reach $R_\imath$ after reflection at S. $SR_\imath(t_3)$ can now be expanded in terms of $ST(t_\imath)$. Since

$$[SR_\imath(t_3)]^2 = (r_e\sin\theta_s\cos\phi_s)^2 + [(r_e\sin\theta_s\sin\phi_s - r\sin\theta_\imath)$$
$$+ (r\sin\theta_\imath - r'\sin\theta_-)]^2$$
$$+[(r_e\cos\theta_s - r\cos\theta_\imath) + (r\cos\theta_\imath - r'\cos\theta_-)]^2 \qquad (3)$$

$$SR_\imath(t_3) \simeq ST(t_\imath) + \frac{(r_e\sin\theta_s\sin\phi_s - r\sin\theta_\imath)(r\sin\theta_\imath - r'\sin\theta_-)}{ST(t_\imath)}$$

$$+ \frac{(r\sin\theta_\imath - r'\sin\theta_-)^2}{2[ST(t_\imath)]} + \frac{(r\cos\theta_\imath - r'\cos\theta_-)^2}{2[ST(t_\imath)]}$$

$$+ \frac{(r_e\cos\theta_s - r\cos\theta_\imath)\ (r\cos\theta_\imath - r'\cos\theta_-)]^2}{[ST(t_\imath)]} \qquad (4)$$

In the last expression of Eq. (3), terms of second order and higher in $|SR_\imath(t_3) - ST(t_\imath)|\ /\ ST(t_\imath)$ have been ignored. For small values of $\omega\Delta_\imath$ and $\delta\theta$ eq. (4) can be further simplified with the aid of the approximations:

$$r\sin\theta_\imath - r'\sin\theta_- \simeq -r(\Delta_\imath\omega - \delta\theta)\cos\theta_\imath \qquad (5)$$

$$r\cos\theta_\imath - r'\cos\theta_- \simeq r(\Delta_\imath\omega - \delta\theta)\sin\theta_\imath \qquad (6)$$

such that after dropping second and higher order terms in $\omega\Delta_\imath$ and $\delta\theta$,

$$SR_\imath(t_3) \simeq ST(t_\imath) - \frac{r_e\sin\theta_s\sin\phi_s \cdot r(\Delta_\imath\omega - \delta\theta)\cos\theta_\imath}{[ST(t_\imath)]}$$

$$+ \frac{r_e\cos\theta_s \cdot r(\Delta_\imath\omega - \delta\theta)\sin\theta_\imath}{[ST(t_\imath)]} \qquad (7)$$

By definition, the time delay Δ_i is given by

$$\Delta_i = (t_3 - t_i) = (t_3 - t_2) + (t_2 - t_i)$$

$$= \frac{SR_i(t_3)}{c} + \frac{ST(t_i)}{c} \qquad (8)$$

where c is the speed of light. From Eqs. (7) and (8) it follows

$$c\Delta_i = 2ST(t_i) + \frac{(\Delta_i\omega - \delta\theta)}{[ST(t_i)]}\, r\, r_e \{-\sin\theta_s \sin\phi_s \cos\theta_i + \cos\theta_s \sin\theta_i\} \qquad (9)$$

If v is the speed of the satellite and d the distance of separation of the phase centers T and R_i i.e. $v = r\omega$, $d = r\delta\theta$, Eq. (9) can be alternatively written as

$$c\Delta_i = 2ST(t_i) + \frac{(v\Delta_i - d)}{[ST(t_i)]}\{-r_e \sin\theta_s \sin\phi_s \cos\theta_i + r_e \cos\theta_s \sin\theta_i\} \qquad (10)$$

A more compact form of Eq. (9) is

$$c\Delta_i = 2ST(t_i) + \frac{(\Delta_i\omega - \delta\theta)}{[ST(t_i)]}\,[(\vec{T}(t_i) \times \vec{S}) \cdot \hat{i}] \qquad (11)$$

where $\vec{S}$ and $\vec{T}(t_i)$ are position vectors and $\hat{i}$ is the unit vector along the x-axis.

Eq. (11) can be solved to give the time delay Δ_i:

$$c\Delta_i = 2ST(t_i)\left\{1 + \frac{[(\vec{T}(t_i) x \vec{S}) \cdot \hat{i}]}{ST(t_i)}\left[\frac{\omega}{c} - \frac{\delta\theta}{2ST(t_i)}\right]\right\} \qquad (12)$$

where second order terms and higher in $(\omega/c)[(\vec{T}(t_i) x\vec{S}) \cdot \hat{i}]/ST(t_i)$ have been dropped.

Similarly, the time delay Δ_2 for a pulse transmitted at t_1 to reach R_2 after a reflction at S is given by

$$c\Delta_2 = 2ST(t_1) \left\{ 1 + \frac{[\vec{T}(t_1) \times \vec{S})] \cdot \hat{i}}{[ST(t_1)]} \left[\frac{\omega}{c} + \frac{\delta\theta}{2ST(t_1)} \right]\right\} \tag{13}$$

If a second pulse is now transmitted, the time delays taken for the pulse to return to the receivers after a reflection at S will be different due to the motion of the satellite. Consider a pulse transmitted at $t=t_1'$. The time delay Δ_1' it takes the pulse to reach R_1 via a reflection at S is given by an expression analogous to Eq. (12)

$$c\Delta_1' = 2ST(t_1') \quad \left\{ 1 + \frac{[\vec{T}(t_1') \times \vec{S}]\ \hat{i}}{ST(t_1')} \left[\frac{\omega}{c} - \frac{\delta\theta}{2ST(t_1')} \right] \right\} \tag{14}$$

Let $t_1' = t_1 + \delta$ and denote $\theta_1 + \omega\delta$ by θ_1'. The distance $ST(t_1')$ can be related to $ST(t_1)$ by the following equation

$$ST(t_1') = \left[ST(t_1)\right] \left\{ 1 + \frac{\omega\delta[\vec{T}(t_1) \times \vec{S}] \cdot \hat{i}}{[ST(t_1)]^2} \right\} \tag{15}$$

After some straightforward algebraic manipulations and with the aid of the assumption that $r\omega\delta \ll ST(t_1)$, we obtain that

$$\frac{\left(\vec{T}(t_1') \times \vec{S}\right) \cdot \hat{i}}{ST(t')} = f(t_1) + \frac{\omega\delta}{ST(t_1)} \left[\vec{S} \cdot \vec{T}(t_1) - f^2(t_1)\right] \tag{16}$$

where

$$f(t) = \frac{(\vec{T}(t) \times \vec{S}) \cdot \hat{i}}{ST(t)} \tag{17}$$

On substituting Eqs. (15) and (16) into Eq. (14) we get

$$c\Delta_1' = 2[ST(t_1)] \left\{ 1 + f(t_1) \left[\frac{\omega\delta}{ST(t_1)} + \frac{\omega}{c} - \frac{\delta\theta}{2ST(t_1)} \right]\right\} \tag{18}$$

If the time separation δ between the two pulses transmitted at $t = t_1$ and $t = t_1'$ respectively are suitably chosen so that

$$\Delta_1' = \Delta_2 \tag{19}$$

then the two phase centers R_1 and R_2 will form a virtual stationary receiver as the time delays of the two distinct pulses (relative to the two phase centers) are equal. In other words, the two pulses are timed appropriately so as to eliminate the motion of the platform.

From Eqs. (13), (18) the time delay matching condition in Eq. (19) requires that

$$1 + f(t_1)\left[\frac{\omega}{c} + \frac{\delta\theta}{2ST(t_1)}\right] = 1 + f(t_1)\left[\frac{\omega}{c} - \frac{\delta\theta}{2ST(t_1)} + \frac{\omega\delta}{ST(t_1)}\right] \tag{20}$$

Eq. (20) is satisfied if

$$\omega\delta = \delta\theta \tag{21}$$

Equation (21) is the two phase center matching condition we wish to derive. This condition requires that, in order for the two pulses to be matched, the time separation δ between them should be equal to half the time the satellite platform takes to cover the angular separation ($2\delta\theta$) of the two phase centers on receive. It is important to note that provided the distance traversed by the satellite in the time interval δ is small compared to $ST(t_1)$, i.e.

$$r\omega\delta << ST(t_1) \quad , \tag{22}$$

the matching condition in eq. (21) is valid regardless of the position of S and the time t_1.

In the derivation of Eq. (20), the first order approximation has been used in the expansions in Eqs. (4), (5) and (6). However, the effect of retaining second order terms in $|SR_1(t_3) - ST(t_1)| \;/\; ST(t_1)$, $\omega\Delta_1$ and $\delta\theta$ on the matching condition has been examined. The mathematics involved is straightforward but rather tedious. Instead of presenting the detailed results we give here only an error bound for a typical case. For an L-band radar on a satellite platform in a circular earth orbit of altitude 10^3 km the mismatch $c(\Delta_1' - \Delta_2)$ is less than 10^{-3}m for values of δ up to a millisecond.

To understand the significance of the matching condition, we may consider the transmission of a pulse train with equal interpulse intervals. The pulse trains received at R_1 and R_2 due to reflections at the point S can be depicted as in Fig. 3. For this illustration we assume that, within the duration of the transmission of the pulse train, the distance between S and T increases with time. It is evident that, as the delay times Δ_1 and Δ_2 are functions of the satellite motion, the received pulse trains are no longer equally spaced in time. In Fig. 3, we also

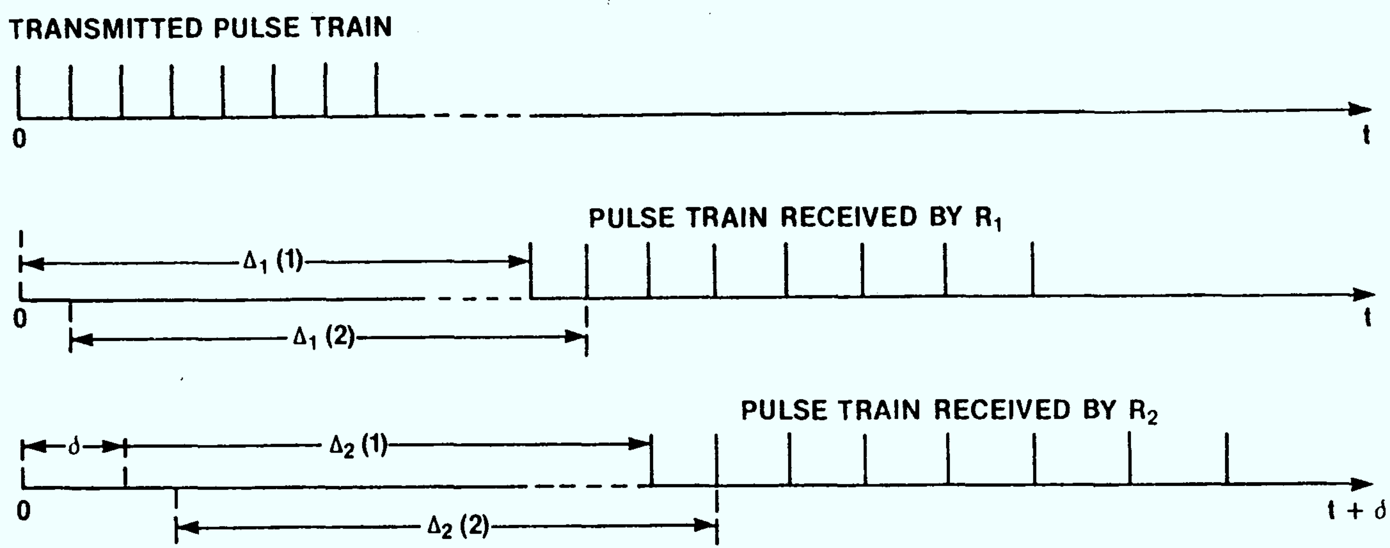

Fig. 3 - Time sequence of the transmitted and received pulse trains under the matching condition.

assume that the matching condition Eq. (21) is satisfied. By shifting the time origin for the pulse train received by R_2 relative to that of R_1 by δ, which equals to an integral multiple of the interpulse period of the transmitted pulse train, it can be seen that subsequences of the two received pulse trains can be synchronized. Consequently, as long as the reflecting property of S remains constant over the duration of the incident pulse train, there exists subsequences of the received pulse trains which will match exactly. In this consideration, an unambiguous range condition is assumed. If multiple time around interference is present, matching will be obtained after the establishment of the steady state. On the other hand, if S is replaced by a moving target, the matching process will break down.

As an example, Eq. (21) can be satisfied by choosing δ to be the pulse repetition interval so that the time delay of the first pulse in the pulse train received by R_2 is equal to the time delay of the second pulse of the pulse train received by R_1. However, the proper choie of δ needs to be considered in conjunction with the appropriate pulse repetition frequency for the detection of the types of targets in a given application. This question will be discussed further in later sections.

3.0 TARGET AND CLUTTER RETURNS

In the previous section we have obtained the matching condition for the two phase centers R_1 and R_2 to form a virtual stationary system with respect to two consecutive pulses reflected from an arbitrary point on the surface of the earth illuminated by the radar. More fundamentally, we have demonstrated that when the matching condition is met there exists subsequences of the received pulse trains at R_1 and R_2 which can be matched very closely by implementing a simple relative time shift equal to δ. In this section we will describe analytically the radar signal back-scattered from a moving target and the total clutter return from the earth background as seen by the two receivers. These results will form the basis for the performance evaluation of the displaced phase center antenna system.

The transmitted signal S(t) is taken to be in the following complex form:

$$S(t) = u(t) \exp [j\omega_c t] \tag{23}$$

where u(t) is a train of N rectangular pulses each of length τ_p such that

$$u(t) = \sum_{n=0}^{N-1} \text{rect} \left[\frac{t - n\delta}{\tau_p}\right] \tag{24}$$

$$\text{and rect }(x) = \begin{cases} 1 & |x| < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

In Eq. (23) ω_c is the radar carrier frequency. It is clear from Eq. (24) that the pulse repetition frequency is given by $1/\delta$. The pulse train arriving at the receivers R_1 and R_2 will be denoted by $r_i(t)$, $(i = 1,2)$. When only a point target is present $r_i(t)$ can be written as

$$r_i(t) = g_i \rho \, u\big(t-\Delta_i(t)\big) \exp \left[j\omega_c(t - \Delta_i(t)\right] \tag{26}$$

where ρ is the complex reflectivity of the point target and $\Delta_i(t)$ is the time delay between the transmitter T and receiver R_i via a reflection at the point target. The factor g_i includes the antenna gain and propagation loss factors.

3.1 The Target Signal

Consider a point target A with a velocity $\vec{v}_A$, located at time t=0 at a point whose spherical coordinates are (r_A, θ_A, ϕ_A). The co-ordinates of the phase centers T, R_1 and R_2 at time t=0 are summarized in Table 2.

Table 2 - Cartesian Coordinates of Phase Centers at Time t=0

Phase Center	Cartesian Coordinates
T	$x = 0,\ y = r \sin\theta_o,\ z = r \cos\theta_o$
R_1	$x = 0,\ y = r'\sin(\theta_o-\delta\theta),\ z = r'\cos(\theta_o-\delta\theta)$
R_2	$x = 0,\ y = r'\sin(\theta_o+\delta\theta),\ z = r'\cos(\theta_o+\delta\theta)$

A pulse emitted at t_1 is intercepted by the target at t_2. Since both the satellite and the target are moving we will denote the distance between $\vec{T}(t_1)$ and $\vec{A}(t_2)$ by $AT(t_2, t_1)$. Then

$$[AT(t_2,t_1)]^2 = (x_A+v_{Ax}t_2)^2 + \{(y_A+v_{Ay}t_2) - r\sin(\theta_o+\omega t_1)\}^2$$

$$+ \{(z_A+v_{Az}t_2) - r\cos(\theta_o+\omega t_1)\}^2$$

$$= [\overrightarrow{TA}]^2 + 2\vec{TA}\cdot\vec{v}_A t_2 + 2\omega t_1[\vec{T}(0) \times \vec{A}(0)] \cdot \hat{i} \tag{27}$$

where

$$\overrightarrow{TA} = \vec{A}(0) - \vec{T}(0) \equiv \vec{R}_o$$

$$\text{and } \vec{v}_A = (v_{Ax}, v_{Ay}, v_{Az})$$

By applying the binomial expansion to Eq. (27) and dropping second and higher order terms we obtain

$$AT(t_2, t_1) = R_o\left[1 + \frac{\vec{R}_o \cdot \vec{v}_A\, t_2}{R_o^2} + \frac{\omega t_1[(\vec{T}(0) \times \vec{A}(0)) \cdot i]}{R_o^2}\right] \tag{28}$$

Assuming that the reflected signal is received by R_i at τ_i then it can be found that

$$R_1A(\tau_1, t_2) = R_o\left\{1 + \frac{\vec{R}_o \cdot \vec{v}_A}{R_o^2} t_2 + \frac{(\omega t_1 - \delta\theta)}{R_o^2}[\vec{T}(0)\times\vec{A}(0)) \cdot \hat{i}]\right\} \tag{29}$$

and

$$R_2A(\tau_2, t_2) = R_o\left\{1 + \frac{\vec{R}_o \cdot \vec{v}_A}{R_o^2} t_2 + \frac{(\omega t_1 - \delta\theta)}{R_o^2}[\vec{T}(0)\times\vec{A}(0)) \cdot \hat{i}]\right\} \tag{30}$$

From Eqs. (28), (29) and (30) the time delay Δ_i defined by

$$\Delta_i(\tau_i) = \tau_i - t_1 \tag{31}$$

can be solved and the result is

$$\Delta_i(t) = \frac{2R_o}{c} - 2\left(\frac{R_o}{c} - t\right)\left[\frac{\omega g}{cR_o} + \frac{\vec{R}_o \cdot \vec{v}_A}{cR_o}\right] + (-1)^i \frac{g\delta\theta}{cR_o} \tag{32}$$

with

$$g = [\vec{T}(0) \times \vec{A}(0)] \cdot \hat{i} \tag{33}$$

It is useful to note that Eq. (32) gives the time delay as a function of the pulse arrival time t. In the derivation of $\Delta_i(t)$ it is required that $|\vec{v}_A| \ll c$ and $r\omega\tau_i \ll R_o$. Hence the target signal

$$r_i(t) = g_i \rho \sum_{n=0}^{N-1} \text{rect}\left[\frac{t - \Delta_i(t) - n\delta}{\tau_p}\right] \exp\left[j\omega_c(t-\Delta_i(t))\right] \tag{34}$$

can be determined.

3.2 The Clutter Return

In order to determine the clutter return from the collection of scatterers on the earth's surface we assume that they are continuously distributed. In addition to the fixed coordinate system Oxyz, it is convenient to introduce a rotating coordinate system Oxy'z' (Fig. 4) with the z'-axis oriented along OT. The coordinates of an arbitrary point C on the surface of the earth in the two coordinate systems are simply related by the matrix equation

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{35}$$

Let (θ_c', ϕ_c') be the polar and azimuth angles of C in the Oxy'z' frame, (Fig. 5). An elemental area δA situated at the point C such that

$$\delta A = r_e^2 \; \delta\theta_c' \sin\theta_c' \; \delta\phi_c' \tag{36}$$

can be written as

$$\delta A = \frac{r_e}{r} R\delta R \; \delta\phi_c' \tag{37}$$

where R is the radar range to the point C. Thus the clutter return from the elemental area δA to the receiver R_i is given by

$$\delta c_i(t) = g_i(R,\gamma,\phi_c')\rho(R,\phi_c') \sum_{k=0}^{N-1} \text{rect}\left[\frac{t - \Delta_i^{(c)}(t) - k\delta}{\tau_p}\right]$$

$$\times \exp\left[j\omega_c(t - \Delta_i^{(c)}(t))\right] \cdot \delta A \tag{38}$$

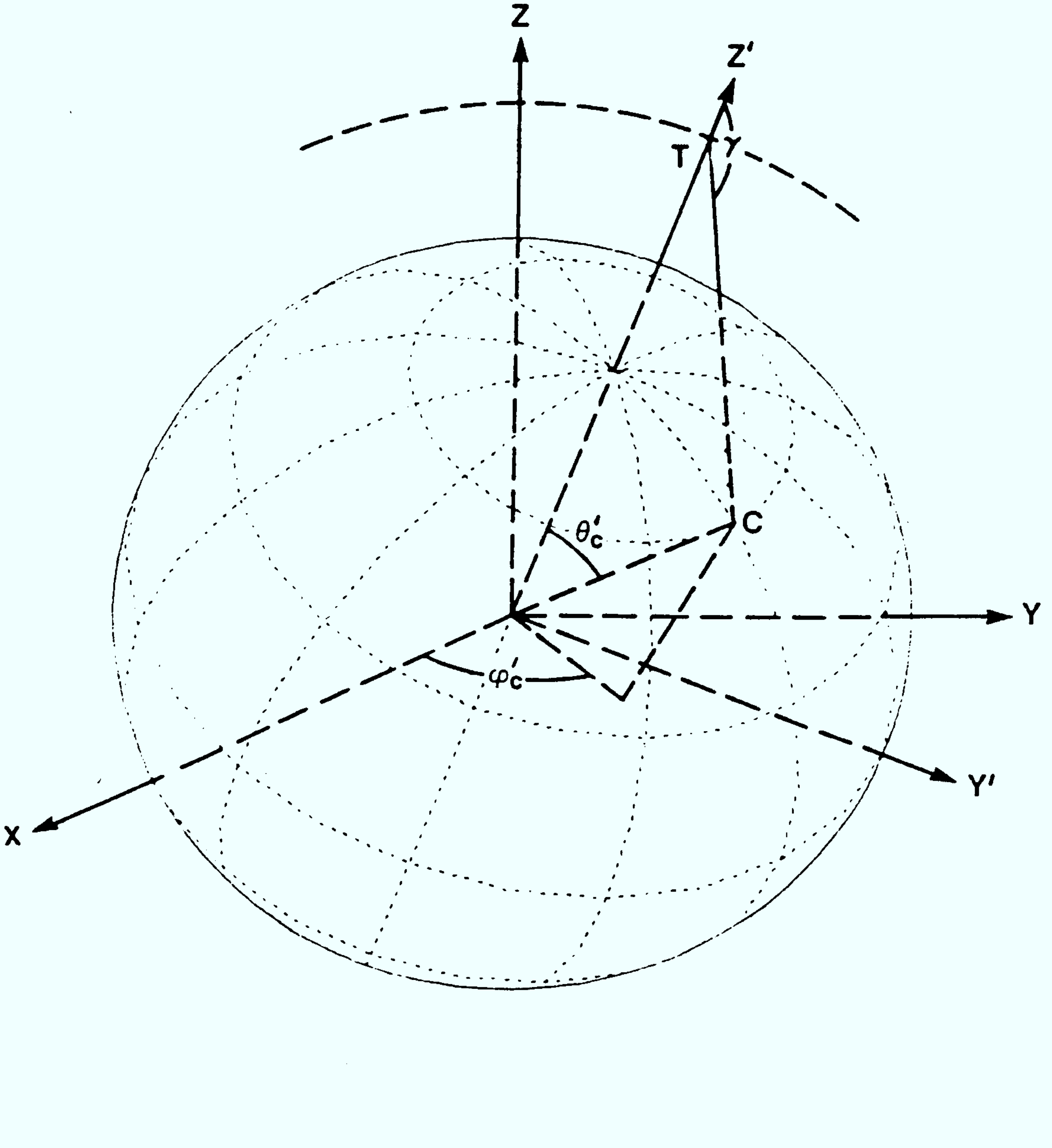

Fig. 4 - Geometrical configuration of the coordinate systems XYZ and X'Y'Z'

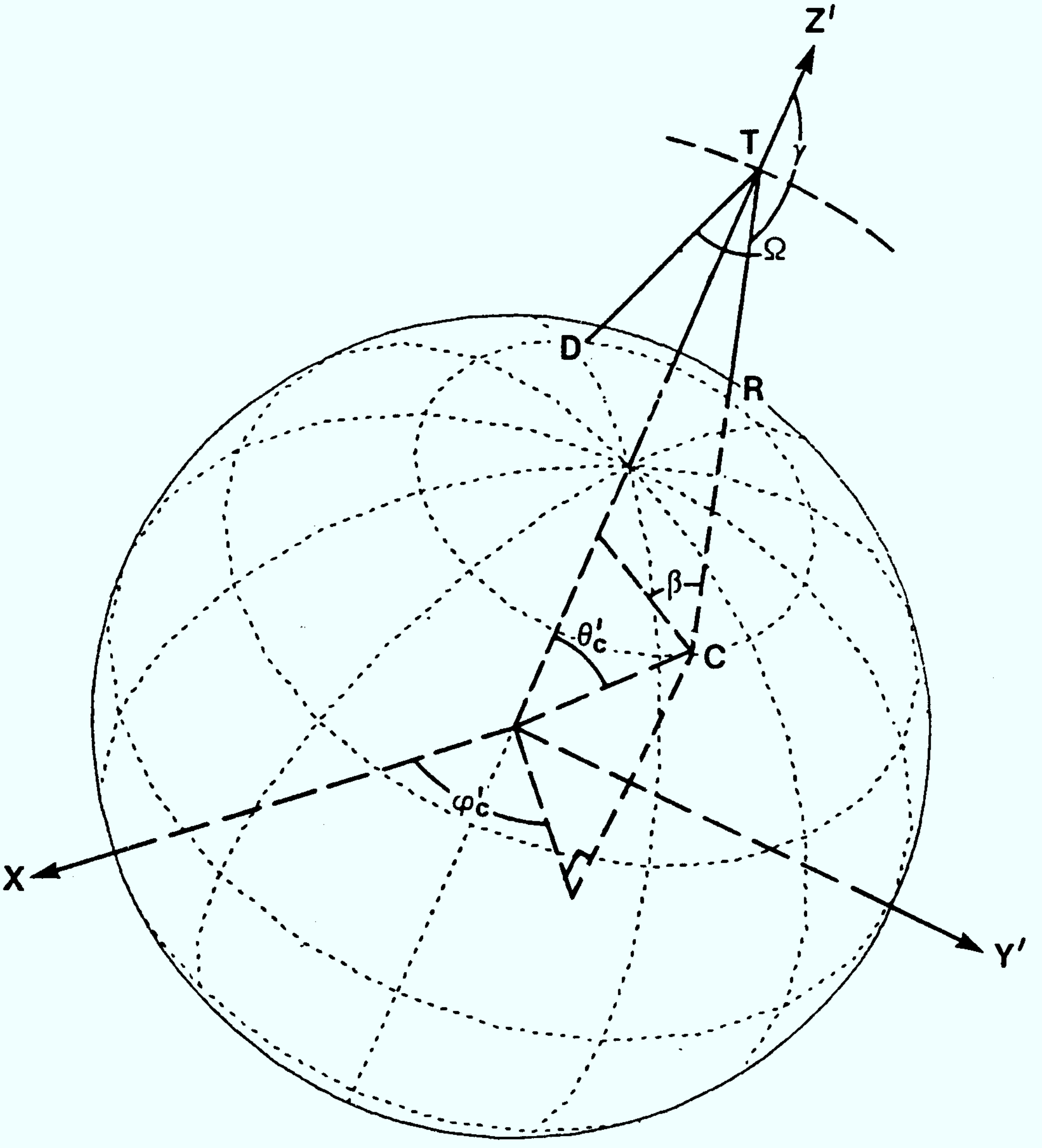

Fig. 5 - Orientation of a typical scatterer C on the surface of the earth relative to the boresight TD of the antenna.

In Eq. (38) $\rho(R,\phi_c')$ is the complex reflectivity per unit area, and $g_i(R,\gamma,\phi_c')$ is the product of the gain functions of the transmitter, the receiver as well as the propagation loss factor.

For the time delay $\Delta_i^{(c)}(t)$ it can be written down in analogy to $\Delta_i(t)$ in Eq. 32:

$$\Delta_i^{(c)}(t) = \frac{2R}{c} + 2(t - \frac{R}{c}) \frac{\omega g_c}{cR} + (-1)^i \frac{\delta\theta \cdot g_c}{cR} \tag{39}$$

with

$$g_c = \left[(\vec{T}(0) \times \vec{C}(0)) \cdot \hat{i}\right] \tag{40}$$

The total clutter return is then found by integrating Eq. (38)

$$c_i(t) = \frac{r_e}{r} \iint R dR d\phi_c' \; g_i(R,\gamma,\theta_c') \rho(R,\phi_c')$$

$$\times \sum_{k=0}^{N-1} \text{rect} \left[\frac{t-\Delta_i^{(c)}(t) - k\delta}{\tau_p}\right] \exp\left[j\omega\,(t-\Delta_i^{(c)}(t))\right] \tag{41}$$

3.3 Matched Filtering

In order to maximize the signal to noise ratio at the output of the receivers the received signal is passed through a matched filter [12]. Following Hofstetter et al [10] we use a filter matched to a signal return corresponding to a single pulse. Because of the very high Doppler shift induced by the satellite motion an angular frequency shift ω_D is included so that the impulse response function of the matched filter $f_m(t)$ is given by:

$$f_m(t) = \text{rect}\left[\frac{t}{\tau_p}\right] \exp\left[-j(\omega_c+\omega_D)t\right] \tag{42}$$

The choice of the frequency shift ω_D is determined by the Doppler frequency of the satallite as observed from a point where the transmitting antenna beam axis intersects the surface of the earth.

The matched filter output $y_i(t)$ corresponding to the signal $r_i(t)$ in Eq. (26) is the convolution of Eq. (42) and Eq. (26), and the result is

$$y_i(t) = \int d\eta \; r_i(\eta) \; \text{rect}\left[\frac{\eta-t}{\tau_p}\right] \exp\left[-j(\omega_c+\omega_D)(\eta-t)\right]$$

$$= g_i \rho \sum_{n=0}^{N-1} \int d\eta \; \text{rect}\left[\frac{\eta - \Delta_i(\eta) - n\delta}{\tau_p}\right] \text{rect}\left[\frac{\eta-t}{\tau_p}\right]$$

$$\times \exp\left[j\omega_c(t-\Delta_i(\eta))\right] \exp\left[-j\omega_D(\eta-t)\right] \qquad (43)$$

To perform the integration in Eq. (43) we replace the time delay $\Delta_i(\eta)$ in the argument of the rectangular function by the approximate expression $\Delta_i(\eta) \approx \frac{2R_o}{c}$. This is legitimate because of the η independent terms ignored are negligible in comparison to $\frac{2R_o}{c}$ and the coefficient of the linear term in η is much smaller than unity. However, in the sensitive phase term the full expression of $\Delta_i(\eta)$ is kept intact. Hence

$$y_i(t) = g_i \rho \sum_{n=0}^{N-1} \int d\eta \; \text{rect}\left[\frac{\eta - \frac{2R_o}{c} - n\delta}{\tau_p}\right] \text{rect}\left[\frac{\eta-t}{\tau_p}\right]$$

$$\exp\left[j(\omega_c+\omega_D)t\right] \exp\left[-j(\omega_c \Delta_i(\eta) + \omega_D \eta)\right] \qquad (44)$$

Consider the integral $i(\alpha,\beta)$ defined by

$$i(\alpha,\beta) = \int_{-\infty}^{\infty} d\eta \; f(\eta) \; \text{rect}\left[\frac{\eta-\alpha}{\tau_p}\right] \text{rect}\left[\frac{\eta-\beta}{\tau_p}\right] \qquad (45)$$

where α, β are positive real constants. It can be readily shown that the following result holds:

$$i(\alpha,\beta) = \text{rect}\left[\frac{\beta-\alpha}{2\tau_p}\right] \int_a^b d\eta \; f(\eta) \qquad (46)$$

where

$$\left\{ {b \atop a} \right\} = \frac{\alpha+\beta}{2} \pm \frac{\tau_p}{2} \mp \left|\frac{\beta-\alpha}{2}\right| \qquad (47)$$

Making use of Eq. (46) the matched filter output of the receivers can be written as

$$y_i(t) = g_i \rho \exp\left[-j\omega_c F_i\right] \exp\left[j(\omega_c+\omega_D)t\right]$$

$$\times \sum_{n=0}^{N-1} \mathrm{rect}\left[\frac{\frac{2R_o}{c} + n\delta - t}{2\tau_p}\right] \exp\left[-j\omega_c\left(G + \frac{\omega_D}{2\omega_c}\right)\left(t+n\delta + \frac{2R_o}{c}\right)\right]$$

$$\times \frac{\sin\left[\omega_c\left(G + \frac{\omega_D}{2\omega_c}\right)\left\{\tau_p - \left|t - \frac{2R_o}{c} - n\delta\right|\right\}\right]}{\omega_c\left(G + \frac{\omega_D}{2\omega_c}\right)} \tag{48}$$

The symbols G and F_i are defined as follows:

$$G = \frac{1}{cR_o}\left(\omega g + \vec{v}_A \cdot \vec{R}_o\right) \tag{49}$$

$$F_i = \frac{2R_o}{c}(1-G) + (-1)^i \frac{g\delta\theta}{cR_o} \tag{50}$$

Let us assume that $\tau_p = \delta/2$ so that the transmitter is operating at the maximum duty cycle of 0.5 to optimize the radio frequency energy within the transmitted pulse train while still allowing the proper functioning of the matched filter.

In Eq. (48), for any given value of t, the term

$$\mathrm{rect}\left[\frac{\frac{2R_o}{c} + n\delta - t}{2\tau_p}\right] \exp\left[-j\omega_c\left(G+ \frac{\omega_D}{2\omega_c}\right)\left(t + n\delta + \frac{2R_o}{c}\right)\right]$$

$$\times \frac{\sin\left[\omega_c\left(G + \frac{\omega_D}{2\omega_c}\right)\left\{\tau_p - \left|t - \frac{2R_o}{c} - n\delta\right|\right\}\right]}{\omega_c\left(G + \frac{\omega_D}{2\omega_c}\right)} \tag{51}$$

makes a non-zero contribution to the matched filtered signal $y_i(t)$ only when the inequality

$$\left| \frac{2R_o}{c} - t + n\delta \right| < \tau_p \tag{52}$$

holds. In other words, the value of n must be such that the conditions

$$- n\delta - \tau_p < \frac{2R_o}{c} - t < - n\delta + \tau_p \tag{53}$$

are fulfilled. From Fig. 6, it can be seen that if

$$- \tau_p < t - \frac{2R_o}{c} < (N-1)\delta + \tau_p \tag{54}$$

there is one and only one value of n which satisfies the conditions of Eq.(53). Let us denote this value of n by n' then Eq. (48) becomes

$$y_i(t) = g_i\rho \exp\left[-j\omega_c F_i\right] \exp\left[j(\omega_c + \omega_D)t\right]$$

$$\times \exp\left[-j\omega_c(G + \frac{\omega_D}{2\omega_c}) (t + n'\delta + \frac{2R_o}{c}) \right]$$

$$\times \frac{\sin\left[\omega_c(G + \frac{\omega_D}{2\omega_c}) \left\{ \tau_p - \left|t - \frac{2R_o}{c} - n'\delta\right| \right\}\right]}{\omega_c\left(G + \frac{\omega_D}{2\omega_c}\right)} \tag{55}$$

If $y_i(t)$ is sampled at the pulse repetition frequency at the range R', it then follows

$$y_i(\frac{2R'}{c} + m'\delta) = g_i\rho \exp\left[-j\omega_c F_i\right] \exp\left[j(\omega_c+\omega_D)(\frac{2R'}{c} + m'\delta)\right]$$

$$\times \exp\left[-j\omega_c(G + \frac{\omega_D}{2\omega})(\frac{2}{c} (R'+R_o) + (m'+n')\delta)\right]$$

$$\frac{\times \sin\left[\omega_c(G + \frac{\omega_D}{2\omega_c})\ \{\tau_p - \left|\frac{2}{c}(R'-R_o) + (m'-n')\delta\right|\}\right]}{\omega_i(G + \frac{\omega_D}{2\omega_c})} \qquad (56)$$

where m' is an integer and $0 \leq m' \leq N-1$. Hence the signal vector at the range R' is given by Eq. (56) and the constraints which n' must satisfy are

$$n'\delta - \tau_p < \frac{2}{c}(R'-R_o) + m'\delta < n'\delta + \tau_p \qquad (57)$$

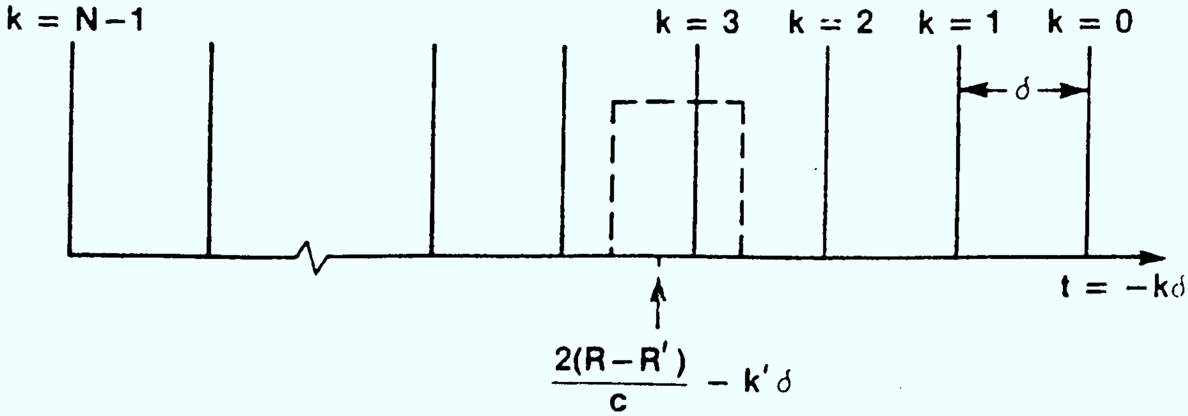

Fig. 6 - Time sequence of a train of N pulses

We turn our attention now to the matched filtered clutter returns. Let $z_i(t)$ be the filtered clutter return of the receiver at R_i. From Eq. (43) we can write

$$z_i(t) = \int d\eta\ c_i(\eta)\ \mathrm{rect}\left[\frac{\eta-t}{\tau_p}\right] \exp[-j(\omega_c+\omega_D)(\eta-t)]$$

Using Eq. (41) we can express $z_i(t)$ as

$$z_i(t) = \left(\frac{r_e}{r}\right)\iint RdRd\phi_c'\ g_i(R,\gamma,\phi_c')\ \rho(R,\phi_c')$$

$$\times \exp[j(\omega_c+\omega_D)t] \sum_{k=0}^{N-1} \int d\eta\ \mathrm{rect}\left[\frac{\eta-t}{\tau_p}\right]$$

$$\times\ \mathrm{rect}\left[\frac{\eta-\Delta_i^{(c)}(\eta) - k\delta}{\tau_p}\right] \exp\left[-j(\omega_c\Delta_i^{(c)}(\eta) + \omega_D\eta)\right] \qquad (58)$$

The integration over the variable η can be carried out as before. With the aid of Eqs. (45-47) the integral I_c defined by

$$I_c = \exp[j(\omega_c+\omega_D)\, t] \int d\eta \; \text{rect}\left[\frac{\eta-t}{\tau_p}\right]$$

$$\times\ \text{rect}\left[\frac{\eta - \Delta_i^{(c)}(\eta) - k\delta}{\tau_p}\right]$$

$$\times\ \exp\left[-j\,(\omega_c\Delta_i^{(c)}(\eta) + \omega_D\eta)\right] \tag{59}$$

can be evaluated. In fact, since from Eq. (39)

$$\omega_c\Delta_i^{(c)}(\eta) + \omega_D\eta$$

$$= \omega_c\left[\frac{2R}{c}\left(1 - \frac{\omega g_c}{cR}\right) + \frac{2\omega g_c}{cR}\,\eta + (-1)^i\, g_c\frac{\delta\theta}{cR}\right] + \omega_D\eta$$

$$= \omega_c\left[\frac{2R}{c}\left(1 - \frac{\omega g_c}{cR}\right) + (-1)^i\,\frac{g_c\delta\theta}{cR}\right] + \frac{2\omega_c\omega}{cR}\, g_D\,\eta \tag{60}$$

where
$$g_D = g_c + \omega_D\frac{cR}{2\omega\omega_c} \tag{61}$$
,

$$I_c = \exp\left[j(\omega_c+\omega_D)t\right] \int d\eta \; \text{rect}\left[\frac{\eta - \Delta_i^{(c)}(\eta) - k\delta}{\tau_p}\right]$$

$$\times\ \text{rect}\left[\frac{\eta-t}{\tau_p}\right] \exp\left[-j\omega_c\left\{\left[\frac{2R}{c}\left(1-\frac{\omega g_c}{cR}\right) + (-1)^i\,\frac{g_c\delta\theta}{cR}\right] + \frac{2\omega g_D}{cR}\,\eta\right\}\right]$$

$$= \text{rect}\left[\frac{\frac{2R}{c} + k\delta - t}{2\tau_p}\right] \exp\left[j(\omega_c+\omega_D)t\right] \exp\left[-j\omega_c \ell_D(R,\gamma,\phi_c')\omega t\right]$$

$$\times\ \exp\left[-j\omega_c\left\{\frac{2R}{c}\left(1-\omega\left[\ell(R,\gamma,\phi_c') - \ell_D(R,\gamma,\phi_c')\right]\right)\right.\right.$$

$$\left.\left. + (-1)^i\,\ell(R,\gamma,\phi_c')\delta\theta + k\ell_D(R,\gamma,\phi_c')\,\omega\delta\right\}\right]$$

$$\times\ \frac{\sin\left[\omega_c\ell_D(R,\gamma,\theta_c')\omega\left\{\tau_p - \left|t - \frac{2R}{c} - k\delta\right|\right\}\right]}{\omega\,\ell\,(R,\gamma,\phi')\omega} \tag{62}$$

In Eq. (62)

$$\ell(R,\gamma,\phi_c') = g_c / cR \tag{63}$$

and $$\ell_D(R,\gamma,\phi_c') = g_D / cR \tag{64}$$

Substituting the result of Eq. (62) into Eq. (58) we obtain

$$z_i(t) = \left(\frac{r_e}{r}\right) \iint R\,dR\,d\phi_c' \; g_i(R,\gamma,\phi_c') \; \rho(R,\phi_c')$$

$$\times \sum_{k=0}^{N-1} \text{rect} \left[\frac{(2R/c) + k\delta - t}{2\tau_p} \exp[j(\omega_c+\omega_D - \omega_c \ell_D \omega)t]\right]$$

$$\times \exp\left[-j\omega \left\{\frac{2R}{c}\left(1-\omega[\ell-\ell_D]\right) + (-1)^i \ell\delta\theta + k\ell_D\omega\delta\right\}\right]$$

$$\times \frac{\sin\left[\omega_c \ell_D \omega \left\{ \tau_p - \left| t - \frac{2R}{c} - k\delta \right| \right\}\right]}{\omega_c \ell_D \omega} \tag{65}$$

For brevity we have suppressed the arguments of the functions $\ell(R,\gamma,\phi_c')$ and ℓ $(R_{\ell}\gamma,\phi')$ in Eq. (63) and no confusion should arise from this abbreviation. The sampled clutter return at the range R' can be written down at once from Eq. (65) to give

$$z_i\left(\frac{2R'}{c} + k'\delta\right) = \left(\frac{r_e}{r}\right) \iint R\,dR\; d\phi_c' \; g_i(R,\gamma,\phi_c')\rho(R,\phi_c')$$

$$\times \sum_{k=0}^{N-1} \text{rect}\left[\frac{\frac{2}{c}(R-R') + (k-k')\delta}{2\tau_p}\right] \exp\left[j(\omega_c+\omega_D+\omega_c \ell_D \omega)\left(\frac{2R'}{c}+k'\delta\right)\right]$$

$$\times \exp\left[-j\omega_c\left\{\frac{2R}{c}\left(1-\omega[\ell-\ell_D]\right) + (-1)^i \ell\delta\theta + k\ell_D\omega\delta\right\}\right]$$

$$\times \frac{\sin\left[\omega_c \ell_D \omega \left\{\tau_p - \left|\frac{2}{c}(R'-R) + (k'-k)\delta\right|\right\}\right]}{\omega_c \ell_D \omega} \tag{66}$$

Let us pause a moment now to examine the integral in Eq. (66) over the range R. Evidently the range of values of R must be such that R should be no less than the satellite altitude and no greater than the distance from the satellite to the horizon. However, much stronger constraints arise from the fact that k and k' should both satisfy the inequalities $0 \leq k, k' \leq N-1$, and the non-vanishing conditions of the rectangular function in Eq. (66). For a given value of R' and k', the maximum and minimum values of R as k varies are given by:

$$R_M = R' + \frac{ck'\delta}{2} + \frac{c\tau_p}{2}$$

$$R_m = R' + \frac{ck'}{2}\delta - (N-1)\delta - \frac{c}{2}\tau_p \tag{67}$$

The total range of values of R is hence $\frac{c}{2}N\delta$ as one would expect.

As before, the conditions that the range of values of R over which the rectangular function in Eq. (66) is non-zero are

$$\left|\frac{2}{c}(R-R') + (k-k')\delta\right| \leq \tau_p \tag{68}$$

Analogous to the case of the signal vector, for given values of R' and k' and a value of R in the range specified by Eq. (67) only one value of k can satisfy Eq. (68). Let us denote this particular value of k by $\bar{k}$ then Eq. (66) can be rewritten as

$$z_i\left(\frac{2R'}{c}+k'\delta\right)=\left(\frac{r_e}{r}\right)\int_{R_m}^{R_M}dR\int_{o}^{2\pi}d\phi_c'\;R\;g_i(R,\gamma,\phi_c')\;\rho\;(R,\phi_c')$$

$$\times\exp\left[\;j(\omega_c+\omega_D-\omega_c\ell_D\omega)\left(\frac{2R'}{c}+k'\delta\right)\right]$$

$$\times\exp\left[-j\omega_c\left\{\frac{2R}{c}\left(1-\omega[\ell-\ell_D]\right)+(-1)^i\ell\delta\theta+\bar{k}\;\ell_D\omega\delta\right\}\right]$$

$$\times\frac{\sin\left[\omega_c\ell_D\omega\left\{\tau_\rho-\left|\frac{2}{c}(R'-R)+(k'-\bar{k})\;\delta\right|\right\}\right]}{\omega\ell_D\omega_c}\tag{69}$$

3.4 The Clutter Covariance Matrices

In studying the effectiveness of the displaced phase center antenna technique in clutter suppression a statistical approach is necessary. Common to a broad range of problems in which an electromagnetic field interacts with matter, two sources of fluctuations are encountered. In the first place, there are the fundamental quantum mechanical fluctuations of a wave source with a finite spectral width [13, 14, 15]. Secondly, under field conditions, physical information of rough surfaces illuminated by the electromagnetic wave, as a rule, allows only a probablistic discription of its reflecting properties [16, 17].

The statistical method commonly used to determine the linear relations between a sequence of observations is the correlation function. To account for the fluctuations of the reflecting properties of the radar background, the covariance matrix [18] is a particularly convenient tool in relating the clutter returns $z_i(t)$ of the two receivers discussed in Section 3.3.

At a given range R' the clutter covariance matrix $C_{ii'}^{kk'}$ is defined in terms of the clutter returns $z_i(t)$ by the relation

$$C_{ii'}^{kk'}=\langle z_i\left(\frac{2R'}{c}+k\delta\right)\;z_{i'}^*\left(\frac{2R'}{c}+k'\delta\right)\rangle\tag{70}$$

where the angular brackets indicate the average over an ensemble of reflecting surfaces. In Eq. (70) the only physical quantity affected by the averaging is the reflection coefficient of the earth's surface. Assuming the surface to be representable by a continuous distribution of uncorrelated scatterers it can be proved (Appendix 2) that

$$\langle \rho(R,\phi)\ \rho^*(R',\phi')\rangle = \left(\frac{r}{r_e}\right) \left(\frac{d\sigma(R,\phi)}{dA}\right) \delta(R-R')\ \delta(\phi-\phi')/R \qquad (71)$$

where $^{d\sigma}/dA$ is the differential cross section, or the cross section per unit area of the reflecting surface usually denoted by $\sigma°$ in radar literature.

From Eqs. (69), (71) the clutter covariance matrix can be simplified to

$$C_{ii'}^{kk'} = \left(\frac{r_e}{r}\right) \int dR \int_0^{2\pi} d\phi_c'\ g_i(R,\gamma,\phi_c')\ g_i(R^*,\gamma,\phi_c')R \quad \sigma°(R,\phi_c')$$

$$x \exp\left[j\omega_c \left\{(1+\omega\ell)\ \left(\frac{2R'}{c} + k\delta\right) + \omega l\bar{k}\delta + (-1)^i \ell\delta\theta\right\}\right]$$

$$x \exp\left[-j\omega_c \left\{(1+\omega\ell)\ \left(\frac{2R'}{c} + k'\delta\right) + \omega l\bar{k}'\delta + (-1)^{i'}\ \ell\delta\theta\right\}\right]$$

$$x \sin\left[\omega\ell_D\omega_c \left\{\tau_p - \left|\frac{2}{c}(R'-R) + (k-\bar{k})\delta\right|\right\}\right]$$

$$x \sin\left[\ \omega\ell_D\omega_c\left\{\ \tau_p - \left|\frac{2}{c}(R'-R) + (k'-\bar{k})\delta\right|\right\}\right]/(\omega\ell_D\omega_c)^2 \qquad (72)$$

In the above equation the integration over R is taken over the intersection of the intervals R_1 and R_2 arising from Eq. (67), i.e.

$$\Gamma_1 : R' + \frac{c}{2}k\delta - \frac{c}{2}(N-1)\delta - \frac{c}{2}\tau_p \leq R \leq R' + \frac{c}{2}k\delta + \frac{c}{2}\tau_p \qquad (73)$$

$$\Gamma_2 : R' + \frac{c}{2}k'\delta - \frac{c}{2}(N-1)\delta - \frac{c}{2}\tau_p \leq R \leq R' + \frac{c}{2}k'\delta + \frac{c}{2}\tau_p \qquad (74)$$

If we denote by k_m and k_M the minimum and the maximum values of k and k' so that

$$k_m = \min(k,k') \quad , \quad k_M = \max(k,k') \qquad (75)$$

then the range of integration over R is:

$$\Gamma_1 \cap \Gamma_2 : R' + \frac{c}{2}\left[k_M\,\delta - (N-1)\delta - \tau_\rho\right] \le R \le R' + \frac{c}{2}\left[k_m\delta + \tau_p\right] \qquad (76)$$

In analogy to the condition of Eq. (66), the integers $\overline{k}$ and $\overline{k}'$ must also satisfy the inequalities

$$\left| \frac{2}{c}(R-R') + (\overline{k} - k)\delta \right| < \tau_p \qquad (77)$$

and

$$\left| \frac{2}{c}(R-R') + (\overline{k}'-k')\delta \right| < \tau_p \qquad (78)$$

It follows immediately that

$$\overline{k} - k = \overline{k}' - k' \qquad (79)$$

and hence

$$C_{ii'}^{kk'} = \left(\frac{r_e}{r}\right) \int_{\Gamma_1 \cap \Gamma_2} dR \int_0^{2\pi} d\phi_c'\; g_i(R,\gamma,\phi_c') g_i' (R,\gamma,\phi_c')\; R\; \sigma^\circ(r,\phi_c')$$

$$\times \exp\left[j\omega_c \left\{(k-k')(1+2\omega\ell)\delta + \left[(-1)^i - (-1)^{i'}\right]\ell\delta\theta\right]\right\}\right] \qquad (80)$$

$$\times \sin^2\left[\omega\ell_D\omega_c\left\{\tau_p - \left|\frac{2}{c}(R'-R) + (k-\overline{k})\delta\right|\right\}\right] / (\omega\ell_D\omega_c)^2$$

The matrix element $C_{ii'}^{kk'}$ can be written explicitly as a sum of contributions from a set of ambiguous range lines. In fact, the expression is

$$C_{ii'}^{kk'} = \sum_{\overline{k}=k-k_m}^{(N-1)-(k_M-k)} \int_{\alpha(\overline{k})}^{\beta(\overline{k})} dR \int_0^{2\pi} d\phi_c'\; g_i(R,\gamma,\phi_c')\; g_i(R,\gamma,\phi_c') R$$

$$\times\ \sigma^\circ(R,\phi_c')\exp\left[j\omega_c\left\{(k-k')(1-2\omega\delta\ell)\delta - \left[(-1)^{i}-(-1)^{i'}\right]\ell\delta\theta\right\}\right]$$

$$\times\ \frac{\sin^2\left[\omega_c\ell_D\omega\left\{\tau_p - \left|\frac{2}{c}(R-R') + (k-\bar{k})\delta\right|\right\}\right]}{(\omega_c\ell_D\omega)^2} \tag{81}$$

and the limits of the integration over the range R are

$$\alpha(\bar{k}) = R' + (k-\bar{k})\frac{c\delta}{2} - \frac{c}{2}\tau_p \tag{82}$$

$$\beta(\bar{k}) = R' + (k-\bar{k})\frac{c\delta}{2} + \frac{c}{2}\tau_p \tag{83}$$

Thus, with Eq. (81), the matrix element $C_{ii'}^{kk'}$ can be determined. Depending on the value of the range R' the matrix element $C_{ii'}^{kk'}$ is a sum of integrals over a number of ambiguous range lines each of width $c\tau_p$ in the direction of R.

Since by definition, the clutter covariance matrix is hermitian the diagonal matrix elements given by Eq. (81) are real as required.

To see the effect of the matching condition (Eq. (21)) on the covariance matrix we can first remove the physically non-essential term $\exp[j\omega_c(k-k')\delta]$ in $C_{ii'}^{kk'}$ due to the constant phase advancement of the individual pulses in the transmitted pulse train by means of a constant phase transformation. Instead of the matrix C we can consider the matrix $\bar{C}$ given by

$$\bar{C} = PCP^{-1} \tag{84}$$

where P is a non singular matrix whose elements $P_{ii'}^{kk'}$ $(i,i' = 1,2;\ k,k' = 0,1,2,\ldots,N-1)$ are defined by the equation

$$P_{ii'}^{kk'} = \delta_{ii'}\delta_{kk'}\exp\left[-j\omega_c k\delta\right] \tag{85}$$

The matrices C and $\overline{C}$ are equivalent with identical eigenvalues. But under the matching condition the matrix elements $\overline{C}_{1,2}^{k,k-1}$ (k=0,1, ..., N-1) and their hermitian conjugates are real. The reality of these set of non diagonal matrix elements is due to the motion compensation of the displaced phase centers which has in effect matched the phase factors occurring in these nondiagonal matrix elements.

4.0 PERFORMANCE EVALUATION OF DISPLACED PHASE CENTER ANTENNA SYSTEMS

In Section 3 we have derived analytical expressions for the pulse trains reflected by the moving point target as well as by an area on the surface of the earth illuminated by the radar. The covariance matrix $C_{ii'}^{kk'}$ which provides a description of the statistical properties of the clutter returns has also been presented. Based on this theoretical framework a performance evaluation can now be carried out. The principal objective of this section is to study the characteristics of an ideal displaced phase center antenna system and to delineate the key parameters which determine its clutter rejection performance. No attempt will be made here to determine the system parameters of a displaced phase center antenna system for a space based radar required to satisfy a specific set of design goals. Such an investigation will be the theme of a forthcoming report.

As we are primarily interested in the performance of the displaced phase center antenna system in the presence of strong clutter we will assume in our present discussion that the radar interference is completely dominated by the radar energy backscattered from the earth's surface. Thus, the effect of system noise is considered to be negligible.

For the extraction of target signals from the interference of background clutter, the radar data collected by the two receivers are processed by a linear signal processor. The signal processor being considered is one which maximizes the signal to clutter ratio. The output of the processor is then compared with a preselected threshold to decide whether a target has been detected.

Since the optimum linear signal processor is discussed in many texts [19, 20] we can directly apply the standard results to the displaced phase center antenna system.

Let $\overline{w} = (w_{ik})$ be the weighting vector which defines the linear signal processor under discussion. At the output of the processor the signal to clutter ratio is given by

$$\left(\frac{s}{c} \right) = \frac{\left| \vec{s}^{*} \cdot \vec{w} \right|^{2}}{\vec{w}^{*} \, \overline{C} \, \vec{w}} \quad . \tag{86}$$

In Eq. (86), $\vec{s} = (s_{ik})$ is the signal vector defined by

$$s_{ik} = \sum_{i'} \sum_{k'} P_{ii'}^{kk'} \; y_i \left(\frac{2R'}{c} + k'\delta \right) \tag{87}$$

with the aid of the results given in Eqs. (56) and (85). The signal to clutter ratio attains a maximum value if the weighting vector $\vec{w}$ is chosen to be

$$\vec{w} = [\,\overline{C}\,]^{-1} \; \vec{s} \tag{88}$$

and hence

$$\left(\frac{s}{c} \right)_{max} = \vec{s}^{\,*} \; [\,\overline{C}\,]^{-1} \, \vec{s} \tag{89}$$

To measure the clutter suppression capability of the displaced phase center antenna system, different figures of merit can be used. Kelly and Tsandoulas [8] have compared the clutter rejection performance of the fully displaced phase center antenna array with that of a single antenna element. Hofstetter et al [10] used instead an improvement factor based on the signal to clutter ratio with reference to that of a single pulse. For the detection of specific target types, a figure of merit based on the minimum detectable velocity [4-6] is probably more useful. For the more generic discussion presented here the improvement factor used in Ref. [10] provides a simple and useful quantity for a qualitative performance assessment. The improvement factor F to be used is defined as the ratio of the quantity in Eq. (89) to the average single pulse signal to clutter ratio at the input of the linear processor. Since the average signal to clutter ratio per pulse at the input of the processor is

$$\left(\frac{s}{c} \right)_p = \frac{1}{2N} \sum_i \sum_k s_{ik}^* \; s_{ik} \Big/ \frac{1}{2N} \sum_i \sum_k \overline{C}_{ii}^{kk} \tag{90}$$

the improvement factor F can be expressed as

$$F = \left(\frac{S}{c} \right)_{max} \Big/ \left(\frac{S}{c} \right)_p$$

$$= \left\{ \vec{s}^{\,*} \; [\,\overline{C}\,]^{-1} \; \vec{s} \right\} \sum_i \sum_k \overline{C}_{ii}^{kk} \Big/ \sum \sum s_{ik}^* \;\; s_{ik} \tag{91}$$

If we denote the signal matrix by $\overline{S}$ such that

$$\overline{S} = \left(\overline{S}_{ii'}^{kk'} \right) \tag{92}$$

and $$\overline{S}_{ii'}^{kk'} = s_{ik} \, s^*_{i'k'} \tag{93}$$

then the improvement factor can be simplified to

$$F = \left\{ \vec{s}^* \left[\overline{C} \right]^{-1} \vec{s} \right\} \text{ trace } \overline{C} \;/\; \text{trace } \overline{S} \tag{94}$$

4.1 Numerical Results for the Improvement Factor

The main task involved in a numerical analysis of the improvement factor is in the computation of the clutter covariance matrix. From Eq. (81) it is evident that, with increasing the number of pulses in the transmitted pulse train, the number of integrals to be evaluated and hence the computational effort increases rapidly. To facilitate the numerical study a number of simplifying assumptions which do not affect the performance characteristics of the displaced phase center antenna system in any fundamental way, have been introduced.

It is assumed that the antennas of the transmitters and receivers are identical in dimensions and gains. This is only a mathematical simplification which ignores the loss of efficiency resulting from not using the full antenna array on signal transmission. Since the separation of the phase centers is, in practice, much smaller than the radar range, the angular differences of any point on the ground to the antennas are small and negligible. In addition, we assume that the antenna aperture functions are Gaussian. Explicitly, we take the functions $g_i(R,\gamma,\phi_c')$ to be given by,

$$g_i(R,\gamma,\phi_c') = g_i^j(R,\gamma,\phi_c')$$

$$= \frac{A_{ant}}{(4\pi)\,\lambda} \; \frac{e^{-\mu\Omega^2}}{R^2} \tag{95}$$

where Ω (see Fig. 5) is the angle between the antenna axis and the position vector of the point (R,ϕ_c'). A_{ant} is the antenna area and $\mu^{-\frac{1}{2}}$ is the antenna beam width.

In Eq. (81), the differential backscattering cross section of clutter σ° is, of course, a function of the terrain type and the angle of incidence. In our computations, however, these variations are ignored i.e. σ° is treated as a constant. Under this assumption the improvement factor F will then be independent of the clutter differential cross section σ°. It is generally true that the target cross-section does not affect the values of F.

In calculating the signal vector and the signal covariance matrix we also assume that the point target is located on the boresight of the transmitting antenna and that it is flying along a trajectory close to the surface of the earth.

The set of parameters common to all the computations presented in this report are summarized in Table 3.

Table 3 Parameters and Relations used in the Numerical Analysts

r_e	: radius of the earth 6.37×10^6m
r	: radius of the satellite orbit 7.37×10^6m
ω_c	: angular frequency of the radar 9.42×10^9 rad/s
ω	: angular velocity of the satellite 9.97×10^{-4} rad/s
δ	: interpulse period 1.0×10^{-4} (unless indicated otherwise)
τp	: pulse width $\tau p = \delta/2$
r_A	: radius of antenna 5m (unless indicated otherwise)
N	: total number of pulses in the pulse train 8 (unless indicated otherwise)
ω_D	: Doppler frequency shift in the matched filter $\omega_D = 2\omega\omega_c r r_e \sin\theta_D' \sin\phi_D'/cR_D$
θ_D', ϕ_D'	: polar and azimuth angle of D (Fig. 5)
R_D	: distance TD (Fig. 5).

In Fig. 7 the variations of the improvement factor is shown as a function of the radial velocity of the target $\vec{v}_A \cdot \hat{R}_D$ i.e. the projection of the target velocity $\vec{v}_A$ along the unit vector $\hat{R}_D$. As the magnitude of the relative velocity of the target increases so does the improvement factor F. Due to the sampling process involved, F is a periodic function in $\vec{v}_A \cdot \hat{R}_D$. The fundamental period is equal to $\frac{\lambda}{2}$ times the pulse repetition frequency. Thus in terms of the Doppler frequency, F has a periodicity equal to the pulse repetition frequency. On the other hand, F has no inversion symmetry about $\vec{v}_A \cdot \hat{R}_D = 0$. In Ref. [9] it has been shown the clutter Doppler spectrum as observed from a space based radar is, in general, asymmetric.

In Fig. 7, the results for the improvement factor are also plotted for different grazing angles. It is seen that the clutter cancellation is significantly more effective at higher grazing angles. This property of the displaced phase center antenna system could be of practical importance. In the absence of any clutter rejection technique, the coverage of a real aperture space based radar is severely limited by the phenomenon of the "nadir hole" [11]. This blind spot of the radar is the result of the reduction of the target relative radial speed as it moves to a region corresponding to a steeper grazing angle and the resulting increase in the intensity of the clutter return.

The effect of changing the pulse repetition frequency on the improvement factor is shown in Fig. 8. The factor F increases with the reduction in the pulse repetition frequency. However, if the clutter spectrum is undersampled degradation due to aliasing can arise. For the displaced phase center antenna system, lowering the pulse repetition frequency would require a larger separation of the phase centers to conserve the matching condition in Eq. (21).

When the number of pulses in the transmitted pulse train is increased there is better clutter cancellation. As can be seen from Fig. 9 the improvement factor F increases as the value for the pulse number N increases. In addition, the oscillations in the magnitude of F become shallower and hence a more uniform clutter cancellation across the target velocity range.

The last parameter we have examined is the antenna size. The improvement factor F has no explicit dependence on the dimension of the antenna. However, through its dependence on the antenna beam width $\mu^{-\frac{1}{2}}$, F increases with antenna dimension. In Fig. 10 the results for circular antennas of different radii are plotted.

5.0 CONCLUSIONS

In this report we have presented a theory of the displaced phase center antenna system for space based radar applications. A matching condition relating the geometrical configuration of the phase centers, the radar pulse repetition rate and the angular velocity of the space platform is derived in order to minimize the effect of platform motion on clutter.

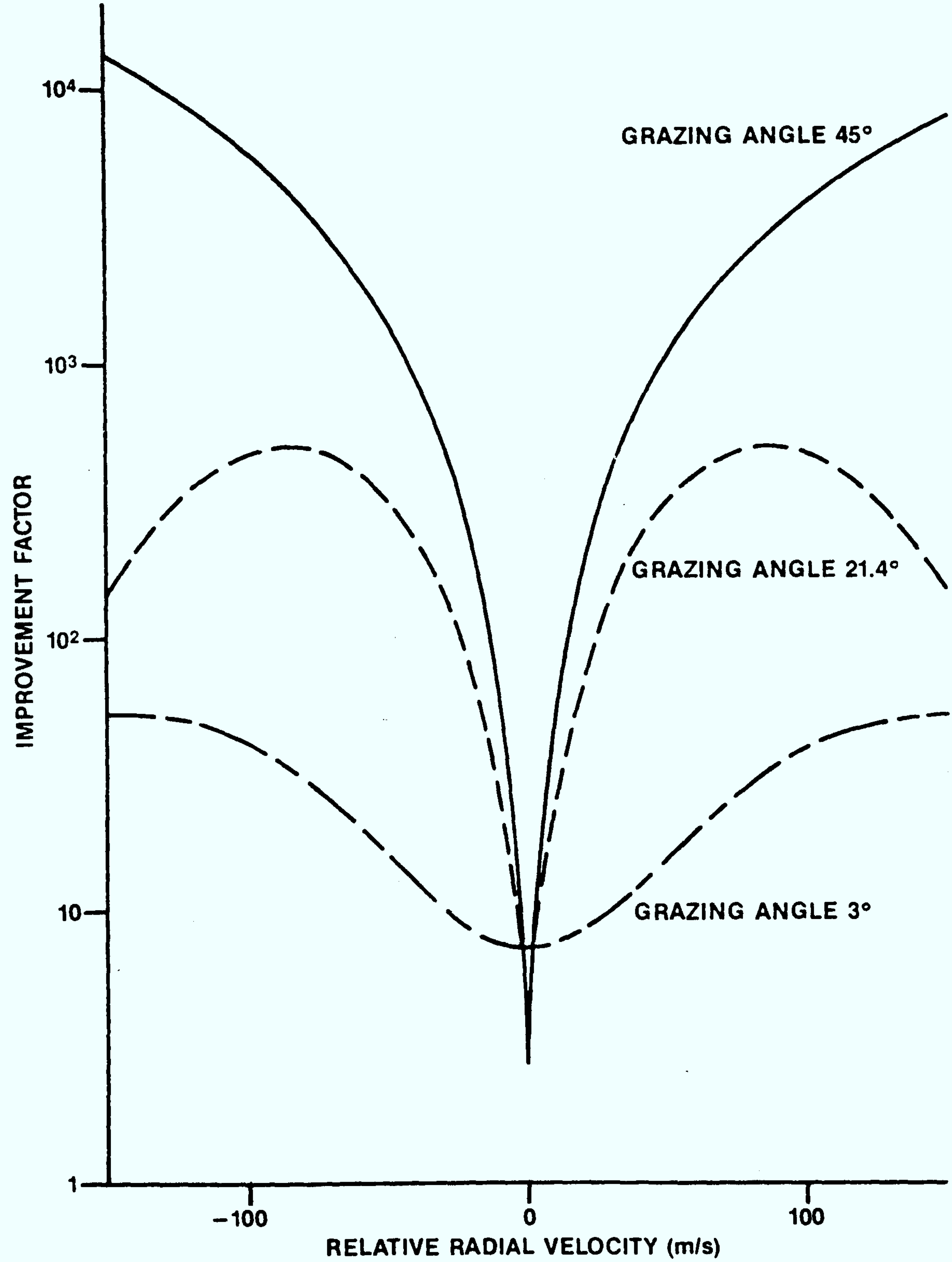

Fig. 7 - Improvement factor F versus the radial velocity of the target relative to the radar at grazing angles of 3°, 21.4° and 45°.

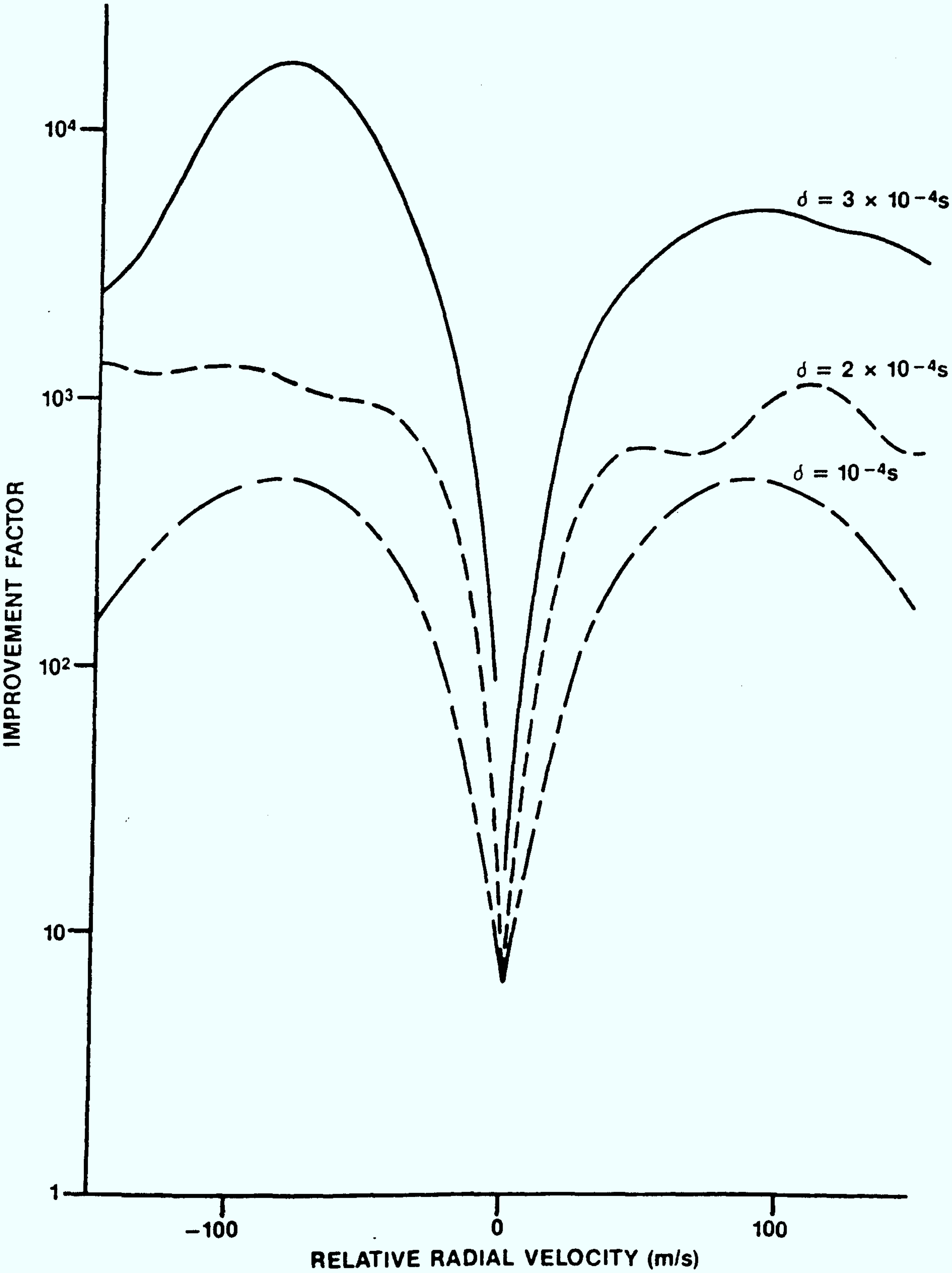

Fig. 8 - Improvement factor F versus the relative radial velocity at interpulse intervals of 10^{-4}s, 2×10^{-4}s and 3×10^{-4}s.

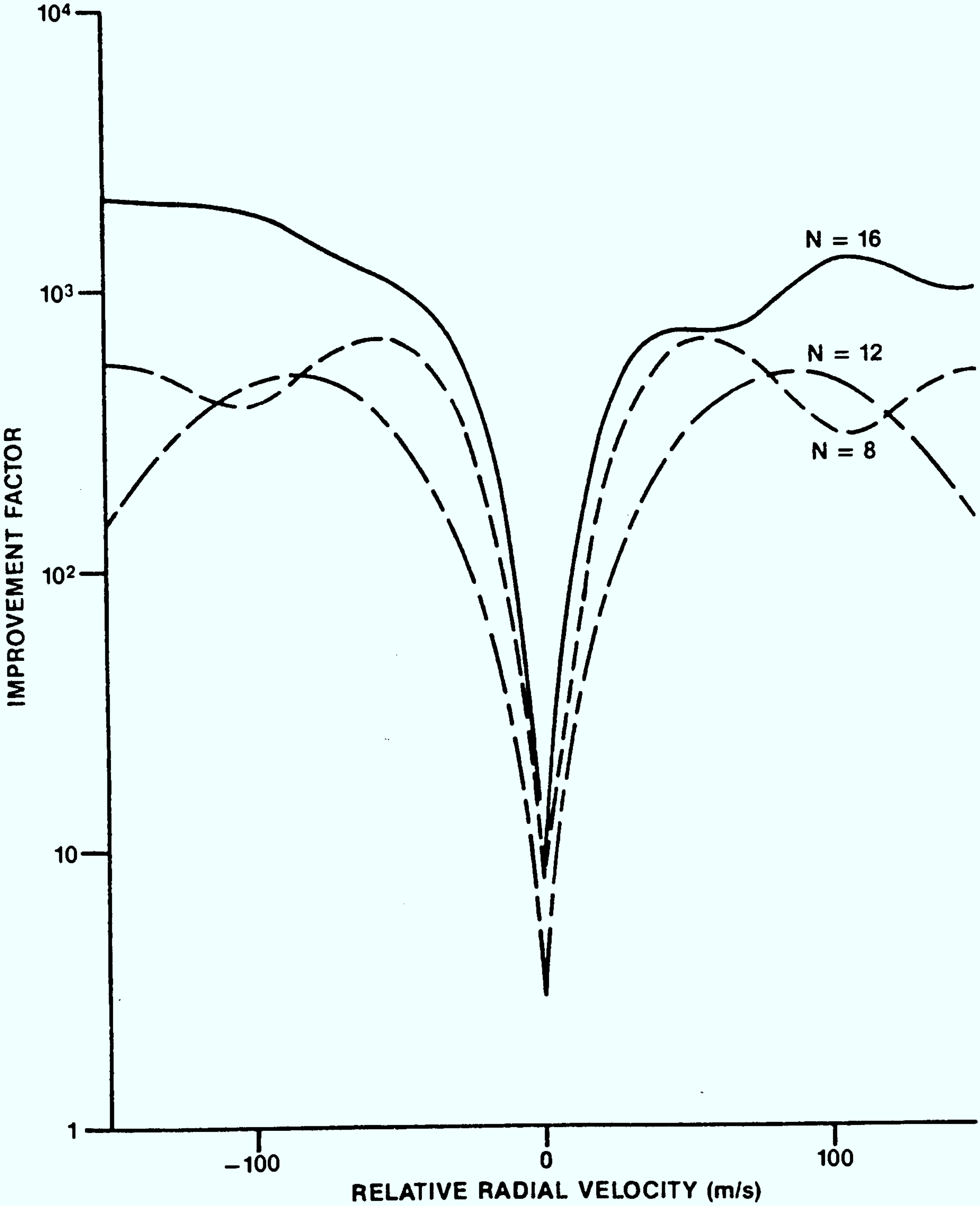

Fig. 9 - Improvement factor F versus the relative radial velocity for trains of 8 pulses, 12 pulses and 16 pulses.

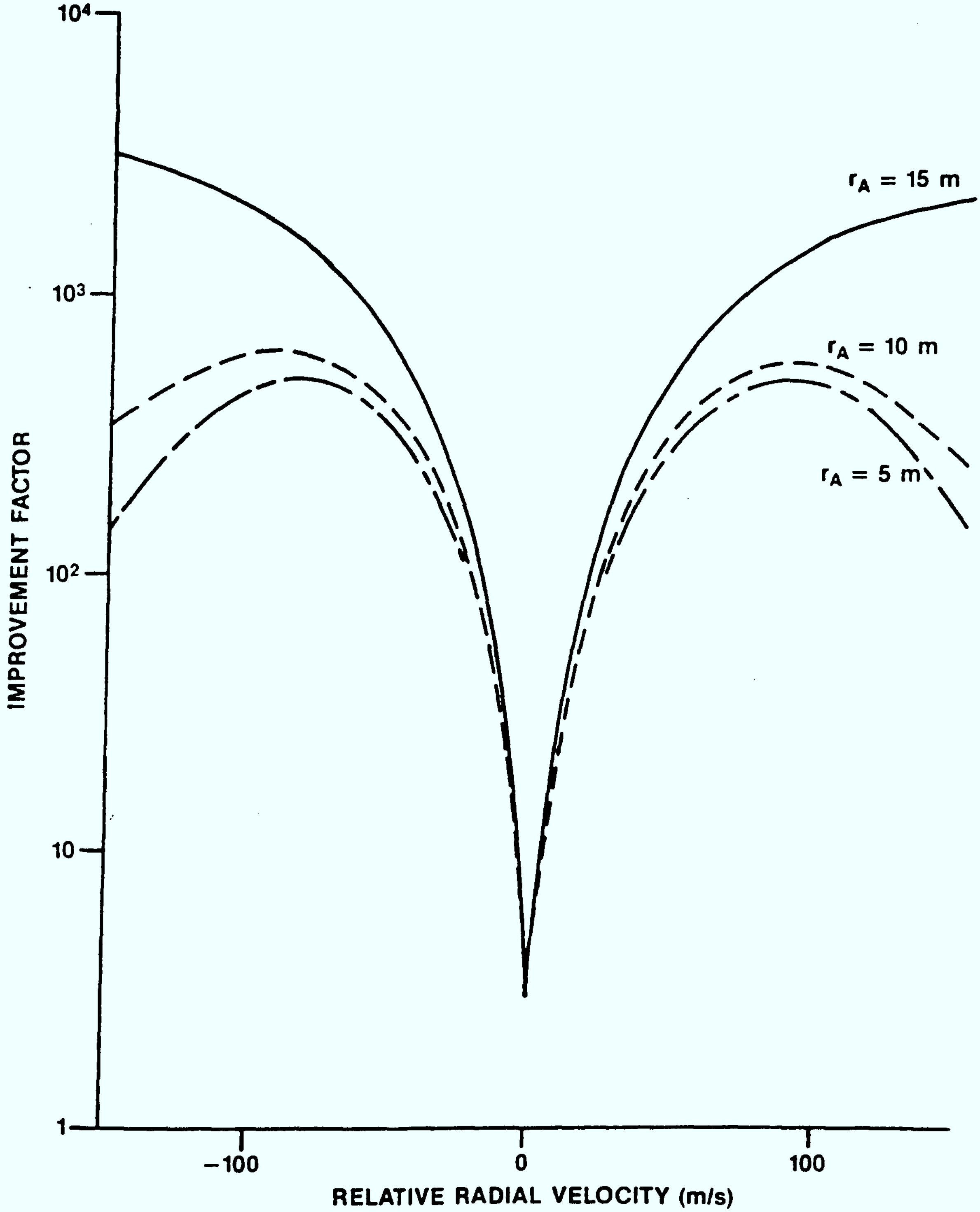

Fig. 10 - Improvement factor F versus the relative radial velocity for antennas of radius equal to 5 m, 10 m and 15 m.

After having established a general expression for both the signal and clutter covariance matrices an optimal signal processor which maximizes the signal to clutter ratio is defined. Numerical results based on a simplified displaced phase center antenna model are given in terms of an improvement factor which highlights the variations of the clutter cancellation capability of the system with changes in the radar grazing angle, pulse train duration, pulse repetition rate, and the antenna dimension.

In the theoretical derivation, the clutter model assumed is one where the backscattered radar energy can be completely determined in terms of the average differential cross section. The effect of scatterer motion on the displaced phase center antenna system is a subject of our current investigation which will also include a study of the modelling of the antenna mismatching.

Although the numerical results in this report are primarily aimed at highlighting the functional relations of the key radar parameters and the level of clutter cancellation, it can already be seen that the displaced phase antenna concept could provide a powerful technique in enhancing space based radar performance in the presence of strong clutter. In a report under preparation, the target detection capability of a displaced phase center space based radar system is studied in a more realistic clutter environment and design specifications.

ANNEX A

Matching Condition for Higher Order Displaced Phase Center Antenna System

In the preceding discussion we have restricted our attention to the simplest displaced phase center antenna system; namely a system with only two phase centers. There is, however, no fundamental reason why higher order systems, that is systems with more than two phase centers, cannot be used. In this appendix the matching condition for the higher order displaced phase center antenna is considered.

To simplify the geometry we assume that the displaced phase center antenna system has an odd number of phase centers which are uniformly distributed as shown in Fig. 11. The phase centers R_ν are labelled by the index ν and $\nu = 0, \pm1, \pm2, \ldots \pm i$. The distance of the phase centre R_ν measured from the geometric center of the full array is d_ν. As before, a geocentric coordinate system with its yz plane coinciding with the orbital plane of the satellite platform is chosen, so that r_ν, θ_ν are the radial distance and the polar angle of the center R_ν respectively. The phase center of the transmitting antenna T is colocated with the phase center R_o.

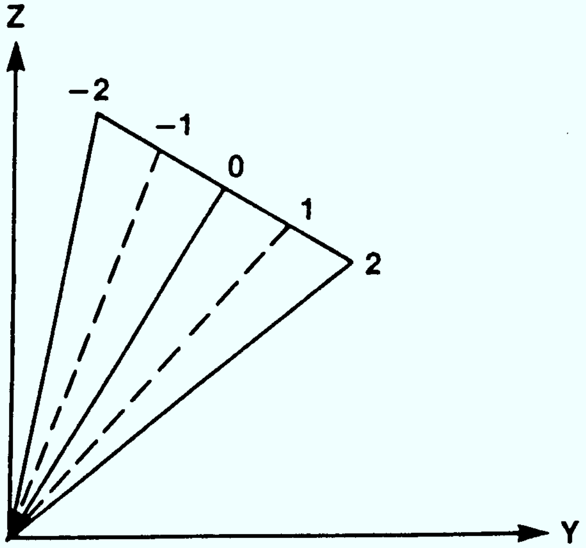

Fig. 11 - Positions of the phase centers for receive antennas.

Let a pulse be transmitted by T at a time $t = \overline{t}_1$. The pulse is assumed to arrive at a point $S(r_e, \theta_s, \phi_s)$ on the surface of the earth at $t = \overline{t}_2$. After reflection at S, the pulse is received by R_ν at $t = t_\nu$. Denoting the difference in polar angles $\theta_\nu - \theta_o$ by $\delta\theta_\nu$, which is a small angle typically of the order 10^{-2} mrad, we can write,

$$r = r_o \left[\cos \delta\theta_\nu \right]^{-1}$$

$$\simeq r_o \left[1 + \frac{1}{2} (\delta\theta_\nu)^2 \right] \qquad (A.1)$$

and

$$\theta_\nu(t_\nu) = \theta_o(t_\nu) + \delta\theta_\nu$$

$$= \theta_o(\bar{t}_1) + \omega(t_\nu - \bar{t}_1) + \delta\theta_\nu \tag{A.2}$$

The distance between S and $R_\nu(t_\nu)$ is given by

$$[SR_\nu(t_\nu)]^2 = (r_e \sin\theta_s \cos\phi_s)^2 + (r_e \sin\theta_s \sin\phi_s - r_\nu \sin\theta_\nu(t_\nu))^2$$
$$+ (r_e \cos\theta_s - r_\nu \cos\theta_\nu(t_\nu))^2 \tag{A.3}$$

From the vectorial relation

$$\overrightarrow{SR_\nu(t_\nu)} = \overrightarrow{ST(\bar{t}_1)} + \overrightarrow{T(\bar{t}_1)R_\nu(t_\nu)} \tag{A.4}$$

Eq. (A.3) can be rewritten as

$$[SR_\nu(t_\nu)]^2 = (r_e \sin\theta_s \cos\phi_s)^2 + [(r_e \sin\theta_s \sin\phi_s - r_o \sin\theta_o(\bar{t}_1))$$
$$+ (r_o \sin\theta_o(\bar{t}_1) - r_\nu \sin\theta_\nu(t_\nu))]^2$$
$$+ [(r_e \cos\theta_s - r_o \cos\theta_o(\bar{t}_1)) + (r_o \cos\theta_o(\bar{t}_1) - r_\nu \cos\theta_\nu(t_\nu))]^2$$
$$= (r_e \sin\theta_s \cos\phi_s)^2 + (r_e \sin\theta_s \sin\phi_s - r_o \sin\theta_o(\bar{t}_1))^2$$
$$+ (r_e \cos\theta_s - r_o \cos\theta_o(\bar{t}_1))^2$$
$$+ 2(r_e \sin\theta_s \sin\phi_s - r_o \sin\theta_o(\bar{t}_1))(r_o \sin\theta_o(\bar{t}_1) - r_o \sin\theta_o(t_\nu))$$

$$+ \; 2(r_e\cos\theta_s \; - \; r_o\cos\theta \;\; (\bar{t}_1))(r_o\cos\theta_o(\bar{t}_1) \; - \; r_\nu\cos\theta_\nu(t_\nu))$$

$$+ \; \left[r_o\sin\theta_o(\bar{t}_1) \; - \; r_\nu\sin\theta_\nu(t_\nu)\right]^2$$

$$+ \; \left[r_o\cos\theta_o(\bar{t}_1) \; - \; r_\nu\cos\theta_\nu(t_\nu)\right]^2 \qquad (A.5)$$

and hence

$$SR_\nu(t_\nu) = \left[ST(\bar{t}_1)\right] \Bigg\{1 + \frac{2(r_e\sin\theta_s\sin\phi_s - r_o\sin\theta_o(\bar{t}_1))(r_o\sin\theta_o(\bar{t}_1) - r_\nu\sin\theta_\nu(t_\nu))}{\left[ST(\bar{t}_1)\right]^2}$$

$$+ \frac{2\left(r_e\cos\theta_s - r_o\cos\theta_o(\bar{t}_1)\right)\left(r_o\cos\theta_o(\bar{t}_1) - r_\nu(\cos\theta_\nu(t_\nu)\right)}{\left[ST(\bar{t}_1)\right]^2}$$

$$+ \frac{\left[r_o\sin\theta_o(t_1) - r_\nu\sin\theta_\nu(t_\nu)\right]^2 + \left[r_o\cos\theta_o(\bar{t}_1) - r_\nu\cos\theta_n(t\;\;)\right]^2}{\left[ST(\bar{t}_1)\right]^2} \Bigg\}^{\frac{1}{2}} \qquad (A.6)$$

Let $\quad D'_\nu = r_o\sin\theta_o(\bar{t}_1) - r_\nu\sin\theta_\nu(t_\nu) \qquad (A.7)$

and $\quad D''_\nu = r_o\cos\theta_o(\bar{t}_1) - r_\nu\cos\theta_\nu(t_\nu) \qquad (A.8)$

With the aid of Eqs. (A.1) and (A.2) we get

$$D'_\nu \simeq - \; r_o\cos\theta_o(\bar{t}_1) \; \left[\omega(t_\nu - \bar{t}_1) + \delta\theta_\nu\right] \qquad (A.9)$$

and $\quad D''_\nu \simeq r_o\sin\theta_o(\bar{t}_1) \; \left[\omega(t_\nu - \bar{t}_1) + \delta\theta_\nu\right] \qquad (A.10)$

Since $\left| \; T\;(t_1)R_\nu(t_\nu) \; \right| / ST(\bar{t}_1)$ we can neglect the higher order terms in the binomial expansion of Eq. (A.6) such that

$$SR_\nu(t_\nu)=ST(\bar{t}_1) - \frac{2\left(r_e\sin\theta_s\sin\phi_s - r_o\sin\theta_o(\bar{t}_1)\right)\ \left\{r_o\cos\theta_o(\bar{t}_1)\left[\omega(t_\nu-\bar{t}_1)+\Delta\theta_\nu\right]\right\}}{ST(\bar{t}_1)}$$

$$+ \frac{(r_e\cos\theta_s - r_o\cos\theta_o(\bar{t}_1))\ \left\{r_o\sin\theta_o(\bar{t}_1)\ \left[\omega(t_\nu-\bar{t}_1) + \delta\theta_\nu\right]\right\}}{ST(t_1)}$$

$$+ \frac{r_o^2\left[\ \omega(t_\nu-\bar{t}_1) + \delta\theta_\nu\right]^2}{2ST(\bar{t}_1)} \qquad (A.11)$$

Dropping the quadratic term in $\left[\omega(t - \bar{t}_1) + \delta\theta_\nu\right]$ which is negligible, we obtain

$$SR_\nu(t_\nu) \simeq ST(t_1) + \frac{r_e r_o\left[\omega(t_\nu-\bar{t}_1) + \delta\theta_\nu\right]}{ST(\bar{t}_1)} - \sin\theta_s\sin\phi_s\cos\theta_o(\bar{t}_1)$$

$$+ \cos\theta_s\sin\theta_o\ (\bar{t}_1)\Big] \qquad (A.12)$$

Denoting the time delay $t_\nu - \bar{t}_1$ by Δ_ν, we have the following equation for Δ_ν:

$$c(t_\nu-\bar{t}_1) = c\Delta_\nu = SR_\nu(t_\nu) + ST(\bar{t}_1)$$

$$= 2ST(t_1) + \frac{r_e r_o[\omega\Delta_\nu + \delta\theta_\nu]}{ST(\bar{t}_1)}\ \Big[\cos\theta_s\sin\theta_o(\bar{t}_1)$$

$$- \sin\theta_s\sin\phi_s\cos\theta_o(\bar{t}_1)\Big] \qquad (A.13)$$

Alternatively, the above equation can be expressed as

$$c\Delta_\nu \simeq 2ST(t_1) + \frac{[\omega\Delta_\nu + \delta\theta_\nu]}{ST(t_1)} \left\{[T(\overline{t}_1) \times \vec{S}] \cdot \hat{i}\right\} \tag{A.14}$$

From Eq. (A.14) an explicit expression can be derived for $c\Delta_\nu$. Ignoring errors of the order $(\omega/c)^2$ and smaller we arrive at the following result

$$c\Delta_\nu \simeq 2ST(\overline{t}_1) \left\{ 1 + \frac{[(\vec{T}(\overline{t}_1) \times \vec{S}) \cdot \hat{i}\,]}{ST(\overline{t}_1)} \left(\frac{\omega}{c} + \frac{\delta\theta_\nu}{2ST(\overline{t}_1)}\right)\right\} \tag{A.15}$$

For a pulse transmitted at $t = \overline{t}'_1$ an analogous expression for the time delay $\Delta' \mathrel{\bar{=}} t' - \overline{t}'_1$ can be written down at once

$$c\Delta'_\nu \simeq 2ST(\overline{t}'_1) \left\{ 1 + \frac{[(\vec{T}(\overline{t}'_1) \times \vec{S}) \cdot \hat{i}\,]}{ST(\overline{t}'_1)} \left(\frac{\omega}{c} + \frac{\delta\theta_\nu}{2ST(\overline{t}'_1)} \right) \right\} \tag{A.16}$$

Let μ be an arbitrary phase center index (μ=0, ±1, ..., ±i).

In order to relate Δ'_μ to Δ_ν we have to express $ST(\overline{t}'_1)$, $\vec{T}(\overline{t}'_1) \times \vec{S}$ in terms of quantities at $t = \overline{t_1}$. Although a certain amount of algebraic manipulation is involved, the approach is similar to that used earlier. Hence, we will only summarize the main results.

Let δ be the interpulse interval so that

$$\overline{t}'_1 - \overline{t}_1 = m_1\delta \tag{A.17}$$

where m_1 is a positive integer and

$$\begin{aligned}\theta_o(\overline{t}'_1) &= \theta_o(\overline{t}_1) + \omega(\overline{t}'_1 - \overline{t}_1) \\ &= \theta\ (\overline{t}_1) + \omega\delta m_1\end{aligned} \tag{A.18}$$

We assume that $\omega \delta m_1 << 1$ then the following expressions can be derived:

$$ST(\overline{t}_1') = ST(t_1) \left\{ 1 + \frac{m_1 \delta\omega \, [(\vec{T}(\overline{t}_1) \times \vec{S}) \cdot \hat{f}]}{[ST(\overline{t}_1)]^2} \right\} \tag{A.19}$$

and $$(\vec{T}(\overline{t}_1') \times \vec{S}) \cdot \hat{f} = (\vec{T}(\overline{t}_1) \times \vec{S}) \cdot \hat{f} + (\vec{T}(\overline{t}) \cdot \vec{S}) \, m_1 \delta\omega \tag{A.20}$$

Retaining only up to linear terms in $m_1 \delta\omega$ and $\delta\theta_\nu$, Eqs. (A.19) and (A.20) lead to the result

$$c\Delta_\nu' = 2ST(\overline{t}_1) \left\{ 1 + f(\overline{t}_1) \left[\frac{\omega}{c} + \frac{\delta\theta_\nu}{2ST(\overline{t}_1)} + \frac{m_1 \delta\omega}{ST(\overline{t}_1)} \right] \right\} \tag{A.21}$$

where

$$f(t) = \frac{[(\vec{T}(t) \times \vec{S}) \cdot \hat{f}\,]}{ST(t)} \tag{A.22}$$

In order to equate the time delays Δ_ν and Δ_μ' the following condition must be valid

$$1+f(\overline{t}_1) \left(\frac{\omega}{c} + \frac{d\theta_\nu}{2ST(\overline{t}_1)} \right) = 1+f(\overline{t}_1) \left[\frac{\omega}{c} + \frac{\delta\theta_\mu}{2ST(\overline{t}_1)} + \frac{m_1 \delta\omega}{ST(\overline{t}_1)} \right] \tag{A.23}$$

which is equivalent to the matching condition

$$m_1 \delta\omega = \delta\theta_\nu - \delta\theta_\mu \tag{A.24}$$

Once again, the matching condition is independent of the position of S and the time $\overline{t}_1$.

If $\Delta\theta$ is the angular separation of two adjacent phase centers then

$$\delta\theta_\nu - \delta\theta_\mu = (\nu-\mu)\ \Delta\theta \tag{A.25}$$

and Eq. (A.24) becomes

$$m_l\omega\delta = (\nu-\mu)\ \Delta\theta/2 \tag{A.26}$$

This condition states that the time lags for the phase centers R_ν and R_μ can be matched if there exists an integer m_l satisfying Eq. (A.26). It is evident that if $\omega\delta$ is itself an integral multiple of $\Delta\theta$ then the time lags between any pair of phase centers can be matched. Apart from considerations related to the level of complexity in implementation, the total number of receive phase centers one can use is only restricted by the condition that $\omega\delta m_l << 1$ such that the approximations invoked in deriving the matching condition will not break down.

APPENDIX B

Statistical Properties of Uniformly Distributed Clutter Elements on a Given Surface Used in 3.4

We first consider a collection of discrete point scatterers distributed in a prescribed manner on a given surface. The total clutter return corresponding to a transmitted waveform $s(t)\, e^{j\omega_c t}$ is given by

$$c(t) = \sum_i \sum_k a_{ik} \frac{G(i,k)}{r_{ik}^2} s(t-\tau_{ik}) \exp[j\omega_c(t-\tau_{ik})] \tag{B.1}$$

where the indices (i,k) identify the point scatterer located at (x_i, y_k), r_{ik} is its distance from the radar, $G(i,k)$ is the two way antenna gain in the direction of (x_i, y_k), and τ_{ik} is the time delay and a_{ik} is the complex reflection coefficient of the scatterer.

The statistical properties we assume for the reflection coefficient of the discrete point scatters are:

$$\langle a_{ik} \rangle = 0 \tag{B.2}$$

$$\langle a_{ik} a_{\ell m} \rangle = 0 \tag{B.3}$$

$$\langle a_{ik} a^*_{\ell m} \rangle = \sigma_{ik} \delta_{i\ell} \delta_{km} \tag{B.4}$$

In Eq. (B.4), σ_{ik} is the backscatter cross section of the scatterer located at (x_i, y_k).

If instead of a collection of discrete point scatterers we consider a model of continuously distributed scatterers we may introduce a quantity β_{ik} associated with an area $\Delta x\ \Delta y$ located at (x_i, y_k) such that

$$\beta_{ik}\ \Delta x\ \Delta y \ = \ a_{ik} \tag{B.5}$$

Now Eq. (B.1) can be rewritten and

$$c(t) = \sum_i \sum_k \Delta x \Delta y\ \beta_{ik} \frac{G(i,k)}{r_{ik}^2}\ s(t-\tau_{ik})\ \exp\left[j\omega_c(t-\tau_{ik})\right] \tag{B.6}$$

with

$$\langle \beta_{ik} \rangle = 0 \tag{B.7}$$

$$\langle \beta_{ik} \beta_{\ell m} \rangle = 0 \tag{B.8}$$

$$\langle \beta_{ik} \beta^*_{\ell m} \rangle = \frac{\beta_{ik}}{\Delta x \Delta y} \frac{\delta_{i\ell}}{\Delta x} \frac{\delta_{km}}{\Delta y} \tag{B.9}$$

In the limit that the number of scatterers approaches infinity and Δx, Δy approach to zero we have

$$\frac{\delta_{i\ell}}{\Delta x} \longrightarrow \delta(x_i - x_\ell) \tag{B.10}$$

and $$\frac{\delta_{km}}{\Delta y} \longrightarrow \delta(y_k - y_m) \tag{B.11}$$

For an elemental area δA on a spherical surface as we have in our problem we can write

$$a_{ik} = a(R_i, \phi_k) \tag{B12}$$

Since $\delta A = \left(\frac{r_e}{r} \right) R dR \delta\phi$, then

$$\beta_{ik} = \beta(R_i, \phi_k) \tag{B.13}$$

is related to a_{ik} by

$$a_{ik} = \beta_{ik} \left(\frac{r_e}{r} \right) R_i \, \delta R_i \, \delta\phi_k \tag{B.14}$$

The correlation function

$$\langle a_{ik} \, a^*_{\ell m} \rangle = \sigma(R_i \, \phi_k) \; \delta R_i , R_l \;\; \delta\phi_k , \phi_m \tag{B.15}$$

implies

$$\langle \beta_{ik} \;\; \beta^*_{\ell m} \rangle = \frac{\sigma(R_i , \phi_k)}{\left(\frac{r_e}{r}\right) R_i \, \delta R_i \, \delta\phi_k} \;\; \frac{\delta_{R_i, R_\ell}}{\left(\frac{r_e}{r}\right) R_\ell \, \delta R_\ell} \;\; \frac{\delta_{\phi k, \phi m}}{\delta\phi_m} \tag{B.16}$$

In the limit that $\delta R \longrightarrow 0$, $\delta\phi \longrightarrow 0$ the Kronecker delta functions again go over to Dirac delta functions in a manner similar to the case of plane geometry (Eqs. (B.10) and (B.11)).

$$\langle \beta_{ik} \beta^*_{\ell m} \rangle = \left(\frac{r}{r_e}\right) \frac{1}{R} \; \delta(R_i - R_\ell) \;\; \delta(\phi_k - \phi_m) \; \frac{d\sigma}{dA} \tag{B.17}$$

or

$$\langle \beta(R, \phi) \;\; \beta^*(R', \theta') \rangle = \left(\frac{r}{r_e}\right) \;\; \frac{1}{R} \; \delta(R-R') \, \delta(\phi - \phi') \;\; \sigma^\circ(R, \phi) \tag{B.18}$$

In Eqs. (B.17) and (B.18) $\frac{d\sigma}{dA} = \sigma^\circ$ is the differential cross section or the cross section per unit area of the surface at the point (R, ϕ). On comparing Eq. (38) with Eq. (B.6) we can write

$$\rho(R, \phi) = \beta(R' \, \phi) \tag{B.19}$$

and hence it follows

$$\langle\rho(R,\phi)\ \rho^*(R',\ \phi')\rangle = \left(\frac{r}{r_e}\right) \frac{1}{R}\ \delta(R-R')\delta(\phi-\phi')\ \frac{d\sigma(R'\phi)}{dA} \qquad (B.20)$$

which is identical to Eq. (71).

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