Communications Research Centre

APPLICATION OF GEOLOCATION TECHNIQUES USING SATELLITES IN GEOSTATIONARY ORBIT (U)

by

Mario Caron
(Communications Technologies Research Branch)

CRC REPORT NO. 1435

June 1991
Ottawa

The work described in this document was sponsored by the Department of National Defence under Tech. Base Activity 1410-102

Government of Canada
Department of Communications
Gouvernement du Canada
Ministère des Communications

Canada

COMMUNICATIONS RESEARCH CENTRE

DEPARTMENT OF COMMUNICATIONS
CANADA

APPLICATION OF GEOLOCATION TECHNIQUES USING SATELLITES IN GEOSTATIONARY ORBIT (U)

by

Mario Caron
(Communications Technologies Research Branch)

CRC REPORT NO. 1435

June 1991
Ottawa

The work described in this document was sponsored by the Department of National Defence under Tech. Base Activities 1410-102

ABSTRACT

This report looks at techniques to perform the localization of SARSAT 406 MHz distress beacons using geostationary satellites. After reviewing the characteristics of the distress beacon transmitters, the main specifications of the GOES satellites are described as they can be assumed as typical geostationary satellites for this application. A brief review of the geolocation techniques is presented and an in-depth analysis is conducted on the time difference of arrival technique. The analysis covers the impact of the Earth's flatness, channel impairments and satellite geometry . The delay estimator performance is expressed as a function of signal-to-noise ratio and circular error probability. The overall system performance is directly related to the performance of the delay estimator.

Results of both theoretical analysis and computer simulations show that the accuracy of the position derived from the time difference of arrival technique is limited by (1) the low SNR of the distress beacons when relayed via geostationary satellites and, (2) by the low transmission rate (and thus bandwidth) of the beacon signal creating "broad" autocorrelation peaks and making the delay estimation process difficult. In the worst case C/No of 30 dB-Hz and with three satellites spaced at 30^{o}, we can say with a 90% confidence level that the circular error probability is between 3.3 and 4.9 km for a beacon in Canada. This is equivalent to an error not exceeding between 8 and 11.8 km for 95% of the time which is comparable to the current SARSAT polar orbit system based on a maximum error of 5 km for 90% of the time.

RÉSUMÉ

Ce rapport examine les techniques permettant d'effectuer la localisation de radiobalises de détresse SARSAT à 406 MHz en utilisant des satellites géostationnaires. Après une revue des caractéristiques des émetteurs des radiobalises de détresse, les spécifications principales des satellites GOES sont décrites puisqu'ils peuvent être considéré comme des satellites géostationnaires typiques pour cette application. Une brève revue des techniques de géolocation est présentée et une analyse en profondeur de la technique des différences de temps est effectuée. L'analyse couvre l'impact de l'applatissement de la Terre, les détériorations du canal et la géométrie des satellites. La performance de l'estimateur de délai est exprimée en fonction du rapport signal-à-bruit et de la probabilité d'erreur circulaire. La performance du système global est directement reliée à la performance de l'estimateur de délai.

Les résultats de l'analyse théorique et des simulations sur ordinateur démontrent que la précision de la position déduite à partir de la technique des différences de temps est limitée par (1) le faible rapport signal-à-bruit des radiobalises de détresse lorsque retransmises par satellites géostationnaires et, (2) par le faible débit de transmission (et ainsi de largeur de bande) du signal de la radiobalise créant de "larges" crêtes d'autocorrelation et rendant le processus d'estimation du délai difficile. Dans le pire cas d'un rapport C/No de 30 dB-Hz et avec trois satellites espacés de 30°, nous pouvons affirmer avec un degré de confiance de 90 % que la probabilité d'erreur circulaire est entre 3.3 et 4.9 km pour une radiobalise au Canada. Ceci est équivalent à une erreur ne dépassant pas entre 8 et 11.8 km pour 95 % du temps qui est comparable au système actuel basé sur une erreur maximale de 5 km pour 90 % du temps.

EXECUTIVE SUMMARY

The current Geostationary Operational Environmental Satellites (GOES's) are equipped with a special repeater allowing 406 MHz SARSAT distress signals to be relayed to the Mission Control Centres (MCC's). When a distress beacon is activated and is within the field of view of the satellite, this special repeater allows for an instantaneous distress alerting. This is particularly important for distress events occurring around the equator where the sole use of the SARSAT/ COSPAS low earth orbit satellites can result in up to one hour delay before the distress event is reported. Geostationary satellites have a continuous coverage of this area and thus can improve significantly the overall response time of the system. The SARSAT/COSPAS system currently relies on the Doppler frequency shift on the beacon signal to determine the beacon position. Geostationary satellites being quasi-motion free relative to the Earth, there is very little Doppler frequency shift that can be used to locate the distress beacons and an alternative technique must be used. This report investigates such alternative techniques.

With geostationary satellites, it is found that only the time difference of arrival technique can be promising. At least three satellites are required for an unambiguous positioning. The analysis shows that the Earth's flatness and the channel (ionosphere and troposphere) delay has little impact on the overall position accuracy of the system. Indeed, it is shown that the accuracy is more limited by the low signal-to-noise ratio of the beacon signal once relayed via a geostationary satellite and of its low bandwidth (or data rate) which creates relatively broad autocorrelation peaks and make the estimation of very precise delay difficult. The theoretical analysis and the computer simulations show that an accuracy comparable to the current SARSAT polar orbit system can be expected i.e. an error less than 5 km in 90 % of the time.

The position estimator can be based on a personal computer equipped with special digital boards to perform the signal processing. The implementation proposed in this report assumed that the position estimation processor is an add-on to the current GOES processor . Implementation should however be deferred until sufficient GOES or other host satellites become available. More advanced techniques to perform the delay estimation could be investigated in the mean time.

ACKNOWLEDGMENTS

I am grateful to DREO for funding this project and in particular to R.J. Keightley from the EHF Satcom division. Thanks also to E.J. Hayes for his support and general guidance, L. Perrier and C. Pike for the information on the current GOES processor and system, Dan Hindson and Norman Secord for the many technical discussions, Dave Andean for his technical guidance in Section 6.2 regarding the satellite positioning techniques and C. Loo for taking the time to carefully review this report.

TABLE OF CONTENTS

1.0 INTRODUCTION

The work reported here has been funded by the EHF Satcom division of the Defence Research Establishment of Ottawa (DREO). Their interest in the application of geolocation techniques using geostationary satellites is twofold. First it provides a tool to locate sources of interference so that, for instance, the satellite antenna/nulling system can generate a null on the interference source. Second, it is a tool to assess the probability that covert terminals can be detected and located, or the success of the electronic counter-countermeasure (ECCM) technique(s) used by the terminal. In general, these signal sources are random, bursty, and have poor signal-to-noise ratios.

The 406 MHz emergency beacons transponded through the Geostationary Operational Environmental Satellite (GOES) satellites have characteristics similar to the signal sources of interest. It is the purpose of this report to investigate the feasibility of developing a system to extract location information from 406 MHz distress signals that have been transmitted by repeaters on GOES or other similar geostationary satellites. Because geostationary satellites offer continuous coverage, they have been considered in various systems for radio position determination applications [1-11]. The major benefit for the search and rescue (SAR) system lies in the possibility of improving the response time of the current system by using geostationary satellites. Indeed, the use of geostationary satellites provides a quasi-instantaneous position reporting for beacons in their field of view while some distress alerts in the current COSPAS-SARSAT system may take up to two hours to be detected and located. This is partly due to the ambiguity of the Doppler positioning technique which requires a second satellite pass to resolve and partly due to the satellite orbit which makes the satellite revisit time relatively long for terminals close to the equator. The use of a geostationary satellite would therefore be particularly useful to remove these latter delays where the geostationary satellite visibility is continuous.

Sections 2.0 and 3.0 of this report review briefly the characteristics of 406 MHz emergency beacons and GOES repeaters respectively. Section 4.0 describes a number of basic geolocation techniques and discusses their applicability to this project. Section 5.0 gives the basic principle of the time difference of arrival (TDOA) technique while Section 6.0 introduces the perturbations to the TDOA principle. Section 7.0 deals exclusively with the design of the time difference estimator. Section 8.0 combines the effects of all perturbations and derives an expression for the overall positioning accuracy of a proposed distress beacon receiver. Section 9.0 describes the circular error probability of the system and Section 10.0 summarizes the receiver structure. Section 11.0 concludes this report.

2.0 406 MHZ EMERGENCY BEACON CHARACTERISTICS

The detailed specifications of the 406 MHz emergency beacon transmitters are given in [12] and this section briefly highlights some of the key signal characteristics.

The beacon transmits a digitally phase modulated carrier burst. The format of the burst is determined by the message type i.e. short (standard) or long (optional). Figure 2.0-1 along with Table 2.0-1 shows the signal format. Basically, the burst duration is either 440 ms (short) or 520 ms (long) and it is repeated every T seconds where T is a random period uniformly distributed between 47.5 and 52.5 seconds. The signal comprises a 160 ms carrier preamble and a 24-bit synchronization word. The protected data field is 61 bits long and the error correcting code is a BCH (127,106) code shortened to (82,61). The following 6 bits are reserved for national use or emergency codes while the last 32 bits are the long message (optional) data. The optional message content is not strictly defined but it is recommended in [12] to maritime users to include the course, speed and time of activation while other users are recommended to include the beacon position derived from an auxiliary radio location system incorporated into the beacon. Of course this study is limited to those beacons which are not equipped with such auxiliary location systems.

The phase modulation is such that a residual carrier is present to ease the signal detection and demodulation. The initial frequency accuracy is ± 2 kHz and the frequency offset does not exceed 5 kHz after 5 years. The power output is nominally 5 watts with an antenna gain between -3 dBi and 4 dBi over 90 % of the hemispherical pattern.

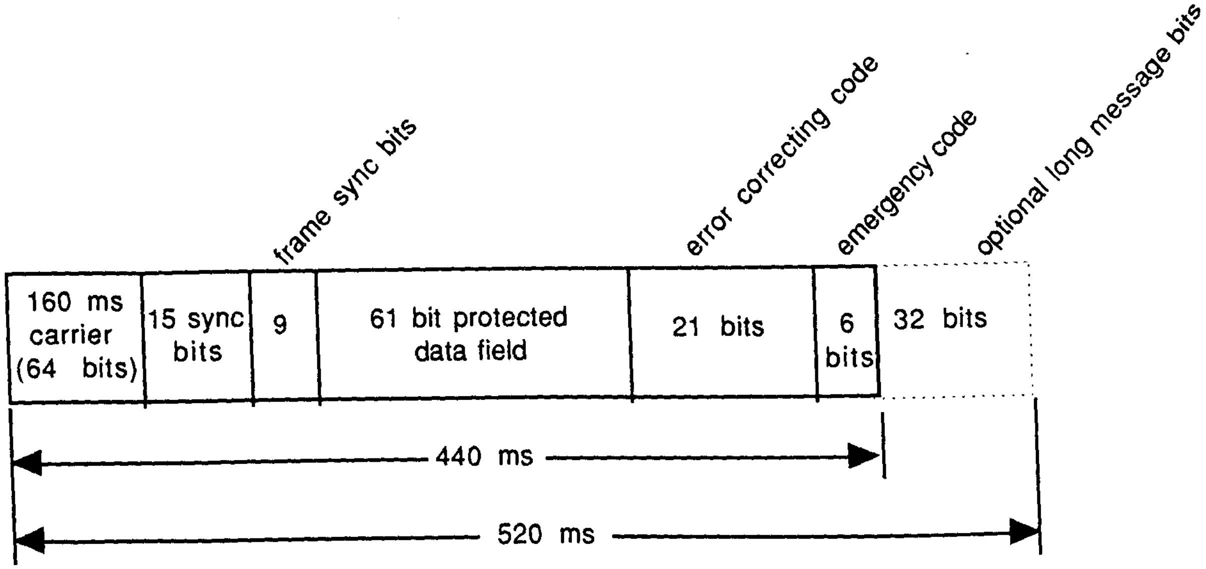


Figure 2.0-1 General 406 MHz Message Format

Parameter	Value
<u>RF Signal</u>	
Carrier frequency (initial)	406.025 ± 0.002 MHz
Carrier frequency (prior to end of useful life)	406.025 ± 0.005 MHz
Frequency stability	
Short term (100 ms)	2×10^{-9}
Medium term	
Mean slope	1×10^{-9}/min
Residual noise	3×10^{-9}
Power output	5 W ± 2 dB into 50 ohm load with VSWR < 1.25:1
Spurious emissions	50 dB below 5 W in 5 MHz bandwidth; carrier harmonics 30 dB below 5 W
Data encoding	Bi-phase L
Modulation	Phase modulation 1.1 ± 0.1 radians peak
Modulation rise and fall times	must be between 50 μs and 250 μs
<u>Digital Message</u>	
Repetition rate	Random. Uniform Distribution Between 47.5 and 52.5 s
Transmission Time	440 ms ± 1% or 520 ms ± 1%
CW preamble	160 ms ± 1 %
Digital message	280 ms ± 1% or 360 ms ± 1%
Bit rate	400 bps ± 1 %
Bit synchronization	15 "ones"
Frame synchronization	000101111
Continuous emission: failure mode	Transmission shall not exceed 45 s

Table 2.0-1 406 MHz Beacon Signal Characteristics

3.0 GOES REPEATER CHARACTERISTICS

The latest Geostationary Operational Environmental Satellite (GOES H) is equipped with a frequency translating repeater which receives signals over a 100 kHz band centered at 406.05 MHz and re-transmits the translated and inverted band at a center frequency of 1698.65 MHz. The 406 MHz repeater on the current GOES H satellite shares circuitry with another GOES subsystem, the Data Collection Platform (DCP) repeater. Both subsystems have different operating bands but a common AGC regulates the total output power of the combined DCP and 406 MHz repeater. Note that the GOES satellites subsequent to GOES H will not have this common AGC and the translation frequency band will be slightly different.

The satellite antenna receives right hand circularly polarized signals and transmits linearly polarized signals. The antenna pattern is hemispheric in both receive and transmit. The repeater gain variation over 406.05 MHz $\pm$ 40 kHz does not exceed 2 dB peak-to-peak.

The minimum satellite G/T is -22 dB/K and the EIRP is +30 dBm shared by 406 MHz distress signals when no transmission occurs on the DCP repeater. During DCP transmission, the 406 MHz beacons available EIRP drops by approximately 6 dB based on experimental results.

Link budgets for a 5^{o} elevation angle and 8 simultaneous distress signals given in [14] for a GOES satellite at either 75^{o} W or 135^{o} W show that the minimum expected C/No is in the order of 30 dB-Hz.

The GOES satellites are maintained in their nominal orbital position within approximately $\pm 0.1^{o}$ in both the East-West and North-South axis.

Note that the current GOES satellite has a CW pilot that can be used for frequency tracking. In addition, time is disseminated at 468.8 MHz. This time code generated by atomic clocks is repeated every 30 seconds. Its accuracy is not currently known but commercial receivers offered by Kinemetrics/Truetime in California give an accuracy of 0.5 ms.

4.0 GEOLOCATION TECHNIQUES

Geolocation refers to the determination of the location of a vehicle, object or person on earth. The location of signal sources is usually based on techniques which rely on one or a combination of frequency, time and spatial information. It is difficult to describe various geolocation techniques that can be based on a combination of this information. The approach taken here is to identify how the information can be used when a limited number of satellites is available. This is realistic and limits the study to practical cases. This section describes techniques applicable to one, two and more than two satellite systems in a general sense. The time difference of arrival technique is discussed in detail in the following sections.

4.1 ONE-SATELLITE SYSTEMS

In general, the geolocation systems based on a single satellite require special circuits to be built into the spacecraft. These circuits are, for instance, an interferometer, a steering antenna with an energy detector scanning the Earth as in [15] or a special antenna mounted on a spacecraft with a specific 3-dimensional motion as in [16,17]. Clearly these special circuits are not available on GOES satellites.

A technique that does not require special circuits is based on the residual velocity of the nearly geostationary satellites. Indeed geostationary satellites are never fully stationary because of the orbit perturbations such as the changes with time of the gravitational pull of the moon and to a lesser extent of the sun [18]. The geostationary satellites are usually maintained in their nominal positions with the East-West and North-South station keeping controls.

The slow geostationary satellite motion creates a small Doppler frequency shift. To have an idea of the order of magnitude of this Doppler shift, Slabinsky [18] gives an example with a $\pm 0.25^{o}$ East-West and $\pm 0.1^{o}$ North-South maximum position error and a satellite eccentricity ε for a station located at 40^{o} North latitude and 50^{o} West of the satellite meridian. The worst case satellite to ground station radial velocity is given by $(0.56+3100\varepsilon)$ m/s. The amplitude of range variation at the ground station is 11.8 km/day about a mean value which changes less than 2.8 km/day. Although the GOES satellite eccentricity is not known, typical values for geostationary satellites are between 4.7×10^{-5} and 3.6×10^{-4}. With an eccentricity of 3.6×10^{-4}, it gives a maximum residual velocity of 1.68 m/s which corresponds to a maximum Doppler frequency shift of 2.27 Hz for a carrier at 406 MHz. This is obviously too small to be used in a positioning system as it would require very good short term oscillator stability on-board the spacecraft and/or complex circuits on the ground station to measure accurately a fraction of this Doppler frequency shift.

In a similar way, the variation of time delay with time could be monitored to derive the source location. Although the knowledge of the time delay variation with time does not give us an unambiguous source position, it delineates an area to search. Based on the example above, the range variation is bounded to 11.8 km/day or 492 m/hour. This corresponds to a time delay variation of 1.6 μs/hour. Given that this is a maximum rate of change which may be difficult to measure over such a long period of time and given that the goal of this system is to provide an "instantaneous" response, this information of the time delay variation is of little use.

4.2 TWO-SATELLITE SYSTEMS

The use of two satellites provides spatial diversity that can be exploited in a number of ways. First, the two satellite system can be seen as a phased array antenna with two elements and interferometer direction finding (DF) techniques can in principle be used. An interferometer relies on the phase or time difference of arrival between two or more antennas. It provides an unambiguous position location provided that the spacing between the two antennas is less than half a wavelength. When the distance exceeds half a wavelength, some techniques usually involving additional antennas must be used to resolve the ambiguity. Because the wavelength at 406 MHz is 0.73 m and the two satellites will likely be many degrees apart to have a maximum coverage area, this technique is not applicable.

An alternate approach would be to measure the time of arrival of the signals from each satellite. This technique has become very popular over the last few years. Several positioning systems based on this principle are currently available or under study [1-11, 19-22]. Although the type of signals and required equipment vary from one system to the other, they are all used to determine the position of the source based on the absolute time of arrival of the signal from two or more satellites or on the time difference of arrival between these signals.

In a two-way system where the source transmits only after being polled by the master station, the time delay is easily obtained from the time the signal left the master station and the time the reply is received. Signals received from two or more satellites are then used to estimate the source location. In a one-way system where the system can transmit at any time, there is no common time base and the position must be determined from the time difference between three or more satellites i.e. given the propagation delay differences between three or more satellites, it is possible to determine where the source is located. For the location of the 406 MHz distress signals, only the one way technique is applicable and with only two satellites, the propagation delay difference from two satellites is insufficient to provide an unambiguous position location even if the satellite positions are known exactly. A minimum of three satellites is required for the one-way technique.

4.3 MORE THAN TWO-SATELLITE SYSTEMS

Basically, the techniques applicable to the two-satellite systems can also be extended to three or more satellites systems. As mentioned before if three satellites are used, then the beacon location can be determined using the time difference of arrival of the signals between each satellite and an estimate of the beacon altitude. The use of a fourth satellite makes the estimate of the altitude unnecessary. For the interferometer technique, the use of more satellites reduces the number of ambiguities but it is unlikely that it will be able to provide a good position location with practical satellite spacing.

5.0 TIME DIFFERENCE OF ARRIVAL BASIC PRINCIPLE

The differential time of arrival technique was described briefly in the previous section. It is worthwhile to review its theory of operation in more detail. Figure 5.0-1 illustrates the scenario. When the beacon is activated, a 440 or 520 ms burst message is transmitted every 50 seconds nominally. The signal is received by two or more satellites, up-converted in frequency to the vicinity of 1.7 GHz and then relayed to a master station. The master station logs at which time the signal from each satellite was received and performs some processing (to be discussed below) to derive the beacon location. Because the beacon can be activated at any time, it is not possible to estimate the full time delay between the transmission from the beacon and the reception at the master station i.e. $T_1 + T_1'$ or $T_2 + T_2'$. This lead us in the previous section to consider the differential techniques. Referring to Figure 5.0-1 and assuming that the master station range to each satellite is known, then it is possible to get an estimate of the time difference of arrival given by : $\Delta T=T_2-T_1$. This time delay corresponds to a differential range $\Delta R=R_2-R_1$. Given this differential range and the satellite locations, it is possible to determine a line of position on the earth where the beacon transmitter could be located. To illustrate this, let us consider a perfectly spherical earth with satellites located above the equator in geostationary orbits at longitudes λ_{0i} where i=1 or 2. Considering the geometry shown in Figure 5.0-2, where the ground station is assumed to be located at latitude φ and longitude λ, the range to the satellite #i is given by :

$$R_i = R_e \frac{\sin \beta_i}{\sin \gamma_i} \quad \text{meters}, \quad i=1,2 \qquad (5.0\text{-}1a)$$

where R_e = equivalent radius of the earth = 6,370,997 meters

$$\beta_i = \cos^{-1}\left[\cos\varphi \cos(\lambda - \lambda_{0i})\right] \qquad (5.0\text{-}1b)$$

$$\gamma_i = \tan^{-1}\left(\frac{\sin \beta_i}{R_s/R_e - \cos\beta_i}\right) \qquad (5.0\text{-}1c)$$

R_s = satellite orbit radius = 42,157,197 meters

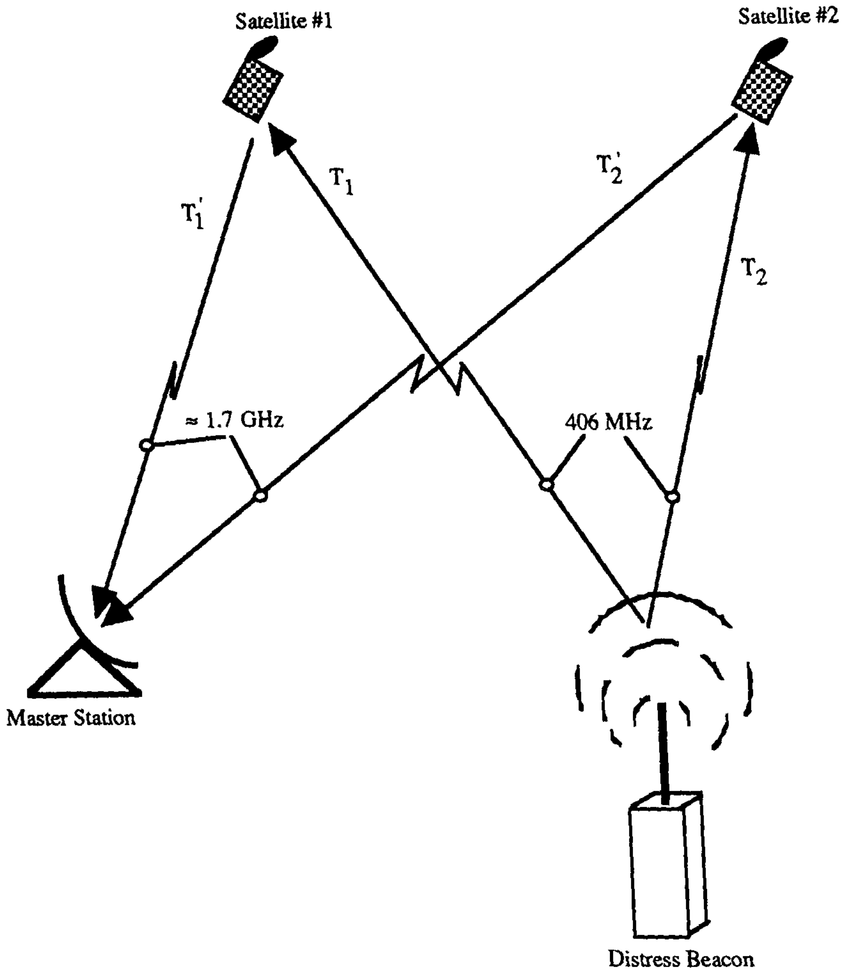


Figure 5.0-1 Overall Block Diagram of Distress Beacon Position Location System

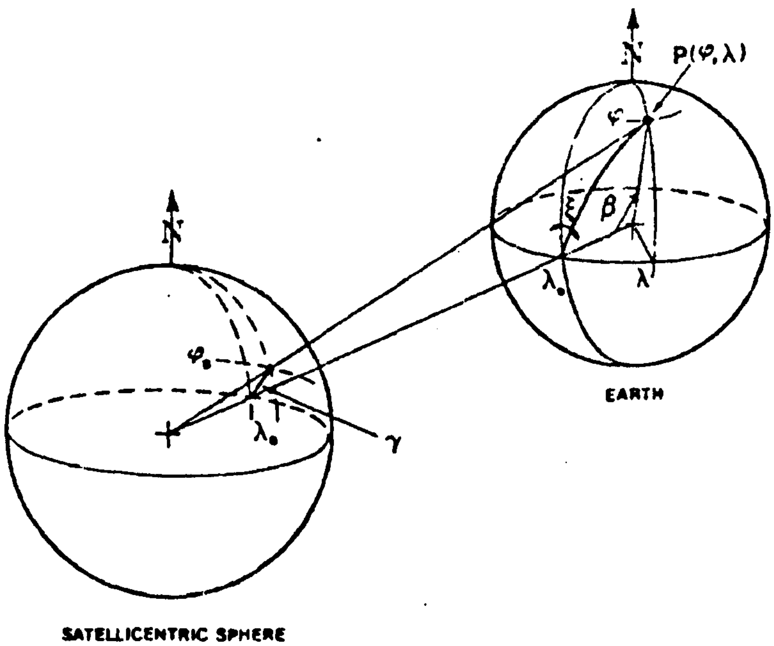


Figure 5.0-2 Basic Geometry for the Computation of Satellite Ranges on a Spherical Earth (from [23])

Note that latitude is positive north and longitude is positive east in all of the above equations and in the remainder of this report. If we define $\Delta\lambda = \lambda - \lambda_{0i}$ and re-arrange, we obtain :

$$R_i = \sqrt{R_s^2 - 2R_sR_e \cos\varphi \cos(\Delta\lambda_i) + R_e^2} \qquad (5.0\text{-}2)$$

and

$$\Delta R_{ij} = R_j - R_i$$

A computer program based on existing software for satellite spherical geometry calculation has been developed to implement these equations. By trial and error, it finds the station longitude that generates a given differential range $\Delta R \pm \delta$ for given satellite locations (λ_{01} and λ_{02}). Figure 5.0-3 illustrates graphically the results for $\lambda_{01} = 135^oW$, $\lambda_{02} = 75^oW$, ΔR=200 km and $\delta = 1$ meter.

To resolve the positioning ambiguity obtained from two satellites, a third satellite must be used to generate at least another positioning line. The intersection of two such lines gives the beacon transmitter location. Figure 5.0-4 illustrates this process for a transmitter located at Ottawa (45.35^{o}N, 75.9^{o}W) and three satellites located respectively at 75^{o}W, 105^{o}W and 135^{o}W. The differential ranges were :

$$\Delta R_{12} = 622{,}078.35 \text{ m}$$
$$\Delta R_{23} = 1{,}724{,}172.32 \text{ m}$$
$$\Delta R_{13} = 2{,}346{,}250.67 \text{ m}$$

Accordingly the accuracy to which we can locate the terminal is limited by the computation accuracy for this ideal case. In the following sections, we introduce some perturbations to this model which will add some errors to the estimated differential ranges. It is interesting to note that in absence of errors we should always have :

$$\Delta R_{12} + \Delta R_{23} - \Delta R_{13} = 0$$

This simple relationship could be used in an operational system as a measure of the quality of the estimate.

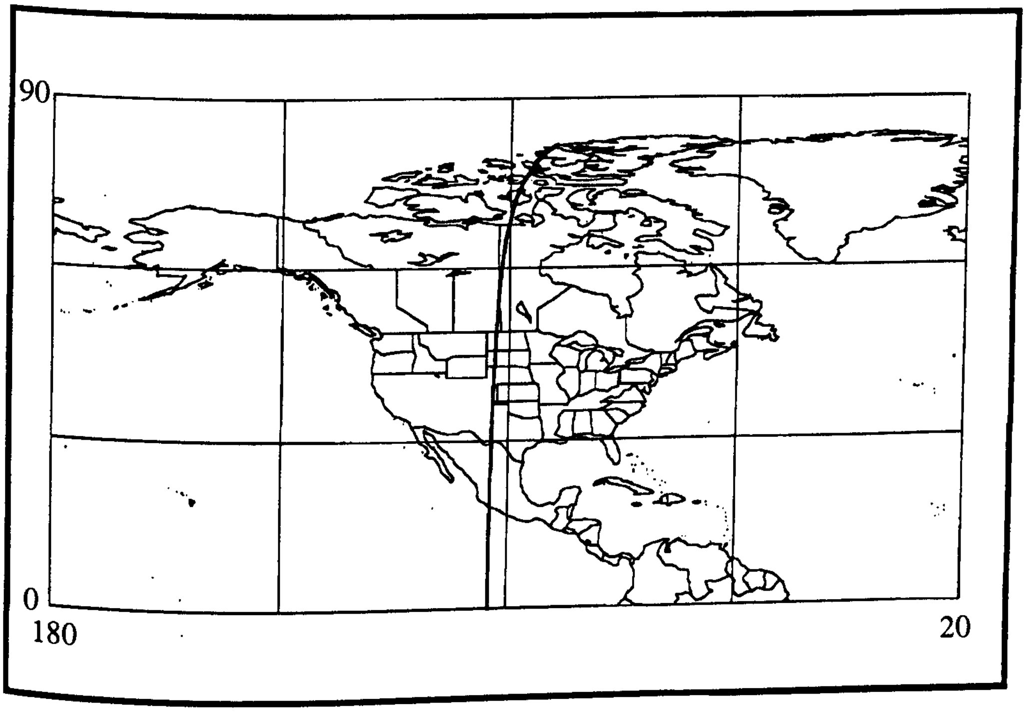


Figure 5.0-3 Position Line Generated with Satellites at 75°W and 135°W and a Differential Range of 200 km.

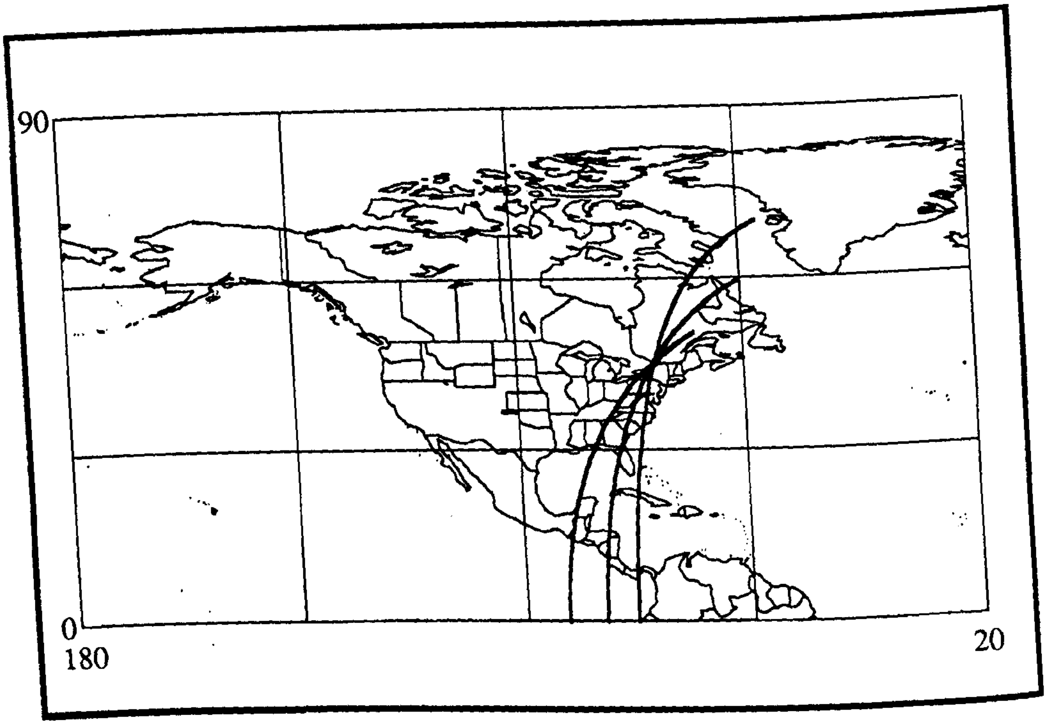


Figure 5.0-4 Position Lines For Three Satellites (75°W, 105°W and 135°W). The Station is at the Intersection of the Lines i.e. in Ottawa.

6.0 TDOA PERTURBATIONS

6.1 NON-SPHERICAL EARTH

In Section 5.0 we assumed a spherical earth. In practice, the earth is better represented as an oblate spheroid[1] with equatorial radius **a**, eccentricity **e** and semi-major axis **b** (see Figure 6.1-1). The semi-major axis **a** and the semi-minor axis **b** are related by :

$$\mathbf{b} = \mathbf{a}\sqrt{\left(1 - \mathbf{e}^2\right)} \qquad (6.1\text{-}1)$$

When using such an earth model, it is important to make the distinction between **geocentric** and **geodetic** latitudes. Figure 6.1-1 shows the point P with geocentric latitude θ and geodetic latitude ϕ. The local height above the surface of the earth is given by the length of the normal to the oblate spheroid. The geodetic (φ) and geocentric (θ) latitudes are related by the following :

$$\tan\theta = (1 - e^2)\tan\varphi \qquad (6.1\text{-}2)$$

To find the point on the earth which generated a given differential range from two given satellites, an approach different than the one in the previous section must be used. Indeed, it is easier to work in what is called the earth centered earth fixed (ECEF) (x,y,z) coordinate system than with spherical coordinates. In the ECEF coordinate system, the origin is at the earth center of mass, the x-axis goes through the Greenwich meridian at the equator, the z-axis is the polar axis and the y-axis completes the right-hand coordinate system.

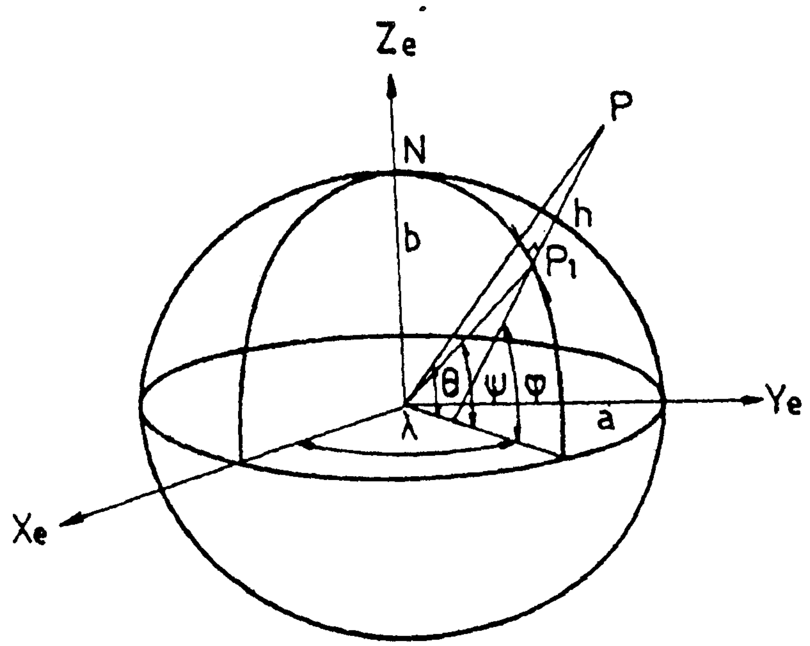
Using the notation of Figure 6.1-1 and assuming that the beacon transmitter is located on the earth (i.e. it is not on a flying aircraft), then the point P corresponds to geodetic latitude φ, and longitude λ. The ECEF coordinates of the point P are given by :

$$X = (r+h)\cos(\varphi)\cos(\lambda) \qquad (6.1\text{-}3a)$$

$$Y = (r+h)\cos(\varphi)\sin(\lambda) \qquad (6.1\text{-}3b)$$

$$Z = [(1-e^2)r+h]\sin(\varphi) \qquad (6.1\text{-}3c)$$

[1] An ellipsoid is an exact Earth model but the use of an oblate spheroid model simplifies significantly the geometry and introduces insignificant errors.



φ : geodetic latitude of point P

λ : geodetic longitude of both P and P_1

h : altitude normal to reference oblate spheroid

a : oblate spheroid equatorial radius = 6,378,137 m based on World Geodetic System 1984 (WGS-84) [43]

e : eccentricity of reference oblate spheroid
e = 0.0818191908426214957 (WGS-84)

b : oblate spheroid polar radius = **a** $\sqrt{1 - e^2}$

θ : geocentric latitude of point P

ψ : geocentric latitude of point P_1

Figure 6.1-1 Oblate Spheroid Earth Model (figure from [24]) where

$$r = \frac{a}{\sqrt{\left(1 - e^2 \sin^2\phi\right)}} \qquad (6.1\text{-}3d)$$

h = 0 for the beacon transmitter

Given the sub-satellite geocentric latitude and longitude (θ_0, λ_0), the satellite ECEF coordinates (X_0, Y_0, Z_0) can be found using equations (6.1-2) and (6.1-3), where the altitude above the earth (h) can be approximated to the difference between the satellite orbit radius and the earth semi-major axis a. Because the sub-satellite latitude is small, this approximation has little impact on the overall result. The range from point P to the satellite is then given by :

$$R = \sqrt{(X_0 - X)^2 + (Y_0 - Y)^2 + (Z_0 - Z)^2} \qquad (6.1\text{-}4)$$

So by trial and error, the point on the earth in terms of latitude and longitude which generates a given differential range for two given satellites can be found using equations (6.1-3) and (6.1-4). A similar approach was taken in [7] to find the user's location based on differential ranging from three satellites and using spherical geometry.

A computer program to perform this computation is given in Appendix A with an example of its output. The accuracy of the solution given by this program is limited to approximately 10^{-7} degrees (3.6×10^{-4} seconds) or by the accuracy imposed by the user in terms of allowed differential range error.

6.2 GEOSTATIONARY SATELLITE MOTION

6.2.1 Impact on Position Accuracy

In practice the geostationary satellites are never perfectly stationary with respect to the rotating earth as discussed in Section 4.1. Even if a satellite could be placed on a synchronous orbit with zero eccentricity, zero inclination and zero longitude drift rate, the satellite motion would soon depart from this geostationary condition because of the orbit perturbations [18]. For instance, the gravitational pull of the sun and moon can change the orbit inclination by 0.005^{o}/day, and the earth's oblateness can give a longitude acceleration of $0.0016^{o}/day^{2}$. In general, the satellites are kept to their nominal position within a given north-south and east-west station keeping errors. As mentioned in Section 3.0, the GOES satellites are maintained at their nominal position to within $\pm 0.1^{o}$. This specification does not however take into account the range variation of the satellite. In practice, the satellite motion prediction and analysis is a very complex task. Slabinsky in [18] analyzed a simplified scenario where some perturbations of the sun and moon were neglected. He basically came out with two useful graphs to estimate the variation of the satellite range as a function of orbit eccentricity and inclination. Figures 6.2.1-1 and 6.2.1-2 report these graphs. They give the range variation for a satellite to ground station longitude difference (L_m-L_e) and ground station latitude. The worst case range variation is given by :

$$\delta R_{max} = |A_i| + A_e$$

where A_i is the range variation due to the orbit inclination and A_e is the one due to the orbit eccentricity. Assuming a worst case inclination of 0.1^{o} and an eccentricity of 3.6×10^{-4} for the GOES satellite as in Section 4.1, the maximum range variation can be found to be given by :

$$(A_i)_{max} = 111.3 * 0.1 = 11.13 \text{ km}$$

$$(A_e)_{max} = 43.58 * 0.36 = 15.69 \text{ km}$$

Having determined the maximum range error when considering the satellites to be perfectly stationary, let us look at the impact on the beacon transmitter location. There are two ways this can be examined. First, a change in the satellite position from the nominal one will result in another position line. So, it is possible to visualize graphically the impact of the satellite motion. However, this will be of little use because the scale used for the map is so large that two position

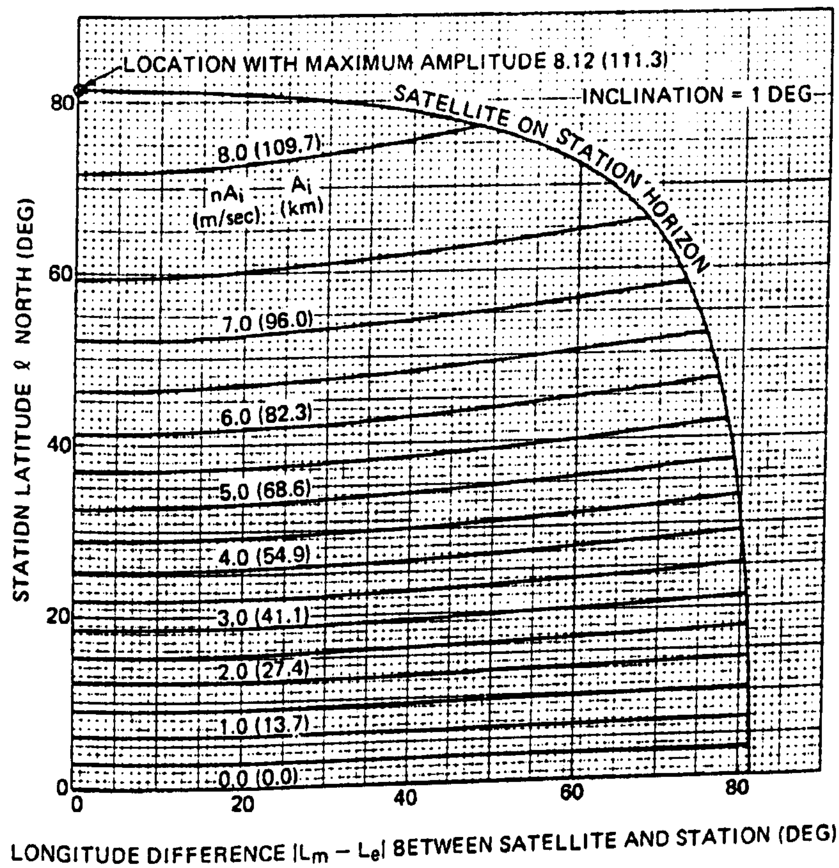


Figure 6.2.1-1 Amplitudes for inclination part of variations, nA_i for range-rate and A_i for range. (Notes : For south latitudes, take negative of quantity for corresponding north latitude. For other inclinations **i**, multiply quantity from graph by **i** degrees.) (from [18])

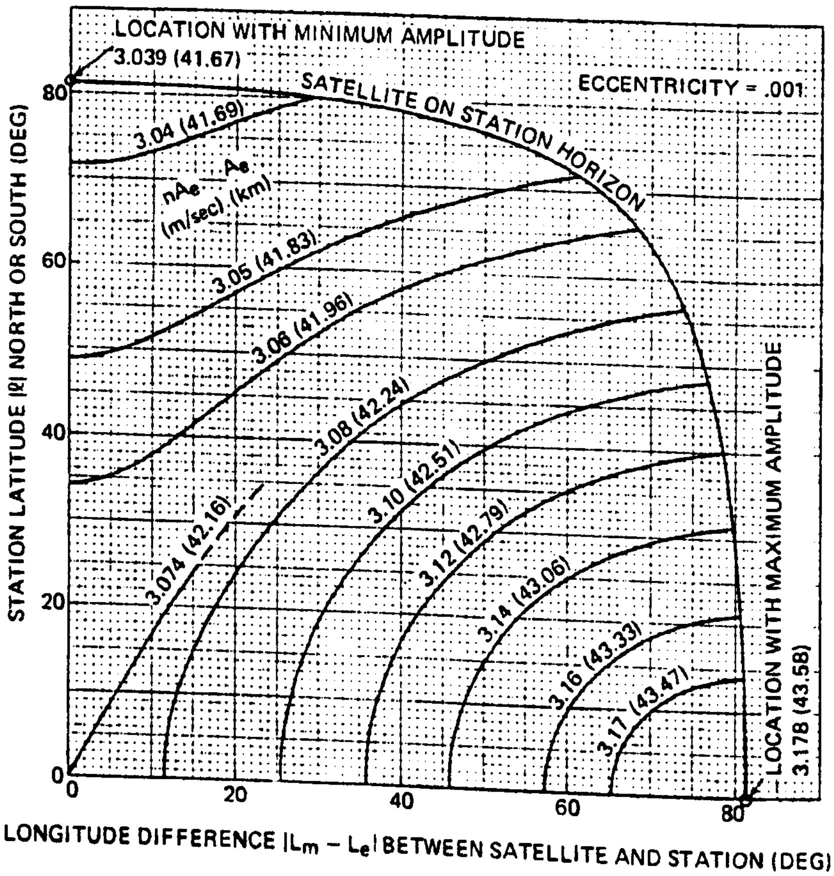


Figure 6.2.1-2 Amplitude for eccentricity part of variations, nA_e for range-rate and A_e for range.
(Notes : For other eccentricities e, multiply quantity from graph by 1000e).
(from [18])

lines a few kilometers away will be very close to each other. A better approach is to compute the distance between the two lines.

The distance between any two points on the earth has been modeled by several investigators. Ludvik [25] gives two Fortran computer programs to compute the geodetic distance between two points for two techniques i.e. Bowring's inverse algorithm and Vincenty's direct algorithm. Both techniques are accurate, but Vincenty's is slightly better. These programs have been adapted to a Macintosh personal computer (see Appendix B) and have been tested using the known true distance between a set of points as described in [26].

With satellites located at 135°W and 75°W, two position lines were generated which corresponded to 200 km and 215 km range difference. The shortest distance between two points on the same parallel and lying on these position lines was computed. Table 6.2.1-1 shows the results for a few points. These results clearly show that the range difference directly impacts on the accuracy of the position location and that the positioning accuracy is on the same order as the range difference for a satellite spacing of 60° and a range difference of 200 km (a detailed analysis of the position dilution of precision (PDOP) factor is presented in Section 8.0). It is clear from the above results that the satellite positions must be monitored at all time.

Latitude (degrees)	Longitude (degrees)		Distance (km)
	0 km range error	15 km range error	
22.96	103.2756348	103.1462402	13.27
45.92	102.6359253	102.4584656	13.77
72.16	99.3142700	98.8860779	14.65

Table 6.2.1-1 Distance Between Position Fixes Generated Using Satellites at 135°W and 75°W with Range Errors of 0 km and 15 km over a Nominal Range Difference of 200 km.

6.2.2 Satellite Position Tracking

In the previous section, it was shown that the geostationary satellite motion could induce a significant amount of errors on the position location of the beacon and it was concluded that the satellite positions must be monitored. There are several ways to track the geostationary satellite motion. The satellite provider usually keeps track on a monthly basis (if not more frequently) of the satellite position and fires the thrusters to maintain the satellite on its orbit every now and then (corrections are typically required every 10-15 days for a station keeping of 0.1° in both axis). Although constant communications could be maintained with the satellite provider, it is worthwhile to look at other techniques which could be used.

First of all, we have available the equipment to perform differential ranging and it is intuitively attractive to look at techniques to use this resource to locate the satellites. If reference beacons with exactly known locations are used, then the measured differential range could be used to determine the satellite positions. There are various techniques that can be used to do so.

A straightforward technique is to revert to the process discussed in the previous sections i.e. use known reference beacon locations to find the position of the satellites. Six reference stations are required because we use two satellites at a time and six equations are required to solve for the six unknown variables i.e. the (x,y,z) coordinates of the two satellites.

If we denote the satellite coordinates by (X_i, Y_i, Z_i), i=1,2 and the six reference station coordinates by (x_i, y_i, z_i) i=1,..,6, then we have :

$$\Delta R_i = \sqrt{(X_2 - x_i)^2 + (Y_2 - y_i)^2 + (Z_2 - z_i)^2} - \sqrt{(X_1 - x_i)^2 + (Y_1 - y_i)^2 + (Z_1 - z_i)^2} \qquad (6.2.2\text{-}1)$$

for i=1,...,6 where ΔR_i is the measured differential range. This set of nonlinear equations can be solved by using a Taylor series expansion and by considering only the first order terms. If we define :

$$F_i = \Delta R_i - \sqrt{(X_2 - x_i)^2 + (Y_2 - y_i)^2 + (Z_2 - z_i)^2} + \sqrt{(X_1 - x_i)^2 + (Y_1 - y_i)^2 + (Z_1 - z_i)^2} \qquad (6.2.2\text{-}2)$$

then in matrix notation, we have to solve the following set of equations :

$$
\begin{bmatrix}
\frac{\partial F_1^0}{\partial X_1} & \frac{\partial F_1^0}{\partial Y_1} & \frac{\partial F_1^0}{\partial Z_1} & \frac{\partial F_1^0}{\partial X_2} & \frac{\partial F_1^0}{\partial Y_2} & \frac{\partial F_1^0}{\partial Z_2} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{\partial F_6^0}{\partial X_1} & \frac{\partial F_6^0}{\partial Y_1} & \frac{\partial F_6^0}{\partial Z_1} & \frac{\partial F_6^0}{\partial X_2} & \frac{\partial F_6^0}{\partial Y_2} & \frac{\partial F_6^0}{\partial Z_2}
\end{bmatrix}
\begin{bmatrix} dX_1 \\ dY_1 \\ dZ_1 \\ dX_2 \\ dY_2 \\ dZ_2 \end{bmatrix}
=
\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix}
\qquad (6.2.2\text{-}3)
$$

where $\partial F_i^0/\partial U_j$ is $\partial F_i/\partial U_j$ evaluated at (X_k^0, Y_k^0, Z_k^0), k=1,2 which is an initial good estimate of the satellite position (e.g. their nominal positions). Solving this equation gives (dX_k, dY_k, dZ_k) k=1,2 which can be used to form the next best estimate i.e. :

$$X_k^1 = X_k^0 + dX_k$$

$$Y_k^1 = Y_k^0 + dY_k$$

$$Z_k^1 = Z_k^0 + dZ_k \qquad k=1,2$$

The process can then be repeated with each new estimate. Because the solution is convergent, the iterations can be stopped when the dX's, dY's and dZ's are less than ε. Based on the computer results of several examples, it has been noticed that only a few repeats are required (e.g. typically 3-5).

When random differential range measurement errors are introduced, the set of equations can still provide a solution but the solution only represents the satellite positions with the erroneous differential ranges. The amount of errors in the satellite position is a function of the differential range error and the geometry made by the satellites and the reference stations. The latter is referred to as the position dilution of precision (PDOP) factor which will be developed later on for the beacons. Briefly, it is the factor that must multiply the range error to obtain the user position error. This factor is completely determined by the geometry of the system.

The analytical equations for determining the PDOP as a function of the satellite and reference station locations have not been derived. However, it is easy to understand conceptually that the PDOP factor will be very large due to the fact that the satellites view the reference stations lying in a very small cone with very little spatial discrimination. Accordingly, even if the reference stations are selected to give the best PDOP, computer simulations of a few cases have showed that the PDOP is between 10-80. So a differential range measurement of 100 meters (about the minimum in the differential mode) would result in a satellite position error of approximately 1-8 km ! Clearly this is not satisfactory and other techniques must be considered.

One possible alternate technique is to use an absolute range measurement instead of a differential one. This requires the reference stations and a master station to be time synchronized which is difficult to achieve but this problem was left aside temporarily. Unfortunately, again it has been found that the large PDOP was the main detrimental element which kept the accuracy above 1 km for a 100 meters range error. Other techniques were also investigated but none provided any better solution than the 1000 meters error on the satellite positions. Such an error is intolerable and we must rely on the satellite provider to make available the satellite position information with enough accuracy.

The satellite provider is equipped with sophisticated tools such as frequent bearing and range measurements, orbit models, Kalman filters, etc. to keep track of the satellite position. For our application, it would be ideal to receive continuously from the satellite (e.g on a beacon signal) its current estimated position. Because the satellite provider has all the tools to compute and predict the satellite position with very high accuracy, it will require a minimum effort on their part to make this information available to all users. As a minimum, the availability of the ephemeris data can be useful for our application but it would still require a significant amount of processing to come out with good predicted satellite positions.

The accuracy to which one can estimate the satellite position depends strongly on how often measurements are made and how sophisticated is the orbit model. In [7] and [21] the satellite position is assumed to be predicted to within 20 m in all axis. Although it is not explicitly explained how this number is derived, we will assume in the following that this accuracy can be achieved.

6.3 CHANNEL DELAYS

The terminal measures the time delay between the signals received from different satellites and to find the range difference, this time delay is divided by the wave velocity in the channel. The wave velocity in the atmosphere is often approximated to 2.9979×10^8 m/s but in practice it is not constant with time. There are two layers of the atmosphere which contribute to this velocity variation. They are the ionosphere and the troposphere.

6.3.1 The Ionosphere Delay

The range error assuming a wave velocity c for the ionosphere is given by [41] :

$$\Delta R = \frac{40.3}{f^2} \, TEC \qquad (6.3.1\text{-}1)$$

where TEC is the total electron content along the path in electrons/m^2, and f is the frequency of interest in Hertz.

Alternately equation (6.3.1-2) can be rewritten as :

$$\Delta R = \frac{40.3}{f^2} f(\theta) \, TEC_v \qquad (6.3.1\text{-}2)$$

where TEC_v is the vertical total electron content and :

$$f(\theta) = \sec\left[\sin^{-1}\left\{0.94792 \cos(\theta)\right\}\right] \qquad (6.3.1\text{-}3)$$

and θ is the elevation angle. The factor $f(\theta)$ varies between 1 at zenith and 3.1 at 0^o elevation angle.

The true range is always less than the one measured assuming a velocity $c = 2.9979 \times 10^8$ m/s and accordingly ΔR is always positive. The TEC is a function of many variables including short and long term changes in solar ionizing flux, magnetic activity, season, time of day, user location and viewing angle. The TEC varies typically between 10^{16} and 10^{19} el/m^2. Figure 6.3.1-1 shows the time delay introduced by the ionosphere for various values of TEC. At 406 MHz, the range error is between 2.44 m and 2.44 km. However it is mentioned

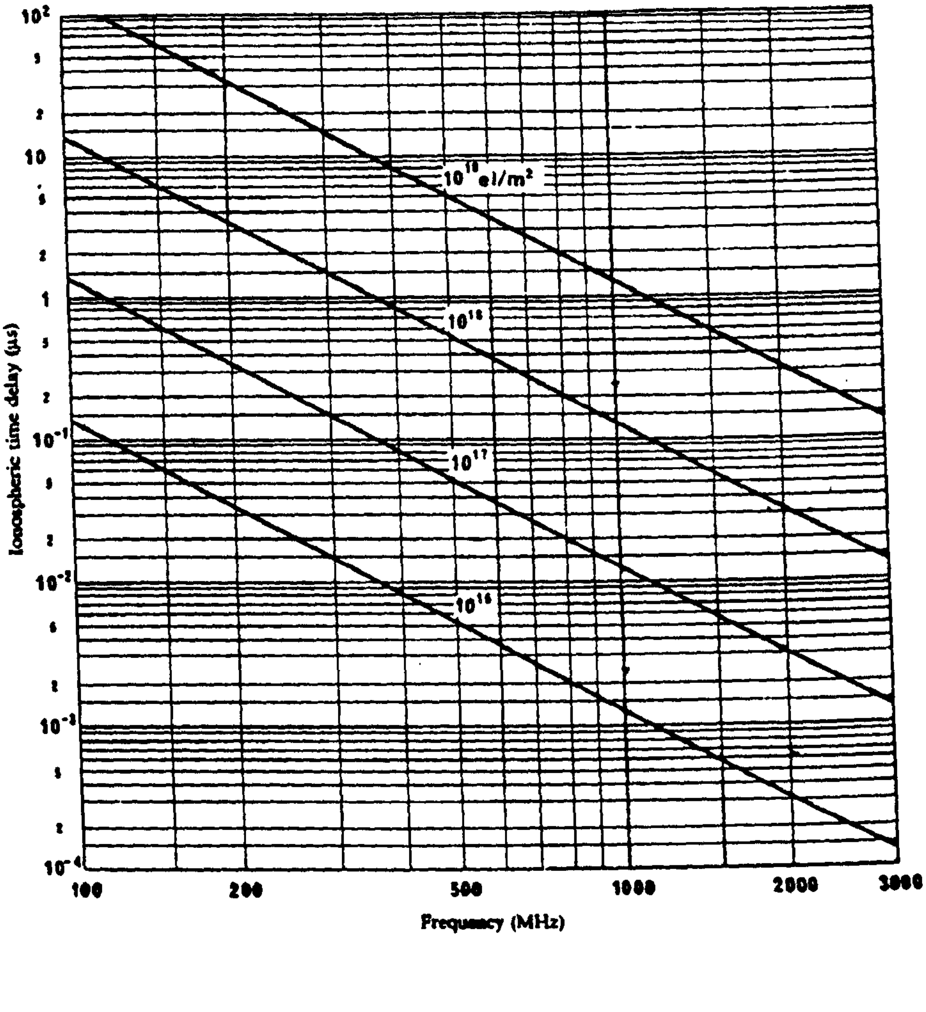


Figure 6.3.1-1 Ionospheric Time Delay Versus Frequency for Various Values of Electron Content (from [27])

in [28] that a typical TEC value is $2x10^{17}$ el/m^2 corresponding to a delay of 49 m. In addition, the TEC exceeds $5x10^{17}$ el/m^2 approximately 10 % of the time and it rarely exceeds 10^{18} el/m^2 which clearly limits the delay to 244 m for all practical cases in the mid to upper latitude regions.

The TEC spatial variation is relatively smooth. Thus if the two satellites are close to each other, the beacon signal for each satellite can be assumed to go through the same ionospheric region with relatively constant TEC. Because we are interested in the time difference of arrival, the ionospheric delay would then cancel out. Table 6.3.1-1 from [29] shows the amount of ionospheric delay that does not cancel out as a function of the station separation when the stations are looking at the same GPS satellite. Our application is slightly different but these results can be used to estimate the amount of delay cancellation using the differential technique.

Using the relationships given in equations (6.3.1-2) and (6.3.1-3), we see that the residual delay after differential timing is given by :

$$r = \frac{TEC_{v2}f(\theta_2)}{TEC_{v1}f(\theta_1)} = \frac{g(d)TEC_{v1}f(\theta_2)}{TEC_{v1}f(\theta_1)} = g(d)\frac{f(\theta_2)}{f(\theta_1)}$$

where g(d) is a factor depending on the distance (d) between the two stations. Whether g(d) is a linear function of distance or not, we can postulate that if the distance between the two signals passing through the ionosphere remains constant, then g(d) is constant and the residual delay relative to the GPS case is increased by the ratio of $f(\theta_2)/f(\theta_1)$. For instance, let us consider a simple case where the beacon is located on the equator exactly between the two satellites spaced by α degrees, then their elevation angles are given by :

$$\theta_i = \tan^{-1}\left[\frac{\cos(\alpha/2) - \frac{R_e}{R_s}}{\sin(\alpha/2)}\right] \qquad i=1,2$$

where R_e and R_s are defined in Section 5.0.

When the beacon is moved along the equator, it can be shown that the difference between the elevation angles will increase up to a maximum when one of the satellite is at the horizon. For a 30^o spacing between satellites, this maximum is approximately 39^o. When the beacon is moved along the 45th parallel, this maximum is reduced to approximately 20^o. So, for location techniques of interest to Canada and for a given distance between the signals passing through the ionosphere,

Station Separation		Residual Delay (%)
(nmi)	(km)	
0	0	0
1	1.852	2
10	18.52	8
50	92.60	17
100	185.2	24
500	926.0	52
1000	1852.0	71
2000	3704.0	91

Table 6.3.1-1 Differential Ionospheric Delay Reduction (from [29])

Distance Between Signals Over the Ionosphere		Residual Delay (%)
(nmi)	(km)	
0	0	0
1	1.852	4.4
10	18.52	17.2
50	92.60	37.4
100	185.2	52.8
500	926.0	100.0
1000	1852.0	100.0
2000	3704.0	100.0

Table 6.3.1-2 Minimum Differential Ionospheric Delay Reduction for the Time Difference of Arrival Location Technique Assuming a 30° Satellite Spacing and for Canada

the residual delay given in Table 6.3.1-1 should be increased by a factor of $f(20^o)=2.2$ and up to a maximum residual delay of 100 % meaning total uncorrelation. Table 6.3.1-2 shows the modified values.

The distance between the two signal paths passing through the ionosphere is a function of the elevation angle and satellite spacing. This distance is minimum for a beacon located on the equator. Referring to Figure 6.3.1-2 which represents the simple case when the beacon is located midway between the two satellites and on the equator, we find that the distance d is given by :

$$d = 2\,\beta\,(R_e + R_{iono})$$

$$\beta = 2\tan^{-1}\left[\frac{\tan\left(\frac{a+b}{2}\right)\cos\left(\frac{A+B}{2}\right)}{\cos\left(\frac{A-B}{2}\right)}\right]$$

$$b = \sin^{-1}\left(\frac{\sin a}{\sin A}\right)$$

$$a = R_{iono}/R_e$$

$$B = 90^o$$

where R_{iono} is the ionosphere mean altitude above the earth (assumed to be 350 km), R_e is the earth radius and A,B are the angles opposite to sides a and b respectively defined in Figure 6.3.1-2. Note that the above trigonometric functions must be performed in radians. With a satellite spacing of 30^o we get $A=72.375^o$, $a=0.05494$, $b=0.057617$, $d=217.24$ km. Table 6.3.1-2 shows that between 52 and 100 % of the delay is not cancelled for a distance of 217 km. This represents the best case for Canada where the distance increases with the latitudes, and it is clear that a model to estimate the ionospheric delay is required.

It is possible to estimate the TEC or to eliminate it in equation (6.3.1-1) using a second signal frequency. Indeed, when we apply equation (6.3.1-1) to two signals with different frequencies, we have :

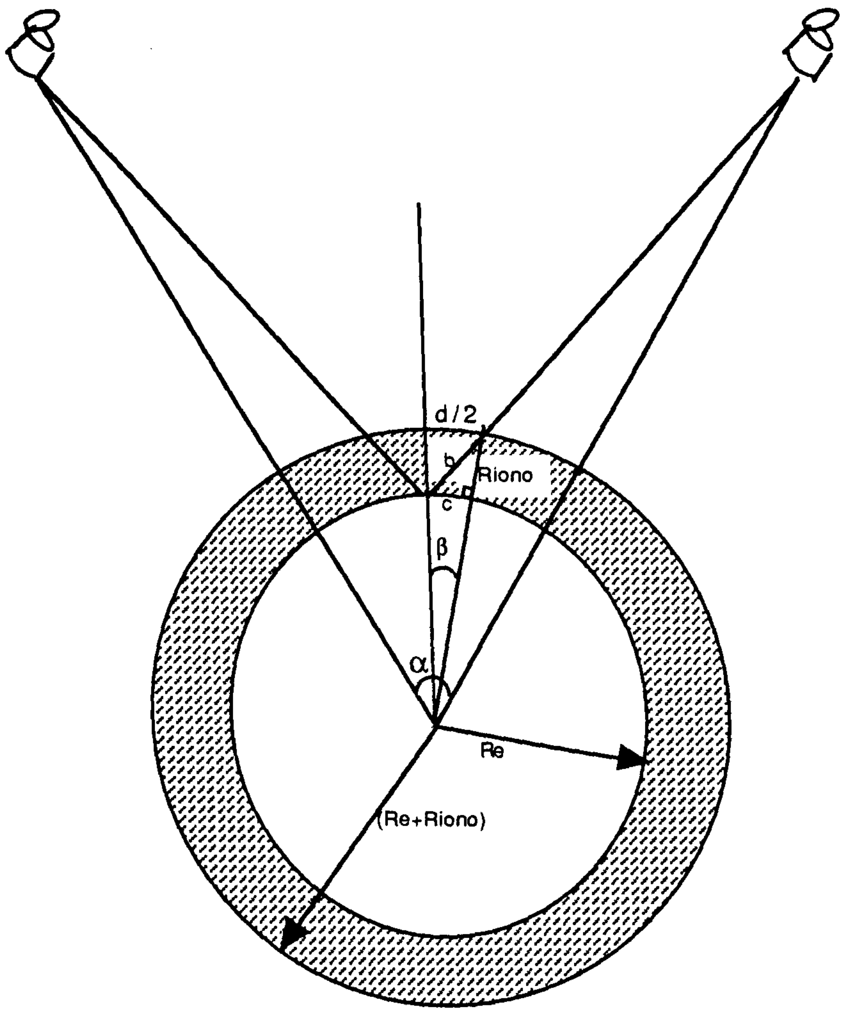


Figure 6.3.1-2 Model for the Discussion of the Ionospheric Delay Spatial Correlation

$$\Delta t_2 = \frac{40.3}{cf_2^2} \text{ TEC}$$

$$\Delta t_1 = \frac{40.3}{cf_1^2} \text{ TEC} \qquad (6.3.1\text{-}4)$$

$$\delta t = \Delta t_2 - \Delta t_1$$

where Δt_1 is the time delay at frequency f_1 and δt is the time delay between the two signal frequencies transmitted by a single receiver at the same time and measured at a receiver. Re-arranging the above equations we find that :

$$\Delta t_1 = \frac{f_2^2}{f_1^2 - f_2^2} \delta t \qquad (6.3.1\text{-}5)$$

Knowing the time delay introduced by the ionosphere (Δt_1), we can find the range error ΔR introduced by the ionosphere and make the appropriate correction. A second frequency is however not available in the search and rescue system.

When only a single frequency signal is available, one is required to predict, as much as possible, the range error based on ionospheric conditions. A great deal of effort has been deployed to find such models for the Global Positioning System (GPS) operating at 1.5 GHz. Klobuchar [30] mentioned the use of a relatively simple model where approximately 50 % of the error introduced by the ionosphere can be corrected. With a state of the art model, up to 70 to 80 % can be compensated. Given that the time delay varies approximately with $1/f^2$ for f>100 MHz, it is possible to use the GPS single frequency correction technique discussed in [30]. This technique however requires the use of some coefficients transmitted by the GPS satellites. These coefficients which required updating about every ten days are used to model the amplitude of the TEC variation. It would therefore be possible to read these coefficients from the GPS signal and apply the correction factor determined by the technique discussed in [30].

From the above discussion, it is clear that the best approach is to rely on the GPS technique and to scale the results by a factor of $(1575.42/406)^2$=15.057. Although this approach means that a GPS receiver must be available, it is considered significantly simpler than trying to re-derive the parameters in the broadcast channel. The GPS system can provide an ionospheric rms range error of 4.5-10 m [31] which translates at 406 MHz to a maximum rms range error of σ_{iono}=150 m. Note again that in the absence of an ionospheric model, we could expect an error of 244 m for most of the time. These errors will be used in Section 9.0 to derive the overall positioning accuracy.

6.3.2 The Troposphere Delay

The troposphere range error is caused by two effects : angular bending of the waves which increases the path length with reference to free space and a decrease in propagation velocity. Both effects result from a change in the refraction index as a function of altitude in the troposphere and are essentially independent of frequency up to 30 GHz. The range correction factor is fully described in [32] and after some simplifications we obtained :

$$\Delta R = f(\theta) * \Delta R(h) \qquad (6.3.2\text{-}1)$$

where $\Delta R(h)$ is given by :

for $0 < h \leq 1$ km

$$\Delta R(h) = \left(2464.4042 - 324.8h - 22.39578h^2\right) \quad \text{mm} \qquad (6.3.2\text{-}1a)$$

for $1 < h \leq 9$ km

$$\Delta R(h) = \left[2283.7805 \exp\left(\frac{1-h}{8.1561}\right) - 124.3926\right] \text{ mm} \qquad (6.3.2\text{-}1b)$$

for 9 km $< h \leq h_{sat}$

$$\Delta R(h) = \left[2656.26 \exp(-0.1424h)\right] \text{ mm} \qquad (6.3.2\text{-}1c)$$

and $f(\theta)$ is given by :

$$f(\theta) = \frac{1}{\sin\theta + \dfrac{0.00143}{\tan\theta + 0.0455}} \approx \frac{1}{\sin\theta} \qquad (6.3.2\text{-}2)$$

where h is the altitude of the transmitter (in kilometers) above the sea level, θ is the elevation angle to the satellite and h_{sat} is the satellite altitude. Because the altitude above the sea level does not exceed approximately 4 km in Canada, only equations (6.3.2-1a) and (6.3.2-1b) are of interest here. In general, h is not known a priori, however, if required it can be estimated using an electronic data base. The position is then estimated using h=0, the altitude for that estimated position is found from the data base and calculations are then repeated with a new h. In the following, we assume h=0 and neglect the error introduced which is about 1 m for the worst case in Canada.

For the tropospheric range error, a residual error of less than 4 m can be expected for 95 % of the time [31] with the above model and exact h. A bias of 1 m can be assumed for the tropospheric model when h is assumed zero for Canada. Looking at the above equation, it can be found that an error as large as 78 meters ($\theta=0$ and h=0) can be expected if the troposphere model is not implemented.

7.0 ESTIMATION OF THE TIME DIFFERENCE OF ARRIVAL

Up to now, we have looked at all the external perturbations of time difference of arrival estimates. It is time to look at the accuracy of the time of arrival estimation itself. The problem consists of determining the delay between a single signal relayed via two satellites to a control ground station. The content of the two received signals is identical but the phase and amplitude may be different. In addition, there may be other signals present with signal characteristics close to the ones of interest. In the following, we first determine the maximum time difference that can be expected. Then, we discuss the problem of time difference estimation relayed via two satellites.

It is assumed that the control ground station receiver for each satellite link is as illustrated in Figure 7.0-1 where the frequency down-converter is locked to the satellite beacon and therefore the frequency offset between two or more satellites can be assumed to be zero. The signal at the input of the delay estimator is assumed centred on a 5 kHz IF which corresponds to the minimum intermediate frequency as required by the long term frequency stability of the beacon transmitter.

7.1 MAXIMUM TIME DIFFERENCE OF ARRIVAL

The maximum time difference of arrival is a function of the satellite spacing. In order to get an idea of the magnitude of the maximum delay, we use the spherical geometry introduced in Section 5.0. The maximum delay between a signal received via two satellites occurs for a station when one of the satellites is seen with the minimum operating elevation angle. The elevation angle when the earth is assumed a sphere is given by :

$$\theta = \tan^{-1}\left[\frac{\cos\beta - R_e/R_s}{\sin\beta}\right] \qquad (7.1\text{-}1)$$

where all variables have been defined in Section 5.0 under equations (5.0-1a), (5.0-1b) and (5.0-1c). Using the following trigonometric identity :

$$\sin\left(\tan^{-1}x\right) = \frac{x}{\sqrt{1+x^2}} \qquad (7.1\text{-}2)$$

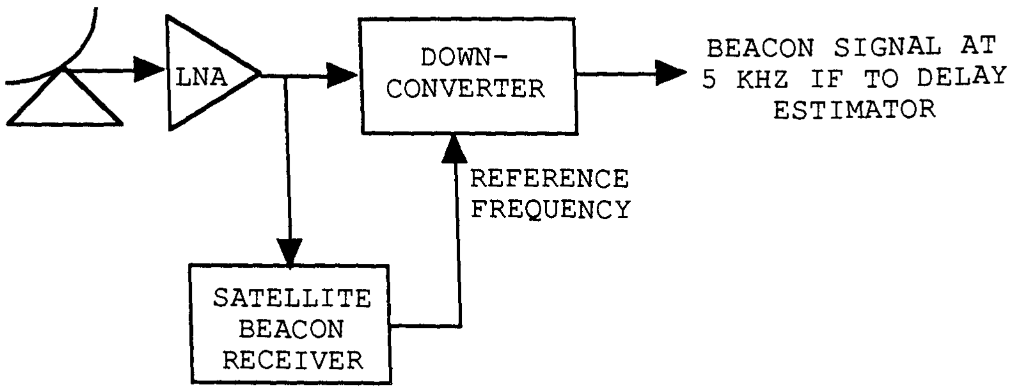


Figure 7.0-1 Basic Satellite Receiver Configuration

and applied to equation (5.0-1a) gives for the range to each satellite :

$$R_i = R_e \sqrt{\left(\frac{R_s}{R_e}\right)^2 - 2\frac{R_s}{R_e}\cos\beta_i + 1} \qquad i = 1,2 \qquad (7.1\text{-}3)$$

where $\cos(\beta_i) = \cos(\varphi)\cos(\lambda - \lambda_i)$

λ_i is the longitude of satellite #i

(φ,λ_i) are the coordinates of the beacon transmitter.

The differential range is then given by :

$$\Delta R = R_1 - R_2 \qquad (7.1\text{-}4)$$

where the satellite #1 is assumed to be seen by the beacon transmitter with an elevation angle $\theta_1 = \theta_{min}$. For a given θ_{min} we can solve equation (7.1-1) to obtain the corresponding β. For $\theta_1 = \theta_{min} = 5^o$ we get $\beta_1 = 76.338855^o$ which when applied to equations (7.1-3) and (7.1-4) gives :

$$R_1 = 6.4544\ R_e$$

$$\Delta R = R_e\left[6.4544 - \sqrt{\left(\frac{R_s}{R_e}\right)^2 - 2\frac{R_s}{R_e}\cos\beta_2 + 1}\right] \qquad (7.1\text{-}5)$$

This last equation is maximized when $\cos(\beta_2)$ is maximized. Using equation (5.0-1b) applied to both satellites, we have :

$$\cos(\beta_1) = \cos(\varphi)\cos(\lambda - \lambda_1) \qquad \text{from which} \qquad \cos(\varphi) = \frac{\cos(\beta_1)}{\cos(\lambda - \lambda_1)}$$

and

$$\cos(\beta_2) = \cos(\varphi)\cos(\lambda - \lambda_2) = \frac{\cos(\beta_1)}{\cos(\lambda - \lambda_1)}\cos(\lambda - \lambda_2)$$

$$= \frac{\cos(\beta_1)}{\cos(\lambda - \lambda_1)}\left\{\cos\lambda\cos\lambda_2 + \sin\lambda\sin\lambda_2\right\}$$

$$= \frac{\cos(\beta_1)}{\cos(\lambda - \lambda_1)}\left\{\left[\cos^2\lambda_1 + \sin^2\lambda_1\right]\left(\cos\lambda\cos\lambda_2 + \sin\lambda\sin\lambda_2\right)\right\}$$

using basic trigonometric identities. After some manipulations we get :

$$\cos \beta_2 = (\cos \beta_1)\left[\cos(\lambda_1 - \lambda_2) - \tan(\lambda-\lambda_1) \sin (\lambda_1 - \lambda_2)\right] \qquad (7.1\text{-}6)$$

The above equation is maximized when $|(\lambda - \lambda_1)|$ is maximized and :

$(\lambda_1 - \lambda_2 < 0$ and $\lambda - \lambda_1 > 0)$ or $(\lambda_1 - \lambda_2 > 0$ and $\lambda - \lambda_1 < 0)$

With $\cos(\beta_1) = \cos(\varphi) \cos(\lambda - \lambda_1)$

the maximization of $|(\lambda - \lambda_1)|$ is equivalent to the minimization of $\cos(\lambda - \lambda_1)$ for a given β_1. This is achieved when $\cos(\varphi)$ is maximum i.e. at φ=0 when the beacon is located on the equator. In this case, $(\lambda - \lambda_1)_{max}=\beta_1$ and equation (7.1-6) becomes :

$$(\cos \beta_2)_{max} = (\cos \beta_1)\left[\cos(\lambda_1 - \lambda_2) + \tan \beta_1 \sin (|\lambda_1 - \lambda_2|)\right] \qquad (7.1\text{-}7)$$

Substituting equation (7.1-7) into equation (7.1-5), we can find the maximum range difference as a function of the satellite spacing $|\lambda_1-\lambda_2|$. Table 7.1-1 shows the maximum time difference of arrival as a function of satellite spacing. Because Canada spans approximately 140^o in longitude, it is fair to assume that the spacing between two adjacent satellites will not exceed 70^o which corresponds to a maximum delay of about 18 ms.

Satellite Spacing (degrees)	Max. Delay (ms)
5	1.84
10	3.65
15	5.41
20	7.11
25	8.74
30	10.28
35	11.71
40	13.02
45	14.20
50	15.22
55	16.09
60	16.79
65	17.31
70	17.64
75	17.79

Table 7.1-1 Maximum Delay as a Function of Satellite Spacing When the Minimum Operating Elevation Angle is Set at 5°.

7.2 DELAY ESTIMATION

There are basically two tasks that must be performed in the time difference of arrival processor. First, we have to detect that a beacon signal is present and second we have to estimate the delay between the three received signals. The detection problem has been addressed in a previous project where a detector based on a spectral estimator has been designed and has been shown to perform very well in an operational scenario (see [33]). Although we could address the problem in a general sense i.e. treat the problem as a detection and estimation problem, it is in general, less difficult to design an optimum processor for either detection or estimation than to design the combined one. The loss of optimality is in general low for such an approach. So in the following it is assumed that the detection process has been performed and we have been given an indication that a signal has been detected. In addition, we will further assume that three detectors are available to process the signal received from each satellite in parallel. The latter assumption allows us to assume that we know which signal is received first, second and third so that the problem is simplified to an estimation of the delay between signals and this eliminates the uncertainty present when both delay and advance can occurred.

It is important in designing a processor to consider what is known. Although in the ideal situation we would like to have a processor which needs to know very little about the signal itself (this is required for the DND's applications where the interfering signal is not known a priori), the complexity of the processor is generally proportional to the amount of unknowns. For the case of distress beacons localization:

a) we know that the signal starts with a 160 ms un-modulated carrier followed by a known and constant 24 bit pattern;

b) we know the signal period and the statistical law that governs the repetition period;

c) we know that in an operational scenario there could be other signals interfering with the one of interest;

d) we have a "rough" estimate of the frequency of the carrier as derived from the detection processor;

e) based on the knowledge of the frequency, we could refine the estimate of the carrier frequency to 1) bring the signal to baseband and reduce the noise bandwidth thus improving the processor input SNR, and 2) to demodulate the signal to recover the modulation bits;

f) we know that the delay is less than 18 ms;

g) finally, we know which signal is in advance relative to the other one.

The first five characteristics have been exploited in the GOES processor [33] to perform distress detection. Thus if a processor equivalent to the GOES one is assumed available, we have some means to recover the information bits in addition to the characteristics of f) and g).

From the parameter estimation theory, the optimum delay estimator in the Maximum Likelihood (ML) sense for signals embedded in additive white Gaussian noise (AWGN) is given by $\hat{T}$ which satisfies the following [44] :

$$\left[\frac{2}{N_o} \int_0^{T_b} \{r_i(t) - s(t,T)\} \frac{\partial s(t,T)}{\partial T} \, dt \right]_{T=\hat{T}} = 0 \qquad (7.2\text{-}1)$$

where $r_i(t)$ is the received signal for satellite #i, s(t,T) is the noise free signal with delay T, T_b is the observation period and N_o is the noise power spectral density. Although this equation does not give too much insight on how to implement the estimator, after some manipulations and approximations, it can be shown that this is equivalent to choosing $\hat{T}$ such that the correlation between $r_i(t)$ and $s(t,\hat{T})$ is maximum.

When applied to the system of interest, the correlation must be performed between the two noisy received signals assuming that we do not know the information in the beacon signal. Figure 7.2-1 shows how such a processor can be implemented digitally. It is clear that the estimator accuracy is limited by the number N of correlators used to cover the 18 ms uncertainty over a certain range of SNR and by noise thereafter. Because the signal SNR of interest is low

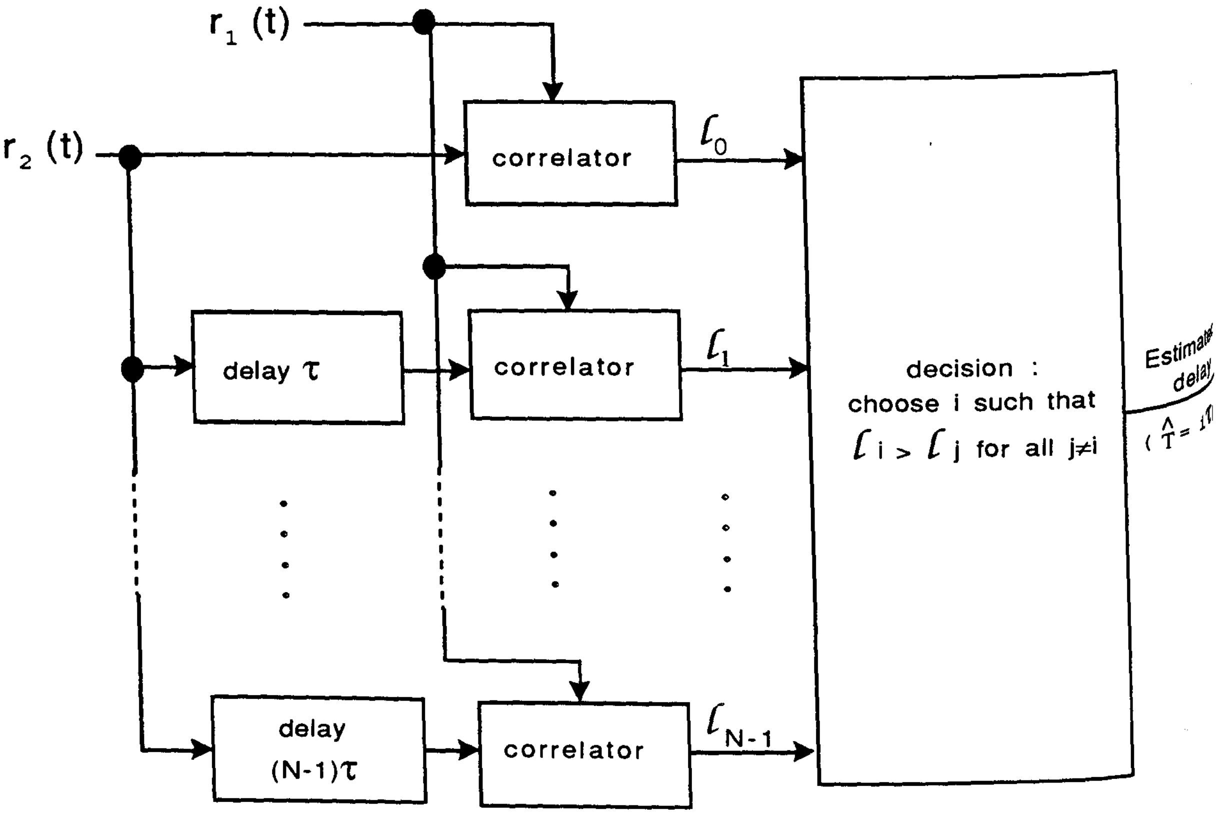


Figure 7.2-1 Block Diagram of Optimum Estimator

(e.g. as low as -10 dB for and input filter bandwidth of 10 kHz), our estimate accuracy will likely be limited by noise.

The above processor is difficult to analyze because we have the product of two noisy terms at the output of the correlator. This problem is similar to the design of an optimum differential demodulator where the approximation used to result in manageable noise statistics consists of assuming that the SNR is large enough to make the product of the noise terms to be negligible relative to the other terms [37]. When making this assumption, the correlator outputs are assumed to contain twice as much noise as in the case of one noisy signal.

The signal-to-noise ratio (SNR) at the input of the correlator is given by :

$$(SNR)_{in} = \frac{C}{NoB_{in}}$$

where B_{in} is the input noise bandwidth. The variance of the noise being the noise power, we have:

$$\sigma_i^2 = NoB_{in} = \frac{1}{(SNR)_{in}}$$

where C=1 has been assumed. The correlator can be seen as a filter which reduces the noise bandwidth by the product $B_{in}T_b$ where T_b is the correlation period. At the output of the correlator, we have a variance equal to :

$$(\sigma_1)^2 = \frac{\sigma_i^2}{(B_{in}T_b)} = \frac{1}{(C/No)T_b}$$

and when the doubling effect is taken into account, the correlator outputs have a variance given by :

$$\sigma^2 = \frac{2}{(C/No)T_b}$$

which is independent of the delay. Defining R_i as the autocorrelation level for a delay of $i\tau$ and assuming for now that both signals are aligned (i.e. they have no delay), the density function of the correlator outputs (ℓ_i's) is given by :

$$f(\ell_i) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{\frac{-(\ell_i - R_i)^2}{2\sigma^2}\right\}$$

The probability that ℓ_i is greater than ℓ_j for all $j \neq i$ is equivalent to the probability that $z_{ij} = \ell_i - \ell_j$ is greater than zero for all $j \neq i$. Both ℓ_i and ℓ_j being Gaussian, z_{ij} is Gaussian with mean $(R_i - R_j)$ and variance $\sigma^2\{1-R_{|i-j|}\}$. So the probability that z_{ij} is greater than zero is given by :

$$Z_{ij} = \text{Prob}(z_{ij}>0) = Q\left\{ \frac{-(R_i - R_j)}{\sigma\sqrt{1-R_{|i-j|}}} \right\} \qquad (7.2\text{-}2)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-y^2/2}\, dy$

In the above equations it was assumed that the signals were aligned i.e.there was no delay. When the delay is non-zero, say $k\tau$, the autocorrelation level indices in equation (7.2-2) are replaced by their absolute difference relative to k i.e. :

$$Z_{ij/k} \equiv \text{Prob}(z_{ij} > 0 \,/\text{delay is } k\tau) = Z_{|k-i||k-j|} \qquad (7.2\text{-}3)$$

The probability of the estimate being within $m\tau$ is then approximated by :

$$\text{Prob}(|\text{error}| \leq m\tau) \approx \sum_{i=k-m}^{k+m} \prod_{\substack{j=0 \\ j\neq i}}^{N-1} \text{Prob}(z_{ij}>0/\text{delay is } k\tau) \qquad (7.2\text{-}4)$$

A computer program to solve equations (7.2-2) and (7.2-4) has been developed (see Appendix C). The autocorrelation level used for the integration was given by a typical beacon identification code and some variation may be obtained for different codes. These differences are however expected to be small because we are only interested in the area close to the main peak i.e. when the two signals are aligned and in this area most beacon identification codes exhibit the same autocorrelation pattern.

Figure 7.2-2 shows the results for the case when the spacing between each correlator is either 12 μs or 24 μs. The figure shows the probability that the estimator will give no error in the estimated delay or an error of one correlator spacing (τ) assuming that the real delay is a multiple of τ. From the figure, it is clear that an accuracy better than 12 μs is unlikely for the minimum C/No of 30 dB-Hz. We can also conclude from the observation of the two sets of curves that sampling faster i.e. taking the correlator spacing smaller than 12 μs will not bring additional confidence in the estimate.

These results have been verified through a Monte Carlo simulation of the estimator (computer program given in Appendix D). For the simulation, it has been assumed that the signal was down-converted to baseband, filtered with a 800 Hz lowpass filter, and sampled at 41.6 kHz (τ = 24 μs) or 83.2 kHz (τ = 12 μs). Then the correlation over 458 ms was computed and the maximum output was selected as representing the estimated delay. Results are shown in Figures 7.2-3 and 7.2-4 for τ=24 μs and τ=12 μs respectively along with the theoretical results. In general the theory and the simulation results agree quite well for C/No in excess of 30 dB-Hz.

Up to now, we considered the case when the delay was a multiple of the correlator spacing. This assumption made the estimate error a multiple of the correlator spacing. In practice the delay could lie anywhere within the correlator spacing such that the previous results indicating *No Error* should be interpreted as $\pm\tau$ and those indicated as being in error by τ should in fact be $\pm 2\tau$ with uniform statistical distribution. In this case, the variance of the estimate is given by :

$$(\sigma_t)^2 = \frac{\tau^2}{3}$$

Also, the confidence level is indicated in Figure 7.2-2 for τ=24 or 12 μs. For τ=12 μs, we have σ_t=6.9 μs with a confidence level of approximately 90 % at a C/No of 30 dB-Hz.

Because the expected delay is bounded by 18 ms, the number of correlation for computation is therefore $\frac{18 \text{ ms}}{24\ \mu\text{s}} = 750$. The correlator spacing of 12 μs defines the minimum sampling rate of 83.2 kHz. The correlation over the 750 lags can be performed efficiently using the Fast Fourier Transform (FFT) technique described in [34,35] and implemented in a Fortran program in [36]. This technique has been used to generate the computer simulation results presented above.

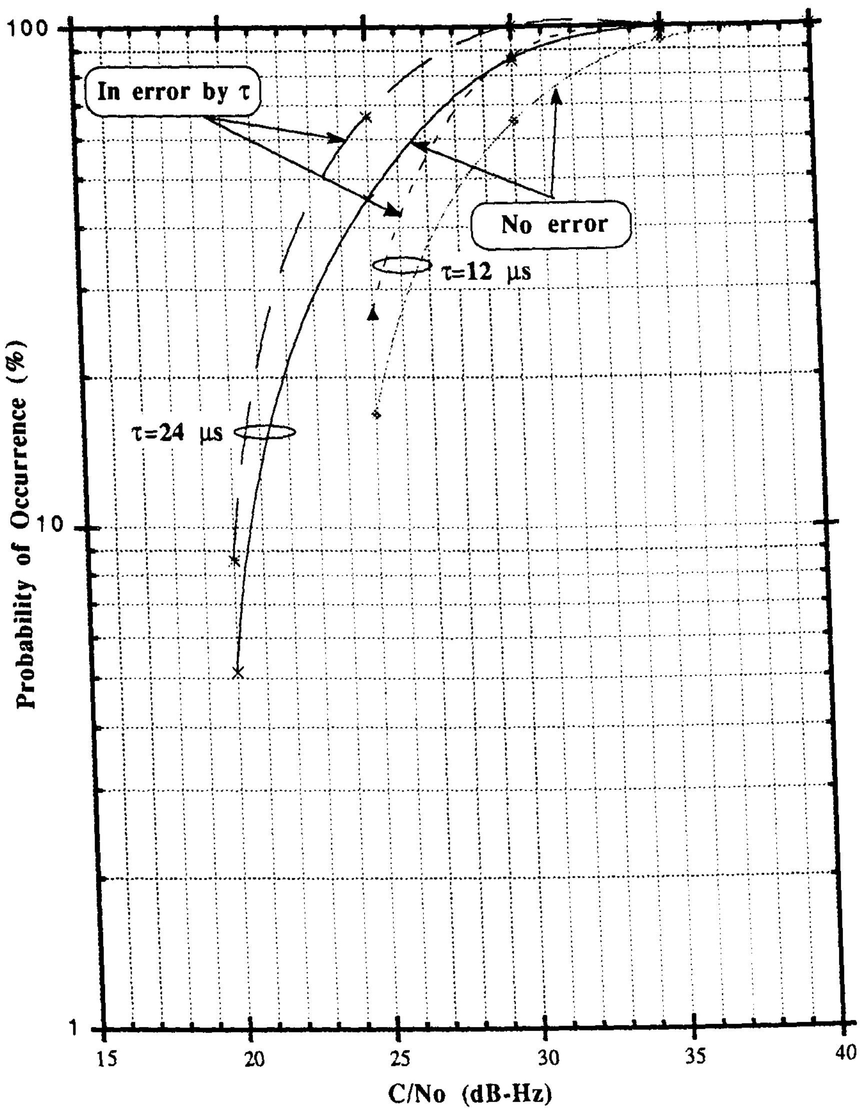


Figure 7.2-2 Probability that the Delay Estimate is Within the Indicated Bounds for Various C/No's and Correlator Spacing of 12 μs and 24 μs.

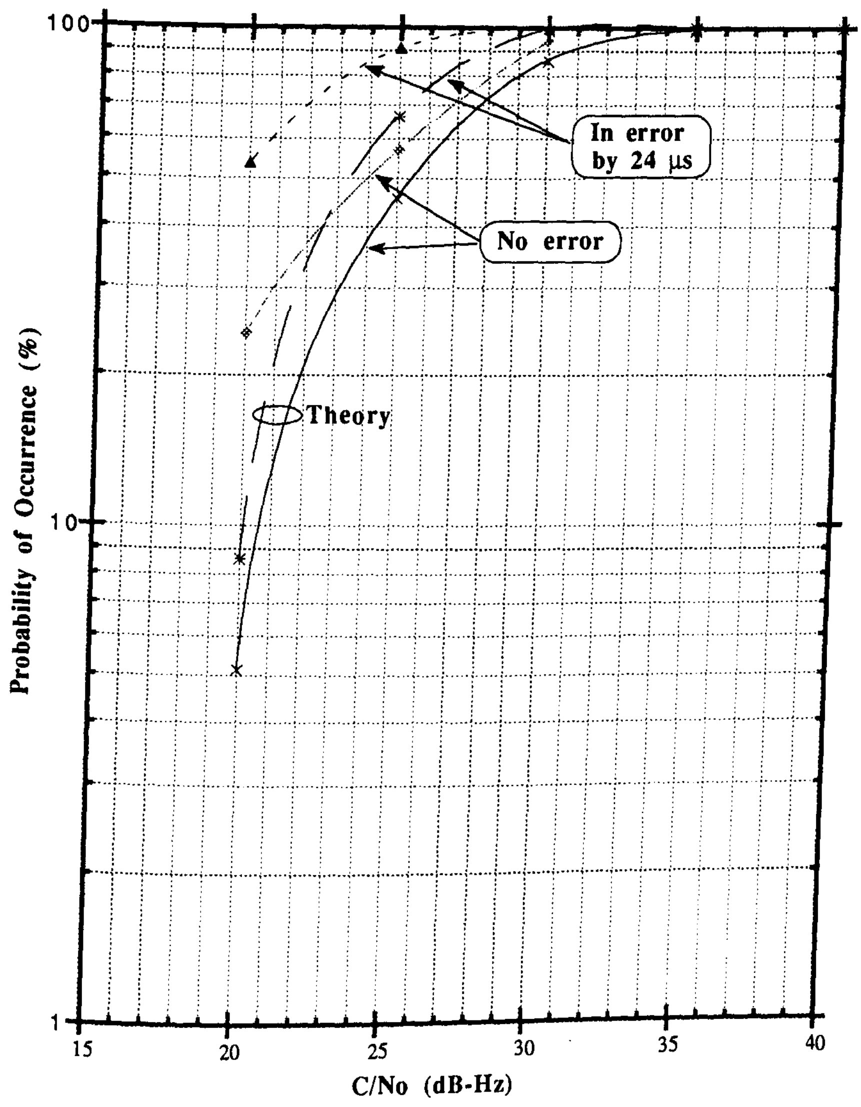


Figure 7.2-3 Simulated and Theoretical Probability that the Delay Estimate is Within the Indicated Bounds for Various C/No's and a Correlator Spacing of 24 µs.

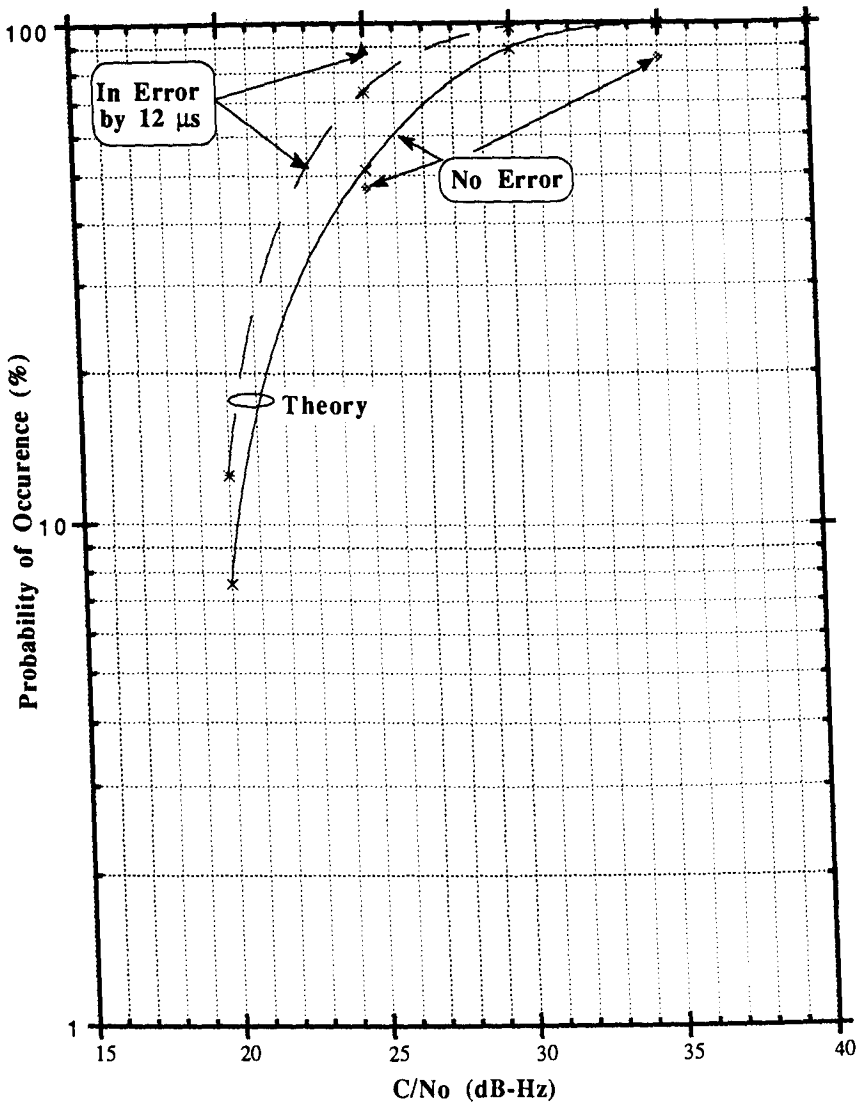


Figure 7.2-4 Simulated and Theoretical Probability that the Delay Estimate is Within the Indicated Bounds for Various C/No's and a Correlator Spacing of 12 μs.

8.0 OVERALL BEACON POSITIONING ACCURACY

Up to now, we discussed the various components of the subsystems which corrupt the signal and/or degrade the accuracy of the positioning technique. In this section, we derive an overall figure that combines all these effects to yield an overall positioning accuracy. There are basically two parameters that define the position accuracy which can be expected from the system : the equivalent range estimate error and the position dilution of precision (PDOP) factor. The former has been discussed in details in the previous sections for each subsystem and it can be assumed that the errors add on a root sum square(rss). Accordingly, the variance of the range error is given by :

$$\sigma_r^2 = c^2 * \sigma_t^2 + \sigma_{iono}^2 + \sigma_{tropo}^2 + \sigma_{sat}^2 \quad m^2 \qquad (8.0\text{-}1)$$

where σ_t = standard deviation of time delay estimate in seconds

σ_{iono} = ionospheric delay standard deviation in meters

σ_{tropo} = tropospheric delay standard deviation in meters

σ_{sat} = satellite position error standard deviation in meters. It is assumed that the standard deviation of the satellite position on each axis is the same i.e. σ_{sat}.

c = speed of light = 2.9979×10^8 m/s

The PDOP for a single position line obtained from two satellites is derived indirectly in [38] and [39] and is given by :

$$PDOP = \frac{1}{4 \sin^2(\theta/2) - \left(\cos \phi_2 - \cos \phi_1\right)^2} \qquad (8.0\text{-}2)$$

where θ and ϕ_i's are defined as shown in Figure 8.0-1. If the coordinates of the two satellites are given by (x_i, y_i, z_i), i=1,2 and the particular point of interest on the earth is defined by (x, y, z), then we have :

$$\cos \theta = \frac{(x_1 - x)(x_2 - x) + (y_1 - y)(y_2 - y) + (z_1 - z)(z_2 - z)}{R_1 R_2} \qquad (8.0\text{-}3)$$

$$\cos \phi_i = \frac{x(x_i - x) + y(y_i - y) + z(z_i - z)}{R_e R_i}$$

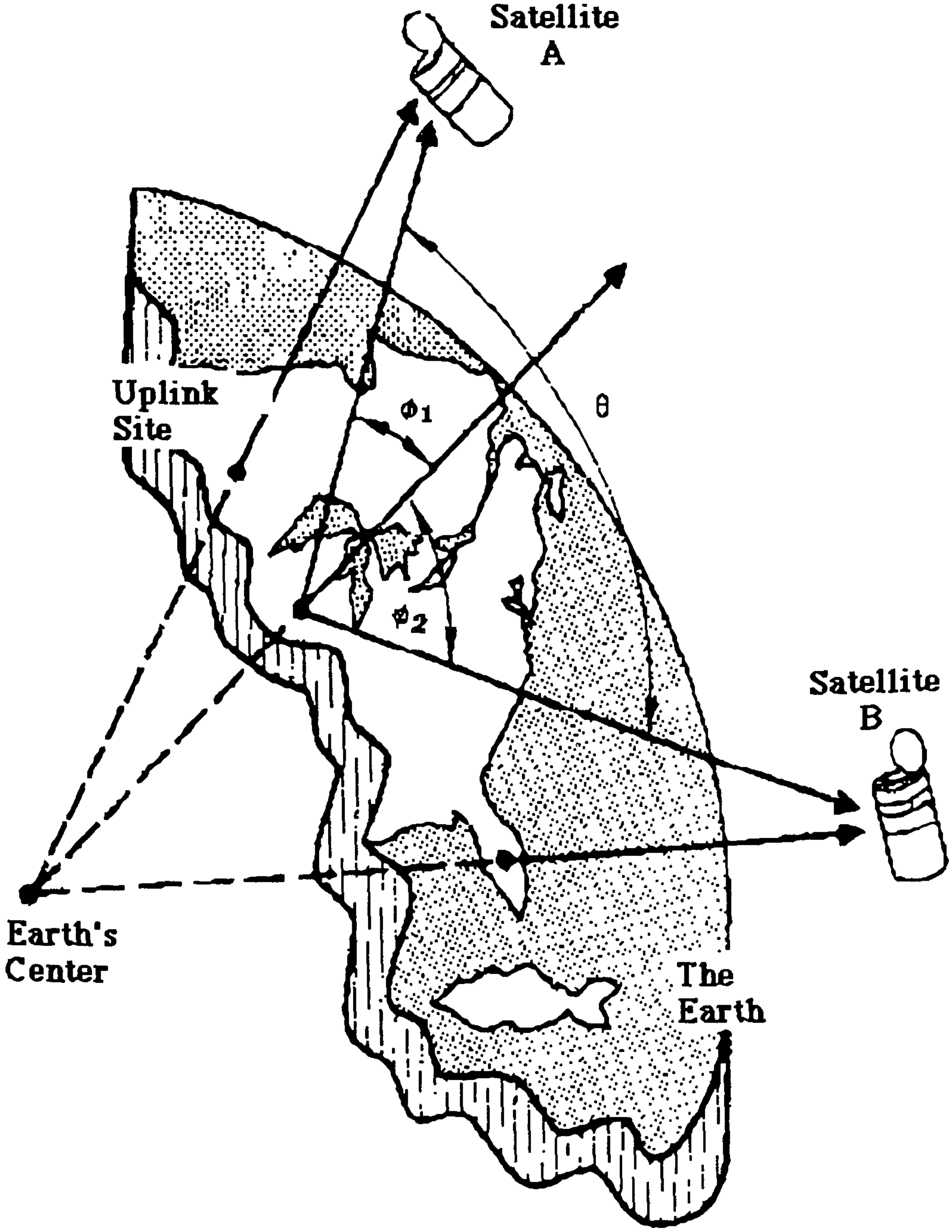


Figure 8.0-1 PDOP Geometry

where R_i is the range to satellite #i and R_e is the range from the point on the earth to its centre and they are defined as :

$$R_i = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} \qquad (8.0\text{-}4)$$

$$R_e = \sqrt{x^2 + y^2 + z^2}$$

The line position accuracy is then defined as :

$$\sigma = (PDOP)\ \sigma_r \qquad (8.0\text{-}5)$$

For the time difference of arrival positioning technique, the intersection of two such positioning lines derived from three satellites defines the beacon location. It can be shown that the best accuracy is achieved when the two lines cross at square angle (see Figure 8.0-2(b)). In this case, it is fair to assume that the position error on each line adds on a root sum square i.e. the accuracy of the position is the rss of the accuracy of the line derived from, let us say, satellites 1 and 2 and the accuracy of the line derived from satellites 1 and 3 i.e. :

$$\sigma = \sqrt{(PDOP_{1,2}\ \sigma_{1,2})^2 + (PDOP_{1,3}\ \sigma_{1,3})^2} \quad \text{m} \qquad (8.0\text{-}6)$$

where $\sigma_{i,j}$ is the standard deviation of the range error as defined in equation (8.0-1) for satellites #i and #j. Because the error made on the range estimate is independent of the satellites, it is fair to assume that $\sigma_{1,2} = \sigma_{1,3} = \sigma_{2,3} = \sigma_r$ and then we have :

$$\sigma = \sigma_r\ \sqrt{(PDOP_{1,2})^2 + (PDOP_{1,3})^2} \qquad (8.0\text{-}7)$$

or equivalently

$$\sigma = \sigma_r\ PDOP_{eq}$$

where $PDOP_{eq} = \sqrt{(PDOP_{1,2})^2 + (PDOP_{1,3})^2}$

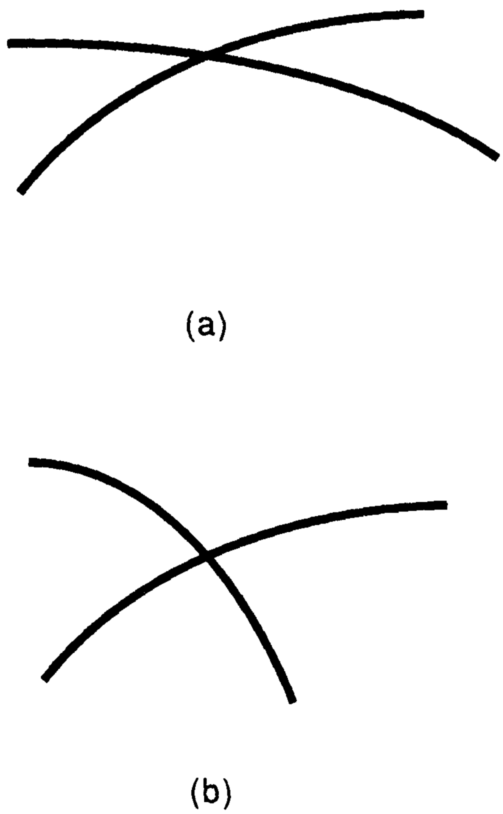


Figure 8.0-2 (a) Bad PDOP, (b) Good PDOP.

Figures 8.0-3 to 8.0-5 show the PDOP for various combinations of two of the three satellites located at 75°W, 105°W and 135°W and for various latitudes. A spherical earth has been assumed to generate these figures. The figures show that the PDOP is minimum around the mid longitude between the two satellites and is symmetric around this point. They also show that the PDOP is slightly better for low latitudes than high latitudes. For high latitudes, the PDOP tends to be more constant than at low latitudes where it increases rapidly as it goes away form the mid longitude point. This is expected as the point lies in the same plane as the geostationary satellite orbit plane. Note the change of scale for Figure 8.0-5 where it is shown that a 60° satellite spacing results in a PDOP of approximately 1.

Figures 8.0-6 to 8.0-8 show the total PDOP when two position lines are combined according to equation (8.0-7). As in the other figures, a minimum is noticed and the curves are symmetric around this point. The same observations as in the previous figures can be made here. In general, it shows that the PDOP for Canada if these three satellites are used is between 1.9 and 2.8 for the best case and between 2.5 and 3.2 for the worst case.

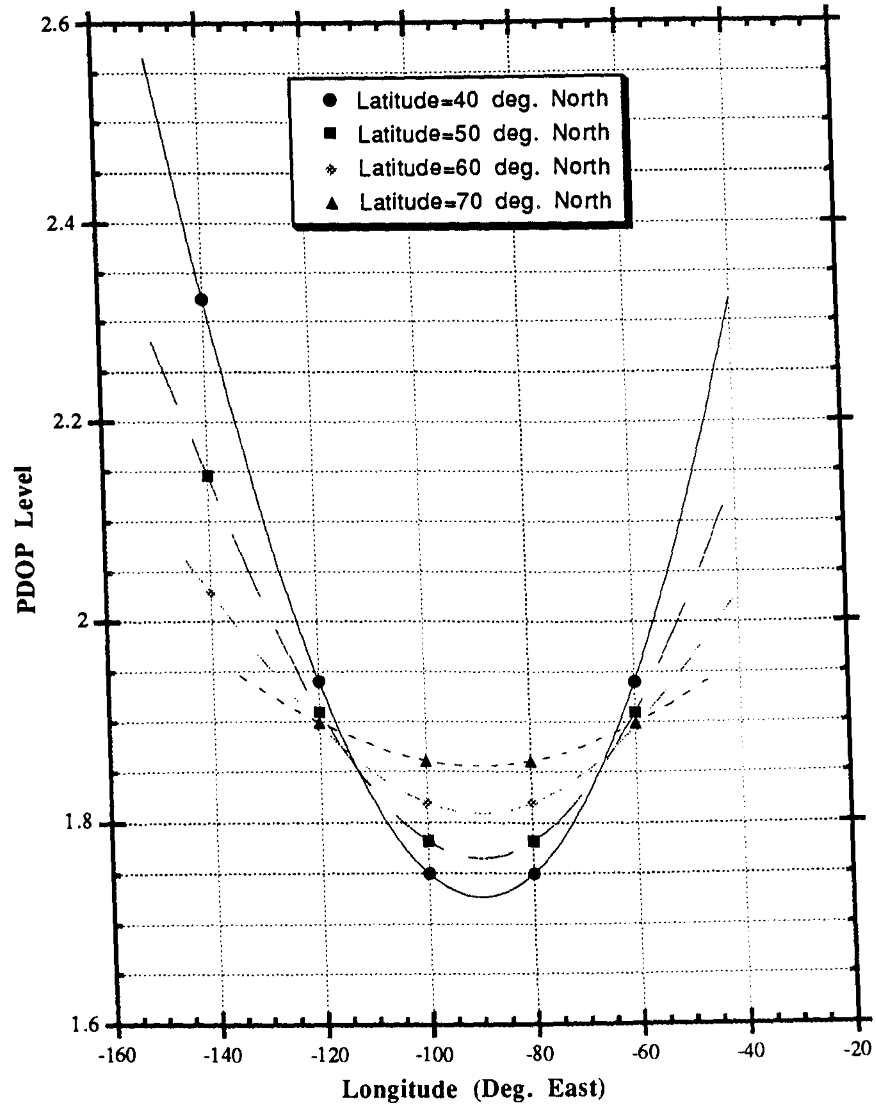


Figure 8.0-3 PDOP for Satellites at 75oW and 105oW.

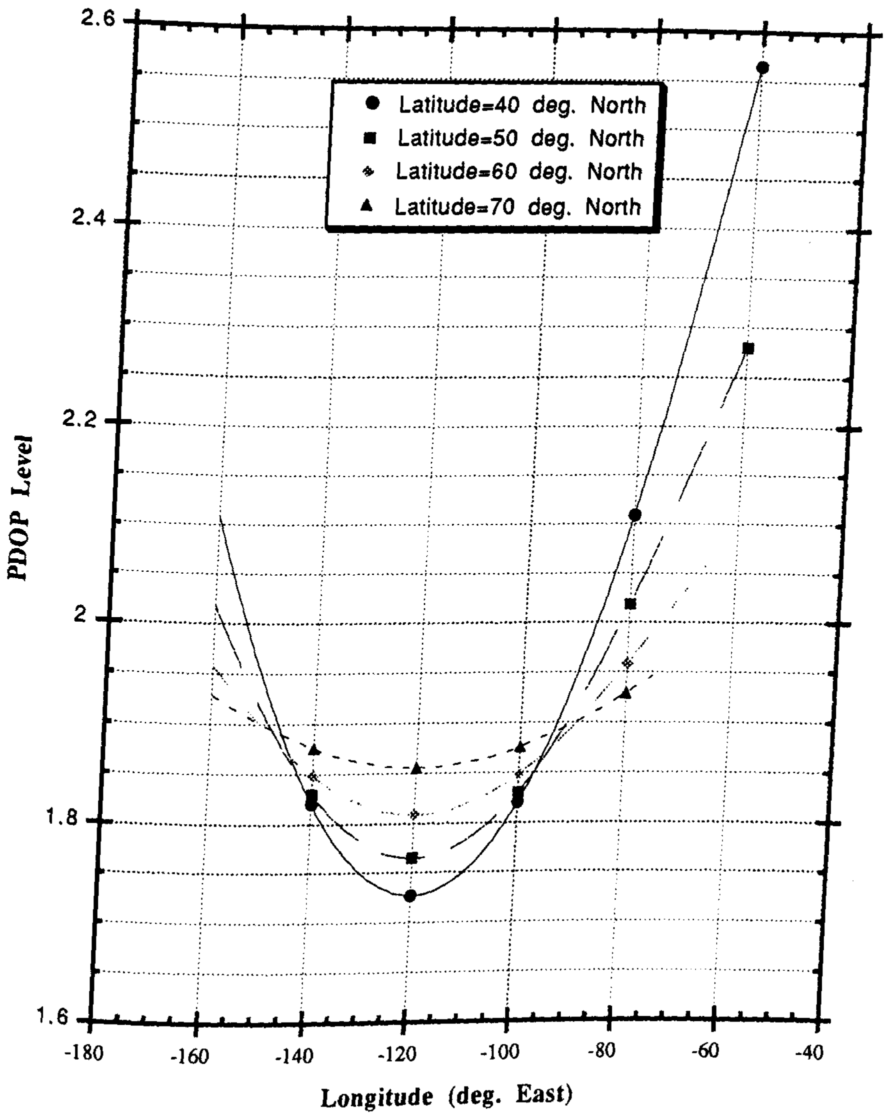


Figure 8.0-4 PDOP for Satellites at 105°W and 135°W.

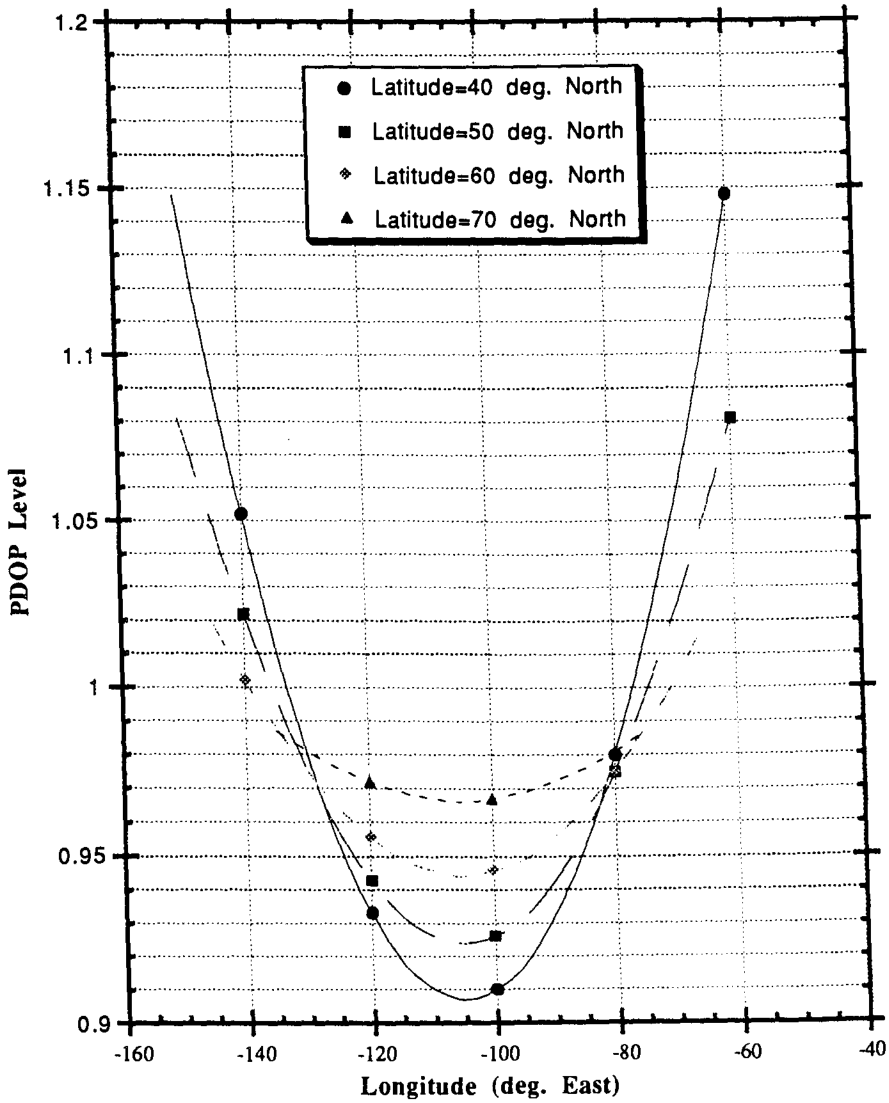


Figure 8.0-5 PDOP for Satellites at 75oW and 135oW.

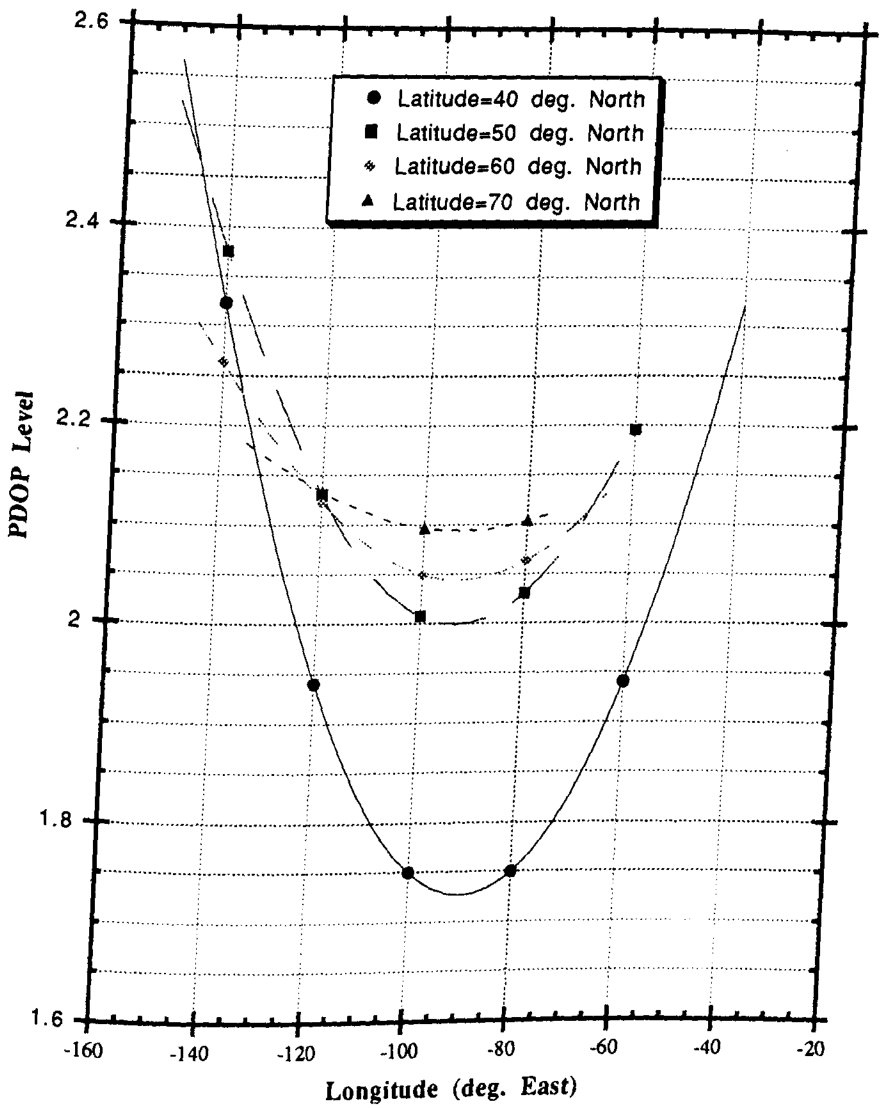


Figure 8.0-6 Resulting Equivalent PDOP for the Combination of the Position Lines from Satellites at 75°W and 135°W and Satellites at 75°W and 105°W

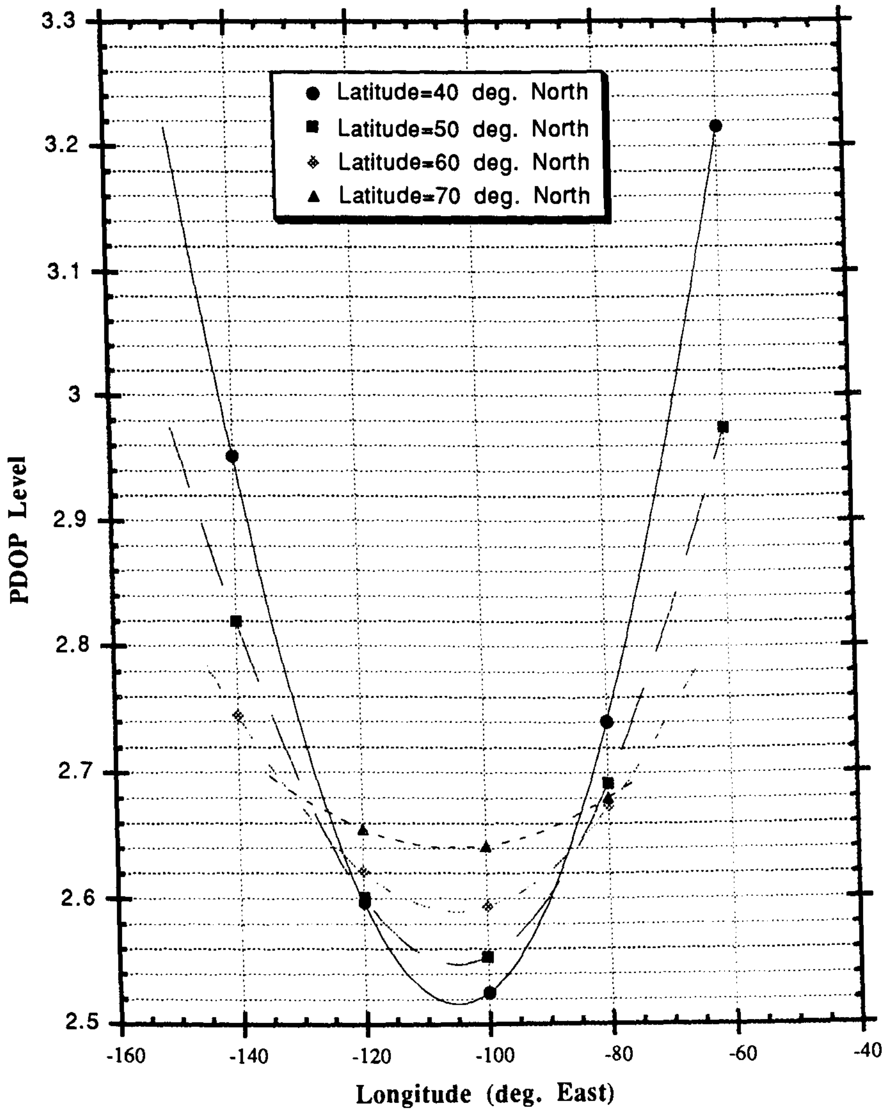


Figure 8.0-7 Resulting Equivalent PDOP for the Combination of the Position Lines from Satellites at 75°W and 105°W and Satellites at 105°W and 135°W

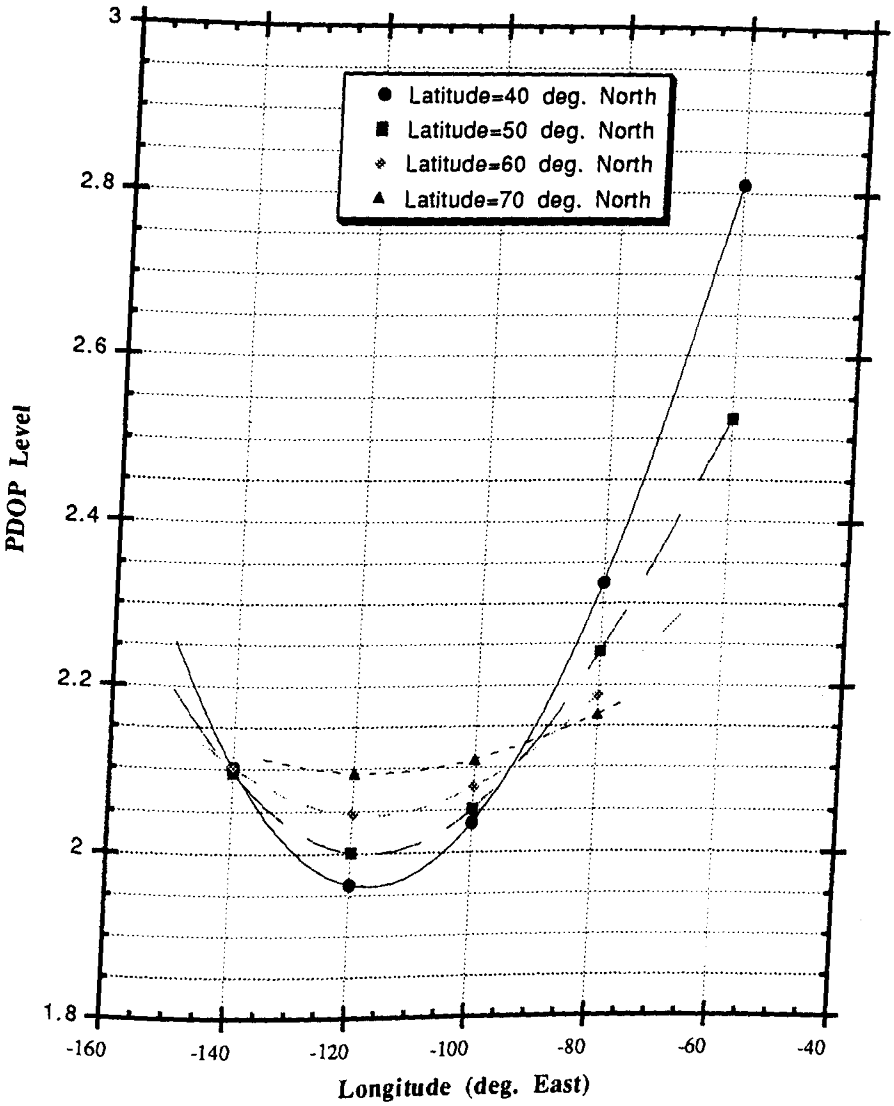


Figure 8.0-8 Resulting Equivalent PDOP for the Combination of the Position Lines from Satellites at 75°W and 135°W and Satellites at 105°W and 135°W

9.0 CIRCULAR ERROR PROBABILITY (CEP)

The accuracy of a positioning technique must always be given with its given degree of confidence. This requires in general the knowledge of the statistical distribution of the position errors which can be obtained only if the statistical distribution of each component of the total position error is known as well as their correlation factor. In practice, these distributions are not known and although one could attempt to develop such a statistical model, experimental data and straightforward approximations are usually preferred.

In [40] numerous methods to express the accuracy of positioning systems are discussed. The circular error probability (CEP) defines the radius of the circle centered on the true position which contains 50 % of the points. It assumes that there are no bias errors present although in practice such bias inevitably do exist due to equipment. The CEP is a two-dimensional measure and is appropriate for the specification of the error in the system of interest.

In Section 8.0 we defined in equation (8.0-7) the equivalent position error to be given by :

$$\sigma = \sigma_r \sqrt{(PDOP_{1,2})^2 + (PDOP_{1,3})^2} = \sigma_r \; PDOP_{eq} \qquad (9.0\text{-}1)$$

which is **equivalent** to the distance rms (**drms**) error discussed in [40]. It is important to emphasize the assumptions made to obtain this equation. First, it is assumed that the error sources on each differential range are uncorrelated and have a zero mean with equal standard deviation (σ_r). Second, it is assumed that a good PDOP is obtained for the location such that the two position lines cross at 90 degrees. Based on these assumptions, it is mentioned in [40] that the CEP is related to the above σ (or drms) by :

$$CEP = \sqrt{\ln(2)}\,\sigma \qquad (9.0\text{-}2)$$

As mentioned in [40], twice the drms represents the 95 % degree of confidence which is given by twice equation (9.0-1) i.e. 2σ.

In Section 8.0, the analysis did not consider the downlink (at 1.5 GHz) effects on the differential range error. In practice, the downlink effects could be estimated using a loopback signal at the master station. However, the ionospheric effects being approximately 15 times less important at L-band than 406 MHz, it will have a small contribution to the total position error and thus can simply be taken into account or neglected.

In order to get an idea of the proposed system performance, let us look at some examples. Let us assume that the satellites are located nominally at 75°W, 105°W and 135°W and that we know their exact positions within 20 meter on each axis. From the end of Section 6.3.1, we find that the ionospheric rms error is 150 m in the uplink. Dividing this number by 15 for the downlink at 1.5 GHz, we get a rms error of 10 m on the downlink. The standard deviation of the differential range error (σ_r) is then given by :

$$\sigma_{iono} = \sqrt{\left(\sigma_{iono}^{up}\right)^2 + \left(\sigma_{iono}^{down}\right)^2}$$

$$= \sqrt{150^2 + 10^2} = 150.33 \text{ m} \quad \text{(see Section 6.3.1)}$$

$$\sigma_{sat} = 20 \text{ m}$$

$$\sigma_t = \frac{12\ \mu s}{\sqrt{3}} = 6.9\ \mu s \text{ (with} \approx 90\% \text{ confidence level at C/No=30 dB-Hz)}$$

$$\sigma_{tropo} = 0 \text{ (negligible relative to other sources of errors)}$$

$$\sigma_r^2 = c^2 * \sigma_t^2 + \sigma_{iono}^2 + \sigma_{tropo}^2 + \sigma_{sat}^2 \quad m^2$$

$$\sigma_r = \sqrt{[(2.9979 \times 10^8)(6.9 \times 10^{-6})]^2 + (150.33)^2 + (20)^2}$$

$$\boxed{\sigma_r = 2.1 \text{ km}}$$

In Section 8.0 it has been seen that the $(PDOP)_{eq}$ was between 1.9 and 2.8 with these satellites and with a good choice of combination of satellites. The drms error and CEP are then bounded to :

$$\sigma_r\,(PDOP)_{eq_{min}} < \sigma < \sigma_r\,(PDOP)_{eq_{max}}$$

$$\boxed{4 \text{ km} < \sigma < 5.9 \text{ km}}$$

$$\boxed{3.3 \text{ km} < \text{CEP} < 4.9 \text{ km}}$$

provided there are no biases in the system.

It is interesting to note that the performance is dictated by the performance of the time difference of arrival estimator. In the above example, ignoring the errors introduced by the troposphere and the ionosphere would have little impact on the overall accuracy of the beacon position. This fact may be used to simplify the receiver design in an operational system.

10.0 RECEIVER STRUCTURE

Due to the low likelihood that three satellites will be available for use in a 406 MHz distress beacon positioning system, very little effort has been spent on the hardware design of the proposed receiver. The intent of this section is to give the reader an idea of the hardware and software required to develop the proposed distress beacon location system.

Figure 10.0-1 shows a block diagram of the receiver. The satellite receiver front ends are similar to the ones being used in the current GOES processor. It includes a low noise amplifier, a frequency down-converter to bring the signal to an intermediate frequency of 5 kHz and a satellite beacon tracker for frequency correction. Note also, it is assumed that the satellite beacon carries the satellite instantaneous position. This information is assumed to be derived by the receiver front end and it will represent a modification to the front end processor in use in the GOES project. The three satellite signals are then sampled at a rate of 83.2 kHz. Several data acquisition systems are available on the market and can be adapted to any personal computers (Macintosh and IBM compatibles). All the shaded area represents the digital processing under the control of a master controller. The detection, frequency estimation and down-conversion to baseband is already performed in the current GOES processor at a sample rate of 20.8 kHz. Accordingly, the same processing algorithms can be used here if they are preceded with a decimation by four (4). The delay estimation refers to the implementation of the correlation computation between each pair of signals. Results are sent to the master controller which interfaces to the user and monitors/controls the operation of each subsystem (e.g. if a signal is detected then freeze the memory, perform the frequency estimation and so on to come out with a position estimate). The GPS receiver is used to get the ionospheric model parameters if required.

The digital processing could be done within a personal computer (IBM 386 or compatible or Macintosh IIfx) equipped with at least one array processor as well as a data acquisition system. The required number of array processors is related to the available time between events. The computer simulation which was generating the signal and noise samples in addition to computing the correlation was taking approximately 2 minutes per signal pair on a Macintosh IIcx computer equipped with a co-processor. Given that array processors provide 40-50 times more computing power than a general purpose computer, it is estimated that no more than 15-20 seconds will be required to process an event i.e. a signal detection event. This assumes that data can be transferred fast enough between each processing subsystems. A more complete software design needs to be done to ascertain these values.

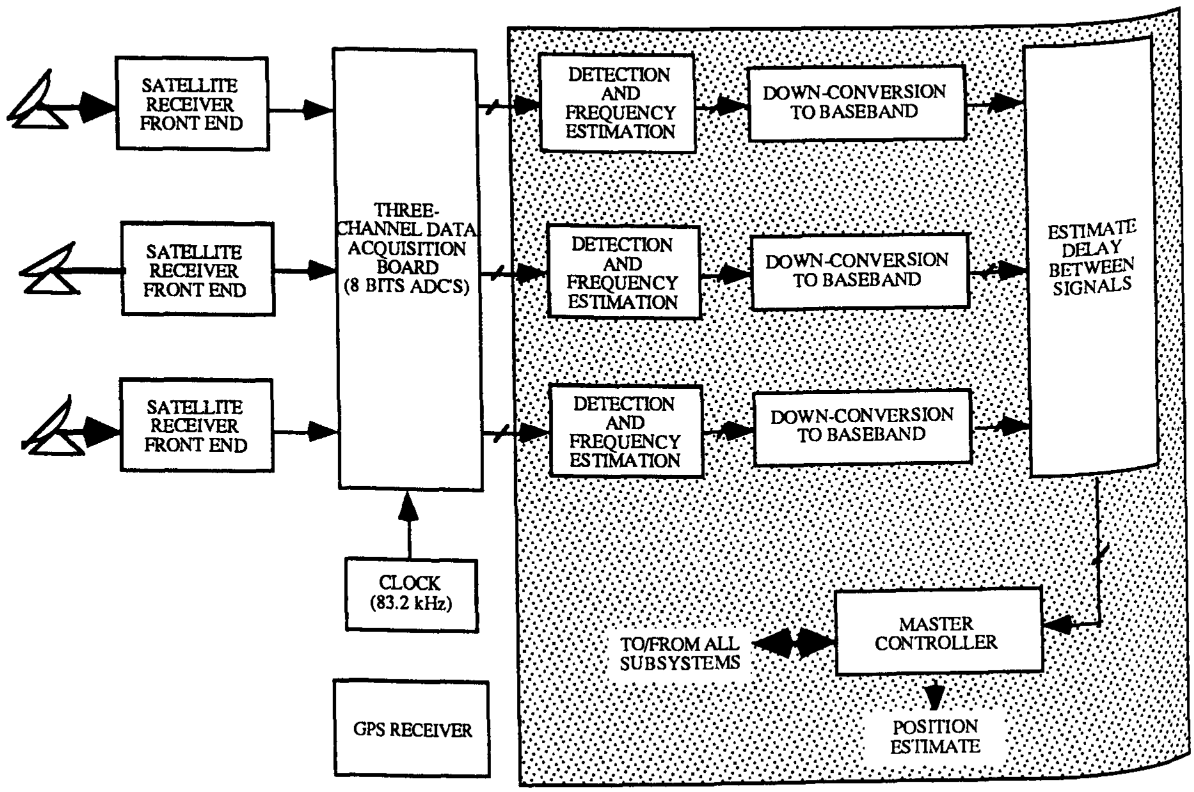


Figure 10.0-1 Block Diagram of Receiver and Estimator

11.0 CONCLUSION

This report presented a technique to locate distress beacons using geostationary satellites. In looking at the various techniques in general, it was found that a minimum of two satellites is required and that the small Doppler frequency shift of the geostationary satellites could in principle be used to resolve ambiguities. In the studied system, this information has not been used because it was believed that it would require sophisticated on-board oscillators that are unlikely going to be available on a transponder type satellite.

The time difference of arrival technique based on three satellites has been fully studied. The effect of the Earth oblateness and the ionosphere and troposphere delays have been modeled. A time difference estimator has been designed and analyzed. Theoretical performance of the estimator supported by computer simulation results demonstrated that the position error and confidence level are similar to the current system. Specifically, it can achieve a CEP between 3.3 and 4.9 km for beacons in Canada and a satellite spacing of 30° under the worst case C/No of 30 dB-Hz obtained when 8 distress beacons are simultaneously activated. The estimator draws a lot from the current GOES processor in order to minimize the amount of effort required to supplement the current distress alerting feature with distress positioning. The resources required to implement the digital processing part of the receiver are estimated to approximately 2 person-year and $75,000 for the hardware and software.

It is obvious that the report did not address all of the techniques in great details and additional analyses should be done. In particular, a better analysis on the use of the geostationary satellite Doppler frequency shift to supplement the time difference of arrival technique with only two satellites must be done. The impact of oscillator instabilities on the overall accuracy must be analyzed and traded off with other system parameters. In addition, the time difference estimator design should be reviewed to include the consideration of correlation in the frequency domain instead of the time domain. Some papers have recently pointed out that correlation in the frequency domain is more robust against noise than in the time domain [42].

12.0 REFERENCES

[1] Rosso R., *LOCSTAR : Un Service de Radiorepérage par Satellite pour l'Europe*, Paper Presented during CNES Course "Location and Navigation Satellite Systems", Toulouse (France), 6-10 March 1989.

[2] Johannsen K.G., *Radio Position Determination by Satellite*, Space Communication and Broadcasting, Vol. 6, No.3, pp. 203-221, July 1988

[3] Johannsen K.G., *Radio Position Determination by Satellite*, 38th IEEE Vehicular Technology Conference, pp. 647-652, 15-17 June 1988

[4] Briskman R.D., *GEOSTAR Initial RDSS System*, AIAA 12th International Communication Satellite Systems Conference, pp. 587-591, March 13-17, 1988.

[5] Ott L.E., *STARFIX - Commercial Satellite Positioning*, IEEE 1988 Position Location and Navigation Symposium Record (PLANS 88).

[6] Troisi S., *Détermination des Coordonnées d'une Station Mobile par la Méthode des Différences de Temps*, Navigation, No. 142, Avril 1988.

[7] Ha T.T., Robertson R.C., *Geostationary Satellite Navigation Systems*, IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-23, No. 2, pp. 247-253, March 1987

[8] Hernandez D., *The Locstar Radio-Determination Satellite System*, Alta Frequenza, Vol. LVI, No.10, Dec. 1987.

[9] Pearson B.S., *LOCSTAR : Europe's Commercial Radiodetermination Satellite Service*, Satellite Communications and Broadcasting, 1987.

[10] Dennis A. R., *STARFIX*, IEEE 1986 Position Location and Navigation Symposium Record, Nov. 4-7, pp. 251-255, 1986

[11] Dennis A.R., *STARFIX - A New High Precision Satellite Positioning System*, Proc. International Symposium on Marine Positioning : "Positioning the Future", INSMAP 86, US, Geological Survey, Reston, VA, Oct. 14-17, 1986.

[12] *Specification for COSPAS-SARSAT 406 MHz Distress Beacons*, COSPAS/SARSAT Steering Committee Document C/S T.001, Issue 2, Revision 0, November 1988

[13] Dumont P., Bonnery M., Marguinaud A., *Receiver-Processor for Detection of 406 MHz SARSAT Distress Messages*, AIAA 12th International Communication Satellite Systems Conference, pp. 350-358, March 13-17, 1988

[14] Keightley R.J., *Simulation of the Canadian GOES Signal Processor*, IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-23, No.3, pp. 361-370, May 1987

[15] Schaefer G.J., *Apparatus and Method for Determining the Position of a Radiant Energy Source*, US Patent 4,276,553, June 30,1981.

[16] Donaldson P., *Navigation by Gyration*, Space, Vol. 4, No.6, Nov./Dec. 1988.

[17] Niemeyer P.H., *STARFIND : Single Satellite Geolocation*, Satellite Communications and Broadcasting, 1987.

[18] Slabinsky V.J., *Variation in Range, Range-Rate, Propagation Time Delay, and Doppler Shift for Nearly Geostationary Satellite*, in Communications Satellite Technology, edited by P.L. Bargellini, Alpine Press Inc., USA, 1974.

[19] Kinal G. V., *Prospects for Integration of Satellite Communications, Navigation and Monitoring*, Paper Presented during CNES Course "Location and Navigation Satellite Systems", Toulouse (France), 6-10 March 1989.

[20] Hershey W.R. et al, *Ranging and Processing Satellite*, IEEE 1988 Position Location and Navigation Symposium (PLANS 88) Record.

[21] Bukhart R.M., Harris R.R., *Geolocation Through Satellite Propagation Delay Difference*, 1987 IEEE Aerospace Applications conference Digest, Vail, CO, USA, 8-13 Feb. 1987.

[22] *Application of Satellite Communication and Position Fixing Techniques to Land Mobile Systems*, General Electric Corp. R&D, New York, NTIS PB-267-019, April 1977.

[23] Chouinard G., *Satellite Beam Optimization for the Broadcasting Satellite Service*, IEEE Trans. on Broadcasting, vol. BC-27, no.1, March 1981.

[24] Nautiyal Atul, *Algorithm to Generate Geodetic Coordinates from Earth-Centered Earth-Fixed Coordinates*, Journal of Guidance, Control, and Dynamics, vol. 11, no. 3, May-June 1988.

[25] Ludvik Pfeifer, *The Use of Bowring's Algorithms for Hydrography and Navigation*, Institute of Navigation, Conference Proceedings of the 39th Annual Meeting, June 1983.

[26] Holmstrom J.S., *A New Approach to the theory of Geodesics on an Ellipsoid*, Navigation:Journal of the Institute of Navigation, Vol. 23, No. 3, Fall 1976.

[27] *Ionospheric Effects Upon Earth-Space Propagation*, CCIR Report 263-5, 1982.

[28] Saint-Etienne J., *Erreurs de Mesure*, Paper presented during CNES Course "Location and Navigation Satellite Systems", Toulouse (France), 6-8 March 1989.

[29] Brown A., *Extended Differential GPS*, Navigation : Journal of the Institute of Navigation, Vol.36, No.3, Fall 1989.

[30] Klobuchar J.A., *Ionospheric Time-Delay Algorithm for Single-Frequency GPS Users*, IEEE Trans. on Aerospace and Electronic Systems, vol. AES-23, no. 3, May 1987.

[31] FLock W.L., Slobin S.D., Smith E.K., *Propagation Effects on Radio Range and Noise in Earth-Space Telecommunications*, Radio Science, vol. 17, no. 6, Nov.-Dec. 1982.

[32] *NAVSTAR Global Positioning System (GPS) System Characteristics - Preliminary Draft*, North Atlantic Treaty Organization (NATO), Military Agency for Standardization (MAS), Standardization Agreement, STANAG 4294, Draft Issue J, 20 April 1989.

[33] Keightley R.J., *Simulation of the Canadian GOES Signal Processor*, IEEE Trans. on Aerospace and Electronic Systems, Vol.AES-23, No. 3, May 1987.

[34] Oppenheim A.V., Schafer R.W., Digital Signal Processing, Prentice Hall Inc., New Jersey, USA, 1975, pp.556-562

[35] Rabiner L.R., Gold B., Theory and Application of Digital Signal Processing, Prentice Hall Inc., New Jersey, USA, 1975, Section 6.18.

[36] Programs for Digital Signal Processing, IEEE Press, 1979, Section 2.2.

[37] Proakis J.G., Digital Communications, MacGraw-Hill Inc., Toronto, 1983, pp. 171-178

[38] Chesnut P.C., *Emitter Location Accuracy Using TDOA and Differential Doppler,* IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-18, No.2, March 1982.

[39] Burkhart R.M., Harris R.R., *Geolocation Through Satellite Propagation Delay Differences,* 1987 IEEE Aerospace Applications Conference Digest, Vail, CO,USA, 8-13 Feb. 1987.

[40] *Method of Expressing Navigation Accuracies*, Nato Standardization Agreement, STANAG 4278 (edition 3), January 1989.

[41] Flock W. L., *Propagation Effects on Satellite Systems at Frequencies Below 10 GHz*, NASA Reference Publication #1108, 1983.

[42] Gardner W.A., *Spectral Correlation of Modulated Signals : Part I I- Digital Modulation*, IEEE Trans. on Communications, vol. COM-35, No. 6, June 1987.

[43] *Department of Defense World Geodetic System 1984 - Its Definition and Relationships with Local Geodetic Systems*, Defense Mapping Agency, Washington, DC, USA, Report # DMA TR 8350.2, September 30, 1987.

[44] Van Trees H.L., Detection, Estimation and Modulation Theory Part I, Wiley, 1968, chap.4

13.0 BIBLIOGRAPHY

An extensive literature search has been conducted at the beginning of the contract. The papers are listed below under five headings :

A. GEOLOCATION TECHNIQUES AND SYSTEMS

B. COSPAS-SARSAT SYSTEM AND 406 MHZ BEACONS

C. DISTRESS SIGNAL PROCESSOR

D. PROPAGATION EFFECTS

E. POSITION ACCURACY AND OTHER TOPICS

The papers under each heading are given in chronological order starting with the most recent.

A. GEOLOCATION TECHNIQUES AND SYSTEMS

[A.1] Portas G., *Les Systèmes de Localisation ARGOS et SARSAT*, Paper Presented during CNES Course "Location and Navigation Satellite Systems", Toulouse (France), 6-10 March 1989.

[A.2] Rosso R., *LOCSTAR : Un Service de Radiorepérage par Satellite pour l'Europe*, Paper Presented during CNES Course "Location and Navigation Satellite Systems", Toulouse (France), 6-10 March 1989.

[A.3] Gal C., *Comparaison des Systèmes de Localisation*, Paper Presented during CNES Course "Location and Navigation Satellite Systems", Toulouse (France), 6-10 March 1989.

[A.4] Kinal G. V., *Prospects for Integration of Satellite Communications, Navigation and Monitoring*, Paper Presented during CNES Course "Location and Navigation Satellite Systems", Toulouse (France), 6-10 March 1989.

[A.5] Donaldson P., *Navigation by Gyration*, Space, Vol. 4, No.6, Nov./Dec. 1988.

[A.6] Johannsen K.G., *Satellite Mobile Communication and Radio Positioning System Planning Aspects*, IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-24, No.4, pp.387-396, July 1988

[A.7] Johannsen K.G., *Radio Position Determination by Satellite*, Space Communication and Broadcasting, Vol. 6, No.3, pp. 203-221, July 1988

[A.8] Johannsen K.G., *Radio Position Determination by Satellite*, 38th IEEE Vehicular Technology Conference, pp. 647-652, 15-17 June 1988

[A.9] Briskman R.D., *GEOSTAR Initial RDSS System*, AIAA 12th International Communication Satellite Systems Conference, pp. 587-591, March 13-17, 1988.

[A.10] Ott L.E., *STARFIX - Commercial Satellite Positioning*, IEEE 1988 Position Location and Navigation Symposium Record (PLANS 88).

[A.11] Brown A.K., Lavrakas J.W., *An Overview of Space-Based Radionavigation Systems*, IEEE EASCON 1988, pp. 161-171

[A.12] Shensa M.J., *Geolocation via Satellite - A Methodology and Error Analysis*, DSIS Technical Report 1224, May 1988, Doc. No. AD-A197 955.

[A.13] Hershey W.R. et al, *Ranging and Processing Satellite*, IEEE 1988 Position Location and Navigation Symposium (PLANS 88) Record.

[A.14] Troisi S., *Détermination des Coordonnées d'une Station Mobile par la Méthode des Différences de Temps*, Navigation, No. 142, Avril 1988.

[A.15] Ha T.T., Robertson R.C., *Geostationary Satellite Navigation Systems*, IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-23, No. 2, pp. 247-253, March 1987

[A.16] Hernandez D., *The Locstar Radio - Determination Satellite System*, Alta Frequenza, Vol. LVI, No.10, Dec. 1987.

[A.17] Pearson B.S., *LOCSTAR : Europe's Commercial Radiodetermination Satellite Service*, Satellite Communications and Broadcasting, 1987.

[A.18] Niemeyer P.H., *STARFIND : Single Satellite Geolocation*, Satellite Communications and Broadcasting, 1987.

[A.19] Bukhart R.M., Harris R.R., *Geolocation Through Satellite Propagation Delay Difference*, 1987 IEEE Aerospace Applications conference Digest, Vail, CO, USA, 8-13 Feb. 1987.

[A.20] Dennis A. R., *STARFIX*, IEEE 1986 Position Location and Navigation Symposium Record, pp. 251-255, Nov. 4-7, 1986

[A.21] Dennis A.R., *STARFIX - A New High Precision Satellite Positioning System*, Proc. International Symposium on Marine Positioning : "Positioning the Future", INSMAP 86, US, Geological Survey, Reston, VA, Oct. 14-17, 1986.

[A.22] Wright T.M.B., *The Doppler Compass : Satellites as source of Course-Keeping Data*, NAV 86 : World Wide Navigation into the 21st Century, 1986 Conf. of the Royal Inst. of Navigation, Brighton, England, 30 Sept.-2 Oct. 1986.

[A.23] Schaefer G.J., *Apparatus and Method for Determining the Position of a Radiant Energy Source*, US Patent 4,276,553, June 30,1981.

[A.24] Lecacheux A., Harvey C.C., Boischot A., *Source Localization and Polarization Determination in Low Frequency Satellite Radio Astronomy*, Annales des Télécommunications, Tome 34, No.3-4, Mars-Avril 1979.

[A.25] Sewards A., Winter A.E., Mamen R., *Satellites for Position Determination*, Canadian Aeronautics and Space Journal, Vol. 24, No.5, pp. 266-273, Sept./Oct. 1978.

[A.26] *Application of Satellite Communication and Position Fixing Techniques to Land Mobile Systems*, General Electric Corp. R&D, New York, NTIS PB-267-019, April 1977.

B. COSPAS-SARSAT SYSTEM AND 406 MHZ BEACONS

[B.1] *Specification for COSPAS-SARSAT 406 MHz Distress Beacons*, COSPAS/SARSAT Steering Committee Document C/S T.001, Issue 2, Revision 0, November 1988

[B.2] *COSPAS-SARSAT 406 MHz Distress Beacons Type Approval Standard*, COSPAS/SARSAT Steering Committee Document C/S T.007, November 1987

[B.3] Kennewell J.A., Davies G.D., *Low Cost Exploration of the SARSAT-COSPAS System*, Third National Space Engineering Symposium 1987, Canberra, 30 June-2 July, 1987.

[B.4] Zurabov Y.G., Bogdanov V.A., *Status of the COSPAS-SARSAT Project and its Possible Operation in Conjunction with INMARSAT System*, IEEE PLANS 80 Position Location and Navigation Symposium Record, pp. 459-467, Dec. 8-11, 1980.

C. DISTRESS SIGNAL PROCESSOR

[C.1] Dumont P., Bonnery M. Marguinaud A., *Receiver-Processor for Detection of 406 MHz SARSAT Distress Messages*, AIAA 12th International Communication Satellite Systems Conference, pp. 350-358, March 13-17, 1988

[C.2] Keightley R.J., *Simulation of the Canadian GOES Signal Processor*, IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-23, No.3, pp. 361-370, May 1987

[C.3] Keightley R.J., *Threshold Testing of The Canadian GOES Experimental Processor*, IEEE Pacific Rim Conference of Communications, Computers and Signal Processing, Jan. 4-5, 1987.

[C.4] El-Naga S., Carter C.R., *Identification Techniques for SARSAT Signals*, IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-23, No.2, pp. 199-214, March 1987

[C.5] Chung T., Carter C. R., *Processing of Real ELT Signals for SARSAT*, Can. Elect. Eng. Journal, Vol. 12, No.1, pp. 33-44, 1987

[C.6] Keightley R.J., King J.V., *The Canadian GOES Experiment : the First Six Months*, CRC Space Systems Tech. Memorandum SS#9/87, Nov. 1987.

D. PROPAGATION EFFECTS

[D.1] Lanyi G.E., Roth T., *A Comparison of Mapped and Measured Total Ionospheric Electron Content Using Global Positioning System and Beacon Satellite Services*, Radio Science, Vol. 23, No. 4,pp. 483-492, July/Aug. 1988.

[D.2] Klobuchar J. A., *Ionospheric Time-Delay Algorithm for Single Frequency GPS Users*, IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-23, No.3, pp. 325-331, May 1987.

[D.3] Feess W. A., Stephens S.G., *Evaluation of GPS Ionospheric Time-Delay Model*, IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-23, No.3, pp. 332-338, May 1987.

[D.4] Lyon G.F., Fulford J.A., Forsyth P.A., *Ionospheric Effects on Space Application Systems*, Canadian Aeronautics and Space Journal, Vol. 29, No.4, pp.315-326, Dec. 1983.

[D.5] Soicher H., *Ionospheric and Plasmaspheric effects in Satellite Navigation Systems*, IEEE Trans. on Antennas and Propagation, Vol.AP-25, No.5, Sept. 1977.

[D.6] Klobuchar J.A., *Ionospheric Time Delay Corrections for Advanced Satellite Ranging Systems*, AGARD Conf. Proc. No. 209 on Propagation Limitations of Navigation and Positioning Systems, 1976.

[D.7] Parkinson B.W., Lassister E.M., Cretcher C.K., *Ionospheric Effects in NAVSTAR GPS*, AGARD Conf. Proc. No. 209 on Propagation Limitations of Navigation and Positioning Systems, 1976

E. POSITION ACCURACY AND OTHER TOPICS

[E.1] Saint-Etienne J., *Erreurs de Mesure*, Paper Presented during CNES Course "Location and Navigation Satellite Systems", Toulouse (France), 6-10 March 1989.

[E.2] Krause L. O., *A Direct Solution to GPS-Type Navigation Equations*, IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-23, No.2, March 1987

[E.3] Edwards A.F., Hope D.R., *Apparent Motion of a Geostationary Satellite*, Third National Space Engineering Symposium 1987, Canberra, ACT, Australia, pp. 225-232, 30 June-2 July 1987.

[E.4] Snively L.O., Osborne W.P., *Analysis of the GEOSTAR Position Determination System*, AIAA 11th International Communication Satellite Systems Conference, pp. 50-58, March 1986.

[E.5] Richards R.T., Snively L.O., *GEOSTAR Positioning Analysis*, 1986 Position Location and Navigation Symposium Record, pp. 13-19, Nov. 4-7, 1986

[E.6] Zarrouati O., *A Simple Analytic Method for the Evaluation of the Performance of a Radiolocation System*, ACTA Astronautica, Vol.15, No.1, pp. 35-44, 1987.

[E.7] Nauck J., *SERES : A Search and Rescue Satellite System in Addition to INMARSAT*, Earth-Oriented Applications of Space Technology, Vol.3, Nos.3/4, pp. 197-202, 1983.

[E.8] Nauck J., *"SERES" - A Polar Orbiting Maritime Search and Rescue Satellite System as a Possible Extension of INMARSAT*, Third Int. Conf. on Satellite Communications and Navigation, London, UK, 7-9 June 1983.

[E.9] Bommas G., Kesenheimer H., Frotsch B.H., *The Implementation of a Satellite EPIRB Within the German SAR-Program*, Third Int. Conf. on Satellite Communications and Navigation, London, UK, 7-9 June 1983.

[E.10] Braff R., Shively C.A., Zelster M.J., *Radionavigation System Integrity and Reliability*, Proc. of the IEEE, Vol. 71, No.10, pp. 1214-1223, Oct. 1983.

[E.11] Goebel W., Landauer G., Messerschmid E.,*Maritime Distress Radio Call Systems Employing Space-Borne Technology*, ACTA Astronautica, Vol.9, No.1, pp. 35-44, 1982.

[E.12] Slabinsky V.J., *Variation in Range, Range-Rate, Propagation Time Delay, and Doppler Shift for Nearly Geostationary Satellite*, in Communications Satellite Technology, edited by P.L. Bargellini, Alpine Press Inc., USA, 1974.

APPENDIX A

COMPUTER PROGRAM TO COMPUTE THE POSITION LINE GENERATED BY A GIVEN DIFFERENTIAL RANGE FOR TWO GIVEN SATELLITES

LISTING OF PROGRAM 'DIFF RANGE XYZ.F'

```
C----------------------------------------------
C
C     Program to compute the line of constant range
C  difference given two satellites on geostationary orbits.
C
C
C  Requires the following files :
C            RANDELT.F : to compute the ground station longitude
C                        given its latitude and its diff. range to two
C                        given satellites
C           'WGS CONVERSION.F' : to convert from WGS-84
C                                (lat., long.) coordinates to (x,y,z)
C           'Diff Range XYZ.make' : to compile and link the program
C
C
C  It is strongly recommended to use the option -extended
C  during the compilation to get the maximum accuracy.
C
C  NOTE : This program works fine assuming the beacon is in Canada
C         and the satellites are over Canada and do not cross the
C         180 degrees meridian
C
C
C                                 M. CARON
C                                 DEC. 1989
C                                 Revised Sept. 1990
C
C----------------------------------------------
C
        PROGRAM DIF_RANGEXYZ
        IMPLICIT DOUBLE PRECISION (A-H,L,O-Z)
        DIMENSION SATX(2),SATY(2),SATZ(2)
        CHARACTER FILEN*80,ANS*1
C
C  this common block passes the semi-major axis and squared eccentricity
C  to the WGS CONVERSION.F routines
C
        COMMON /DISTDATA/ AE,ECC2
C
C Semi-major axis and squared eccentricity according to WGS-84
C npt : number of points on the line crossing the North hemisphere
C
        AE = 6378137.D+00
        ECC2 = 6.694379990D-3
        npt=100
C
        WRITE (6,1)
  1     FORMAT (/,/,/,T16,51('*'),/,T16,'*',49X,'*',/,T16,
     &  '* POSITION ESTIMATION BASED ON DIFFERENTIAL RANGE *',
     &/,T16,'*',49X,'*',/,T16,51('*'),/,/,/,/,T5,
     & 'NOTE : Longitudes must be within [-180,180] (+ve=East)',/
     &,T5,'        Latitude must be within [-90,90] (+ve = North)'
     & ,/,/,/)
C
C INPUT VARIABLES
C
  5    WRITE (6,10)
  10   FORMAT (/,T3,'Satellite lat., long. #1 and #2'
```

```
     &    ,/,T3,'Satellite #1 must be west of satellite #2')
      READ (5,*) SLAT1,SLON1,SLAT2,SLON2
      IF ((DABS(SLON1) .GT.180.D+00).OR.(DABS(SLON2).GT.180.D+00)) THEN
            WRITE (6,*) '?? Longitudes must be within +/- 180 deg. ??'
            GOTO 5
       END IF
       IF ((DABS(SLAT1).GT.90.D+00).OR.(DABS(SLAT2).GT.90.D+00)) THEN
            WRITE(6,*) ' ?? Latitudes must be within +/- 90 deg. ??'
            GOTO 5
      END IF
C
 20   WRITE (6,30)
 30   FORMAT (T3,'Range difference (#2 minus #1) in meters')
      READ (5,*) RDELTA
       IF(RDELTA.EQ.0.D+00) THEN
            WRITE(6,*)'?? CANNOT BE ZERO ??'
            GOTO 20
       END IF
C
 40   WRITE (6,50)
 50   FORMAT (T3,'Range difference accuracy in meters')
      READ (5,*) ACC
      IF (ACC.LE.0.D+00) THEN
            WRITE (6,*) '?? MUST BE GREATER THAN ZERO ??'
            GOTO 40
      END IF
C
 51   WRITE (6,52)
 52   FORMAT (T3,'File name where to dump data')
      READ (5,*) FILEN
      OPEN (UNIT=98,FILE=FILEN,STATUS='UNKNOWN')
      REWIND (98)
C
C  compute the satellite altitude above the earth
C  approximate to geo. orbit minus earth semi-major axis
C
        HSAT = 42157197.D+00-AE
C
C  convert satellite lat, long. coordinates to (x,y,z)
C
        CALL XYZ (SLAT1,SLON1,HSAT,SATX(1),SATY(1),SATZ(1))
        CALL XYZ (SLAT2,SLON2,HSAT,SATX(2),SATY(2),SATZ(2))
C
      WRITE (6,60)
 60   FORMAT (/,/,T3,'  #  ',5X,'LATITUDE',5X,'LONGITUDE',/)
C
C  knowing that the beacon is on the north hemisphere
C  scan the north hemisphere in 'npt' latitude steps
C
        flatinc = 82.d+00/float(npt-1)
        WRITE (98,*) NPT
       DO 100 I=0,npt-1
C
C  set the initial guess for the longitude to the west most point
C  The subroutine RANDELT will look for the point 10 degrees around the
C  initial guess
C  xlat is the ground station latitude
C
```

LISTING OF PROGRAM 'DIFF RANGE XYZ.F' CONTINUED

```
         XLON = -170.D+00
         XLAT = DFLOAT(I)*flatinc
C
C  find the longitude of the station that generates the range difference
C  of interest for the latitude given by xlat
C
   65   CALL RANDELT (RDELTA,1,2,SATX,SATY,SATZ,XLAT,
     &    ACC,XLON,GLON,IER)
C
C  if IER = 2 then the subroutine could not reach the required accuracy
C   with step size as small as 1.d-10 degree
C  if IER=3 that means that the routine failed to find a solution
C   (this should not happen unless there is erronous entries)
C  if IER=0 means no error occured
C  If the accuracy has not been reached, then set IC = 1
C
        IF(IER.LT.3 .AND. IER.NE.0) THEN
             WRITE (6,*) 'IER=',IER
             IC=1
        ELSE
             IC=0
        END IF
        IF(IER.EQ.3) GOTO 100
C
C  compute the longitude in deg. min. sec.
C
        XLON = GLON
        XDEG=DFLOAT(IDINT(DABS(XLON)))
        XMIN = DFLOAT(IDINT((DABS(XLON)-XDEG)*60.))
        XSEC = (DABS(XLON) - XDEG - XMIN/60.)*3600.
        WRITE (6,70) (I+1),XLAT,XLON,XDEG,XMIN,XSEC,ic
   70   FORMAT (T3,I5,5X,F8.2,5X,F15.7,3X,F5.0,' DEG.   ',
     &     F3.0,' min. ',F8.4,' sec.',2x,i2)
C
        WRITE (98,75) XLAT,XLON
   75     format(f,2x,f)
C
  100   CONTINUE
C
        WRITE (98,62) SLAT1,SLON1,SLAT2,SLON2,RDELTA,ACC
   62   FORMAT (/,T3,'DIFF RANGE XYZ',/,T3,
     &   'Sat. geocentric lat., long. #1, #2 : ',2(F7.2,',',F8.2),/,
     $   T3,'Range differences (meters) : ',F15.3,/,T3,
     $      'Accuracy Required (meters) : ',F15.7,/)
C
C  write an end of file to the file and close it
C
          ENDFILE(98)
        CLOSE (98)
        WRITE (6,200)
  200   FORMAT (T3,'Another run ? (Y or N)')
        READ (5,*) ANS
        IF(ANS.NE.'Y' .AND. ANS.NE.'y') STOP 'Tourlou !'
        GOTO 5
        END
```

```
C-----------------------------------------------
C
C      SUBROUTINE TO COMPUTE THE LONGITUDE OF THE GROUND
C   STATION LOCATED AT LATITUDE XLAT WHICH GIVES A DIFFERENTIAL
C   RANGE OF RIJ BETWEEN THE TWO (I AND J) SATELLITES LOCATED AT
C   SATX(I),SATY(I),SATZ(I) AND SATX(J),SATY(J),SATZ(J).
C
C        RIJ  : range difference in meters = Range to
C               sat.#j - range to sat#i
C        I,J  : satellite numbers
C        SATX,SATY,SATZ : (x,y,z) coordinates of the two satellites
C        XLAT : latitude in degrees   (+ve North, -ve = South)
C        ACC : required range difference accuracy in meters
C        XLON : initial guess on longitude in degrees
C               ( +ve=East, -ve=West)
C               It is suggested to use XLON = -170. if a good guess
C               is not known
C        GLON : the longitude of the point which generated the
C               required differential range with accuracy ACC
C               at latitude XLAT
C        IER  : is an error code
C               set to 0 when no errors,
C                   to 1 when there is (are) error(s) in the arguments
C                   to 2 when the accuracy could not be reached with
C                        step size as small as 1.d-10 degree,
C                   to 3 when no solutions could be found.
C
C   NOTE : All real variables are double precision
C
C                                         M. CARON
C                                         MAY 1989
C
C-----------------------------------------------
C

       SUBROUTINE RANDELT (RIJ,I,J,SATX,SATY,SATZ,XLAT,
     &                     ACC,XLON,GLON,IER)
C

       IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DIMENSION SATX(1),SATY(1),SATZ(1),SLAT(2),SLON(2)
C
C   The following function computes the straight line distance
C   between two points given by (sx,sy,sz) and (x,y,z)
C

       DISTANC(SX,SY,SZ,X,Y,Z)=DSQRT((SX-X)**2+(SY-Y)**2+(SZ-Z)**2)
C
C   check for argument errors
C

       IER = 0
       IF(RIJ.EQ.0.D+00 .OR. DABS(XLAT).GT.90.D+00 .OR.
     & DABS(XLON).GT.180.D+00 .OR. ACC.LE.0.D+00) THEN
            IER = 1
            RETURN
       END IF
C

       DELTA = RIJ
C
C   MAKE SURE THE SAT.#IS IS WEST TO THE SECOND SAT.
C   the following test works only when the line joining the
```

```
C  two satellites does not cross the 180 deg. meridian line
C  First convert the satellite (x,y,z) coordinates to geodetic
C  lat., long coordinates
C
         CALL GEODETIC (SATX(I),SATY(I),SATZ(I),SLAT(1),SLON(1),HT)
         CALL GEODETIC (SATX(J),SATY(J),SATZ(J),SLAT(2),SLON(2),HT)
c
c set IS to the west most satellite
C
         IS = I
         IFF =J
         IF(SLON(1).GT.180.D+00) SLON(1)=SLON(1)-360.D+00
         IF(SLON(2).GT.180.D+00) SLON(2)=SLON(2)-360.D+00
         IF (SLON(1).GT.SLON(2)) THEN
              IS=J
              IFF=I
         END IF
C
C        WRITE (6,*) 'is,iff,i,j =',IS,IFF,I,J
C
C if satellites have been interchanged then change the sign
C of the difference
C
         IF(IS.NE.I) DELTA = -DELTA
C
C  set the step size to 1 degree initially
C  DLON is set to 10 degrees west of the initial guess longitude
C
        DSTEP = 1.D+00
        DLON=XLON - 10.D+00
C
 50     CONTINUE
C
        GLON=DLON
C
C COMPUTE THE RANGES
C First compute the (x,y,z) coordinates of the location given
C by XLAT and DLON
C The XYZ routine works with longitudes defined between
C 0 to 360 degrees
C
         TLON = DLON
         IF(DLON.LT.0.D+00) TLON=DLON+360.D+00
         CALL XYZ (XLAT,TLON,0.D+00,X,Y,Z)
C
         RI=DISTANC (SATX(IFF),SATY(IFF),SATZ(IFF),X,Y,Z)
         RJ=DISTANC (SATX(IS),SATY(IS),SATZ(IS),X,Y,Z)
C
        DIF2 = RJ - RI
C
C        write (6,*) 'dif2, rj, ri = ',DIF2,RJ,RI
C        PAUSE
C
C  because satellite #IFF is east of satellite #IS and
C  we take R(is) - R(iff) and the longitude is scanned
C  from west to east, the differential range will
C  be negative and will gradually increase with the
C  increase of longitude
```

```
C
        IF (DIF2.GE.0.D+00 .AND. DELTA.GT.0.D+00) THEN
C
C If differential range greater than the required one
C then we passed the point.  Come back on the previous point,
C decrease the step
C size and increase the longitude by the new step size
C otherwise check if less than the required one
C if less then increase the longitude.
C
             IF(DIF2.GT.(DELTA+ACC)) THEN
                  DLON=DLON-DSTEP
                  DSTEP=DSTEP/2.D+00
                  IF(DSTEP.LT.1.D-10) THEN
                       IER=2
                       RETURN
                  END IF
                  DLON = DLON + DSTEP
             ELSE IF (DIF2.GE.(DELTA-ACC)) THEN
                  RETURN
             ELSE
                  DLON=DLON+DSTEP
             END IF
C
        ELSE IF (DIF2.LT.0.D+00 .AND. DELTA.GT.0.D+00) THEN
C
C if outside the allowed accuracy then increase the longitude
C to converge toward positive DIF2
C
             IF (DIF2.LT.(DELTA-ACC)) THEN
                  DLON = DLON + DSTEP
             ELSE
                  RETURN
             END IF
C
        ELSE IF(DIF2.GE.0.D+00 .AND. DELTA.LT.0.D+00) THEN
C
C if outside the allowed accuracy, then we passed the point
C  Come back on the previous point, decrease the step size and
C  increase the longitude by the new step size
C
             IF(DIF2.GT.(DELTA+ACC)) THEN
                  DLON=DLON-DSTEP
                  DSTEP=DSTEP/2.D+00
                  IF(DSTEP.LT.1.D-10) THEN
                       IER=2
                       RETURN
                  END IF
                  DLON = DLON + DSTEP
             ELSE
                  RETURN
             END IF
C
        ELSE IF (DIF2.LT.0.D+00 .AND. DELTA.LT.0.D+00) THEN
C
C if outside the allowed accuracy and the differential range is
C less than the required one, then increase the longitude
C if the differential range is greater than the required one
```

```
C  then we passed the point.  Come back on the previous point,
C  decrease the step size and increase the longitude by the new
C  step size
C
            IF(DIF2.LT.(DELTA-ACC)) THEN
                 DLON=DLON+DSTEP
            ELSE IF (DIF2.GT.(DELTA+ACC)) THEN
                 DLON=DLON-DSTEP
                 DSTEP=DSTEP/2.D+00
                 IF(DSTEP.LT.1.D-10) THEN
                      IER=2
                      RETURN
                 END IF
                 DLON = DLON + DSTEP
             ELSE
                    RETURN
             END IF
        END IF
C
C if longitude is greater than 180 degrees then we failed to
C find a solution
C
         IF(DLON.LE.180.0D+00) GOTO 50
        IER = 3
        RETURN
        END
```

```
c==========================================================================
c
      subroutine xyz(lat,long,ht,x,y,z)
c
c       This subroutine converts WGS84 lat,long,height
c       above ref. elipsoid to EFEC x y z co-ord.
c       Based on Olsen, Journal of Guidance, Control,
C         & Dynamics, Mar/Apr 1988, pp 188-190.
c
c       lat, long   : geodetic  latitude and longitude of point in
c                     WGS-84 latitudes must be within +/- 90 degrees
c                     positive North longitudes must be within 0 to
C                     360 degrees east
c       ht          : height above the reference ellipsoid in meters
c       x,y,z       : coordinates of the point in meters from center
C                     of the Earth
c
c       written by D. Hindson Oct 1989
c
c Modified by M. Caron  Dec. 1989
c
      implicit double precision (a-z)
        COMMON /DISTDATA/ ae,e2
C
      dtor=datan(1.D0)/45.D0
      latr=lat*dtor
      longr=long*dtor
C
      re=ae/dsqrt(1.D0-e2*(dsin(latr))**2.D0)
      x=(re+ht)*dcos(latr)*dcos(longr)
      y=(re+ht)*dcos(latr)*dsin(longr)
      z=((1.D0-e2)*re+ht)*dsin(latr)
      return
      end
C--------------------------------------------------------------------------
      subroutine geodetic(x,y,z,ylt,xln,ht)
c
c       This subroutine converts from x y z ECEF co-ords to WGS84
c       geodetic position.
c       Algorithm from Olsen Jornal of Guidance, Control,
C         and Dynamics, Mar/Apr 1988 pp188-190.
c
c       written by D. Hindson  Oct 1989
c
      implicit double precision (a-z)
        COMMON /DISTDATA/ ae,e2
C
c WGS84 constants
c
c equatorial radius
c
C       ae=6378137.D0
c
c elipsicity squared
c
C       e2=.006694379990D0
C
      al=ae*e2
```

```
      a2=e2/2.D0
      a3=a2/2.D0
      a4=a1**2.D0/2.D0
      a5=a1/2.D0
      a6=1.D0-e2
      xln=datan2(y,x)
      w2=x**2.D0+y**2.D0
      w=dsqrt(w2)
      r2=w2+z**2.D0
      r=dsqrt(r2)
      s22=z**2.D0/r2
      d1=a1/r
      d2=d1+1.D0
      c=a5*s22*(d2+s22*(a3-d1))
      ht=r-ae+c
      s2=z/r
      c2=w/r
      s1=d1*s2*c2*(d2-s22*(2.D0*d1-a2))
      c1=1.D0-(a4/r2)*s22*(1.D0-s22)
      s=s1*c2+c1*s2
      ylt=dasin(s)
      ss=s**2.D0
      rr=1.D0-e2*ss
      re=ae/dsqrt(rr)
      rf=a6*re
      c=dsqrt(1.D0-ss)
      w1=(re+ht)*c
      z1=(rf+ht)*s
      dw=w-w1
      dz=z-z1
      dl=-s*dw+c*dz
      dh=c*dw+s*dz
      ylt=ylt+dl/(rf/rr+ht)
      ht=ht+dh
        dtor = datan(1.0d+0)/45.d+00
        ylt = ylt / dtor
        xln = xln / dtor
      return
      end
```

LISTING OF SCRIPT FILE TO COMPILE AND LINK

```
# File:     'Diff Range XYZ.make'
# Target:   'Diff Range XYZ'
# Sources:  'DIFF RANGE XYZ.F' randelt.f 'WGS84 CONVERSION.F'
# Created:  Friday, February 2, 1990 10:27:15 AM

'DIFF RANGE XYZ.F.o' ƒ 'Diff Range XYZ.make' 'DIFF RANGE XYZ.F'
        FORTRAN -3 -mc68881 -opt=3 -extended 'DIFF RANGE XYZ.F'
randelt.f.o ƒ 'Diff Range XYZ.make' randelt.f
        FORTRAN -3 -mc68881 -opt=3 -extended randelt.f
'WGS84 CONVERSION.F.o' ƒ 'Diff Range XYZ.make' 'WGS84 CONVERSION.F'
        FORTRAN -3 -mc68881 -opt=3 -extended 'WGS84 CONVERSION.F'

SOURCES = 'DIFF RANGE XYZ.F' randelt.f 'WGS84 CONVERSION.F'
OBJECTS = 'DIFF RANGE XYZ.F.o' randelt.f.o 'WGS84 CONVERSION.F.o'

'Diff Range XYZ' ƒƒ 'Diff Range XYZ.make' {OBJECTS}
        Link -ad 4 -w -t APPL -c '????' ∂
                {OBJECTS} ∂
                "{Libraries}"Runtime.o ∂
                "{Libraries}"Interface.o ∂
                "{FLibraries}"FORTRANlib.o ∂
                "{FLibraries}"IntrinsicLib.o ∂
                -o 'Diff Range XYZ'
```

RUN EXAMPLE

```
*******************************************************
*                                                     *
* POSITION ESTIMATION BASED ON DIFFERENTIAL RANGE *
*                                                     *
*******************************************************
```

NOTE : Longitudes must be within [-180,180] (+ve=East)
Latitude must be within [-90,90] (+ve = North)

Satellite lat., long. #1 and #2
Satellite #1 must be west of satellite #2
0,-135,0,-75
Range difference (#2 minus #1) in meters
215000
Range difference accuracy in meters
1
File name where to dump data
demo.dat

#	LATITUDE	LONGITUDE				
1	0.00	-103.3149719	103. DEG.	18. min.	53.8989 sec.	0
2	0.83	-103.3147736	103. DEG.	18. min.	53.1848 sec.	0
3	1.66	-103.3141479	103. DEG.	18. min.	50.9326 sec.	0
4	2.48	-103.3131256	103. DEG.	18. min.	47.2522 sec.	0
5	3.31	-103.3116913	103. DEG.	18. min.	42.0886 sec.	0
6	4.14	-103.3098297	103. DEG.	18. min.	35.3870 sec.	0
7	4.97	-103.3075714	103. DEG.	18. min.	27.2571 sec.	0
8	5.80	-103.3048859	103. DEG.	18. min.	17.5891 sec.	0
9	6.63	-103.3017731	103. DEG.	18. min.	6.3831 sec.	0
10	7.45	-103.2982483	103. DEG.	17. min.	53.6938 sec.	0
11	8.28	-103.2942810	103. DEG.	17. min.	39.4116 sec.	0
12	9.11	-103.2899017	103. DEG.	17. min.	23.6462 sec.	0
13	9.94	-103.2850800	103. DEG.	17. min.	6.2878 sec.	0
14	10.77	-103.2798004	103. DEG.	16. min.	47.2815 sec.	0
15	11.60	-103.2740936	103. DEG.	16. min.	26.7371 sec.	0
16	12.42	-103.2679291	103. DEG.	16. min.	4.5447 sec.	0
17	13.25	-103.2612915	103. DEG.	15. min.	40.6494 sec.	0
18	14.08	-103.2541962	103. DEG.	15. min.	15.1062 sec.	0
19	14.91	-103.2466278	103. DEG.	14. min.	47.8601 sec.	0
20	15.74	-103.2385712	103. DEG.	14. min.	18.8562 sec.	0
21	16.57	-103.2300262	103. DEG.	13. min.	48.0945 sec.	0
22	17.39	-103.2209778	103. DEG.	13. min.	15.5200 sec.	0
23	18.22	-103.2114105	103. DEG.	12. min.	41.0779 sec.	0
24	19.05	-103.2013245	103. DEG.	12. min.	4.7681 sec.	0
25	19.88	-103.1907043	103. DEG.	11. min.	26.5356 sec.	0

RUN EXAMPLE (CONTINUED)

26	20.71	-103.1795197	103. DEG.	10. min.	46.2708 sec.	0
27	21.54	-103.1677856	103. DEG.	10. min.	4.0283 sec.	0
28	22.36	-103.1554718	103. DEG.	9. min.	19.6985 sec.	0
29	23.19	-103.1425781	103. DEG.	8. min.	33.2812 sec.	0
30	24.02	-103.1290588	103. DEG.	7. min.	44.6118 sec.	0
31	24.85	-103.1149292	103. DEG.	6. min.	53.7451 sec.	0
32	25.68	-103.1001434	103. DEG.	6. min.	0.5164 sec.	0
33	26.51	-103.0846863	103. DEG.	5. min.	4.8706 sec.	0
34	27.33	-103.0685730	103. DEG.	4. min.	6.8628 sec.	0
35	28.16	-103.0517273	103. DEG.	3. min.	6.2183 sec.	0
36	28.99	-103.0341644	103. DEG.	2. min.	2.9919 sec.	0
37	29.82	-103.0158539	103. DEG.	0. min.	57.0740 sec.	0
38	30.65	-102.9967499	102. DEG.	59. min.	48.2996 sec.	0
39	31.47	-102.9768524	102. DEG.	58. min.	36.6687 sec.	0
40	32.30	-102.9561157	102. DEG.	57. min.	22.0166 sec.	0
41	33.13	-102.9344940	102. DEG.	56. min.	4.1785 sec.	0
42	33.96	-102.9119873	102. DEG.	54. min.	43.1543 sec.	0
43	34.79	-102.8885193	102. DEG.	53. min.	18.6694 sec.	0
44	35.62	-102.8640900	102. DEG.	51. min.	50.7239 sec.	0
45	36.44	-102.8386536	102. DEG.	50. min.	19.1528 sec.	0
46	37.27	-102.8121338	102. DEG.	48. min.	43.6816 sec.	0
47	38.10	-102.7845154	102. DEG.	47. min.	4.2554 sec.	0
48	38.93	-102.7557526	102. DEG.	45. min.	20.7092 sec.	0
49	39.76	-102.7257690	102. DEG.	43. min.	32.7686 sec.	0
50	40.59	-102.6945190	102. DEG.	41. min.	40.2686 sec.	0
51	41.41	-102.6619568	102. DEG.	39. min.	43.0444 sec.	0
52	42.24	-102.6279907	102. DEG.	37. min.	40.7666 sec.	0
53	43.07	-102.5925598	102. DEG.	35. min.	33.2153 sec.	0
54	43.90	-102.5556030	102. DEG.	33. min.	20.1709 sec.	0
55	44.73	-102.5170288	102. DEG.	31. min.	1.3037 sec.	0
56	45.56	-102.4767456	102. DEG.	28. min.	36.2842 sec.	0
57	46.38	-102.4346924	102. DEG.	26. min.	4.8926 sec.	0
58	47.21	-102.3907166	102. DEG.	23. min.	26.5796 sec.	0
59	48.04	-102.3447571	102. DEG.	20. min.	41.1255 sec.	0
60	48.87	-102.2966614	102. DEG.	17. min.	47.9810 sec.	0
61	49.70	-102.2463379	102. DEG.	14. min.	46.8164 sec.	0
62	50.53	-102.1936035	102. DEG.	11. min.	36.9727 sec.	0
63	51.35	-102.1383667	102. DEG.	8. min.	18.1201 sec.	0
64	52.18	-102.0804138	102. DEG.	4. min.	49.4897 sec.	0
65	53.01	-102.0195618	102. DEG.	1. min.	10.4224 sec.	0
66	53.84	-101.9556580	101. DEG.	57. min.	20.3687 sec.	0
67	54.67	-101.8884888	101. DEG.	53. min.	18.5596 sec.	0
68	55.49	-101.8177795	101. DEG.	49. min.	4.0063 sec.	0
69	56.32	-101.7433167	101. DEG.	44. min.	35.9399 sec.	0
70	57.15	-101.6647949	101. DEG.	39. min.	53.2617 sec.	0
71	57.98	-101.5819092	101. DEG.	34. min.	54.8730 sec.	0
72	58.81	-101.4942932	101. DEG.	29. min.	39.4556 sec.	0
73	59.64	-101.4015808	101. DEG.	24. min.	5.6909 sec.	0
74	60.46	-101.3033447	101. DEG.	18. min.	12.0410 sec.	0
75	61.29	-101.1991272	101. DEG.	11. min.	56.8579 sec.	0
76	62.12	-101.0883789	101. DEG.	5. min.	18.1641 sec.	0
77	62.95	-100.9704895	100. DEG.	58. min.	13.7622 sec.	0
78	63.78	-100.8447876	100. DEG.	50. min.	41.2354 sec.	0
79	64.61	-100.7105408	100. DEG.	42. min.	37.9468 sec.	0

RUN EXAMPLE (CONTINUED)

80	65.43	-100.5668335	100. DEG.	34. min.	0.6006 sec.	0
81	66.26	-100.4127197	100. DEG.	24. min.	45.7910 sec.	0
82	67.09	-100.2470703	100. DEG.	14. min.	49.4531 sec.	0
83	67.92	-100.0685425	100. DEG.	4. min.	6.7529 sec.	0
84	68.75	-99.8756409	99. DEG.	52. min.	32.3071 sec.	0
85	69.58	-99.6666260	99. DEG.	39. min.	59.8535 sec.	0
86	70.40	-99.4394226	99. DEG.	26. min.	21.9214 sec.	0
87	71.23	-99.1916504	99. DEG.	11. min.	29.9414 sec.	0
88	72.06	-98.9203491	98. DEG.	55. min.	13.2568 sec.	0
89	72.89	-98.6221924	98. DEG.	37. min.	19.8926 sec.	0
90	73.72	-98.2929688	98. DEG.	17. min.	34.6875 sec.	0
91	74.55	-97.9277344	97. DEG.	55. min.	39.8437 sec.	0
92	75.37	-97.5201416	97. DEG.	31. min.	12.5098 sec.	0
93	76.20	-97.0626221	97. DEG.	3. min.	45.4395 sec.	0
94	77.03	-96.5454102	96. DEG.	32. min.	43.4766 sec.	0
95	77.86	-95.9561157	95. DEG.	57. min.	22.0166 sec.	0
96	78.69	-95.2786255	95. DEG.	16. min.	43.0518 sec.	0
97	79.52	-94.4915771	94. DEG.	29. min.	29.6777 sec.	0
98	80.34	-93.5660400	93. DEG.	33. min.	57.7441 sec.	0
99	81.17	-92.4617920	92. DEG.	27. min.	42.4512 sec.	0
100	82.00	-91.1215820	91. DEG.	7. min.	17.6953 sec.	0

Another run ? (Y or N)
n

STOP Tourlou !

```
 100
0.000000000000000000     -103.314971923828125000
0.828282828282828283     -103.314773559570312500
1.656565656565656566     -103.314147949218750000
2.484848484848484848     -103.313125610351562500
3.313131313131313131     -103.311691284179687500
4.141414141414141414     -103.309829711914062500
4.969696969696969696     -103.307571411132812500
5.797979797979797980     -103.304885864257812500
6.626262626262626262     -103.301773071289062500
7.454545454545454546     -103.298248291015625000
8.282828282828282828     -103.294281005859375000
9.111111111111111112     -103.289901733398437500
9.939393939393939393     -103.285079956054687500
10.767676767676767680    -103.279800415039062500
11.595959595959595960    -103.274093627929687500
12.424242424242424240    -103.267929077148437500
13.252525252525252530    -103.261291503906250000
14.080808080808080810    -103.254196166992187500
14.909090909090909090    -103.246627807617187500
15.737373737373737370    -103.238571166992187500
16.565656565656565660    -103.230026245117187500
17.393939393939393940    -103.220977783203125000
18.222222222222222220    -103.211410522460937500
19.050505050505050500    -103.201324462890625000
19.878787878787878790    -103.190704345703125000
20.707070707070707070    -103.179519653320312500
21.535353535353535350    -103.167785644531250000
22.363636363636363640    -103.155471801757812500
23.191919191919191920    -103.142578125000000000
24.020202020202020200    -103.129058837890625000
24.848484848484848480    -103.114929199218750000
25.676767676767676770    -103.100143432617187500
26.505050505050505050    -103.084686279296875000
27.333333333333333330    -103.068572998046875000
28.161616161616161620    -103.051727294921875000
28.989898989898989900    -103.034164428710937500
29.818181818181818180    -103.015853881835937500
30.646464646464646460    -102.996749877929687500
31.474747474747474750    -102.976852416992187500
32.303030303030303030    -102.956115722656250000
33.131313131313131310    -102.934494018554687500
33.959595959595959600    -102.911987304687500000
34.787878787878787880    -102.888519287109375000
35.616161616161616160    -102.864089965820312500
36.444444444444444440    -102.838653564453125000
37.272727272727272730    -102.812133789062500000
38.101010101010101010    -102.784515380859375000
38.929292929292929290    -102.755752563476562500
39.757575757575757580    -102.725769042968750000
40.585858585858585860    -102.694519042968750000
41.414141414141414140    -102.661956787109375000
42.242424242424242420    -102.627990722656250000
43.070707070707070710    -102.592559814453125000
```

43.898989898989898990	-102.555603027343750000
44.727272727272727270	-102.517028808593750000
45.555555555555555560	-102.476745605468750000
46.383838383838383840	-102.434692382812500000
47.212121212121212120	-102.390716552734375000
48.040404040404040400	-102.344757080078125000
48.868686868686868680	-102.296661376953125000
49.696969696969696960	-102.246337890625000000
50.525252525252525260	-102.193603515625000000
51.353535353535353540	-102.138366699218750000
52.181818181818181820	-102.080413818359375000
53.010101010101010100	-102.019561767578125000
53.838383838383838380	-101.955657958984375000
54.666666666666666660	-101.888488769531250000
55.494949494949494940	-101.817779541015625000
56.323232323232323240	-101.743316650390625000
57.151515151515151520	-101.664794921875000000
57.979797979797979800	-101.581909179687500000
58.808080808080808080	-101.494293212890625000
59.636363636363636360	-101.401580810546875000
60.464646464646464640	-101.303344726562500000
61.292929292929292920	-101.199127197265625000
62.121212121212121220	-101.088378906250000000
62.949494949494949500	-100.970489501953125000
63.777777777777777780	-100.844787597656250000
64.606060606060606060	-100.710540771484375000
65.434343434343434340	-100.566833496093750000
66.262626262626262620	-100.412719726562500000
67.090909090909090900	-100.247070312500000000
67.919191919191919200	-100.068542480468750000
68.747474747474747480	-99.875640869140625000
69.575757575757575760	-99.666259765625000000
70.404040404040404040	-99.439422607421875000
71.232323232323232320	-99.191650390625000000
72.060606060606060600	-98.920349121093750000
72.888888888888888880	-98.622192382812500000
73.717171717171717160	-98.292968750000000000
74.545454545454545460	-97.927734375000000000
75.373737373737373740	-97.520141601562500000
76.202020202020202020	-97.062622070312500000
77.030303030303030300	-96.545410156250000000
77.858585858585858580	-95.956115722656250000
78.686868686868686860	-95.278625488281250000
79.515151515151515160	-94.491577148437500000
80.343434343434343440	-93.566040039062500000
81.171717171717171720	-92.461791992187500000
82.000000000000000000	-91.121582031250000000

DIFF RANGE XYZ
Sat. geocentric lat., long. #1, #2 : 0.00, -135.00 0.00, -75.00
Range differences (meters) : 215000.000
Accuracy Required (meters) : 1.0000000

APPENDIX B

COMPUTER PROGRAM TO COMPUTE THE SHORTEST DISTANCE BETWEEN TWO POINTS ON THE EARTH

LISTING OF PROGRAM DISVINCENTY.F

```
      program disvincenty
c
c     This program calculates geodetic distance using Vincenty's Inverse
c     algorithm. Based on paper presented at 39th annual meeting of ION
c     by L. Pfeifer pp515-524.
c
c     written by D. Hindson
c     May 1 1990
c

      implicit double precision (a-z)
      integer ans,fdeg,fmin,bdeg,bmin
      character*1 msym,ssym
      msym=char(39)
      ssym=char(34)
      pi=4.d0*datan(1.d0)
      dtor=pi/180.d0
      write(6,*)'Do you want to use WGS84 reference ellipsoid (y=1)'
      read(5,*) ans
      if (ans.eq.1) then
c
c     WGS84 parameters
c

            a=6378137.d0
            finv=298.257223563d0
      else
            write(6,*)'Enter ellipsoid semi-major axis (m)'
            read(5,*) a
            write(6,*)'Enter 1/flattening of ellipsoid'
            read(5,*) finv
      endif
c
c     Input section
c
300   write(6,*)'Do you want to enter lat. longs. in degrees (1) or d m s (2)'
      read(5,*) ans
      if (ans.eq.1) then
            write(6,*)'Input start latitude (deg. N +ve)'
            read(5,*) lats
            write(6,*)'Input start longitude (deg. E +ve)'
            read(5,*) longs
            write(6,*)'Input finish latitude (deg. N +ve)'
            read(5,*) latf
            write(6,*)'Input finish longitude (deg. E +ve)'
            read(5,*) longf
      else
            write(6,*)'Input start latitude (deg. min. sec.  N +ve)'
            read(5,*) lats,min,sec
            if (lats.lt.0.d0) then
                  lats=lats-min/60.d0-sec/3600.d0
            else
                  lats=lats+min/60.d0+sec/3600.d0
            endif
```

```
              write(6,*)'Input start longitude (deg. min. sec.  E +ve)'
              read(5,*) longs,min,sec
              if (longs.lt.0.d0) then
                    longs=longs-min/60.d0-sec/3600.d0
              else
                    longs=longs+min/60.d0+sec/3600.d0
              endif
              write(6,*)'Input finish latitude (deg. min. sec.  N +ve)'
              read(5,*) latf,min,sec
              if (latf.lt.0.d0) then
                    latf=latf-min/60.d0-sec/3600.d0
              else
                    latf=latf+min/60.d0+sec/3600.d0
              endif
              write(6,*)'Input finish longitude (deg. min. sec.  E +ve)'
              read(5,*) longf,min,sec
              if (longf.lt.0.d0) then
                    longf=longf-min/60.d0-sec/3600.d0
              else
                    longf=longf+min/60.d0+sec/3600.d0
              endif
        endif
c
c       convert lat long to radians
c       note: algorithm uses W +ve thud -ve sign on long
c
        lats=lats*dtor
        longs=-longs*dtor
        latf=latf*dtor
        longf=-longf*dtor
c
c       call subroutine dis to calc distance and azimuths
c
        call dis(a,finv,lats,longs,latf,longf,faz,baz,dist)
        call d_dms(faz,fdeg,fmin,fsec)
        call d_dms(baz,bdeg,bmin,bsec)
        write(6,*)'  '
        write(6,*)' Distance between points = ',dist,' m'
        write(6,*)' '
        write(6,*)' Forward azimuth         = ',faz,' degrees'
        write(6,110),fdeg,fmin,msym,fsec,ssym
        write(6,*)' '
        write(6,*)' Back azimuth            = ',baz,' degrees'
        write(6,110),bdeg,bmin,msym,bsec,ssym
110     format(26x,'=  ',4x,i3,1x,i2,a1,f7.3,a1)
        write(6,*)' '
        write(6,*)' '
        write(6,*)' Do you want to run again? (y=1)'
        read(5,*) ans
        if (ans.eq.1) goto 300
        stop
        end
```

```

      subroutine dis(a,finv,glat1,glon1,glat2,glon2,faz,baz,s)
      Implicit double precision (a-z)
      integer i,ans
      tol=0.3d-11
      twopi=6.283185307179586d0
      r=-1.d0/finv+1.d0
      tu1=dtan(glat1)*r
      tu2=dtan(glat2)*r
      cu1=1.d0/dsqrt(tu1*tu1+1.d0)
      su1=cu1*tu1
      cu2=1.d0/dsqrt(tu2*tu2+1.d0)
      s=cu2*cu1
      baz=s*tu2
      faz=baz*tu1
      x=glon2-glon1
      i=0
100   i=i+1
      if(i.gt.1000) then
            write(6,*)' 1000 itterations performed continue? (y=1)'
            read(5,*) ans
            if (ans.ne.1) return
      endif
      sx=dsin(x)
      cx=dcos(x)
      tu1=cu2*sx
      tu2=-su1*cu2*cx+baz
      sy=dsqrt(tu1*tu1+tu2*tu2)
      cy=s*cx+faz
      y=datan2(sy,cy)
      sa=s*sx/sy
      c2a=-sa*sa+1.d0
      cz=faz+faz
      if(c2a.gt.0.d0) cz=-cz/c2a+cy
      e=cz*cz*2.d0-1.d0
      c=((-3.d0*c2a+4.d0)/finv+4.d0)*c2a/finv/16.d0
      d=x
      x=((e*cy*c+cz)*sy*c+y)*sa
      x=(1.d0-c)*x/finv+glon2-glon1
      if(dabs(d-x).gt.tol*y) goto 100
      faz=datan2(-tu1,tu2)
      if(faz.lt.0.d0) faz=faz+twopi
      baz=datan2(cu1*sx,su1*cu2-baz*cx)
      if(baz.lt.0.d0) baz=baz+twopi
      x=dsqrt((1.d0/r/r-1.d0)*c2a+1.d0)+1.d0
      x=(x-2.d0)/x
      c=1.d0-x
      c=(x*x/4.d0+1.d0)/c
      d=(0.375d0*x*x-1.d0)*x
      x=e*cy
      s=1.d0-e-e
      s=((((sy*sy*4.d0-3.d0)*s*cz*d/6.d0-x)*d/4.d0+cz)*sy*d+y)*c*a*r
      return
      end
```

```
      subroutine d_dms(angle,deg,min,sec)
c
c     routine convert angle in radians to decimal degrees and degrees min sec
c
      implicit double precision (a-z)
      integer deg,min
      pi=4.d0*datan(1.d0)
      angle=angle*180.d0/pi
      deg=int(angle)
      min=int((angle-dfloat(deg))*60.d0)
      sec=(angle-dfloat(deg)-dfloat(min)/60.d0)*3600.d0
      return
      end
```

SCRIPT FILE TO COMPILE AND LINK

```
# File:     Disvincenty.make
# Target:   Disvincenty
# Sources:  disvincenty.f
# Created:  Tuesday, October 2, 1990 4:15:27 PM


OBJECTS = disvincenty.f.o



Disvincenty ƒƒ Disvincenty.make {OBJECTS}
        Link -f -srt -ad 4 -ss 1000000 -w -t APPL -c '????' ∂
                {OBJECTS} ∂
                "{Libraries}"Runtime.o ∂
                "{Libraries}"Interface.o ∂
                "{FLibraries}"FORTRANlib.o ∂
                "{FLibraries}"IntrinsicLibFPU.o ∂
                "{FLibraries}"FSANELibFPU.o ∂
                -o Disvincenty
disvincenty.f.o ƒ Disvincenty.make disvincenty.f
        FORTRAN  -mc68020 -mc68881 -opt=3 -extended disvincenty.f
```

```
Do you want to use WGS84 reference ellipsoid (y=1)
1
Do you want to enter lat. longs. in degrees (1) or d m s (2)
1
Input start latitude (deg. N +ve)
46
Input start longitude (deg. E +ve)
-66
Input finish latitude (deg. N +ve)
45
Input finish longitude (deg. E +ve)
-75

Distance between points =    711748.631389547865    m

Forward azimuth       =    264.252737733617996    degrees
                      =    264 15'  9.856"

Back azimuth          =    77.8267293949662975    degrees
                      =    77 49' 36.226"


Do you want to run again? (y=1)
N

STOP
```

APPENDIX C

PROGRAM TO COMPUTE THE ACCURACY OF THE DELAY ESTIMATOR

```
C=================================================================
C
C PROGRAM TO COMPUTE THE PROBABILITY THAT A GIVEN ACCURACY
C IS REACHED IN THE DELAY ESTIMATOR PROCESSOR
C
C                         M. Caron
C                         AUGUST 1990
C
C  Note : It is suggested to use the "-extended" option when
C        compiling to obtain better accuracy on the calculation
C
C          RunMacII Accuracy4.f -extended -opt=3
C
C       Only the first 512 correlation lags are assumed to be
C       of interest.
C
C=================================================================
C Load trap code for Macintosh system routines
!!m inlines.f
      PROGRAM ACCURACY4
      real*8 CORREL(0:512)
      DOUBLE PRECISION SUM,PROD,FCN,U,sigma,AA,Q
      EXTERNAL FCN,Q
      CHARACTER*80 FILEN,outf*80
C
          do i=0,512
                 correl(I)=0.
          end do
c
      WRITE(6,10)
   10 FORMAT (/,/,/,T10,'DELAY ESTIMATION ACCURACY',/,T10,
     &               '=========================',/,/)
C
   50 WRITE (6,60)
   60 FORMAT (T3,'FILE NAME WHERE TO GET THE',
     &   ' AUTOCORRELATION LEVELS')
       OPEN (UNIT=10,FILE=*,STATUS='OLD',readonly)
          inquire(unit=10,name=filen)
          write(6,*) filen
c
          call f_drawoutpwindow
C
C read autocorrelation levels over the first NCOR lags
C
        READ (10,20) NCOR
   20   FORMAT(i)
        IF(NCOR.GT.513) THEN
             WRITE(6,25) NCOR
   25         FORMAT(T3,'?? NCOR GREATER THAN THE MAXIMUM LIMIT.',
     &               'NCOR=',I6,'??')
             STOP
        END IF
       READ (10,28)(IA,CORREL(I),I=0,NCOR-1)
   28 FORMAT (i,f)
```

```
      CLOSE(10)
C
 80   WRITE (6,85)
 85   FORMAT (T3,'C/NO IN DB-HZ')
      READ (5,*) CNO
C
 120  WRITE (6,130)
 130  FORMAT(T3,'Enter the estimate error in number of correlators')
      READ(5,*) MDEL
      IF(MDEL.LT.0 .OR. MDEL.GT.NCOR) GOTO 120
c
 182  CONTINUE
C
      SUM = 0.0D+00
        lim= MIN(ncor,512)/10
c
c compute variance
c
        SIGMASQ= 10.d+00**(-CNO/10.d+00)*2.d+00/0.44d+00
        write (6,*) 'sigmasq=',sigmasq
c
c assume that the response is symmetric around the real delay
c so do from 1 to mdel and multiply the prob. by 2 for the
c two side effect
c
      DO 300 KMI=1,MIN(MDEL,NCOR-1)
        PROD=1.0D+00
C
        DO 200 KMJ=0,MIN(NCOR-1,511)
          IF(KMJ.EQ.KMI) GOTO 200
                  SIGMA = DSQRT(SIGMASQ*(1.D+00-CORREL(IABS(KMI-KMJ))))
                  U=-(CORREL(KMI)-CORREL(KMJ))/SIGMA
                  AA=Q(U)
               PROD=PROD*AA
X                 WRITE (6,185) U,Q(U),PROD
X185              FORMAT (X,F,X,F,X,F)
C
 200    CONTINUE
            write (6,205) KMI,PROD*100.
 205        FORMAT (T3,'Prob. that lag #',I2,' is the highest is ',f10.4,' %')
        SUM=SUM+PROD
        WRITE (6,210) 200.D+00*SUM
 210      FORMAT(T3,'Cumulative Probability =',f10.4,' %',/)
c
c if probability that the output KMI samples away from the true output
c is less than 10**(-5),  then the prob that the output KMI+n samples
c away from the true output is negligible (n>0)
c
          IF(PROD.LT.1.D-5) LEAVE
C
 300  CONTINUE
X      PAUSE
    KMI=0
```

```
      PROD=1.0D+00
      DO 400 KMJ=1,MIN(NCOR-1,512)
              SIGMA = DSQRT(SIGMASQ*(1.D+00-CORREL(IABS(KMI-KMJ))))
              U=-(CORREL(KMI)-CORREL(KMJ))/SIGMA
              AA=Q(U)
              PROD=PROD*AA
X             WRITE (6,185) U,Q(U),PROD
 400  CONTINUE
        write (6,205) KMI,PROD*100.
      SUM=2.D+00*SUM+PROD
      WRITE(6,210) SUM*100.d+00
C
        WRITE (6,*) '1 FOR MORE '
        READ(5,*) I
        IF(I.EQ.1) GOTO 80
      STOP
      END
C
c function to compute the Q-function
c
c  program derived from the TI-58 pocket calculator
c  internal program
c
c                    M. Caron June 1990
c
c
        double precision function Q(x)
        implicit double precision (a-h,o-z)
c
        data a1,a2,a3,a4,a5,a6 /0.2316419,1.330274429,1.821255978,
     &  1.781477937,0.356563782,0.31938153/
        DATA PI/3.14159265358979324/
c
        y=x
        minus = 0
        if(y.le.0.d+00) then
              minus = 1
              y=dabs(y)
        end if
c
        deno = a1*y + 1
        b = a2/deno**4 - a3/deno**3 + a4/deno**2 - a5/deno + a6
        b = b/deno/dsqrt(2.*pi)*DSQRT(exp(-(y**2)))
        if(minus.eq.0) then
              Q=b
        else
              Q=1.d+00-b
        end if
        return
        end
```

```
DELAY ESTIMATION ACCURACY
========================

FILE NAME WHERE TO GET THE AUTOCORRELATION LEVELS
correl200.dat

C/NO IN DB-HZ
30
Enter the estimate error in number of correlators
3
sigmasq=   4.54545454545454545454E-003
Prob. that lag # 1 is the highest is   11.3654 %
Cumulative Probability =  22.7307 %

Prob. that lag # 2 is the highest is    1.0957 %
Cumulative Probability =  24.9222 %

Prob. that lag # 3 is the highest is    0.0603 %
Cumulative Probability =  25.0429 %

Prob. that lag # 0 is the highest is   64.8936 %
Cumulative Probability =  89.9365 %

1 FOR MORE
2

STOP
```

APPENDIX D

COMPUTER PROGRAM TO SIMULATE THE DELAY ESTIMATION PROCESSOR

Listing of Program 'Delay Estimator.f'

```
C-----------------------------------------------------
C
C      Program to simulate the estimation of the delay between
C an EPIRB signal relayed via two satellites. It is based partly
C on software developed by R.J Keightley on the GOES project and
C the correlation program is based on the one in : "Programs for
C Digital Signal Processing", IEEE Press, 1979, chapter 2.2.
C
C      This version use a lowpass baseband signal sampled at multiple
C of 1.6 kHz
C
C
C                                M. Caron Sept 1990
C
C-----------------------------------------------------
C
       PROGRAM DELEST
C
C Define parameters that are system dependent
C NBIT : number of beacon message bits
C FSMPL  : sampling frequency in Hz
C DRATE : beacon data rate in bps
C DELMX  : maximum delay to be simulated in seconds
C CARRIER : pure carrier duration preceding the beacon message (seconds)
C LAGMX : maximum number of lag interested in
C IMULMX : maximum multiplication factor for the sampling frequency
C
       PARAMETER (NBIT=112,FSMPL=1.6E+3,DRATE=400.0,DELMX=18.E-3,
     &       CARRIER=0.16,LAGMX=2049,IMULMX=52)
C
C Define parameter related to the above ones
C NDIM :  number of samples defining the signal (including max. delay)
C FSMUL : sampling frequency multiplied by the maximum factor IMULMX
C The following are based on the basic sampling frequency
C NCARR : number of samples of pure carrier
C NSMPL : number of samples per beacon message bit
C NSIG  : number of samples to define the total beacon burst
C
C      PARAMETER (FSMUL=FSMPL*IMULMX,NCARR=CARRIER*FSMPL+0.1,
C  &      NSMPL=(FSMPL/DRATE/2.+0.1),NSIG = NSMPL*NBIT*2+NCARR,
C  &      NDIM=NSIG*IMULMX+(DELMX*FSMUL+0.1)+1)
C
       PARAMETER (FSMUL=83.2E+3,NCARR=256,NSMPL=2,NSIG = 704,
     & NDIM=38106)
C
C SIGNAL1() and SIGNAL2() contain the samples of the signal from satellite
C #1 and #2 respectively.
C CROSS(i) is the cross-correlation values at lag (i-1) corresponding
C       to a delay of (i-1)/fsmpl  (seconds)
C FRAME() is the array containing the beacon message bits
C
       DIMENSION SIGNAL1(NDIM),SIGNAL2(NDIM),CROSS(LAGMX)
       INTEGER FRAME(NBIT)
       CHARACTER FILEN*80,ANS*1
```

```
      INTEGER*4 SEED1,SEED2
      REAL*4 XMEAN,SIGMA1,SIGMA2,R1,R2,Q1,Q2
C
      DATA XMEAN /0.0/
C
C SEED1 and SEED2 are the two seeds for the random number generator
C
      SEED1 = 4
      SEED2 = 3141592
C
C validate some of the parameter values that couldn't be inter-related
C in the above PARAMETER statements
C compute the maximum number of lags of interest (LAGSHOW)
C Compute the number of lags to be computed (power of two)
C
      A = DELMX*FSMUL
      LAGSHOW = IFIX(A)
      IF(FLOAT(LAGSHOW).NE.A) LAGSHOW = LAGSHOW+1
      I=ALOG(A)/ALOG(2.)
      IF(FLOAT(2**I).NE.A) I = I + 1
      LAG = 2**I + 1
      IF(LAG.GT.LAGMX) STOP 'DELMX IS TOO LARGE FOR LAGMX. ABORT'
C
C compute the size of the FFT to be used
C compute the number of noise samples required
C compute the total number of samples required to represent the signal
C
      NSIZE = 2**(I+1)
C
      NOISE = IFIX(A)
      IF(FLOAT(NOISE).NE.A) NOISE = NOISE + 1
C
      NTOT = NSIG*IMULMX + NOISE
      IF(NTOT.GT.NDIM) STOP 'NTOT IS GREATER THAN NDIM. PROG. ABORTED'
C
C INPUT VARIABLES
C
1     WRITE (6,2)
2     FORMAT  (/,/,/,/,T20,35('*'),/,T20,'*',33X,'*',/,T20,
     &  '* DISTRESS SIGNAL DELAY ESTIMATOR *',/,T20,
     &      '*',33X,'*',/,T20,35('*'),/,/,/,T3,
     & 'C/No of signal #1 and #2 (dB-Hz)')
      READ (5,*) CNO1,CNO2
C
10    WRITE (6,*) 'Sampling rate multiplication factor ( > 0; < ',IMULMX,')'
      READ(5,*) IMUL
      IF(IMUL.LE.0) GOTO 10
C
20    WRITE (6,22) DELMX*1.E+3
22    FORMAT (T3,'Delay to be simulated (milliseconds) (must be ',
     & 'less than ',F8.2,' ms )')
C
      READ (5,*) DELAY
      DELAY=ABS(DELAY)*1.E-3
```

```
        IF(DELAY.GT.DELMX) GOTO 20
C
c get the file names and open the files to store the result
c summary and the compiled list of errors
c
        write (6,*) 'File name where to store summary results'
        OPEN (10,FILE=*'File name for summary',STATUS='UNKNOWN')
        INQUIRE (10,NAME=FILEN)
        WRITE (6,*) FILEN
        REWIND (10)
c
        write (6,*) 'File name where to store error'
        open (20,file=*'File name for error',status='unknown')
        rewind (20)
        inquire (20,name=filen)
        write(6,*) filen
c
c refresh the output window
c
        call f_DrawOutpWindow
c
25      write (6,*) 'How many runs do you want ? '
        read (5,*) NRUNS
        if(nruns.le.0) goto 25
c
        write (6,*) 'Do you want to enter the random number generator seeds (Y or N)'
        read (5,*) ANS
        if(ans.eq.'y' .or. ans.eq.'Y') then
                WRITE (6,*) 'ENTER SEED1, SEED2'
                READ (5,*) SEED1,SEED2
        END IF
C
 28     WRITE (6,30)
 30     FORMAT (/,/,T3,'...computing',/,/)
C
C compute the new sampling frequency FS
C Compute the standard deviation of the noise to generate the
C the appropriate C/No's
C Compute the number of samples of delay
C Compute the number of signal samples
C Compute the total number of samples including noise
C Compute the number of samples per bit
C generate the signals. No phase advance for the first signal (ADV=0)
C
        FS=FSMPL*FLOAT(IMUL)
        CALL SDEV (FSMPL,CNO1,SIGMA1)
        CALL SDEV (FSMPL,CNO2,SIGMA2)
C
        NDELAY = IFIX((DELAY)*FS+0.1)
        NSIGNAL = NSIG*IMUL
        A = DELMX*FS
        NOISE = IFIX(A)
        IF(FLOAT(NOISE).NE.A) NOISE = NOISE + 1
        NTOT = NSIGNAL + NOISE
```

```
        IF(NTOT.GT.NDIM) STOP 'NTOT IS GREATER THAN NDIM. PROG. ABORTED'
        NSAM = IFIX(FS/DRATE/2.0+0.1)
c
        ibegin = jsecnds(0)
        do 1000 ir=1,nruns
        ADV = 0.

        CALL ELTLowpass (NSIGNAL,NBIT,NSAM,FS,SIGMA1,SEED1,
     &               ADV,FRAME,SIGNAL1)

C
C add noise at the end of the first signal because it is
C assumed that signal#1 is in advance to signal #2
C
        CALL GAUSSRN (XMEAN,SIGMA1,SEED1,R1,Q1)
        IBEEN = 0
        DO I=NSIGNAL+1,NTOT
              SIGNAL1(I) = R1
              IF(IBEEN.EQ.1) THEN
                    SIGNAL1(I)=Q1
                    CALL GAUSSRN (XMEAN,SIGMA1,SEED1,R1,Q1)
                    IBEEN = 0
              ELSE
                    IBEEN = 1
              END IF
        END DO

C
C DEFINE SIGNAL NUMBER 2
C Compute required phase advance in terms of fraction of sample
C add noise preceding the signal by the amount of delay
C
        ADV = DELAY*FS - FLOAT(NDELAY)
x       write (6,*) 'Phase advance in samples = ',ADV
        CALL GAUSSRN (XMEAN,SIGMA2,SEED2,R2,Q2)
        IBEEN = 0
        DO I=1,NDELAY
              SIGNAL2(I) = R2
              IF(IBEEN.EQ.1) THEN
                    SIGNAL2(I)=Q2
                    CALL GAUSSRN (XMEAN,SIGMA2,SEED2,R2,Q2)
                    IBEEN = 0
              ELSE
                    IBEEN = 1
              END IF
        END DO
C
C add the signal itself
C
        N = NDELAY + 1

        CALL ELTLowpass (NSIGNAL,NBIT,NSAM,FS,SIGMA2,SEED2,
     &               ADV,FRAME,SIGNAL2(N))

```

```
C
C Fill up rest of vector with noise samples
C
      N = N+NSIGNAL
      DO I=N,NTOT
            SIGNAL2(I) = R2
            IF(IBEEN.EQ.1) THEN
                  SIGNAL2(I)=Q2
                  CALL GAUSSRN (XMEAN,SIGMA2,SEED2,R2,Q2)
                  IBEEN = 0
            ELSE
                  IBEEN = 1
            END IF
      END DO
C
C signals are now both defined
C
x     write (6,*) 'Save the signal on a file (Y/N)'
x     READ (5,*) ANS
C
x     IF(ANS.EQ.'Y' .OR. ANS.EQ.'y') THEN
x           write (6,*) 'from what to what (maximum=',ntot,')'
x           read(5,*) in,is
x           in=max(in,1)
x           is=min(ntot,is)
x           is=max(is,in)
x           WRITE (6,*)' FILE NAME where to save data'
x           OPEN (98,FILE=*,STATUS='UNKNOWN')
x           inquire (98,name=filen)
x           write (6,*) FILEN
x           REWIND (98)
x           WRITE (98,'(I8,A1,F15.7,A1,F15.7)')(I,CHAR(9),SIGNAL1(I)
x  &                     ,CHAR(9),SIGNAL2(I),I=in,is)
x           ENDFILE(98)
x           CLOSE (98)
x     END IF
C
C compute cross-correlation
C FFT size = NSIZE
C
      CALL CORRELATE (SIGNAL1,SIGNAL2,NTOT,LAG,CROSS,NSIZE,
   &        IER)
C
      IF (IER.NE.0) THEN
            WRITE (6,*) 'ERROR CODE FROM CORRELATE = ',IER
            STOP 'PROGRAM ABORTED'
      END IF
C
x     write (6,*) 'set 1 to store correlation levels'
x     read (5,*) i
x     if(i.eq.1) then
x           write (6,*) 'file name where to store correlation levels'
x           open (2,file=*,status='unknown')
x           inquire (2,name=filen)
```

```
x               write (6,*) filen
x               rewind (2)
x               write (2,170) (j,cross(J),j=1,nsize/2+1)
x170            format (i,x,f)
x               endfile(2)
x               close(2)
x       end if
C
C find the maximum correlation level over all the lags
C
        CMX = 0.
        J=0
        DO I=1,LAGSHOW
             IF(CROSS(I).GT.CMX) THEN
                  CMX = CROSS(I)
                  J = I-1
             END IF
        END DO

C
        estdel = (FLOAT(J)/FS)*1.E+3
        esterr = (estdel/1000.-delay)*1.0e+6
        if(ir.eq.1) then
             write (20,189)
189          format(' C(Ndelay) Maximum_Correlation Lag_Number Delay Error')
             WRITE (10,190) CNO1,CNO2,DELAY*1.E+3,NDELAY,
     &       CMX,J,estdel,esterr,seed1,seed2
             WRITE (6,190) CNO1,CNO2,DELAY*1.E+3,NDELAY,
     &      CMX,J,estdel,esterr,seed1,seed2

190          FORMAT (/,/,/,T3,'C/No of signal #1, #2 = ',F8.2,2X,F8.2,
     &  ' (dB-Hz)',/,T3,'Simulated Delay =',F10.2,' (ms) = ',I5,
     &    ' (samples)',/,T3,
     &  'Maximum Cross-Correlation = ',F15.4,' and',/,T9,
     &  'occured at samples #',I5,
     &  ' corresponding to a delay of ',F15.7,' ms',/,
     &        t3,'Delay error = ',F9.3,'µs  Next seeds (1 and 2 ) = 'i,x,i,/)

       else
             WRITE (10,195) CMX,J,estdel,esterr,SEED1,SEED2
195          FORMAT (t3,'Maximum Cross-Correlation = ',F15.4,
     &       ' and',/,T9,
     &       'occured at samples #',I5,
     &             ' corresponding to a delay of ',F15.7,' ms',/,
     &      t3,'Delay error = ',F9.3,'µs  Next seeds (1 and 2 ) = 'i,x,i,/)

             WRITE (6,196) ir,CMX,J,estdel,esterr,SEED1,SEED2
196          FORMAT (t3,'Run no.:',I5,/,t3,'Maximum Cross-Correlation = ',F15.4,
     &       ' and',/,T9,
     &       'occured at samples #',I5,
     &             ' corresponding to a delay of ',F15.7,' ms',/,
     &      t3,'Delay error = ',F9.3,'µs  Next seeds (1 and 2 ) = 'i,x,i,/)
      end if
      write (20,200) cross(ndelay),CMX,J,estdel,esterr
```

```
200     format(2(2x,f),2x,i,2(2x,f))
        itime=jsecnds(ibegin)
        if(itime.gt.3600) then
                do k=10,20,10
                        inquire (unit=k,name=filen)
                        close (unit=k,status='keep')
                        open (unit=k,file=filen,status='old',access='append')
                end do
                ibegin=jsecnds(0)
        end if
c
1000    continue
C
        endfile(20)
        close(20)
C
        ENDFILE(10)
        CLOSE(10)
        STOP 'TOURLOU !'
        END
C-----------------------------------------------------------
C Program to compute the cross-correlation between two signals
C
C Based on :"Programs for Digital Signal Processing",
C         IEEE Press, 1979, chapter 2.2.
C
C X(),Y() : arrays of N samples to be correlated
C CROSS(i) : arrays of LAG values giving the correlation
C          of X() and Y() at lag (i-1)
C NSIZE is the FFT size and is related to the required
C     number of lags using LAG=NSIZE/2+1
C IER is the error code = 0 indicates no errors
C                       = 1 means NSIZE is greater than the
C                         maximum size allowed by the program
C                       = 2 means the dimension i.e. LAG of CROSS is not
C                         large enough for the requested NSIZE
C
C
C                                       M. Caron Jan 1990
C
C-----------------------------------------------------------
C
        SUBROUTINE CORRELATE (X,Y,N,LAG,CROSS,NSIZE,IER)
C
        PARAMETER (MXSIZ=4096)
        DIMENSION X(N),Y(N),CROSS(LAG)
        COMPLEX XX(MXSIZ),XMN,XI,YI,Z(MXSIZ/2+1)
C
C Check for errors
C
        IER = 0
        IF(NSIZE.GT.MXSIZ) THEN
                IER=1
                RETURN
```

```
      ELSE IF (LAG.LT.NSIZE/2+1) THEN
            IER=2
            RETURN
      END IF
C
C define variables
C LSHFT = overlap factor per section
C MHLF1 = maximum shift of interest in samples
C NSECT = number of sections of NSIZE samples with overlap of NSIZE/2
C NRD = number of samples to reaf each time
C IW+1 = index of next sample to be read
C NRDY = number of samples to be read on Y( )
C NRDX = number of samples to be read on X( )
C
      LSHFT = NSIZE/2
      MHLF1 = LSHFT+1
      NSECT = FLOAT(N+LSHFT-1)/FLOAT(LSHFT)
      NRD = LSHFT
      IW = 0
      NRDY = NSIZE
      NRDX = LSHFT
C
C initialize the temporary cross-correlation array
C
      DO I=1,MHLF1
            Z(I) = CMPLX(0.,0.)
      END DO
C
x     WRITE (6,*) 'NSECT = ',NSECT
      NSECT1 = NSECT - 1
C
C compute the number of sections between each prompt to the user to
C let him know the current status
C
x     ND =(FLOAT(NSECT)/10.+0.5)
C
C Compute the FFT of each section of X and Y.  Use the odd/even technique
C to compute both over a single FFT for each section
C Accumulate in the frequency domain and then inverse FFT to obtain the
C correlation
C
      DO 190 K=1,NSECT

C
C PRINT K FROM TIME TO TIME TO MONITOR PROGRESS
C
x           IF((K/ND)*ND .EQ.K) WRITE (6,*) 'K=',K
C
C IF THE LAST SECTION, THEN CHECK IF THE NUMBER OF SAMPLES REMAINING
C IS EEQUAL TO NSIZE.  IF NOT, FILL IN WITH ZEROS.
C
            IF(K.GE.NSECT1) THEN
                  NRDY = N - (K-1)*LSHFT
                  IF(K.EQ.NSECT) NRDX = NRDY
```

```
                        IF(NRDY.NE.NSIZE) THEN
                                NRDY1 = NRDY+1
                                DO 100 I=NRDY1,NSIZE
 100                            XX(I) = CMPLX(0.,0.)
                        END IF
                END IF
C
C read NRDY data starting at sample #IW+1
C
                DO 120 I=1,NRDY
                        J=(I+IW)
 120                    XX(I) = CMPLX(X(J),Y(J))
C
                NRDX1 = NRDX+1
                DO 170 I=NRDX1,NSIZE
 170            XX(I) = CMPLX(0.,AIMAG(XX(I)))
C
C correlate X and Y and accumulate CONJG(X)*Y
C

                CALL FFT (XX,NSIZE,0)

C
                DO 180 I=2,LSHFT
                        J=NSIZE+2-I
                        XI=(XX(I)+CONJG(XX(J)))/2.
                        YI=(XX(J)-CONJG(XX(I)))/2.
                        YI = CMPLX(AIMAG(YI),REAL(YI))
                        Z(I) = Z(I) + CONJG(XI)*YI
 180            CONTINUE
C
                XI = XX(1)
                Z(1)=Z(1)+CMPLX(REAL(XI)*AIMAG(XI),0.)
                XI = XX(MHLF1)
                Z(MHLF1)=Z(MHLF1)+CMPLX(REAL(XI)*AIMAG(XI),0.)
                IW = IW + LSHFT
 190    CONTINUE
C
C COMPUTE INVERSE DFT FOR CORRELATION
C
        DO 200 I=2,LSHFT
                J=NSIZE+2-I
                XX(I)=Z(I)
                XX(J) = CONJG(Z(I))
 200    CONTINUE
C
        XX(1) = Z(1)
        XX(MHLF1) = Z(MHLF1)
C

        CALL FFT(XX,NSIZE,1)

C
C normalize the results
```

```
C
        FN = FLOAT(N)
        DO I=1,MHLF1
              CROSS(I) = REAL(XX(I))/FN
        END DO
        RETURN
        END
C-----------------------------------------------------
C Program to compute the DFT AND IDFT
C
C Based on :"Programs for Digital Signal Processing",
C       IEEE Press, 1979, chapter 2.2.
C
C INV = 0 ==> DFT,  INV=1 --> IDFT
C
C X() : COMPLEX ARRAY OF N NUMBERS
C
C                                         M. Caron Jan 1990
C
C-----------------------------------------------------
C
        SUBROUTINE FFT (X,N,INV)
        COMPLEX X(N),U,W,T
        DATA PI/3.14159265358979324/
C
        M=IFIX(ALOG(FLOAT(N))/ALOG(2.)+0.1)
        NV2 = N/2
        NM1 = N-1
        J=1
        DO 40 I=1,NM1
              IF(I.LT.J) THEN
                    T=X(J)
                    X(J)=X(I)
                    X(I)=T
              END IF
 10           K=NV2
 20           IF(K.LT.J) THEN
                    J=J-K
                    K=K/2
                    GOTO 20
              END IF
 30           J=J+K
 40     CONTINUE
        DO 70 L=1,M
              LE=2**L
              LE1 = LE/2
              U = CMPLX(1.,0.)
              W = CMPLX(COS(PI/FLOAT(LE1)),-SIN(PI/FLOAT(LE1)))
              IF(INV.NE.0) W=CONJG(W)
              DO 60 J=1,LE1
                DO 50 I=J,N,LE
                  IP=I+LE1
                  T=X(IP)*U
                  X(IP)=X(I)-T
```

```
                  X(I)=X(I)+T
   50           CONTINUE
                U=U*W
   60         CONTINUE
   70     CONTINUE
          IF(INV.EQ.0) RETURN
          DO 80 I=1,N
   80     X(I) = X(I)/CMPLX(FLOAT(N),0.)
          RETURN
          END
```

Listing of Subroutine 'EltLowpass.f'

```
      SUBROUTINE ELTLowpass (NSIG,NBIT,NSMPL,FSMPL,SIGMAI,SEED,
     &                     ADV,FRAME,SIGNAL)
C
C     This program generates the samples of a simulated ELT/EPIRB
beacon.
C     A sampling rate of FSMPL Hz is used.
C
C
C     The received signal is of the form:
C
C          y(i)=A sin ((2*pi*fc*delt) + phi(i))
C
C     where:
C
C     A       = signal amplitude
C     fc      = nominal carrier frequency
C     delt    = 1/sample frequency
C     phi(i)  = phase change as a result of the modulation
C             = + 1.1,-1.1 radians for a data "1"
C             = - 1.1,+1.1 radians for a data "0"
C     * note that Manchester Coding is used
C
C      NSIG is the number of samples to define the signal
C             NSIG=(signal duration*FSMPL)
C     NBIT : number of message bits in the burst
C     NSMPL : number of samples per message bit
C     FSMPL : sampling frequency in Hz
C     SIGMAI : standard deviation of the noise
C      SEED : the random number generator to be used
C     ADV : phase advance in terms of fraction of a sample
C     FRAME : array of NBIT integers returning the message bits
C     SIGNAL : array of at least NSIG real numbers defining the
sampled
C             signal
C
C
C     Progammer R.J. Keightley
C
C
C
C                    Modified by  M. Caron   Jan 1990
C
C
      INTEGER FRAME(NBIT),MESSAGE(112)
      REAL SIGNAL(1),INC
      REAL*4 R,Q,SIGMA,XMEAN
      INTEGER*4 SEED
      DATA XMEAN /0.0/
      DATA PI/3.14159265358979324/
      DATA MESSAGE /1,1,1,1,1,1,1,1,1,1,1
     &  ,1,1,1,1,0,0,0,1,0,1,1
     &  ,1,1,0,1,1,0,1,0,1,1,1
     &  ,1,0,0,1,1,1,0,0,0,0,1
     &  ,1,1,0,0,0,0,0,1,0,0,1
```

```
     &  ,1,1,0,0,1,0,1,0,0,0,0
     &  ,0,1,0,0,0,1,0,0,0,1,0
     &  ,1,0,0,0,0,0,1,0,0,0,1
     &  ,1,1,0,0,1,0,1,1,0,0,0
     &  ,0,1,1,1,0,0,0,0,0,1,0
     &  ,1,0 /
C
      PIBY4=PI/4.
      TWOPI=2.0*PI
C
C     A frame of ELT signal will be simulated.  First, read from
file
C  FRAME.DAT the precomputed data bits and store them in array
FRAME.
C
c     OPEN(UNIT=99,FILE='FRAME.DAT',STATUS='OLD')
c     READ(99,*)  (FRAME(I),I=1,NBIT)
c     CLOSE(UNIT=99,STATUS='KEEP')
      do i=1,nbit
           frame(i)=message(I)
      end do
C
C  SUM and NO are used to compute the generated standard deviation
C  and to compare it with the requested one at the end
C
      SIGMA = SIGMAI
x     SUM = 0.
x     NO = 0
x     SUM2=0.
C
C ISMPL+1 : next sample index for array SIGNAL
C OMEGA = 2*pi*frequency
C compute the phase increment per sample
C compute the number of samples for the pure carrier
C preceding the message
C compute random phase shift between 0 and 2pi(constant for the
burst)
C set first phase value (PARAM) such that the first sample starts
C with ADV*INC plus random phase shift
C
      ISMPL=0
      NCARR=IFIX(160.E-3*FSMPL+0.1)
c     R=0.
c     Q=TWOPI
c     CALL UNIFORM (R,Q,SEED,Q)
c     PARAM= Q
      param = 0.
C
C     Now, generate 160 ms (+ or - 1%) (or 160.E-3 * FSMPL = NCARR)
C  sample intervals.
C
      PARAM = AMOD(PARAM,TWOPI)
      SS = sin(param+piby4)
      DO ICARR=1,NCARR,2
```

```
            ISMPL=ISMPL+1
            CALL GAUSSRN (XMEAN,SIGMA,SEED,R,Q)
            SIGNAL(ISMPL)=SS+R
            ISMPL=ISMPL+1
            SIGNAL(ISMPL)=SS+Q
x           SUM2 = SUM2 + R**2 + Q**2
x           SUM = SUM + R + Q
x           NO = NO + 2
         END DO
         ISMPL = NCARR
C
C  IC is a correction factor required because the random number
C  generator produces two random numbers per call
C  IC=0 if NSMPL is even IC=1 otherwise
C
         IC = 1
         IF((NSMPL/2)*2.EQ.NSMPL) IC = 0
C
C    There will be a nominal (FSMPL/800) = NSMPL samples per phase
C  of the Manchester symbol where the data rate is 400 bps
C
C    Now modulate the carrier with the frame data bits
C
         SS=PARAM+PIBY4
         DO IBIT=1,NBIT
           IF (FRAME(IBIT).EQ.1) THEN
C
C when bit is 1, modulation is +1.1, -1.1 radians
C
                 DO JSMPL=1,NSMPL,2
                       ISMPL=ISMPL+1
                       CALL GAUSSRN (XMEAN,SIGMA,SEED,R,Q)
                       SIGNAL(ISMPL)=SIN(1.1+SS)+R
                       ISMPL=ISMPL+1
                       SIGNAL(ISMPL)=SIN(SS+1.1)+Q
x                      SUM2 = SUM2 + R**2 + Q**2
x                      SUM = SUM + R + Q
x                      NO = NO + 2
                 END DO
C
                 ISMPL = ISMPL - IC
                 DO JSMPL=1,NSMPL,2
                       ISMPL=ISMPL+1
                       CALL GAUSSRN (XMEAN,SIGMA,SEED,R,Q)
                       SIGNAL(ISMPL)=SIN(SS-1.1)+R
                       ISMPL=ISMPL+1
                       SIGNAL(ISMPL)=SIN(SS-1.1)+Q
x                      SUM2 = SUM2 + R**2 + Q**2
x                      SUM = SUM + R + Q
x                      NO = NO + 2
                 END DO
                 ISMPL = ISMPL - IC
         ELSE
C
```

```
C  when the bit is a zero, the modulation is -1.1, +1.1 radians
C
                 DO JSMPL=1,NSMPL,2
                        ISMPL=ISMPL+1
                        CALL GAUSSRN (XMEAN,SIGMA,SEED,R,Q)
                        SIGNAL(ISMPL)=SIN(SS-1.1)+R
                        ISMPL=ISMPL+1
                        SIGNAL(ISMPL)=SIN(SS-1.1)+Q
x                       SUM2 = SUM2 + R**2 + Q**2
x                       SUM = SUM + R + Q
x                       NO = NO + 2
                 END DO
                 ISMPL = ISMPL - IC
C
                 DO JSMPL=1,NSMPL,2
                        ISMPL=ISMPL+1
                        CALL GAUSSRN (XMEAN,SIGMA,SEED,R,Q)
                        SIGNAL(ISMPL)=SIN(SS+1.1)+R
                        ISMPL=ISMPL+1
                        SIGNAL(ISMPL)=SIN(SS+1.1)+Q
x                       SUM2 = SUM2 + R**2 + Q**2
x                       SUM = SUM + R + Q
x                       NO = NO + 2
                 END DO
                 ISMPL = ISMPL - IC
         ENDIF
       END DO
C
C  compute the standard deviation produced by the random number
C  generator and write the absolute error relative to the
requested one
C
x      SMEAN = SUM/FLOAT(NO)
x      STDEV=SQRT(SUM2/FLOAT(NO) - SMEAN**2)
C
x      WRITE (6,100) SMEAN,SIGMA,STDEV,ABS(SIGMA-STDEV)/SIGMA*100.
x 100       FORMAT(/,T3,'FROM ELTSIG',/,T5,'Mean = ',F15.7,/,/,T5,
x      &   'Required standard Deviation = ',
x      &  F15.7,/,T5,'Standard Deviation Obtained = ',F15.7,/,/,T5,
x      &  'Absolute error (%) = ', F7.2,/,/)
c      WRITE (10,100) SMEAN,SIGMA,STDEV,ABS(SIGMA-STDEV)/SIGMA*100.
C
       RETURN
       END
```

Listing of File 'Random Number.f'

```
C-----------------------------------------------------------
C
C     Subroutine to generate a random number between
C A and B with uniform distribution.
C
C     ISEED is the seed to be used at input and is
C            the next seed for the next call at output
C     RAND is the random number
C
C     It is suggested to use an initial seed of 4 for
C  the first call.
C
C     Ref.  "Mathematical Methods for Digital Computers Vol II',
C            John Wiley and Sons, Inc., 1967, Chap.12
C
C                          M. Caron
C                          Jan 1990
C
C-----------------------------------------------------------
C
      SUBROUTINE UNIFORM (A,B,ISEED,RAND)
      REAL*4 A,B,RAND
      INTEGER*4 ISEED
C
C     ISEED = ISEED*65539
C     RAND = FLOAT(IABS(ISEED))/(2.0**31)
C
C  use fortran routine to generate the number
C
      RAND = RAN(ISEED)
      RAND = RAND*(B-A) + A
      RETURN
      END
C-----------------------------------------------------------
C
C     Subroutine to generate a pair of random numbers with
C  Gaussian distribution with mean XMEAN and standard
C  deviation DEV.
C
C     ISEED : is the seed to be used and the new one is
C             returned
C     DEV   : is the standard deviation
C     XMEAN : mean
C     R     : is the first random number
C     Q     : is the second random number
C
C     It is suggested to used ISEED=4 for the first call
C
C     Ref.  "Mathematical Methods for Digital Computers Vol II',
C            John Wiley and Sons, Inc., 1967, Chap.12
C
C
C
C                          M. Caron
C
```

```
C                                         Jan 1990
C
C-----------------------------------------------------------------
C
      SUBROUTINE GAUSSRN (XMEAN,DEV,ISEED,R,Q)
C
      INTEGER*4 ISEED
      REAL*4 DEV,XMEAN,R,Q
      DATA TWOPI/6.28318530717959/
C
C      PI = 3.14159265358979324  TWOPI = 2.0*PI
C
c     CALL UNIFORM (0.,1.,ISEED,R)
c     CALL UNIFORM (0.,1.,ISEED,Q)
C
C  use Fortran routine to generate the uniformly
C  distributed numbers
C
      R=ran(iseed)
      Q=ran(iseed)
      S = SQRT(-2.0*ALOG(R))
      TWOPIQ = TWOPI*Q
      R = S*COS(TWOPIQ) * DEV + XMEAN
      Q = S*SIN(TWOPIQ) * DEV + XMEAN
      RETURN
      END
```

Listing of File 'SDEV.f'

```
      SUBROUTINE SDEV(FSMPL,CNO,SIGMA)
C
C    This subroutine determines SIGMA, the standard deviation of
the
C  noise that when added to the ELT signal yields a signal of the
C  specified C/No.
C
C    FSMPL is the sampling frequency.
C   CNO is the required C/No in dB-Hz
C
      REAL SIGMA,SIGMSSQ,CNO,FSMPL
C
C    Calculate the noise variance (SIGMASQ) and standard
C  deviation (SIGMA).  Sigmasq is the power of the noise in
receiver
C  bandwidth.
C
C    Let power spectrum, PS, the power in the DFT frequency bin,
C  be given by
C
C    PS(k)    = 1/N**2 |X(k)|**2
C
C    sigmasq  = 1/N sum |x(i)|**2 , where sum is over N points
C           = 1/N**2 sum |X(k)|**2, where sum is over N points
C           = sum PS(k), where sum is over N points
C
C    For white gaussian noise
C
C    No/2 = power/bandwidth
C         = sum PS(k)/(N*fres), where the sum is over N points and
fres
C           is the frequency resolution of the DFT = fsmpl/N
C         = 1/N**2 * 1/fsmpl * sum |X(k)|**2, where sum is over N
points
C         = sigmasq/fsmpl
C
C    C = signal energy
C      = 1/N * sum |y(i)|**2 (time domain)
C      = .5 (sinusoidal signal)
C
C    C/No = fsmpl/(4*sigmasq)
C
C    sigmasq = fsmpl/(4*(alog((C/No)/10.))
C
C
C  BASED ON SOFTWARE DEVELPED BY KEIGHTLEY
C
C                  M. CARON  JAN 1990
C
C
      SIGMASQ=FSMPL/(10.**(CNO/10.)*4.)
      SIGMA=SQRT(SIGMASQ)
      RETURN
      END
```

Listing of Script File to Compile and Link

```
#   File:        'Delay Estimator.make'
#   Target:      'Delay Estimator'
#   Sources:     'Delay Estimator.f' ELTLowpass.F 'random number.f'
SDEV.F
#   Created:     Friday, September 14, 1990 8:24:27 AM


OBJECTS = 'Delay Estimator.f.o' ELTLowpass.F.o 'random number.f.o'
SDEV.F.o



'Delay Estimator' ƒƒ 'Delay Estimator.make' {OBJECTS}
     Link -f -srt -ad 4 -ss 1000000 -w -t APPL -c '????' ∂
          {OBJECTS} ∂
          "{Libraries}"Runtime.o ∂
          "{Libraries}"Interface.o ∂
          "{FLibraries}"FORTRANlib.o  ∂
          "{FLibraries}"IntrinsicLibFPU.o ∂
          "{FLibraries}"FSANELibFPU.o ∂
          -o 'Delay Estimator'
'Delay Estimator.f.o' ƒ 'Delay Estimator.make' 'Delay Estimator.f'
      FORTRAN  -mc68020 -mc68881 -opt=3 'Delay Estimator.f'
ELTLowpass.F.o ƒ 'Delay Estimator.make' ELTLowpass.F
      FORTRAN  -mc68020 -mc68881 -opt=3 ELTLowpass.F
'random number.f.o' ƒ 'Delay Estimator.make' 'random number.f'
      FORTRAN  -mc68020 -mc68881 -opt=3 'random number.f'
SDEV.F.o ƒ 'Delay Estimator.make' SDEV.F
      FORTRAN  -mc68020 -mc68881 -opt=3 SDEV.F
```

Run Example

```
          **************************************
          *                                    *
          * DISTRESS SIGNAL DELAY ESTIMATOR    *
          *                                    *
          **************************************


 C/No of signal #1 and #2 (dB-Hz)
30,30
Sampling rate multiplication factor ( > 0; <       52)
26
 Delay to be simulated (milliseconds) (must be less than    18.00 ms )
5
File name to store summary results
example.dat

File name where to store error
error.dat

How many runs do you want ?
1
Do you want to enter the random number generator seeds (Y or N)
N


...computing





C/No of signal #1, #2 =    30.00    30.00 (dB-Hz)
Simulated Delay =     5.00 (ms) =  208 (samples)
Maximum Cross-Correlation =        0.4816 and
    occured at samples #  208 corresponding to a delay of      5.0000000 ms
Delay error =    0.000µs  Next seeds (1 and 2 ) =  1548668310  1015696458


STOP TOURLOU !
```

UNCLASSIFIED

SECURITY CLASSIFICATION OF FORM

(highest classification of Title, Abstract, Keywords)

DOCUMENT CONTROL DATA

(Security classification of title, body of abstract and indexing annotation must be entered when the overall document is classified)

1. ORIGINATOR (the name and address of the organization preparing the document. Organizations for whom the document was prepared, e.g. Establishment sponsoring a contractor's report, or tasking agency, are entered in Section 8)

Communications Research Centre
Ottawa, Ontario K2H 8S2

2. SECURITY CLASSIFICATION (overall security classification of the document including special terms if applicable)

UNCLASSIFIED

3. TITLE (the complete document title as indicated on the title page. Its classification should be indicated by the appropriate abbreviation (S,C,R or U) in parentheses after the title)

Application of Geolocation Techniques Using Satellites in Geostationary Orbit (U)

4. AUTHORS (Last name, first name, Middle initial)

Caron, Mario

5. DATE OF PUBLICATION (month and year of publication of document)

June 1991

6a. NO. OF PAGES (total containing information. Include Annexes, Appendices, etc.)

132

6b. NO. OF REFS. (total cited in document)

44

7. DESCRIPTIVE NOTES (the category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.)

Technical Report

8. SPONSORING ACTIVITY (the name of the department project office or laboratory sponsoring the research and development. Include the address.)

Defence Research Establishment Ottawa (DREO)
3701 Carling Ave
Ottawa, Ontario K1A 0Z4

9a. PROJECT OR GRANT NO. (if appropriate, the applicable research and development project or grant number under which the document was written. Please specify whether project or grant)

1410-102

9b. CONTRACT NO. (if appropriate, the applicable number under which the document was written)

10a. ORIGINATOR'S DOCUMENT NUMBER (the official document number by which the document is identified by the originating activity. This number must be unique to this document)

CRC Technical Report 1435

10b. OTHER DOCUMENT NOS. (Any other numbers which may be assigned to this document either by the originator or the sponsor)

CRC Report Number 1435

11. DOCUMENT AVAILABILITY (any limitations on further dissemination of the document, other than those imposed by security classification)

(x) Unlimited distribution
( ) Distribution limited to defence departments and defence contractors; further distribution only as approved
( ) Distribution limited to defence departments and Canadian defence contractors; further distribution only as approved
( ) Distribution limited to government departments and agencies; further distribution only as approved
( ) Distribution limited to defence departments; further distribution only as approved
( ) Other (please specify)

12. DOCUMENT ANNOUNCEMENT (any limitation to the bibliographic announcement of this document. This will normally correspond to the Document Availability (11). However, where further distribution (beyond the audience specified in 11) is possible, a wider announcement audience may be selected.)

13. ABSTRACT (a brief and factual summary of the document. It may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall begin with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (S), (C), (R), or (U). It is not necessary to include here abstracts in both official languages unless the text is bilingual).

This report looks at techniques to perform the localization of SARSAT 406 MHz distress beacons using geostationary satellites. After reviewing the characteristics of the distress beacon transmitters, the main specifications of the GOES satellites are described as they can be assumed as typical geostationary satellites for this application. A brief review of the geolocation techniques is presented and an in-depth analysis is conducted on the time difference of arrival technique. The analysis covers the impact of the Earth's flatteness, channel impairments and satellite geometry . The delay estimator performance is expressed as a function of signal-to-noise ratio and circular error probability. The overall system performance is directly related to the performance of the delay estimator.

Results of both theoretical analysis and computer simulations show that the accuracy of the position derived from the time difference of arrival technique is limited by (1) the low SNR of the distress beacons when relayed via geostationary satellites and, (2) by the low transmission rate (and thus bandwidth) of the beacon signal creating "broad" autocorrelation peaks and making the delay estimation process difficult. In the worst case C/No of 30 dB-Hz and with three satellites spaced at 30°, we can say with a 90% confidence level that the circular error probability is between 3.3 and 4.9 km for a beacon in Canada. This is equivalent to an error not exceeding between 8 and 11.8 km for 95% of the time which is comparable to the current system based on a maximum error of 5 km for 90% of the time.

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thesaurus, e.g. Thesaurus of Scientific Terms (TEST) and that thesaurus-identified. If it is not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title.)

Geolocation
Position location
Geostationary satellite
SARSAT/COSPAS
Satellite Search and Rescue
Time difference of arrival
GOES satellite

DATE DUE

OCT 1 9 1992			
OCT 1 6 1994			