WORKING PAPER

Meters on the Roof

A Statistical Examination of the Project Results

CONSUMER AND
CORPORATE AFFAIRS CANADA
LEGAL METROLOGY BRANCH

Canada

METERS ON THE ROOF

A Statistical Examination of the Project Results

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The analysis and conclusions of this study do not necessarily reflect those of the department

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INTRODUCTION

To understand the reasons why the experimental project analyzed in this report was undertaken, it is useful to highlight the events that led to the project and to review the Legal Metrology Branch's policies during the early 1960's regarding meter verification, sealing, and reverification.

The Electricity Inspection Act specified that, before being put into service, all measuring devices used for revenue purposes be verified for accuracy against regulated limits of error and then sealed by a government inspector to prevent tampering. The common household meter is such a device.

Because no meter can be expected to remain accurate indefinitely, the Act also specified that each meter be "reverified" at prescribed intervals. A verification seal period of six years was originally prescribed by regulation for all electricity meters, but this was subsequently extended to eight years for domestic meters. Consequently, until the early 1960's it was common practice for utilities to remove from service all those meters that had reached the end of the seal period, routinely inspect and service them in a meter shop and then have a government inspector reverify and reseal them.

Technological developments in the design and constructon of meters -- particularly the advent of magnetic suspension meters -- led both utilities and manufacturers to demand that seal periods be extended. The rationale for this request was that better quality meters would maintain accuracy within prescribed tolerances over longer periods of time.

During the same period, the Legal Metrology Branch was in the process of introducing and refining statistical sampling plans for meter verification and reverification. Several staff members decided to carry out a number of longrange experiments that would test the validity of the verification solutions being adopted and also help establish Branch policies regarding the metrology aspects of domestic watt hour meters. In addition, the generalized use of magnetic suspension meters was encouraged by the notion that an appropriately constructed meter could be placed either indoors or outside with no significant difference in performance over time. Branch staff felt that this concept, however reasonable and attractive, should be tested experimentally in a natural environment over a number of consecutive years. Thus was born the "Meters on the Roof" project of 1968.

A total of 144 meters, 36 meters from each of the then four manufacturers were installed on specially constructed racks on the roof of the Legal Metrology Branch Building. Each meter was calibrated and adjusted as near as was practicable to zero error at 2.5 and 25 amperes current before installation for the test program. In 1970, an analysis of the first full year's quarterly observations indicated that the findings were inconclusive because each of the three tiers of meters had been energized from a separate phase of a three-phase system. It was therefore decided to test all meter in series and on single phase. Three calibrated "monitor" meters located inside the building were also placed on the same phase. These meters registered what was deemed to be, for statistical calculations, the "true" number of watt hours consumed by the test meters on the roof.

Following this modification, observations taken quarterly during 1971 were analyzed and described in a Technical Memorandum published in 1975. Quarterly recordings of registration errors continued uninterrupted until October 1979, when the project was terminated. Direct meter readings taken during the months of May and October over the eight-year period (a total sample of 2252) were later converted to the more convenient percentage form.

In 1982 the Branch decided to undertake a statistical analysis of variance with respect to the criteria "manufacturer" and "orientation".

The reader, upon examining the results of this analysis of variance, will note the amount and variety of information that the authors have been able to extract from the experimental data. It was possible to break down the 2252 observations into clear-cut subsamples according to precise criteria. It was possible to consider, for instance, observations made on all meters from a particular manufacturer, or observations made on all meters facing north, or observations made during warm seasons. Consequently, the sample was well suited to both an analysis of variance and to a study of the "theory of mixtures" -- one of the major fields of research of the authors of this report (see Note, p. 22).

In conclusion, it can be considered a lucky coincidence that while the analysis of variance was being adapted to the "Meters on the Roof" project, the theory of mixtures was studied and developed, totally independently, at the National Research Council's Division of Physics. The final juncture of these two domains of statistics, and the resulting success, demonstrates the benefit that two institutions can gain through cooperation and mutual assistance.

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Richard G. Knapp
Director
Legal Metrology

1. OUTLINE OF THE PROJECT

A brief history describing the original objectives and experimental arrangements is contained in the "Introduction" prepared by Richard G. Knapp, Director, Legal Metrology Branch.

From a purely statistical point of view, each year of the eight-year long experiment is represented by four samples of TRE (Total Registration Errors); each sample is constituted by the recordings performed on the totality of 144 meters under test. Each of four manufacturers (General Electric, Ferranti-Packard, Westinghouse, Sangamo) is represented by a group of 36 nominally identical meters, each group being divided into four equal "cells". A cell (9 meters) is thus the smallest set of meters where any two meters can be interchanged without affecting the analysis.

The recordings were performed four times a year: in February, May, August and November. If none of the meters had been eliminated, this would have led to a total of 4608 TRE. In reality, this population was somewhat reduced as the initial number of 144 decreased gradually in the course of the years to 136.

Each "recording" operation, i.e., the reading of all meters represents for the team of two of the Electrical Laboratory staff members 2 or 3 days of work. The reduction of these recordings, i.e., the expressing of them as "percentage errors" is time-consuming work. Because of the shortage of man-power (technical and clerical) initially only the recordings made in November could be reduced. The analysis of variance was thus applied to the yearly results of eight consecutive years (November to November), and the number of the year, e.g., 1972, indicates the TRE (in % form) related to the period extending from November 1971 to November 1972., Table IA.

TABLE IA (1972)

	GE $i = 1$	FP $i = 2$	W $i = 3$	S $i = 4$	$h = 4$
SOUTH	Y_{11} = - 0.01	$Y_{2,1}$ = + 0.72	$Y_{3,1}$ = + 0.62	$Y_{4,1}$ = + 1.24	
	- 0.20	+ 0.56	+ 0.60	+ 1.40	
	+ 0.12	- 0.95	+ 0.09	+ 0.72	
	+ 0.39	+ 0.83	+ 0.76	+ 1.15	
	+ 0.07	+ 0.24	+ 0.14	+ 0.80	
	- 0.17	- 0.40	- 0.72	+ 0.62	
	+ 0.16	+ 0.44	+ 0.73	+ 0.64	
	- 0.01	- 0.91	+ 1.06	+ 1.02	
	Y_{19} = + 0.54	- 0.49	+ 0.87	+ 0.95	
EAST	$Y_{1,10}$ = + 0.31	$Y_{2,10}$ = - 0.13	$Y_{3,10}$ = + 0.05	$Y_{4,10}$ = + 0.70	
	+ 0.30	+ 0.27	- 0.29	+ 0.64	
	+ 0.11	+ 0.43	- 0.21	+ 1.20	
	+ 0.65	+ 0.34	- 0.38	+ 0.57	
	+ 0.36	+ 0.07	+ 1.17	+ 0.89	
	+ 0.58	- 0.07	+ 0.60	+ 0.75	
	+ 0.56	+ 0.40	- 0.14	+ 0.92	
	+ 0.65	+ 0.53	- 0.18	+ 0.87	
	$Y_{1,18}$ = + 0.47	+ 0.05	+ 0.24	+ 0.81	
NORTH	$Y_{1,19}$ = + 0.32	$Y_{2,19}$ = + 0.26	$Y_{3,19}$ = + 0.39	$Y_{4,19}$ = + 0.51	
	+ 0.11	+ 0.69	- 0.07	+ 0.60	
	+ 0.07	+ 0.98	+ 0.68	+ 0.98	
	+ 0.65	+ 0.55	- 0.18	+ 0.44	
	+ 0.72	+ 0.83	+ 0.59	+ 0.72	
	+ 0.69	+ 0.30	- 0.03	+ 0.25	
	+ 0.68	+ 0.62	+ 0.98	+ 0.92	
	+ 0.62	+ 0.31	+ 0.31	+ 0.81	
	+ 0.35	+ 0.42	+ 0.03	+ 1.20	
WEST	$Y_{1,27}$ = + 0.22	$Y_{2,27}$ = + 0.50	$Y_{3,27}$ = + 0.27	$Y_{4,27}$ = + 0.27	
	+ 0.46	+ 0.66	+ 0.40	+ 2.50	
	+ 0.36	+ 0.30	+ 0.03	+ 0.59	
	+ 0.05	+ 0.77	+ 0.28	+ 0.45	
	+ 0.42	- 0.03	+ 0.15	+ 0.47	
	+ 0.99	+ 0.50	- 0.43	+ 0.56	
	+ 0.40	+ 0.04	+ 0.56	+ 1.65	
	+ 0.40	+ 0.20	+ 0.57	+ 0.88	
	$Y_{1,36}$ = + 0.63	$Y_{2,36}$ = + 0.32	$Y_{3,36}$ = - 0.08	$Y_{4,36}$ = + 0.45	
	t_1 = +13.02	t_2 = +10.20	t_3 = + 9.46	t_4 = +30.14	t_0 = 62.82
	t_1^2 = 169.52	t_2^2 = 104.04	t_3^2 = 89.49	t_4^2 = 908.42	
					t_0^2=3946.35

After some calculations had been carried out, it became obvious that for statistical investigations other than the analysis of variance, larger samples would be more appropriate. The decision was then taken to reduce also the recordings of the observations made in May. The calculations presented in Section 4 are performed on samples that, by a general consensus, can be termed "large": their sizes are of the order of two thousand elements. The conclusions drawn from samples of such sizes are considered as sufficiently reliable to lead to practical conclusions. Before applying the methods of statistical analysis let us first examine the nature of several general questions which may be asked and to which the analysis that follows may propose more or less complete answers.

a) General consistency and reliability of the instruments when they are used under special conditions and, by extension, the likelihood that they will behave similarly when placed in conditions in which they are used in practice. It is obviously of considerable importance, for the manufacturer, for the official metrological institutions and, of course, for the consumer of electrical energy to determine what kind of atmospheric conditions influence the functioning of a meter. Is it possible to foresee whether the location of a meter on a house or factory can influence the amount of money a customer has to pay? Does the meter located on a sunny side of a house favour the producer or the user of electrical power?

b) Influence of the criterion of classification "manufacturer".*1 Although all meters are assumed to be "identically constructed" it is obvious that each manufacturer is likely to introduce some small specific factors the presence of which may not be directly perceptible but may be nevertheless evidenced statistically.

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*1 In statistics, terms such as "manufacturer", "direction", etc., designate "criteria" according to which a sample can be classified.

c) Influence of the criterion "geographical direction". This is actually one of the major reasons why the experiment has been undertaken. It is impossible to assert a priori that a meter facing South will register electricity in the same manner and with the same accuracy as a meter facing North. Orientation may act not only through temperature but also through such factors as wind, humidity, etc.

d) Influence of the criterion "season". This poses problems which are, from the standpoint of statistical calculations, identical to those of "direction".

The information supplied through the analysis of these problems is equally important by its implications of scientific, technical and economic nature.

The mathematical method used to compare the behaviour of the meters by taking into account a certain criterion of classification is that of "analysis of variance". The principles of this powerful statistical tool have been established by R.A. Fisher and are, in general, presented in all text books on statistics.

For the convenience of the Reader, however, the fundamentals of the analysis of variance, as they are used in this report, are briefly outlined in the following section.

2. OUTLINE OF THE ANALYSIS OF VARIANCE

In this report (as in all previous works on similar matters) the symbol Y (capital letter with appropriate subscripts) designates the so-called "Total Registration Error" (TRE) calculated on the yearly basis, i.e., from November to November. A number such as Y_k ranging from 1 to 144 is the "percentage" by which the registration of the k^{th} meter differs from that of the monitors.*2 If, for instance, $Y_{10} = -0.23$, this means that the meter of rank 10 registers (after twelve months of being energized) an amount of electricity which is 0.23% smaller than that registered by the monitors*. The latter is sometimes, but not very correctly called "true" registration, in kWh.

To make this report more easily accessible to non-statisticians, only the analysis of varience with one single criterion will be used: the 144 meters will be divided into four classes either according to the criterion "manufacturer" or according to the criterion "direction". At the beginning of the experiment (1972) all classes were exactly of the same size (36 meters). As it has been mentioned above, the initial sizes were not rigorously maintained because in the course of the years some meters were, for various reasons, eliminated but not replaced.

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*2 Monitors are the reference standards located in the laboratory (not outdoors). They form the base for the evaluation of the registration of the meters under test.

If a sample of N elements Y_k (k = 1, 2, ... N) is considered in its unclassified form, the mean is given by the formula

$$\overline{Y} = \frac{1}{N} \sum_{k=1}^{N} Y_k. \qquad \ldots(1)$$

An important and basic relation in all the calculations that follow is the expression of the sum of squares of the deviations of sample elements from any number A in terms of the sum of squares of the deviations from the sample mean $\overline{Y}$:

$$\sum_k (Y_k - A)^2 = \sum_k (Y_k - \overline{Y})^2 + N(\overline{Y} - A)^2. \qquad \ldots(2a)$$

For A = 0, this reduces to

$$\sum_k Y_k{}^2 = \sum_k (Y_k - \overline{Y})^2 + N\overline{Y}^2. \qquad \ldots(2b)$$

If the sample is distributed into "classes", these classes will be denoted by the subscript i, the total number of elements in the i^{th} class will be denoted by the symbol n_i. The rank in the class will be indicated by the subscript j so that j = 1, 2, 3, ... n_i. An element will therefore have two subscripts, e.g., Y_{ij}:

Y_{ij} with: i = 1, 2, ...h (i.e. h classes; here h = 4)
j = 1, 2, ...n_i (n_i elements in the i^{th} class)

The i^{th} class constitutes in itself an unclassified sample of n_i elements so that it is possible to apply to it the relation (1) in which a) the subscript k is replaced by the subscript j that takes on all values from j = 1 to j = n_i, and b) Y is replaced by the class mean $\overline{Y}_i$ which is equal to

$$\overline{Y}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} Y_{ij}. \qquad \ldots(3)$$

This gives

$$\sum_{j=1}^{n_i}(Y_{ij} - A)^2 = \sum_{j=1}^{n_i}(Y_{ij} - \overline{Y}_i)^2 + n_i(\overline{Y}_i - A)^2.$$

If in this relation A is made equal to the general mean $\overline{Y}$, then

$$\sum_{j=1}^{n_i}(Y_{ij} - \overline{Y})^2 = \sum_{j=1}^{n_i}(Y_{ij} - \overline{Y}_i)^2 + n_i(\overline{Y}_i - \overline{Y})^2. \qquad \ldots(4)$$

Finally, the summation with respect to all h classes leads to the fundamental formula of <u>decomposition of the sum of squares</u>:

$$\sum_{i=1}^{h}\sum_{j=1}^{n_i}(Y_{ij} - \overline{Y})^2 = \sum_{i=1}^{h}\sum_{j=1}^{n_i}(Y_{ij} - \overline{Y}_i)^2 + \sum_{i=1}^{h}(\overline{Y}_i - \overline{Y})^2 n_i. \qquad \ldots(5)$$

A particularly important feature of this expression is that a close inspection of its three terms reveals that each of them (provided we take into account the appropriate number of degrees of freedom) can lead to an estimate of the variance of the sample. For the calculation of an estimate, the term must be divided by its own specific number of degrees of freedom. The estimates are all unbiased, i.e., they all converge to the same value if the sample size grows to infinity.

The expressions for the estimates are presented below. The symbols for the estimates of variances are s^2, s'^2, s''^2 and the corresponding degrees of freedom are denoted by ν, ν', ν'', respectively.

$$s^2 = \frac{1}{N-1}\sum_{ij}(Y_{ij} - \overline{Y})^2 ; \qquad \nu = N-1. \qquad \ldots(6a)$$

$$s'^2 = \frac{1}{N-h}\sum_{ij}(Y_{ij} - \overline{Y}_i)^2 ; \qquad \nu' = N-h. \qquad \ldots(6b)$$

$$s''^2 = \frac{1}{h-1} \sum_{ij} n_i(Y_i - \overline{Y})^2 \; ; \qquad \nu'' = h-1. \qquad \ldots(6c)$$

The foundation of the analysis of variance can now be outlined as follows. If the universe of elements (i.e., a very large number of observations) from which a sample of 144 elements was drawn was homogeneous then any classification according to any external formal criterion would not introduce any statistical bias and the three estimates of the variance would not be significantly different from each other. If, however, the estimates s'^2 and s''^2 are significantly different, then this difference points to the existence of a systematic reason, i.e., to a real "cause of variation" (term adopted by statisticians). Those causes which disturb the value of s'^2 are termed "causes of variation within classes", for instance variations within the criterion of classification "manufacturer". The causes affecting s''^2 are termed "causes of variation between class means".

In mathematical statistics, the most commonly used ratio of variances is s'^2/s''^2 or its inverse. It is designated by the symbol F, to honour R.A. Fisher who created and fully developed the analysis of variance. The definition of F is therefore the following:

$$\text{if } s'^2 > s''^2, \quad \text{then } F = \frac{s'^2}{s''^2},$$

$$\text{if } s'^2 < s''^2, \quad \text{then } F = \frac{s''^2}{s'^2}. \qquad \ldots(7)$$

In most of the text books on statistics the F tables are presented in an abridged form. A more complete form may be found in CRC-Standard Math Tables. In the present case, there are h = 4 classes, hence $\nu_1 = 4-1 = 3$; the value of ν_2 is $\nu_2 = N-4 = 144-4 = 140$. In the F table, for $\nu_1 = 3$, $\nu_2 = 140$, there are two values of F: an upper value so-called "5% point" which is smaller than the lower value termed "1% point" (often printed in bold-face type). With respect to these "points", here equal to 2,4 and 3.4 respectively, the numberical values of F can be

a) below the 5% point,
b) above the 1% point,
c) between these two points.

Suppose that the hypothesis that is tested is that the variances s'^2 and s''^2 correspond to samples that are drawn from a completely homogeneous, normally distributed (or near-normally distributed) universe. The upper value a) is that value which in repeated sampling would be exceeded with a probability equal to 0.05 (i.e., 5%). The lower value b) would be, on the other hand, exceeded with a probability of 0.01 (i.e., 1%). If the value of F is below the "5% point" then this value can be considered as not significant and this would be favourable to the formulated hypothesis, i.e., to assuming the homogeneity of the universe.

This section can be summarized colloquially as follows: if the sample is classified according to the criterion "manufacturer" and if such a classification has no systematic effect (or, what is the same, the meters of one manufacturer are not distinguishable from those of other manufacturers) the variances s'^2 are s''^2 are likely to be close to each other. A similar statement can be made concerning the criterion direction. If F is large, i.e., if s'^2 is acutually very different from s''^2 then the criterion has a significant effect on the behaviour of the meters. As it has been said above, the convenient points are those which are termed 5% point and 1% point. The values that are located between these points cannot be securely assessed and may require some other investigations.

It must be finally and strongly emphasized, as the conclusion of this theoretical outline of the principles of the analysis of variance, that this method, however powerful (for testing the likelihood of an assumption) does not suggest for what real reason the hypothesis should be accepted or rejected. It is left to the experience and the imagination of the observer-calculator to uncover the reasons which produce the observed effects.

3. NUMERICAL CALCULATIONS

The form of equations (6) is not very well adapted to numerical calculations. To illustrate how these calculations should be performed, the results of the observations made in November 1972 will be treated in detail. The principal steps of the calculations are also reproduced, in a more condensed form, in the "outputs" of the computer, the meaning of the symbols used being everywhere the same. The calculations of all other years are then presented only under the form of the computer "outputs". Appendix I.

The 1972 observations (TRE in % form) are displayed in Table IA. They are classified vertically according to the criterion "manufacturer": General Electric (GE), Ferranti-Packard (FP), Westinghouse (W), Sangamo (S). The same table can also be used horizontally, the classification being made according to the criterion "direction": South (S), East (E), North (N), West (W). The calculations will be described for the vertical classification. Their sequence is the following:

a) Class totals $t_i = \sum_{j=1}^{36} Y_{ij}$. This gives t_1, t_2, t_3, t_4. ...(8)

Grand total t_0 : $t_0 = t_1 + t_2 + t_3 + t_4$.

Squares: t_1^2, t_2^2, t_3^2, t_4^2 and $t_0^2 = (t_1 + t_2 + t_3 + t_4)^2$

b) General "sum of squares" S:

$$\Sigma = \sum_{k=1}^{144} Y_k^2 = \sum_{i=1}^{4}\sum_{j=1}^{36} Y_{ij} \text{ ; } k = 1.\ 2, \ldots .144 \begin{cases} i = 1,\ 2,\ 3,\ 4 \\ j = 1,\ 2 \ldots 36 \end{cases} \quad \ldots(9)$$

c) Class means squares μ_i: $\mu_i = \frac{t_i^2}{n_i}$, $\mu_1 = \frac{t_1^2}{36}$, $\ldots \mu_4 = \frac{t_4^2}{36}$. ...(10)

d) Total class mean square μ_0 : $\mu_0 = \sum_i \mu_i = \mu_1 + \mu_2 + \mu_3 + \mu_4$. ...(11)

e) General mean square M : $M = \frac{t_0^2}{N} = \frac{(t_1 + t_2 + t_3 + t_4)^2}{144}$ ...(12)

The equation (6a) can be, in conformity with (2), put under the form

$$\sum_{ij}(Y_{ij} - \overline{Y})^2 = \sum_{ij} Y_{ij}^2 - N\overline{Y}^2 = S - N(\frac{t_0}{N})^2$$

$$= S - \frac{t_0^2}{N} = S - M = [A] \text{ (output)} \qquad ...(13)$$

This is also valid for (6b) except that the symbol M must be replaced by μ_0:

$$\sum_{ij}(Y_{ij} - \overline{Y}_i)^2 = S - \mu_0 = [B] \text{ (output).}$$

Finally, the relation (6c) leads to

$$\sum_i n_i(\overline{Y}_i - \overline{Y})^2 = \mu_0 - M = [C] \quad \text{(output)}$$

The numerical values in the above relations are

$\mu_1 = 4.71$, $\mu_2 = 2.89$, $\mu_3 = 2.49$, $\mu_4 = 25.23$, $\mu_0 = 35.32$.
S = 58.23. M = 27.41.
[A] = 30.82, [B] = 22.91, [C] = 7.91.

Table of final formulae

$$s^2 = \frac{1}{N-1} \sum_{ij}(Y_{ij} - \overline{Y})^2 = \frac{[A]}{N-1} = \frac{30.82}{143} = 0.22.$$

$$s'^2 = \frac{1}{N-h} \sum_{ij}(Y_{ij} - \overline{Y}_i)^2 = \frac{[B]}{N-h} = \frac{22.91}{140} = 0.16.$$

$$s''^2 = \frac{1}{h-1} \sum_i ni(\overline{Y}_i - \overline{Y})^2 = \frac{[C]}{h-1} = \frac{7.91}{3} = 2.64.$$

Fisher's Index F:

$$F = \frac{s''^2}{s'^2} = \frac{2.64}{0.16} = \underline{16.2}$$

According to the meaning of F, this shows that the value of F is very significant as, in the table, the 5% point is equal to 2.4 (for $\nu_1 = 140$, $\nu_2 = 3$). This is interpreted as being strongly against the assumption that the sample is homogenous with respect to manufacturers.

The above calculations are also summarized in the upper half of Table IB. The lower half shows the results of the calculations concerning the criterion "direction". Here, F = 1.56 which, with respect to 1% point (3.4) is very small. Hence, the assumption that the sample is homogeneous with respect to the geographical orientation of the meters is very likely to be correct.

The examination of all eight tables (I to VIII) confirms the conclusion yielded by the year 1972. The Reader is now justified to ask whether similar conclusions could be reached by our examining directly the numerical data presented in the tables. Consider the values of class totals $t_i(i = 1, 2, 3, 4)$:

	t_1	t_2	t_3	t_4
Manuf.	13.02	10.20	9.46	30.14
Direct.	13.62	14.09	18.35	16.76

The total "Manuf. $t_4 = 30.14$" is very different from other totals on the same horizontal line but no such remark can be made concerning the second line (Direct.). Thus by classifying meters according to the criterion "manufacturer" we form a group of 36 meters (with $t_4 = 30.14$) which is substantially different from the other three analogous groups. On the other hand, the classification according to directions produces four groups that are very similar to each other (second line in the table above).

A few conclusions of a general nature which result from the analysis of variance are joined to those which will result from the analysis performed in the next section.

4. OTHER STATISTICAL INVESTIGATIONS

In the "analysis of variance" a sample is represented by its mean and variance. No attention is paid to its actual internal frequency distribution. However, it is necessary to check that this distribution is not too far from normality (bell-shaped) as the establishment of Fisher's Tables is founded on the postulate that the distribution of the universe is normal or close to normal. The Reader can easily check that such is the case by examining the data presented in the computer "outputs" (Tables I and VIII). In the present section, the internal structure of the samples will play an important role, particularly in relation with the structure of the mixtures of partial subsamples into which the total populations can be subdivided.

Before performing this analysis, let us first emphasize that a sample of 144 elements is actually rather a "small" sample. If it is subdivided into parts, these parts are so small that statistical calculations can lead only to unreliable conclusions. In order to operate on larger samples, all observations made during the eight consecutive years were lumped into one single sample. In addition, it was decided to recalculate all TRE anew but on a six-month basis. This has led to a total population of 2252 elements which, because of eliminations, is a little smaller than the theoretical maximum of 2 x 144 x 8 = 2304. The loss of 52 TRE is due to the elimination of 7 meters (indicated by asterisks in the outputs). The total population consists therefore of:

a) Eight "May Tables" (analogous to Table IA) containing 1128 TRE calculated for six-month periods, November to May;

b) Eight "November Tables" containing 1124 TRE calculated for six-month periods, May to November.

The errors recorded in May belong to the period that contains winter months so that this period is called "cold season" and is designated by the subscript "c". The period extending from May to November contains summer months and is called "warm season" (subscript "w"). The individual values of the 2252 TRE are not given in the present memo; however, they figure in classified form in both Fig. 1 and Fig. 2: in Fig. 1 the total (2252) is considered as a "mixture" of two subsamples, subsample "cold season" and subsample "warm season"; in Fig. 2 the total is a mixture of two subsamples, "cold direction" and "warm direction". The statistics of the subdivision into "seasons" are the following:

Cold season (Nov-May):	$N_c = 1128$,	$\overline{m}_c = 0.57$,	st. dev.	$s_c = 0.52$
Warm season (May-Nov):	$N_w = 1124$,	$\overline{m}_w = 0.22$	" "	$s_w = 0.59$
Total:	$N_t = 2252$,	$\overline{m}_t = 0.39$,	" "	$s_b = 0.58$

Before drawing conclusions from Fig. 1 and 2 it is necessary to mention again, the important fact that, in the course of the eight-year long experiment, seven meters have been eliminated for malfunctioning. The manner in which the registrations have been recorded (the control is not continuous) does not provide a better insight on the manner in which malfunctionings have actually started to appear. In about half of the cases, the meter was found to have simply stopped (and remained stopped for the rest of the experiment). In other cases, the reading was found to be totally erroneous. Only an experienced manufacturer and an expert in electrical measurements can judge the quality of the meters that were tested in the manner described in this memorandum. If the cases of malfunctioning are considered as not significantly numerous, one can say that the meters have been working satisfactorily. A meter seems to be, within the accepted tolerance limits, a reliable instrument in spite of the hard climate of the Ottawa region.

Concerning frequency distribution, it is possible to make the following general remark: all five observed diagrams are reasonably regular, symmetric and bell-shaped but four of them are not really well represented by Gaussian curves. They are leptokurtic with different degrees of leptokurtosis.*3 In the modern statistical theory of errors the presence of leptokurtosis is simply treated as an additional characteristic of a sample without any specific meaning so far as the quality of observations is concerned.

The general impression (as it may result from a superficial look) produced by Fig. 1 is that of "disperion" while that produced by Fig. 2 is that of "compactness". This suggests the idea that the criterion "season" influences the behaviour of meters more strongly than the criterion "direction". A closer examination shows that in both figures the means with subscript w are close to 0 than the means with subscript c: it seems therefore that in warmer conditions (South and Summer) the registration errors have the tendency to decrease while in colder conditions there is a tendency in the opposite sense. In addition, the means of the subsamples are farther apart in Fig. 1 than in Fig. 2 and the difference in the former is not negligible:

$$\overline{m}_c - \overline{m}_w = 0.35$$

As s_c and s_w are both of the order of 0.55, the difference is of the order of 0.6 standard deviation. This is not an insignificant difference. This can be expressed colloquially as follows: it is to the advantage of the consumer to place his meter on the Southern side of the building and to use

---

*3 An observed diagram of class frequencies is said to be leptokurtic (K. Pearson's term) if, in its central portion, the observed class frequencies (represented by dots . in both "totals") are greater than the corresponding theoretical class frequencies of the Gaussian curve that fits into the diagram. In the curve representing the total sample (2252 elements) there are nine dots . that are above the Gaussian curve. This central excess is compensated by the corresponding lowering of . dots on both sides of the central part.

the electricity mostly during the warm part of the year. However, his gain would be only "probable" and would be only very slight!

Note

The sample of Total Registration Errors (TRE) analyzed above is also used to illustrate the theory of "mixtures", particularly when the components are of the modulated normal type. The article on this subject is published in the Bulletin Géodésique, 53, No. 1, 1983, p. 62 and can be obtained from the authors (MR or EG) at NRC, Division of Physics, Ottawa, Ontario, Canada K1A OR6.

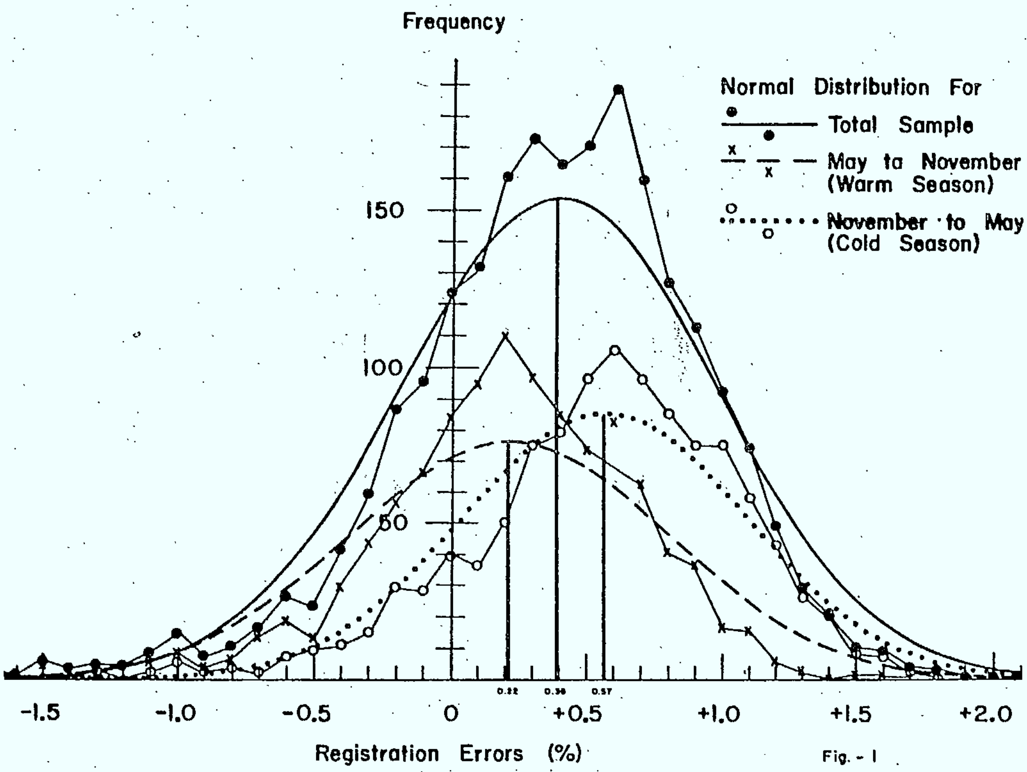
Frequency
150
100
50
Normal Distribution For
Total Sample
May to November
(Warm Season)
November to May
(Cold Season)
-1.5
-1.0
-0.5
0
+0.5
+1.0
+1.5
+2.0
Registration Errors (%)

Fig. - I

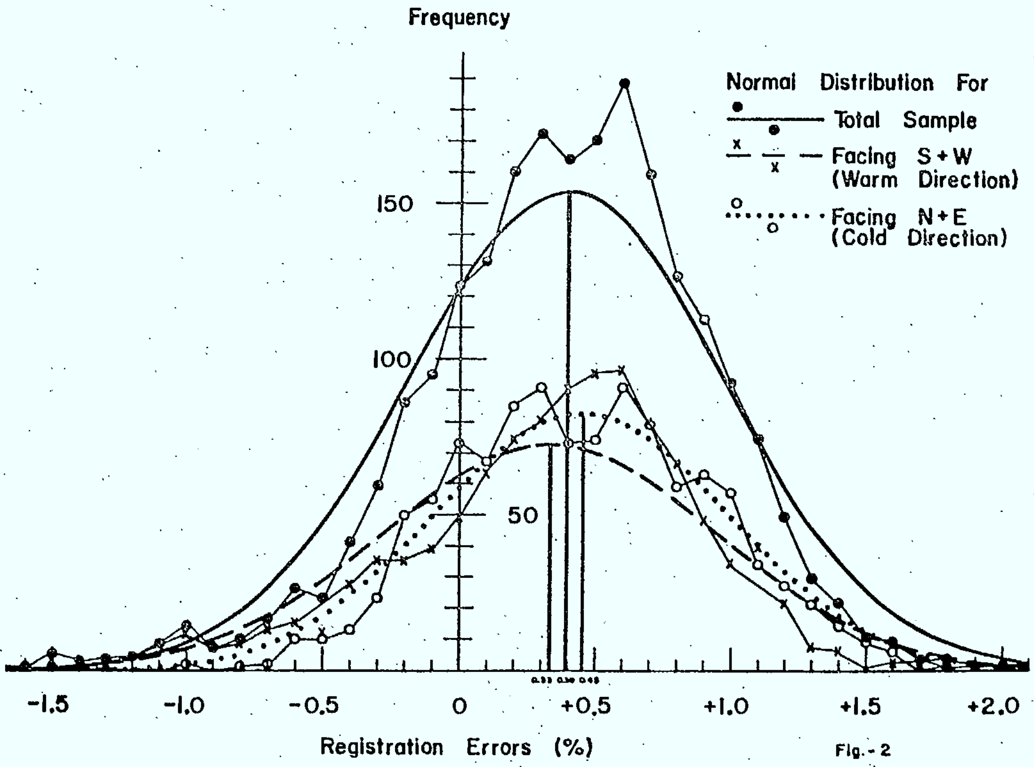
Frequency
150
100
50
Normal Distribution For
Total Sample
Facing S + W
(Warm Direction)
Facing N + E
(Cold Direction)
-1.5
-1.0
-0.5
0
+0.5
+1.0
+1.5
+2.0
Registration Errors (%)

Fig. - 2

APPENDIX I

This appendix contains the tables of the observed data and the computer "outputs" for each of the years 1972 - 1979. In these tables the observed data are represented as in Table IA.

The reader will notice that the numerical results, although somewhat different from year to year, lead to the same conclusions as those presented at the end of Section 2.

Date of Data is 1972 NOV

	GE	FP	W	S
SOUTH	-.01	.72	.62	1.24
	-.20	.56	.60	1.40
	.12	-.95	.09	.72
	.39	.83	.76	1.15
	.07	.24	.14	.80
	-.17	-.40	-.72	.62
	.16	.44	.73	.64
	-.01	-.91	1.06	1.02
	.54	-.49	.87	.95
EAST	.31	-.13	.05	.70
	.30	.27	-.29	.64
	.11	.43	-.21	1.20
	.65	.34	-.38	.57
	.36	.07	1.17	.89
	.58	-.07	.60	.75
	.56	.40	-.14	.92
	.65	.53	-.18	.87
	.47	.05	.24	.81
NORTH	.32	.26	.39	.51
	.11	.69	-.07	.60
	.07	.98	.68	.98
	.65	.55	-.18	.44
	.72	.88	.59	.72
	.69	.30	-.03	.25
	.68	.62	.98	.92
	.62	.31	.31	.81
	.35	.42	.03	1.20
WEST	.22	.50	.27	.27
	.46	.66	.40	2.50
	.36	.30	.03	.59
	.05	.77	.28	.45
	.42	-.03	.15	.47
	.99	.50	-.43	.56
	.40	.04	.56	1.65
	.40	.20	.57	.88
	.63	.32	-.08	.45
	t_1	t_2	t_3	t_4

TABLE I-B

Date of Data is 1972 NOV

Criterion- MANUFACTURER

S= 58.23   N= 144.00
M= 27.41   h=4

Manufacturer	GE	FP	W	S
Totals (t)	13.02	10.20	9.46	30.14
Totals squared	169.52	104.04	89.49	908.42
Mu	4.71	2.89	2.49	25.23

t zero	=	62.82	N-h= 140.00
t zero squr.	=	3946.35	h-1=3
Mu zero	=	35.32	
A	=	30.82	
B	=	22.91	
C	=	7.91	
s' square	=	.16	
S'' square	=	2.64	
F	=	16.12	
(v' 3, v''=		140.00)	

Criterion- DIRECTION

S= 58.23   N= 144.00
M= 27.41   h=4

Direction	SOUTH	EAST	NORTH	WEST
Totals (t)	13.62	14.09	18.35	16.76
Totals squared	185.50	198.53	336.72	280.90
Mu	5.15	5.51	9.35	7.80

t zero	=	62.82	N-h= 140.00
t zero squr.	=	3946.35	h-1=3
Mu zero	=	27.82	
A	=	30.82	
B	=	30.40	
C	=	.42	
s' square	=	.22	
s'' square	=	.14	
F	=	1.56	
(v'=3 v''=		140.00)	

Date of Data is 1973 NOV

.18	.93	1.46	1.39
-.09	.64	.82	.87
.18	-.52	.25	.94
.55	.88	.89	1.00
.14	.48	.18	1.10
-.03	-.37	-.53	.68
.36	.53	.66	.93
.07	-.88	1.16	1.07
.68	-.56	.90	1.00
.37	.01	.23	1.04
.36	.51	-.10	.72
.21	.77	-.11	1.21
.72	.43	-.19	.67
.43	.12	1.27	.94
0.00	.96	.64	.80
.63	.56	-.03	.94
.73	.62	-.16	1.12
.51	.04	.33	.81
.31	.35	.41	.53
.13	1.14	-.04	.56
.04	1.09	.67	1.04
.65	.56	.02	.52
.78	.91	.60	.73
.71	.43	-.07	.27
.70	.79	.99	1.01
.59	.38	.31	1.03
.38	.35	.21	1.29
.23	.59	.35	.39
.50	.07	.57	-1.15
.34	1.40	.08	.65
.07	.85	.35	.50
-.06	.05	.20	.52
1.04	.59	-.32	.55
.47	.05	.61	-.07
.47	.19	.67	.95
.71	.43	-.01	.35

Date of Data is 1973 NOV

TABLE II

Criterion- MANUFACTURER

S= 63.06 N= 144.00
M= 33.64 h=4

Manufacturer	GE	FP	W	S
Totals (t)	14.06	15.37	13.27	26.90
Totals squared	197.68	236.24	176.09	723.61
Mu	5.49	6.56	4.89	20.10

t zero	=	69.60	N-h= 140.00
t zero squr.	=	4844.16	h-1=3
Mu zero	=	37.05	
A	=	29.42	
B	=	26.01	
C	=	3.41	
s' square	=	.19	
S'' square	=	1.14	
F	=	6.11	
(v' 3,	v''=	140.00)	

Criterion- DIRECTION

S= 63.06 N= 144.00
M= 33.64 h=4

Direction	SOUTH	EAST	NORTH	WEST
Totals (t)	17.94	18.11	20.37	13.18
Totals squared	321.84	327.97	414.94	173.71
Mu	8.94	9.11	11.53	4.83

t zero	=	69.60	N-h= 140.00
t zero squr.	=	4844.16	h-1=3
Mu zero	=	34.40	
A	=	29.42	
B	=	28.66	
C	=	.76	
s' square	=	.20	
s'' square	=	.25	
F	=	.81	
(v'=3	v''=	140.00)	

Date of Data is 1974 NOV

-.10	.60	.88	.84
-.27	.29	.63	.56
.03	-1.26	0.00	.56
.35	.42	.64	.81
-.13	.40	.06	.80
-.48	-.81	1.02	.47
.05	.34	.51	.56
-.61	-1.20	.08	.85
.44	-.76	.63	.77
.04	-.31	.15	.99
.19	.29	.01	.55
.06	.50	-.07	1.09
.57	.23	.28	.43
.30	-.02	1.38	.79
-.17	.83	.54	.63
.52	.38	-.13	.90
.57	.46	-.26	.87
.60	-1.01	.25	.70
.19	.09	.28	.39
0.00	.81	-.18	.38
-.06	.86	.46	.86
.53	.50	-.12	.39
.61	.76	.61	.60
.60	.19	-.16	.13
.50	.61	1.00	.84
.42	.15	1.11	.95
.23	.43	.12	1.22
.06	.29	.26	.22
.32	1.22	.44	.50
.22	1.03	-.18	.43
-.10	.65	.16	.29
***	-.23	0.00	.30
.87	.26	-.46	.39
.25	-.25	1.46	.15
.21	-.08	.49	.72
.50	.33	.33	.37

Date of Data is 1974 NOV

TABLE III

Criterion- MANUFACTURER

S= 47.27 N= 143.00
M= 16.67 h=4

Manufacturer	GE	FP	W	S
Totals (t)	7.31	6.99	12.22	22.30
Totals squared	53.44	48.86	149.33	497.29
Mu	1.53	1.36	4.15	13.81

t zero =	48.82	N-h= 139.00
t zero squr.=	2383.39	h-1=3
Mu zero =	20.85	
A =	30.60	
B =	26.42	
C =	4.18	
s' square =	.19	
S'' square =	1.39	
F =	7.33	
(v' 3, v''=	139.00)	

Criterion- DIRECTION

S= 47.27 N= 143.00
M= 16.67 h=4

Direction	SOUTH	EAST	NORTH	WEST
Totals (t)	7.97	13.13	16.30	11.42
Totals squared	63.52	172.40	265.69	130.42
Mu	1.76	4.79	7.38	3.73

t zero =	48.82	N-h= 139.00
t zero squr.=	2383.39	h-1=3
Mu zero =	17.66	
A =	30.60	
B =	29.61	
C =	.99	
s' square =	.21	
s'' square =	.33	
F =	.64	
(v'=3 v''=	139.00)	

Date of Data is 1975 NOV

-.03	.76	.96	.70
-.19	.21	.72	.62
.06	-.16	.05	.59
.42	.37	.79	.81
-.07	.58	***	1.02
-.42	-.72	.34	.47
.08	.29	.63	.62
-.05	-.27	1.02	.84
.47	-.94	.62	.78

-.51	.29	.35	1.16
.24	.42	.11	.61
.11	.50	.09	1.09
.55	.25	-.14	.46
.31	-.01	1.55	.80
.37	.83	.65	.67
.49	.41	-.02	.96
.53	.49	-.11	.89
.44	-.14	.19	.83

.13	.05	.30	.39
-.06	.74	-.10	.31
-.08	.78	.44	.83
.40	.50	-.01	.36
.49	.79	.67	.61
.47	.13	.15	.09
.39	.59	-1.00	.81
.36	.14	.26	.90
.21	.51	.24	1.16

.02	.28	.41	.17
.28	.37	.53	.53
.18	-.83	-.11	.37
-.18	.46	.26	.25
***	-.29	.03	.3
.72	.05	-.26	.33
.22	-.36	.44	.60
.10	-.15	.59	.66
.47	-.02	***	.51

Date of Data is 1975 NOV

TABLE IV

Criterion- MANUFACTURER

S= 40.12 N= 141.00
M= 16.10 h=4

Manufacturer	GE	FP	W	S
Totals (t)	6.94	6.90	10.64	23.16
Totals squared	48.16	47.61	113.21	536.39
Mu	1.38	1.32	3.33	14.90

t zero	=	47.64	N-h= 137.00
t zero squr.	=	2269.57	h-1=3
Mu zero	=	20.93	
A	=	24.03	
B	=	19.19	
C	=	4.83	
s' square	=	.14	
S'' square	=	1.61	
F	=	11.50	
(v' 3,	v''=	137.00)	

Criterion- DIRECTION

S= 40.12 N= 141.00
M= 16.10 h=4

Direction	SOUTH	EAST	NORTH	WEST
Totals (t)	11.99	15.71	12.95	6.99
Totals squared	143.76	246.80	167.70	48.86
Mu	4.11	6.86	4.66	1.44

t zero	=	47.64	N-h= 137.00
t zero squr.	=	2269.57	h-1=3
Mu zero	=	17.06	
A	=	24.03	
B	=	23.06	
C	=	.96	
s' square	=	.17	
s'' square	=	.3	
F	=	.52	
(v'=3	v''=	137.00)	

Date of Data is 1976 NOV

.19	1.05	1.23	.85
-.05	.37	.74	.69
.18	-.59	.23	.75
.56	.61	.86	.87
-.01	.80	-.08	1.07
-.21	-.09	.33	.61
.32	.35	.61	.79
.02	-1.16	1.05	.95
.64	-.95	.61	.92

***	-.15	.67	1.18
.36	.55	.02	.70
.24	.68	.11	1.07
.69	.36	-.18	.52
.37	.11	1.50	.86
***	.91	.72	.78
.61	.45	-.05	1.10
.66	.44	-.16	1.11
.57	-.16	.21	1.21

.29	.25	.46	.49
.09	.95	.05	.46
.03	.98	.67	1.01
.53	.49	.05	.45
.68	.93	.74	.68
.66	.31	.26	.21
.60	.72	.82	1.00
.54	.21	.39	1.15
.28	.60	.30	1.31

.14	.46	.57	.37
.45	.76	.70	.57
.27	1.33	.08	.62
.01	.70	.29	.44
***	-.21	.36	.52
.97	.33	-.23	.55
.36	-.30	.35	.86
.34	-.17	.60	.88
.67	.13	***	.82

Date of Data is 1976 NOV

TABLE V

Criterion- MANUFACTURER

S= 58.24   N= 140.00
M= 32.45   h=4

Manufacturer	GE	FP	W	S
Totals (t)	12.05	12.05	14.88	28.42
Totals squared	145.20	145.20	221.41	807.70
Mu	4.40	4.03	6.33	22.44

t zero = 67.40   N-h= 136.00
t zero squr.= 4542.76   h-1=3
Mu zero = 37.20
A = 25.79
B = 21.05
C = 4.75
s' square = .15
S'' square = 1.58
F = 10.23
(v' 3, v''= 136.00)

Criterion- DIRECTION

S= 58.24   N= 140.00
M= 32.45   h=4

Direction	SOUTH	EAST	NORTH	WEST
Totals (t)	15.11	18.06	19.64	14.59
Totals squared	228.31	326.16	385.73	212.87
Mu	6.34	9.59	10.71	6.26

t zero = 67.40   N-h= 136.00
t zero squr.= 4542.76   h-1=3
Mu zero = 32.91
A = 25.79
B = 25.33
C = .46
s' square = .19
s'' square = .15
F = 1.21
(v'=3 v''= 136.00)

Date of Data is 1977 NOV

.19	1.12	1.31	.86
-.05	.33	.84	.66
.04	-.54	.27	.78
.55	.58	.90	.89
-.03	.87	-.04	.89
-.17	0.00	.39	.64
.28	.38	.72	.82
.03	-1.20	1.11	.96
.64	-.98	.77	.91
***	-.16	.78	1.22
.30	.55	.05	.73
.22	.66	.23	1.31
.69	.40	-.14	.50
.54	-.12	1.55	.93
***	.95	.76	.81
.59	.44	.02	1.13
.65	.42	-.12	1.16
.59	-.20	.25	1.39
.30	.24	.54	.51
.10	.97	.13	.48
.04	.99	.70	1.09
***	.50	.13	.46
.66	1.00	.79	.65
.66	.32	.30	.24
.61	.77	1.22	1.02
.52	.15	.47	1.20
.30	.62	.30	1.36
.05	.50	.60	.38
.45	.75	.78	.59
.28	1.42	.19	.66
.01	.73	.42	.47
***	-.20	.49	.59
.97	.35	0.00	.58
.36	-.34	.41	.89
.33	-.18	.67	.85
.69	.13	***	.81

Date of Data is 1977 NOV

TABLE VI

Criterion- MANUFACTURER

S= 63.97　　N= 139.00
M= 36.08　　h=4

Manufacturer	GE	FP	W	S
Totals (t)	11.39	12.22	17.79	29.42
Totals squared	129.73	149.33	316.48	865.54
Mu	4.05	4.15	9.04	24.04

t zero	=	70.82	N-h= 135.00
t zero squr.	=	5015.47	h-1=3
Mu zero	=	41.29	
A	=	27.89	
B	=	22.68	
C	=	5.20	
s' square	=	.17	
S'' square	=	1.73	
F	=	10.33	
(v' 3,	v''=	135.00)	

Criterion- DIRECTION

S= 63.97　　N= 139.00
M= 36.08　　h=4

Direction	SOUTH	EAST	NORTH	WEST
Totals (t)	15.72	19.08	20.34	15.68
Totals squared	247.12	364.05	413.72	245.86
Mu	6.86	10.71	11.82	7.23

t zero	=	70.82	N-h= 135.00
t zero squr.	=	5015.47	h-1=3
MU zero	=	36.62	
A	=	27.89	
B	=	27.35	
C	=	.54	
s' square	=	.20	
s'' square	=	.18	
F	=	1.12	
(v'=3	v''=	135.00)	

Date of Data is 1978 NOV

-.41	.60	.82	.41
-.52	-.16	.47	.27
***	-1.35	-.18	.36
.02	.12	.42	.54
-.49	.43	-.18	.43
-.75	-.64	.04	.24
-.30	0.00	.44	.36
-.41	-1.77	.73	.53
.16	-1.36	.48	.46
***	-.79	.43	.73
-.10	-.01	-.14	.39
-.23	.04	-.05	.8
.29	-.04	-.48	.12
.06	-.28	1.31	.67
***	.60	.35	.46
.22	-.03	-.29	.79
.21	.09	-.39	.80
.19	-.46	-.19	1.06
***	-.17	.27	.22
-.23	.52	-.14	.21
-.28	.60	.42	.79
***	.18	-.08	.20
.33	.71	.43	.43
.36	-.04	-.03	-.01
.22	.42	.75	.71
.19	-.23	.19	.91
-.04	.12	.06	1.05
-.13	.10	.31	.06
.06	.27	.43	.20
-.06	.93	-.15	.29
-.41	.32	.18	.15
***	-.59	.12	.30
.62	-.01	.32	.20
.01	-.75	.12	.57
-.06	-.54	.37	.50
.36	-.32	***	.53

Date of Data is 1978 NOV

TABLE VII

Criterion- MANUFACTURER

S= 32.69 N= 137.00
M= 2.73 h=4

Manufacturer	GE	FP	W	S
Totals (t)	-1.12	-3.49	7.16	16.79
Totals squared	1.25	12.18	51.27	281.90
Mu	.04	.34	1.46	7.83

t zero	=	19.34	N-h= 133.00
t zero squr.=		374.04	h-1=3
Mu zero	=	9.68	
A	=	29.9	
B	=	23.01	
C	=	6.95	
s' square	=	.17	
S'' square	=	2.32	
F	=	13.38	
(v' 3,	v''=	133.00)	

Criterion- DIRECTION

S= 32.69 N= 137.00
M= 2.73 h=4

Direction	SOUTH	EAST	NORTH	WEST
Totals (t)	-.19	6.19	9.04	4.30
Totals squared	.04	38.32	81.72	18.49
Mu	.00	1.13	2.40	.54

t zero	=	19.34	N-h= 133.00
t zero squr.=		374.04	h-1=3
Mu zero	=	4.08	
A	=	29.96	
B	=	28.61	
C	=	1.35	
s' square	=	.22	
s'' square	=	.45	
F	=	.48	
(v'=3	v''=	133.00)	

Date of Data is 1979 NOV

.09	1.16	1.25	.71
-.14	.21	.80	.56
***	-.69	.27	.70
.54	.45	.92	.84
-.11	.83	0.00	.77
-.96	-.03	.40	.57
.18	.38	.75	.79
.02	-1.46	1.09	.8
.63	-1.15	.60	.85
***	-.48	.87	1.01
.37	.53	.22	.73
.21	.63	.34	1.19
.67	.31	-.12	.44
.52	.07	1.64	1.02
***	1.00	.82	.78
.60	.36	.05	1.16
.63	.39	-.10	1.14
.59	-.28	.26	1.38
.34	.22	.54	.58
.10	.94	.15	.47
.03	.95	.73	1.11
***	.49	.08	.43
.63	1.08	.81	.69
.64	.25	.17	.25
.56	.73	1.09	.98
.51	.04	.50	1.21
.28	.58	.32	1.32
.12	.54	.61	.34
.36	.71	.90	.51
.25	1.38	.19	.60
-.06	.66	.50	.41
***	-.28	.38	.58
.93	.29	-.03	.51
.36	-.48	.43	.89
.28	-.24	.70	.85
.69	-.03	***	.85

Date of Data is 1979 NOV

TABLE VIII

Criterion- MANUFACTURER

S= 63.47 N= 138.00
M= 31.70 h=4

Manufacturer	GE	FP	W	S
Totals (t)	9.86	10.06	18.13	28.09
Totals squared	97.22	101.20	328.70	789.05
Mu	3.14	2.81	9.39	21.92

t zero	=	66.14	N-h= 134.00
t zero squr.	=	4374.50	h-1=3
Mu zero	=	37.26	
A	=	31.77	
B	=	26.21	
C	=	5.56	
s' square	=	.20	
S'' square	=	1.85	
F	=	9.47	
(v' 3,	v''=	134.00)	

Criterion- DIRECTION

S= 63.47 N= 138.00
M= 31.70 h=4

Direction	SOUTH	EAST	NORTH	WEST
Totals (t)	12.69	18.95	19.80	14.70
Totals squared	161.04	359.10	392.04	216.09
Mu	4.60	10.56	11.20	6.36

t zero	=	66.14	N-h= 134.00
t zero squr.	=	4374.50	h-1=3
Mu zero	=	32.72	
A	=	31.77	
B	=	30.75	
C	=	1.02	
s' square	=	.23	
s'' square	=	.34	
F	=	.67	
(v'=3	v''=	134.00)	

APPENDIX II

Note on the correlation between the Total Registration Errors (TRE) and the Disk Errors (DE) in the Project "Meters on the Roof".

This note has been prepared in coauthorship with B. Fulsom, Head, Electrical Laboratory and J. Samuel, summer student (1982).

DISC ERRORS IN THE PROJECT "METERS ON THE ROOF"

I. Introduction

The basic feature of "Total Registration Errors" (TRE) is that they require that the measurements be made at long intervals, at least several months. This implies the consumption of large quantities of electrical energy and the maintenance, during long periods, of appropriate circuitry and installations. One of the reasons for such a situation is that the mechanism of a meter gives reliable readings only when relatively large quantities of watt hours are considered. Nevertheless, for the manufacturer and for the user who consumes large amounts of electrical energy, TRE are obviously of the greatest importance and must be investigated.

In current practice, however, one of the routine operations is the determination of the meter's error over a very short time e.g. only a few minutes. It is obvious that such a determination cannot be as reliable as that of the total registration error but, for obvious reasons, it must be performed. When a user finds that a meter requires to be checked, it is transferred to an appropriate laboratory in which the so-called Disk Error (DE) is determined.

The fundamental property of an induction type meter is that a turn of its rotating disk corresponds to a constant number of units of energy (joule or watt hour). This constant, denoted by the symbol "Kh", is somewhat disturbed by the conditions in which the measurement is performed. The determination of the TRE benefits from the averaging effect due to the very large number of turns which take place during long periods of time.

Another important point that must be mentioned concerns the current used for DE determinations. The influence of its intensity on Kh is not simple and in any case, is far from linear. In common practice, Kh is determined with the current being equal to 2.5 A and 25 A. All manufacturers aim at producing meters that possess prescribed values for disk constants and a meter is considered (by the manufacturer) as acceptable for domestic electrical installation if its Disk Error is not larger than 1%. In fact, such a limit is narrower than the legal tolerance (2%) because the checking of new meters is based on sampling procedures and not on the examination of every individual instrument.

The reference for all DE is an instrument called "Transfer standard". It generates pulses, the time between two consecutive pulses corresponding to a constant amount of electrical energy. It is used in the laboratory of Legal Metrology to determine the constant Kh of the "Reference Watt hour Standard". The constant Kh is adjusted so that one watt hour corresponds to three revolutions of the disk. In other words, Kh = 1/3 and legally a meter is judged acceptable if it is not in error by more than 2%.

II. Experimental Work and Calculation of Correlation:

It is obvious that even on a high quality meter a linear relationship between its TRE and DE will not be absolutely perfect. In statistical language we can however expect that these two kinds of errors will be, although not "perfectly", but "very highly correlated". In the course of the eight-year long experiment already described, a certain number of determinations of the Disk Errors have been also performed. However, for practical reasons, the population of DE thus constituted could not be treated in its entirety and only three separate "periods" could be calculated and statistically analysed. The first and the third periods extend over a year, the second period extends over two consecutive years. The DE for a period is the average of the observations performed at the beginning, the middle and the end of the period. This average is denoted by the symbol X_i so that, in each period, to a TRE Y_i corresponds a Disk Error X_i. The tables of the X_i are not presented in this memorandum and the data concerning the periods are summarized as follows:

First period: 1 Nov 1971 to 11 Oct 1972, N = 144 meters.
Second " : 15 Oct 1974 to 15 Oct 1976, N = 139 meters.
Third " : 12 Oct 1975 to 15 Oct 1979, N = 138 meters.

The sequence of operations leading to the value of the "correlation index" r can be presented as follows:

A) Calculation of means $\overline{Y}$ and $\overline{X}$:

$$\overline{Y} = \frac{1}{N}\sum_{i-1}^{N} \overline{Y}_i \; ; \; \overline{X} = \frac{1}{N}\sum_{i-1}^{N} X_i$$

B) Calculation of the deviations y_i and x_1:

$$y_i = Y_i - \overline{Y} \; ; \; x_i = X_i - \overline{X}.$$

C) Calculation of the variances s_y^2 and s_x^2:

$$s_y^2 = \frac{1}{N}\sum_{i=1}^{N} y_i^2 \; ; \; s_x^2 = \frac{1}{N}\sum_{i=1}^{N} x_i^2.$$

D) Calculation of the covariance:

$$\mu_{11} = \frac{1}{N}\sum_{i=1}^{N} y_i x_i.$$

E) The correlation index r, as it results from these values, is:

$$r = \frac{\mu_{11}}{s_y s_x}$$

F) The significance of the obtained numerical value of r is obtained by referring this value to the corresponding Student's t-variate. The relation between t and r is:

$$t = \frac{r}{1-r^2} (N - 2)$$

A strong correlation between y and x (as it is expected here) leads to values of r that are close to unity; that in its turn leads to large values of t. With, for instance, r = 0.5 (N = 144), t = 6.9. This is a very large t and it indicates the existence of a strong correlation.

The numerical values of various quantities mentioned above are, in present analysis, given in the following table.

	FIRST PERIOD (1972)	SECOND PERIOD (1976)	THIRD PERIOD
μ_{11} =	0.170	0.180	0.250
s_y^2 =	0.215	0.185	0.236
s_x =	0.464	0.430	0.486
s_x^2 =	0.301	0.324	0.361
s_x =	0.549	0.569	0.601
r =	0.667	0.736	0.855

It is to be noted that all values of r are larger than 0.5. For r = 0.667, t is larger than 10. The correlations between X and Y in all three periods are therefore extremely strong. The overall conclusion of the experiment and of its statistical analysis is that there is no reason to invalidate the traditional use of the Disk Error test to check the correctness of an electrical meter.

If the circumstances permit (i.e. if the appropriate manpower becomes available) it may become possible to apply the Analysis of Variance directly to Disk Errors. It would be interesting to see how this analysis agrees, in its conclusions, with that of the Analysis of Variance applied to the Total Registration Errors (TRE).