CONSUMER INFORMATION, ADVERTIZING AND QUALITY VARIATIONS

PART 1 of 2

SUMMARY

APPENDIX A - D

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SUMMARY

This project provides some theoretical foundation necessary to evaluate the role of advertising in consumer goods markets. In particular we supply partial answers to two sets of related quations which must determine the appropriateness and form of government intervention. First: does the market operate efficiently to supply the correct amount and type of information to consumers? Do sellers have an incentive to supply misleading or irrelevant information, or are they disciplined by market forces to eliminate this incentive? Can the conditions conducive to such departures be identified? And second: does advertising create and promote monopoly either in the form of barriers to entry or in the form of "artificial product differentiation" which insulates firms from existing competition?

Economists and regulators have frequently posed these questions. There exists a vast body of empirical economic research which attempts to evaluate the second set of questions. However, the theoretical foundation of most empirical analysis is so faulty as to make much of it virtually meaningless. In the absence of a meaningful theory, the empirical debate has centered on definitions of profit and expense and consequently yielded widely divergent results, frequently derived from the same data.

The problem appears to be a lack of economic theory explaining consumer and producer behaviour under conditions where information is

scarce and costly. A thorough search of economics and marketing literature (a brief summary of which constitutes appendix A) yielded very little insight into the problem. Most of the theoretical work in economics is restricted to a study of the optimal decision rules for consumer sampling strategies when searching for the lowest price of a specific good. The marketing literature is composed mostly of case studies reporting the results of specific marketing strategies. It is not very useful in the absence of a unifying theoretical framework.

Our main effort is directed towards the development of consumer and producer behavioral models under conditions of uncertainty. Our work is detailed in the following seven appendices, each of which concentrates on different aspects of the problem. While the appendices are tied together by a common theory of consumer behavior, they involve different simplifying assumptions designed to throw light on special problems. We summarize these appendices in this report, stressing possible policy conclusions.

In Appendix B we concentrate on the problems of pre purchase search for both price and quality. As the problem of simple price search has been thoroughly investigated in the economics literature, we focussed our attention on two problems: (a) <u>efficient</u> price sampling and (b) the <u>pre-purchase evaluation</u> of quality by sampling or other informational sources. Our models suggest that efficient sampling strategies for different types of products vary depending on purchase pattern. As costs

of sampling (mainly time) are high relative to value of purchase, there is a tendency to combine sample and purchase activity and to economize on search, by searching (shopping) jointly for groups of commodities. This is the rationale for the marketing classification of goods into convenience goods, which are sampled and purchased together, and shopping goods, which are sampled separately prior to purchase.

This differential behavior has important implications with respect to the organization and location of retailing. Convenience goods tend to be sold in the same location (shopping center) and sometimes in the same store (supermarket). At the same time, an extensive network of locationally convenient low volume retail outlets develops. These cater to the occasional purchaser at higher prices. The higher prices in these outlets depend on shopping and sampling costs (i.e. on mobility and time costs) of the purchasers.

Because of the difficulties associated with comparison shopping, where prices and baskets vary from time to time, consumers rely heavily on sample goods to yield information about average supermarket prices. This leads to the possibility of misleading advertizing via reduced prices on "loss leaders" which are designed to yield the impression that the market in question is cheaper than others. The misleading nature of such advertising is, of course, extremely subtle and not easy to detect.

Analysis of the determinants of shopping frequency and location suggests that there is no simple answer to the question: do the poor pay more? Our analysis suggests that for convenience goods, lack of transport

facilities as well as a higher level of ignorance are likely to lead the very poor to pay higher prices than middle income groups, for equal quality goods. This may be exacerbated by lower physical mobility of the old and/or chronically disabled who constitute a large segment of the very poor. However, lower time costs relative to savings from shopping may increase shopping activity and therefore offset the other factors.

To tackle the problem of quality evaluation we concentrate on the risk aspect of this evaluation. As a result, our analysis highlights the role of market organization to minimize such risk. This is the main reason for the importance of the brand name and, possibly, for increased concentration both within and across industry lines. The development of franchises is a good example of this process. The franchise operates as a quality controller on behalf of the consumer, reducing the risk of bad quality.

In evaluating the results of the preceding models, a major deficiency, common to all economics models in this area, emerged. The role of learning from personal experience, and hence, the changes in consumer behavior over time are not properly incorporated. Moreover, the way in which information, in particular, advertizing, is incorporated into consumers' decisions, was very unsatisfactory for the purpose of welfare evaluation.

The problem is that if consumers can validate claims of advertizers quickly and without substantial cost, experience will immediately reinforce correct pre purchase information and invalidate incorrect information.

Therefore, the market immediately disciplines all false advertizing by suspending belief in the offending source, rendering future advertizing ineffective. This is, essentially, the theory proposed by Nelson [1974], Stigler [1961] and Telser [1964]. In this view, advertizing is basically an efficient means of disseminating information about product characteristics. Because the information transmitted is accurate, the firms which offer the best deal advertize the most. Casual observation, and the lists of convictions for false advertizing suggest that this view is not consistent with reality.

At the other extreme, one finds those (e.g. Galbraith [1967]) who believe that no validation from personal experience occurs at all, because advertizing affects consumers tastes. While the evidence for this theory is questionable, we choose to reject it, because it is not possible to make any welfare judgements within this framework, without superimposing one's personal tastes on the analysis.

We analyze the decision process of a rational, albeit ignorant, consumer. Using Bayesian statistical analysis, we derive optimal consumption decisions on the basis of available pre purchase information and rules for the continuous updating of information and decisions due to experience and other stimulae (e.g. advertizing, word-of-mouth, etc.) over time. The learning process yields two relatively simple equations for the interaction between prior beliefs and new information, particularly information acquired through personal consumption of the product in question. False

advertizing claims tend to be invalidated. However, the degree and speed of such invalidation depend on the consumer's ability to detect false claims. This ability is very seriously limited for many relatively complex goods such as medicines, repair services, performance of durables, or even nutritional qualities of different foods. The profitability of pursuing false advertizing depends on the importance of qualities which are difficult to verify for the product in question and the ability of the advertizer to convince the consumer that the product indeed contains these qualities.

To illustrate these problems in a more precise way, we construct two alternative models of advertizing. In each, we analyze and contrast optimal producer policies to maximize profits with the socially optimal policies.

Appendix D presents the results for "disseminative" advertizing. Disseminative advertizing is defined as that portion of advertizing which informs consumers about the existence and observable characteristics of the product. It is assumed that all product characteristics are observable before purchase so that experience does not invalidate prior evaluations. The prime observable characteristic is, of course, price. As most search models in economics involve price only, advertizing models to date have all been pure disseminative models. The model shows that for a growing population, the optimal policy for a monopolist calls for a high level of advertizing at the time of introduction of a new product, falling over time to a steady state level at which a constant proportion

of the population is informed of the product and its characteristics. The steady state level of advertizing is proportional to the rate of population growth plus the rate at which information is lost over time. Exactly the same level and path of advertizing is shown to be socially optimal for the levels of quality and price determined by the optimizing monopolist. This price is, of course, too high, but the quality is optimal, given price, and independent of advertizing.

Appendix E presents the results for "persuasive" advertizing. Persuasive advertizing is defined as advertizing which provides information to consumers about product characteristics which cannot be observed and verified prior to use. As the information is favourable to the product, it is assumed to raise the consumers' evaluation of its quality. We abstract from the disseminative function of advertizing, which is likely to be performed by all types of advertizing, by assuming that all relevant consumers know about the product and its observable characteristics. Using a slightly simplified version of the learning model developed in Appendix C to describe the interaction between actual experience and prior beliefs, we again analyze the optimal policies from a monopoly and social welfare points of view.

Under most circumstances the optimal policy for a monopolist is shown to involve a high level of true product quality coupled with a high volume of advertizing upon the introduction of a new product. At this point, consumers' perceived product quality underestimate the true quality. The

effects of cumulative experience and advertizing reinforce each other and raise perceived product quality over time. (This explains the notion of "goodwill".) At the same time, the monopolist gradually reduces both real quality and advertizing to their steady state levels. At these levels, perceived product quality exceeds true product quality and this perceptual overestimate is sustained by continuous advertizing. Again, the firm's ability to maintain the effectiveness of its advertizing in the face of adverse experience depends on the ability of the firm to advertize attributes which are not easily validated and which, therefore, implies a slow speed of consumer learning.

Comparing the quality advertizing packages of the monopolist with that which is required to maximize social welfare, (taking the monopoly price as given) shows that the steady state quality offered by a monopolist falls short of that which maximizes welfare and the steady state advertizing for a welfare maximum is zero. This is, of course, because steady state advertizing is essentially misleading advertizing. Note, however, that during the early phases of the product life some persuasive advertizing is optimal even if it is exaggerated, because it leads consumers who undervalue the true quality to consume it and hence, make the correct decision in the light of their experience.

Appendix F extends the results of appendices C, D and E to oligopoly markets. We first extend the consumer behavior theory developed in Appendix C to situations where there are several competing brands of the same product. We show that the extent of experimentation with alternative

brands depends negatively on the risk associated with choosing the wrong brand and the degree of risk aversion of the consumer, and positively on the ease of identifying the relevant product characteristics in use. Price cross elasticities of demand are a positive function of the degree of experimentation. Therefore, high perceived risk, complex and seldom bought products, such as medical goods, consumer durables and personal expert services involve little experimentation and hence, strong brand loyalties (low price cross elasticities). It is clear that advertizing which increases the consumer's perceived risk and reduces his estimate of his ability to evaluate quality in use, increases brand loyalty. We call this type of advertizing defensive advertizing.

Defensive advertizing is likely to be undertaken by large established firms in oligopoly. By increasing brand loyalty, it creates "artificial product differentiation" which enables these firms to enjoy an increased degree of monopoly even in the absence of a formal cartel agreement. Such advertizing also exists even with formal cartel agreements where it supplies a form of insurance by reducing vulnerability to cartel breakdown, as well as serving as a barrier to entry. Therefore, there is no incentive to curtail them in the interest of joint profit maximization for the cartel. This type of advertizing is similar to that of the steady state monopolist of Appendix E.

It is clear that such advertizing, which reinforces conservatism in consumption is not suited to the new product or brand. New entrants or

small firms, bent on increasing their market share, must adopt different strategies. As shown in appendices D and E, the optimal strategy for a newcomer is to start with a high quality product supported by high advertizing. Such advertizing, which we call offensive advertizing, must persuade consumers to try the new brand. To do this, it must decrease his risk perception and stress qualities which are easily validatable. Note that while the quality of the new brand in terms of easily validatable characteristics must exceed that of the established brands, this may not be the case for qualities which are difficult to verify. Such advertizing is likely to be effective where the elements creating strong brand loyalty are absent, so consumers may experiment at little cost. Most goods which are purchased frequently in small amounts are likely to be in this category. Offensive advertizing campaigns are likely to come in spurts associated with development of new easily validatable qualities. Their incidence is high in industries with fast technical progress and large potential customer turnover.

An examination of the relative efficiency of offensive relative to defensive advertizing based on psychological theories of signal detection suggests that offensive advertizing is considerably less efficient than defensive advertizing per dollar. This is because the "noise" of defensive advertizing masks the "signal" transmitted by offensive advertizing. As advertizing takes a considerable time to execute, established firms must maintain a constant level of defensive advertizing as a barrier to

entry. This reduces the level and frequency of entry by price cutting or offensive advertizing, even if no economies of scale in production exist.

The social loss from advertizing clearly depends on its nature. As shown in Appendix E, offensive advertizing is socially useful in both its disseminative and persuasive functions. In addition, it is an important competitive tool which reduces the degree of monopoly of established firms. This is not so for defensive "steady state" advertizing; not only does the advertizing itself constitute social waste, it also enables established firms which possess some degree of monopoly to supply products of lower true quality at higher price/quality ratios than those which maximize social welfare. In addition, such advertizing creates barriers to entry and increases monopoly power, with concomitant adverse effects on prices and output.

Appendix G extends out analysis to monopolistically competitive industries. We show that imperfect ability to judge quality by inspection or experience may enable firms supplying inferior products to exist over significant segments of the market simultaneously with those supplying better quality at the same prices. The proportion of inferior firms in the market is shown to depend positively, mainly on the inability of consumers to evaluate quality prior to purchase and the rate of population growth. It is shown that in these circumstances inferior firms are likely to advertize more than superior ones, even if the advertizing is disseminative only. The effects of advertizing on market shares is not determinate.

This model appears to fit markets for professional services, e.g. physicians, lawyers, dentists, in which price competition is severely limited by professional associations and product characteristics. The analysis reveals that the effects of advertizing prohibition common in such markets are not easily determined.

APPENDIX A

SURVEY OF THE LITERATURE

The following is a selective critical survey of the literature. As there are good recent surveys which cover many of the issues of consumer information, we decided not to duplicate this effort. Rather, we try to summarize the relevent issues in the literature and report on some research published recently and hence excluded from these surveys.

The best surveys are : (a) Hirshleiffer [1973] on the general problem of information in economics ; (b) Comanor and Wilson [1974, pp. 1-53] on the economics of advertizing ; (c) Montgomery and Urban [1969, pp. 28-94] on marketing approach to consumer theory.

Do markets for the production of information fail? This question requires an affirmative answer to justify government intervention. Until recently, the standard economic answer was that such markets fail because they yield an underallocation of resources to the production of information and consequently, too small an output of information. This market failure could be traced to eigher economies of scale in the production of information, a non-appropriability of return from producing information or some aspect of uncertainty about the value of the information once it is produced. The initial setting for this analysis of information is inventive activity.[1/] Recently, it has been argued that, on the contrary, certain

informational activities generate an overallocation of resources. For example, Fama and Laffer [1971] argue that there is an overallocation of resources in information production motivated by the prospect of speculative trading in a company's stock, as such trading is merely redistributive in impact. Arrow [1962] and Spence [1973] argue that there is a waste of resources on educational expenditures where education serves only as a filter to prospective employees and contributes nothing to productivity. Higher quality workers then acquire education only as a signal. Signalling in this fashion appears to be a very inefficient informational device. The point is that extremely general statements about the failure of markets for information are not possible. Rather, additional specification becomes necessary.

Most of the studies to date on consumer goods markets by economists have restricted their attention to the search by consumers for lowest prices for a homogeneous commodity. These models posit an initial dispersion of prices by the producers of the commodity. The propositions about search in these models deal with the characteristics of the consumer performing the search.[2/] Hirshleiffer points out [1973, pp. 36-7], that there is not a very satisfactory resolution of the persistence of price inequalities in these models with the notion of a long-run market equilibrium. The models are cast in terms of a once-for-all search so that there is no treatment of learning and repeat purchase.

The earlier paper by Stigler [1961] formalizes the price search problem in this simple setting and derives the conventional marginal cost-marginal benefit rule to determine the size of the search. Stigler argues that as cost of information is independent of its ues, there is a tendency for a monopoly to emerge with growth in the size of the market. Stigler's model of advertizing is a diffusion model of information spread over the population. J. Gould [1970] has formalized and expanded the Stigler model together with two other models of advertizing, initially specified in papers by S. Ozga [1960] and M. Nerlove and K. Arrow [1962]. Gould's analysis indicates that the models imply different optimal advertizing strategy over time. All of these models are highly aggregated and offer little insight into the role of advertizing in the consumer decision process. We argue that only by a careful specification of the way in which expectations are formulated at the individual consumer level can interesting and rich hypotheses by formulated about the role of advertizing in influencing the expectations variables and the consumer learning process. These in turn, determine, amongst other things, the level of consumption of various goods by the consumer. Such understanding is a necessary prerequisite to a more complete theory of the factors influencing the price, advertizing and quality decisions of firms at a more aggregated market level.

Two recent papers by J. Hirshleiffer [1973] and M. Rothschild [1974] survey the work done by economists on price search. Since the Hirshleiffer

and Rothschild surveys, two additional pieces on search by R. Kihlstrom [1974] and M. Kohn and S. Shavell [1974] have appeared in the literature. Kohn and Shavell maximize the present value of the future stream of utilities discounted by a pure time preference rate by deciding on the correct amount of sampling for a commodity where there are non-negative and possibly random search costs. Recall of a previous offer is possible in their model, although they claim their results are robust with respect to changes in this assumption.

Furthermore, they consider both sampling from independent populations and adaptive learning in their model. Their principal result concerns the existence of an optimal decision rule which terminates search and the comparative dynamics behavior of the optimal decision rule for changes in type of sampling, the discount rate, search costs and increases in risk.

Kihlstrom's paper [1974] analyzes the quality aspects of goods in the consumption set. Prices are assumed to be known. Quality for each consumption good enters as a good-augmenting variable. Consumers are uncertain about quality for a subset of goods which independent distributions of quality across goods. Information search in Kihlstrom's model consists of payments for observations or bits of data correlated with attributes or quality of the commodity. Kihlstrom demonstrates that all of the usual neo-classical properties of demand for conventional goods hold for the demand for information (i.e. negative definite substitution effects, equality of cross-substitution effects and homogeneity of degree zero of the demand relationship).

Two common deficiencies of most theoretical search models are a neglect of consumers risk aversion and of the difficulties of evaluating quality. Exceptions are Nelson [1970], Comonar and Wilson [1974] and Darby and Karni [1973]. Nelson concentrates on the problems of evaluating quality. He defines a distinction between "search" and "experience" goods. Search goods are those which can be evaluated by inspection -- hence price search models are appropriate. Experience goods can only be evaluated after purchase. Therefore price search models are inappropriate. A learning model is obviously relevant -- but he does not develop one.

Darby and Karni and Spence are interested in the question of the consequences of the difficulties of evaluating quality to the optimal provision of quality by the market. They find that the asymmetric availability of information to producers -- but not to consumers, lends the former to supply lower than optimal quality. They ignore, however, the problem of consumer learning. Therefore, it is not clear that such asymmetries, in fact, persist.

Comanor and Wilson's study concentrates on the role of advertizing as a determinant of market structure. They survey at length the issues involved in advertizing as a provider of information in the context of consumer uncertainty about quality and risk aversion.[3/] However, they do not rigorously incorporate the learning process and therefore are not able to evaluate the dynamic elements of the problem. The learning models which we propose in Appendix C correct this deficiency and yield more rigorous statements of the issues.

The marketing literature is extremely extensive, reporting mostly on the results of specific marketing strategies. In the absence of a unifying theory, it is very difficult to summarize in a concise fashion. For an extensive excellent summary, the reader is referred to Montgomery and Urban [1969].

One exception to the foregoing is the theory of brand choice developed by Kuehn [1961] and extended by Haines [1969]. Haines [1969] also contains an extensive survey. This theory which is based on the learning models of Bush and Mosteller [1955] yields linear adaptive brand choice models which can be summarized in the form of first order mark-off chains. These models do not take account of risk and therefore cannot be properly used. However, the mark-off chain approach turns out to be useful for the analysis of market effects. One such application is demonstrated in Appendix B.

APPENDIX B

SEARCH PRIOR TO PURCHASE

Prior to the purchase of a commodity, consumers may seek information about price or quality or both by (a) collecting observations by sampling along the lines of the economic models and/or (b) collecting information through alternative sources such as the media or some formal educational experience. The question is whether or not the market organizes efficiently for these activities.

The economic models emphasize that there are rules for optimal sample sizes, which may change depending on the ability of the consumer to recall previous offers not accepted. These rules generally require the consumer to search until the expected benefit equals the expected search costs for the marginal unit. (This defines what Kohn and Shavell [1974] label as a switchpoint.) The calculation of expected benefits varies with the rules of the search.

One phenomenon overlooked by the economic models to date concerns joint search and purchase costs. This jointness occurs because while shopping for one commodity the consumer can sample other commodities sold at the same location for prices and other observable characteristics. The marginal costs of sampling additional commodities at the same location are low relative to those encountered in sampling from human populations. The theory of efficient sampling from such populations is applicable to

our problem.

Efficient sampling is obtained by subdividing the commodities to be sampled into groupings according to two principles:

(a) Strata: groupings contain similar items but which are different from each other so that all groups must be sampled.

(b) Clusters: groupings which contain diverse items, but are similar to each other. Thus, sampling one cluster gives a good picture of other clusters. If clusters are chosen so that members of a cluster are geographically close, sampling efficiency is increased by sampling a cluster intensively, rather than sampling across clusters.

The application of these principles to the consumer problem is illustrated with the following example. Consider the problem of shopping for two standard grocery items, such as laundry detergent and frozen orange juice. If the consumer's brand preference were not very strong, he would sample different brands in different supermarkets in order to find the lowest price of each commodity. However, sampling across supermarkets is very expensive, relative to sampling across brands or items in the same supermarket. If the consumer believes that the relative prices of brands are basically the same for all supermarkets, but the average brand price varies among supermarkets, and that this is true for most items in his shopping basket, he can reduce search costs by sampling one item only in each supermarket, to determine which supermarket is cheapest. He can then sample different brands within the chosen supermarket to obtain the cheapest. As transportation costs between markets are high, there is

a tendency to economise on direct market sampling. The consumer therefore tends to shop repeatedly in the same supermarket, shopping intensively within markets for the cheapest brand of any item in his basket. In this respect, the markets are analogous to the clusters discussed before. This provides a rationale to the often used classification of convenience goods, which are sampled and purchased together at the same store, versus shopping goods, which are sampled and purchased separately in different stores. Hence, convenience goods are characterised by a high degree of shop loyalty rather than brand loyalty. Consequently, retailing outlets organize themselves so as to minimize the transaction cost.

Convenience goods tend to be sold in the same location (shopping center) and sometimes in the same store (supermarket). At the same time an extensive network of locationally convenient low volume retail outlets develops. These cater to the occasional purchaser at higher prices. The higher prices in these outlets depend on shopping and sampling costs. These costs are shopping time costs and direct costs, e.g. transportation. Of these, the major component is clearly the time cost. These costs depend on the distance from shopping and on the mode and ease of transport. As shown before, there is a tendency to concentrate purchases of convenience goods in one or two stores. There is also a tendency to minimize the planned number of trips per period subject to limitations imposed by costs of storage and the availability of funds. Such storage and availability of funds are clearly a function of income. Therefore, the poor are likely to shop more often. This is reinforced by lower time costs per

unit time, which tend to reduce the costs of shopping relative to gains from purchasing different items at the cheapest store. Other things equal, this is likely to lead to patronage of more than one supermarket by the poor. As small neighbourhood stores are more expensive than supermarkets, the poor can be expected not to frequent them unless compelled to do so by limits on their mobility. It should be stressed, however, that the relevant value of time is the value of time to that member of the family doing the shopping. It is clearly higher for families where all adults work relative to those in which one adult -- usually the housewife -- does not work. Consequently, working families are more likely to confine their major shopping to one supermarket and to patronize neighbourhood stores more heavily than other families. Liberal opening hours of neighbourhood stores are designed to encourage this tendency.

The effects of geographical patterns on shopping habits must also be taken into consideration. Low population density in the suburbs, leads to heavy reliance on the automobile for shopping purposes, and therefore to a concentration of shopping facilities in shopping centers. The marginal cost of moving between shopping centers is high, but the marginal cost of comparison shopping within a center is low. Therefore, if more than one supermarket is available in a given shopping center relative price uniformity should be expected between them. Alternatively they may specialise in quality or service to reduce price competition. Neighbourhood stores cannot easily survive because economies of scale require a relatively high density to yield sufficient volume from occasional convenience shopping. Because the automobile must be used for most shopping trips, the

time saving due to a small saving in distance is small. As a result, the price advantage of the supermarket dominates. In addition, suburban families are more likely to contain a non-working mother, whose time valuation may not be high. All these factors tend to reduce the incidence of neighbourhood stores in suburban areas (aside from any zoning regulations).

In city core areas, the situation is different. The difficulty of using the automobile due to congestion and parking problems make the marginal time saving due to proximity considerably greater. High land costs reduce the density of supermarkets with parking lots. At the same time, the high population density enables neighbourhood stores drawing pedestrian customers to survice in greater numbers. The population in inner city areas tends to be composed of singles or childless working couples in high-rise apartments whose time is valuable and whose space is limited. Therefore they tend to frequent neighbourhood stores. The situation is likely to be similar for low income inner city dwellers who, because of the lack of private cars are confined to shop relatively frequently in their own neighbourhood. This problem is particularly severe for the old and handicapped whose mobility is severely limited.

There is evidence to suggest that sociological considerations may be important in determining shopping habits, e.g., Dixon and McLachlin [1971] show that poor Puerto-Ricans pay higher prices than poor blacks in central Philadelphia because they frequent small Puerto-Rican neighbourhood stores rather than the local supermarkets. There is no evidence on ethnic group behaviour in Canada in this respect.

Another consequence of the high sampling costs between markets is that consumers resort to methods of collecting information other than direct sampling. This is one of the reasons why price advertising is common in supermarket advertising, but not in general brand advertising of grocery items, which are easily sampled within the market. The role of this advertising is to convince the consumers that the advertising supermarket is cheaper than its competitors. This can be done by demonstrating that a sample basket is cheaper in the advertising store, if consumers believe the advertised basket to be representative of the "true" price level. It is interesting to note that as the validation of this belief about the representativeness of the advertised prices is difficult,[3/] an avenue for gains from "misleading" advertising is opened. This explains the phenomenon of "loss leader," in which an advertised item is sold below cost in order to attract customers, who then purchase other items at higher prices. It is important to note that the "misleading" nature of such advertisements is very subtle, as it occurs by inference only. If this hypothesis is correct, it suggests that the often used distinction, that price advertising is informative rather than persuasive, is not valid.

An additional characteristic of information concerns the spillovers of information from one commodity to another. For example, it may be inexpensive to determine the lowest price or highest quality for a convenience item, but much more expensive to determine the characteristics of a durable commodity such as a stove or refrigerator,

where the cash outlay is a significant proportion of either wealth or current income. Consumers may infer higher quality or lower price to the durable based on successful experience or search for the convenience item, both sold at the same retail outlet or both with the same brand name. Then the optimal search rule becomes search until the marginal cost from one more search for a commodity equals the direct marginal utility on this commodity plus the indirect marginal benefits on other commodities. 4/

As a result, it pays large department stores to internalize these informational spillovers by selling these different commodities at the same location or by labelling diverse items with a store label (e.g. Sears, Simpsons or Eatons own labels) which creates an even stronger association. Stores are then acting as quality control agents on behalf of consumers. This provides both the foundation for an economic theory of private brands and an explanation for quality specialization in department stores.

The same phenomenon yields an advantage to diversified producers. A consumer who has satisfactory or unsatisfactory experience with a given producer's electric stove is likely to transfer the information to other electric appliances manufactured by the same company. Such transfer of information may occur even to completely unrelated commodities manufactured by the same producer. Where consumers are very risk averse, the value of any positive information is very high. This makes the transfer of information about

quality of items which are sampled infrequently (durables, travel) relatively important and yields a significant advantage to diversification. It also induces firms to maintain relatively uniform quality so that such information transfer is confirmed by experience.

Furthermore, the general propositions on diversification of financial portfolios may be applicable to uncertainty in consumer goods. Therefore, given finite variances in the distribution of prices (qualities), consumers should be able to diversify away any systamatic risk by diversifying over brands, leaving them with only covariability between brands. The difficulty is that with limited budgets, this requires extreme divisibility of commodities.

As consumers may require only small amounts of some consumer goods that come only in integer amounts, it is not possible to accomplish this diversification for all goods. Thus, there is a justification for warranties and guarantees to avoid systematic risk, especially on durables. The question to be investigated in our future research is whether the market produces an optimal amount and duration of this producer liability.

Prior to purchase, consumers may acquire information through the media or some formal educational experience. F. Mathewson, a co-researcher on this project, has analyzed the impact of consumer characteristics on the patterns of information seeking through the media (G.F. Mathewson [1972]). This model treats the pay-off from

information acquired at a cost through the media as a form of self-insurance and direct benefit to the searcher. In the absence of this information, consumers may make budget allocations which turn out to be errors, after the fact. This may be true either because they purchase the wrong good or they pay too high a price. Mathewson deals with only the quality search aspect although the price phenomenon is easily included. The mould for this model is one characterized by increasing real wages for consumers over time and an improving quality of goods over time. Thus, information may be dated because it is relevant only for yesterday's goods. Education enters the model to increase the wage rate for consumers and improve their ability to filter out desired information from the data offered through the media. Increases in education make the consumer more efficient in his information search but at the same time increase the cost of any time element in informational search. The net effect upon the demand for information by changes in education depends upon the relative magnitudes of these two effects. This model was tested with two samples of consumer survey data. The qualitative predictions of the model on the demand for information by changes in the consumer characteristics of wage, education, and age were empirically validated.

Amongst other things, the specification of this consumer demand model suggests that there is a case for government protection of those not capable of either optimizing or processing information, for example, children, the mentally retarded, or consumers operating under stress. These considerations appear to be the motivating force behind most of the government's consumer protection legistlation. The point is that although a case can be made for this action, it should not be oversold.

One problem with all of these models is that it is very difficult to include advertising in any meaningful way. Does advertising promote or impede the search for lowest price and highest quality or the production process for information? Is it possible to deceive consumers continuously? Does advertising encourage or discourage consumer inertia?

We argue that it is not possible to analyze or evaluate advertising without a model of learning on the part of consumers. On the basis of their prior search consumers form expectations which are adjusted on the basis of their own (and others) experience. Advertising can influence this learning process. We attack this problem next.

APPENDIX C

LEARNING MODELS

I. INTRODUCTION

The purpose of this section is to outline a model of consumer choice in the presence of uncertainty about quality. Our approach stresses the role of experience in the learning process in addition to other sources of information - such as word of mouth, advertising and inspection. Therefore, the process of validation of prior beliefs plays a critical role. Our model is a dynamic learning model in which opinions change over time in response to various stimulae. While this aspect of consumer behaviour is frequently mentioned in the economic literature, to our knowledge, it has not been rigorously explored to this point. 5/

The model is deliberately simplified to eliminate inter-temporal utility maximization, although a properly informed consumer would logically pursue such maximization. Specifically, consumers are assumed to maximize expected utility at each point in time independent of the effects of their decision on future satisfaction. This specification facilitates the evaluation of producer policies in determining an optimal program of quality advertising and price policies through time. Our purpose is therefore, to stress the major element of unequal availability of information about product quality

between producers and consumers. [6] The advantage this yields to producers is one of the major sources of producer market power and of socially nonoptimal quality, advertising and pricing policies. The model also sheds light on the very contentious problem of advertising as a barrier to entry.

Consider the problem facing an individual consumer in determining budget allocation during any given period between his consumption of a product, about the quality of which he is uncertain, and the bundle of all other commodities, the quality of which is known with certainty. For simplicity, we assume that the consumer knows the true variance of the uncertain quality, but is uncertain about its mean. [7] The quality indexes are assumed to be measured in scales which reflect the subjective evaluation of quality by the individual in question.

Quality for each good is assumed to be fixed scaler independent of quantity of the good in question or the other goods. In this sense, the measure of quality is similar to that proposed by Fisher and Shell [1971]. Quality is treated as a scaler multiplying the quantity. An increase in quality is therefore equivalent to an increase in quantity in terms of utility. In this context, the elements of quality which determine the scale are thought of as performance characteristics associated with the specific product. Thus,for example, if we are evaluating a car the characteristics which determine the scale of quality may be comfort, power, durability, frequency of repair etc.

The functions of the good are determined in advance and each good is evaluated in terms of these functions regardless of the quantity or quality of other goods. This implies separability in the utility function and eliminates complimentarity. An alternative way to look at the same problem is to evaluate the quality indices in terms of the production parameters transforming goods into characteristics in the model of Lancaster [1966]. In this model as well, it is necessary to assume that these production parameters are fixed and independent of the level of consumption of all goods. Otherwise, no unique quality index can be assigned to any individual good without knowing the qualities and quantities of all goods consumed. Note that the Lancaster approach involves more fundamental characteristics than the ones we have assumed. Lancaster's characteristics - such as for the example of the car: transportation, time-saving, comfort etc. - may be supplied by other commodities than the one in question. In many cases these characteristics are very difficult to measure and the common practice is to use proxy variables.

In practice, both approaches use essentially observed proxy variables to measure the quality of individual goods without reference to other goods. However, this is obtained at the cost of rather stringent assumptions about the nature of the utility function of the individual.[8/] The quality measures we use, involve a mixture of subjective and objective elements. The index is a weighted average of objective elements but the weights are subjective. The utility derived from a given product of a specified quality is

given by the utility function, which is not separable. Thus, the utility of the combination of quality and quantity depends on the quantity-quality combination of all other goods.

It is important to note that the measure of quality [9] is a post experience measure, i.e. it is the quality index which would be assigned to the product by the individual if he possessed all the relevant information about the performance of the product. The variability in pescribed quality may arise from objective or subjective factors. Objective factors are, for example, frequency of repair of a durable, probabilities of complete failure, variations in physical performance characteristics. These are dependent on the state of technology and on the care and cost of manufacturing the product. Subjective factors are associated with difficulties of the consumer in evauluating quality. This may be because condition of usage vary. Hence, the evaluation of performance may vary. Or the individual's mood and hence his own objective valuation may vary from time to time yielding different evaluations of the same product. [9] The distinction between objective and subjective variance is important because the first is a policy variable of the producer, but the second is not. Also,as will be shown later, they affect consumer decisions in different ways. Note that quality may be positive or negative. In fact in some cases quality may be negative and large, as in the case where product failure is very hazardous and causes substantial loss.

In order to make a decision about the product, the individual must form some opinion about the average quality of the uncertain product. This opinion may be gleaned from word of mouth, from news or advertising media, or from personal search. [10/] We assume that the individual formulates this opinion in the form of a Bayesian prior probability distribution of the mean quality. For illustrative purposes, we restrict our attention to one uncertain good. We assume the prior is normal, and that the true probability distribution of quality is also normal. [11/] The consumer's problem may now be formalized as follows: maximize expected utility at time t, where expected utility may be defined as

$$Eu(zx, I-px) \qquad (C\text{-}1)$$

where x = quantity of the uncertain product at time t

z = index of quality of each unit purchased at time t (z is a normal random variable with unknown mean Q and known objective variance r)

I = real income expressed in terms of the alternative basket at time t.

$(I - px)$ = the quantity of the alternative basket at time t.

Note that t subscripts are dropped for convenience. The prior distribution of the unknown quality index mean Q is assumed to be normal with mean $\bar{Q} > 0$ and variance v.

The subjective evaluation of the distribution of z is :

$$f(z) = \int_{-\infty}^{\infty} f(z|Q)\ f(Q)dQ = \frac{1}{2\pi\ rv} \int_{-\infty}^{\infty} \exp\left\{ -\frac{1}{2}\left[\left(\frac{z - Q}{r}\right)^2 + \left(\frac{Q - \bar{Q}}{v}\right)^2\right]\right\} dQ \quad (C\text{-}2)$$

$$= \frac{1}{\sqrt{2\pi(r+v)}} \exp\left\{ -\frac{1}{2}\left[\frac{(z - \bar{Q})^2}{r + v}\right]\right\}$$

Hence

$$Eu = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u(zx,\ I - px)\ f(z,Q)\ dzdq$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u[wr + yv + \bar{Q})\ x,\ I - px]\ f(w,y)\ dwdy \quad (C\text{-}3)$$

where w,y are standard normal variates

$$w = \frac{z - Q}{r}, \qquad y = \frac{Q - \bar{Q}}{v}.$$

To maximize Eu set

$$\frac{\partial(Eu)}{\partial x} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [u_{zx}(wr + yv + \bar{Q}) - pu_2]\ f(w,y)\ dwdy = 0 \quad (C\text{-}4)$$

Equation (C-4) states that in equilibrium the expected marginal utility of a dollar expenditure on good 1 must equal to the expected utility of a dollar expenditure on the other good -- a result which is not surprising.

The terms in equation (C-4) may be interpreted as follows:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_{zx}(wr + yr + \bar{Q})\, f(w,y)\, dwdy \qquad \text{(C-5)}$$

is the expected marginal utility of good 1 evaluated according to the distribution of quality embedded in the prior; pu_2 is the foregone marginal utility of the last dollar spent on the good x.

The demand functions for both goods at time t are therefore functions of the relative price of good x, the consumer's real income, the mean and variance of the prior as well as the true variance of quality.

$$x = x(P, I, \bar{Q}, (r+v)) \qquad \text{(C-6)}$$

where $r+v$ is the variance of the expected quality. We assume that the consumer is risk averse and therefore buys less x as $(r+v)$ increases. As v is a decreasing function of purchases in the previous period an element of habit persistence is introduced.

So far, we have discussed the <u>expected</u> utility of the consumer, as a means to determining his demand for the risky commodity. His realized <u>actual</u> utility, however, depends on the outcome of his decision i.e., on the quantity of x he purchases and on the <u>actual</u> sample value of z. This z is a "state of nature" and independent of consumer action. However, because x depends on the consumer's

prior, the expected value of his actual utility depends on his prior. To maximize the expected realized utility, the individual should choose a prior mean, $\bar{Q}$, which is as close to the true value of Q as his information allows. He should also choose the variance of his prior to reflect as closely as possible the risk he associates with his estimate of $\bar{Q}$. His estimate of the prior should contain all the relevant information he possesses prior to purchase. The payoff from additional information is a better decision about x and a higher realized utility. As a consequence, the nature of the initial prior formation is of great interest. We shall come back to this equation.

Before we turn to analyzing the process of incorporating the experience garnered through use of the product in previous periods into decisions about future consumption, we must analyze the nature of the experience. As suggested before, observation on the performance of a product at any time is subject to variance arising from subjective reasons, in addition to the objective variance of the product in question. This subjective variance arises from difficulties in evaluating the product even after use, and may be conveniently thought of as measurement errors. Even very knowledgeable consumers may find it difficult to evaluate the quality of products which are used in conjunction with other products or services. For example, the failure of a drug to cure may be

a result of insufficient medical knowledge to cure the particular disease, or alternatively,it may be the result of poor quality of the specific brand, insufficient dosage or incorrect diagnosis. The consumer is clearly not in a position to evaluate which of the foregoing reasons account for his predicament. His observation is therefore that a specific medical service was unsatisfactory, but he cannot assign specific responsibility for the failure, or even know with certainty that failure has occurred. The situation is more complex in the case where the treatment worked. In this case recovery may have occurred spontaneously. The medicine was therefore redundant or excessive. The "satisfied" consumer may be worse off due to the cost of the unnecessary medicine aside from any possible side effects. Consumers may, of course, hire experts to evaluate proposed services either before service is rendered, or after. However, this is an expensive proposition, and even if undertaken, there is the problem of evaluating the expert. This means that the error in measurement of quality after use is likely to be greater, the less knowledgeable a consumer is and the more complex the product or the productive process into which the product is an input. 12/

In other cases such errors of measurement occur because the product yields an intangible output which is difficult to quantify directly. In this case measurement of the output must be made according to proxy variables associated with the output. Thus taste or vitamin content may be used to measure nutritional value of foods, taste or colour to measure value of tooth paste etc.

It is important to distinguish between known and unknown errors of measurement. The first occurs when the consumer is sufficiently knowledgeable to realize that his evaluation is imperfect so that the consumer is likely to take it into consideration in modifying future consumption decisions in the light of experience. In the case of unknown measurement erros, however, the individual may draw the wrong conclusions from his experience without realizing this and pursue suboptimal policies. Errors of this nature are particularly prevalent in the evaluation of alternative brands of the same product. Extensive tests by Consumer's Reports and others suggest that many consumers cannot tell the difference in use between alternative brands of many consumer goods ranging from detergent, liquor and cigarettes to high fidelity equipment. However, the majority of those tested claimed to be able to do this. It is clear that the cross price elasticities are reduced substantially for such consumers. We shall come back to this point later.

We now proceed to include experience in the formal model; define q as the perceived quality in use. q is a measure of the true quality z. While q may in fact be biased, the consumer cannot know this, for otherwise he would revise q so as to make it unbiased.[13/] We assume that q is (generally) unbiased. However, we later indicate some of the modifications to the analysis where q is biased. Assume that the variance of the measurement error (Var (q/z)) is fixed for all levels of z, then

$$\text{Var}(q) = \text{Var}(z) + 2\,\text{covar}(z,q|z) + \text{Var}(q|z)$$

Define s as the consumer's estimate of Var(q). The known time quality variance (Var z = r) is then the lower bound on s. Note that in contrast to parameters of the prior the estimate of s, even if erroneous, cannot be refuted by experience as the error is not observable. Thus, s is fixed for each individual even if erroneous.

Let us now turn to the process of incorporating the actual experience garnered during period t in the prior of period $t+1$. For this purpose, we use Bayes' theorem to answer the question: What is the most likely prior probability distribution to have given rise to the actual observation in the sample?

$$f(Q|\bar{q},x) = f\,\frac{(\bar{q}|Q,x)\ f\ (Q)}{f(\bar{q})} \qquad \text{(C-7)}$$

where $\bar{q} = \frac{\Sigma q}{x}$, i.e. $\bar{q}$ is the average perceived quality in the sample. The distribution $f(Q|\bar{q},x)$ is the posterior distribution for time t and therefore, the prior for $t+1$. Under the assumptions of normality of the prior at time t and normality of underlying quality, the posterior distribution is also normal with mean

$$Q_{t+1} = \frac{\bar{s}}{v+\bar{s}}\,Q + \frac{v}{v+\bar{s}}\,\bar{q} \qquad \text{(C-8)}$$

and variance $$v_{t+1} = \frac{v\bar{s}}{v+\bar{s}} \qquad \text{(C-9)}$$ [14/]

where $\bar{s} = s/x$

The mean of the posterior distribution is a weighted average of the prior and the sample means where the weights are the variances of the sample and the prior means respectively. The less the variance of the sample relative to that of the prior, the stronger is the weight of the sample experience in modifying the posterior. The variance of the posterior is always smaller than that of the prior as the new information from the sample reduces the degree of uncertainty expressed by the posterior variance.

The consumer in the next period maximizes his expected utility given the new prior in exactly the same way as in the previous period. Equations (C-1) to (C-6) now hold for any period. However, the nature of v and $\bar{Q}$ change continuously. It is convenient to think of the process of prior modification in continuous terms. Then the modification procedure may be expressed in the form of two differential equations:

$$\frac{dQ}{dt} \equiv \dot{Q} = m(\bar{q}-\bar{Q}) \qquad \text{(C-10)}$$

and

$$\frac{dv}{dt} \equiv \dot{v} = -mv \qquad \text{(C-11)}$$

where $m \equiv x/(x+s/v)$

These equations describe a relatively simple adaptive process where expectations about the mean are adjusted according to the gap between expectation and experience, and the degree of certainty about expectations rises with experience. Unlike the usual adaptive behaviour

in economics and in marketing,[15/] the rate of adaptation varies over time, assuming prices, income, and quality remain unchanged. This is because the level of uncertainty (v) falls over time yielding a higher value of x so that the rate of adaptation of $\bar{Q}$ to experience rises or falls as the product of xv rises or falls. But as x at any point of time is bounded by the budget constraint and as $v \to 0$ as $t \to \infty$, the rate of adjustment will slow down with time.

Note that the consumer's estimate of the variance of the measurement errors (s) affects consumer demand only indirectly through its influence on the speed of learning parameter (m). As s rises, m falls. This means that the speed by which consumers learn from experience falls as the consumer's estimate of his ability to evaluate his experience accurately decreases. Lack of knowledge may lead consumers to estimate s incorrectly (the bias may go either way) and therefore to adapt at the wrong speed. Such adaptation affects consumption through its effects on both the prior mean and variance. The effect on the prior variance (v) dissipates rapidly with experience. This is because $v \lesseqgtr \frac{s}{nx}$ where n is the number of periods of experience. Thus even a small amount of experience reduces the degree of uncertainty relevant for decisions. The risk of sampled goods is considerably lower than that of unsampled ones. An important source of habit persistence.

The utility of improved quality and of risk may, or may not be related to the individual's ability to measure quality without error. If an individual cannot easily distinguish between two

different brands of the same good which yields a specific kind of utility even though the goods are indeed objectively different, the utility loss of buying a lower quality one is negligible, provided there are no other unidentifiable effects. This, for example, would be the case for high quality high fidelity equipment, where a consumer cannot distinguish between sets which a more sensitive or better trained individual could easily rank. The quality of the sets may therefore be quite different, but the consumer's inability to distinguish the difference makes the valuation of the difference in terms of utility very low. As a consequence,his price cross elasticity between brands is high. On the other hand, if the measurement error occurs because of jointness or complexity, so the effect of improved quality of one input is not easily identified, measurement error is not necessarily associated with low marginal utility of additional quality. The example of the medicine cited previously is a good illustration of this point. Although the consumer cannot evaluate the quality of medical care with any precision, his marginal valuation of improved care and of risk may be high. In this case cross price elasticity may be quite low.

Equations (C-6), (C-10) and (C-11), together with the initial conditions on Q and v describe the consumer demand over time. The consumption path will change over time as experience is accumulated, eventually yielding an optimal level of consumption x^* (where * denotes the steady state level of the variables) which depends on

accumulated past experience which is embodied in the prior mean, but disregards all current observations

$$x^* = x(\bar{Q}, r, P_t, y_t). \qquad \text{(C-12)}$$

This is because, as time goes on, the variance of the prior shrinks until as $t \to \infty$, $v \to 0$. At this point, the consumer believes that he knows the value of the true mean of q with certainty and therefore, ignores the signals from current samples which are pure random variation of q.

Up to now, we have assumed that information acquired is maintained over time without any loss. If the product were not consumed over any given period, the prior remains unchanged during the period. The economics and marketing literature [16/] suggests that information is lost by the passage of time, i.e. that the stock of information depreciates with time. The usual procedure in learning models and in the derived marketing literature is to assume a steady rate of depreciation of the expected mean quality $\bar{Q}$. This is a consequence of the fact that variance is not considered so that the form taken by the learning priors is linear. As a result equation (C-10) becomes:

$$\dot{\bar{Q}} = m(\bar{q} - \bar{Q}) - \delta\bar{Q} \qquad \text{(C-10a)}$$

where m is the speed of adjustment coefficient and δ is the depreciation of accumulated stock of knowledge.

In the context of our rational decision model, this formulation makes no sense at all. There is no reason why the passage of time should induce consumers to assume that the average quality of any given product has deteriorated. Even if technological progress is allowed, so that all goods continuously improve in quality, there is no reason to believe, in the absence of contrary evidence that the quality of the good in question does not improve in the same way as those of other goods. In our model, as q is measured as the relative quality of good x relative to that of the basket of all other goods, an equal improvement in quality over time will leave q unchanged. 17/ Thus, this formulation of "forgetting" cannot be adopted.

A much more attractive way to tackle this problem is through the modification of the motion of the variance of the prior over time. The fact that information becomes dated or is forgotten does not change the prior conception of the average quality of the good in question. It does change the consumer's estimate of the reliability of his prior beliefs and modifies his estimate of the prior's variance. As the reliability of prior beliefs is likely to decline with the passage of time, the variance of the prior must rise with time. We shall assume this process occurs in a linear fashion so that equation (11) is modified to include a "coefficient of information depreciation" (δ) in the form:

$$\dot{v} = (-m + \delta)v \qquad \text{(C-12a)}$$

The effect of "forgetting" is to counteract the effect of experience on the prior variance and to decrease the rate at which the prior variance is reduced with experience. Thus, it reduces consumer demand for x as a function of time due to the increased perceived risk associated with the product. In addition, the effect of the increased uncertainty operates through equation (C-11) to speed up the rate at which experience is incorporated into the demand function. The net effect of δ on the speed at which the consumer will move towards his steady state consumption level of x^* is not determinate. Note that as $t \to \infty$, v approaches a finite positive limit. This is because $\dot{v}$ approaches zero before v approaches zero. As the rate of reduction in uncertainty due to accumulated experience falls to the level of the rate of information depreciation, the level of uncertainty stabilizes at the steady state level v^*. Thus, the level of prior uncertainty is not eliminated from the steady state demand function for x.

$$x^* = x(\bar{Q}, (r+v^*), p, y) \tag{C-13}$$

where

$$v^* = \frac{\delta s}{x^*(1-\delta)} \tag{C-14}$$

As $\frac{\partial v}{\partial X} < 0$, an element of interdependence is introduced into the demand function. The utility of the uncertain good is derived not only from its consumption but also from its informational value, which reduces

the risk associated with the good. Because the consumer is assumed to ignore the value of the information gained through experience the informational value is underestimated.

Note also that x^* is a random variable dependent on q_t. This is because the variance of the prior does not vanish and hence, the consumer continues to adjust the mean of his prior according to current experience. The coefficient of adjustment is now a constant equal to δ. The steady states adjustment function (equation (C-10)) degnerates into equation (C-15) to yield the standard adaptive expectations model.

$$\dot{\bar{Q}} = \delta(\bar{q} - \bar{Q}) \qquad \text{(C-15)}$$

The "forgetting" coefficient yields a lower bound to the speed of adjustment in the learning equation.

III. The Acquisition of Information

We are now ready to proceed to evaluate the nature and process involved in the acquisition of information in a more rigorous fashion. In the context of our model, almost all information is embodied in the prior. Any addition to the information stock of the consumer must be translated into a change in the parameters of the prior. This enables us to quantify and evaluate the effects of information on consumer behaviour. It is also possible to establish the precise value of this information in terms

of utility and, therefore, to evaluate its welfare implications.

One limitation must be pointed out at the outset. Our definition of information excludes variations in stimulae which affect consumers taste directly. This may be a serious deficiency in our analysis, as many authorities claim that advertising changes consumers' tastes. 18/

However, even among psychologists, there is no general agreement about the validity of such claims, nor the precise way in which such advertising affects the utility function. Economists in general tend to discount these claims, 19/ citing as evidence advertising fiascos such as the Edsel. Because of our doubt about the validity of these claims, and the intractability of the problem of incorporating taste changes into consumer theory, and deriving welfare judgements in this case, we choose not to pursue this approach. 20/ This does not mean that information (including advertising) cannot affect consumer behaviour in our model -- but that such behaviour modification must enter through changes in the prior.

One valid exception to this rule may be the investigation of the effects of some forms of information on the consumer's ability to perceive and correctly measure quality. As mentioned before, this ability is one of the determinants of the perceived quality variances. Additional information may translate itself into a correct evaluation of s and therefore into a more optimal speed of learning.

All general forms of information,such as education or general experience as a consumer,have this effect. Of course,misinformation may have the opposite effect of increasing the error of s . It is important to note that while misinformation which affects the prior is corrected by experience, misinformation causing bias in s is not corrected by experience as measurement errors are not observable.

Returning now to information as a prior, we must analyze the way in which different forms of information change the consumer's mean and/or variance of the prior.

Consider, first, objective sources of information. These may take the form of search and personal inspection, canvassing friends and acquaintances, reading government and other testing organizations reports or hiring consultants to evaluate the product -- e.g. a doctor to recommend a brand of medicine. Information obtained by all of these methods will generally affect both the mean and variance of the individual's prior. If there is small variance in the opinions of the individual's sources, the effect is to reduce the variance of his prior, as well as to change the mean in the direction of the general consensus. The main marginal benefit from further search is in the reduction of the variance of the prior which, in turn, increases product demand due to higher expected utility. The effect on the speed of his preference

adjustment is ambiguous and depends on the product of the quantity purchased and the prior variance. However, as we have seen before, the effects of the prior variance generally dominate. Therefore, the effect of a consensus of outside sources is to reduce the speed at which the consumer makes up his own mind.

If there is a lack of consensus among information sources, the prior variance may increase, as a result of search -- the consumer is more "confused" after the search than before. In this sense search yields "too much information". This may very well be the case for information about the stock market, new technology and controversial social issues. These conditions are likely to occur in situations where it is difficult to evaluate quality or where the intrinsic product variance is large. They are also likely under conditions where interpersonal differences in taste are great, so that the same objective quality is evaluated differently by different consumers. As the subjective expected utility before trial is clearly reduced due to further search, the individual is likely to avoid increased search. He attempts to evaluate the reliability of his sources of information, and restrict his search to these sources. This activity may be defined a process of "filtering" the information. Because of this, it is not likely that the variance of the prior is increased as a result of search. We can generally treat objective information as a process which reduces prior variance. Note, however, that inefficient filtering may yield a reduction of

variance around the "wrong" mean. Such a situation also yields slow adjustment and thus explains stubborn brand loyalty.

The effect of additional information is also likely to change the prior mean. In the case of a consensus, it is likely that the prior mean of any individual consumer will move closer to the true mean. For the majority of consumers, such a move is almost certain, for the existence of a consensus suggests fairly uniform judgement without great individual disparities in subjective evaluation. This is particularly true for a product with little intrinsic variability (small s) and extended experience. The value of such information is clearly positive, as it leads the consumer to make the "right" decisions and not to modify his judgements too fast because of random sample information.

The situation is quite different in the case where a diversity of opinion exists. Here the effect of additional information depends on the efficiency of the filtering process. This is particularly important where subjective evaluations vary a great deal, e.g. personal services, food, cosmetics. Here, the individual must collect the relevant objective information on the basis of which to make his own subjective evaluation, as well as subjective information from people with similar tastes. Efficiency in filtering depends on the individual's prior knowledge which enables him to sift the relevant objective information. This knowledge

is usually the result of education and general experience, which is likely to be a function of age and affluence. The efficiency also depends on having access to and identifying sources of information, who have had personal experience of the product and whose subjective judgement is correlated [21] with that of the consumer in question. Such sources are known as "opinion leaders" in the marketing and sociology literature about diffusion processes. [22] Again, the ability to identify these sources is likely to increase with education and experience, particularly for new products.

The combined effects of inefficiency in filtering generally yield a biased prior with small variance. Such consumers not only buy the wrong amount of the product, they also learn from experience too slowly. The value of this information to these consumers is clearly negative.

The incidence of this problem is likely to be higher among the poorly educated, inexperienced consumers, as well as among those who are limited in their capacity to make sound judgements, such as the very young and very old.

As is obvious from the preceding discussion, there is no simple unambiguous way of introducing modifications in the prior information into the model. While it is clear that the effects operate on the prior mean and variance, the direction of change in these is not easily specified in the general case, and must be

evaluated carefully for each product and consumer. We shall therefore not introduce their effect explicitly into the model at this stage. We shall, however, use this framework in our further research in expanding the model to analyze pre-purchase search.

We may now apply this framework to the main variable of interest: information supplied by the seller.

IV. Information Supplied by the Seller

The prime form of such information is media advertising. However not all "selling" expenses fall in this category. Which form of selling expenses is undertaken depends on the relative efficiency of the different forms. Where potential customers are geographically diffuse and not easily identifiable, media advertising is most efficient. A small value of purchase of each customer per unit time also improves the efficiency of the mass media, relative to other "selling" methods. On the other hand, personal selling is more efficient where the value of the purchases is high and purchasers easily identified. This is particularly common in the case of selling to middlemen who resell the product to the consumers, or who act as advisers, eg. doctors, etc.

To introduce advertising into our model we must modify our equations of motion (equations (C-10) and (C-11a)).

The simplest way to introduce the effects of advertising on the variance is to postulate that advertising reduces the rate of forgetting and possibly even overrides it altogether. Equation (C-12a) will now change to

$$\dot{v} = (-m + \delta + g(a))v \quad \text{where} \quad g(a), \frac{\partial g}{\partial a} > 0 \tag{C-12b}$$

where a is advertising outlay during the period. [23/]
Either as well or as an alternative, advertising may affect the rate of adjustment of the prior mean. Equation (C-10) may be written

$$\dot{\bar{Q}} = m(\bar{q} - \bar{Q}) + f(a) \quad \text{where} \quad f(a), \frac{\partial f}{\partial a} > 0 \tag{C-10b}$$

We assume decreasing returns to advertising in both forms, so that

$$\frac{\partial^2 g}{\partial a^2}, \quad \frac{\partial^2 f}{\partial a^2} < 0.$$

The effects of seller supplied information which we will henceforth call "advertising" are similar to those of any other information favourable to the product. It tends to move the prior mean up and/or reduce prior variance. In this sense it creates "loyalty" in the form of increased purchase and lower reliance on experience in the modification of the Prior. Whether advertising has the effect of raising the prior mean or reducing the prior variance is of great importance, as this will determine the profitability of false advertising.

The main aspect of advertising is that its objectivity is suspect in the eyes of the consumer, because of the obvious gain the producer derives, if it is accepted by the consumer. Such gains may be realized only in the short run -- until the consumer can validate the information through experience. However, where validation is slow, the temporary gains may be significant.[24/] This yields an incentive for the producer to disguise the source of information he supplies so that consumers believe that it is supplied by objective sources. Or at least to supply "objective" endorsements such as "good housekeeping seal" or "expert" opinion ("more doctors recommend...") with the message. It also pays the seller to "bribe" objective advisers to give advice favourable to him, hence, under-the-table discounts to retailers, or free samples to doctors and university professors. Note that such practices yield positive payoff to the seller when the information supplied is correct, because, to the extent that they succeed in changing the individual's prior in the correct direction, they accelerate his progrress towards the steady state x^*.

In the case where the information supplied is false such practices, if undertaken, are presumably to the benefit of the seller, but are detrimental to the buyer. This device essentially reduces the efficiency of the individual's filter and therefore, has the same effect.

It must be pointed out, however, that the device itself does not necessarily reduce consumer welfare. Its effect depends

on whether it induces the consumer to make the "right" decision. Regulation of promotion techniques frequently takes the form of prohibition of such claims to objectivity. As shown above, the welfare effect of such regulation is not always clear.

Note that the advertising discussed here is of a nature which many people indentify as "persuasive", i.e. touting the advantages of a particular product, as distinct from informing the consumer about product availability or price. Such advertising is considered by many economists to be socially undesirable in that it induces the consumer to pursue a source of action favourable to the seller. 25/ We have shown clearly that such blanket condemnation is not justified. The value of such advertising to the consumer depends on the extent that producer and consumer interests coincide. The real problem in the welfare evaluation of advertising is, therefore, to investigate the extent to which such coincidence occurs.

To investigate this problem it is necessary to evaluate the optimal policy of firms facing consumers whose behavior is described by our model. However, before turning to this task we must aggregate our consumers into markets. This is far from a trivial task.

V. Aggregation

A common procedure in economics is to treat the individual consumer as a "representative" consumer, so that market reactions are equal to a constant multiple of the indivdual's reaction. This procedure is not satisfactory for our purposes as it does not allow for the introduction of changes in the size and composition of the market. On the other hand, a specification which allows for the full diversity of consumer tastes and reactions appears intractable. Our compromise is to aggregate consumers in two different ways in order to shed light on the difference between "disseminative" and "persuasive" advertising. While, of course,the same advertisement is likely to perform both functions, this distinction is important. It is implicitly at the heart of the debate about the social value of advertising, and therefore the question of regulation. Unfortunately, the literature does not provide a clear definition of these advertising functions. We define disseminative advertising as that portion of advertising which informs the consumer about the existence and observable characteristics of the product. It is assumed that all product characteristics are observable before purchase so that experience does not invalidate prior evaluations - "search goods" in Nelson's [1970] terminology. The prime search characteristic is, of course,price. As most search models in economics involve price only,advertising models have all been pure disseminative models.

Persuasive advertising is defined as advertising which affects the consumers' priors. It is assumed that consumers know about the existence and price of the product and they have already formed a prior on its quality. Persuasive advertising involves information about product characteristics which cannot be observed prior to purchase and use -- i.e. experience characteristics in Nelson's terminology. Note that there is no presumption of truth or untruth about persuasive advertising. This is a matter to be determined by the optimal policy of the producer.

The aggregation procedures for the two models are described in detail in Appendices D and E. These appendices also analyze and assess the producers' optimal advertising, quality and price policies.

APPENDIX D

A MODEL OF DISSEMINATIVE ADVERTIZING

I. INTRODUCTION

In this section we consider optimal firm policy for disseminative advertising and evaluate the social efficiency of this policy. Persuasive advertising clearly has some disseminative properties in that it informs potential consumers who do not know about the product, as well as persuades them to buy it. However, in this Appendix we concentrate on the disseminative function. We define disseminative advertising as that portion of advertising which informs potential consumers of the existence and/or observable characteristics of the product. All characteristics are assumed to be verifiable prior to purchase so that experience does not invalidate prior evaluation.

In the disseminative model, the quantity purchased by consumers who are familiar with the product due to this form of advertising, depends on their tastes, the price and quality of the product, all known with certainty. This process yields a conventional market demand curve for the product which shifts to the right over time due to disseminative advertizing and left over time due to forgetting and the consequent non-purchase by some consumers. For simplicity, we ignore word-of-mouth

advertizing, the introduction of which complicates the mathematics considerably without adding much of substance. Therefore, our disseminative model is in the spirit of other economic models of advertising, e.g. Arrow and Nerlove [1962], Gould [1970], Schmalansee [1972].

Section II analyzes optimal firm policy. Section III evaluates this policy from a social welfare point of view.

II OPTIMAL FIRM POLICY

We begin by positing a natural net rate of growth for the entire population (N) of α . The growth of population is represented as:

$$\dot{N} = \alpha N \tag{D1}$$

(Time subscripts are omitted for convenience.) Define M to be that subset of the population that are aware of the product, and βM to be that portion of the informed who forget in each instance of time. If we further define a to be a measure of the effectiveness of advertising dollars at a moment of time in informing the uninformed, then changes in the informed group may be represented as:

$$\dot{M} = a(N - M) - \beta M \tag{D2}$$

Finally, define total demand for the product to be $M \cdot h(\mu, P)$ where μ represents quality of the good and P represents the per unit price, $C(\mu)$ represents the per unit cost of quality, and $w(a)$

represents the per capita expenditure advertizing by the firm, where $w_a, w_{aa} > 0$. The firm in this model is assumed to behave as if it were a monopoly, i.e. it ignores rivals' reaction, if any.

These components may be assembled into a profit relationship for the firm as follows:

$$\Pi = Ph(\mu, P)M - Mh(\mu, P)C(\mu) - Nw(a) \tag{D3}$$

This monopoly pursues a goal of the maximization of the discounted stream of its profits.

This is formally defined as:

$$J = \int_0^\infty e^{-\rho t} \Pi dt \tag{D4}$$

where $\rho > \alpha > 0$ and ρ is the rate of discount which is time invariant.

For convenience, define $G(\mu, P) \equiv (P - C(\mu)h(\mu, P)$. Deflate the variables in the sytem by N , the population. Then (D2) and (D3) become, respectively:

$$\dot{m} = a(1 - m) - (\alpha + \beta)m \tag{D5}$$

$$\Pi = e^{\alpha t}[G(\mu, P)m - w(a)] \tag{D6}$$

where N_o is subsumed in the Π notation.

In this model, the firm has three decision variables - price, quality of the product, and advertizing. The set of first-order conditions for each of these variables that accomplishes the profit

maximization goal is described as:

$$G_p \equiv (Ph_p + h) - h_p\, C(\mu) = 0 \qquad (m \neq 0) \qquad \text{(D7)}$$

$$G_\mu \equiv Ph_\mu - (h_\mu\, C + h\, C_\mu) = 0 \qquad (m \neq 0) \qquad \text{(D8)}$$

$$-w_a + \gamma(1 - m) = 0 \qquad \text{(D9)}$$

$$\dot{\gamma} = \gamma(\rho + \beta + a) - G(\mu, P) \qquad \text{(D10)}$$

$$\dot{m} = a(1 - m) - (\alpha + \beta)m \qquad \text{(D11)}$$

Each of these equations has an economic interpretation. (D7) and (D8) represent the conventional marginal revenue-marginal cost equality for price and quality. Observe that these optimal levels of P and μ are time invariant as both (D7) and (D8) are independent of m, the proportion of the total population that is informed.

Equation (D9) says that advertizing should be increased until the marginal cost, w_a, equals the marginal revenue to the firm, of an increase in the proportion of the population informed, i.e. γ (the value of an additional unit of population) times $(1 - m)$ (the marginal productivity of advertizing).

(F10) and (F11) represent the two dynamic equations. (D11) is the equation that describes the changes in the informed group in the population. (D10) may be most easily interpreted at rest (i.e. $\dot{\gamma} = 0$). Upon substitution from (D9), (D10) at rest, becomes :

$$\gamma = [G(\mu, P) - \frac{aw_a}{1-m}]/(\rho + \beta) \qquad \text{(D12)}$$

$[G_m(\mu, P) - \frac{aw_a}{1-\bar{m}}]$ measures the marginal profit from increasing m by one unit. Therefore, in equilibrium, the value of an additional unit of the informed proportion of the population should equal the discounted impact on net profits of an additional unit of the informed proportion of the population.

(D9) may be solved for $a = a(\gamma, m)$. Further analysis reveals that $\partial a/\partial \gamma > 0$, while $\partial a/\partial m < 0$. [26/] These are important for analyzing the dynamic system. The dynamic system permits us to understand the evolution of advertizing and informed population through time which are in the interests of long-run profit maximization for the firm. This is accomplished through a phase diagram. (Figure D-1)

Investigation reveals that $\partial \dot{\gamma}/\partial \gamma > 0$, $\partial \dot{\gamma}/\partial m < 0$, $\partial \dot{m}/\partial \gamma > 0$, $\partial \dot{m}/\partial m < 0$.

Therefore,

$$\frac{d\gamma}{dm}\Big|_{\dot{\gamma}=0} \gtrless 0$$

and (D13)

$$\frac{d\gamma}{dm}\Big|_{\dot{m}=0} \gtrless 0 .$$

Both the $\dot{\gamma}$ and $\dot{m}$ locus at rest have positive slopes. As a result, there is a question of the existence and uniqueness of a long-run steady state equilibrium. Existence is guaranteed if the two schedules, $\dot{\gamma} = 0$ and $\dot{m} = 0$, cross at least once. Uniqueness is guaranteed if the two schedules, $\dot{\gamma} = 0$ and $\dot{m} = 0$, cross only once.

An interior long-run solution exists if $[G_m(\mu, P) - (\alpha+\beta)m^\infty/(1-m^\infty)]/(\rho+\beta)$, the discounted net profit from maintaining a fixed long-run steady state proportion of informed consumers relative to the total population, is a positive number. We assume this to hold. Otherwise, no advertising or production occurs.

Uniqueness is guaranteed if the $\dot{\gamma} = 0$ and $\dot{m} = 0$ schedules cross only once. These schedules cross only once if the slope of one always exceeds the slope of the other. To see this, we need to substitute specific expressions for the formulae labelled (D13).

The question is the sign of the following expression :

$$\Phi \equiv \frac{d\gamma}{dm}\Big|_{\dot{\gamma}=0} - \frac{\partial\gamma}{dm}\Big|_{\dot{m}=0}$$

Upon substitution Φ becomes :

$$\Phi = (\gamma^\infty)^2(1 - m^\infty)(-m^\infty) - \gamma^\infty(1 - m^\infty)(\alpha + \beta + \alpha^\infty)w_{aa} - w_{aa}^2(\alpha + \beta + \alpha^\infty)^2 - \gamma^\infty(\alpha + \beta + \alpha^\infty)w_{aa} \qquad \text{(D14)}$$

where γ^{∞}, m^{∞}, a^{∞} are the long-run levels of the respective variables. By previous assumption, each of these is positive and $0 < m \leq 1$. As a result, $\Phi < 0$ and $\dot{m} = 0$ cuts $\dot{\gamma} = 0$ from below.

This property guarantees the existence of a stable trajectory as shown in Figure D-1.

Thus, firms starting with a small proportion of the population informed of their product, specifically, with $m_o < m^{\infty}$, conduct an advertizing policy so that the proportion of the population that are informed increases to a steady state level m^{∞}. With population growing exponentially at a rate α, then, this monopoly in the long-run would wish the group knowing of the product's existence to grow at a rate α.

What does this imply about the firm's advertizing policy? Take the time derivative of the $a = a(\gamma, m)$ relationship to obtain :

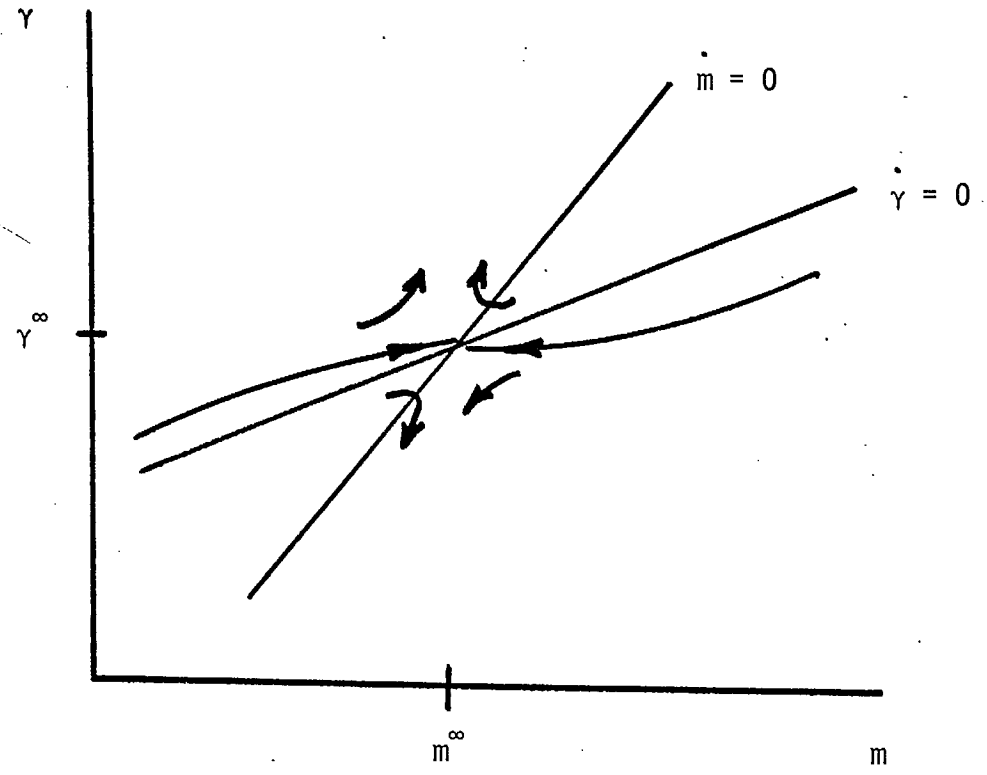
$$\dot{a} = a_{\gamma} \dot{\gamma} + a_{m} \dot{m} \qquad \text{(D15)}$$

We cannot determine a unique sign for $\dot{a}$, knowing that $a_{\gamma} > 0$, $a_{m} < 0$, $\dot{\gamma} > 0$ and $\dot{m} > 0$. Substitution from the first-order conditions yields :

$$\dot{a} = \frac{w_a(\rho + \beta)}{w_{aa}} + \frac{w_a(\alpha + \beta)}{w_{aa}} \frac{m}{1-m} - \frac{G(\mu, P)}{w_{aa}} (1 - m) \qquad \text{(D16)}$$

Figure D-1

Phase Diagram for Disseminative Advertizing



From (D16), we determine that $\frac{d\dot{a}}{dm} > 0$, $\frac{d^2\dot{a}}{dm^2} > 0$. As $\ddot{a} \equiv \frac{d\dot{a}}{dt} \equiv \frac{d\dot{a}}{dm}\dot{m}$, and as $\frac{d\dot{a}}{dm} > 0$ and $\dot{m} > 0$ along the accumulative trajectory, then $\ddot{a} > 0$.

When $\gamma = \gamma^{\infty}$, $m = m^{\infty}$, then $a = a^{\infty}$ and from (D15), $\dot{a} = 0$. As $\ddot{a} > 0$, then for periods of time when $m < m^{\infty}$ and $\gamma < \gamma^{\infty}$, we may infer that $\dot{a} < 0$.

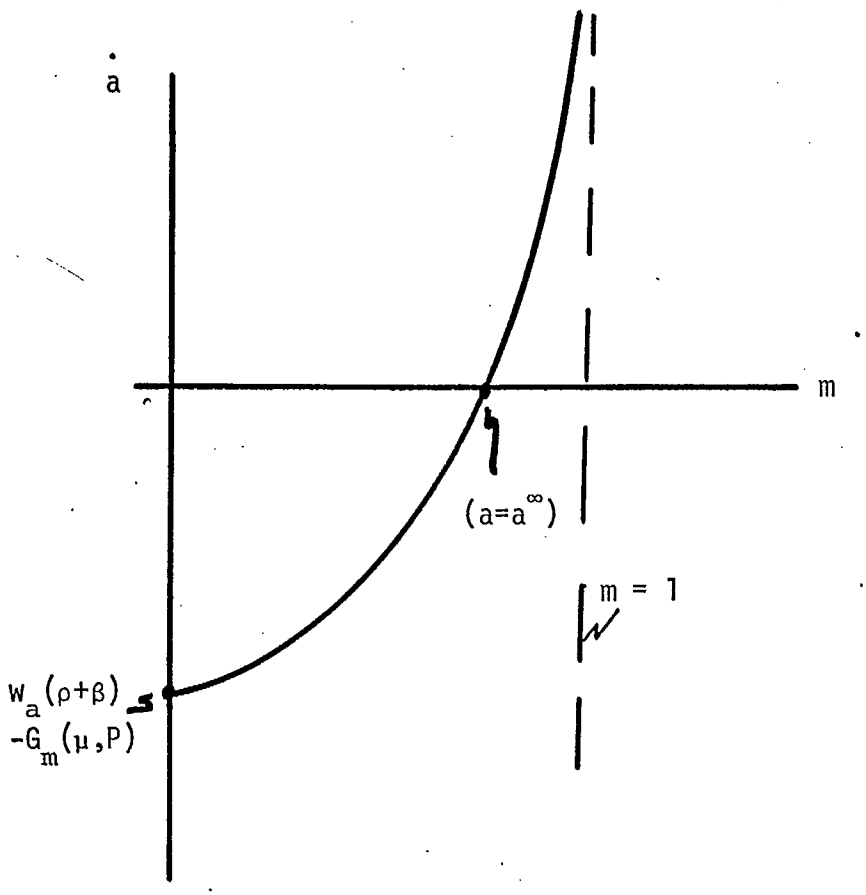
Figure D-2 depicts the relationship for $\dot{a}$ as a function of m. Thus, the advertizing policy corresponding to the policy of accumulation of informed consumers is a high initial advertizing expenditure, followed by declining advertizing expenditures at an increasing rate until the long-run steady state level of $a^{\infty} = \frac{(\alpha + \beta)m}{1-m} = \frac{(\alpha + \beta)M}{N-M}$ is achieved.

III DISSEMINATIVE ADVERTIZING AND WELFARE

The monopolist engages in the socially optimal amount of disseminative advertizing for the given price and quality level established by the monopolist. It is straightforward to demonstrate this claim. Define the solution to (D7) and (D8), the monopolist's price and quality level, as $*$ and μ^* , respectively. Holding this fixed, define the net benefits from

Figure D-2

Relationship Between $\dot{a}$ and m for Disseminative Advertizing



the informed group of the population as :

$$\int_0^{M^{**}} P^* \, h(\mu^*, P^*)M \, dM - Mh(\mu^*, P^*)C(\mu) - Nw(a) \qquad (D16)$$

Deflating by N , the population size, yields :

$$e^{\alpha t} N_0 \left[\int_0^{m^{**}} P^* \, h(\mu^*, P^*)m \, dm - mh(\mu^*, P^*)C(\mu) - w(a)\right] \qquad (D17)$$

The social objective function is the discounted stream of consumer benefits which is written as :

$$J^s = \int_0^{\infty} e^{-(\rho-\alpha)t}\left[\int_0^{m^{**}} P^* \, h(\mu^*, P^*)m dm - mh(\mu^*, P^*)C(\mu) - w(a)\right]dt \qquad (D18)$$

(D18) is to be maximized subject to :

$$\dot{m} = a(1 - m) - (\alpha + \beta)m$$

The necessary conditions for this maximization are identical to the set (D7) to (D11), so that for a fixed price and quality, the monopolist follows the social optimal amount of advertizing. This occurs because the monopolist's profit function is linear in the state variable m , i.e. the marginal revenue from increasing m equals the marginal valuation to society from increasing m . Of course, the monopolist produces with too high a price in the usual sense of equating respective marginal costs to marginal revenue rather than marginal valuations. By restricting resources in this fashion, the monopolist generates rents for itself at

the expense of resource allocation. It is interesting to note that, contrary to the persuasive advertizing model that maximizes realized social welfare, the long-run steady state for a social optimum calls for a positive advertizing policy for disseminative advertizing.

MATHEMATICAL APPENDIX TO APPENDIX D

To maximize (D4) where π is described in (D6) subject to (D5), form the current-valued Hamiltonian:

$$H \equiv G(\mu,P)m - w(a) + \lambda\,[a(1-m) - (\alpha+b)m]$$

First order conditions are stated in the text. Quasi-concavity assumptions on H mean that these first order conditions are sufficient for a maximum.

As stated in the text, (D7) and (D8) yield the optimal values for P and μ which are time invariant, as (D7) and (D8) are independent of the dynamic variables λ and m. (D9) yields $a = a(\lambda,m)$. Differentiation of (D9) yields:

$$\frac{\partial a}{\partial \lambda} = \frac{1-m}{w_{aa}} > 0$$

and

$$\frac{\partial a}{\partial m} = \frac{\lambda}{w_{aa}} < 0$$

For the phase diagram depicted in Figure D-1, consider $\dot{\lambda}$ and $\dot{m}$ at rest. Then

$$\frac{d\lambda}{dm}\Big|_{\dot{\lambda}=0} = \frac{\lambda^2}{w_{aa}(p+\alpha+b+a)+\lambda(1-m)} > 0$$

and

$$\frac{d\lambda}{dm}\Big|_{\dot{m}=0} = \frac{\lambda+(a+ +b)w_{aa}}{(1-m)^2} > 0$$

Consequently, both $\dot{\lambda}=0$ and $\dot{m}=0$ are positively sloped. The question is which one is more positively sloped than the other. The significance of this is revealed in the following analysis which analyzes the roots of a linear approximation to the dynamic system around the steady state solutions.

Approximate $\dot{\lambda}$ and $\dot{m}$ by a Taylor series expansion as:

$$\dot{\lambda} = \frac{\partial\dot{\lambda}}{\partial\lambda}\ (\lambda-\lambda^{\infty}) + \frac{\partial\dot{\lambda}}{\partial m}\ (m-m^{\infty})$$

$$\dot{m} = \frac{\partial\dot{m}}{\partial\lambda}\ (\lambda-\lambda^{\infty}) + \frac{\partial\dot{m}}{\partial m}\ (m-m^{\infty})$$

The corresponding quadratic equation is

$$\mu^2 - \left(\frac{\partial\dot{\lambda}}{\partial\lambda} + \frac{\partial\dot{m}}{\partial m}\right)\mu + \left(\frac{\partial\dot{\lambda}}{\partial\lambda}\frac{\partial\dot{m}}{\partial m} - \frac{\partial\dot{m}}{\partial\lambda}\frac{\partial\dot{\lambda}}{\partial m}\right) = 0$$

Conventional stability occurs for real roots with one positive root corresponding to the unstable trajectory and one negative root corresponding to the stable trajectory.

This condition is met if and only if:

$$\frac{\partial\dot{\lambda}}{\partial\lambda}\frac{\partial\dot{m}}{\partial m} - \frac{\partial\dot{m}}{\partial\lambda}\frac{\partial\dot{\lambda}}{\partial m} < 0$$

or if

$$\frac{\partial\lambda}{\partial m\big|\dot{\lambda}=0} \equiv -\frac{\frac{\partial\dot{\lambda}}{\partial m}}{\frac{\partial\dot{\lambda}}{\partial\lambda}} < -\frac{\frac{\partial\dot{m}}{\partial m}}{\frac{\partial\dot{m}}{\partial\lambda}} \equiv \frac{d\lambda}{dm\big|\dot{m}=0}$$

This may be stated equivalently by defining Φ as

$$\Phi \equiv \frac{d\lambda}{dm\big|_{\dot{\lambda}=0}} - \frac{d\lambda}{dm\big|_{\dot{m}=0}}$$

and requiring that $\Phi < 0$.

Substitution of specific terms into Φ yields expression (D14) in the text.