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CONSUMER INFORMATION, ADVERTIZING AND QUALITY VARIATIONS

PART 2 of 2

APPENDIX E - G

FOOTNOTES

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The Department of Consumer and Corporate Affairs
Government of Canada

Y. Kotowitz and F. Mathewson

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APPENDIX E

PERSUASIVE ADVERTISING MODEL

I. INTRODUCTION

In this Section we consider the role of advertizing in the learning process of consumers who have already been informed of the product and therefore have formed a prior about the product quality. If the prior mean is high and the variance wave is low enough, consumers would purchase some units of the good. Based on this sample, consumers learn and accordingly adjust their priors. Given this learning process and the effect of advertizing, the problem is to analyze the optimal advertizing policy of a firm interested in achieving the goal of maximizing the rate of return to its shareholders and to compare it to the social optimum.

The consumer learning model described in Appendix C, indicates that advertizing has two roles, not necessarily mutually exclusive. First, advertizing could affect the consumer's rate of adjustment of his prior. Specifically, when the true quality of the goods determined by a sample falls short of the consumer's prior, advertizing could reduce the speed of adjustment of the consumer's prior. In a setting where the experienced quality exceeds the prior, advertizing could enhance the learning effect. As well, advertizing may increase the confidence a consumer has in his prior by reducing the variance of the prior.

We focus on the role of advertizing in affecting the adjustment of the consumer's prior evaluation of the mean quality of the product. The central issue is the determination of the relationship amongst true product quality, perceived product quality and advertizing in this context. Section II analyses the optimal policy for persuasive advertizing for a monopoly. Section II analyses the socially optimal policy for persuasive advertizing.

II OPTIMAL FIRM POLICY

The seller is assumed to face a market composed of many consumers, each of whom buys x_i units of the product and learns according to equations (C-10 and C-11). There are two avenues we may pursue in presenting the market demand and learning equations. The first is to assume that all consumers have identical tastes so that market equations are just sums of individual demand curves and learning equations are averages of individual learning equations. This approach is not attractive as the assumption of homogeneity of the population is unreasonable, and in any case, the solutions are intractable where confidence in priors changes over time (i.e. $\dot{v} \neq 0$).

The second approach, which we chose, is to assume that the market is composed of individuals whose tastes and/or prior perceptions are different. Each of these individuals is assumed to consume a fixed quantity of the product in question, if the utility it yields is greater than its cost, given the individual's perception of the product's quality; or not to consume the product at all, if the utility it yields is insufficient to compensate him for its cost. Aggregating across all individuals is assumed to yield a well-behaved demand function of the form

$$P = P(N\bar{x}, Q, s+v) \qquad \text{(E1)}$$

where N is the number purchasing the product, and $\bar{x}$ is the fixed quantity consumed of the product.

The relevant derivatives are a function of the tastes and perceptions of the marginal individuals, i.e. those who are on the margin between buying or not buying the product. Unfortunately, even under the simplified assumption that quantity bought by each individual is fixed, the problem is mathematically intractable. Therefore, we add the assumption that $\dot{v} = 0$. This assumption is not unreasonable because the $\dot{v}$ equation and therefore m, the speed of learning, are only relevant for consumers who have purchased the product. As we have shown before, v converges to its steady state level δ, quite rapidly, so the majority of those who consume the product are likely to be close to their steady state level of v. Changes in true quality are only relevant for those who already consume the product. The majority of old customers are therefore likely to be sufficiently experienced to have a fairly stable v.

Because there are many consumers on the demand side of the market, the actual sample quality mean $\bar{q}$ may be replaced by μ, the actual mean quality, a decision variable for the producer(s) of this good.

Under this specification, the profit relationship for the firm at time t may be defined as :

$$\Pi \equiv R(N\bar{x}, Q, s+v) - N\bar{x} \cdot C(\mu) - a \qquad \text{(E2)}$$

where $C(\mu)$ is the cost of production for each physical unit of output, with $C_{\mu} > 0$, $C_{\mu\mu} > 0$

a is the annual advertizing expenditure,

$\bar{x}$ is the constant quantity purchased per shopping trip,

$R(N\bar{x}, Q, s+v)$ is the gross revenue function, with R assumed to be quasi-concave in its arguments, specifically $P_{QQ} < 0$

As well, the firm needs to take account of the evolution of the mean of the priors according to

$$\dot{Q} = m(\mu - Q) + f(a) \qquad \text{(E3)}$$

where $m \equiv \frac{x}{x + s/v} \equiv \delta$

$f_a > 0 \, , \; f_{aa} \leqslant 0 \, .$

To achieve increased sales in our model the producer can increase advertizing or reduce prices. This induces the marginal customers to try the product and depending on their accumulated experience with it, they either continue or cease to purchase the product. The effects of a change in true product quality cannot, by itself, attract new customers. It only affects the repeat purchase pattern. As a result, the effectiveness of advertizing depends on the true quality and the speed of learning. "True" advertizing, which conforms with the true quality of the product, is reinforced by experience but "exaggerated" advertizing is discounted by experience. The extent to which this discounting occurs depends on the speed of learning.

Because advertizing expenditures today affect the adjustment of the quality priors by consumers, and therefore, the expected quality of future purchases, and, in turn, the future profitability of the firm, current advertizing decisions need to include the future benefits received from these advertizing dollars.

More formally, this is accomplished by having the firm maximize its discounted stream of profits, defined as

$$J = \int_0^\infty e^{-\rho t} \Pi \, dt \tag{E4}$$

subject to the equation of motion for Q , equation (E3). This maximization yields a set of first-order conditions for each of the firm's policy variables, N , μ and a .

These necessary conditions may be written as follows :

$$R_{N\bar{x}} - C(\mu) = 0 \tag{E5}$$

$$\frac{-N\bar{x}\ C_\mu}{\delta} + \lambda = 0 \tag{E6}$$

$$-1 + \lambda f_a = 0 \tag{E7}$$

$$\dot{\lambda} = \lambda(\rho + \delta) - R_Q \tag{E8}$$

$$\dot{Q} = \delta(\mu - Q) + f(a) \tag{E9}$$

where λ is the current valued shadow price on Q . Equations (E5) through (E7) describe optimality conditions for each of the firm's decision variables -- N[7], μ and a , respectively. Each equation affords an economic interpretation. The first equation in the set describes the optimal output condition (number of customers) and is the usual marginal revenue, marginal cost equality.

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The second equation is the marginal condition for true product quality. According to this condition, true quality should be expanded until the marginal cost of achieving a unit of perceived quality via quality change $(N\bar{x}\, C_\mu/\delta)$ just equals the marginal value of perceived quality (λ).

The third equation is the marginal condition for advertizing expenditures. It states that advertizing should be expanded until the marginal cost of advertizing (which is one dollar) equals the marginal value of advertizing, which is equal to the marginal effect of advertizing on perceived quality (f_a) times the marginal value of perceived quality (λ).

If we combine these last two equations we obtain

$$\frac{N\bar{x}\, C_\mu}{\delta} = \frac{1}{f_a} \qquad \text{(E10)}$$

This marginal condition requires in equilibrium, an equality between the marginal cost of true quality $(N\bar{x}\, C_\mu/\delta)$ and the marginal cost of achieving the same increment in perceived quality through advertizing.

The final two equations describe the evolution of the two dynamic variables, λ and Q, over time. These equations are most easily interpreted in the long-run steady state, i.e. $\dot{\lambda} = \dot{Q} = 0$. Then, advertizing acts to maintain a gap between actual and perceived quality. The value of perceived quality (λ) equals the discounted marginal revenue from a change in perceived quality $(R_Q/(\rho + \delta))$.

This system of equations may be analyzed more easily by eliminating the auxiliary variable λ . Successive substitution yields the following equivalent set of equations for optimal levels of the instruments, N and μ :

$$R_{N\bar{x}}\, C(\mu) = 0 \tag{E11}$$

$$-N\bar{x}\, C_{\mu} + \delta/f_{a} = 0 \tag{E12}$$

These two equations may be solved for $N = N(\bar{x}, \delta, Q, a)$ and $\mu = \mu(\bar{x}, \delta, Q, a)$. Further analysis reveals that $\partial N/\partial a$, $\partial\mu/\partial Q < 0$ and $\partial N/\partial Q$, $\partial\mu/\partial a > 0$.[28] These partial effects are used in a more complete analysis of the system but yield little direct insight. With the elimination of λ and the solution chosen here for the problem, the two dynamic equations become :

$$\frac{f_{aa}}{f_{a}}\dot{a} = R_{Q}f_{a} - (\rho + \delta) \tag{E12a}$$

and

$$\dot{Q} = \delta(\mu - Q) + f(a) \tag{E9}$$

(E12a)describes the changes in advertizing policy through time (E9) remains unchanged.

By examining each of these equations at rest with N and μ evaluated at their optimal levels, it is possible to determine the optimal trajectories of a and Q .

Analysis reveals that :

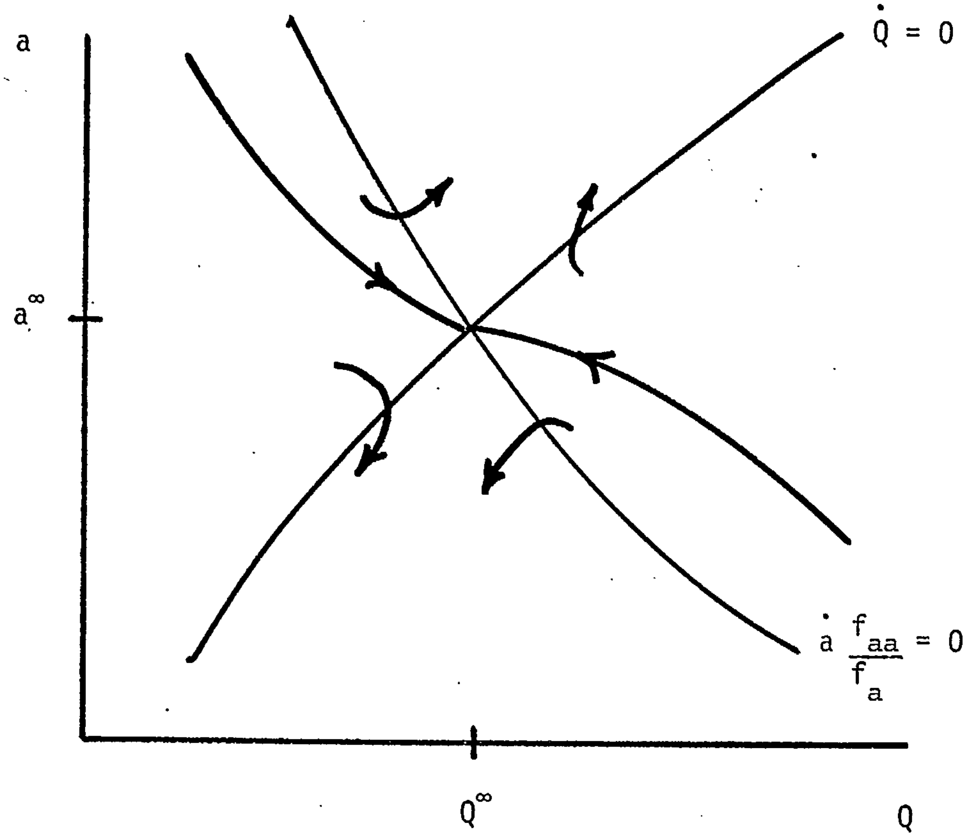
$$\frac{da}{dQ}\Big|_{\dot{a}=0} = -\frac{\frac{\partial \dot{a}}{\partial Q}}{\frac{\partial \dot{a}}{\partial a}} < 0 \quad \text{and} \quad \frac{da}{dQ}\Big|_{\dot{Q}=0} = -\frac{\frac{\partial \dot{Q}}{\partial Q}}{\frac{\partial \dot{Q}}{\partial a}} > 0 \qquad \text{(E13)}$$

These facts may be assembled into a phase diagram which describes the movement of a and Q through time. The equations labelled $\dot{Q} = 0$ and $\dot{a}\, f_{aa}/f_a = 0$ depict the two dynamic equations at rest. Their slopes are described by (E13). Notice that the slopes of the two dynamic equations are such that they intersect only once. Appropriate concavity conditions guarantee the existence and uniqueness of a long-run equilibrium. Investigation of the movement in this phase diagram reveals that the dynamic system has two roots, both real, one positive and the other negative. The stable trajectory pictured in Figure E-1 corresponds to the negative and consequently, stable root. The long-run equilibrium values of a and Q are a^{∞} and Q^{∞}, respectively, with corresponding values of $\mu = \mu^{\infty} \equiv \mu(\bar{x}, \delta, Q^{\infty}, a^{\infty})$ and $N = N^{\infty} \equiv N(\bar{x}, \delta, Q^{\infty}, a^{\infty})$.

If the initial perceived level of quality falls below Q^{∞} , then there is a unique initial level of advertizing when the firm pursues its long-run profit maximizing goals. There is a corresponding initial level of N and μ . Over time, Q increases while μ and a fall. The initial level of mean true quality must then exceed perceived quality, given the definition of changes in Q over time from the consumer learning model.

Figure E-1

Phase Diagram for Persuasive Advertizing


a
$\dot{Q} = 0$
a^{∞}
$\dot{a}\,\frac{f_{aa}}{f_a} = 0$
Q^{∞}
Q

Alternatively, the initial level of perceived quality may exceed the long-run steady state level. There is a corresponding unique level of advertizing expenditures, quantity purchased and mean true quality that services the firm's interest in maximizing profits. Over time, Q falls while μ and a increase. The initial level of true quality must accordingly fall short of perceived quality, given the definition of changes in Q over time from the consumer learning model.

An interesting point is that this model predicts that profit-maximizing firms select policies which cause true quality and perceived quality to move in opposite directions through time. Note also, that advertizing and true quality tend to move together through time. In the more common case, initial perceived quality of a new product is likely to be low relative to the steady state because consumers are ignorant of a new product, and therefore are cautious in assigning it a high prior quality (Q). In this case, the optimal profit maximizing policy on the part of the seller yields high true quality and advertizing at the time of introduction of the new product, but as perceived quality rises, true quality and advertizing fall in response. This picture seems to be consistent with the casual observation of advertizing and quality upon the introduction of new products.

We gain further insight into the behaviour of this firm by examining the changes through time of (a) the price-cost margin and (b) the firm's profits.

(a) The price-cost margin :

Define the price-cost margin (the mark-up) as

$$k \equiv \frac{[R(N\bar{x},\ Q,\ s+v)/N\bar{x}] - C(\mu)}{[R(N\bar{x},\ Q,\ s+v)/N\bar{x}]}$$

At the optimum N , $P = R_{N\bar{x}}((\eta+1)/\eta) = C(\mu)((\eta+1)/\eta)$ where η is the demand elasticity, so that substitution into the above definition of k yields

$$k = -1/\eta \tag{E14}$$

This is the well-known result that k is a measure of monopoly power. Whether k increases or decreases through time depends on the impact of changes in perceived quality on demand elasticity. If increases in perceived quality make demand more inelastic, then for optimal advertizing-sales-quality programs with $Q_0 < Q^{\infty}$, mark-up increases with time while with $Q_0 > Q^{\infty}$, mark-up decreases with time. Obviously, the opposite results hold if demand becomes more elastic with increases in perceived quality.

In the context of our model, note that as Q/μ rises over time, if $Q_0 < Q^{\infty}$, $P/c(\mu)$ rises for a given N . This is because the elasticity of demand of old customers decreases over time. However, the elasticity of demand of potential new customers may be lower so it may pay the monopolist to reduce prices over time in order to attract new customers, in spite of the decrease in the demand elasticity of every customer.

(b) Profits :

Instantaneous profits are defined as

$$\Pi = R(N\bar{x},\ Q,\ s+v) - N\bar{x}\ C(\mu) - a$$

Differentiation of Π with respect to time reveals that :

$$\dot{\Pi} \equiv (R_{N\bar{x}} - c(\mu))\ \dot{N\bar{x}} + R_Q\ \dot{Q} - N\bar{x}\ C_\mu\ \dot{\mu} - \dot{a}$$

Using the same information provided in the previous section together with the first order condition for optimal number of consumers $(R_{N\bar{x}} = C(\mu))$, we see that

$$\dot{\Pi} > 0 \quad \text{as} \quad Q_0 < Q^\infty$$

$$\text{and} \quad \dot{\Pi} < 0 \quad \text{as} \quad Q_0 > Q^\infty$$

The interpretation of these results is straightforward. When $Q_0 < Q^\infty$ and consumers hold priors on mean quality lower than true mean quality, then the firm's initial profits are smaller than their long-run profits. Through time, as perceived quality increases and true quality and advertizing decrease, profits rise. The converse holds for conditions where $Q_0 > Q^\infty$. In both cases, the trajectories defined in Figure 1 are optimal because they both maximize the discounted sum of profits, given the initial conditions.

III PERSUASIVE ADVERTIZING AND WELFARE

How does the quality-advertizing package used by the monopolist compare with the quality-advertizing package that maximizes social welfare? First, there is the problem that the private firm model is a monopoly model so that we would expect too few resources to be devoted towards economic activity in the traditional sense that monopolists in an attempt to collect rents are inefficient producers in the Pareto sense. Therefore, to the extent that monopolists equate the marginal revenue from quality instead of the marginal valuation from quality to the marginal costs of quality, quality is too low. This is also true for the quantity produced.

As well, there is the problem of differentiating between "true" and "perceived" quality for welfare purposes. As the monopoly model emphasizes, firms in the private sector in the face of our consumer learning model, can use advertizing to perpetuate a state where expected quality exceeds actual quality. For the purposes of welfare evaluation and social welfare maximization, it is realized and not expected quality which is the measure of well-being.[29/] Thus, we would not expect advertizing to have any role in the long-run in the state where social welfare is measured as realized rather than expected utility.

To illustrate these points more precisely, we compare our previous model of the firm to two other models. In the first model, expected social welfare is maximized; in the second model, realized social welfare is maximized.

We employ the consumer surplus measure of net social welfare for evaluating the optimal state. To avoid the aggregation difficulties that arise from simultaneously defining the optimal number of consumers (N) and consequently, under our assumptions the optimal quantity sold $(N\bar{x})$, we fix N at the level that is optimal for the monopolist.

Denote this level as N^m . We then wish to inquire for this level of market activity whether the quality level for the monopolist is or is not socially optimal.

To facilitate this analysis define expected average consumer benefits per unit of the good purchased as:

$$B^e \equiv \int_0^{Q^*} P(\bar{x}, Q, s + v)/Q \, dQ$$

Define realized average consumer benefits per unit of the good purchased as:

$$B^a \equiv \int_0^{\mu^*} P(\bar{x}, \mu, s + v)/\mu \, d\mu$$

where Q^* and μ^* are, respectively, the socially optimal level of expected and realized quality.

B^e and B^a are the conventional measures of consumer benefits as areas under demand curves and, as such, are subject to the usual waivers about constant marginal utilities of income for consumers.

Given our assumption about market size, the corresponding levels of total benefts are $N^m\bar{x}B^e$ and $N^m\bar{x}B^a$.

We begin by calculating the socially optimal quality-advertising program where social welfare is measured as expected social welfare. Then the objective functional is defined as:

$$J^e \equiv N^m\bar{x}\int_0^{\infty} e^{-\rho t} \left(\int_0^{Q^*} P(\bar{x}, Q, s + v)/Q \, dQ \right.$$
$$\left. - N^m x \cdot C(\mu) - a\right)dt \qquad (E15)$$

The maximization of expected social welfare subject to the evolution of Q over time as defined in (E3) (and continuing to assume that $\dot{v} = 0$) is described by the following first order conditions:

$$-N^m\bar{x}\,C\mu + \lambda\delta = 0 \tag{E16}$$

$$-1 + \lambda f_a = 0 \tag{E17}$$

$$\dot{\lambda} = \lambda(\rho + \delta) - N^m\bar{x}P/Q \tag{E18}$$

$$\dot{Q} = \delta(\mu - Q) + f(a) \tag{E19}$$

All variables here have the same interpretation as equations in the set (E5) to (E9). There is no equation for N as N is fixed at the level N^m. Inspection reveals that these equations are identical to the set for the monopolist, save that in (E18) R_Q, the marginal revenue from an additional unit of quality, is replaced by P/Q, the marginal valuation of an additional unit of quality. Thus, in long-run steady state, i.e. $\dot{\lambda} = 0$, (E18) states that the value of perceived quality (λ) should equal the discounted marginal value from a change in perceived quality ($N^m\bar{x}P/Q(\rho+\delta)$).

The comparison between the advertising-perceived quality programs generated by these two sets of equations may be most easily conducted by eliminating λ.

Then, as before, upon substitution

$$\frac{N^m\bar{x}\,C_\mu}{\delta} = \frac{1}{f_a} \tag{E20}$$

$$\frac{f_{aa}}{f_a}\,\dot{a} = N^m\bar{x}\frac{p f_a}{Q} \tag{E21}$$

$$\dot{Q} = \delta(\mu - Q) + f(a) \tag{E22}$$

If we graph (E21) and (E22) at rest, then examination of this model reveals that, provided that $P_Q < P/Q$, which is guaranteed by the concavity of P , there is no change in any of the qualitative predictions of the monopoly model, i.e. $\left.\frac{da}{dQ}\right|_{\dot{a}=0} < 0 \quad \frac{da}{dQ}_{\dot{Q}=0} > 0$.

Consequently, the phase diagram for this case resembles the diagram generated by the monopoly case. There is however, a quantitative distinction between the two cases.

This distinction is apparent from examining (E12) and (E21) at rest.

(monopoly) $\quad R_Q f_a = \rho + \delta \quad$ or $\quad f_a N^m\bar{x}\, P_Q = \rho + \delta \qquad$ (E23)

(expected social welfare) $\quad f_a N^m\bar{x}\, P/Q = \rho + \delta \qquad$ (E24)

First, ρ and δ are positive constants. Fix Q at any arbitrary level. Then, given the concavity of P , $P/Q > P_Q$. Then from (E23) and (E24) f_a exceeds f_a under expected social welfare. Given diminishing marginal productivity to advertising, a under expected social welfare must a under monopoly. Figure E-2 portrays the two-phase diagrams.

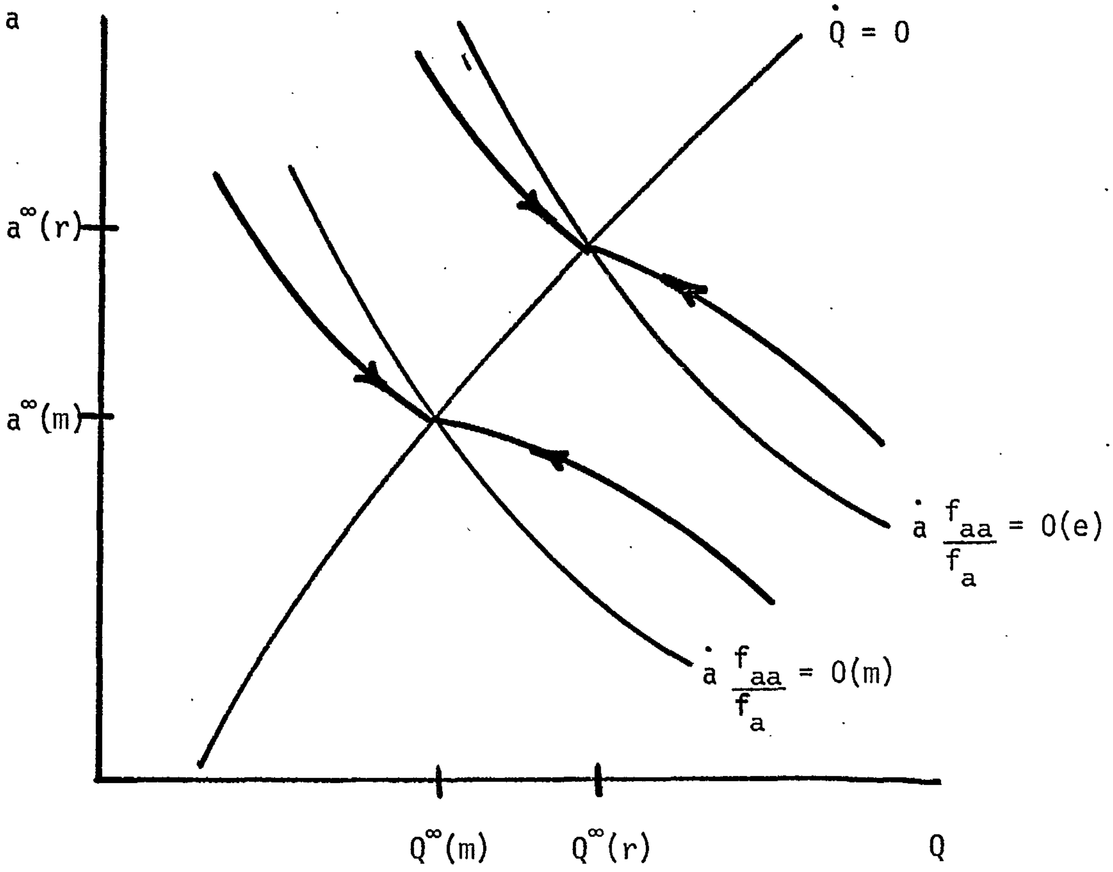
For any given market size $(N^m\bar{x})$ and given perceived quality (Q) , this diagram reveals that the monopoly model produces an advertising budget that is too small from a social point of view.

In fact, the price and quality are both lower in the monopoly case than in the expected social welfare maximization case, although the price-quality ratio in the monopoly case exceeds the price-quality ratio in the expected social welfare maximization case.

At first glance, this may appear counter-intuitive. It is easily explained and illustrates the inappropriateness of the expected social welfare maximization criterio. Because the expected social welfare criterion

Figure E-2

Comparison of Phase Diagrams Under Monopoly (m) and Expected Social Welfare Maximization (e)


a
$\dot{Q} = 0$
$a^{\infty}(r)$
$a^{\infty}(m)$
$\dot{a}\frac{f_{aa}}{f_a} = 0(e)$
$\dot{a}\frac{f_{aa}}{f_a} = 0(m)$
$Q^{\infty}(m)$
$Q^{\infty}(r)$
Q

permits consumers to experience lower realized than expected utility, it legitimizes the role of advertising as an instrument that retards, in this case, the evolution through time of the prior on quality held by the consumer. The observation that for a fixed Q, $P/Q > P_Q$ means that in the long-run, under expected social welfare, a higher Q and, not surprisingly, a higher advertizing expenditure are optimal. We reserve any comparison of actual quality until we present the socially optimal solution under the realized social welfare criterion.

Then, the objective functional is defined as:

$$J^r \equiv N^m\bar{x} \int_0^\infty e^{-\rho t} \left(\int_0^{\mu^*} P(\bar{x}, \mu, s + V) d\mu \quad - N^m\bar{x}\, C(\mu) - a \right) dt \qquad \text{(E25)}$$

Again, following the same procedure, the maximization of realized social welfare subject to the evolution of Q over time as defined in (E3) (and maintaining the assumption that $\dot{V} = 0$) is described by the following first order conditions:

$$N^m\bar{x}P - N^m\bar{x}\, C_\mu + \lambda\delta = 0 \qquad \text{(E26)}$$

$$-1 + \lambda f_a = 0 \qquad \text{(E27)}$$

$$\dot{\lambda} = \lambda(\rho + \delta) \qquad \text{(E28)}$$

$$\dot{Q} = \delta(\mu - Q) + f(a) \qquad \text{(E29)}$$

Again, all variables maintain their previous interpretation. Comparisons between this solution set and the others are more effectively made by eliminating λ. Thus, the equations become

$$\frac{N^m\bar{x}\,C_\mu - N^m\bar{x}\,P/Q}{\delta} \qquad (E30)$$

$$\frac{f_{aa}}{f_a}\,\dot{a} = -\frac{\rho+\delta}{f_a} \qquad (E31)$$

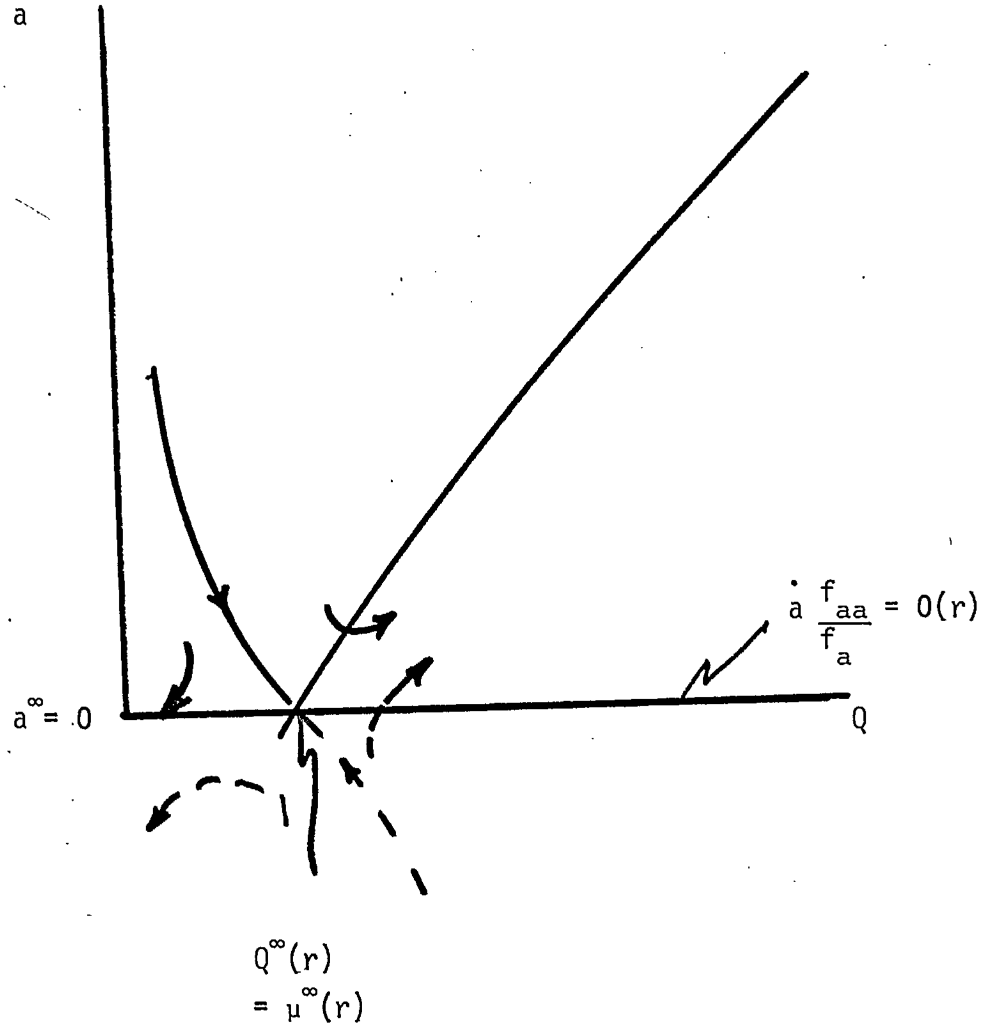
$$\dot{Q} = \delta(\mu - Q) + f(a) \qquad (E32)$$

Comparison of (E30) through (E32) with the monopoly solution, (E20) through (E22) reveals some differences. First, the current $\dot{a}$ equation (E31) is independent of Q. Furthermore, setting $\dot{a} = 0$ reveals that $a = 0$ for $\dot{a} = 0$ as $\lim f_a \to \infty$ as $a \to 0$. (E32) remains unchanged. Figure E-3 portrays the corresponding phase diagram for this case.

The long-run steady state is characterized by zero advertising and a level of perceived quality equal to actual quality. This occurs because consumers receive value only for "true" and not perceived quality. If the initial perceived quality falls short of the long-run perceived (actual) quality (i.e., $Q_o < Q^{\infty}r) = \mu^{\infty}(r))$, then perceived quality is less than "true" quality and advertising expenditures are necessary at the outset to persuade consumers to sample the good. However, if the initial perceived quality exceeds the long-run perceived (actual) quality, then perceived quality is greater than "true" quality, and advertising is unnecessary at the outset as consumers are induced by their own perceptions to sample the product.

Figure E-3

Phase Diagram for Realized Social Welfare Maximixation (r)


a
$a^{\infty} = 0$
Q
$\dot{a}\ \frac{f_{aa}}{f_a} = 0(r)$
$Q^{\infty}(r)$
$= \mu^{\infty}(r)$

Finally, equation (E30) is different from (E20) and (E6). These equations may be graphed to compare the long-run actual quality in each of these three models. For purposes of comparison, these equations may be rewritten as :

$$N^m\bar{x}\, C_\mu = \delta/f_a \qquad \text{(for monopoly and expected social welfare maximization)} \qquad \text{(E33)}$$

and

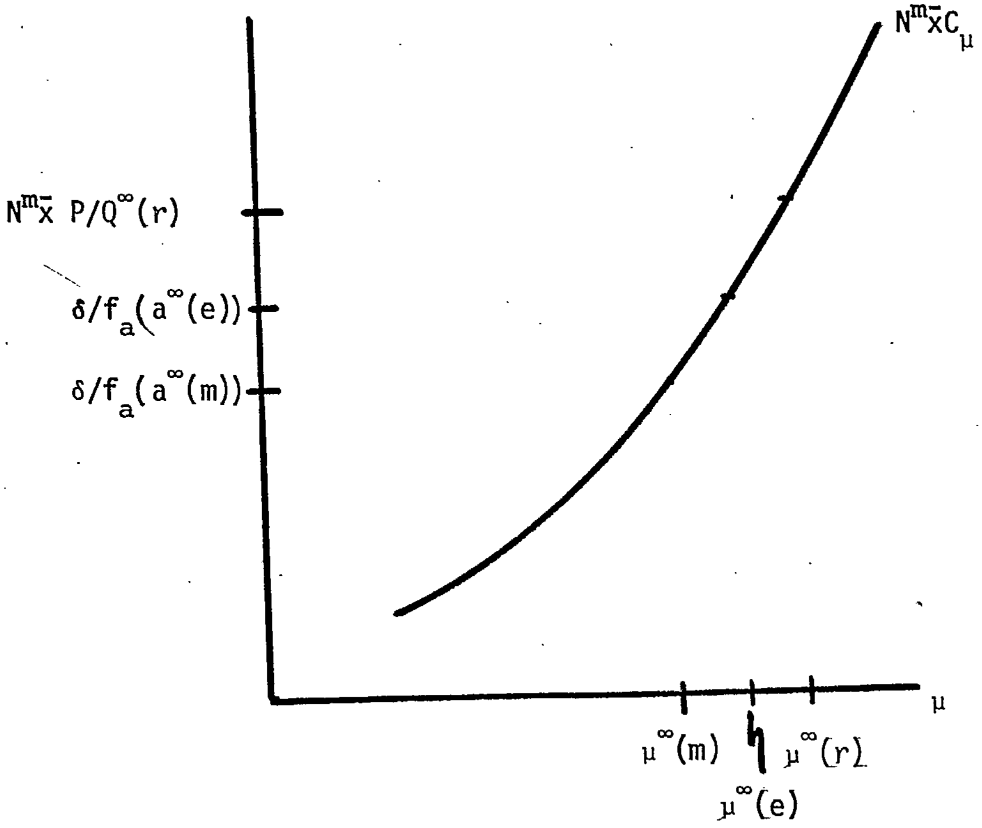
$$N^m\bar{x}\, C_\mu = N^m\bar{x}\, P/Q^\infty(r) + \delta/f_a \qquad \text{(for realized social welfare maximization)} \qquad \text{(E34)}$$

Figure E-4 compares the long-run solution values for μ from the models. $N^m\bar{x}\, C_\mu$ is the marginal cost of quality. This curve slopes upwards to the right. For the monopoly and expected social welfare maximization model, in long-run equilibrium $N^m\bar{x}\, C_\mu$ is equated to δf_a . As $a^\infty(e) > a^\infty(m)$ (see Figure E-2), $f_a(a^\infty(e)) < f_a(a^\infty(m))$. This implies from (E33), for a fixed N at N^m , that $\mu^\infty(e) > \mu^\infty(m)$. This is illustrated in Figure E-4.

For the realized social welfare maximization, $a^\infty(r) = 0$ so that $\delta/f_a(a^\infty(r))$ and $N^m\bar{x}\, C_\mu = N^m\bar{x}\, P/Q^\infty(r)$. However, $P/Q^\infty(r) > P_Q(Q^\infty(m))$, as $Q^\infty(v) < Q^\infty(m)$ and from equation (E23) $N^m\bar{x}\, P_Q\, (Q^\infty(m)) = (\rho + \delta)/f_a$ for the monopoly solution. Therefore, $P|Q^\infty(r) > \delta/f_a(a^\infty(m))$. Similarly, for the expected social welfare maximization, from equation (E21), $N^m\bar{x}\, P/Q^\infty(e) = (\rho + \delta)/f_a$ so that $P/Q^\infty(r) > \delta/f_a(a^\infty(e))$, as $Q^\infty(r) < Q^\infty(m)$. Therefore, long-run actual quality for the realized social welfare maximization exceeds long-run actual quality for the expected social welfare maximization which, in turn, exceeds long-run actual quality for the monopolist. This is illustrated in Figure E-4.

Figure E-4

Comparison of the Long-Run Values of μ From the Models



MATHEMATICAL APPENDIX TO APPENDIX E

(1) Monopoly Model

To maximize (E4) subject to (E3) form the current-valued Hamiltonion:

$$H = R(N\overline{x}, Q, s + v) - N\overline{x}\, C(\mu) - a + \lambda[\mu - Q) + f(a)]$$

First order conditions are stated in the text. Quasi-concavity assumptions on H mean that these first order conditions are sufficient for a maximum.

(E11) and (E12) are solved for $N = N(\overline{x}, \delta, Q, a)$ and $\mu = \mu(\overline{x}, \delta, Q, a)$. To determine signs of $\partial N/\partial \overline{x}$, etc., totally differentiate (E11) and (E12) to yield:

$$\overline{x}\, R_{N\overline{x}N\overline{x}}\, dN + R_{N\overline{x}Q}\, dQ + R_{x\overline{v}}\, d\overline{v} + R_{xw} dw - C_{\mu} d_{\mu} + NR_{N\overline{x}N\overline{x}}\, d\overline{x} = 0$$

$$- \overline{x}\, C_{\mu} dN + 1/f_{a}\, d\delta - \delta f_{aa}/f_{a}^{2}\, da - N\overline{x}\, C_{\mu\mu} d\mu - NC_{\mu} d\overline{x} = 0$$

Define

$$D \equiv \begin{vmatrix} \overline{x}\, R_{N\overline{x}N\overline{x}} & - C_{\mu} \\ - \overline{x}\, C_{\mu} & - N\overline{x}\, C_{\mu\mu} \end{vmatrix} > 0$$

Solving this system yields:

$$\frac{\partial N}{\partial a} = \frac{1}{D} \frac{\delta f_{aa}}{f_{a}^{2}} C_{\mu} < 0$$

$$\frac{\partial N}{\partial Q} = \frac{1}{D} R_{N\overline{x}Q}\, N\overline{x}\, C\mu\mu > 0$$

$$\frac{\partial \mu}{\partial a} = \frac{1}{D} \frac{\delta R_{N\bar{x}N\bar{x}} f_{aa}}{f_a^2} > 0$$

$$\frac{\partial \mu}{\partial Q} = -\frac{1}{D} \bar{x} R_{N\bar{x}Q} C_\mu < 0$$

For phase diagram (Figure 1) consider $\dot{a} = 0$ and $\dot{Q} = 0$ (i.e. equations (E12a) and (E9), respectively).

$$\frac{da}{dQ} \Big| \dot{a} = 0 = - \frac{\frac{f_a}{N\bar{x} R_{N\bar{x}} N\bar{x} C_{\mu\mu} + \bar{x} C_\mu^2} [N\bar{x} C_{\mu\mu}(P_{QQ} R_{N\bar{x}N\bar{x}} - R_{QNx}^2) + \bar{x}^2 R_{QNx} C_\mu^2]}{R_Q f_{aa} + \bar{x} R_{QN\bar{x}} \frac{\partial N}{\partial a}} < 0$$

$$\frac{da}{dQ} \Big| \dot{Q} = 0 = - \frac{\delta(\frac{\partial \mu}{\partial Q} - 1)}{\delta \frac{\partial \mu}{\partial a} + f_a} > 0$$

(2) Social Welfare Maximization (Expected and Realized)

Results are analogous here except for the substitution of particular terms, i.e. P/Q for P_Q in the $\dot{\lambda}$ equation for expected social welfare maximization, and μ for Q in the demand expression in the realized social welfare case.

Appendix F

CONSUMER AND PRODUCER BEHAVIOUR - MULTIPLE BRANDS

Appendices C, D and E analyze consumer and producer behaviour for the case of a single brand of an uncertain good. In this appendix, we extend some of the analysis to the case of several brands of the same good. The extension is straightforward for the case of the consumer. Indeed, Appendix C already treats many of the problems associated with multiple brands. However, the extension is less straightforward for the analysis of producers where the number of producers and brands is small. Because the theory of oligopoly is sadly incomplete, competitive and collusive behaviour in such markets is very difficult to forecast and evaluate. Nonetheless, our theory of consumer behaviour enables us to make at least some reasonable observations about the nature of advertising as a competitive tool and as a barrier to entry.

I Consumer Behaviour

The utility function of the consumer (equation C1) must be extended to include several brands as follows:

$$\text{Max}_{x} \; u\Big(\big(\sum_{i=1}^{m} z_i x_i\big),\; x_n\Big)$$

$$\text{subject to} \quad x_n \equiv I - \sum_{i=1}^{m} P_i x_i \qquad \text{(F1)}$$

where z_i and x_i are the quality and quantity of brand i . z_i's are random variables as before with unknown mean Q_i and known objective variances r_i ; I is income and P_i is the price of brand i . For simplicity, we assume mean brand qualities may be different but their variances are equal $v_i = v_j = v$.

The prior distribution of the unknown quality mean for each brand i (Q_i) is assumed to be normal with mean $\bar{Q}_i$ and variance v_i . Q_i is assumed to be independent of Q_j.

Following the procedure outlined in Appendix C, (equations (C2) to (C4)) the maximization of expected utility yields the optimization rule for all i.

$$\frac{\partial(Eu)}{\partial x_i} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[u_{zx}(wr + yv_i + \bar{Q}_i) - P_i u_{xn}]dwdy \leq 0 \qquad \text{(F2)}$$

where w and y are standard normal variables.

As the utility function is linearly separable in the brands of the uncertain product, it follows that if for a certain brand k

$$\frac{\iint u_{zx}(wr + yv_k + \bar{Q}_k)dwdy}{\iint u_{zx}(wr + yv_i + \bar{Q}_i)dwdy} > \frac{P_r}{P_i} \qquad \text{(F3)}$$

i.e. if the marginal expected utility from consumption of one dollar's worth of brand k exceeds that of the other brands, only brand k will be purchased. This occurs if the prior mean of brand k, $\bar{Q}_i$, is greater and the prior variance of brand $k(v_k)$ is smaller than that of all other brands. An increase in a brand's expected quality or a decrease in its variance due to advertising or to accumulated experience, or a decrease in its price may cause some consumers to switch to it from other brands, as well as induce consumers who previously did not buy the product at all to do so. Only in the unlikely event that marginal expected utilities of expenditure on different brands are the same, will the consumer buy more than one brand. This result of exclusive brand purchase, as well as brand switching in response to changes in prices or other variables, appears to be in conformity with consumer behaviour.

As in the case of the single brand, the demand for the chosen brand is then a function of the price, the expected quality (represented by the prior mean) and the expected quality variance which is the sum of the true quality variance and the prior variance.

The prior is adjusted over time in the same way as the single product variance: (equations (C10b) and (C12b)).

$$\frac{d\bar{Q}_i}{d_t} \equiv \dot{\bar{Q}} = m\,(\bar{q} - \bar{Q}) + f(a_i, a_j) \quad \text{where} \quad \frac{\partial f}{\partial a_i} > 0, \quad \frac{\partial f}{\partial a_j} < 0 \qquad \text{(F4)}$$

$$\frac{dv_i}{d_t} \equiv \dot{v} = (-m_i + \delta - g(a_i, a_j))v \quad \text{where } \frac{\partial g}{\partial a_i} > 0 \, , \quad \frac{\partial g}{\partial a_j} < 0 \qquad \text{(F5)}$$

m is defined as $m \equiv \frac{x_i}{x_i + s_i/v_i}$

where a_j is the sum of the rival brands advertising outlay, and s_i is the individual's estimate of his ability to measure quality correctly.

As will be shown later, there is good reason to believe that rival advertising does not only affect the brand's prior negatively, but it also reduces the effectiveness of the brand's own advertising. Therefore,

$$\frac{\partial^2 f}{\partial a_i \partial_j} \equiv f_{ij} < 0 \quad \text{and} \quad \frac{\partial^2 g}{\partial a_i \partial_j} \equiv g_{ij} < 0 \qquad \text{(F6)}$$

The demand by each consumer for a specific brand may be volatile, changing abruptly between zero and some positive amount as price or product evaluation change. However, the market demand may be smooth if there are many consumers. Given our assumptions about the myopic nature of consumer decision-making, experimentation on the part of consumers is limited. Thus, the consumer is assumed to form a prior about the quality of the variance of the brands and choose the brand which satisfies equation (F3), disregarding the value of the information which may be forthcoming from the experience from consumption of each brand. There will be no deliberate experimentation whereby a brand, which does not satisfy equation (F3), is chosen because of its informational value. Brand switching occurs only as prices or priors about the various brands change in response to advertising and other information. In particular, unfavourable experience with the chosen brand, which reduces the mean of the prior may induce switching to another brand if the reduction of the prior mean is great enough to offset the smaller prior variance of the familiar brand. This reduction of prior variance is one of the main sources of brand loyalty. This brand loyalty is strong for products for which risk aversion is particularly strong.

Products affecting health or those which have other irreversible consequences -- such as safety -- appear to belong in this group. As previously shown in Appendix C, such products are likely to have low brand price cross-elasticities of demand. Another group of products for which our myopic model is appropriate, are those for which experimentation is costly or where rapid technological progress outdates the information garnered through consumption. In these cases, the value of experimentation to discover relative brand qualities through experience is low relative to its cost, which is the risk of spending one's money on unknown quality brands. It is clear that in such cases risk aversion dominates and experimentation to accumulate information does not occur. Consumer durables are good examples.

At the other extreme, we find goods for which risk aversion is weak and the costs of sampling low. In this case the value of experimentation to obtain information cannot be ignored. The model must be modified to include the value of such information for improved decisions in the future. 38/ The expected value of this information consists of corrections of any bias in the prior and reduction of prior variance with experience. Rather than develop a formal model of stochastic optimal control, which is mathematically complex and not easy to interpret, we illustrate the problem by way of an example. Consider the case of two brands, the variances in the quality of which are known to be equal. The first brand's average quality is known with certainty, the second brand is believed to be of equal average quality to the first, but this belief is uncertain. The expected variance of the second therefore includes the variance of the prior. As we have already shown, a risk averse consumer prefers the known item to the unknown if he considers his utility in the current period only. However, if he considers future utility, he may prefer the unknown brand. The reason is that if he discovers by experience that the unknown brand is in fact better than he thought, he benefits from the discovery by consuming it for a long period. If, on the other hand, it is in fact worse, he can cut his losses by switching back to the known brand.

The possible loss from experimentation is limited to the loss in utility from consuming the inferior brand over the experimentation period but the possible gain may extend over many future periods.

In the case of a normal prior, the probability that the product is in fact better than expected is the same as the probability that it is worse. It is clear that the expected gains exceed the losses. A risk neutral consumer therefore always prefers the unknown brand under these circumstances. Risk aversion may, however, offset the informational value and lead the consumer to choose the known brand. The value of information acquired by experimentation falls as experience is accumulated because, as shown in Appendix C, the variance of the prior decreases very rapidly. Unknowledgeable consumers experiment until experience is accumulated and then cease experimenting and follow the rule of equation (F3). As suggested before, the experimentation is limited if the risk of a current loss in utility from consuming the inferior brand is high, or if the information acquired by experimentation is valid for a short time only.

II The Effects of Advertising

An interesting result of the foregoing analysis is that increased uncertainty about product quality may lead to more experimentation with it. This is perhaps the reason for the periodic introduction of "new" products and brands coupled with exaggerated -- often irrelevant -- advertising in the market for goods, such as detergent and household cleaners, which are bought frequently and which do not involve serious risk in consumption. Such introduction, however, must not occur too often in order not to reduce excessively the value of the information acquired by experimentation, by reducing the time span over which it is useful. Note that experimentation depends on s , the estimated subjective measure of the variance of measurement error. The larger s , the less the consumer experiments, other things being equal, because experimentation yields little information per experiment. Advertising which increases s therefore reduces experimentation. Established brands attempt to increase their consumers' estimate of s by "clouding the issue", i.e. by making advertising claims which cannot be easily validated and by persuading consumers that judgement is difficult. Experimentation with new brands is not expected to reveal quality differences, so that experimentation is reduced. The qualities claimed and the difficulty in evaluating them must, of course, be important, so that the individual retains brand preference in the face of increased rival advertising or price reductions. Patent medicines are a good example of this type of good. This type of advertising is equivalent to the creation of random noise in the perception processes, which also operates to restrict efficiency of rival advertising, as well as restrict ability to validate. We return to this question later.

In contrast to advertising by established brands, which attempts to prevent experimentation, the role of advertising of new brands is to promote experimentation. It is therefore in the interest of new brand advertising to reduce s in order to make experimentation more attractive and to induce consumers to make decisions after experimentation. We have shown in Appendix E that new product true quality is likely to be high at the time of introduction and decline thereafter. If advertising succeeds in decreasing s, it is likely that decisions to adopt the new brand after experimentation are accelerated. So, the new brand advertising is likely to stress easily validatable qualities of the new product, which are indeed superior. Then, consumers are induced to try the product with the promise of fast accumulation of information, which is indeed corrrect. However, as the brand becomes established we have shown that true quality and advertising fall. As this occurs. the role and nature of advertising change. Advertising changes from an offensive tool attempting to induce consumers to change behaviour, into a defensive one trying to maintain inertia. Advertising claims become more difficult to validate and the difficulty of validation is stressed.

The situation regarding experimentation and validation is very different for the case of goods which are risky or in which information is rapidly dated. Examples of such goods are consumer durables, schooling, hospital care, etc. Because experimentation is very limited, validation from personal experience is very difficult. In particular, comparative valuation which requires the experience of several brands is almost nonexistent. In the case of consumer durables, for instance, experience with a given brand yields very little information on the performance of a later model of the same brand, due to frequent model changes. The only continuity may be in the assumption that a firm specialises in a given quality, so that an indication of the quality of one model or product carries over to other models and products of the same firm. Because the costs of a mistake may be high, even this degree of assurance creates strong brand loyalty. Because these goods are also complex, measurement error in the evaluation of quality is likely to be high, and known to be so, i.e. s is large. The consequent slow speed of learning means that only serious disappointment in the performance of the experienced brand leads to brand switching. As a result, defensive advertising is likely to reinforce this natural tendency.

The task of offensive advertising in this case is very much more difficult. It must induce re-evaluation of relative priors which is sufficiently strong to overcome risk aversion and to induce the consumer to shift his pattern of consumption permanently, rather than to induce him to experiment only. Because of the risk involved, the consumer is not likely to accept advertising claims without checking them independently through other sources. Advertising claims should be a much less efficient offensive weapon. Other methods of persuasion -- such as warranties, guarantees, personal selling efforts of salesmen -- msut be substituted for media advertising or must reinforce it. The role of advertising in this case as an offensive tool for new brands, is to attract attention to these other modes of persuasion and to disseminate technical information which is easily verified, e.g. size. As a defensive tool, its role is to reinforce the effect of a large s and to assure the consumers of the continuity of the different brands and models of a given firm i.e. to assure them that their experience can be extrapolated to the future. Note that advertising claims in this capacity are likely to be in the nature of vague assurances, e.g. "you can rely on Westinghouse", "Fords are built better", etc.

One important exception to the discussion of defensive and offensive advertising, is advertising which is designed to create "cognitive dissonance". Such advertising tries to bias the perception of quality after use in the direction of the prior, i.e. to assure consumers that they have made the correct decision in purchasing the product or brand of the advertiser. This reinforces the natural bias of people to make errors in the direction of their prior biases. In terms of our model, such advertising biases q as a measure of z and reduces s. The individual is persuaded of his ability to distinguish the advertised brand favourably from other brands even when, in fact, he cannot do so. Such advertising is likely to stres seemingly validatable properties: taste, smell, etc. The advantage of such advertising, if effective, is that as it affects unobservable errors of measurement, it eliminates experience as a guide.

The example cited in Appendix C, of cigarettes and liquor may well illustrate such "artificial product differentiation". Note that such advertising is equally effective in either an offensive or defensive role for the case of goods with which consumers experiment. However, it is only suitable as a defensive tool for goods with which consumers do not experiment because its purpose is to reinforce past decisions to buy rather than to induce consumption changes.

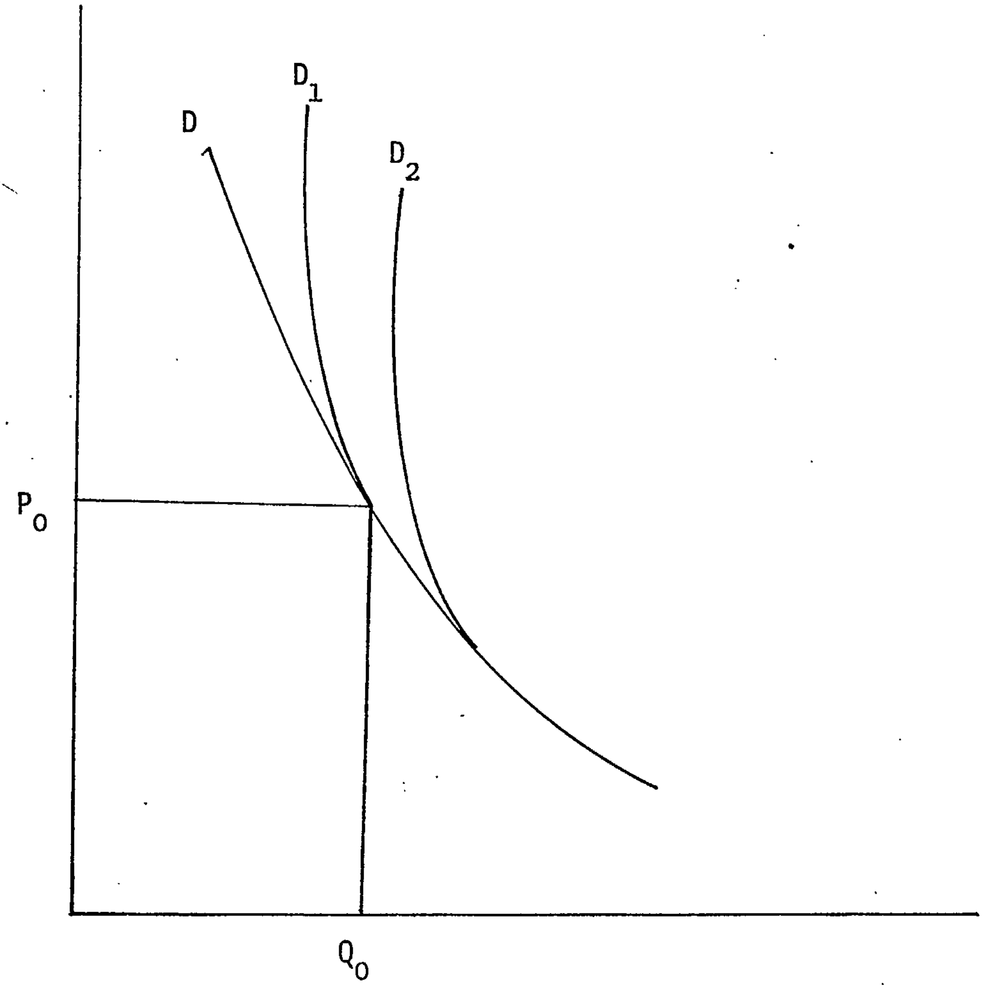
IV Advertising in Stationary Oligopoly

The distinction we have drawn between offensive and defensive advertising is useful in analysing the volume of advertising under conditions of oligopoly. There is a great deal of debate about whether an oligopolistic industry is likely to advertise more or less than a monopolistic one. The reason for the indeterminacy is that the marginal effects of advertising on profits depend on the price cost margins, on the one hand, and on the degree of competition, particularly non-price competition, on the other. Ignoring the investment aspects of advertising, Dorfman and Steiner [1959] have shown that optimal advertising for a monopoly requires that the advertising to sales ratio equals the ratio of the advertising elasticity to the absolute value of the price elasticity of demand. As the price elasticity of demand falls, the profitability and consequently, the amount of advertising increases. This is because price-cost margins are higher -- the lower the elasticity of demand, so the marginal contribution to profits of additional sales due to advertising is greater.

Under conditions of oligopoly with price and advertising competition, the price elasticity of demand for an individual firm's product rises but the elasticity of advertising remains unchanged if all firms advertise. On the assumption that each firm gets a share of the new customers acquired by the joint industry advertising equal to its advertising share, advertising falls.
However, if price competition were avoided by tacit or explicit agreement, advertising would be at least as high as in the monopoly, provided prices are maintained at the monopoly level. This is because as price competition is avoided, the returns to successful non-price competition increase. It is contended (see, for example, Schmalensee [1972a], Cable [1972], Simon [1970]) that such arrangements to restrict price competition but not non-price competition is common in oligopoly. It is not immediately obvious why collusion to restrict competition in prices is not extended to non-price competition. Stigler [1954] suggests that the ability to collude depends on unanimity of opinion about the best policy, e.g. about what is best price to maximize industry profit and on the ability to detect breaches of the agreement. Both of these clearly vary directly with the number of firms, but inversely with the inequality of their size distribution. The problem is that these same variables are likely to affect the ability to collude in advertising, as advertising is such a visible medium. Such collusion leads oligopolists to maintain the same level of advertising as that of a monopolist.

However, our analysis may explain why advertising in oligopoly might exceed that of a monopoly even in the absence of collusion. In fact, advertising may be helpful in reaching and maintaining relatively high price-cost margins. The key is that established firms in oligopoly mainly engage in defensive advertising. As we have shown, the purpose of such advertising is to increase and maintain brand loyalty and to discourage experimentation, i.e. to create and reinforce product differentiation. The main effect is the reduction in the price cross elasticity of each brand's customers relative to other brands. While such advertising may also induce customers of rival brands, or new customers to try the brand in question, this is not the main function of this type of advertising. The effects of advertising on consumer demand can be illustrated in Figure F-1. D is the demand curve of the representative consumer who has been buying the brand in question. Defensive advertising induces him to revalue the advertised brand relative to other brands. It may induce him to increase his purchase of the good in question and therefore his purchase of the advertised brand, but the main effect is to shift his demand curve upwards, i.e. to increase his valuation of consumer surplus. This effect will cause the consumers' demand curve to shift to D_1, assuming no increase in quantity purchased at the going price (P_0), or to shift to D_2, otherwise. In any case, it is very likely that for customers of this particular brand defensive advertising reduces the price elasticity of demand at P_0. To the

Figure F-1



extent that advertising attracts new customers or the customers of other brands, it shifts the aggregate demand curve further to the right, but the demand of these customers is not likely to be as inelastic as that of the original brands customers, because it is not yet reinforced by long experience. Because of this, the total demand shift affects the demand elasticity in an unknown way. However, if defensive advertising follows the principles dicussed, its main effect is likely to be on the original customers of the brand in the way described in Figure F-1. The reduction in the elasticity of demand of the customers of the original brand now enables the advertiser to raise the product price as advertising is increased. The result of the increased advertising and price of a brand on sales of rival brands is likely to be small because the customers attracted away from rival brands by advertising are likely to return because of the higher price. Consequently, rival firms are not worse off and have no incentive to retaliate. In fact, if all firms pursue this policy, each discovers that further unilateral defensive advertising and price increases are profitable. The process continues in decreasing steps to an equilibrium level of price and advertising. It can be shown that the resulting price is lower than the monopoly price under plausible conditions but that advertising may well exceed that of a monopoly. This is because even in a cartel which fixes prices, firms cannot be sure that the cartel will hold. Defensive advertising may then be undertaken in order to differentiate the product and ensure reduced vulnerability to price or offensive advertising. This reduction in vulnerability reduces the potential gains from breaking the agreement and so increases the stability of the cartel.

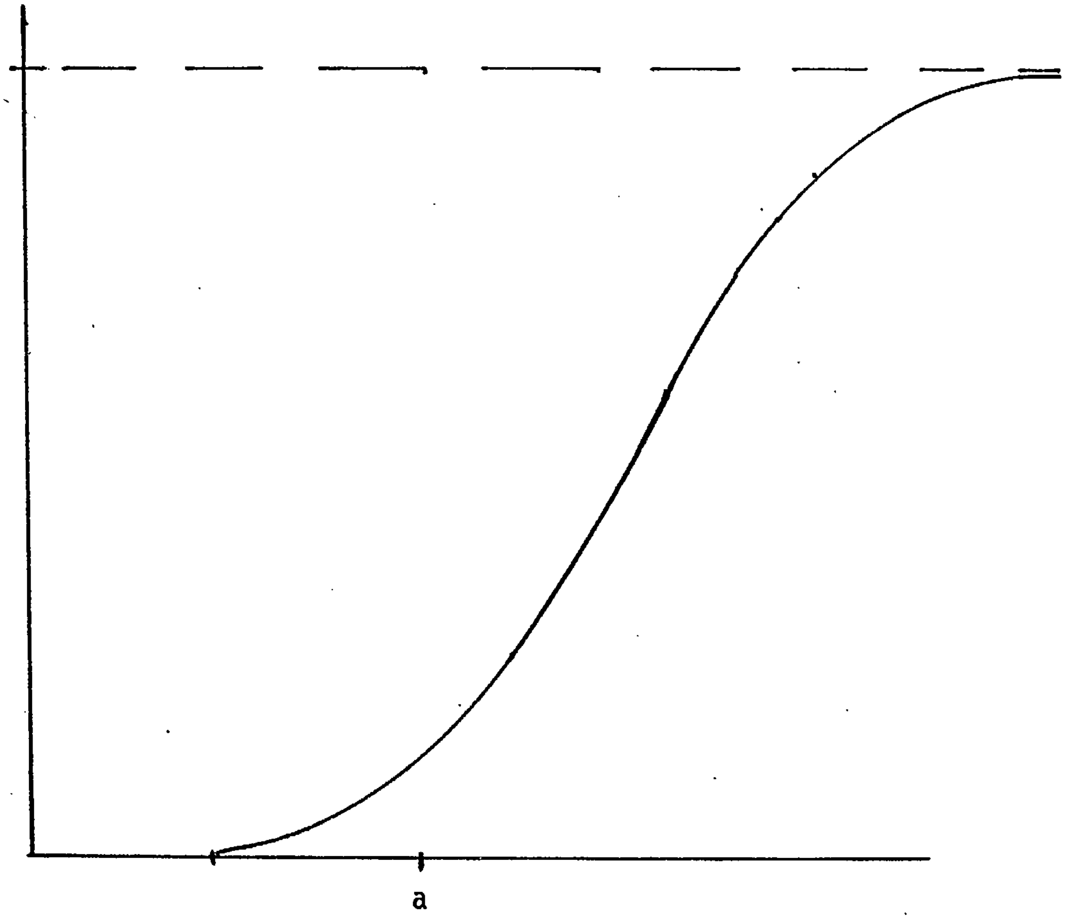
Note that offensive advertising, in common with competitive price cutting, is not as attractive for established firms in oligopoly because offensive devices mainly attract rival customers and invite retaliation in kind. If the offending firm is sufficiently large relative to the market, such retaliation is quite likely. The offensive firm therefore takes this probable retaliation into consideration, so that the expected productivity of price cutting or offensive advertising are likely to be low. As the probability of detection and rival reaction increases with the offender's size, one would expect that firms which account for a small share of the market are more likely to pursue offensive policies. If the product lends itself to offensive advertising, it is likely to be used in preference to across-the-board price-cutting. The reason is that price-cutting is a two-edged sword. It is just as effective defensively as offensively and can be applied in retaliation very quickly. Offensive advertising is similar to, and indeed complementary with, promotional temporary price-cutting. Because increased experimentation can be induced only in products and for qualities which are relatively quickly validated, the promotion campaign can be short-lived. Such campaigns, labelled "impulse advertising" in the marketing literature also have the advantage of being difficult to counter by retaliatory advertising or even pricing because of the time lead needed to prepare and implement an advertising campaign. Because such campaigns are generally more effective if the product possesses easily validatable qualitites, they are likely to occur when a firm is able to develop such properties in its product. Therefore, these campaigns are likely to occur in spurts accompanying the introduction of new products or new qualities of older products. Their incidence rises with the rate of technological progress and they are likely to be conducted by brands which account for a relatively small fraction of the market. In contrast, established brands are likely to engage in defensive advertising, which requires maintenance of relatively stable outlays overy long perjods. These are undertaken in order to protect established brands from potential competition, from established rivals, or from new entratns. These levels of advertising correspond to the steady state of our model of Appendix E.

An important element adding to the efficiency of defensive advertising arises from the limits of human perception indicated by psychological experiments in signal detection. [39/] These experiments indicate that the probability of signal detection increases non-linearly as the signal to noise ratio rises. Figure F-2 describes the general shape of the relation.

When the volume of a signal is equal to that of the noise, the probability of signal detection is zero. The signal is obscured by noise. As the volume of signal relative to the noise rises, the probability of detection increases slowly. At some point, (a) the marginal gain in detection from increased signal level rises rapidly, and then falls as the probability of detection approaches one. The marginal returns to increased signal level are first low and fairly stable, then rapidly increasing, and then decreasing again.

For our purposes, an obvious correspondence suggests itself. Offensive advertising which attempts to convey new information requires the detection and separation of such information from the mass of irrelevant messages transmitted in the market place. Its analysis is analogous to that of signal detection in the presence of noise. In fact, the non-linearities described above are exacerbated by the fact that consumer action on the basis of the detected signal is not assured. The consumer must not only detect the message, he must also believe in it. In the presence of conflicting claims from a variety of sources, consumers have learned to be skeptical. This may cause a shift to the right of the detection curve of Figure F-2, increasing the difficulties of market expansion or penetration by offensive advertising. By the same token, defensive advertising can be thought of as noise, the purpose of which is mostly to prevent additional information reaching the consumer and influencing him to change his established patterns of behaviour. Evaluating advertising in this fashion

FIGURE F-2



throws some light on the role and nature of advertising as a competitive tool.

Consider first the widely debated issue of returns to scale in advertising. Assuming a given level of defensive advertising, it appears that offensive advertising has first increasing returns and then decreasing returns in terms of perception. This is exactly the form which, in the absence of retaliation, yields both advantages of size over some range, together with eventual diminishing returns. According to Stigler [1968, Chapter 3], these are the conditions required to yield both barriers to entry, as well as monopoly returns. Increasing returns act as a barrier to entry, which enables firms to set price above cost. But decreasing returns at the higher advertising levels must occur to yield an upper limit on advertising and realise monopoly returns. Otherwise, non-price competition among existing brands eliminates all monopoly profits.

The element of barriers to entry is strongly reinforced by the nature of the detection curve. Defensive advertising can keep newcomers from reaching high marginal productivity of offensive advertising by maintaining a level of defensive advertising which keeps offensive to defensive advertising ratio at, or below, the point (a) in Figure F-2. At this point the marginal increment in defensive advertising needed to offset an increase in offensive advertising is low, conferring tremendous advantages on the established firms. It should be pointed out that the same effect operates to reduce the efficiency of offensive advertising by insiders. This reinforces our argument on the prevalance and role of defensive advertising on the part of established firms in oligopoly situations.

Note that because of the lead time required to raise advertising and because of the short intensive nature of offensive advertising, it is necessary to maintain some level of defensive advertising at all times to reduce the efficiency of offensive advertising and thereby, reduce the probability of such advertising. In this respect, because of the lag in response, defensive advertising, above the optimal level at steady state described in Appendix D and E, may be maintained by a monopolist as a means of preventing entry.

It is frequently argued by economists, e.g. Modigliani [1958] that monopolists or oligopolists will also maintain prices below the short-run profit maximizing level in order to discourage entry. This "limit pricing" appears at first glance similar to our analysis of defensive advertising. However, the similarity is misleading. The limit pricing theory is logically flawed because as prices can be adjusted rapidly, the threat of a price decrease with entry is enough to keep entry out. So there is no need to forego current profit to convince potential entrants of the threat. Defensive advertising, however, must be maintained at all times because changes in its level cannot be instituted in time to counter potential entry unless such entry is known well in advance.

V Population Turnover

So far we have restricted our analysis to stationary markets in which there is no significant customer turnover. The picture may change when account is taken of such turnover, in which some customers leave the market and new ignorant customers are added continuously. The rate of turnover and the way in which new customers acquire information has a profound effect on the model. Consider first the case where the product is sufficiently important or consumed within the family in such a way, that new potential buyers enter the market with well-defined narrow priors based on friends' or parents' experience. In this case, the stationary model analysed before is still valid, because new consumers are not very different from old consumers. Their distribution across brands will be close to that of older consumers, and their priors may be fairly similar as well, reflecting the priors of their sources of information.

Consider, however, the case where new consumers are relatively uninformed and where they cannot accept their elders' examples, either because information becomes quickly dated, or because their tastes are different than those of the older consumers. If turnover rates are high, established firms must advertise offensively to capture the new consumers. In this case, the established firm may not possess advantages relative to new firms because both must attract undecided new consumers via offensive advertising. Only if defensive advertising by existing firms has spillover effects in attracting new customers as well, do the established firms have an advantage relative to new ones. To the extent that turnover rates are high, it might pay established firms to modify the nature of their defensive advertising in the direction of higher specificity. However, this reduces the degree of substitutability between quality and defensive advertising and hence, causes the industry to become more competitive in product design and advertising, removing the advantages of defensive advertising. This policy will pay only as the advantage of size in advertising due to the non-linearity in perception is sufficiently high so that newcomers must operate at inefficiently low levels of advertising, limited by the small size of the uninformed population. It is clear that the higher is the rate of turnover of this population -- the lower is the advantage, as the fraction of the population which can be affected by offensive advertising rises as turnover rates rise. Economies of scale in production will have similar effects to economies of scale in advertising and hence, tend to reinforce them. At the extreme, where turnover rates are very high, the market may consist mostly of new undecided customers, such as is the case for "fad" items, goods consumed by a narrow age group -- e.g. teenagers -- or some infrequently purchased service goods which are subject to wide taste differences. In such markets, defensive advertising with the advantages it confers on existing firms vanishes. These markets are characterised by a greater degree of competition by way of offensive advertising or even price-cutting. Such competition is limited only by the economies of scale in production or advertising.

In some cases, it may be possible to differentiate the product and tailor the advertising in order to segment the market between old and new consumers. In this case defensive advertising coupled with the appropriate product characteristics is undertaken for established consumers, while offensive competitive advertising is aimed at the new uncommitted consumers. The old market segment is characterised by relative market stability, low advertising stressing relatively general qualities such as reliability, safety, etc. The new market segment is characterised by competition in the form of heavier advertising relative to sales and relative instability in advertising expenditure and market shares. Advertising and product characteristics take offensive forms with easily validatable claims and qualities. Such segmentation is particularly attractive to existing firms when rates of turnover are not very high and there exist economies of scale in production. Thus, existing firms enjoy an advantage in production which enables them to give up the advantage of scale in advertising to the new customers.

APPENDIX G*

COMPETITIVE MARKET EQUILIBRIUM WITH QUALITY UNCERTAINTY AND ADVERTISING

I. INTRODUCTION

Appendices C-E reported on our research on the dynamics of producer and consumer behaviour under conditions of uncertainty about quality for the case of a monopoly. Appendix F extended the analysis, albeit in a heuristic manner to oligopolistic markets. In this appendix we propose to extend the analysis to competitive markets. Such extension is important because traditionally competition is assumed to eliminate imperfections and lead to Pareto efficient markets. The term competitive is not to be confused with perfectly competitive markets which assume perfect information. Our model is therefore a dynamic analog of the monopolistic competition model, where a limited temporary degree of monopoly power is conferred upon each firm by limited knowledge on the part of consumers. Consumers are assumed to behave in accordance with the theory described in appendices C and F. They have difficulty in identifying quality by inspection and only learn gradually and imperfectly from experience. Therefore, in the presence of brands with different attributes they may make mistakes in brand selection. The main questions we attempt to answer are: will a market equilibrium exist, in which firms selling inferior quality goods survive? And if so, what are the determinants of the proportion of such firms? In particular, we shall focus on the role of advertising in this respect.

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* This appendix reports on work carried out mainly by Professor Krashinsky.

The market structures we describe are somewhat simplified versions of widely prevalent markets in the economy. In particular they cover those industries in which products are relatively unstandardised: e.g., housing whether rental or construction, repair services, professional services such as medicine or law, schools, etc.

Section II defines the terms used in the model and discusses the determinants of the behavioural parameters of consumers and producers. Section III then derives the steady state solution for the model and the conditions under which inferior quality goods will persist over time. Some numerical examples are pursued. Section IV examines the effects of advertising. Finally Section V attempts to give an overview to some of the work in this appendix.

II. THE MODEL OF PRODUCERS AND CONSUMERS UNDER QUALITY UNCERTAINTY

We will first introduce the general form of the model we shall use, in order to make the reader familiar with the terms we shall use. We will then examine briefly, the actions of consumers and producers in a market under the conditions of quality uncertainty.

The model we use is essentially a Markov chain model, in which actions in any period depend only on the value of variables determined in the previous period. We can use it to examine the conditions under which learning by consumers will not be sufficient to drive low quality firms from the market.

a) The General Form of the Model

For convenience, assume that there are only two levels of quality provided by firms: "good" quality and "bad" quality. The model may be extended to include a greater number of degrees of quality, but little of substance is added to the conclusions achieved in the simpler model. We may view quality differences as referring to differences in "quality per dollar". High quality firms produce goods that sell at a particular price and that are superior in quality to goods produced by low quality firms to sell at the same price. This assumption removes the need to consider price directly in this model.[30]

The model is discrete with respect to time, although the continuous model may be viewed as a limiting case. At this point, I will introduce the symbols of the model and characterize the steady state solution. For the purposes of definition, the key adjustment parameters are shown initially as constants. They are more properly seen later as functions whose value is determined by the behavioural characteristics of producers and consumers in the market as well as by the distribution of good and bad products for sale in the market at any particular time. This will permit a more meaningful discussion of the steady state.

Call X_t and Y_t the quantity of goods of good and bad quality, respectively, sold at the beginning of time period t, and set x_t and y_t equal to the proportion of good and bad quality output, respectively, at time t (thus $x_t = X_t/(X_t + Y_t)$ and $y_t = (1 - x_t)$. Assume that each consumer purchases exactly one unit of the good. X_t and Y_t thus represent both the numbers of goods offered for sale and the number of consumers purchasing those goods.[31]

We will define the terms which describe behaviour on both the demand and supply side of the market. First, consider the demand side. In a given time period (between t and $t+1$) a certain proportion $\beta (0 \leq \beta \leq 1)$ of all consumers leave the market altogether (they lose interest in the particular good, move away, etc.). For many goods this proportion may be fairly large. At the same time, some consumers will switch the firms that they patronize, in each case attempting to improve the quality of their purchase: call bY_t the net switch of consumers from bad quality firms.[32/] Alternately, we may call m the fraction of consumers of bad products that switch to the products of good firms, and n the fraction of consumers of the good products that mistakenly switch to the output of bad firms. In this case bY_t is identical to the expression $mY_t - nX_t$.

At this point in the process, new consumers enter the market. Suppose that they represent a fraction α of the previous total number of consumers at time t (that is, $X_t + Y_t$), and that the new consumers purchase good and bad quality goods in the proportions p and $1-p$ $(0 \leq p \leq 1)$. The terms b and p encompass the behaviour of consumers and will be made variable later.

Now, since X_{t+1} and Y_{t+1} are the numbers of goods sold at time $t+1$ after all consumers, new and old, have completed their purchases, then:

$$X_{t+1} = (1 - \beta)X_t + bY_t + \alpha(X_t + Y_t)p \qquad (G-1)$$

$$Y_{t+1} = (1 - \beta)Y_t - bY_t + \alpha(X_t + Y_t)(1-p) \qquad (G-2)$$

Adding, we obtain

$$X_{t+1} + Y_{t+1} = (1 - \beta + \alpha)(X_t + Y_t) \qquad (G-3)$$

Applying this result to equation (1), we may rewrite the relationship in terms of the proportions of good and bad goods purchased in each time period:

$$(1 - \beta + \alpha)x_{t+1} = (1 - \beta)x_t + by_t + p\alpha \qquad \text{(G-4)}$$

Substituting in $y_t = 1 - x_t$ and simplifying, this becomes

$$x_{t+1} = \frac{b + p\alpha}{1-\beta+\alpha} + \frac{1 - \beta - b}{1 - \beta + \alpha} x_t \qquad \text{(G-5)}$$

Alternately, this may be written as

$$x_{t+1} = \frac{m + p\alpha}{1 - \beta + \alpha} + \frac{1 - \beta - m - n}{1 - \beta + \alpha} x_t \qquad \text{(G-6)}$$

The steady state value of x (call it x^*), at which $x^* = x_t = x_{t+1}$, occurs if

$$x^* = \frac{b + p\alpha}{b + \alpha} \qquad \text{(G-7)}$$

This may be rewritten as

$$x^* = \frac{m + p\alpha}{m + n + \alpha} \qquad \text{(G-8)}$$

We will use b rather than m and n in our analysis.

Before continuing, it is worth examining the solution in equation (7). First, x^* will be less than one, and hence bad quality will persist in the steady state, if and only if α exceeds zero and p is less than one. Thus bad quality depends for its continuation upon the influx of new consumers who can be deceived as to quality. If there are no new consumers $(\alpha = 0)$, then $b > 0$ will ensure that progressive learning by experienced consumers must eventually eradicate bad quality. On the other hand, gullible new consumers will not be enough by themselves to ensure that bad quality remains. If there is no bad quality for sale, then p must be one (whatever consumer gullibility), guaranteeing $x^*=1$. And, as we shall see, bad quality will be for sale only if consumers are sufficiently gullible to support the activities of bad quality firms at an acceptable level of profit.

Also, it may be seen that

$$\frac{\partial x^*}{\partial b} = \frac{\alpha(1-p)}{(b+\alpha)^2} > 0 \qquad (\text{if } \alpha > 0,\ p < 1) \qquad \text{(G-9)}$$

$$\frac{\partial x^*}{\partial p} = \frac{\alpha}{b+\alpha} > 0 \qquad (\text{if } \alpha > 0) \qquad \text{(G-10)}$$

$$\frac{\partial x^*}{\partial \alpha} = \frac{b(1-p)}{(b+\alpha)^2} < 0 \qquad (\text{if } b > 0,\ p < 1) \qquad \text{(G-11)}$$

Equations (9) and (10) are straightforward -- increased perception of quality by either experienced consumers (a larger b) or new consumers (a larger p) will increase x^*, the proportion of the market controlled by good quality firms. Equation (10) states that if b and p are held constant, an increase in the rate of entry of new consumers will favour bad quality (reduce x^*). This occurs because of the fact that in the steady state, a greater proportion of experienced consumers than new consumers choose good quality.

Now turn to the supply side. In each time period, new firms spring up to compete for the new consumers as well as to attract those experienced consumers in the market looking for change. The new firms may enter providing either good or bad quality. Assume initially that all firms that sold output at the beginning of period t produce the same quantity of output of the same quality for sale at the beginning of period $t+1$.

Then call $\gamma_g(X_t + Y_t)$ and $\gamma_b(X_t + Y_t)$ the quantities of additional products of good and bad quality, respectively, produced for sale at the beginning of time $t+1$. The terms γ_g and γ_b represent the firms' behavioural parameters, determined partially by the conditions of the market. The new output may be the result of new firms entering the market or the expansion of old firms.

Now after consumers have chosen their goods, some of the goods produced may be left unsold. These unsold goods are assumed to disappear at the end of each period with a significant cost to their producers. Call $V_{x(t+1)}$ and $V_{y(t+1)}$ the unsold amounts of good and bad quality goods, respectively. Then

$$V_{x(t+1)} = X_t + \gamma_g(X_t + Y_t) - X_{t+1} \qquad \text{(G-12)}$$

$$V_{y(t+1)} = Y_t + \gamma_b(X_t + Y_t) - Y_{t+1} \qquad \text{(G-13)}$$

The assumption of the disappearance of unsold goods is again not crucial, although it does actually apply in most of the service sector. We might also consider the unsold goods to be retained at some cost in inventory, to reappear the next period through the terms γ_g and γ_b.

Now the terms p and b, which represent consumer behaviour and determine x^*, will depend upon the number and types of goods available to consumers, which depend upon the entry rates γ_g and γ_b, which will usually be determined by the sales success enjoyed by good and bad firms. And this success is expressed negatively in equations (12) and (13). It will be useful to express the quantities of unsold goods as a proportion of output:[33]

$$v_{x(t+1)} = \frac{V_{x(t+1)}}{X_t} \qquad \text{(G-14)}$$

$$v_{y(t+1)} = \frac{V_{y(t+1)}}{Y_t} \qquad \text{(G-15)}$$

The equilibrium in this model will be determined by the interaction of supply and demand. On the demand side, old and new customers are faced by goods of different quality in amounts determined by the entry rates γ_g and γ_b. The consumers then set p and b in ways discussed elsewhere. This in turn determines the unsold goods ratios $v_{x(t+1)}$ and $v_{y(t+1)}$. And these expectations are connected to the firms' estimates of consumer behaviour (in the terms b and p).

The model is definitional until the determination of the behavioural parameters in motivated. At that point, mechanisms to deal with definitional problems (ensuring that $v_{x(t+1)}$ be non-negative, for example) may be suggested. The model may also be made continuous without changing its conclusions.[34/]

b) Consumers and Uncertainty

In the model consumers are divided into two groups, old consumers and new consumers. Old consumers have purchased the commodity in the previous time period. New consumers enter the market with no previous experience. Clearly, however, there are markets in which experienced consumers leave and then re-enter several periods later. This may be included by regarding β and b as net shifts of experienced consumers. Alternately, the re-entrants may be considered as new consumers whose knowledge goes into the determination of the parameter p.

Together, the two groups of consumers define the environment into which firms consider entry. We shall examine the motivations of old and new consumers separately.

New Consumers: New consumers must choose from among the good arrayed before them in the market place. Call G_t and B_t the number of good and bad quality goods available to new consumers purchasing the commodity at the beginning of period t. Both G_t and B_t are fixed from the viewpoint of any individual consumer. If the consumers simply choose randomly, they would act so that $p = G_t/(G_t + B_t)$. However, we do assume that consumers will have some information and that more information will be acquired through search, implying that $p \geq G_t/(G_t + B_t)$. It will be convenient to set $p = (G_t + \Pi B_t)/(G_t + B_t)$, so that p will run from $G_t/(G_t + B_t)$ to 1 as Π is varied from 0 to 1.

G_t and B_t may be defined in a fashion appropriate to the particular market being examined. I shall examine two approaches. In the first, we assume that each firm produces one unit of output and that new consumers thus may purchase only from firms that have not sold their output to old consumers (who purchase before new consumers). Thus

$$G_{t+1} = \beta X_t - bY_t + \gamma_g(X_t + Y_t) \qquad \text{(G-16)}$$

$$B_{t+1} = \beta Y_t + bY_t + \gamma_b(X_t + Y_t) \qquad \text{(G-17)}$$

If we relax the assumption that each firm produces only one unit of output, then there will be more units of output per firm available to new consumers among bad firms than among good firms, which would tend to raise Π above zero even for consumers who choose randomly among brands. This will not be a concern if we then reduce the length of time of a period, so that few established firms, whether good or bad, ever have more than one unsold unit available for new consumers in any one period.

The second approach assumes that new consumers may purchase from any firm, irrespective of whether it has unsold inventory, by waiting for a period of time (or, we may allow firms in demand to expand as part of γ_g or γ_b). Thus

$$G_{t+1} = X_t + \gamma_g(X_t + Y_t) \qquad \text{(G-18)}$$

$$B_{t+1} = Y_t + \gamma_b(X_t + Y_t) \qquad \text{(G-19)}$$

The first alternative might be more appropriate to the apartment market, or the nursing home market (for example) where new consumers may only purchase where there are vacancies. The second alternative might better suit markets like hair-styling or the restaurant business where past patronage need not be a guarantee of a reservation.

Now consider the search behaviour of consumers. Suppose first that consumers know that the true distribution of good and bad goods available is $(g_0, 1-g_0)$ and that consumers can identify quality when they search. Let $U(Q, I)$ be the utility function of the consumer, where Q is the quality of the good purchased -- $Q = 1$ represents good quality and $Q = 0$ represents bad quality -- and I is the income remaining for the purchase of other goods. If the consumer initially comes across a bad quality product, his utility without further search will be

$$(EU)_1 = U(0, I_0) \qquad \text{(G-20)}$$

His expected utility after one additional search will be

$$(EU)_2 = g_0\, U(1, I_0 - C) + (1-g_0)U(0, I_0 - C) \qquad \text{(G-21)}$$

where C is the money cost of one search (monetizing time as well as any direct costs). The consumer will search if $(EU)_2 > (EU)_1$. If the marginal utility of income is constant (and equal to λ), we may write

$$(EU)_2 = g_0\, U(1, I_0) + (1-g_0)U(0, I_0) - \lambda C \qquad \text{(G-22)}$$

and the consumer will search if

$$g_0[U(1, I_0) - U(0, I_0)] > \lambda C \qquad \text{(G-23)}$$

This is the normal result that search will occur as long as the expected benefits exceed the expected search costs.

In this model, any consumer who decides search is worthwhile will continue to search until he finds good quality (unless the marginal utility of income or the costs of search change). The value of Π will be determined by the proportion of consumers who engage in search.

However, as was discussed in more detail elsewhere in this report, models of pre-purchase search can be much more complex when we relax the strict assumptions made above. Informational spillovers of search are one factor. In addition, consumers may only be able to evaluate the true quality of their initial purchase after several searches to view the alternatives. In this case, search will again depend upon g_0 and the potential gain in utility in moving from a bad firm to a good one. For example, a high value of g_0 will reduce search, since most consumers will choose an expected utility without search that is clost to $U(1, I_0)$ rather than sustain the cost of search.

Furthermore, with quality uncertainty, consumers may err in their judgements of quality. This will tend to reduce Π from that in our original discussion in two ways: fewer consumers will search, because the risk of error reduces the efficacy of search; and those consumers who do search will be less likely to correctly discover the goods with the highest quality.

It is not our aim at this point to discuss in too much detail the theories of pre-purchase search. These are discussed at some length elsewhere in this report and in the literature. Consumer search will determine the value of Π, which directly determines x^*. The value of Π is raised (which raises x^*) by reducing the cost of search and raising the perceived gains of successful search. The relationship between Π and g_0 is ambiguous. Finally, if consumers do not perceive the potential harm of low quality (if they are not aware of the existence of medical quacks, for example), search will be lower than it might otherwise be, and public programmes to alert consumers to the danger can raise Π substantially.

Old Consumers: The term b represents the additional knowledge acquired by consumers through experience. We have discussed this elsewhere in this report as post-purchase validation of prior beliefs about quality. Experience enables consumers to better evaluate the quality they are receiving (as compared to what is being sold by other firms) and detect any original errors in judgement. They will have time to compare the quality they are purchasing with that received by other consumers they meet. And, hopefully, they will have some better facility for judging other goods when they repurchase in the following period.

But if the old consumer has acquired more information about what makes for good quality, switching brands may also now involve costs that were not present for the new consumer. For many products, there are start-up costs whenever a consumer begins purchasing from a producer (for example, an auto repair shop takes time to become familiar with the particular problems of any given car, or a tenant purchasing rental housing makes friends among his neighbours and learns the locations of the local shops -- in the latter case, moving itself is a large cost). Thus search is easier, but the potential gains in switching from bad to good quality are less.

The term b now represents both the degree of consumer dissatisfaction with bad quality and the cost (both in terms of information search and of brand switching) of acting on that dissatisfaction. Although b will normally be non-negative, one can imagine the perverse situation in which elaborate advertising by low quality firms induces massive switches by the consumers of good quality.

We might expect b to depend strongly upon the value of x_t. Generally, we would expect $\frac{db}{dx_t}$ to be positive. An increase in x_t increases the odds that a particular consumer of bad quality will accidentally discover better quality (through the media, by talking to friends, etc.). In addition, a larger proportion of good firms makes search much more productive for consumers of bad quality.

The function b is increased (for any value of x_t) by a policy that provides information to consumers to help them evaluate the relative quality they are receiving. This is especially true for goods that normally are hard for the consumer to assess until well into the future (types of education, for example). And naturally b increases as the cost of search is lowered.

c) Firms and Uncertainty

Firms may enter the market providing either good or bad quality output. It is possible that in some environments, firms may not consciously choose their level of quality, but simply are attracted to the industry and produce quality that expresses their particular talents.35/

In this case, γ_g and γ_b would be constants determined by the scarcity of requisite talent. While we can analyze this case, it is more interesting to allow various quality firms to enter according to profitability.

We assume then that firms maximize profits and take as given consumer behaviour, about which they have perfect information. If all firms sold all their output, bad quality firms by definition, would earn higher rates of profit than good quality firms (since the motive for bad quality is lower costs to gain higher profits). But assuming that both these full sales profit rates exceed some normal market rates, then entry in any period will leave some goods of each quality level unsold and lower profit rates to the market rate. At this point, the marginal firm will be indifferent among entering as a good quality firm, entering as a bad quality firm, and staying out of the market.

Of course, we need to know how profit rates are affected by the unsold good rates v_{xt} and v_{yt}. Certainly, as v rises, profits will fall, but the exact relationship will depend upon the form of the cost function. We require only the inverse relationship between v and profits. Now in order to set the expected rates of return of all firms equal to some market rate, entry must be such that the expected unsold goods ratio for firms of a given quality must be equal to some constant. Call v_{x0} that constant unsold goods ratio for good firms and v_{y0} that rate for bad firms. It is not unreasonable to expect that $v_{y0} > v_{x0}$, since bad firms operate under a larger profit rate when all goods are sold.[36/]

It may be shown that for a given x_t, there exists at least one pair (γ_g, γ_b) that will generate the unsold goods rates v_{x0} and v_{y0} in good and bad firms, respectively. If any additional firm enters at that point, it will drive down the rate of return to all firms of that quality below the market rate.[37/]

For $v_{xt} = v_{x0}$ and $v_{yt} = v_{y0}$ at all points in time, firms must not only know what consumers will do, firms must also know how other firms will act (since γ_g and γ_b are set by numerous firms deciding upon entry, exit, expansion and contraction). We may stylize the process as follows. In each time period t, γ_g and γ_b are set initially. Firms then fortell the vacancy rates $v_{x(t+1)}$ and $v_{y(t+1)}$ that will result. If $v_{x(t+1)} > v_{x0}$, γ_g is lowered; if $v_{x(t+1)} < v_{x0}$, γ_g is raised. The same process occurs for γ_b. This stops when $v_{x(t+1)} = v_{x0}$ and $v_{y(t+1)} = v_{y0}$. Clearly, this is an idealization when the system is in transition, but it is not an unrealistic way to view the steady state.

We may expect that v_{x0} and v_{y0} represent only averages for firms. New elements may expect to have above average unsold goods ratios (since they initially have no old customers), to be compensated for later when the ratios drop. And v_{y0} may itself change; if consumers become more perceptive about quality, bad firms will find that there is less they can get away with (and still attract any customers). Finally, equilibrium at any point in time (that is, $v_{xt} = v_{x0}$ and $v_{yt} = v_{y0}$) may require that one or both entry rates (γ_g and γ_b) be negative. This would imply exit or contraction by existing firms. Of course, if $x_t = 1$, we cannot allow $\gamma_b < 0$.

III. THE STEADY STATE

a) The Existence of Equilibrium

The system that we have described is a Markov process, in which x_{t+1} depends ultimately only on x_t (and on the parameters of the model). This may be shown by bringing together our work above.

First, we described how b was a function only of the relative number of good products. Hence

$$b = b(x_t) \tag{G-24}$$

Then we described p as a function only of $G_{t+1}/(G_{t+1} + B_{t+1})$. Using equations (16) and (17), we see that

$$\frac{G_{t+1}}{G_{t+1} + B_{t+1}} = \frac{\gamma_g - b + (\beta + b)x_t}{\beta + \gamma_g + \gamma_b} \tag{G-25}$$

Or, if we use equations (18) and (19), we obtain

$$\frac{G_{t+1}}{G_{t+1} + B_{t+1}} = \frac{\gamma_g + x_t}{1 + \gamma_g + \gamma_b} \tag{G-26}$$

In either case, $G_{t+1}/(G_{t+1} + B_{t+1})$ is a function only of γ_g, γ_b, x_t and b, so we have

$$p = p(\gamma_g, \gamma_b, b, x_t) \tag{G-27}$$

Now the entry rates of firms are established so as to generate unsold goods ratios (v_{x0} and v_{y0}) that result in market rates of profit. From equations (14), (12) and (1) we can obtain

$$v_{xt+1} = \beta + b + \frac{\gamma_g - \alpha p - b}{x_t} \tag{G-28}$$

Setting this equal to v_{x0}, we see that γ_g is a function only of b, p, and x_t. Similarly, using equations (15), (13) and (2) we can obtain

$$v_{yt+1} = \beta + b + \frac{\gamma_b - \alpha(1-p)}{1 - x_t} \tag{G-29}$$

Again, by setting this equal to v_{y0}, we obtain γ_b as a function only of b, p and x_t and, of course, the constant parameters β and α). We have

$$\gamma_g = \gamma_g(b, p, x_t) \tag{G-30}$$

$$\gamma_b = \gamma_b(b, p, x_t) \tag{G-31}$$

Now equations (24), (27), (30) and (31) are a system that solves to yield b, p, γ_g and γ_b in terms only of x_t. But this tells us that equation (5) can be reduced to expressing x_{t+1} only as a function of x_t. This Markov process maps the unit interval into the unit interval. If all our functions are continuous, this process will have at least one steady state (using the fixed point theorem). At least one steady state will be stable if time is made continuous.

Several points are worth noting. First, in the discrete time Markov model, the process need not converge, but can oscillate about a steady state. Second, more than one point can be stable, even in this simple model. Of course, one of these stable points would result in higher overall quality than the others, even though at all points, every firm's expected rate of return conforms to some market average.

If we relax the assumption that firms may perfectly judge the actions of consumers and other firms, we complicate the model. In this case γ_g and γ_b would just be estimates of the "correct" entry rates (the rates that generate the market return) and a certain randomness would be injected into the adjustment mechanism. It would still over time move close to the equilibrium derived above.

b) The Derivation of the Steady State

Define $p = (G_{t+1} + \Pi B_{t+1})/(G_{t+1} + B_{t+1})$. Using equations (G-16) and (G-17) to define G_{t+1} and B_{t+1} (the first approach), we may write

$$1-p = \frac{(1 - \Pi)[\gamma_b + (\beta + b)(1 - x_t)]}{\beta + \gamma_g + \gamma_b} \qquad \text{(G-32)}$$

If we now set $v_{xt+1} = v_{x0}$ and $v_{yt+1} = v_{y0}$ in equations (G-28) and (G-29) we may now write

$$x_t\, v_{x0} = \beta x_t - b(1 - x_t) + \gamma_g - p\alpha \qquad \text{(G-33)}$$

$$(1 - x_t)v_{y0} = (\beta + b)(1 - x_t) + \gamma_b - (1-p)\alpha \qquad \text{(G-34)}$$

We can solve equations (32), (33) and (34) to eliminate γ_g and γ_b and write

$$p = \frac{x_t\, v_{x0} + \Pi(1 - x_t)v_{y0} + \Pi\alpha}{x_t\, v_{x0} + (1 - x_t)v_{y0} + \Pi\alpha} \qquad \text{(G-35)}$$

Substitute this expression into the equation (7) for the steady state and solve, we obtain

$$x^* = \frac{(b + \alpha)\alpha\Pi + (b + \alpha\Pi)v_{y0}}{(b + \alpha)(v_{y0} - v_{x0})} \qquad \text{(G-36)}$$

If this equation yields $x^* > 1$, set $x^* = 1$.

Equation (36) is still an implicit equation for x^* , as we have not specified the form of the function b (in fact Π may also vary with x). The reader may imagine b to be constant in the range of x_t close to x^* .

Some values for x^* are calculated in Table 1. It is seen that when information is poor, bad firms will continue to survive. But bad firms may be driven out of business when information is somewhat less than perfect. Bad firms disappear as long as "enough" customers can recognize their true quality ("enough", of course, depends on the relative profitability of being bad). The condition for $x^* = 1$ is

$$(\alpha + b)(\alpha\Pi + v_{x0}) \geq \alpha\, v_{y0}(1 - \Pi) \qquad \text{(G-37)}$$

However, as most bad firms disappear, $G_t/(G_t + B_t)$ becomes close to one, and Π may well fall (since consumers have less incentive to search). This makes condition (37) less likely, though it may still hold.

By differentiating equation (36), we may show that

$$\frac{\partial x^*}{\partial b} = \frac{v_{y0}\, \alpha(1 - \Pi)}{(\alpha + b)^2\,(v_{y0} - v_{x0})} \geq 0 \qquad \text{(G-38)}$$

$$\frac{\partial x^*}{\partial \Pi} = \frac{\alpha(v_{y0} + \alpha + b)}{(\alpha + b)(v_{y0} - v_{x0})} \geq 0 \qquad \text{(G-39)}$$

$$\frac{\partial x^*}{\partial v_{x0}} = \frac{b v_{y0} + \alpha\Pi(v_{y0} + \alpha + b)}{(\alpha + b)(v_{y0} - v_{x0})^2} \geq 0 \qquad \text{(G-40)}$$

$$\frac{\partial x^*}{\partial v_{y0}} = \frac{-(b + \alpha\Pi)v_{x0} - \alpha\Pi(\alpha + b)}{(\alpha + b)(v_{y0} - v_{x0})^2} \leq 0 \qquad \text{(G-41)}$$

This shows what we expect: that rises in b or Π (more consumer information) will increase x^*. A rise in v_{y0}, which allows bad firms to

Table I

Values of x^* for varying α, v_{y0}, Π and b; v_{x0} set at 0.05

		$v_{yo} = 0.10$				$v_{yo} = 0.15$				$v_{yo} = 0.20$				$v_{yo} = 0.25$			
		$v_{xo} = 0.05$				$v_{xo} = 0.05$				$v_{xo} = 0.05$				$v_{xo} = 0.05$			
	b→	.02	.06	.25	.75	.02	.06	.25	.75	.02	.06	.27	.75	.02	.06	.25	.75
	π ↓																
$\alpha = 0.1$	.00	.333	.750	1	1	.250	.563	1	1	.222	.500	.952	1	.208	.469	.893	1
	.05	.517	.913	1	1	.363	.659	1	1	.311	.575	1	1	.285	.533	.936	1
	.10	.700	1	1	1	.475	.756	1	1	.400	.650	1	1	.363	.597	.979	1
	.25	1	1	1	1	.813	1	1	1	.667	.875	1	1	.594	.789	1	1
	.50	1	1	1	1	1	1	1	1	1	1	1	1	.979	1	1	1
	.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	π ↓																
$\alpha = 0.2$	.00	.182	.462	1	1	.136	.346	.833	1	.121	.308	.741	1	.114	.289	.694	.987
	.05	.473	.739	1	1	.305	.504	.967	1	.249	.426	.837	1	.221	.387	.772	1
	.10	.764	1	1	1	.473	.662	1	1	.376	.544	.933	1	.327	.485	.850	1
	.25	1	1	1	1	.977	1	1	1	.758	.897	1	1	.648	.779	1	1
	.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	π ↓																
$\alpha = 0.3$	.00	.125	.333	.909	1	.094	.250	.682	1	.083	.222	.606	.952	.078	.208	.568	.893
	.05	.519	.717	1	1	.314	.463	.873	1	.246	.378	.742	1	.212	.335	.677	.986
	.10	.913	1	1	1	.534	.675	1	1	.408	.533	.879	1	.345	.463	.786	1
	.25	1	1	1	1	1	1	1	1	.896	1	1	1	.746	.844	1	1
	.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	π ↓																
$\alpha = 0.5$	.00	.077	.214	.667	1	.058	.161	.500	1	.051	.143	.444	.800	.048	.134	.417	.750
	.05	.673	.804	1	1	.380	.478	.800	1	.282	.369	.656	.993	.233	.315	.583	.900
	.10	1	1	1	1	.702	.795	1	1	.513	.595	.867	1	.418	.496	.750	1
	.25	1	1	1	1	1	1	1	1	1	1	1	1	.974	1	1	1
	.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Symbols defined in the text.

tolerate higher levels of "non-patronage" (higher unsold goods ratios), will cause more bad firms to enter and lower x^* . And a rise in v_{x0} , which makes good firms more robust, will raise x^* .

It is also possible that all good firms will be eliminated, setting $x^* = 0$. This does not occur in Table 1, because we keep b constant, while in fact, as x_t falls we may expect b to decline (because there are fewer good products to discover, even accidentally).

x^* will be zero if $x_{t+1} < x_t$ for all $x_t > 0$. It may be shown, using equations (5) and (35), that $x_{t+1} < x_t$ if and only if

$$x_t(b + \alpha)(v_{y0} - v_{x0}) > (b + \alpha)\Pi\alpha + v_{y0}(\Pi\alpha + b) \qquad \text{(G-42)}$$

If Π and b are quite small to begin with, and decline as x_t approaches zero, then equation (42) may well hold.

We may also re-derive the equation for x^* using the second approach to define G_t and B_t . Applying equations (18) and (19), we can show that

$$x^* = \frac{(b + \alpha\Pi)(1 + \alpha - \beta + v_{y0})}{(b + \alpha)(v_{y0} - v_{x0})} \qquad \text{(G-43)}$$

This expression is uniformly larger than the x^* derived in equation (36). Otherwise, it has similar properties. Knowledge need not be perfect to eliminate all bad firms, but when information is bad enough (b and Π very low), bad firms will continue to exist.

c) A Sample Path to the Steady State

It may be interesting to see how the steady state is reached when we begin with $x_{t=1} \neq x^*$. Set $\alpha = 0.2$, $\beta = 0.2$, $\Pi = b = 0.1$,

Table II

Sample Path to Steady State: $\alpha = \beta = 0.2$, $\pi = b = 0.1$, $v_{yo} = 0.15$, and $v_{xo} = 0.05$ ($x^* = 0.8$, $x_1 = 0.4$)

t	1	2	3	4	5	6	7	8
x_t	.4000	.4554	.5006	.5380	.5693	.5956	.6181	.6373

t	9	10	15	20	25	30	40	50
x_t	.6450	.6685	.7188	.7474	.7649	.7761	.7886	.7944

t	60	70	80	90	100
x_t	.7972	.7986	.7993	.7997	.7998

$v_{y0} = 0.15$, $v_{x0} = 0.05$. Use equation (36), which yields $x^* = 0.8$. Set $x_1 = 0.4$. It may be shown that in this particular case

$$x_{t+1} = \frac{0.24 + x_t(1.16 - 0.7\,x_t)}{1.7 - x_t} \qquad \text{(G-44)}$$

Some values of x_t are shown in Table II. In this case x_t is 75% of the way to steady state after twelve time periods and 95% of the way after thirty-two time periods.

IV. ADVERTISING

Thus far in the model, firms have not engaged specifically in advertising. Good quality firms simply produce the highest possible quality per dollar and then wait for consumers to arrive. Bad quality firms produce a lower quality per dollar and can tolerate a higher ratio of unsold goods. We have suggested that bad firms may attempt to fool consumers by producing lower quality goods disguised to resemble higher quality goods. The degree to which quality may be eroded without becoming obvious to consumers will determine the size of the gap between v_{y0} and v_{x0} : the more bad firms can lower quality per dollar, the higher will be v_{y0} , the tolerable level of unsold goods.

One active way for all firms to promote their goods is through advertising. While this can enter the model implicitly through p , v_{x0} and v_{y0} (advertising by good firms raises p and lowers v_{x0} ; by bad firms lowers p and lowers v_{y0}), we prefer to examine it as a separate issue in a later section. To do so, we call A_G and A_B the advertising measured in dollars done by individual good and bad firms (assuming that in the steady state all firms of one quality advertise to the same extent).

Advertising is assumed to have no effect on old consumers. New consumers are influenced to visit early in the search procedure, the particular firm whose advertisement they notice. Once at the firm, they purchase or continue search dependent upon the factors discussed earlier (the cost of continued search, perceived relative quality at the initial firm, etc.). If we set $\bar{p} = (G_t + \Pi B_t)/(G_t + B_t)$, this may be added by writing $\Pi = \Pi_0 + f(A_G, A_B)$, where Π_0 is value of Π in the absence of advertising, and f is a function of advertising defined such that $f = 0$ if $A_G = A_B$, and $\frac{\partial f}{\partial A_G} > 0$, $\frac{\partial f}{\partial A_B} < 0$ for all values of A_G and A_B.

Advertising will also reduce the tolerable levels of unsold goods. This may be added by writing $v_{x0} = \bar{v}_{x0} - C_G(A_G)$ and $v_{y0} = \bar{v}_{y0} - C_B(A_B)$, where $\bar{v}_{x0}$ and $\bar{v}_{y0}$ are the tolerable levels of unsold goods ratios for good and bad firms when advertising expenditures are zero, and C_G and C_B are functions of dollar advertising costs, showing the effects of increased advertising in lowering the unsold goods ratios that firms can tolerate. C'_G and C'_B are the first derivatives of the respective functions and are assumed to be positive, while $C_G(0) = C_B(0) = 0$. If all firms have identical loss functions for unsold goods, then $C'_G = C'_B =$ constant.

If good firms each spend A_G on advertising and bad firms each spend A_B on advertising, then we may rewrite equation (G-36) for the steady state as

$$x^* = \frac{(b+\alpha)\alpha(\pi_o+f) + [b+\alpha(\pi_o+f)][\bar{v}_{yo}-C_B(A_B)]}{(b+\alpha)[\bar{v}_{yo}-\bar{v}_{xo}-C_B(A_B) + C_G(A_G)]} \qquad \text{(G-45)}$$

In order that this be a steady stae, no firm must have an incentive to increase advertising. If an individual firm increases its advertising expenditures above that of other firms in its quality class, the high advertising firm will reduce its expected unsold goods ratio below that of the other firms in its quality class. If the ith good firm and the jth bad firm increase advertising expenditures by ΔA_G and ΔA_B above A_G and A_B, respectively, then $v^i_{x(t+1)}$ and $v^j_{y(t+1)}$ fall by $g(\Delta A_G, A_G, A_B)$ and $h(\Delta A_B, A_B, A_G)$, respectively, where $v^i_{x(t+1)}$ and $v^j_{y(t+1)}$ are the unsold goods ratios of the ith good firm and jth bad firm. Define g and h such that $g(0, A_G, A_B) = h(0, A_G, A_B) = 0$; $h_1, g_1 > 0$; $h_2, g_2 < 0$; $h_3, g_3 < 0$ where g and h define the effects of advertising on sales of individual good and bad firms respectively. g_1, h_1, define the marginal effectiveness of own advertising and g_2, h_2, g_3 and h_3 define the effects of increased advertising by the respective class (including the firm in question) on the sales of the individual.

For the ith good firm, we know that

$$v^i_{x(t+1)} = \beta + b + \frac{\gamma_g - \alpha p - b}{x_t} - g(\Delta A_G, A_G, A_B) \qquad \text{(G-46)}$$

and for the jth bad firm, we know that

$$v^j_{y(t+1)} = \beta + b + \frac{\gamma_b - \alpha(1+p)}{1+x_t} - h(\Delta A_B, A_B, A_G) \qquad \text{(G-47)}$$

Now for the ith good firm, the tolerable level of unsold goods will be

$$v^i_{xo} = \bar{v}_{xo} - C_G(A_G, \Delta A_G) \tag{G-48}$$

and for the jth bad firm, tolerable unsold goods will be

$$v^j_{yo} = \bar{v}_{yo} - C_B(A_B, \Delta A_B) \tag{G-49}$$

Given specific levels of advertising A_G and A_B, the firm entry rates γ_g and γ_b adjust so as to produce an equilibrium x^* given in equation (G-45). This is a steady state if every individual firm then chooses not to increase or decrease its advertising. The first order conditions for this to occur are

$$\frac{\partial v^i_{x(t+1)}}{\partial(\Delta A_G)} = \frac{\partial v^i_{xo}}{\partial(\Delta A_G)} \tag{G-50}$$

and

$$\frac{\partial v^i_{y(t+1)}}{\partial(\Delta A_B)} = \frac{\partial v^i_{yo}}{\partial(\Delta A_B)} \tag{G-51}$$

This occurs when

$$g_1 = c^1_G \text{ if } V_{xo} \geq 0 \text{ , or set } V_{xo} = 0 \text{ if } g > c^1_G \text{ for } \tag{G-52}$$

[illegible]

and

$$h_1 = c^1_B \text{ if } V_{yo} \geq 0 \text{ , or set } V_{yo} = 0 \text{ if } h > c^1_B \text{ for } \tag{G-53}$$

where g_1 and h_1 are equal to $\frac{\partial g}{\partial(\Delta A_G)}$ and $\frac{\partial h}{\partial(\Delta A_B)}$ respectively.

Equations (G-52) and (G-53) are what we would expect. C_G^1 and C_B^1 represent the marginal cost of one more unit of advertising to the firm in question, where we measure marginal cost in terms of the decrease of the unsold goods ratio needed to pay for the advertising. g_1 and h_1 represent the marginal benefits to the firm of one more unit of advertising, where we measure marginal benefits in terms of the fall in unsold goods engendered by more advertising. If the vacancy rate drops to zero before this marginal equality is achieved, advertising will be stopped at that point short of the marginal equality becuase no negative vacancies are allowed.

We are interested in the relative sizes of A_G and A_B, because they determine the distribution of the steady state market shares of good and bad firms. Assume first that advertising is equally effective for both types of firms because new consumers cannot distinguish quality upon inspection. Therefore $g_1 = h_1$ for the same levels of A_G and A_B. Therefore advertising will be the same for both types of firms if $C_G^1 = C_B^1$ and if the marginal relations (equations (G-52), (G-53)) hold with strict equality.

It is possible to argue that $C_G^1 = C_B^1$; that is, that the cost of an extra dollar of advertising, in terms of the required reduction in unsold goods, is identical for good and bad firms. If unsold goods cannot be stored and disappear at no cost (e.g., vacancies in schools), then the marginal cost (i.e., the alternative cost) of these goods is zero, regardless of their quality or cost of production, i.e. $C_B^1 = C_G^1 = {}^1/P$ (P = output price).

However, if the unsold goods do not disappear, then the alternative cost of an unsold good is the value of this good in the next period, i.e., the cost saving of not producing the good in the next period less the storage and deterioration costs that occur between periods. If storage costs are zero for good and bad firms, the cost of unsold goods is equal to the discounted present value of the cost of producing the goods in the next period. Clearly these costs are higher for the high quality good than for the low quality one. These alternative costs must be added to the direct costs of advertising in calculating C. Therefore $C_G^1 > C_B^1$. Under these conditions bad firms will find it more advantageous to advertise than good firms.

A further reason why bad firms are likely to advertise more under these circumstances is because $\bar{V}_{xo} < \bar{V}_{yo}$, therefore if the returns to advertising do not fall rapidly, good firms may reach V_{xo} before the marginal equality of equation (G-52) is reached. In this case even if $h = g$ and $C_B^1 = C_G^1$, bad firms will advertise more.

We must also consider g_1 and h_1, the relative effectiveness of $1 in advertising in reducing the actual unsold goods of good and bad firms. One way to view advertising is as purely informative. It simply informs consumers of the location, hours, prices, etc., of the firm. Assume that this advertising succeeds in attracting a uniform number of extra potential consumers (per dollar of additional advertising) during the search period. Since more of these extra searchers would actually buy high quality output than would buy low quality output ($\pi_o>0$), we might conclude that $g_1>h_1$ for $A_G=A_B$. This effect operates to increase the advertising of good firms relative to bad ones.

Therefore for purely disseminative advertising, these are conflicting effects. The higher productivity of advertising of good firms may be offset by the lower cost of goods sold (i.e., higher profit margins) of the bad firms. If, however, advertising is relatively effective, it is likely that bad firms will advertise more because $V_{xo}=0$ for relatively small amounts of good firm advertising.

The case for greater advertising by bad firms becomes stronger when persuasive advertising is considered. Because bad firms sell the illusion of quality, they may be just as effective in advertising as good firms, leading to greater advertising.

In any case, the effects of increased advertising by good or bad firms on the proportion of good and bad quality goods sold in steady state (x^*) is not determinate. Thus consider an increase in advertising by good firms due to an increase in g. This will raise the numerator of equation 45 because the proportion of new customers attracted to good firms rises. However, it will also increase the denominator, because good firms' advertising expenses rose, so that each must have a lower vacancy rate. The lower vacancy rate causes less new good firm entry, lowering the share of good quality goods on the market.

V. Some Conclusions about the Model

The model suggests that while a small amount of consumer uncertainty will not allow bad firms to exist, too much uncertainty will allow bad firms to exist over time in the steady state. It is interesting that in the steady state, the bad firms do not earn profits any different than those earned by good firms. Quality uncertainty represents a real inefficiency in this sense, since no firm would be worse off if we set $\gamma_b = 0$ and $x^* = 1$, while consumers would clearly be better off. Such a solution is, however, unstable, as there exists the incentive for bad firms to enter the market.

It is also possible that the steady state solution may change over time. If consumer information increases as public experience in the market develops, then Π and b will increase and x^* will rise. For many new markets, the introduction of a product to the public results at first in a large number of "fast buck" operators (bad quality firms). Consumer sophistication increases (in part through the media), and the market develops into a safer one for consumers.

Where quality uncertainty results in $x^* < 1$, governmental intervention can take various forms. One alternative is to attempt to increase information flows within the market. Publicity about the potential danger in low quality firms can increase public awareness and consumer search (raising b and p and thus increasing x^*).

The government may choose to intervene more directly in the control of output sold on the market. Licensing and regulation of firms may attempt to keep low quality firms at a minimum. These methods have met with mixed success in the past. Regulatory agencies may serve to protect the industry rather than the consumer. Perhaps more important, in many cases of quality uncertainty, the market is itself ill-suited to regulation. The very factors that make consumer evaluation difficult frequently will make it impossible to specify a set of written regulations that ensure high quality.

There are also methods that the private market itself has evolves for handling quality uncertainty. Warranties offer the consumer protection against bad quality. Department stores suggest that they are acting as agents for the consumer in testing out products that they carry. And brand names represent a claim by a producer that his reputation is at stake on a wide variety of products (so that shoddy merchandise in one market will lose him business in all his other markets).

These methods do work quite well in many markets, but they cannot eliminate quality uncertainty. Much quality uncertainty occurs in the service sector, where quality depends highly on local personnel. Franchising is not effective when quality control for the franchising agent are too costly.

We can best conclude this section by recapitulating some of the conditions that will increase quality uncertainty and reduce x^*. Quality uncertainty is high when consumers cannot evaluate the quality of the goods they purchase, either before they but, or even after they have consumed the product (where bad quality in a car repair manifests itself in a higher likelihood of accident). Quality uncertainty is effective when there are frequent changes of consumers (new consumers continually entering), when the costs of brand switching are high, and when there are long time intervals between purchases. We believe that uncertainty is more of a problem in evolving markets and in the service sector.

FOOTNOTES

1/ See Arrow, K.J. [1971]. H. Demsetz [1969] disagrees with the policy implications of Arrow's analysis even where markets fail as argued.

2/ Examples of such propositions are:

(1) There will be less search and greater equilibrium price variability for goods taking a small part of the budget.

(2) If the cost of search is proportional to income, the rich will search more and pay less for goods, with income elasticity greater than one.

3/ There is strong evidence of serious misperceptions of price rankings of supermarkets. See e.g. Brown [1971].

4/ More formally, this rule may be written in the case of price search as:

$$\int_0^\infty U_1 \left[X_k \frac{\partial \phi_k}{\partial V_k} + \sum_{j \neq k} X_j \frac{\partial \phi j}{\partial V_k} + z \frac{X_k^2 \sigma_k \frac{\partial \sigma k}{\partial V_k}}{(\sigma(\tilde{Z}))^{1/2}}\right] f(z) dz = 1$$

where X_k is the commodity whose price is being searched.

X_1 a good whose price, $P_1 = 1$ is known.

P_j is the price of X_j, a random variable, $j \geq Z$, distributed normally.

FOOTNOTES
(continued)

4/ (continued)

$$E(\tilde{P}_j) \equiv \phi_j(ECP_j), V_2, V_3, \ldots, V_n)$$

$V_j \equiv$ dollars of expenditure on sample S good.

$X_j (j \geq Z)$ to discover the lowest price of X_j .

$$\tilde{Z}_j \equiv \sum_{j=z}^{N} \tilde{P}_j X_j \quad j \quad E(\tilde{Z}_j) \equiv \sum_{j=z}^{N} X_j E(\tilde{P}_j)$$

$$\sigma(\tilde{Z}) \equiv \Big[\sum_{j=z}^{N} X_j^2 \, \sigma_j^2 + \sum_{i}^{N} \sum_{j=i}^{N} X_i X_j \sigma_{ij} \Big]^{1/2}$$

σ_j^2 is the variance on sampled prices for X_j .

$$\sigma_j \equiv \sigma_j(\sigma_{Pj}, V_j)$$

σ_{Pj} is the variance on actual prices for X_j

$$\sigma_{ij} \equiv \sigma_{ij} \, (\sigma_{PiPj})$$

σ_{PiPj} is the covariance of actual prices.

$z \equiv \dfrac{\tilde{Z} - E(\tilde{Z})}{\sigma(\tilde{Z})}$, a standard normal variable.

$f(z)$ is the density function of z .

$$U = U[Y - \sum_{j=z}^{N} X_j \phi_j - z\sigma(\tilde{Z}) - \sum_{j=z}^{N} V_j, X_z, \ldots, X_N)$$

5/ This approach is similar to the adaptive learning models that appear in the marketing literature. For example, see G. Hains [1969] and R. Bush and F. Mosteller [1955]. However, all of these exclude the element of risk.

6/ Drugs, physicians' services, durable repairs are examples of such commodity markets.

7/ A more realistic assumtion is that the consumer is uncertain about the quality variance as well. This does not pose insurmountable problems in the analysis of consumer behaviour. However the analysis of producer behaviour becomes too complex.

8/ For a detailed analysis and evaluation of these problems see Muellbauer [1975]. As shown by Muellbauer, the empirical measurement of quality by the characteristics approach involves very serious difficulties. We have therefore chosen the Fisher-Shell approach.

9/ This phenomenon will be familiar to anyone who has graded any tests.

10/ A negative prior on quality $\bar{Q} < 0$ will, of course, insure that the consumer will not even consider buying the product.

11/ The results are very robust to violations of this assumption. See Schlaifer [1961] p. 309.

12/ Darby and Karni [1973] coined the term "credence goods" for products of this nature where information and product are sold together as a package because the joint costs of producing both are considerably lower than the sum of the costs of producing them separately. They focus their analysis on services where diagnosis and service are rendered jointly such as in cases of medicine, and repair services. They show that the incentive to sell unnecessary services or to generally defraud customers is particularly high in such circumstances. Their analysis is therefore a special case of our general model.

13/ Psychological research suggests that individuals tend to exhibit cognitive dissonance, i.e. that experience tends to confirm prior belief more often than the objective evidence would suggest. The bias of measurement is therefore likely to be in the direction of the prior mean $(\bar{Q})$. This bias is likely to be greater for consumers who underestimate the variance of their measurement error(s).

14/ For proof see Raiffa and Schlaifer [1961] p. 295.

15/ See e.g. Muth [1965], Hains [1969].

16/ See e.g. Comanor and Wilson [1974], Stigler [1961], Haynes [1969].

17/ Note that such neutral improvement in the quality of all goods is simply equivalent to an increase in real income, which is easily handled in the model.

18/ See e.g. Galbraith [1967], Kaldor [1950].

19/ See e.g. Johnson [1964], Stigler [1961].

20/ For futile efforts in this direction see e.g. Kaldor [1950].

21/ Examples are movie or book critics. Note that negative correlation is as good as positive as a source of information.

22/ See e.g. Hains [1969], Montgomery and Urban [1969], Coleman [1964]. Note that the efficiency of sales effort for new products depends on the sellers' ability to identify and persuade these opinion leaders. This gives rise to the common practice of "endorsement" by known personalities.

23/ The effect of $g(a)$ is similar to an effect of advertising on s. Note that in common with a change in s the effects of advertising are not corrected by experience.

24/ The exact incentives of producers to supply false information will be analysed later.

25/ For a discussion of this point as well as an attempt to estimate it quantitatively see Boyer [1974]. The common distinction is between "persuasive" (or "good-will" in Boyer's terminology) and "informative" advertising. Empirically the distinction is made according to media. Newspaper advertising is considered informative and radio-T.V. advertising "persuasive". As pointed out later, this distinction is not satisfactory, because the same advertisement provides both functions in all cases.

26/ See mathematical appendix to Appendix D for detail.

27/ It is convenient to set the problem in terms of N- the number of purchasers rather than in terms of price, although of course price is the policy variable of the firm. Note that $R_{N\bar{x}} = P(1+1/\eta)$, where η is the price elasticity of demand.

28/ See mathematical appendix to Appendix E for detail.

29/ This point is not generally understood. For an inappropriate use, see e.g. Spence[1974].

30/ Various characteristics of consumer search may make it easier to sell low quality in one price range than another. For the purposes of this model we will assume that the distinctions between good and bad quality are similar at all prices.

31/ This assumption serves to simplify the discussion that follows without compromising the results. This enables us to discuss learning behaviour by consumers without explicit concern for the number of items consumed by any single individual. In fact, that datum could be incorporated into the determination of the behavioural parameters of the model.

32/ Normally b is positive and less than $(1-\beta)$. Under strange circumstances, b may be negative, in which case $-bY_t$ cannot exceed $(1-\beta)X_t$.

33/ It might seem more natural to express $v_{x(t+1)}$ as the ratio of unsold goods to all goods produced for sale, so that $v_{x(t+1)} = V_{x(t+1)}/\{X_t + \gamma_g(X_t + Y_t)\}$. This makes the algebra less understandable, so we use the version in the text.

34/ X and Y become instantaneous functions of t, and the parameters p, b, β, α, γ_g and γ_b become rates of change. Then we may write

$$dX = [-\beta X + bY + p\alpha(X + Y)]dt$$

and $dY = [-\beta Y - bY + (1-p)\alpha(X + Y)]dt$. Since $x = X/(X + Y)$, we can derive $dx = \frac{YdX - XdY}{(X + Y)^2}$. By substitution, this yields

$$\frac{dx}{dt} = b + p\alpha - (b + \alpha)x$$

35/ This would be the charitable view of the medical profession, for example. Lower quality medical care in this view is the result of the lesser competence of some doctors, not a conscious attempt to defraud patients.

36/ In the case of many service industries, unsold products may not be stored, and reduced sales do not significantly cut production costs (the barber incurs virtually the same costs if his shop sells no haircuts or operates at capacity). Call AC average cost and P the price per unit. Then the zero profit condition will be $P(1-v_o) = AC$, or $v = 1 - \frac{AC}{P}$. Since $\frac{AC}{P}$ is lower for bad firms, v_{yo} will be higher.

37/ Given X_t and Y_t, setting $v_{x(t+1)}$ and $v_{y(t+1)}$ equal to v_{xo} and v_{yo}, respectively, is equivalent to setting $V_{x(t+1)}$ and $V_{y(t+1)}$ (since $V_{xt} = X_t\, v_{xo}$ and $V_{yt} = Y_t\, v_{yo}$). Now we know that

$$V_{x(t+1)} = \beta X_t - bY_t + (\gamma_g - p\alpha)(X_t + Y_t)$$

$$V_{y(t+1)} = \beta Y_t + bY_t + \{\gamma_b - (1-p)\alpha\}(X_t + Y_t)$$

Therefore $V_{x(t+1)} + V_{y(t+1)} = (\beta + \gamma_g + \gamma_b - \alpha)(X_t + Y_t)$

and we can write $\gamma_g + \gamma_b = \dfrac{V_{x(t+1)} + V_{y(t+1)}}{X_t + Y_t} + \alpha - \beta$

so that $\gamma_g + \gamma_b = \text{constant} = K$

Now the first equation above shows that $V_{x(t+1)}$ is a function of constants $(\beta, X_t, Y_t, b, \alpha)$ and p and γ_g. But p is a function of G_{t+1} and B_{t+1} which in turn depend only on constants and γ_g (and $\gamma_b = \text{constant} - \gamma_g$). Thus in this context, $V_{x(t+1)}$ depends only on γ_g, and we want to find γ_g so that $v_{x(t+1)} = X_t v_{xo}$. Now, when $\gamma_g = by_t - \beta x_t$ (which is frequently negative), $G_{t+1} = 0$, and there will be no unsold goods produced by good firms. Somewhere in between $\gamma_g = by_t - \beta x_t$ and $\gamma_g = K + (b+\beta)y_t$ there will be a value of γ_g that sets $V_{x(t+1)} = X_t v_{xo}$, since at $\gamma_g = by_t - \beta x_t$ we know that $V_{x(t+1)} = 0$, and at $\gamma_g = K + (b+\beta)y_t$, we know that $V_{x(t+1)} = X_t v_{xo} + Y_t v_{yo}$.

38/ For a detailed model and proof of a similar problem see Rothchild [1974].

39/ See Green and Swets [1966], Chapter 8.

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