ENVIRONMENT CANADA

Environmental Conservation Service

WATER USE OPTIMIZATION MODEL

DRAFT

BASIC CONCEPTS IN MODEL DEVELOPMENT

PREPARED BY:

R. McNEILL
INLAND WATERS DIRECTORATE
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I. INTRODUCTION

In a recent Departmental Discussion Paper,[1] emphasis was placed on methods of addressing the growing imbalance of water supply and demand in a number of regions in Canada. The paper outlined the need for Inland Waters Directorate to develop programs to facilitate water management and policies aimed at resolving water shortages. A number of national and regional programs are now being developed in order to meet this need. One of these programs in Pacific and Yukon Region is the development of a water use optimization model. This model will provide a framework for analyzing current and future water supply and demand within a river basin. It will identify water use conflicts, optimal use of available supply and economic values associated with increasing or decreasing water supply. As such, the model will provide information useful for water management and planning and for water demand forecasting. The basic problems, concepts and methodology used in developing the model are described in this paper.

A related program, aimed at prediction of water shortages, is the development of a national-regional water use forecasting model. The basic structure for a forecasting model has been developed by headquarters personnel and it is now proposed that regional staff participate in the application of this model to river basins in their respective regions. Data requirements for this forecasting exercise will, for a large part, overlap the requirements for the water use optimization model. Thus, development and application of both the forecasting and optimization models can take place concurrently. It should also be noted that the objectives of the

[1] "Discussion Paper; Towards a Federal Freshwater Strategy" Environment Canada, Inland Waters Directorate. November 1982 (Draft).

two types of models are complimentary. The forecasting model is aimed at identifying basins and sub-basins where water shortages will arise, while the optimization model provides a framework for testing management alternatives for resolving water shortages and for allocating water in short supply. As water shortages increase, the development of both forecasting and optimization models will be essential for the management of the water resource and resolution of water use conflicts.

The optimization model will be developed in two phases.

Phase 1 Develop the model structure and apply it to the Okanagan River Basin. Because of previous work done under a Canada Water Act Study, the Okanagan River Basin was found to have the best data base for development of the model. Initial application to this basin will aid in developing a consistent structure for the model and will identify problem areas.

Phase 2 Adapt the model so that it can be applied to any river basin or sub-basin where data are available. A procedure will be developed whereby basic data for any river basin can be entered in a standard format, and the optimization model generated from a computer program. The model will therefore have to be flexible so that it can include various water uses and hydrological relationships that occur in different river basins.

II. OUTPUT FROM THE MODEL

When the model is applied to a specific basin or sub-basin, the optimum economic allocation of water within the basin will be computed. For example, in a certain time period and reach of the basin, the model could determine that 50 percent of the available run-off be allocated to agriculture, 20 percent to domestic use and 20 percent to reservoir storage. The complete model solution would show the amount of water allocated to each use, in each time period and reach. In a later section of this paper, an example problem illustrates the output from the model in more detail.

In solving for the optimal solution, the model will show whether water is in surplus or is in short supply in each time period and reach. When there is a shortage of water, the model will specify a shadow price for an additional unit of water. This shadow price represents the net economic value that could be obtained if an additional unit of water were available. If water is in surplus, the shadow price will be zero, and the amount of surplus water will be designated as a "slack resource".

The model will provide a framework for analyzing management alternatives for resolving water shortages, such as increasing storage, improving irrigation efficiency or implementing conservation measures. For example, a new storage development could be evaluated by running the model at current and proposed storage levels and comparing the level of benefits generated at each storage level.

Inter-basin transfers can also be examined using the optimization model. The benefits from increased water supply to a river basin could be evaluated by running the model at both the current and augmented levels of supply for the receiving basin. Likewise, the benefits lost in the exporting basin could be determined by applying the model to the exporting basin before and after the proposed diversion.

III. THE PROBLEM SETTING

There are a number of possible uses for a natural flow of water in a river basin. The kind of uses will depend on the location and seasonal timing of the flow and on the geographical features of the basin. For example, spring run-off into a tributary of a mainstem river could be;

1) diverted for irrigation, domestic or industrial use,
2) left in stream for fisheries purposes,
3) left in stream and diverted at a lower point in the system,
4) stored for use in a later time period.

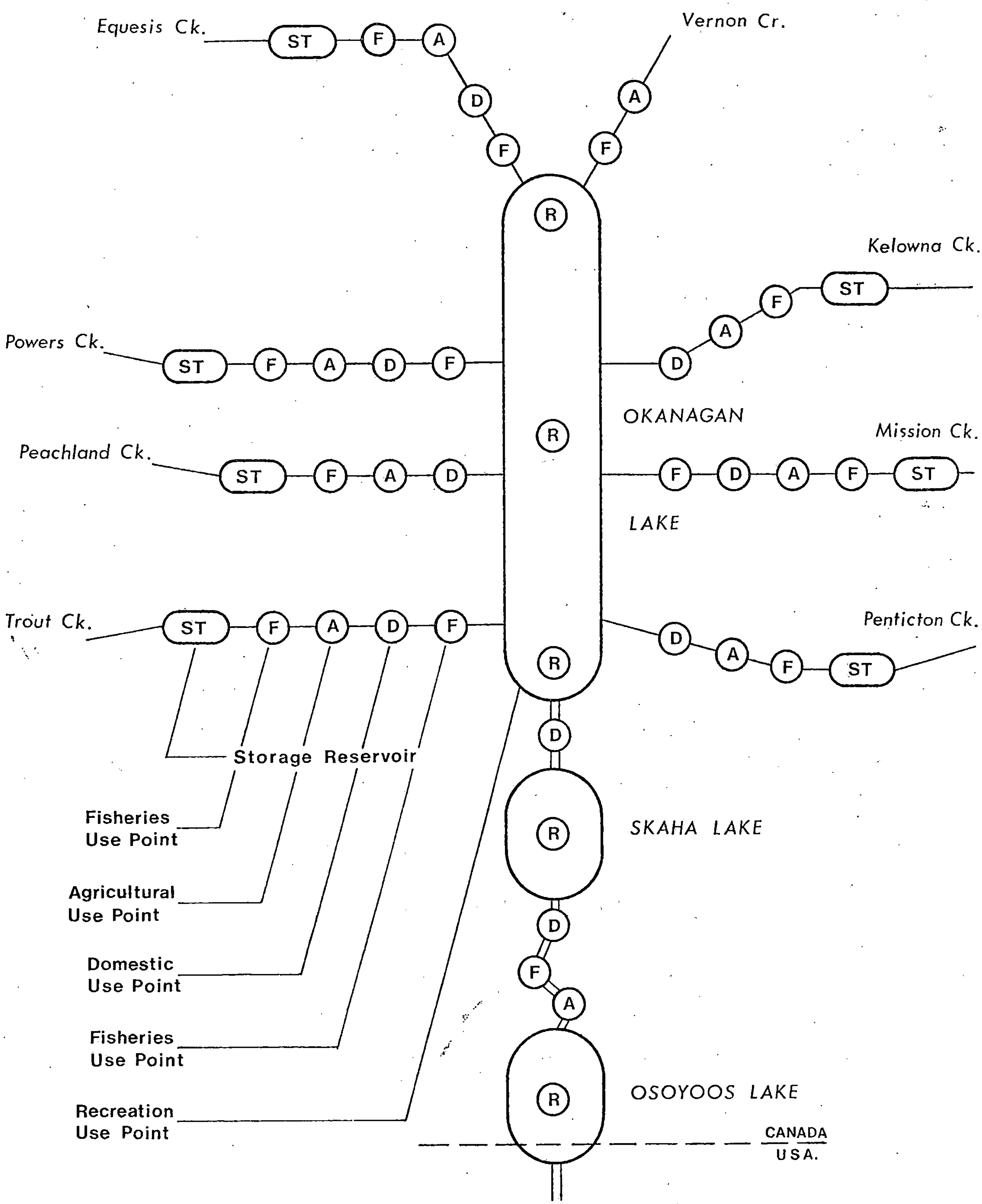
The basic problem is to choose <u>how</u> to use the water, <u>where</u> to use it and <u>when</u> to use it in order to get the greatest economic benefit from its use. This choice is constrained by a number of factors such as storage capacity, land use capability and fish habitat. For example, the choice of water storage could not be considered if storage capacity were not available.

Figure 1 shows a simplified schematic diagram of the Okanagan River Basin. It can be seen that there are a number of choices of how the water might be used including fisheries, domestic, agriculture and recreation. In general, the further upstream the water supply, the greater the number of possible uses it has. Storage is available on both the tributary reservoirs and the main valley lakes.

An optimization model is a useful tool for determining the best allocation of water among possible uses. When constructing such a model, two basic points must be considered. First, the best use and value of water at one point in a system should not be calculated independently from its use and value in other parts of the system. Thus a basin-wide model is needed, incorporating the linkages between various parts of the system. Secondly, the best use and value of water can vary according to where and when it occurs in the

FIGURE 1

OKANAGAN RIVER BASIN WATER USE AND STORAGE



basin. The model should therefore be disaggregated by sub-basins or reaches and by time periods.

The data requirements for modelling will have to be broken down in the same classification, i.e. by reach and time period. Appendix One shows the data requirements for the model using the Okanagan River Basin as an illustrative case.

IV. METHODOLOGY

A) Defining Optimal Use of Water

The objective of the model is to assist in determining the optimal allocation of water in a river basin. For the purposes of this study, the allocation of water among possible uses is considered to be optimal if it results in the maximum possible economic benefits, given the physical constraints which exist. Economic benefits are defined in the same way as in traditional benefit-cost analysis.[2]

It is recognized that maximization of economic benefits is not the sole criterion for determining the utilization of the water resource. For example, this criterion does not consider the equity of distribution of benefits that occur from water use. Achieving maximum economic benefits may also be constrained by legal and institutional factors, such as tenured rights to water use or international agreements on flows and water levels.

Although the model considers only economic benefits, it can be used to provide information to assist the policy maker in determining a socially acceptable allocation of water. This can

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[2] For example, refer to Treasury Board Secretariat, Benefit-Cost Analysis Guide (Hull, Quebec; Ministry of Supply an Services, Canada, 1978)

be done by imposing restrictions on the model to represent non-economic values, equity considerations and legal constitutional constraints. For example, minimum acceptable flows for fisheries or minimum lake levels for recreationists can be imposed in order to protect these user interests. Given such restrictions, the model can then solve for the most efficient solution. Comparisons between the non-restricted and the restricted solutions will show the economic benefits that are lost. The policy maker would then have some measure of the economic opportunity costs of various alternatives based on non-economic criteria.

B) Modelling Technique

The modelling technique used in this project is linear programming. A number of features of linear programming[3] make it suitable for modelling water use optimization.

1) Linear programming is a technique which maximizes or minimizes a mathematical 'objective function' specified by the model builder. In the water use optimization model, the objective function will represent the economic benefits associated with water use activities in a basin. The model will allocate water to various uses so as to maximize the value of the objective function.
2) The optimization procedure is carried out subject to a number of constraints which are expressed as linear equations. In the water use optimization model, the set of constraints will incorporate water supply activities such as run-off, storage capacity and storage release; and water requirement activities such as irrigation and household

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[3] A substantial body of literature exists on the theory and application of linear programming. A useful introductory discussion is contained in Alpha C. Chiang, Fundamental Methods of Mathematical Economics (New York, McGraw-Hill, 1974) chapters 18-19.

requirements. These factors can be fairly easily represented as linear equations.

3) A number of well documented and accessible computer packages for linear programming are available. The MPSX (Mathematical Programming System Extended) package will be utilized for the water use optimization model. This package is capable of handling large models and is available at many major computer systems. As such, it is well suited to the water use optimization model which will be very large for river basins where there are numerous tributaries and many demands on the water resource.

In summary, a linear programming model is simply a system of equations comprised of an objective function and a set of linear constraints. The general form of a linear programming model can be represented as follows;

maximize the objective function,

$$c_i x_i + c_2 x_2 + \cdots\cdots c_n x_n \tag{1}$$

subject to the linear constraints,

$$a_{ii} x_i + a_{i2} x_2 + \cdots\cdots a_{in} x_n \sim b_1$$

$$a_{2i} x_i + a_{22} x_2 + \cdots\cdots a_{2n} x_n \sim b_2$$

$$a_{mi} x_i + a_{m2} x_2 + \cdots\cdots a_{mn} x_n \sim b_m$$

Where ~ represents $\leqq$, =, or $\geqq$,

x_i to x_n = levels of various activities,

c_i to c_n = coefficients respresenting value of the activities in the objective function,

a_{ii} to a_{mm} = coefficients respresenting relationships between various activities,

b_1 to b_m = right hand side values of linear constraints.

The model will choose levels of the activities x_1 to x_n so that the value of the objective function is maximized and the constraint equations satisfied.

In the water use optimization model, the variables x_i to x_n represent activities associated with water supply and demand in a basin. Some activities represent the final use of water and result in economic benefits; irrigated acreage or sport fish population are examples. A value is assigned to the final use activities in the objective function through the c_i coefficients. For activities which do not represent final use of the water, the c_i coefficients will be equal to zero. These activities fall under the categories of water requirement activities (such as irrigation requirements) and water supply activities (such as run-off and storage). The constraint equations and a_{ii} coefficients represent basic physical or hydrological relationships between activities. For example, the relationship between storage levels, release from storage and run-off would be specified as a linear constraint. More details on the fomulation of the constraints and the objective function are given in the example problem in the next section.

V. A SAMPLE PROBLEM FOR A SINGLE SUB-BASIN

The water use optimization model will be fairly large and complex when applied to a river basin where there are numerous tributaries and water uses. For explanatory purposes, a smaller sample model of a single sub-basin was constructed using linear programming. This sample problem will serve to illustrate most of the relationships and techniques that will be used in a full river basin model. An understanding of the sample problem will result in an easier comprehension of the structure and capabilities of the full river basin model.

The sample model is based on a number of basin assumptions which serve to simplify the problem.

1) Only a single sub-basin with one storage reservoir is considered.
2) There are three final use activities in the sub-basin. These are irrigated acreage, domestic consumption and sport fish population.
3) The only significant run-off occurs in the reaches above the reservoir while all demand points are below the reservoir.
4) The model has a time horizon of one year which is divided into three periods. Period one represents spring, period two represents summer and period three represents fall and winter.

A) Specifying the Objective Function

The three final use activities each have value associated with them. It can be a complex task to determine the economic benefits of each use, but as a first approximation we can use some fairly simple procedures. The residual value approach is a suitable method for irrigated acreage at this stage. In this approach the net value of the crop per irrigated acre is taken as the value in the objective function. If for example, this value is $1000, then the objective function will increase by $1000 for every acre that is irrigated.

The user-day approach can be used for sport fish population. In British Columbia this is a fairly common approach where a base population of a species is assumed to support a number of fishing days. For example, if a stream population of 10 sport fish can support one fishing day, which has a value of $10.00, then a value of $1.00 can be assigned to each fish in the stream population. Thus for every extra fish produced in the stream, the value of the objective function will increase by $1.00.

Placing a value on domestic use of water is a more difficult task, and very few studies on this subject have been carried out in Canada.[4] At this stage of model development an assumption is made that a unit of water has a much higher average value for domestic use than for irrigation or the sport fishery. Therefore the model is constrained to meet all domestic water requirements before any commitment is made to agriculture or the sport fishery, and domestic use of water is not assigned a c_i value in the objective function.

The objective function can be simply formulated as:

(2)

ECONOMIC BENEFITS = $1000xIRRIGATED ACRES + $1.00xFISH POPULATION

The objective is to maximize this function subject to a number of constraints. The model will find the feasible quantities of irrigated acreage and fish population that will give the maximum dollar return calculated in equation (2).

B) Specifying the Constraints

Water supply and water-use activities form the basis of the constraints in the model. The constraints are in the form of linear equations which define relationships between activities. Limits to the levels of certain activities are also defined as linear constraints.

Agricultural Water Requirements

In each period, each acre under population requires a prescribed amount of water. If for example, each acre requires .5' of water in the first period, 1.5' in the second period and .5' in

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4 The most recent example of such a study is by C. Macerollo and M. Ingram, "The Value of Water in the Grand River Basin: An Estimate of the Demand for Water in Ontario" Canada Water Resources Journal Vol. 6 No. 1 (1981) pp. 51-63.

the third period, we can then define three variables; AGREQ 1, AGREQ 2 and AGREQ 3 corresponding to requirement in each time period. The relationship between the number of irrigated acres and agricultural water requirements can be expressed as;

$$\text{AGREQ 1} = .5 \times \text{IRRIGATED ACRES} \quad (3)$$

$$\text{AGREQ 2} = 1.5 \times \text{IRRIGATED ACRES} \quad (4)$$

$$\text{AGREQ 3} = .5 \times \text{IRRIGATED ACRES} \quad (5)$$

Transforming these into the form of the contraints for the general linear programming form shown in equation (1) gives;

$$\text{AGREQ 1} - .5 \times \text{IRRIGATED ACRES} = 0 \quad (6)$$

$$\text{AGREQ 2} - 1.5 \times \text{IRRIGATED ACRES} = 0 \quad (7)$$

$$\text{AGREQ 3} - .5 \times \text{IRRIGATED ACRES} = 0 \quad (8)$$

There will be a further agricultural constraint in that only a limited amount of irrigable land is available in the sub-basin. If the limit is 800 acres we can express this as another constraint;

$$\text{IRRIGATED ACRES} \leq 800 \quad (9)$$

Sport Fish Water Requirements

It is assumed that a relationship exists between the amount of water flowing in the stream and the size of the fish population. This relationship can be expressed as a per fish water requirement similar to the per acre water requirement for agriculture.

If it is assumed that each fish requires .1, .05 and .05 acre feet in the three time periods, then we can define three new variables and express the requirements as;

$$\text{FISH REQ 1} - .1 \times \text{FISH POPULATION} = 0 \quad (10)$$

$$\text{FISH REQ 2} - .05 \times \text{FISH POPULATION} = 0 \quad (11)$$

$$\text{FISH REQ 3} - .05 \times \text{FISH POPULATION} = 0 \quad (12)$$

Because of habitat constraints, the maximum fish population cannot exceed 10,000 in the stream. Therefore another constraint must be entered as;

$$\text{FISH POPULATION} \leq 10{,}000 \quad (13)$$

Domestic Water Requirements

As mentioned earlier, the model will be constrained to supplying domestic requirements. If domestic requirements are 100 acre feet in each time period, we can define the requirements as constraints:

DOMREQ 1 = 100 (14)

DOMREQ 2 = 100 (15)

DOMREQ 3 = 100 (16)

Supply Constraints

Supply can also be defined as a linear equation in each time period. We first define three variables representing run-off in each time period. This runoff occurs above the storage reservoir. For example:

RUN-OFF 1 = 1200 (17)

RUN-OFF 2 = 600 (18)

RUN-OFF 3 = 400 (19)

The storage capacity can be expressed as a constraint in each time period. If the capacity cannot exceed 1500 acre ft. then the constraints are expressed as:

STORAGE 1 $\leq$ 1500 (20)

STORAGE 2 $\leq$ 1500 (21)

STORAGE 3 $\leq$ 1500 (22)

Suppose that at the beginning of the three time periods there are 500 acre ft. of storage left over from the previous year. This amount of water is available for use before any run-off occurs. It is defined as an activity in the model called STORAGE 0.

STORAGE 0 = 500 (23)

Now we can define the relationship between run-off, storage and release from storage. The release is simply the change in storage levels plus the run-off. For the first time period the following equation represents release from storage.

RELEASE 1 = STORAGE 0 - STORAGE 1 + RUN-OFF 1 (24)

For time periods two and three, the equations are similar;

RELEASE 2 = STORAGE 1 - STORAGE 2 + RUN-OFF 2 (25)

RELEASE 3 = STORAGE 2 - STORAGE 3 + RUN-OFF 3 (26)

In equations (24), (25) and (26) evaporation is assumed to be insignificant. To put these equations in the standard format for constraints we simply rearrange them as follows;

STORAGE 0 - STORAGE 1 + RUN-OFF 1 - RELEASE 1 = 0 (27)

STORAGE 1 - STORAGE 2 + RUN-OFF 2 - RELEASE 2 = 0 (28)

STORAGE 2 - STORAGE 3 + RUN-OFF 3 - RELEASE 3 = 0 (29)

Constraints on Supply and Demand Balance

A major constraint that must now be considered is that total requirements (demands) in each time period must not exceed the release in each time period. For period one this constraint is expressed as;

AGREQ 1 + FISHREQ 1 + DOMREQ 1 $\leq$ RELEASE 1 (30)

where total requirements are the sum of agricultural, fish and domestic requirements. Specifying this constraint for each time period and rearranging into standard constraint form gives;

AGREQ 1 + FISHREQ 1 + DOMREQ 1 - RELEASE 1 $\leq$ 0 (31)

AGREQ 2 + FISHREQ 2 + DOMREQ 2 - RELEASE 2 $\leq$ 0 (32)

AGREQ 3 + FISHREQ 3 + DOMREQ 3 - RELEASE 3 $\leq$ 0 (33)

Contraint on Outflow

To complete the model, net outflow from the tributary is specified as an inflow to the next part of the system. Ignoring return flows, net outflow is equal to release minus consumptive use. It should be noted that fisheries requirements are a non-consumptive use.

OUTFLOW 1 = RELEASE 1 - AGREQ 1 - DOMREQ 1 (34)

In the standard constraint form, the outflow constraints are;

RELEASE 1 - AGREQ 1 - DOMREQ 1 - OUTFLOW 1 = 0 (35)

RELEASE 2 - AGREQ 2 - DOMREQ 2 - OUTFLOW 2 = 0 (36)

RELEASE 3 - AGREQ 3 - DOMREQ 3 - OUTFLOW 3 = 0 (37)

Table 1 - Linear Programming Tableau for Example Problem

		ACRE	FISH	AGREQ1	AGREQ2	AGREQ3	FISHREQ1	FISHREQ2	FISHREQ3	DOMREQ1	DOMREQ2	DOMREQ3	RELEASE1	RELEASE2	RELEASE3	STORAGE0	RUNOFF1	STORAGE1
RENT		1000.0	1.0000															
FISHC	LE		1.0000															
ACREC	LE	1.0000																
IC1	EQ	-0.50000		1.0000														
IC2	EQ	-1.5000			1.0000													
IC3	EQ	-0.50000				1.0000												
FC1	EQ		-0.10000				1.0000											
FC2	EQ		-5.00000E-02					1.0000										
FC3	EQ		-5.00000E-02						1.0000									
DC1	EQ									1.0000								
DC2	EQ										1.0000							
DC3	EQ											1.0000						
WWC1	LE			1.0000			1.0000			1.0000			-1.0000					
WWC2	LE				1.0000			1.0000			1.0000			-1.0000				
WWC3	LE					1.0000			1.0000			1.0000			-1.0000			
BGNSTC	EQ																	
ROA1C	EQ																	
BAL1C	EQ												1.0000			-		
ST1C	LE																	
REL1C	GE												1.0000					
ROA2C	EQ																	
BAL2C	EQ													1.0000				
ST2C	LE																	
REL2C	GE													1.0000				
ROA3C	EQ																	
BAL3C	EQ														1.0000			
ST3C	LE																	
REL3C	GE														1.0000			
OUT1C	EQ			-1.0000						-1.0000			1.0000					
OUT2C	EQ				-1.0000						-1.0000			1.0000				
OUT3C	EQ					-1.0000						-1.0000			1.0000			

This completes the specification of the sample problem as a linear programming model. The objective function is represented by equation (2) and the linear constraints are represented by equations (3) to (37). In total, 28 activities have been entered in the model. These activities fall under three general categories; final use activities or final products which utilize the water, water input activities which define the water requirements for the final products, and water supply activities which define the amounts of water available for use.

The whole model can be summarized in tableau form as in table 1. The column headings represent the various activities and the rows represent constraints. The first row, labelled as rent, is the objective function. The tableau entries are the coefficients of the objective functions and constraints. The right hand side values of the constraint equations are shown under the column headings RHS.

C. Solution to Example Problem

The model is solved by determining the levels of the 28 activities such that;

a) all constraint equations are satisfied,
b) the value of the objective function is at its maximum possible.

This type of linear programming problem is solved by the simplex method, which is a standard algorithm used in most linear programming software packages. The solution levels of all the model activities are shown in table 2.

From table 2, it can be seen that the optimal solution to the sample problem allocates 100 acre ft. of water to domestic consumption in each time period as required. Furthermore, all 800 available acres or irrigable land are supplied with water and the remaining supply is used to maintain 2000 fish. In period one, storage is brought to a level of 1000 acre ft., while 700 acre ft. are released. In period two, the storage is depleted to 200 acre ft. and a large release of 1400 acre ft. occurs in order to satisfy the heavy irrigation requirements in this period. By the end of period three, storage is completely depleted.

For certain constraint equations, the simplex method will impute a shadow price which is the value by which the objective function would increase if the constraint equation were increased by one unit. The significance of the shadow prices can be illustrated for equations (17), (18) and (19) which define the run-off in each time period.

$$\text{RUN-OFF } 1 = 1200 \qquad (17)$$

$$\text{RUN-OFF } 2 = 600 \qquad (18)$$

$$\text{RUN-OFF } 3 = 400 \qquad (19)$$

As shown in Table 2 the shadow price is $5.00 for each of the three constraints. Increasing the right hand side of any of these three equations by one acre ft. could thus result in an increase of $5.00 in the objective function. Therefore the economic value of an additional acre ft. of water in any time period is $5.00. Since the shadow price is positive, water is in short supply and any additional water has an economic value. From Table 2 it can be seen that all available water is allocated to the various uses in the reach and that no excess runoff occurs. The only flow available for the next reach is the water that was supplied for fisheries in the model.

The maximum storage capacity utilized was 1000 acre feet in period one while the maximum reservoir capacity is 1500 feet as specified in constraint equation (20). Since the reservoir capacity is not fully utilized, increasing the capacity would not alleviate the current water shortage or result in a better allocation of water. Thus other alternatives for increasing the

TABLE 2 Optimal Solution to Sample Problem

	Final Use Activities		Water Requirements Activities (acre feet)			Water Supply Activities (acre feet)			Run-off to Next Reach (acre-feet)
	Irrigated Acres	Fish Population	Irrigation Requirement	Fish Requirement	Domestic Requirement	Runoff	Storage Level	Storage Release	
Period 1	800	2,000	400	200	100	1200	1000	700	200
Period 2	800	2,000	1200	100	100	600	200	1400	100
Period 3	800	2,000	400	100	100	400	0	600	100

Shadow Prices

Period 1 water run-off = $5.00
Period 2 water run-off = $5.00
Period 3 water run-off = $5.00

Value of Objective Function = $82,000

water supply, such as diverting water from another basin, would have to be examined.

In summary, the solution to the sample problem provides several pieces of information useful for water planning and management. First, it shows the optimal allocation of water among competing uses and the optimal storage pattern. Second, it indicates that water is in short supply and gives the economic value of an additional unit of water in each time period. Finally, it shows that there is excess storage capacity which implies that water shortages cannot be resolved by increasing storage capacity.

VI. CONSTRUCTING THE MODEL FOR THE OKANAGAN RIVER BASIN

The Okanagan River Basin model will be based on the same relationships outlined in the sample problem. Sub-basin models, similar to the sample problem, will be developed for each major tributary or reach of the Basin. These sub-models will then be linked to form a single large model for the whole Basin. The linking procedure is fairly straightforward; the net run-off from each sub-basin is added to the natural run-off in the sub-basin immediately downstream, increasing the available supply in the downstream sub-basin. The single composite model will then allocate water between and within all of the sub-basins included in the analysis.

A) Additional Features of the Okanagan River Basin Model

Some additonal activities and features that were not included in the sample problem will be included in the Basin model. These additions will not change the basic structure of the model from the sample problem, but will allow a more detailed analysis of the water allocation problem.

Additional activities entered in the basin model will still fall under the same general categories of final use activities, water requirement activities and water supply activities.

The Basin model will include a more detailed seasonal analysis than the sample problem. This will be accomplished by dividing the year into seven periods rather than only three periods. The seven periods will include the six individual months from May to October and the remaining months aggregated into a single period. This break-down is necessary in order to incorporate the important monthly variations in water requirements for agriculture and the sport fish population during the summer and early fall months.

Water-based recreation will be added to the Okanagan River Basin model as a final use activity. A value will be given to recreation in the objective function and various constraints will define the relationship between lake elevations and levels of recreational activity. This feature will allow explicit consideration of the conflict between recreational lake level requirements and agricultural water-use requirements which could occur in severe drought years.

The sport fishery population will be expanded into a number of final use sectors including resident stream trout, stream spawning trout and kokanee salmon populations. The detailed analysis is necessary because of the different seasonal flow requirements of these three species. The extra effort in modelling the sport fishery sector is warranted since this sector has been substantially depleted and continues to suffer because of heavy agricultural water-use requirements on the tributaries.[5]

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5 See Main Report of the Consultative Board (Chapter 8). Canada-British Columbia Okanagan Basin Agreement, March 1974.

Some additional activities affecting water supply will be included in the Basin model. In some areas of the Basin, the supply of water is reduced because of evaporation from the main valley lakes. Seasonal evaporation losses will therefore be included as activities in the model. In other areas of the basin, supplies are augmented by agricultural and domestic return flows from upstream reaches. Return flows will be included as water supply activities in the receiving sub-basins or reaches.

In the Okanagan River Basin model it will be necessary to incorporate some non-linear relationships in the analysis. The main example is in the sport fishery sector where a non-linear relationship exists between instream flows and fish population. Because of the non-linear relationship, a single set of linear constraints cannot be used to relate seasonal flows to fish population, as was done in the sample problem. The non-linear relationship will be incorporated by approximating it with a series of linear functions or segments. A set of stream flow requirements and constraints will then be defined for each linear segment.

Another important feature which will be included in the Okanagan River Basin model is a constraint on end of year storage levels. Note that in the example problem, storage was completely depleted by the end of the final time period. Since no constraint or value was attached to remaining water in storage that could be utilized in the following year, the tendency was to deplete the water in storage in order to meet current year requirements. In the Basin model a constraint will be added that end of year storage levels must equal beginning of year storage levels. Thus the model will represent an equilibrium situation where stocks of water are not being depleted or increased over the long term, although they may fluctuate within each year.

B. Simplifying Procedures Used in Model Construction

Only the eight major tributaries and the mainstem are modelled in detail. These areas account for most of the water withdrawn for agriculture and domestic use. Net run-off under historical operating conditions from the unmodelled tributaries is included as an exogenous water supply. On tributaries where a number of reservoirs exist and are operated as a system, the individual reservoirs and linkages will not be modelled. Instead the individual reservoirs are aggregated and considered as a single large reservoir.

Industrial water requirements are aggregated with domestic requirements into a single use category. As in the sample problem, no value is given to this activity in the objective function. Instead, a constraint is built into the model that all domestic-industrial water requirements must be met before any water is allocated to other uses. It should be noted that domestic-industrial requirements are relatively small compared to agricultural requirements in the Okanagan River Basin.

VII. REFINING THE MODEL

Some improvements will be made to the model as data become available and techniques are developed. These are listed in order of priority below.

A) Agricultural Sub-Model

This sub-model will be developed in order to show how water-use can be efficiently allocated on agricultural land and the results will be incorporated in the Okanagan River Basin model. Development of this sub-model is considered as a high priority because there is considerable scope for agricultural conservation of water in response to shortages and conflicts. At the present time, a significant conflict exists between agricultural requirements and fishery requirements in most of the major tributaries of the Basin. Shortages in some areas are becoming more serious as municipalities expand and place heavier demands on water supplies which currently supply agricultural needs. Agricultural conservation practices could reduce the need for construction of expensive storage and diversion systems aimed at resolving these water-use conflicts.

In the simplified model and in the initial development of the Okanagan River Basin Model a fixed requirement for water per unit of irrigated land is assumed (The fixed requirement per acre can vary in different regions of the Basin). In order to remove the assumption of a fixed requirement of water per acre, the economics of on-farm water use must be examined. At present, users are charged for water on a per acre basis and have little incentive to conserve water. However, in the optimization model, water has value for non-agricultural uses; and conservation methods to reduce water consumption should be considered as an alternative. Any resulting costs or production losses associated with agricultural conservation of water must be evaluated. This is the objective of the agricultural sub-model.

The results from the agricultural sub-model can be incorporated into the Okanagan River Basin model allowing the removal of the assumption of a fixed water requirement per acre. In its first stage of development, the Basin model will allocate water to agricultural uses simply by increasing or decreasing the number of irrigated acres in the solution. Once the results of the agricultural sub-model have been incorporated, the Basin model will also be capable of determining the optimal per acre rate of application for agricultural uses. The techniques and the general structure in the agricultural sub-model will also be applicable to water use optimization models for other river basins in Canada.

Development of the agricultural sub-model will be contracted to the University of British Columbia, with completion scheduled for December 1984. The first stage of the project will be to develop the general structure of the sub-model to ensure that it can be linked or incorporated into the Okanagan River Basin model. A report outlining the general structure and data requirements will be prepared by March 31, 1984. The second stage of the project is the collection of data and detailed construction of the model which will be carried out in the subsequent spring and summer. The results from the agricultural sub-model will be incorporated into the Okanagan River Basin model during the fall of 1984. The final report from the contractor will be ready in December 1984.

B) Dynamic and Stochastic Elements

The model as described solves for optimal management of the water resource for a single year for a given water supply. The problem of optimal management over a period longer than one year was not considered beyond the imposing of a constraint that end of year storage must be equal to beginning year storage. This constraint ensures that the model will not reach a solution where the basic stocks of water are depleted over time. However, in low runoff years it may be desirable to draw down the lake and reservoirs to lower-than-average levels, in effect "mining" the stock of water. This may result in decreased benefits in the following years, depending on the future runoff levels. When the trade-off between using a stock of water in one year versus saving it for later years is considered, dynamic and stochastic elements must be incorporated in the model.

There are several multi-period optimization techniques which have been applied with varying degrees of success. The technique most suitable for this analysis is dynamic programming which is a general procedure to optimize a series of decisions over a specified time frame. When applied to the Okanagan River Basin, a dynamic programming model would have the objective of maximizing the present value of the benefits of water use over the time frame of the analysis. The final output of this model would be a table of annual decision rules relating annual withdrawal and end-of-year storage levels to annual run-off. Based on this output, the water manager would, in any given year, observe the run-off and choose the optimum amount of water to be withdrawn in the current year and the optimum amount to be left in storage for following years.

The theory and methodology of dynamic programming are complex and a discussion of the concepts is beyond the scope of this paper. A general discussion of the theory and applications can be found in the work of the Bellman[6] who first formalized the concepts of dynamic programming. An example of the application of dynamic programming to water resource management is the work of Burt[7] who examined the economics of multi-period storage of ground and surface water.

Construction of a dynamic programming model for the Okanagan River Basin would require substantial work effort and computer resources. The linear programming model will have to be solved for numerous run-off and storage scenarios and the results used as the basis of the dynamic programming model. A probability

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6 R.Bellman, Dynamic Programming (Princeton University Press, 1976).

7 O.R. Burt, "The Economics of Conjunctive Use of Ground and Surface Water" Hilgardia Vol. 6, No. 2, December (1964)

distribution of future water run-off in the basin will then have to be estimated and incorporated in the model. Computer solutions of the dynamic programming model will be expensive because of the multiple possible future scenarios of water use which will have to be evaluated.

VIII. GENERALIZING THE MODEL

Generalizing the model involves making it flexible so that it can be applied to any river basin where sufficient data are available. The model builder will not have to specify the model structure for a given river basin, but would only have to enter specific data which would be used to modify the structure of the generalized model. This input data would be in a similar format to the example data in Appendix One.

A computer program will be written that will generate the model structure for any river basin given a standard set of input data. The program will construct an objective function and set of constraints similar to those in the Okanagan Basin Model. Some additional constraints and activities may have to be added to account for different types of water uses or for different numbers of reaches and sub-basins. Any other differences would be in the right hand side values of some constraints.

The computer program will save a great deal of time when the model is applied to a new river basin since the constraints and activities will not have to be specified by the program user. Without the computer program, the model specification would be time consuming because of the large numbers of constraints and activities which would have to be entered. In the Okanagan River Basin Model, for example, there are about 1,000 constraints to be specified. Furthermore, the objective function and constraints must be entered in a standard format required by the MPSX conputer package for linear programming. This process can take several days for a large model. In contrast, less than a day would be required to enter the standard input data required by the computer program.

IX. The capability and reliability of the water use optimization model can be improved in a number of areas. The model, as it is presently being developed, is most applicable to river basins such as the Okanagan where there are high agricultural demands and relatively low industrial and domestic demands. Consequently, less emphasis is placed on the economics of domestic and industrial water use. Studies in these areas could improve the applicability of the model to basins where domestic and industrial water use are relatively large.

As previously mentioned, few studies have been carried out in the area of domestic demand for water in Canada. Most forecasts and studies of water use have simply assumed a fixed water requirement per household, and have not examined the value of water to the consumer. In order to determine this value, it would be necessary to estimate a consumer demand function for water, where quantity demanded is a function of price and other variables. This demand function would have to be specific to different river basins, taking into account regional differences such as climate and income. To date, statistical estimation of such demand functions has been hindered by a lack of water-price data and by problems in disaggregating domestic use and industrial use. A thorough examination of the existing water-price data base and a study of alternative methods of estimating domestic demand for water are recommended.

Further information on the economics of industrial water use would be required for model development in some river basins where industrial demands are significant. Most studies to date have assumed a fixed proportion between industrial output and water input, and have not examined in detail the relationship between water input and industrial production. A production function approach where output is expressed as a function of water use and other inputs would allow the assumption of fixed proportions to be dropped. This approach would also allow derivation of the industrial demand function for water and calculation of its value to industry. It is recommended that this approach be considered in further studies, particularly for industries which are large consumers of water.

Additional studies are also required in the general area of demand for recreational activities which depend on the water resource. Although substantial work has been done on the techniques for estimating demand functions for recreation, there is a lack of information on user-values specific to various river-basins in Canada. Work is also needed in relating changes in user-values to changes in water levels and flows that occur when water is diverted for consumptive uses.

One final area requiring further study and program development is the whole process of data collection on water use. Data requirements are intensive for both optimization and forecasting models, and most river basins do not have a sufficient data base for development of either type of model. Improvement in data collection methods and coordination of present efforts in data collection will have to be made before there can be widespread application of forecasting and optimization models.

X. TIMETABLE FOR FURTHER WORK

Work on the initial application of the model to the Okanagan River Basin is underway, using data from the 1970 data. More recent data will be incorportaed as it becomes available. Development of a computer program for model generation for other basins will commence in December. Refinement of this program will continue as necessary.

The estimated timetable for the complete project is shown below:

TIMETABLE

Work	Start	Finish
1) Phase One		
- application to Okanagan Basin using 1970 data	Already in progress	Jan. 15, 1983
- update of Okanagan model with more recent data as it becomes available	Jan. 15, 1983	Continuing phase
2) Phase Two		
- development of computer program for model generation	Dec. 15, 1983	Mar. 15, 1984
- refinement of program as necessary	Mar. 15, 1984	Continuing phase
3) Agricultural Sub-model		
- development by U.B.C.	Jan. 1, 1984	Oct. 15, 1984
- final report on sub-model	Oct. 15, 1984	Feb. 28, 1985
4) Dynamic Stochastic Extension	Jan. 15, 1984	Sept. 1, 1984

APPENDIX ONE

Data Requirements for Water Use Optimization Model

This appendix outlines data requirements for the water use optimization model using the Okanagan Basin as an illustrative case. The data given in this report are from the Okanagan Basin Study (1974), so updating will be necessary.

1. DEGREE OF AGGREGATION

The data are disaggregated into a number of classifications.

The basic breakdown is by:

A) Region: supply and demand for eight major tributaries and four sub-basins of mainstem system,

B) Season: supply and demand for seven periods during the year,

C) Sector: supply and demand for major sector including agriculture, industrial, fisheries and recreation.

A) Regional Breakdown

The eight tributaries were selected because of data availability and importance in terms of water requirements. Together, the eight tributaries account for about 48% of the natural inflow into Okanagan Lake in an average year. The other tributaries to Okanagan Lake are aggregated into a single net run-off figure for the seven periods. The mainstem system is broken down into Okanagan Lake; Skaha Lake, Okanagan River and Osoyoos Lake.

It should be noted that the regional breakdown is dependent on the characteristics of the basin. In the Okanagan Basin it is necessary to break the area into major tributaries and sub-basins because of physical characteristics affecting water supply and demand. In other basins a different breakdown might be required to get meaningful results from the model.

B) Seasonal Breakdown

A seven period breakdown is used for the Okanagan. The months of May, June, July, August, September and October each comprise a single period while the remaining months are aggregated into one period. This breakdown was selected in order to represent critical periods for the various demands. For example, October is a critical month for kokanee spawning, May is critical for stream trout spawning and July and August are critical for irrigation. A different seasonal breakdown may be required for other basins.

C) Sectoral Breakdown

1) Consumptive Uses - In the Okanagan Basin Study, consumptive uses of the water were disaggregated into three sectors; agriculture, domestic and industrial. For the purpose of the model, industrial and domestic uses were aggregated into a single category as they are relatively small compared to agriculture. Considerably more effort was made in estimating agricultural demands since they accounted for about 80% of the consumptive use. A different sectoral breakdown might be desirable in other basins, particularly if domestic and industrial uses are relatively larger than in the Okanagan Basin.

2) Non Consumptive Uses - These are broken down into recreation and fishery requirements. The primary recreational requirement is that lakes be kept above a minimum elevation to ensure that boat launches can be used and that lake bottom is not exposed around swimming areas. There are at least four different types of fishery requirements; lake level requirements for kokanee spawning, flow requirements in the tributaries for spawning rainbow trout and kokanee, and flow requirements in Okanagan River for spawning

sockeye salmon. In the Okanagan Basin Study, fishery requirements were usually defined as minimum instream flows or lake levels required to sustain a predetermined fish population. In some cases, relationships are given between flows and fish population. These relationships are used in the optimization model.

2. EXAMPLES OF DATA USED FOR THE MODEL

Examples of the data at 1970 levels of development are shown below. Trout Creek is used to illustrate the data requirements for tributaries. Data for each of the sub-basins of the mainstem are also shown. From these data it is possible to derive most of the coefficients used in the optimization model.

In these examples, recreational requirements are not shown for the main valley lakes. As mentioned earlier, these recreational requirements generally concern lake levels; if the lakes fall below certain levels, then there will be losses in recreational values because of exposure of lake bottom, reduction in boating area and inoperable boat launches. However, only very rough estimates of the values lost are available from the 1974 Okanagan Basin Study. Some better and more recent estimates are available for Osoyoos Lake, which can be modified for use in the model. Further work will have to be done at estimate losses in recreational value for the other main valley lakes at low lake levels.

A) Trout Creek Sub-Basin

Agricultural Requirement

Irrigated acreage 4306

Water Duty 3.03 acre ft. per acre

Monthly Requirement per acre (acre feet)

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
0	0	0	0	.4545	.7575	.7575	.7575	.303	0	0	0

Population 5960

Domestic requirements (acre feet)

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
29	29	32	29	29	32	29	29	32	29	29	32

Industrial Requirement not significant

Fisheries Requirements

Resident trout; monthly requirements

16.3 acre ft. per 100 population from May to September 12.6 acre ft. per 100 population from October to April	to maximum of 3986 population
31.3 acre ft. per 100 population from May to September 62.6 acre ft. per 100 population from October to April	from 3986 to 5580 population
90.0 acre ft. per 100 population from May to September 180.0 acre ft. per 100 population from October to April	from 5580 to 6132 population

Rainbow Spawners: Monthly requirement

May, 322 acre ft. per 100 population

All other months: 213 acre ft. per 100 population

Kokanee Spawners:

November to May 13.3 acre ft per 100 population
June to August 100 acre ft. per 100 population
Sept. 11.1 acre ft. per 100 population
Oct. 20.0 acre ft. per 100 population

Water Supply

Natural flow above storage points (acre ft.) - average yr.

May	June	July	August	Sept.	Oct.	Other 6 months
25,104	15,283	1831	888	755	755	5,065

Natural Flow between storage points (acre ft.) average yr.

May	June	July	August	Sept.	Oct.	Other 6 months
7794	388	127	36	36	36	346

Total Available Storage 9307 acre ft.

B) Okanagan Lake Sub-Basin

Agricultural Requirement: Non-tributary

1911 irrigated acres, irrigation duty 2.4 ft. per acre

Monthly Requirements (ft. per acre)

May	June	July	August	September
.36	.60	.60	.60	.24

Domestic Requirement: Non-tributary

Monthly Requirement (acre ft.)

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
260	260	260	313	469	729	833	729	469	365	260	260

Industrial Requirement: Non-tributary

Monthly Requirement (acre ft.)

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1300	1300	1560	1920	2860	3380	3900	3380	2340	1560	1300	1300

Net Supply - excluding 7 tributaries modelled (1000's of acre ft.)

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
8.13	15.53	20.87	53.04	91.91	71.72	42.5	23.55	18.01	19.93	10.49	7.59

Evaporation (1000's of acre ft.)

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
.8	6.4	9.6	20.0	29.9	38.7	47.0	45.1	33.8	21.6	7.3	2.0

C) Skaha Lake Sub-basin

Agricultural Requirement; Lake and Tributaries

1871 irrigated acres, irrigation duty 2.6 ft. per acre

Monthly Requirement (ft. per acre)

May	June	July	August	September
.39	.65	.65	.65	.26

Domestic Requirement; Lake and Tributaries

Monthly Requirement (ft. per acre)

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
20	20	25	28	45	53	56	53	37	25	20	2

Industrial Requirement; Lake and Tributaries

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
35	35	35	35	40	45	50	55	60	40	35	35

Water Supply

Natural run-off excluding release from Okanagan Lake
(1000's of acre ft.)

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1.0	1.76	1.42	2.89	14.44	11.6	4.96	2.91	1.20	.86	.84	.84

Evaporation (1000's of acre ft.)

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
.08	.26	.57	1.16	1.74	2.20	2.45	2.44	1.85	1.18	.47	.18

D) Okanagan River Sub-basin

Agricultural Requirement; River and Tributaries

7597 irrigated acres, water duty 4.85 ft.

Monthly Requirement (ft. per acre)

May	June	July	August	September
.73	1.21	1.21	1.21	.48

Domestic Requirement; River and Tributaries (acre ft.)

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
54	54	64	75	117	139	160	139	96	64	54	54

Industrial Requirement; River and Tributaries (acre ft.)

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
70	70	70	70	80	90	100	110	0	80	70	70

Water Supply

Natural run-off excluding release from Skaha Lake
(1000's of acre ft.)

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1.56	.96	1.33	2.03	20.59	17.17	8.44	7.72	6.88	2.2	1.97	1.39

E) Osoyoos Lake Sub-basin

Agricultural Requirement

2767 irrigated acres, irrigation duty 4.9 ft. per acre

Monthly Requirement (ft. per acre)

May	June	July	August	September
.74	1.23	1.23	1.23	.49

Domestic Requirement;

Monthly Requirement (acre ft.)

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
30	30	36	42	66	72	90	72	54	36	30	30

Industrial Requirement; not significant

Water Supply

Natural run-off excluding release from Okanagan River (1000's of acre ft.)

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
3.31	2.41	2.76	5.55	12.38	10.17	8.75	8.88	4.82	1.31	.89	1.72

Evaporation (1000's of acre ft.)

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
.07	.31	.80	1.61	2.33	2.91	3.33	3.01	2.48	1.56	.61	.16

25378