APPLICATION OF THE LOGARITHMIC AND STAGE DISCHARGE EQUATION METHODS FOR RATING CURVE EXTENSION

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Water Resources Branch
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Summary

Two methods of rating curve extension - the logarithmic and stage-discharge equation (as developed by Dr. B.P. Sangal) - have been applied to eleven rating curves from ten streams gauging stations in the Pacific and Yukon Region. The data from these stations was readily available from previous studies. The results are comparable, within 10 percent, for six rating curve extensions and of the remaining four the logarithmic method provides a better fit in two. One rating curve was not sufficiently defined for the reliable application of either method. This is not to be considered as a definitive study with so small a sample of rating curves. The recommendation of this study is to apply both methods and then select the one that gives the best results.

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1.0 INTRODUCTION

The satisfactory determination of peak flood discharge at a gauging station by extension of the rating curve requires that the rating curve be developed by actual measurements to a stage where the differences between successive rates of change in discharge with respect to change in stage have become fairly constant. The conditions of the channel that are most favourable for an accurate extension of the rating curve consist of well-defined rapids of riffles below the gauge at all stages and a uniform increase of channel cross section as the stage increases, with no abrupt changes in area or addition of overflow channels.

The technique of rating curve extension is illustrated using eleven rating tables from streamflow stations in the Pacific and Yukon Region. The data for this study was readily available from previous studies of station evaluation and flood analysis. Only a small sample of rating curves has been analyzed and thus this report cannot be considered a definitive study on the subject. It is left up to those concerned with the quality of data to satisfy themselves on the appropriate method of extension.

2.0 PURPOSE

The objective of this study is to illustrate the procedure for selecting either the logarithmic or stage-discharge equation method for rating curve extension and to obtain some experience in judging the acceptability of each method in its ability to fit the rating curves.

3.0 <u>Methods of Rating Curve Extension</u>

3.1 Logarithmic

The criterion for a precise extension by the logarithmic method is fulfilled when the equation $G = a + b\ (Q/\Delta Q)$ $(b = S\ \Delta G = \text{constant})$[1] plots as a straight line on rectangular coordinates. The accuracy of the extension depends on the straightness of the line of relationship. Some segments of each channel cross section may fit a logarithmic curve but seldom would the entire cross section. To obtain a valid extension the highest measured flow defining the rating curve must be included in that portion of the section used for fitting.

3.2 Extension by the Stage-Discharge Equation Developed by Dr. B.P. Sangal (1986)

The criterion for a precise extension by the stage-discharge equation method is fulfilled when there is a straight line relationship between G and ΔQ^2 in the equation $G = g + c^1 \Delta Q^2$ ($c^1 = 1/c$ = constant)[1]. The same comments made about channel cross sections in the logarithmic extension section also hold true for the stage-discharge extension.

A straight line relationship rarely exists and the goodness of fit is always less than precise.

3.3 Rating Curve Analysis

In the following section illustrations of the relationship between stage and the incremental increase of the rating curve and as applied to the logarithmic and stage-discharge equation methods of extension are shown and discussed. The cross section of the stream channel in the vicinity of the gauge is also shown to illustrate the relationship of the channel configuration to the method of rating curve extension. At some stations the high water measuring facility is some distance from the gauge. In these cases wading measurements in the vicinity of the gauge are used to plot the channel cross section.

Comparison of R^2 values are made but it must be remembered that a log transformation of the data is performed in the application of the logarithmic extension.

Comparisons of the extended curves are also illustrated. The curves are extended to the same stage as that of the original rating curve with no attention paid to whether overbank flow conditions existed.

[1] Personal Communication from Dr. B.P. Sangal

The values for the best fit equations are listed in Tables 1 and 2 for the logarithmic and stage-discharge equation respectively.

(a) Barlow Creek near Quesnel (08KH018)

Rating Table #7 was defined by measurements to a stage of 5.5 feet. Neither method of curve extension plots as a straight line in any segment of the above stage range as shown on Figure 1. A segment of stage between 4.8 and 5.5 feet will be used and extended by the logarithmic method. Fitting of the stage-discharge equation is done by iteration and the range of rating curve is selected automatically with the upper limit being at the highest defined stage.

(b) Chapman Creek above Sechelt Diversion (08GA060).

Rating Table #23 was defined by measurements to a stage of 2.2 metres. Neither curve analysis gives a straight line relationship in this range as shown in Figure 2. However, the logarithmic analysis appears to give a straight line segment between the stages of 2 and 2.4 metres. Even though this is above the defined range it is useful for extension purposes. The stage-discharge method has the best range of stage between 1.7 and 2.2 metres.

(c) Forrest Kerr Creek above 460 m Contour (08CG005)

Rating Table #7 was defined by measurements to a stage of 1.6 metres. The logarithmic analysis gives a straight line relationship from 1 to 1.8 metres as shown in Figure 3. Use the range from 1 to 1.8 metres. The stage-discharge method indicates a suitable range from 0.6 to 1.6 metres.

(d) Illecillewaet River at Greeley (08ND013)

Rating Table #17 was defined by measurements to a stage of 2.6 metres. The logarithmic method does not indicate a straight line relationship although the range from 1.4 to 2.6 metres would be acceptable. The stage-discharge analysis has the best relationship between 1.4 and 2.6 metres as shown in Figure 4.

(e) Illecillewaet River at Greeley (08ND013)

Rating Table #18 was defined by measurements to a stage of 2.4 metres. Neither curve displays a straight line relationship within this range of stage. The segment best representative of a straight line for the logarithmic extension is from 1.8 to 2.6 metres as shown on Figure 5. The range selected for the stage-discharge extension is between 1.4 and 2.2 metres.

(f) Lardeau River at Marblehead (08NH007)

Rating Table #22 was defined by measurements to a stage of 8.5 feet. Both methods of extension are represented by apparently straight line segments within the measured range of stage. The segment for the logarithmic extension is from 6.5 to 8.5 feet as shown in Figure 6. The stage-discharge equation is best represented from 5.5 to 8.5 feet.

(g) Lillooet River near Pemberton (08MG005)

Rating Table #16 was defined by measurements to a stage of 4.2 metres. Neither method of extension is represented by any straight line segments as shown in Figure 7. The section used for the logarithmic extension is from 2 to 4.2 metres. The range best representing the stage-discharge equation is from 1.8 to 4.0 metres.

(h) Lubbock River near Atlin (09AA007)

Rating Table #11 has been defined by measurements to a stage of 8.4 feet. The logarithmic method of extension is represented by a straight line segment from 6.5 to 8.5 feet as shown in Figure 8. The segment best representing the stage-discharge equation is from 3.5 to 5.5 feet.

(i) More Creek near the Mouth (08CG005)

Rating Table #5 was defined by measurements to a stage of 6.2 feet. Neither method of extension is represented by a straight line segment within the range of definition as shown in Figure 9. Since the rating curve has been defined for only such a short range of the total extension, a segment of from 3 to 7 feet is used for the logarithmic method. The stage-discharge equation also has the best range from 3.0 to 7.0 feet.

(j) Nation River near Fort St. James (07ED001)

Rating Table #8 was defined by measurements to a stage of 3.4 metres. Both methods of extension display straight line segments within this range as shown in Figure 10. Although the straight line segment for the logarithmic extension is between 2 and 2.8 metres use the segment 2 to 3.4 metres for extension in order to gain the influence of the highest measurement. The stage-discharge method represents a straight line between the range 1.2 to 2.0 metres.

(k) North Alouette River at 232nd Street, Maple Ridge (08MH006)

Rating Table #28 was defined by measurements to a stage of 1.2 metres. Neither method of extension displays any straight line segments within the defined range as shown in Figure 11. The segment from 0.8 to 1.2 metres was used for the logarithmic extension. The stage discharge equation uses the range from 0.3 to 1.2 metres.

4.0 Results of the Analysis

The results of the application of the two methods of rating curve extension are discussed in this section for each rating curve.

(a) Barlow Creek near Quesnel (08KH018)

Neither method of extension produces any straight line segment although both methods appear to have adequate relationships. The results are, however, strikingly different with a 12.5 percent difference in discharge at the maximum extended stage. The stage-discharge method of extension gives a slightly better fit (Table 3) over the coincident section of the rating curve and a better coefficient of determination based on natural numbers as shown in Table 2. A comparison of the extended curves is shown in Figure 12.

(b) Chapman Creek above Sechelt Diversion (08GA060)

The logarithmic analysis gives a straight line segment part with the defined stage and part above it. Using this range of stage, which is at the point where the difference between successive rates of change in discharge with respect to change in stage have become fairly constant, the logarithmic method of extension has a better fit as shown in Table 3 and indicated by a better coefficient of determination based on logarithmic transformation. The stage-discharge method would be expected to slightly overestimate as the discharge value at highest defined stage is above that of the rating curve.

A comparison of the extended rating curves is shown in Figure 13.

Figure 14 shows that the fitted curves give smooth incremental graduation in the rating curve whereas the eye-ball method is quite irregular in certain ranges.

(c) Forrest Kerr Creek above 460 m Contour (08CG005)

Both methods appear to give similar fits as shown in Table 3 and as also shown by the coefficient of determination. The logarithmic method would be expected to slightly overestimate as the discharge value at the highest defined stage is higher than that of the rating curve value.

A comparison of the extended rating curves is shown in Figure 15.

(d) Illecillewaet River at Greeley (08ND013) Rating Curve #17

Both methods give the same results within 1 percent as shown in Table 4.

A comparison of the extended rating curve is shown in Figure 16.

(e) Illecillewaet River at Greeley (08ND013) Rating Curve #18

The logarithmic method appears to have a better fit to the rating curve as shown in Table 3 although the coefficients of determination are comparable. The coefficients of determination cannot be compared directly as one is computed from natural numbers and one from log transformed data. The results for maximum stage are within 9.6 percent as shown in Table 4.

A comparison of the extended rating curves is shown in Figure 17.

(f) Lardeau River at Marblehead (08NH007)

Both methods give the same results within 1.7 percent as shown in Table 4. The logarithmic method appears to fit better as shown in Table 3 although the coefficients of determination are similar. The stage-discharge method would be expected to slightly underestimate compared with the logarithmic method as its values as (shown in Table 3) are lower.

A comparison of the extended rating curves is shown in Figure 18.

(g) Lillooet River near Pemberton (08MG008)

The stage-discharge equation analysis gives a line that is nearly straight at the top portion of the defined rating curve. This method has a better fit as shown in Table 3 and a better coefficient of determination. The logarithmic method would be expected to be overestimated as the value at the highest defined stage is higher than the rating curve value.

A comparison of the extended rating curves is shown in Figure 19.

(h) Lubbock River near Atlin (09AA007)

The logarithmic method gives a straight line segment at the top end of the defined range of the rating curve. This method has a better fit as shown in Table 3 and a better coefficient of determination. The discharge values at the maximum stage are within 3 percent as shown in Table 4. The stage-discharge method would be expected to slightly overestimate as the value at the highest defined stage is higher than the rating curve value.

A comparison of the extended rating curves is shown in Figure 20.

(i) More Creek near the Mouth (08CG005)

This rating curve has not been defined to the stage where the difference between successive rates of change in discharge with respect to change in stage have become fairly constant. The logarithmic method, however, gives a better fit to the rating curve as shown in Table 3 and has a slightly better coefficient of determination. The stage-discharge equation method would be expected to underestimate as the discharge value at the highest defined stage is lower than the rating curve value.

A comparison of the extended rating curves is shown in Figure 21.

(j) Nation River near Fort St. James (07ED001)

Both methods give straight line segments that do not include the highest defined stage. The fit for both methods is comparable as shown in Table 3. The coefficient of determination is slightly better for the logarithmic method. It would be expected that the logarithmic method would slightly underestimate and the stage-discharge method would slightly overestimate as the discharge value at the highest defined stage is lower in one case and higher in the other. The difference in discharge estimates are 5.6 percent as shown in Table 4.

A comparison of the extended rating curves is shown in Figure 22.

(k) North Alouette River at 232nd Street Maple Ridge (08MH006)

The logarithmic method appears to have a better fit to the rating curve as shown in Table 3 and as also shown by the coefficient of determination. The stage-discharge equation method would be expected to underestimate the discharge value at the maximum stage as the discharge value at the highest defined stage is lower than that of the rating curve.

A comparison of the extended rating curves is shown in Figure 23.

Figure 24 shows that the fitted curves give smooth incremental graduations in the rating curve whereas the eye-ball method is quite irregular in certain ranges of stage.

5.0 Conclusions

The results are comparable within 10 percent in six cases, and in the remaining four each method fits two of them better. An attempt should not be made to extend the More Creek rating curve.

One method of rating curve extension should not be chosen over the other. The criteria curves are very sensitive to any changes in slope and may very well exaggerate any anomaly in the construction of the rating curves.

In order for the hydrographer to select a method of curve extension, an analysis should be carried out on each method. A decision will then be made on how well the rating curve is fitted and on the coefficient of determination.

Both methods of extension when applied to the rating curves produce a smooth progression of increments in flow. These curve fitting methods should be adapted to produce smooth rating curves unless natural conditions indicate otherwise.

An analysis of area and velocity tends to confuse or disguise any changes taking place in the stream channel as there tends to be so much scatter in the plotted points.

One advantage of the logarithmic method of curve extension is being able to select a definite segment of the rating curve for extension.

BIBLIOGRAPHY

SANGAL, Dr. B.P. Extending a Rating Curve December 5, 1986. Memorandum to D.R. Kimmett, Director, Water Resources Branch, Ottawa.

SMITH, A.G. Comparison of Methods of Extension of Rating Curves. Internal Report. Vancouver B.C. Water Resources Branch, Planning and Studies Section, 1987.

Tables 1 to 4

TABLE 1

COEFFICIENTS FOR THE LOGARITHMIC EQUATION
Q = c (G-a) **s
(From "HQ Curve" Computer Program)

STATION	Coefficients c	a	s	R^2 LOG TRANSFORMED DATA
Barlow Creek	78.9452	4.58	1.9200	0.99993
Chapman Creek	102.0558	1.71	1.0394	1.00000
Forrest Kerr Creek	33.35328	0.00	2.2512	0.99992
Illecillewaet River (Rating Curve #17)	108.74249	1.04	1.5676	0.99999
Illecillewaet River (Rating Curve #18)	146.95197	1.06	1.3442	0.99999
Lardeau River	444.74649	3.12	1.71680	0.99998
Lillooet River	35.05311	0.68	2.1342	0.99975
Lubbock River	108.88075	3.18	1.0042	1.00000
More Creek	34.28712	-2.38	2.2072	0.99997
Nation River	118.81822	0.84	1.2348	0.99999
North Alouette River	0.31176	-1.33	5.5800	0.99993

TABLE 2

COEFFICIENTS FOR STAGE-DISCHARGE EQUATION
Q = a + b (G - g) **1.5
(From "HQ Curve OPT" Computer Program)

STATION	Coefficients a	b	g	R^2 FROM UNTRANSFORMED DATA
Barlow Creek	11.363	122.245	4.905	0.99998
Chapman Creek	9.249	113.224	1.705	0.99955
Forrest Kerr Creek	12.681	84.705	0.610	0.99997
Illecillewaet River Rating Curve #17)	7.266	120.931	1.150	0.999987
Illecillewaet River Rating Curve #18)	18.627	134.253	1.095	0.999906
Lardeau River	617.637	791.792	4.080	0.999978
Lillooet River	51.453	121.470	1.790	0.99986
Lubbock River	70.501	33.990	2.330	0.99975
More Creek	174.649	247.857	0.100	0.99927
Nation River	8.287	80.494	0.600	0.99971
North Alouette River	12.310	88.643	0.600	0.99849

TABLE 3

COMPARISON OF RATING CURVE AND FITTED CURVE DISCHARGE VALUES

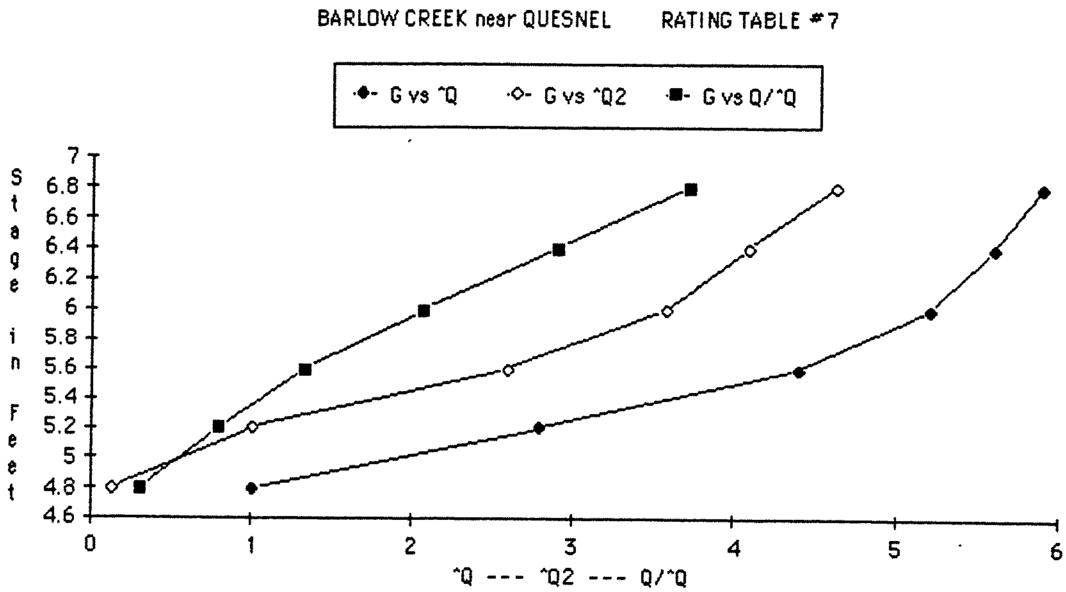
STATION NAME	RATING CURVE VALUES - DEFINED STAGE in Decreasing Order				
	Highest Defined Stage	1	2	3	4
Barlow Creek					
Rating Curve #7	67.5 cfs	54	41.5	31	22
Logarithmic	67.0	53.7	41.8	31.3	22.3
Stage Discharge	67.5	53.9	41.7	31	21.9
Chapman Creek					
Rating Curve #23	48.2 m^3s	38	27.8	18.6	12.6
Logarithmic	48.2	38	27.8	17.7	
Stage-Discharge	48.7	37.4	27.4	19.1	12.6
Forrest Kerr Creek					
Rating Curve #7	96.5 m^3s	83.5	72	61	51.2
Logarithmmic	96.8	83.7	71.7	60.7	50.8
Stage-Discharge	96.1	83.8	72.2	61.2	51.1
Illecillewaet River					
Rating Curve #17	219 m^3s	197	176	156	137
Logarithmic	218	197	176	156	137
Stage-Discharge	218	197	176	156	137
Illecillewaet River					
Rating Curve #18	218 m^3s	196	175	155	135
Logarithmic	218	196	175	155	135
Stage-Discharge	219	196	175	154	136
Lardeau River					
Rating Curve #22	8000 cfs	7750	7500	7250	7000
Logarithmic	7993	7740	7490	7244	7001
Stage-Discharge	7975	7727	7482	7238	7000
Lillooet River					
Rating Curve #16	502 m^3s	476	450	424	399
Logarithmic	513	483	453	424	397
Stage-Discharge	506	478	451	424	398
Lubbock River					
Rating Curve #11	572 cfs	561	550	539	523
Logarithmic	572	561	550	539	528
Stage-Discharge	579	566	554	542	529
More Creek					
Rating Curve #5	3950 cfs	3850	3750	3650	3550
Logarithmic	3940	3840	3741	3643	3546
Stage-Discharge	3909	3817	3727	3637	3548
Nation River					
Rating Curve #8	382 m^3/s	372	343	325	307
Logarithmic	380	361	343	325	308
Stage-Discharge	385	365	346	326	308
North Alouette River					
Rating Curve #28	54 m^3/s	43.6	34.2	26.8	20.8
Logarithmic	54.2	43.4	34.3	26.9	20.8
Stage-Discharge	53.5	43.7	34.7	26.9	20.2

TABLE 4

Comparison of Extended Rating Curve Values

STATION NAME	RATING TABLE NUMBER	MAXIMUM STAGE OF EXTENSION	RATING TABLE (Q)	LOGARITHMIC EXTENSION (Q)	STAGE DISCHARGE EQUATION	DIFFERENCE BETWEEN METHODS IN PERCENT
Barlow Creek near Quesnel	#7	7.0 feet	354 cfs	430 cfs	382 cfs	13.0
Chapman Creek above Sechelt Diversion	#23	2.7 m	103 m^3s	101 m^3s	122 m^3s	21.0
Forest Kerr Creek above 460 m Contour	#7	2.5 m	244 m^3s	264 m^3s	233 m^3s	6.0
Illecillewaet River at Greeley	#17	4.1 m	618 m^3s	628 m^3s	620 m^3s	1.0
Illecillewaet River at Greeley	#18	4.1 m	673 m^3s	655 m^3s	718 m^3s	10.0
Lardeau River at Marblehead	#22	10.0 feet	11760 cfs	12190 cfs	11990 cfs	2.0
Lillooet River near Pemberton	#16	6.5 m	1420 m^3s	1500 m^3s	1290 m^3s	16.0
Lubbock River near Atlin	#11	9.0 feet	640 cfs	638 cfs	656 cfs	3.0
More Creek near the Mouth	#5	20.0 feet	21630 cfs	32700 cfs	22200 cfs	47.0
Nation River near Fort St. James	#8	4.3 m	614 m^3s	550 m^3s	581 m^3s	6.0
North Alouette River	#28	1.7 m	166 m^3s	148 m^3s	113 m^3s	31.0

Figures 1 to 24



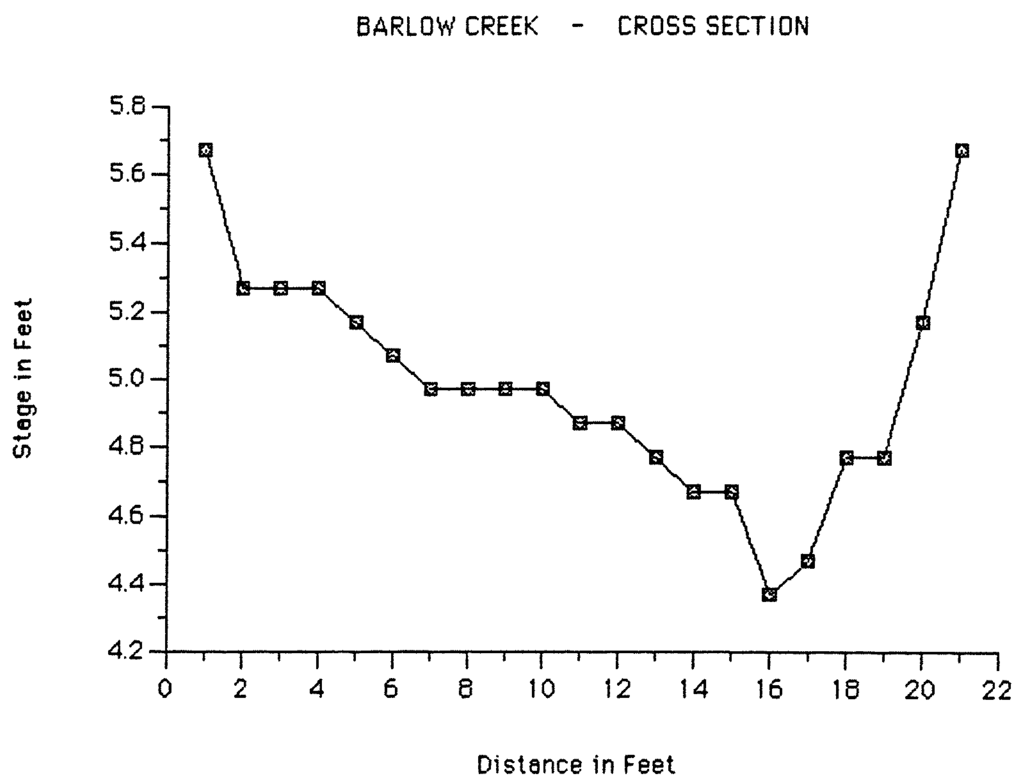
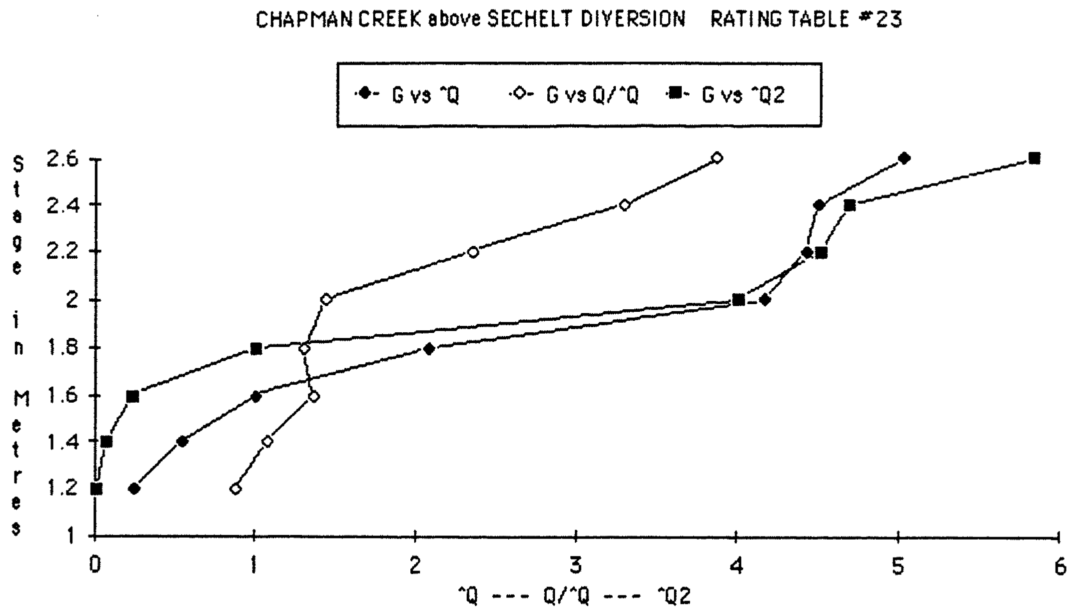


Figure 1 Stage versus Incremental Discharge of the Rating Curves as Applied to the Logarithmic and Stage-Discharge Equations and Cross Section at Metering Site for Barlow Creek near Quesnel



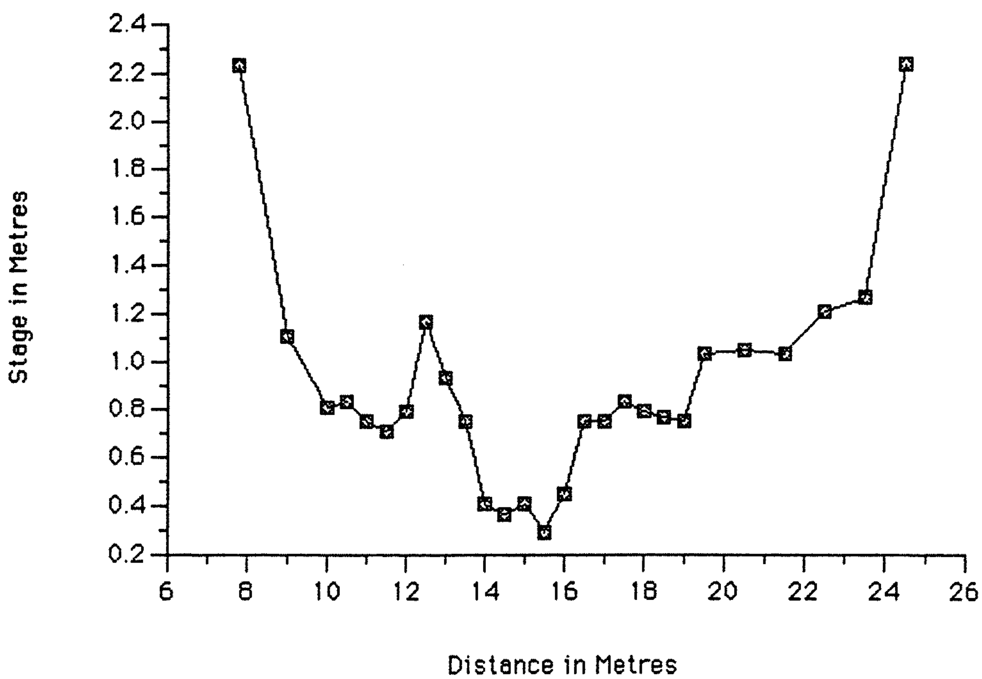
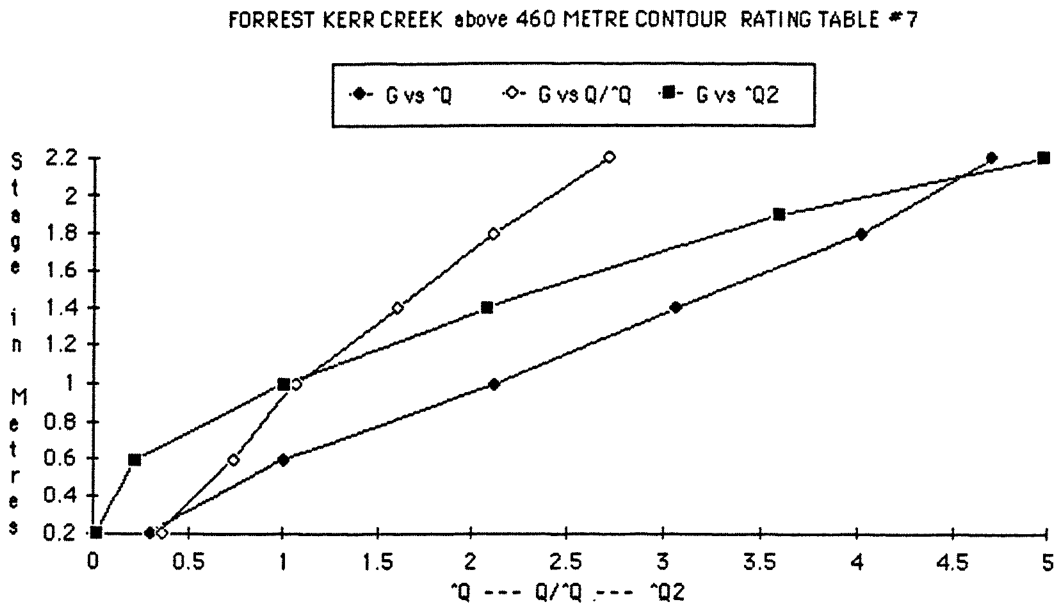


Figure 2 Stage versus Incremental Discharge of the Rating Curves as Applied to the Logarithmic and Stage-Discharge Equations and Cross Section at Metering Site for Chapman Creek above Sechelt Diversion



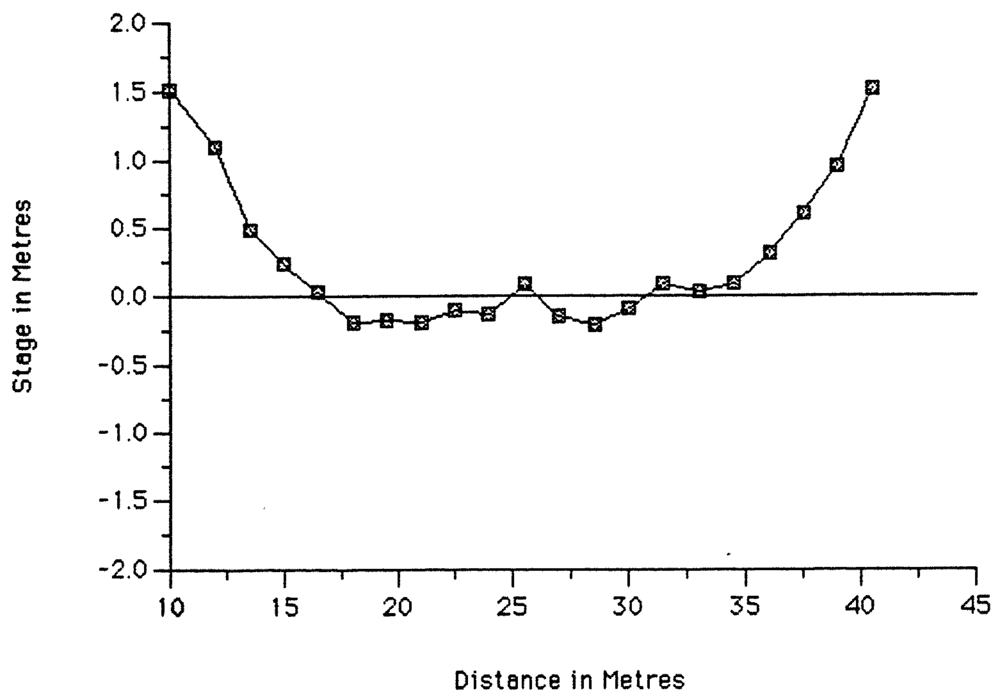
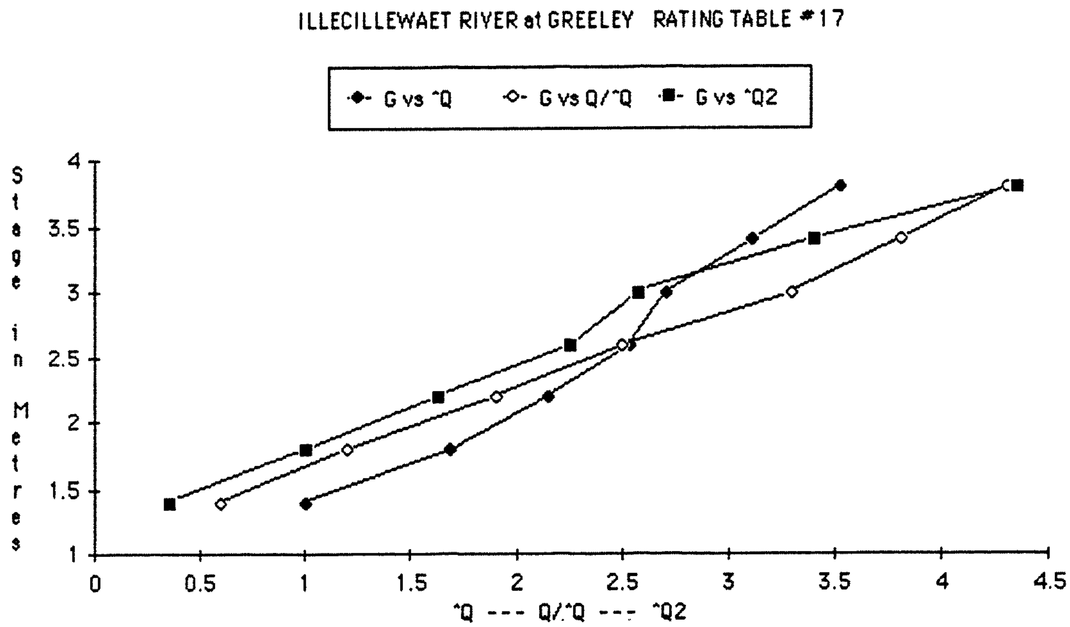


Figure 3 Stage versus Incremental Discharge of the Rating Curves as Applied to the Logarithmic and Stage-Discharge Equations and Cross Section at Metering Site for Forrest Kerr Creek above 460 m Contour



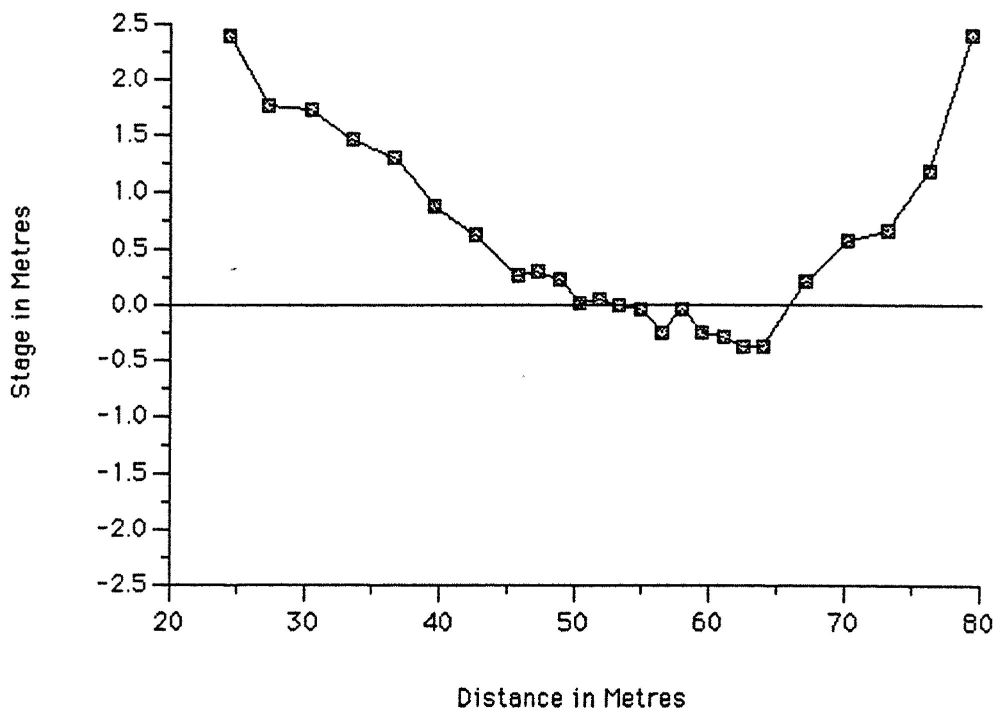
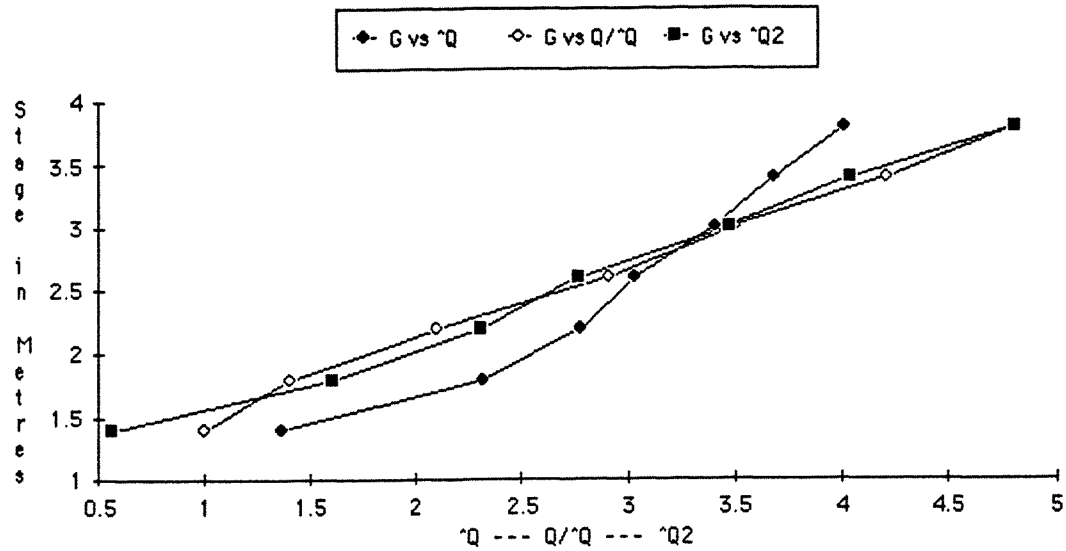


Figure 4 Stage versus Incremental Discharge of the Rating Curves as Applied to the Logarithmic and Stage-Discharge Equations and Cross Section at Metering Site for Illecillewaet River at Greeley (Rating Table #17)



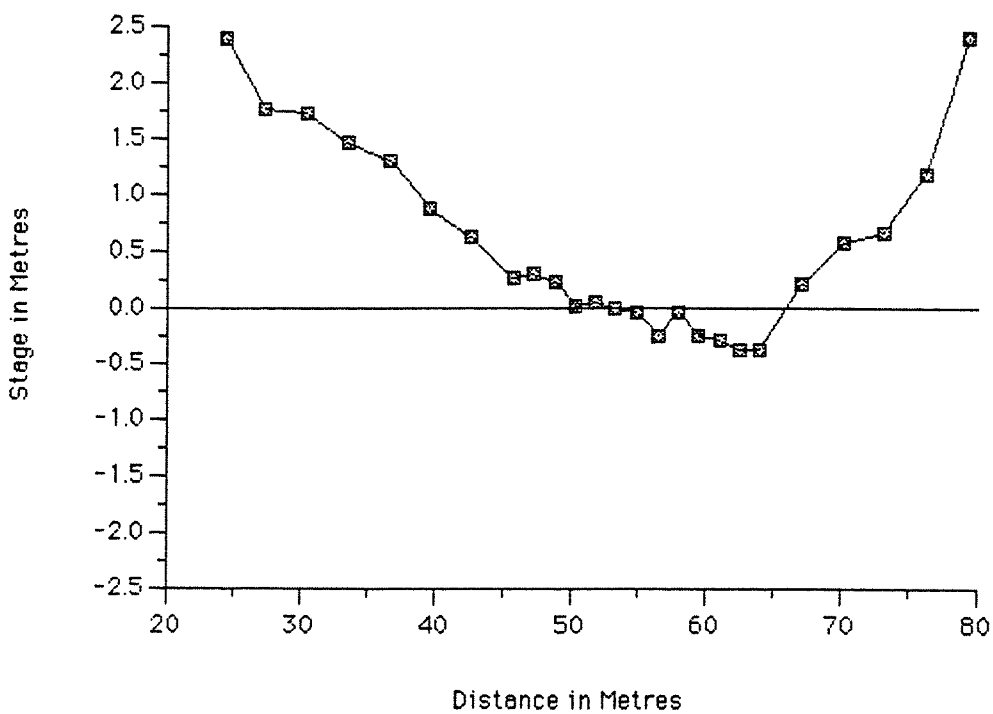
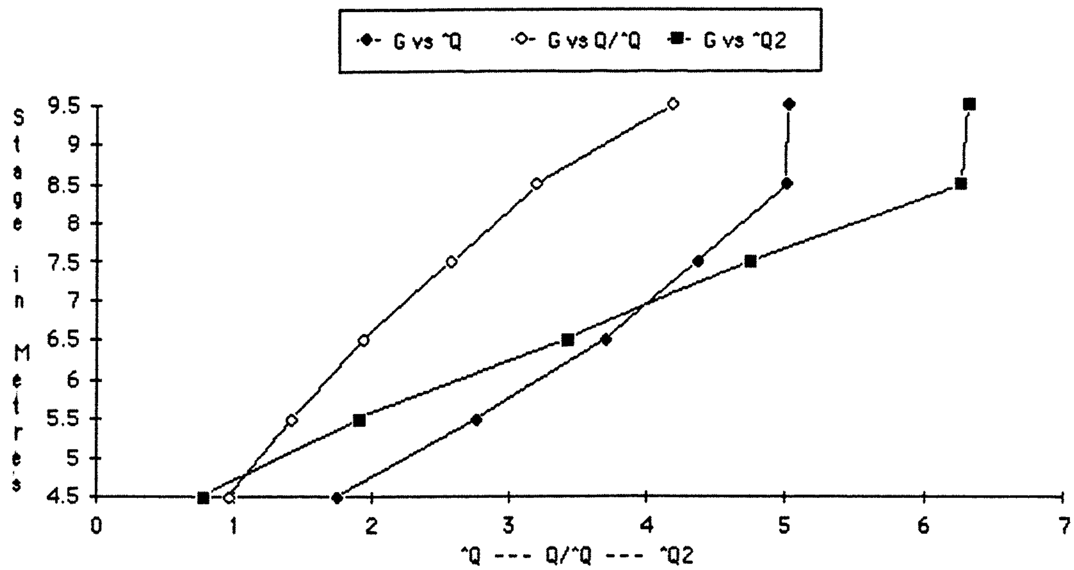


Figure 5 Stage versus Incremental Discharge of the Rating Curves as Applied to the Logarithmic and Stage-Discharge Equations and Cross Section at Metering Site for Illecillewaet River at Greeley (Rating Table #18)



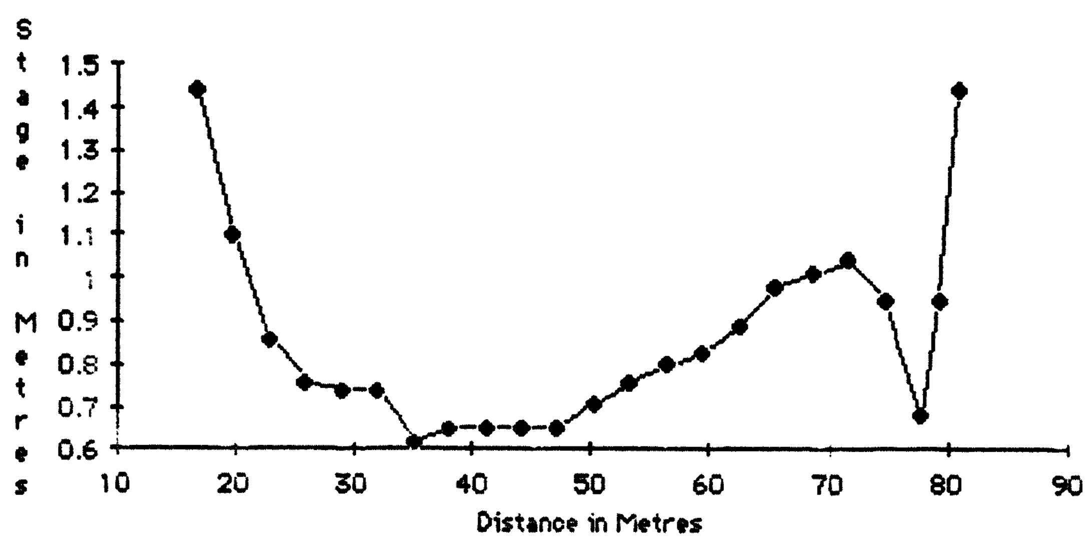
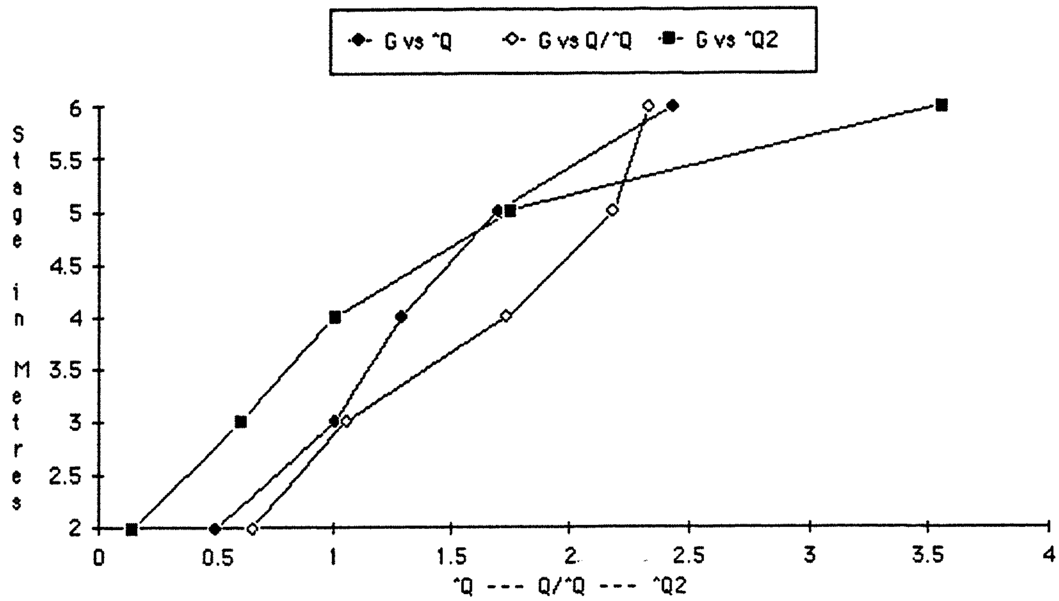


Figure 6 Stage versus Incremental Discharge of the Rating Curves as Applied to the Logarithmic and Stage-Discharge Equations and Cross Section at Metering Site for Lardeau River at Marblehead



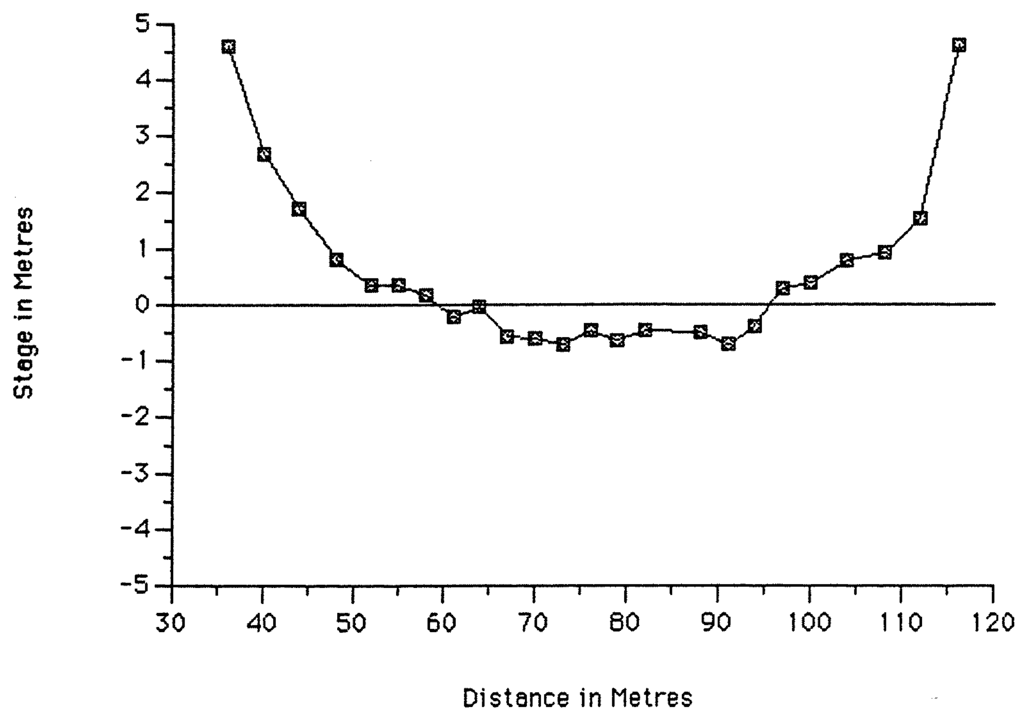
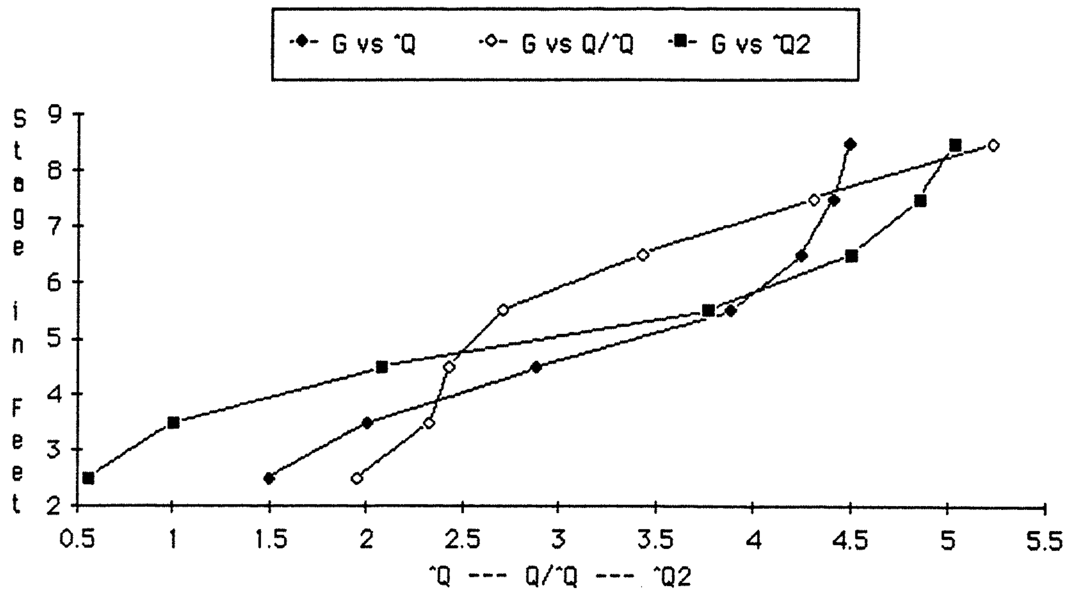


Figure 7 Stage versus Incremental Discharge of the Rating Curves as Applied to the Logarithmic and Stage-Discharge Equations and Cross Section at Metering Site for Lillooet River near Pemberton



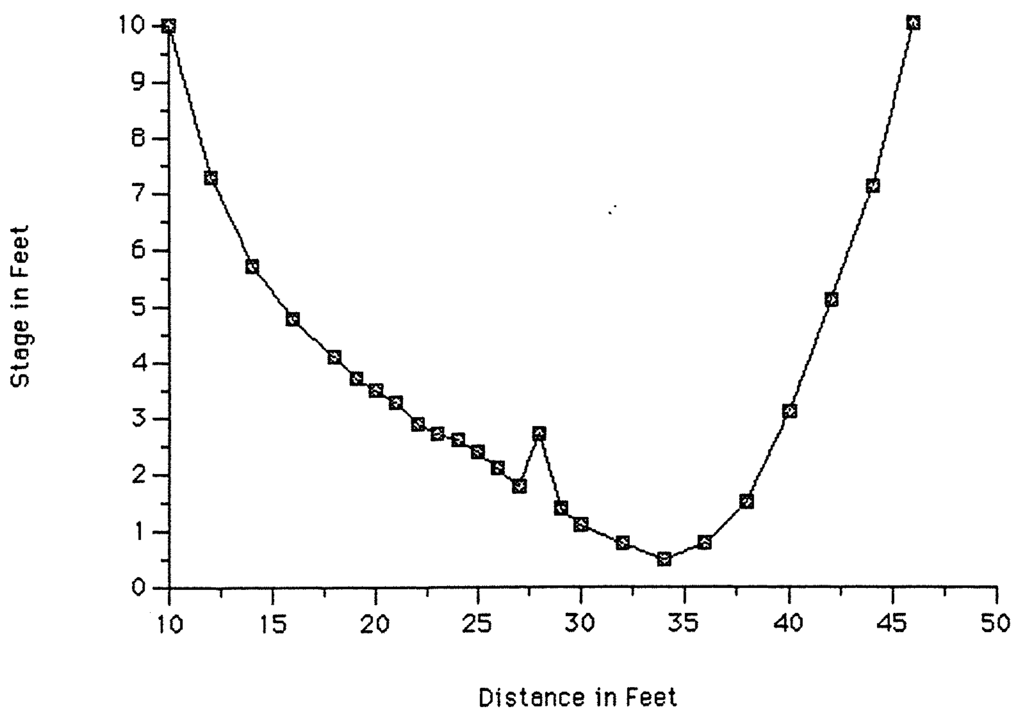
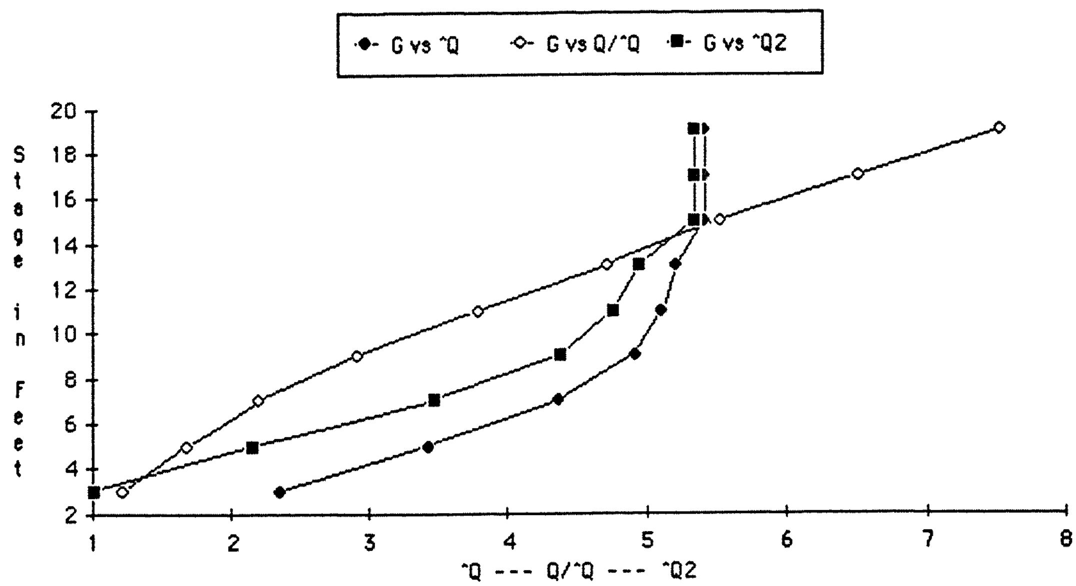


Figure 8 Stage versus Incremental Discharge of the Rating Curves as Applied to the Logarithmic and Stage-Discharge Equations and Cross Section at Metering Site for Lubbock River near Atlin



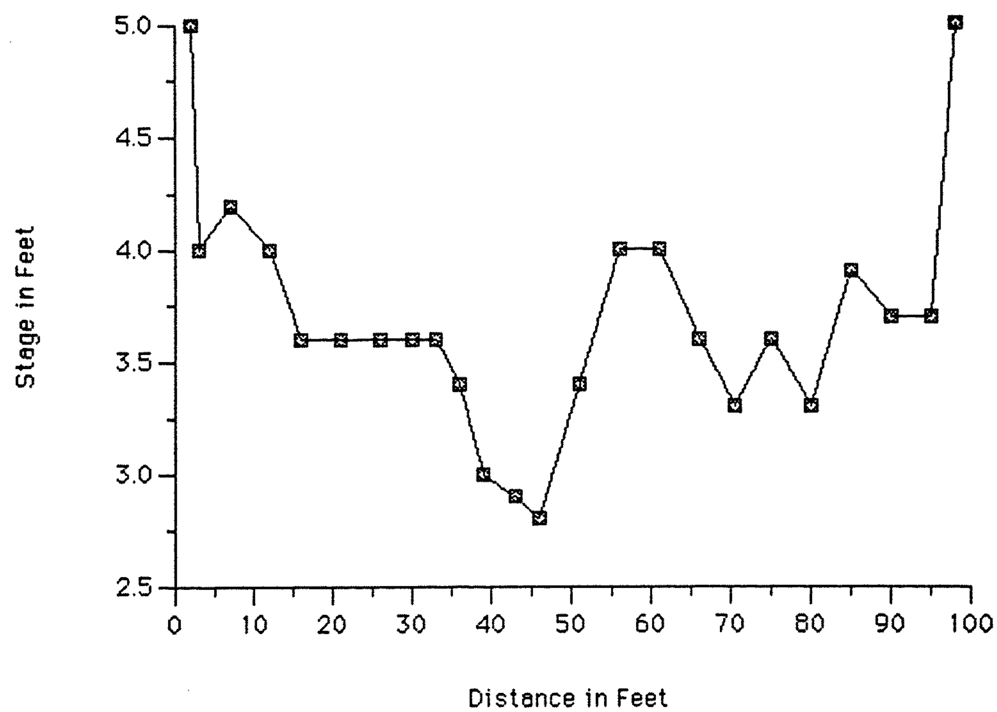
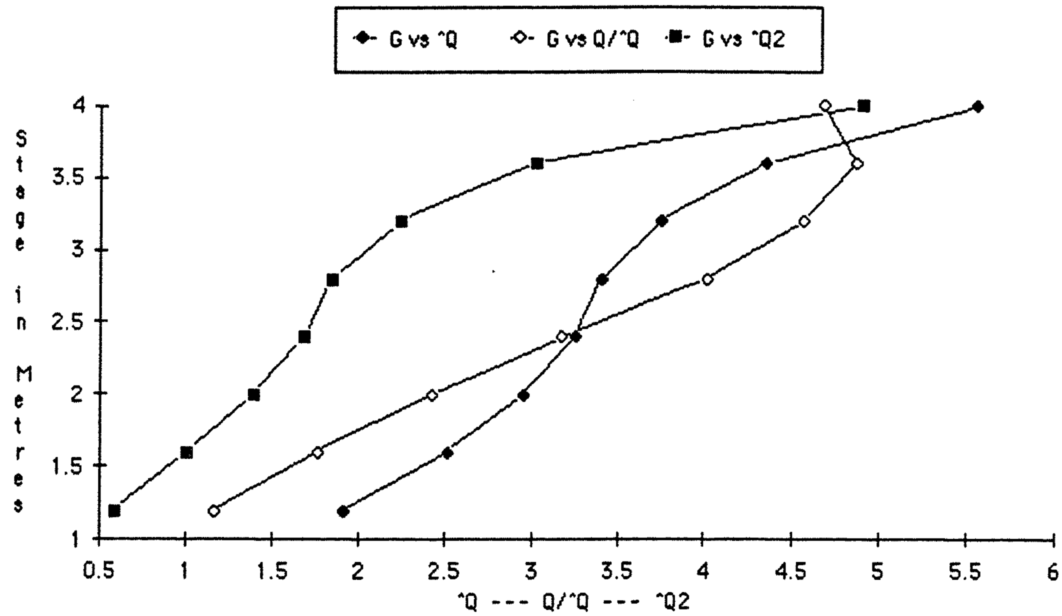


Figure 9 Stage versus Incremental Discharge of the Rating Curves as Applied to the Logarithmic and Stage-Discharge Equations and Cross Section at Metering Site for More Creek near the Mouth



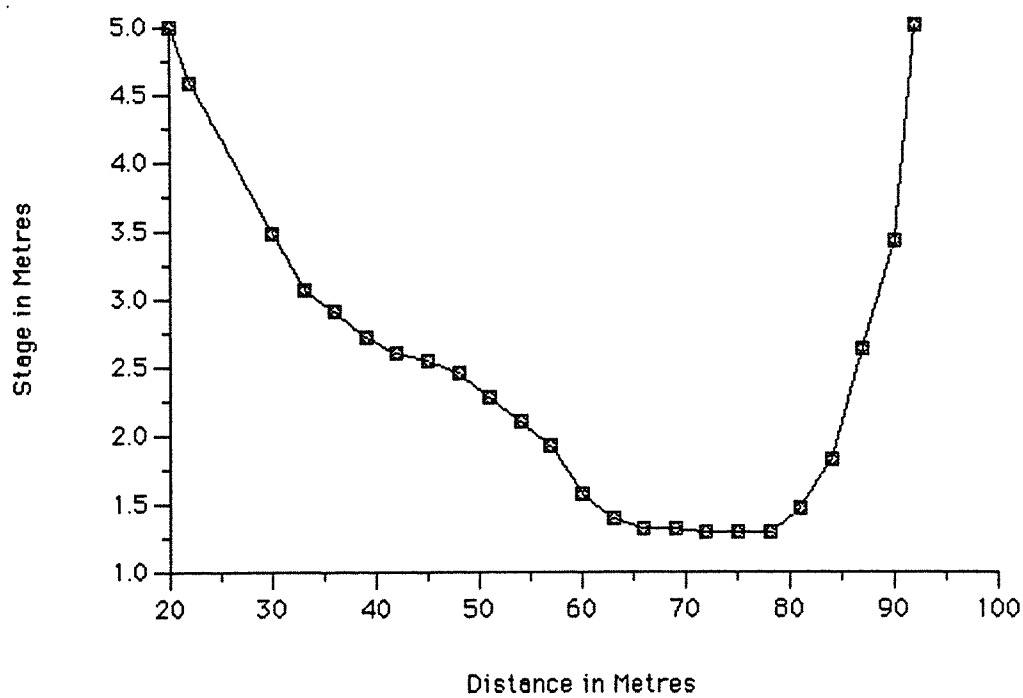
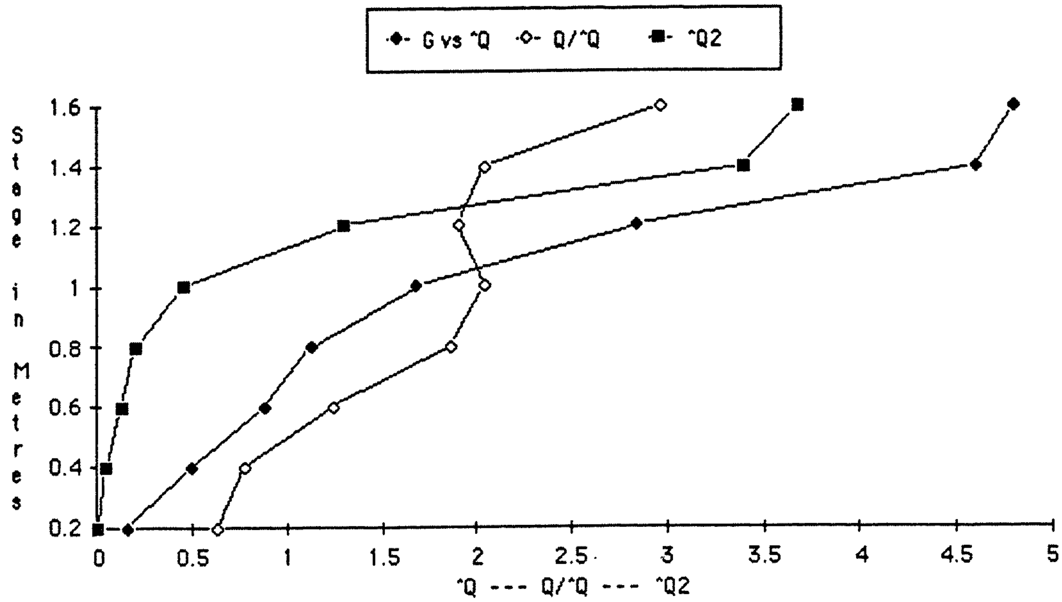


Figure 10 Stage versus Incremental Discharge of the Rating Curves as Applied to the Logarithmic and Stage-Discharge Equations and Cross Section at Metering Site for Nation River near Ft. St. James



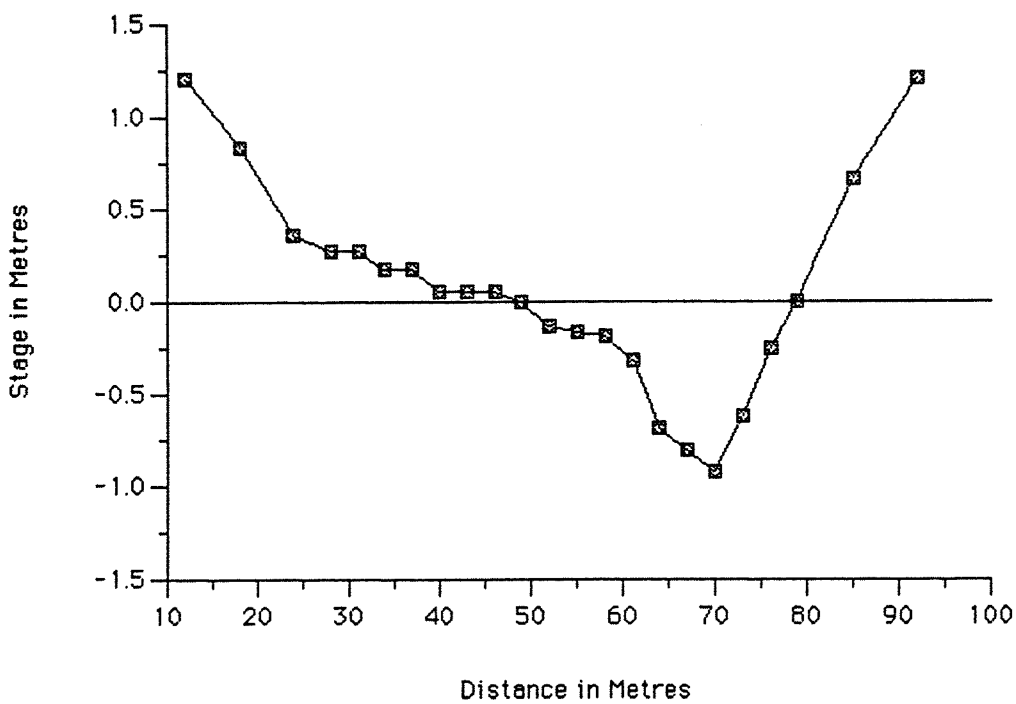


Figure 11 Stage versus Incremental Discharge of the Rating Curves as Applied to the Logarithmic and Stage-Discharge Equations and Cross Section at Metering Site for North Alouette R. at 232nd Street

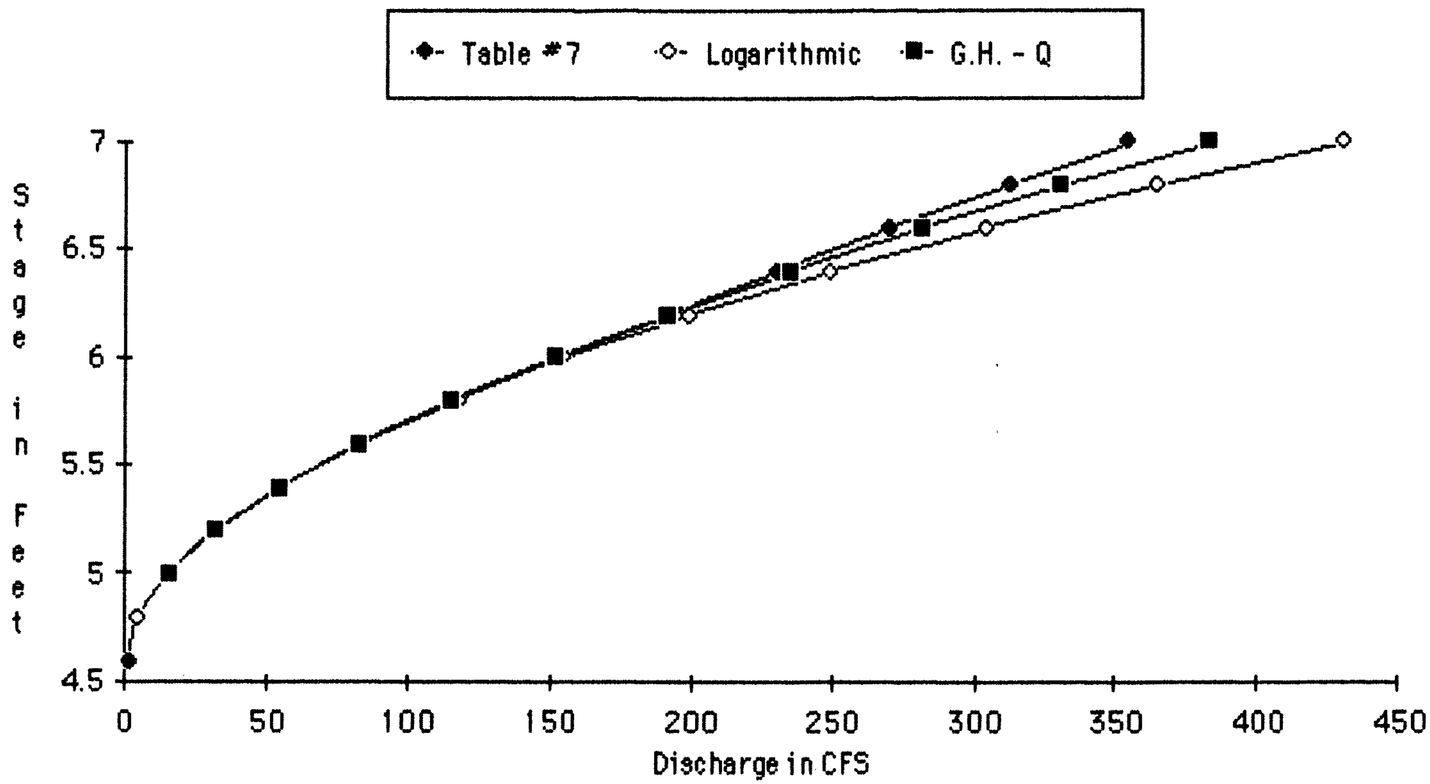


Figure 12 Comparison of Extended Rating Curves - Barlow Creek

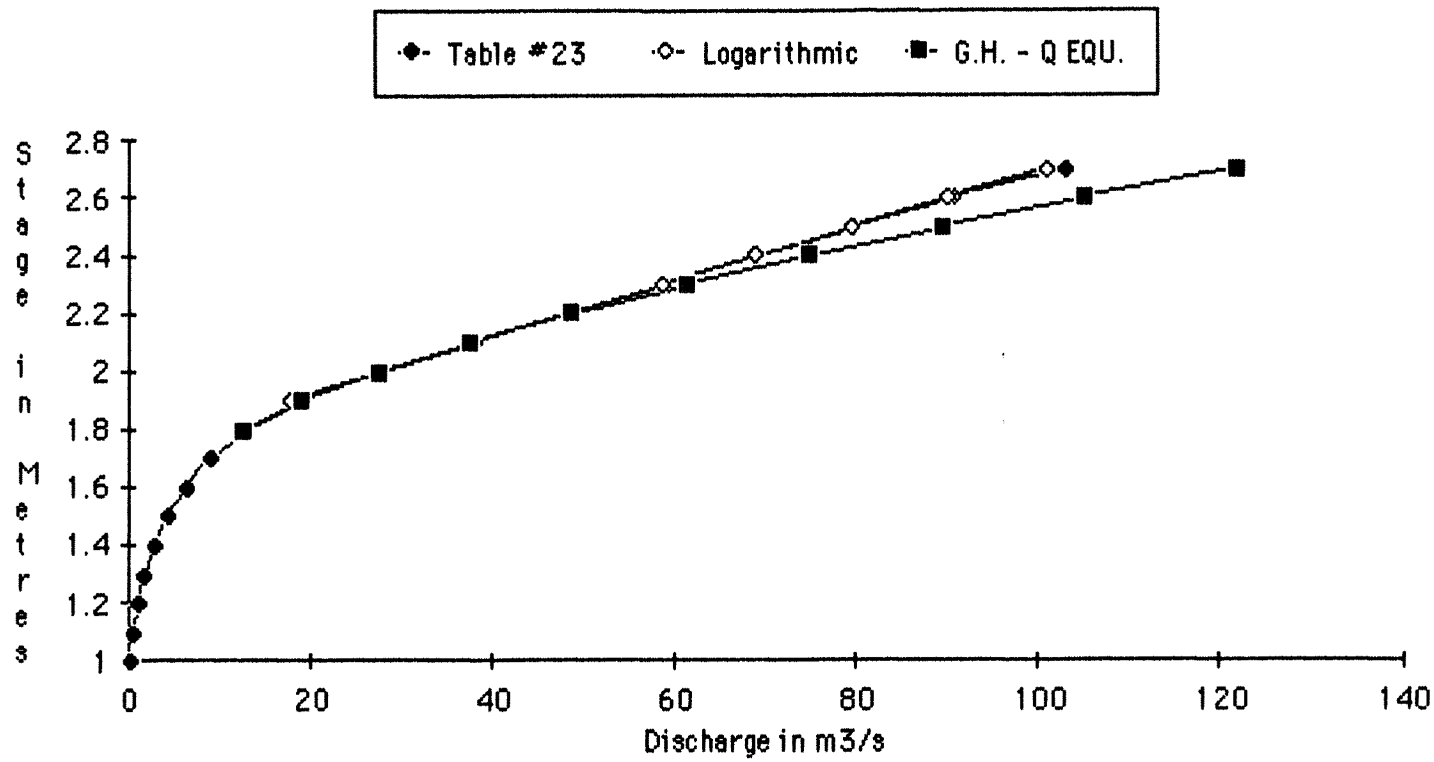


Figure 13 Comparison of Extended Rating Curves - Chapman Creek

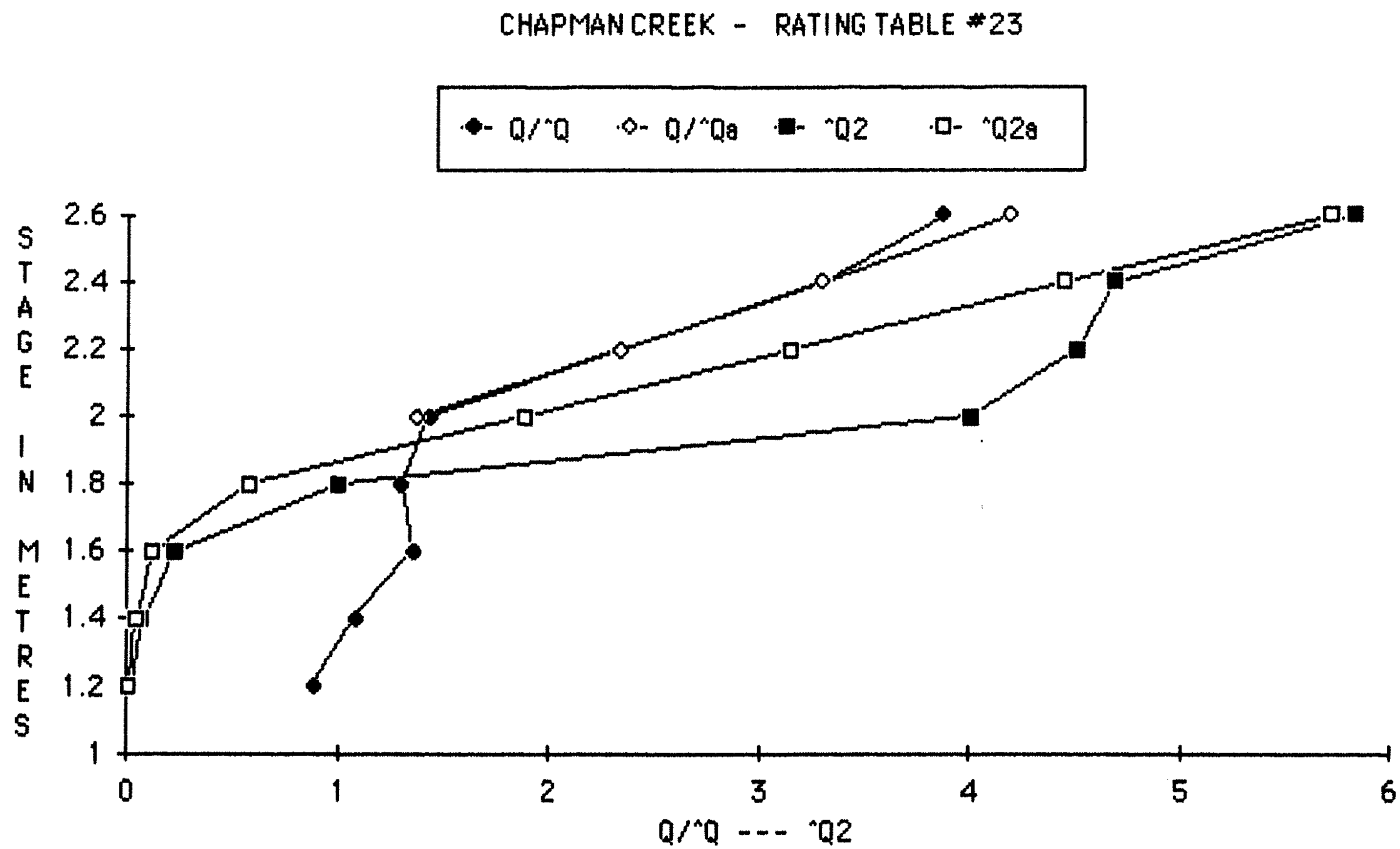


Figure 14 Stage versus Incremental Discharge as Fitted to the Rating Curve by the Logarithmic and Stage-Discharge Equation Curve - Chapman Creek

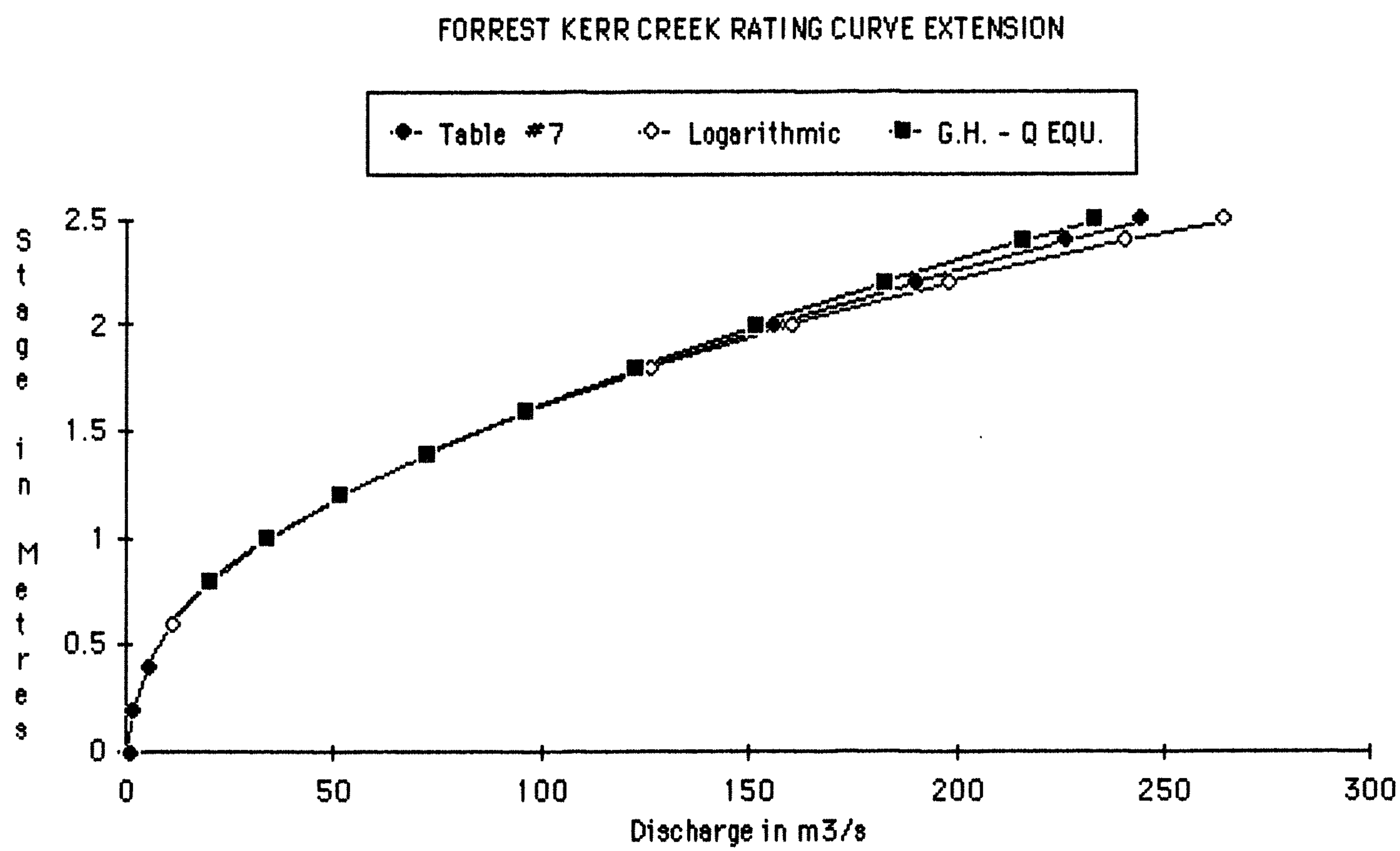


Figure 15 Comparison of Extended Rating Curves - Forrest Kerr Creek

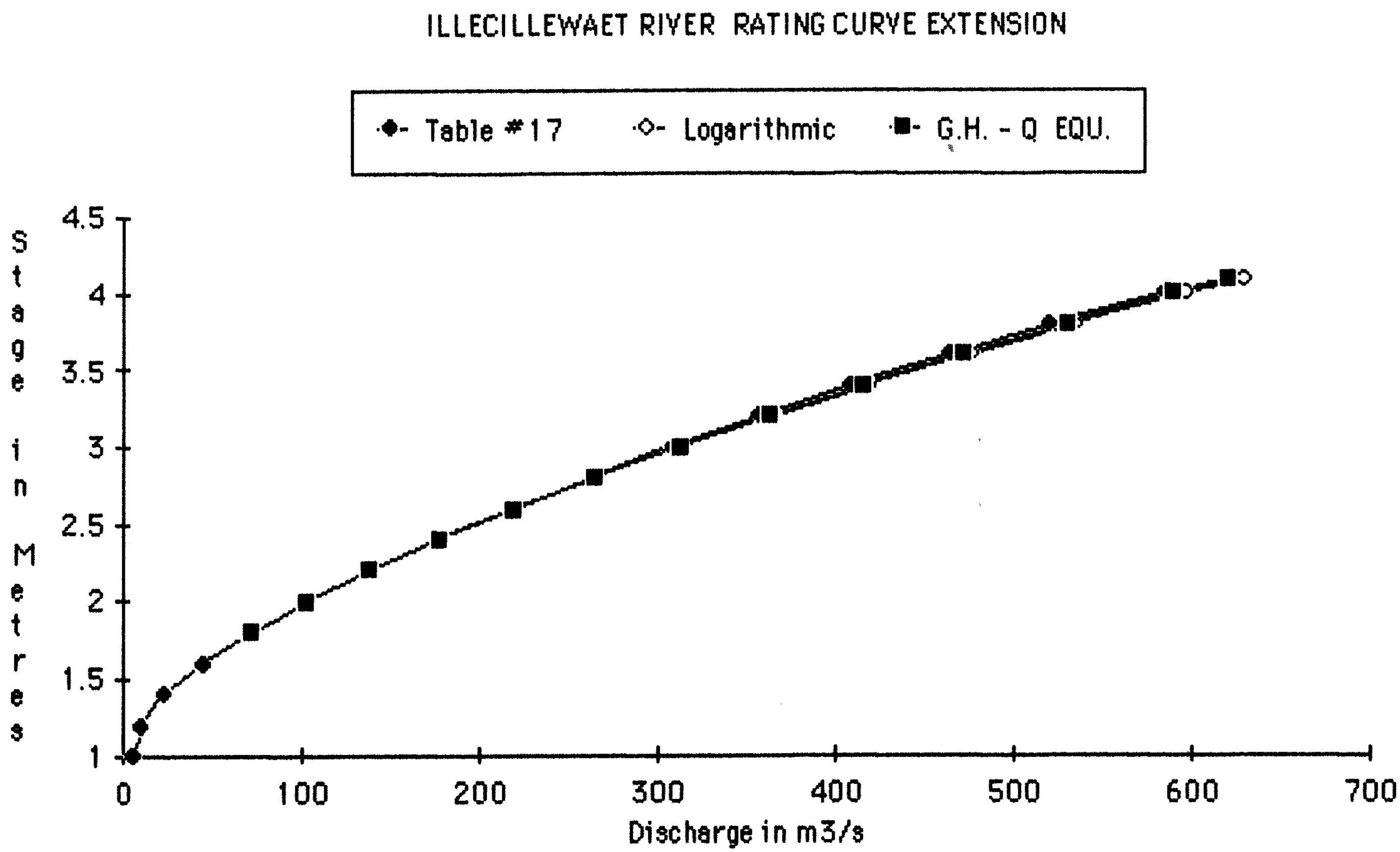


Figure 16 Comparison of Extended Rating Curves - Illecillewaet River - Table #17

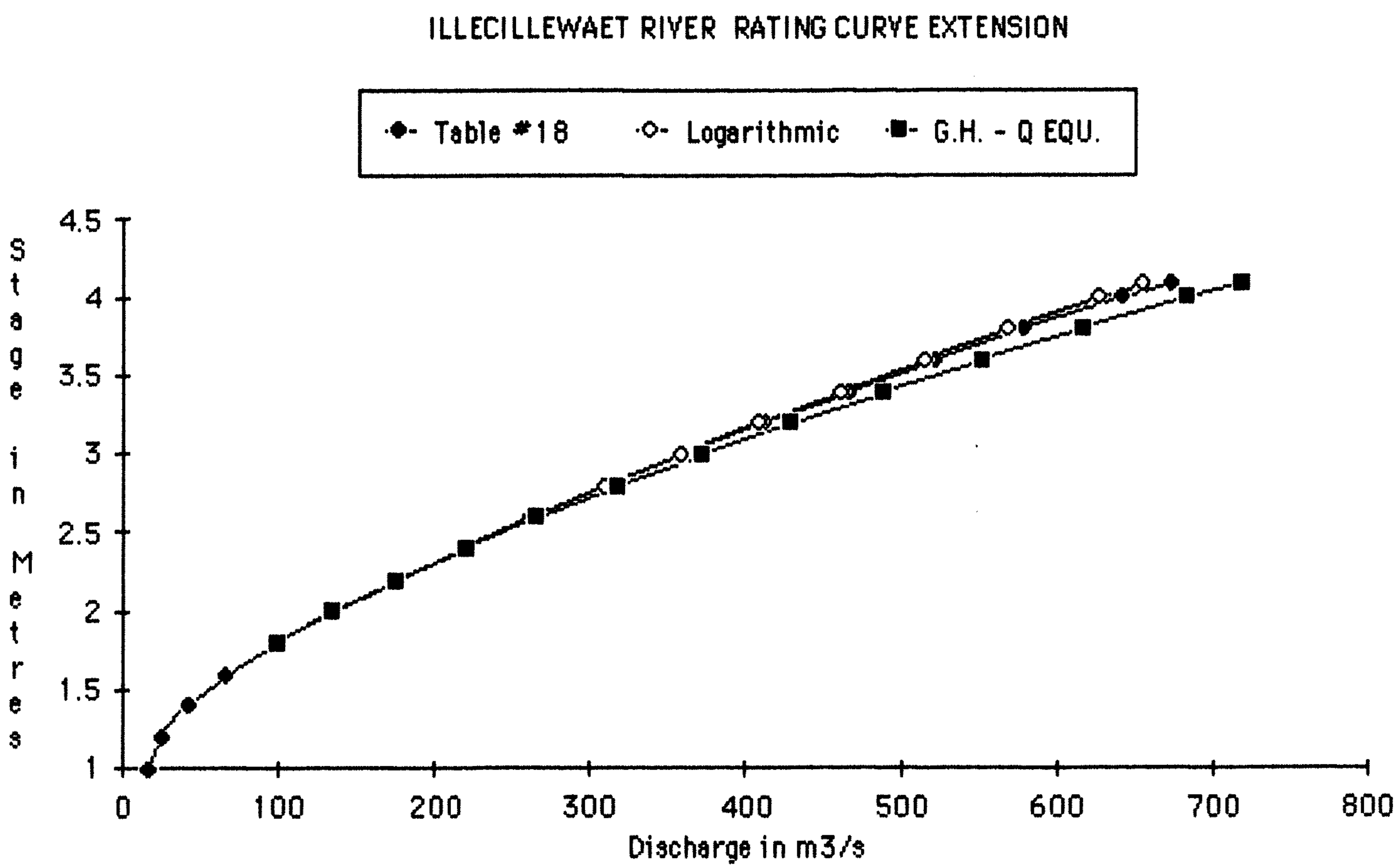


Figure 17 Comparison of Extended Rating Curves - Illecillewaet River - Table #18

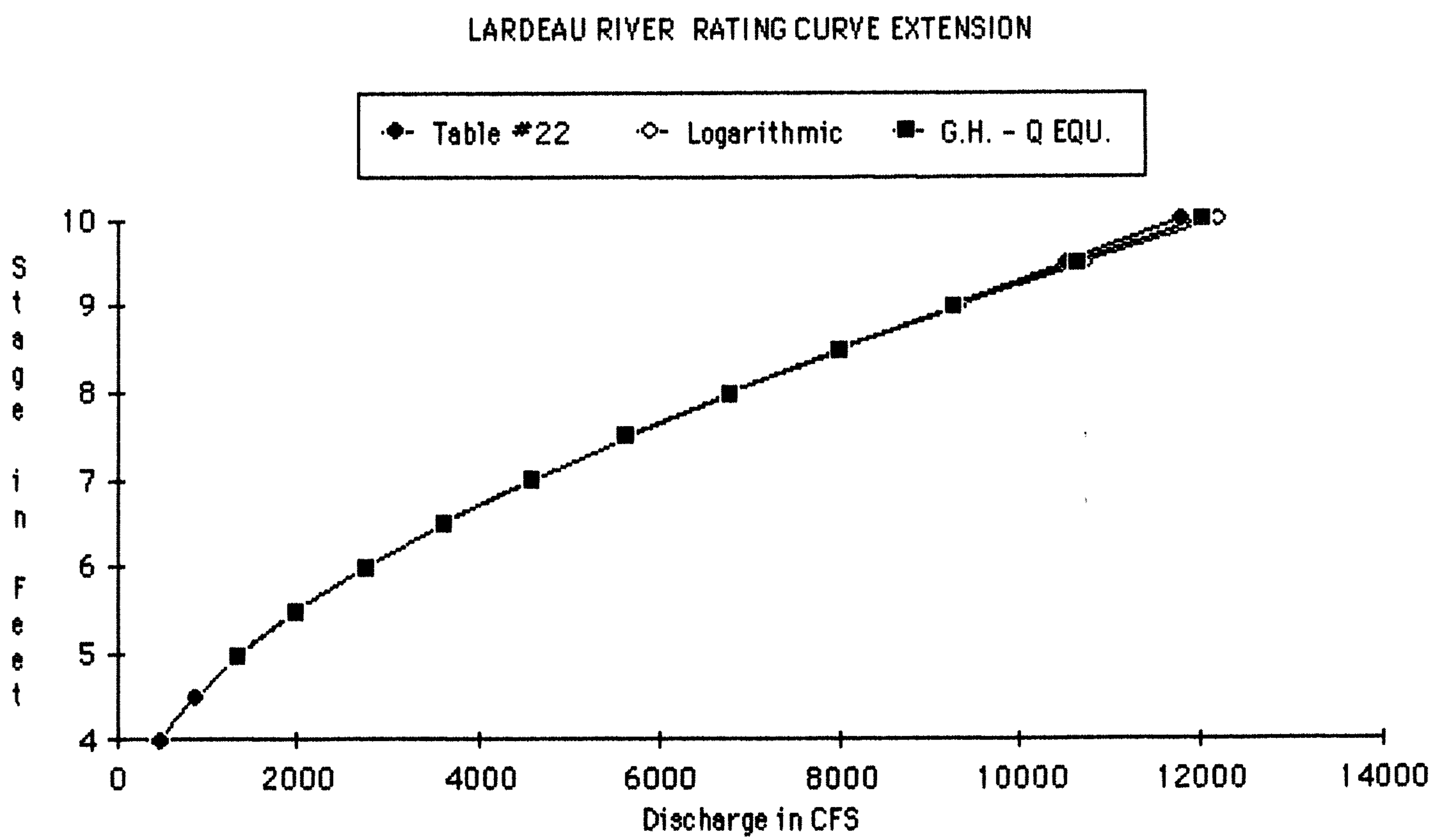


Figure 18 Comparison of Extended Rating Curves - Lardeau River

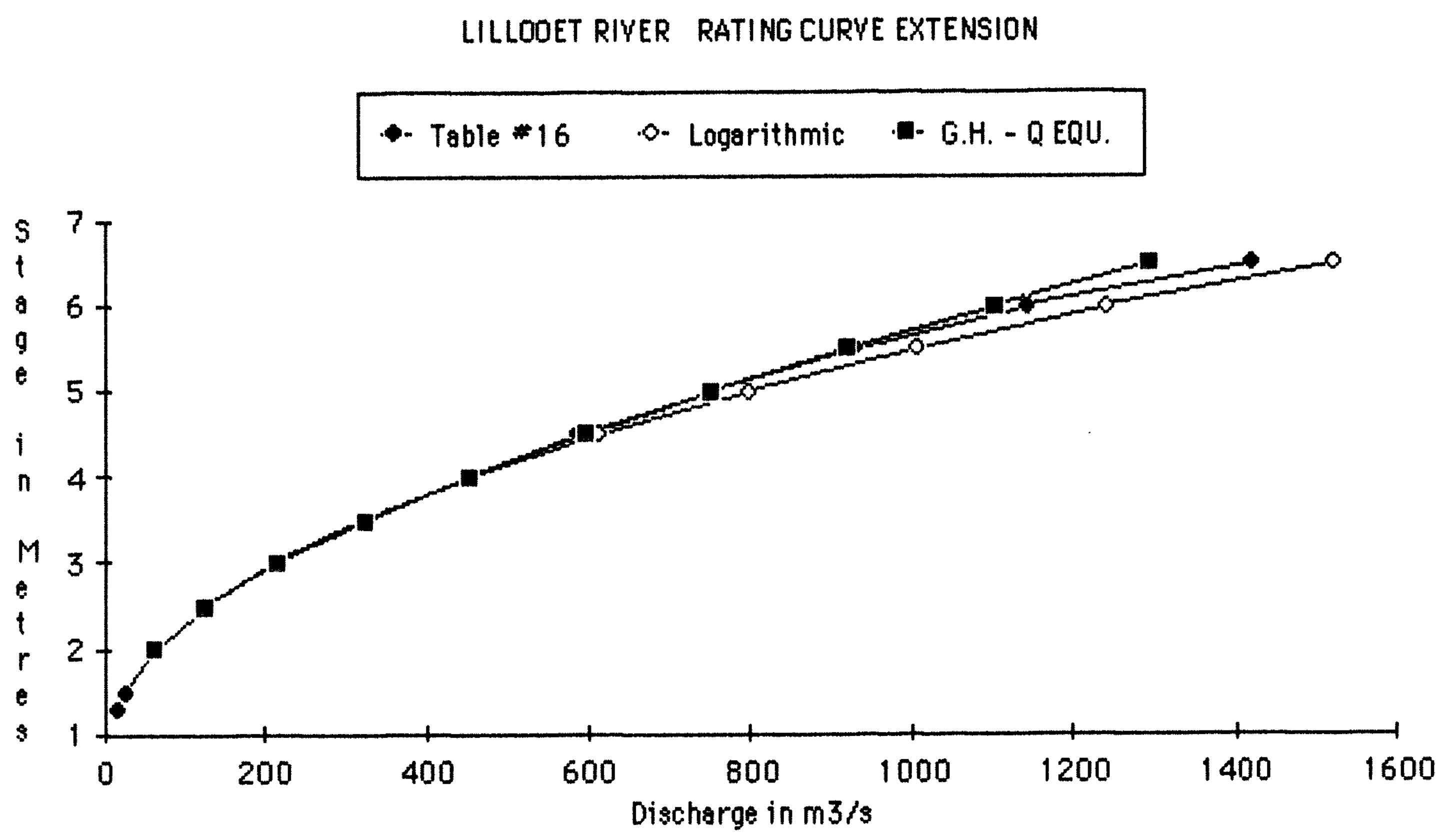


Figure 19 Comparison of Extended Rating Curves - Lillooet River

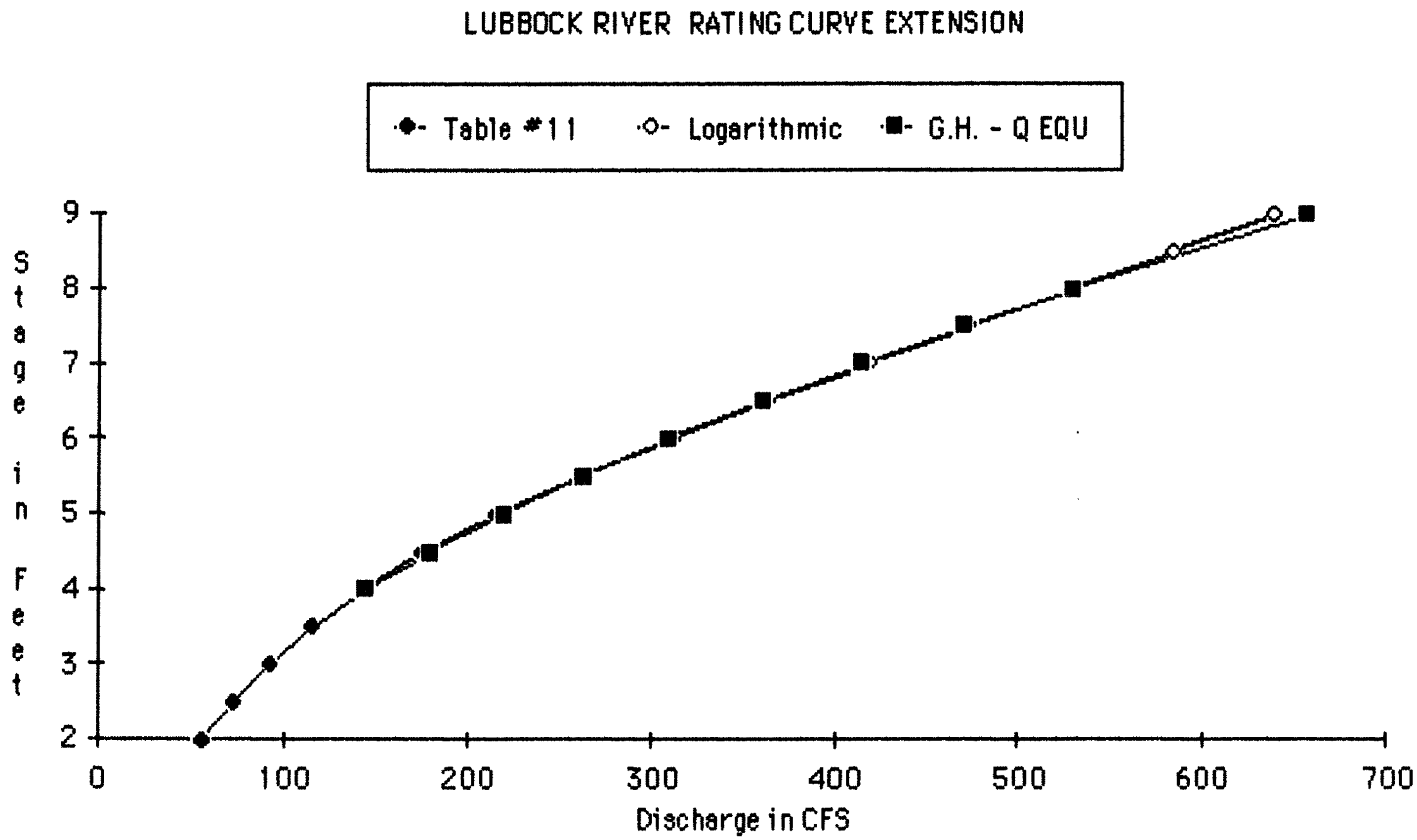


Figure 20 Comparison of Extended Rating Curves - Lubbock River

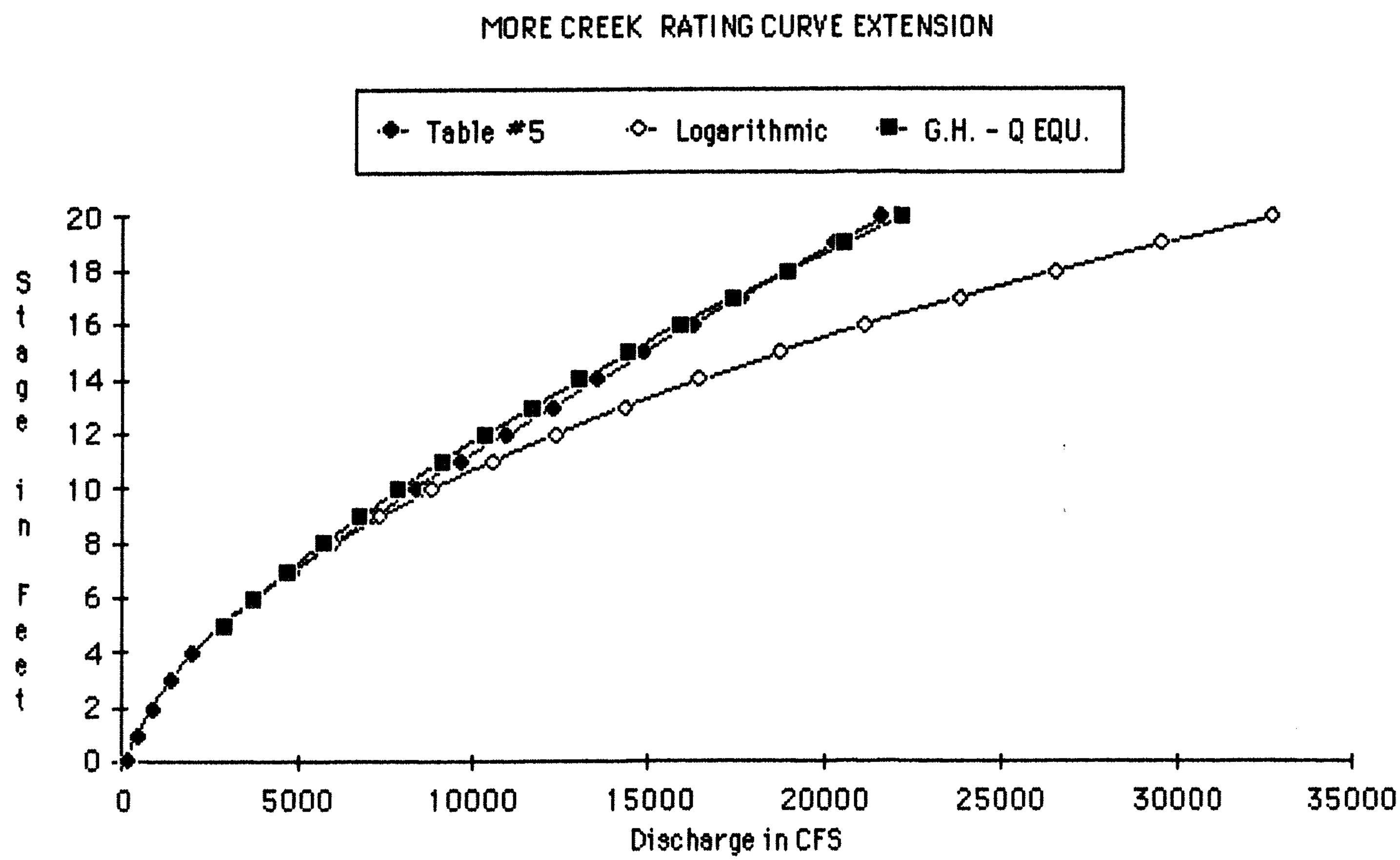


Figure 21 Comparison of Extended Rating Curves - More Creek

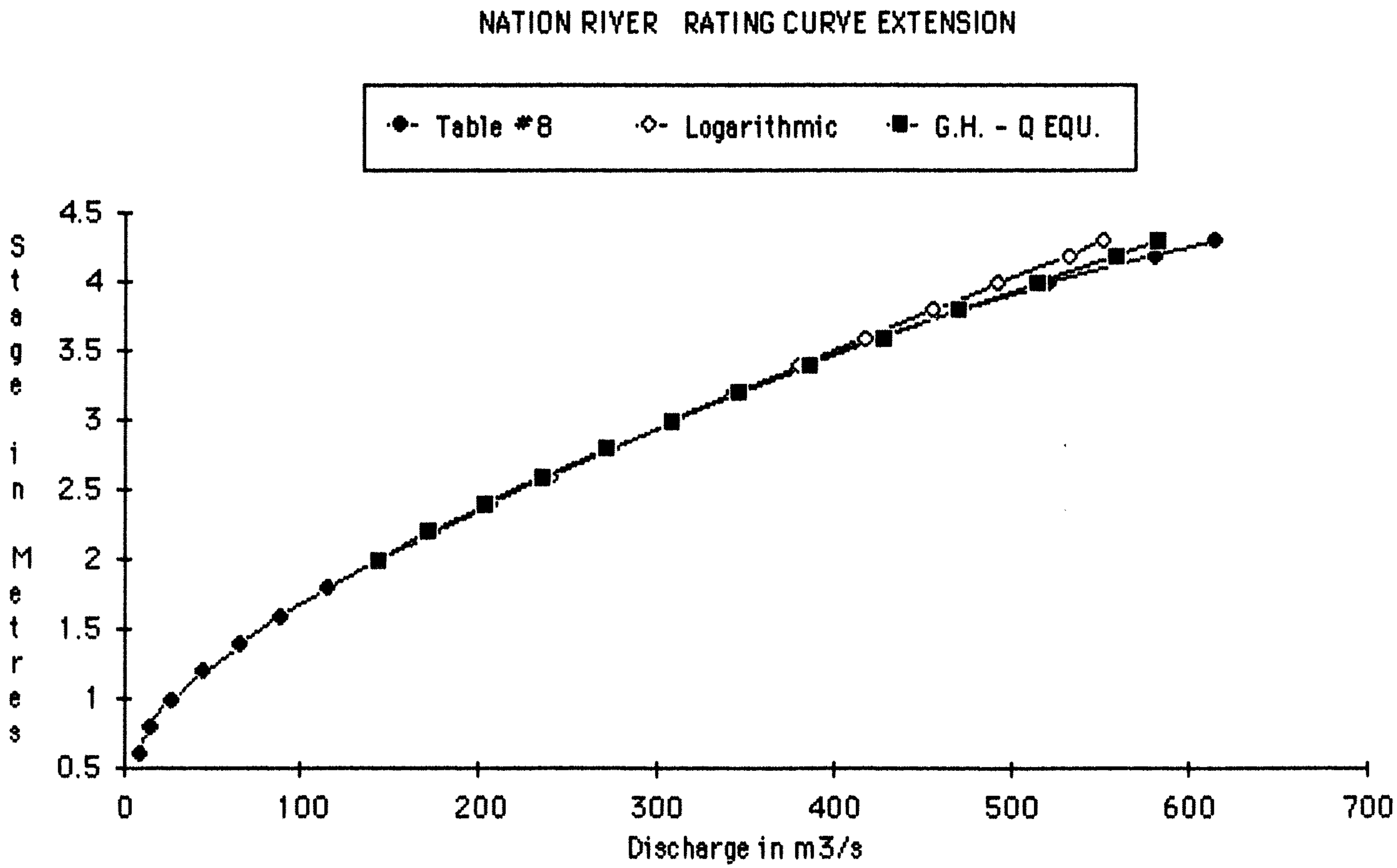


Figure 22 Comparison of Extended Rating Curves - Nation River

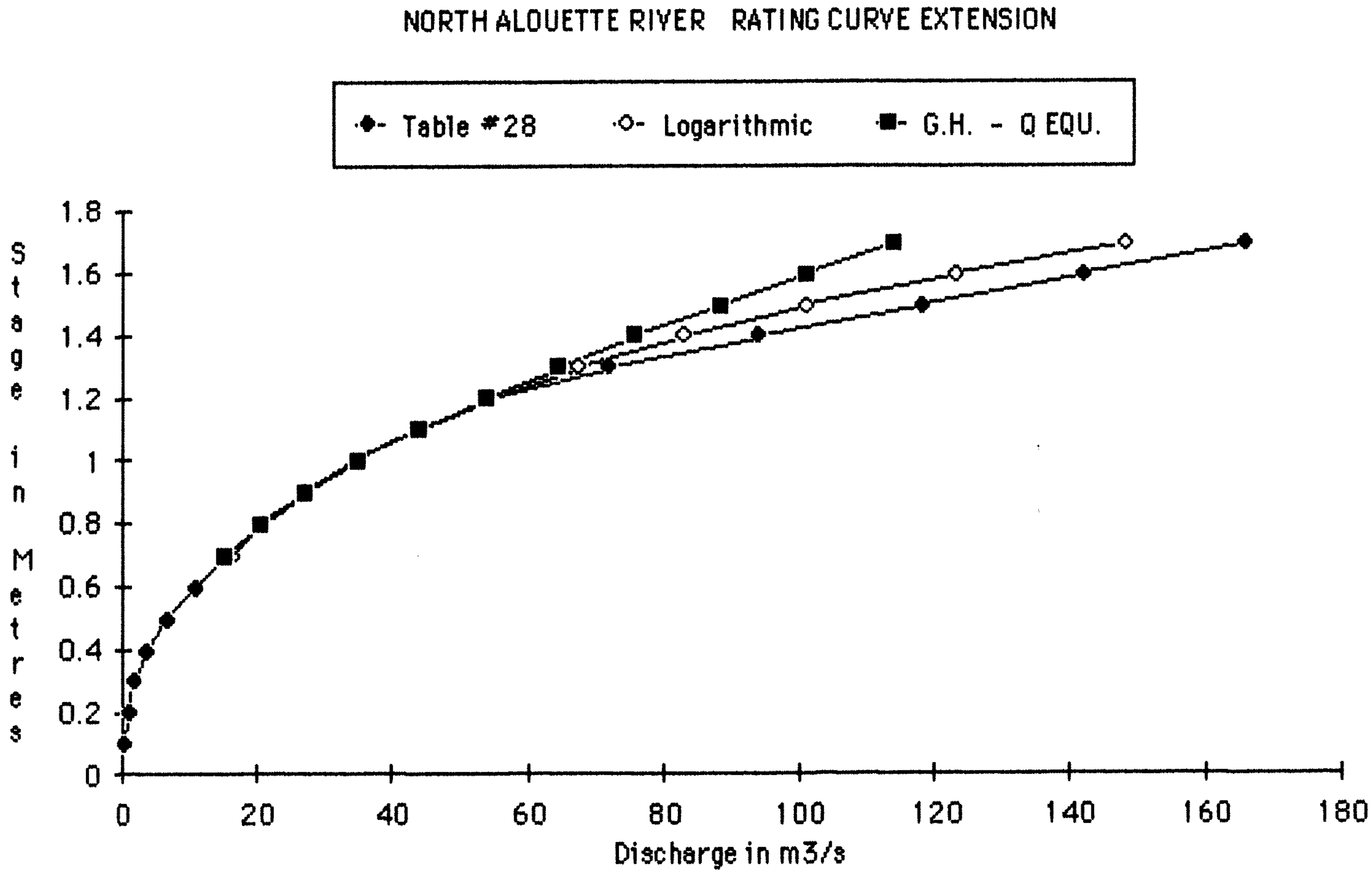


Figure 23 Comparison of Extended Rating Curves - North Alouette River

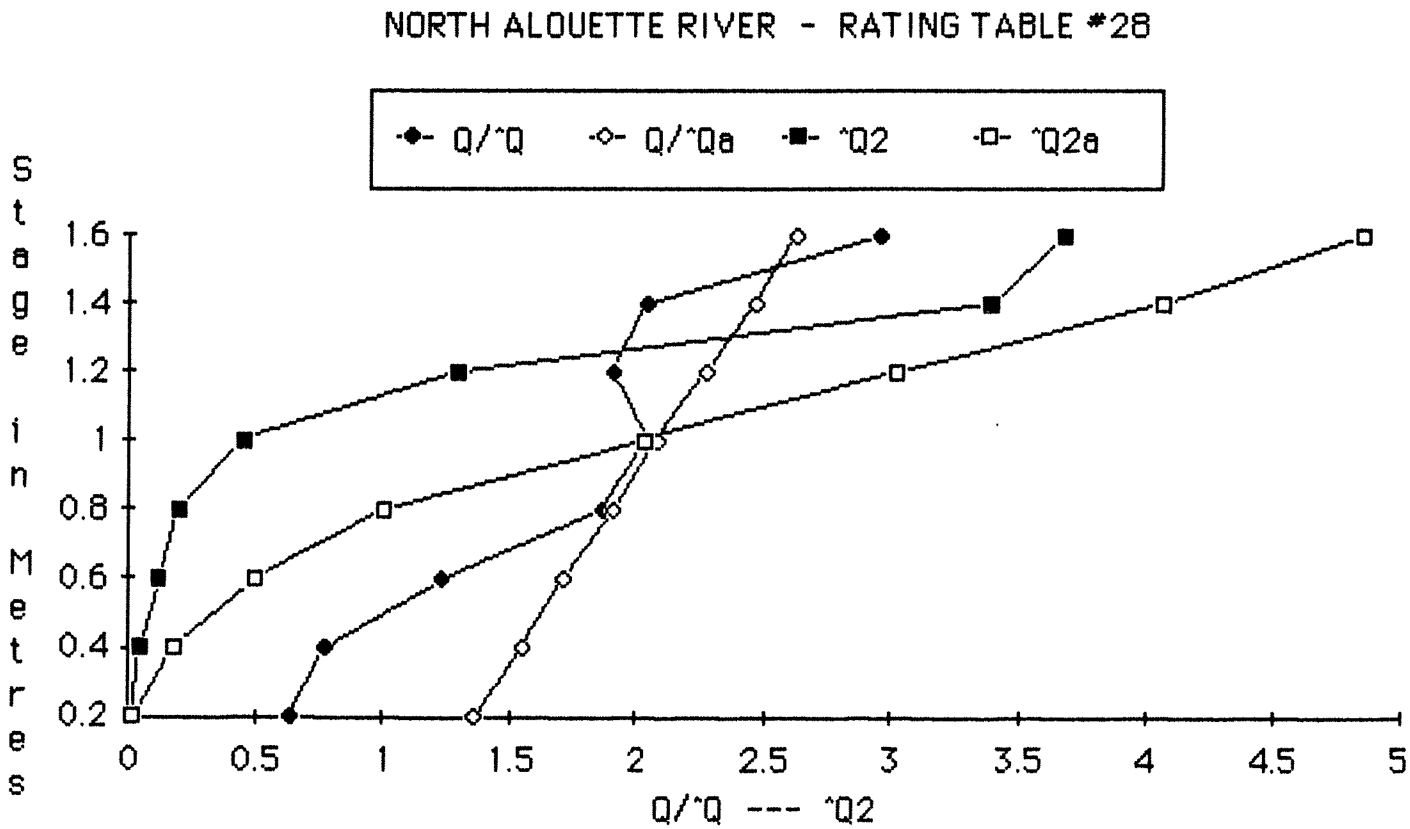


Figure 24 Stage versus Incremental Discharge as Fitted to the Rating Curve by the Logarithmic and Stage-Discharge Equation Curves for North Alouette River