COMMUNICATIONS
RESEARCH
LABORATORY

FRACTAL MODELLING OF FOREST SURFACES FOR ELECTROMAGNETIC WAVE SCATTERING RESEARCH

by

Wei Zhang
Tony Tam
Titus Lo
and
John Litva

Report # CRC CR-95-003

Faculty of Engineering
McMaster University
Hamilton, Ontario
Canada L8S 4K1

CRL Report No. 299
March 1995

Fractal Modelling of Forest Surfaces for Electromagnetic Wave Scattering Research

Wei Zhang, Tony Tam, Titus Lo and John Litva
Communications Research Laboratory
McMaster University, Hamilton, Ontario, Canada

Report# CRC CR-95-003

March 31, 1995

Abstract

This report presents the development of a new model for forest surfaces based on fractal geometry, or more specifically, a two-scale fractal concept. The model is designed for studying the electromagnetic wave scattering of communications signals from a forest. Based on the considerations of forest canopy surface features, the model consists of two parts: Fractional Brownian Motion(FBM) and Bandlimited Weierstrass Function(BWF). Theoretical analysis and computer generated samples of the model are introduced, and surface parameters related to scattering are calculated. Finally, we discuss the potential applications and extensions to the new model.

Contents

1 BACKGROUND

In the research areas of communications and remote sensing, electromagnetic wave scattering from forests and vegetation environments have been intensively studied for a long time. The remote sensing community is interested in relating the field scattered from a forest or vegetation canopy to its geometric and biophysical characteristics. Studies have been conducted on a wide frequency band and at the higher frequencies, for example, from 1GHz to 100GHz. Modelling based on electromagnetic field theory is the general research method. The research carried out by The Wave Scattering Research Center at the University of Texas at Arlington is typical of work that has been carried out in this area. It has resulted in some useful models [1].

The communications community, on the other hand, is interested in studying radio-link performance, and how it is affected by wave attenuation, fading, and co-channel interference, due to the presence of a forest canopy somewhere along the path of linkage. One study for PCS communications at VHF and UHF frequency bands used ray tracing based on the Geometrical Theory of Diffraction. In this method, ray tracing was applied to the forest, which was represented by a Forest Macro Model(FMM). This model regarded the forest as a dielectric layer with an air layer on top and a ground layer on the bottom [2].

The applications of interest in this paper, are land-mobile communications, e.g. cellular telephones, as well as radio and television broadcasting. In these situations, the transmitter is usually elevated well above the trees, and a few kilometers away. Meanwhile, the user is usually at a lower altitude, as in the case of cellular telephones where the receiving antenna is only about 2 meters above the ground. A common situation would involve a vehicle on the road with a forest or bushes on the sides of the road. Trees and bushes may interfere with the propagation of the radio signal in the form of multipath, diffraction, and attenuation.

In order to study the effects of trees and bushes on radio signals, the Communications Research Center, in conjunction with Communications Research Laboratory, is developing models for radio signal propagation through forests with a rough structure. The first phase of our research is devoted to developing a model of the forest canopy, upon which, scattering analysis can be subsequently carried out.

Generally speaking, all existing forest models can be divided into two categories, either phenomenological or physical. The phenomenological models are based on the intuitive understanding of the relative importance of different forest components, such as branches, twigs, trunks, leaves and soil. A scattering model is then constructed by summing up the contributions from each forest component which is believed to be important. On the other hand, physical models are based on the interaction of electromagnetic waves with forest or vegetation canopies. The canopy can be modelled as a discrete or a continuous random medium. The scattered field is evaluated from this random medium. However, the parameters of this random medium must be pre-determined.

The scattering medium model should characterize the effect of a forest environment on electromagnetic signals. Tree models are frequently based on either/or measurements and analytical descriptions. Both random and periodic surfaces, such as Gaussian and sinusoidal surfaces, have been used to model rough surfaces[3]. However, in nature, a rough surface like a forest is neither totally random nor totally deterministic; therefore, these models do not provide a good description of a naturally occurring surface.

In analyzing and generating the complex shapes or structures of natural objects, the most important problem is how to describe the shape of an object efficiently. Usually, most man-made objects, e.g., industrial parts and buildings, can be adequately described by using a set of simple shape primitives such as spheres, cylinders, cones, and cubes. However, such an approach is not suitable for describing natural shapes, since it requires a huge amount of data and computation to describe them in detail, and furthermore, the result is very complex. Thus, an effective modelling scheme which requires less storage and computation is needed to describe naturally occuring shapes. With the advent of fractal geometry theory, accurate descriptions of naturally occurring structures becomes available[4].

Fractals are a family of mathematical functions which are characterized by fractional dimension and self-similarity. A fractal set has a dimension which is typically called Hausdoff-Besicovich dimension and strictly exceeds the topological dimension. Since the theory was founded[4], a considerable number of applications using fractal geometry has been studied in many fields in the past years and achieved great success. It has been pointed out that we can find many fractals in nature, i.e., coastlines, mountains, solid surfaces, branching pattern of trees and rivers. Fractal geometries have also been noticed in the lungs of the human body, clouds, relationships between the magnitude of earthquakes and their frequency, the aggregation of galaxies, and so on[14].

Because of the ability of fractals to model naturally occurring structures, they are being increasingly used in the analysis of complex natural shapes such as ocean surfaces[5], mountain terrains[13], grasslands, clouds[6], and lung tree[15]. Similarly, we have incorporated the concept of fractals into our forest canopy model to obtain a good approximation to the natural forest surface features.

2 RATIONALES AND ASSUMPTIONS

As mentioned before, in the study of electromagnetic wave propagation for communications systems, the forest is often simply modelled as a uniform dielectric layer between an air and ground layer. The most significant ray on the forest is the so-called lateral ray. This ray is propagated along the surface of the forest canopy and it represents a whole class of rays. This model is widely used in the calculation of wave propagation becacuse of its simplicity and clear physical concept. However, at high frequencies, the incident wavelength could be comparable with the surface roughness of a forest, the scattering from the forest components, (for example, the leaves, the twigs, the branches, the trunks), must be considered. A more suitable model is a multi-layered scattering medium model consisting of a crown layer, a trunk layer and a ground interface[16, 17, 18]. The geometry of wave scattering from a forest is shown in Fig.1.

The scattered field at the reciever may include the contributions from a number of multipath signals such as those coming from the forest canopy, the trunks, the ground and combinations of them. In the first phase of our research, we will just consider the scattering from the forest canopy surface.

The forest canopy surface is a special kind of naturally occurring rough surface with some special features and would be different from case to case according to a variety of forest parameters such as: the density of trees, the average height, the species, the season, and so on. From observations, and for convienence, some of the basic features and assumptions of forest surfaces that are considered in our research are as follows:

1. Different trees have different basic shapes, depending on their species. We consider forests which consists of a similar type of trees.

2. Forest surfaces exibit, to some degree an element of random periodicity. This can be explained by the fact that all forests consist of many individual trees, which are separated according to a certain natural rule. Furthermore, their spatial volume is dependent on their age and species.

3. The canopy surfaces are assumed to be continuous, in other words, we don't consider

the surface of a single tree or of a sparse forest.

4. The surface roughness of a single tree comes from the random growing process. The spatial random distribution of trees in a forest is dominated by a random process based on competition among trees. We assume that there are two stochastic independent processes.

5. In our research, we will first consider one typical geometry of wave scattering from a forest enviroment in communications. In this scenario, both the transmitter and the receiver are over the surface of the forest as shown in Fig.1.

3 MODELLING FOREST SURFACES

3.1 Two scale composite model

Based on the preceding analysis and considerations about the basic features of a forest, the surface is modelled as two independent components: a small-amplitude, high-frequency roughness superimposed on a low-frequency variation of larger amplitude. Such a technique is called a two-scale model of roughness. The high frequency component results from the random growing process of a single tree, so that the spatial frequency can be related to the wavelength of the incident wave. The low frequency component is due to the random process of tree distribution and should appear as random in amplitude and periodicity. A more thorough treatment on the theory and applications of a composite model can be found in references[3,8]. The references describe the general method used for generating composite surfaces by superimposing two simple surfaces which are formed by filtering Gaussian noise. In our research, the Fractional Brownian Motion(FBM) and Bandlimited Weierstrass Function(BWF) are used separately and in combination, to model a forest canopy surface. FBM is especialy suited to describing natural fractals with random self-similarity. Conversely, the BWF efficiently creates natural fractals with random periodicity. Both functions have good analytical properties in mathematics, and have been successfully applied in a number of fields and applications[4, 5, 6, 7].

In the one dimensional case, we have the general expression of a forest surface as

$$f(x) = B(x) + S * W(x) \tag{1}$$

where B(x) is the FBM, W(x) is the BWF with mean zero, and S is the scale factor to be adjusted for different applications. Because W(x) is independent of B(x), we can easily obtain all of the statistics for f(x) by summing the statistics of W(x) and B(x). Thus, f(x) has two fractal dimensional parameters, H_b and H_w, which correspond to FBM and BWF respectively, and they reflect the roughness of the surface on two different scales. We can define the fractal dimension H of f(x) with a set, or a pair, of parameters as: $H = (H_b, H_w)$. In this sense, the surface function, f(x), can be called a two-scale fractal.

3.2 Bandlimited Weierstrass Function(BWF)

The BWF is often used as the model of a rough surface because of its good analytical properties such as, a finite spatial frequency band and self-similarity over the corresponding finite range of resolution. We use it to model the basic part of the forest surface. The typical BWF is expressed as a weighted sum of periodic functions[9]

$$W(x) = A \sum_{m=N_1}^{N_2} C_m \lambda^{-mH_w} \sin(K\lambda^m x + \phi_m), \tag{2}$$

where $0 < H_w < 1$ is the roughness fractal dimension of the BWF, K is the fundamental spatial wave number, $\lambda > 1$ is the spatial frequency scaling parameter, ϕ_m is an arbitrary phase with uniform distribution on $[0, 2\pi]$. The number of tones or spectral lines is given by $N = N_2 - N_1 + 1$. The amplitude control factor A is

$$A = \sqrt{\frac{2\sigma_w^2(1-\lambda^{-2H_w})}{\lambda^{-2N_1H_w}(1-\lambda^{-2(N_2-N_1)H_w})}} \tag{3}$$

where σ_w is the given variance. The correlation and power spectrum are

$$C_w(\tau) = E[W(x+\tau)W(x)] = \frac{A^2}{2} \sum_{m=N_1}^{N_2} \lambda^{-2mH_w} \cos(K\lambda^m \tau), and \tag{4}$$

$$P(\omega) = \frac{A^2}{2} \sum_{m=N_1}^{N_2} \lambda^{-2mH_w} [\delta(\omega - K\lambda^m) + \delta(\omega + K\lambda^m)]. \tag{5}$$

From references[5,7], we can find a similar discussion dealing with these parameters. Here the random amplitude C_m, with standard normal distribution, does not affect the statistical properties, but will improve the surface randomness.

3.3 Fractional Brownian Motion(FBM)

The one dimensional function FBM, B(x), is a random function of coordinate x, and is defined by [4] as

$$P\{\frac{B(x+dx) - B(x)}{|dx|^{H_b}} < y\} = \frac{1}{\sqrt{2\pi}\sigma_b} \int_{-\infty}^{y} exp(\frac{-t^2}{2\sigma^2})dt, \tag{6}$$

where $0 < H_b < 1$ is the fractal dimension of FBM, σ_b is the given normal distribution of the variance, dx is an increment. Sometimes, the increment process of FBM, $\Delta B(x)$ is also defined as $\Delta B(x) = B(x + dx) - B(x)$. Here are other statistical features:

1. Mean: $E[B(x + dx) - B(x)] = 0$
2. Variance: $E[|B(x + dx) - B(x)|^2] = \sigma^2 |dx|^{2H_b}$
3. Covariance: $E[B(x)B(t)] = \frac{1}{2}\sigma^2[|x|^{2H_b} + |t|^{2H_b} - |x - t|^{2H_b}]$
4. Correlation function:

$$C_{\Delta B}(\tau) = E[\Delta B(x + \tau)\Delta B(x)] = \sigma^2 |dx|^{2H_b} r(dx, \tau) \tag{7}$$

   where, when $\tau \ll dx$, the normalized correlation function $r(dx, \tau) = 1 - |\tau/dx|^{2H_b}$

5. Power spectrum:

$$P(\omega) = \sigma_b^2 |dx|^{2H_b} \delta(\omega) + \frac{\sigma 2}{\omega^{2H_b+1}} \Gamma(2H_b + 1) \tag{8}$$

   where $\Gamma(\bullet)$ is the Gamma function. $\delta(\omega)$ is a unit pulse function at zero.

4 COMPUTER SIMULATED SURFACES

The formulation and analysis for the one-dimensional(1D) case can be easily extended to the two-dimensional(2D) case. Thus, with the aid of a computer, we are able to create a 2D fractal surface.

The basic Weierstrass surface is shown in Fig.3. When N1 and N2 are adjusted, the basic spatial frequency of the surface, or the field of view of the incident wave, will also change. If other periodic functions are used instead of the sine function in (2), different basic shapes can be generated to represent different tree species. Two examples are shown in Fig.4.

For the approximation of the FBM surface, the most straightforward method is the Random Midpoint Displacement(RMD)[10]. This algorithm is simple and suitable for execution on a computer, but unfortunately, the process generated by the RMD does not have stationary increments, thereby preventing it from yielding a true FBM. To obtain a better approximation of the FBM surface, we developed a new method called the Related RMD[11,12], which has not only the same advantages as RMD, but can also allow the incorporation of a particular correlation function in the surface model.

The related midpoint displacement method can work with a square lattice of points. If the mesh size dx denotes the resolution of such a grid, we obtain another square grid of resolution $dx/\sqrt{2}$ by adding the midpoints of every square. At the same time, the orientation of every square lattice is rotated by 45 degrees. The two types of interpolation lattice are shown in Fig.2.

The interpolation formulas used here are

$$f(x_0) = \frac{1}{2}(1-2^{-H})[f(x_{11})+f(x_{21})] + \frac{1}{2}(2^{-H})[f(x_{12})+f(x_{22})] + 2^{1-\frac{H}{2}}\sqrt{1-r}\sigma|dx|^H g_1 \quad (9)$$

, and

$$f(x_0) = \frac{1}{2}(1-2^{-H})[f(x_{11})+f(x_{21})] + \frac{1}{2}(2^{-H})[f(x_{12})+f(x_{22})] + 2^{1-H}\sqrt{1-r}\sigma|dx|^H g_2 \quad (10)$$

for the type 1 and type 2 respectively, where r is the correlation factor between the midpoint and its adjacent points, H is the fractal dimension, σ is the variance of the FBM and $g_1 and g_2$ are the samples of a Gaussian random variable with mean 0 and variance 1 added on the midpoint. This process is continued until we obtain the expected resolution. In each stage we also use the successive random addition method to all of the points after the midpoint interpolation to get a better stationarity. By choosing r according to different small scale corralation functions, different surface roughness can be obtained. Two samples generated using RRMD are shown in Fig.5. The composite surface is derived by superimposing B(x) with W(x) and is shown in Fig.6.

When carrying out the superimposing, different scales of surface roughness can be obtained by adjusting the scale factor S. At the same time, the random periodicity on a surface will disappear with decreasing S. In this way, the modelling program can also generate periodically random surfaces for different cases such as a bushy or mountainous region.

The statistics of a sample surface (mean, variance, correlation function and correlation length, power spectrum, slope variance, etc.) were calculated and are compared with the results of theoretical analysis in Fig.7 to Fig.10. Fig.8 shows the correlation functions of the FBM, BWF and composite surface. It is apparent that the FBM and BWF have different fractal features in small scale so that the composite surface will have different statistics depending on the different S value. From Fig.9, we find that the BWF and FBM have different contributions to the final composite surface in terms of frequency. Fig.10 shows that the composite surface has the typical power law with the fractal between $1/f$ and $1/f^2$. These results show that the theoretical analysis and calculation (dot and dash lines) are consistent to the statistics of the sample surface(solid lines).

5 CONCLUSIONS AND RECOMMENDATIONS

In this report, a new two scale fractal model of a forest canopy has been developed as the first phase of our research on electromagnetic wave scattering from a forest environment. Following is a summary of some of the major aspects of the canopy model:

1. In our new model, the basic features of a forest surface, such as the random growth of trees and the periodically random distribution of trees in a forest, are reflected by combining two typical fractal functions, namely the FBM and BWF.

2. The composite fractal model is described by using a pair of fractal dimensions and is much more versatile than using a single fractal model. A number of user selectable parameters allows the creation of not only forest canopy surfaces, but other natural surfaces such as mountain regions.

3. Along with the fractal surface, we have also derived the basic statistics of the FBM and BWF, specifically, the correlation function and power spectrum. These statistics allow the model to be analyzed in a statistical manner.

4. The realization of a computer generated two dimensional surface is achieved by using the RRMD and BWF calculation algorithms. A comparison between theoretical analysis and statistics calculated from generated surfaces show a good consistency.

Thus, we now have a fractal-based forest canopy model which allows us to efficiently generate a canopy surface for a variety of forest situations. Currently, work is being carried out on the next phase of research which involves both analyzing the scattering from the fractal surface as well as improving the model itself. Following is a list of topics which will be investigated to improve the model.

1. An investigation on how the main parameters, such as N, S, H_w, H_b, which change the basic fractal shape, can be changed so that the model can be extended to different applications such as bushes or mountain areas.

2. Currently, the canopy model is a static one, more research is required in terms of tree movements so that we can add a time varying characteristic to the model. One approach which is being investigated, is the use of spatial filtering techniques. Such techniques may be used to take into account the variability of a forest with time and seasons

3. We will consider the vertical density characteristics as well as the dielectric properties of the surface itself.

In terms of analyzing the scattering from the fractal model, a future report will detail the investigation. In particular, for scattering analysis, the fractal model is broken up into planar facets which at first will be considered to be conducting. Preliminary analysis will also only consider 1st order reflections and neglects shadow region effects. Furthermore, we are considering the case where both the transmitter and receiver are above the canopy.

Thus far, a computer program has been written which calculates the orientation of each facet in terms of its relation to the transmitter and receiver. Based on its orientation, or if the facet resides in a shadow region, a facet is kept if a 1st order reflection is possible. A summation of the scattered fields from the eligible facets is then made to determine the scattering effect of the fractal surface. To date, such an analysis is not applicable for a forest environment because of the assumption of a conducting surface. The next step in the scattering analysis will be to treat the model as a dielectric surface.

References

[1] A.Karam, F.Amar and A.K.Fung, "Electromagnetic wave scattering from a forest or vegetation canopy: ongoing research at th University of Texas at Arlington," IEEE Antennas and Propagation Magazine, Vol.35, No.2, pp18-26, 1993

[2] T.Kurner, J.Cichon and W.Wiesbeck, "Concepts and results for 3D digital terrain-based wave propagation models: an overview," IEEE Journal on selected area in comm., Vol.11, No.7, pp1002-1012, 1993

[3] J.A.Ogilvy, *Theory of Wave Scattering From Random Rough Surfaces*, Adam Hilger, Bristol, 1991

[4] B.B.Mandelbrot, *The Fractal Geometry of Nature*, freeman, San Fransisco, Calif. 1983

[5] J.Chen, "Scattering of electromagnetic wave from a time-varying fractal surface," Microwave and Optical Tech. Letters., pp87-91, 1993

[6] A.P.Pentland, "Fractal-based description of nature scene," IEEE Trans. Pattern Analysis and Machine Intelligent, pp661-674, 1984

[7] D.L.Jaggard, "Scattering from bandlimited fractal fibers," IEEE Trans. Antenna and Propagation, Vol.37, No.12, pp 1591-1597 1989

[8] A.K.Fun and M.F.Chen, "Numerical simulation of scattering from simple and composite random surface," J.Opt.Soc.Am. Vol.2, No.12, pp2274-2284, 1985

[9] K.Falconer, *Fractal Geometry: Mathematic Foundations and Applications,* John Wieley and Sons press, 1990

[10] A.Fournier, D.Fussel and L.Carpenter, "Computer rendering of stochastic models," Comm. ACM, Vol.25, No.6, pp371-384, 1982

[11] W.Zhang, Ph.D Thesis, South China University of Technology, Guangzhou, P.R.China, 1992

[12] W.Zhang and Y.Yu, "A fractal based method for image compression," Proc.of ICSP'90, pp931-934, Beijing, 1990

[13] N.Yokoya and K.Yamamoto, "Fractal-based analysis and interpolation of 3D natural surface shapes and their application to terrain modeling," Computer Vision, Graphis and Image Processing, Vol.46, pp284-302, 1989.

[14] M.F.Barnsley, *Fractals Everywhere,* Academic Press, 1988.

[15] T.R.Nelson and D.K.Manchester, "Modeling of lung morphogenesis using fractal geometries," IEEE Trans. on Medical Imaging, Vol.7, No.4, pp321-327, 1988.

[16] S.S.Seker, "VHF/UHF radiowave propagation through forests: modelling and experimental observetions," IEE Proceedings-H, Vol.39, No.1, pp72-78, Feb.1992.

[17] M.A.Karam, "A miccrowave sccattering model for layered vegetation," IEEE Trans.Geoscience and Remote sensing, Vol.30, No.4, pp767-784, 1992.

[18] F.T.Ulaby, "Michigan microwave canopy scattering model," Int.J.Remote sensing, Vol.11, No.7, pp1223-1253, 1990.

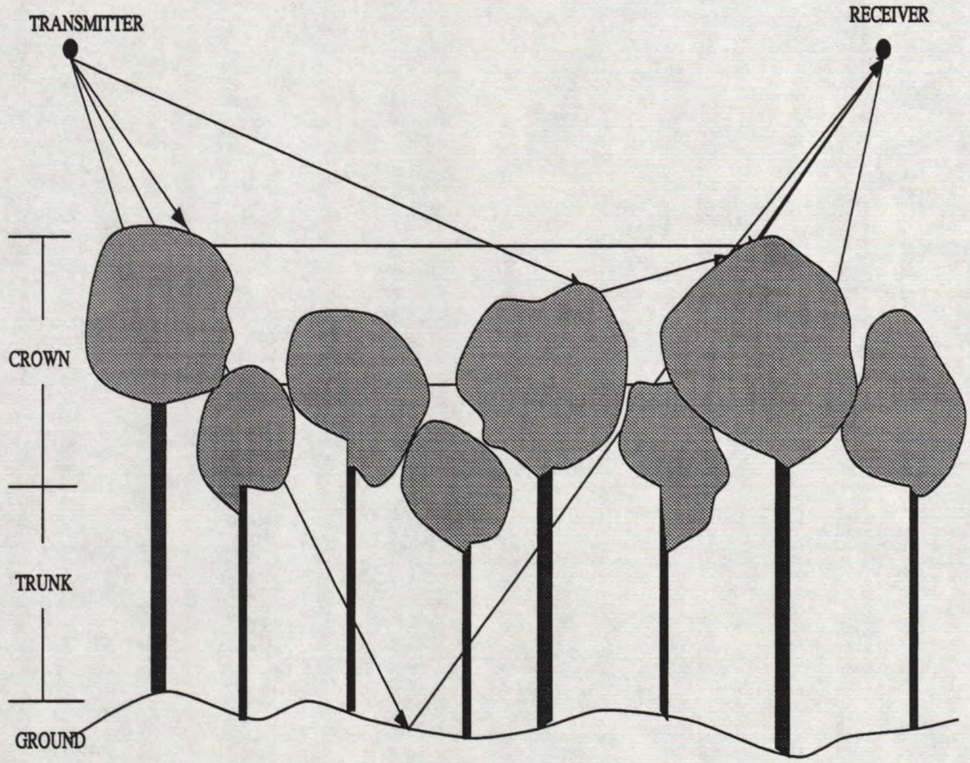

Figure 1: The geometry of wave scattering from a forest.

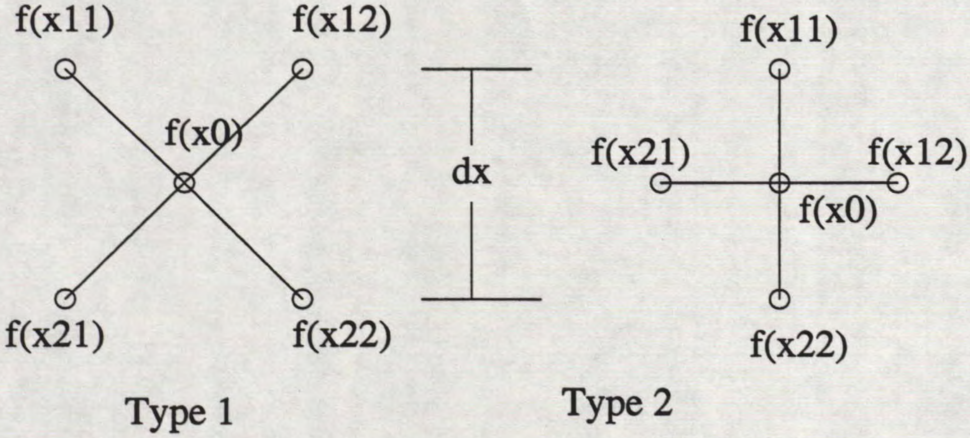

Figure 2: Grid types for 2-D Related Random Midpoint Displacement method.

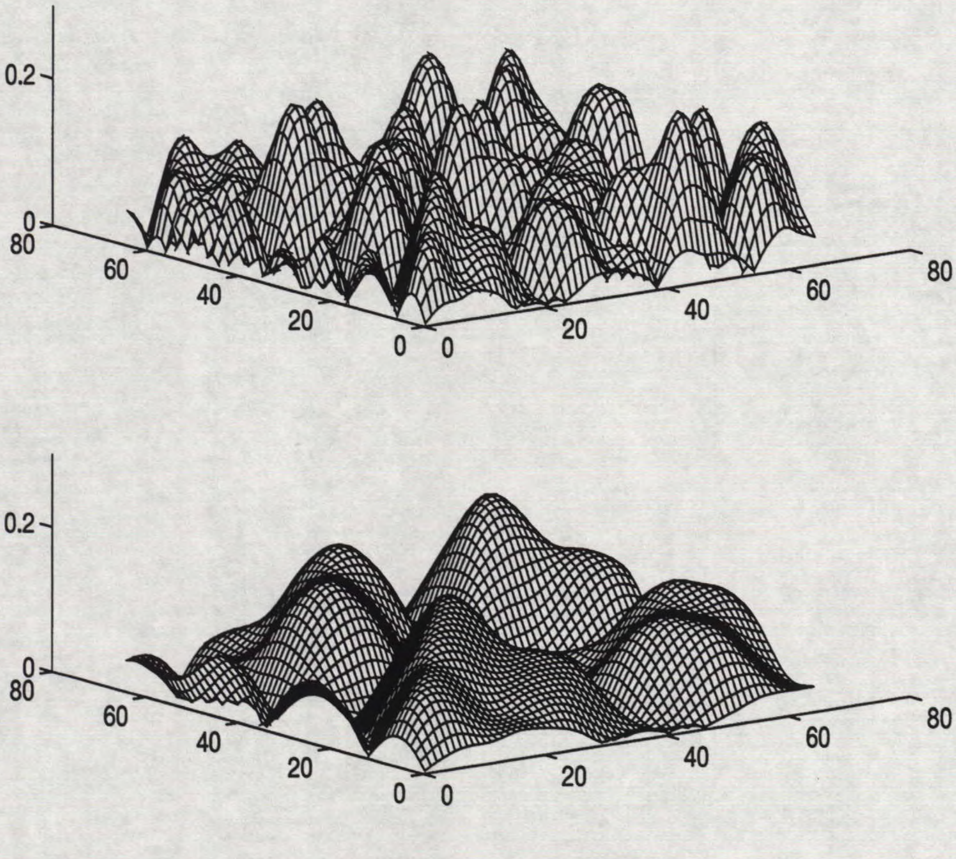

Figure 3: The BWF sample surfaces with the different field of view, where $H_w = 0.5, \sigma = 2.0, \lambda = 1.5$.

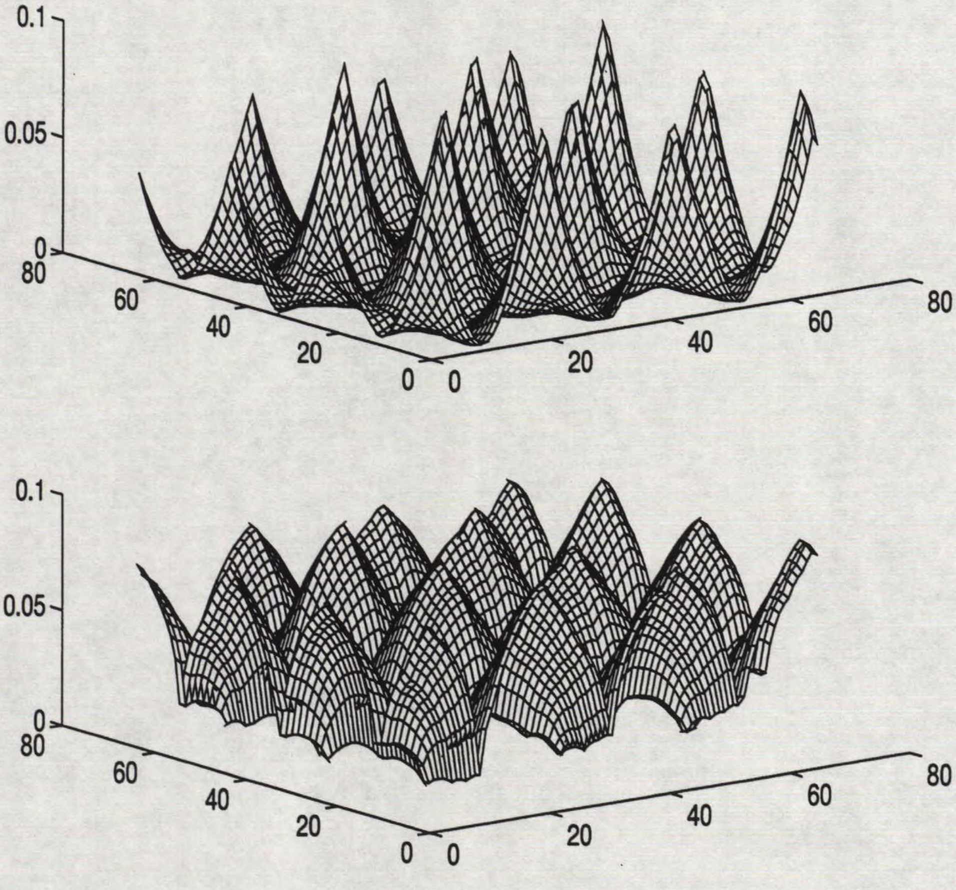

Figure 4: The BWF sample surfaces with the different basic functions.

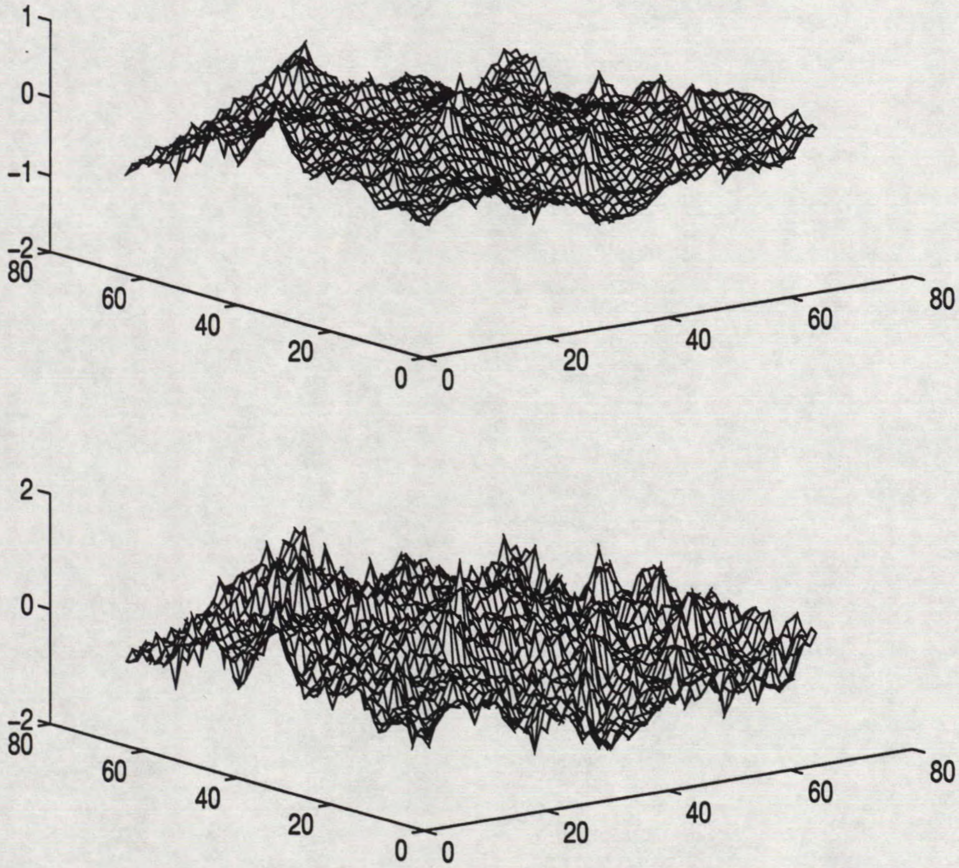

Figure 5: The sample surfaces of FBM with the different fractional dimensions of $H_b = 0.2$ (the upper) and $H_b = 0.5$ (the lower) respectively.

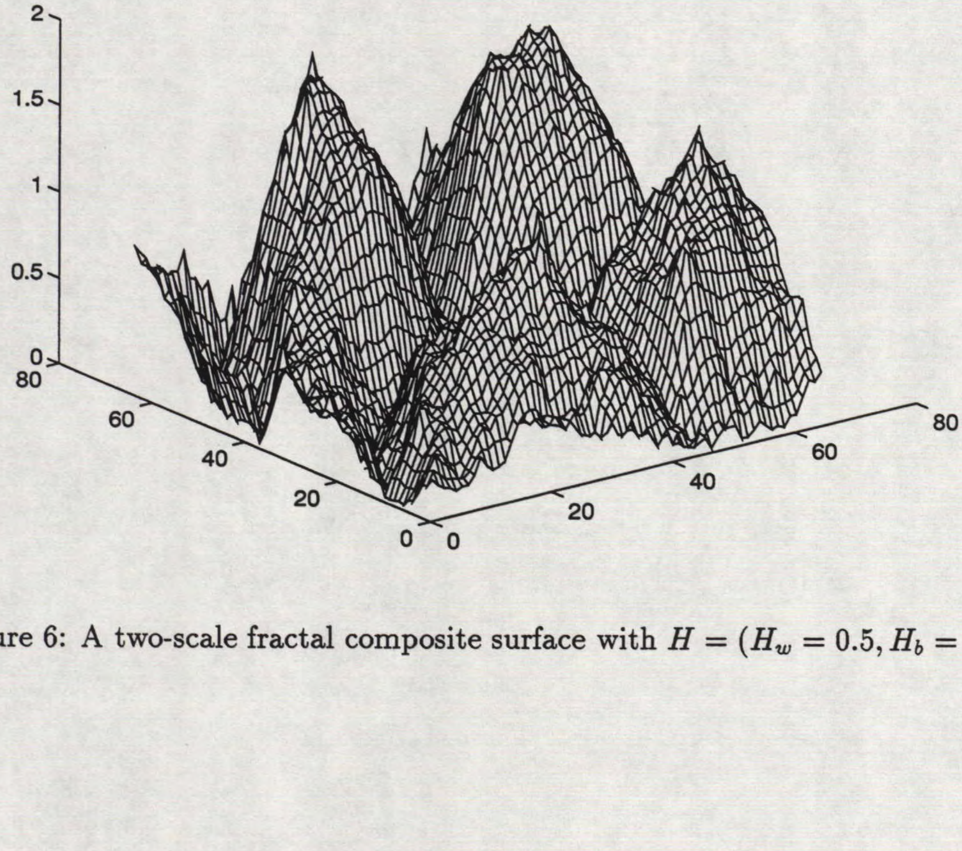

Figure 6: A two-scale fractal composite surface with $H = (H_w = 0.5, H_b = 0.5)$.

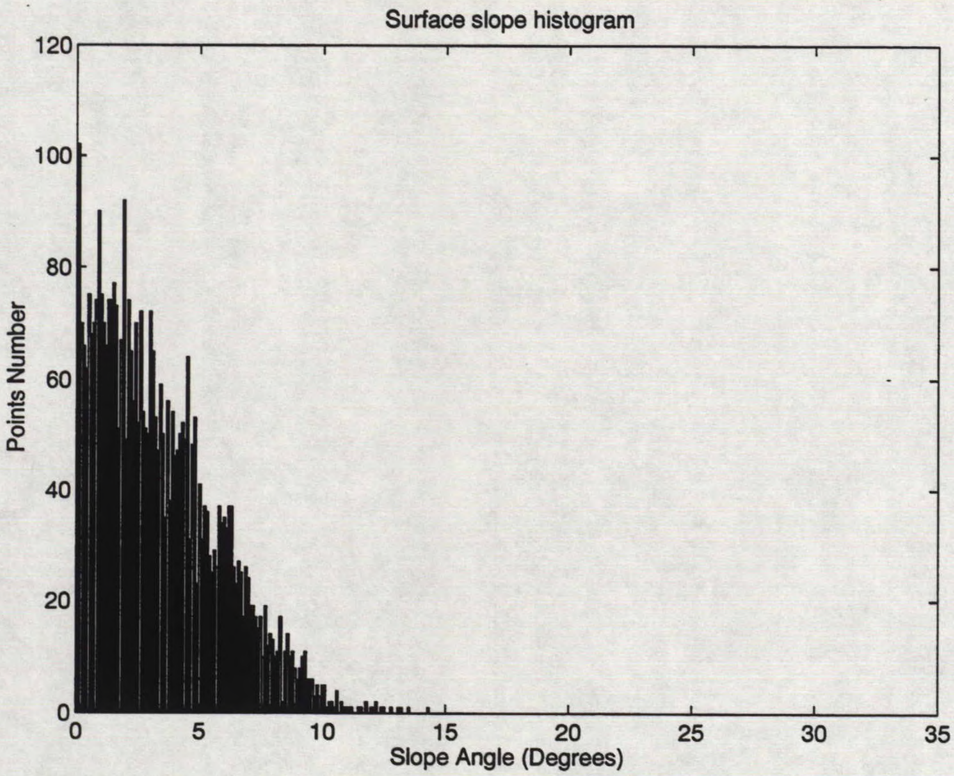

Figure 7: The slope histogram of the composite surface.

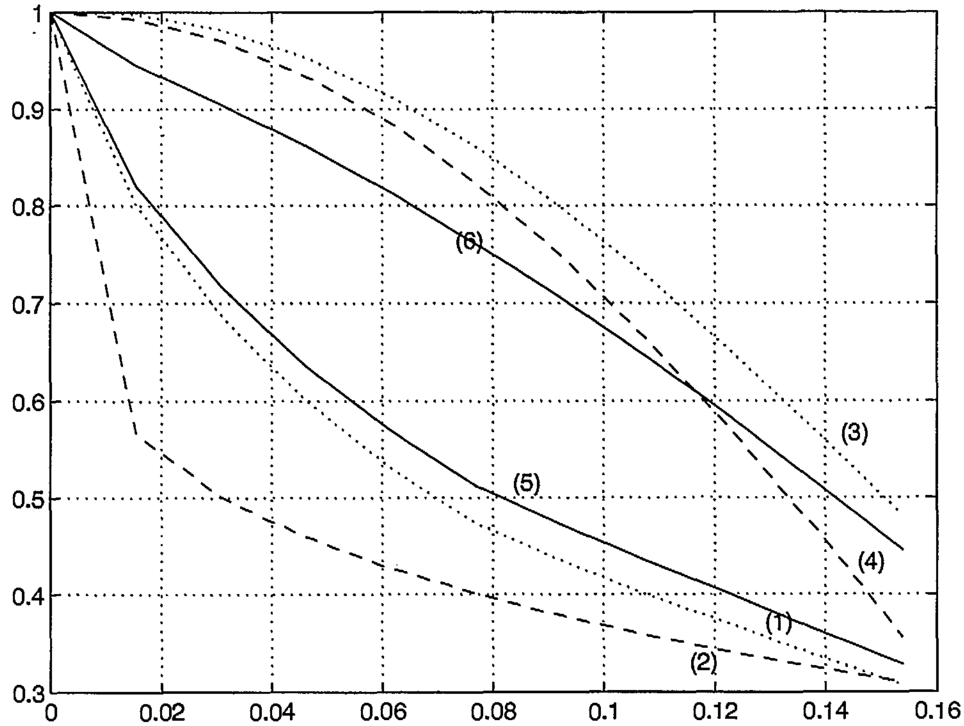

Figure 8: Correlation function: (1) and (3) are statistical results from samples of FBM and BWF respectively, (2) and (4) are calculated results from their formulas. (5) and (6) are statistical results from the composite sample surface, but with the different values of S.

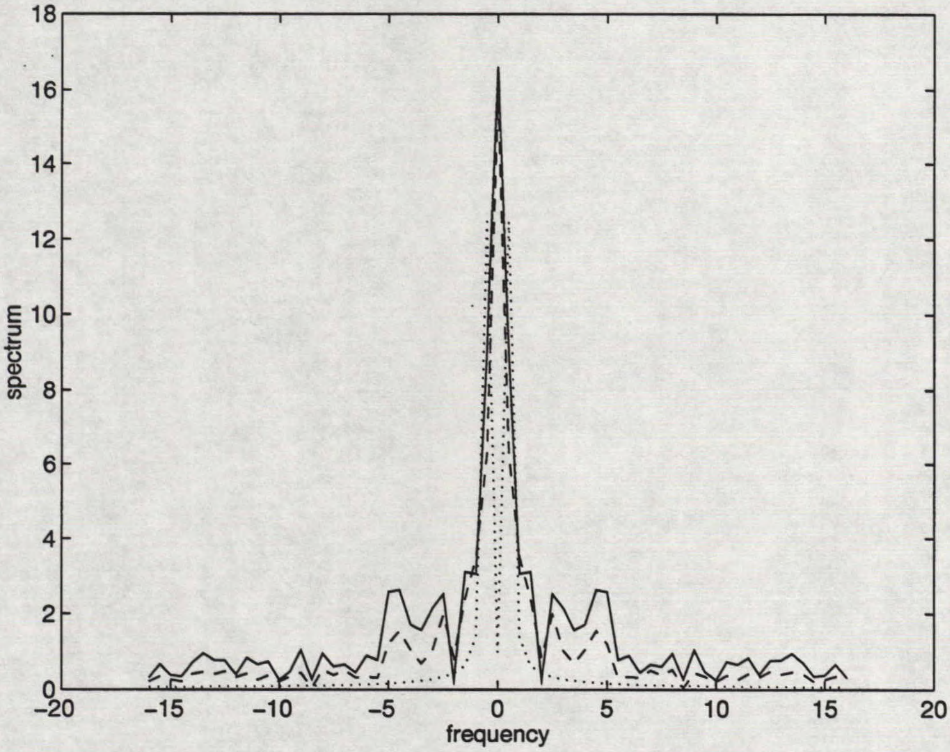

Figure 9: Power spectrum: the dotted line and dashed line are calculated results for the BWF and FBM surfaces respectively, and they are superimposed to obtain the composite surface result(solid line), from which the different parts of frequency contributions can be found.

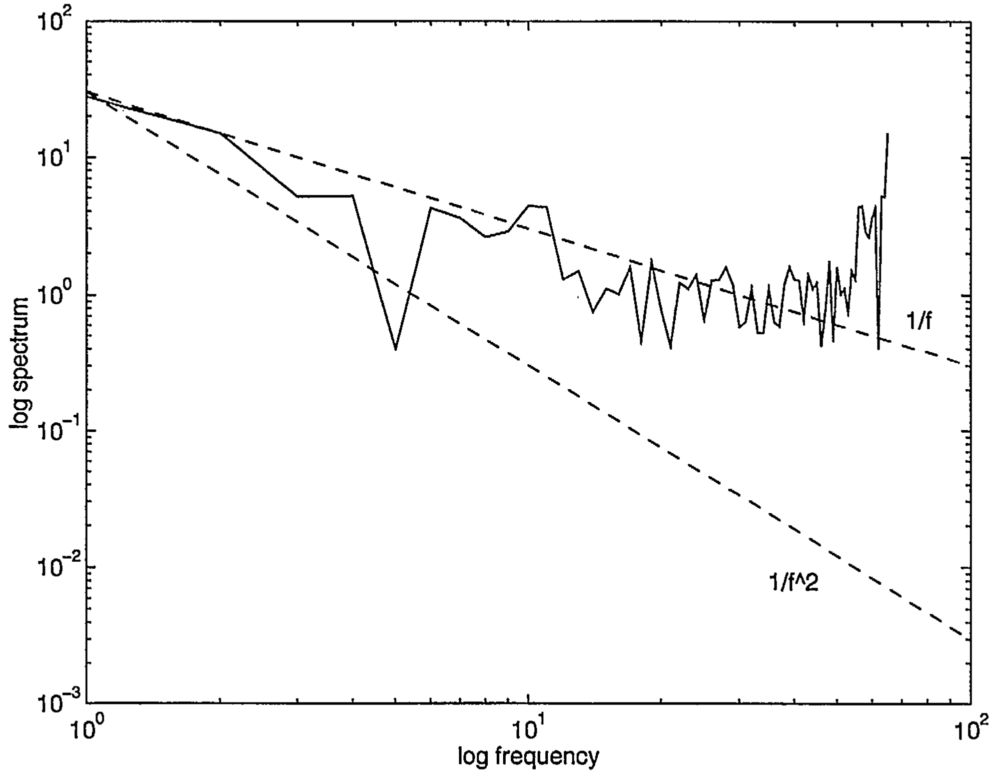

Figure 10: Log power spectrum: the log power spectrum of composite surface shows that the typical fractal feature by the power law lies between $1/f$ and $1/f^2$ spectrums.

ΤΑ ΠΑΝΤΑ · ΕΝ · ΧΡΙΣΤΩΙ · ΣΥΝΕΣΤΗΚΕΝ ·