GUIDE LINES FOR PACKERS

IN ORDER TO MEET
THE NET CONTENTS REQUIREMENTS
OF THE
WEIGHTS AND MEASURES ACT
AND
CONSUMER PACKAGING AND LABELLING ACT

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PREFACE

These "Guide Lines" have been prepared in order to explain to the packaging industry what the effect will be upon filling practices and "target" weights or volumes of the tolerances (or limits of error) and of the acceptance sampling plans established under the authority of the Weights and Measures Act or of the Consumer Packaging and Labelling Act.

An attempt has also been made to make clear the principles upon which the acceptance plans have been based and the assumptions underlying the prescribing of the tolerances. It will be noted that the prescribed tolerances are to some extent arbitrary but it is believed that in general they are consistent with good packaging practice as it exists at the present time.

It is thought appropriate at this point to emphasize that if advances in technology are such that the prescribed tolerances should have become unduly generous or if evidence is found that a packer is attempting to "exploit the tolerance" then the tables of tolerances may be reviewed. It should be noted that the Regulations can be amended by Order in Council, so that closer tolerances might be prescribed in future in order to eliminate what would be clearly an undesirable situation.

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Introduction

The regulations published under the authority of the Weights and Measures Act (1971) or of the Consumer Packaging and Labelling Act (1971) will, by March 1976 at the latest, effectively control nearly all pre-packaged commodities in so far as the accuracy of the declaration of net contents is concerned. The only exceptions will be those commodities which are subject to the specific requirements of other federal legislation but where other legislation makes no reference to the accuracy of declaration of net contents, then the provisions of the Weights and Measures Act or of the Consumer Packaging and Labelling Act, as appropriate, will prevail.

For uniformity of inspection practices, the "tolerances" (as they are referred to in the Consumer Packaging and Labelling Act) will be the same as the "limits of error" prescribed in the Weights and Measures Regulations and the same "acceptance sampling plans" will be employed under each Act. (For convenience of expression, we will hereinafter use the word "tolerance" as synonymous with "limit of error".)

The provisions relating to statements of quantity, limits of error, inspection and inspection by sample of commodities are outlined in Part II of the Weights and Measures Regulations; Schedule II gives tables for limits of error and Schedule III gives tables for the sampling inspection of lots of pre-packaged commodities. (Similar provisions will appear in the Consumer Packaging and Labelling Regulations when they are finalized.) Reference should be made to these regulations for complete details but for the purposes of this paper (which is primarily intended for the guidance of packers who generally declare net contents in terms of weight or volume) we are reproducing Parts 3, 4, 5 and 6 of Schedule II and Parts 1 and 2 of Schedule III from the Weights and Measures Regulations.

Many packers who pack on a large scale and who are experienced in the techniques of quality control will have no difficulty after studying the prescribed tolerances in calculating the "target value" for the net contents of any particular product line or commodity which they pack. By "target value" is meant the average net contents of the packages in any particular lot which the packer must aim to achieve if he wishes to be reasonably sure that the lot will be considered acceptable when checked for the correctness of the declaration of net contents by the inspection staff of the Department of Consumer and Corporate Affairs.

On the other hand there are undoubtedly many packers operating on a relatively small scale who would appreciate some guidance in setting target values which would be such that the majority of their lots of packages would be acceptable upon inspection. This paper is intended principally for the guidance of such packers and it is hoped that they will find it of some assistance in controlling their production lines and in determining to what extent they may be required to "over fill" in order to meet the requirement of the Weights and Measures Act and of the Consumer Packaging and Labelling Act.

Tolerances - General Considerations

The setting of tolerances is to a large degree an arbitrary act, but one which to a very considerable extent must reflect the "state of the art" as it exists at a given time and which represents the practices of conscientious packers using reasonably modern equipment.

For these reasons, the Department of Consumer and Corporate Affairs (hereinafter referred to as CCA), studied the tolerances employed in various countries, including the United States and certain of the countries of the European Common Market which together constitute a considerable proportion of our trading partners. (The United Kingdom does not recognize the existence of tolerances in its legislation pertaining to Weights and Measures although the need for tolerances is tacitly admitted in several semi-official publications intended as "guide-lines" for government inspectors and for packers.)

When these tolerances were plotted on log-log paper, it was found that although most countries had established tolerances which were "step-functions", yet it was possible to draw two straight lines (on log-log paper) which effectively represented close approximations to the tolerances established for "easy to pack commodities" and for "difficult to pack commodities". Unfortunately no very clear directions were given as to the method of distinguishing between these two categories of commodities in the European legislation so that apparently the appropriate category for any commodity could only be established by studying the data obtained after analysing the statistics of random samples taken by the government inspectors. This could have the effect of encouraging careless filling operations, for the greater the standard deviation (or alternatively, the greater the "range") of the filling operation, the greater would be the likelihood of the commodity being processed to be declared a "difficult to pack commodity" and thus have the advantage of a more generous tolerance.

After considering the implications of the foregoing remarks and the limits of error for various classes of weighing and measuring devices approved for use in trade (and which are prescribed in the Weights and Measures Regulations), the decision was made to establish for packages the net contents of which were given in terms of weight, tolerances which were approximately one and one-half (1.5) times the limits of error prescribed for weighing devices approved for use in trade under the Weights and Measures Act. For packages the net contents of which were given in terms of volume, the somewhat arbitrary decision was made that the tolerances should be double the tolerances prescribed for packages with a net contents declaration in terms of weight. This means, in effect, that the tolerances for packages with a net contents declaration in terms of volume will be approximately equal to the limits of error prescribed for such volumetric measuring devices as static measures, measuring tanks and liquid meters which have been approved for use in trade.

Prescribed Tolerances

The prescribed tolerances as given in the tables, although "official", are really only approximations (although very close approximations in most cases) to the "theoretical" tolerances which were based upon the foregoing general considerations and the following specific decisions:

(i) that the tolerance for a 500 gram package should be exactly 8.0 grams;

(ii) that the tolerance for a 500 millilitre package should be exactly 16.0 millilitres;

(iii) that for packages with net contents not greater than 20 kg or 20 litres, the tolerances should be such that tripling the weight or volume should result in doubling the tolerance.

The tolerances derived from the foregoing considerations can be expressed in mathematical terms as follows, for values up to 20 kg or 20 litres.

Let "C" be the declared net contents and "T" the prescribed tolerance or limit of error (in "absolute" units).

If C and T are in grams, then T $= 0.15857\ C^{0.63093}$

If C and T are in millilitres, then T $= 0.31715\ C^{0.63093}$

If C and T are in ounces (av.), then T $= 0.04615\ C^{0.63093}$

If C and T are in fluid ounces, then T $= 0.09222\ C^{0.63093}$

The tolerances or limits of error (T) may also be expressed as percentages of C,

i.e. $\%\ \text{tolerance} = \frac{T}{C} \times 100.$

If C and T are in grams, % tolerance $= \frac{15.857}{C^{0.36907}}$

If C and T are in millilitres, % tolerance $= \frac{31.715}{C^{0.36907}}$

If C and T are in ounces (av.), % tolerance $= \frac{4.615}{C^{0.36907}}$

If C and T are in fluid ounces, % tolerance $= \frac{9.222}{C^{0.36907}}$

The appended graph (Fig. 1) illustrates the tolerances, when the values of C and T are given in the metric system, both in "absolute" units and as "% tolerances".

As mentioned above, the tables which appear in the regulations and which are reproduced below, can be considered to be "official". If tolerances are required for intermediate values not appearing in the tables, they can be obtained by the use of the formulae given above or by linear interpolation within the tables.

Use of Linear Interpolation to Obtain Tolerances

As an illustration of the use of linear interpolation, suppose it is required to obtain the tolerance for a declared net content of 750 ml which is not a value appearing in the tables.

However, for 600 ml the tolerance is 18.0 ml and for 800 ml the tolerance is 22.0 ml.

Then the tolerance for 750 ml would be 21.0 ml for

$$\frac{750-600}{800-600} = \frac{21.0-18.0}{22.0-18.0}$$

In algebraic terms, suppose the table is set up as below:

Declared Net Content	Tolerance
X_1, X_2 > X_3 (intermediate value)	Y_1, Y_2 > Y_3 (unknown tolerance)

and suppose we wish to find the tolerance Y_3 corresponding to a declared net content X_3 which lies between X_1 and X_2. Then the value of Y_3 is obtained (using linear interpolation) from the equations:

$$\frac{X_3 - X_1}{X_2 - X_1} = \frac{Y_3 - Y_1}{Y_2 - Y_1}$$

or $$Y_3 = Y_1 + \frac{(Y_2 - Y_1) \cdot (X_3 - X_1)}{(X_2 - X_1)}$$

Definitions and Symbols Used in this Paper

The following definitions and symbols will be used throughout this paper.

x = the "net contents" of an individual package of the lot (whether x is directly measured or calculated).

C_d = "declared net contents" (i.e. the statement on the label of each and every package in the lot which purports to indicate the actual net contents).

C_m = "minimum net contents" (i.e. the minimum value of the net contents of a package which, if a "tolerance" is permitted, would not cause that package to be considered a "defective" and thus subject to rejection or to sanctions under the law).

e = "error permitted" or "tolerance" (i.e. $e = C_d - C_m$).

def. = "defective" (i.e. an individual package, the net contents of which are less than C_m).

marg. = "marginal" (i.e. an individual package, the net contents of which are greater than C_m but less than C_d).

μ = "average net contents", (i.e. arithmetic mean or average of the net contents of all the packages in the lot or complete population under consideration).

$$\mu = \frac{1}{N} \sum_{i=1}^{N} x_i$$

σ = "lot standard deviation" of the net contents of all the packages in the lot or complete population under consideration

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2}$$

where N = total number of packages in the lot or population, (assumed to be "indefinitely large").

n = "sample-size" (i.e. the number of packages in a random sample of packages taken from the complete lot).

$\bar{x}$ = "sample average net contents" or "sample mean"

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

s = "sample standard deviation" or "sample s.d." of net contents

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

R_g = "sample range" of net contents

= $x_{max.} - x_{min.}$ (over a group of "g" packages in the sample - "g" is generally taken as 5 or 10 sample units).

$\bar{R}$ = "average sample range" of net contents

$$= \frac{1}{m} (R_{g_1} + R_{g_2} + \ldots + R_{g_m})$$

where m x g = n (i.e. the sample consists of "m" groups of "g" units in each group).

Determination of "Population Parameters" and "Sample Statistics" for the Net Contents of Packaged Commodities

Before a "target value" for any particular product line can be established, it is necessary for the packer to determine the "average net contents" (μ) and the "standard deviation" (σ) for the particular packaging process under consideration.

Now by definition, μ and σ, are so-called "population parameters" and theoretically can only be determined if the net content is known for each and every item in the population of packages in the complete production of the particular product line being studied.

However, estimates of μ and σ can be made by determining the "sample mean", $\bar{x}$, and the "sample standard deviation", s, as defined above. The closeness of the estimates to the "true values" is in large measure dependent upon the size of the sample (or samples) chosen so that large samples should be taken (i.e. "n" should be of the order of at least 100) in order to have a reasonable level of confidence in the accuracy of the estimates of μ and σ.

Although the formula given above for the sample deviation, s, gives what the statisticians call a "consistent" estimate of σ (and the sample variance, s^2, gives an "unbiased" estimate of the population variance σ^2), yet the arithmetic calculation involved may not prove to be easy to perform on the floor of a busy packing plant. Consequently, an alternative method is often employed using the so-called "range method" of estimating the value of σ, which is arithmetically simpler to use although it is not as "efficient" as the sample standard deviation (s) for the estimation of σ. (It might be noted that if the test-method employed requires the destruction of relatively expensive items in the sample, then the range method would not normally be used.)

Range Method for Estimation of Lot Standard Deviation

It has been shown mathematically that the "efficiency" of estimation (using the range method) of the lot standard deviation does not, as might be assumed intuitively, increase with the size of the sample if the range is taken to be the difference between the largest and smallest values occurring in the sample as a whole. It is found that it is often best to divide the total group of sample values into a number of equal-size sub-groups and then to calculate the average (denoted by $\overline{R}$) of the several sub-group ranges. Although the optimum size of sub-groups is 7 or 8, yet for convenience a sub-group size of 5 items is generally employed.

As an illustrative example, suppose that the net content of each item in a sample of size n=15 has been determined and the observed values were as follows, in the order in which the observations were actually made:

17.0 g, 16.5 g, 15.5 g, 16.0 g, 15.5 g, 16.5 g,
17.0 g, 17.5 g, 16.5 g, 17.0 g, 15.5 g, 15.5 g,
16.5 g, 16.5 g, 16.0 g.

These observed values would then be divided into 3 sub-groups of 5 items each as follows, without altering the order in which the original observations were made:

	17.0	grams
	16.5	"
	15.5	"
	16.0	"
	15.5	"
R_1 =	1.5	"

16.5	grams
17.0	"
17.5	"
16.5	"
17.0	"
$R_2 = 1.0$	"

15.5	grams
15.5	"
16.5	"
16.5	"
16.0	"
$R_3 = 1.0$	

$$\overline{R} = \frac{R_1 + R_2 + R_3}{3} = \frac{1.5 + 1.0 + 1.0}{3} = 1.2 \text{ grams}$$

(If we had simply taken the range over the complete set of readings without forming sub-groups, the range would have been 17.5 - 15.5 = 2.0 grams.)

It has been shown by L.H.C. Tippett in a paper entitled "On the Extreme Individuals and the Range of Samples Taken from a Normal Population" (Biometrika, Vol. 17 (1925), p.364) that the standard deviation of the lot, σ, is related to the range, R, by the equation:

$$\sigma = R/d$$

where the factor d depends upon the sub-group size, g. Certain typical values of "d" are given below:

Size of Sub-group (g)	Factor (d)
2	1.128
3	1.693
4	2.059
5	2.326
6	2.534
7	2.704
8	2.847
9	2.970
10	3.078

Hence for the example given above, the estimated standard deviation for the lot

$$= \sigma = \overline{R}/d = \frac{1.2}{2.326} = 0.516 \text{ grams}$$

Discussion of Sampling Plans - General

Before acceptance sampling plans can be produced it is necessary to be clear as to what quality of lot will be considered to be acceptable "on the average" or "in the long run".

Various alternative definitions are possible--e.g. it might be maintained that a lot would only be considered acceptable if there were no defectives whatsoever (the definition adopted, at least theoretically, in the United Kingdom) or it might be held that a lot would be considered acceptable as long as the declared net contents were present on the average, with no restriction on the number of defective packages that might be in the lot.

The definition of an acceptable lot which underlies the acceptance sampling plans specified in the Weights and Measures Regulations is that a lot of prepackaged commodity will be considered "barely acceptable" if:

(i) the number of "good" packages (i.e. packages which contain at least the declared net content) is not less than 50% of the number of packages in the lot, and

(ii) the number of "defective" packages (i.e. packages which contain less than the declared net contents by an amount greater than the appropriate tolerance) does not exceed 1% of the number of packages in the lot.

Figure 2 illustrates what is considered to be a "barely acceptable lot" (shown by the solid line) whereas the dotted line illustrates a lot which would be considered as an example of "systematic exploitation of the tolerance".

The use of the term "barely acceptable" is intended to convey the meaning that a packer who "targets" for a lot average (μ) which is barely equal to the declared net contents (Cd) and who produces lots which contain 1% defectives, should not enjoy the advantage of a very low producer's risk (α), which is often set at α=0.01 or α=0.05. On the contrary, if the packer targets for lots of "barely acceptable quality" it is felt that he and the consumer should "share the risk", i.e. that the acceptance sampling plans should be such that $\alpha \doteq \beta \doteq 0.50$.

It should be noted that the Weights and Measures Regulations make no explicit reference to a "barely acceptable lot" as defined above due to certain legal subtleties in the wording of the Act. Thus Section 9 of the Act states, in effect, that no trader shall sell or offer for sale <u>any</u> commodity that is not marked accurately within prescribed limits of error (or tolerances) and Section 33 of the Act states that it is an offence to sell or offer for sale <u>any</u> commodity if the quantity of commodity is, subject to prescribed limits of error, less than the quantity purported to be sold or offered for sale or that should be delivered. Thus if even <u>one</u> defective package exists in a lot which is sold or offered for sale, the trader would strictly speaking be guilty of an offence. However Paragraph 10(d) of the Act states that "the Governor in Council may make regulations prescribing the manner in which an inspector shall determine whether any lot, shipment, proposed shipment or identifiable quantity of any commodity meets the requirements of this Act and the regulations". Thus the regulations are written so that if a sample of 37 or fewer items is taken, then <u>no</u> (0) defective may be permitted, regardless of the average or mean of the sample. On the other hand, if a sample of 38 or more items is taken then one (1) defective (but no more than 1 defective) will be permitted provided the number of "short" items (i.e. marginals plus defectives) does not exceed the number shown in Schedule III, Part 2, Column I. (The defective item found in the sample should, of course, not be returned to the lot but should be re-packed or marked correctly.)

Types of Sampling Plans

Because of the legal restrictions referred to in the preceding paragraphs, we are forced, in effect, to employ two types of sampling plans:

(i) 3-class attributes plans based upon the paper by Bray, Lyon and Burr "Three Class Attributes Plans in Acceptance Sampling" (TECHNOMETRICS, Vol. 15, (August 1973), p.575).

Any such plan is usually designated as (n, c_1, c_2)

where n = number of items in the sample

c_1 = maximum permitted number of marginals plus defectives in the sample

c_2 = maximum permitted number of defectives in the sample.

Thus a 3-class plan designated as (38, 19, 1) would be a plan such that if a random sample of 38 items were taken from a lot, then the lot would be accepted if not more than 19 items in the sample were below the declared net contents (i.e. the total of marginals plus defectives did not exceed 19) and not more than 1 defective was found in the sample.

The numbers (38, 19, 1) were chosen in such a way that a large lot of packages containing 50% "good" packages and 1% "defective" packages would have a probability of acceptance, Pa, equal to or slightly greater than 0.5 or a "50-50" chance of being accepted by an inspector.

(ii) 2-class attributes plans similar to those employed in the Canadian Government Specifications Board "Standard on Inspection by Attributes", 105-GP-1, which is essentially the same as the U.S. Military Standard MIL-STD-105.

Any such plan is usually designated as (n, c)

where n = number of items in the sample

c = maximum permitted number of defectives in the sample

However, in order to maintain consistency with the symbolism employed for 3-class plans we will designate a 2-class plan as (n, n, c_2) where the first "n" is the number of items in the sample and the second "n" indicates that all of the items in the sample may contain less than the declared net contents provided not more than c_2 defectives are found in the sample.

Thus a 2-class plan designated as (38, 38, 0) would be a plan such that if a random sample of 38 items were taken from a lot, then the lot would be accepted if no defective ($c_2 = 0$) were found in the sample even though all 38 items were "marginal packages".

The numbers (38, 38, 0) were chosen in such a way that a large lot containing 1% defective packages (regardless of the percentage of marginal packages) would have a probability of acceptance, Pa, equal to or slightly greater than 0.5 or a "50-50" chance of being accepted by an inspector.

This would be equivalent to stating that the "Indifference Quality" (I.Q.) would be 1% or $p_o = 0.01$ for $\alpha \doteq \beta \doteq 0.50$ using the terminology employed in such books on acceptance sampling as:

"Sampling Inspection" by Freeman, Friedman, Mosteller and Wallis (McGraw-Hill - New York, 1948) or "Selected Techniques of Statistical Analysis" by Eisenhart, Hastay and Wallis (McGraw-Hill - New York, 1947).

Definition of "Lot" for Acceptance Sampling Plans

It is important to be clear as to the meaning to be attached to the word "lot" when used in connection with acceptance sampling. The meaning of "lot" when used herein is "the collection of items (generally packages) from which a random sample may be drawn and which will be accepted or rejected as a whole on the basis of the statistics of the sample".

Thus for inspection purposes a "lot" is not necessarily the same as a manufacturer's lot or shipping lot or order lot, etc. An inspection lot, for example, may be only part of a shipping lot or may embrace two or more shipping lots.

It should be noted that the lot must be such that a "random sample" can be drawn from it. Now a random sample is such that any member of the lot has, a priori, an equal chance of being the first member of the sample to be drawn; any remaining member of the lot has, a priori, an equal chance of being the second member of the sample to be drawn, and so on.

It is obvious that to treat a large collection of packages (which in turn may be contained in many cartons) as one homogeneous lot would require that the sample be drawn entirely at random from anywhere in the whole lot. In a warehouse which is filled from wall to wall and from floor to ceiling with packages, the drawing of a truly random sample would pose major problems for the inspector and could lead to complete chaos in so far as the manufacturer is concerned.

It is likely, therefore, that the inspector will "define" the lot for the purposes of his inspection as that "layer" of cartons or of packages which is most readily accessible for the drawing of the sample. The layer may be the top layer or one of the side layers or possibly the lot might be defined as just the collection of packages on a pallet.

At all events the lot, however defined, must be sampled in a random fashion and the statistics of the sample studied. Depending upon the sample statistics, the defined lot will be accepted or rejected -- but only the defined lot. In other words, the results of a sample which was drawn at random from only one layer would not constitute grounds for accepting or rejecting the remaining layers. However, if one layer were to be rejected and the contents of the sample appeared to be seriously deficient, the inspector would probably request that the next layer be inspected and so on if layer after layer appeared to contravene the Act. Even though the inspector might terminate his inspection after inspecting only one layer, it should be noted that the obligation remains with the packer to give full weight or measure within prescribed tolerances.

Another point should be noted. If the inspection is done on a "layer basis", the layer may or may not represent a single batch, code, shipping lot, etc. but may embrace several batches, codes, shipping lots, etc. A sample from such a layer might therefore be more representative of "average production" than would a sample which came from a layer containing only a single batch, code, shipping lot, etc. This would probably have the effect of protecting manufacturers whose quality was consistently satisfactory while revealing the lack of control on the part of manufacturers whose quality varied from run to run or from day to day.

Instructions to Inspectors Respecting Sample Sizes

It will be seen that Schedule III, Part 1, gives the minimum size of sample (n) to be drawn from a lot of any specified size (N), in accordance with subsection 52(2) of the Weights and Measures Regulations or section 40 of the Consumer Packaging and Labelling Regulations. There appears to be no general agreement as to the appropriate relationship between N and n except that as N becomes larger, the ratio n/N becomes smaller. The values given in Schedule III, Part 1, are arbitrary to a large extent -- although they were established after studying the suggested sample sizes versus lot sizes as given in several well-known manuals on acceptance sampling.

It should be noted that there is nothing to prevent the drawing of a sample of larger size if in the opinion of the inspector this appears to be desirable. Suppose, for example, that from a lot consisting of $N = 200$ items, the inspector drew the suggested sample of size $n = 10$ and that in the sample there were no (o) defectives but that all 10 sample-items were "marginals". In this case it would not be unreasonable to assume that the packer was attempting to "exploit the tolerance" by aiming at a lot average net contents (μ) which was such that $Cm \leq \mu < Cd$. Consequently the inspector would be expected to draw a further sample of 28, for a total sample size of $n = 38$. Should a <u>single</u> defective be found in the sample of 38, then the lot would be rejected if the total number of marginals <u>plus</u> defectives exceeded 19. However, if there was no defective in the sample of 38, then the lot would have to be accepted regardless of the number of marginals, because of the legal restrictions mentioned above.

If, on the other hand, in the initial sample of 10 there was no defective and the number of marginals did not exceed 5, then the inspector would probably be prepared to accept the lot without taking a further sample.

Graphs Illustrating the Relationships Between Lot Average ("Target") μ and Declared Net Weight or Volume (Cd) for Various Probabilities of Acceptance (Pa)

A series of graphs A-1 to A-20 and B-1 to B-20, (not illustrated herein) were drawn which showed the lot average ("target") weight or volume (μ) versus the probability of acceptance (Pa) for a selected series of values of Cd and of the lot standard deviation (σ). The graphs were based upon computations for the 2-class attributes plan (38, 38, 0) and for the 3-class attributes plan (38, 19, 1) where the assumption was made that the net contents of the items in the lot which is being inspected follow a "normal probability" or "Gaussian" distribution, N (μ, σ). (A "normal probability curve" is a bell-shaped curve such as that illustrated in Figure 2.)

By cross-plotting, four other series of graphs (C-1 to C-3, D-1 to D-6, E-1 to E-3 and F-1 to F-6) were obtained and which are appended hereto.

These graphs can be used for the approximate estimation of the necessary extent of overfilling (or underfilling), expressed as a percentage of Cd, for certain probabilities of acceptance in a manner which is best illustrated by a specific example.

Suppose a packer wishes to ensure that his production run will be such that it will have a 90% probability of acceptance if inspected by a C.C.A. inspector using a (38, 38, 0) plan. Further, let us suppose that the declared net weight is Cd = 250 grams and the standard deviation of the packaging process is $\sigma = 7.749$ grams.

From the equations given for the "theoretical" values of the tolerance versus Cd we obtain for Cd = 250 grams, the value $e = T = 0.15857 \times 250^{0.63093} = 5.166$ grams.

Hence the ratio $\sigma/e = 7.749/5.166 = 1.5$ so that $\sigma = 1.5e$.

From Figure C-3, for the value of Cd = 250 grams, read the values of "% overfill" (or "% underfill") corresponding to $\frac{1}{8}$.e, $\frac{1}{4}$.e, $\frac{1}{2}$.e, 1.e, 2.e, 3.e and 4.e.

These values are as follows:

σ	"% of Cd"
$\frac{1}{8}$.e	- 1.5%
$\frac{1}{4}$.e	- 0.75%
$\frac{1}{2}$.e	+ 1.0%
1.e	+ 3.75%
2.e	+ 9.5%
3.e	+15.25%
4.e	+21.0%

The values for "$k = \sigma/e$" and "% of Cd" should be plotted on ordinary square graph paper, as in Figure 3 below, and a straight line drawn which is a "best fit" through the points.

The value corresponding to $\sigma = 1.5e$ will be seen to be approximately 6.5% overfill. Hence for a value of Cd = 250 grams, the packer should aim for a lot average of $\mu = 1.065$ X 250 = 266.25 grams if he wishes to have a 90% probability of acceptance of the lots being sampled and inspected. (An exact computation gives the probability of acceptance as 89.70% which is extremely close to the desired value of 90%.)

It is of interest to note that for the (38, 38, 0) sampling plan it can be shown mathematically that there must be a linear relationship between the values of "$k = \sigma/e$" and

"% of Cd". Again, in the limiting case when $\sigma \to 0$, it is obvious that the value of "% of Cd" is $\left(\frac{-e}{Cd} \times 100\% = \frac{Cm-Cd}{Cd} \times 100\%\right)$ regardless of the value of Pa selected.

Thus if $\sigma \to 0$ and the lot average $\mu \to Cm$ "from above" then all of the lot must lie above Cm and there are no defectives. Hence there must be a probability of acceptance of 100% using a (38, 38, 0) sampling plan. On the other hand, if $\sigma \to 0$ and the lot average $\mu \to Cm$ "from below", then all of the lot must lie below Cm and must consist entirely of defectives. In this case, there must be a probability of acceptance equal to zero (0) using a (38, 38, 0) sampling plan.

This is illustrated by Figure 3 in which straight lines have been drawn corresponding to values of Pa = 90%, 95% and 99%, based on values read from Figures C-1, C-2 and C-3 for Cd = 250 grams. It will be seen that in all cases as $\sigma \to 0$ all of the lines pass through a single point "A" with abscissa equal to $\frac{-5.166}{250} \times 100 = -2.07\%$ of Cd.

As another example, suppose a packer wishes to ensure that his production run will be such that it will have a 95% probability of acceptance if inspected by a C.C.A. inspector using a (38, 19, 1) plan. Further, let us suppose that the declared net weight is Cd = 75 grams and the standard deviation of the packaging process is $\sigma = 1.813$.

From the equations given for the "theoretical" values of the tolerance versus Cd we obtain for Cd = 75 grams the value

$$e = T = 0.15857 \times 75^{0.63093} = 2.417 \text{ grams}$$

Hence $\sigma/e = 1.813/2.417 = 0.75$

so that $\sigma = 0.75e = \frac{3}{4}.e$

From Figures D-3 and D-4, for the value of $Cd = 75$ grams read the values of "% overfill" (% of Cd) for the various values of "$k = \sigma/e$" and plot on squared paper as in Figure 4. The value corresponding to $k = \frac{3}{4}$ is 2.5% overfill. Hence the packer should aim for a lot average, $\mu = 1.025 \times 75 = 76.875$ grams if he wishes to have a 95% probability of acceptance when a (38, 19, 1) sampling plan is used. (An exact computation gives a value of $Pa = 95.44\%$.)

In Figure 4 are shown the curves for $Pa = 90\%$, 95% and 99%. It will be seen that for these three cases the points appear to lie on straight lines for values of $\sigma > \frac{1}{2} \cdot e$ whereas for values of $\sigma < \frac{1}{2} \cdot e$ the points lie on curves which all converge to the point "B", with abscissa equal to 0% of Cd.

Unlike the case above (Figure 3) where the sampling plan was (38, 38, 0), the author is unable to find a mathematical explanation for the shape of these curves nor for the apparent linear relationship between "$k = \sigma/e$" and "% Cd" for $\sigma > \frac{1}{2} \cdot e$.

However it can be easily shown that for any selected value of Cd, all curves for all values of Pa must pass through the point "B" as $\sigma \to 0$. Thus if $\sigma \to 0$ and the lot average $\mu \to Cd$ "from above", then all of the lot must lie above Cd and there are 0 marginals and 0 defectives. Hence there is a 100% probability of acceptance using a (38, 19, 1) sampling plan. On the other hand if $\sigma \to 0$ and the lot average $\mu \to Cd$ "from below", then all of the lot must lie below Cd and the sum of the marginals plus defectives will be equal to $n = 38$. In this event the probability of acceptance will be zero (0) using a (38, 19, 1) sampling plan.

It will be noted that all of the graphs appended hereto are given in terms of metric units. It will therefore be necessary for a packer who wishes to declare the net contents of packages in terms of Canadian units of measure (e.g. ounces, pounds, fluid ounces) to convert both the declared weight or volume and the appropriate tolerance to metric units before using the method outlined above for estimating the lot average ("target") weight or volume as <u>% of Cd</u>, overfill or underfill.

Extracts from the WEIGHTS AND MEASURES REGULATIONS

SCHEDULE II

Limits of Error for Commodities

Part 3

LIMITS OF ERROR FOR QUANTITIES MEASURED IN METRIC UNITS OF MASS OR WEIGHT FOR COMMODITIES OTHER THAN INDIVIDUALLY MEASURED COMMODITIES

Item	Column I Stated Mass or Weight	Column II Limits of Error
1	1 g	0.16 g
2	1.5 g	0.20 g
3	2 g	0.25 g
4	3 g	0.32 g
5	4 g	0.38 g
6	5 g	0.44 g
7	6 g	0.50 g
8	8 g	0.59 g
9	10 g	0.68 g
10	15 g	0.88 g
11	20 g	1.05 g
12	30 g	1.36 g
13	40 g	1.62 g
14	50 g	1.87 g
15	60 g	2.10 g
16	80 g	2.50 g
17	100 g	2.90 g
18	150 g	3.80 g
19	200 g	4.50 g
20	300 g	5.80 g
21	400 g	7.00 g
22	500 g	8.00 g
23	600 g	9.00 g
24	800 g	11.00 g
25	1 kg	12.5 g
26	1.5 kg	16.0 g
27	2 kg	19.4 g
28	3 kg	25.0 g
29	4 kg	30.0 g
30	5 kg	34.0 g
31	6 kg	39.0 g
32	8 kg	46.0 g
33	10 kg	53.0 g
34	15 kg	68.0 g
35	20 kg	80.0 g
36	Over 20 kg up to and including 100 kg	0.40% of the stated mass or weight
37	Over 100 kg up to and including 500 kg	0.32% of the stated mass or weight
38	Over 500 kg	0.20% of the stated mass or weight

Part 4

LIMITS OF ERROR FOR QUANTITIES MEASURED IN CANADIAN UNITS OF MASS OR WEIGHT FOR COMMODITIES OTHER THAN INDIVIDUALLY MEASURED COMMODITIES

Item	Colum I Stated Mass or Weight	Column II Limits of Error	
1	0.1 ounce	0.011 ounce	0.0007 pound
2	0.2 ounce	0.017 ounce	0.0011 pound
3	0.5 ounce	0.030 ounce	0.0019 pound
4	1.0 ounce	0.048 ounce	0.0030 pound
5	2.0 ounces	0.070 ounce	0.0044 pound
6	5.0 ounces	0.14 ounce	0.0088 pound
7	10.0 ounces	0.20 ounce	0.0125 pound
8	1.0 pounds	0.26 ounce	0.016 pound
9	1.5 pounds	0.34 ounce	0.021 pound
10	2.0 pounds	0.42 ounce	0.026 pound
11	3.0 pounds	0.54 ounce	0.034 pound
12	4.0 pounds	0.64 ounce	0.040 pound
13	5.0 pounds	0.73 ounce	0.046 pound
14	6.0 pounds	0.83 ounce	0.052 pound
15	8.0 pounds	1.00 ounce	0.063 pound
16	10.0 pounds	1.15 ounces	0.072 pound
17	15.0 pounds	1.50 ounces	0.094 pound
18	20.0 pounds	1.75 ounces	0.109 pound
19	25.0 pounds	2.10 ounces	0.131 pound
20	30.0 pounds	2.30 ounces	0.144 pound
21	35.0 pounds	2.50 ounces	0.156 pound
22	40.0 pounds	2.75 ounces	0.172 pound
23	45.0 pounds	3.00 ounces	0.188 pound
24	Over 45.0 pounds up to and including 225 pounds	0.40% of the stated mass or weight	
25	Over 225 pounds up to and including 1000 pounds	0.32% of the stated mass or weight	
26	Over 1000 pounds	0.20% of the stated mass or weight	

Part 5

LIMITS OF ERROR FOR QUANTITIES MEASURED IN METRIC UNITS OF VOLUME OR CAPACITY FOR LIQUID COMMODITIES AND FOR LIQUID FOODS THAT ARE SOLD IN THE FROZEN STATE

Item	Column I Stated Volume or Capacity	Column II Limits of Error
1	1 ml	0.32 ml
2	1.5 ml	0.40 ml
3	2 ml	0.50 ml
4	3 ml	0.64 ml
5	4 ml	0.76 ml
6	5 ml	0.88 ml
7	6 ml	1.00 ml
8	8 ml	1.18 ml
9	10 ml	1.36 ml
10	15 ml	1.76 ml
11	20 ml	2.10 ml
12	30 ml	2.72 ml
13	40 ml	3.24 ml
14	50 ml	3.74 ml
15	60 ml	4.20 ml
16	80 ml	5.00 ml
17	100 ml	5.80 ml
18	150 ml	7.60 ml
19	200 ml	9.00 ml
20	300 ml	11.6 ml
21	400 ml	14.0 ml
22	500 ml	16.0 ml
23	600 ml	18.0 ml
24	800 ml	22.0 ml
25	1 l	25.0 ml
26	1.5 l	32.0 ml
27	2 l	38.8 ml
28	3 l	50.0 ml
29	4 l	60.0 ml
30	5 l	68.0 ml
31	6 l	78.0 ml
32	8 l	92.0 ml
33	10 l	106 ml
34	15 l	126 ml
35	20 l	160 ml
36	Over 20 litres up to and including 50 litres	0.75% of stated volume or capacity
37	Over 50 litres up to and including 250 litres	0.6% of stated volume or capacity
38	Over 250 litres up to and including 500 litres	0.5% of stated volume or capacity
39	Over 500 litres	0.4% of stated volume

Part 6

LIMITS OF ERROR FOR QUANTITIES MEASURED IN CANADIAN UNITS OF VOLUME OR CAPACITY FOR LIQUID COMMODITIES AND FOR LIQUID FOODS THAT ARE SOLD IN THE FROZEN STATE

Item	Column I Stated Volume or Capacity	Column II Limits of Error
1	0.1 fluid ounce	0.022 fluid ounce
2	0.2 fluid ounce	0.034 fluid ounce
3	0.5 fluid ounce	0.060 fluid ounce
4	1.0 fluid ounce	0.092 fluid ounce
5	2.0 fluid ounces	0.14 fluid ounce
6	5.0 fluid ounces (1 gill)	0.25 fluid ounce
7	10.0 fluid ounces	0.39 fluid ounce
8	16.0 fluid ounces	0.52 fluid ounce
9	20.0 fluid ounces (1 pint)	0.60 fluid ounce
10	24.0 fluid ounces	0.64 fluid ounce
11	32.0 fluid ounces	0.80 fluid ounce
12	40.0 fluid ounces (1 quart)	0.93 fluid ounce
13	48.0 fluid ounces	1.05 fluid ounces
14	60.0 fluid ounces	1.20 fluid ounces
15	80.0 fluid ounces (2 quarts)	1.45 fluid ounces
16	100.0 fluid ounces	1.70 fluid ounces
17	120.0 fluid ounces (3 quarts)	1.85 fluid ounces
18	128.0 fluid ounces	2.00 fluid ounces
19	160.0 fluid ounces (1 gallon)	2.25 fluid ounces
20	2 gallons	3.50 fluid ounces
21	3 gallons	4.60 fluid ounces
22	4 gallons	5.40 fluid ounces
23	5 gallons	6.20 fluid ounces
24	Over 5 gallons up to and including 10 gallons	0.75% of stated volume or capacity
25	Over 10 gallons up to and including 50 gallons	0.6% of stated volume or capacity
26	Over 50 gallons up to and including 100 gallons	0.5% of stated volume or capacity
27	Over 100 gallons	0.4% of stated volume or capacity

SCHEDULE III

Samples

Part 1

Item	Column I Number of Units in the Lot	Column II Number of Units in the Sample
1	2 or more, not exceeding 10	2
2	11 or more, not exceeding 25	4
3	26 or more, not exceeding 60	6
4	61 or more, not exceeding 100	8
5	101 or more, not exceeding 300	10
6	301 or more, not exceeding 500	12
7	501 or more, not exceeding 1,000	14
8	1,001 or more, not exceeding 2,000	16
9	2,001 or more, not exceeding 5,000	18
10	5,001 or more, not exceeding 10,000	20
11	10,001 or more, not exceeding 25,000	25
12	25,001 or more, not exceeding 50,000	30
13	50,001 or more, not exceeding 100,000	38
14	100,001 or more, not exceeding 200,000	50
15	200,001 or more, not exceeding 500,000	60
16	500,001 or more, not exceeding 1 million	80
17	more than 1 million	100 for each million items in the lot

Part 2

Item	Column 1 Number of Units in the Sample	Column II Number of Units in the Sample Containing less than the Stated Quantity
1	38	19
2	40	20
3	50	25
4	60	31
5	80	41
6	100	52

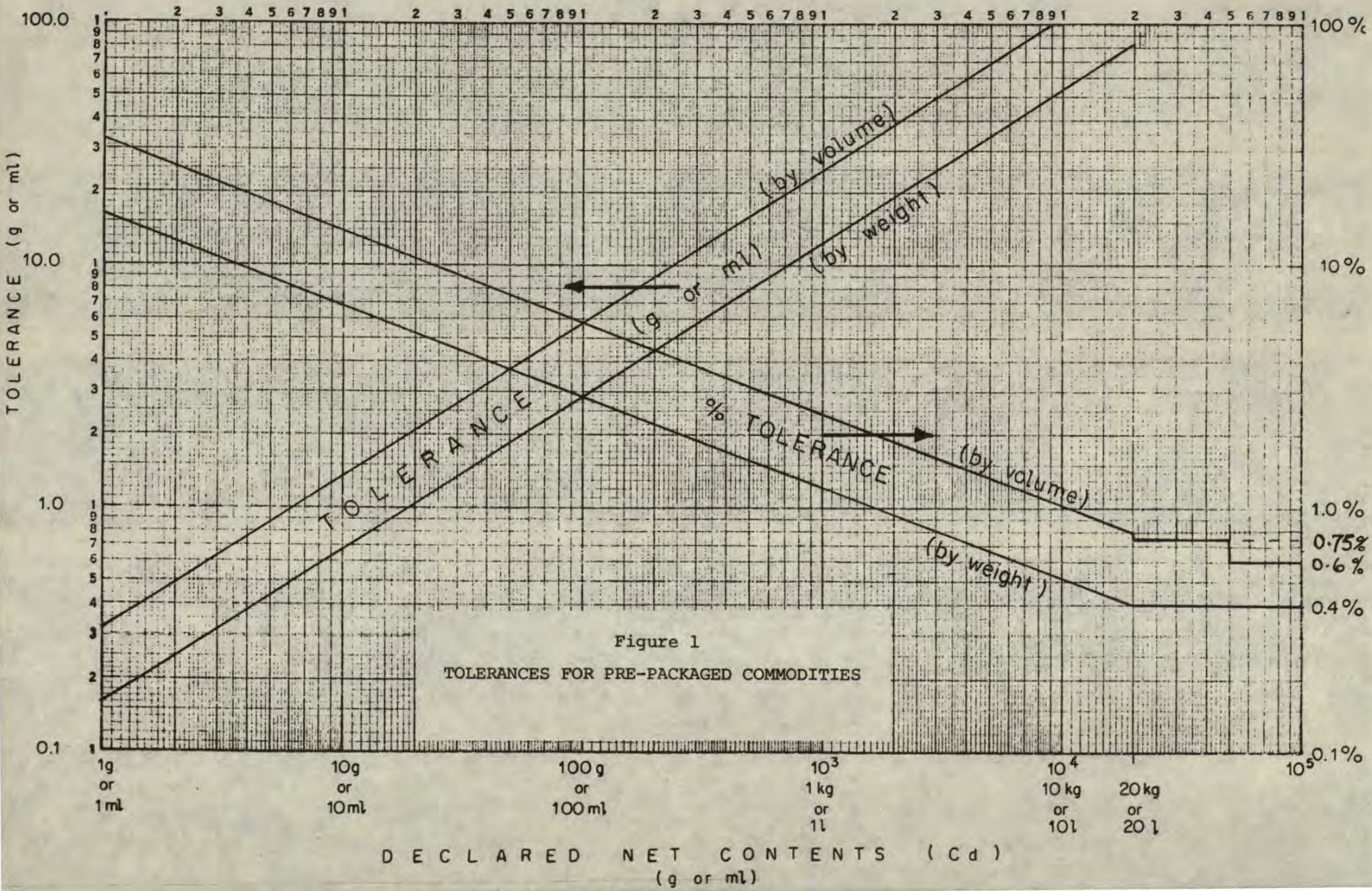

Figure 1

TOLERANCES FOR PRE-PACKAGED COMMODITIES

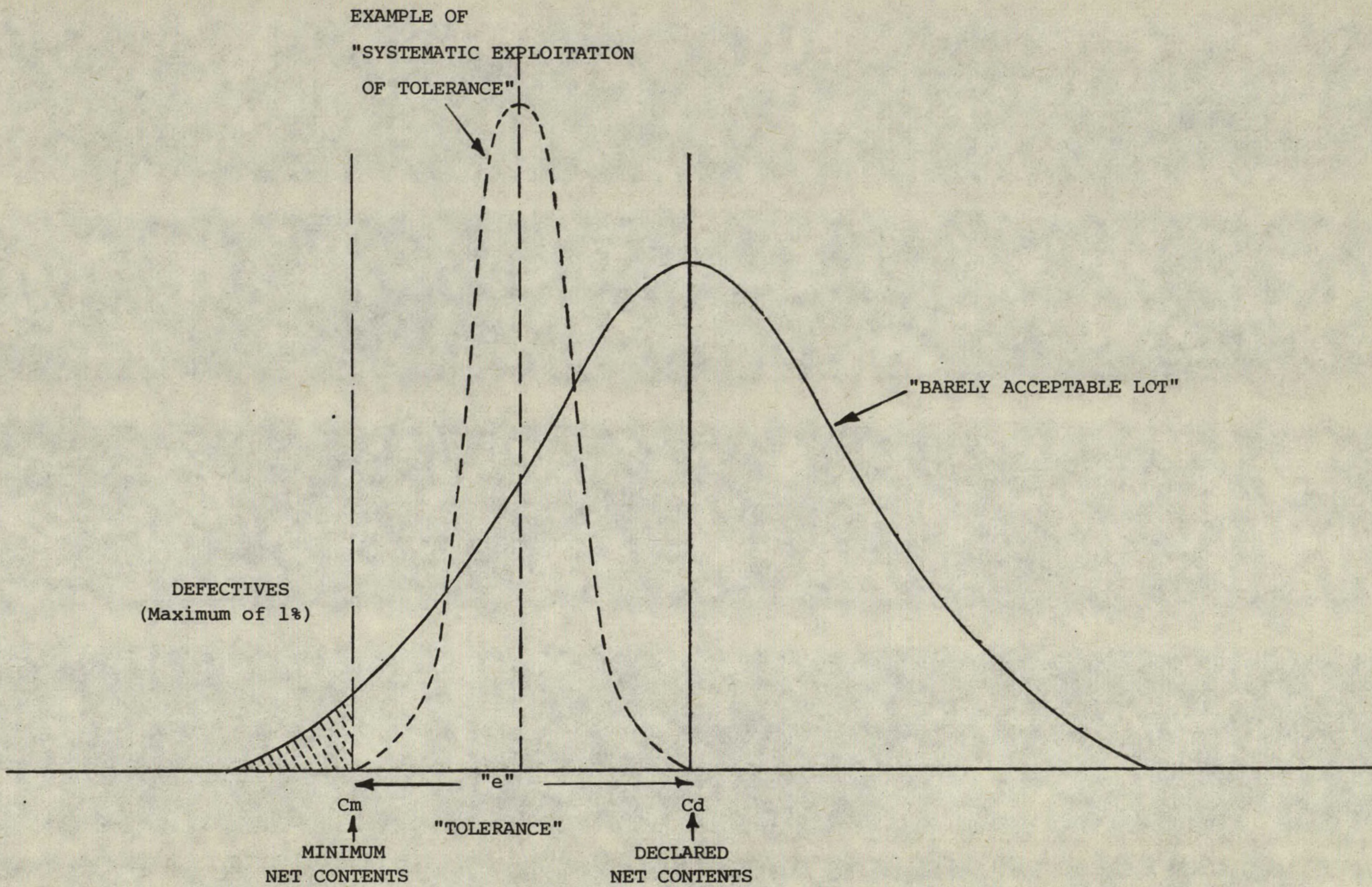

Figure 2
Illustrations of "Barely Acceptable Lot"
and of "Systematic Exploitation of Tolerance"

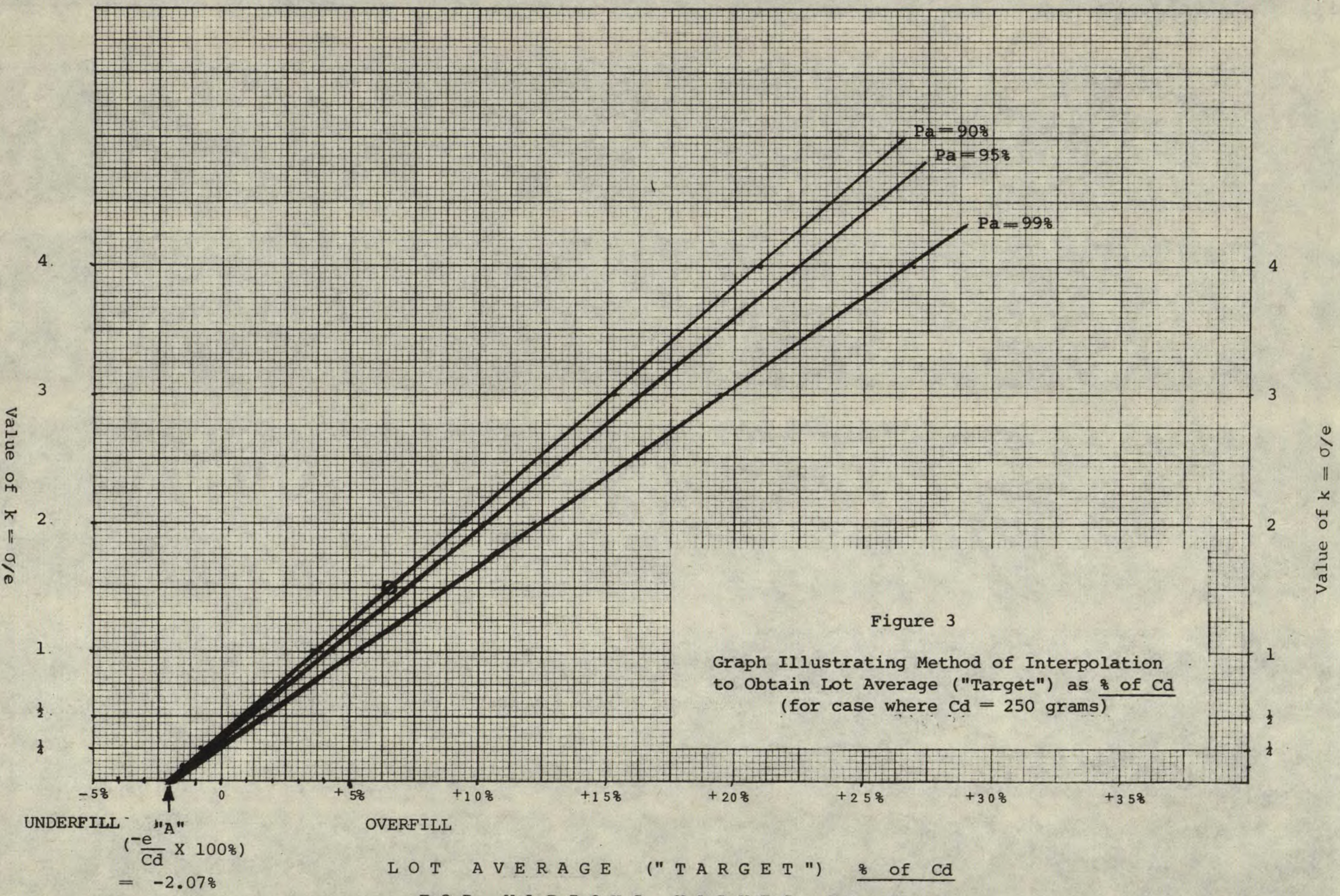

Figure 3

Graph Illustrating Method of Interpolation to Obtain Lot Average ("Target") as <u>% of Cd</u> (for case where Cd = 250 grams)

LOT AVERAGE ("TARGET") <u>% of Cd</u>
FOR VARIOUS VALUES OF Pa
USING {38, 38, 0} SAMPLING PLAN

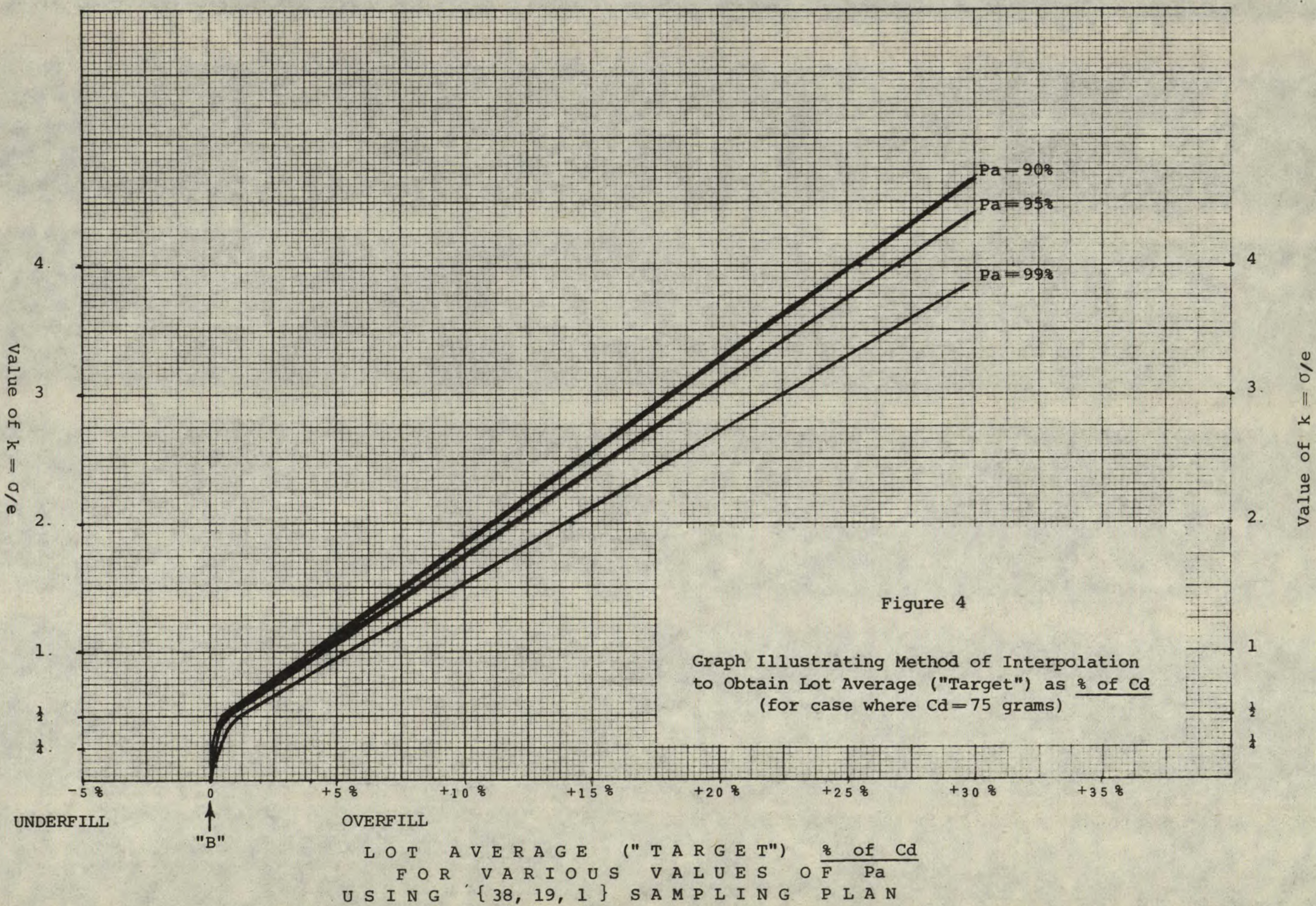

Figure 4

Graph Illustrating Method of Interpolation to Obtain Lot Average ("Target") as % of Cd (for case where Cd = 75 grams)

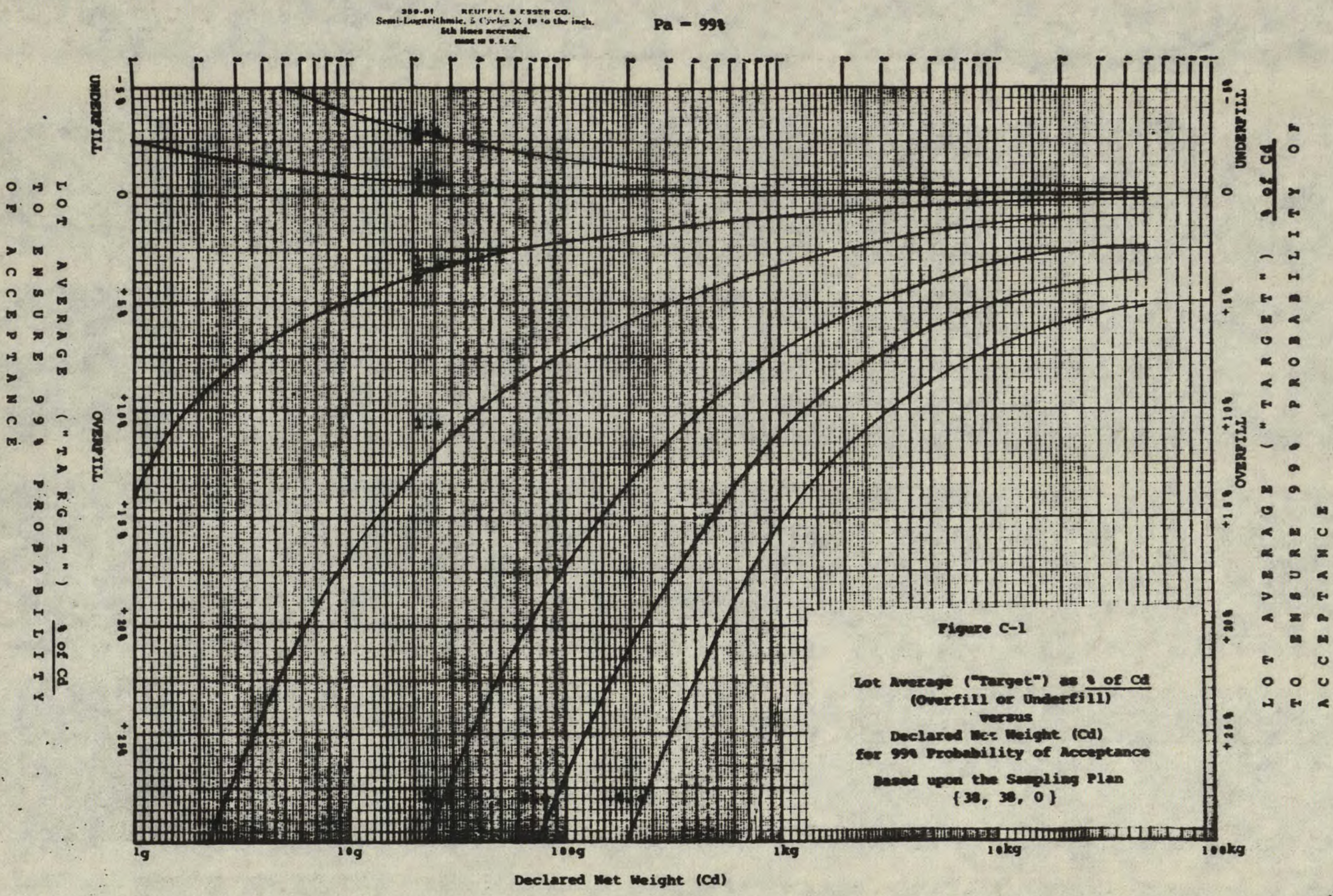

Figure C-1

Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Weight (Cd)
for 99% Probability of Acceptance

Based upon the Sampling Plan
{38, 38, 0}

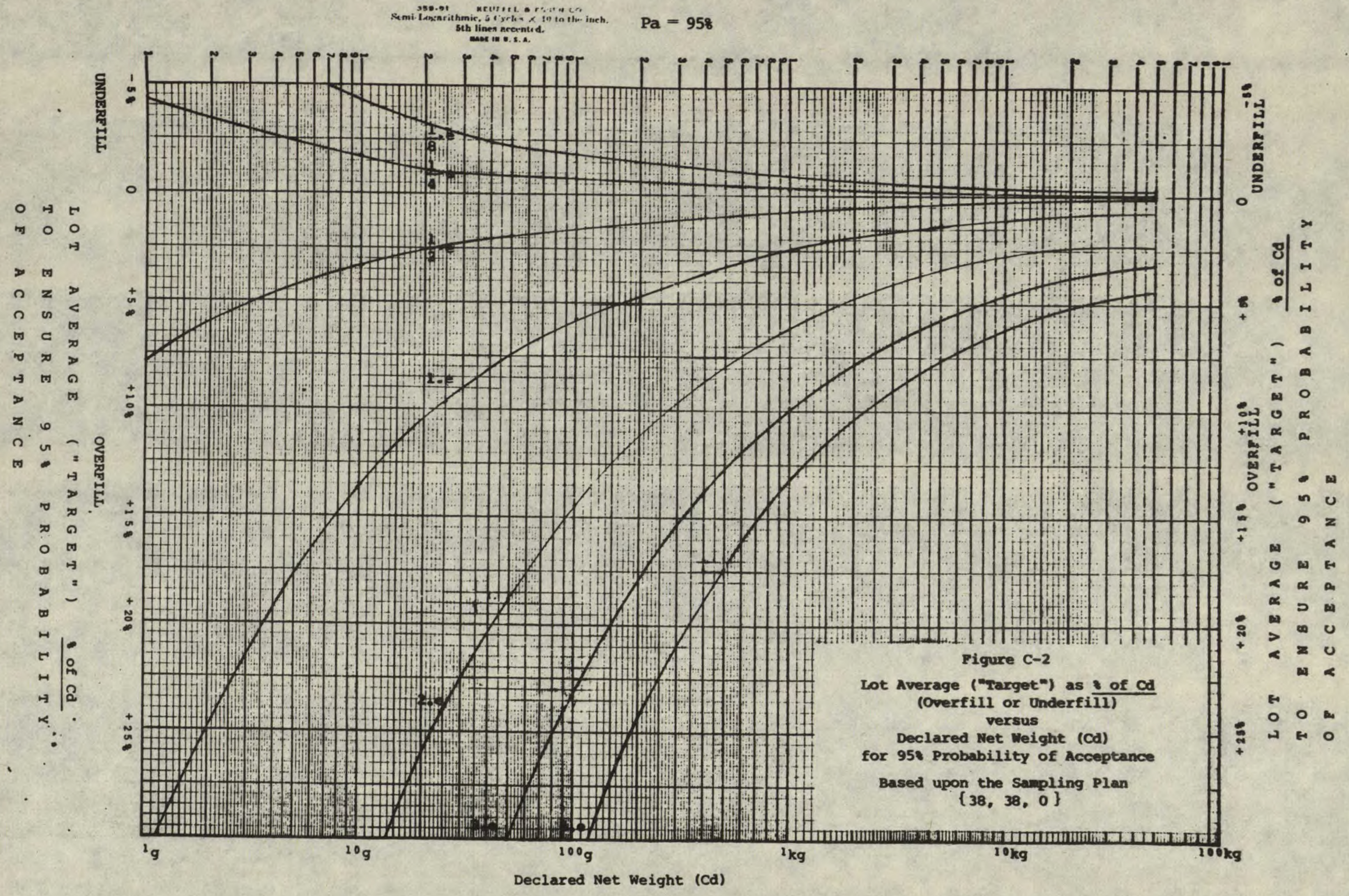

Figure C-2

Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Weight (Cd)
for 95% Probability of Acceptance

Based upon the Sampling Plan
{38, 38, 0}

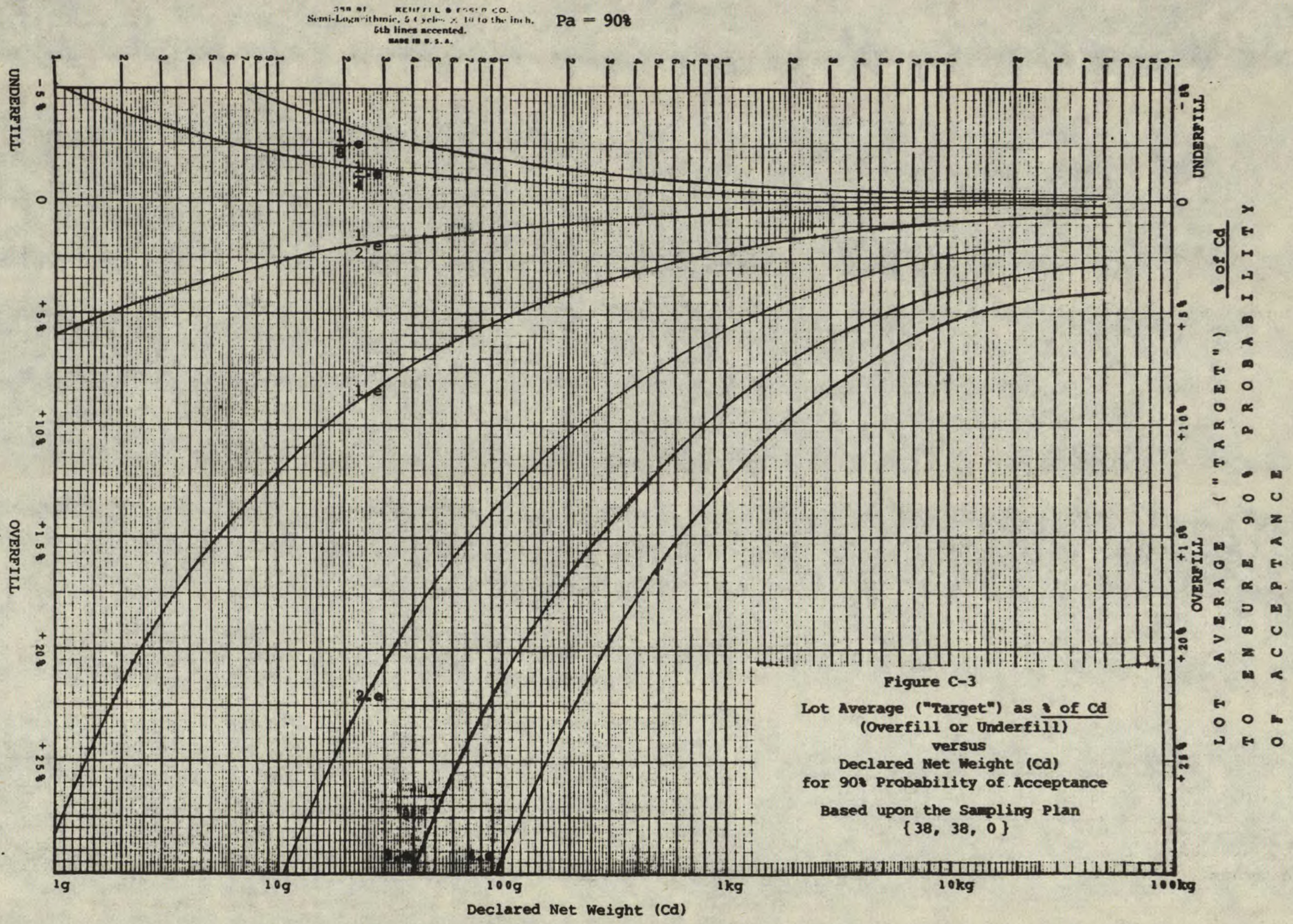

Figure C-3

Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Weight (Cd)
for 90% Probability of Acceptance

Based upon the Sampling Plan
{38, 38, 0}

Pa =99%

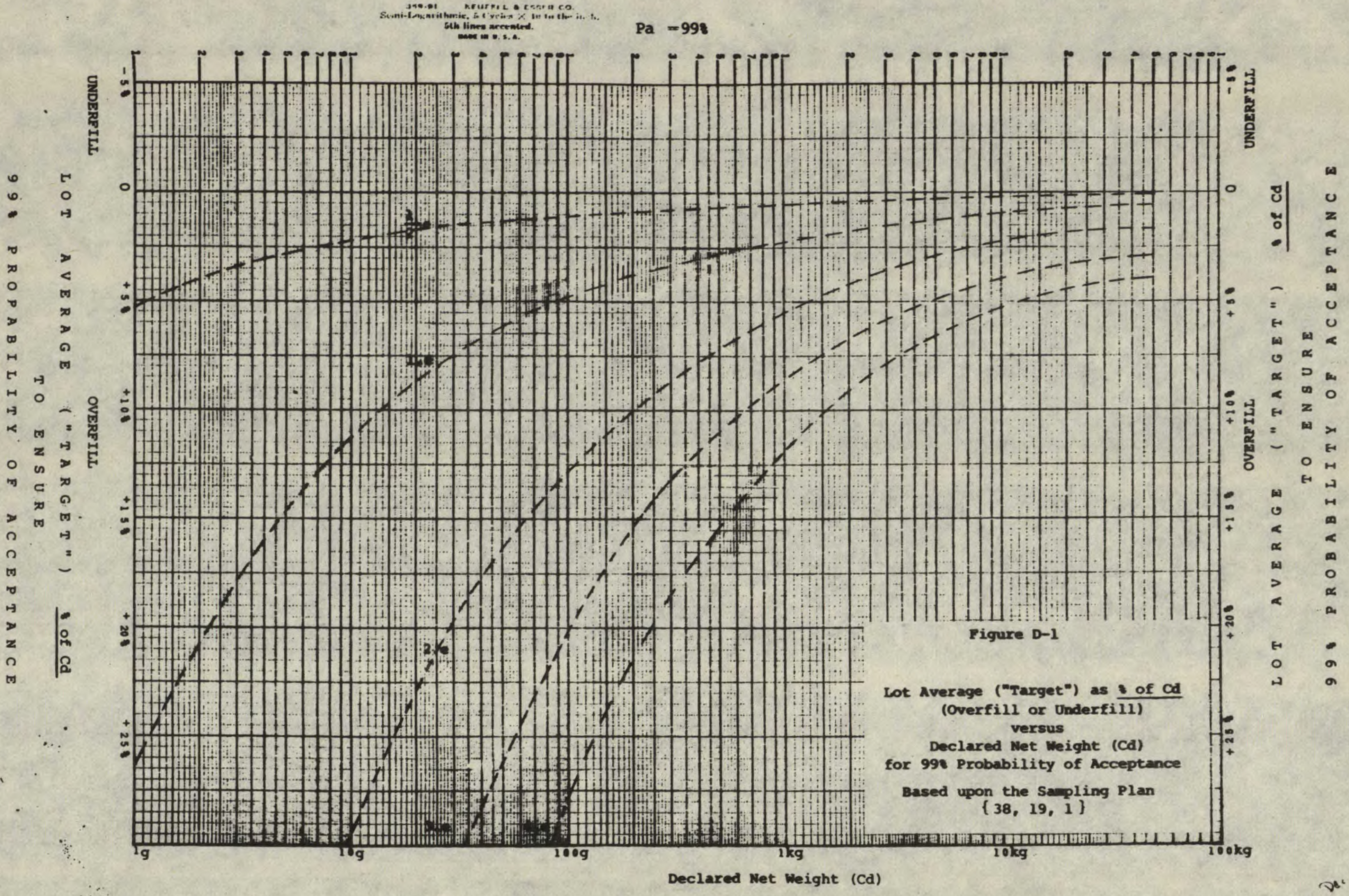

Figure D-1

Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Weight (Cd)
for 99% Probability of Acceptance

Based upon the Sampling Plan
{38, 19, 1}

Pa = 99%

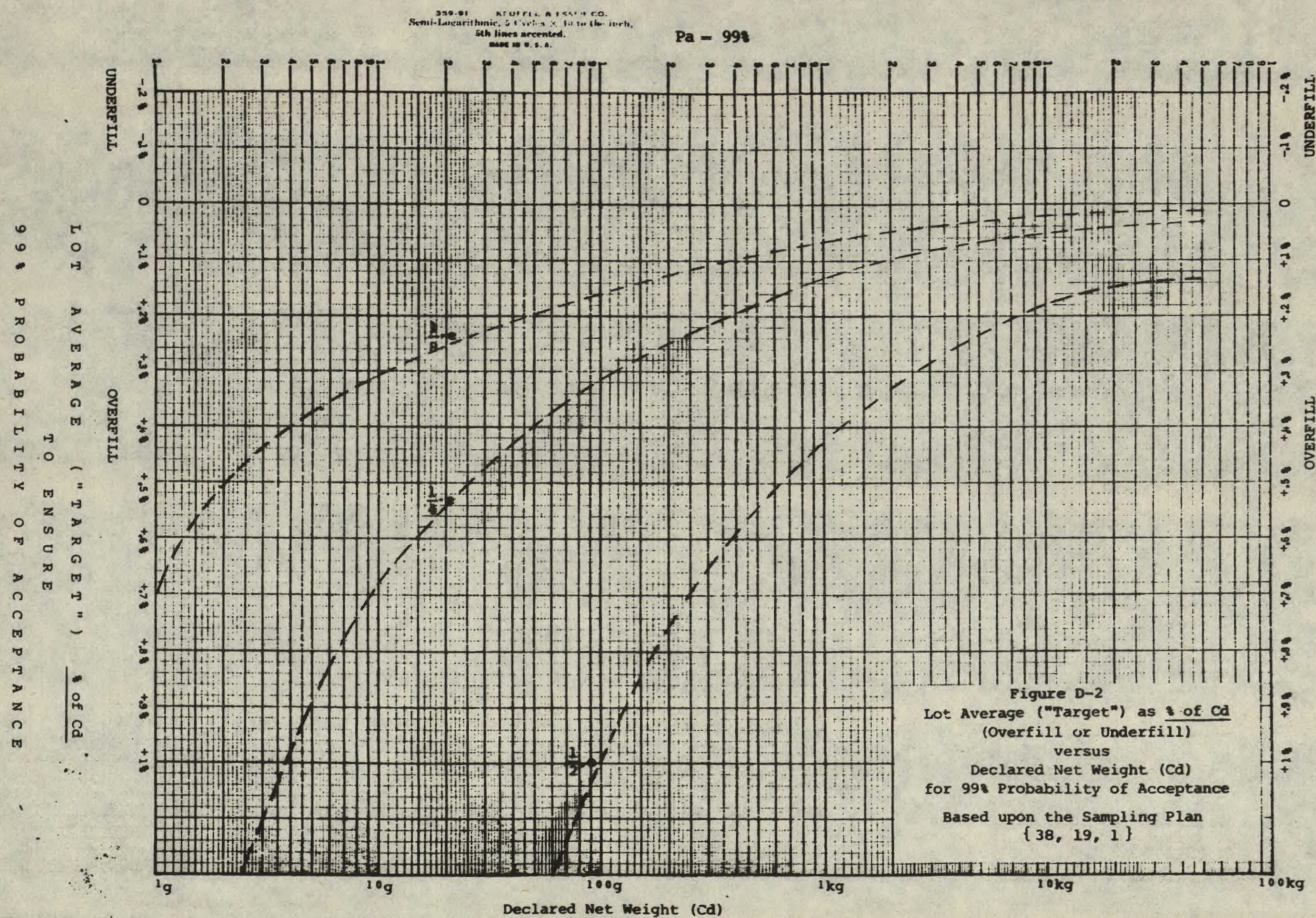

Figure D-2
Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Weight (Cd)
for 99% Probability of Acceptance

Based upon the Sampling Plan
{38, 19, 1}

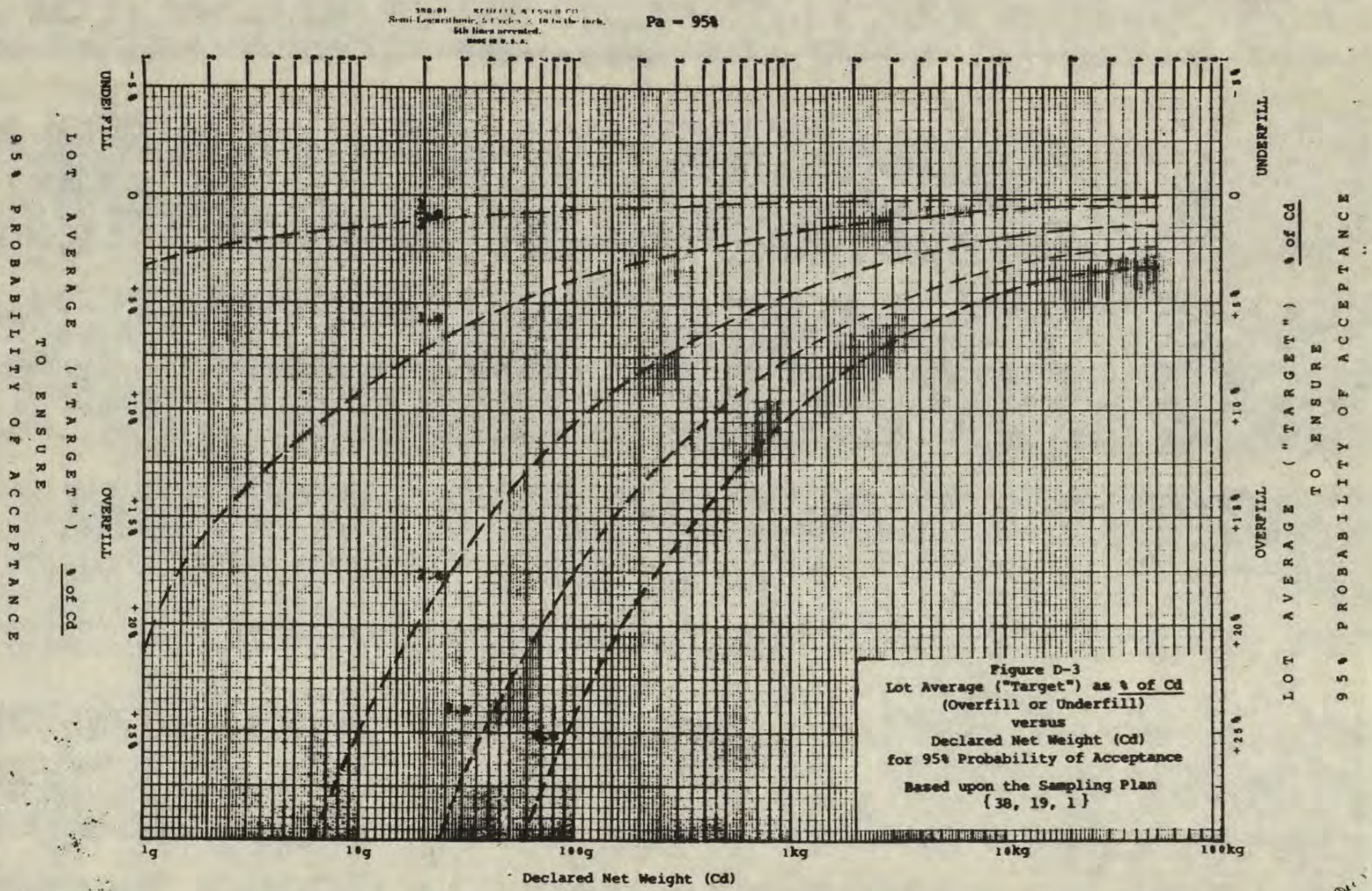

Figure D-3
Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Weight (Cd)
for 95% Probability of Acceptance

Based upon the Sampling Plan
{38, 19, 1}

Pa = 95%

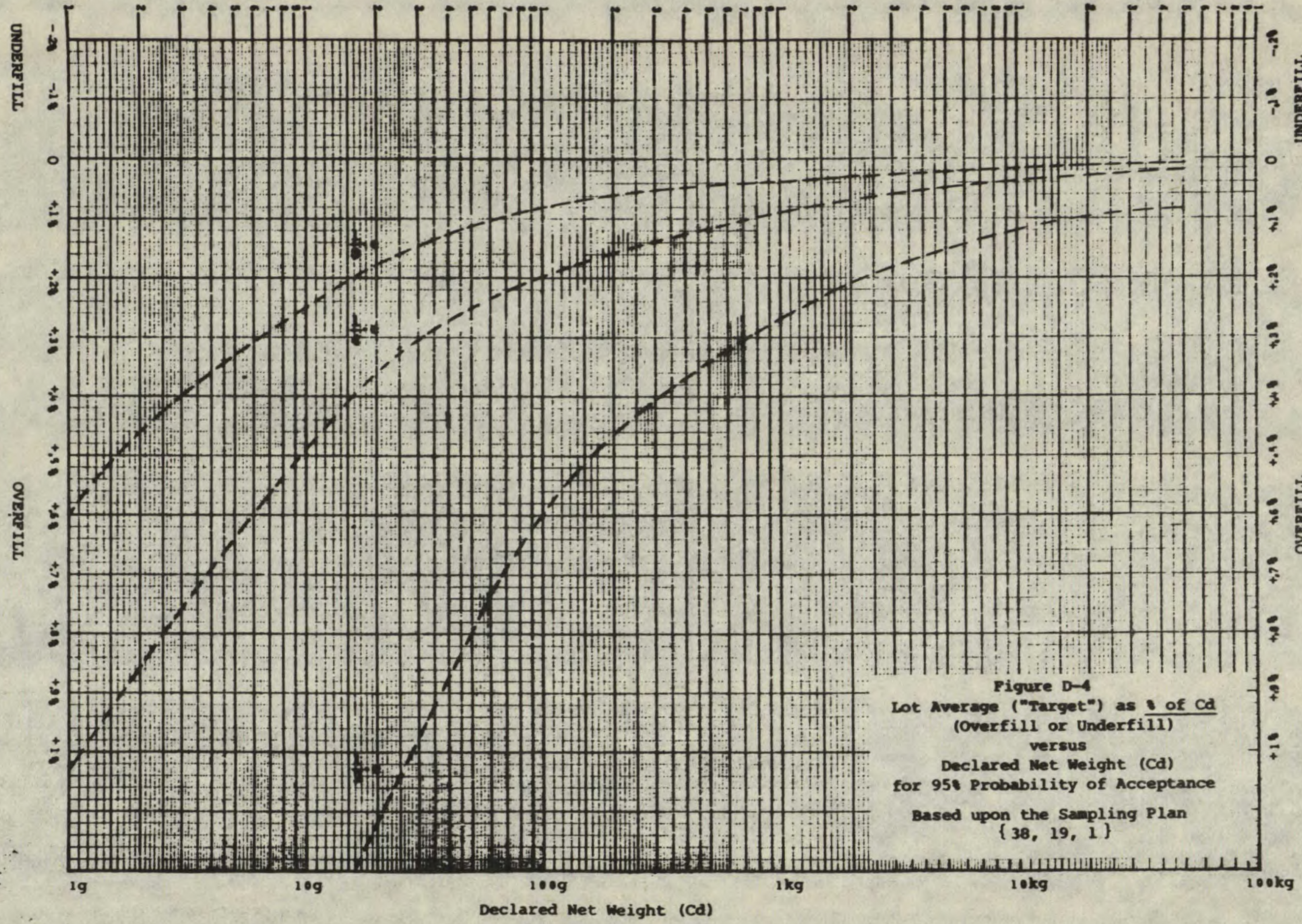

Figure D-4
Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Weight (Cd)
for 95% Probability of Acceptance

Based upon the Sampling Plan
{38, 19, 1}

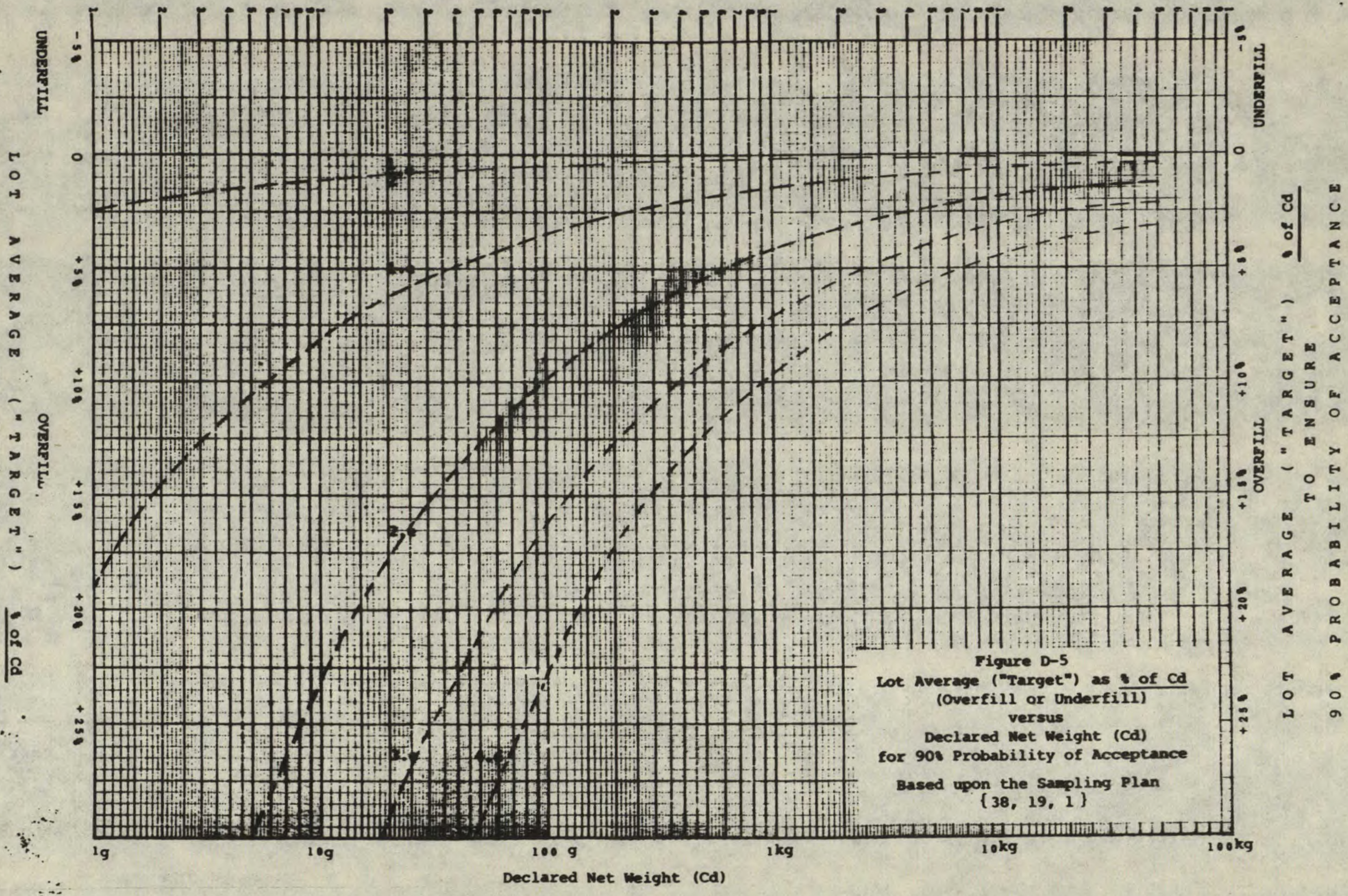

Figure D-5
Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Weight (Cd)
for 90% Probability of Acceptance

Based upon the Sampling Plan
{38, 19, 1}

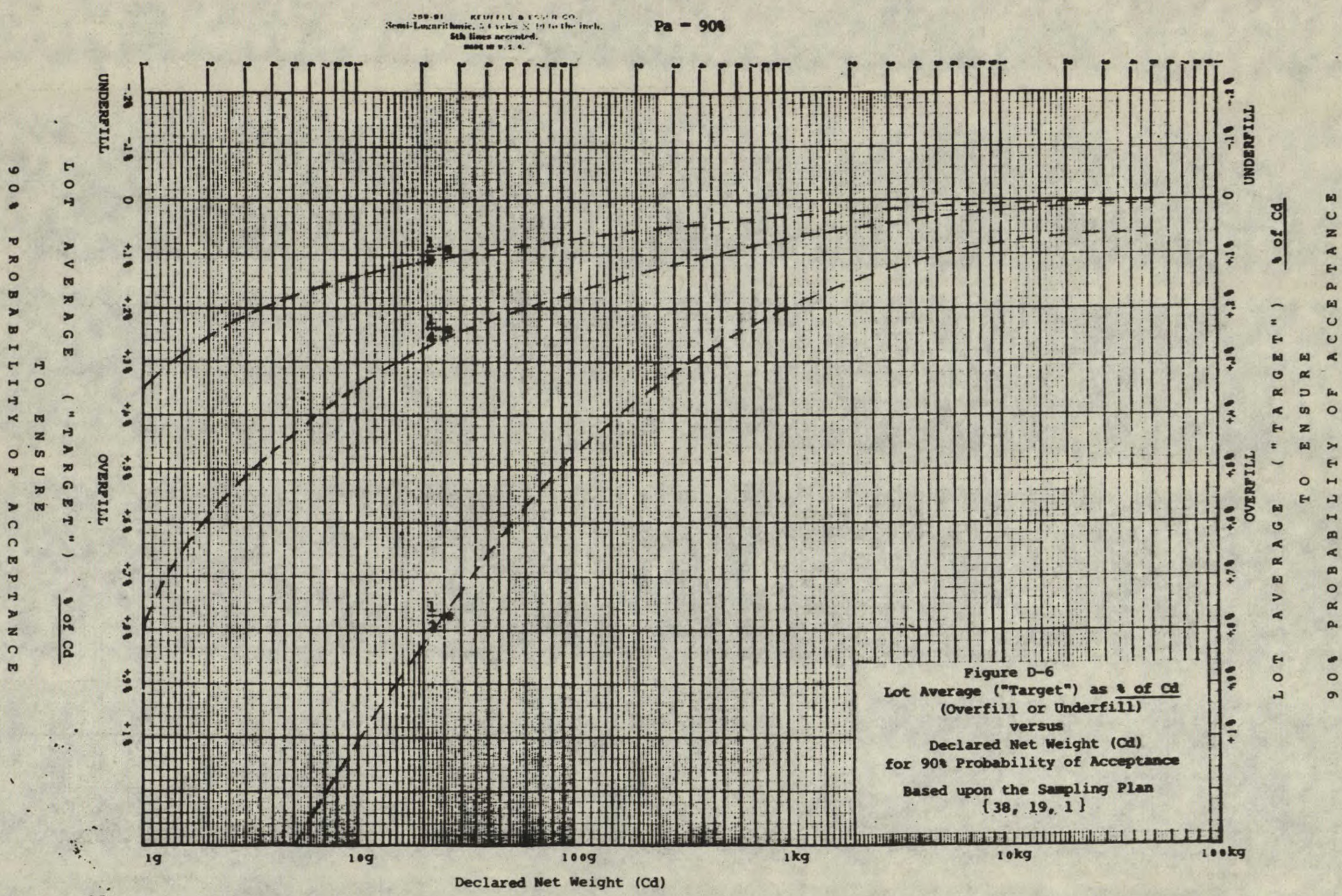

Figure D-6
Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Weight (Cd)
for 90% Probability of Acceptance

Based upon the Sampling Plan
{38, 19, 1}

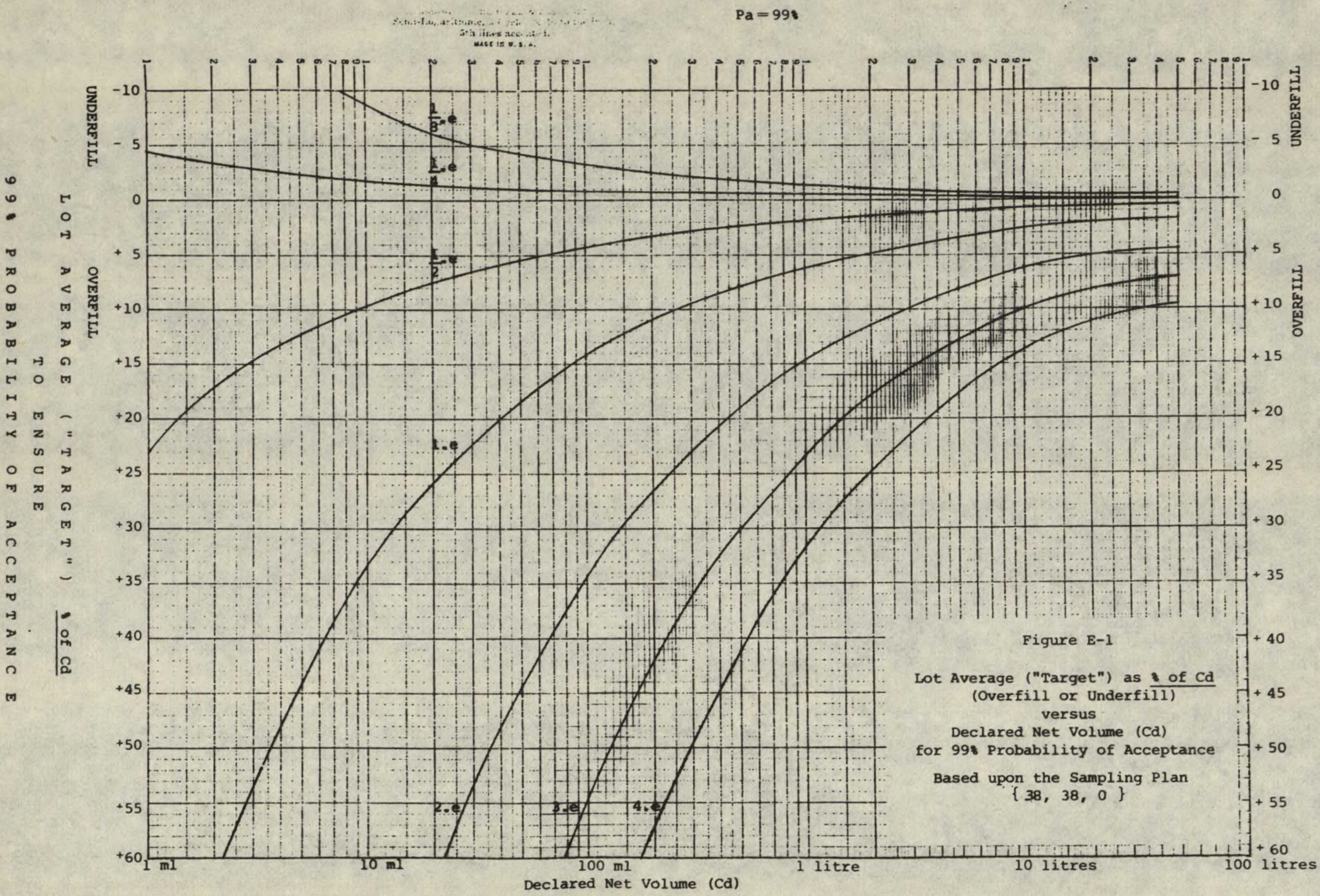

Figure E-1

Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Volume (Cd)
for 99% Probability of Acceptance

Based upon the Sampling Plan
{ 38, 38, 0 }

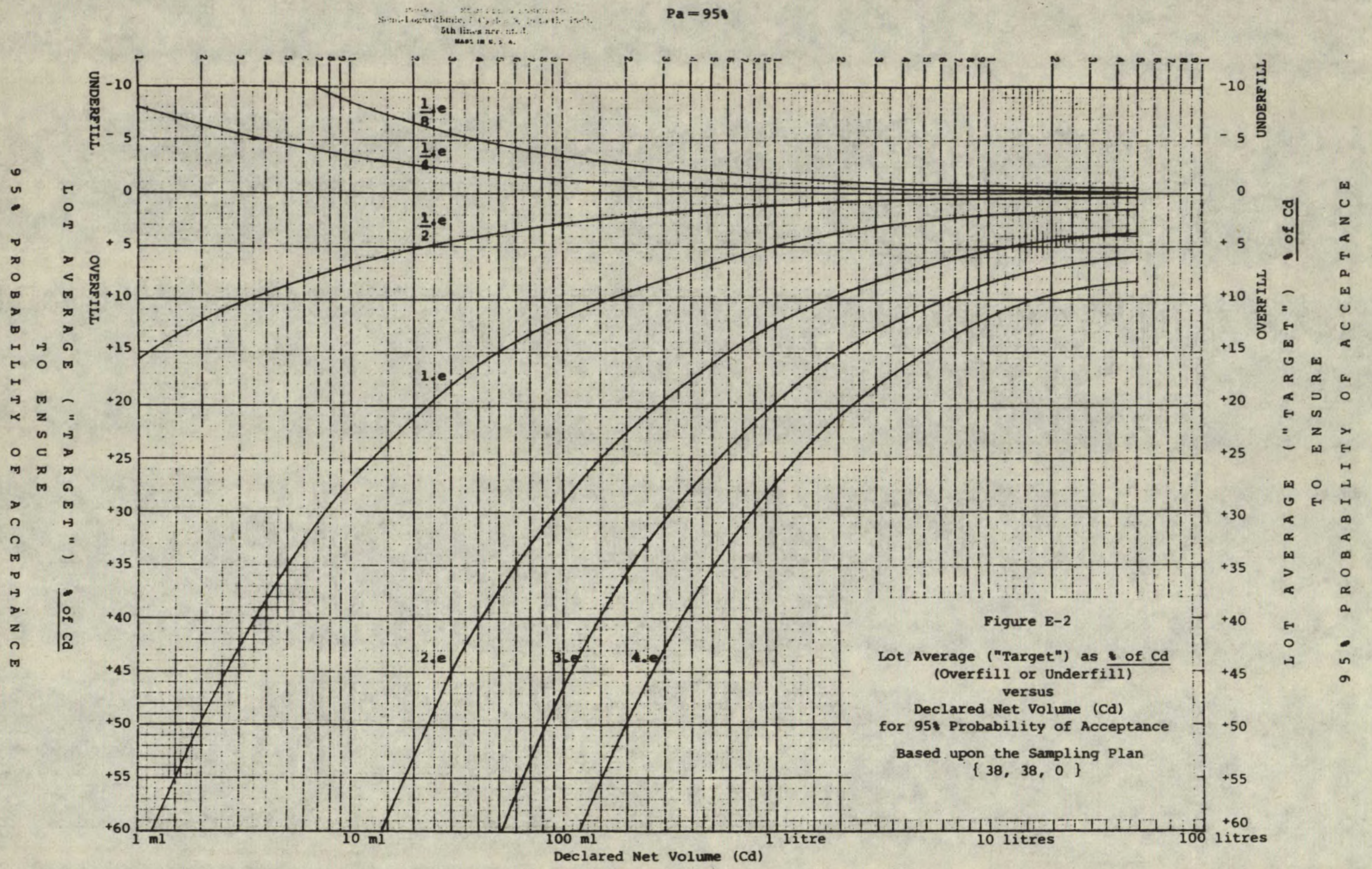

Figure E-2

Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Volume (Cd)
for 95% Probability of Acceptance

Based upon the Sampling Plan
{ 38, 38, 0 }

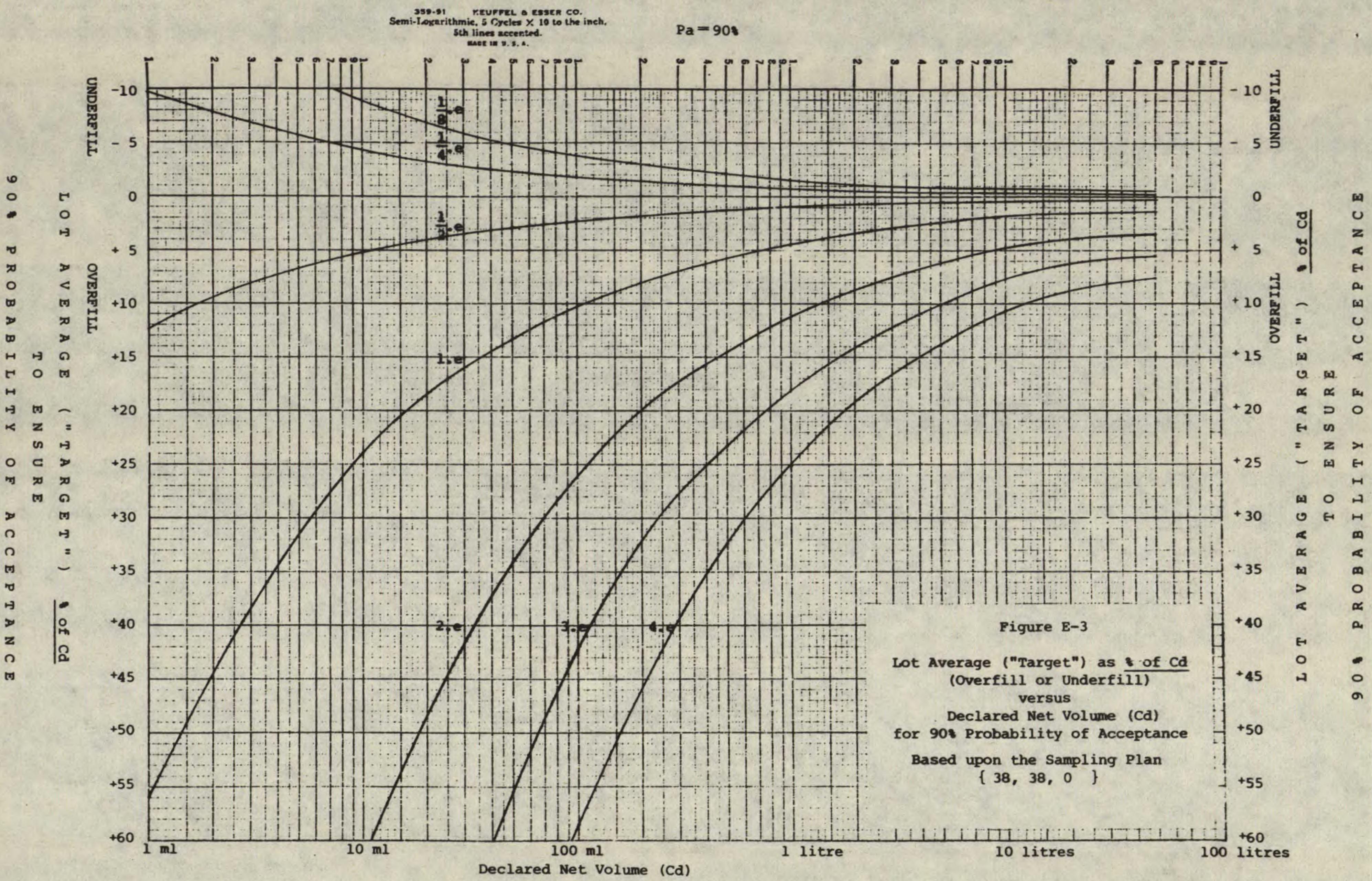

Figure E-3

Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Volume (Cd)
for 90% Probability of Acceptance

Based upon the Sampling Plan
{ 38, 38, 0 }

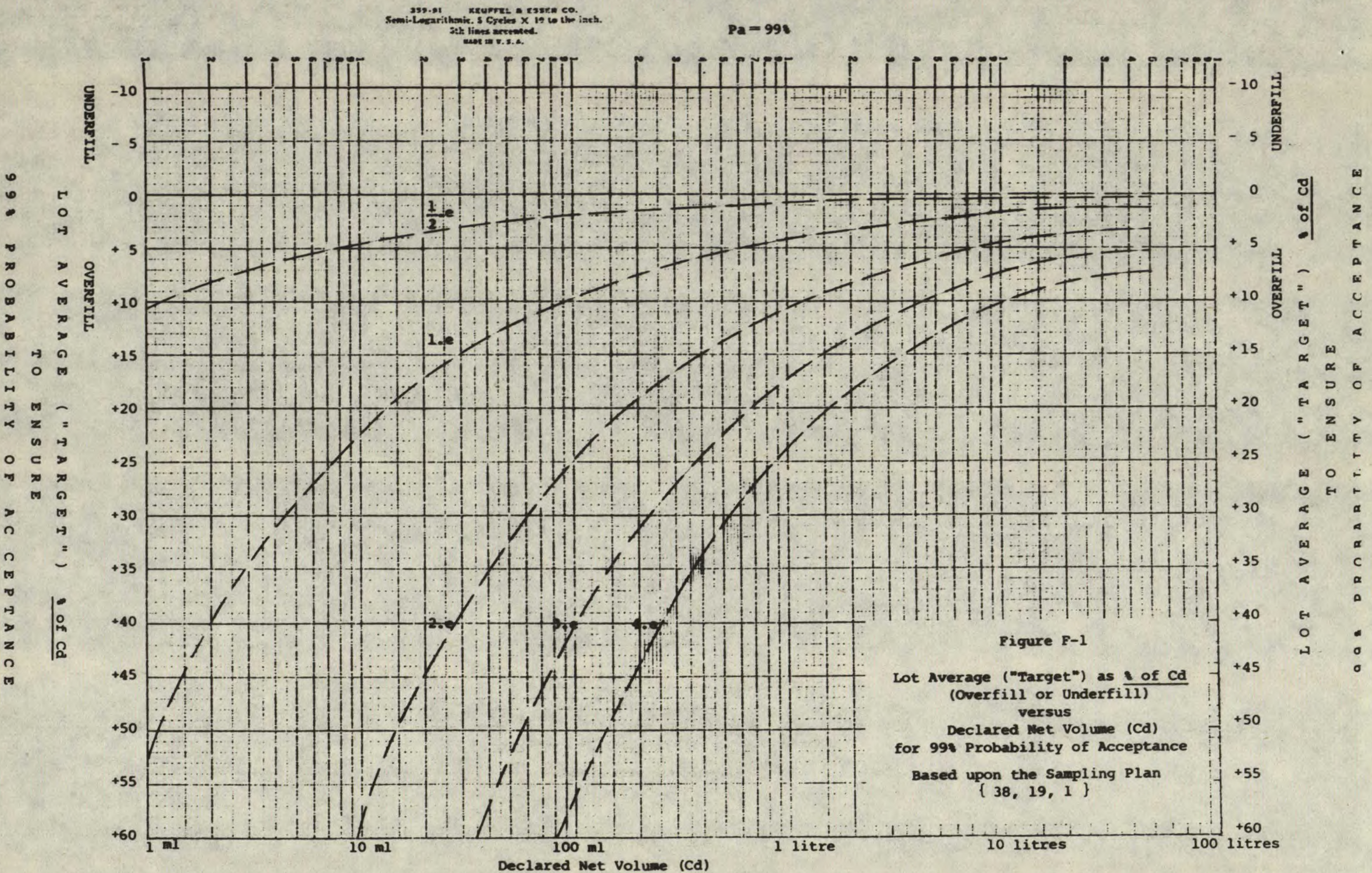

Figure F-1

Lot Average ("Target") as <u>% of Cd</u>
(Overfill or Underfill)
versus
Declared Net Volume (Cd)
for 99% Probability of Acceptance

Based upon the Sampling Plan
{ 38, 19, 1 }

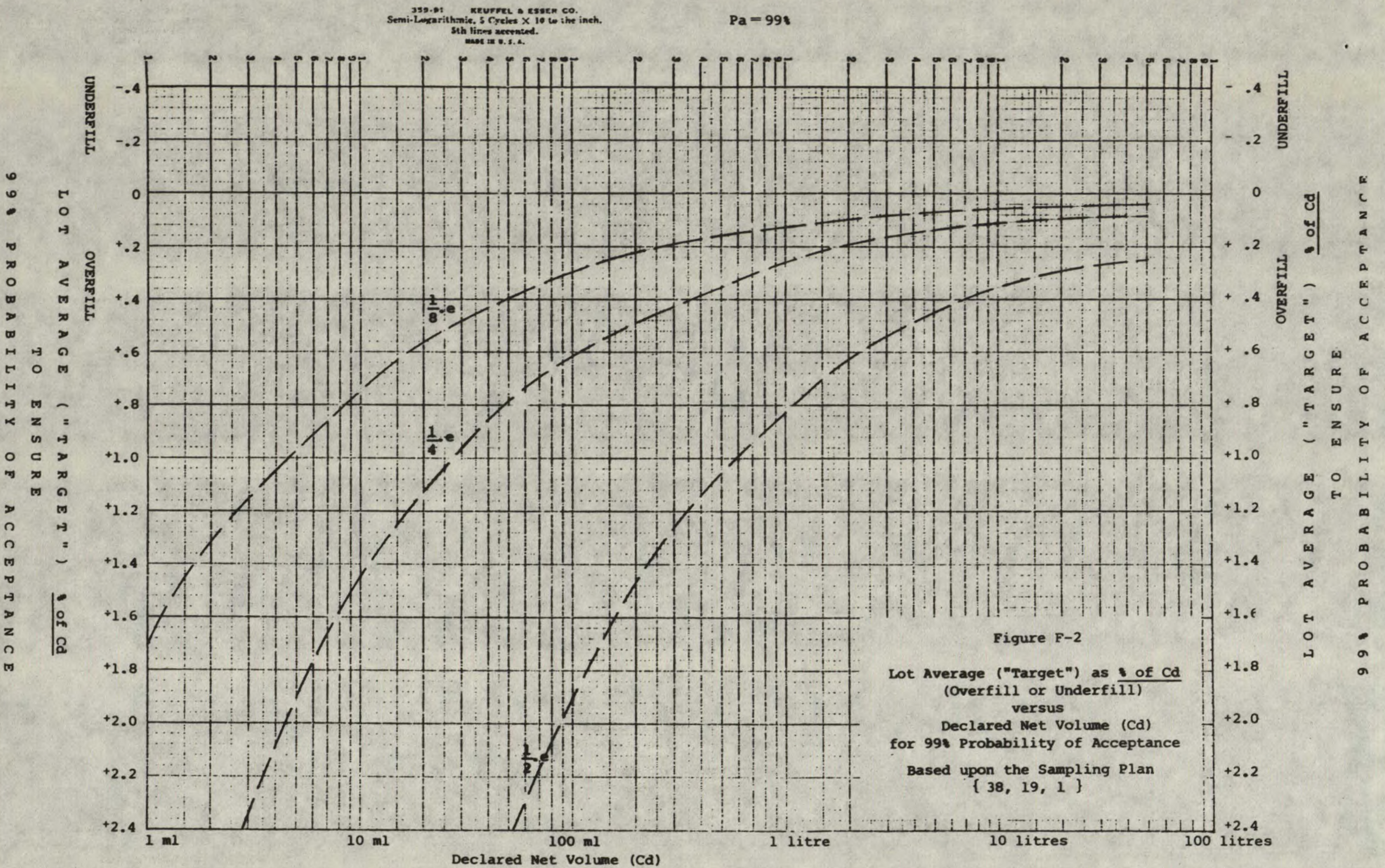

Figure F-2

Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Volume (Cd)
for 99% Probability of Acceptance

Based upon the Sampling Plan
{ 38, 19, 1 }

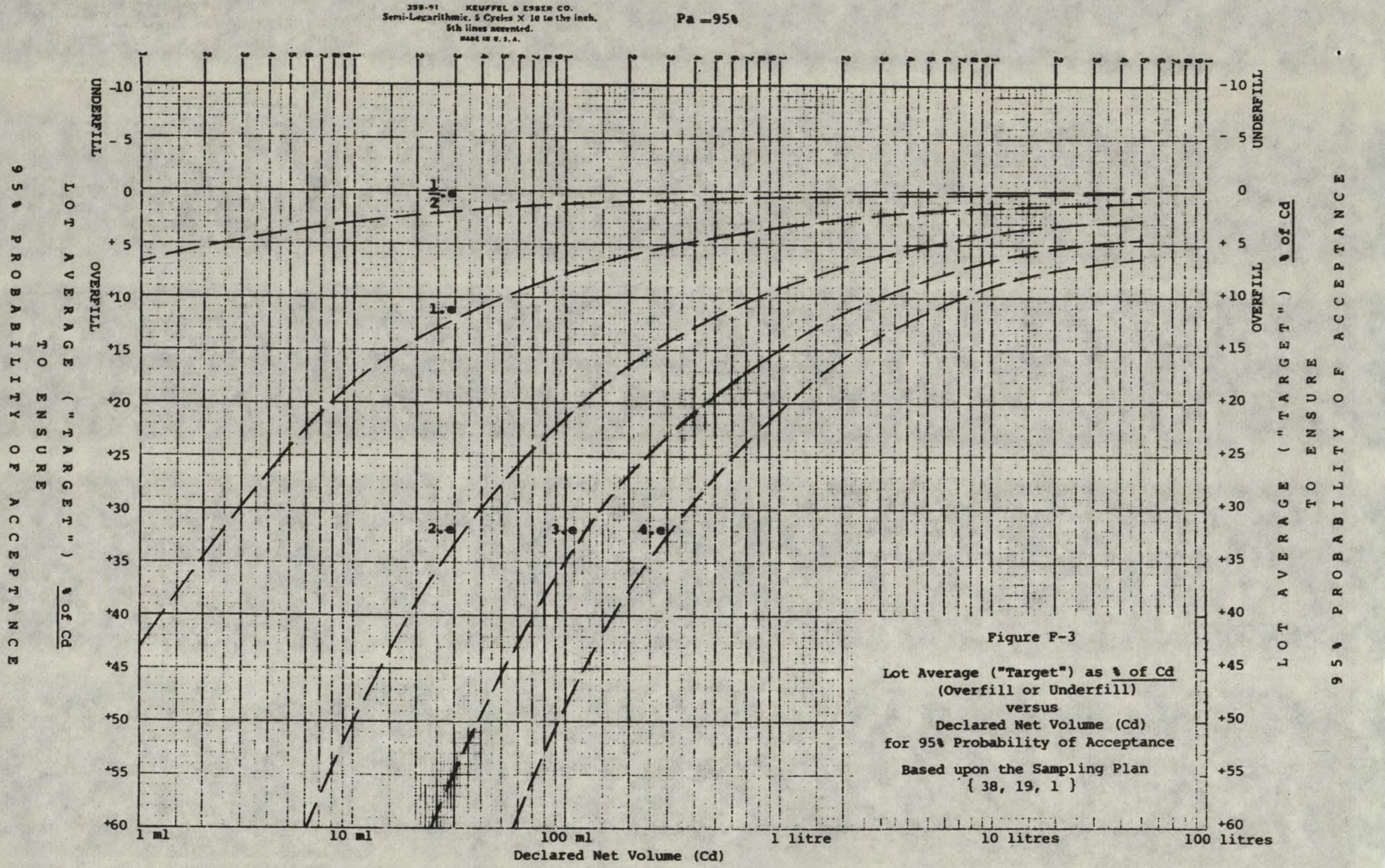

Figure F-3

Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Volume (Cd)
for 95% Probability of Acceptance

Based upon the Sampling Plan
{ 38, 19, 1 }

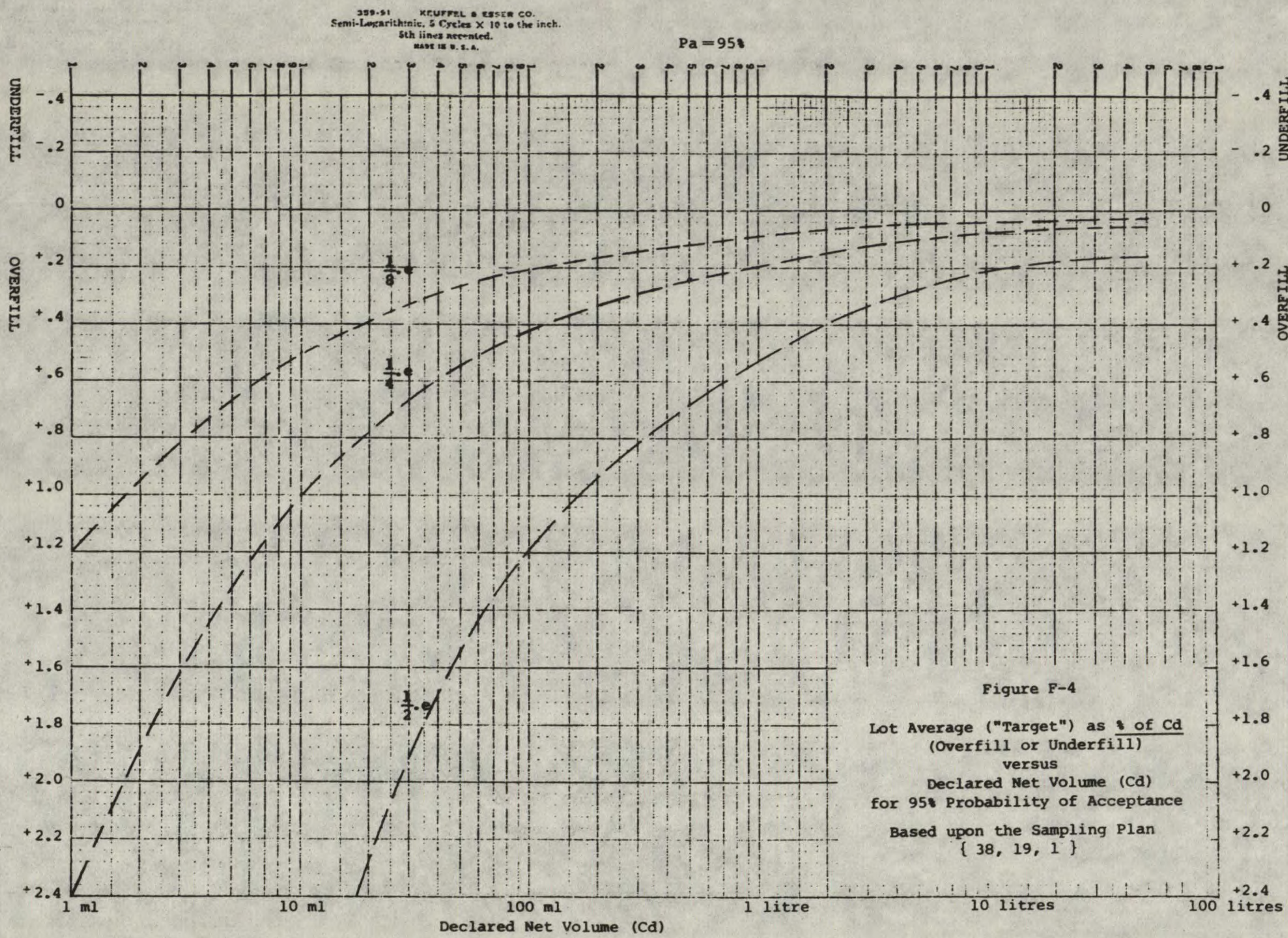

Figure F-4

Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Volume (Cd)
for 95% Probability of Acceptance

Based upon the Sampling Plan
{ 38, 19, 1 }

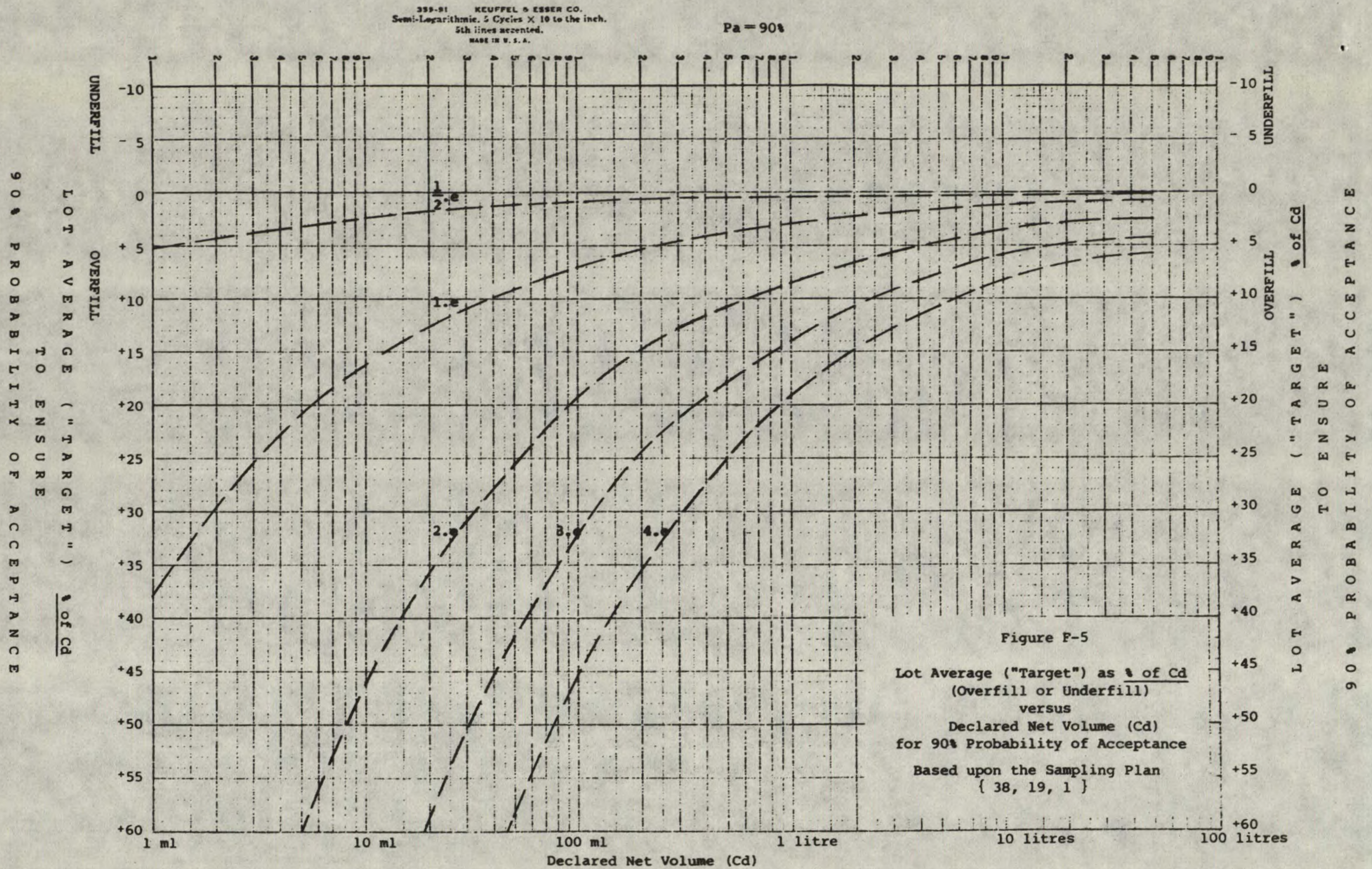

Figure F-5

Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Volume (Cd)
for 90% Probability of Acceptance

Based upon the Sampling Plan
{ 38, 19, 1 }

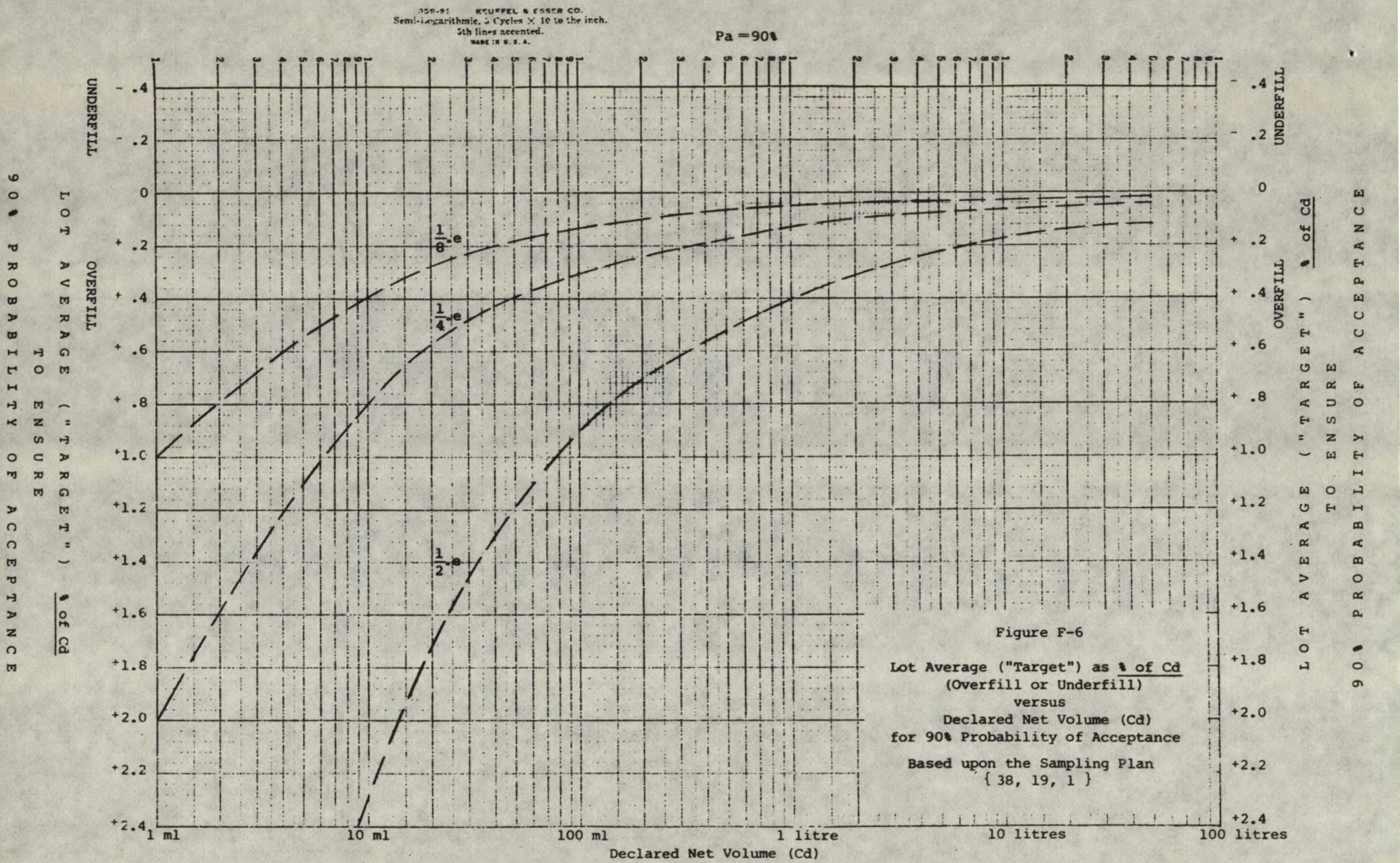

Figure F-6

Lot Average ("Target") as % of Cd
(Overfill or Underfill)
versus
Declared Net Volume (Cd)
for 90% Probability of Acceptance

Based upon the Sampling Plan
{ 38, 19, 1 }