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FORECASTING TECHNIQUES AND THE APPLICATIONS TO THE TELECOMMUNICATIONS INDUSTRY

J.I. Bernstein, V. Corbo, R.S. Pindyck
Institute of Applied Economic Research
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J.I. Bernstein, V. Corbo and R.S. Pindyck
Institute of Applied Economic Research
Concordia University

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I. QUANTITATIVE APPROACHES TO FORECASTING

I. INTRODUCTION

Forecasting is not new to the telecommunications industry; in that industry, like any other, producers must plan ahead, and this involves in some way forecasting such variables as future demand for telecommunications equipment and its response to price changes, future usage of existing equipment, future costs of capital and labor, etc. What is new is that planners are beginning to base their forecasts on more quantitative and (we hope) more reliable methods. In particular, forecasts based on simple extrapolations or intuitive "finger in the wind" methods are being replaced by forecasts based on empirically estimated models.

Why use models to make forecasts? All forecasts are in effect based on some kind of model, explicit or otherwise. Suppose one wanted to forecast future installations of telephone equipment. One might make an "intuitive" forecast of a 5 percent growth in installations over the next year, and this might be based on the belief that expected growth in GNP and in residential and business construction, with relative constancy in equipment prices, will yield thi kind of demand growth. In effect, such an intuitive forecast is based on an implicit model, since one variable is being implicitly related to other variables. Even an "intuitive" forecast of 5 percent growth based on an average of annual growth rates over the past ten years in effect relies on an implicit model - basically an extremely simple time-series model that extrapolates past trends into the future.

There are considerable advantages, however, to constructing explicit models for forecasting purposes. Model building forces the individual to think clearly about and account for all the important interrelationships involved in a problem. The reliance on intuition can be dangerous at times because of the possibility that important relationships will be ignored or improperly used. In addition, it is important that individual relationships be tested empirically or validated in some way or another. Unfortunately, this is usually not done when intuitive forecasts are made. In the process of building a model, however, the individual must test or validate not only the model as a whole, but also the individual relationships that make up the model.

When making a forecast, it is also important to provide a statistical measure of confidence to the user of the forecast, i.e. some measure of how accurate one might expect the forecast to be. The use of purely intuitive methods usually precludes any quantitative measure of confidence in the resulting forecast. The statistical analysis of the individual relationships that make up a model, and of the model as a whole, makes it possible to attach a measure of confidence to the model's forecasts. We will discuss this issue of forecast confidence in more detail later, but for now we simply point out that it provides an important advantage for the use of explicit models.

Once a model has been constructed and fitted to data, a sensitivity analysis can be used to study many of its properties. In particular, the effects of small changes in individual variables in the model can be evaluated. For example, in the case of a model that describes and predicts installations of telephone equipment, one could measure the effect on installations of a change in interest rates or GNP. This type of quantitative sensitivity study, which is important both in understanding and in using a model, can only be done if the model is an explicit one.

Types of Models.

There are three general classes of models that can be constructed for purposes of forecasting and analysis. Each involves a different degree of model complexity and structural explanation, and each presumes a different level of comprehension about the real world processes that one is trying to model. The three classes of models are as follows:

A. Time-Series Models. In this class of models we presume to know nothing about real world causal relationships that affect the variable we are trying to forecast. Instead we examine the past behavior of a time-series in order to infer something about its future behavior. The time-series model used to produce a forecast might involve the use of a simple linear extrapolation model, or the use of a more complex stochastic model for adaptive forecasting.

One example of the use of time-series analysis would be the simple extrapolation of a past trend in predicting population growth. Another example would be the development of a complex linear stochastic model to forecast passenger loads on an airline. Models such as this have been developed and used to forecast the demand for airline capacity, seasonal telephone demand, the movement of short-term interest rates, as well as other economic variables. Time-series models are particularly useful when little is known about the underlying process that one is trying to forecast. The limited structure in time-series models makes them most reliable only in the short run, but they are nonetheless rather useful.

B. Single-Equation Regression Models. In this class of models the variable under study is explained by a single function (linear or non-linear) of explanatory variables. The equation will often be time-dependent (i.e. the time index will appear explicitly in the model), so that one can predict the response over time of the variable under study to changes in one or more of the explanatory variables.

An example of a single-equation regression model might be an equation that relates a particular variable, such as installations of telephone equipment, to a set of explanatory variables such as GNP, a price index of telephone equipment, a rate of inflation, etc.

C. Multi-Equation Simulation Models. In this class of models the variable to be studied may be a function of several explanatory variables, but now these explanatory variables are related to each other as well as to the variable under study through a set of equations. The construction of a simulation model begins with the specification of a set of individual relationships, each of which is fitted to available data. Simulation is the process of solving these equations simultaneously over some range in time.

An example of a multi-equation simulation model would be a complete model of the demand and supply of telephone equipment. Such a model might contain equations that explain the cost of production of equipment, the supply of equipment as a function of price, demand for equipment as a function of price, GNP, and other variables, and changes in the stock of equipment. These endogenous variables would be related to each other and to other exogenous variables (such as GNP, the consumer price index, interest rates, etc.), through a set of linear or non-linear equations. Given assumptions about the future behavior of the exogenous variables (i.e. those variables determined outside of the model), one could simulate this model into the future and obtain a forecast for each of the model's endogenous variables. A model such as this can be used to analyze the impact on the industry of changes in external economic variables.

Multi-equation simulation models presume to explain a great deal about the structure of the physical process that is being studied. Not only are individual relationships specified, but the model accounts for the interaction of all these interrelationships at the same time. Thus, a five equation simulation model actually contains more information than the sum of five individual regression equations. The model not only explains the five individual relationships, but it also describes the dynamic structure implied by the simultaneous operation of these relationships.

How does one choose which type of model to construct? The choice can be a difficult one, involving tradeoffs among time, energy, cost, and desired forecast precision. The construction of a multi-equation simulation model might require large expenditures of time and money, not only in terms of actual work, but also in terms of computer time. The gains that result from this effort might include a better understanding of the relationships and structure involved as well as the ability to make a better forecast. However, in some cases these gains may be small enough so that they are outweighed by the heavy costs involved. Because the multi-equation model necessitates a good deal of knowledge about the process being studied, the construction of such models may be extremely difficult.

The decision to build a time-series model usually occurs in cases when little or nothing is known about the determinants of the variable being studied, when a large number of data points are available (thus making some kind of statistical inference feasible), and when the model is to be used largely for short-term forecasting. Given some information about the processes involved, however, it may not be obvious whether a time-series model or a single equation regresion model is preferable as a means of forecasting. It may be reasonable for a forecaster to construct both types of models and compare their relative performances. Or, as we will discuss later, a time-series model can be constructed as part of a single equation regression model or multi-equation simulation model.

Since all of these models are important in terms of their potential application to forecasting in the telecommunications industry, we will discuss each of them in somewhat more detail.

Single-Equation Regression Models.

Much of the substance of modern econometrics is based on the construction and testing of single-equation models. The single-equation regression model is probably the most widely used model for forecasting, it is the basic building block of the multi-equation simulation model, and the statistical techniques used in its estimation are basic to time-series models as well.

The construction of a single-equation regression model begins with the specification of a relationship between a dependent variable and a set of independent variables. The relationship need not be linear in the variables themselves, but it is usually linear in the unknown parameters that are to be estimated.[1] For example, we might specify the following relationship between sales of telephone equipment S, GNP, an average price index of equipment, P, a residential construction index C, and past sales:

$$\log S_t = a_0 + a_1 \log GNP_{t-1} + a_2 \log P_t + a_3 \log C_t + a_4 \log S_{t-1} \quad (1)$$

The idea here is that sales depends directly on not only current but also past values of GNP, price, and construction. The introduction of the lagged sales variable imposes a long geometrically declining lag on the relationship between sales and the other independent variables. Logarithms are taken since it is believed (for purposes of this example) that the relationship between percentage changes is the same for every level of the variable.

The next step is to estimate the unknown parameters a_0, a_1, a_2, a_3, and a_4. This is usually done using a variation of least-squares estimation. The idea is to choose the parameters so as to minimize the sum of the squared differences between the actual data points (for the dependent variable S) and the predicted values from the regression equation. We will not go into the details of this technique here, as it is rather standard, and used not only in econometric applications but also in other statistical applications.

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[1] Models that are non-linear in the parameters are called inherently non-linear More complicated estimation techniques must be used to fit these models to data, and the statistical testing of these models is likewise more complicated. Inherently non-linear models can be constructed, however, and their use is becoming more prevalent as computer capacity becomes more available for their estimation.

We should point out that the use of ordinary least-squares estimation implies certain assumptions about the equation to be estimated and the implicit error of the equation. Clearly no regression equation will be sufficient to explain all of the variation in a dependent variable; there are always some factors that are not included or that cannot be included because data does not exist. For example, it may be that sales of equipment are also dependent on the weather (which cannot be predicted), people's moods (which cannot be measured), and other variables that we have failed to take into account. All of these unexplained effects can be summarized in the form of an implicit additive error term for the regression. In order to obtain unbiased and efficient estimates of the parameters (i.e. estimates which on the average can be expected to equal the true values of the parameters, and estimates that have the smallest possible variance), certain assumptions regarding the statistical properties of the implicit error term and its relationship to the independent variables must hold. Again, we do not have the time or space to go into those assumptions here, but suffice it to say that in many cases where those assumptions fail to hold, alternative estimation procedures can be used to yield unbiased and efficient estimates of the parameters.

Once a model has been estimated, we would like to attach some quantitative measures to its validity. In testing a single equation regression model, we are interested in the statistical significance of the individual estimated coefficients and the significance of the equation as a whole. Standard errors can be computed for the individual coefficients, and these indicate the variability of the estimated values of the coefficients from the true values, i.e. they provide a measure of fit for the individual coefficients. In addition, we can obtain an estimate of the standard deviation of the implicit additive error term, and we call this the standard error of the regression. The standard error of the regression and the standard errors of the individual coefficients are used to determine confidence intervals on forecasts generated from the equation. Again, we will not go into the details of computing standard errors and other test statistics at this point; it is standard material that can be found in most textbooks on econometrics.

Once a single equation regression model has been estimated and statistically tested, it can be used to forecast future values of the dependent variable. We can distinguish between two types of forecasts generated by a model, _ex post_ and _ex ante_. Both forecasts predict values of the dependent variable beyond the time period over which the model was estimated, but in the _ex post_ case the forecast period is such that observations on both endogenous variables and the exogenous explanatory variables are known with certainty. Thus, _ex post_ forecasts can be checked against existing data and provide a means of evaluating a forecasting model. An _ex ante_ forecast predicts values of the dependent variable beyond the estimation period, using independent variables which may or may not be known with certainty, depending on the nature of the data and the length of the lags associated with the explanatory variables.

We may also distinguish between _conditional_ and _unconditional_ forecasts. In an unconditional forecast, values for all the independent variables in the forecasting equation are known with certainty. Any _ex post_ forecast is, of course, an unconditional forecast, but _ex ante_ forecasts may also be unconditional.

Suppose, for example, that for some industry, monthly sales, S, are related linearly to two variables X_1 and X_2, but with lags of three months and four months respectively:

$$S(t) = a_0 + a_1X_1(t-3) + a_2X_2(t-4). \qquad (2)$$

If the coefficients of this equation were estimated, the equation could be used to produce unconditional forecasts of S one month, two months, and three months into the future.

In a conditional forecast, values for one or more independent variables are not known with certainty, so that guesses (or forecasts) for them must be used to produce the forecast of the dependent variable. If we wanted to use the equation above to forecast S four months into the future, we would also have to forecast X_1 one month into the future, making our forecast of S *conditional* upon our forecast of X_1. Of course, if the right-hand side of the forecasting equation contained no lags, e.g. if it was of the form

$$S(t) = a_0 + a_1X_1(t) + a_2X_2(t) \tag{3}$$

then every *ex ante* forecast generated by the equation would be a conditional forecast.

Of course after making a forecast we would also like to have some measure of forecast accuracy, i.e. a margin of error to attach to the forecast. Such a measure is the *standard error of forecast*, which is an estimate of the standard deviation of the forecast error, and provides a confidence bound for the forecast. We will discuss the computation and use of the standard error of forecast after we describe multi-equation models and time-series models.

Mutli-equation Simulation Models.

One problem with the single equation regression model is that it does not explain the interdependencies that may exist among the independent variables themselves, or how these independent variables are related to other variables. In addition, the single equation model explains causality in only one direction;

i.e., independent variables determine a dependent variable, but there is no "feedback" relationship between the dependent variable and the independent variables. Multi-equation simulation models, on the other hand, allow us to account simultaneously for all of the interrelationships between a set of variables. Often these models consist of a set of regression equations which, after having been estimated, are solved simultaneously on a computer. However, some of the equations that comprise a simulation model might not be estimated, but might be accounting identities, or even behavioral 'rules of thumb' that specify how particular variables will change under different conditions.

Since many, if not all, of the equations that comprise a multi-equation simulation model are themselves single-equation regression models, the methodological issues discussed earlier also apply here. In addition, however, there are two sets of methodological issues that are of particular relevance to multi-equation simulation models. The first set of issues has to do with some particular problems that arise in the estimation of these models. The second set of issues has to do with the dynamic structure of the models, i.e. the dynamic behavior that arises from the numerous feedback loops that arise when individual equations are solved simultaneously over time.

Methodological issues in estimation arise for two reasons. First, in a multi-equation model there is a question about whether or not individual equations can be identified, i.e. whether the structure of the model allows for estimation of the equations' parameters, even if there is plenty of data available. To illustrate this problem, consider a model that consists of two equations, one that relates quantity supplied to price:

$$Q^S = a_0 + a_1 P \qquad (4)$$

and one that relates quantity <u>demanded</u> to price:

$$Q^D = b_0 + b_1P. \tag{5}$$

Since supply must always equal demand, all of the data will represent the intersection of the two curves (supply curve and demand curve) represented by these two equations. Both equations will be unidentified, so that even if data is available for price and quantity, it will not be possible to estimate the parameters a_0, a_1, b_0, and b_1. If, on the other hand, supply depended on an additional variable C (a cost index), and demand depended on an additional variable Y (per capita income) then our equations would be given by

$$Q^S = a_0 + a_1P + a_2C \tag{6}$$

and

$$Q^D = b_0 + b_1P + b_2Y \tag{7}$$

and both equations would now be identified. Movements in the variable C would cause the supply curve to trace out the demand curve, by moving along the demand curve over time. Similarly, changes in the variable Y would cause the demand curve to trace out the supply curve over time. Thus, if data were available for Q, P, C, and Y, it would be possible to estimate all of the parameters of the equations. The set of methodological techniques associated with the identification problem makes it possible to determine whether or not individual equations are identified.

The construction of a simulation model involves much more than simply putting together several individually estimated single equations. When individual regression equations, which may fit the historical data very well, are combined to form a simultaneous equation model, simulation results may bear little resemblance to reality. The difficulty arises because the construction of the simulation model often involves understanding the dynamic structure of the system that results when individual equations are combined, and this may not be a straightforward process. The second set of methodological techniques have to do with analyzing the dynamic structure of the simulation model, and using that analysis to modify the model so that it provides a better representation of the real world. Since a simulation model is basically a set of <u>difference equations</u> that are solved simultaneously, the methodological techniques (which were developed some time ago by mathematicians and engineers) basically provide ways of studying the characteristics of difference equations solutions.

As is the case with a single equation regression model, before a simulation model is used for forecasting one must be able to evaluate it and compare it to alternative models of the same physical process. Model validation and testing is more complicated for a simulation model than it is for a single-equation regression model. One can begin, as in the single-equation case, to look at the set of available statistics (standard errors and the like) for each individual equation to make a judgment about the goodness of fit of that equation. However, in a simulation model each individual equation may have a good statistical fit, but the model as a whole may do a poor job in reproducing the historical data. The converse may also be true; the individual equations of a simulation model may have a poor statistical fit, but the model when taken as a whole may reproduce the historical data quite closely. Thus, additional criteria must be used in the evaluation of simulation models. Usually these criteria involve statistics that describe the simulation performance of the model under different conditions. A typical statistic is the root-mean square (RMS) simulation error, which is given by:

$$\text{RMS error} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(Y_t^s - Y_t^a)^2} \quad (8)$$

where Y_t^s = simulated value of Y_t

Y_t^a = actual value of Y_t

T = number of periods in simulation.

RMS errors can be computed for each endogenous variable of the model by simulating the entire model over some historical time period. These RMS errors, together with the individual equation statistics, can be used to evaluate the model as a whole. Other criteria involve the ability of the model to pick up turning points in the data, i.e. a sudden change in the historical data, and, of course, the ability of the model to produce small errors in an ex post forecast.

Time-Series Models.

A time-series model is quite different in nature from the single-equation and multi-equation models described above. In a time-series model we do not predict future movements in a variable by relating it to a set of other variables in a causal framework; instead we base out predictions solely on the past behavior of the variable and that variable alone. Consider, for example, a data series y(t) representing the historical performance of some economic variable, e.g. a production index, or perhaps the daily sales volume for some commodity. We may or may not be able to explain (based on economic theory, intuitive reasoning, etc.) why y(t) behaved the way it did. If y(t) represents the sales volume some good, for example, it may have moved up or down partly in response to changes in prices, personal income, and interest rates (or so we might believe). However, much of its movement may have been due to factors that we may simply not be able to explain, such as the weather, changes in consumer taste, or simply seasonal cycles in consumer spending.

It may be difficult or impossible to explain the movement of y(t) through the use of a structural model, i.e. by relating it explicitly to other economic variables. It may be that data are not available for those explanatory variables which are believed to affect y(t), or if data were available, the estimation of a regression model might result in standard errors that are so large as to make most of the estimated coefficients statistically insignificant so that forecast confidence intervals would be unacceptably large. Even if we could estimate a statistically significant regression equation for y(t), the result may not be useful for forecasting purposes. To obtain a forecast for y(t) from a regression equation, those explanatory variables that are not lagged must themselves be forecasted, and this may be more difficult than forecasting y(t) itself. The standard error of forecast for y(t) with future values of the independent variables known may be small (if the regression equation fits well), but the forecast error for the independent variables themselves may be so large as to make the total forecast error for y(t) too large to be acceptable.

Clearly, then, situations may exist where it is impossible or undesirable to "explain" y(t) using a structural model, and we might ask whether there is an alternative means of obtaining a forecast. Are there ways in which we can observe the data series for y(t) and draw some conclusions about its <u>past</u> behavior that would allow us to infer something about its probable <u>future</u> behavior? For example, is there some kind of overall upward trend in y(t) which, because it has dominated the past behavior of the series, might dominate its future behavior? Or does the series exhibit some kind of cyclical behavior which we could extrapolate into the future? If some kind of systematic behavior of this type is present, we can attempt to construct a model for the time series which does not offer a structural explanation for its behavior in terms of other variables, but does replicate its past behavior in a way that might help us forecast its future behavior. On this basis the time-series model accounts for patterns in the past movements of a particular variable, and uses that information to predict future movements of the variable. In a sense a time-series model is just a sophisticated method of extrapolation, and yet it may often provide a very effective tool for forecasting.

Most of the models that we work with belong to the class of linear time-series models introduced by G.E.P. Box and G.M. Jenkins that have recently found wide application to economic and business forecasting.[2] The most basic linear time-series model is the mixed auto-regressive moving average model, which is represented by the following equation:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \delta + \varepsilon_t - \Theta_1 \varepsilon_{t-1} - \ldots - \Theta_q \varepsilon_{t-q} \quad (9)$$

where

$y_{t-1}, \ldots, y_{t-p}$ are the autoregressive terms, δ represents the mean value of the series, and the remaining terms represent a moving average error process, i.e. a weighted average of current and lagged values of a random error term. The estimation of the parameters of this equation is somewhat difficult, since it involves the use of a nonlinear estimation process. However, given the availability of a computer facility and the appropriate software, estimation does not impose any particular problem.

In order for the above equation to be a valid forecasting tool, the time-series to which it applies must be <u>stationary</u>. This means that the mean value of the series, and the expected variation around that mean value, should be the

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[2] These models were introduced by Box and Jenkins in their book, Time Series Analysis (San Francisco; Holden-Day, 1970).

same at any point in time. Most economic variables violate this condition of stationarity, since they tend to grow over time (so that the mean value is an increasing function of time). Time-series models can, however, be constructed for non-stationary series in most cases. To do this, the time series is <u>first differenced</u> one or more times, i.e. we construct a new series w_t from

$$w_t = \Delta y_t = y_t - y_{t-1}. \qquad (10)$$

Convenient tests can be used to determine whether w_t is stationary; if it is not, then the series w_t is differenced and the process is repeated until a stationary series results. An autoregressive moving average model can then be constructed for the stationary series. This model is then used to produce forecasts, and the forecasted series is <u>integrated</u> (i.e. summed up one or more times) to yield a forecast of the original series y_t.

Time-series models have already found considerable application to forecasting in the telecommunications industry. In one recent example, time-series models were constructed and used to forecast the inward and outward station movements of the Wisconsin Telephone Co. using monthly data from January 1951 to October 1966.[3] Inward and outward station movements in any month simply represent the number of telephone installations or disconnects respectively. The problem of obtaining forecasts of station movements is rather important to the telephone industry, since these forecasts are used as fundamental inputs for both short and long-term company planning. The difference between inward and outward station movements represents the net increase (or decrease) of telephones in service, so that an expected positive difference would lead to a sequence of

[3] See H.E. Thompson and G.C. Tiao, "Analysis of Telephone Data: A Case Study of Forecasting-Seasonal Time Series," <u>Bell Journal of Economics and Management Science</u>, Vol. 2, No. 2, Autumn 1971.

capital expenditures. Underestimating the difference might create a shortage in the supply of telephones and associated facilities, while overestimating it would result in a premature expansion of facilities and thus added cost to the company. In this study it was demonstrated that time-series models could provide better forecasts of the difference between inward and outward station movements than had been possible earlier.

One very interesting application of time-series models is in combination with a single-equation regression model. Suppose, for example, that we would like to forecast the variable y_t using a regression model. Presumably such a model would include all those independent variables that could provide an explanation for movements in y_t. Let us suppose that the best regression model contains two independent variables, x_1 and x_2, as follows:

$$y_t = a_0 + a_1 x_{1t} + a_2 x_{2t} + \varepsilon_t \quad . \tag{11}$$

Note that this equation has an implicit additive error term that accounts for unexplained variance in y_t; i.e. it accounts for that part of the variance of y_t that is not explained by x_1 and x_2. (As we mentioned earlier, the additive error term is implicit in any regression model.) The equation can be estimated to obtain values of the parameters a_0, a_1, and a_2, and it can then be used to forecast y_t. However, one source of forecast error would come about from the additive error term ε_t whose future values cannot be predicted.

By subtracting the estimated values of y_t from the actual values, we can calculate a residual series u_t which represents unexplained movement in y_t, i.e. pure noise. One effective application of time-series analysis is to construct a time-series model for the residual series u_t of the regression. We would then substitute the time-series model for the implicit error term in the original regression equation. When using the equation to forecast y_t we would also be able to make a forecast of the error term ε_t using the time-series model. The time-series model provides some information as to what future values of ε_t are likely to be; i.e. it helps "explain" the unexplained variance in the regression equation. The resulting "combined" model is likely to provide much better forecasts than the regression equation alone, or a time-series model alone, since it includes a structural (economic) explanation of that part of the variance of y_t that can be explained structurally, and a time-series "explanation" of that part of the variance y_t that cannot be explained structurally. We think that this combined use of time-series models and regression models could provide an extremely powerful forecasting tool for the telecommunications industry.

Forecast Confidence Intervals.

As we said earlier, it is very important when producing a forecast to also estimate a standard error of forecasts that can be used to provide a confidence interval around the forecast. Without this, the user of the forecast has no way of knowing how reliable it is likely to be, and how much "confidence" he can place in it. It is important that the "margin of error" for the forecast be quantitatively determined.

In the case of a single equation regression model, the error associated with a forecast can come about from a combination of four distinct sources. First, the random nature of the additive error process in the linear regression model guarantees that forecasts will deviate from true values, even if the model is specified correctly (i.e. its basic structure correctly represents the structure of the real world), and its parameter values are known with certainty. Second, the process of estimating the regression parameters introduces error because estimated parameter values are random variables which may deviate from the true parameter values. The estimated parameters are unlikely to equal the true values of the parameters for the model, even though they will (if they are unbiased) equal those parameters on the average. Third, in the case of a conditional forecast, errors are introduced when calculated guesses or forecasts are made for the values of the explanatory variables in the period in which the forecast was made. Finally, errors may be introduced because the model specification may not be an accurate representation of the "true" real world process. This last source of error can be quite important, although it is often ignored in forecasting applications.

A set of widely used techniques exists to compute standard errors of forecast for a regression model that take into account the first three sources of error. We will not describe those techniques here since that would involve too much technical detail.[4] Accounting for specification error is more problematical; no simple techniques exist, although sensitivity studies can be performed to determine the effects of alternative model specifications.

Straightforward techniques also exist to compute standard errors of forecast for time-series models. In the case of a time-series model, statistical tests are also available to determine whether or not the model is correctly specified. Given that it is correctly specified, simple formulas can be used to compute a standard error of forecast that takes into account the first two sources of error described above (the third source of error is not applicable since there are no independent variables in a time-series model).

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[4] The computation of standard errors of forecast is described in some detail in Chapter 6 of R.S. Pindyck and D.L. Rubinfeld, <u>Econometric Models and Economic Forecasts</u>, McGraw-Hill, New York, 1976.

In the case of a multi-equation simulation model the calculation of standard errors of forecast (for each of the endogenous variables of the model) is much more difficult. The problem is that the forecast values for the endogenous variables are determined not only by the additive error terms and estimated coefficients of each equation in the model, but also by the dynamic interaction of the equations when the model is simulated over time. As a result there are no simple formulas that can be used to compute standard errors of forecast for a multi-equation simulation model. However, an alternative approach does exist, and that is <u>stochastic</u> (or, Monte Carlo) <u>simulation</u>. A stochastic simulation is performed by specifying, for each equation of the model, a probability distribution for the additive error term and for each estimated coefficient. Next, a large number (say 50 or 100) of simulations are performed, and in each simulation values for the additive error terms and estimated coefficients are chosen at random from the corresponding probability distributions. For any particular endogenous variable, the results of the simulation (i.e. the resulting set of forecasts) yield points that trace out a probability distribution of that variable's forecasted value. Thus the dispersion of the forecasts about their mean value can be used to define a forecast confidence interval. We must point out, however, that this can be a computationally expensive process, and for this reason forecasters frequently ignore the calculation of confidence intervals when working with multi-equation simulation models.

II. FORECASTING WITH SINGLE EQUATION MODELS

II.1. THE DEMAND FOR LOCAL TOLL AND TOTAL TELEPHONE SERVICES*

*This section draws on a study entitled The Captra Model, for the Department of Communications of the Canadian Government and developed by the Institute of Applied Economic Research under the direction of J.I. Bernstein. In the project A. Anastasopoulos and V. Corbo participated as consultants and G. Tsoublekas as researcher.

1. Introduction

In this section we describe the demand characteristics for the telephone services of the Trans-Canada Telephone System (TCTS companies and including Edmonton Telephones). In describing the demand, conditions for the system, we formulate a model which estimates the historical structure. This structural specification is then utilized to forecast the future trends of the carriers revenues.

The study of demand behaviour for telephone services is an important undertaking, because of its role in determining company revenues. Indeed, demand systems already exist depicting the Canadian telephone industry, in general; for example. R. Dobell et. al. [5] and L. Waverman [14]. Moreover, their important works have focused on particular demand aspects, as in V. Corbo [4], and I.I.Q.E. [6]. Our immediate interest is in the general structural form of the telephone demand relations for total, local, and toll services.

Before proceeding to formulate the module, we must determine the appropriate aggregations across economic agents (in this case carriers) and commodities (which are total, local and toll). The demand module disaggregates carriers into four categories. We treat Bell Canada and British Columbia Telephone separately; we aggregate Alberta Government Telephones, Edmonton Telephones, Saskatchewan Telecommunications and the Manitoba Telephone System into one category called public companies; we aggregate Maritime Telegraph and Telephone, New Brunswick Telephone and Newfoundland Telephone into one category called private companies. The rationale for this aggregation is based on three fundamental reasons. Firstly, Bell Canada and B.C. Telephone are the leaders, in terms of market shares, of the industry and so are dealt with individually. Secondly the public companies, as their name suggests, are government owned while the three remaining companies are privatly controlled. Finally, locational considerations suggest that the western carriers be separated from the eastern area. Hence, our transactor disaggregations are derived from the market share, legal and spatial characteristics of the industry.

The explication of our work is divided into three further sections. In section 2 we develop the theoretical framework and its rationale, in section 3 we present the estimated results and their evaluation; and lastly we forecast until 1985 the total, local, and toll demands of the TCTS companies.

2. The Theoretical Models

The theoretical basis for the demand model which is utilized in the econometric investigations is discussed in this section. The economic theory that we draw upon is largely the analysis of the individual household and also from the firm.

In developing the model, the first question to be answered is who are the demanders of telephone services. Manifestly, both households and firms are the demanders, since the telephone is a consumption product to households and a factor of production (part of intermediate inputs) to firms. Ideally, then, we would desire to construct demand equations disaggregated, not only along supplier and service categories, but, in addition, along demander groups. However, because of data limitations, we follow the usual route and aggregate the household and firms' demand for each revenue category into a single aggregate. We therefore assume that although the motivations and constraints of consumers and producers are different the ultimate elements affecting their telephone service demand are the same.

Individual demand behavior, according to economic theory, suggests that given the objectives of the demanders (preferences for consumers and generally profits for firms), that the quantity demanded of the i^{th} service by the k^{th} household in period t (X^k_{it}) depends on the nominal income of the k^{th} household in period t (Y^k_t), the price of the i^{th} service in period t (P_{it}), and the price of other commodities demanded and supplied by the household (P_{jt}; j=1,...,n and j≠i). In a functional form we find that,

$$(1) \qquad X^k_{it} = h^k_{it}\ (P_{1t},\ldots,P_{nt},Y^k_t)$$

where h^k_{it} is the demand function of the i^{th} service for the k^{th} household in period t.

To derive the aggregate household demand for any service i, in any period t, we must sum equation (1) over all households who are demanding the service.

$$(2) \qquad \sum_{k=1}^{J} X^k_{it} = \sum_{k=1}^{J} h^k_{it}\ (P_{1t},\ldots,P_{nt},Y^k_t)$$

where J is the number of household demanders. So then,

$$(3) \qquad X^H_{it} = h_{it}\ (P_{1t},\ldots,P_{nt},Y^1_t,\ldots,Y^J_t)$$

where $X^H_{it} = \sum_{k=1}^{J} X^k_{it}$ and $h_{it}\ (P_{1t},\ldots,P_{nt},Y^1_t,\ldots,Y^J_t)$ $= \sum_{k=1}^{J} h^k_{it}\ (P_{1t},\ldots,P_{nt},Y^k_t)$. Notice that in the aggregate demand function the income terms for each household enter separately and not as an aggregate. This fact takes into consideration that the distribution of income among households is not fixed. If we assume that the distribution of income among households in any period of time is fixed then we can write equation (3) as,

$$(4) \qquad X^H_{it} = h_{it}\ (P_{1t},\ldots,P_{nt},Y^H_t)$$

where $Y^H_t = \sum_{k=1}^{J} Y^k_t$ is the aggregate income of the households.

Moreover, let us assume that the form of the demand function does not depend on the time period and so

(5) $$X^H_{it} = h_i \ (P_{1t},\ldots,P_{nt},Y^H_{it})$$

For the producers, these demands for telephone services are derived, not from utility maximization procedures as in the case of households, but from cost minimization techniques. The quantity demanded of the i^{th} telephone service by the ℓ^{th} firm in period t (X^{ℓ}_{it}) depends on the nominal income (since output is given) of the ℓ^{th} firm in period t (Y^{ℓ}_t), the price of the i^{th} service in period t (P_{it}), and the price of all other commodities demanded and supplied by the firm (P_{jt};j=1,...,m and j≠i). Hence we have

(6) $$X^{\ell}_{it} = g^{\ell}_{it} \ (P_{1t},\ldots,P_{mt},Y^{\ell}_t)$$

where g^{ℓ}_{it} is the ℓ^{th} firms' demand function for the i^{th} service in period t. Summing over all the firms yields,

(7) $$X^F_{it} = g_{it} \ (P_{1t},\ldots,P_{mt},Y^1_t,\ldots,Y^I_t)$$

where I is the number of firms, $X^F_{it} = \sum_{\ell=1}^{I} X^{\ell}_{it}$ and

$$g_{it} \ (P_{1t},\ldots,P_{mt},Y^1_t,\ldots,Y^I_t) = \sum_{\ell=1}^{I} g^{\ell}_{it} \ (P_{1t},\ldots,P_{mt},Y^{\ell}_t).$$

Again it is not aggregate output which affects the aggregate producer demand function for the i^{th} service in period t, but rather all the outputs separately which reflects the size and composition of output levels for firms demanding telephone services. By assuming the output composition is fixed in every period and the demand functions do not change over time we get,

(8) $$X_{it}^{F} = g_i \ (P_{1t},\ldots,P_{mt},Y_t^F)$$

where $Y_t^F = \sum_{\ell=1}^{I} Y_t^{\ell}$.

To derive the consumer and producer demand for the i^{th} service in period t we must sum equations (5) and (8).

(9) $$x_{it} = H_i \ (P_{1t},\ldots,P_{rt},Y_t^H,Y_t^F)$$

where $x_{it} = X_{it}^H + X_{it}^F$, $H_i \ (P_{1t},\ldots,P_{rt},Y_t^H,Y_t^F) = h_i \ (P_{1t},\ldots,P_{nt}, Y_t^H) + g_i \ (P_{1t},\ldots,P_{mt},Y_t^F)$ and so H_i is the aggregate (consumer and producer) demand function for the i^{th} telephone service.

Once again by assuming the distribution of income between households and firms are fixed and by letting the prices of all commodities other than the i^{th} service be represented by a price index in period t (P_t), we can write equation (9) as,

(10) $$x_{it} = H_i \ (P_{it},P_t,Y_t)$$

where $Y_t = Y_t^H + Y_t^F$.

Now that we have arrived at the aggregate demand function for any telephone service, we are able to impose the 'a priori' restrictions from economic theory. Economic theory does not predict the form of the demand function (H_i), but the theory does impose restrictions on the pattern of price and income effects in systems of demand behavior. Firstly, household and firm behavior is such that the demand function should be homogeneous of degree zero in the prices and income. In other words, if there is an equiproportionate change in all prices and income then the cost minimizing producer demand and utility maximizing consumer demand do not change. Consequently, the aggregate demand is not affected. This result implies that we can write equation (10) as,

(11) $$x_{it} = H_i \ (p_{it}, y_t),$$

where $p_{it} = P_{it}/P_t$, and $y_t = Y_t/P_t$. The variable p_{it} is the relative price of the i^{th} service in period t and y_t is the real income in period t.

The second proposition pertains to the nature of the effects of a change in the relative price and the real income on demand. Economic theory states that if the effect of a change in y_t is to increase the quantity demanded then it must be true that the effect of a change in p_{it} is to decrease the quantity demanded. Therefore the negativity condition is;

if $\frac{\partial x_{it}}{\partial y_t} > 0$ then it must be the case that $\frac{\partial x_{it}}{\partial p_{it}} < 0$.

The last restriction, in this context, is the so-called adding-up condition which states that the sum of the proportion of expenditure on all commodities out of income (or output) must equal unity. This means that if $p_{it} \ x_{it}$ is the expenditure on the

i^{th} service and there are r commodities then $\sum_{i=1}^{r} \frac{p_{it}x_{it}}{y_t} = 1$.

This restriction, however, is not as important as the previous two because we are aggregating across households and firms. The reason is that, in general, this third condition holds for consumers, but does not for producers, unless their production functions exhibit constant returns to scale. Since the nature of the production functions for the producers who demand telephone services is outside the purview of our study, we shall develop demand models which do and do not incorporate this last condition. Moreover, whether this last condition is satisfied or not will not be a prerequisite for the acceptance or rejection of a particular functional form.

We have now described the relevant features of our specification which are derivable from the theory, for the empirical applications of equation (11), it is necessary to specialize the general form of the demand relation and to account for stochastic phenomena.

2.1 The Linear Demand Model

The linear demand model assumes that the form of the aggregate demand function (H_i) is linear, so that,

(12) $$x_{it} = \beta_0 + \beta_1 p_{it} + \beta_2 y_t + e_t ,$$

where e_t represents the disturbance that can occur because H_t may not be strictly linear or there may exist measurement errors in the dependent variable and also other minor variables may have been omitted from the equation.

We must also find that if $\beta_2 > 0$ then it should be the case that $\beta_1 < 0$. This means that if increases in income tend to increase demand then increases in the service's price tend to decrease demand. It bears mentionning that equation (12) satisfies the homogeneity and negativity conditions but does not satisfy the adding-up restriction. Nevertheless, in light of the coveat stated at the end of section 2, concerning the adding-up condition, the linear model should not be dismissed outright on these grounds.

2.2 The Double-Log Demand Model

In the double-log model we begin with the general demand equation, but instead of assuming that it is linear, we assume that it is multiplicative,

(13) $$x_{it} = \alpha_0 \, p_{it}^{\beta_1} \, y_t^{\beta_2} \, u_t \, ,$$

where u_t represents the error term and α_0 the constant. By taking logarithms of equation (13) we arrive at,

(14) $$\log x_{it} = \beta_0 + \beta_1 \log p_{it} + \beta_2 \log y_t + e_t \, ,$$

where $\log \alpha_0 = \beta_0$ and $\log u_t = e_t$.

The double-log formulation, as in the linear case, incorporates the homogeneity condition. Moreover, if $\beta_2 > 0$ then we should expect $\beta_1 < 0$. Notice that the magnitudes of β_0, β_1, and β_2 will be different from the linear model but the signs of the coefficients should be the same. The reason for this is that we are specifying an alternative hypothesis concerning the true structural form and in this case β_1 and β_2 are partial price and partial income elasticities rather than partial rates of change. Finally, the double-log equation does not incorporate the adding-up condition.

3. <u>The Empirical Results</u>

In a study of the demand for telephone services, the quantity demanded should be measured in some homogeneous unit such as minutes of calls. Unfortunately, we do not have data at such a disaggregated level. Therefore, we used a variant of revenue deflated by its price. We took revenue for any service i (including uncollectables, since they represent unpaid output) and substracted from it the indirect taxes associated with that particular revenue category.

The price indexes for the total, local,and toll services were obtained from the Bell Canada Rate Hearings Exhibits [3]. Since we had these price indexes only for Bell Canada, we assume that the price index for any category are in a fixed proportion across all carriers in the industry. If this assumption does not hold then the consequences of the error, in the measurement of the indexes, are unknown, with regards to the bias and the inconsistency of the estimates obtained in the demand equations. Nevertheless, our assumption is reasonable because of the fact that Bell is the accepted market leader in the industry. Thus proceeding with this assumption, we deflated the current approriate revenues by the relevant price index to obtain a measure of quantity demanded for any service.

To define the relative price regressor we divided the price index for any service by the consumer price indexes of large metropolitan areas, within the region, in which the carrier has the jurisdiction to operate. For Bell Canada, we considered the weighted average of the consumers price indexes for Toronto and Montreal; for B.C. Telephone, we used the index for Vancouver; for the public carriers, we used the index for Winnipeg; and for the private carriers, we used the weighted average of the indexes for St. John's, St. John, and Halifax.

Real income was defined as the sum of the nominal gross provincial products in which the company has jurisdiction, deflated by the appropriate consumer price index.

In performing our regressions we tried the linear, and double-log models for all these categories in each company. We also used ordinary and generalized least squares as the estimation techniques. The best results are reported in the following tables (the sample period for Bell Canada is 1950-1975, for all other carriers 1961-1975 and the industry, with β_0 the constant, β_1 the relative price coefficient, β_2 the real income coefficient, ρ_1 and ρ_2 refer to the coefficients arising from adjusting for autocorrelation once and twice).

From Table 1 we find the results for Bell Canada. We found that the double-log adjusted once for autocorrelation performed best. We also see that $\beta_1<0$, and $\beta_2>0$ for all the services, as we would expect. Moreover, since Bell is essentially a monopolist in its jurisdiction, then economic theory tells us that the carrier cannot be operating on the inelastic segment of its demand curve. This means that it is not sufficient for β_1 (the partial price elasticity of demand) to be negative, it must be smaller than -1. In all three categories, our results are consistent with the theory.

Table 2 presents the adjusted double-log results for B.C. Telephone Company. We can observe that the fit is very good, in terms of the significance of the coefficients, R^2 and the Durbin-Watson statistic. In addition, $\beta_1 < 0$, $\beta_2 > 0$, and of course in all the categories the price elasticity is smaller (or equal to) -1.

In Table 3 the public carriers' results are presented. These estimates we obtained from the linear model, adjusted once for autocorrelation in the cases of total and toll, and adjusted twice for the local telephone services. Once again the estimates are significant and have the correct signs. However, because the model is linear, β_1 is not the price elasticity of demand but rather the elasticity is now variable over the sample period. Computing the price elasticities which was defined in section 2, yields the average values of the elasticities over the period:

Table 1

Bell Canada Demand Equations
(t-values in parentheses)

Demand Category	β_0	β_1	β_2	ρ_1	R^2	D.W.
Total	-2.156 (-2.00)	-1.325 (-8.32)	.816 (8.04)	.81 (6.95)	.99	1.3
Local	-1.825 (-1.74)	-1.061 (-7.02)	.734 (7.45)	.82 (7.19)	.99	1.2
Toll	-7.311 (-5.71)	-1.567 (-7.63)	1.205 (10.03)	.69 (4.80)	.99	1.6

Table 2

British Columbia Telephone Demand Equations
(t-values in parentheses)

Demand Category	β_0	β_1	β_2	ρ_1	R^2	D.W.
Total	-5.417 (-9.61)	-1.069 (-7.86)	1.152 (18.11)	.18 (.69)	.99	2.0
Local	-3.319 (-4.74)	-1.000 (-5.29)	.841 (10.63)	.03 (.12)	.99	2.0
Toll	-10.209 (-13.04)	-1.000 (-5.19)	1.600 (18.20)	.34 (1.33)	.99	1.6

Table 3

Public Carriers' Demand Equations

(t-values in parentheses)

Demand category	β_0	β_1	β_2	ρ_1	R^2	D.W.
Total	524.163 (4.74)	-482.819 (-6.34)	.014 (4.30)	.55 (2.49)	.99	1.6
Local	-12.010 (-1.82)	-97.27 (-6.57)	.002 (3.01)	1.11* (3.44)	.99	2.1
Toll	231.815 (2.63)	-253.830 (-4.1)	.011 (4.42)	.47 (2.00)	.98	1.8

* ρ_2 = -.01 (-.03)

Table 4

Private Carriers' Demand Equations

(t-values in parentheses)

Demand Category	β_0	β_1	β_2	ρ_1	ρ_2	R^2	D.W.
Total	-8.566 (-3.27)	-1.347 (-5.75)	1.329 (12.24)	.33 (1.01)	-.59 (-2.32)	.99	1.9
Local	-5.087 (-2.28)	-1.233 (-2.70)	1.104 (4.95)	.68 (2.00)	-.58 (-1.80)	.99	1.9
Toll	-9.880 (-3.55)	-1.457 (-6.83)	1.530 (14.67)	.46 (2.00)	-.54 (-2.50)	.99	2.3

Table 5

Industry Demand Equations

(t-values in parentheses)

Demand Category	β_0	β_1	β_2	ρ_1	R^2	D.W.
Total	-4.435 (-4.63)	-1.353 (-11.68)	1.026 (11.94)	.61 (2.90)	.99	1.8
Local	-3.457 (-3.38)	-1.073 (-9.10)	.882 (9.61)	.63 (3.10)	.99	1.6
Toll	-9.709 (-8.08)	-1.462 (-8.57)	1.423 (13.18)	.42 (1.80)	.99	1.9

For total services -3.1, for local services -1.4, for toll services -3.1. These numbers are consistent with our preconceived upper bounds.

Finally Tables 4 and 5 present the results for the private companies and the industry as a whole. We can observe that the descriptive power, for the double-log twice adjusted model for private group and the double-log once adjusted for the industry, is very good by all the usual statistical and economic tests. Indeed, we find that $\beta_1 < -1$ and $\beta_2 > 0$ in both Tables 4 and 5 for all categories of telephone services.

4. The Forecast Results

In this segment we present the demand forecasts based on the estimated results found in the previous section. From Tables 1 - 5 we can obtain the estimated coefficients from which to base our forecasts on demand. However, we must exogeneously forecast the relative price and real income variables. To perform this task we utilized a first-order autoregressive structure and estimated this structure by employing the Cochrane-Orcutt least squares method, in order to adjust for the presence of autocorrelation.

For Bell Canada; the relative price for total services was derived from,

$BLPDTS_t - .51\ BLPDTS_{t-1} = .98(BLPDTS_{t-1} - .51\ BLPDTS_{t-2})$,

for local services,

$BLPDLS_t - .44\ BLPDLS_{t-1} = .98\ (BLPDLS_{t-1} - .44\ BLPDLS_{t-2})$,

for toll services,

$BLPDTT_t - .46\ BLPDTT_{t-1} = .97\ (BLPDTT_{t-1} - .46\ BLPDTT_{t-2})$,

for the real income variable,

$BLGPD_t - .73\ BLGPD_{t-1} = 1.03\ (BLGPD_{t-1} - .73\ BLGPD_{t-2})$.

For B.C. telephone; the relative price for total service was derived from,

$BCPDTS_t - .84\ BCPDTS_{t-1} = .95\ (BCPDTS_{t-1} - .84\ BCPDTS_{t-2})$, for local services,

$BCPDLS_t - .75\ BCPDLS_{t-1} = .96\ (BCPDLS_{t-1} - .75\ BCPDLS_{t-2}),$

for toll services,

$BCPDTT_t = .96\ BCPDTT_{t-1},$

for the real income variable,

$BCGPD_t = 1.06\ BCGPD_{t-1}.$

For the publicly-owned companies we found, for the relative price of total services,

$TGPDTS_t - .78\ TGPDTS_{t-1} = .96\ (TGPDTS_{t-1} - .78\ TGPDTS_{t-2}),$

for local services,

$TGPDLS_t - .71\ TGPDLS_{t-1} = .96\ (TGPDLS_{t-1} - .71\ TGPDLS_{t-2}),$

for toll services,

$TGPDTT_t = .96\ TGPDTT_{t-1},$

and for the real income variable,

$TGGPD_t = 1.06\ TGGPD_{t-1}.$

For the private compaies (other than B.C. Telephone and Bell Canada) we derive for the relative price variables,

$OPPDTS_t - .75\ OPPDTS_{t-1} = .96\ (OPPDTS_{t-1} - .75\ OPPDTS_{t-2}),$

for local services,

$OPPDLS_t - .64\ OPPDLS_{t-1} = .96\ (OPPDLS_{t-1} - .64\ OPPDLS_{t-2}),$

for toll services,

$OPPDTT_t = .97\ OPPDTT_{t-1},$

for the real income variable we have,

$OPGPD_t - .59\ OPGPD_{t-1} = 1.05\ (OPGPD_{t-1} - .59\ OPGPD_{t-2}).$

Finally in dealing with the complete telephone industry we derive; for total services,

$IDPDTS_t - .81\ IDPDTS_{t-1} = .96\ (IDPDTS_{t-1} - .81\ IDPDTS_{t-2})$,

for local services,

$IDPDLS_t - .71\ IDPDLS_{t-1} = .96\ IDPDLS_{t-1} - .71\ IDPDLS_{t-2})$,

for toll services,

$IDPDTT_t = .97\ IDPDTT_{t-1}$,

and finally for the real income variable,

$IDGPD_t - .86\ IDGPD_{t-1} = 1.01\ (IDGPD_{t-1} - .86\ IDGPD_{t-2})$.

Thus with the forecasted values of the exogeneous variables which we may obtain from the preceeding equations and combining the information from Tables 1 - 5, we obtain the forecasted values for the carrier's services from 1976-1985, which are given in Tables 6 - 10 (the units are in millions).

Table 6

Forecasted Values of Demand for Bell Canada (1976-1985)

Total	Local	Toll
1461.181	760.518	644.194
1533.028	783.679	677.223
1598.786	806.739	711.233
1665.699	830.477	749.196
1737.677	856.625	791.556
1808.042	884.480	838.823
1885.597	914.155	890.693
1968.447	946.717	947.664
2051.992	980.439	1009.288
2149.520	1017.394	1075.994

Table 7

Forecasted Values of Demand for Bell Canada Telephone
(1976-1985)

Total	Local	Toll
354.603	145.039	180.008
413.642	162.390	204.588
480.583	180.910	233.224
557.242	201.140	265.868
644.839	222.739	303.384
743.969	246.164	346.194
856.625	271.510	395.045
984.368	293.242	450.789
1128.901	317.666	514.399
1293.362	345.157	586.985

Table 8

Forecasted Values of Demand for the Public Companies
(1976 - 1985)

Total	Local	Toll
453.462	155.170	258.024
493.909	168.294	273.435
532.013	181.488	291.156
568.387	195.005	310.288
603.565	209.051	330.438
638.009	223.810	351.455
672.111	239.442	373.301
706.204	256.102	395.994
740.576	273.943	419.580
775.478	293.116	444.120

Table 9

Forecasted Values of Demand for the Private Companies
(1976-1985)

Total	Local	Toll
218.984	90.107	109.727
247.646	101.291	119.224
276.442	110.388	131.631
314.820	119.941	148.117
362.491	132.158	167.335
412.403	147.083	188.105
463.126	163.204	210.819
520.089	179.648	236.749
586.399	196.763	266.667
661.163	215.940	300.666

Table 10

Forecasted Values of Demand for the Industry
(1976-1985)

Total	Local	Toll
2519.964	1163.281	1157.479
2787.778	1247.629	1216.825
3077.891	1332.750	1285.625
3388.017	1420.835	1363.759
3718.219	1510.204	1449.538
4072.450	1605.194	1543.797
4455.970	1702.750	1647.476
4861.002	1806.235	1758.119
5292.257	1914.095	1879.949
5761.770	2026.367	2010.221

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II.2. THE DEMAND FOR INTERPROVINCIAL FLOWS OF TELEPHONE SERVICES*

* This section is based on a larger study entitled Interprovincial Flows of Telephone Services done under the direction of Vittorio Corbo for the Department of Communications of the Canadian government.

In this section we present a model that was designed to forecast annually the flow of telephone services originating in the provinces of Ontario and Quebec (Bell Canada territories) and destined to each Canadian province.

We formulate econometric models of demand for telephone services between two points, allowing for differences in the demand equation depending on the direction of the call. Thus, the demand equation for calls from Ontario to, say, British Columbia is regarded as different from the one for telephones calls from British Columbia to Ontario. Five types of sectors are distinguished and separate demand equations and forecasts are presented for each. These sectors are the Production, Service, Government, Public and Household sectors.

In the overall study, on which this section draws, three different theoretical formulations are used for the specification of the demand equation. A "simple demand" model in which the quantity demanded of telephone services is expressed as a function of overall economic activity (usually the variable chosen in Gross Provincial Product) and of relative prices.

The second model considered in the study was the so called "flow adjustment" model[1]. In this type of model the "desired level" of telephone services is a function of the variables used in our "simple demand model", while the actual level of services demanded moves towards the desired level in some specific way to be studied below.

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1 This term is due to H. S. Houthakker and L.D. Taylor [1]

The third model studied was a so called "stock adjustment" model. Here the quantity demanded is a function of the stock of services already held as well as a function of income and relative prices. The change in the stock of services, on the other hand, is a function of the current purchases of the service and the rate at which the stock of services depreciates. In our case the stock of telephone services is taken to be the accumulated value of the quantity demanded of the service and it is assumed to represent the "habit" of telephone use. That is, the more accustomed one is to the use of the telephone (ceteris paribus) the higher will be the quantity demanded of telephone services.

The best results, in terms of the statistical properties of the function fitted and of the forecast performances, were obtained for the flow adjustment model. Therefore here we will summarize only the results of that model.

The "flow adjustment model"

In the flow adjustment model, we begin by assuming that the "desired level" of telephone services (q^*_t) is a linear function of real income (y_t) and relative prices (p_t). In addition, it is assumed that the adjustment of the actual level of telephone service (q_t) toward the desired level follows a partial adjustment process.

Specifically, the Houthakker-Taylor "flow adjustment" model is given by the following two equations.

$$(1) \quad q^*_t = b_1 + b_2 \; y_t + b_3 \; p_t$$

and

$$(2) \quad \dot{q}_t = \theta (q^*_t - q_t)$$

where $\dot{q}_t$ is the time differential of q_t.

Obviously q^*_t is an unobservable quantity. Furthermore, in this form the model is expressed in continuous time, and therefore to work in descrete time,it is necessary to make some approximations. After quite a substantial amount of manipulations, Houthakker and Taylo [1] obtained the following reduced form for equation (1) and (2) expressed in discrete time.

$$(3) \quad q_t = c_0 + c_1 \; q_{t-1} + c_2 \; (y_t + y_{t-1}) + c_3 \; (p_t + p_{t-1})$$

The type of equation estimated is of the form:

$$Y_{ij,k}(t) = \gamma_0 + \gamma_1 \left[CRPP_{ij}(t) + CRPP_{ij}(t-1)\right]$$
$$+ \gamma_2 \left[\frac{PT(t)}{CIP_{ij}(t)} + \frac{PT(t-1)}{CIP_{ij}(t-1)}\right] + \gamma_3 Y_{ij,k}(t-1)$$

where:

$Y_{ij,k}(t)$ = Telephone service type k (where the types are: Production Sector, Service Sector, Government Sector, Public Sector and Household) originating in Province i (i.e. Ontario and Quebec) and destined to province j in period t (in number of calls).

$CRPP_{ij}(t)$ = Composite of Real Provincial Product in Provinces i and j in period t. This composite is formed by weighing by 3/4 the Real Provincial Product RPP of the Province where the call originates and by 1/4 of the Province where the call is received.

$PT(t)$ = Price Index for Trans-Canada long distance telephone calls.

$CIP_{ij}(t)$ = Composite price Index. This variable is a weighted average of the Retail Price Index of the Province where the calls originate and the Retail Price Index of the Province where the call is received. The weights are equal to each province's share of the total RPP of both provinces.

A priori, we expect the coefficients of the above equation to exhibit the following sign pattern:

$\gamma_1 > 0$, $\gamma_2 < 0$ and $\gamma_3 > 0$.

Due to the presence of a lagged endogenous variable in the right hand side of the equation, we used the Hildreth-Lu scanning technique to estimate the coefficient of the first order autoregressive process in the disturbance. We should also mention that all these regressions were estimated with a sample extending from the fourth quarter of 1967 to the first quarter of 1973.

The results obtained when this type of equation was estimated appear in Tables 6 to 10.

From these tables we see that our *a priori expectations* regarding the sign pattern of the coefficients is satisfied.

This completes our discussion of the estimation results and we are now in a position to turn to the main purpose of this study, namely, the forecast for the demand for telephone services for the period 1974 to 1978.

Production Sector "Flow Adjustment Model"

Ontario to:	γ_0	γ_1	γ_2	γ_3	ρ	R^2
British Columbia	-61.162 (-1.011)	0.006 (2.858)	5.005 (0.168)	0.707 (4.870)	-0.20 (-0.957)	0.982
Alberta	-61.971 (-0.854)	0.006 (2.408)	4.582 (0.133)	0.690 (4.773)	-0.00 (-0.00)	0.978
Saskatchewan	-7.266 (-0.219)	0.002 (1.996)	-6.098 (-0.399)	0.342 (1.735)	-0.10 (-0.471)	0.913
Manitoba	-49.280 (-0.627)	0.008 (2.957)	-5.713 (-0.149)	0.545 (3.441)	-0.30 (-1.475)	0.966
Ontario	2041.403 (0.247)	0.872 (4.083)	-4246.410 (-1.096)	-0.516 (-3.243)	0.60 (3.518)	0.960
Quebec	3765.521 (2.558)	0.035 (1.052)	-2317.543 (-2.915)	-0.413 (-2.343)	0.80 (6.254)	0.939
New Brunswick	-14.234 (-0.498)	0.003 (2.788)	-6.202 (-0.459)	0.475 (2.857)	-0.20 (-0.957)	0.968
Maritimes	-17.529 (-0.351)	0.005 (2.489)	-11.944 (-0.506)	0.461 (2.590)	0.20 (0.957)	0.973
Newfoundland	-14.690 (-1.167)	0.001 (2.130)	3.165 (0.565)	0.750 (4.832)	-0.40 (-2.047)	0.952

Production Sector "Flow Adjustment Model"

Quebec to	γ_0	γ_1	γ_2	γ_3	ρ	R^2
British Columbia	37.039 (1.100)	0.002 (1.533)	-29.772 (-1.824)	0.488 (3.172)	-0.10 (-0.471)	0.953
Alberta	70.658 (2.608)	0.0004 (0.390)	-43.897 (-2.961)	0.439 (2.647)	0.30 (1.475)	0.978
Saskatchewan	27.959 (2.476)	-0.0002 (-0.482)	-16.219 (-2.792)	0.284 (1.397)	0.30 (1.475)	0.929
Manitoba	63.023 (1.851)	0.002 (0.957)	-42.036 (-2.545)	0.344 (2.238)	0.10 (0.471)	0.93
Ontario	5083.258 (5.048)	-0.017 (-0.705)	-2914.161 (-4.909)	-0.255 (-1.228)	0.80 (6.254)	0.952
Quebec	12731.469 (3.227)	0.307 (2.502)	-8119.633 (-3.281)	-0.604 (-3.097)	0.80 (6.254)	0.840
New Brunswick	105.881 (2.504)	0.0009 (0.422)	-64.643 (-3.090)	0.436 (3.083)	0.20 (0.957)	0.95
Maritimes	95.449 (2.775)	-0.0005 (-0.283)	-54.510 (-3.313)	0.514 (3.924)	-0.20 (-0.957)	0.95
Newfoundland	100.227 (4.014)	0.001 (1.094)	-65.964 (-4.317)	-0.042 (-0.207)	0.70 (4.598)	0.96

Service Sector "Flow Adjustment Model"

Ontario to:	γ_0	γ_1	γ_2	γ_3	ρ	R^2
British Columbia	579.215 (3.521)	0.002 (0.450)	-373.774 (-4.017)	-0.273 (-1.082)	0.80 (6.254)	0.973
Alberta	440.006 (2.813)	0.0004 (0.115)	-282.010 (-3.170)	0.073 (0.277)	0.80 (6.254)	0.968
Saskatchewan	40.924 (1.137)	0.003 (2.627)	-39.452 (-2.185)	-0.277 (-1.447)	0.70 (4.598)	0.960
Manitoba	161.426 (1.519)	0.005 (1.251)	-120.815 (-2.341)	0.311 (1.616)	0.30 (1.475)	0.976
Ontario	9177.207 (0.640)	0.932 (2.607)	-9148.805 (-1.373)	-0.329 (-1.653)	0.40 (2.047)	0.939
Quebec	3331.396 (3.255)	0.010 (0.374)	-1999.829 (-3.619)	-0.134 (-0.666)	0.80 (6.254)	0.95[illegible]
New Brunswick	39.229 (0.755)	0.004 (2.286)	-43.428 (-1.758)	-0.267 (-1.271)	0.60 (3.518)	0.94
Maritimes	147.884 (1.856)	0.006 (2.681)	-122.939 (-2.823)	-0.361 (-2.020)	0.80 (6.254)	0.96
Newfoundland	18.877 (0.593)	0.001 (1.122)	-18.926 (-1.279)	0.169 (0.717)	0.60 (3.518)	0.93

Service Sector "Flow Adjustment Model"

Quebec to:	γ_0	γ_1	γ_2	γ_3	ρ	R^2
British Columbia	277.888 (7.804)	-0.0004 (-0.380)	-173.241 (-7.633)	-0.399 (2.242)	0.80 (6.254)	0.97[illegible]
Alberta	247.300 (7.041)	-0.001 (-0.894)	-153.710 (-6.967)	-0.494 (-2.521)	0.80 (6.254)	0.972
Saskatchewan	33.005 (4.108)	0.0001 (0.342)	-20.547 (-4.441)	-0.269 (-1.356)	0.70 (4.598)	0.90[illegible]
Manitoba	137.014 (6.028)	-0.001 (-1.465)	-77.787 (-6.105)	-0.069 (-0.378)	0.60 (3.518)	0.95[illegible]
Ontario	1136.368 (2.034)	-0.030 (-1.282)	-557.093 (-2.222)	0.955 (9.414)	-0.20 (-0.957)	0.957
Quebec	13162.496 (3.642)	-0.399 (-2.480)	-6276.055 (-3.846)	0.790 (6.722)	-0.00 (-0.00)	0.914
New Brunswick	222.496 (4.927)	-0.0006 (-0.296)	-132.355 (-5.363)	0.026 (0.156)	0.60 (3.518)	0.94[illegible]
Maritimes	276.000 (6.187)	-0.003 (-1.930)	-159.410 (-6.092)	-0.183 (-0.928)	0.70 (4.598)	0.96[illegible]
Newfoundland	79.781 (3.709)	-0.003 (-2.693)	-38.705 (-3.772)	0.795 (6.001)	-0.10 (-0.471)	0.944

TABLE 8

Government Sector "Flow Adjustment Model"

Ontario to:	γ_0	γ_1	γ_2	γ_3	ρ	R^2
British Columbia	4.960 (0.428)	0.0001 (0.204)	-3.610 (-0.741)	0.719 (4.080)	0.10 (0.471)	0.880
Alberta	3.450 (0.399)	0.0002 (0.512)	-3.209 (-0.905)	0.699 (5.087)	-0.20 (-0.957)	0.928
Saskatchewan	0.754 (0.167)	0.0002 (1.041)	-1.220 (-0.629)	0.077 (0.397)	0.20 (0.957)	0.803
Manitoba	21.860 (1.635)	0.0004 (0.765)	-14.990 (-2.445)	-0.280 (-1.241)	0.60 (3.518)	0.898
Ontario	3354.061 (2.086)	0.032 (0.659)	-2064.044 (-2.746)	-0.205 (-0.751)	0.60 (3.518)	0.940
Quebec	116.679 (0.964)	0.007 (2.243)	-114.413 (-1.763)	-0.504 (-3.221)	0.80 (6.254)	0.84
New Brunswick	1.387 (0.179)	0.0004 (1.615)	-3.133 (-0.877)	-0.442 (-2.017)	0.60 (3.518)	0.81
Maritimes	0.261 (0.016)	0.0005 (0.808)	-2.237 (-0.317)	0.026 (0.107)	0.40 (2.047)	0.65
Newfoundland	17.562 (2.035)	-0.00005 (-0.175)	-11.581 (-2.560)	-0.139 (-0.489)	0.80 (6.254)	0.78

Government Sector

Québec to:	γ_0	γ_1	γ_2	γ_3	ρ	R^2
British Columbia	7.259 (2.882)	-0.0003 (-1.959)	-3.365 (-3.190)	0.596 (2.672)	-0.30 (-1.475)	0.65[illegible]
Alberta	12.042 (4.622)	-0.001 (-3.445)	-5.741 (-5.033)	0.860 (7.842)	-0.40 (-2.047)	0.90[illegible]
Saskatchewan	0.591 (0.650)	-0.00002 (-0.406)	-0.208 (-0.510)	-0.057 (-0.287)	-0.10 (-0.471)	0.06[illegible]
Manitoba	5.497 (2.640)	-0.0002 (-1.397)	-2.764 (-2.775)	0.098 (0.647)	0.40 (2.047)	0.54[illegible]
Ontario	63.044 (1.100)	0.002 (0.913)	-40.609 (-1.545)	-0.143 (-0.670)	0.50 (2.708)	0.757
Quebec	3122.881 (4.431)	0.064 (3.145)	-2112.994 (-4.735)	-0.463 (-2.680)	0.80 (6.254)	0.950
New Brunswick	10.248 (2.100)	0.0005 (2.489)	-7.980 (-2.684)	-0.631 (-3.263)	0.80 (6.254)	0.7[illegible]
Maritimes	8.073 (1.832)	-0.0004 (-1.335)	-3.703 (-1.889)	0.614 (2.975)	0.10 (0.471)	0.5[illegible]
Newfoundland	6.207 (1.112)	0.0005 (1.795)	-5.462 (-1.723)	-0.144 (-0.648)	0.70 (4.598)	0.80[illegible]

Public Sector "Flow Adjustment Model"

Ontario to:	γ_0	γ_1	γ_2	γ_3	ρ	R^2
British Columbia	-69.246 (-1.020)	0.004 (1.920)	22.194 (0.673)	-0.024 (-0.108)	0.70 (4.598)	0.770
Alberta	-42.184 (-0.704)	0.004 (1.975)	5.037 (0.182)	-0.070 (-0.318)	0.50 (2.708)	0.820
Saskatchewan	-73.556 (-2.171)	0.003 (3.118)	31.038 (1.734)	-0.121 (-0.602)	0.80 (6.254)	0.734
Manitoba	-417.393 (-2.079)	0.017 (3.033)	174.791 (1.648)	-0.272 (-1.381)	0.80 (6.254)	0.691
Ontario	-45931.020 (-2.641)	1.332 (3.541)	20578.426 (2.253)	-0.298 (-1.594)	0.80 (6.254)	0.515
Quebec	N.C.	0.076 (2.730)	-555.871 (-0.452)	-0.345 (-1.631)	1.00 (3396.221)	0.643
New Brunswick	-131.607 (-2.010)	0.005 (2.697)	60.161 (1.742)	-0.117 (-0.546)	0.80 (6.254)	0.668
Maritimes	-201.142 (-2.089)	0.008 (3.051)	85.166 (1.681)	-0.106 (-0.513)	0.80 (6.254)	0.720
Newfoundland	13.115 (0.226)	0.0006 (0.314)	-10.384 (-0.402)	0.103 (0.402)	0.20 (0.957)	0.499

N.C. = Non Computed

Quebec to:	γ_0	γ_1	γ_2	γ_3	ρ	R^2
British Columbia	10.228 (0.674)	0.0005 (0.483)	-7.716 (-1.189)	0.041 (0.121)	0.30 (1.475)	0.671
Alberta	2.610 (0.220)	0.001 (1.030)	-4.314 (-0.833)	-0.135 (-0.397)	0.40 (2.047)	0.677
Saskatchewan	8.147 (1.846)	-0.0004 (-1.186)	-3.840 (-2.098)	0.520 (1.690)	-0.10 (-0.471)	0.505
Manitoba	9.032 (0.845)	0.0001 (0.153)	-5.646 (-1.240)	0.130 (0.400)	0.20 (0.957)	0.600
Ontario	756.449 (1.196)	-0.018 (-0.644)	-360.029 (-1.361)	0.409 (1.574)	-0.00 (-0.00)	0.534
Quebec	N.C.	0.628 (3.365)	-7764.594 (-1.069)	-0.845 (-3.321)	1.00 (3396.221)	0.294
New Brunswick	-19.649 (-0.504)	0.005 (2.277)	0.152 (0.007)	-0.418 (-1.461)	0.70 (4.598)	0.703
Maritimes	-1.286 (-0.032)	0.004 (1.145)	-9.048 (-0.540)	-0.288 (-0.742)	0.30 (1.475)	0.625
Newfoundland	N.C.	0.004 (0.507)	-575.068 (-2.389)	0.068 (0.229)	1.00 (3396.221)	0.647

N.C. = Non Computed

Household Sector "Flow Adjustment Model"

Ontario to:	γ_0	γ_1	γ_2	γ_3	ρ	R^2
British Columbia	16.985 (0.077)	0.017 (4.038)	-89.962 (-0.768)	0.337 (1.714)	-1.000 N.C.	0.970
Alberta	-134.208 (-0.957)	0.016 (4.564)	10.530 (0.146)	0.323 (1.690)	-0.90 (-8.259)	0.950
Saskatchewan	-19.366 (-0.203)	0.006 (2.695)	-10.747 (-0.228)	0.275 (1.218)	-0.90 (-8.259)	0.900
Manitoba	338.565 (1.674)	0.018 (3.810)	-271.055 (-2.430)	-0.343 (-1.185)	-0.70 (-3.921)	0.955
Ontario	-18844.316 (-0.624)	2.027 (2.529)	2265.051 (0.162)	0.266 (1.112)	-0.00 (-0.00)	0.937
Quebec	-356.590 (-0.164)	0.116 (1.962)	-235.866 (-0.230)	0.096 (0.325)	0.20 (0.816)	0.915
New Brunswick	-32.495 (-0.201)	0.015 (2.966)	-37.225 (-0.485)	-0.073 (-0.309)	-0.40 (-1.746)	0.897
Maritimes	115.228 (0.402)	0.022 (2.513)	-156.581 (-1.132)	-0.186 (-0.738)	-0.30 (-1.258)	0.903
Newfoundland	115.175 (0.808)	0.009 (2.329)	-116.278 (-1.581)	-0.166 (-0.594)	-0.30 (-1.258)	0.932

N.C. - Non Computed

Household Sector "Flow Adjustment Model"

Quebec to:	γ_0	γ_1	γ_2	γ_3	ρ	R^2
British Columbia	12.674 (0.196)	0.005 (1.848)	-19.551 (-0.620)	0.446 (2.521)	-0.60 (-3.00)	0.89[illegible]
Alberta	49.080 (1.465)	0.006 (4.171)	-38.513 (-1.920)	-0.489 (-2.936)	0.70 (3.921)	0.9[illegible]
Saskatchewan	-8.600 (-0.855)	0.001 (2.286)	3.192 (0.7[illegible]3)	0.346 (2.056)	-0.70 (-3.921)	0.6[illegible]
Manitoba	96.733 (3.696)	0.002 (1.564)	-54.609 (-3.948)	-0.669 (3.173)	0.50 (2.309)	0.90[illegible]
Ontario	877.600 (0.491)	0.127 (1.904)	-841.798 (-0.939)	-0.048 (-0.153)	0.50 (2.309)	0.896
Quebec	29629.613 (1.710)	1.136 (2.714)	-20041.914 (-1.786)	-0.141 (-0.484)	0.80 (5.333)	0.932
New Brunswick	244.278 (3.135)	0.019 (4.114)	-169.667 (-4.130)	-0.909 (-3.798)	0.30 (1.258)	0.9[illegible]
Maritimes	108.218 (1.832)	0.013 (2.592)	-84.721 (-2.948)	-0.790 (-2.512)	0.30 (1.258)	0.8[illegible]
New foundland	156.319 (2.044)	0.003 (0.617)	-101.008 (-2.839)	-0.453 (-1.352)	-0.10 (-0.402)	0.765

FORECAST OF DEMAND FOR TELEPHONE SERVICES FOR THE PERIOD 1974 - 1978

In this section we make use of the equation estimated in the previous section to forecast the demand for the period 1974-1978. Before presenting the forecasts, however, we will describe the procedure we used to forecast the exogenous variables in our model.

Forecast of the exogenous variables

In our models there are two main exogenous variables. The Real Provincial Product (RPP) and the Relative Price of Telephone Services (TC/IP). In the case of RPP, the best results in terms of overall fit were obtained by fitting the following extended autoregressive equation:

$$RPP_i(t) = \delta_0 + \delta_1 D_1(t) + \delta_2 D_2(t) + \delta_3 D_3(t) + \delta_4 . t + \delta_5 \; RPP_i(t-1)$$

where

$RPP_i(t)$ = Real Provincial Product in province i in period t.

$D_1(t)$ = Seasonal Dummy variable, equal to one in the first calendar quarter and zero otherwise.

$D_2(t)$ = Seasonal Dummy variable, equal to one in the second calendar quarter and zero otherwise.

$D_3(t)$ = Seasonal Dummy variable equal to one in the third calendar quarter and zero otherwise.

t = Time in quarters (t = 1 in the first quarter of 1966 and increasing by one unit per quarter).

The second major exogenous variable that has to be forecasted is the relative price variable. Here, three avenues are open to us. The first is to assume that during the forecast period (1974-1978) the nominal price of telephone services is constant; consequently requiring only that we forecast the appropriate retail price index. The second alternative is to assume that during the forecast period relative price of telephone service (TP/IP) is constant. The third alternative, and the one for which the results appear most reasonable, is to forecast the relative price of telephone services by making use of an autoregressive process. For this purpose, the following equation was used:

$$\ln \frac{PT(t)}{CIP_{ij}(t)} = \theta_0 + \theta_1 t + \theta_2 \ln \frac{PT(t-1)}{CIP_{ij}(t-1)}$$

Now we will present the forecasts obtained when the equations of Table 6 to 10 were used for the structural equations and the independent variables were forecasted using the equations just described.

First, we will present the forecasts and then we will comment on them. In Table 14 we present the forecasts, in this table we have actual figures for 1972, the year which was used as the initial point for our projections. We also have actual figures for the first quarter of 1973.

Here, due to space limitations, we will present only the forecast for Business telephone services originating in Ontario and with destination British Columbia and Alberta[1].

Conclusions

In this paper, structural models were estimated and then used to make forecasts of the interprovincial flows of telephone services. When the structural models were studied, the empirical evidence did not reject the hypothesis that flow of services can be explained by structural demand equations. These results allowed us to use regression techniques to make forecasts. An alternative method of forecasting is the use of Time-Series Techniques. Time Series Techniques are useful for the forecasting of time series, where the only information which exists is the time series itself. In cases where systematic relations exist between economic variables, it is more fruitful to use these latter types of relations for the forecasts. The second method was the one followed here.

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1 The study in which this section is based was carried completed in April 1974 and at that moment only the actual data up to the first quarter of 1973 were available.

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II.3. THE DEMAND FOR CANADIAN INTERNATIONAL TELECOMMUNICATION SERVICES *

* This section is based on a larger study entitled the Canadian International Telecommunications Demand Model (CINTEL). The model was developed by the predecessor of the IAER, the International Institute of Quantitative Economics (IIQE) for the Department of Communications of the Canadian federal government. The authors are grateful to Marcel Dagenais, Oli Hawrylyshyn and Mohan Munasinghe who contributed at various stages to the overall IIQE study.

During the last decade, the rapid growth of international telecommunications services, as well as the advent of new technologies such as satellite and terrestial microwave links, have given impetus to the investigation of the demand for such services. Several recent studies (4), (5), (6), (8) have examined the impact of a number of economic and demographic variables on international telecommunications demand in various countries, particularly in the U.S.

The purpose of this section is to study the demand for flows of Canadian international telecommunications. We are especially interested in forecasting the effect of key policy variables such as price and quality of service on the volumes of telecommunications flows (i.e. telephone, telegraph and telex).

DEMAND MODEL

Telecommunications services are separated into three modes with different characteristics: telephone, telegraph and telex. Here, unlike other studies, a distinction is made between demands for outgoing and incoming services, since the former depends on the preferences of Canadian users of these services, whereas the latter reflects the economic behaviour of more heterogeneous group of users in other countries[1]. In this paper due to space limitations we will deal only with outgoing services for details of the results for incoming services refer to the overall study in which this section of the paper is based (reference 2 below).

As shown elsewhere (2), using standard microeconomic consumer theory and beginning from the very disaggregated level of individual households and business firms, it is possible to arrive at an aggregate demand function for each of the outgoing modes of telecommunications services of the form:

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1 Although desirable, a further disaggregation of users into household and business (including government) categories is not possible due to the lack of appropriate data, but this separation is useful conceptually, to identify the set of possible explanatory variables.

$$V_j^i(t) = D_{jt}^i\left[\underline{z}_j(t)\right] \qquad i = 1,2,3 \qquad (1)$$

where

$V_j^1(t) = TFOC_j(t)$ = volume of telephone traffic flowing out of Canada to country j in year t in units of thousands of minutes[1];

$V_j^2(t) = TGOC_j(t)$ = volume of telegraph traffic flowing out of Canada to country j in year t in units of thousands of words;

$V_j^3(t) = TXOC_j(t)$ = volume of telex traffic flowing out of Canada to country j in year t in units of thousands of minutes;

D_{jt}^i = the form of the corresponding demand function for Canadian users; and

$\underline{z}_j(t) = \left[z_j^1(t),\ldots,z_j^k(t),\ldots,z_j^K(t)\right]$ is a set of K explanatory variables, to be discussed below in more detail.

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1 From now on, whenever a new variable is defined, the actual units used in the estimations will also be given for convenience.

For a given type of telecommunications service (i.e. a fixed value of i), the model can be simplified by assuming that the form of the demand function D^i_{jt} is the same either: (a) across all countries of destination (i.e. as j varies), for a given year t; or (b) over the years (i.e. as t varies), for a given foreign country j; or (c) over all values of j and t. Since the data is limited to a time series over only 3 years, we are forced to adopt at the very outset the cross-section approach (a). Thus equation (1) may be rewritten:

$$V^i_j(t) = D^i_t \left[\underline{z}_j(t) \right] \tag{2}$$

Subsequently however, because D^i_t does not vary over time (a hypothesis which is tested empirically and accepted), the observations can be pooled according to approach (c), yielding the simplest model:

$$V^i_j(t) = D^i \left[\underline{z}_j(t) \right] \tag{3}$$

From theoretical considerations concerning the structure of household and business demand for telecommunications services discussed in (2) the following set of explanatory variables arose:

$$\underline{z}_j(t) = \Big[P_j^F(t),\ P_j^G(t),\ P_j^X(t),\ \overline{P}(t),\ I_c(t),\ IMP_j(t),\ EXP_j(t),$$
$$CINV_j(t),\ CTOUR_j(t),\ IMM_j(t),\ CSTR_j(t),\ QS_j(t)\Big] \qquad (4)$$

where

$P_j^F(t)$, $P_j^G(t)$, $P_j^X(t)$ = the Canadian prices of telephone, telegraph and telex services to country j in year t, in current dollars per thousand minutes, per thousand words, and per thousand minutes respectively;

$\overline{P}(t)$ = the aggregate Canadian price index in year t;

$I_c(t)$ = the income of Canada in year t, in millions of current dollars;

$IMP_j(t)$ = the level of Canadian imports from country j in year t, in millions of current dollars;

$EXP_j(t)$ = the level of Canadian exports to country j in year t, in millions of current dollars;

$CINV_j(t)$ = the level of Canadian investment in country j in year t, in millions of current dollars;

$CTOUR(t)$ = the flow of Canadian tourists to country j in year t, in thousands of persons;

$IMM_i(t)$ = the stock of immigrants in Canada from country j in year t, in thousands of persons;

$CSTR_j(t)$ = the structural parameter for Canadian users communicating with country j in year t; and

$QS_j(t)$ = a measure of the quality of service of telecommunications services between Canada and country j in year t: index increasing from 3 to 9 with decreasing quality;

The justification for this choice of explanatory variables is given in (2).

Estimations and Results

We will work with functions linear in the logarithms of the variables. Besides having constant elasticities, these functions possess the advantage that the magnitudes of the variables are considerably reduced, so that the assumption of homoscedasticity is more plausible.

Even after the logarithmic transformations, collinearity in the sample is severe between imports and exports of commodities, as well as between immigrant stock and the flow of Canadian tourists. Therefore, it is impossible to estimate accurately the individual contribution made by each of these variables to the quantity demanded of telecommunications services. This is not too much of a problem in our case because we are interested mainly in forecasting the impact of changes in key policy variables such as price and quality of service, on the volume of telecommunication. In our estimations we used imports and exports together as a trade flow variable ($IMEX_j(t)$), and retained the Canadian tourist variable[1].

Outgoing Flows of Telephone Services[2]

The demand equation (3) presented in Section A1 can be further restricted to be homogeneous of degree zero in the money variables P^F, P^G, P^X, $\overline{P}$, IMEX and I_C, as well as linear in the logarithms:

$$\begin{aligned} \text{Log } TFOC_j(t) = {} & \beta_{1,1} + \beta_{1,2} \text{ Log } \frac{P^F_j(t)}{\overline{P}(t)} + \beta_{1,3} \text{ Log } \frac{P^G_j(t)}{\overline{P}(t)} \\ & + \beta_{1,4} \text{ Log } \frac{P^X_j(t)}{\overline{P}(t)} + \beta_{1,5} \text{ Log } \frac{IMEX_j(t)}{\overline{P}(t)} + \beta_{1,6} \text{ Log } CTOUR_j \\ & + \beta_{1,7} \text{ Log } \frac{I_C(t)}{\overline{P}(t)} + \beta_{1,8} \text{ Log } TD_j(t) + \beta_{1,9} \, WHC_j \\ & + \beta_{1,10} \, LC_j + \beta_{1,11} \text{ Log } QS_j(t) + \varepsilon_{1j}(t) \end{aligned} \tag{6}$$

[1] Obviously, from the specification error theorem (7), the coefficient of this last variable in the regression will include also the contribution of the tourist flow variable to the total flow of telecommunication services.

[2] For a list of data sources, see the Appendix to (2).

where the new symbols introduced are:

$IMEX_j(t) = IMP_j(t) + EXP_j(t)$;

$TD_j(t)$ = the telephone density in country j in year t, in telephones per one hundred inhabitants;

WHC_j = the working hours commonality index between Canada and country j: index increasing from 1 to 10 with increasing commonality;

LC_j = the language commonality index between Canada and country j: index increasing from 1 to 4 with increasing commonality; and

$\varepsilon_{1j}(t)$ = the random error of the regression.

In Table 1, the estimated results of this equation for a sample of 40 countries for the year 1969, 1970 and 1971 are given in lines 1.1, 1.2 and 1.3. Here the most significant coefficients (trade and tourist variables) as well as the error variances $\hat{\sigma}_E^2$ are very stable[1]. Using a Chow test

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[1] Another business related variable CINV, should also be included as one of the regressors, but this was not possible since data was readily available only for a very small group of countries. However, it is reasonable to expect a high degree of collinearity between Log $\frac{IMEX}{\overline{P}}$ and Log $\frac{CINV}{\overline{P}}$, and therefore the coefficient of the former variable should include most of the contribution from the latter. The economic variables Log $\overline{P}(t)$ and Log $I_c(t)$ both of which refer to Canada are constant for a given year, and therefore the contribution of Log $\frac{I_c}{\overline{P}}$ is mixed with the constant in lines 1.1 to 1.3.

((1), (3)), the null hypothesis of equality in the full set of coefficients in these regression equations 1.1 , 1.2 and 1.3 is indeed accepted. Therefore, we may pool the data over all three years to obtain the results in line 1.4.

We now summarize the main characteristics of the results in line 1.4.

1. Excluding the constant term, the price of telegraph and the price of telex variables, the coefficients of this equation, are significantly different from zero at a 5% level with the exception of the coefficient of WHC which is significant at a 6% level.

2. For the price variables, only the own price elasticity is significantly different from zero, indicating that the demand for telephone services is independent of the price of telegraph and telex services, a result that is also found in a corresponding study for telecommunications flows into and out of the United States (lumped together) made by Lago (5). The own price elasticity is -1.391 but not statistically different from -1, indicating that revenues from international telephone services are fairly independent of its price.

Table 1: Demand for outgoing flows of telephone services [1]
Dependent variable Log $TFOC_j(t)$ (explanatory variables deflated)

Explanatory Variables

Equation Number	Constant	Log $\frac{P_j^F(t)}{\bar{P}(t)}$	Log $\frac{P_j^G(t)}{\bar{P}(t)}$	Log $\frac{P_j^X(t)}{\bar{P}(t)}$	Log $\frac{IMEX_j(t)}{\bar{P}(t)}$	Log$CTOUR_j(t)$	Log$\frac{L(t)}{\bar{P}(t)}$	Log $TD_j(t)$	WHC_j	LC_j	Log $QS_j(t)$	$\bar{R}^2$	F	$\hat{\sigma}_\epsilon^2$	N	[illegible]
1.1	4.713 (.371)	-.706 (-.555)	-.297 (-.271)	.503 (.264)	.275* (3.053)	.525* (4.823)	---	.289 (1.907)	.036 (.578)	.198 (1.796)	-.822 (-1.455)	.849	23.49	.432	37	[illegible]
1.2	16.936 (1.359)	-1.512 -1.222	.581 (.553)	-.724 (-.397)	.293* (3.263)	.475* (4.644)	---	.194 (1.173)	.100 (1.546)	.202 (1.858)	-1.090 (-1.967)	.856	24.92	.416	37	[illegible]
1.3	22.757 (1.819)	-1.926 (-1.671)	.440 (.448)	-.843 (-.457)	.281* (3.232)	.471* (4.527)	---	.039 (.218)	.032 (.453)	.157 (1.603)	-.906 (-1.683)	.859	22.77	.319	33	[illegible]
1.4	-31.986 (-1.620)	-1.391* (-2.127)	.252 (.450)	-.168 (-.173)	.269* (5.714)	.503* (9.077)	4.004* (2.664)	.200* (2.366)	.066 (1.932)	.183* (3.220)	-.970* (-3.291)	.877	76.91	.337	107	[illegible]

[1] t-statistic in parenthesis; an asterisk next to a coefficient denotes significance at the 5% level for a two tail test.

3. Among the levels of activity variables, the trade and tourist variables (and the variables collinear with them) are the two most important ones explaining the international demand for outgoing telephone services. The first one enters into the equation through the explanation of business demand in our dissagregated model, and the second via the explanation of household demand.

4. The inclusion of the quality variables (Log QS and Log TD) improves the $\bar{R}^2$ and makes the coefficient of the price of telephone variable more significant. This result implies that there is evidence of a capacity limitations constraint in the international telephone network. The coefficient of the own price variable may have become more significant because of the sample is made more homogeneous through the inclusion of the quality of service variable.

Outgoing Flows of Telegraph Services

For the outflow of telegraph services, we again restrict the demand equation to be homogeneous of degree zero in the monetary variables, yielding an equation similar to (6) except that TFOC is replaced by TGOC, and the variables TD and QS are suppressed because we assume that there is no capacity limitations with the telegraph system. In the different regressions for this equation, the coefficients of the variables involving language and working hours commonality (LC and WHC) were found to be statistically insignificant in the preliminary regressions, and these regressors were suppressed also[1]. The final results that were obtained from these estimations appear in Table 2.

Here again, the trade and tourist flow variables have substantial explanatory power, and furthermore the coefficients are very stable from regression to regression. It is important to note that in this set of equations, the real income variable is not significant, according to the standard t-test. This can be due to two factors, the first of which is the small variability in Log $\frac{I_c}{\overline{P}}$. The second point is related to the possibility that a service like telegraph could be demanded mainly by business and in that case the quantity demanded would be very closely related to Log $\frac{IMEX}{\overline{P}}$. The other important point to note is that the coefficients of the price variables are now highly significant for the price of telegraph and the price of telex. The coefficient of the telephone price does not have the expected sign, but is not significant.

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1 Such effects may be expected 'a priori', because while telephone services demand practically instantanous access as well as common language between callers, telegraph (and also telex) services emphasise more leisurely contact in which LC and WHC play a relatively less important role.

Table 2: Demand for outgoing flow of telegraph services

Dependent variable Log $TGOC_j(t)$ (explanatory variables deflated)[1]

Explanatory Variables

Equation number	Constant	Log $\frac{P_j^F(t)}{\bar{P}(t)}$	Log $\frac{P_j^G(t)}{\bar{P}(t)}$	Log $\frac{P_j^X(t)}{\bar{P}(t)}$	Log $\frac{IMEX_j(t)}{\bar{P}(t)}$	Log$CTOUR_j(t)$	Log $\frac{I_c(t)}{\bar{P}(t)}$	$\bar{R}^2$	F	$\hat{\sigma}_\varepsilon^2$	N	Year
2.1	-4.194 (-.781)	-.266 (-.311)	-1.745* (-3.482)	2.341* (2.690)	.512* (9.085)	.224* (3.207)	-	.855	43.65	.203	37	1969
2.2	-.232 (-.040)	-.893 (-.965)	-1.991* (-3.619)	2.656* (2.708)	.449* (6.791)	.279* (3.662)	-	.817	33.24	.251	37	1970
2.3	-4.340 (-.663)	-.623 (-.623)	-1.581* (-2.932)	2.811* (2.628)	.470* (6.654)	.267* (3.280)	-	.814	29.09	.266	33	1971
2.4	-7.299 (-.504)	-.627 (-1.234)	-1.846* (-6.050)	2.596* (4.889)	.476* (13.564)	.257* (6.196)	.407 (.345)	.843	96.23	.216	107	1969 1970 1971
2.5	-2.419 (-.778)	-.637 (-1.261)	-1.850* (-6.098)	2.575* (4.902)	.476* (13.661)	.256* (6.219)	-	.844	116.47	.215	107	1969 1970 1971

1 See footnote in Table 1

The coefficients from lines 2.1 to 2.3 are very similar and when we test the null hypothesis for equality of the complete set of coefficients using a Chow test, it is accepted. The pooled regression results appear in lines 2.4 and 2.5 In line 2.4 we include the real income variable but it is not significant and therefore we leave it out in the final regression. In line 2.5 the own price elasticity is -1.850 and it is significantly lower than -1. This result implies that a substantial revenue increase could be obtained by lowering the price of telegraph services. The coefficient of the price of telex variable indicates that a cut in telex price will shift some of the demand away from telegraph; an important consideration to be taken into account by telecommunications regulatory authorities.

Outgoing Flows of the Telex Services

The results for the telex equation which is very similar to the telegraph equation of the previous Section, appear in Table 3. Here, we again accept the hypothesis of equality of coefficients over the years and therefore we can pool the observations. One of the most interesting features of these results is the stable character of the coefficient of Log $\frac{IMEX}{P}$. This is expected because telex is mostly used by business and therefore its demand is more business oriented, the level of activity of international business being measured by the value of total trade. The coefficient of income is not significant also, confirming the hypothesis that telex services are demanded mainly by business involved in international trade.

The results of Table 3 also show a very high (negative) own price elasticity for telex services. This result is substantially higher than values reported in previous studies (5), (8). The implication of this high elasticity is that there is room for a substantial increase in the revenue of the telex industry through the reduction of prices. Judging by the sign of the coefficient of Log $\frac{P^G}{\bar{P}}$, telegraph is a gross substitute of telex, as expected 'a priori'. The coefficient of telephone price is not significant. One of the implications of these results together with those for telephone and telegraph is that telegraph and telex services are substitutes. Therefore price regulation of any one of the two modes would have important consequences for the other mode.

Table 3: Demand for outgoing flow of telex services

Dependent variable Log $TXOC_j(t)$

Explanatory Variables[1]

Equation number	Constant	Log $\frac{P_j^F(t)}{\bar{P}(t)}$	Log $\frac{P_j^G(t)}{\bar{P}(t)}$	Log $\frac{P_j^X(t)}{\bar{P}(t)}$	Log $\frac{IMEX_j(t)}{\bar{P}(t)}$	Log $\frac{I_c(t)}{\bar{P}(t)}$	$\bar{R}^2$	F	$\hat{\sigma}_\varepsilon^2$	N	Year
3.1	28.330 * (3.571)	-.256 (-.170)	1.129 (1.313)	-4.176* (-2.923)	.843 * (9.426)	–	.801	37.38	.628	37	1969
3.2	38.041 * (6.105)	-.723 (-.606)	.732 (1.044)	-4.666* (-3.976)	.855 * (11.456)	–	.854	53.67	.418	37	1970
3.3	30.024 * (4.662)	-.110 (-.099)	.864 (1.234)	-4.377* (-3.903)	.916 * (12.199)	–	.889	65.08	.332	33	1971
3.4	10.850 * (1.542)	-.412 (-.574)	.915 * (2.150)	-4.393* (-6.264)	.865 * (19.566)	1.906 (1.147)	.857	128.88	.433	107	1969 1970 1971

1 See footnote in Table 1.

CONCLUSIONS AND POLICY IMPLICATIONS

Here, we will concentrate on the implications of the results for outgoing flows of telecommunications, with respect to tariff structure policy. The first important conclusion which may be deduced is that the demand for telegraph and especially telex services are substantially price elastic, indicating that a price reduction policy by regulatory authorities would result in important increases in revenues from these two services.

The second noteworthy result of this study is the high cross price elasticity between telegraph and telex. Therefore, any price regulation for telex services should take into account, its impact on the demand for telegraph traffic and vice versa. The third major result is that the cross price elasticity between telephone and either of the other two modes of telecommunications is insignificant. The above conclusion does not support the contention that international telephone rates cannot be decreased by the regulatory authority because of its possible negative effect on the demand for telegraph. This argument was used during the late sixties in the U.S., to propose merging of record carriers in order to internalize the cross price effects of the lowering of telephone prices. Our results indicate clearly that changes in telephone price do not have a statistically significant effect on the flows of the other two types of telecommunications.

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II.4 THE DEMAND FOR CATV SERVICES IN CANADA*

* This section is based on a larger economic study of the market and financial characteristics of the 16 largest CATV companies in Canada, which was carried out by our predecessor the IIQE on behalf of the Department of Communications of the Canadian government. In the overall project participated Vittorio Corbo and Mohan Munasinghe as investigators and Roger Morin as financial consultant. Mohan Munasinghe was responsible for the developing of the demand model presented here.

The purpose of this section is to examine some of the main determinants of demand for CATV services in Canada. It is anticipated that more precise information on the relative attractiveness to Canadian viewers of TV signals imported from the U.S., live programming services etc. will assist decision makers in the framing of a more coherent long term regulatory policy. The study covers the 16 largest CATV companies operating 30 cable systems, which accounted for about 55% of all cable TV subscribers in Canada, during 1972.

The growth of a typical CATV system may be characterised by a number of "state" variables such as number of subscribers, total subscriber revenue, total expenditure etc. However, in the present model, Penetration Ratio defined as the ratio of actual to potential subscribers was chosen as the key variable since it is already normalised for "size" (i.e. variables such as total subscriber revenue would depend on the absolute size of the system). The model used in this study is a modified version of the one developed by Noll, Peck and Mc Gowan[1] (21). It is outlined below:

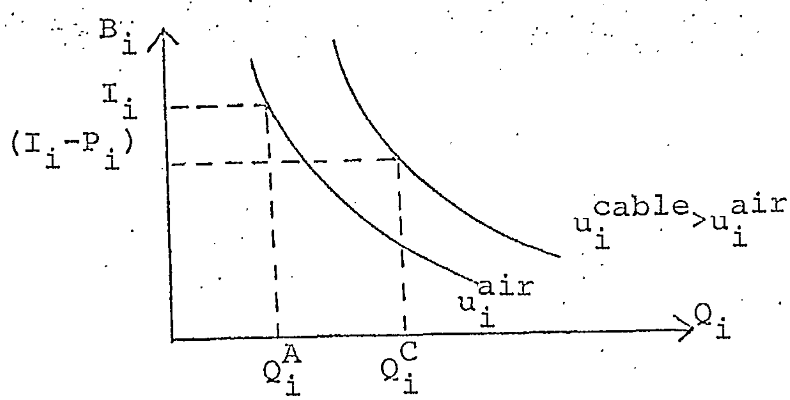

FIGURE 1. CATV subscription condition: $u_i^{cable} > u_i^{air}$

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[1] The form of the function used to represent quality of CATV service in our model incorporates several new variables such as advertising and sales promotion costs etc.

We assume that a log-linear utility function u represents the i^{th} consumer's preferences between television quality Q_i and all other types of goods (measured by his budget B_i):

$$u_i = \alpha_{i0} (B_i)^{\alpha_{i1}} (Q_i)^{\alpha_{i2}}$$

As shown in Figure 1, a consumer having annual income I_i has access to free over-the-air TV of quality Q_i^A, whereas if he accepts CATV services of quality Q_i^C for an annual fee of P_i his budget for other goods will be reduced to (I_i-P_i). Therefore he will increase his utility by subcribing to CATV if $\alpha_{i0}(I_i-P_i)^{\alpha_{i1}}(Q_i^C)^{\alpha_{i2}} > \alpha_{i0} I_i^{\alpha_{i1}}(Q_i^A)^{\alpha_{i2}}$

By assuming that within the operating region of a given system, all potential CATV subscribers have the same mean annual income I, subscription fee P, over-the-air TV quality Q^A and CATV quality Q^C, it can be shown that the critical condition for the i^{th} consumer to subscribe is:

$$\beta_i > \overline{\beta} = \frac{\ell n(1-P/I)}{\ell n(Q^A/Q^C)} \qquad (1)$$

Where $\beta_i = \frac{\alpha_{i2}}{\alpha_{i1}}$ represents the importance of TV quality relative to other goods, according to the tastes of the i^{th} consumer. If the distribution of potential subscribers with respect to the parameter β is $h(\beta) = \mu e^{-\mu\beta}$, $\mu > 0$;

where $h(\beta)d\beta$ represents the fraction of the total potential subscriber population having taste parameter lying between β and $\beta + d\beta$, we find that the penetration ratio (i.e. ratio of actual to potential subscribers) is given by:

$$PR = e^{-\mu\overline{\beta}} \qquad (2)$$

From (1) & (2): $$\frac{\ell n(1-P/I)}{\ell n(PR)} = \frac{1}{\mu} \ell n \left(\frac{Q^C}{Q^A}\right) \qquad (3)$$

Next we assume that Q^A can be represented as a function of the different types of TV channels available over-the-air (on an A or B contour basis)[1].

$$\therefore \quad Q^A = \gamma_0^A (1+P_C^A)^{\gamma_1} (1+D_C^A)^{\gamma_2} (1+I_C^A)^{\gamma_3} (1+E_C^A)^{\gamma_4} (1+P_{US}^A)^{\gamma_5} (1+D_{US}^A)^{\gamma_6} (1+I_{US}^A)^{\gamma_7} (1+E_{US}^A)^{\gamma_8} u^A$$

where the superscript A refers to over-the-air service,

the subcript C identifies Canadian stations,

the subscript US identifies U.S. stations,

P = number of primary network stations

D = number of duplicate network stations

I = number of independent stations

E = number of educational stations

u = random disturbances term

A slightly different expression may be developed for Q^C:

$$Q^C = \bar{Q}^C . e^{\phi_1 . DBC} . e^{\phi_2 . DAP} . (1+C_A)^{\phi_3} . (1+C_P)^{\phi_4} . f(T) \qquad (5)$$

where $\bar{Q}^C = \gamma_0^C (1+P_C^C)^{\gamma_1} (1+D_C^C)^{\gamma_2} (1+I_C^C)^{\gamma_3} (1+E_C^C)^{\gamma_4} (1+P_U^C)^{\gamma_5} (1+D_U^C)^{\gamma_6} (1+I_U^C)^{\gamma_7} (1+E_U^C)^{\gamma_8} . u^C$;

the superscript C refers to CATV services;

DBC = 1 for big city with population > 200,000 (including Ottawa)

= 0 otherwise

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[1] This point is discussed in greater detail in the Data Appendix to the main study (6).

DAP = 1 if automated programming is provided

= 0 otherwise

C_A = advertising and sales promotion cost/potential subscriber

C_p = live programming cost/potential subscriber

f(T) = function of system age T.

The dummy DAP has been introduced into Q^C, to represent additional services such as automated news/weather service, etc., whereas DBC captures the effect of a variety of others forms of entertainment such as cinema, theatre, etc. which compete with CATV, in large metropolitan areas. $\dot{Q}^C$ depends also on the amount of advertising and sales promotion (C_A) carried out, and on the cost of live programming (C_p) provided. The function f(T) captures the effect of system age on PR. Ceteris paribus, one may expect that the penetration ratio will increase as the system ages.

Substituting equations (4) and (5) in (3) yields:

$$Y = \lambda_0 + \lambda_1 \ln X_{PC} + \lambda_2 \ln X_{DC} + \lambda_3 \ln X_{IC} + \lambda_4 \ln X_{EC} + \lambda_5 \ln X_{PU}$$

$$+ \lambda_6 \ln X_{DU} + \lambda_7 \ln X_{IU} + \lambda_8 \ln X_{EU} + a_1 \cdot DBC + a_2 \cdot DAP$$

$$+ a_3 \ln \overline{C}_A + a_4 \ln \overline{C}_p + \frac{1}{\mu} \ln f(T) + V \qquad (6)$$

where $Y = \frac{\ell n(1-P/I)}{\ell n(PR)}$; $\lambda_o = \frac{1}{\mu}\ell n \frac{\gamma_o^C}{\gamma_0^A}$; $\lambda_i = \frac{\gamma_i}{\mu}$ for i=1,...,8;

$a_j = \frac{\phi_j}{\mu}$ for j=1,...,4;

$X_{PC} = \frac{1+P_C^C}{1+P_C^A}$; $X_{DUS} = \frac{1+D_{US}^C}{1+D_U^A}$ etc.;

$\overline{C}_A = 1 + C_A$; $\overline{C}_p = 1 + C_p$ and $V = \ell n(\frac{u^C}{u^A})$

Thus assuming that an equation such as (6) applies to all the systems, its parameters can be estimated from a sample consisting of a cross section of CATV systems.

Two simple forms for f(T) are used in our estimations:

(i) $f(T) = e^{\rho T}$ so that $\ln f(T) = \rho T$

or (ii) $f(T) = (T)^{\phi_5}$ so that $\ln f(T) = \phi_5 \ln T$

Equation (6) is the final form of the model to be estimated[1].

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[1] Clearly, it would be interesting to examine the effect of the increasing ownership of colour TV sets on penetration ratio (i.e. ceteris paribus, we would expect PR to increase as the ratio of colour TV sets to black and white sets increased), but lack of data precludes such a study.

Results

The results to be presented here are based on the 20 larger CATV systems, since these were found to form a relatively more homogeneous group from the point of view of the overall characteristics of the systems. The estimations were based on pooled information for 1971, 1972 and 1973 for these twenty systems[1].

The regression results with YN as the dependent variable[2] are shown in Table   Equation (1.1) is the basic model; the t-values for the coefficients of the variables involving primary Canadian network, duplicate U.S. network, and independent U.S. stations (i.e. ℓnX_{PC}, ℓnX_{DU} and ℓnX_{IU} respectively) are quite significant. When we add the big city dummy DBC in equation (1.2), its negative coefficient is highly significant and the $\bar{R}^2$ increases dramatically. We continue adding the variables for automated programming, advertising cost and live programming cost (i.e. DAP, $\ell n\bar{C}_A$ and $\ell n\bar{C}_P$ respectively) through equations (1.3) to (1.5). In each case the t-statistic for the coefficient of the new variable added is found to be significant, and the

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1 Ideally, household and multi-outlet (e.g. apartment) subscribers should have been treated separately due to differences in cable price, tastes etc., but this was not possible owing to the lack of disaggregated data.

2 YN corresponds to the variable Y defined in equation (6), using income I_N which is net of taxes. The results obtained with the dependent variable YG (using gross income I_G) were not as good as the results for YN. A more detailed description of these and other variables used in the regressions is given in the Data Appendix.

TABLE III

Results of the Penetration Ratio Model[1]

Dependent Variable: Y

(52 observations during 1971-7[illegible])

EQUATION NUMBER	COMMENT	lnX_{PC}	lnX_{DC}	lnX_{IC}	lnX_{EC}	lnX_{PU}	lnX_{DU}	lnX_{IU}	lnX_{EU}	lnP	DDC	DAP	$ln\bar{C}_A$	$ln\bar{C}_P$	CONSTANT	$\bar{R}^2/F$
1.1	Base Equation for 33 Large Systems	.0209* (2.0512)	-.0073 (-.7273)	-.0081 (-.7543)	-.0157 (-1.3308)	.0151 (1.3150)	.0157* (2.1634)	.0438* (3.9542)	-.0355 (-.8714)	.0066 (1.6611)	–	–	–	–	-2.5415 (-2.4560)	.6225/ 5.3607
1.2	Add DDC to (1.1)	.0227* (2.5253)	-.0016 (-.2775)	-.0033 (-.1759)	-.0104 (-1.1322)	-.0004 (-.0420)	.0210* (4.1382)	.0241* (2.5724)	-.0022 (-.4407)	-.0000 (-.0119)	-.0232* (-5.3009)	–	–	–	3.7425 (1.7095)	.7553/ 17.7723
1.3	Add DAP to (1.2)	.0251 (1.4332)	-.0027 (-.2243)	-.0102 (.1248)	-.0071 (-.7695)	-.0041 (-.4393)	.0221* (4.2034)	.0201* (2.8223)	-.0019 (-.3533)	.0002 (.0472)	-.0250* (-5.7098)	-.0061 (-1.6480)	–	–	1.5815* (2.3734)	.7742/ 27.2527
1.4	Add $ln\bar{C}_A$ to (1.3)	.0345 (1.5947)	-.0037 (-.6222)	-.0250 (-.4329)	-.0012 (-.2614)	-.0015 (-.4615)	.0219* (5.2437)	.0229* (3.0542)	-.0003 (-.0742)	.0040 (1.4682)	-.0260* (-7.3528)	-.0059* (-2.3134)	.0197* (4.7363)	–	1.4225 (.7425)	.8242/ 25.5274
1.5	Add $ln\bar{C}_P$ to (1.4)	.0272 (1.3272)	-.0022 (-.3924)	-.0036 ([illegible])	-.0049 (-.6532)	-.0032 (-1.1942)	.0217* (5.4599)	.0204* (2.8182)	.0001 (.0262)	.0033 (1.3637)	-.0260* (-7.7003)	-.0075* (-2.6272)	.0172* (4.3115)	.0285* (2.2304)	1.9015 (1.0722)	.8271/ 26.2322
1.6	Remove X_{EU} from (1.5)	.0372 (1.8743)	-.0219 (-.4019)	-.0036 (-.3440)	-.0012 (-.4009)	-.0032 (-1.2428)	.0717* (5.5346)	.0203* (3.0665)	–	.0033 (1.2834)	-.0260* (-7.3083)	-.0075* (-2.6635)	.0172* (4.3777)	.0059* (2.2162)	1.9513 (1.2572)	.8725/ 29.2222
1.7	Remove X_{EC} from (1.6)	.0303 ([illegible])	[illegible]	-.0045 (-.7825)	–	-.0247 (-1.3730)	.0233* (5.6639)	.0207* (3.1345)	–	.0031 (1.2193)	-.0265* (-8.2373)	-.0079* (-2.5900)	.0172* (4.7380)	.0093* (2.3342)	2.2547 (1.2263)	.8723/ 22.64[illegible]

[1] t = statistic given within parenthesis. An asterisk next to a coefficient indicates significance at a 5% level for a two-tail test.

[2] See text for description of variables.

overall fit improves ($\bar{R}^2$ rises). Note that the addition of the highly significant advertising cost variable $\ell n\bar{C}_A$ from equation (1.3) to (1.4) reduces considerably the coefficient of the primary Canadian Network Stations variable(ℓnX_{PC})[1].

In the case of the variables X_{EC} and X_{EU}, involving educational TV services in Canada and the U.S. respectively, the number of observations for which these variables take a value different from zero is small. The effect of dropping ℓnX_{EU} (which is not significant anyway), from the regression is shown in equation (1.6). The other coefficients hardly change, but in general the t-statistics and $\bar{R}^2$ improve. By contrast, as seen in equation (1.7) most of the impact of removing ℓnX_{EC} is absorbed by an increase in the value of the coefficient of ℓnX_{PC}; the overall fit improves slightly. One reason for this is that the primary Canadian network and educational Canadian station variables (i.e. ℓnX_{PC} and ℓnX_{EC} respectively) have the highest value of "simple" correlation coefficient (0.63) between any two variables in the model.

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[1] If the former variable is left out as in previous studies (5), (22), the importance of ℓnX_{PC} would be overestimated.

Consider the pattern of coefficients in equations (1.6) and (1.7). Of the TV station variables, as expected, primary Canadian network service has the biggest coefficient (significantly different from zero at the 7% level), followed by duplicate U.S. network and independent U.S. stations (highly significant and stable). Although contrary to 'a priori' expectations, the coefficients of the other TV station variables are negative, they are not significantly different from zero. For some of these variables, there is little variability among the observations in the sample (e.g. X_{PU}). From general considerations, it may be expected that the coefficients of the primary Canadian and U.S. network station variables (i.e. ℓnX_{PU} and ℓn^{x}_{PC} respectively) are comparable; the same may be said for duplicate Canadian and U.S. network stations represented by ℓnX_{PU} and ℓnX_{DU} respectively.

From the statistical point of view, the most significant explanatory variable is the big city dummy DBC, where the negative coefficient could be interpreted as indicating that the effect of competing entertainments such as cinema and theatre would be to reduce the penetration ratio. Surprisingly, automated programming DAP (news, weather, etc.) has a significant negative effect on penetration ratio. However, this may be just a statistical "mirage" because in our sample DAP is negatively correlated with most of the TV station variables (especially X_{PC} and X_{PU}). Thus, due to unavailability of more CATV channel capacity, if automated programming is being provided instead of another more highly valued station, then this could tend to lower the observed PR for such a system. Both advertising and live programming expenditures per potential subscriber, $\ln\overline{C}_A$ and $\ln\overline{C}_P$ respectively, have a significant positive impact on the penetration ratio PR. The former is roughly twice as important as the latter, and its t-statistics is higher also.

The penetration ratio increases with the age of the system $\ln T$, but this coefficient is not significantly different from zero. This may be due to the typically "step alike" manner in which a system finances its own growth (discussed in greater detail in the IIQE study (9)). Thus a system could start by wiring the most promising section of its total franchise area. At this stage penetration would be low, but after some time PR would begin to increase. Using the revenues so generated, the company would proceed to wire another portion of its franchise area, causing the overall penetration ratio to drop initially and then rise again with time. Thus as long as the system is growing, the observed value of penetration would depend on what stage of this cycle the system was in.

Furthermore, the older systems may have chosen the most promising markets yielding better values of PR (for a given level of service and system age).

Next we study the implications of these results in terms of elasticities of the penetration ratio. In Table we present the elasticities computed from equation (1.6) of Table at the mean values of $\frac{P}{I}$ and Y[1].

Clearly, as expected, the proportional change in penetration ratio (PR) to a change in the availability of primary Canadian network service is substantially higher (almost double) than the response of PR to changes in either the duplicate U.S. network or the independent U.S. service variables. Similarly, an additional dollar spent on increased advertising and sales promotion is likely to have twice the impact on the penetration ratio as the additional dollar allocated to live programming. The dramatic effect of the big city dummy (DBC) on penetration ratio is also shown in Table V. Taking average values for a typical large system, PR would be equal to 0.713 outside a big city environment, but this value would fall to 0.476 in a metropolitan area.

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1 Changes in the values of variables such as X_{PC} are not truly continuous. Thus from equation (6), $X_{PC} = \frac{1+N^{cable}}{1+N^{air}}$ and assuming that $N^{cable} = 1$, and $N^{air} = 2$, we get $X_{PC} = 0,67$. If we increase the number of primary Canadian network stations provided via cable by 1 (i.e. $N^{cable} = 2$) then X_{PC} would increase by 50% (i.e. now $X_{PC} = 1$). Hence, we cannot [illegible] relative importance of variables such as X_{PC} in terms of their impact on the penetration ratio.

TABLE V

Effect of changes in the explanatory variables on the Penetration Ratio.

Explanatory Variable X_i	Elasticity of the Penetration Ratio (PR) with respect to the Explanatory Variable.[1] ϵ_{PR,X_i}
Primary Canadian Network Stations (X_{PC})	0.500
Duplicate U.S. Network Stations (X_{DU})	0.286
Independent U.S. Stations (X_{IN})	0.271
Advertising and Sales Promotion Cost/Potential Subscriber ($\overline{C}_A$)	0.227
Live Programming Cost/ Potential Subscriber ($\overline{C}_P$)	0.131
Big City Area (DBC=1) :	PR = 0.476
Non-Big City Area (DBC=0) :	PR = 0.713

[1] Elasticities were computed at the mean values of $\frac{P}{I}$ and Y, using the coefficients of regression equation 1.6 in Table III.

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III. FORECASTING WITH MULTI-EQUATION MODELS

III.1. FORECASTING THE EFFECT OF RATE OF RETURN REGULATION ON THE CAPITAL INTENSITY OF A CARRIER: A MODEL OF BELL CANADA *

* This section draws on IAER (1976). That study was carried out under the direction of Vittorio Corbo and with the participation of M. Munasinghe and R. Morin as consultants and G. Tsoublekas as researcher.

The purpose of this section is to develop a model of a regulated carrier, which is then estimated and used to forecast the effect of regulation on the capital/labor ratio of the carrier.

The model that we use is of the Averch-Johnson type. In these models financial variables enter only through the cost of capital services, and the model is solved only for real variables: output and primary factor inputs. For a model in which financial variables are dealt with explicitly, see the next section.

The model that we use is the Averch-Johnson family. In this type of models three main assumption are made:

(i) The firm seeks to maximize profits;

(ii) the decision process is constrained by the available technology and by the imposition of an upper limit on the rate of return that it can earn on its capital. This rate of return is the "allowed rate of return". It is further assumed that the "allowed rate of return" is greater than the firm's cost of capital but lower than the rate of return that the firm would achieve if it were to operate as an unregulated monopoly;

(iii) no regulatory lags exist.

If this model is an appropriate description of the behaviour of a regulated company then the following two main propositions follow:

(1) At the output selected by the regulated firm the capital-labour ratio chosen is greater than that which minimizes cost.

(2) The output of the regulated firm would not rise above that of the unregulated profit-maximizing firm, except in the unusual circumstance in which capital is an inferior input. That is, when an increase in output is accompanied by a decrease in capital used.

Now we are going to present a model of a regulated firm. The notation used is as follows:

Q = the firm's value added in real terms

K = the amount of physical capital employed by the firm

L = the amount of labour employed by the firm

P = the price of value added

w = the wage rate

P_K = the rental price of a unit of physical capital

s = the "allowed rate of return" on a unit of physical capital, $s > P$

Π = profits

R = P.Q = total value added, at current prices

$\frac{\partial R}{\partial L}$ = marginal revenue product of labour, i.e. increase in revenue due to a (small) increase in the labour hired

$\frac{\partial R}{\partial K}$ = marginal revenue product of capital

The regulated firm then seeks to maximize profits

(1) $\Pi = PQ - wL - P_K K$

subject to the rate of return constraint

(2) $s \geq \frac{PQ - wL}{K}$

and to the production technology constraint

(3) $Q(L,K) \geq Q$

If the regulatory and production function constraints are assumed to be binding, that is if the inequalities (2) and (3) are satisfied with equalities, then this maximization problem implies the following first order conditions for choosing inputs and the level of output.

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1 For the derivation of this relations, see Appendix A.

(4) $\frac{\partial R}{\partial L} = w$ (5) $\frac{\partial R}{\partial K} = P_K - \frac{\lambda}{1-\lambda}(s-P_K)$

(6) $R(L,K) = wL + sK$

where the only new symbol introduced is λ which is a Lagrangian multiplier from the maximization problem. Furthermore, Baumol and Klevorick (1970) have shown that $1 > \lambda > 0$. Let us see the meaning of each of these equations required to be fulfilled for the firm to maximize profits subject to the regulatory constraint. Equation (4) states the condition that the marginal revenue product of labour has to be equal to the wage rate. This equation takes the same form as in the case of unregulated monopolies, although this does not mean that the regulated firm will choose the same level of labour input as the unregulated monopoly, because the level of factor inputs is obtained from the simultaneous solution of equations (4), (5) and (6) above. Now, for the case of an unregulated monopoly, $\lambda = 0$ and therefore in (5) above we will have $\frac{\partial R}{\partial K} = P_K$, the well known equality between the marginal revenue product of capital and the price of capital. While for the regulated monopoly, $P_K - \frac{\lambda}{1-\lambda}(s-P_K) = \frac{P_K-\lambda s}{1-\lambda} < P_K$ and from here the over capitalization for the chosen value of output follows.

An Econometric Model of Bell Canada and a Test of the Over-capitalization Hypothesis

The Production Function

In order to test the Averch-Johnson hypothesis we need to specify the production function relevant for Bell Canada. We reviewed the existing production functions for Bell Canada, i.e. Dobell et al (1970), Millen (1974), and we concluded that there is no clear-cut evidence with respect to what is the appropriate specification of technology. An attractive avenue for further empirical research is to assume a more general form of production function which can be considered as a second order approximation to any production function around a point in which the logarithms of each of the inputs are made equal to zero. This form of production function is called the Transcendental Logarithmic Production Function (translog) due to Christensen et al (1971, 1973) and it has the advantage that it reduces to a Cobb-Douglas form as a special case.

We write the translog production function as:

$$(8) \quad \ell n\, Q_t = \alpha_0 + \alpha_D D_t + \alpha_1 \ell n\, L_t + \alpha_2 \ell n\, K_t + \tfrac{1}{2} \gamma_{11} (\ell n\, L_t)^2 + \tfrac{1}{2} \gamma_{22} (\ell n\, K_t)^2 + \gamma_{12} (\ell n\, L_t)(\ell n\, K_t) + \varepsilon_t$$

where the only new variables introduced are D_t which is an index of technology and ε_t which is the random error of the regression. We have further assumed here Hicks-neutral technical change.

The following definitions were used for the different variables.

Q = Total value added (including uncollectibles) minus Indirect taxes and raw materials in millions of 1967 dollars re-escalated to make the average equal to one during the sampling period.

L = Weighted man-hours, were the weights are the relative hourly wage rate of the different labour categories, re-escalated as above.

K = Net capital stock in millions of 1967 dollars, re-escalated as above.

s = Actual rate of return defined as total revenue minus indirect taxes, plus uncollectibles, minus cost of materials, rent and supplies, minus the wage bill, all in current dollars divided by the value of the net capital stock in millions of 1967 dollars.

P_K = Price of capital services, computed using the Jorgenson formula.[1]

D_t = Percentage of calls direct distance dialled

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[1] In this formula

$$P_K = [\delta * q_t + CC * q_{t-1} - (q_t - q_{t-1})] * \frac{(1-u*z)}{1-u}$$

where q = Price index of capital goods 1967 = 1.00

CC = Cost of capital

u = Income tax rate

δ = Rate of replacement

z = Present value of depreciation deductions on a dollar's investment in plant

The source of each of these elements appears in the Appendix C to IAER (1976).

First we will estimate a general translog model with Hicks-neutral technical change. The results, after correcting for autocorrelation are the following (the figures in parentheses are the t-ratios):

$$\ell n\, Q_t = \underset{(-.162)}{-.020} \underset{(.760)}{+.172}\, \ell n\, L_t \underset{(4.077)}{+1.230}\, \ell n\, K_t \underset{(.882)}{+2.125}\, (\ell n\, L_t)^2$$

$$\underset{(2.999)}{+.559}\, (\ell n\, K_t)^2 \underset{(-1.639)}{-.929}\, \ell n\, L_t\, \ell n\, K_t \underset{(-.091)}{-.041}\, D_t$$

$$\hat{\rho} = \underset{(1.230)}{.265},\ SSR = .003789,\ D.W. = 1.89,\ \text{Years: } 1953\text{-}1972$$

$$R^2 = .9993$$

When we test for constant returns to scale and then for complete global separability in a sequencial test in both cases the null hypothesis is accepted and therefore we conclude that Bell Technology can be approximate by a Cobb-Douglas production function of the following form[1]:

When this model is estimated we obtain the following results

$$\ell n\, (Q_t/L_t) = \underset{(-9.356)}{-.330} \underset{(6.486)}{+.391}\, \ell n\, (K_t/L_t) \underset{(11.826)}{+1.136}\, D_t$$

$$\hat{\rho} = \underset{(1.900)}{.391},\ SSR = .007050,\ D.W. = 1.83,\ \text{Years: } 1953\text{-}1972$$

$$R^2 = .9984$$

1 For details of the tests performed, see IAER (1976, 14-17).

We should also mention that we estimated also the Cobb-Douglas function with gross production as the dependent variable and three factors of production (labor, capital and raw materials) but the coefficient of the raw materials variable was never statistically significant. Therefore the hypothesis of fixed coefficient for raw materials and a constant returns Cobb-Douglas function for value added is shown out by our data.[1]

One could claim that the poor showing of the general translog model is due to the strong collinearity among the regressors in equation (8) and that therefore the translog function should not be estimated directly but from side conditions for profit maximization.

Thus as a further search into the technology of Bell Canada we will take the unrestricted translog as a general production function describing the technology of Bell and then we will use the side conditions for profit maximization subject to a rate of return constraint to identify the parameters of the production function.

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[1] The measure of raw materials available is the one reported in R. Millen (1974) and it includes rents and other supplies besides raw materials and therefore part of its poor performance in the equation could be due to an error in the variables problem.

When translog model was estimated in this fashion, independent of the definition for the cost of capital (see IAER (1976, Appendix C)), the production function was not well-behaved. It was nor monotonic neither quasi-concave (see Appendix B of IAER (1976) for details of these concepts). Therefore the general translog model cannot be considered as a proper description of the technology used by Bell-Canada. We also estimated a simultaneous model with a C.E.S. function but the elasticity of substitution was always negative. Therefore the C.E.S. model was also rejected by the data.

We conclude therefore that the technology of Bell Canada can be approximated by a Cobb-Douglas production function.

- The Demand Equation

The demand equation estimated is of the partial adjustment type and is given by:

(12) $\ell n\ Q_t = a_1\theta + a_2\theta\ \ell n\ y_t + a_3\theta\ \ell n\ p_t + (1-\theta)\ \ell n\ Q_{t-1}$

In equation (12) a_3 is the long run price elasticity of demand.

When we estimated this last equation we obtained[1]:

$$\ell n\, Q_t = \underset{(-1.058)}{-.920} \; \underset{(1.223)}{+.154}\; \ell n\, Y_t \; \underset{(-1.123)}{-.160}\; \ell n\, P_t \; \underset{(11.765)}{+.899}\; \ell n\, Q_{t-1}$$

$$\hat{\rho} = \underset{(-1.229)}{-.271}\;,\; R^2 = .999\;,\; \text{D.W.} = 1.98\;,\; \text{Years } 1954\text{-}1972$$

where the only new variable introduced is:

Y_t = real gross domestic product in Ontario and Quebec

From this demand equation we compute a long-run price elasticity (η) of -1.58. This value is then used in the rest of the model.

Testing for an Averch-Johnson Effect

Now we will perform a test of the Averch-Johnson hypothesis by estimating λ from equation (5) in section . For easier reference we reproduce that equation below

$$(5) \quad \frac{\partial R}{\partial K} = \frac{P_K - \lambda s}{1 - \lambda}$$

But we also know that:

$$\frac{\partial R}{\partial K} = P(1+\frac{1}{\eta}) \cdot \frac{\partial Q}{\partial K}$$

where

η = price elasticity of demand

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[1] We also used as an explanatory variable the log of P_t divided by a combined consumer price index of Toronto and Montreal but the results were inferior.

From our estimated Cobb-Douglas production function we obtain:

$$\frac{\partial Q}{\partial K} = .391 \frac{Q}{K}$$

From the demand equation we had $\eta = -1.58$. Therefore using equation (5) we obtain:

$$(.365)(.391) \quad \frac{PQ}{K} = \frac{P_K - \lambda s}{1 - \lambda}$$

Finally we obtain:

$$\frac{P_K K}{PQ} = .1428 \ (1-\lambda) + \lambda \frac{sK}{PQ}$$

That is:

$$M_K = .1428 \ (1-\lambda) + \lambda Z$$

This equation was estimated for the three different definitions of the cost of capital obtaining:

(13) $$M_{K1} = .1428 \underset{(7.510)}{(1-.715)} \underset{(7.510)}{+.715} Z$$

$$\hat{\rho} = \underset{(3.189)}{.592}, \ R^2 = .290, \ D.W. = 1.77, \ \text{Years } 1954\text{-}1972$$

(14) $$M_{K2} = .1428 \underset{(11.395)}{(1-.762)} \underset{(11.395)}{+.762} Z$$

$$\hat{\rho} = \underset{(2.576)}{.487}, \ R^2 = .108, \ D.W. = 1.89, \ \text{Years } 1954\text{-}1972$$

(15) $M_{K3} = .1428\ (1-.785) + .785\ Z$
$\qquad\qquad\qquad\quad (12.184)\quad (12.184)$

$\hat{\rho} = \underset{(2.479)}{.473}$, $R^2 = .097$, D.W. = 1.89 , Years 1954-1972

Therefore from the three different definitions of the cost of capital we conclude that λ is statistically significant and a number between zero and one as predicted by the theory of a regulated firm.

To measure the effect of regulation on the capital-labor ratio we compute this ratio under regulation and without regulation.

With regulation

$$\frac{K}{L} = \frac{\alpha_2}{1-\alpha_2} \frac{w}{\frac{P_K - \lambda s}{1 - \lambda}}$$

without regulation ($\lambda = 0$)

$$\frac{K}{L} = \frac{\alpha_2}{1-\alpha_2} \frac{w}{P_K}$$

In the first order condition λ is a variable, therefore in the above equations λ is a sort of an average value, therefore we will compute K/L for the case in which the right hand side variables take values equal to their average for the period 1952-1972, to obtain some measure of the over-capitalization in Bell Canada.

Effect of Regulation on the Capital-Labor Ratio

	Observed	Regulation Model	No-Regulation
PK1	33.44	34.4	16.9
PK2	33.44	30.5	16.1
PK3	33.44	28.8	15.8

From these results we conclude that there is strong evidence of over-capitalization in Bell Canada as predicted by the theory of regulation.

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III.2. AN INTEGRATED MODEL OF REAL AND FINANCIAL DECISIONS OF A REGULATED FIRM*

*This section draws on a study entitled Future Financial Needs in the Telecommunications Sector, which was developed for the Department of Communication of the Canadian Government by the Institute of Applied Economic Research, under the direction of J.I. Bernstein. In the project, A. Anastasopoulos and V. Corbo participated as consultants and G. Tsoublekas as researcher.

1. Introduction

The development of models integrating a firm's output supply, factor demand and financial decision processes has only recently become the focus of research on both the theoretical and empirical levels. Indeed, one may find this occurrence rather puzzling given the fundamental nature of the questions to be answered by such an undertaking. Questions such as: If a firm has many financing instruments, for instance long-term debt and common equity, what is the effect on the quantity of output of a change in the relative cost of debt to equity? If a firm has a fixed debt-equity ratio, what are the consequences for physical capital and labor requirements of possible changes in this ratio? The answers to these questions obviously encompass important policy implications for corporate and industrial behavior. In this section of the paper we construct a model which may be utilized to answer these and many more questions. In particular, our model proceeds to the complex context of regulated firms and thus highlights the interplay between financial and real phenomena in this type of environment.

This analysis builds on the works of Lermer and Carleton [8], Robichek and Myers [10], Turnovsky, [11], Vickers [12], [13].

However, our model differs from these previous researches in four important respects. Firstly, we deal with market values, that is market prices, rather than book values.[1] Secondly, we introduce money into the problem because firms operate in monetary as opposed to barter economies. Thirdly, we extend the framework to infix rate of return regulation. This addition is particularly relevant for applications of the model to the telecommunications industry, in which government play an important role in determining the size, composition and other elements of the industry. Finally, our analysis considers an essential part of the characterization of a firm, which typically is missing from the usual models. We explicitely bring in the balance sheet (in terms of market prices) for our regulated corporation.

The implications of incorporating the balance sheet brings to the forefront the essential nature of integrating real and financial decisions. In particular, it becomes obvious that firms can decide either on their financial capital budget (including net money holdings) or on their demand for physical capital. This is because, given prices, determining the financial capital budget is synonymous, with ascertaining physical capital requirements.

In the sections that follow we delimit the general model and then specialize it to fit many different circumstances in which a firm may operate.

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[1] Although using market prices rather than book values does not, in general, introduce theoretical stumbling blocks, the estimation and simulation results could be quite different.

2. The General Model

Let us begin by introducing the production function, which is defined by equation (1),

(1) $$y = F(K,L)$$

where y is output, K is capital services, L is labor services, F represents the technology, with the marginal products of capital and labor being positive.[2]

Demand behavior may be summarized by the inverse demand function represented by equation (2),

(2) $$p = D(y), \frac{dp}{dy} = D_y < 0$$

where p is the price of the output and D is the demand function and since increases in output decrease the price we are assuming that the product is normal and the firm has monopoly power in its output markets. The fact that we are assuming that the firm has some degree of monopoly power is consistent for corporations operating in the telecommunications industry.

If we combine equations (1) and (2) then we can define the pure profits for this firm not only as,

(3) $$\Pi = py - w_\ell L - w_k K$$

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[1] The marginal products are $\frac{\partial F}{\partial k} = F_k > 0$, $\frac{\partial F}{\partial L} = F_\ell > 0$, for capital and labor respectively. We also omit the time variable for notational simplicity. Technological change can also be introduced into the model.

where Π are profits, w_ℓ is the factor price of labour, and w_k is the factor price of capital, but by 4

$$(4) \qquad \Pi = D(F(K,L))\, F(K,L) - w_\ell L - w_k K$$

In addition the factor price of capital is defined as $w_k = p_k(r+\delta)$, where p_k is the price of the physical capital stock, r is the rate of return on financing the capital and δ is the rate of depreciation.[3]

In regard to the factor markets we assume (for simplicity) that the firm is a price-taker (i.e. perfect competitor) in the labor market, but has some degree of monopsony power in the real capital market.[4] Consequently, equation (4), the pure profits equation, (i.e. the market value of the firm's income statement), summarizes the product and factor market relations.

The financial structure of the firm may be represented by a set of relations, the first of which is the market value of the balance sheet denoted by equation (5),

$$(5) \qquad M + p_k K = p_b B + p_s S$$

where M is net money balances, p_b is the price of bonds, p_s is the price of shares, B is the number of bonds and S is the number of shares.[5] Next we define the rates of return on debt, to be

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[3] We are assuming the absence of corporate taxes, which, of course may be incorporated into our model by a suitable redefinition of w_k; see Jorgenson [7].

[4] The exact nature of this monopsony power will be specified below when we discuss the components of the rate of return on capital.

[5] M can be thought of as a composite of such things as cash plus accounts receivable minus accounts payable, B is a composite bond and S is a composite type of equity. The analysis can be disaggregated to permit different debt and equity instruments.

the difference between the value of debt in period t+1 minus the value of debt in period t divided by the value in period t, and the same type of formula applies to equity and money. Thus with these definitions of the rates of return (letting r_b be the bond rate, r_s the equity rate, and r_m the rate of return on M), then,

$$(6) \qquad (1+r) = \frac{p_b B}{p_k K}(1+r_b) + \frac{p_s S}{p_k K}(1+r_s) - \frac{M}{p_k K}(1+r_m),$$

and using equation (5), (6) transforms to

$$(7) \qquad r = \frac{p_b B}{p_k K} r_b + \frac{p_s S}{p_k K} r_s - \frac{M}{p_k K} r_m .$$

It is important to realize, that the fact that the rate of return on physical capital is a weighted average of the rates of return on the different types of financial capital, arises not from any ad-hoc definition of r, but rather from the correct procedure of introducing and specifying the form of the balance sheet (in market value terms) for the firm.

The rates of return for a particular commodity is not a constant number but, of course, depends on the spot and forward prices, and the quantity of that commodity. However, if the rate of return changes solely because the quantity of the financial commodity changes, while the firm cannot influence spot and forward prices, then the firm cannot be said to have any price-setting power in the financial markets. On the other hand, if the quantities of other commodities influence the rate of return then it must be the case that spot on forward prices are, at least partially, influenced by the firm's behavior, and so there is some degree of price-setting power. In our general model we assume that the firm cannot influence spot prices but only forward prices and so there are imperfections in the financial capital markets.[6] These imperfections are manifested by the following equation,

(8.1) $r_b = D(p_b B, p_s S, M)$

(8.2) $r_s = E(p_b B, p_s S, M)$

(8.3) $r_m = M(p_b B, p_s S, M)$

where the rates of return depend on the values of debt, equity and money capital.

Equations of similiar but less general form can be found in the literature. Indeed, it is often expressed that the rates of return on debt and equity depend on the debt-equity ratio. If we assume that the equations in (9) are homogeneous of degree zero in debt, equity and money i.e. proportional changes in the composition of the firm's portfolio do not affect the rates of return, then we can write equations (9) in terms of the debt-equity ratio $\frac{p_b B}{p_s S}$ and the money-equity ratio $\frac{M}{p_s S}$.[7] While this may be an interesting property to test empirically, for the theoretical

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6 Later in the paper we deal with the case where the financial capital markets are perfectly competitive, i.e. the firm cannot affect spot and forward prices.

7 For example, similiar equations appear in the paper by Turnovsky [14]. He imposes the homogeneity condition and there is no money balances in the domain of the function.

formulation we see no reason to 'a priori' impose such a restriction. Finally, then we can interpret the equations in (8) as inverse conjectural debt, equity and net money equations for they exhibit the market imperfections, in our partial equilibrium framework.

The regulatory environment is typically characterized by the constraint,

$$\Pi + p_k rK \leqq p_k sK \tag{9}$$

where s is the allowed rate of return on capital after taking account of depreciation.[8]

Before discussing the objective of the firm, it bears mentioning that, in our context, we view the factor price of labour, the price of the physical capital stock, the price of debt, the price of equity and the depreciation rate as random variables. Thus, since the price of debt and equity capital are random, the rates of return on debt and equity are also stochastic variables. Due to the presence of uncertainty, the objective criteria of the firm must embrace the manner in which the owners-managers react to uncertain events. We assume that the firm maximizes the expected value of profit.[9] This implies that the firm is risk neutral, in that its goal is to maximize the expected value of profit, irrespective

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8 There are numerous excellent papers on regulation, for example Bailey [1], Baumol and Klevorick [2] and Johnson [7].

9 We can alternatively assume the firm maximizes the expected utility of profit. However, our ultimate purpose is to estimate the derived relationships and since virtually nothing is known concerning the utility function for a corporation we have elected to assume that $U(\Pi) = \Pi$ where U is the utility function.

of the variance of the distribution (or for that matter, other moments of the distribution). Therefore using equations (4), (5), and (7) we can write the expected value of profits as,

$$(10) \quad E(\Pi) = E[D(F((p_bB+p_sS-M)p_k^{-1},L))F((p_bB+p_sS-M)p_k^{-1},L) - w_\ell L - (r_b+\delta)p_bB - (r_s+\delta)p_sS - (r_m+\delta)M].$$

In our model we maximize equation (10), with respect to B, S, M and L, and subject to equations (8.1), (8.2), (8.3) and the regulatory constraint (9). We accordingly define the Lagrangian function to be,

$$(11) \quad \mathcal{L} = E\ [(1-\lambda)[D(F((p_bB+p_sS-M)p_k^{-1},L))F((p_bB+p_sS-M)p_k^{-1},L) - w_\ell L - \delta(p_bB+p_sS-M)] - \mathcal{D}(p_bB,p_sS,M)p_bB - \mathcal{E}(p_bB,p_sS,M)p_sS - \mathcal{M}(p_bB,p_sS,M)M + \lambda s(p_bB+p_sS-M)],$$

where λ is the Lagrangian multiplier associated with the regulatory constraint. The optimality conditions for this system are,

$$\frac{\partial\mathcal{L}}{\partial L} = F_\ell(D'F+D) - E(w_\ell) = 0$$

$$(12) \quad \frac{\partial\mathcal{L}}{\partial B} = (1-\lambda)[F_k(D'F+D) - E(\delta p_k)] + \lambda sE(p_k) - E[p_k(\frac{\partial\mathcal{D}}{\partial B}p_bB+\frac{\partial\mathcal{E}}{\partial B}p_sS+\frac{\partial\mathcal{M}}{\partial B}M)] - E(p_kr_b) = 0$$

$$\frac{\partial\mathcal{L}}{\partial S} = (1-\lambda)[F_k(D'F+D) - E(\delta p_k)] + \lambda sE(p_k) - E[p_k(\frac{\partial\mathcal{D}}{\partial S}p_bB+\frac{\partial\mathcal{E}}{\partial S}p_sS+\frac{\partial\mathcal{M}}{\partial S}M)] - E(p_kr_s) = 0$$

$$\frac{\partial\mathcal{L}}{\partial M} = (1-\lambda)[F_k(D'F+D) - E(\delta p_k)] + \lambda s\ E(p_k) - E[p_k\ (\frac{\partial\mathcal{D}}{\partial M}p_bB+\frac{\partial\mathcal{E}}{\partial M}p_sS+\frac{\partial\mathcal{M}}{\partial M}M)] - E(p_kr_m) = 0$$

$$\frac{\partial L}{\partial \lambda} = E\big[-D(F((p_bB+p_sS-M)p_k^{-1},L))F((p_bB+p_sS-M)p_k^{-1},L) - w_\ell L - \delta(p_bB+p_sS-M) + s(p_bB+p_sS-M)\big] = 0.$$

With these five equations we can determine the optimal quantities of labour, debt, equity, money and the Lagrangian multiplier. The optimal equations may be interpreted as; the marginal revenue product of labour equals the expected value of the wage rate, the marginal revenue product of capital under regulation plus the expected marginal cost of regulation equals the expected value of the marginal cost of debt capital, which equals the expected marginal cost of equity capital, which equals the expected marginal cost of money capital. In our model, therefore, there is the simultaneous determination of real and financial decisions, where both the optimal capital budget and debt-equity ratios (as well as other important aggregations) are determined from the four fundamental control variables (L, B, S, and M).

It is quite clear, that the equations describing the different rates of return play an important role in determining the equilibrium for the firm. This means that changes in these functional relationships (we should note that the relationships depicted in equation set 8 are very general) due to changes in information or market power will affect our results. For instance, if the rates of return depended solely on physical capital, which implies (from the balance sheet) that the rates depend additively on debt, equity and money capital, then the capital budget and labour are determined, while the debt-equity ratio is indeterminate. This result, while often recognized in perfect capital markets, has not been stressed as a possible solution in imperfect markets (as it is in this model). In addition, if labour is an argument of the rate of return equations, the general nature of the solution, as we presented in equations (12) are not altered, but given the introduction of the complex interaction between the financial capital and labour markets the number of empirical restrictions arising out of the model will decrease. Consequently, our model is consistent with a varied aray of equation forms of differing degrees of generality. Two special cases, with respect to the rates of return, have often appeared in the literature; one dealing with fixed rates of return, and the other incorporating a fixed debt-equity ratio.

3. Fixed Rates of Return

In this section we analyse the case where the firms operate in essentially perfectly competitive capital markets. The results of this change in market structure is to drop equation set (8).

If we performed the same exercises as in section 2 we would find that our optimality conditions require that the expected value of the rates of return on debt, equity, and money capital be equalized, which is not surprising given the perfect markets and risk neutrality of the firm. What is also obvious is that the new equations defined by $\frac{\partial L}{\partial B} = \frac{\partial L}{\partial S} = \frac{\partial L}{\partial M} = 0$ are now three linearly dependent equations. This obviously means that we should treat $p_b B + p_s S - M$ as a composite variable and so solve the model for the capital budget and labour. Therefore we find that the composition of the budget, as shown by the relevant ratios of debt to equity and money to equity are indeterminate. This does not mean that we have a corner solution, in the sense that the quantity of at least one of the components of financial capital are zero; the solution only states that the composition is indeterminate.

4. Fixed Debt-Equity Ratio

Some firms, rather than determining certain decisions through optimal behaviour models, apply "rules of thumb" with regard to various control variables. One example, of a rule of thumb approach, is to set a fixed debt-equity ratio and determine the remaining control variables conditional on this exogeneous ratio.

Suppose we define the fixed debt-equity ratio to be

(13) $$\frac{p_b B}{p_s S} = \Lambda$$

where Λ is the exogeneous leverage. Manifestly, the firm has the choice of determining debt or equity but not both. Therefore utilizing our Lagrangian given by equation (11) we can substitute $p_s S\Lambda$ for $p_b B$ and so the resulting optimality conditions are the same for labour, and money capital, but for equity we get,

$$(14) \quad \frac{\partial L}{\partial S} = (1-\lambda)\left[F_k(D'F+D) - E(\delta p_k)\right](1+\Lambda) + \lambda s E(p_k)(1+\Lambda)$$
$$- E\left[p_k(1+\Lambda)\left(\frac{\partial D}{\partial S} p_s S + \frac{\partial E}{\partial S} p_s S + \frac{\partial M}{\partial S} M\right)\right]$$
$$- E\left[p_k(r_b \Lambda + r_s)\right] = 0 .$$

In equation (14) we can observe that the marginal costs of financing capital through equity includes the marginal costs of adjusting debt so that the debt-equity remains constant.

5. A Final Word on Estimation and Simulation

The implementation of this model for the purposes of historical description and forecasting future trends necessitates the division of the system into demand, production, and financial sub-modules. This segmentation enables us to estimate the relevant price elasticities of demand, marginal products of capital and labour, and the marginal costs of the alternative financing instruments. Substituting,into the optimality conditions these estimated values,permits us to compute the value of λ (which according to the theory of regulation should lie between zero and one) and the physical capital-labour ratios, both within regulated and nonregulated environments.

Having completed the iterations for estimation we envision at least four important areas in which simulation exercises can be performed. The first one pertains to the regulatory aspect. What is the impact on output, debt, equity, money, and the factors of production, when the firm faces a market rate of return on its capital rather than a regulated rate. Secondly, what are the effects of an exogeneous change in the production capabilities of the firm, for example a change in product mix, or a change in factor intensity, such as in the case of a firm becoming more labour-intensive. Thirdly, what is the impact on the firm's behaviour if the company maintains a fixed debt-equity ratio, rather than one which is endogeneous to the decision-making. Finally, what is the effect of an institutionally fixed level of investment, in the context of a variable debt-equity ratio.

These and many more simulation experiments may be carried out using our model depicting corporate market behaviour.

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