Report No. 1 to
Department of Supply and Services, Ottawa
on Contract No. 01 GR 36001 - 1 - 0519
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PROPAGATION CHARACTERISTICS OF OPTICAL FIBRES

Covering Period May 1st, 1971 to March 31st, 1972.

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Department of Electrical Engineering

McGILL UNIVERSITY
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Abstract

A preliminary investigation of the propagation characteristics of the cladded fibre optical waveguide is carried out. The mode spectrum of the various circularly symmetric and hybrid surface wave modes, their group velocities and the radial dependence of their field components are presented for the case of "Leaky Modes". The work for the calculation of the attenuation coefficients of the various surface wave mode on a lossy cladded fibre is still going on. The corresponding work for the "propagating modes" which are guided by total internal reflection inside the core, is also to be pursued. It is believed that this study provides the necessary pre-requisite knowledge for the subsequent investigation of the effects of localized inhomogeneities inside the cladded fibre waveguide. A bibliography on fibre optical waveguides is provided at the end of this report.

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1. Introduction

Recent advances in low-loss optical fibers and in solid-state optical sources make optical-fibre transmission very attractive for such applications as on-premises interconnections, medium-capacity interoffice trunks, and large-capacity intercity routes etc. Many review articles have appeared, discussing various practical and theoretical aspects of optical-fibre waveguides (See Section 1. Bibliography).

Various waveguiding structures have been proposed for optical communications. A thin dielectric film guiding structure was proposed by Karbowiak.[1] With films of thickness a fraction of the operating wavelength and width some 10,000 wavelengths, the mode supported by the film is essentially a plane surface wave, symmetrical about the mid-plane of the film and decaying exponentially in amplitude away from the film. Some modified forms were examined by Kawakami and Nishizawa[2] and also Larsen.[3] These examples of thin film dielectric waveguides are aimed at achieving low loss waveguides by ensuring that adequate guidance is provided by the structure but that the major part of the energy propagates in the low lossy medium adjacent to the structure. Hence, relatively lossy dielectric material can be tolerated. Kao[4] suggested the cladded fibre as a suitable waveguiding structure. It consists of a cylindrical fibre core of circular cross-section cladded by a coaxial dielectric of lower dielectric constant. This structure is designed to produce a single mode, i.e. the dominant HE_{11} mode, by reducing the core diameter sufficiently to cut off all higher order modes. The thickness of the cladding is chosen to make sure that the field at the core will decay sufficiently inside the cladding so that the outer boundary can be handled without affecting wave propagation. This, together with the fact that glass is noninductive, will virtually eliminate cross-talk when a bundle of fibres is used. Since the energy is mainly confined within the dielectric, low loss material is therefore required to ensure low power loss. Still another structure, the self-focussing fibre, was proposed by Kawakami and Nishizawa.[5] This is a dielectric cylindrical waveguide of circular cross-section with a refractive index given by $\sqrt{\epsilon(r)} = \sqrt{\epsilon_o}\,(1 + k r^2)^{-1/2}$, k = a constant and r = radius, so that the paraxial solution gives equal velocity of propagation for all rays lying within the paraxial ray region. Here again, low loss material is required. Investigation of the optical fibre in single mode operation for component application was made by Schineller.[6] The structures investigated were mainly of planar form.

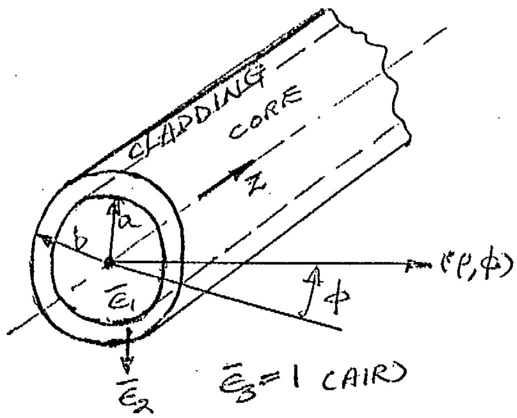
The present study is primarily concerned with the propagation characteristics of cladded optical fibre as proposed by Kao. In particular, we are interested in the attenuation of the optical waves caused by losses in the fibre. Losses in the fibre arise because of absorption and scattering. Absorption loss is caused essentially by traces of metallic ions in the fibre glass. They have their peak absorptions within the visible and near-infrared part of the spectrum. Scattering losses are mainly caused by Raleigh scattering and scattering due to imperfections in the bulk of the core and in the "waveguide" imperfections. The former is due to minute dielectric inhomogeneities frozen in the glass, while the latter may be introduced by such fabrication-induced scatterers as bubbles, crystallites, dust particles, cracks, core-cladding irregularities etc. Our objective is to, hopefully, formulate a theoretical model to account for losses due to discrete scattering centres within the cladded fibre, and to correlate the physical model to such measurable quantities as e.g. attenuation on a optical fibre. The information thus gained might possibly be used to suggest improvements in methods of fabricating these fibres and to understand the basic limitations of optical fibres as transmission lines.

The present work reports the preliminary study of propagation characteristics of cladded optical fibre which is free from any inhomogeneities. Although, some of the work has been done before, it was felt necessary to have certain repetition of work for the following reasons : (1) This would afford one of the authors (J. Marucci) an educational opportunity to acquaint himself with the methods of analysis used in fibre optical waveguide studies. (2) To assess the effects caused by the presence of discrete inhomogeneities, a clear understanding of the propagation characteristics of the cladded fibre without any inhomogeneities is clearly essential. Having the analysis in hand, it would be possible to examine those cases, for which no previous data are available. Further, it is suggested that the analysis used in evaluating attenuation coefficients for optical waves propagating in lossy fibres (assuming both the core and cladding to be lossy) can also be used for calculating gain coefficients if either the core or the cladding is made of an amplifying medium of liquid. We understand that this problem is currently under investigation by CRC scientists . .

The methods of analysis followed mainly the works of Astrahan, Kiely, Kharadly and Lewis, and Roberts.[8-11] However, a new method was used for group velocity calculations.

2. Derivation of the Characteristic Equation for Propagating Surface Wave Modes

The waveguide under consideration is the cladded fibre, as shown below.



$$\bar{\mu}_1 = \bar{\mu}_2 = \bar{\mu}_3 = 1$$

$$c \equiv \frac{a}{b}$$

$$\bar{\epsilon}_1 > \bar{\epsilon}_2$$

FIGURE 1. GEOMETRY OF THE PROBLEM.

It consists of a core of relative permittivity $\bar{\epsilon}_1$, which is embedded in a second dielectric ($\bar{\epsilon}_2$), called the cladding. The third region is air. In the following analysis, we will seek source-free wave solutions for the waveguide of infinite extent in the z-direction.

Solving Maxwell's source free wave equations, i.e. $\nabla^2 \left\{ \begin{matrix} \underline{E} \\ \underline{H} \end{matrix} \right\} = -\omega^2 \mu_o \epsilon \left\{ \begin{matrix} \underline{E} \\ \underline{H} \end{matrix} \right\}$ in each region we arrive at the following solutions for the fields :

Region I (core) : $\rho \leq a$

$$E_{z1} = a_{n1} J_n (k_1 \rho) F_n \quad , \quad F_n = e^{j(n\phi + \omega t - \gamma z)} \qquad (1)$$

$$H_{z1} = b_{n1} J_n (k_1 \rho) F_n \qquad (2)$$

$$E_{\phi 1} = \frac{1}{k_1^2} \left\{ \frac{\gamma n a_{n1}}{\rho} J_n(k_1 \rho) + j \omega \mu_1 k_1 b_{n1} J_n'(k_1 \rho) \right\} F_n \tag{3}$$

$$H_{\phi 1} = -\frac{1}{k_1^2} \left\{ j \omega \epsilon_1 k_1 a_{n1} J_n'(k_1 \rho) - \frac{\gamma n b_{n1}}{\rho} J_n(k_1 \rho) \right\} F_n \tag{4}$$

$$E_{\rho 1} = -\frac{1}{k_1^2} \left\{ j \gamma k_1 a_{n1} J_n'(k_1 \rho) - \frac{\omega \mu_1 n b_{n1}}{\rho} J_n(k_1 \rho) \right\} F_n \tag{5}$$

$$H_{\rho 1} = \frac{1}{k^2} \left\{ -\frac{\omega \epsilon_1 n a_{n1} J_n(k_1 \rho)}{\rho} - j \gamma k_1 b_{n1} J_n'(k_1 \rho) \right\} F_n \tag{6}$$

where $\epsilon = \bar{\epsilon}\, \epsilon_o$, $\gamma = k_o \bar{\gamma}$, and $k_1 = \sqrt{\omega^2 \mu_1 \epsilon_1 - \gamma^2} = (\frac{2\pi}{\lambda_o})^2 \sqrt{\bar{\mu}_1 \bar{\epsilon}_1 - \bar{\gamma}^2}$

Region II (cladding) : $a \le \rho \le b$

$$E_{z2} = \left\{ a_{n2} J_n(k_2 \rho) + a_{n3} Y_n(k_2 \rho) \right\} F_n \tag{7}$$

$$H_{z2} = \left\{ b_{n2} J_n(k_2 \rho) + b_{n3} Y_n(k_2 \rho) \right\} F_n \tag{8}$$

$$E_{\phi 2} = \frac{1}{k_2^2} \left\{ \left(\frac{\gamma n}{\rho}\right) \left[a_{n2} J_n(k_2 \rho) + a_{n3} Y_n(k_2 \rho) \right] + j \omega \mu_2 k_2 \left[b_{n2} J_n'(k_2 \rho) + b_{n3} Y_n'(k_2 \rho) \right] \right\} F_n \tag{9}$$

$$H_{\phi 2} = -\frac{1}{k_2^2} \left\{ j \omega \epsilon_2 k_2 \left[a_{n2} J_n'(k_2 \rho) + a_{n3} Y_n'(k_2 \rho) \right] - \frac{\gamma n}{\rho} \left[b_{n2} J_n(k_2 \rho) + b_{n3} Y_n(k_2 \rho) \right] \right\} F_n \tag{10}$$

$$E_{\rho 2} = -\frac{1}{k_2^2} \left\{ j \gamma k_2 \left[a_{n2} J_n'(k_2 \rho) + a_{n3} Y_n'(k_2 \rho) \right] - \frac{\omega \mu_2 n}{\rho} \left[b_{n2} J_n(k_2 \rho) + b_{n3} Y_n(k_2 \rho) \right] \right\} F_n \tag{11}$$

$$H_{\rho 2} = -\frac{1}{k_2^2}\left\{\frac{\omega \epsilon_2 n}{\rho}\left[a_{n2} J_n(k_2\rho) + a_{n3} Y_n(k_2\rho)\right] + i\gamma k_2\left[b_{n2} J_n'(k_2\rho) + b_{n3} Y_n'(k_2\rho)\right]\right\} F_n \qquad (12)$$

where $k_2 = \frac{2\pi}{\lambda_o}\sqrt{\bar{\mu}_2 \bar{\epsilon}_2 - \bar{\gamma}^2}$

Region III (outer medium): $\rho \geq b$

$$E_{z3} = a_{n4} H_n^{(1)}(k_3\rho)\, F_n = \frac{2}{\pi}(-i)^{n+1} a_{n4} K_n(W_3\rho)\, F_n \qquad (13)$$

$$H_{z3} = b_{n4} H_n^{(1)}(k_3\rho)\, F_n = \frac{2}{\pi}(-i)^{n+1} b_{n4} K_n(W_3\rho)\, F_n \qquad (14)$$

$$E_{\phi 3} = \frac{1}{k_3^2}\left\{\frac{\gamma n}{\rho} a_{n4} H_n(k_3\rho) + i\omega\mu_3 k_3 b_{n4} H_n'(k_3\rho)\right\} F_n \qquad (15)$$

$$H_{\phi 3} = -\frac{1}{k_3^2}\left\{i\omega\epsilon_3 k_3 a_{n4} H_n'(k_3\rho) - \frac{\gamma n}{\rho} b_{n4} H_n(k_3\rho)\right\} F_n \qquad (16)$$

$$E_{\rho 3} = -\frac{1}{k_3^2}\left\{i\gamma k_3 a_{n4} H_n'(k_3\rho) - \frac{\omega\mu_3 n}{\rho} b_{n4} H_n(k_3\rho)\right\} F_n \qquad (17)$$

$$H_{\rho 3} = \frac{1}{k_3^2}\left\{-\frac{\omega\epsilon_3 n}{\rho} a_{n4} H_n(k_3\rho) - i\gamma k_3 b_{n4} H_n'(k_3\rho)\right\} F_n \qquad (18)$$

where $k_3 = i W_3 = i\left(\frac{2\pi}{\lambda_o}\right)\sqrt{\bar{\gamma}^2 - 1}$ pure imaginary and ' denotes differentiation with respect to the argument.

It is to be noted that Equations (1) - (18) give the most general field solutions for the cladded dielectric structures. If, however, we are only interested in

$$M\underline{v} = \underline{0} \qquad c = a/b\,,\; x = k_1 a\,,\; v = k_2 b\,,\; cV = k_2 a\,,\; N = k_3 b = jW$$

Boundary Condition									$\underline{v}$		$\underline{0}$
$E_{z1}=E_{z2}$ at $\rho=a$	$J_n(x)$	$-J_n(cv)$	$-Y_n(cV)$	0	0	0	0	0	a_{n1}		0
$H_{z1}=H_{z2}$ at $\rho=a$	0	0	0	0	$J_n(x)$	$-J_n(cV)$	$-Y_n(cV)$	0	a_{n2}		0
$E_{\phi1}=E_{\phi2}$ at $\rho=a$	$\frac{\gamma n}{x^2}J_n(x)$	$\frac{-\gamma n}{(cV)^2}J_n(cV)$	$\frac{-\gamma n}{(cV)^2}Y_n(cV)$	0	$\frac{j\omega\mu_1}{x}J'_n(x)$	$\frac{-j\omega\mu_2}{(cV)}J'_n(cV)$	$\frac{-j\omega\mu_2}{(cV)}Y'_n(cV)$	0	a_{n3}		0
$H_{\phi1}=H_{\phi2}$ at $\rho=a$	$\frac{j\omega\epsilon_1}{x}J'_n(x)$	$\frac{-j\omega\epsilon_2}{(cV)}J'_n(cV)$	$\frac{-j\omega\epsilon_2}{(cV)}Y'_n(cV)$	0	$\frac{-\gamma n}{x^2}J_n(x)$	$\frac{\gamma n}{(cV)^2}J_n(cV)$	$\frac{\gamma n}{(cV)^2}Y_n(cV)$	0	a_{n4}	$=$	0
$E_{z2}=E_{z3}$ at $\rho=a$	0	$J_n(V)$	$Y_n(V)$	$-H_n(N)$	0	0	0	0	b_{n1}		0
$H_{z2}=H_{z3}$ at $\rho=b$	0	0	0	0	0	$J_n(V)$	$Y_n(V)$	$-H_n(N)$	b_{n2}		0
$E_{\phi2}=E_{\phi3}$ at $\rho=b$	0	$\frac{\gamma n}{V^2}J_n(V)$	$\frac{\gamma n}{V^2}Y_n(V)$	$\frac{-\gamma n}{N^2}H_n(N)$	0	$\frac{j\omega\mu_2}{V}J'_n(V)$	$\frac{j\omega\mu_2}{V}Y'_n(V)$	$\frac{-j\omega\mu_3}{N}H'_n(N)$	b_{n3}		0
$H_{\phi2}=H_{\phi3}$ at $\rho=b$	0	$\frac{j\omega\epsilon_2}{V}J'_n(V)$	$\frac{j\omega\epsilon_2}{V}Y'_n(V)$	$\frac{-j\omega\epsilon_3}{N}H'_n(N)$	0	$\frac{-\gamma n}{V^2}J_n(V)$	$\frac{-\gamma n}{N^2}Y_n(V)$	$\frac{\gamma n}{N^2}H_n(N)$	b_{n4}		0

M (8 × 8) $\qquad \underline{v}$ (8×1) $\qquad \underline{0}$

(19)

the low-order modes which are propagated in the core by total internal reflection, we should seek solutions in such a form that the radial wavenumber k_2 in the cladding is purely imaginary. Field solutions for this special case are given in Appendix I.

Equating the E_ϕ, E_z components at $\rho = a$ and $\rho = b$, we get eight equations in eight unknowns. This is the result of imposing the boundary condition of the continuity of tangential electric and magnetic field components across each interface. Since we are not considering sources of excitation, the set of eight equations in eight unknowns is homogeneous, and hence, for non-trivial solutions, the determinant of the system must be set equal to zero. This gives the characteristic equation for propagating modes i.e. det $M = 0$. However, we shall first reduce this (8 x 8) matrix to a (4 x 4) matrix, and then take the determinant. The procedure is as follows : From Equations (1) and (2), we solve for $\{a_{n1}, b_{n1}\}$.

$$a_{n1} = \{ J_n (cV)\, a_{n2} + Y_n (cV)\, a_{n3} \} / J_n (x) \tag{20}$$

$$b_{n1} = \{ J_n (cV)\, b_{n2} + Y_n (cV)\, b_{n3} \} / J_n (x) \tag{21}$$

From Equations (5) and (6) we solve for $\{a_{n4}, b_{n4}\}$,

$$a_{n4} = \{ J_n (V)\, a_{n2} + Y_n (V)\, a_{n3} \} / H_n (N) \tag{22}$$

$$b_{n4} = \{ J_n (V)\, b_{n2} + Y_n (V)\, b_{n3} \} / H_n (N) \tag{23}$$

Substituting a_{n1}, b_{n1}, a_{n4}, b_{n4} into the remaining four equations, we get after some algebra and elementary operations :

$$\underbrace{\begin{bmatrix} n\,\Delta_3 S & n\,S & -C^2 F\mu_o \Delta_3 B_{19} & -C^2 F\mu_o B_{79} \\ C^2 F\epsilon_o \Delta_3 A_{19} & C^2 F\epsilon_o A_{79} & -n\,\Delta_3 S & -n\,S \\ n\,T\,\Delta_4 & n\,T & -F\mu_o \Delta_4 B_{25} & -F\mu_o B_{85} \\ F\Delta_4 \epsilon_0 A_{25} & F\epsilon_o A_{85} & -n\,T\,\Delta_4 & -n\,T \end{bmatrix}}_{A} \begin{bmatrix} a_{n2} \\ a_{n3} \\ (\frac{b_{n2}}{i}) \\ (\frac{b_{n3}}{i}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (24)$$

where

$$\left.\begin{aligned} A_{19} &= (\bar\epsilon_2 \Delta_1 - \bar\epsilon_1 \Delta_9) & B_{19} &= (\bar\mu_2 \Delta_1 - \bar\mu_1 \Delta_9) \\ A_{85} &= (\bar\epsilon_2 \Delta_8 - \bar\epsilon_3 \Delta_5) & B_{85} &= (\bar\mu_2 \Delta_8 - \bar\mu_3 \Delta_5) \\ A_{25} &= (\bar\epsilon_2 \Delta_2 - \bar\epsilon_3 \Delta_5) & B_{25} &= (\bar\mu_2 \Delta_2 - \bar\mu_3 \Delta_5) \\ A_{79} &= (\bar\epsilon_2 \Delta_7 - \bar\epsilon_1 \Delta_9) & B_{79} &= (\bar\mu_2 \Delta_7 - \bar\mu_1 \Delta_9) \end{aligned}\right\} \qquad (25)$$

$$\left.\begin{aligned} S &= (1/V)^2 - (C/X)^2 \\ T &= (1/V)^2 + (1/W)^2 \\ Q &= \frac{\omega^2 \mu_o \epsilon_o}{\gamma^2} = (\frac{\lambda_g}{\lambda_o}) = \frac{1}{\bar\gamma^2} \\ W &= 2\pi(\frac{b}{\lambda_o})\sqrt{\bar\gamma^2 - 1}\ , \quad \bar\gamma = \frac{\gamma}{k_o} \\ F &= \frac{\omega}{\gamma} = \frac{\lambda_g}{\lambda_o\sqrt{\mu_o \epsilon_o}} \\ (Q &= F^2 \mu_o \epsilon_o) \end{aligned}\right\} \qquad (26)$$

$$c = (a/b)\,, \quad X = k_1 a \,, \quad V = k_2 b \,, \quad N = k_3 b = j W \tag{27}$$

$$\left.\begin{aligned}
\Delta_1 &= \frac{J'_n(cV)}{cV\,J_n(cV)} & \Delta_5 &= \frac{H'_n(N)}{N\,H_n(N)} \\
\Delta_2 &= \frac{J'_n(V)}{V\,J_n(V)} & \Delta_7 &= \frac{Y'_n(cV)}{(cV)\,Y_n(cV)} \\
\Delta_3 &= \frac{J_n(cV)}{Y_n(cV)} & \Delta_8 &= \frac{Y'_n(V)}{V\,Y_n(V)} \\
\Delta_4 &= \frac{J_n(V)}{Y_n(V)} & \Delta_9 &= \frac{J'_n(X)}{X\,J_n(X)}
\end{aligned}\right\} \tag{28}$$

Now, for non-zero solutions of a_{n2}, a_{n3}, $(b_{n2/i})$, $(b_{n3/i})$ in the 4 equations, we require det $A = 0$. This determinant takes the following form :

$$\det A = [\, Y_1 + Y_2 + Y_3 + Y_4 + Y_5 \,] = 0 \qquad \text{Dispersion Equation} \tag{29}$$

where

$$\left.\begin{aligned}
Y_1 &= -n^4 S^2 T^2 \,[\Delta_3 - \Delta_4]^2 \\
Y_2 &= -c^4 Q^2 \,[\Delta_3(A_{19})(A_{85}) - \Delta_4(A_{25})(A_{79})][\Delta_3(B_{19})(B_{85}) - \Delta_4(B_{25})(B_{79})] \\
Y_3 &= -2n^2 S T Q c^2 \Delta_3 \Delta_4 \,[\Delta_1 - \Delta_7][\Delta_2 - \Delta_8]\,\bar{\epsilon}_2 \bar{\mu}_2 \\
Y_4 &= n^2 S^2 Q \left[\bar{\mu}_2(C_{84}) - \bar{\mu}_3(C_{54})\right]\left[\bar{\epsilon}_2(C_{84}) - \bar{\epsilon}_3(C_{54})\right] \\
Y_5 &= n^2 T^2 Q c^4 \left[\bar{\mu}_2(D_{14}) - \bar{\mu}\,(D_{94})\right]\left[\bar{\epsilon}_2(D_{14}) - \bar{\epsilon}_1(D_{94})\right]
\end{aligned}\right\} \tag{30}$$

$$C_{84} = (\Delta_8 \Delta_3 - \Delta_2 \Delta_4)$$

$$C_{54} = \Delta_5 (\Delta_3 - \Delta_4)$$

$$D_{14} = (\Delta_1 \Delta_3 - \Delta_7 \Delta_4)$$

$$D_{94} = \Delta_9 (\Delta_3 - \Delta_4)$$

In what follows, we take $\bar{\mu}_1 = \bar{\mu}_2 = \bar{\mu}_3 = \bar{\epsilon}_3 = 1$. Thus the dispersion equation is a function of several variables, namely,

$$\det A = f_n (\bar{\gamma}, b/\lambda_o, n, c, \bar{\epsilon}_1, \bar{\epsilon}_2) = 0 \tag{31}$$

The cases of usual interest are when $n = 0$, $n = 1$.

Case 1 : $n = 1$ (hybrid modes)

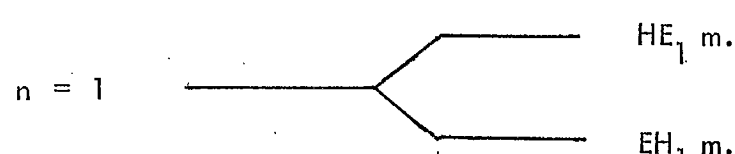
Then,

$$\det A\big|_{n=1} = f_1 (\bar{\gamma}, b/\lambda_o, 1, c, \bar{\epsilon}_1, \bar{\epsilon}_2) = 0$$

By fixing c, $\bar{\epsilon}_1$, $\bar{\epsilon}_2$, we finally get

$$\det A\big|_{n=1} = f_1 (\bar{\gamma}, b/\lambda_o) = 0 \quad (\omega - \beta \text{ diagram}) \tag{32}$$

Next, we use a root-searching technique for solving $f(\bar{\gamma}, b/\lambda_o) = 0$, and we find two types of hybrid modes (HE and EH) :



HE_{11} is called the fundamental mode, because it has no low-frequency cutoff. For our dispersion curves, $c = 0.5$, $\bar{\epsilon}_1 = 2.56$, $\bar{\epsilon}_2 = 2.53$. $(\bar{\epsilon}_1 > \bar{\epsilon}_2)$.

<u>Case 2</u>: $n = 0$ <u>(circularly symmetric modes)</u>

$$\det A\big|_{n=0} = f_o(\bar{\gamma}, b/\lambda_o, 0, c, \bar{\epsilon}_1, \bar{\epsilon}_2)$$
$$= Y_2$$
$$= -c^4 Q^2 \left[\Delta_3 A_{19} A_{85} - \Delta_4 A_{25} A_{79}\right]\left[\Delta_3 B_{19} B_{85} - \Delta_4 B_{25} B_{79}\right]$$
$$= 0 \qquad (33)$$

and we can show that for

<u>E_{om} (TM) mode :</u> $\left[\Delta_3 A_{19} A_{85} - \Delta_4 A_{25} A_{79}\right] = 0 = f_o(\bar{\gamma}, b/\lambda_o)\Big|_{n=0}^{TM}$ (34)

<u>H_{om} (TE) mode :</u> $\left[\Delta_3 B_{19} B_{85} - \Delta_4 B_{25} B_{79}\right] = 0 = f_o(\bar{\gamma}, b/\lambda_o)\Big|_{n=0}^{TE}$ (35)

Here again, we employ a root-searching technique for each equation (E_{om}, H_{om}), and take $c = 0.5$, $\bar{\epsilon}_1 = 2.56$, $\bar{\epsilon}_2 = 2.53$ as before.

$n = 0$ → E_{om} (TM) $[H_z = 0]$

$n = 0$ → H_{om} (TE) $[E_z = 0]$

3. Phase Velocity : (v_p)

Let

$$\gamma = \bar{\gamma}\, k_o \quad , \quad \omega = k_o v_o \quad , \quad v_o = \frac{1}{\sqrt{\mu_o \epsilon_o}} = \text{velocity of light in vacuum}$$

then

$$v_p = \frac{\omega}{\gamma} = \frac{k_o v_o}{\bar{\gamma}\, k_o} = \frac{v_o}{\bar{\gamma}}$$

therefore

$$\left(\frac{v_p}{v_o}\right) = \frac{1}{\bar{\gamma}} \quad , \quad \text{normalized phase velocity.} \tag{36}$$

Thus, instead of plotting directly $\bar{\gamma}$ vs. (b/λ_o) as found from the root-searching technique, we plot $(\frac{1}{\bar{\gamma}})$ vs $(\frac{b}{\lambda_o})$ from which we can read off the phase velocity $(\frac{1}{\bar{\gamma}} = v_p/v_o)$ immediately.

4. Group Velocity : (v_g)

The group velocity is obtained in a novel way as follows.

$$v_g = \frac{d\omega}{d\gamma} = \frac{d\,(k_o v_o)}{d\,(k_o \bar{\gamma})} \quad , \quad \text{using} \quad \begin{array}{l} \omega = k_o v_o \\ \gamma = k_o \bar{\gamma} \end{array}$$

now

$$\frac{d\gamma}{d\omega} = \frac{d\,(k_o \bar{\gamma})}{v_o\, d k_o} = \frac{1}{v_o}\left[\, \bar{\gamma} + \frac{k_o\, d\bar{\gamma}}{d k_o} \right]$$

therefore

$$\frac{v_o}{v_g} = \frac{v_o}{(\frac{d\omega}{d\gamma})} = v_o \frac{d\gamma}{d\omega} = \bar{\gamma} + k_o \frac{d\bar{\gamma}}{dk_o}$$

$$= \bar{\gamma} + \frac{1}{x} \qquad , \quad x \equiv \frac{dk_o}{k_o d\bar{\gamma}}$$

hence

$$(\frac{v_g}{v_o}) = \frac{1}{\bar{\gamma} + (\frac{1}{x})} \tag{37}$$

Now the dispersion equation is

$$f_n(\bar{\gamma}, b/\lambda_o) = 0$$

or

$$f_n^{(1)}(\bar{\gamma}, b\,\omega) = 0 \quad , \quad \text{using} \quad \frac{1}{\lambda_o} = \frac{\omega}{2\pi v_o}$$

or

$$f_n^{(2)}(\bar{\gamma}, b\,k_o) = 0 \quad , \quad \text{using} \quad \omega = k_o v_o$$

Therefore, differentiating implicitly $f_n^{(2)}(\bar{\gamma}, b\,k_o) = 0$ with respect to $\bar{\gamma}$, we get a new equation, namely,

$$g_n^{(2)}(\bar{\gamma}, b\,k_o, \frac{dk_o}{d\bar{\gamma}}) = 0$$

or

$$g_n(\bar{\gamma}, b/\lambda_o, x) = 0 \tag{38}$$

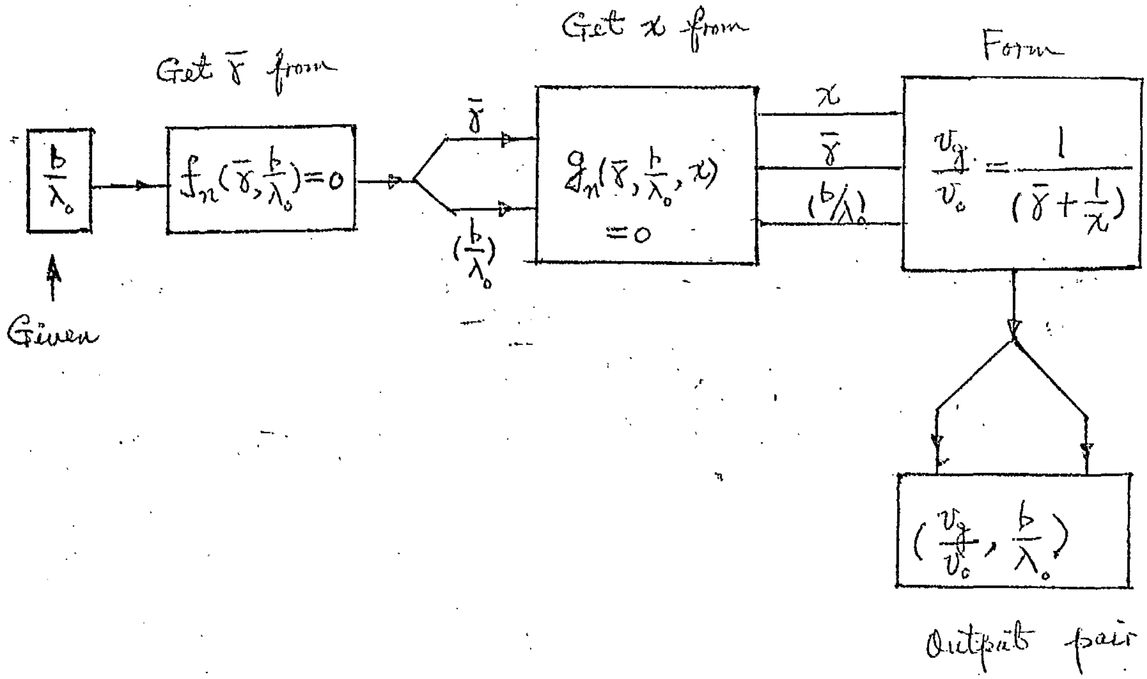
using $\quad b\,k_o = \dfrac{b\,2\pi}{\lambda_o} \quad$ and $\quad x = \dfrac{d\,k_o}{k_o\,d\,\gamma}$.

For a given pair of $(\bar{\gamma}, b/\lambda_o)$ corresponding to a point on the dispersion curve of a particular mode, we can solve $g_n(\bar{\gamma}, b/\lambda_o, x) = 0$ for x by using the root-searching technique. Having found the value of x corresponding to a given $(\bar{\gamma}, b/\lambda_o)$, we can find

$$\left(\frac{v_g}{v_o}\right) = \frac{1}{\left(\bar{\gamma} + \frac{1}{x}\right)} \Bigg|_{(\bar{\gamma},\, b/\lambda_o)}$$

Doing this for other values of $(\bar{\gamma}, b/\lambda_o)$ on the particular dispersion curve, we get pairs of $\left(\frac{v_o}{v_o}, \frac{b}{\lambda_o}\right)$ from which we plot the group velocity curves.

In <u>summary</u> :



5. Field Expressions

The field solutions as given before have 8 unknown coefficients, namely, $\{a_{n1}, a_{n2}, a_{n3}, a_{n4}, b_{n1}, b_{n2}, b_{n3}, b_{n4}\}$. However, only one of these coefficients is independent, and its magnitude is determined by the sources of excitation (which we are not considering). We will arbitrarily take a_{n2} to be the independent coefficient, and express the remaining ones in terms of a_{n2}. We do this as follows :

Case 1 : n = 1 (hybrid modes)

When n = 1, the 4 equations and 4 unknowns on page 9 become:

$$\left.\begin{aligned}
&(1)\quad \Delta_3 S a_{12} + S a_{13} - [c^2 F\mu_o \Delta_3 B_{19}](b_{12}/i) - [c^2 F\mu_o B_{79}](b_{13}/i) = 0 \\
&(2)\quad c^2 F\epsilon_o \Delta_3 A_{19} a_{12} + c^2 F\epsilon_o A_{79} a_{13} - \Delta_3 S(b_{12}/i) - S(b_{13}/i) = 0 \\
&(3)\quad T\Delta_4 a_{12} + T a_{13} - F\mu_o \Delta_4 B_{25}(b_{12}/i) - F\mu_o B_{85}(b_{13}/i) = 0 \\
&(4)\quad F\epsilon_o \Delta_4 A_{25} a_{12} + F\epsilon_o A_{85} a_{13} - T\Delta_4 (b_{12}/i) - T(b_{13}/i) = 0
\end{aligned}\right\} \quad (39)$$

Now, the first 3 equations can be rewritten in this form :

$$\underbrace{\begin{bmatrix} (-1/\Delta_3) & (c^2 F\mu_o B_{19}/S) & (c^2 (F\mu_o) B_{74}/\Delta_3 S) \\ (-A_{74}/\Delta_3 A_{19}) & (S/c^2 F\epsilon_o A_{19}) & (S/c^2 F\epsilon_o \Delta_3 A_{19}) \\ (-1/\Delta_4) & (F\mu_o B_{25}/T) & (F\mu_o B_{85}/T\Delta_4) \end{bmatrix}}_{A\ (3\times 3)} \begin{bmatrix} a_{13} \\ (\frac{b_{12}}{i}) \\ (\frac{b_{13}}{i}) \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{12} \\ a_{12} \end{bmatrix} \quad (40)$$

By using Cramer's rule, we can solve for $\{a_{13}, \frac{b_{12}}{i}, \frac{b_{13}}{i}\}$ in terms of a_{12}. The result is :

$$\left.\begin{aligned} a_{13} &= B_6\, a_{12} \\ (b_{12}/i) &= G_4\, a_{12} \qquad\qquad A \equiv [c_{ij}] \\ (b_{13}/i) &= G_5\, a_{12} \end{aligned}\right\} \tag{41}$$

where

$$B_6 = \frac{1}{\det A}\{(c_{22}c_{33}-c_{32}c_{23})-(c_{12}c_{33}-c_{32}c_{13})+(c_{12}c_{23}-c_{22}c_{13})\} \tag{42}$$

$$G_4 = \frac{1}{\det A}\{(c_{11}c_{33}-c_{31}c_{13})-(c_{21}c_{33}-c_{31}c_{23})-(c_{11}c_{23}-c_{21}c_{13})\} \tag{43}$$

$$G_5 = \frac{1}{\det A}\{(c_{21}c_{32}-c_{31}c_{22})-(c_{11}c_{32}-c_{31}c_{12})+(c_{11}c_{22}-c_{21}c_{12})\} \tag{44}$$

$$\det A = c_{11}(c_{22}c_{33}-c_{32}c_{23}) - c_{21}(c_{12}c_{33}-c_{32}c_{13}) + c_{31}(c_{12}c_{23}-c_{22}c_{13}) \tag{45}$$

and c_{ij} , $i = 1, 2, 3$, $j = 1, 2, 3$, are the matrix elements of A (e.g. $c_{31} = -1/\Delta_4$). Recalling from before that, for $n = 1$,

$$\begin{aligned} a_{11} &= \{J_1(cV)\, a_{12} + Y_1(cV)\, a_{13}\}/J_1(x), \quad \text{use } a_{13} = B_6\, a_{12} \\ &= \{J_1(cV) + B_6\, Y_1(cV)\}a_{12}/J_1(x) \equiv G_1\, a_{12} \end{aligned} \tag{46}$$

$$\begin{aligned} b_{11} &= \{J_1(cV)\, b_{12} + Y_1(cV)\, b_{13}\}/J_1(x) \\ &= i\,\{J_1(cV)\, G_4 + Y_1(cV)\, G_5\}a_{12}/J_1(x) \equiv i\, G_3\, a_{12} \end{aligned} \tag{47}$$

Similarly,

$$\bar{a}_{14} = \frac{2}{\pi}(-j)^2 a_{14} = \{J_1(V) + B_6 Y_1(V)\} a_{12}/K_1(W)$$

$$= G_2 a_{12} \tag{48}$$

$$\bar{b}_{14} = \frac{2}{\pi}(-j) b_{14} = -\{J_1(V) G_4 + Y_1(V) G_5\} a_{12}/K_1(W)$$

$$= -G_6 a_{12} \tag{49}$$

where we have used $H_n(N) = H_n(jW) = \frac{2}{\pi}(-j)^{n+1} K_n(W)$ and set $n = 1$.

Thus, the seven coefficients expressed in terms of a_{12} are :

$$\left.\begin{array}{llll} a_{11} = G_1 a_{12} & & b_{11} = j G_3 a_{12} \\ a_{13} = B_6 a_{12} & & b_{12} = j G_4 a_{12} \\ \bar{a}_{14} = G_2 a_{12} & & b_{13} = j G_5 a_{12} \\ & & \bar{b}_{14} = -G_6 a_{12} \end{array}\right\} \tag{50}$$

Case 2 : $n = 0$ (circularly symmetric modes)

(a) E_{om} modes (TM) :

Here, $b_{01} = b_{02} = b_{03} = b_{04} = 0$ ($H_{z1} = H_{z2} = H_{z3} = 0$)

On using this condition and setting $n = 0$, the 4 equations in 4 unknowns (p.9) reduce to the following :

$$\left.\begin{aligned} \Delta_3 A_{19} a_{02} + A_{79} a_{03} &= 0 \\ \Delta_4 A_{25} a_{02} + A_{85} a_{03} &= 0 \end{aligned}\right\} \quad (51)$$

From (51),

$$a_{03} = -\Delta_3 \left(\frac{A_{19}}{A_{79}}\right) a_{02} \quad (52)$$

Hence

$$a_{01} = \{J_o (cV) a_{02} + Y_o (cV) a_{03}\}/J_o (X)$$

$$= \left[\{J_o (cV) - \Delta_3 (A_{19}/A_{79}) Y_o (cV)\}/J_o (X) \right] a_{02} \quad (53)$$

$$\bar{a}_{04} = -\frac{2}{\pi} i a_{04} = \left[\frac{J_o (V) - \Delta_3 (A_{19}/A_{79}) Y_o (V)}{K_o (W)} \right] a_{02} \quad (54)$$

In summary :

$$\left.\begin{aligned} b_{o1} &= b_{02} = b_{03} = b_{04} = 0 \\ a_{01} &= \left\{ \frac{J_o (cV) - \Delta_3 (A_{19}/A_{79}) Y_o (cV)}{J_o (X)} \right\} a_{o2} = h_4 a_{02} \\ a_{03} &= -\Delta_3 \left(\frac{A_{19}}{A_{74}}\right) a_{02} = h_5 a_{02} \\ \bar{a}_{04} &= \left\{ \frac{J_o (V) - \Delta_3 (A_{19}/A_{79}) Y_o (V)}{K_o (W)} \right\} a_{02} = h_6 a_{02} \end{aligned}\right\} \quad (55)$$

(b) H_{om} modes (TE) :

Here, $a_{01} = a_{02} = a_{03} = a_{04} = 0.$

Again, putting these conditions in the 4 equations and 4 unknowns, we find :

$$\left.\begin{aligned} b_{o1} &= \left\{\frac{J_o(cV) - \Delta_3 (B_{19}/B_{79})\, Y_o(cV)}{J_o(X)}\right\} b_{02} = h_1 b_{02} \\ b_{03} &= -\Delta_3 \left(\frac{B_{19}}{B_{79}}\right) b_{02} = h_2 b_{02} \\ \bar{b}_{04} &= \left\{\frac{J_o(V) - \Delta_3 (B_{19}/B_{74})\, Y_o(V)}{K_o(W)}\right\} b_{02} = h_3 b_{02} \end{aligned}\right\} \qquad (56)$$

where b_{02} is chosen as the independent variable.

Note : On the angular ($\emptyset$) variation of fields :

Consider $E_{z1} = a_{n1} J_n (k_1 \rho)\, e^{jn\emptyset}$. However, a general solution is given by

$$E_{z1} = \left[a_{n1} J_n(k_1 \rho)\, e^{jn\emptyset} + A_{-n1} J_{-n}(k_1 \rho)\, e^{-jn\emptyset} \right]$$

$$= J_n (k_1 \rho) \left[a_{n1} e^{jn\emptyset} + (-1)^n a_{-n1} e^{-jn\emptyset} \right], \text{ on using } J_{-n}(y) = (-1)^n J_n(y)$$

Since we are considering wave propagation in the z direction, the factors describing the variation of the fields with θ must be real.

By taking $a_{n1} = (-1)^n a_{-n1}$, $E_{z1} = J_n (k_1 \rho)\, a_{n1}\, 2 \cos n \emptyset$. From $a_{n1} = (-1)^n a_{-n1}$, we can derive :

$$a_{-n2} = (-1)^n a_{n2} , \quad a_{-n3} = (-1)^n a_{n3} , \quad \bar{a}_{-n4} = \bar{a}_{n4}$$

$$b_{-n1} = -(-1)^n b_{n1} , \quad b_{-n2} = -(-1)^n b_{n2} , \quad b_{-n3} = -(-1)^n b_{n3} , \quad \bar{b}_{-n4} = -\bar{b}_{n4}$$

Again, a general solution for H_{z1} is

$$H_{z1} = \{ b_{n1} J_n (k_1 \rho) e^{jn\phi} + b_{-n1} J_{-n} (k_1 \rho) e^{-jn\phi} \}$$

$$= \{ b_{n1} J_n (k_1 \rho) e^{jn\phi} + (-1) (-1)^n b_{n1} (-1)^n J_n (k_1 \rho) e^{-jn\phi} \}$$

$$= b_{n1} J_n (k_1 \rho) \{ e^{jn\phi} - e^{-jn\phi} \}$$

$$= j 2 b_{n1} J_n (k_1 \rho) \sin n\phi$$

Similarly, by using a combination of $+n$ and $-n$ and the relations between coefficients of $+n$ and $-n$ as given above , we can establish if the angular variation of the remaining field components have a $\cos n \phi$ or $\sin n \phi$.

Final Field Expressions :

On setting

$$n = 1, \quad s = (\frac{\rho}{b}) , \quad Z_o = \sqrt{\frac{\mu_o}{\epsilon_o}} , \quad F = e^{j(\omega t - \gamma z)}, \quad \bar{b} = (b/\lambda_o)$$

and using :

$$a_{11} = G_1 a_{12} , \quad a_{13} = B_6 a_{12} , \quad \bar{a}_{14} = G_2 a_{12} , \quad b_{11} = j G_3 a_{12} ,$$

$$b_{12} = j G_3 a_{12} , \quad b_{12} = j G_4 a_{12} , \quad b_{13} = j G_5 a_{12} , \quad \bar{b}_{14} = -G_6 a_{12} ,$$

the original field solutions take the following form $(n = 1)$:

Electric

$$E_{z1} = \cos\phi\left[G_1 J_1(X s/c)\right] F_o a_{12} \quad , \quad s = \rho/b \qquad (\bar{b} = \frac{b}{\lambda_o})$$

$$E_{z2} = \cos\phi\left[J_1(V s) + B_6 Y_1(V s)\right] F_o a_{12}$$

$$E_{z3} = \cos\phi\left[G_2 K_1(W s)\right] F_o a_{12}$$

$$E_{\phi 1} = j \sin\phi\,(2\pi/X^2)\,\bar{b}\left[\bar{\gamma}(c^2/s)\,G_1 J_1(X s/c) - Z_o X \epsilon\, G_3 J_1'(X s/c)\right] F_o a_{12}$$

$$E_{\phi 2} = j \sin\phi\,(2\pi/V^2)\,\bar{b}\left\{\bar{\gamma}(1/s)\left[J_1(V s) + B_6 Y_1(V s)\right] - V Z_o\left[G_4 J_1'(V s) + G_5 Y_1'(V s)\right]\right\} F_o a_1$$

$$E_{\phi 3} = j \sin\phi\,(2\pi/W^2)\,\bar{b}\left[-\bar{\gamma}(1/s)\,G_2 K_1(W s) + W Z_o G_6 K_1'(W s)\right] F_o a_{12}$$

$$E_{\rho 1} = j \cos\phi\,(2\pi/X^2)\left[Z_o(c^2/s)\,G_3 J_1(X s/c) - \bar{\gamma} X c\, G_1 J_1'(x c/s)\right] \bar{b} F_o a_{12}$$

$$E_{\rho 2} = j \cos\phi\,(2\pi/V^2)\left\{Z_o(1/s)\left[G_4 J_1(V s) + G_5 Y_1(V s)\right] - \bar{\gamma} V\left[J_1'(V s) + B_6 Y_1'(V s)\right]\right\} \bar{b} F_o a_{12}$$

$$E_{\rho 3} = j \cos\phi\,(2\pi/W^2)\left[\bar{\gamma} W G_2 K_1'(W s) - Z_o(1/s)\,G_6 K_1(W s)\right] \bar{b} F_o a_{12}$$

(57

where a_{12} is the independent coefficient.

Magnetic (n = 1)

$$H_{z1} = -\sin\phi \left[G_3 J_1 (X s/c)\right] F_o a_{12}$$

$$H_{z2} = -\sin\phi \left[G_4 J_1 (V s) + G_4 Y_1 (V s)\right] F_o a_{12}$$

$$H_{z3} = -\sin\phi \left[G_6 K_1 (W s)\right] F_o a_{12}$$

$$H_{\phi 1} = j \cos\phi \, (2\pi/X^2)\, \bar{b} \left[\bar{\gamma}(c^2/s)\, G_3 J_1 (X s/c) - (\bar{\epsilon}_1/Z_o)\, X c \, G_1 J_1' (X s/c)\right] F_o a_{12}$$

$$H_{\phi 2} = j \cos\phi \, (2\pi/V^2)\, \bar{b} \left\{(\bar{\gamma}/s)\left[G_4 J_1 (V s) + G_5 Y_1 (V s)\right] - \bar{\epsilon}_2 (V/Z_o)\left[J_1' (V s) + B_6 Y_1' (V s)\right]\right\} F_o a_{12}$$

$$H_{\phi 3} = j \cos\phi \, (2\pi/W^2)\, \bar{b} \left[-(\bar{\gamma}/s)\, G_6 K_1 (W s) + (W/Z_o)\, G_2 K_1' (W s)\right]$$

$$H_{\rho 1} = j \sin\phi \, (2\pi/X^2)\, \bar{b} \left[-\bar{\epsilon}_1/Z_o\, (c^2/s)\, G_1 J_1 (X s/c) + \bar{\gamma}\, X c \, G_3 J_1' (X s/c)\right]$$

$$H_{\rho 2} = j \sin\phi \, (2\pi/V^2)\, \bar{b} \left\{-\bar{\epsilon}_2/Z_o (1/s)\left[J_1 (V s) + B_6 Y_1 (V s)\right] + \bar{\gamma}\, V \left[G_4 J_1' (V s) + G_5 Y_1' (V s)\right]\right\} F_o a_{12}$$

$$H_{\rho 3} = j \sin\phi \, (2\pi/W^2)\, \bar{b} \left[1/Z_o \; (1/s)\, G_2 K_1 (W s) - \bar{\gamma}\, W \, G_6 K_1 (W s)\right] F_o a_{12}$$

where

$$s = (\rho/b) \quad , \quad \bar{b} = (b/\lambda_o) \qquad (58)$$

Field Plots : (n = 1)

In plotting the field components as a function of $S = (\rho/b)$, we have normalized all the components by $(F_o\, a_{12})$, and also taken out the factor $\cos \phi$, $j \cos \phi$ and $j \sin \phi$ wherever it appeared. For example, in plotting E_ϕ what is actually plotted is $\dfrac{E_\phi}{(F_o\, a_{12})\, j \sin \phi}$. We have not included this normalization factor in the field curves.

6. Attenuation

For a lossy dielectric $\gamma = \beta - j\,\alpha$, where α is the attenuation coefficient. The axial power flow becomes

$$P(z) = P_o\, e^{-2\alpha z} \tag{59}$$

where P_o is the initial power flow at $z = 0$.

Differentiating both sides :

$$L \equiv -\frac{\partial P}{\partial z} = 2\,\alpha\, P(z)$$

Therefore

$$\alpha = \frac{L}{2\,P(z)} \;. \tag{60}$$

Now for small losses $P(z) \simeq P_o$, hence

$$\alpha \simeq \frac{L}{2\,P_o} \quad \text{approximately} \tag{61}$$

where L = power loss per unit length

P_o = power flow for a lossless waveguide

Next, we derive an expression for L. Set $L = \sum_{i=1}^{3} L_i$, where L_i is the loss per unit length in medium i ($i = 1, 2, 3$). It is given by

$$L_i = \frac{1}{2} \int_{S_i} \sigma^i_{eff} \, (\underline{E}_i \cdot \underline{E}_i^*) \, dS_i \tag{62}$$

where $\underline{E}_i$ = total electric field in medium i

S_i = cross-sectional area of i^{th} medium

and σ^i_{eff} = effective conductivity of the i^{th} dielectric medium

$= (\omega \tan \delta_i)\, \epsilon_i' \quad , \quad \epsilon_i' = \text{Re}\,(\epsilon_i) = \text{Re}\,(\epsilon_i' - j\,\epsilon_i'')$

To show that $\sigma^i_{eff} = (\omega \tan \delta_i)\, \epsilon_i'$:

$$\epsilon_i = \epsilon_i' - j\,\epsilon_i'' = \epsilon_i' (1 - j \tan \delta_i)$$

where $\tan \delta_i = \dfrac{\epsilon_i''}{\epsilon_i'}$

For time-harmonic fields we have

$$\nabla \times \underline{H}_i = (\sigma + j\,\omega\,\epsilon_i)\,\underline{E}_i \quad , \quad \sigma \text{ conductivity of the dielectric medium}$$

$$\simeq j\,\omega\,\epsilon_i\,\underline{E}_i \quad , \quad \sigma \simeq 0 \text{ for dielectric}$$

$$= j\,\omega\,\epsilon_i' (1 - j \tan \delta_i)\,\underline{E}_i$$

$$= \left[(\omega \tan \delta_i)\, \epsilon_i' + j\,\omega\,\epsilon_i' \right] \underline{E}_i$$

Therefore $\nabla \times \underline{H}_i = \left[\sigma^i_{eff} + j\omega\epsilon'_i \right] \underline{E}_i$

where $\sigma^i_{eff} = (\omega \tan \delta_i)\, \epsilon'_i$ (63)

Hence,

$$L_i = \frac{1}{2} \int_{S_i} \sigma^i_{eff} (\underline{E}_i \cdot \underline{E}_i^*)\, d\, S_i \quad , \quad |E_i| = \text{peak electric field}$$

$$= (\omega \tan \delta_i) \frac{\epsilon'_i}{2} \int_{S_i} (\underline{E}_i \cdot \underline{E}_i^*)\, d\, S_i$$

$$= (\omega \tan \delta_i)\, W_i \qquad (64)$$

where

$$W_i = \frac{\epsilon'_i}{2} \int_{S_i} (\underline{E}_i \cdot \underline{E}_i^*)\, d\, S_i = 2 \left[\frac{\epsilon'_i}{4} \int_{S_i} (\underline{E}_i \cdot E_i^*)\, d\, S_i \right]$$

$$= 2\, W_E \qquad (65)$$

W_E = time-averaged stored electric energy per unit length

therefore

$$L = \sum_{i=1}^{3} L_i = \omega \sum_{i=1}^{3} (\tan \delta_i)\, W_i \qquad (66)$$

then

$$\alpha \simeq \frac{L}{2 P_o} = \frac{\omega \sum_{i=1}^{3} (\tan \delta_i)\, W_i}{2 P_o} \qquad (67)$$

For convenience, we normalize α with α_2 which is the attenuation coefficient of a TEM wave propagating in an infinite medium with dielectric constant $\bar{\epsilon}_2 = \bar{\epsilon}_2' - j\,\bar{\epsilon}_2''$. For small losses, we can show that

$$\alpha_2 \simeq \frac{1}{2}\,\frac{(\omega)}{v_o}\,\sqrt{\bar{\epsilon}_2}\,\tan\delta_2 \tag{68}$$

$$\left(\frac{\alpha}{\alpha_2}\right) = \frac{v_o \sum_{i=1}^{3} (\tan\delta_i)\; W_i / P_o}{\sqrt{\bar{\epsilon}_2'}\,\tan\delta_2} \quad \text{normalized attenuation coefficient} \tag{69}$$

Since the expression for the attenuation coefficient α involves the energy and power expressions, the latter will now be derived.

(1) Energy (per unit length)

$$W_T \equiv \sum_{i=1}^{3} W_i = W_1 + W_2 + W_3$$

where

$$W_1 = \frac{1}{2}\epsilon_1 \int_0^{2\pi}\int_0^a \underline{E}_1 \cdot E_1^* \, \rho \, d\rho \, d\phi$$

$$= \frac{1}{2}\epsilon_1 \int_0^{2\pi}\int_0^a \left[E_{z1}\cdot E_{z1}^* + E_{\phi 1} E_{\phi 1}^* + E_{\rho 1}\cdot E_{\rho 1}^*\right] \rho \, d\rho \, d\phi$$

Similarly

$$W_2 = \frac{1}{2}\epsilon_2 \int_0^{2\pi}\int_a^b \left[E_{z2}\cdot E_{z2}^* + E_{\phi 2}\cdot E_{\phi 2}^* + E_{\rho 2}\cdot E_{\rho 2}^*\right] \rho \, d\rho \, d\phi$$

$$W_3 = \frac{1}{2}\epsilon_3 \int_0^{2\pi}\int_b^{\infty} \left[E_{z3}\cdot E_{z3}^* + E_{\phi 3}\cdot E_{\phi 3}^* + E_{\rho 3}\cdot E_{\rho 3}^*\right] \rho \, d\rho \, d\phi \qquad (70)$$

Closed form expressions of W_1, W_2, W_3 are given later.

(2) Axial Power Flow

$$P_o \equiv \sum_{i=1}^{3} P_i = P_1 + P_2 + P_3$$

where

$$P_1 = \frac{1}{2}\int_0^{2\pi}\int_0^a (E_{\rho 1} H_{\phi 1}^* - E_{\phi 1} H_{\rho 1}^*) \, \rho \, d\rho \, d\phi$$

$$P_2 = \frac{1}{2}\int_0^{2\pi}\int_a^b (E_{\rho 2} H_{\phi 2}^* - E_{\phi 2} H_{\rho 2}^*) \, \rho \, d\rho \, d\phi \qquad (71)$$

$$P_3 = \frac{1}{2}\int_0^{2\pi}\int_b^{\infty} (E_{\rho 3} H_{\phi 3}^* - E_{\phi 3} H_{\rho 3}^*) \, \rho \, d\rho \, d\phi$$

Closed form expressions for P_1, P_2, P_3 are given later.

From (70), detailed expressions for energy per unit length for $n = 1$ can be shown to be given by,

$$W_1 = \left(\frac{\pi b^2}{v_o}\right) |a_{12}|^2 \frac{c^2 \bar{\epsilon}_1}{2 Z_o} \left\{ G_1^2 \; T1(1) + \frac{2\pi^2}{X^2} c^2 \bar{b}^2 \left[\bar{\gamma}^2 G_1^2 + Z_o^2 G_3^2\right]\left[T1(2) + T1(0)\right] \right.$$

$$\left. - \left(\frac{2\pi}{X^2}\right)^2 4 c^2 Z_o \bar{\gamma} \bar{b}^2 G_1 G_3 \; T4(1) \right\}$$

$$W_2 = \left(\frac{\pi b^2}{v_o}\right) |a_{12}| \frac{\bar{\epsilon}_2}{2 Z_o} \left\{ TJ(1) + B_6^2 \; TY(1) + 2 B_6 \; YJ(1) \right.$$

$$+ \frac{2\pi^2}{V^2} \bar{b}^2 \left[TJ(2) + TJ(0)\right]\left[\bar{\gamma}^2 + Z_o^2 G_4^2\right] + \frac{2\pi^2}{V^2} \bar{b}^2 \left[TY(2) + TY(0)\right]\left[\bar{\gamma}^2 B_6^2 + Z_o^2 G_5^2\right]$$

$$+ \frac{4\pi^2}{V^2} \left[YJ(2) + YJ(0)\right]\left[\bar{\gamma}^2 B_6 + Z_o^2 G_4 G_5\right] \bar{b}^2 - 2\left(\frac{2\pi}{V^2}\right)^2 \bar{\gamma} Z_o \bar{b}^2 \left\{ 2 G_4 [S1] + 2 B_6 G_5 [S2] \right.$$

$$\left. \left. + (B_6 G_4 + G_5) \; [S34] \right\} \right\}$$

$$W_3 = \left(\frac{\pi b^2}{v_o}\right) |a_{12}|^2 \frac{1}{2 Z_o} \left\{ G_2^2 \; K1(1) - \left(\frac{2\pi}{W^2}\right)^2 4 \bar{\gamma} \bar{b}^2 Z_o G_2 G_6 \; K4(1) \right.$$

$$\left. + \frac{2\pi^2}{W^2} \bar{b}^2 \left[\bar{\gamma}^2 G_2^2 + Z_o^2 G_6^2\right]\left[K1(2) + K1(0)\right] \right\} \qquad (72)$$

where

T1(1), T1(2), T1(0), T4(1), TJ(1), TY(1), YJ(1), TJ(2), TJ(0), TY(2), TY(0), YJ(2), YJ(0), S1, S2, S34, K1(1), K1(2), K1(0), K4(1)

are algebraic expressions involving Bessel, Neumann and modified Hankel functions, but not given explicitly here for the sake of brevity.

Similarly, the detailed expressions for the axial power flow can also be obtained as follows (n = 1):

$$P_1 = \pi b^2 |a_{12}|^2 \frac{c^2}{2} (\frac{2\pi}{X^2})^2 \bar{b}^2 \Big\{ \frac{X^2 c^2}{2} \bar{\gamma} \Big[Z_o G_3^2 + (\bar{\epsilon}_1/Z_o) G_1^2 \Big] \Big[T1(2) + T1(0) \Big]$$

$$- 2c^2 G_1 G_3 \Big[\bar{\gamma}^2 + \bar{\epsilon}_1 \Big] \; T4(1) \Big\}$$

$$P_2 = \pi b^2 |a_{12}| (\frac{2\pi}{V^2})^2 \frac{\bar{b}^2}{2} \Big\{ \bar{\gamma} \frac{V^2}{2} \Big[Z_o G_4^2 + (\bar{\epsilon}_2/Z_o) \Big] \Big[TJ(2) + TJ(0) \Big]$$

$$+ \frac{\bar{\gamma} V^2}{2} \Big[Z_o G_5^2 + (\bar{\epsilon}_2/Z_o) B_6^2 \Big] \Big[TY(2) + TY(0) \Big] + \bar{\gamma} V^2 \Big[Z_o G_4 G_5 + (\bar{\epsilon}_2/Z_o) B_6 \Big] \Big[YJ(2) + YJ(0) \Big]$$

$$- \Big[\bar{\epsilon}_2 + \bar{\gamma}^2 \Big] \Big\{ 2 G_4 \, [S1] + 2 B_6 G_5 \, [S2] + (G_5 + B_6 G_4) \, [S34] \Big\} \Big\}$$

$$P_3 = \pi b^2 |a_{12}|^2 (\frac{2\pi}{W^2})^2 \bar{b}^2 \Big\{ \frac{W^2}{2} \bar{\gamma} \Big[Z_o G_6^2 + (1/Z_o) G_2^2 \Big] \Big[K1(2) + K1(0) \Big]$$

$$- 2 G_2 G_6 \Big[1 + \bar{\gamma}^2 \Big] K4(1) \Big\} \qquad (73)$$

By glancing at the energy and power expressions, we see that they contain $\bar{\gamma}$ and $\bar{b} = (b/\lambda_o)$. Now a particular mode is characterized by a set of values for $(\bar{\gamma}, \bar{b})$, which can be found from the dispersion equation as discussed before. Thus, to find the attenuation for HE_{11} mode, we must put in the corresponding values $(\bar{\gamma}, \bar{b})$ in the energy and power expressions, since

$$(\frac{\alpha}{\alpha_2})\Big|_{(\bar{\gamma},\bar{b})} = \frac{v_o}{\sqrt{\epsilon_2'} \tan\delta_2} \left[\frac{(\tan\delta_1) W_1(\bar{\gamma},\bar{b}) + (\tan\delta_2) W_2(\bar{\gamma},\bar{b}) + (\tan\delta_3) W_3(\bar{\gamma},\bar{b})}{P_o(\bar{\gamma},\bar{b})} \right] \qquad (69)$$

Similarly for the other modes (EH_{11}, H_{12}, ...), we supply the values of $(\bar{\gamma}, \bar{b})$ belonging to the mode in question.

7. Discussion of Numerical Results

Using the dispersion equations derived previously, the mode spectrum of the guided waves on the cladded fibre waveguide was plotted in Figure 2. The parameters used are as indicated i.e. $c = a/b = .5$, $\bar{\epsilon}_1 = 2.56$ and $\bar{\epsilon}_2 = 0.99\,\bar{\epsilon}_1$. These parameters have been used throughout all the calculations. The mode designation follows the usual one used for dielectric surface waveguides (for example, see Ref. 10). It is seen that the dominant hybrid mode HE_{11} has no low frequency cutoff. This mode is followed by the next higher order circularly-symmetric modes H_{o1} and E_{o1}. These are in return followed by higher order hybrid modes EH_{11}, HE_{12} etc. All the higher order modes have finite low frequency cutoff. For large values of (b/λ_o), the dispersion curves for all the modes all approach the value $1/\bar{\gamma} = 1/\sqrt{\bar{\epsilon}_1} = 0.629$. Note that those portions of the dispersion curves shown in Figure 2 represent the modes for which the angles of incidence θ of the guided waves at the core-cladding interface are less than the critical angle, θ_c $(\theta < \theta_c)$. Consequently, a portion of the waves leak into the cladding region and travel inside the cladding. They might appropriately be called "Leaky Modes".[6] The portions of the dispersion curves which produce waves guided by total internal reflection (i.e. $\theta > \theta_c$) in the core and hence evanescent fields in the cladding have yet to be mapped out. Since evanescent fields are produced in the cladding, the radial wave number there, $k_o\sqrt{\bar{\epsilon}_2 - \gamma^2}$, must be imaginary. This would necessitate changes in the field expressions and also dispersion equation as indicated in the Appendix.

The group velocities of the various modes have been calculated, using a novel method, and are shown in Figure 3. The group velocity of all the modes is less than v_o , the velocity of light in vacuum, but can be made arbitrarily close to $v_o / \sqrt{\bar{\epsilon}_1}$ for large diameter fibres. The lowest order mode, the HE_{11} mode, is an exception in that its group velocity can be very closed to v_o , if the fibre is made sufficiently thin. In general, for a given diameter of the fibre, the group velocities of the various modes are not equal. This gives rise to delay distortion for propagated signals. At the two extremes when the group velocity is v_o or $v_o/\sqrt{\bar{\epsilon}_1}$ the dispersion is smallest and signal distortion at a minimum. It is under either of these two conditions that the ibre can be considered to be suitable for transmission over long distances. The former corresponds to single mode operation, the latter multi-mode propagation.

The radial dependence of the various field components is shown in Figure 4. For the HE_{11} mode, two points on the dispersion curve were chosen, one near cutoff where $\bar{\gamma} = 1.010962$ and the other far away from cutoff where $\bar{\gamma} = 1.582547$. For the HE_{12} mode, a point close to cutoff was chosen, where $\bar{\gamma} = 1.001572$, while, for the EH_{11} mode, a point far away from cutoff was chosen, where $\bar{\gamma} = 1.562437$. The field components computed at these operating points are for illustrative purposes. It is seen that for the EH_{11} and HE_{12} modes, a radial node exists in the Ez and Hz component in the cladding region. It can be expected that, as the mode order increases, the fields inside the core and cladding would become more oscillatory so that the number of radial nodes in Ez and Hz will increase. Whereas the fields in the free-space region decay exponentially in the outward radial direction, the fields inside the cladding do not decay as rapidly as might be expected. The reason is that the fields plotted are for the "Leaky Modes" only. Further computations will be performed for the "Propagating Modes" in order to show the "evanescent fields" in the cladding, which can be expected to decay rapidly.

No conclusive remarks can yet be made about the attenuation coefficients. The plots in Figure 5 are only preliminary in the sense that the program for the normalized attenuation coefficient (α / α_2) is still being debugged (α_2 is the attenuation coefficient for an infinite plane wave in an infinite dielectric medium with the same dielectric constant as the cladding).

8. Conclusions

A preliminary study of the propagation characteristics of the surface wave modes on a cladded fibre optical waveguide has been carried out. It is felt this is a necessary prerequisite for studying the effects of inhomogeneities or dust particles imbedded in the cladded fibre. All the computations were performed for the "Leaky Modes". Further computations will be performed for the "Propagating Modes" with evanescent fields in the cladding. This involves the case, in which the normalized axial propagation coefficient $\bar{\gamma}$ lies in the range $\sqrt{\varepsilon}_1 < \bar{\gamma} < \sqrt{\varepsilon}_2$. A novel way of calculating the group velocity was used. The calculations of the attenuation coefficients are still being carried out. Although some preliminary results were shown. Their accuracies need to be established. However, it is felt that the method of analysis used for calculating the attenuation coefficients may profitably be used to calculate the gain coefficients if either the core is surrounded by an amplifying medium in the cladding on the core itself is made of an amplifying medium.

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APPENDIX I

FIELD SOLUTIONS FOR THE SURFACE WAVE MODES GUIDED IN THE CORE BY TOTAL INTERNAL REFLECTION

Let m represent the m^{th} region. Then, for the m^{th} region,

$$E_{\phi m} = \frac{1}{k_m^2}\left[-i\,\frac{\gamma_m}{\rho}\,\frac{\partial E_{zm}}{\partial \phi} + i\,\omega\,\mu_m\,\frac{\partial H_{zm}}{\partial \rho}\right] F_n \qquad (1)$$

$$H_{\phi m} = -\frac{1}{k_m^2}\left[i\,\omega\,\epsilon\,\frac{\partial E_{zm}}{\partial \rho} + i\,\frac{\gamma_m}{\rho}\,\frac{\partial H_{zm}}{\partial \phi}\right] F_n \qquad (2)$$

$$E_{\rho m} = -\frac{1}{k_m^2}\left[i\,\gamma_m\,\frac{\partial E_{zm}}{\partial \rho} + i\,\frac{\omega\,\mu_m}{\rho}\,\frac{\partial H_{zm}}{\partial \phi}\right] F_n \qquad (3)$$

$$H_{\rho m} = \frac{1}{k_m^2}\left[i\,\frac{\omega\,\epsilon}{\rho}\,\frac{\partial E_{zm}}{\partial \phi} - i\,\gamma_m\,\frac{\partial H_{zm}}{\partial \rho}\right] F_n \qquad (4)$$

where E_{zm}, H_{zm} are given by

Region I:

$$E_{z1} = a'_{n1}\,J_n\,(k_1\,\rho)\;F_n \qquad (5)$$

$$H_{z1} = b'_{n1}\,J_n\,(k_1\,\rho)\;F_n \qquad (6)$$

Region II:

$$E_{z2} = \left[a'_{n2}\,I_n\,(|k_2|\,\rho) + a'_{n3}\,K_n\,(|k_2|\,\rho)\right] F_n \qquad (7)$$

$$H_{z2} = \left[b'_{n2}\,I_n\,(|k_2|\,\rho) + b'_{n3}\,K_n\,(|k_2|\,\rho)\right] F_n \qquad (8)$$

Region III:

$$E_{z3} = a'_{n4}\,K_n\,(|k_3|\,\rho)\;F_n \qquad (9)$$

$$H_{z3} = b'_{b4}\,K_n\,(|k_3|\,\rho)\;F_n \qquad (10)$$

FIGURE CAPTIONS

Figure 2. Mode Spectrum (Phase Velocity) for Cladded Fibre.

$c = 0.5, \quad \bar{\epsilon}_1 = 2.56, \quad \bar{\epsilon}_2 = .99\,\bar{\epsilon}_1 \simeq 2.53$

Figure 3. Group Velocity for Cladded Fibre.

$c = 0.5, \quad \bar{\epsilon}_1 = 2.56, \quad \bar{\epsilon}_2 = .99\,\bar{\epsilon}_1 \simeq 2.53$

Figure 4(a). NEAR CUT-OFF radial dependence of Hz, HØ, Hr for HE_{11} mode.

$(b/\lambda_o) = 0.15, \quad \bar{\gamma} = 1.010962$

Figure 4(b). NEAR CUT-OFF radial dependence of Ez, EØ, Er for HE_{11} mode.

$(b/\lambda_o) = 0.15, \quad \bar{\gamma} = 1.010962.$

Figure 4(c). FAR FROM CUT-OFF radial dependence of Hz, HØ, Hr for HE_{11} mode.

$(b/\lambda_o) = 1.70, \quad \bar{\gamma} = 1.582547$

Figure 4(d). FAR FROM CUT-OFF radial dependence of Ez, EØ, Er for HE_{11} mode.

$(b/\lambda_o) = 1.70, \quad \bar{\gamma} = 1.582547$

Figure 4(e). NEAR CUT-OFF radial dependence of Hz, HØ, Hr for HE_{12} mode.

$(b/\lambda_o) = 0.52, \quad \bar{\gamma} = 1.001572$

Figure 4(f). NEAR CUT-OFF radial dependence of Ez, EØ, Er for HE_{12} mode.

$(b/\lambda_o) = 0.52, \quad \bar{\gamma} = 1.001572$

Figure 4(g). FAR FROM CUT-OFF radial dependence of Hz, HØ, Hr for EH_{11} mode.

$(b/\lambda_o) = 2.50, \quad \bar{\gamma} = 1.562437$

Figure 4(h). FAR FROM CUT-OFF radial dependence of Ez, EØ, Er for EH_{11} mode.
$(b/\lambda_o) = 2.50$, $\overline{\gamma} = 1.562437$

Figure 5. Normalized Attenuation Coefficient (α/α_2) .
$\tan\delta_1 = 0.0005$
$\tan\delta_2 = 0.0005$
$\tan\delta_3 = 0.0$

MODE SPECTRUM FOR CLADDED FIBRE

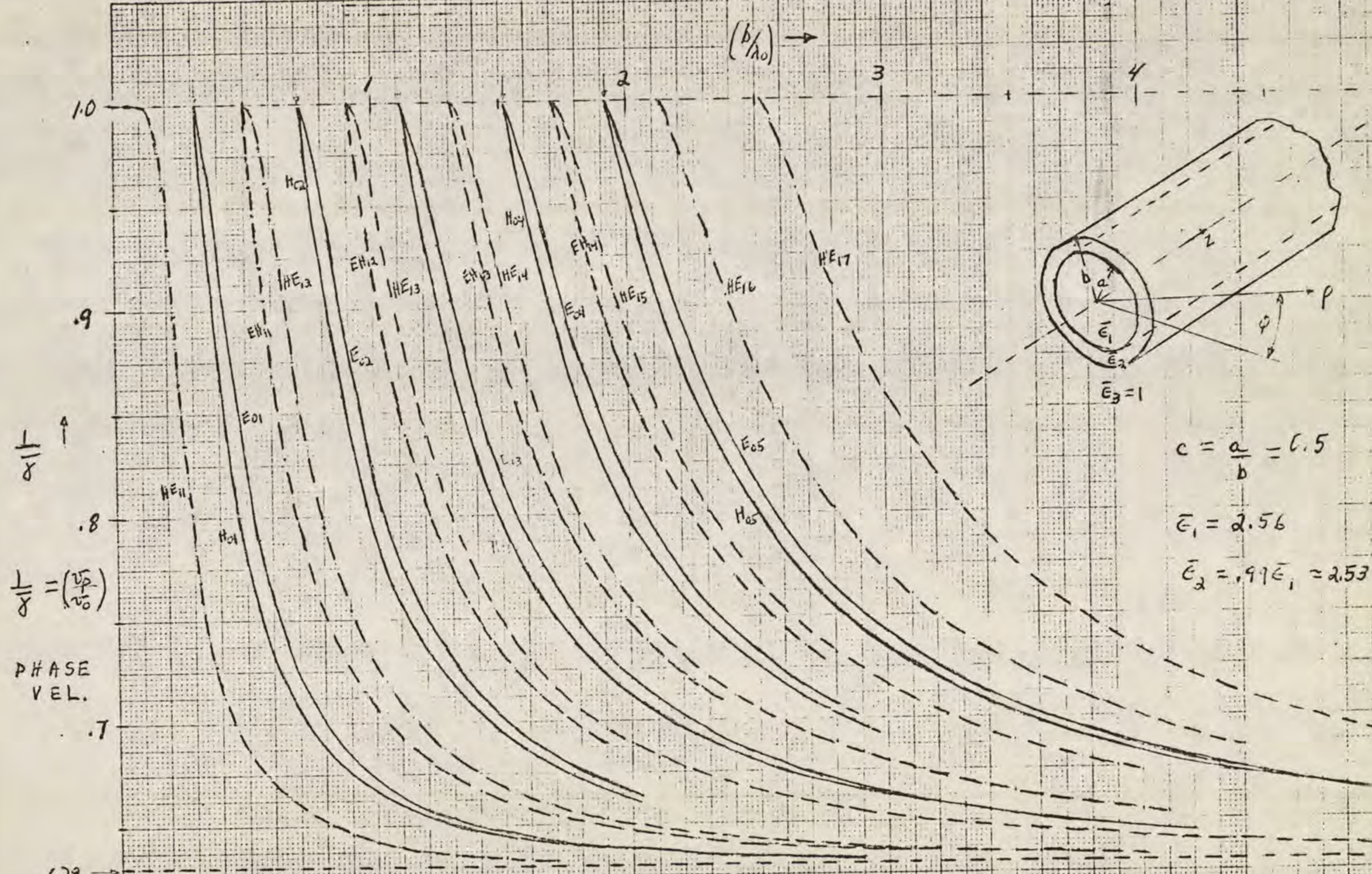


FIG. 2

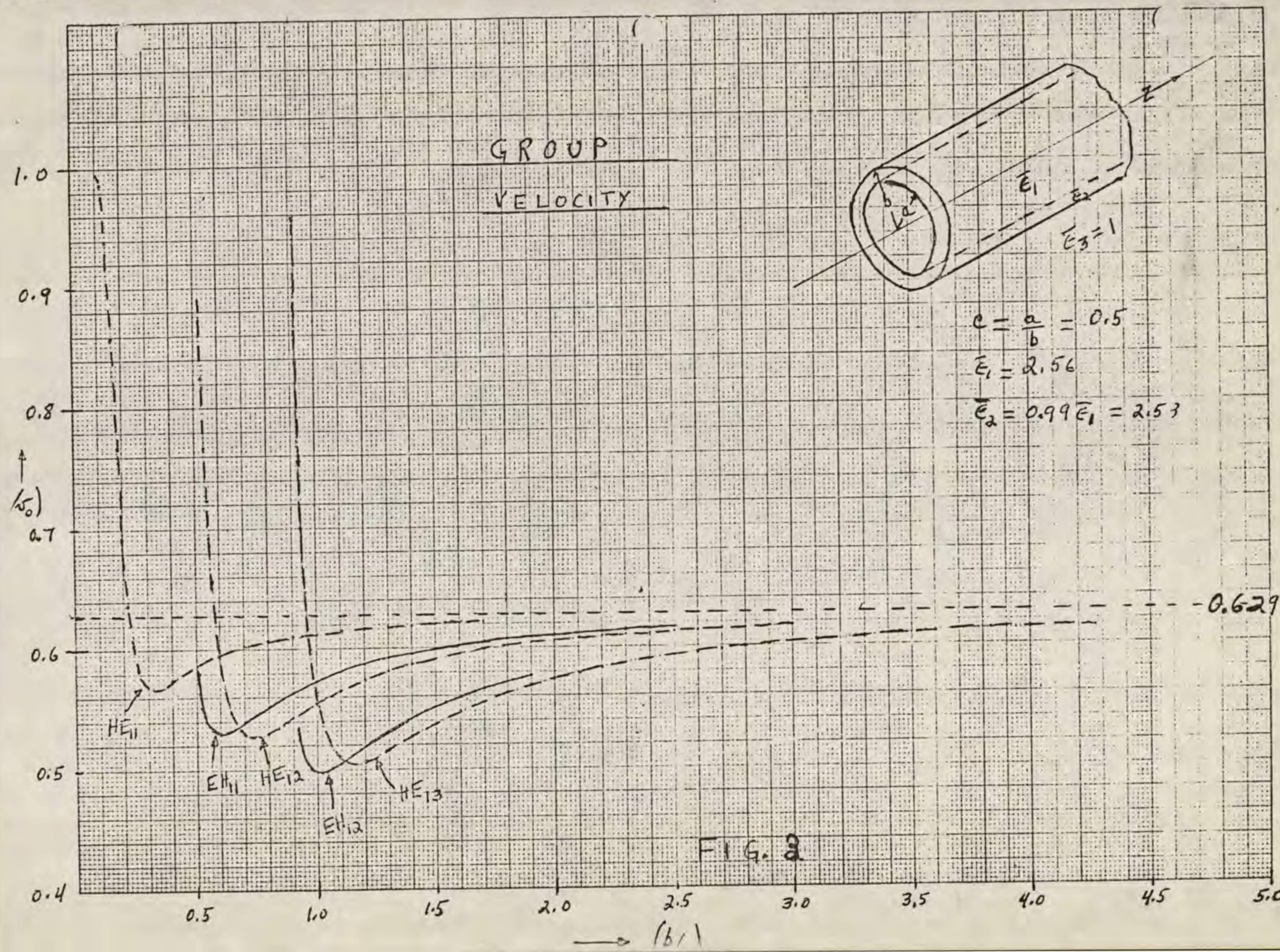
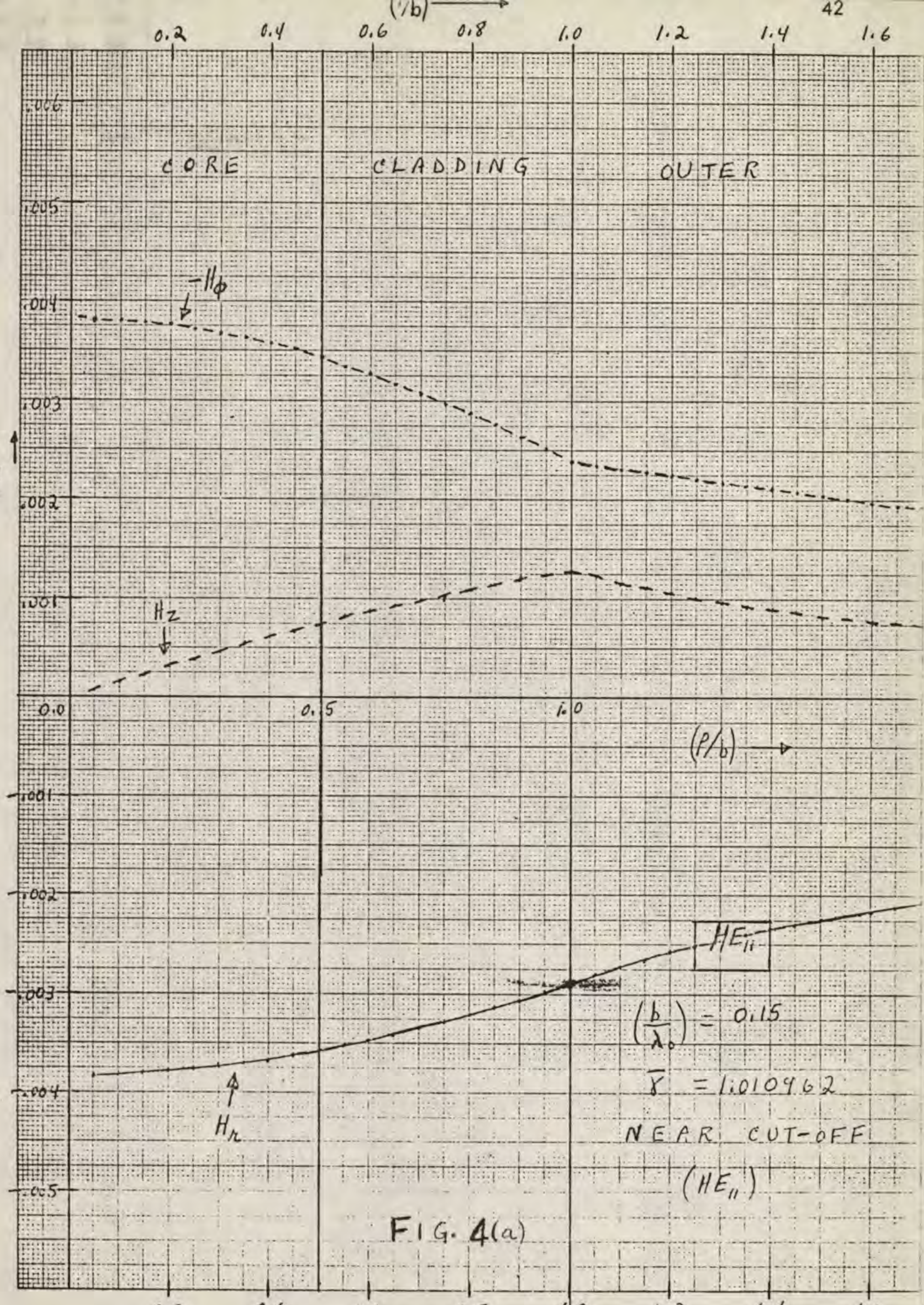

GROUP
VELOCITY
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
0.629
HE11
EH11
HE12
EH12
HE13
z
b
a
ε1
ε2
ε3 = 1
c = a/b = 0.5
ε1 = 2.56
ε2 = 0.99 ε1 = 2.53

FIG. 2



(ρ/b)
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
.006
CORE
CLADDING
OUTER
.005
.004
$-H_\phi$
.003
H
.002
.001
H_z
0.0
0.5
1.0
(ρ/b)
-.001
-.002
HE_{11}
-.003
$\left(\frac{b}{\lambda_0}\right) = 0.15$
$\bar{\gamma} = 1.010962$
-.004
H_r
NEAR CUT-OFF
(HE_{11})
-.005
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6

FIG. 4(a)

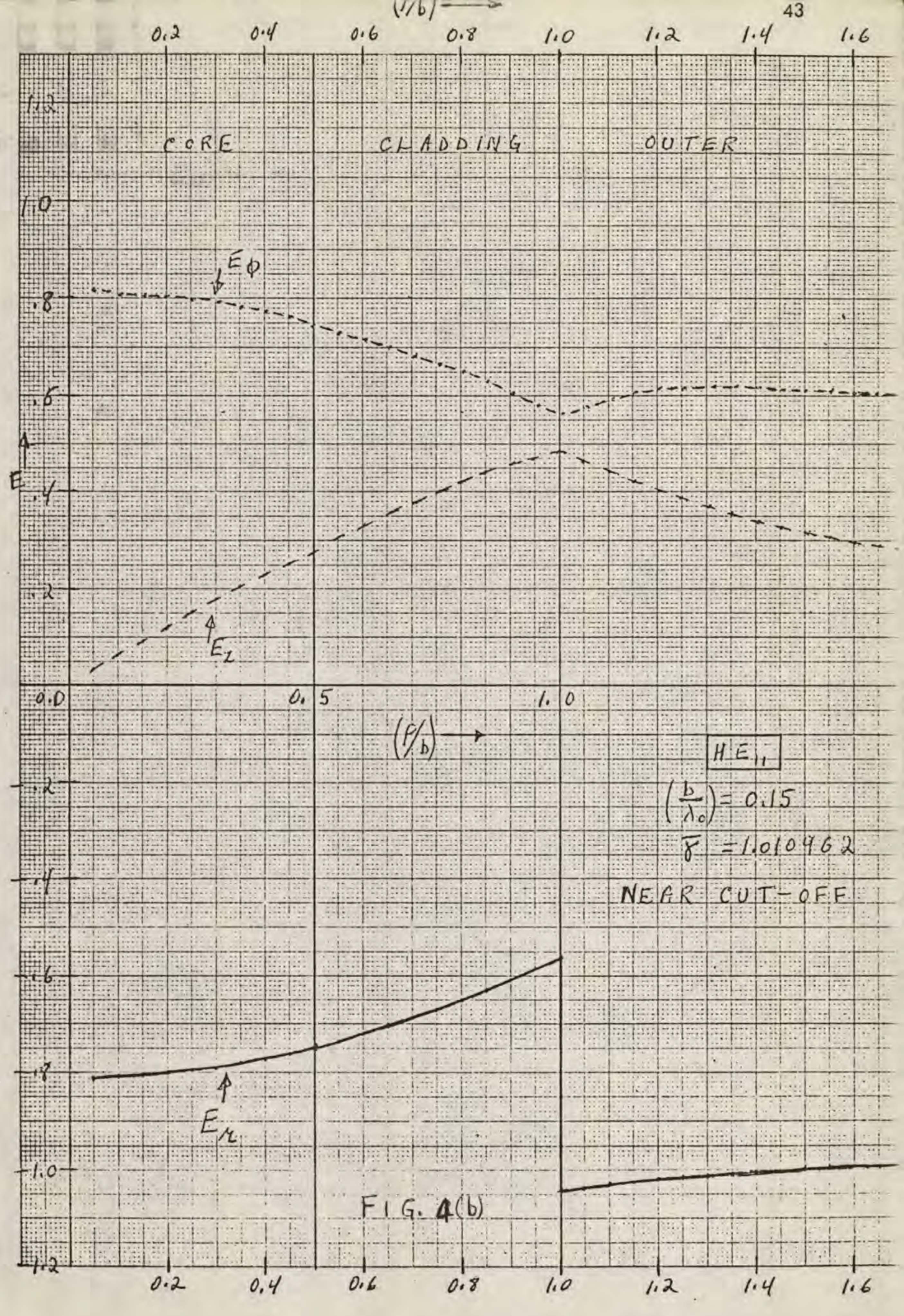

(ρ/b) →
CORE
CLADDING
OUTER
Eφ
Ez
Er
E
(ρ/b) →
HE11
(b/λo) = 0.15
γ̄ = 1.010462
NEAR CUT-OFF

FIG. 4(b)

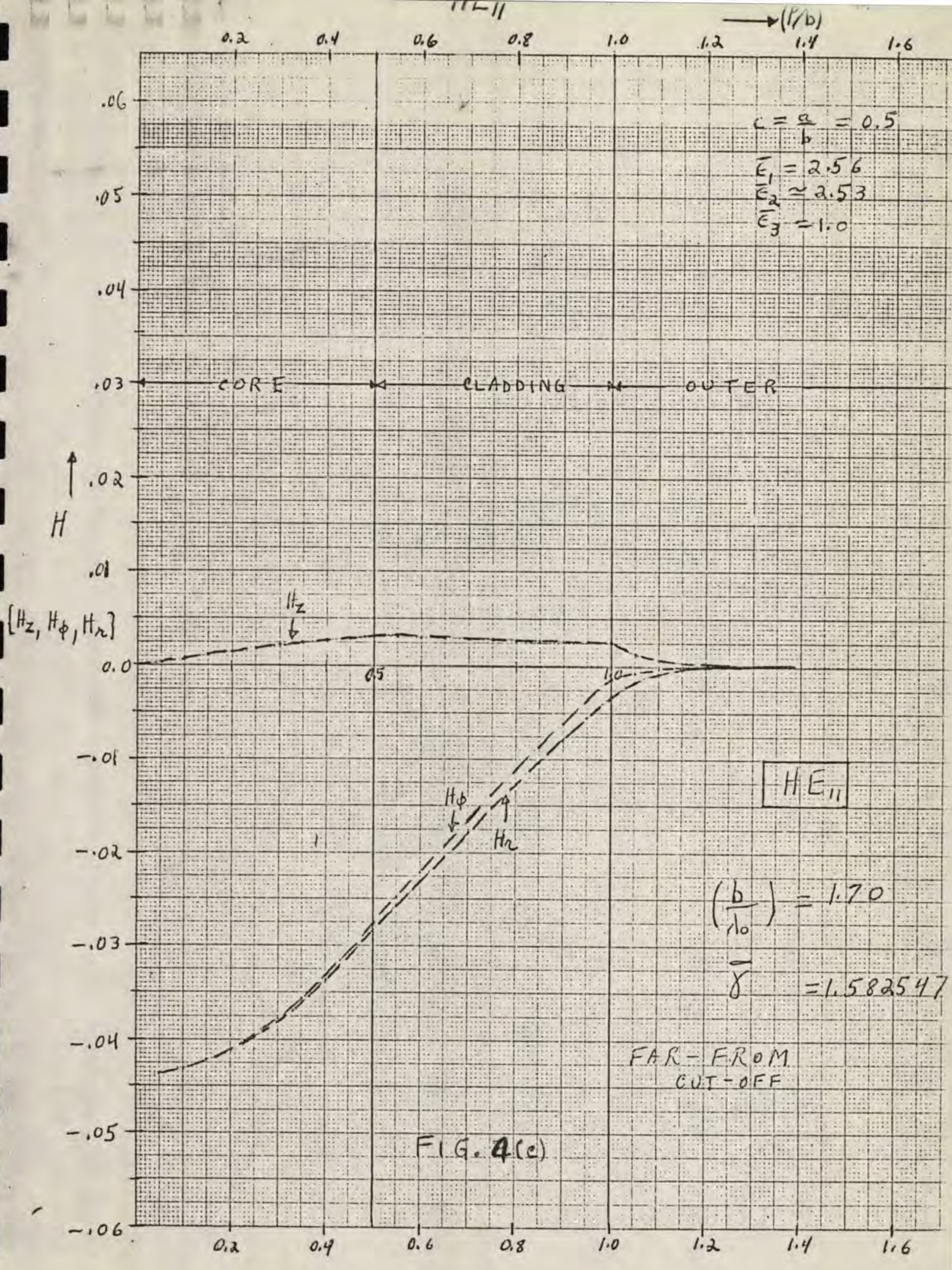

HE11
(ρ/b)
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6
c = a/b = 0.5
ε̄1 = 2.56
ε̄2 ≃ 2.53
ε̄3 = 1.0
CORE
CLADDING
OUTER
H
{Hz, Hφ, Hr}
Hz
Hφ
Hr
.06
.05
.04
.03
.02
.01
0.0
-.01
-.02
-.03
-.04
-.05
-.06
0.5
1.0
HE11
(b/λ0) = 1.70
γ̄ = 1.582547
FAR-FROM CUT-OFF

FIG. 4(c)

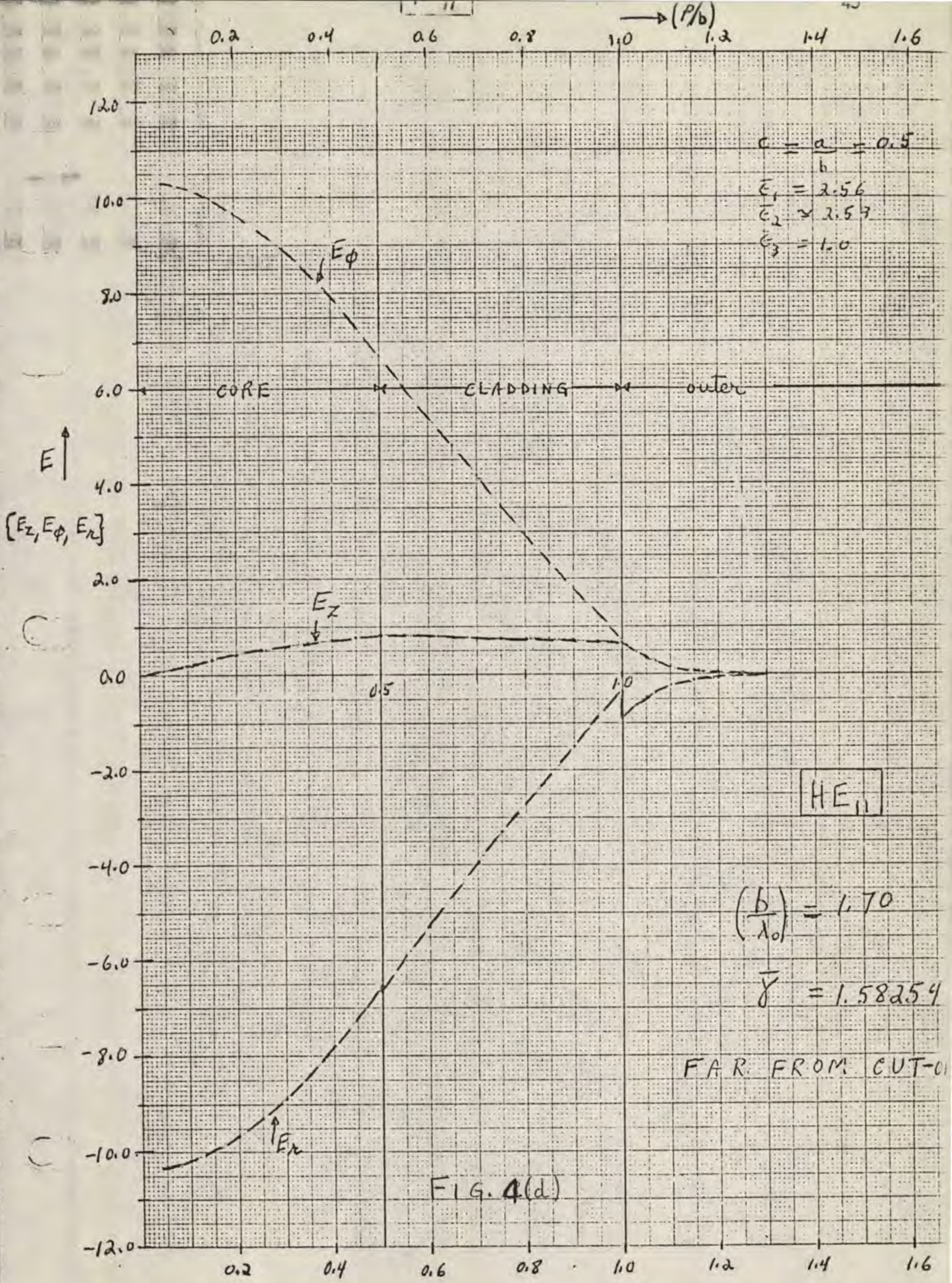

→ (ρ/b)
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
12.0
10.0
8.0
6.0
4.0
2.0
0.0
-2.0
-4.0
-6.0
-8.0
-10.0
-12.0
E
{Ez, Eφ, Er}
Eφ
Ez
Er
CORE
CLADDING
outer
0.5
1.0
c = a/b = 0.5
ε1 = 2.56
ε2 ≃ 2.53
ε3 = 1.0
HE11
(b/λ0) = 1.70
γ = 1.58254
FAR FROM CUT-O

FIG. 4(d)

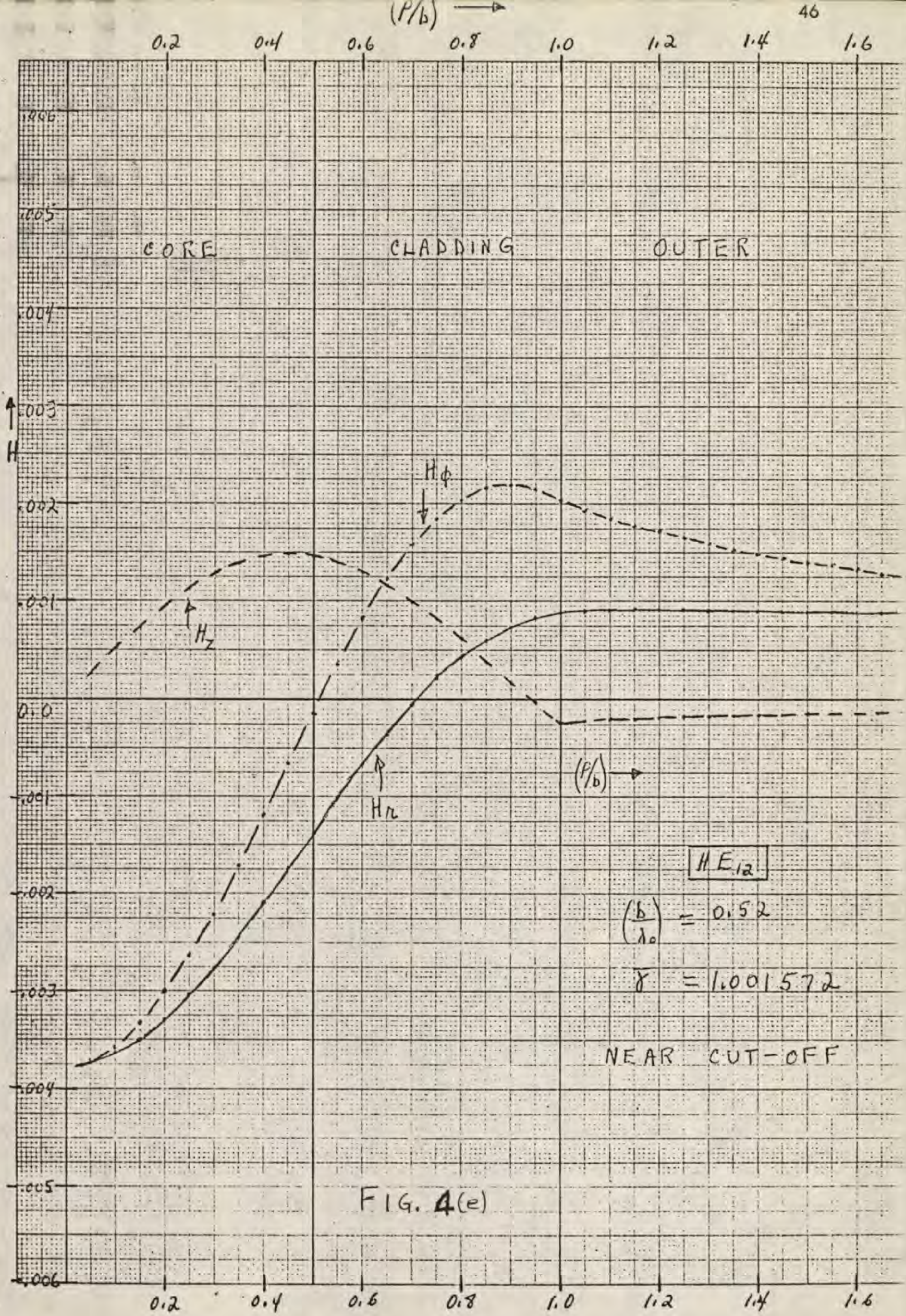

(P/b)
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
.006
.005
.004
.003
.002
.001
0.0
-.001
-.002
-.003
-.004
-.005
-.006
H
CORE
CLADDING
OUTER
H_ϕ
H_z
H_r
(P/b)
HE_{12}
$\left(\frac{b}{\lambda_0}\right) = 0.52$
$\gamma = 1.001572$
NEAR CUT-OFF

FIG. 4(e)

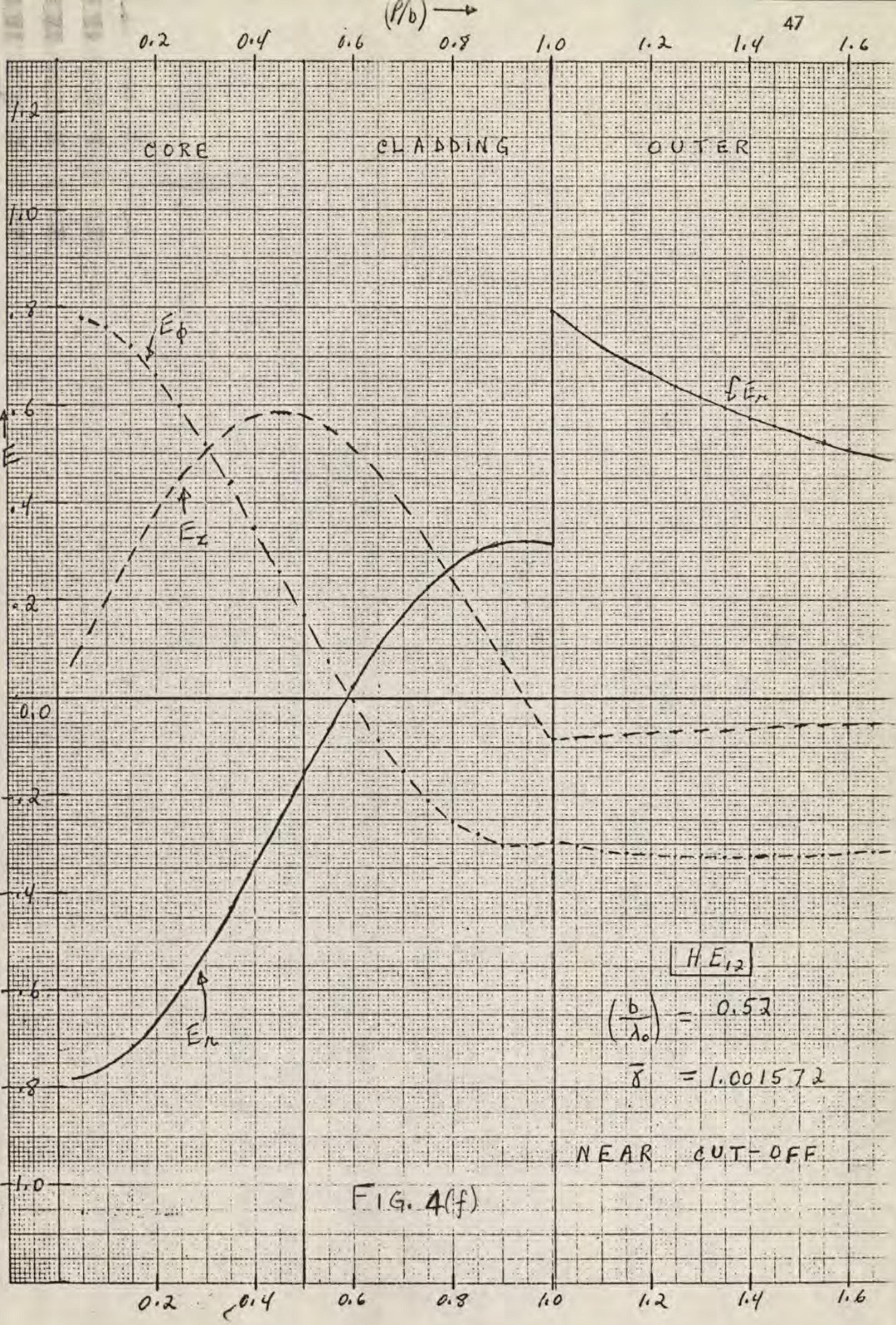

(ρ/b) →
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
CORE
CLADDING
OUTER
E
1.2
1.0
.8
.6
.4
.2
0.0
-.2
-.4
-.6
-.8
-1.0
Eφ
Ez
Er
HE12
(b/λ0) = 0.52
γ̄ = 1.001572
NEAR CUT-OFF

FIG. 4(f)

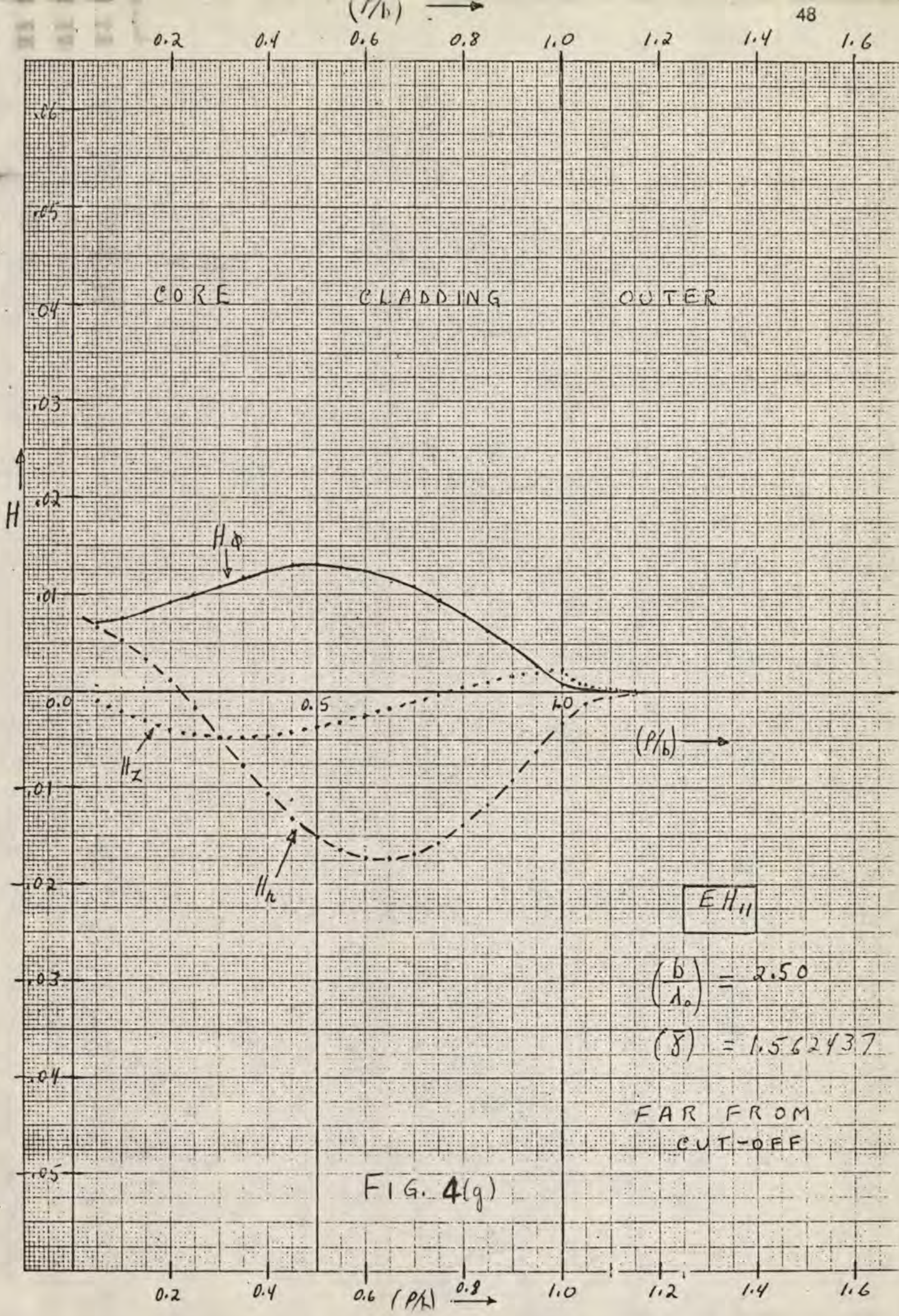
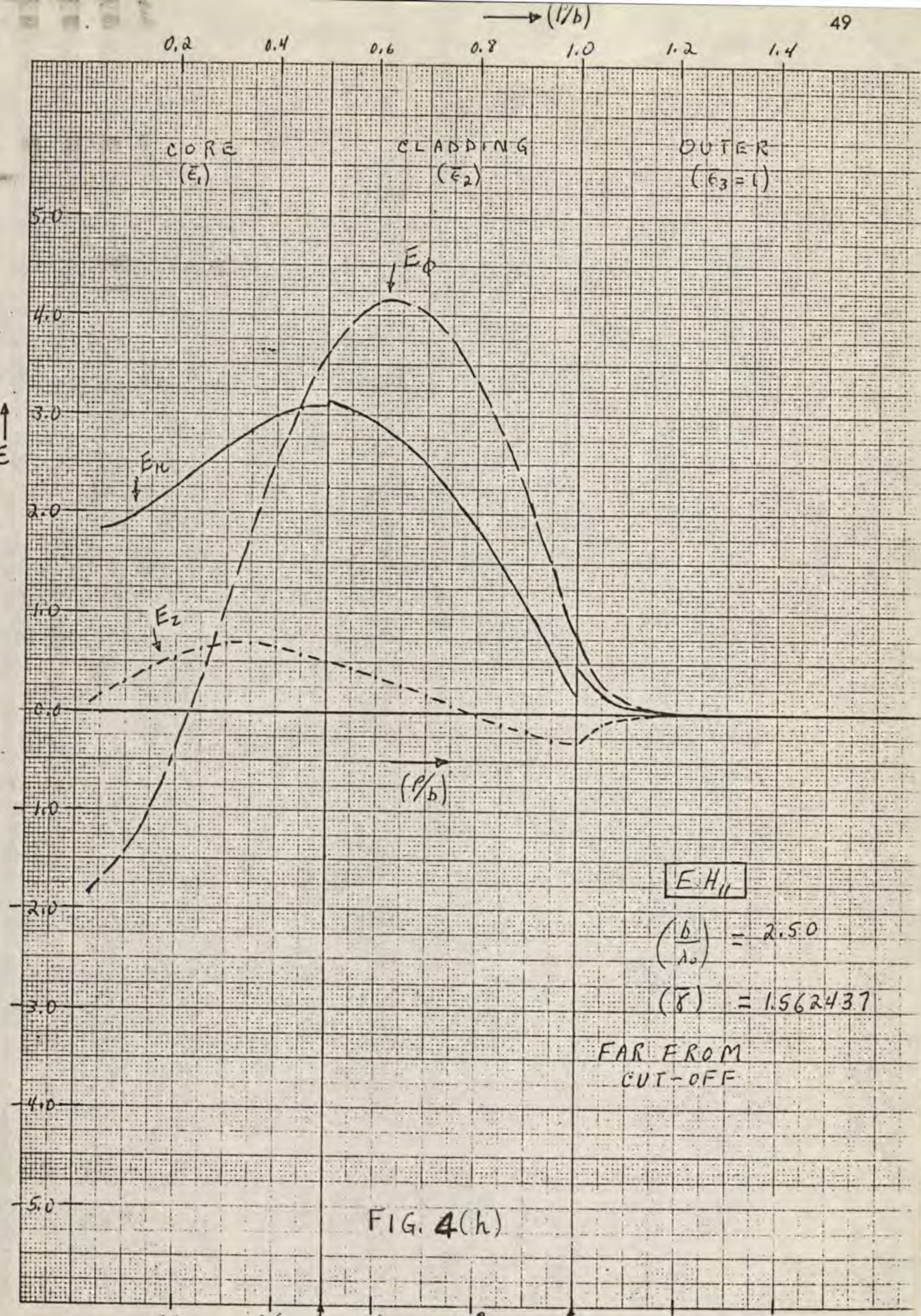

(ρ/b)
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
.06
.05
.04
.03
.02
.01
0.0
-.01
-.02
-.03
-.04
-.05
H
CORE
CLADDING
OUTER
Hφ
Hz
Hr
0.5
1.0
(ρ/b)
EH11
(b/λ₀) = 2.50
(γ) = 1.562437
FAR FROM CUT-OFF
0.2
0.4
0.6
(ρ/b)
0.8
1.0
1.2
1.4
1.6

FIG. 4(g)



→ (ρ/b)
0.2
0.4
0.6
0.8
1.0
1.2
1.4
CORE
(ϵ_1)
CLADDING
(ϵ_2)
OUTER
($\epsilon_3 = 1$)
5.0
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
E
E_ϕ
E_ρ
E_z
(ρ/b)
EH_{11}
$\left(\frac{b}{\lambda_0}\right) = 2.50$
$(\bar{\gamma}) = 1.562437$
FAR FROM
CUT-OFF
0.2
0.4
0.5
0.6
0.8
1.0
1.2
1.4

FIG. 4(h)

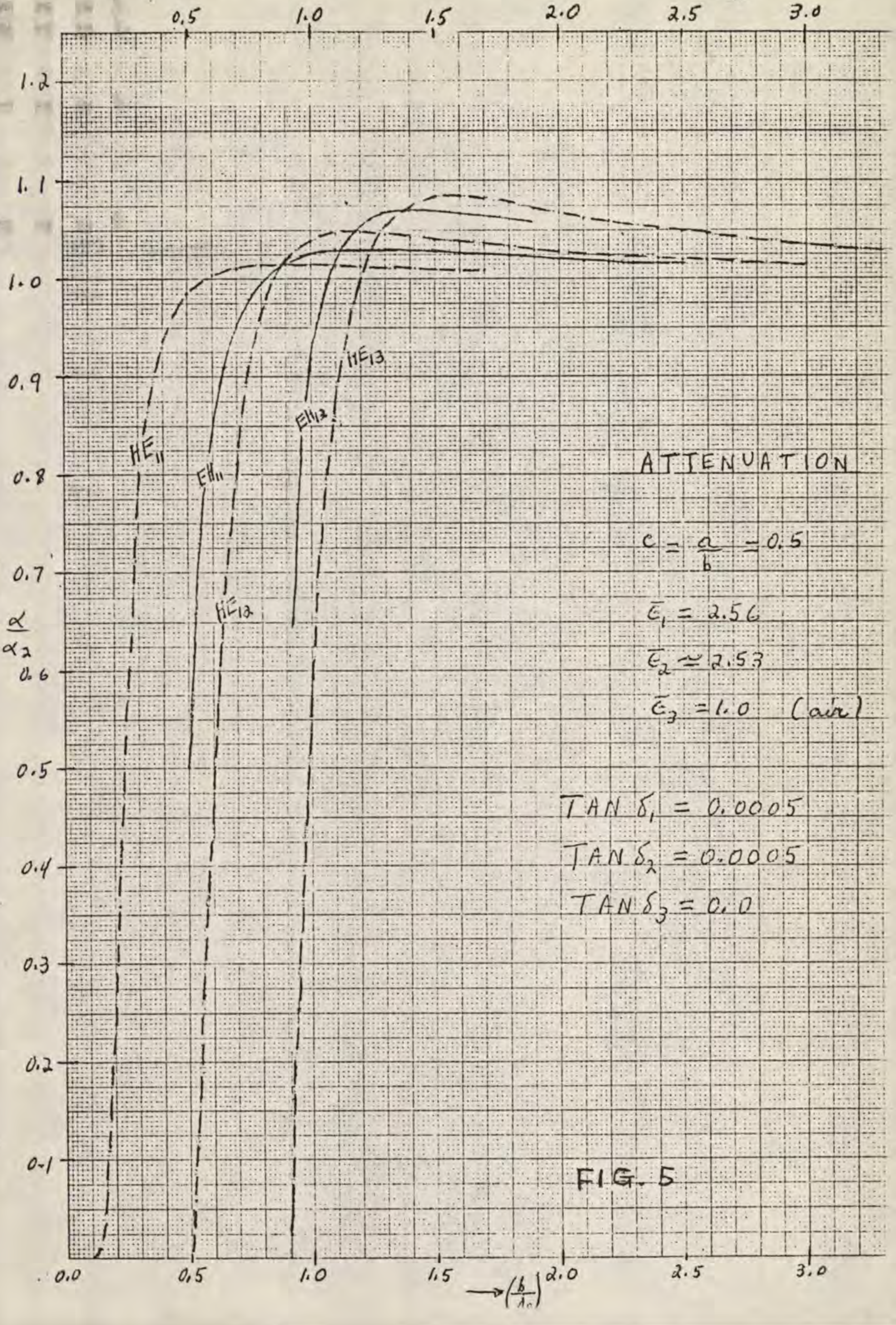

0.5
1.0
1.5
2.0
2.5
3.0
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
α / α₂
HE11
EH11
HE12
EH12
HE13
ATTENUATION
c = a/b = 0.5
ε̄1 = 2.56
ε̄2 ≃ 2.53
ε̄3 = 1.0 (air)
TAN δ1 = 0.0005
TAN δ2 = 0.0005
TAN δ3 = 0.0
(b/λ0)

FIG. 5

PART 2

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