MORTGAGE OPTIONS AND MORTGAGE RATE INSURANCE

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Since the late 1970s borrowers and lenders in Canadian mortgage markets have experienced a financial environment characterized by highly volatile interest rates. During the period of October 1979 to September 1982 interest rates on five-year, conventional mortgages rose from 13.5% to 21.5% (September 1981) and then fell back to the 15% level, a standard deviation of monthly rates of 2.6% for the period. Such interest rate volatility increases the risk of residential mortgages by causing the borrowing costs of debt to be unpredictable.

One approach that has been proposed for protecting mortgage borrowers from the risks resulting from interest rate volatility is mortgage rate insurance (MRI).[1] In essence with MRI a premium is paid to an insurer in return for its acceptance of all or a portion of the interest rate risk inherent in a mortgage renewal or commitment. Under MRI renewal coverage if interest rates rise, the insurer pays the borrower the difference between the mortgage payments at a specified contract rate and the higher payments required by the prevailing market interest rate at the time of renewal. With MRI commitment policies developers/investors in new real estate projects would receive protection against the risk of mortgage rates at the time of exercise (takedown) of a floating-rate loan commitment being at a higher level than the commitment rate stipulated in the insurance contract.

Another risk adjustment mechanism that is also under consideration is mortgage rate options. The basic idea in this case is to create a financial contract on a mortgage instrument whose value rises (falls) with increases (decreases) in mortgage interest rates. For a mortgage market participant acquiring such an option through an exchange, higher market interest rates result in a rising value of the option and a gain on the options trade to offset the higher mortgage payments required at renewal or takedown of a mortgage commitment. While mortgage lenders and borrowers could directly utilize mortgage rate options to protect against the risks created by volatile interest rates, a MRI insurer as a risk intermediary is a major potential user of these options. By buying selected mortgage or mortgage-related options, the insurer could substantially layoff the interest rate risks inherent in mortgage renewal and commitment insurance.

This study analyzes the hedging potential for a MRI insurer of a mortgage rate option as envisaged by the Toronto Stock Exchange as well as the Treasury bond options being traded in the United States. Section I describes the primary characteristics of these option markets. The following Section II explores the likely relationship of mortgage rate options to mortgage rate insurance. Section III develops a hedging strategy for laying off in these option markets the interest rate risk covered in MRI commitment and renewal policies. Section IV concludes the study by presenting the basic structure of a portfolio hedging model that includes mortgage options and could be constructed to assist a MRI insurer in overall risk management.

I. Option Markets

An option contract gives the purchaser for the price of the option (often called the "premium") the right, but not the obligation, to buy (call option) or sell (put option) a specified amount of a commodity at a fixed price (the exercise or striking price) for a given length of time. The owner of the option can exercise only at maturity (called a European option) or at any time up to the maturity date (American option). In an option market organized by an exchange, the option contracts are standardized in terms of the deliverable commodity and the maturity date and the option exchange acts as a guarantor of all transactions.

Informal option trading has existed in a variety of economic markets for many years including call and put options on real and financial assets.[2] Starting in 1973 options exchanges were created for the trading of listed common stock options in both the United States and Canada. An option contract in mortgages that has been available in the U.S. is the purchase commitment offered by the Federal National Mortgage Association (FNMA). The FNMA commitment gives the purchaser (mortgage lender) for a fee the right to sell a set quantity of mortgages to FNMA at a fixed price at any time during a four-month period. Thus, the commitment is essentially an American put option.

Mortgage Rate Options

The Toronto Stock Exchange (TSE) is presently considering the possibility of starting a market for mortgage rate options (MRO). The preliminary specifications of the MRO are in terms of contracts conveying the right to buy or sell a designated mortgage "commodity" at a specified price (determined by an interest rate level designated as the strike rate) at any time until its expiration date (an American-type option). The commodity underlying this proposed TSE option is a Canadian mortgage of either $10,000 or $100,000 face value having a certain interest rate (the option's strike rate) with monthly blended payments of interest and principal over a 25-year amortization period. The market value of this commodity is tied to a mortgage rate index and the actual delivery if the option is exercised is through cash settlement.

The writer (seller) of a put MRO is obligated to deliver to the buyer of the put a sum of money equal to any positive difference between the face value of the commodity (mortgage) and the market value of the commodity based on the level of the mortgage rate index when the option is exercised. A put option is therefore only exercised if mortgage rates (as measured by the index) are higher than the strike rate. Potential MRO put buyers are mortgage market participants seeking protection from the impact of rising interest rates. Conversely, a MRO call requires the writer to pay the buyer of the call, if mortgage rates fall below the strike rate, the difference between the market value of the mortgage as determined by the mortgage rate index at the time of exercise and the face value of the mortgage. Likely MRO call buyers are option traders from the mortgage market who suffer losses from falling interest rates.[3]

Three categories of mortgage rate put and call options are being planned corresponding to mortgage terms of 1, 3 and 5 years. It is presently anticipated that three indices (one for each mortgage term) would be complied on a daily basis from a survey of origination rates on closed conventional mortgages. The proposed specifications call for new options to be created monthly in both puts and calls for each mortgage term having strike rates in .25% intervals on either side of the prevailing levels of the corresponding mortgage rate index. The option maturity is initially 12 months and the expiration date of an option is the first Friday of the maturity month.

Tables 1, 2 and 3 show the underlying values at exercise of mortgage rate options at varying strike rates and mortgage rate index levels. These tables are derived based on a $10,000 mortgage with monthly payments, semi-annual compounding of interest, 25-year amortization and mortgage terms of 1, 3 or 5 years. The upper, right-hand wedges in these tables are the exercise values of MRO puts while the lower, left-hand areas relate to MRO calls.

To illustrate, assume a trader purchases 10 mortgage rate put options on one-year term mortgages with a strike rate of 12%.[4] Three months later interest rates rise and the mortgage rate index is at the 15% level. If the trader exercises the options, the put writer through the exchange pays the trader $262.05 per option (Table 1) or a total of $2,620.50 for the 10 MRO puts. This total exercise value is equivalent to the present value at the time of exercise of the differences in monthly payments on a $100,000 mortgage over one year when interest rates are at 15% compared to 12%.

U.S. Treasury Bond Options

In October, 1982 the Chicago Board of Trade (CBT) initiated trading in put and call options on U.S. Treasury bond futures contracts followed by the Chicago Board Option Exchange (CBOE) starting an option market for Treasury bonds themselves. Similar to the MRO, both types of Treasury bond options give the option buyer the right to sell to or buy from the option writer a particular commodity at any time prior to the expiration of the option. The basic difference between the CBOE option on the actual Treasury bond and the CBT option on the bond futures contract is what is delivered when the option is exercised.

When a put option on a U.S. Treasury bond is exercised, the option writer receives the actual bond and pays the option buyer the strike (exercise) bond

TABLE 1

MORTGAGE RATE OPTIONS
UNDERLYING VALUE AT EXERCISE
FOR VARYING MORTGAGE RATE INDEX LEVELS

(1 YEAR TERM)

STRIKE RATE	INDEX RATE 0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18
0.08	0.00	91.57	181.79	270.88	358.64	445.22	530.52	614.66	697.78	779.57	860.37
0.09	92.12	0.00	90.76	180.38	268.68	355.78	441.59	526.25	609.87	692.16	773.45
0.10	183.95	91.29	0.00	90.15	178.97	266.58	352.90	438.06	522.17	604.95	686.73
0.11	275.66	182.47	90.66	0.00	89.33	177.45	264.27	349.91	434.51	517.77	600.02
0.12	367.02	273.32	181.00	89.82	0.00	88.61	175.92	262.05	347.13	430.86	513.58
0.13	458.12	363.91	271.09	179.41	89.09	0.00	87.79	174.40	259.95	344.14	427.32
0.14	548.84	454.12	360.80	268.64	177.84	88.26	0.00	87.08	173.09	257.74	341.37
0.15	639.26	544.05	450.24	357.59	266.31	176.27	87.54	0.00	86.47	171.57	255.65
0.16	729.49	633.79	539.50	446.37	354.62	264.10	174.91	86.92	0.00	85.55	170.07
0.17	819.19	723.00	628.23	534.64	442.42	351.44	261.80	173.35	85.99	0.00	84.95
0.18	908.68	812.01	716.77	622.71	530.03	438.60	348.50	259.61	171.80	85.38	-0.0

TABLE 2

MORTGAGE RATE OPTIONS
UNDERLYING VALUE AT EXERCISE
FOR VARYING MORTGAGE RATE INDEX LEVELS

(3 YEAR TERM)

INDEX RATE

STRIKE RATE	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18
0.08	0.00	248.82	489.59	723.16	949.22	1168.32	1380.43	1586.07	1785.69	1978.79	2166.29
0.09	253.04	0.00	244.88	482.45	712.41	935.31	1151.12	1360.36	1563.50	1760.02	1950.86
0.10	506.10	248.91	0.00	241.51	475.30	701.93	921.38	1134.16	1340.76	1540.64	1734.78
0.11	759.53	498.27	245.38	0.00	237.57	467.88	690.91	907.19	1117.20	1320.41	1517.79
0.12	1012.54	747.26	490.47	241.28	0.00	233.93	460.49	680.21	893.59	1100.07	1300.64
0.13	1265.25	996.02	735.38	482.43	237.50	0.00	230.04	453.16	669.85	879.55	1083.29
0.14	1517.20	1244.09	979.67	723.02	474.48	233.46	0.00	226.46	446.42	659.31	866.15
0.15	1768.59	1491.64	1223.49	963.19	711.09	466.61	229.76	0.00	223.19	439.23	649.14
0.16	2019.63	1738.89	1467.04	1203.13	947.52	699.61	459.41	226.38	-0.0	219.15	432.11
0.17	2269.30	1984.83	1709.34	1441.88	1182.80	931.50	688.00	451.75	222.22	0.00	215.96
0.18	2518.48	2230.31	1951.22	1680.24	1417.73	1163.07	916.30	676.86	444.20	218.94	-0.0

TABLE 3

MORTGAGE RATE OPTIONS
UNDERLYING VALUE AT EXERCISE
FOR VARYING MORTGAGE RATE INDEX LEVELS

(5 YEAR TERM)

	INDEX RATE										
STRIKE RATE	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18
0.08	0.00	376.30	734.41	1076.11	1401.46	1711.71	2007.29	2289.30	2558.77	2815.38	3060.73
0.09	386.59	0.00	367.99	719.19	1053.67	1372.69	1676.68	1966.80	2244.07	2508.18	2760.75
0.10	774.48	377.73	0.00	360.59	704.07	1031.75	1344.07	1642.20	1927.20	2198.72	2458.45
0.11	1164.01	757.22	369.86	0.00	352.40	688.65	1009.22	1315.29	1607.93	1886.82	2153.64
0.12	1553.74	1137.07	740.20	361.19	0.00	344.73	673.43	987.34	1287.56	1573.72	1847.56
0.13	1943.70	1517.27	1111.02	722.97	353.09	0.00	336.76	658.43	966.14	1259.50	1540.30
0.14	2333.06	1897.00	1481.50	1084.53	706.07	344.71	0.00	329.34	644.45	944.93	1232.61
0.15	2721.96	2276.40	1851.76	1445.98	1059.04	689.51	336.92	-0.0	322.43	629.97	924.46
0.16	3110.64	2655.67	2221.98	1807.48	1412.14	1034.52	674.14	329.70	0.00	314.53	615.77
0.17	3497.45	3033.21	2590.60	2167.49	1763.86	1378.25	1010.17	658.30	321.43	0.00	307.93
0.18	3883.65	3410.23	2958.79	2527.15	2115.31	1721.78	1346.07	986.84	642.84	314.55	0.00

price specified in the option. Such a put option, therefore, is only exercised if the strike price is greater than the existing market price of the bond. That is, Treasury bond interest rates are higher than the yield corresponding to the strike price of the put option. For a call option on an actual Treasury bond, the buyer of the call receives the bond in return for paying the call writer the strike price of the option. A call option is exercised by the buyer when, due to falling interest rates, the market value of the bond is greater than the option strike price.

When an option on a U.S. Treasury bond futures is exercised, both the buyer and seller of the option receive positions in a specified Treasury bond futures contract listed in the futures market of the Chicago Board of Trade.[5] With a put option, the option buyer upon exercise assumes a short position in a Treasury bond futures while the put writer takes a corresponding long position. For the buyer of the put option, the short futures position means he agrees to sell to the long position (put writer) a Treasury bond for the strike price of the option with actual delivery of the bond at the maturity date of the futures contract. Comparable to the option on the actual bond, the put option on the Treasury bond futures is only exercised if the option strike price is greater than the prevailing market price of the futures contract. Conversely, when a call option on a Treasury bond futures is exercised, the option buyer acquires a long position and the option writer a short position in the futures contract. The buyer of the call option thus benefits by the amount of the positive difference between the going futures price and the strike price of the option.

The CBOE options on U.S. Treasury bonds are denominated in either $20,000 or $100,000 increments with the option written on an identified outstanding long-term bond. For example, at the beginning of January 1983, options are being traded on two bond issues, one having a 14% coupon maturing in 2011 and the other having a 10 3/8% coupon with a maturity date in 2012. The CBT futures options are written on specific CBT Treasury bond futures contracts. The futures contracts are in terms of a $100,000 bond having an 8% coupon with a minimum maturity of the deliverable security being at least 20 years.[6]

Both exchanges intend to establish trading in options expiring in four common months: March, June, September and December. In the case of the CBT futures option, the actual expiration date is the third Friday of the preceding month (e.g., a March option actually expires in February). This procedure is necessary since exercise of the option leads to the creation of a futures position in a contract which matures in the specified month. Both option exchanges have extablished clearinghouses to serve as guarantors of contract performance. The fulfillment of option obligations (on the part of the writer) is guaranteed by the clearinghouse assuming the opposite side of each trade in the option market (as well as requirements for option writers concerning security deposits and daily settlement of any losses on the writer's position).

At the present time daily sales volume of U.S. Treasury bond puts and call options combined are in the range of 2,000 contracts ($100 million in Treasury bonds) on the CBT and 200 contracts on the CBOE. Open interest levels are around 14,000 outstanding CBT options and 3,000 CBOE options at any given time.[7] The longest maturity of the outstanding option contracts is approximately 6 months.

From a general interest rate hedging perspective, straight Treasury bond options and Treasury bond futures options are essentially equivalent hedging vehicles. In both cases put (call) buyers are able to purchase a financial instrument which has a payout under conditions of rising (falling) market interest rates. As an example, assume a trader purchases either a CBOE or CBT put option on a $100,000 Treasury bond or Treasury bond futures with a striking price of $60,000 (60-00). Over a three month period market interest rates rise to the 16% level and prices on Treasury bonds fall to $52,094 (52-03). If the trader exercises the CBOE option, he receives a payment of $60,000 and delivers to the option writer a Treasury bond which he can purchase in the current bond market for $52,094, resulting in a net gain of $7,906 (ignoring transaction costs). If the trader exercises the CBT futures option, he assumes a short position in the Treasury bond future and receives a cash settlement of $7,906 (less margin requirements on short position). The settlement is the difference between the cost to the option writer of his matching long position of $60,000 and the existing futures price of $52,094. The trader can then either close out the short futures position or retain it as a further interest rate hedge.

Premiums

Similar to prices in other competitive markets, option premiums (the market prices of options) for both mortgage rate and Treasury bond options are determined by buyers who represent the demand for options and sellers (writers) who represent the supply. The actual dollar amount of the premium for a given option is primarily a reflection of the option's <u>intrinsic</u> value and its <u>time</u> value at that moment. Intrinsic value is any amount that the market value of the commodity (mortgage, Treasury bond or Treasury bond futures) is currently <u>below</u> the option strike price in the case of a put option or <u>above</u> the strike price in the case of a call option.[8] Time value is whatever amount over and above any intrinsic value that results from the possibility that at some point prior to expiration subsequent changes in interest rates and commodity prices will further increase the value of the rights conveyed by the option.

To help demonstrate the concepts, Table 4 shows the closing premiums on two selected days for the outstanding CBT U.S. Treasury bond futures options. The premiums are quoted on a par basis in 64ths of a point (one 64th = $15.625 per $100,000 futures contract). For example, the premium at the close of trading on November 5, 1982 for a December call option with a strike price of 68 was 10-27 or $10,421.875 per $100,000 U.S. Treasury bond futures.

On November 5th the settlement price on the March U.S. Treasury bond futures contract was 77-30 (in 32nds of a point). Therefore, all March call options with a strike price from 68 to 76 are "in-the-money" options since the existing futures price is greater than the strike price. Similarly, the March put option with a strike price of 78 is in-the-money given in this case the strike price is greater than the futures price. These in-the-money options all have an intrinsic value. For instance, the March call option with a strike price of 76 has an intrinsic value of 1-30 (in 32nds) or $1,937.50 per $100,000 option. The buyer of this option could immediately exercise it and assume a long futures position, establishing a $1,937.50 gain on the transaction. The additional value present in the premium on the call above the intrinsic value (2-17 or $2,531.25) is a reflection of the time value of the option.

Since "out-of-the-money" options have no intrinsic value, the premiums on November 5th for the March call options with a strike price of 78 or 80 and the March puts with strike prices less than 78 consist solely of time value. The time value is influenced by how much time remains until expiration. For an American-type of option, the less time remaining, the less time value. The out-of-the-money March put option with a strike price of 72 has a premium of 1-00 ($1,000) on November 5, 1982; however, by January 5, 1983 the premium fell to only 0-11 ($171.875). The explanation for the declining time value is that as expiration approaches there is less likelihood that an out-of-the-money option will ever become profitable to exercise.

TABLE 4

Premiums

U. S. Treasury Bond Futures Option

November 5, 1982

Strike Price	Call Options Dec	Call Options Mar	Put Options Dec	Put Options Mar
68	10-27	10-00	0-01	0-32
70	8-27	8-20	0-01	0-42
72	6-27	6-56	0-02	1-00
74	4-38	5-29	0-10	1-32
76	2-56	4-30	0-27	2-20
78	1-28	3-00	1-02	3-20
80	0-30	2-27	2-20	--

January 5, 1983

Strike Price	Call Options Mar	Call Options Jun	Put Options Mar	Put Options Jun
68	8-54	8-43	0-02	--
70	6-57	7-01	0-04	0-45
72	4-56	5-33	0-11	1-07
74	3-16	4-13	0-30	1-44
76	1-60	3-05	1-12	2-36
78	0-61	2-18	2-14	3-47
80	0-32	1-25	3-40	4-53

Source: Selected issues of *Wall Street Journal*.

Besides the time remaining to expiration, the strike price and the prevailing market price of the underlying commodity, the premium on a MRO or Treasury bond option is also a function of the expected volatility in the market price of the commodity as well as the relationship between the interest yield of the instrument and the short-term, risk-free interest rate. The greater the volatility of interest rates and therefore the market prices of the particular financial instrument, the greater the likelihood of a payout on the option and the higher the premium. Variations of the original Black-Scholes option pricing model can be formed for deriving the expected premiums on a MRO and both types of U.S. Treasury bond options.[9]

II. Relationship to Mortgage Rate Insurance

Option markets offering either mortgage rate options or the two types of U.S. Treasury bond options potentially could be utilized by a mortgage rate insurer to layoff the interest rate risk inherent in MRI. A MRI insurer can be viewed as essentially a put writer selling to mortgage borrowers an insurance contract that is equivalent to a European put option on a mortgage. Similar to the put options previously discussed, the MRI insurer (put writer) receives a premium in return for paying to the insured (put buyer) at renewal or commitment (exercise of the option) if mortgage rates rise above the deductible (strike rate) the difference in monthly mortgage payments over a specified term (total claim equal to the difference between the face value of the mortgage based on the deductible and the market value of the mortgage given prevailing interest rates). By also becoming put buyers in the available option markets, the MRI insurer can in essence offset or hedge the insurance position and pass through the risk (and most of the insurance premium) to the put writers in the option markets. The next section of this study considers the appropriate option hedging strategy for a MRI insurer.

An outstanding question that first should be addressed is whether the proposed TSE mortgage rate options would be a complementary or competing good for MRI. Instead of using MRI, mortgage borrowers could directly acquire MRO puts for the amounts of their outstanding loan balances once the TSE market is operating. They could thereby lock in up to 12 months before renewal or takedown of a floating-rate commitment the strike rate of the MRO for a future mortgage term of 1 to 5 years. Through choice of the option strike rate, borrowers could effectively add a deductible to their interest rate protection (strike rate greater than existing mortgage rate) or even buydown the future renewal/commitment rate (strike rate less than existing mortgage rate).

There are at least three factors which suggest that MROs and MRI are complementary goods and that there will be an ongoing need in at least the near future for an intermediary such as a MRI insurer acting between most mortgage borrowers and the MRO market.

1. product acceptance;

Given the direct analogy to existing forms of insurance, mortgage borrowers should have little difficulty understanding MRI as a consumer product. Most borrowers, however, have had little previous experience with financial options and, especially in the case of homeowners, may have substantial problems accepting MROs. While commercial borrowers could be expected over time to become familiar with MROs and acquire them in commitment situations as a substitute for commitment insurance, it is unlikely that many homeowners would be willing to directly purchase MROs without substantial "repackaging" of the product in the form of renewal insurance.

2. market depth;

Eventhough present specifications call for the creation of new MRO put options in at least three strike rates each month for all three mortgage terms (a total of not less than 108 put options outstanding at any time), it is unlikely in the foreseeable future that a MRO market in Canada would be able to maintain sufficient liquidity in such a large number of option contracts. Probably only a more limited number of options will be actively traded and many mortgage borrowers will not be able to find a MRO which exactly matches the timing of their renewals or commitments. Given that the MROs are American options, borrowers could acquire put options with maturities beyond their renewal/commitment and simply exercise them before expiration. As noted in Section I, however, option premiums are directly related to the time to expiration and these borrowers would be paying an additional premium for unused interest rate protection.

3. residual risk;

Since the planned MROs are to have a maximum maturity of 12 months, borrowers having mortgage renewals or commitments beyond one year would need to "rollover" their options as they approach expiration. With each rollover the additional premium is partially a function of the expected interest rate volatility at that time. If interest rate volatility is not constant, the total premium cost is uncertain and there is a resulting residual interest rate risk that cannot be hedged in the MRO market.

III. Hedging Strategy

When compared to alternative hedging vehicles such as financial futures markets, the risk management by a MRI insurer using option markets requires a less complicated hedging strategy. Utilizing futures markets to hedge interest rate risk is difficult with MRI since the insurer's position is basically one-sided while the futures position is symmetric. For the insurer there is no gain if interest rates rise eventhough there would be losses in short futures positions. An options hedge is not symmetric, the downside loss of a put buyer is limited to the option premium. The hedge position available through put options therefore corresponds quite well to the insurer's position in MRI. This section of the study examines MRI hedging positions in the MRO and Treasury bond option markets and considers the potential effectiveness of such hedging.

MRI Hedging Positions

Assuming the objective of a MRI insurer is to maximize the laying off of the interest rate risk in its insurance portfolio, the basic hedging strategy is to try to perfectly match the insurance and option positions. In other words, for each $1 of mortgage rate insurance written with a given coverage (renewal term), deductible and renewal date, the MRI insurer acquires $1 put option that expires on the renewal date on an instrument having a term equivalent to the coverage with a strike rate matching the deductible. For instance, assuming in January 1983 a MRI policy is written on a $50,000 mortgage having a 12% interest rate that is subject to renewal in January 1984. The MRI coverage is for 3 years after renewal and the policy contains a 2% deductible. In this example the matching option position is for the MRI insurer to buy a $50,000 put option expiring in January 1984 with a strike rate of 14% on a mortgage or mortgage-related instrument having a 3-year term.

When a perfect match is possible, the insurance hedge ratio (ratio of the size of the put option position to the size of the insurance position) for a MRI insurer is simply 1. The insurance hedge ratio (IHR) can be represented as:

$$\text{IHR} = \frac{\partial I/\partial r}{\partial O/\partial r} \tag{1}$$

where $\partial I/\partial r$ is the change in the insurance value (I) for a given change in interest rates (r) and $\partial O/\partial r$ is the change in the option value (O) for a given change in interest rates.[10] This ratio can also be broken down into two components:

$$\frac{\partial I/\partial r}{\partial O/\partial r} = \frac{\partial I/\partial M_i}{\partial O/\partial M_o} \times \frac{\partial M_i/\partial r_i}{\partial M_o/\partial r_o} \tag{2}$$

with $\partial I/\partial M_i$ and $\partial O/\partial M_o$ being the changes in the insurance and option values for a given change in the market value of the instruments underlying the insurance (M_i) and the option (M_o); and, $\partial M_i/\partial r_i$ and $\partial M_o/\partial r_o$ are the changes in instrument values for each change in interest rates (r_i and r_o). The numerator and denominator of the first component in equation (2) are the option hedge ratios that are derived from the option pricing model. When there is a perfect match, the option hedge ratios for the insurance and the option are equal and $\partial M_i/\partial r_i = \partial M_o/\partial r_o$, causing IHR to equal 1.

Adjustments are necessary in the hedging strategy and in the insurance hedge ratio to recognize the effect of any mismatches between the insurance and option positions. As noted in the previous section of this study, option prices (values) are basically a function of the characteristics of the underlying instrument (e.g., term to maturity (t) and the expected volatility of its market value (σ)), the strike or exercise price (s) and the time remaining to option expiration (e). Differences between the option and insurance positions in these determinants of value are therefore potential sources of hedging adjustments and a IHR not equal to 1.

First, consider the situation where the mortgage term of the MRO (t_o) is not equivalent to the MRI coverage (t_i). An example is a MRI insurer hedging a policy covering the renewal of a mortgage for a 4-year term with a MRO on a 3-year mortgage. In the form of equation (2) if interest rates on mortgages of different terms move together, IHR becomes

$$\frac{\partial I(t_i)/\partial r}{\partial O(t_o)/\partial r} = \frac{\partial I(t_i)/\partial M_i}{\partial O(t_o)/\partial M_o} \times \frac{\partial M_i/\partial r}{\partial M_o/\partial r} \qquad (3)$$

Since all other determinants are the same for both the MRO and the MRI, $\partial I(t_i)/\partial M_i = \partial O(t_o)/\partial M_o$ and equation (3) reduces to

$$\frac{\partial I(t_i)/\partial r}{\partial O(t_o)/\partial r} = \frac{\partial M_i/\partial r}{\partial M_o/\partial r} \qquad (4)$$

IHR is simply the relative change in mortgage values for a given change in interest rates.[11]

With respect to the example, a change in mortgage interest rates has a greater impact on the present value of a 4-year mortgage when compared to a 3-year mortgage, so the IHR > 1. Table 5 shows the approximate IHRs for various combinations of insurance coverages and the MRO mortgage terms assuming all other premium determinants are equivalent and renewal interest rates for different terms are perfectly correlated with the mortgage rate index.[12] In the example of 4-year MRI coverage being hedged by a MRO on a 3-year mortgage, the IHR=1.25. This ratio value means that for each $1 of MRI written, the insurer would take a $1.25 position in a MRO put.

The second mismatch to be considered is the case where the insurance strike price (s_i) is not the same as the option strike price (s_o). For example, assume that interest rates are at the 12% level and the MRI insurer is writing policies with 2% deductibles (a MRI strike rate of 14%). However, the highest MRO put strike rate is only 13%. In this example the strike price of the MRI is lower than the strike price of the MRO put option. Holding all other determinants equal, the IHR in this case is

$$\frac{\partial I(s_i)/\partial r}{\partial O(s_o)/\partial r} = \frac{\partial I(s_i)/\partial M}{\partial O(s_o)/\partial M} \times \frac{\partial M/\partial r}{\partial M/\partial r} = \frac{\partial I(s_i)/\partial M}{\partial O(s_o)/\partial M} \qquad (5)$$

The IHR in equation (5) reduces to the relative option hedge ratios of the MRI and MRO positions with different strike prices. If the $s_o > s_i$ (as in the above example), the IHR < 1. Conversely, if $s_o < s_i$ (option strike rate greater than the insurance strike rate), the IHR > 1.

The third possible adjustment in hedging strategy is for differences in the volatilities of the market values of the financial instruments underlying the insurance (σ_i) and the option (σ_o). Such a mismatch is likely if a MRI insurer utilized either type of U.S. Treasury bond option.[13] Alternative levels of this measure for MRI policies and U.S. Treasury bond options can result from differences in the volatilities of Canadian and U.S. interest rates and from differences in the terms to maturity of the underlying instruments.

The IHR can be shown under conditions of differing volatilities to be

$$\frac{\partial I(\sigma_i)/\partial r}{\partial O(\sigma_o)/\partial r} = \frac{\partial I(\sigma_i)/\partial M_i}{\partial O(\sigma_o)/\partial M_o} \times \frac{\partial M_i/\partial r}{\partial M_o/\partial r} \qquad (6)$$

TABLE 5

MRI/MRO Hedge Ratios

MRO Terms	MRI Renewal Terms 1	2	3	4	5
1	1.00	1.87	2.63	3.28	3.85
3	.38	.71	1.00	1.25	1.51
5	.26	.49	.66	.85	1.00

The first component on the right-hand side of equation (6) is the relative option hedge ratios of the MRI and the Treasury bond option positions. These ratios are derived from the outputs of the option pricing model using as inputs the respective volatilities of the instrument values. The second component of equation (6) can be separated into two factors

$$\frac{\partial M_i/\partial r}{\partial M_o/\partial r} = \frac{\partial M_i/\partial r_i}{\partial M_o/\partial r_o} \times \frac{\partial r_i/\partial r}{\partial r_o/\partial r} \tag{7}$$

The first element in equation (7) is the comparative changes of the mortgage and bond values for a given change in their respective interest rates. This factor reflects the differences in the MRI term coverages (1 to 5 years) and the maturities of U.S. Treasury bonds (20-30 years). The second element is the relative change in mortgage and bond interest rates.

The second component in equation (6) can be estimated through time series regressions of changes in the market values of actual U.S. Treasury bonds or Treasury bond options on the market values of Canadian mortgages. Table 6 shows the results of such a regression comparing Canadian mortgages with 5-year terms and U.S. Treasury bond futures for the period of August 1977 through December 1980 over different hedge terms. The regression coefficients in this table are estimates of the ratio of $\partial M_i/\partial r$ to $\partial M_o/\partial r$ for a hedging strategy incorporating Treasury bond futures.

The final mismatch can affect hedging with either MROs or U.S. Treasury bond options. It requires an adjustment in the IHR to reflect any differences between the insurance period, e_i (period from writing MRI policy until mortgage renewal) and the option period, e_o (time remaining to exercise of the

TABLE 6

Canadian Mortgages/Treasury Bond Futures

Hedge	Regression Coefficient	R^2
2 Weeks	.044 (.053)	.005
4 Weeks	.266 (.068)	.089
12 Weeks	.211 (.057	.085
26 Weeks	.263 (.048)	.183
52 Weeks	.269 (.038)	.306

Figures in parentheses under estimated coefficients are standard errors.

All coefficients are significantly different from 1 at .05 level.

hedging option). Assuming all other characteristics of the MRI and the option are equivalent, the IHR is determined by the relationship between the option hedging ratios of the insurance and option positions.

$$\frac{\partial I(e_i)/\partial r}{\partial O(e_o)/\partial r} = \frac{\partial I(e_i)/\partial M}{\partial O(e_o)/\partial m} \times \frac{\partial M/\partial r}{\partial M/\partial r} \qquad (8)$$

This timing mismatch means the MRI insurer may be required to purchase a put option with an exercise date beyond the mortgage renewal date. In equation (8) if $e_o > e_i$, IHR < 1. Also, it is likely that given the short maturities of the available options an insurer will be writing policies for periods beyond the outstanding option maturities. If $e_o < e_i$, IHR > 1 and the MRI insurer needs to rollover the options as they expire. If the volatilities of the option instrument change over time, there is a residual risk that option premiums would rise and the sum of these premiums over the insurance period would be greater than the MRI premium.

Potential Effectiveness

The potential effectiveness of hedging in a MRO market by a MRI insurer is generally quite high. Since the mortgage rate put option and mortgage rate insurance are essentially comparable financial instruments and they both are tied to Canadian mortgage interest rates, MROs having expiration dates that coincide with the renewal dates covered in MRI policies can be utilized to perfectly hedge the interest rate risk. A minor concern would be how closely the mortgage rate indices match the renewal rates on the MRI mortgages. Small differences are likely and would slightly reduce hedging effectiveness. This concern, of course, could be alleviated by the MRI insurer directly tying the insurance claim to the index level rather than the actual renewal rate of the borrower.[14]

As discussed previously, the hedging of MRI policies with renewal dates beyond the maturities of the outstanding MROs requires the insurer to rollover the options as they expire. To the degree that interest rate volatility is unstable from one period to the next, there are fluctuations in option premiums and a reduction in hedging effectiveness. The estimated monthly volatilities of the instrument values of mortgages with 1-year terms over the period of 1978 to 1982 are presented in Table 7. As shown in this table there has been substantial variation in recent years in these volatility measures. These data suggest that while a substantial portion of the MRI risk can be shifted to the proposed MRO market, there is a residual risk that will be absorbed by the insurer.

In the case of U.S. option markets, the effectiveness of interest rate hedging by the MRI insurer is further affected by any lack of correlation in the movements of Canadian and U.S. interest rates. For U.S. Treasury bond options to be effective hedging mechanisms it is necessary for Canadian mortgage rates and U.S. Treasury bond rates to generally move together over the term of the hedge. The R^2s in Table 6 measure the degree to which changes in the market values of Canadian mortgages over different hedge terms are correlated with changes in the prices of Treasury bond futures. The R^2s for the shorter-term hedges (2-week through 12-week) are quite low, meaning U.S. Treasury bond options are of little assistance in hedging MRI positions of such durations. In the longer 26-week and 52-week hedges, the R^2s are higher and, when combined with an exchange rate hedge, the relationship is sufficient for U.S. Treasury bond options to offer worthwhile longer-term hedging opportunities.[15]

TABLE 7

Mortgage Volatility Estimates*

	Year				
	1978	1979	1980	1981	1982
January	.0039	.0053	.0108	.0159	.0218
February	.0038	.0049	.0103	.0144	.0199
March	.0044	.0047	.0116	.0134	.0183
April	.0044	.0042	.0144	.0131	.0166
May	.0039	.0039	.0229	.0173	.0146
June	.0036	.0042	.0228	.0156	.0134
July	.0032	.0053	.0206	.0142	.0127
August	.0031	.0064	.0186	.0218	.0121
September	.0038	.0095	.0170	.0197	.0152
October	.0061	.0132	.0159	.0204	.0263
November	.0066	.0121	.0144	.0263	.0247
December	.0059	.0121	.0175	.0241	-

* The volatility measures are estimates of the annualized standard deviations of mortgage values for one-year term instruments based on the last week of the month.

IV. Portfolio Hedging Model

As shown in the previous section, it is not necessary for a hedging instrument utilized by a MRI insurer to exactly match the characteristics of the MRI policy. By adjusting the insurance hedge ratio, it is possible to hedge a MRI position with any highly correlated instrument. Given the wide variety of hedging alternatives available to the insurer and the likelihood that the liquidity of any one specific market will not be sufficient to totally satisfy the potential MRI hedging requirements, it is useful for the insurer to have a hedging model to assist in portfolio decisions. In this final section the study outlines the basic structure of such a MRI portfolio hedging model.

Let I_r^j be defined as the the relationship between a change in the value of a specific type j of MRI policy (e.g., 5-year coverage with 2% deductible) and a change in the mortgage interest rate, r.

$$I_r^j = \frac{\partial I^j}{\partial r} \tag{9}$$

If w^j policies of type j have been written, then the combined hedging requirement for the MRI policies is $w^j I_r^j$. The total hedging position across all types of policies is simply the sum for each policy type.

$$I_r = \sum_{j=1}^{J} w^j I_r^j \tag{10}$$

I_r is the total dollar amount that should be hedged to offset the interest rate risk in all the MRI policies.

Similarly, from the hedging instrument or asset perspective, we can define the relationship between the asset, k, and the mortgage interest rate to be

$$A_r^k = \frac{\partial A^k}{\partial r} \tag{11}$$

where A^k is an asset whose value is correlated with mortgage rates and could include options, financial futures and other hedging instruments.

To be perfectly hedged against changes in interest rates, the total hedging positions in assets A_r^k should equal the MRI risk exposure, I_r.

$$\sum_{k=1}^{K} a^k A_r^k = A_r = I_r \tag{12}$$

where the a^k are the dollar amounts of the individual hedge assets.

While the w^js are given by the amounts of each type of insurance written, the a^ks are decision variables for the MRI insurer. If the insurer's only concern is to eliminate interest rate risk, then the portfolio weights a^k can be chosen to minimize losses on the hedge positions. It can be shown that the optimal portfolio weights are those values where the price of a unit of risk reduction is equalized across the hedging assets.[16]

Capital markets in a world without transaction, liquidity or information costs would equalize these prices in equilibrium. If markets were always in such an equilibrium, an MRI insurer could choose any of the hedging assets with identical results. However, these costs can be of significance in the short run. In such a world the objective is to choose the portfolio of assets which minimizes these costs.

Transaction and liquidity costs can arise from two sources. First, there are direct costs to the hedger from executing the trade: commissions, carrying charges, bid-ask spreads. These direct costs are easily calculated. The second component is the implicit costs (positive or negative) arising from the difference between an asset's price and the costless arbitrage price. That is, even where arbitrage is possible (e.g., between futures and spot markets), prices do not completely equalize to their theoretical values because arbitragers only enter when trades occur outside a price interval determined by arbitrage costs. Since the option or futures asset can trade above or below the costless arbitrage price, these implicit costs can be either positive or negative.

As a simple method of estimating these implicit costs, we develop the concept of the implied risk free rate. As discussed earlier, the price of an option depends upon the option exercise price, the asset price, the expected variance, the time to expiration and the risk free rate (r^f). If we have observed values for the five determinants, we can derive the option premium. In addition, if we can observe the option premium and four of the determinants, it is possible to calculate the value of the fifth determinant implied by the option pricing model. The implied risk free rate (r^i) can be derived in this manner. The implied risk free rate is the rate that causes the option to be worth its market price. If the implied rate is above the existing risk free interest rate (e.g., the rate on Treasury bills), then the put (call) option is selling for less (more) than its theoretical arbitrage price. Therefore, buying the put (shorting the call) is more advantageous than shorting the underlying asset. A measure of this advantage is $r^f - r^i$.

Similarly in the case of financial futures contracts, arbitrage implies that futures and spot prices are related by

$$F = B(1 + r^f - c)t \qquad (13)$$

where F = futures price; B = asset price; c = yield on the asset; and, t = time to expiry of the futures contract. In this case the implied risk free rate is

$$r^i = \frac{F - B(1-c)t}{Bt} \qquad (14)$$

When the implied risk free rate in futures markets is above the market rate, the futures price is above its theoretical value and taking a short position in the futures is superior to shorting the spot instrument.

It is now possible to sketch the framework of a portfolio hedging strategy. Suppose asset k has an annualized rate of transaction costs of T_k and an annualized rate of liquidity costs of L_k. The total cost of hedging with asset k then is

$$H_k = T_k + L_k + (r^f - r^i_k) \qquad (15)$$

At any given moment for a particular set of prices, the H_ks can be ordered from low to high and the lowest cost asset can be choosen to hedge the MRI positions. If the market activity of the MRI insurer is significant in a certain hedging instrument, the activity may begin to influence the price of the lowest cost asset. If so, the change in price may alter the ranking and create an alternative lowest cost hedge. In such a case the insurer could cease activity in the first asset and switch to the new low cost asset.

By following such portfolio strategy it should be possible to minimize the cost of hedging the interest rate risk inherent in MRI. This basic model can be extended to incorporate other factors, as well as computerized to simply the hedging procedures of the insurer. Such a portfolio hedging model should improve the overall effectiveness of the MRI risk management process.

FOOTNOTES

1. An extensive analysis of mortgage rate insurance is contained in G. W. Gau and D. R. Capozza, "Mortgage Rate Insurance: Overview, Risk Management and Pricing", study prepared for Canada Mortgage and Housing Corporation, June, 1982. Two background references regarding the overall concept of mortgage rate insurance are G. G. Kaufman, "The Case for Mortgage Rate Insurance," Journal of Money, Credit and Banking, 7 (November 1975): 515-519; and R. Edelstein and J. Guttentag, "Interest Rate Change Insurance and Related Proposals to Meet the Needs of Home Buyers and Home Mortgage Lenders in an Inflationary Environment," Capital Markets and the Housing Sector: Perspectives on Financial Reform, edited by R. M. Buckley, J. A. Tuccillo, and K. E. Villani (Cambridge, Massachusetts: Ballinger Publishing Company, 1977).

2. For an excellent background discussion of options, see Chapter 14 in W. F. Sharpe, Investments (Englewood Cliffs, N.J.: Prentice-Hall, 1978).

3. Examples of mortgage market participants who might seek protection from falling mortgage interest rates include: (1) mortgage bankers undertaking forward commitments to sell mortgages at a specified rate; (2) financial intermediaries with short-term mortgages; (3) property owners planning to sell their assets in the future with vendor financing; and, (4) prospective home buyers attempting to hedge against rising house prices resulting from declining mortgage rates.

4. It should be noted that the purchaser of an option pays a single premium to the writer and receives only one payout when the option is exercised. Unlike a financial futures hedge there are no margin requirements and no daily "marking to the market" of the position of the option buyer (both do apply, however, to the option writer).

5. A general analysis of options on financial futures contracts can be found in J. C. Sinquefield, "Understanding Options on Futures", Mortgage Banking, 42 (July 1982): 34-40.

6. It should be noted that U.S. Treasury bond prices are quoted as a percent of par with a minimum price movement being 1/32 of a point (one 32nd = $31.25 per $100,000 bond). For instance, a price quotation of 97-06 means a market price of $97,187.50 for a $100,000 bond.

7. Open interest is the number of option contracts at any given time which have not yet been exercised or offset by an opposite option transaction.

8. The values shown in Tables 1-3 are the intrinsic values of MROs at various strike rates assuming the specified index rate is the current market level of the index.

9. F. Black and M. Scholes, "The Pricing of Options and Corporate Liabilities", Journal of Political Economy, 91 (May-June 1973): 637-654. For a discussion of adaptations of the basic model for the pricing of options on various types of commodities, consult F. Black, "The Pricing of Commodity Contracts", Journal of Financial Economics, 3 (January/March

1976): 167-179; and M. R. Asay, "A Note on the Design of Commodity Option Contracts", Journal of Futures Markets, 2 (Spring 1982): 1-7.

10. The option value (O) is the exercise value of the put option and the insurance value (I) is the claim amount paid at renewal.

11. This relationship also assumes that the mortgage rate utilized in deriving MRI claims is equivalent to the mortgage rate index that determines MRO exercise values.

12. These IHRs are based on the changes in the present values of a mortgage with a 15% contract rate given a 1% change in mortgage interest rates.

13. This analysis also applies to other U.S. options on debt instruments such as the GNMA option that has been proposed by the CBOE. For a description of this option, see W. E. Long and T. N. Rzepski, "The Exchange-Traded GNMA Option", Mortgage Banking, 40 (September 1980): 34-38.

14. Such a procedure might also be benefical to a MRI insurer for moral hazard reasons. There is the potential in MRI for collusion between the mortgage lender and borrower whereby the renewal rate on a covered mortgage is set higher than market rates in return for the borrower receiving some other contractural benefits. The tying of MRI claims to a mortgage rate index reduces this hazard.

15. When hedging Canadian mortgage positions with U.S. Treasury bond futures, empirical tests indicate a significant improvement in hedging effectiveness

is possible through the addition of a currency hedge. For example in the instance of a 26-week hedge, the R^2 increases from .183 to .317 when a Canadian dollar futures position is combined with a Treasury bond futures position. For further informaton on these empirical tests, see G.W. Gau and D.R. Capozza, "Mortgage Rate Insurance: Overview, Risk Management, and Pricing".

16. Let R^k be the expected return on A_k. Then the total expected return on A_r is

$$R = \sum_{k=1}^{K} R^k a^k$$

Our problem is to choose a^k to maximize R subject to $I_r=A_r$. The k+1 first order conditions of such a maximization imply

$$\frac{R^1}{A_r^1} = \frac{R^2}{A_r^2} = \ldots\ldots = \frac{R^k}{A_r^k}$$

The optimal portfolio weights, a^k, therefore are those values such that the ratios of the expected returns to the asset interest rate sensitivity are equal. This condition can be interpreted as saying that the price of a unit of risk reduction among the assets should be equalized.