© Journal of Business & Economic Statistics, Vol. 2, No. 4, October 1984

Comment

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1. INTRODUCTION

This is a very interesting article that gives an excellent historical survey on seasonal adjustment. Bell and Hillmer pose several important questions on the validity of criteria used to evaluate seasonal-adjustment methods belonging to different classes, particularly the common practice of measuring revisions. In this regard, one of us (Dagum 1981) wrote, "the comparison of revisions of seasonal adjustment methods is useless if the methods have very dissimilar central filters. We must first decide which is the optimal central filter according to some well defined criteria" (p. 34). We are, therefore, very pleased to see that the authors share a similar concern and make a contribution in this direction by proposing a criterion to evaluate the consistency of the basic assumptions of a method with the information content in the data.

There are, however, limitations—some more fundamental than others—on the validity and applicability of their criterion that we would like to comment on.

First, the criterion proposed in this article evaluates the basic assumptions implied by *only the symmetric filters of a method and ignores how consistent its asymmetric filters are with the information in the series.* The asymmetric filters are used for the adjustment of current observations and those of recent years, which are the most important for current economic analysis (one of the main reasons for seasonal adjustment).

A second important constraint is that the proposed criterion *can never be achieved in practice* and thus is of little value to assess the empirical performance of seasonal-adjustment methods.

Bell and Hillmer doubt the usefulness of criteria based only on adjusted data and support the view that it is preferable to spend efforts in evaluating the assumptions behind the methods. Our view on this matter is that theory and practice should not be treated as conflicting in nature but rather as complementary, and any criterion or a set of criteria would have to take both into account to be useful to assess the adequacy of seasonal-adjustment methods.

A third limitation is introduced in the application of the criterion due to the arbitrary assumptions made on the behavior of the seasonal component. The definition of this component restricts the class of ARIMA models that can be decomposed by their signal-extraction method to the class with a $(0, 1, 1)_s$ seasonal part.

Finally, we found some errors in the illustrative example as follows:

1. Their signal-extraction approximation of the weights of the nonseasonal component of the central X-11 gives a frequency-response function of the filter that is different from the one of Cleveland and Tiao (1976) and, a fortiori, from that of X-11.
2. Their canonical decomposition of the $(1, 1, 0)(0, 1, 1)_{12}$ model fitted to the example does not satisfy the conditions for a canonical decomposition as given in Hillmer and Tiao (1982).
3. We found that an ARIMA (1, 1, 0)(0, 1, 2) model approximates the information content of the example series better than the $(1, 1, 0)(0, 1, 1)_{12}$ model identified by the authors.

2. DEFINITION OF COMPONENTS, MODEL IDENTIFICATION, AND THE CRITERION OF CONSISTENCY WITH THE DATA

The arbitrary assumptions 7 and 8 (Section 4.2) limit the ARIMA models that can be adequately decomposed by the authors' signal-extraction procedure to the class with an IMA $(0, 1, 1)_s$ seasonal part. If the ARIMA model for a series Z_t is, for example, a $(0, 1, 1)(0, 1, 2)_{12}$, it can be shown that the authors' decomposition method gives the *unreasonable result of a nonseasonal component N_t with seasonality.* In effect, the model decomposition must be such that

$$f_Z(B) = f_S(B) + f_N(B); \qquad (1)$$

that is,

$$\frac{\theta^*(B)\theta^*(F)\sigma_a^2}{\phi^*(B)\phi^*(F)} = \frac{\theta_S(B)\theta_S(F)\sigma_b^2}{\phi_S(B)\phi_S(F)} + \frac{\theta_N(B)\theta_N(F)\sigma_c^2}{\phi_N(B)\phi_N(F)}.$$

For the $(0, 1, 1)(0, 1, 2)_{12}$ model, it becomes

$$\frac{(1-\theta B)(1-\theta F)(1-\Theta_1 B^{12}-\Theta_2 B^{24}) \cdot (1-\Theta_1 F^{12}-\Theta_2 F^{24})\sigma_a^2}{(1-B)(1-F)(1-B^{12})(1-F^{12})}$$

$$= \frac{\theta_S(B)\theta_S(F)\sigma_b^2}{(1+B+\cdots+B^{11})(1+F+\cdots+F^{11})} + \frac{\theta_N(B)\theta_N(F)\sigma_c^2}{(1-B)^2(1-F)^2}. \quad (2)$$

After multiplying both sides of Equation (2) by the denominator of the left side, it becomes

$$(1-\theta B)(1-\theta F)(1-\Theta_1 B^{12}-\Theta_2 B^{24}) \cdot (1-\Theta_1 F^{12}-\Theta_2 F^{24})\sigma_a^2 = (1-B)^2(1-F)^2\theta_S(B)\theta_S(F)\sigma_b^2 + (1+B+\cdots+B^{11}) \cdot (1+F+\cdots+F^{11})\theta_N(B)\theta_N(F)\sigma_c^2. \quad (3)$$

The left side of (3) is a polynomial of degree 25 in B and F. Because of arbitrary assumption 8, the first member of the right side can be at maximum a polynomial of degree 13. Therefore, for the equality to hold, $\theta_N(B)$ must be of degree 14 in B, which implies that the nonseasonal component would have seasonality.

Similarly, it can be easily shown that because of assumption 7, the autoregressive operator of N_t will be seaonal if the ARIMA model is such that $P + D > 1$.

The authors give explanations for their preferences for the strong assumptions 7 and 8, but it would be interesting to know what the consequences for their decomposition method will be if assumption 7 is changed to the order of $\phi_s(B) \leq s(P + D) - 2$ and assumption 8 is changed to $\theta_s(B) \leq sQ - 1$. These two new assumptions will avoid the presence of seasonality in the nonseasonal component and thus eliminate what we consider unnecessary restrictions on the use of their decomposition method for the criterion.

It is true that a large number of series will be well fitted by ARIMA models that have an IMA (0, 1, 1)$_s$ seasonal part, but the existence of series that follow an IMA (0, 1, 2)$_s$ for the seasonal is also not too uncommon—at least not so infrequent as to be ignored. In

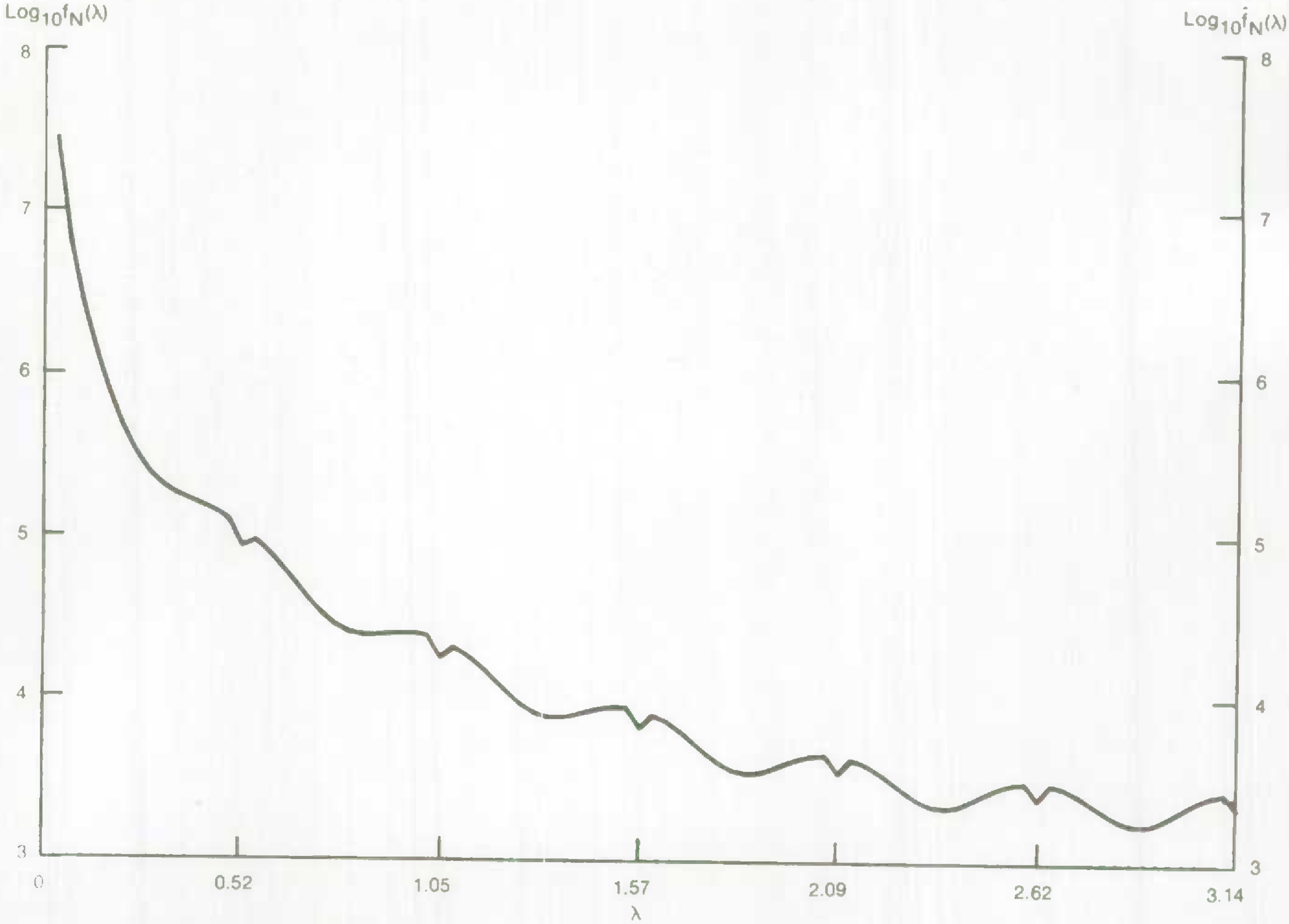

Figure 1. Canonical Implied Spectral Density for the (1, 1, 0) × (0, 1, 2)$_{12}$ Model (ϕ = .26, Θ_1 = .76, and Θ_2 = .12) of the Employed Nonagricultural Males, 20 and Older, Series.

fact, the series of Employed Nonagricultural Males, 20 and Older, chosen for the example is a case in point. A $(2, 0, 1)(0, 1, 2)_{12}$ ARIMA model was fitted to this series for the period January 1964–December 1977 by Dagum (1978). We were rather surprised to see that the seasonal part of the model changed to a $(0, 1, 1)_{12}$ during the period January 1965–August 1979. We knew through our experience that series extended with a few years can easily change the nonseasonal part of the model (particularly if the economic cycle enters a new phase) but not so the seasonal part (note that here we are referring to the model and not to the parameter estimates that can be sensitive to new observations). We then estimated the $(1, 1, 0)(0, 1, 1)_{12}$ model identified by Bell and Hillmer and a $(1, 1, 0)(0, 1, 2)_{12}$ for the example series using unconditional least squares. The results for the $(1, 1, 0)(0, 1, 1)_{12}$ model are very close to those given in the article—that is,

$$(1 - .26B)(1 - B)(1 - B^{12})Z_t = (1 - .88B^{12})a_t$$
$$\hat{\sigma}_a^2 = 16028,\ Q(12) = 9.8,$$
$$Q(24) = 19.1,\ Q(36) = 31.6. \quad (4)$$

The results for the $(1, 1, 0)(0, 1, 2)_{12}$ model are

$$(1 - .26B)(1 - B)(1 - B^{12})Z_t = (1 - .76B^{12} - .12B^{24})a_t$$
$$\hat{\sigma}_a^2 = 15732,\ Q(12) = 6.6,$$
$$Q(24) = 16.2,\ Q(36) = 24.8. \quad (5)$$

None of the estimated autocorrelations of the residuals are larger than two standard errors (.157) in both cases. Clearly, model (5) fits better and thus we can say that it approximates the information content of the series better than model (4). If we now test the proposed criterion of consistency with the data using (a) the $(1, 1, 0)(0, 1, 2)_{12}$ model for Z_t and (b) an approximation of the frequency-response function of the seasonal-adjustment filter corresponding to a $(1, 1, 0)(0, 1, 1)_{12}$ (assuming that $\Theta_2 = .12$ can be ignored), the implied spectral density of the nonseasonal component (see Figure 1) shows undesirable peaks at each band of the seasonal frequencies—an indication that the nonseasonal component has seasonality. Consequently, if the ARIMA model that best approximates the information content of a series does not belong to the class that can be adequately decomposed by Bell and Hillmer's signal extraction procedure, their criterion of consistency will no longer be applicable.

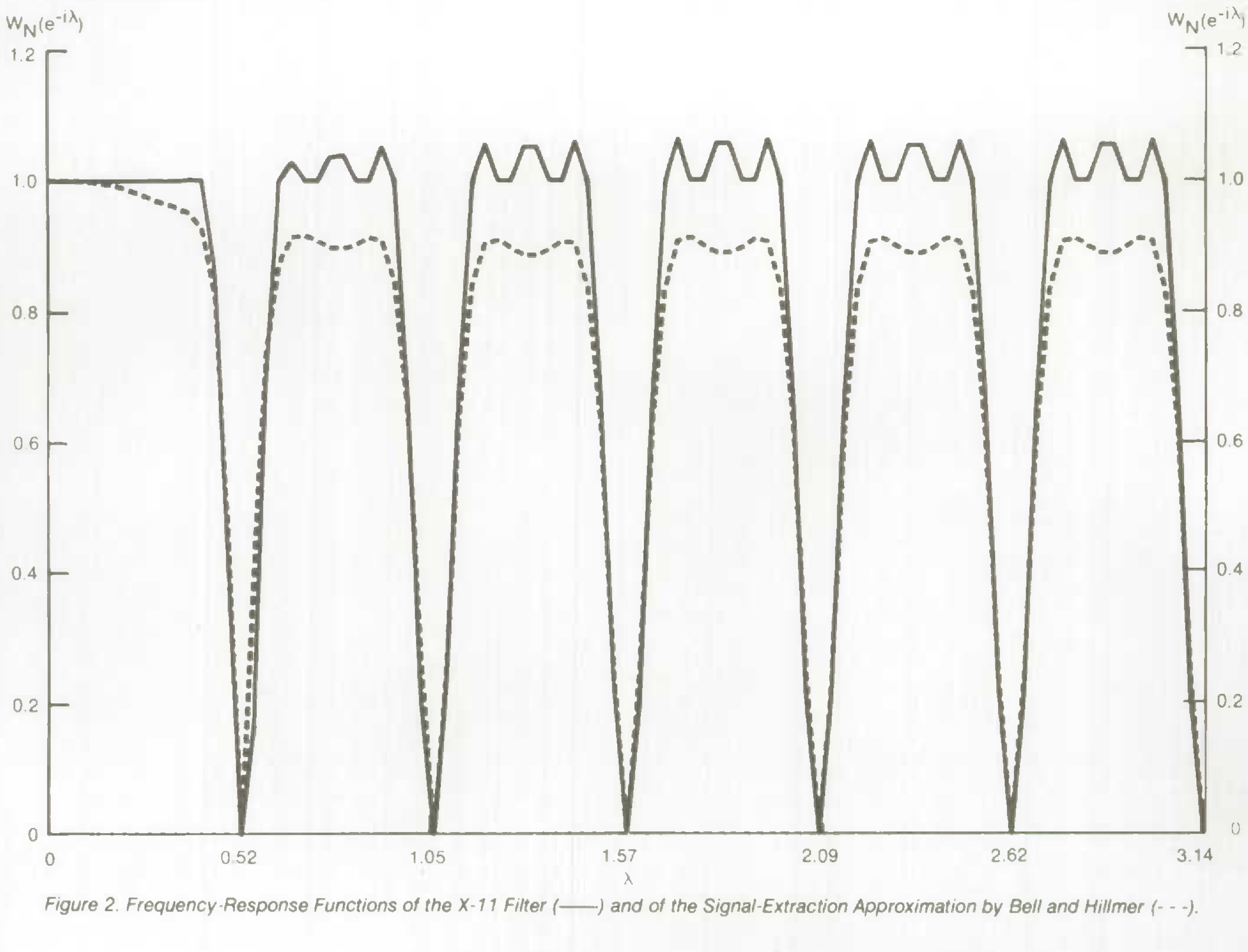

Figure 2. Frequency-Response Functions of the X-11 Filter (——) and of the Signal-Extraction Approximation by Bell and Hillmer (- - -).

3. SIGNAL EXTRACTION APPROXIMATIONS OF THE X-11 CENTRAL FILTERS

The approximation of the nonseasonal-component extraction filter of X-11 implicit in Bell and Hillmer's equation (8) differs significantly from the one given by Cleveland and Tiao (1976). The frequency-response functions of both approximations and of the central X-11 filter (13-term Henderson and 3 × 5 moving averages) are shown in Figures 2 and 3. These frequency-response functions were calculated using the ratio

$$\frac{f_n(\lambda)}{f_z(\lambda)},$$

where $f_z(\lambda)$ is obtained from Bell and Hillmer's (9). The corresponding root mean squared errors are given in Table 1.

The presence of dips in the implied spectral density for the nonseasonal component of the central X-11 filter shown in the example results mainly from the fact that the 3 × 5 seasonal filter implies a seasonality that moves faster than the one corresponding to the ARIMA model fitted by Bell and Hillmer with $\Theta = .88$. We conjecture that the use of the central X-11 filters resulting from a longer seasonal moving average (MA) such as the 3 × 9 MA will eliminate the dips. Furthermore, it should be pointed out that the standard option of X-11 for this series is the combination of the 3 × 5 MA with the nine-term Henderson filter, which is more flexible than the 13-term Henderson filter used in both approximations.

Table 1. Root Mean Squared Errors of Standard X-11 Central-Filter Approximations

X-11 Central Filters Approximation From Signal Extraction Theory	RMSE of the Frequency Response Functions
Bell and Hillmer	.105
Cleveland and Tiao (1976)	.051

4. CANONICAL DECOMPOSITION OF THE $(1, 1, 0)(0, 1, 1)_{12}$ MODEL

The canonical decomposition of the $(1, 1, 0)(0, 1, 1)_{12}$ model does not satisfy the conditions for the canonical decomposition as given by equation 4.2 in

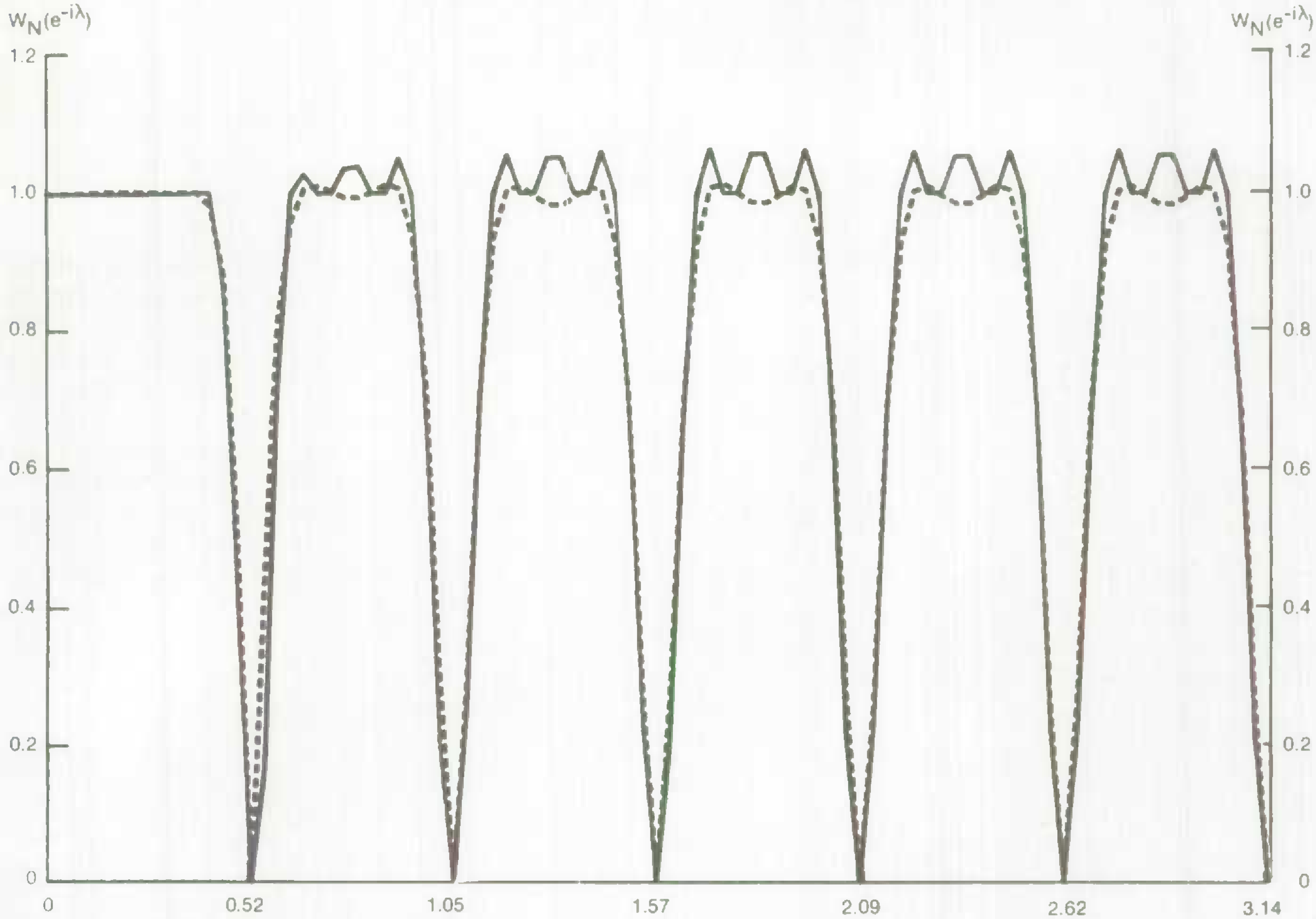

Figure 3. Frequency-Response Functions of the X-11 Central Filter (——) and of the Signal-Extraction Approximation by Cleveland and Tiao (1976) (- - -).

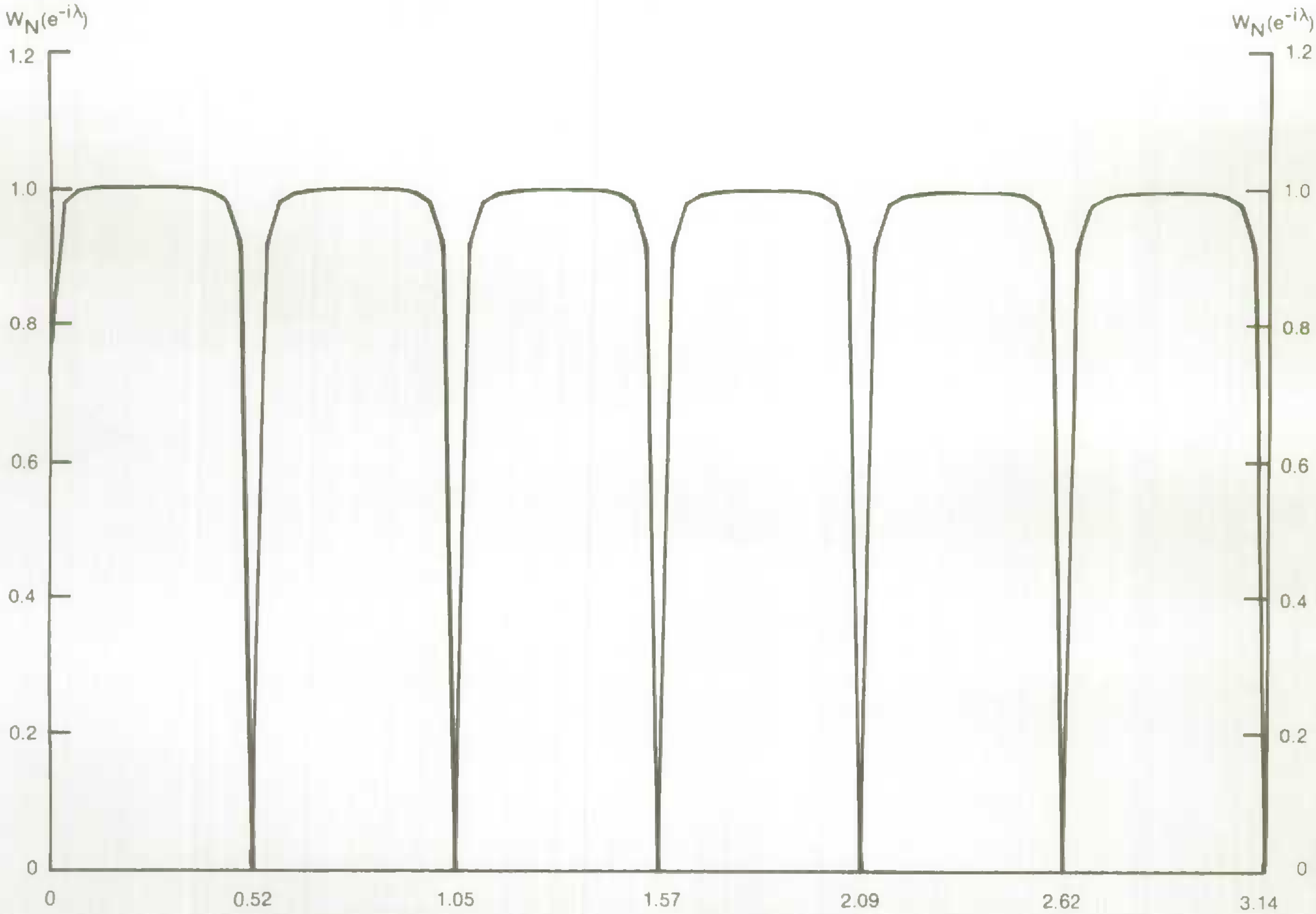

Figure 4. Frequency-Response Function of the Canonical Seasonal-Adjustment Filter of Bell and Hillmer for the (1, 1, 0) × (0, 1, 1)₁₂ Model (φ = .26, Θ = .88).

Hillmer and Tiao (1982). In effect, using this latter equation and after some algebraic transformations, the canonical decomposition must satisfy

$$\frac{144\theta_N^2(1)\sigma_c^2}{\theta^{*2}(1)\sigma_a^2} = 1 \qquad \text{at } \lambda = 0 \tag{6}$$

and

$$\frac{16\phi^2(-1)\theta_S^2(-1)\sigma_b^2}{\theta^{*2}(-1)\sigma_a^2} = 1 \qquad \text{at } \lambda = \pi . \tag{7}$$

Condition (6) implies that the sum of the weights of the seasonal-adjustment filter must be equal to one to preserve the level of the original series. Using the values of the estimated ARIMA models

$$Z_t = \frac{(1 - .88B^{12})a_t}{(1 - .26B)(1 - B)(1 - B^{12})}, \; \hat{\sigma}_a^2 = 16, 150,$$

and

$$N_t = \frac{(1 - .990B - .001B^2)}{(1 - .26B)(1 - B)^2} C_t, \; \hat{\sigma}_c^2 = 14, 412,$$

in the example, we obtain .722831, which means that the level of the seasonally adjusted series by the canonical decomposition given in Bell and Hillmer's (13) will be 72.3% of the original series. Figure 4 shows this value for the frequency-response function of the author's filter at $\lambda = 0$. Furthermore, distortions will be introduced in the estimates of the trend cycle because its frequency-response function differs significantly from the unity at the low frequencies. Similarly, it can be shown that condition (7), which implies that the frequency response of the seasonal filter is equal to one at $\lambda = \pi$, is not satisfied.

ADDITIONAL REFERENCE

DAGUM, E. B. (1981), "Comments on A Comparative Study of the X-11 and BAYSEA Procedures of Seasonal Adjustment," by H. Akaike and M. Ishiguro, in *Proceedings of the Conference on Applied Time Series Analysis of Economic Data, American Statistical Association, U.S. Bureau of the Census, and National Bureau of Economic Research*, 31–35.