BANK OF CANADA
BANQUE DU CANADA

Working Paper/Document de travail
2013-3

Real-financial Linkages through Loan Default and Bank Capital

by Tamon Takamura

Bank of Canada Working Paper 2013-3

January 2013

Last updated: July 2020

Real-financial Linkages through Loan Default and Bank Capital

by

Tamon Takamura

Financial Stability Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
ttakamura@bankofcanada.ca

Bank of Canada working papers are theoretical or empirical works-in-progress on subjects in economics and finance. The views expressed in this paper are those of the author. No responsibility for them should be attributed to the Bank of Canada.

ISSN 1701-9397 © 2013 Bank of Canada

Acknowledgements

This paper was circulated under the title “A General Equilibrium Model with Banks and Default on Loans." I am indebted to Sami Alpanda, Gino Cateau, Pablo D'Erasmo, Bill Dupor, Paul Evans, Aubhik Khan, Yaz Terajima and Julia Thomas for helpful discussion, comments and suggestions.

Abstract

Many studies in macroeconomics argue that financial frictions do not amplify the impacts of real shocks. This finding is based on models without endogenous default on loans and bank capital. Using a model featuring endogenous interactions between firm default and bank capital, this paper revisits the propagation mechanisms of real and financial shocks. The model, calibrated to the US economy, shows that real shocks translate into a financial problem and cause persistent business cycle fluctuations through counter-cyclical firm default and interest-rate spread. Consistent with the previous studies, financial shocks lead the economy into booms and recessions, notably during the US financial crisis. Capital injections to banks through the Troubled Asset Relief Program were an effective policy response for mitigating the vicious cycle between loan default and interest-rate spread.

Topics: Financial institutions; Financial stability; Financial system regulation and policies; Interest rates
JEL codes: E32, E44, E69

Résumé

D'après de nombreuses études macroéconomiques, les frictions financières n'amplifieraient pas les répercussions des chocs réels. Cependant, cette constatation est fondée sur des modèles qui ne tiennent pas compte des défauts de paiement et des fonds propres bancaires endogènes. À partir d'un modèle intégrant des interactions endogènes entre défaillances d'entreprises et fonds propres bancaires dans l'économie américaine, la présente étude revisite les mécanismes de propagation des chocs réels et des chocs financiers. Les résultats montrent que les chocs réels deviennent un problème financier donnant lieu à des fluctuations persistantes du cycle économique du fait de défaillances d'entreprises et d'écarts de taux d'intérêt contracycliques. Conformément aux conclusions des autres études, les chocs financiers entraînent l'économie dans des cycles d'expansion et de récession, comme pendant la crise financière aux États-Unis. Les injections de capitaux que le gouvernement a alors offertes aux banques par l'intermédiaire du Troubled Asset Relief Program ont été efficaces pour combattre le cercle vicieux où les défauts de paiement mènent à des écarts de taux d'intérêt et vice-versa.

Sujets : Institutions financières; Stabilité financière; Réglementation et politiques relatives au système financier; Taux d'intérêt
Codes JEL : E32, E44, E69

1 Introduction

Many studies in macroeconomics argue that the effects of real shocks do not get amplified through financial frictions, whereas those of financial shocks do. Among others, Kocherlakota (2000) demonstrates that the financial frictions discussed in Kiyotaki and Moore (1997) only weakly propagate the effects of real income shocks under plausible parameter values. Khan and Thomas (2013), who generalize Kiyotaki and Moore (1997) to quantitatively examine the amplification of a large collateral (financial) shock, also find the same result for aggregate productivity shocks. Jermann and Quadrini (2009, 2012) show a striking result that financial shocks that affect firms' incentive constraints for taking loans account for a much larger fraction of business cycle fluctuations in aggregate output and hours worked than aggregate productivity shocks.[1]

However, these findings are based on models that do not take into account firm default on loans and bank capital. An important objective of this paper is to revisit the propagation mechanisms of real and financial shocks by adding an endogenous interaction between loan default and bank capital to a dynamic stochastic general equilibrium (DSGE) model.[2] Through this channel, the effect of real shocks can translate into financial problems. More specifically, a fall in the average efficiency of production increases the loan default rate, which impairs bank capital. The reduced bank capital, in turn, could increase the interest-rate spread, which further raises the equilibrium rate of loan default.

The consideration of both bank capital and loan default is important for analyzing both business cycle fluctuations of the US economy and policy responses in recent decades. In particular, during the Great Recession, commercial banks sharply reduced new business lending[3] amid grow-

[1]Iacoviello (2015) also highlights the importance of financial shocks during the Great Recession. Christiano et al. (2014) find that time-varying uncertainty in the cross-sectional distribution of entrepreneurs' productivities is the most important driver of the business cycle fluctuations.

[2]An exception is Dib (2010). This paper builds upon the costly-state-verification model of Bernanke et al. (1999) and introduces bank net worth. Dib (2010) analyzes the propagation mechanism in the environment where banks hold equity to meet the regulatory requirement and shows that the effects of shocks are attenuated through financial frictions.

[3]Ivashina and Scharfstein (2010) document that the total amount of US corporate loans issued by large commercial banks fell sharply after mid-2007. Moreover, Koepke and Thomson (2011) explain that credit channels declined sharply in the banking sector in 2008 and 2009, followed by a sluggish recovery in 2010.

ing concerns about loan default and the resulting under-capitalization of banks. By 2009, the failure rate on business loans had jumped to almost 3.6%, more than double the historical average of 1.76%, while bank net worth fell to more than 5% below trend during the second half of 2008.[4] On the production side, the number of firms in business fell substantially, in part due to unfavorable financial market conditions.[5] The sharp decline in business lending and bank capital prompted immediate and unprecedented policy responses from the authorities. By the end of 2009, the US Treasury Department had provided banks with more than 200 billion US dollars in capital through the Troubled Asset Relief Program (TARP).

In order to analyze the implications of the endogenous interaction between loan default and financial frictions for the propagation of the effects of real and financial shocks in the US economy, this paper poses the following questions. First, how do the effects of real and financial shocks propagate through financial frictions? Second, what are the contributions of real and financial shocks in explaining the observed US business cycle fluctuations before and after the financial crisis? Finally, what are the implications of this interaction for the capital injections to banks through TARP?

To answer these questions, I develop a model which introduces the interaction between loan default and bank capital by building upon Gertler and Karadi (2011). Specifically, there are three major types of agents–households, banks and non-financial firms–where banks raise deposits from households and provide firms with loans for setting up projects in the next period. As in Gertler and Karadi (2011), a lack of commitment on the part of banks to repay depositors requires banks to hold net worth against deposits. These deposit frictions lead to an inefficiently low supply of business loans, which drives loan rates above deposit rates. In this framework, real shocks affect the average efficiency of firms to produce goods while financial shocks directly impact the deposit frictions for banks, reducing the collateral value of bank net worth. My paper distinguishes itself

[4]The loan failure rate in this paper is measured by the non-current rate of commercial and industrial (C&I) loans, while bank net worth is measured by real Tier 1 capital of institutions affiliated to the Federal Deposit Insurance Corporation (FDIC). Loans with the non-current status are 90 days or more past due or are no longer accruing.

[5]Siemer (2019) finds that the number of firms cumulatively declined by about 5% during the financial crisis. He also shows that financial constraints had an important impact on young firms and firms in the extensive margins.

from Gertler and Karadi (2011) in that business loan contracts between banks and firms are used for project formation and involve the risk of default. Endogenous default leads to a risk premium, which increases the interest-rate spread even when the demand for loans decreases. Moreover, bank net worth changes in response to loan default, which further affects the interest-rate spread through deposit frictions and, thus, loan default in equilibrium. It is this interaction between loan default and deposit frictions that allows the effects of shocks originated in the real sector to propagate into the financial sector, and vice versa.

Using this framework, this paper conducts the following analyses. First, I derive responses that follow a real shock and a financial shock, which endogenously change default rates and banks' capital requirements, to examine the propagation mechanism of these two shocks. Second, I estimate real and financial shocks from the US data using a Bayesian estimation method and examine the quantitative importance of real and financial shocks in driving business cycles. When estimating the model, policy shocks represent capital injections, which are identified by matching the data on TARP. This allows me to evaluate, in the next step, the effectiveness of TARP in mitigating the adverse effects of real and financial shocks during the financial crisis. More specifically, a counter-factual simulation is conducted, in which policy shocks do not materialize during the financial crisis. The contribution of TARP is measured by comparing the counter-factual output and investment series with simulated data using both structural and policy shocks. To the best of my knowledge, this is the first quantitative evaluation of TARP from a macroeconomic perspective using a general equilibrium framework.[6]

The main findings of this paper are as follows. First, I find that the interaction between loan default and bank capital plays a critical role in amplifying the effects of both shocks. In particular, endogenous loan default has an important implication for real shocks. If the loan default channel is eliminated, the absence of a default premium dampens the effects of real shocks. Moreover, without the feedback effect between loan default and deposit frictions, bank net worth recovers

[6]Gertler and Kiyotaki (2010) provide a qualitative analysis of capital injections to financial intermediaries in parallel to unconventional monetary policies. See Calomiris and Khan (2015) and the references therein for studies assessing TARP assistance.

more rapidly than in the model with default, weakening the propagation mechanism of the model even further. As a result, the interest rate spread following a real shock becomes *pro-cyclical*, in contrast to the counter-cyclical response in the model with loan default, and the peak responses in output and investment shrink substantially.

Second, through an estimation of real and financial shocks, I find that the effects of real shocks can explain long-lasting booms and recoveries in aggregate output and investment, and contribute to a large share of business cycle variations in these variables. This result that real shocks are an important source of business cycle fluctuations is in contrast to the findings of Jermann and Quadrini (2009, 2012). Even though financial frictions are central to their paper and mine, the difference in my results is driven by endogenous default on loans, which does not exist in their study. While a lower efficiency of production relaxes the incentive constraints in their paper, the loan default channel in my model reduces bank net worth and tightens deposit frictions in equilibrium. Together with the default premium, this leads to an increase in the interest-rate spread and amplifies the effect of real shocks. Regarding financial shocks, I find that these shocks are an important driving force leading business cycles to change at critical junctures. In particular, financial shocks account for a large share of the declines from trend in aggregate output and investment in early quarters of 2009. This finding is consistent with that of Jermann and Quadrini (2009, 2012) as well as other papers in the literature documenting that shocks to financial arrangements are an important driver of business cycles.

Third, I find that the interaction between loan default and bank capital has an implication for evaluating capital injection policy. The counter-factual analysis shows that real GDP would have decreased further by 7% of the actual decline in data in the second quarter of 2009, had it not been for TARP. In contrast, the measured policy effect declines significantly if the loan default is eliminated from the model. This implies that taking into account the possibility of loan failure is crucial for measuring the effectiveness of the capital injection policy, and also that such a policy is most valuable when impaired bank capital exacerbates the effect of the vicious cycle between loan default and bank capital.

This paper is part of the literature stressing the importance of financial frictions and the supply of credit. Carlstrom and Fuerst (1997) show that an aggregate productivity shock is propagated through financial frictions on capital production. However, in their model, the default rate is procyclical and any amplification is through entrepreneur net worth. Meh and Moran (2010) develop a model with bank net worth. Their paper embeds the double moral hazard problem developed by Holmstrom and Tirole (1997) in a New Keynesian DSGE framework and analyzes the propagation of shock to net worth, productivity and monetary policy. My model differs in its underlying frictions and, crucially, in the role of endogenous default in driving results. Gomes and Schmidt (2009) obtain a counter-cyclical default in the absence of financial intermediaries and focus on credit spreads for long-term bonds. Pioneering works of DSGE models with financial frictions include Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999). The latter introduces standard debt contracts based on the costly-state verification model of Townsend (1979). Iacoviello (2015) and Nuño and Thomas (2014) develop DSGE models with banks intermediating between depositors and borrowers. However, none of these papers studies the interaction between endogenous default on loans and bank net worth. A more recent paper by Ferrante (2019) has both endogenous default and deposit frictions as in my paper. My paper differs from his in at least two dimensions. First, my model features variable-interest business loans extended by commercial banks, whereas his paper focuses on marketable, long-term corporate and mortgage bonds with fixed coupon rates that are held by a broader range of financial institutions. Long-term loans with fixed interest rates imply a larger impact of shocks on bank capital as lenders bear the interest-rate risk. This would strengthen the effects of both real and financial shocks in my paper. However, the empirical evidence documented by Kumbhat et al. (2017) shows that, in the US, the majority of corporate *loans*, which are examined in this paper, offers variable interest rates.[7] Moreover, most small- and medium-scale enterprises and start-up firms have limited access to the bond market. Thus, my model is applicable to a broad range of non-financial firms that rely on bank loans for funding. This difference between his paper and mine naturally leads me to study the effect of

[7]In contrast, the majority of corporate *bonds*, issued by firms directly in the bond market, bears fixed interest rates.

financial frictions on the entry decisions of firms, which is not discussed in his paper. Second, in addition to shocks that affect deposit frictions, my paper analyzes the propagation mechanism of real shocks with a particular emphasis on the interaction between loan default and deposit frictions, whereas his model features uncertainty shocks affecting the distribution of idiosyncratic shocks on borrowers. None of the papers cited above provides the quantitative assessment of TARP.

This paper is organized as follows. Section 2 constructs the model. Section 3 presents calibration and estimation results. Section 4 discusses the model mechanics through impulse response functions. Section 5 explains the estimated shocks and examines the relative contribution of real and financial shocks. Section 6 evaluates the effectiveness of capital injections by TARP. Finally, Section 7 concludes.

2 Model

There are four types of private agents in the economy: households, firms, commercial banks and non-bank financial companies. Households earn wages from firms, rental income from capital, interest income from bank deposits and dividends from firms, banks and non-bank financial companies. They purchase goods for consumption from firms and save through bank deposits or by holding physical capital. Firms operate one-period projects in different locations by renting capital, k_f, and labor, l_f, to produce output, y_f, in competitive factor and output markets. The mass of potential firms has a unit measure. All projects shut down after production and are replaced by new ones. Firms at each project location decide whether to implement their projects in the next period. Project implementation requires paying a fixed cost, κ, at each location and banks are assumed to be the only entity that can finance this cost. In addition, new projects must pay a random administrative labor cost that is funded by the shareholders of firms, *i.e.*, households. Banks intermediate between households and firms, and they finance loans using their net worth and bank deposits made by households. Once firms decide to operate, a lack of credibility on the part of firms makes it necessary for each of them to contract with a non-bank financial company, which requires the firm to pay the rental fee and return the undepreciated portion of capital through monitoring on behalf of the owner.

In this model, there are three types of financial frictions. First, banks have a limited commitment to repay households' deposits. This constrains the extent to which banks are leveraged. Second, bank loans have a risk of default and banks cannot fully seize the profit from a project when a firm fails to repay its debt. These frictions create wedges between the bank lending rate and the deposit rate, affecting the number of firms investing in new projects. Third, the inability of firms to make a commitment on paying rental capital costs requires an intermediation by non-bank financial companies, which generates a wedge between the rental price of capital paid by firms and that received by households. I assume that financial shocks directly affect both banks and non-bank financial companies as described below.[8]

Firms hold a continuum of ex-ante identical potential project locations. Let b denote the gross debt payment to a bank for a project that is funded. Given the state of the economy, a firm anticipates the profit of a project next period net of debt and weigh it against the administrative labor cost, $wh(\xi)$, where w is the wage rate and $h(\xi)$ is the quantity of employment that depends on a draw of an idiosyncratic random variable ξ. Only projects with $wh(\xi)$ lower than an endogenously determined threshold level, $wh(\bar{\xi})$, will be implemented. Firm projects are heterogeneous, *ex-post*, in terms of their productivity levels. Let ε be an idiosyncratic productivity level and z represent the aggregate productivity level. Given ε and aggregate states of the economy, firms in each project location produce output with a technology, $y_f = \varepsilon z F(k_f, l_f)$. Because of decreasing returns to scale, projects make profits after wages and capital rental costs, including the intermediation fee to the non-bank financial company, are paid. But since debt is predetermined, projects with low idiosyncratic productivity levels will default on loans. Insolvent projects will have zero value after

[8]As in standard financial frictions in the literature, this friction provides a channel through which financial shocks directly affect firms' capital demand, independently of the financial conditions of banks. As examined in Section 5, ignoring loan default and its impact on bank lending from my model could result in attributing much of the observed investment dynamics to changes in the rental capital wedge rather than the bank lending rate spread. This has an important implication for the analysis of relative contributions of financial shocks to real shocks.

banks confiscate any remaining gross profits.

Each bank starts each period with a number of loans made in the previous period, χ_i, and the volume of deposit, s_i. After agents learn aggregate and idiosyncratic productivity levels, financial transactions on existing loan contracts are settled and bank net worth, n, is determined. During this process, solvent projects repay b while banks liquidate insolvent projects and seize a fraction $\lambda \in (0, 1)$ of their profits, where $1 - \lambda$ represents the costs of liquidation. Before banks make new loans, some die (fraction $1-\theta$) and are replaced by new banks. At this point, the government may inject additional capital to surviving banks to support their resilience during crises. On the other hand, the start-up funds for θ new banks are provided by households. This assumption ensures that banks do not over-accumulate net worth to self-finance new loans.

Although individual banks collect deposits, s'_i, from households, banking requires net worth due to a limited commitment to repay depositors.[9] Following Gertler and Karadi (2011), I assume that banks may abscond with a fraction ψ of their funds, $s'_i + n$, if the amount of borrowing is very large relative to their net worth. This implies that banks must possess a sufficiently large stake in their assets so as to convince depositors that banks' cost of foregoing the value of implementing their business is large. Gertler and Karadi's financial friction represents the banking sector's capital requirement in a convenient way. Given the amount of funds in hand, banks choose the volume of new loans, χ'_i. Thereafter, the gross repayment amount, b', balances the supply and demand for loans.

A non-bank financial company is associated with a particular firm project and sets the intermediation fee per unit of capital to cover the cost of monitoring. I assume that the efficiency of monitoring depends on the exogenous aggregate state of the financial system, ψ, to capture potential linkages across financial institutions in a tractable way. This provides a direct channel through which financial shocks affect the intensive margin of production.

A unit measure of households derives utility from consumption and leisure and discounts future utility by $\beta \in (0, 1)$. They own firms and banks and have access to a complete set of

[9] A prime indicates a variable in the subsequent period.

state-contingent claims. The representative household's expected discounted lifetime utility is $\Sigma_{t=0}^{\infty}\beta^t u\left(C_t, 1-L_t\right)$, where C and L denote consumption and hours worked, respectively. Given an aggregate state of the economy, the household chooses consumption, hours worked and savings through deposit and capital. The representative household's individual state variables are deposit, s, and capital, k.

2.1 Firm projects

Each project operated involves renting capital and labor in competitive factor markets to produce final goods. Given productivity levels, the wage rate, and the effective rental price of capital, a firm maximizes profit subject to the decreasing returns to scale production function, $y_f = \varepsilon z F\left(k_f, l_f\right)$. Here, ε is assumed to be an $i.i.d.$ random variable and $\log\left(z\right)$ follows an AR(1) process. Since every project is one-period lived, firms solve a static optimization problem: $\max_{k_f,\, l_f}\left\{y_f - \widetilde{r}_k k_f - w l_f\right\}$, subject to the production function, where $\widetilde{r}_k$ is the effective rental price of capital for firms including the cost of monitoring by a non-bank financial company. Let $f\left(\varepsilon; \mathbf{x}\right)$ be the profit function before debt repayment and $\mathbf{x}$ be a vector of aggregate state variables. Idiosyncratic shocks cause some firms to default on their debt. More specifically, a project involves default if $f\left(\varepsilon, \mathbf{x}\right) < b$. A threshold level of default, $\underline{\varepsilon}$, is the level of idiosyncratic productivity at which projects break even after repaying their loans:

$$f\left(\underline{\varepsilon}, \mathbf{x}\right) = b.$$

After all financial transactions are made, solvent projects pay their net profit to households, while insolvent projects surrender $f\left(\varepsilon, \mathbf{x}\right)$ to banks, leaving no value to shareholders. Since profits are distributed to households only when projects are solvent, the final profit of a project is expressed as $1_{[\varepsilon>\underline{\varepsilon}]}\left(f\left(\underline{\varepsilon}, \mathbf{x}\right) - b\right)$, where $1_{[\varepsilon\geq\underline{\varepsilon}]}$ is an indicator function that takes the value of 1 if $\varepsilon \geq \underline{\varepsilon}$ and 0 otherwise.

The current generation of projects ends with production. Thereafter, firms will have a unit of new potential projects and decide whether or not to produce next period. In doing so, they compare the value of implementing a project with a random labor cost, $wh(\xi)$. A project will be implemented if the former is greater than or equal to the latter. Notice that the value of implementation involves the debt repayment, b, for a start-up loan, κ, and the interest cost of borrowing. Since the value of implementation is the expectation of a discounted final profit of a project, a threshold level, $\bar{\xi}$, is defined as

$$wh(\bar{\xi}) = \mathbb{E}\left[\beta \frac{P'}{P} \int_{\varepsilon > \underline{\varepsilon}'} \left(f\left(\varepsilon; \mathbf{x}'\right) - b'\right) d\Pi\left(\varepsilon\right)\right], \tag{1}$$

where $\Pi(\varepsilon)$ is the probability distribution of ε, P is the household's marginal utility of consumption, and $\mathbb{E}$ is an expectation over aggregate states conditional on $\mathbf{x}$.[10] The right-hand side of this equation is integrated over idiosyncratic shocks above the threshold ($\varepsilon > \underline{\varepsilon}$) because insolvent projects have no value to their owners. The condition, (1), implies a demand for loans. Let $J(\xi)$ be the probability distribution of ξ. As projects with $\xi < \bar{\xi}$ will be implemented, it follows that the demand for new loans (equivalently, the measure of firms setting up projects) is $\chi' = J\left(\bar{\xi}\right)$. Moreover, the total amount of administrative labor cost paid by households is $w \int_{\xi < \bar{\xi}} h(\xi) dJ(\xi)$.

2.2 Banks

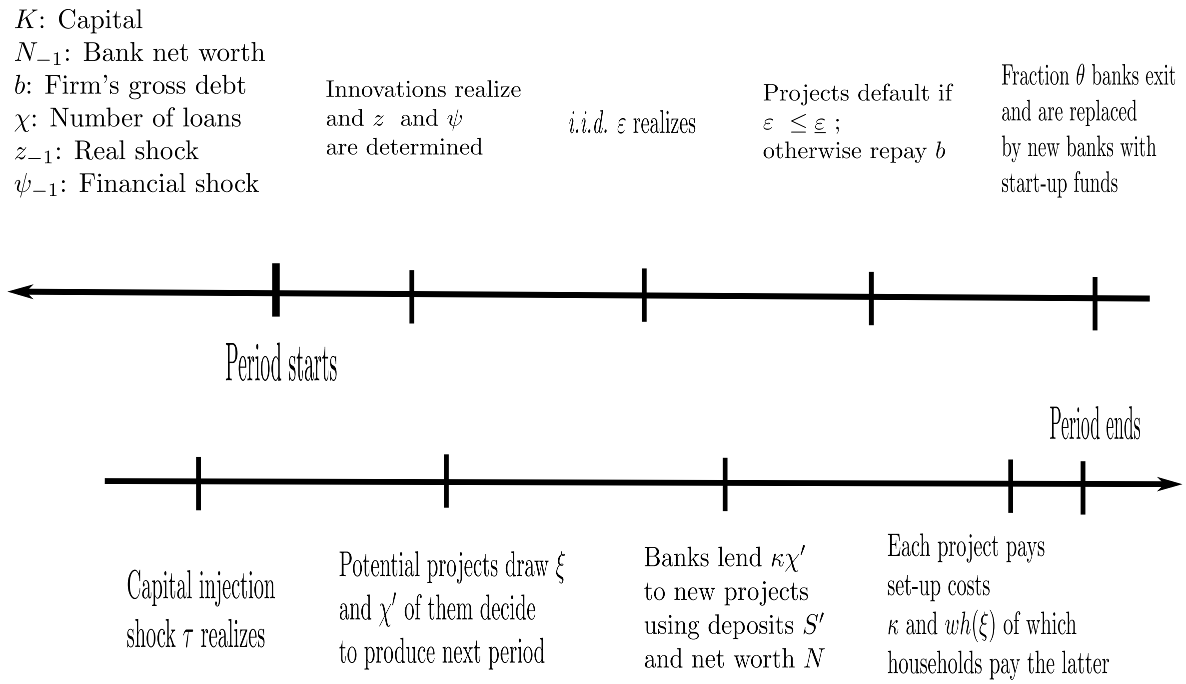
The characterization of banks in this model builds upon Gertler and Karadi (2011). The main difference is that, in this paper, banks make loans for project set-up costs subject to default risk, while, in the model of Gertler and Karadi (2011), banks hold claims on the state-contingent returns to capital held by firms. Despite the fact that banks in practice provide long-term loans, I assume one-period loans in this paper as a tractable approximation of the fact that, in the US, a significant fraction of business *loans*, which are the focus of this paper, offer variable interest rates (Kumbhat et al., 2017).[11]

The timing of events in Figure 1 is useful for understanding a bank's problem. Every period

[10] $\beta P'/P$ is the stochastic discount factor.

[11] See Ferrante (2019) for the analysis of financial institutions holding assets with longer maturity and fixed interest rates.

Figure 1: Timing of events



begins with realizations of aggregate and idiosyncratic productivity levels. The ability of firm projects to repay debt depends crucially on these levels. Since ε is $i.i.d.$, the average revenue from a loan is $V(\mathbf{x}) = [1 - \Pi(\underline{\varepsilon})]\, b + \lambda \int_{\varepsilon<\underline{\varepsilon}} f(\varepsilon, \mathbf{x})\, d\Pi(\varepsilon)$. Thus, a bank's retained earnings can be expressed as the gross interest revenue minus gross interest payments to depositors and dividend payouts to the shareholders: $V(\mathbf{x})\,\chi_i - R(\mathbf{x})\, s_i - d_B$, where R and d_B are the gross deposit rate and dividend payouts, respectively. Next, a fraction θ of banks will exit the industry and their retained earnings are distributed to shareholders. Any transfer of funds from the government, τ_i, will be made available to surviving banks, determining their net worth in this period:[12]

$$n = V(\mathbf{x})\,\chi_i - R(\mathbf{x})\, s_i - d_B + \tau_i. \tag{2}$$

As explained below, the government's capital injection to banks is a lump-sum transfer using taxpayers' money, and banks take such policy as given. I assume that τ_i is a zero-mean $i.i.d.$

[12] Technically, it is possible to consider default of banks when their net worth drops to a negative value. As we will see later, banks differ only in their size in this model, and returns to their assets are common. Thus, when bank default occurs, all banks must fail at the same time. I exclude this possibility by focusing on the local dynamics around the steady state.

idiosyncratic shock to capture the way individual troubled banks in the US received public funds through the TARP. Using the net worth and borrowings from depositors, s_i', banks finance new loans $\kappa\chi_i'$. That is, the balance sheet identity of a bank is

$$s_i' + n = \kappa\chi_i'. \tag{3}$$

Equation (3) implies that information on n_{-1} and χ_i is sufficient to know s_i. Given (3) and (2), the individual law of motion of net worth simplifies as follows:

$$n = \rho(\mathbf{x})\,\kappa\chi_i + R(\mathbf{x})\,n_{-1} - d_B + \tau_i, \tag{4}$$

where $\rho \equiv V/\kappa - R$ is the ex-post net return on a loan.

As described above, banks are not able to borrow as much as they wish because of the endogenous capital requirement, $\psi(s_i' + n) \leq B(n, \chi_i'; \mathbf{x})$, where B represents the end-of-period value of a bank while, from (3), the left-hand side of the inequality is equivalent to the value of assets a bank can seize by reneging on the deposit contract. This capital requirement can be expressed as

$$\kappa\chi_i' \leq \psi^{-1} B(n, \chi_i'; \mathbf{x}). \tag{5}$$

(5) states that banks must hold sufficient net worth relative to their assets to guarantee that deposits are risk-free in equilibrium. Here, ψ is a stochastic variable that affects the financial capital required by depositors. When ψ increases, banks are required to hold more net worth against loans. In this paper, ψ is regarded as a financial shock, and I examine how such a shock, hitting the banking sector, affects business cycle fluctuations.[13]

Given the law of motion of net worth and the capital requirement, the Bellman equation of a

[13] Jermann and Quadrini (2009, 2012) consider a similar type of incentive constraint between households and firms.

bank's problem is as follows.

$$B\left(n_{-1},\chi_i;\mathbf{x}_{-1}\right)=\mathbb{E}_{-1}\beta\frac{P}{P_{-1}}\max_{d_B,n}\left\{d_B+(1-\theta)\left(n-\tau_i\right)+\max_{\chi_i'}\theta B\left(n,\chi_i';\mathbf{x}\right)\right\} \tag{6}$$

subject to (4), (5), $d_B\geq 0$ and the law of motion of aggregate states, $\mathbf{x}=\Xi\left(\mathbf{x}_{-1}\right)$. At the end of the period, every bank has loans χ_i and net worth n_{-1}. In the next period, the bank chooses the levels of dividend payout, d_B, and net worth, n, anticipating the possibility of stochastic death, which occurs with probability θ. Capital injection takes place only when the bank survives, and, if it does, the bank chooses the quantity of new loans, χ_i'. Below, I briefly characterize the solution of a bank's problem. As shown in Appendix A, as long as (5) is expected to bind, banks do not pay out dividends: $d_B=0$. Intuitively, banks are expected to obtain returns higher than the deposit rate when the capital requirement limits the supply of loans below the efficient level. Because I focus on dynamics in the neighborhood of the steady state where (5) binds, this result always holds.

Then, as the capital requirement binds in equilibrium, I can exploit the linearity of the bank's problem to guess the solution to the value function as $B\left(n,\chi_i';\mathbf{x}\right)=g_n\left(\mathbf{x}\right)n+g_\chi\left(\mathbf{x}\right)\chi_i'$. Substituting this expression into (5) proves that the total value of loans is proportional to the bank's net worth:

$$\kappa\chi_i'=\phi\left(\mathbf{x}\right)n, \tag{7}$$

where ϕ is the leverage ratio (asset-to-net-worth ratio) that is defined as $\phi\equiv g_n/\left(\psi-g_\chi/\kappa\right)$. Given that the value function can be written as $B\left(n,\chi_i';\mathbf{x}\right)=\left[g_n\left(\mathbf{x}\right)+g_\chi\left(\mathbf{x}\right)\phi\left(\mathbf{x}\right)/\kappa\right]n$ using (7), it is convenient to define the price of bank net worth as $G\left(\mathbf{x}\right)\equiv g_n\left(\mathbf{x}\right)+g_\chi\left(\mathbf{x}\right)\phi\left(\mathbf{x}\right)/\kappa$. Then, we can show that

$$\phi\left(\mathbf{x}\right)=\psi^{-1}G\left(\mathbf{x}\right). \tag{8}$$

Substituting (4) and (7) into (6), it is straightforward to show that G has a recursive representation:

$$G\left(\mathbf{x}\right)=\mathbb{E}\beta\frac{P'}{P}\left(1-\theta+\theta G\left(\mathbf{x}'\right)\right)\left(\rho\left(\mathbf{x}'\right)\phi\left(\mathbf{x}\right)+R\left(\mathbf{x}'\right)\right).$$

If there was no capital requirement, banks would break even in expectation, and G would always be equal to one. That is, bank net worth is no more valuable than a unit of consumption goods. In this economy, however, the price of bank net worth is greater than one in the neighborhood of the steady state.

Finally, the aggregate law of motion of banks' net worth must be derived. Let N_{-1} and χ denote the aggregate quantities of net worth and loans, respectively, at the beginning of the current period. The stochastic death of banks forces a measure $1 - \theta$ of banks to be replaced by new banks. Following Gertler and Karadi (2011), the aggregate start-up fund is a fraction of existing aggregate bank assets, $\omega\kappa\chi$, where $\omega > 0$ is a constant. Because individual bank net worth before the government capital injection is $\rho\kappa\chi_i + Rn_{-1}$ from (4) with $d_B = 0$ and the probability of survival from stochastic death is θ, the total volume of net worth held by continuing banks is $\theta\left[\rho\kappa\chi + RN_{-1}\right] + T_B$. Adding the aggregate start-up funds provided to new banks established in the same period, the aggregate law of motion of net worth is

$$N = \theta\left[\rho\kappa\chi + RN_{-1}\right] + \omega\kappa\chi + T_B. \tag{9}$$

2.3 Non-bank financial companies

There is a continuum of non-bank financial companies. One non-bank financial company is attached to a firm operating in the economy. The role of non-bank financial companies is to intermediate between households and firms in the rental capital market to ensure that the rental price as well as the undepreciated value of physical capital are paid to households by using their technologies to monitor firms' conditions and collect amounts payable to households. I assume that monitoring a healthy firm is costless, whereas taking an enforcement action against a failed firm is costly. In addition, the financial condition of a firm is private information to the firm before monitoring takes place. Thus, to secure the resources for monitoring, each non-bank financial company charges a premium per unit of capital over the rental price received by households. As a result, the effective rental price of capital for firms, $\widetilde{r}_k$, is higher than the rate received by households, r_k, by the potential cost of monitoring, z_m:

$$\widetilde{r}_k = r_k + z_m.$$

I characterize the dynamics of intermediation charge per unit of capital around the steady state by an exogenous process that depends on the financial shock, ψ. That is, in a linearized form,

$$\widehat{z}_m = \gamma \widehat{\psi},$$

where $\widehat{x}$ is the deviation of variable x from its steady-state value, and γ is a parameter that indicates the extent to which the intermediation cost changes with the health of the financial system. This assumption is motivated by the observation that the enforcement of financial contracts becomes more costly during periods of financial stress. In equilibrium, any remaining proceeds collected from firms are transferred to shareholders, *i.e.*, households, as lump-sum profits, net of expenses incurred to monitor failed firms.

2.4 Households

Households hold a non-negative amount of deposit, s, and capital, k, and receive gross returns of $(r_k + 1 - \delta)$ on capital and R on deposits. Here, δ is the depreciation rate of capital. Additional sources of income are wages and dividends from firms and banks. The household expenditure involves consumption and savings through deposits and capital. The utility maximization problem of households is:

$$H(s, k; \mathbf{x}) = \max_{c,\ L,\ s',\ k'} \left\{ u(C, 1 - L) + \beta \mathbb{E}\left[H(s', k'; \mathbf{x}') \right] \right\}$$

subject to

$$
\begin{aligned}
c + s' + k' &\leq w\left(\mathbf{x}\right) L + Rs + \left(r_k + 1 - \delta\right) k + \pi, \\
k',\ s' &\geq 0, \\
\mathbf{x}' &= \Xi\left(\mathbf{x}\right),
\end{aligned}
$$

where π includes profits from firms and non-bank financial companies, dividend payments from banks and a net transfer from the government. Taking the first order condition, we obtain a standard consumption Euler equation,

$$
1 = \mathbb{E}\beta \frac{P'}{P} R',
$$

where $P = D_1 u\left(C, 1 - L\right)$. Since bank deposits and capital are perfect substitutes for households, an arbitrage condition holds: $R = r_k + 1 - \delta$.

2.5 Government

I take into account the government's capital injection to banks in order to evaluate the effectiveness of TARP. For simplicity, I characterize TARP as a series of exogenous net capital flows from the government, which are financed by lump-sum transfers from households. More specifically, I assume that τ_i is an idiosyncratic, zero-mean *i.i.d.* shock. This may be a reasonable approximation of the dynamics of TARP's assistance to banks through the Capital Purchase Program (CPP) and Targeted Investment Program (TIP)[14] in that, for most recipients of CPP/TIP assistance, capital injection as well as principal repayment was a one-time event.[15]

[14]The Capital Purchase Program and Targeted Investment Program were subprograms of TARP that aimed at assisting the recapitalization of banks following the financial crisis in the US. Other TARP programs include assistance to the AIG, the automobile industry, etc.

[15]See Section 6 for more details on the analysis of TARP.

2.6 Market clearing conditions

The market clearing condition in the final goods market is

$$Y = C + I + \chi (1-\lambda) \int_{\varepsilon<\underline{\varepsilon}} f(\varepsilon, \mathbf{x})\, d\Pi(\varepsilon) + \Pi(\underline{\varepsilon})\, z_m K,$$

where $I = K' - (1-\delta) K + \kappa\chi'$ is the aggregate investment, $Y = \chi \int y_f(\varepsilon, \mathbf{x})\, d\Pi(\varepsilon)$ is the aggregate output, and the last term on the right-hand side is the resources consumed by non-bank financial companies for monitoring defaulted firms. The aggregate output is equal to the sum of consumption, investment and the total costs of default. In the labor market, households' labor supply must be equal to the sum of labor demand across projects and the administrative labor for setting up projects.

$$L = \chi \int l_f(\varepsilon, \mathbf{x})\, d\Pi(\varepsilon) + \int_{\xi<\overline{\xi}} h(\xi) dJ(\xi).$$

Finally, the capital rental market and the bank loan market must clear:

$$K = \chi \int k_f(\varepsilon, \mathbf{x})\, d\Pi(\varepsilon),$$

$$\chi' = J\left(\overline{\xi}\right). \tag{10}$$

2.7 Recursive competitive equilibrium

A recursive competitive equilibrium is a set of functions, $\left(k_f, l_f, \underline{\varepsilon}, \overline{\xi}, G, \chi_i', n, H, c, L, s', k', b, R, w\right)$, satisfying the following conditions. First, economic agents solve their problems: a) firms solve their respective problems, and $\left(k_f, l_f, \underline{\varepsilon}, \overline{\xi}\right)$ describes the associated decision rules for firms; b) banks solve their respective problems, and (χ_i', n, d_B) describes the associated decision rules for banks; c) households solve their respective problems, and (c, L, s', k') describes the associated decision rules for households. Second, markets for final goods, labor, capital and bank loans clear. Third, laws of motion for aggregate state variables are consistent with individual decisions: $K' = k'(K, S; \mathbf{x})$; $S' = s'(K, S; \mathbf{x})$; N is given by (9); χ' is given by (10).

3 Calibration and estimation

This section explains the procedures of calibration and estimation of parameters and exogenous shocks. To solve the model numerically, it is assumed that households' instantaneous utility stems from indivisible labor (Hansen, 1985; Rogerson, 1988), $u(c, L) = \log c + \eta_\nu (1 - L)$, and that the production function is Cobb-Douglas, $\varepsilon z F(k, l) = \varepsilon z k^\alpha l^\nu$, where $\alpha + \nu < 1$. In addition, idiosyncratic productivity, ε, follows the Pareto distribution with the probability distribution function, $\Pi(\varepsilon) = 1 - (\varepsilon/\varepsilon_{\min})^{-k_\varepsilon}$, and ξ follows the log-normal distribution with the location parameter μ_ξ and the shape parameter σ_ξ. The administrative work for setting up a project is assumed to be a quadratic function of ξ, *i.e.*, $h(\xi) = \xi^2$.

3.1 Calibration and estimation methodology

The time frequency is quarterly and I set most structural parameters of my model to match calibration targets in the steady state. However, using the Bayesian method, I estimate the parameters that determine the characteristics of aggregate shock processes, such as persistence and the standard deviations of innovations, as well as γ, which measures the sensitivity of spread on the rental price to financial shocks outside the steady state. Moreover, the shape parameters of idiosyncratic shock distributions, k_ε and σ_ξ, are chosen to match the volatility of cyclical fluctuations in the aggregate output and the number of firms, respectively, based on the estimation results.

3.2 Calibration

The subjective discount factor, β, is chosen to generate a 4% real interest rate per year. The probability of a bank's stochastic death $1 - \theta$ is inferred from what fraction of commercial banks' net worth is paid out to households in the form of dividends, salaries and employee benefits.[16] The steady-state spread in the rental price of capital, $z_{m,ss}$, matches the average spread between Baa

[16] In this model, banks distribute funds to households only when they exit due to stochastic death.

Table 1: Calibrated parameters

Parameter	Value	Target moment
β	0.99	Real interest rate of 4% per year
θ	0.96	Ratio of bank dividends, salaries and employee benefits to net worth
$z_{m,ss}$	0.0025	Baa-Aaa bond yield spread of 1% per year
z_{ss}	1	Normalization
k_ε	21.25	Volatility of detrended output
σ_ξ	0.0077	Volatility of detrended number of firms
ψ_{ss}	0.23	Bank leverage of 10
ω	0.0008	Bank-net-operating-income-to-asset ratio of 23 basis points per quarter
δ	0.012	Investment-to-output ratio of 0.23
α	0.19	Annualized-capital-to-output ratio of 2
ν	0.62	Net operating surplus to output ratio of 0.186
η_ν	2.6	Hours worked of 1/3
μ_ξ	-1.9	Labor-income-to-output ratio of 2/3
$\varepsilon_{\min}$	0.95	$\mathbb{E}[\varepsilon] = 1$
κ	0.09	Spread between Baa bond yield and federal funds rate of 4% per year
λ	0.57	Commercial and industrial loan failure rate of 1.8%

Notes: a) Net operating surplus is for non-financial corporate businesses; b) ψ_{ss} and ω are jointly calibrated. Similarly, parameters from δ to λ are calibrated jointly to match the corresponding targets.

and Aaa bond yields of 1% per year. The steady-state level of real shock, z_{ss}, is normalized to 1. The values of ψ_{ss} and ω are chosen to match the banks' leverage ratio of 10 and banks' quarterly net-operating-income-to-asset ratio of 23 basis points.

Given k_ε and σ_ξ, parameters δ, α, ν, η_ν, μ_ξ, $\varepsilon_{\min}$, κ, and λ are chosen to match the following targets: (i) the annualized-capital-output ratio of 2; (ii) the investment-to-output ratio of 0.23; (iii) the average hours worked of one-third of normalized available hours of one; (iv) the labor-income-to-output ratio of 2/3; (v) the ratio of net surplus of non-financial businesses to their value added of 0.186; (vi) the annualized spread of 4% between the bank lending rate and the funding rate, proxied by the difference between the yield on Baa bonds and the federal funds rate; (vii) the unconditional expectation of ε normalized to 1; and (viii) the average loan failure rate of 1.8%. The calibrated parameters are summarized in Table 1.

3.3 Estimation

The Bayesian estimation is implemented by including four types of data in observable equations: (a) real private fixed investment, (b) real bank net worth, (c) real net capital injection flows through TARP and (d) the spread between Baa and Aaa corporate bond yields. Roughly, I extract information on real shocks, z, and financial shocks, ψ, through the first two series while controlling for the capital injection shocks, T_B, through the third series. The inclusion of the capital injection series allows me to evaluate the macroeconomic effects of TARP by conducting a counter-factual analysis later on. The interest rate spread series are used for extracting information on the sensitivity of the rental price wedge on financial shocks, γ. For identification, at least four types of shocks are necessary to match four data series. To satisfy this requirement, I include *i.i.d.* measurement errors in the interest rate spread series, which capture the movements in this series that are unexplained by the changes in financial shocks. I assume an AR(1) structure for real and financial shocks and estimate their persistence parameters, ρ_z and ρ_ψ, as well as the standard deviation of *i.i.d.* normal innovations to these shocks, σ_z and σ_ψ. For the *i.i.d.* normal policy shocks and measurement errors in the interest rate spread, their standard deviations, σ_{T_B} and σ_{ME}, are estimated.

Data on the private fixed investment are taken from the National Income and Product Accounts. Tier 1 capital of financial institutions affiliated with the FDIC is used to represent bank net worth.[17] Data on TARP capital injection flows are publicly available from the TARP Investment Report by the US Treasury. However, the majority of TARP assistance was provided through bank holding companies, and the information on when and how much of these funds were transferred to FDIC-affiliated financial institutions is not readily available. Using call reports of all commercial banks and savings institutions, I collect the data on capital transfers from parent holding companies to their group financial institutions around the periods of TARP capital injections. The capital transferred to FDIC-affiliated financial institutions is regarded as the gross capital injection to banks. Repayment dates of TARP funds by each recipient are available from

[17] Tier 1 capital is deflated by the GDP deflator to convert it to a real series.

the TARP Investment Report. The principal repayment dates by commercial banks and savings institutions in my data set are set to these dates. In addition, the recipients of TARP were required to pay dividends to the Treasury, which were equivalent to the annual rate of 5% of the outstanding balance. The dividend payment series are constructed at the commercial and savings-institution levels by computing the dividend payments up to the reported repayment quarters. Finally, all the above series are aggregated across banks to construct the aggregate net capital injection, T_B.[18]

All data series start from 1990Q1 and end in 2016Q4 as determined by the availability of the Tier 1 capital series. The series for investment, bank net worth and interest rate spreads are detrended using the Hodrick-Prescott filter with the smoothing parameter of $1,600$ for quarterly data. To be consistent with the log-linearized version of my model, the aggregate net capital injection series is divided by the trend of aggregate bank net worth and expressed in percentage deviation from the trend.

Estimation is implemented in two steps. First, the model is solved using a log-linear approximation around the steady state. Since only the mean of the bank distribution is required to aggregate bank net worth and the number of loans provided by banks, this method delivers a convenient and accurate approximation to local dynamics in the neighborhood of the steady state. Then, information from prior distributions of estimated parameters and the log-likelihood implied by the data are combined to find the mode of the log posterior density. Second, this information is used to propose draws for simulating the posterior distribution. The Metropolis-Hastings Algorithm is used to implement the simulation step. Simulations of $200,000$ Monte Carlo Markov Chain draws characterize the posterior distribution, which results in an acceptance rate of 22%. Table 2 summarizes the estimation results.

4 Mechanism

In this section, I analyze the mechanism behind the effect of endogenous loan default in my model. I also consider the interaction of this default with bank capital, which propagates the effect of real

[18]This series is deflated by the GDP deflator.

Table 2: Estimated parameters

	Prior [mean,std]	Posterior mode	5%	95%
γ	Beta$[0.5, 0.25]$	0.10	0.06	0.13
ρ_z	Beta$[0.5, 0.25]$	0.91	0.87	0.95
ρ_ψ	Beta$[0.5, 0.25]$	0.67	0.58	0.76
σ_z	IGamma$[0.1, \infty]$	0.09	0.08	0.11
σ_ψ	IGamma$[0.4, \infty]$	0.26	0.19	0.26
σ_{T_B}	IGamma$[1.0, \infty]$	0.86	0.78	0.97
σ_{ME}	IGamma$[0.4, \infty]$	0.07	0.06	0.08

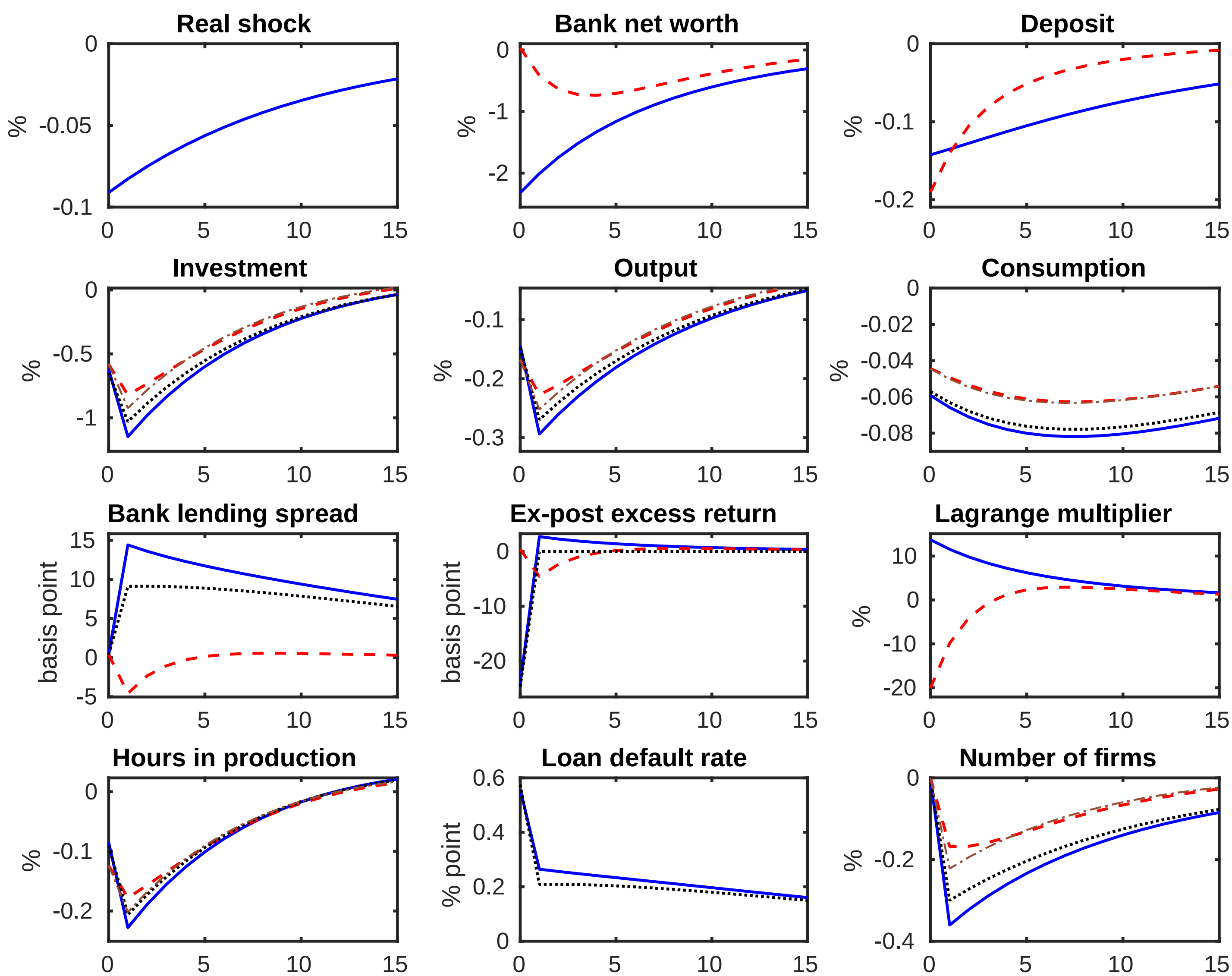
shocks. This result is in contrast to the findings in many influential papers in macroeconomics (Kocherlakota, 2000; Khan and Thomas, 2013; and Jermann and Quadrini, 2009, 2012). This interaction is also at work for financial shocks. As shown below, the counter-cyclical default rate and interest rate spread are the key to understanding this result. The counter-cyclical pattern in these variables is empirically plausible, in contrast to that obtained through the propagation mechanism of real shocks studied in Carlstrom and Fuerst (1997). To explain these results in detail, I examine impulse response functions following unexpected real and financial shocks that independently give rise to a recession.

4.1 Real shock

Figure 2 shows impulse response functions to an unexpected decline in aggregate productivity. In period 0, the aggregate efficiency of production falls by one standard deviation from its steady-state value and gradually returns there with the persistence of 0.91. These values are based on the estimation results discussed in Section 3.3. Solid lines (blue) in the figure show the responses in the benchmark model. The model is successful in producing counter-cyclicality of the default rate and interest rate spread, as well as the procyclicality of number of loans,[19] output, investment, hours worked and consumption. Importantly, these movements are consistent with what we observe in data.

[19]In my model, the number of loans and the number of firms implementing projects are equivalent. These terms are used interchangeably in this paper.

Figure 2: Impulse responses to real shock



Notes: This figure shows impulse responses to a one-standard-deviation decline in the real shock. Solid lines (blue) are responses in the benchmark model, dashed lines (red) show responses in the model without loan default, dotted lines (black) show responses in the model without deposit frictions, and dash-dotted lines (brown) show responses in the model without any financial frictions. Responses in the alternative models are shown only when applicable and meaningful.

Among these variables, movements in the interest rate spread conveniently explain the amplification mechanism of this model. A direct impact of the fall in the efficiency of production is a deterioration of the overall profit levels of firms, which, in turn, increases the number of firms defaulting on their loans. This leads to a decrease in the demand for loans, which would relax financial constraints other things being equal. The equilibrium interest rate spread, however, rises in my model for two reasons. First, the counter-cyclical loan default generates upward pressure on the loan rate, which offsets the downward pressure. Second, the counter-cyclical loan default impairs bank net worth and *tightens* deposit frictions despite the fall in loan demand. Moreover, the tightened deposit frictions cause a higher equilibrium loan default rate, generating a feedback effect between the two forces at work.

To illustrate these points further, the figure also displays responses from three alternative models that eliminate either the loan default channel, deposit frictions or both from the baseline model: in Model A, banks face deposit frictions but firms do not have the option to default on loans due to unlimited liabilities; in Model B, deposit frictions do not exist but loans are defaultable; and in Model C, neither loan default nor deposit friction exists.[20] Dashed lines (red) indicate responses implied by Model A. In contrast to the benchmark model, the bank lending rate spread *decreases*, reflecting lower demand for loans. As a result, the responses of aggregate quantities are much less pronounced than those of the baseline model. In fact, these responses are even smaller than those of the frictionless model (Model C) that are shown by dash-dotted lines (brown). This dampened effect of real shocks in Model A parallels Jermann and Quadrini's (2012) finding. By contrast, as shown by the dotted line (black), if loans are defaultable as in Model B, the default premium *raises* the interest rate spread. Given the results from Models A and B, one might be tempted to infer that, if both loan default and deposit frictions operate at the same time, an increase in the bank lending rate spread would be smaller than that of Model B. However, the figure shows that the combined effect in the baseline model is *larger*. Similarly, the responses

[20]Given the general-equilibrium nature of the feedback effect in my model, an analytical exposition is non-trivial. Simulations of alternative models, in spirit, capture a graphical representation of the key mechanism of my model. Each simulation in Figures 2 and 3 is based on the parameter values used in the baseline model. Appendix A provides details of these variants of the baseline model.

in the number of firms (loans) and the aggregate quantities such as output and investment are largest in the baseline model. This amplification arises from the two-way interactions between endogenous loan default and deposit frictions, as mentioned above. On the one hand, a higher loan default generates a higher funding premium by tightening deposit constraints, as indicated by an *increase* in the Lagrange multiplier on deposit constraints. This is in contrast to a decrease in the same variable in Model A. On the other hand, the tighter deposit constraints induce an even higher rate of loan failure through a higher lending rate, which is implied by the difference in loan default rates between the baseline model and Model B. In this way, even if a shock originates in the real sector, its effect translates into financial issues in a non-trivial way and generates an additional impact on both the real and financial sector outcomes.

Quantitatively, the total effect of featuring loan default in the baseline model relative to Model A is large. Comparing the solid and dashed lines (blue versus red) in Figure 2, the declines in output and investment are not very different between the two cases on impact of the shock. However, the peak output and investment responses in the benchmark model are larger than those in Model A by almost 30% and 40%, respectively. Moreover, as the number of active firms diverges across these two cases over time, partly due to the slow recovery in bank capital in the baseline model, it takes longer for the output and investment responses of the full model to return to the level in period 0 than those of Model A.

4.2 Financial shock

Real shocks affect the profit of firms and the likelihood of repaying their debt. While there is no doubt that the deteriorated performance of underlying assets is the fundamental problem for banks during recessions, the fear of systemic risk can make even relatively sound banks suffer by making it hard for them to raise funds. Although the fire sale of assets to deleverage balance sheets was particularly prominent in investment banks (Adrian and Shin, 2009), Ivashina and Scharfstein (2010) point out that commercial banks also cut new business loans during the recent financial crisis. In the light of this evidence, I try to capture the effect of exogenous variation in bank creditworthiness through changes in ψ.

Figure 3 shows impulse response functions to an unexpected one-standard-deviation increase in the financial shock, ψ, which gradually returns to the steady-state level with a persistence of 0.67. Again, the parameters characterizing the exogenous stochastic processes are based on estimates in Section 3.3. Similar to the real shock, in the benchmark model, the default rate and interest rate spread exhibit a counter-cyclical pattern while output, investment, hours worked, consumption and the number of loans respond procyclically.[21] A comparison between solid lines (the baseline model) and dashed lines (the model without loan default) in Figure 3 shows that an endogenous default on loans leads to a greater increase in the bank lending rate spread, which, in turn, generates sharper declines in output, investment, consumption, hours worked and the number of loans than in the model without the risk of loan default.[22] For example, the peak responses in output and investment are, respectively, 26% and 40% larger in the benchmark model than those in the model without loan default. Moreover, even though the initial declines in output and investment are similar in both cases, it takes longer for these responses to return to the same levels if loan default is at play.

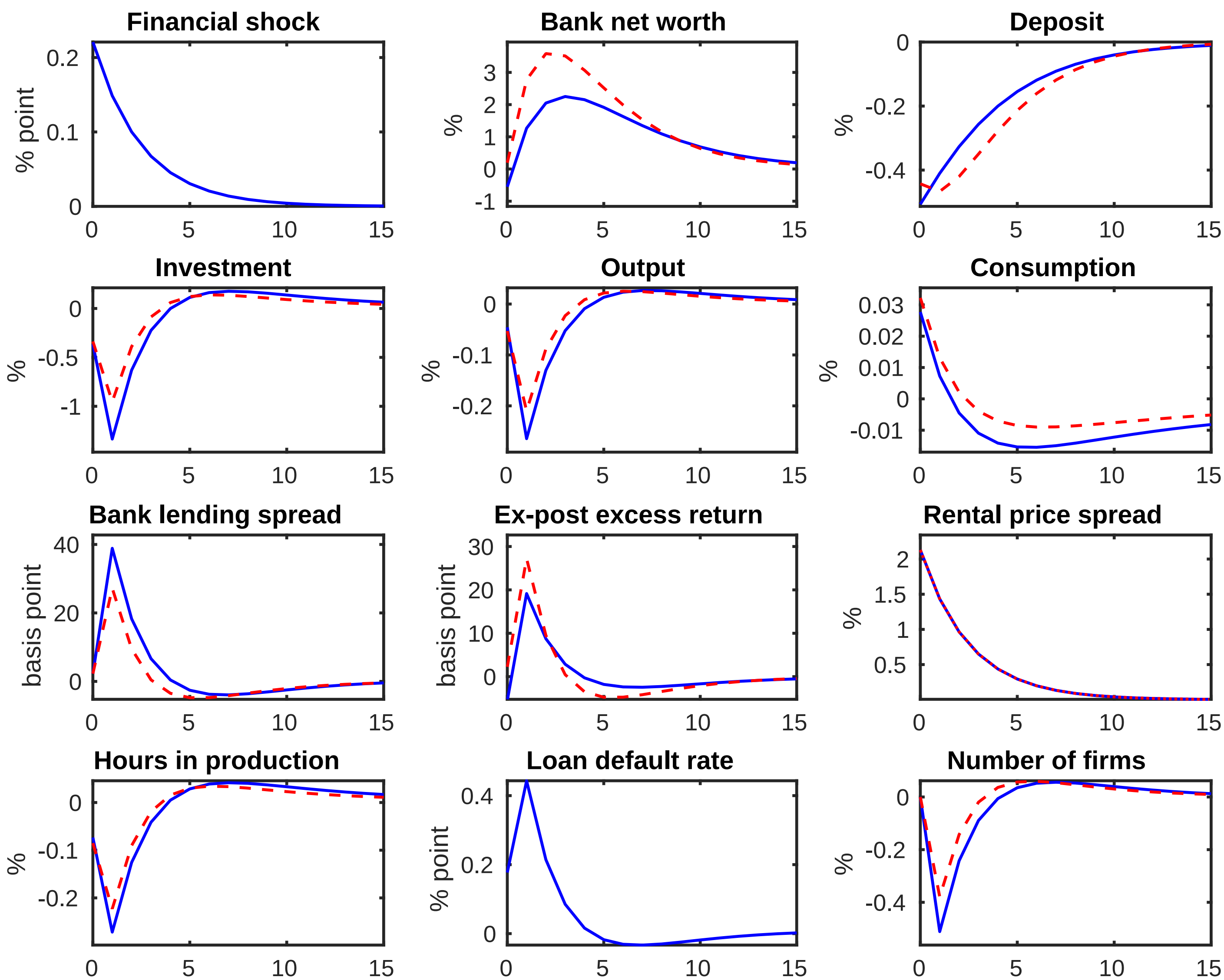
To better understand the mechanism behind these results, I highlight two channels through which the financial shock affects the economy. First, the spread on the rental price of capital widens persistently as the efficiency of monitoring by non-bank financial companies deteriorates. This reduces the profit of firms and the level of production. As a result, the loan default rate increases in the initial period given the predetermined loan repayment, b. This leads to a reduction in the bank net worth on impact of the financial shock. Moreover, the persistent increase in the rental price spread reduces future firm profits, which, in turn, limits the demand for production input over time.

Second, the financial shock affects the supply of loans. As the public confidence in banks

[21] Although there is an initial increase in consumption after the shock, it persistently falls below the steady-state level after period 1. The initial increase in consumption is related to a rise in the rental price spread, which decreases the demand for capital and thus the equilibrium amount of savings in capital stock. Figure 3 shows that the addition of loan default in the model weakens this effect.

[22] Since the financial shock is relevant only when deposit frictions bind, the figure shows an alternative simulation in which firms have no option to default.

Figure 3: Impulse responses to financial shock



Notes: This figure shows impulse responses to a one-standard-deviation increase in the financial shock. Solid lines (blue) are responses in the benchmark model while dashed lines (red) show responses in the model without loan default.

deteriorates, the deposit frictions get tighter and the bank lending spread increases. This is because banks' funding is more limited and loans are in short supply. In the absence of new innovations to shocks, the higher lending rate spread helps banks reaccumulate the diminished net worth in an effort to attract more funding.[23] However, loan default in my model limits the extent to which banks can do so. Relative to the model without loan default, the recovery of bank net worth is much more modest in the benchmark model. As a result, the bank lending rate spread and loan default rate remain high for a protracted period of time, in part due to the interaction between loan default and deposit frictions. The sharp and persistent increase in the bank lending rate spread leads to a larger and longer decline in aggregate quantities than in the case of no loan default.

Another important implication of financial shocks is that the leverage of banks declines following a negative financial shock. Recall that (8) implies a negative correlation between the financial shock and the leverage of banks, other things being equal. As shown in Section 5, this feature of the model helps explain an important aspect of the financial sector response during the recent financial crisis in the US.

Given the results in this section, I can proceed further to examine the contribution of the two types of shocks in explaining business cycle fluctuations in the US and the effectiveness of capital injection.

5 Measurement of shocks and their contributions

In this section, I measure the cyclical components of real and financial shocks to evaluate their contributions in the business cycle.

[23]This pricing effect would be smaller if bank assets had unhedged fixed interest rates and longer maturities, as in Ferrante (2019).

5.1 Estimated structural shocks

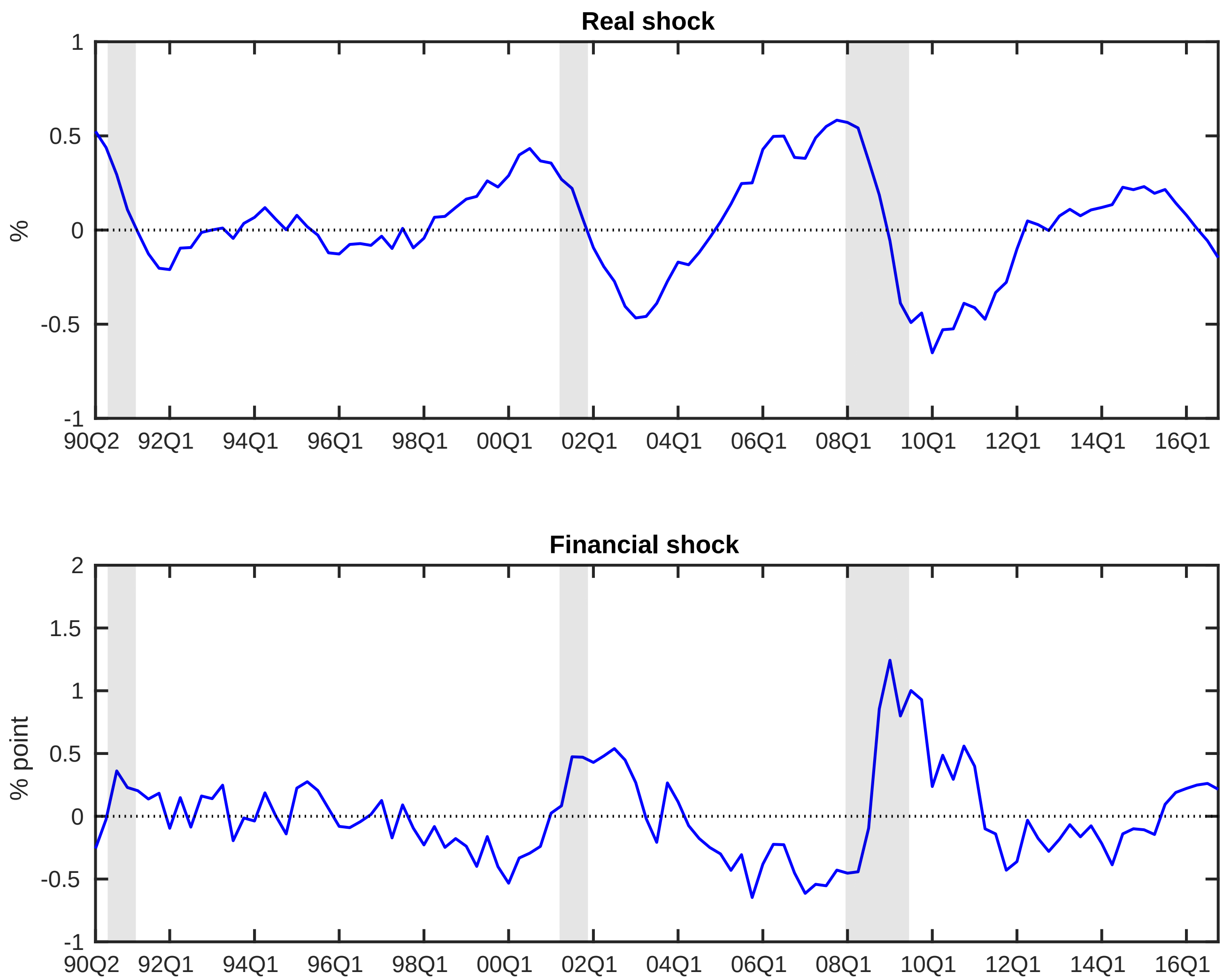
There is no unique method to measure latent variables from data, especially financial shocks. Jermann and Quadrini (2009, 2012) use an enforcement constraint that corresponds to (5) in my model to measure financial shocks, given the Solow residual series for aggregate productivity shocks. They then use these recovered observations to estimate the innovations to shocks and simulate their model. Jermann and Quadrini (2012) also use a Bayesian estimation method to estimate these shocks. They find that financial shocks are the leading force in business cycle fluctuations in the US. I do not use their first approach because the standard Solow residual method is not consistent with aggregate supply in this paper. Instead, in an effort to match the model to data, I use the Bayesian method to estimate the persistence and standard deviation of the underlying aggregate shocks along with the parameter determining the responsiveness of rental price spread to financial shocks, as explained in Section 3.3.

Figure 4 plots real and financial shocks that are computed using the Kalman smoother. Notice that there are elevated increases in financial shocks from 2008Q4 to 2009Q4. This captures liquidity problems in the banking sector around the time when Lehman Brothers failed. Prior to the financial crisis, the estimated financial shocks capture the National Bureau of Economic Research (NBER) recession periods in 1990 and 2001. These movements in financial shocks are broadly consistent with our prior knowledge. The fluctuations in real shocks are also in line with the boom and bust of economic conditions. The beginnings of NBER recession dates are close to the turning points at which real shocks start to decline precipitously, which are a precursor to periods when borrowers are less able to repay debt.

5.2 Contributions of real and financial shocks

Given the estimated series of all structural shocks, I generate model predictions by feeding in these recovered shocks. More specifically, the simulation starts from 1990Q2 using estimated endogenous-state variables in 1990Q1. Thereafter, endogenous-state variables are determined within the model over time. Even though the whole sequence of shocks is fixed, agents do not

Figure 4: Estimated real and financial shocks



Notes: Shaded areas indicate NBER recession dates. All variables are expressed in terms of deviation from the steady state.

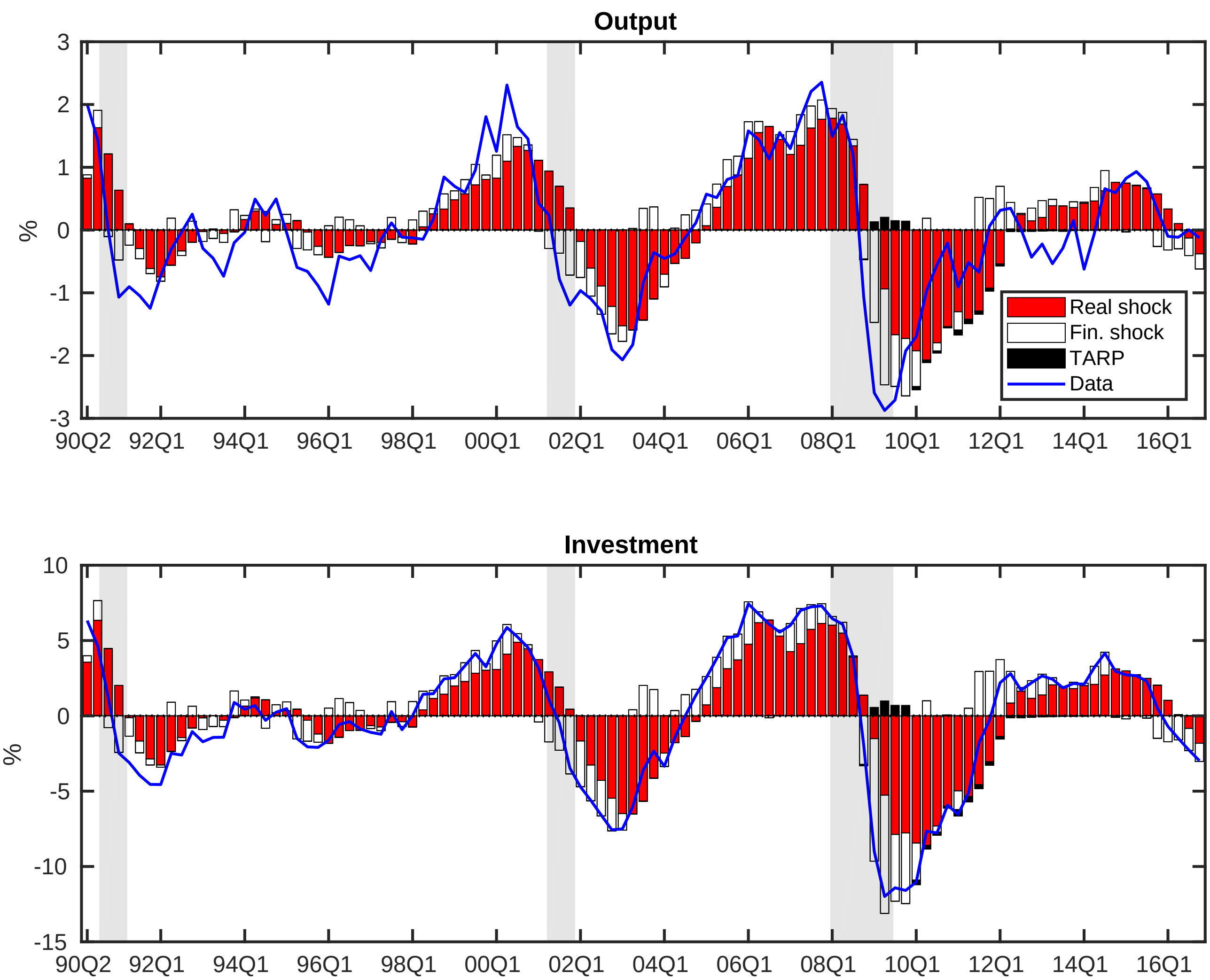
know the realizations of shocks *ex-ante* and hold rational expectations of future states of the economy given the current state. Figure 5 shows the actual and model-implied data on aggregate output and investment in the form of historical decomposition to explain the relative contribution of real and financial shocks. Recall that I use the data on investment in the observation equations; any discrepancies between the data (solid line) and the combined effects of real and financial shocks (bars) are due to the effects of initial-state variables, which shrink over time. On the other hand, only the volatility of the output series is matched through calibration and estimation. The upper panel shows that the simulated output path tracks the observed path fairly well.

Two observations stand out from this result. First, real shocks contribute to long-lasting effects of booms and recessions in output and investment, accounting for a large share of the fluctuations in mid to late phases of business cycles. This historical importance of real shocks is in contrast to the findings in the macroeconomics literature. Especially, using the sample period of 1984Q1-2009Q4, Jermann and Quadrini (2009, 2012) find that their model's prediction worsens with aggregate productivity shocks. Aggregate productivity shocks work poorly in their model because a reduction in loan demand relaxes the key financial friction in their paper, making it easier for firms to borrow working capital during recessions. In this paper, reduced firm profits also imply a higher rate of loan default, which makes deposit frictions tighter by eroding bank capital. Thus, the propagation mechanism in my model hinges critically on financial frictions, which tightly connect vulnerabilities in the real and financial sectors by creating the feedback effect between the two.

Second, while real shocks account for persistent cyclical movements, financial shocks play a particularly important role at the onset of recessions and booms, especially during the financial crisis starting in 2008Q4. For example, in 2008Q4, the declines in output and investment are entirely driven by financial shocks. In 2009Q1, the same is still true for output, while 84% of the decline in investment is due to financial shocks.[24] Similar patterns are observed for the 1990 and 2001 recessions.

[24]Section 6 discusses the effects of TARP.

Figure 5: Historical decomposition of output and investment



Notes: Shaded areas indicate NBER recession dates. All variables are expressed in terms of deviation from the steady state.

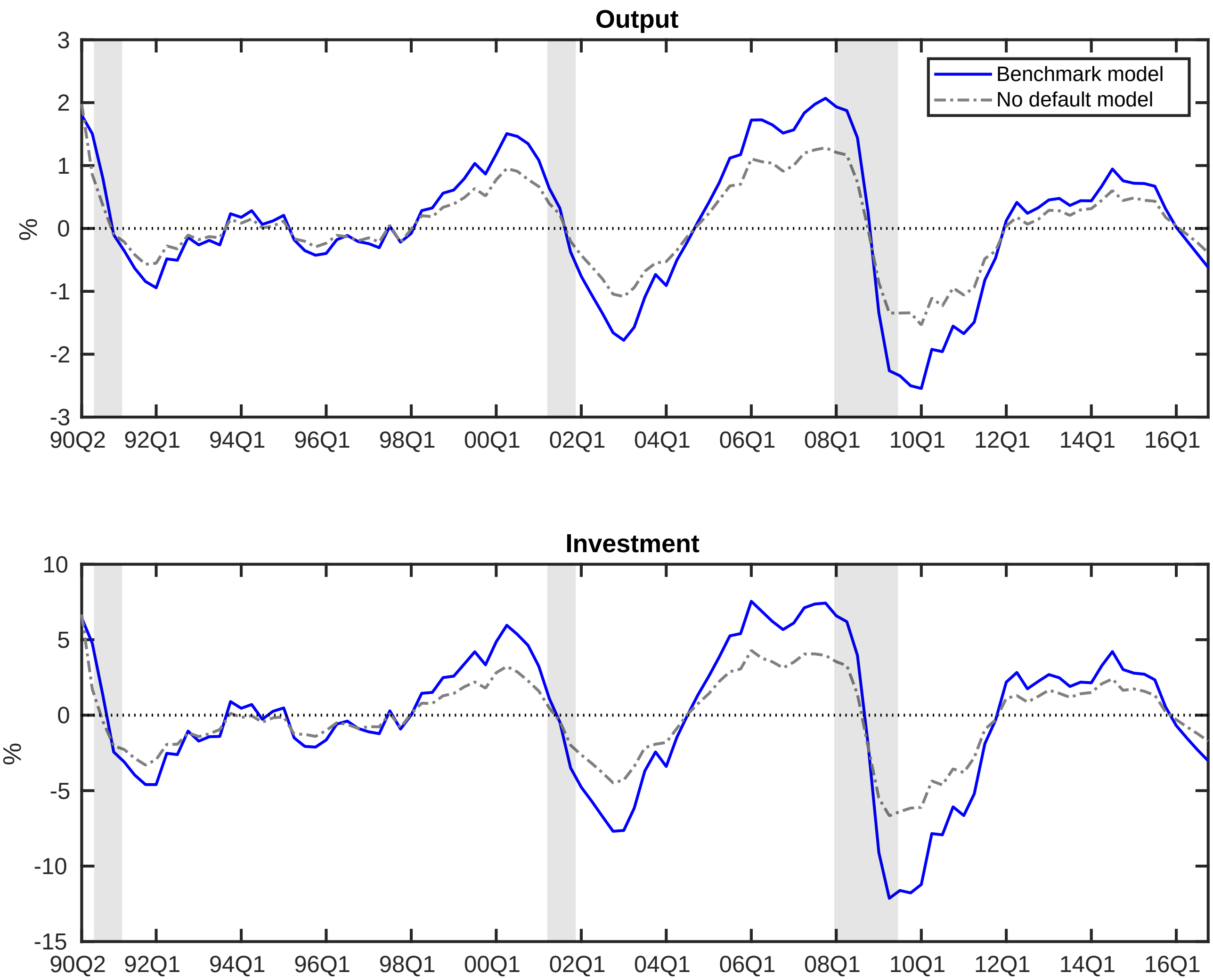
To highlight the quantitative importance of the role played by loan default as well as its interaction with deposit frictions in the context of the current exercise, I run a counter-factual simulation to discuss their implications. Specifically, I compare the simulated output and investment series in my model versus the same series generated by the model without loan default. For comparison, the simulations use the same structural shocks and the parameters governing the evolution of both aggregate and idiosyncratic shocks. These parameters include the persistence and the standard deviation of aggregate shocks; the sensitivity of rental price spread on financial shocks, γ; and the location and shape parameters of idiosyncratic shock processes, k_ε, $\varepsilon_{\min}$, μ_ξ and σ_ξ. Other parameters in the alternative model are re-calibrated to match calibration targets that are relevant for the economy with no default risk. Figure 6 reveals that the amplification through loan default and its interaction with deposit frictions is quite significant, especially around the peaks and troughs of business cycles. The standard deviations of output and investment generated by the model with loan default are approximately 59% and 73% larger, respectively, than those of the model without.

5.3 Untargeted financial variables

In addition to output and investment, I present the overall movements of key financial variables such as the bank lending rate spread, loan failure rate[25] and bank leverage. Figure 7 plots these variables. Even though the movements in these variables are not targeted or matched through estimation and calibration, the model-implied paths capture the overall movements in these variables reasonably well. Table 3 reports correlations between the simulated and observed paths of each of these variables. The correlations are all positive and reasonably high especially after the turn of the century.

[25] For comparison, the figure presents the observed loan failure rate for all loans in addition to that of commercial and industrial loans. These loan failure rates have similar cyclical patterns.

Figure 6: Amplification through loan default

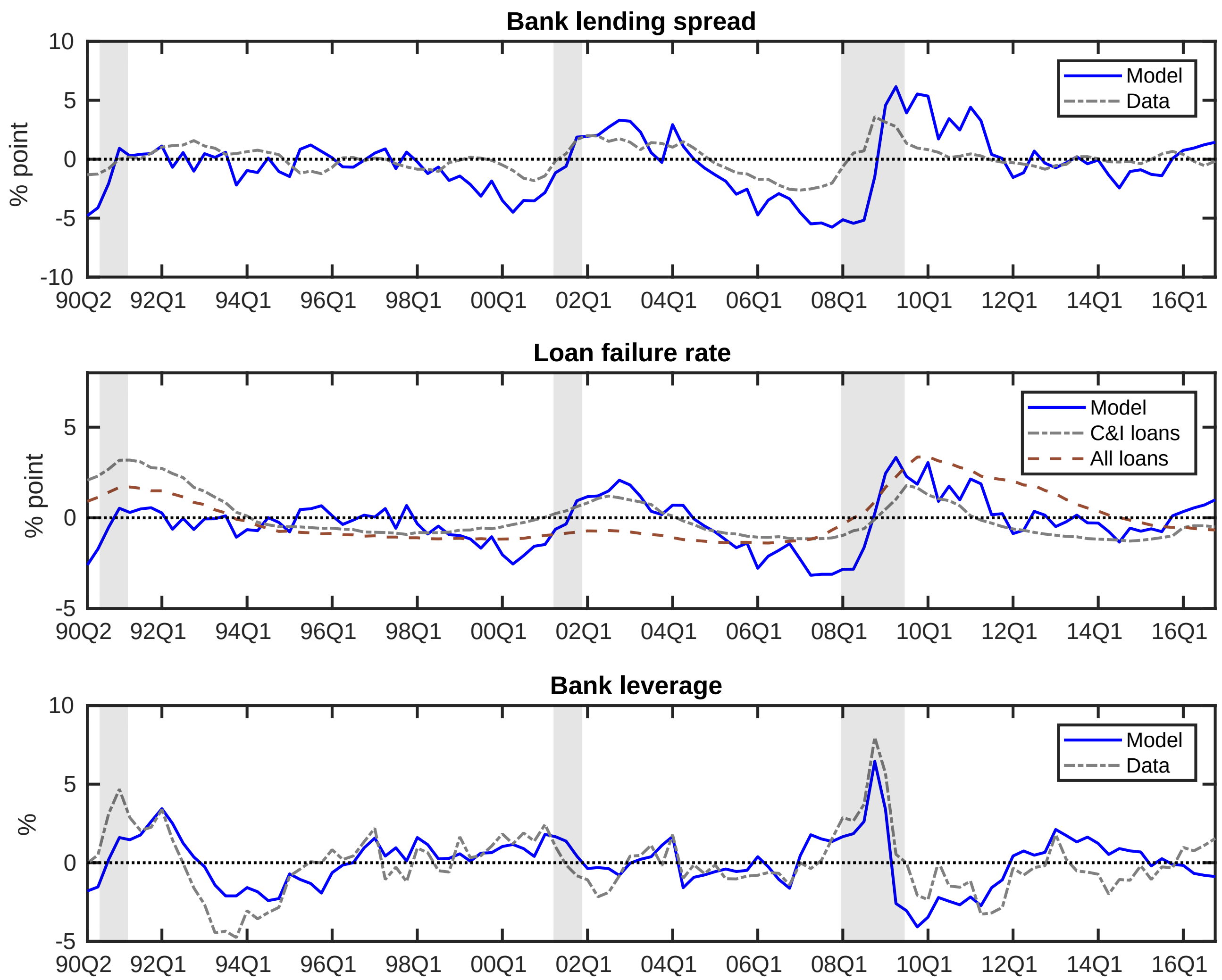


Notes: Shaded areas indicate NBER recession dates. All variables are expressed in terms of deviation from the steady state.

Table 3: Correlation between simulated and observed paths

	full sample	2000Q1-2016Q4
interest rate spread	0.64	0.70
loan failure rate		
C&I loans	0.40	0.73
all loans	0.48	0.55
bank leverage	0.72	0.71

Figure 7: Simulated financial variables



Notes: Shaded areas indicate NBER recession dates. All variables are expressed in terms of deviation from the steady state.

5.4 The loan default channel and the relative contribution of shocks

The results so far indicate that the interaction between the risk of loan default and deposit friction helps explain the dynamics of output, investment and some key financial variables associated with banks. The next analysis shows that whether to take into account loan default has an implication for the relative contribution of real and financial shocks to explain output and investment dynamics. That is, a model without the loan default channel and its interaction with bank capital would require a higher fraction of business cycle variations explained by financial shocks, given that the effects of real shocks are dampened in such a model.

As an example, I use the model without loan default shown in Section 5.2 and recalibrate the value of γ to match the volatility of investment, leaving all the other parameter values unchanged. Recall that this version of the model features a wedge in the rental price of capital on which financial shocks have a direct impact. In spirit, this friction resembles the standard financial friction commonly assumed in the literature, which generates a wedge in production factor prices.

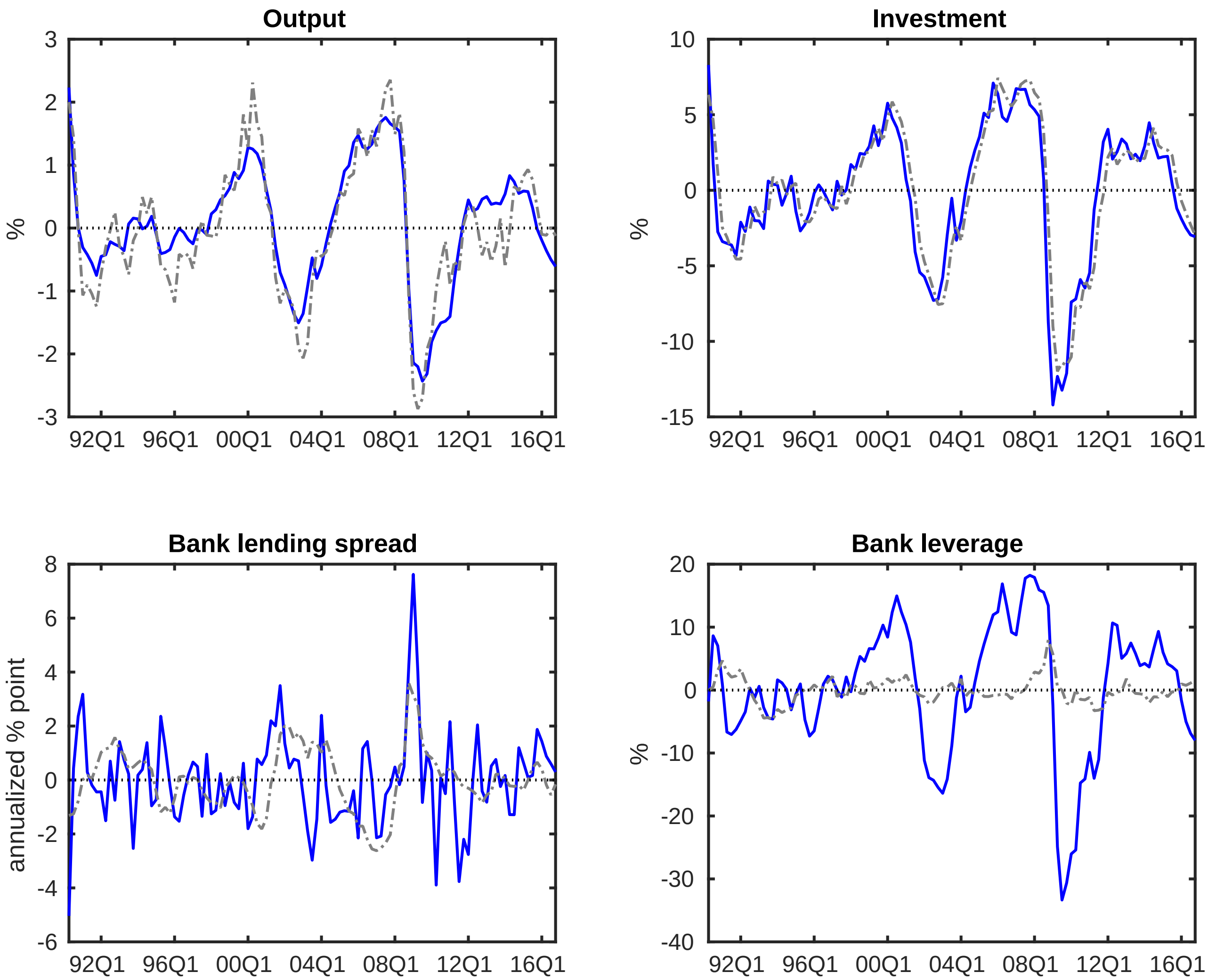
Table 4 reports the limit of variance decomposition of simulated output and investment for the benchmark model and the model featuring no risk of loan default as well as the adjustment for γ. The results in the table clearly show that, for both variables, the contributions of financial shocks in the model without the risk of loan default are much larger than those in the benchmark model. For investment, in particular, the relative contribution of financial shocks exceeds that of real shocks when loan default does not play a role. This result implies that taking into account the interaction between the real and financial sectors through loan default could be a critical factor in determining the relative contribution of real and financial shocks.

In this example, the volatility of investment is matched by choosing a suitable value of γ. Its effect on other variables are shown in Figure 8. Not surprisingly, the upper panels indicate that simulated output and investment paths are much more amplified. However, as shown in the lower panels, the bank leverage and the bank lending rate spread series fit very poorly to the data: the full-sample correlation between the model-implied path and the data falls to 0.33 for the bank lending rate spread and 0.27 for the bank leverage. Thus, to the extent that banks

Table 4: Variance decomposition

	Benchmark model	No loan default (with higher γ)
output	z : 81%	z : 52%
	ψ : 19%	ψ : 48%
investment	z : 67%	z : 23%
	ψ : 33%	ψ : 77%

Figure 8: Matching investment in the model without loan default



Notes: Solid lines indicate model-implied paths while dash-dotted lines show paths observed in data. All variables are expressed in terms of deviation from the steady state.

were an integral part of the financial crisis, ignoring the risk of loan default when accounting for the observed fluctuations in aggregate quantities comes at the cost of unreliable predictions for important financial variables related to banking. Such a model is unlikely to provide useful guidance on how a policy like capital injection affects the real economy through the banking sector.

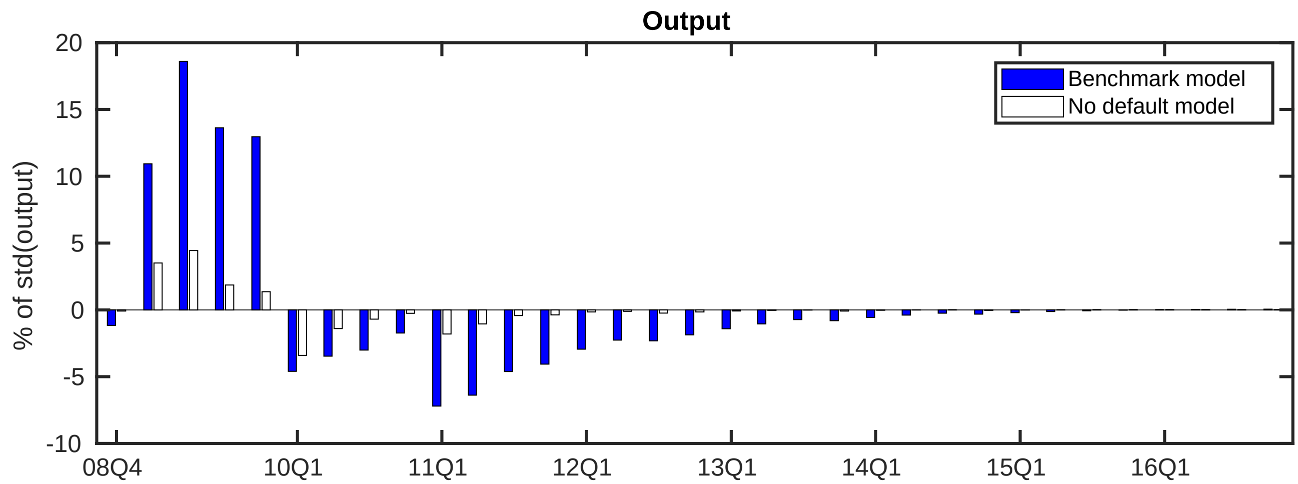
6 Capital injection to banks

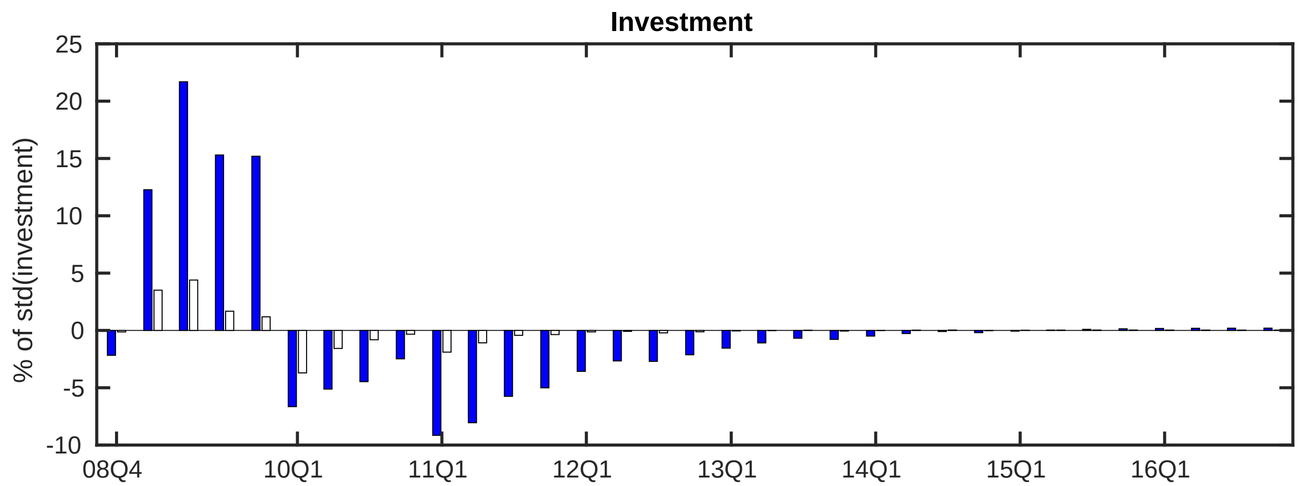
After the failure of Lehman Brothers in 2008, a series of policy actions were taken to assist the financial sector, which included, among others, capital injections to large banks through TARP. In this section, I examine the effectiveness of capital injections to commercial banks in mitigating the negative impacts of the financial crisis on the aggregate economy. More specifically, the quantitative importance of CPP and TIP in stimulating real output and investment is the focus of evaluation. These rescue programs were large-scale but short-lived to minimize both the opposition to bailing out troubled banks and any possible moral hazard problems. Moreover, TARP resulted in increased dividend payment costs for banks and political pressure against bonus payments to bank executives. For these reasons, many banks did not hold the injected capital for a long time, and the net capital injection turned negative in 2009Q4.

For evaluating this policy, a counter-factual analysis is implemented in two steps. First, a model simulation uses all structural shocks, *i.e.*, real, financial and capital injection shocks, as in Section 5. Second, the resulting responses in output and investment are compared to those obtained through simulations that counter-factually omit the policy shocks. The differences between the two simulations can be attributed to the effect of capital injections.

In Figure 9, colored bars show the result of the counter-factual analysis. In the figure, I measure to what extent capital injections contributed in mitigating fluctuations in output and investment in percentage of the standard deviation of each series observed in the data. Positive values imply that the policy was stimulative relative to the counter-factual economy, whereas negative values mean the opposite. In the figure, the policy's positive contributions are concentrated in the four

Figure 9: The impact of TARP on output and investment





quarters of 2009. The magnitude of contributions ranges from 11% to 19% of a typical variation of output, and from 12% to 22% of that of investment. Equivalently, the peak contribution of TARP in 2009Q2 amounts to 7% of the actual cyclical fall in output and 8% of that of investment. Note that capital injections are funded by lump-sum taxes to households, while firms start business operations with a one-period delay in my model. This explains why the initial positive contribution of TARP appears in 2009Q1 instead of 2008Q4. My model does not capture the potential impact of TARP in saving banks from bankruptcy, for example, during the financial crisis. Thus, these numbers could be interpreted as a conservative measure of the macroeconomic impact of TARP.

The figure also shows the cost of implementing capital injections. As stated above, the net capital injection turned negative in 2009Q4 as more and more banks repaid the funds to the government. This generates a negative impact on the economy as bank net worth is pulled out from the banking sector.[26] Quantitatively, each of these negative effects was smaller than the positive effects during 2009, with the largest negative impacts being -7% and -9% of typical variations in output and investment, respectively, in 2011Q1. The negative impact of TARP due to repayment of funds continued as smaller banks tended to hold injected capital on their balance sheet, but the aggregate effect decays over time after 2011Q1. Overall, the sum of present values of all the effects is positive for both output and investment, indicating that the positive effect of TARP exceeds the negative effect.

TARP was implemented during periods in which the risk of loan default increased and concerns for under-capitalization prevailed in the banking sector. Therefore, a natural question arises as to whether and to what extent the amplification mechanism through endogenous default on loans and bank net worth is important for measuring the effectiveness of capital injection. To answer this question, I conduct an analogous counter-factual analysis in the economy where business loans are not subject to the risk of default. That is, I feed the same structural shocks used in the counter-factual simulation above into the model without the risk of default. The results are shown by uncolored bars in Figure 9. A comparison with the previous counter-factual results suggests

[26] The negative impact in 2008Q4 is due to the funding through lump-sum transfers from households. Quantitatively, this effect is small.

that the measured effects of the capital injection policy is much smaller in the environment where endogenous loan default does not exist. For example, in 2009Q2, the magnitude of effects were only 24% of that in the benchmark model for output and 20% for investment. The intuition behind this result is that the effect of the capital injection policy is amplified through the endogenous loan default channel and its interaction with bank net worth: when capital injection starts, it relaxes deposit frictions and attenuates upward pressures on interest rate spread during the crisis. In turn, this will reduce the loan default rate and mitigate a decline in bank net worth. This result implies that, for evaluating the effectiveness of the capital injection policy, it is crucial to take into account the endogenous mechanism that propagates the policy effect on financial intermediation through the economy.

7 Conclusion

A general equilibrium model presented in this paper features deposit frictions between banks and depositors as well as the risk of default on business loans. Consistent with data, this model generates counter-cyclical loan default rates and interest rate spreads as well as procyclical aggregate quantities such as output and investment. These results hold for both real and financial shocks. Eliminating the loan default channel would lead to *procyclical* interest rate spreads following real shocks, which attenuates the effects of shocks.

One important implication of this result is that the relative contribution of real shocks might be underestimated in the literature. In particular, I find that real shocks can potentially explain a larger fraction of fluctuations in output and investment than what the recent literature suggests they could. An important mechanism that generates this outcome is the existence of loan default, which translates deterioration in firm profits into banking instability and vice versa. This interaction between the real and financial sectors leads to an increase in the bank lending rate spread to a larger extent than that achieved independently by either of the financial frictions–loan default or deposit frictions–without such a feedback effect. In this sense, financial frictions lie at the heart of my analysis, and my finding on real shocks serves to buttress their importance even further. Of course, real shocks cannot, by themselves, explain all movements in output and investment. Historical decompositions indicate that financial shocks of the type I consider in this paper are important for steering the economy to recessions and booms. In particular, financial shocks account for a large share of business cycle fluctuations in output and investment in the subsequent four quarters after Lehman Brothers failed in late 2008. If loans were risk-free, the model would require a different propagation mechanism for matching the aggregate quantities, which might give financial shocks more room to play. This implies that a careful modeling of the interaction between the real and financial sectors is important when evaluating the contributions of real and financial shocks.

Moreover, I show that my model is capable of explaining cyclical changes in the bank leverage, the loan failure rate and the bank lending rate spread. This is an important feature, which allows me to use it as a vehicle to assess the effectiveness of the capital injection policy that took place during the US financial crisis. Through counter-factual simulations, I find that the TARP program mitigated the catastrophic impact of the financial crisis by offsetting 7% of the actual decline in output from trend and 8% for investment. Again, the endogenous default on loans plays a critical role for the measurement of policy effects. The model without the risk of loan default gives more modest numbers because additional bank net worth is not as important in that model as it is when banks suffer from loan failures.

Even though this paper provides a simple framework to analyze the interaction between endogenous default and bank net worth, many questions are not addressed here. First, the model has one-period-lived projects and loans. Introducing multi-period projects is an important direction of extension as it allows me to analyze how the life span of firms is related to financial frictions, including the ones discussed in this paper, and how the time-varying life span of firms affects entry decisions of new firms. Second, my model abstracts from loans that carry fixed interest rates such as mortgage loans. In practice, banks manage the interest-rate risk associated with maturity mismatch between their assets and liabilities. The literature documents some evidence that commercial banks, especially large ones, can mitigate the interest-rate risk through interest-rate swaps and securitization (Purnanandam, 2007; DeYoung and Yom, 2008) or by exerting the monopoly power over deposits to better match the *effective* maturity of liabilities to that of assets (Drechsler et al., 2018). However, it is challenging to identify to what extent the interest-rate risk remains in the banking sector as a whole and who, other than banks themselves, are sharing the risk with banks. Any unhedged interest-rate risk can increase the impact of real and financial shocks on bank net worth and reinforce the propagation mechanism shown in this paper. Third, banks in my model accumulate financial capital through retained earnings. An important next step is to introduce outside equity and inter-bank lending to examine how the use of these financial instruments affects the performance of banks as well as the propagation mechanism through the financial sector.[27] Such an extension is relevant for the analysis of regulatory capital requirements such as Basel III. Fourth, bank failure is not explained in this model. Although this extension is non-trivial, entry and exit of banks may have important implications on business cycle fluctuations, as discussed by Corbae and D'Erasmo (2011). Finally, this paper abstracts from nominal frictions, such as nominal debt contracts and sticky prices, and cannot speak to the effects of monetary policy. Similarly, this paper does not include the unconventional monetary policies that were implemented after the Great Recession.[28] These issues are beyond the scope of this paper and are left for future research.

[27]Gertler and Kiyotaki (2010) consider an inter-bank loan market where lending banks limit the amount of loans to borrowing banks.

[28]Ferrante (2019) studies the effects of the Fed's asset-purchase programs.

Appendix A: Derivation of solutions

This section provides more detailed derivation of the solutions to firms' and banks' problems and lists the system of equilibrium conditions.

A.1 Firms

A.1.1 Existing firm projects

Given idiosyncratic and aggregate states, existing firm projects choose capital and labor:

$$f(\varepsilon, z) = \max_{k,\ell} \left\{ \varepsilon z k^{\alpha} \ell^{\nu} - \widetilde{r}_k k - w\ell \right\}.$$

First-order conditions with respect to k and ℓ are

$$\begin{aligned} \alpha \varepsilon z k^{\alpha-1} \ell^{\nu} &= \widetilde{r}_k, \\ \nu \varepsilon z k^{\alpha} \ell^{\nu-1} &= w. \end{aligned}$$

Let $\Lambda \equiv 1/(1-\alpha-\nu)$, $\Gamma \equiv \alpha/\widetilde{r}_k$ and $\Omega \equiv \nu/w$. The optimal employment of labor and capital are

$$\begin{aligned} \ell &= (\varepsilon z)^{\Lambda} \Gamma^{\alpha\Lambda} \Omega^{(1-\alpha)\Lambda}, \\ k &= (\varepsilon z)^{\Lambda} \Gamma^{(1-\nu)\Lambda} \Omega^{\nu\Lambda}. \end{aligned}$$

The profit before loan repayment is

$$f(\varepsilon, z) = (1-\alpha-\nu)(\varepsilon z)^{\Lambda} \Gamma^{\alpha\Lambda} \Omega^{\nu\Lambda}.$$

Given what firms owe to banks, b, projects with low values of ε default while those with high values of ε repay the debt and distribute remaining profits to households. The threshold value of

$\underline{\varepsilon}$ below which firms default is defined as

$$(1-\alpha-\nu)(\underline{\varepsilon}z)^{\Lambda}\Gamma^{\alpha\Lambda}\Omega^{\nu\Lambda} = b.$$

A.1.2 New firm projects

Before drawing idiosyncratic productivity shocks, firms are identical. Anticipating the economic conditions in the next period, including the likelihood of default, firms decide whether to set up a new project. Only firms drawing a low administrative labor cost relative to an expected profit set up a project. The threshold level of entry, $\bar{\xi}$, is defined as

$$w\bar{\xi}^2 = \beta\mathbb{E}\frac{P'}{P}\int_{\underline{\varepsilon}(z')}^{\infty}\left[f(\varepsilon',z')-b'\right]d\Pi(\varepsilon'),$$

where $P = D_1 u\,(C, 1-L)$ is the marginal utility of consumption generated by households. Given $d\Pi(\varepsilon) = (\underline{\varepsilon}/\varepsilon_{\min})^{-k_\varepsilon}\,d\varepsilon$, this can be expressed as

$$w\bar{\xi}^2 = \beta\mathbb{E}\frac{P'}{P}\left[(1-\alpha-\nu)(\varepsilon_{\min}z)^{\Lambda}(\Gamma')^{\alpha\Lambda}(\Omega')^{\nu\Lambda}\frac{k_\varepsilon}{k_\varepsilon-\Lambda}\left(\frac{\underline{\varepsilon}'}{\varepsilon_{\min}}\right)^{-k_\varepsilon+\Lambda} - b'\left(\frac{\underline{\varepsilon}'}{\varepsilon_{\min}}\right)^{-k_\varepsilon}\right].$$

As ξ follows the log-normal distribution with parameters (μ_ξ, σ_ξ), the measure of firms operating in the next period, χ', is determined as follows:

$$\chi' = \int_0^{\bar{\xi}}\frac{1}{\xi\sqrt{2\pi\sigma_\xi^2}}\exp\left\{-\frac{1}{2}\left(\frac{\log(\xi)-\mu_\xi}{\sigma_\xi}\right)^2\right\}d\xi.$$

A.2 Banks

A.2.1 Current-period profits and net worth

At the beginning of the period, a bank has a net worth, n_{-1}, and loans, χ_i, from the previous period. The loans consist of a continuum of *ex-ante* identical firm projects. *Ex-post*, idiosyncratic productivity shocks and aggregate shocks materialize, which determine a bank's gross revenue per loan, V. More specifically, V is the sum of repayment from solvent firm projects and liquidation values of defaulted firm projects,

$$V = b\left[1 - \Pi(\varepsilon)\right] + \lambda F = b\left(\frac{\varepsilon}{\varepsilon_{\min}}\right)^{-k_\varepsilon} + \lambda F,$$

where $\lambda \in (0,1)$, and F is the total profit of failed firms before repayment:

$$F = \int_{\varepsilon_{\min}}^{\varepsilon} f(\varepsilon, z)\, d\Pi(\varepsilon) = (1-\alpha-\nu)\frac{k_\varepsilon}{k_\varepsilon - \Lambda}(\varepsilon_{\min} z)^{\Lambda}\Gamma^{\alpha\Lambda}\Omega^{\nu\Lambda}\left[1 - \left(\frac{\varepsilon}{\varepsilon_{\min}}\right)^{-k_\varepsilon+\Lambda}\right].$$

Since the size of each loan is κ, the ex-post (net) return on loans, ρ, is

$$\rho = \frac{V}{\kappa} - R.$$

Let the bank net worth before the stochastic death be $\widetilde{n}$. A bank generates $\widetilde{n}$ through the gross revenue, $V\chi_i$, minus the sum of repayments to depositors, Rs_i, and any dividend payout, d_B.

$$\widetilde{n} = V\chi_i - Rs_i - d_B.$$

Using the bank's balance sheet identity, $\kappa\chi_i = s_i + n_{-1}$, this can be expressed as

$$\widetilde{n} = \left(\frac{V}{\kappa} - R\right)\kappa\chi_i + Rn_{-1} - d_B = \rho\kappa\chi_i + Rn_{-1} - d_B.$$

The government may inject capital to banks, τ_i, which is characterized by an *i.i.d.* random shock. Therefore, the net worth of banks after a capital injection, n, is

$$n = \widetilde{n} + \tau_i.$$

A.2.2 Dynamic problem of banks

Since capital injection is an *i.i.d.* shock, a bank's value at the end of the period is a function of individual states, (n_{-1}, χ_i), and a vector of aggregate states, $\mathbf{x}_{-1}$. The bank's dynamic problem is to maximize (6) subject to (4), (5) and $d_B \geq 0$.

First, I show that banks do not pay out dividends. First-order conditions with respect to d_B, n and χ_i' are

$$1 - \eta_B + \lambda_B = 0, \tag{A.1}$$

$$1 - \theta + \theta D_1 B\left(n, \chi_i'; \mathbf{x}\right) + \psi^{-1} D_1 B\left(n, \chi_i'; \mathbf{x}\right) \mu_B = \eta_B, \tag{A.2}$$

$$\theta D_2 B\left(n, \chi_i'; \mathbf{x}\right) + \left(\psi^{-1} D_2 B\left(n, \chi_i'; \mathbf{x}\right) - \kappa\right) \mu_B = 0, \tag{A.3}$$

where η_B, μ_B and λ_B are the Lagrange multipliers associated with (4), (5) and $d_B \geq 0$, respectively. The envelope conditions are

$$D_1 B\left(n_{-1}, \chi_i; \mathbf{x}_{-1}\right) = \mathbb{E}_{-1} \beta \frac{P}{P_{-1}} R \eta_B, \tag{A.4}$$

$$D_2 B\left(n_{-1}, \chi_i; \mathbf{x}_{-1}\right) = \mathbb{E}_{-1} \beta \frac{P}{P_{-1}} \rho \kappa \eta_B. \tag{A.5}$$

If $\mu_B = 0$ for all periods, (A.2) and (A.4) imply that $\eta_B = (1-\theta)\left(1 + \theta + \theta^2 + \cdots\right) = 1$. But if (5) is binding or binds in the future, $\eta_B > 1$. This implies that $\lambda_B > 0$ from (A.1), which means that banks do not pay out dividends and retain all the earnings. This is because the expected net marginal value of loans over the risk-free rate, $\mathbb{E}\left(\beta P'/P\right) \rho' \eta_B'$, is positive when $\mu_B > 0$ as implied by (A.3) and (A.5). In this paper, I consider dynamics around the steady state in which (5) binds.

The next step is to solve the bank's problem. Notice that the linearity of the problem implies that the value function can be written as

$$B\left(n, \chi_i'; \mathbf{x}\right) = g_n\left(\mathbf{x}\right) n + g_\chi\left(\mathbf{x}\right) \chi_i'. \tag{A.6}$$

Because (5) is binding in the neighborhood of the steady state, substitute (A.6) into (5). This yields $\kappa\chi_i' = \psi^{-1}\left(g_n(\mathbf{x})\, n + g_\chi(\mathbf{x})\, \chi_i'\right)$, or equivalently,

$$\kappa\chi_i' = \frac{g_n(\mathbf{x})}{\psi - g_\chi(\mathbf{x})/\kappa} n = \phi(\mathbf{x})\, n,$$

as in (7), where

$$\phi \equiv g_n / \left(\psi - g_\chi/\kappa\right). \tag{A.7}$$

Substituting this result back into (A.6), I obtain

$$B\left(n, \chi_i'; \mathbf{x}\right) = \left[g_n(\mathbf{x}) + \frac{g_\chi(\mathbf{x})}{\kappa}\phi(\mathbf{x})\right] n = G(\mathbf{x})n, \tag{A.8}$$

where $G = g_n + (g_\chi/\kappa)\phi$ can be interpreted as the price of bank net worth. From (A.7) and the definition of G, the leverage ratio can be expressed as in (8):

$$\phi(\mathbf{x}) = \psi^{-1}\left(g_n(\mathbf{x}) + \frac{g_\chi(\mathbf{x})}{\kappa}\phi(\mathbf{x})\right) = \psi^{-1} G(\mathbf{x}).$$

Using (A.8), (7), $d_B = 0$, and $\mathbb{E}_{-1}\tau_i = 0$, the Bellman equation is expressed as

$$G(\mathbf{x}_{-1})\, n_{-1} = \mathbb{E}_{-1}\beta\frac{P}{P_{-1}}\left[1 - \theta + \theta G(\mathbf{x})\right] n = \left[\mathbb{E}_{-1}\beta\frac{P}{P_{-1}}\left(1 - \theta + \theta G(\mathbf{x})\right)\left(\rho(\mathbf{x})\,\phi(\mathbf{x}_{-1}) + R(\mathbf{x})\right)\right] n_{-1}.$$

Because this must hold for all n_{-1}, G satisfies the following equation:

$$G(\mathbf{x}) = \mathbb{E}\beta\frac{P'}{P}\left(1 - \theta + \theta G(\mathbf{x}')\right)\left(\rho(\mathbf{x}')\,\phi(\mathbf{x}) + R(\mathbf{x}')\right).$$

In the system of equations, the solution for ϕ and G imply g_n and g_χ.

A.3 Equilibrium conditions

To summarize, a set of conditions below constitutes a recursive competitive equilibrium.

Households:

$$P = C^{-1},$$

$$w = \eta_\nu C,$$

$$C^{-1} = \beta \mathbb{E} C'^{-1} R',$$

$$1 - \delta + r_k = R.$$

Firms:

$$\left(\frac{\varepsilon}{\varepsilon_{\min}}\right)^{1/(1-\alpha-\nu)} (1-\alpha-\nu)\, h = b, \tag{A.9}$$

$$h = (\varepsilon_{\min} z)^{1/(1-\alpha-\nu)}\, \Gamma^{\alpha/(1-\alpha-\nu)} \Omega^{\nu/(1-\alpha-\nu)},$$

$$\Gamma = \frac{\alpha}{\widetilde{r}_k},$$

$$\Omega = \frac{\nu}{w},$$

$$Y = \frac{k_\varepsilon}{k_\varepsilon - 1/(1-\alpha-\nu)} \chi h,$$

$$w\bar{\xi}^2 = \mathbb{E}\beta \frac{P'}{P}\left[\frac{k_\varepsilon}{k_\varepsilon - 1/(1-\alpha-\nu)}(1-\alpha-\nu)\, h' \left(\frac{\varepsilon'}{\varepsilon_{\min}}\right)^{-(k_\varepsilon - 1/(1-\alpha-\nu))} - b'\left(\frac{\varepsilon'}{\varepsilon_{\min}}\right)^{-k_\varepsilon}\right],$$

Banks:

$$G = \mathbb{E}\beta \frac{P'}{P}(\rho'\phi + R')\left\{(1-\theta) + \theta G'\right\}, \tag{A.10}$$

$$\phi = \psi^{-1} G, \tag{A.11}$$

$$\kappa \chi' = \phi N,$$

$$N = \theta\left[\rho\kappa\chi + R N_{-1}\right] + T_B + \omega\kappa\chi,$$

$$\rho = V/\kappa - R,$$

$$V = \left(\frac{\varepsilon}{\varepsilon_{\min}}\right)^{-k_\varepsilon} b + \lambda F,$$

$$F = \frac{k_\varepsilon}{k_\varepsilon - 1/(1-\alpha-\nu)}(1-\alpha-\nu)\, h \left[1 - \left(\frac{\underline{\varepsilon}}{\varepsilon_{\min}}\right)^{-(k_\varepsilon - 1/(1-\alpha-\nu))}\right].$$

Non-bank financial companies

$$\widetilde{r}_k = r_k + z_m,$$

$$z_m - z_{m,ss} = \gamma(\psi - \psi_{ss}).$$

Market-clearing conditions:

$$Y = C + I + \chi(1-\lambda) F + \left[1 - \left(\frac{\underline{\varepsilon}}{\varepsilon_{\min}}\right)^{-k_\varepsilon}\right] z_m K,$$

$$I = K' - (1-\delta) K + \kappa \chi',$$

$$K = \frac{k_\varepsilon}{k_\varepsilon - 1/(1-\alpha-\nu)} \chi (\varepsilon_{\min} z)^{1/(1-\alpha-\nu)} \Gamma^{(1-\nu)/(1-\alpha-\nu)} \Omega^{\nu/(1-\alpha-\nu)},$$

$$L = \frac{k_\varepsilon}{k_\varepsilon - 1/(1-\alpha-\nu)} \chi (\varepsilon_{\min} z)^{1/(1-\alpha-\nu)} \Gamma^{\alpha/(1-\alpha-\nu)} \Omega^{(1-\alpha)/(1-\alpha-\nu)} + \mathbb{E}\left[\xi^2 \mid \xi < \bar{\xi}\right] \chi,$$

$$\chi' = \int_0^{\bar{\xi}} \frac{1}{\xi\sqrt{2\pi\sigma_\xi^2}} \exp\left\{-\frac{1}{2}\left(\frac{\log(\xi) - \mu_\xi}{\sigma_\xi}\right)^2\right\} d\xi.$$

Laws of motion for exogenous variables:

$$\begin{bmatrix} \log z' \\ \psi' - \psi_{ss} \\ T_B' \end{bmatrix} = \begin{bmatrix} \rho_z & 0 & 0 \\ 0 & \rho_\psi & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \log z \\ \psi - \psi_{ss} \\ T_B \end{bmatrix} + \begin{bmatrix} e_z' \\ e_\psi' \\ e_T' \end{bmatrix}.$$

A.4 The model without the risk of loan default

Since banks are fully protected from default by firms, this is effectively equivalent to the case where $\underline{\varepsilon} = \varepsilon_{\min}$. Thus, $(\underline{\varepsilon}/\varepsilon_{\min}) = 1$ holds for the equilibrium conditions in Section A.3, where applicable, and the definition of $\underline{\varepsilon}$, (A.9), becomes irrelevant.

A.5 The model without deposit frictions

Since deposit frictions do not exist, financial shocks do not affect banks. Hence, (A.11) is irrelevant. Since the value of banks does not matter, (A.10) is also irrelevant.

A.6 The model without financial frictions

This is a combination of the previous two cases. After setting $\underline{\varepsilon} = \varepsilon_{\min}$, (A.9), (A.10), and (A.11) are irrelevant.

Appendix B: Data

This section explains the sources of data used for the analyses in this paper.

B.1 Calibration and estimation

Ratios of dividends, salaries and employee benefits paid by banks to equities are taken from call report data. The codes for dividends are RIAD4160 and RIAD4170, salaries and employee benefits are RIAD4135, and equities after the payment of dividends, salaries and employee benefits are RIAD4135. Baa and Aaa bond yields are taken from Moody's seasoned corporate bond yields. The federal funds rate is available from the Board of Governors of the Federal Reserve System. Real GDP and private fixed investment are from Table 1.1.3 of the National Income and Product Accounts (NIPA). The net operating surplus and value added of non-financial businesses are taken from Table 1.14 of NIPA. The bank leverage is measured as the sum of loans, leases, and securities divided by Tier 1 capital, based on the aggregated bank balance sheet data from the FDIC Quarterly Banking Profile (Balance Sheet). The net operating income of banks is taken from the FDIC Quarterly Banking Profile (Income and Expense). The non-current rate of C&I loans is taken from the FDIC Quarterly Banking Profile (Loan Performance Indicators). The number of firms is taken from the US Census Business Dynamics Statistics (Firm Characteristics Data Table).

B.2 TARP

The TARP Investment Program Transaction Reports (TIPTR) provide information on each transaction of the TARP programs, including the Capital Purchase Program and the Targeted Investment Program, which are the focus of this paper. If a recipient of TARP is a bank holding company, call report data are used to infer the actual funds transferred to subsidiary banks. The principal of TARP funds injected to a subsidiary bank is measured by the RIAD4415 series, "other transactions with stock holders including parent companies," corrected for any misreportings, RIADB507, close to the capital injection date. If a recipient is not a bank holding company, the principal amount and date are available on the TIPTR. I apply repayment dates of a bank holding company reported on the TIPTR to its subsidiary banks for computing outstanding balances. Each bank's dividend payments are one-fourth of the annualized rate of 5% of its outstanding balance in each quarter. The net flow of capital to each bank in each quarter is computed as principal received minus the sum of principal repaid and dividend payments. Aggregating the individual net flow of capital across banks that received TARP funds in the form of preferred stocks generates the aggregate net flow of TARP funds.

References

[1] Adrian, Tobias and Hyun Song Shin, 2009, "Money, Liquidity and Monetary Policy," *American Economic Review,* 99, 600-9.

[2] Bernanke, Ben, Mark Gertler and Simon Gilchrist, 1999, "The Financial Accelerator in a Quantitative Business Cycle Framework," in *Handbook of Macroeconomics,* Volume 1c, 1341-93 (Amsterdam, Elsevier Science).

[3] Calomiris, Charles and Urooj Khan, 2015, "An Assessment of TARP Assistance to Financial Institutions, " *Journal of Economic Perspective*, 29, 53-80.

[4] Carlstrom, Charles and Timothy Fuerst, 1997, "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review,* 87, 893-910.

[5] Christiano, Lawrence, Roberto Motto and Massimo Rostagno, 2014, "Risk Shocks, " *American Economic Review*, 104(1), 27-65.

[6] Corbae, Dean and Pablo D'Erasmo, 2011, "A Quantitative Model of Banking Industry Dynamics," unpublished paper, University of Wisconsin - Madison.

[7] DeYoung, Robert and Chiwon Yom, 2008, "On the Independence of Assets and Liabilities: Evidence from U.S. Commercial Banks, 1990-2005, " *Journal of Financial Stability*, 4, 275-303.

[8] Dib, Ali, 2010, "Banks, Credit Market Frictions, and Business Cycles," Bank of Canada Working Paper 2010-24.

[9] Drechsler, Itamar, Alexi Savov and Philipp Schnabl, 2018, "Banking on Deposits: Maturity Transformation without Interest Rate Risk, " NBER Working Paper 24582.

[10] Ferrante, Francesco, 2019, "Risky Lending, Bank Leverage and Unconventional Monetary Policy, " *Journal of Monetary Economics*, 101, 100-27.

[11] Gertler, Mark and Peter Karadi, 2011, "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics,* 58, 17-34.

[12] Gertler, Mark and Nobuhiro Kiyotaki, 2010, "Financial Intermediation and Credit Policy in Business Cycle Analysis," in *Handbook of Monetary Economics*, Volume 3, 547-599 (Amsterdam, Elsevier Science).

[13] Gomes, Joao and Lukas Schmidt, 2009, "Equilibrium Credit Spreads and the Macroeconomy," unpublished paper, Wharton School of Business, University of Pennsylvania.

[14] Hansen, Gary, 1985, "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics*, 16, 309-27.

[15] Holmstrom, Bengt and Jean Tirole, 1997, "Financial Intermediation, Loanable Funds, and the Real Sector," *Quarterly Journal of Economics,* 112, 663-91.

[16] Iacoviello, Matteo, 2015, "Financial Business Cycles," *Review of Economic Dynamics*, 18, 140-63.

[17] Ivashina, Victoria and David Scharfstein, 2010, "Bank Lending During the Financial Crisis of 2008," *Journal of Financial Economics,* 97, 319-38.

[18] Jermann, Urban and Vincenzo Quadrini, 2009, "Macroeconomic Effects of Financial Shocks," NBER Working Paper No. 15338.

[19] ——, ——, 2012, "Macroeconomic Effects of Financial Shocks," *American Economic Review* 102, 238-71.

[20] Khan, Aubhik and Julia Thomas, 2013, "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity," *Journal of Political Economy*, 121, 1055-107.

[21] Kiyotaki, Nobuhiro and John Moore, 1997, "Credit Cycles," *Journal of Political Economy*, 105, 211-48.

[22] Kocherlakota, Narayana, 2000, "Creating Business Cycles Through Credit Constraints," *Federal Reserve Bank of Minneapolis Quarterly Review,* 24, 2-10.

[23] Koepke, Matthew and James Thomson, 2011, "Bank Lending," *Economic Trends,* March, Federal Reserve Bank of Cleveland.

[24] Kumbhat, Ashish, Francisco Palomino and Ander Perez-Orive, 2017, "The Potential Increase in Corporate Debt Interest Rate Payments from Changes in the Federal Funds Rate, " *FEDS Notes*, November 15, 2017.

[25] Meh, Cesaire and Kevin Moran, 2010, "The Role of Bank Capital in the Propagation of Shocks," *Journal of Economic Dynamics and Control*, 34, 555-76.

[26] Moody's, 2007, *Moody's Ultimate Recovery Database*, available at http://www.moodys.com/sites/products /DefaultResearch/2006600000428092.pdf.

[27] Nuño, Galo and Carlos Thomas, 2014, "Bank Leverage Cycles," *American Economic Journal: Macroeconomics*, 9, 32-72.

[28] Purnanandam, Amiyatosh, 2007, "Interest Rate Derivatives at Commercial Banks: An Empirical Investigation, " *Journal of Monetary Economics*, 54, 1769-1808.

[29] Rogerson, Richard, 1988, "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics*, 21, 3-16.

[30] Siemer, Michael, 2019, "Employment Effects of Financial Constraints During the Great Recession, " *Review of Economics and Statistics*, 101(1), 16-29.

[31] Takamura, Tamon, 2013, "A General Equilibrium Model with Banks and Default on Loans," *Bank of Canada Staff Working Paper*, 2013-3.

[32] Townsend, Robert, 1979, "Optimal Contracts and Competitive Markets with Costly State Verification," *Journal of Economic Theory*, 21, 265-293.