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Labour Supply and Firm Size

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Abstract

Larger firms feature i) longer hours worked, ii) higher wages, and iii) smaller (larger) wage penalties for working long (short) hours. We reconcile these patterns in a general equilibrium model, which features the endogenous interaction of hours, wages, and firm size. In the model, workers willing to work longer hours sort into larger firms that offer a wage premium. Complementarities in hours worked generate wage penalties that increase with the distance from the average firm hours. We use the model to argue about the importance of the interaction between hours, wages, and firm size on inequality.

Topics: Firm dynamics; Labour markets
JEL codes: E24, J2, J31

Résumé

Les grandes entreprises se caractérisent par i) des heures travaillées plus longues, ii) des salaires plus élevés et iii) des désavantages salariaux moins (plus) importants pour les heures de travail plus longues (courtes). Nous concilions ces tendances dans un modèle d'équilibre général intégrant l'interaction endogène entre heures, salaires et taille des entreprises. Dans ce modèle, les travailleurs désireux de faire plus d'heures s'orientent vers les grandes entreprises qui offrent un avantage salarial. Les complémentarités entre les heures travaillées génèrent des désavantages salariaux qui augmentent avec l'écart par rapport au nombre moyen d'heures travaillées dans l'entreprise. Nous utilisons le modèle pour montrer que l'interaction entre les heures, les salaires et la taille des entreprises a un effet important sur l'inégalité.

Sujets : Dynamique des entreprises; Marchés du travail
Codes JEL : E24, J2, J31

1 Introduction

There exists significant variation in workers' labor supply. This variation, and its interaction with firm-level heterogeneity, has important implications for inequality. For instance, if workers in high-paying firms also work longer, variation in hours amplifies income inequality, while inequality is mitigated if workers in high-paying firms work fewer hours. Thus, understanding how hours vary across firms of different characteristics is crucial for assessing the role of worker and firm heterogeneity on inequality.

In this paper, we study the relationship between firm size, hours, and wages and find that workers in larger firms, which tend to pay higher wages, tend to work longer hours. Moreover, we show that the relationship between hours and wages differs systematically by firm size. Motivated by these findings, we develop a general equilibrium framework with heterogeneous workers and firms to show that incorporating the joint relationship between hours, wages, and firm size has important aggregate implications for earnings inequality that are absent in canonical models that abstract from this relationship.

The first part of the paper uses data from the US Current Population Survey (CPS) to document three motivating facts on the relationship between hours and wages by firm size. To begin with, we present a relatively understudied empirical pattern: average hours worked increases with firm size. Second, we revisit the well-established size-wage premium, wherein average hourly wages are found to increase with the size of the firm (Brown and Medoff 1989, Oi and Idson 1999). The third fact we present is novel. Our data reveal that the wage penalties associated with relatively long and short working hours vary by firm size. Specifically, we find that workers at larger firms experience smaller wage penalties for working long hours but are subject to greater wage penalties for working shorter hours. Collectively, these three facts build upon the well-documented size-wage premium, offering a more comprehensive perspective of how wages, hours, and their relationship vary across firms of different sizes.

Motivated by our empirical findings, the second part of the paper introduces a theoretical framework to examine the interplay of hours, wages, and firm heterogeneity. In the model, firms differ in their exogenous productivity and decide on their labor input.[1] Aligning with recent findings in Shao et al. (2023) and Kuhn et al. (2023) indicating that working hours are complements and coordinated in production, our model's production function allows for complementarities between workers' hours. Specifically, a firm's labor input is a non-linear aggregate of the hours worked by all its employees, implying workers are more productive if their hours are similar to those of their co-workers. In addition, workers differ in their value of leisure and have additional preferences for working in firms of different productivity levels. Given these sources of heterogeneity, workers decide their labor supply and which firm to work for.

Despite its minimal structure, our model, which is calibrated to match key features of the US economy, successfully replicates all three motivating facts. First, the size-wage premium is generated by an interaction of firm-level heterogeneity in productivity and workers' preferences over working in firms of different characteristics. As in Card et al. (2018), with such idiosyncratic tastes, workers view firms of different productivity as imperfect substitutes. This results in a less pronounced increase in firm employment with productivity, allowing the marginal productivity of labor (wages) to rise with firm productivity, and hence size, in equilibrium. We introduce heterogeneity in tastes as a simple and relatively standard feature, aiming to generate a size-wage premium that interacts with other mechanisms in the model to generate our other two motivating facts. Importantly, this modeling choice for generating the size-wage premium does not, on its own, generate increasing hours with firm size, nor does it cause the size-dependent wage penalties for short and long hours. Indeed, as we discuss below, differences in firm productivity and endogenous sorting between workers and firms remain critical for reconciling the data.

Second, the positive correlation between firm size and worker hours can be attributed to the interaction between the size-wage premium and workers' decisions about labor supply. Since income is the product of hourly wage and working hours, a size-wage premium suggests that those who work longer hours will see larger income gains when employed by larger firms. Furthermore, the higher wages offered by large firms can enhance income returns

[1]Several papers document a robust positive correlation between measures of firm productivity and the number of employees in a firm (see, for example, Leung et al., 2008 and Bartelsman et al., 2013). Consistent with this, we use the number of employees in a firm to proxy for productivity.

from working longer hours. In line with this mechanism, workers who prefer longer hours sort into larger firms, and for any given type of worker, they tend to work longer hours in these larger firms. Therefore, average working hours increase with firm size.

Third, differences in short- and long-hour wage penalties across firms result from endogenous worker sorting decisions under the presence of complementarities in working hours. Consistent with the data, complementarities in workers' hours ensure that worker productivity declines as the gap between their hours and a firm's usual working hours widens, and consequently they suffer more significant wage losses as they deviate from usual hours worked. Since larger firms feature longer average hours worked, long (short) hours are less (more) heavily penalized compared to small firms. Importantly, even without complementarities between workers' hours, the size-wage premium will lead to a positive relationship between hours and firm size. Instead, we introduce complementarities in hours because it is empirically supported and endogenously converts the size-hour patterns into the size-dependent short- and long-hour penalties.

Our baseline model is kept intentionally simple to illustrate the key mechanisms reconciling our main motivating facts. We also present an extended model that includes factors such as worker efficiency, risk aversion, wealth accumulation, and friction in switching employers. This extended model allows us to show the robustness of our findings in a richer environment and sets the stage for two important exercises. First, we use this model to discuss a testable implication about workers sorting into different-sized firms. Second, we conduct an accounting exercise to quantify the various model mechanisms in shaping wage and income inequality.

The model has strong implications for worker sorting based on desired hours. Specifically, it predicts that workers who work fewer hours than their co-workers are more likely to sort into smaller firms where their desired hours align more closely with their average co-workers. Conversely, those working longer hours than their co-workers are more likely to sort into larger firms. We find support for this prediction in the data. By tracking CPS respondents over a span of 12 months, we show that workers with relatively shorter hours are more likely to transition into smaller firms and less likely to move to larger firms—just as the theory predicts. This finding complements existing literature on labor market sorting, highlighting the importance of sorting based not only on worker skills and firm heterogeneity but also on workers' desired hours.

We also use our extended model to quantify the role of model mechanisms in shaping worker inequality. First, we examine the role of heterogeneity in working hours in shaping wage inequality, both in the data and in the model. In line with the data, the model features wage dispersion due to heterogeneity in worker skill, firm productivity, and, notably, working hours. While the impact of worker and firm heterogeneity on wage inequality has been extensively studied, the role of heterogeneity in hours remains largely unexplored. Our findings suggest that variation in hours accounts for 16% of the *explained* wage dispersion, compared to a 19% contribution observed in the data. Interpreted through the lenses of the model, working hours influence wage dispersion due to complementarities in workers' hours. Indeed, without these complementarities, wages within firms would be unrelated to working hours, thus making the heterogeneity of hours immaterial to wage inequality after controlling for firm productivity.

Finally, we explore the role of hours on inequality in income—the product of hours and wages. A variance decomposition exercise reveals that hours' dispersion and its covariance with wages account for approximately 20% of overall income dispersion in the data and model. Furthermore, we highlight the role of complementarity in working hours in influencing income inequality. Although complementarities generate wage dispersion, they also introduce incentives for workers to work similar hours through wage penalties for working dissimilar hours, compressing the distribution of hours in the economy. Quantitatively, we find that the latter channel dominates, and income inequality decreases in the degree of complementarity in hours.

Taken together, this paper highlights the endogenous interaction between hours, wages, and firm-level heterogeneity—an interaction that is currently under-emphasized in the literature but, as we argue, has important implications for aggregate outcomes.

Related literature. This paper is closely related to several strands of literature studying the interaction of firm characteristics, wages, and hours worked. Among our three motivating facts, the size-wage premium has been most extensively documented and studied. No consensus on the determinant of the size-wage premium exists, and our model generates it through workers' heterogeneous preferences over potential employers, closely following the approach in Card et al. (2018) and Lamadon et al. (2022). Such heterogeneity prevents worker flows from equalizing wages across firms and generates higher wages at larger, more productive firms.

In contrast, the positive relationship between hours worked and firm size that we document is relatively under-studied. Montgomery (1988) and Headd (2000) touch on this relationship by showing that larger firms have a lower fraction of part-time workers. Our analysis moves beyond distinguishing between part-time and full-time workers and considers the entire distribution of hours across firms. Accordingly, our model is rich enough to capture differences between small and large firms that are not driven by discontinuities in part-/full-time hiring costs or workers' productivity.

Our third fact, the variation in the long- and short-hours penalties across size categories of firms, is novel and naturally relates to the literature that documents the presence of such penalties across the economy. Recent work by Yurdagul (2017) and Bick et al. (2022) have documented a hump-shaped relationship between wages and hours in the aggregate, while Shao et al. (2023) document such a relationship within establishments. We build on these findings by providing new evidence showing that the hump-shaped wage-hours profile varies with firm size.

Our paper joins a growing literature studying heterogeneous agent macroeconomic models that aim to be consistent with micro-level evidence on wages and the labor supply of workers. Much of this literature has focused on the response of labor supply to business cycle or life-cycle fluctuations (see, for example, Heathcote et al. 2014, Erosa et al. 2016, and Chang et al. 2020). Instead, we focus on the cross-sectional relationship between hours, wages, and firm characteristics. Bick et al. (2022) study the relationship between hours and wages, using an exogeneously specified non-linear wage schedule to replicate the hump-shaped wage-hours profile. In contrast, we generate the non-linear wage schedule endogenously in a general equilibrium model, emphasizing the interaction between hours, wages, and firm-level heterogeneity as well as its implications for inequality.

A distinguishing feature of our model is the presence of complementarities in the hours of different workers. Our modeling of complementarity is closely related to Yurdagul (2017) and Shao et al. (2023).[2] The feature aims to capture the necessity for workers to coordinate their work schedule to produce output and is supported by empirical evidence. For instance, recent work by Shao et al. (2023) and Kuhn et al. (2023) use matched employer-employee data to show that workers' hours within the same establishment are gross complements and coordinated. As such, our model relates to the literature exploring the impact of constraints on working hours resulting from coordination (see, for example, Altonji and Paxson 1988, Chetty et al. 2011, Labanca and Pozzoli 2021, and Cubas et al. 2019). We contribute to this strand of literature by explicitly incorporating inflexible working hours (via complementarities) in a theoretical model and studying its interaction with firm heterogeneity and its implications for worker sorting across firms and inequality.

Our analysis of inequality is related to recent empirical work, such as Song et al. (2019) and Barth et al. (2016), that explores the role of worker and firm heterogeneity in generating wage dispersion. We contribute to this literature by documenting the role of hours heterogeneity in wage and income inequality. Blau and Kahn (2011) and Checchi et al. (2016) document the relationship between heterogeneity in hours and income inequality. We relate to this literature by highlighting the importance of complementarity in hours.

An outline of the paper is as follows. Section 2 describes our motivating facts in detail, and Section 3 outlines our model. In Section 4 we calibrate the model and discuss how it reconciles the motivating facts. In Section 5 we extend our baseline model with realistic features to make it more suitable for quantitative analysis. Section 6 explores the aggregate implications of our theoretical framework for wage and income inequality. Section 7 concludes.

2 Motivating facts

This section documents three motivating facts about the distribution of hours and wages across firm size. First, we establish a robust positive relationship between firm size and average worker hours. Workers in the smallest firms (1 to 9 employees) work 7% fewer

[2] Battisti et al. (2021) also utilize a similar production function to analyze the correlation between estimates of the Frisch elasticity and the extent of hour complementarities.

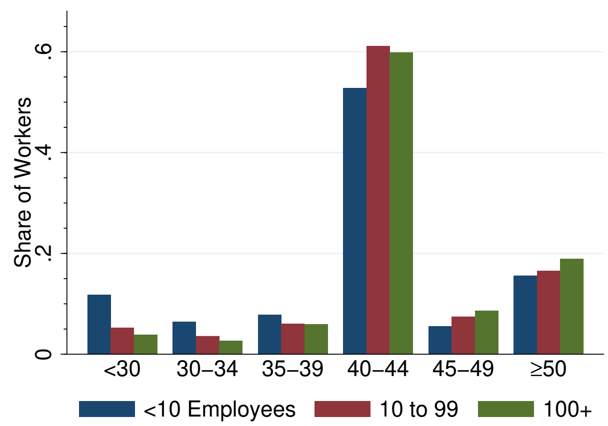
hours per week than those in the larger firms. Second, we confirm the existence of a size-wage premium. Third, we show that workers face penalties for working either short or long hours in all firms. However, the magnitude of these penalties vary systematically across size categories. In particular, we find that the penalty for working long hours decreases with size while the penalty for working short hours increases with size.

Data description. To establish these facts, we use data from the Annual Social and Economic Supplement (ASEC) of the CPS covering information from 1991 to 2018. This supplement to the CPS contains detailed information on respondents' economic activity for the past year. Importantly, it includes information on worker earnings, usual weekly hours worked, and firm size.[3] The partitioning of size bins has varied over time, so for clarity and consistency, we report three categories of firm size: i) small (1 to 9 employees), ii) medium (between 10 and 100 employees), and iii) large firms (over 100 employees). We restrict attention to individuals aged between 25 and 64 who worked with a single private employer in the previous year, and exclude those who usually work less than 10 hours a week or earn less than half the federal minimum wage. Respondents with imputed values for firm size, hours worked, or weeks worked are also dropped. The final sample includes just over 1 million respondents.

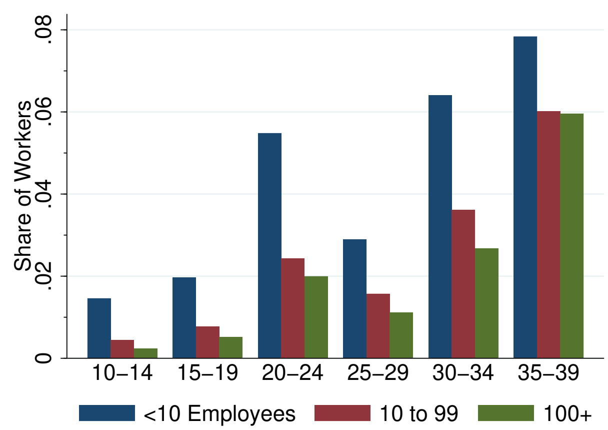
We report a number of additional empirical results in Appendix B. Specifically, Appendix B.1 provides robustness checks for our findings using data from the CPS. Appendix B.2 argues that our findings are not driven by measurement error in hours, while Appendix B.3 replicates our empirical analysis at both the establishment and firm level using analogous data from the 1997 to 2018 Canadian Labour Force Surveys (LFS).

[3] Data is extracted from IPUMS and described in Flood et al. (2020). Firm size is reported in bins and records the total number of employees that a worker's employer has at all establishments. For example, the 2019 ASEC, which contains information for the reference year 2018, asks the following: "Counting all locations where this employer operates, what is the total number of persons who work for your employer?" We start our sample in 1991 since, prior to this, the smallest reported firm size category was firms with under 25 employees.

(a) Overall



(b) Short hours



(c) Long hours

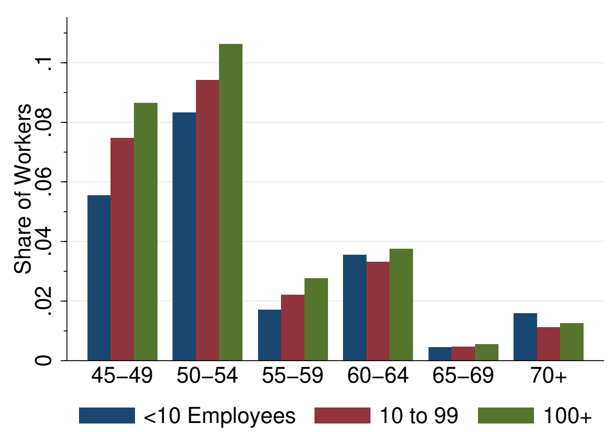


Figure 1: Distribution of working hours by firm size

Notes: The figure reports the share of workers by their usual weekly hours worked and firm size using data from the CPS.

<u>Fact 1</u> Average hours increase with size.

We begin by studying the relationship between firm size and hours worked. Figure 1 reports the distribution of usual weekly hours worked by firm size, with Panel (a) showing the overall distribution of usual hours worked. While the median weekly hours across all firms is between 40 and 44 hours, there are important differences in the share of short and long hours worked across firm sizes. This is evident in Panels (b) and (c), which report, respectively, the distribution of the right and left tails of the hours distribution. Panel (b) shows that workers in small firms are much more likely to work shorter (<40) hours than their counterparts in larger firms. For example, around 3% of employees in larger firms work 30 to 35 hours, while the analogous share in small firms is around 6%. Panel (c) shows that employees in medium and large firms are more likely to work between 45–59 hours with a similar likelihood of working very long (≥ 60) hours. For example, just around 8% of employees in small firms (less than 10 employees) work between 50 and 55 hours, while the analogous share in larger firms is 10%.

As suggested from Figure 1 and confirmed in Figure 2, there is a positive relationship between average hours worked and firm size. On average, workers in the largest firms work around 3 hours longer than workers in the smallest firms.

While informative, these cross-sectional averages do not control for confounding factors, such as industry, that might impact both firm size and hours worked. To control for such factors, we estimate the following regression:

$$h_i = \alpha + \left(\sum_{f \in F} \beta_f \mathbb{I}_{i,f} \right) + \delta X_i + \epsilon_i \tag{1}$$

where h_i are the usual weekly hours worked. X_i is a vector of individual-level controls, which includes a quadratic in years of experience and dummies for the race, education, gender, marital status as well as state, year, and industry fixed effects. The variable $\mathbb{I}_{i,f}$ is an indicator variable that is equal to one if an individual is employed in a firm of size $f \in F$ so that the coefficient β_f captures the additional number of hours worked by firm size.

Table 1 reports this coefficient and shows that when excluding controls for industry or demographics, workers in the firms with 10 to 99 employees work, on average, around 1.8 and

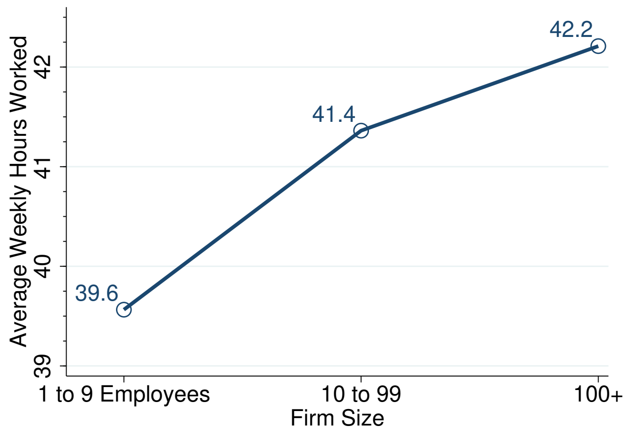


Figure 2: Average weekly hours by firm size

Notes: The figure reports the average usual weekly hours worked by firm size using data from the CPS.

1.5 hours longer per week than workers in firms with 1 to 9 employees, while workers in firms with over 100 employees work between 2.6 and 2.2 hours longer per week. Controlling for industry and demographic characteristics explains some of the differences in hours worked between medium and larger firms and implies that workers in firms with 10 to 99 employees work around 1 hour and 15 minutes longer per week and workers in firms with over 100 employees work 2 hours and 5 minutes longer per week than workers employed in the smallest firms.

In addition, the differences between medium and large firms are statistically significant, confirming that average weekly hours are positively related with firm size even after controlling for observable characteristics. These differences are also economically significant and could amount to as much as an additional 2 to 3 weeks of work per year in medium and large firms relative to small firms.

We report a number of robustness exercises in Appendix B.1. Specifically, we use the more detailed firm size categories reported in the CPS to show that average weekly hours tend to increase monotonically with firm size. We also show that hours increase with size when Equation (1) is estimated with additional controls, including occupation fixed effects and controls for hourly worker status. Building on Montgomery (1988) and Headd (2000), who show that smaller firms have a higher share of part-time workers, we find a positive relationship between weekly hours and firm size even after controlling for worker's part-

/full-time status or focusing on workers that work within a narrow window of hours (35 to 45 hours per week).

Table 1: Firm size and hours worked

	(1)	(2)	(3)
10 to 99 Employees	1.801***	1.524***	1.263***
	[+2.3 weeks/yr]	[+2.0 weeks/yr]	[+1.6 weeks/yr]
	(0.041)	(0.039)	(0.040)
100+ Employees	2.640***	2.228***	2.097***
	[+3.4 weeks/yr]	[+2.9 weeks/yr]	[+2.7 weeks/yr]
	(0.038)	(0.037)	(0.040)
Year, State FE	Y	Y	Y
Demographic Controls	N	Y	Y
4-digit Industry FE	N	N	Y
N	1,000,820	1,000,820	1,000,820
R^2	0.013	0.102	0.129

Notes: The table reports the coefficient β_f estimated from Equation (1) where the reference size category is firms with 1 to 9 employees. The brackets report the additional number of weeks worked per year implied by the estimated regression coefficient. For example, an additional, relative to small firms, 2 hours worked per week over 52 weeks implies an additional 104 hours worked per year. Given that the median workweek consists of 40 hours, this suggests an additional 2.6 (104/40) weeks worked per year. Standard errors are reported in parentheses. *** indicates statistical significance at the 1% level.

Fact 2 Average wages increase with size.

The wage premium in large firms has been studied extensively (see, for example, Oi and Idson 1999). We establish the size-wage premium in our data by estimating the following regression:

$$\log(w_i) = \alpha + \left(\sum_{f\in F} \beta_f \mathbb{I}_{i,f}\right) + \left(\sum_{h\in H} \gamma_h \mathbb{I}_{i,h}\right) + \left(\sum_{f\in F}\sum_{h\in H} \theta_{f,h}\left[\mathbb{I}_{i,f} \times \mathbb{I}_{i,h}\right]\right) + \delta X_i + \epsilon_i \qquad (2)$$

where $\log(w_i)$ is the log hourly wages of individual i. Hourly wages are computed as the ratio of annual earnings, usual weekly hours and weeks worked. As in Equation (1), X_i is a vector of individual-level controls, which includes demographic controls, state, year, and industry fixed effects. The indicator variable $\mathbb{I}_{i,f}$ is equal to one if an individual is employed in a firm of size $f \in F$. Similarly, $\mathbb{I}_{i,h}$ is equal to one if an individual works h hours.

We partition weekly hours into a set H by grouping hours in 5-hour bins. The partitioned set is $H = \{10-14, 15-19, \ldots, 65-69, 70-99\}$. The final bin, $70-99$, is larger as there are relatively few workers working over 70 hours. As most workers work 40 hours, we choose the category $40-44$ as the reference category for hours and omit the coefficients γ_{40} and $\theta_{40,f}$ for all f. The reference size category is firms with 1 to 9 employees.

The regression in Equation (2) extends the specification in Bick et al. (2022) by also controlling for firm size and an interaction term between firm size and usual weekly hour bins. Including these regressors allows us to study i) the size-wage premium (fact 2) and ii) the relationship between hours and wages by firm size (fact 3, discussed below).

The coefficient that captures the firm size-wage premium (for workers that work in the 40 hours bin) is β_f. Table 2 reports β_f and shows that it increases monotonically in size. Indeed, the wage premium between the largest and smallest size categories is around 23%.

Table 2: The size-wage premium

	(1)	(2)	(3)
10 to 99 Employees	0.122***	0.106***	0.095***
	(0.003)	(0.003)	(0.003)
100+ Employees	0.317***	0.254***	0.211***
	(0.003)	(0.003)	(0.003)
Year, State FE	Y	Y	Y
Demographic Controls	N	Y	Y
4-digit Industry FE	N	N	Y
N	1,000,820	1,000,820	1,000,820
R^2	0.125	0.348	0.417

Notes: The table reports the coefficient β_f estimated from Equation (2) where the reference size category is the smallest size firms/establishments—that is, firms with 1 to 9 employees. The reference hours bin is $40-44$ hours. Data are from the CPS sample. Standard errors are reported in parentheses. *** indicates statistical significance at the 1% level.

<u>Fact 3</u> Long-hour (short-hour) penalty decreases (increases) with size.

Next, we study the relationship between wages by hours worked and firm size. Panel (a) of Figure 3 plots the unconditional average hourly earnings of workers by hours worked and firm size. Two salient features are evident from these cross-sectional averages. First, the size-wage premium exists across the entire distribution of working hours. Second, we find a hump-shaped relationship between hours and earnings across all firm sizes. That is, there appears to be a wage penalty resulting from working either longer or shorter hours than the modal hours worked.

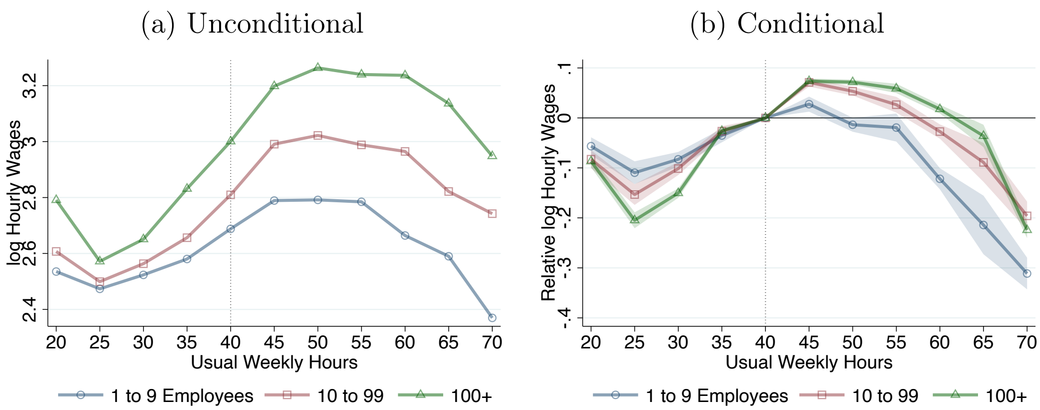


Figure 3: The relationship between wages and hours by firm size

Notes: Panel (a) plots the unconditional average of log hourly wages by usual weekly hours and firm size. Panel (b) reports the sum of coefficients ($\gamma_h + \theta_{f,h}$) estimated from Equation 2. The reference category for hours worked is $40 - 45$ and the reference category for firm size is firms with 1 to 9 employees. The shaded regions are the 95% confidence intervals. Data are from the CPS sample.

Panel (b) reports the same relationship while also controlling for observable characteristics. In particular, it reports the sum of the coefficients γ_h and $\theta_{f,h}$ estimated from Equation (2). This sum captures the wage penalty of working outside of the 40–44 hours bin by size category.[4] An aggregate hump-shaped wage-hours relationship has been documented by Yurdagul (2017) and Bick et al. (2022) in the US. Panel (b) shows that this hump-shaped relationship varies with firm size.

[4]Strictly speaking, the coefficient γ_h captures the wage penalty of working away from the 40 hours bin for small firms. While $\theta_{f,h}$ captures the difference in penalty relative to small firms. Recall that $\theta_{f,h}$ for small firms is zero, as small firms are the reference size category.

Specifically, the conditional hourly wages in Figure 3 suggest that the penalty for working shorter hours is more severe in larger firms while the penalty for working longer hours is less severe. For example, relative to working in a firm with 1 to 9 employees, working around 25 hours in a firm with 10 to 99 (over 100) employees results in around a 4 (10)% *increase* in wage penalty. On the other hand, working longer hours—say around 60 hours—results in a roughly 9 (14)% *decrease* in wage penalty relative to working the same hours in a small firm with 1 to 9 employees.[5]

Together, the three empirical facts highlighted in this section motivate our theoretical analysis. A focus of our framework is the causal link between differential wage penalties and average hours across firms. We generate a link between wage penalties and average hours by allowing for complementarities in hours worked in our theoretical model. A natural consequence of such complementarities is that wage penalties are increasing in the distance from the usual hours worked within a workplace. Since larger firms feature longer average hours, longer hours are penalized less severely compared to smaller firms. Conversely, smaller firms feature shorter hours, and hence shorter hours are penalized less severely compared to larger firms.[6]

Having discussed our primary motivating facts, we now move on to describe our theoretical framework.

3 Model

In this section, we present our baseline model. The setup exhibits a minimal framework in order to better highlight the model mechanisms. There are two types of agents: firms and households, and we discuss their decisions below.

[5]This difference in relative penalties is captured explicitly by the coefficient $\theta_{f,h}$ in Equation (2). Figure A.1 plots this coefficient and shows that there are indeed statistically significant differences in wage penalties by size. Further, in Appendix B.1, we use the Outgoing Rotation Group of the CPS to control for the hourly pay status of workers—that is, whether they earn an hourly wage or are salaried. We find that the composition of hourly earners does not generate qualitatively different implications for wage penalties by firm size. We also argue that our results are robust to controlling for occupations and are not driven by measurement error in hours.

[6]This intuition implies that average hours not only affect the wage-hours menu faced by workers but are also affected by these wage-hours menus because they alter workers' labor supply decisions. This feedback mechanism will also be present in our model.

3.1 Firms

There is a continuum of firms with unit mass. For simplicity, we assume that firms' only endogenous input is labor. Production of all the firms in the economy can be represented by $Y = zL^{\theta}$, where L denotes the effective labor input. Firms differ in their exogenous productivity, z, a discrete random variable distribution represented by $F(z)$, which can take J different values. We think of the productivity term z as broadly capturing all non-labor inputs of the firm as well as its technology. In what follows, we will denote the index of a firm's productivity level by j and its productivity by z_j.

As in Yurdagul (2017) and Shao et al. (2023), we allow for complementarities between hours of workers:

$$L = \underbrace{\left(\frac{\int_{i \in N} l_i^{\rho} di}{\int_{i \in N} di} \right)^{\frac{1}{\rho}}}_{\text{average}} \underbrace{\left(\int_{i \in N} di \right)}_{\text{scale}}, \tag{3}$$

where N is the set of workers, and $\{l_i\}_{i \in N}$ is their hours worked. The first term of labor aggregation is an average hours measure in CES form, where the total weight of each individual is normalized to unity, i.e., a scaling up of the labor unit maintaining the distribution of hours would not alter this term. The second term is meant to scale up the labor aggregation depending on the number of workers involved. Accordingly, the labor aggregation boils down to the standard linear aggregation of hours when $\rho = 1$. In our model, we allow for $\rho < 1$, which captures imperfect substitutability between the working hours of co-workers in a unit.

We abstract from the indices of workers by rewriting the aggregation in terms of the measure of workers employed at each level of hours, and also collect the scale component in one term:

$$L = \left(\int_0^1 \mu(l) l^{\rho} dl \right)^{\frac{1}{\rho}} \left(\int_0^1 \mu(l) dl \right)^{1 - \frac{1}{\rho}}, \tag{4}$$

where $\mu(l)$ is the measure of workers working l hours.

Labor markets are perfectly competitive within each productivity level z_j of firms, with the firms taking the wage schedule $w_j(l)$ as given. Given the wage schedule, firms must decide on the measure, $\mu_j(l)$, of workers from a given hours level to maximize their static profits. It will become apparent later that, in equilibrium, not all $l-$types might be available on the $j-$market, and we will only derive equilibrium wages for those that are in positive supply. In addition to wages, firms also take as given the available hours in the market, z_j, denoted by $[\underline{l}_j, \bar{l}_j]$ in their optimization problems.

Firms in market j maximize their profit by choosing a labor demand schedule μ_j:

$$\pi_j = \max_{\mu_j} \; Y - \int_{\underline{l}_j}^{\bar{l}_j} w_j(l)\mu_j(l) l dl \tag{5}$$

$$s.t. \, Y = z_j \left[\left(\int_{\underline{l}_j}^{\bar{l}_j} \mu_j(l) l^\rho dl \right)^{\frac{1}{\rho}} \left(\int_{\underline{l}_j}^{\bar{l}_j} \mu_j(l) dl \right)^{1-\frac{1}{\rho}} \right]^\theta$$

$$\mu_j(l) \in [0, \bar{\mu}_j] \;\; \forall l \in [\underline{l}_j, \bar{l}_j].$$

Here we restrict the demand schedule to be between zero and $\bar{\mu}_j$, a scalar that is sufficiently large and positive, for each level of working hours. The upper bound will be far from binding in the equilibrium but is imposed in order to avoid corner solutions in which each firm hires workers with only one level of hours.[7] The optimal measure of labor is denoted by $\mu_j^*(l)$. We assume that firms are held by absentee entrepreneurs that are outside of our model.

3.2 Workers

There is a continuum of workers with static decisions. Preferences are given by:

$$c_{it} - v_{it} \frac{l_{it}^{1+\phi}}{1+\phi}. \tag{6}$$

The value of leisure is log-normally distributed, $\log v_{it} \sim N(\log v_0, \sigma_v)$. We denote the set of value of leisure shocks by B_v.

[7] This $\bar{\mu}_j-$constraint limits firms' profit under the corner solutions. It helps rule out the corner solutions and allows us to focus on the symmetric equilibrium where firms in the same market demand a uniform share of the total supply of a given worker type. Not only is the symmetric solution tractable, but it is also consistent with the existence of within-firm hours' dispersion as observed in the data. As such, we view the $\bar{\mu}_j-$constraint as capturing some aspects missing from our model that prevent firms from hiring only certain hour types, including labor market frictions, regulatory restrictions, heterogeneity in the hours required for each task, and diminishing returns to each hours group, etc.

Each period, a worker receives shocks to the value obtained in each firm productivity group. Formally, there is a vector $\boldsymbol{\epsilon} = \{\epsilon_1, \epsilon_2, ..., \epsilon_J\}$ with as many components as the number of different firm-level productivities. This vector follows a distribution $G(\boldsymbol{\epsilon})$ and is drawn independently every period. These shocks capture workers' taste for the other factors locating a worker into a firm not featured in our model, and their role is discussed in more detail below.

Once all of the shocks are realized, and the worker knows the firm-level productivity that she will work for, she chooses the labor supply. Accordingly, her value function is:

$$V(v, \boldsymbol{\epsilon}) = \max_j \{V^G(z_j, v) + \epsilon_j\},$$

where the value conditional on working in a firm of productivity j is:

$$V^G(z_j, v) = \max_{l \geq 0} \; w_j(l)l - v\frac{l^{1+\phi}}{1+\phi}.$$

Without loss of generality, one can think of the ϵ-shocks as being realized after the value of leisure shock of a worker. The probability of a worker choosing a firm productivity level z_j is denoted by $\mathbf{o}(z_j, v) \in \{1, ..., J\}$. Meanwhile, $\mathbf{l}(z_j, v)$ denotes the labor supply policy function.

Equilibrium. The equilibrium consists of a set of policy functions: $\mu_j^*(l)$ for firms $j \in \{1, .., J\}$ and $\mathbf{l}(z_j, v)$ and $\mathbf{o}(z_j, v)$ for workers in firm group j, wage functions $w_j(l)$, and a time-invariant distribution of workers $\varphi(z_j, v)$ over the type of their employer (z_j) and the value of leisure v, such that:

(i) The policy functions solve the problems of workers and firms.

(ii) Labor markets clear. The total measure of workers demanded by all firms for each level of firm productivity z_j and working hours $l \in [0, 1]$ is equal to the corresponding labor supply:

$$\mu_j^*(l)F(z_j) = \int_{v \in B_v} \varphi(z_j, v)\mathbb{1}[\mathbf{l}(z_j, v) = l]dv, \;\; \forall j \in \{1, 2, .., J\}$$

(iii) The evolution of the distribution across workers satisfies, for each $v \in B_v$ and $j \in \{1, 2, .., J\}$:

$$\varphi(z_j, v) = \int \Gamma_v(\tilde{v})\mathbf{o}(z_j, v) \times \sum_{\tilde{j}=1}^{J} \varphi(z_{\tilde{j}}, \tilde{v}) d\tilde{v}.$$

Wage schedule. The model equilibrium can only be solved numerically, but we can analytically characterize the wage schedule in equilibrium. We focus on the symmetric equilibrium where firms in each z_j market demand a uniform share of the total labor supply of a given hour. That is, the demand schedule for every z_j-firm is $\mu_{a,j}(l) \equiv \mu_j^s(l)/F(z_j)$ in this equilibrium. In Appendix C.3, we solve for the optimization problem of the firms and show that, under fairly relaxed conditions, this demand schedule is optimal for firms. In that case, we can substitute the distribution of l within the z_j-market to the optimality conditions of labor demand and derive the equation for the equilibrium wages:

$$w_j(l) \quad = \quad \theta z_j L_{a,j}^{\theta-1} E_{a,j}(l^\rho)^{\frac{1}{\rho}} \left[\frac{1}{\rho} \frac{l^{\rho-1}}{E_{a,j}(l^\rho)} + (1 - \frac{1}{\rho}) l^{-1} \right], \tag{7}$$

where on the right-hand side we have the marginal productivity of a worker with l hours. In particular, $E_{a,j}(l^\rho) \equiv \big(\int_{\underline{l}_j}^{\bar{l}_j} \mu_{a,j}(l) l^\rho dl\big) / \big(\int_{\underline{l}_j}^{\bar{l}_j} \mu_{a,j}(l) dl\big)$ is the "weighted average" of l^ρ, and $L_{a,j}$ is the labor aggregation implied by the symmetric demand schedule $\mu_{a,j}(l)$. Equation (7) shows that a worker's wage depends on her hours of work relative to fellow workers in the same firm. The maximum hourly wage is achieved at $l_j^* = E_{a,j}(l^\rho)^{\frac{1}{\rho}}$. Wages decrease as working hours get further away from l_j^*. This will be key in generating not only a hump-shaped wage-hours relationship overall, but also in making the penalties for short and long hours depend on the usual hours in each firm, and hence will change across firms, as we highlight in our motivating facts.

3.3 Discussion

Before discussing our quantitative analysis, we provide more detail on some of the non-standard features of our model and the role they play.

Complementarities in hours. Unlike most macroeconomic models of production, we allow for complementarities in the hours of different workers by aggregating hours in a non-linear manner. Intuitively, such complementarity captures the need for workers to coordinate, which can arise naturally when workers' tasks require coordination. The need for such coordination has long been recognized since at least Adam Smith's early discussions of assembly line production. Recently, Bick et al. (2022) and Labanca and Pozzoli (2021) document suggestive evidence supporting the presence of complementarities in hours. More direct evidence is presented in Shao et al. (2023), who use matched employer-employee data from Canada to estimate the elasticity of substitution between hours worked and find that working hours are indeed gross complements in production.

Given the evidence supporting complementarities in working hours, we explicitly incorporate them into our production function. As we discuss below, these complementarities play a crucial role in generating the observed hump-shaped relationship in wages and hours. However, the presence of complementarities do not, on their own, generate differences in wage penalties by firm size or the patterns of sorting and income inequality that we highlight. Indeed, we will specifically highlight how alternative levels of complementarities impact the model's implications in our quantitative analysis below.

Structure of the labor market. We assume that markets are segmented by firm type z with perfect competition for labor within each segment. Perfect competition within each market guarantees a direct mapping between wages and the marginal productivity of workers. The assumption of segmentation generates heterogeneous equilibrium wage schedules between firms of different types—as is observed in the data. Without segmentation, perfect competition in an aggregate market would result in a uniform, economy-wide wage schedule. A more general segmentation whereby firms and workers are randomly allocated into an arbitrary number of sub-markets, with perfect competition within each sub-market, would not change the results of our analysis.

Taste shocks. Workers are assumed to have heterogeneous preferences over their workplaces, which are captured by the vector ϵ. We introduce these preferences in order to generate higher wages in larger firms—that is, the size-wage premium. In the absence of a consensus explanation for the size-wage premium, we interpret this heterogeneity in tastes as capturing a number of (non-wage) factors that affect individuals' sorting into firms of different size and productivity. Such factors include differences in non-pecuniary benefits like workplace safety, childcare, or sick leave provisions, as well as differences in technology.[8]

Intuitively, the presence of preferences for workplace prevents wages from equalizing across firms of different productivity and generates higher wages in more productive firms that desire to be larger. Our use of taste shocks to generate the size-wage premium follows the approach of recent work such as Card et al. (2018) and Lamadon et al. (2022). Taste shocks have the benefit of being a simple way to generate a size-wage premium and, importantly, these shocks do not by themselves generate the other empirical facts that we focus on. If anything, the presence of taste shocks adds noise to the sorting decisions of workers and weakens the positive relationship between firm size and hours. For instance, in the extreme case of very large taste shocks, workers would sort based primarily on their idiosyncratic preferences for workplace, resulting in similar hours workers across firm size.

Taste shocks also provide a computational advantage as they effectively "convexify" the occupational choice decision of workers by introducing randomness. This transforms workers' policy function to a probability between 0 and 1 rather than a binary of 0 or 1 and facilitates convergence in the model solution.

4 Quantitative analysis

This section describes the quantitative implications of the model. We begin by detailing our calibration strategy and then show that the calibrated model can match our three main motivating facts. Following this, we highlight the mechanisms in the model that result in outcomes that are consistent with the data.

[8] We discuss the literature documenting such differences across firms, as well as further details on the role of the taste shocks, in Appendix C.2.

4.1 Calibration

We describe the calibration exercise in three parts: the functional forms, the parameters calibrated outside the model, and the parameters calibrated targeting features in the data.

Functional forms. Firm productivity is assumed to follow a Pareto distribution with shape parameter λ, and the lowest productivity is normalized to 1. This distribution is approximated using 24 grid points. Our results are robust to increasing this number.

We assume that the ϵ-shocks affecting workers' value in each firm follow a Generalized Extreme Value distribution:

$$G(\boldsymbol{\epsilon}) = \exp\left[-\sum_{j=1}^{J} \exp\left(-\frac{\epsilon_j}{\sigma_\epsilon}\right)\right].$$

The parameter σ_ϵ determines the variance of these shocks.

Parameters calibrated outside the model. The model is calibrated to match key features of the US data.

Our model is intentionally simple, and in particular, it assumes linear utility in consumption, hence no income effects. Only one parameter captures the substitution between consumption and leisure, namely ϕ. While the standard values of this parameter in the labor-macro papers are below 5, the corresponding studies typically allow for income effects. In order to avoid hours that increase steeply with wages (hence firm productivity and size) in our simple setup, we adopt a rather high value of ϕ at 10 in our baseline.[9]

The parameter governing the complementarities between working hours, ρ, is set by using results from Shao et al. (2023). Using matched employer-employee data from Canada, Shao et al. (2023) first provide evidence consistent with the presence of complementarities in hours in production, and then, using a generalized version of the production in this paper, they estimate the substitution parameter ρ to be around -0.46. Based on these results, and with the underlying assumptions that the production technologies adopted by US and Canadian

[9] In our extended model calibration, we will set this parameter at a standard value of 2.

firms are similar, we set the value of ρ to be -0.46.[10]

We set the labor share of output equal to the standard value of 0.67.

Parameters targeting features in the data. The remaining parameters are calibrated to match specific targets in the data. To compare model implications to the data, it is useful to construct model-implied firm size categories to match those in the CPS sample—the source of our motivating facts. These size categories are firms with 1 to 9 employees (small firms), firms with 10 to 99 employees (medium firms), and firms with more than 100 employees (large firms). According to the Business Dynamics Statistics (BDS), the fraction of firms in these three categories in 2015 is 77%, 21%, and 2%, respectively. Firms in our model are categorized as "small", "medium" and "large" so that they replicate this distribution—once firms are sorted by size. We then calibrate the shape parameter of the firm TFP distribution, λ, to match the employment share of the largest firm size category. Table 3 summarizes the statistics for the three size categories in the data and the model. In the last column, we provide information on how the average size in the model increases across categories. The fit of our firm grouping to the data is assuring: not only is our firm grouping similar to the data in terms of the percentage of firms in each size category, but also in terms of how average employment increases over the size categories.

For the value of leisure distribution, we target the observed average and dispersion of hours in the data. In particular, we set the level parameter (v_0) to match the average weekly hours of workers. The standard deviation (σ_v) is calibrated to match the standard deviation in log-hours.

The preference shocks for working at different productivity levels of firms generate noise in workers' choices over firms and prevent sorting from being driven entirely by pecuniary returns. As such, changes in the size of these shocks change the steepness of the wage profiles across firm productivity and size groups. Given this, we set σ_ϵ to match the ratio between the average wages in firms with over 100 employees and firms with under 100 employees.

Table 4 presents the parameter values in our calibration. It also reports the implied

[10] We justify the assumption of similarity between the US and Canadian economies by replicating, in Appendix B.3, our motivating facts using data from the Canadian Labour Force Survey (LFS). An alternative calibration targeting moments from the Canadian economy gives qualitatively similar implications, reconciling these facts.

Table 3: Firm size categorization

	Data		
	Firm Share	Employment Share	Avg. employment (log, rel. to small firms)
Small	0.77	0.11	0
Medium	0.21	0.23	2.1
Large	0.02	0.66	5.4

	Model		
	Firm Share	Employment Share	Avg. employment (log, rel. to small firms)
Small	0.77	0.10	0
Medium	0.21	0.24	2.2
Large	0.02	0.66	5.5

Notes: The size categories in the data follow the categorization in the CPS. Small firms are firms with 1 to 9 employees, medium firms are those with between 10 and 99 employees, and large firms are those with over 100 employees. Data are from the 2015 Business Dynamics Statistics. The average size of small firms in 2015 was 3.4 employees. We follow the observed fraction of firms in each category to construct the same groupings in our model.

moments of the model against the data for the targets. The model performs well in matching the features of the data.

4.2 Model implications

This section compares the model's implications with the data. We begin by showing that the model generates empirical patterns consistent with the motivating facts detailed in Section 2, and then discuss the relevant features of the model that generate these patterns.

Figure 4 reports the relationship of average wage (solid line) and hours worked (dashed line) by firm size category and shows that the model successfully replicates our first two motivating facts. As the size-wage premium is a target in our calibration, the model quantitatively matches the increase in wages over firm size; as in the data, wages increase by around 25% from small to large firms.

The model also replicates the (non-targeted) positive relationship between average hours worked and firm size as average hours increase monotonically with firm size. Quantitatively, the model predicts a steeper relationship between hours and size with an increase in average hours from the smallest to largest firms of around 6 hours per week in the model compared to a 3 hours increase in the data. In the extended model of Section 5, this fit will improve

Table 4: Model parameters

Panel A: Outside the model

Parameter	Value	Basis
θ	0.67	Labor share of output
ϕ	10	Low elasticity
ρ	-0.46	Shao et al. (2023)

Panel B: Calibrated

Parameter	Value	Target	Data	Model
λ	2.02	Employment share (large firms)	0.66	0.66
v_0	15.6	Average weekly hours	41.7	42.0
σ_v	2.625	SD log hours	0.23	0.23
σ_ϵ	0.25	Wage gap, large to rest	0.26	0.25

Notes: Panel A reports parameters that are set following the literature. Panel B reports the parameters that are calibrated to match specific data features with the model. The last two columns in Panel B report the data target and model-implied value. The employment share of firms with over 100 employees is computed from the 2015 Business Dynamics Statistics. Measures of hours and wages are calculated using the pooled CPS sample. The data-target for the wealth-income ratio is from Erosa et al. (2016).

when allowing for the combination of risk aversion and wealth accumulation.

Figure 5 shows that the model also generates our third and final motivating fact. In particular, for each size category, the model features i) a hump-shaped relationship between hours worked and wages, ii) highest wages close to average hours worked, and iii) an increasing penalty as a worker's hours deviate from the usual hours in the firm. These patterns are not only present in the wage schedules that workers take as given (Panel (a)), but also in the observed relationships between wages and hours (Panel (b)). In addition, the model reconciles our empirical finding that the long-hours penalty is higher in smaller firms, and the short-hours penalty is higher in larger firms.

4.3 Accounting for the motivating facts

Having shown that the model successfully replicates our three motivating facts, we explore, in turn, the mechanisms that generate each fact.

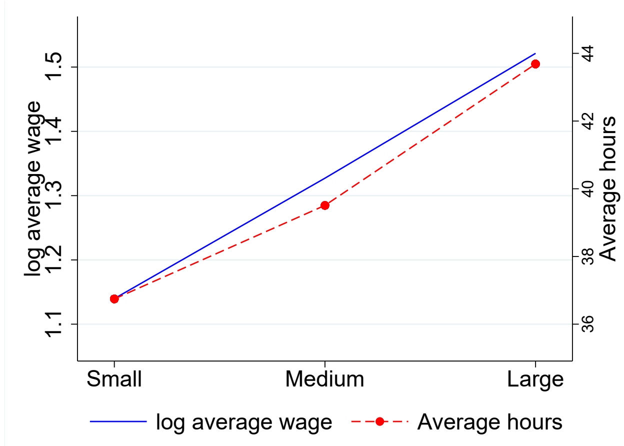


Figure 4: Wages and hours over size in the model

Notes: The figure plots the log average wages (solid line, left axis) and average weekly hours worked (dashed line, right axis) for each size group in the model. Section 4.1 describes the construction of size categories in the model.

(a) Wage schedule (b) Equilibrium wages

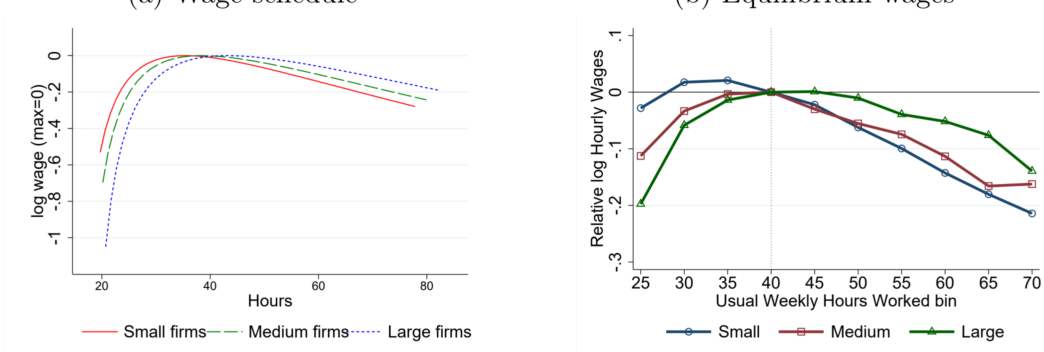


Figure 5: Wage-hours relationship and firm size; wage schedule and equilibrium wages

Notes: Panel (a) reports, for each firm size category, the relative wage schedule by hours worked. To construct the relative wage schedule, we first take an average of the wage function $w_z(l, x)$ across efficiency x, weighted by the measure of each efficiency group. We then plot the logarithm of the average in difference to its maximum level against working hours. Panel (b) plots, for each size category, the sum of coefficients $(\gamma_h + \theta_{e,h})$ estimated from Equation 2 (only controlling for size group, hours bin, and their interactions) using simulated data from the model. This exactly replicates the construction of Figure 3. Section 4.1 describes the construction of size categories in the model.

4.3.1 Wage differentials between small and large firms

In the absence of heterogeneous preferences for firms, workers are sorted into firms based on wages alone; therefore, they would flow into the highest-paying firm until wages are equalized across firms. Workers' random preference for firms (i.e., taste shock) introduces noise to their sorting and allows the model's equilibrium to sustain a positive wage gap between high- and low-productivity firms. Indeed, the larger the taste shocks, the less important wage differentials are to workers' sorting decisions, and the larger the size-wage premium. Figure 6 illustrates that the increase in average wage from small to large firms increases by two-thirds when we double the standard deviation of the taste shocks.

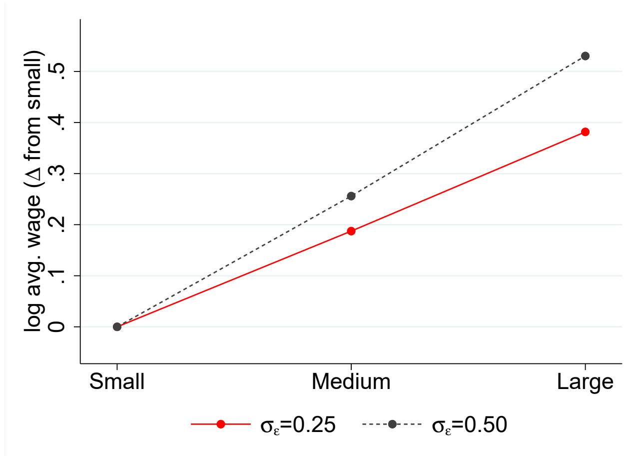


Figure 6: Size-wage premium, role of heterogeneity in tastes

Notes: The figure log average wages (relative to small firms) for each size group in a model. The solid red line plots the results from the benchmark calibration where $\sigma_\epsilon = 0.25$, the short-dashed grey line is the model with $\sigma_\epsilon = 0.50$, which means the standard deviation of taste shocks that are twice as high as the benchmark value. We do not recalibrate any other parameter of the model. Section 4.1 describes the construction of size categories in the model.

4.3.2 Increasing hours over firm size

In the model, the presence of a size-wage premium results in longer average hours worked in larger firms. This outcome is, in part, a result of the sorting of workers with different desired hours into different firms. In general, there are two opposing forces that shape the pattern of sorting. The first force pushes workers with longer desired hours to work in larger firms. Specifically, since income is the product of hours and wages, the income gains from working longer are higher in larger (higher-wage) firms, so workers with longer desired hours have a higher propensity to work in larger firms. The second force pushes in the opposite direction by incentivizing workers with shorter desired hours to work in larger firms. To understand this, notice that employment in smaller (lower-wage) firms will result in a lower income and hence higher marginal utility for any given level of hours. The marginal utility will be higher for workers with shorter desired hours, and hence these workers' value increases in income more than workers with longer desired hours.[11]

In addition to the sorting of workers with different desired hours into firms of different sizes, there is an intensive margin channel that generates longer hours in larger firms. This incentive margin channel encourages all workers to work longer hours in large (high-wage) firms regardless of their characteristics. As with sorting, which can be considered an extensive margin channel, two opposing forces determine the ultimate effect of the intensive margin channel. On the one hand, higher wages in large firms incentivize workers to work longer hours due to higher income gains. On the other hand, the marginal utility of the same worker in small firms is higher due to a lower income, which encourages them to work longer hours.

The relative magnitude of these two opposing forces depends on how quickly marginal utility decreases as income increases and determines the net effects of the extensive and intensive margins. These, in turn, will pin down the overall relationship between hours worked and firm size. In our baseline model, we assume linear utility in consumption, hence the second force is not present. As a result, workers with longer desired hours self-select into larger firms, and for any given type, workers also work longer hours in large firms. Therefore, average hours increase with firm size in equilibrium, a result of both extensive and intensive margin effects.

Figure 7 explores the importance of the mechanisms behind the pattern of increasing hours over firm size. We highlight the role of sorting in Panel (a). In particular, we compare the benchmark model's size-hours relationship (solid red line) to a counterfactual relationship computed by assuming that all workers of the same characteristics work the same hours as their counterparts in small firms (dashed blue line). This captures the size-hours relationship due only to differences in the selection of workers based on their characteristics. Since larger firms attract workers with longer desired hours, the counterfactual also features a positive relationship between firm size and average hours. Panel (a) suggests that sorting accounts for about two-thirds of the positive size-hours relationship in the baseline model. The remaining one-third can be attributed to the intensive margin channel, namely that workers of the same characteristics work longer hours in larger firms.

[11]This is akin to the substitution and income effect following an increase in wages in a standard labor supply decision. However, unlike the standard problem, in our model, agents make the decision to choose their workplace and hence their hourly wages.

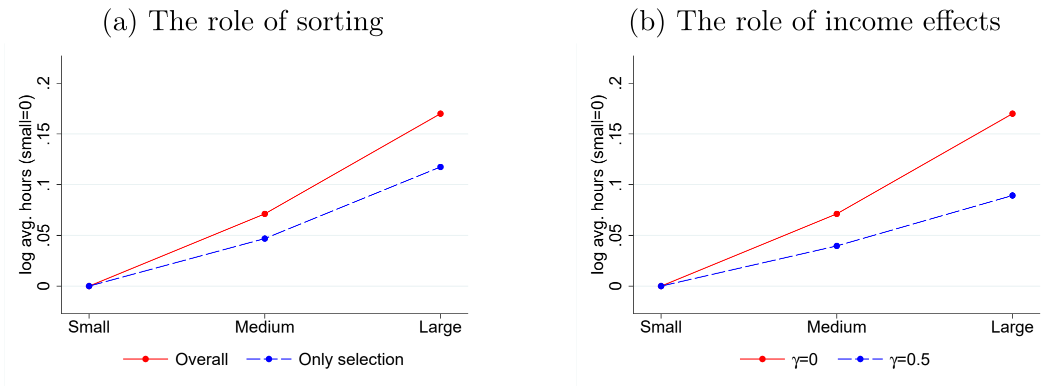


Figure 7: The relationship between hours worked and firm size

Notes: Panel (a) plots log average hours (in difference to small firms) by firm size in the benchmark model (solid red line) and in an alternative version of the model that only allows selection between firm groups (dashed blue line). To construct average hours in the alternative version, we set hours worked for each worker with the same state variable—value of leisure—to be equal to the average hours in small firms. We then compute the average hours in each size group according to the resulting distribution of these states across firms. Panel (b) plots the log of average hours (relative to small firms) across firm size for the benchmark calibration with linear preferences over consumption (solid red line) to those in the counterfactual with risk aversion, namely, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and $\gamma = 0.5$ (dashed blue line).

In Panel (b), we bring in the income effects in labor supply by allowing for concave utility in consumption, with CRRA preferences, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, and $\gamma = 0.5$. We compare the hours-size patterns in this case to that of the baseline with linear utility, $\gamma = 0$. Intuitively, higher risk aversion makes the second force highlighted above stronger, causing long-hour workers to sort less into high-wage firms and discouraging workers from significantly extending their working hours in the face of higher wages. Accordingly, the steepness of the size-hours relationship decreases to half with $\gamma = 0.5$.

4.3.3 Hump-shaped wage schedule

The model generates a hump-shaped relationship between wages and working hours due to complementarities between workers' hours. Such complementarity maximizes an individual worker's marginal productivity when she works the same hours as the rest of her production unit. Any hours worked beyond those of her co-workers results in diminishing marginal productivity for the worker. By the same token, when a worker works shorter hours, she pulls her co-workers' productivity down, which is reflected in her marginal productivity, hence in her wages. Accordingly, there is a penalty for working shorter and longer than the

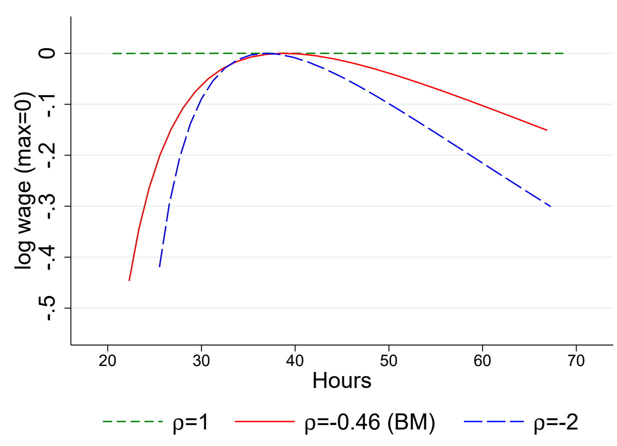


Figure 8: The relationship between wages and hours, role of complementarities

Notes: The figure plots the relative wage schedule by hours worked for different values of ρ for a medium productivity grid (13th out of 24). To construct the relative wage schedule, we plot the logarithm of $w_z(l)$ in difference to its maximum level against working hours. The three lines correspond to the benchmark model ($\rho = -0.46$), as well as the alternatives changing the substitutability parameter to values of 1 and -2. In the alternative computations, we do not recalibrate any other parameter. Section 4.1 describes the construction of size categories in the model.

usual hours in the firm. Figure 8 clearly illustrates the role of complementarities, by plotting the wage schedule faced by a worker in a firm of an intermediate productivity as ρ changes. As working hours become less complementary (higher ρ), the link between hours—within a production unit—becomes weaker, and penalties become less severe for both short and long hours.

The amplification effect of hours complementarity. Complementarities between workers' hours amplify the positive hour-size relationship. To see this, suppose that, for some reason, a firm features longer average hours than all other firms. Complementarity in hours implies that, compared to another firm, workers with longer desired hours will be penalized less in this firm while workers with shorter desired hours will be penalized more. As a result, long-hour workers will wish to sort into this firm (an extensive margin effect). This sorting will, in turn, result in similar workers of that firm working longer (an intensive margin effect). Both these extensive and intensive margin effects, driven by complementarities, will amplify the positive relationship between hours and size.

We highlight this amplification by comparing the benchmark calibration to an alternative version of the model that does not feature complementarities. Figure 9 compares the size-hours relationship in the two versions of the model. It highlights that the slope of the hours-size relationship decreases by about 10% in the counterfactual with perfect substitutability, i.e., $\rho = 1$. Notice also that even in the absence of complementarities, the equilibrium sorting of agents is such that average working hours increase with firm size.

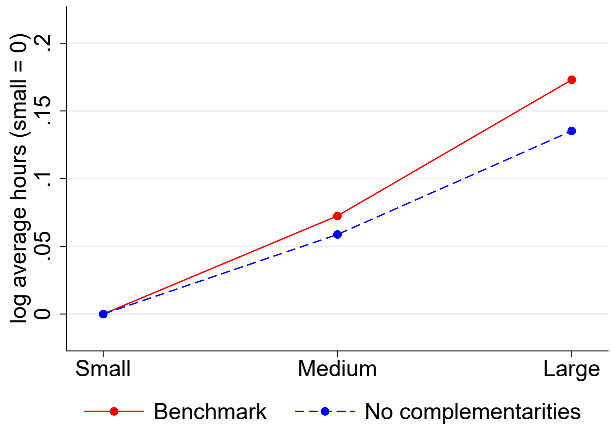


Figure 9: The relationship between hours worked and firm size, role of complementarities
Notes: The figure plots log average hours (in difference to small firms) in the benchmark calibration (solid red line) and an alternative calibration that changes only the substitution parameter $\rho = 1$ (dashed blue line).

5 Extended model

The baseline model presented in the previous section is rich enough to illustrate the main mechanisms that we argue to be behind the observed motivating facts. However, it abstracts from important channels that can be quantitatively and qualitatively important for these facts, as well as for other important aggregate implications of the model. In this section, we enrich the baseline framework towards a more realistic and full-fledged model for two main reasons. The first is to show that the main implications of the model are robust to such extensions. The second is to later use the extended model to quantify the role of model mechanisms in shaping wage and income inequality, as well as sorting, for which the added features will be first-order relevant.

The first important change we introduce in the extended model is heterogeneity in the idiosyncratic worker efficiency. We model idiosyncratic efficiency as a persistent shock,

$\Gamma_x(x'|x)$. This is important because, in addition to generating additional wage heterogeneity, the efficiency of workers can potentially affect the working hours of workers and their sorting into small and large firms.[12]

The second (duo of) changes with the extended model are to allow for risk aversion in preferences for consumption and, to complement that, allow for wealth accumulation of workers. As we noted earlier, the race between income and substitution effects is key in shaping how hours correlate with wages and firm size; hence, setting up the model to have a more realistic preference structure is important. Moreover, to let the trade-off between income and substitution effects shape endogenously, we also allow for savings.

Finally, we include friction in the choice among different firms. In particular, we assume that only with probability $s < 1$ in each period can a worker decide on the productivity type z of the firm that she will work for. Otherwise, she needs to remain with the employer type of the last period. With this feature, we aim to capture many different types of rigidities in employer selection that the workers face, which can be important in shaping the observed dynamics in terms of transitions between employers as well as volatility in wages and income.

Ultimately, the firms' labor aggregation now allows for heterogeneity of skills in the production unit in the same way as Shao et al. (2023):

$$L = \left(\int_{x \in B_x} \int_0^1 x \mu(l,x) l^{\rho} dl dx \right)^{\frac{1}{\rho}} \left(\int_{x \in B_x} \int_0^1 x \mu(l,x) dl dx \right)^{1-\frac{1}{\rho}},$$

where $\mu(l,x)$ is the measure of workers of efficiency x, working l hours. Note that the efficiency units are introduced in the labor aggregation such that all workers of different types have similar complementarity links between each other. The efficiency of each worker only affects (i) the weight of her hours in shaping the "average" hours in the unit, and (ii) the scale of the labor aggregation.[13]

[12]The value of leisure shocks, v, are also assumed to be persistent shocks with a transition matrix Γ_v.

[13]An alternative would be to have the non-linear aggregation only happen within efficiency units, i.e.,

$$L = \int_{x \in B_x} x \tilde{L}(x) dx, \text{ where } \tilde{L}(x) \equiv \left(\int_0^1 \mu(l,x) l^{\rho} dl \right)^{\frac{1}{\rho}} \left(\int_0^1 \mu(l,x) dl \right)^{1-\frac{1}{\rho}}.$$

Workers' optimization is now dynamic due to frictions in the choice of firms and the wealth accumulation:

$$V(a,x,v,\boldsymbol{\epsilon}) = \max_j \{V^G(a,x,z_j,v) + \epsilon_j\}$$

where:

$$\begin{aligned} V^G(a,x,z_j,v) &= \max_{a'\geq 0, l\geq 0} \frac{c^{1-\gamma}}{1-\gamma} - v\frac{l^{1+\phi}}{1+\phi} \\ &\quad +\beta E_{x',v',\boldsymbol{\epsilon}'|x,v}\left[sV(a',x',v',\boldsymbol{\epsilon}') + (1-s)\left(V^G(a',x',z_j,v') + \epsilon'_j\right)\right] \\ \text{s.t} &\quad c = w_j(l,x)l + a(1+r) - a'. \end{aligned}$$

Finally, the wage equation implied by the extended model is very similar to that in the baseline, except it captures the heterogeneous skills of workers:[14]

$$w_j(l,x) = \theta z_j x L_j^{\theta-1} E_j(l^\rho)^{\frac{1}{\rho}}\left[\frac{1}{\rho}\frac{l^{\rho-1}}{E_j(l^\rho)} + (1-\frac{1}{\rho})l^{-1}\right],$$

where

$$E_j(l^\rho) \equiv \left(\int_{x\in B_x}\int_{\underline{l}_j}^{\bar{l}_j} x\mu_j(l,x)l^\rho dldx\right) \div \left(\int_{x\in B_x}\int_{\underline{l}_j}^{\bar{l}_j} x\mu_j(l,x)dldx\right).$$

Note that, just like in our simpler baseline, the highest wage is achieved at hours equal to $l_j^* = E_j(l^\rho)^{\frac{1}{\rho}}$ for all the workers independently of their type x, and wages decrease as working hours get further from l_j^*.

5.1 Calibration of the extended model

We calibrate this model following the same strategy (i.e., same targets) as in the baseline for the parameters that also appeared in that model. Among the new parameters in the extended

While this alternative does not significantly affect our results, we choose not to use it for two reasons. First, it requires keeping track of the average hours for each worker efficiency-firm productivity pair (rather than only for each firm productivity). Second, we do not think of the complementarities in hours, or the coordination in tasks, as only happening within efficiency groups but potentially in the whole production unit with workers of different skills, occupations, and hierarchies interacting.

[14] We outline the definition of the stationary general equilibrium for the extended model in Appendix C.1.

Table 5: Parameters of the extended model

Panel A: Outside the model

Parameter	Value	Basis
ϕ	2	Standard
γ	2	Standard
θ	0.67	Labor share of output
ρ	-0.46	Shao et al. (2023)

Panel B: Calibrated, old parameters, new values

Parameter	Value	Target	Data	Model
λ	2.05	Employment share (large firms)	0.66	0.63
σ_ϵ	0.8	Wage gap, large to rest	0.26	0.27
v_0	3.77	Average weekly hours	41.7	41.9
σ_v	0.56	SD log hours	0.23	0.23

Panel C: Calibrated, new parameters

Parameter	Value	Target	Data	Model
ρ_v	0.62	Autocorr. log hours	0.67	0.67
σ_x	0.38	SD log wages	0.63	0.63
ρ_x	0.74	Autocorr. log wages	0.70	0.72
β	0.92	Wealth-income ratio	2.5	2.7
s	0.25	Avg. tenure (years)	4	4

Notes: Panel A reports parameters that are set following the standard values. Panel B reports the calibration of the parameters that are also in the baseline model, but recalibrated in the extended model following the same targets that are calibrated to match specific data features with the model. Panel C reports the calibration of the parameters that only appear in the extended model. The last two columns in Panels B and C report the data target and model-implied value. The employment share of firms with over 100 employees is computed from the 2015 Business Dynamics Statistics. Measures of hours and wages are calculated using the pooled CPS sample. The data-target for the wealth-income ratio is from Erosa et al. (2016).

model, we set the risk aversion to the standard value of 2. For the rest, we calibrate the parameters to match specific targets.

Even though we assume a discrete Markov process for the value of leisure and idiosyncratic efficiency shocks, we parameterize the grids and the evolution of these shocks to resemble AR(1) processes:

$$\log(v_{i,t+1}) = (1-\rho_v)\log(v_0) + \rho_v \log(v_{i,t}) + \xi_{i,t}, \;\; \xi_{i,t} \sim N(0, \sigma_v)$$

$$\log(x_{i,t+1}) = \rho_x \log(x_{i,t}) + \psi_{i,t}, \;\; \psi_{i,t} \sim N(0, \sigma_x)$$

This way, we boil down the corresponding parameters to ρ_v, σ_v, and v_0 for the value of leisure, and ρ_x and σ_x for worker efficiency. We follow Tauchen (1986) to map these AR(1) processes to the discrete Markov processes assumed in the model. We calibrate the persistence of the value of leisure, ρ_v, to match that of the log-hours in the data. The dispersion and persistence of the efficiency shock, σ_x and ρ_x, are calibrated to match those of the log-wages.

The discount factor, β, is set to calibrate the average wealth-to-income ratio. Finally, we set the friction in switching employers, s, to match the average job tenure in years. Table 5 summarizes the calibration for the extended model.

5.2 Implications of the extended model for the motivating facts

The calibrated version of the extended model matches the motivating facts in a similar fashion to the baseline model. Figure 10a shows that the average wage and the hours increase over the three firm size categories. Out of these two, the slope of the first one is a target for our calibration, whereas the second one is a non-targeted moment. In the latter, the increase in hours from the smallest to the largest category is about 4 hours, whereas the data counterpart is 2.5 hours.

Figure 10b shows the replication of our third motivating fact with the extended model. The hump-shaped relationship between hours and wages is still evident. Importantly, the property that long- (short-) hour penalties are more severe in small (large) firms still holds and is still explained jointly by the i) longer average hours in larger firms and ii) complementarities between working hours.

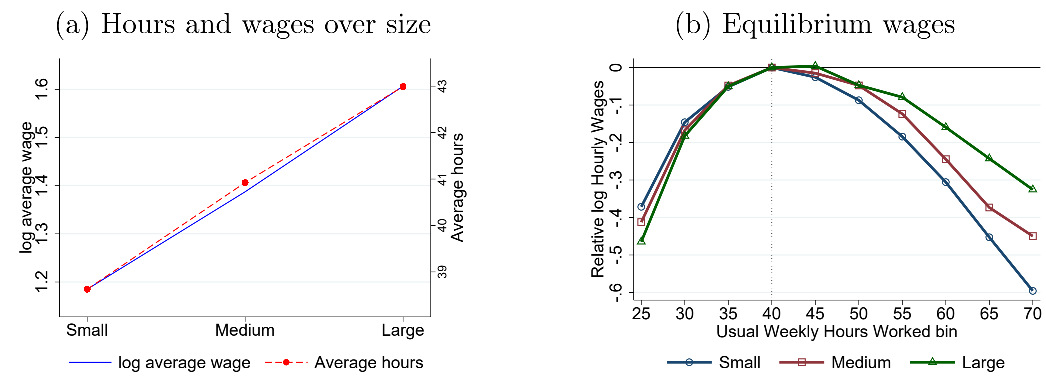


Figure 10: Motivating facts predicted by the extended model

Income vs. substitution effects and the worker heterogeneity in efficiency. In our simple model, we highlighted the trade-off between income and substitution effects in i) making long- and short-hour workers sort into smaller and larger firms (extensive margin), and ii) making workers work longer or shorter hours across smaller and larger firms (intensive margin). In particular, we noted that in our baseline calibration with only the substitution effects, long-hour workers sort more into higher-wage, large firms, and workers work longer hours in these firms than in small firms because the income gains from doing so are larger. In our extended model, where the income effects are also present, the substitution effects are still dominating the income effects to deliver similar patterns as in the simple model, with the average hours increasing over firm size. While the concavity in preferences pushes for stronger income effects, endogenous wealth accumulation brings the levels of consumption to a sufficiently high level to limit the changes in the marginal utilities of consumption from low- to high-income levels.

An additional remark is due regarding the role of heterogeneity in efficiencies at the worker level in shaping hours versus size patterns. Note that similar logic to the patterns of hours across firm size applies to the patterns of average worker efficiency across firm size. In particular, when substitution effects dominate income effects, high-efficiency workers sort more into larger firms, because the income gains from extracting the size-wage premium are higher with high efficiency. Similarly, working longer hours brings higher income gains for high-efficiency workers. Accordingly, heterogeneity in idiosyncratic efficiency contributes to longer hours and higher wages in larger firms. Nevertheless, we find that sorting on ability plays a quantitatively insignificant role in generating the size-wage and size-hours relationships. Indeed, average efficiency changes by 3% between small and large firms, which is insufficient to make a quantitative impact on the hours and wage steepness over firm size.

5.3 Sorting on hours

In our model, theoretically, both worker skill and desired working hours play roles in the allocation of workers to firms. While the literature has typically focused on sorting based on worker skill and firm productivity (see, for example, Eeckhout, 2018), we argue that sorting based on desired hours is an essential yet under-studied factor. In this section, we document that our model's predictions about sorting based on desired hours are supported by empirical evidence.

Through the lens of our model, workers sort into firms based on skills x, preferences for workplaces, and desired working hours. Desired hours are influenced by value of leisure and marginal utility and play a significant role in the allocation process. The relationship between hours and firm size, combined with complementarities in hours, causes workers with longer or shorter desired working hours to prefer larger or smaller firms, respectively. According to our model, workers who work fewer or more hours than their co-workers will seek to transition to smaller or larger firms, respectively, where their hours are more similar to their co-workers.

We test this implication by comparing worker transitions in the model to those observed in the data. Specifically, we test whether there are systematic differences in the rates at which workers move between different firm size groups based on their work hours relative to their co-workers. We construct these transition matrices using data from the CPS by tracking respondents over 12 months, which corresponds to one period in the model.

We categorize a worker as working shorter hours if their hours in period t, h_t, is at least 10% less than the average for their firm size group, $\bar{h}_{f,t}$. On the other hand, a worker is classified as working longer hours if their hours, h_t, are at least 10% more than the average for their firm size group, $\bar{h}_{f,t}$. We then compute the transition matrices for transitions between firm sizes between t and $t+1$ separately for the short- and long-hour workers.

Table 6 shows the model and data transition matrices. As a non-targeted moment, the model matches well the qualitative patterns of how the transitions between size groups of firms relate to the previous relative hours of workers.

To begin with, our model predicts that workers working shorter hours ($h_t < \bar{h}_{f,t}$) are more likely to transition into smaller firms. In the model, around 7% and 3% of short-hour employees in large firms transition to medium and small firms, respectively, and around 4% of short-hour employees in medium-sized firms transition to small firms. Sorting based on hours would imply fewer transitions towards smaller firms when $h_t > \bar{h}_{f,t}$. This is indeed the case: only 6% and 2% of workers in large firms transition to medium and small firms, respectively, when they work longer hours than their average co-worker. Similarly, only

Table 6: Transitions across firm size based on hours worked

	Model			Data		
	Small	Medium	Large	Small	Medium	Large
		$h_t < \bar{h}_{f,t}$			$h_t < \bar{h}_{f,t}$	
Small	0.79	0.07	0.14	0.73	0.17	0.10
Medium	0.04	0.82	0.14	0.15	0.62	0.23
Large	0.03	0.07	0.90	0.04	0.10	0.86
		$h_t > \bar{h}_{f,t}$			$h_t > \bar{h}_{f,t}$	
Small	0.77	0.06	0.17	0.58	0.22	0.19
Medium	0.02	0.81	0.17	0.08	0.61	0.31
Large	0.02	0.06	0.92	0.02	0.09	0.89

Notes: The table reports transition matrices computed in the benchmark model (left column of matrices) and the CPS data (right column of matrices). The first row of matrices reports the transition rates for workers that work at least 10% shorter hours than the average hours in their firm size group in period t. We denote this average as $\bar{h}_{f,t}$, where f indicates firm size group and a worker's own hours at h_t. The second row of matrices show the analogous transition rates for workers that work at least 10% longer hours than the average hours in their firm size group in period t, that is, $h_t > \bar{h}_{f,t}$. Table A.1 in the Appendix reports the model and data transition matrices for workers whose hours are within 10% of their firm size group average. To construct transition matrices in the CPS, we track respondents over a 12-month period and identify the size group in which they are employed in the first t and second $(t+1)$ observations. Workers are grouped based on their firm size group and hours worked in the first period t. Each entry in a transition matrix reports the share of workers that transition from the size group in the matrix row in period t to the size group in the matrix column in period $t+1$.

2% of long-hour workers in medium-sized firms transition to small firms. Instead, long-hour workers are much more likely to switch to large firms. For instance, 17% of long-hour workers employed in small firms switch to large firms. The analogous transition share for short-hour workers in small firms is 14%.[15]

This pattern is also present in the data. The matrices on the right side of the table show the observed transition matrices in the CPS. Although the model does not perfectly match the data, the pattern consistent with sorting based on hours is evident. For example, when $h_t < \bar{h}_{f,t}$, 17% and 10% of workers in small firms switch to medium and large firms, respectively. When $h_t > \bar{h}_{f,t}$, these rates are 22% and 19%. Transitions from large firms also are in line with what is predicted by the theory. When $h_t < \bar{h}_{f,t}$, 10% and 4% of workers in large firms switch to medium and small firms over a 12-month period, and when $h_t > \bar{h}_{f,t}$ these rates are 9% and 2%.

[15]For the sake of brevity, we omit the transition matrices for workers who worked similar hours to their size group, i.e., $h_t \approx \bar{h}_{f,t}$). These are reported in Appendix Table A.1, which shows that transition rates for this group are roughly in between those of workers with shorter and longer hours.

Taken together, Table 6 reveals that in the data, working shorter hours is associated with subsequent movements toward smaller firms, while working longer hours is associated with subsequent movements toward larger firms. This is consistent with workers sorting based on hours. These findings complement our discussion of the positive cross-sectional relationship between firm size and average hours by showing this relationship also holds dynamically in the data, which our model performs well in replicating.

6 Implications for inequality

Our theoretical framework features the interaction of hours, wages, and firm productivity. Here, we show that this interaction has important implications for inequality in both hourly wages and income—that is, the product of wages and hours.

Wage inequality. The extended model exhibits wage inequality due to heterogeneity in three factors: worker skills, firm-level productivity, and working hours. The role of heterogeneity in worker and firm productivity in contributing to inequality is well-studied. Our novel insight is to highlight the role of hours. To do this, we explore the contribution of each of the above three factors to wage inequality and then compare it to the data.

Table 7 summarizes the results of our analysis. The first row reports the contribution of worker skills, firm-level productivity, and working hours to wage dispersion in the data. As emphasized in existing work, much of the observed dispersion in wages is driven by unobservable worker characteristics (Abowd et al. 1999). Accordingly, we find that worker skill (proxied by education and years of experience), firm productivity (proxied by firm size), and hours worked only account for 20% of the observed dispersion in wages (first column).[16]

The remaining columns report the contribution of explained dispersion that is due to worker skill, firm productivity, and hours worked, respectively. We find that 73% of explained dispersion is due to worker skill while firm size accounts for only 8% of hourly wage dispersion.

[16]To compute this share, we compared the overall standard deviation of observed (log) hourly wages (0.63) to the weighted average of within-group standard deviations, which are computed separately for each group where a group is defined by worker skill, firm productivity, and hours worked, and the weights are number of observations in a group. We find that the weighted average dispersion is 0.50. Hence, the difference (0.63-0.50, or 20% of 0.63) is *explained* by worker skill, firm productivity, and hours worked.

The remainder, 19%, is due to dispersion in hours worked. Strikingly, dispersion in hours contributes more than twice as much to raising wage dispersion than firm productivity and is around 1/4 as important as worker skills.

The second row of Table 7 conducts a similar breakdown using simulated data from the benchmark model. Notice that there is no notion of unobservable characteristics in the model. So, controlling for x, z, and l accounts for all wage dispersion in the model. Heterogeneity in hours worked accounts for 16% of the entire wage dispersion—close to the 19% contribution in the data. The contribution of hours arises due to complementarities, which we will discuss in detail later.

Our model implies a much larger contribution of firm productivity on wage dispersion than in the data (17% vs. 8%) and understates the contribution due to worker skills (67% vs. 73%). This is due to a coarser division of firms in the data, as firm size groups are split into three in the data. In order to make our analysis of the simulated data more comparable to the observed data, we group firm productivity into the same three size categories as in the data.

The third row of Table 7 reports the decomposition results using three size categories to proxy firm productivity. With the coarser division of firm types, the model no longer explains the entirety of wage dispersion. With this change, the model more closely matches the data with 9% of explained wage dispersion being due to firm productivity. Worker skill plays a larger role in accounting for wage dispersion due to the correlation between x and z. In the absence of information on the exact firm type z, a larger share of the wage dispersion is attributed to x. It is also worth noting that the share explained by working hours decreases. According to our theory, individual hour effects on wages can properly be captured only if we use the right reference hours for each worker. The non-linear relationship between hours and wages gets blurred when we bunch more firms of different productivities (hence reference hours) together. By using a coarser firm categorization, we get a reduced explanatory power of hours on wages.[17]

[17]This same logic suggests that using finer firm groupings in the data should increase the observed contribution of hours in wage dispersion. We test this by using the full range of firm size categories reported in the CPS and find that using finer size categories results in the contribution of hours to wage dispersion increasing by 3 percentage points to 22%. Recall that the reporting of firm size categories in the CPS has varied over time. For clarity and consistency, we focused on reporting three categories of firm size.

Table 7: Accounting for dispersion in hourly wages in the data and model

	Share Overall Dispersion Explained by Skills, Firm Productivity, & Hours	Contribution due to		
		Skills	Firm Productivity	Hours
Data	0.20	0.73	0.08	0.19
Model (fully observed z)	0.99	0.67	0.17	0.16
Model (grouping z)	0.83	0.80	0.09	0.11
Model w/o compl. (fully observed z)	1.00	0.71	0.29	0.00
Model w/o compl. (grouping z)	0.82	0.87	0.13	0.00

Notes: The table reports the results from an exercise that decomposes wage dispersion in the data (first row) and model-simulated data (remaining rows). The first column reports the share of unconditional standard deviation of log hourly wages that is *explained* by worker skill, firm productivity, and hours worked. To compute this share, we compare the overall standard deviation (σ_w) to a weighted average of within-group standard deviations ($\sigma_w^{x,z,l}$), which are computed separately for each group where a group is defined by worker skill (x), firm productivity (z), and hours worked (l), and the weights are number of observations in a group. The share explained is $\frac{\sigma_w - \sigma_w^{x,z,l}}{\sigma_w}$. To proxy for worker skill and firm productivity in the data, we use four education bin and years of experience, and three firm size categories, respectively. The remaining columns report the contribution of skills, firm productivity, and hours worked, respectively. These are computed as $\frac{\sigma_w - \sigma_w^{x}}{\sigma_w - \sigma_w^{x,z,l}}$, $\frac{\sigma_w^{x} - \sigma_w^{x,z}}{\sigma_w - \sigma_w^{x,z,l}}$, and $\frac{\sigma_w^{x,z} - \sigma_w^{x,z,l}}{\sigma_w - \sigma_w^{x,z,l}}$, respectively.

In the data, heterogeneity in hours accounts for almost one-fifth of explained dispersion in wages. Through the lens of the model, such a contribution to dispersion in wages is driven by the presence of complementarity in workers' hours. To illustrate this, the last two rows of Table 7 repeat the decomposition analysis in a version of the model where there are no complementarities. When we assume that firm type is fully observed, hours worked play no role in generating wage dispersion. This is to be expected since, without complementarities, wages within firms are independent of working hours. That is, there is no hump-shaped relationship between wages and hours. The conclusion is similar when we group firm types, with the bulk (87%) of dispersion being accounted for by the worker skills.

Taken together, the analysis reported in Table 7 highlights a number of important findings. First, in the data, variation in hours is a significant contributor to wage inequality. Second, our model qualitatively matches the data with respect to decomposing the contribution of wage dispersion. Finally, we show that complementarities in hours are essential for heterogeneity in hours to contribute to wage dispersion. Next, we explore the model's implications for income inequality—that is, the product of wages and hours.

Income inequality. Variation in hours naturally generates variation in income, not only mechanically but also due to the correlation between hours and wages. While hours inequality contributes to income inequality positively, the correlation between wages and hours can mitigate or amplify the overall income inequality. For example, if high-wage workers work more hours than low-wage workers, then overall income inequality would be greater. This can be seen clearly through a simple variance decomposition of (log) income,

$$\begin{aligned}\text{Var(income)} &= \text{Var(wage)} + \text{Var(hours)} + 2 \times \text{Cov(wage,hours)} \\ &= \text{Var(wage)} + \text{Var(hours)} + 2 \times \text{Corr(wage,hours)} \times \sqrt{\text{Var(wage)} \times \text{Var(hours)}}\end{aligned} \tag{8}$$

Motivated by this, we explore the drivers of variance in income, emphasizing the endogenous correlation between hours and wages. Table 8 reports each component on the right-hand side of Equation (8) for both the data and model (first two rows). The third and fourth columns of the table show that hours dispersion and the covariance of hours with wages account for a significant portion of overall income dispersion (around 20% in total) in both the data and the model. Recall that, in our calibration, we only targeted dispersion in hours and wages and did not target the covariance between hours and wages nor the variance in income. Despite this, the model gives a decent fit to both the dispersion in income and, importantly, the covariance between hours and wages.

Table 8: Decomposing the variance in income, the role of hours and wages

	Var(income)	Var(wage)	Var(hours)	2 × Cov(wage,hours)	Corr(wage,hours)
Data	0.50	0.40	0.05	0.06	0.18
Model	0.51	0.40	0.05	0.06	0.20
Model w/o comp.	0.59	0.37	0.14	0.08	0.18

Notes: The table reports the results from a variance decomposition of log income following Equation (8) in the data (first row), benchmark model (second row), and a version of the model without complementarities (last row). The first column reports the total variance in log-income while the next three columns report each of the three components of the right-hand side of Equation (8). The last column reports the correlation between hours and wages.

As with wage inequality, complementarities play an important role in influencing income inequality. In fact, there are three main channels through which complementarities alter the right-hand side of Equation (8). First, complementarities generate wage dispersion for a given worker-firm combination. Indeed, if hours were perfectly substitutable across workers, wages would be identical for all workers in a firm (of a given skill x) regardless of their hours worked. Thus complementarities raise the variance in wages.

Second, complementarities introduce an incentive for workers to work similar hours to their co-workers (through hump-shaped wage profiles). This compresses the distribution of hours worked in an economy, and hence complementarities also serve to reduce the variance in hours pushing towards lowering income inequality.

Third, complementarities strengthen the positive relationship between hours and firm size. We have highlighted this before in Figure 9, where the change in hours from small to large firms, relative to the dispersion of hours in a given model, is weaker without complementarities. Accordingly, complementarities also raise the *correlation* between hours and wages.

To see the overall impact of these three channels on income inequality, the last row of Table 8 reports the terms in Equation (8) in a version of the model without complementarities. First, removing complementarities raises income dispersion in our model from 0.51 to 0.59. Decomposing this dispersion reveals that the increase in income dispersion is driven primarily by an increase in hours dispersion. That is, the second channel discussed above dominates the other two. The compression of the hours distribution due to complementarities is quite stark, with the hours dispersion in our benchmark model being around 35% of the counterpart model without complementarities.

The table also highlights the aforementioned channels through which complementarities contribute positively to income dispersion, even though these effects are ultimately offset with compression in hours distribution. First, the model with complementarities features higher wage dispersion. Second, the correlation between hours and wages is stronger in our model. However, the latter effect does not translate into an increase in income inequality because the reduced hours dispersion with complementarities shrinks the magnitude of the covariance between hours and wages. In fact, the covariance is lower in our baseline than in the counterfactual without complementarities. In sum, the reduction in hours dispersion is strong enough to cancel the opposing positive effects of complementarities in income inequality.

Consistent with existing empirical literature, the model predicts that heterogeneity in hours is an essential contributor to income inequality (see, for example, Blau and Kahn, 2011 and Checchi et al., 2016). Since a role for hours arises endogenously, our theoretical framework provides insights into the mechanisms by which hours impact income inequality. Indeed, we find that complementarity in working hours plays a crucial role by compressing the distribution of hours while raising wage inequality and driving a more pronounced sorting of hours across firms.

7 Conclusion

This paper studies the relationship between hours, wages, and firm-level heterogeneity—specifically firm size. Using micro-data from the US, we document that workers' average wages and average hours increase with firm size, and, novel to the literature, that wage penalties for long (short) hours are larger in smaller (larger) firms.

Motivated by this evidence, we develop a general equilibrium model of heterogeneous firms and workers. Our framework generates a size-wage premium through heterogeneity in workers' preferences for the workplace. The size-wage premium leads workers willing to work longer hours to endogenously sort into larger (more productive) firms, as well as making similar workers work longer hours in larger firms. The existence of complementarities between workers' hours combined with the longer hours in larger firms results in less severe long-hour wage penalties and more severe short-hour wage penalties in larger firms—as observed in the data.

We use our model to study the aggregate implications of the interaction of hours, wages, and firm size. We argue that this interaction has important consequences for earnings inequality. We show that, consistent with the data, the model suggests a significant role for heterogeneity in hours in driving wage and income dispersion. We also study the importance of complementarities in working hours for inequality.

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Online Appendix for:

Labor Supply and Firm Size

Lin Shao   Faisal Sohail   Emircan Yurdagul

A Additional figures and tables

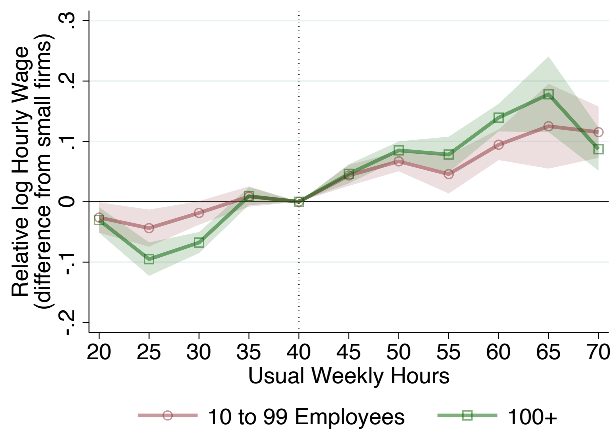


Figure A.1: Wage penalty relative to small firms

Notes: The figure reports the coefficient $\theta_{e,h}$ estimated from Equation 2 where the reference group for hours worked is workers that work $40-44$ hours. The reference group for size is firms with under 10 employees. The shaded regions are the 95% confidence intervals. Data from the pooled CPS sample.

Table A.1: Transitions across firm size based on hours worked for workers working similar to average hours

	Model			Data		
	Small	Medium	Large	Small	Medium	Large
		$h_t \approx \bar{h}_{f,t}$			$h_t \approx \bar{h}_{f,t}$	
Small	0.78	0.07	0.15	0.56	0.25	0.18
Medium	0.03	0.81	0.16	0.09	0.62	0.29
Large	0.03	0.06	0.91	0.02	0.10	0.87

Notes: The table reports two transition matrices computed in the benchmark model (left matrix) and the CPS data (right matrix). The matrices report the transition rates for workers that work within at least 10% of the average hours in their firm size group in period t. We denote this average $\bar{h}_{f,t}$, where f indicates firm size group and a worker's own hours at h_t. Table 6 in the main text reports the model and data transition matrices for workers who work 10% shorter or longer hours than their firm size group average. To construct transition matrices in the CPS, we track respondents over a 12-month period and identify the size group in which they are employed in the first t and second $(t+1)$ observations. Workers are grouped based on their firm size group and hours worked in the first period t. Each entry in a transition matrix reports the share of workers that transition from the size group in the matrix row in period t to the size group in the matrix column in period $t+1$.

B Data appendix

In this appendix, we report supplementary empirical results including robustness exercises using the CPS. We also explore the role of measurement error in hours for our empirical findings related to hours and replicate our motivating facts (at both the establishment and firm levels) using the Canadian Labour Force Survey (LFS).

B.1 Additional evidence from the CPS

In this section we use the CPS to provide additional empirical results and robustness to our main motivating facts using the CPS.

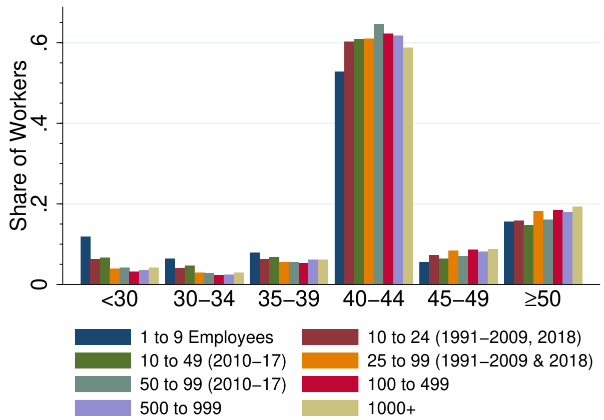
Average hours by detailed firm size categories. For expositional clarity, in the main text we explored the distribution of firm size using only three size categories. Here, we report the same results using the more detailed firm size categories included in the CPS. Indeed, for most of our sample, CPS respondents have reported their firm size in one of seven categories. These are firms with i) 1 to 9 (under 10), ii) 10 to 24, iii) 25 to 49, iv) 50 to 99, v) 100 to 499, vi) 500 to 999, and vii) 1000+ employees. Between 2010 and 2017, the size categories of 10 to 24 and 25 to 99 were instead reported as 10 to 49 and 50 to 99.

Panel (a) of Figure B.2 reports the distribution of hours worked using these detailed firm size categories. As with the more course firm size categories, Panel (a) makes clear that even with more detailed firm size categories employees in smaller firms tend to work shorter hours compared to employees in larger firms. Focusing on the share of workers working the modal number of hours (40-44), we find that this share is lowest for firms with under 1 to 9 employees at 53% and tends to increase with firm size, with 62% of employees in firms with 100 to 499 and 500 to 499 working 40 to 44 hours. The share is slightly lower for the largest firms with over 1000 employees at around 59%. Panels (b) and (c) focus on the distribution of short and long hours worked, respectively, and show that the entire distribution of hours is shifted to the left for smaller firms and to the right for larger firms. Indeed, the share of workers that work under 25 hours is around 4% for firms with over 1000 employees and 12% for firms with 1 to 9 employees, while the analogous shares of workers that work over 55 hours for these firms is 9% and 7%.

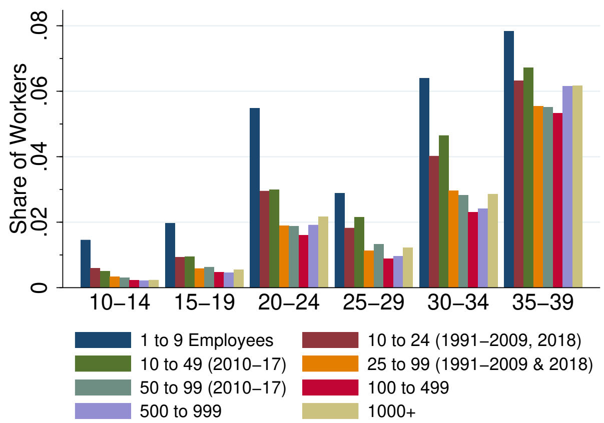
Consistent with the difference in hours distributions by firm size, Figure B.3, which reports the unconditional average of weekly hours worked, shows that workers in larger firms tend to work longer. Indeed, employees in firms with over 1000 employees work, on average, 42.2 hours per week while employees in firms with 1 to 9 employees work 39.6 hours, 10 to 24 employees work 41 hours, and 25 to 99 employees work 42.0 hours. Overall, using more detailed firm size categories reveals that average hours worked increase in a concave manner with firm size.

Finally, we consider the conditional average of hours worked by detailed firm size categories by estimating the coefficient β_f in Equation (1), where f now comprises the finer firm size categories in the CPS. Table B.2 reports the estimates of β_f. The first three columns report this estimate when using all available data. To accommodate the change in firm size categories over CPS years, the fourth column reports β_f using the 2010 to 2017 CPS and the last column uses the 1991 to 2009 and 2018 CPS. Regardless of the size categories reported,

(a) Overall



(b) Short hours



(c) Long hours

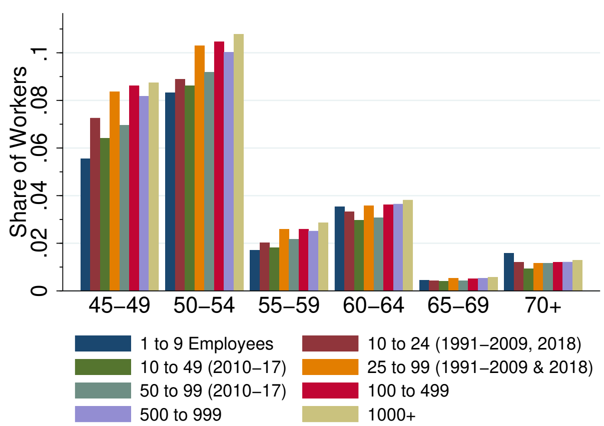


Figure B.2: Distribution of working hours by detailed firm size categories
Notes: The figure reports the share of workers by their usual weekly hours worked and detailed firm size categories reported in the CPS. The firm size categories 10 to 49 and 50 to 99 are only available in the CPS from 2010 to 2017 and replace the categories 10 to 24 and 25 to 99 in these years.

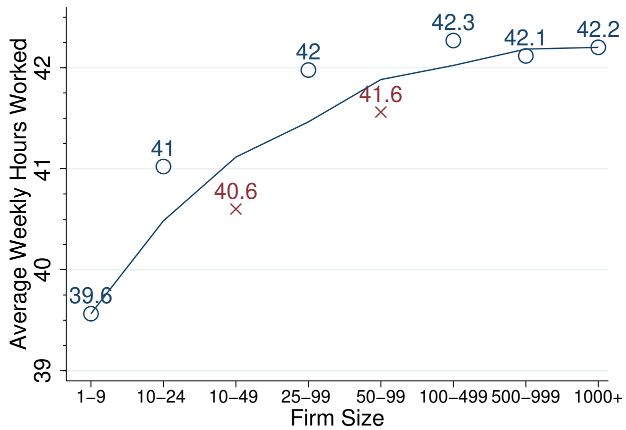


Figure B.3: Average weekly hours by detailed firm size categories

Notes: The figure reports the average usual weekly hours worked by detailed firm size categories reported in the CPS. The line is a locally weighted regression line. The × markers indicate average weekly hours for firm size categories 10 to 49 and 50 to 99. These categories are only available in the CPS from 2010 to 2017 and replace the categories 10 to 24 and 25 to 99 in these years.

we find that the coefficient β_f increases with firm size. For example, relative to firms with 1 to 9 employees, column (3) shows that employees in firms with 10 to 24 employees work almost 1 hour longer per week, 25 to 99 employees work 1 hour and 36 minutes longer, 100 to 499 employees work 2 hours 4 minutes longer, 500 to 999 employees work 2 hours and 7 minutes longer, while workers in firms with over 1000 employees worker 2 hours and 9 minutes longer. These weekly differences are economically significant and amount to workers in firms with over 1000 employees working an additional 2.8 weeks per year compared to employees in firms with 1 to 9 employees.

It is important to note that though the increase in hours with firm size is monotonic, it is not linear. This can also be seen in the unconditional averages reported in Figures 2 and B.3. Instead, the coefficient β_f increases linearly between smaller firm size categories of 10 to 24 employees and 100 to 499 employees, but increases relatively modestly for larger firms. Indeed, while the estimates of β_f are statistically significantly different from each other for size categories 10 to 24 and 25 to 99 as well as 25 to 99 and 100 to 499, the estimates of β_f for 100 to 499 and larger firm sizes are not statistically significant from each other.

Hourly and salaried workers. As highlighted in Bick et al. (2022), workers that are paid by the hour experience a relatively stable penalty when working over 60 weekly hours. In contrast, salaried workers experience much larger penalties when working long hours above 60. Given this, our empirical finding that the long- (and short-) hour penalty varies with firm size could follow simply due to differences in the compositions of workers across firms. For example, if larger firms feature a higher share of hourly workers working longer hours than smaller firms, this could generate the relatively flatter long-hours penalty. Figure B.4 tests whether this is the case by plotting the share of workers that are paid hourly by firm size and usual hours worked bins. The figure shows that the share of hourly workers declines as hours worked increase across all firm sizes. Further, the share of hourly workers is relatively

Table B.2: Detailed firm size categories and hours worked

	(1)	(2)	(3)	(4)	(5)
10 to 24 employees	1.347***	1.135***	0.951***	-	0.853***
	(0.053)	(0.050)	(0.050)	-	(0.055)
10 to 49	1.298***	1.102***	0.951***	1.120***	-
	(0.062)	(0.059)	(0.059)	(0.077)	-
25 to 99	2.279***	1.929***	1.602***	-	1.522***
	(0.048)	(0.045)	(0.046)	-	(0.052)
100 to 499	2.699***	2.354***	2.074***	2.232***	1.986***
	(0.043)	(0.042)	(0.043)	(0.080)	(0.051)
500 to 499	2.545***	2.244***	2.111***	2.319***	2.004***
	(0.053)	(0.051)	(0.052)	(0.097)	(0.062)
1000+	2.634***	2.179***	2.155***	2.380***	2.039***
	(0.040)	(0.038)	(0.042)	(0.075)	(0.050)
Year, State FE	Y	Y	Y	Y	Y
Demographic Controls	N	Y	Y	Y	Y
4-digit Industry FE	N	N	Y	Y	Y
CPS Samples	All	All	All	2010-2017	1991-2009, 2018
N	1,000,820	1,000,820	1,000,820	305,556	695,264
R^2	0.014	0.102	0.129	0.124	0.134

Notes: The table reports the coefficient β_f estimated from Equation (1) where the reference size category is firms with 1 to 9 employees. Standard errors are reported in parentheses. *** indicates statistical significance at the 1% level.

similar across firm size bins for usual hours above 40, suggesting that the composition of workers is likely not the primary driver of the flatter long-hours penalty in larger firms.

Having said this, larger firms feature a relatively higher share of short-hour, hourly workers than smaller firms. To concretely test whether differences in composition drive the differences in the short- and long-hour penalties by firm size, we re-estimate the regression in Equation 2 while also including an indicator for whether workers are salaried or paid by the hour. Figure B.5 reports the sum $(\gamma_h + \theta_{e,h})$ (Panel (a)) and the coefficient $\theta_{e,h}$ (Panel (b)) as estimated from this regression. Due to the smaller sample size when restricting attention to respondents with information on hourly or salaried status, we group usual hours worked into 10-hour bins and include 2-digit industry fixed effects. The reference group for usual hours worked in the regression is workers that work $40 - 49$ hours.

Panel (a) shows that the hump-shaped nature of the wage-hours profile remains unchanged when controlling for the salaried status of workers. Also persisting are apparent differences in the wage penalties between the smallest firm size categories and larger firms. This can be seen more clearly in Panel (b), which shows that medium and large firms exhibit more severe short-hour and less severe long-hour penalties compared to small firms. However, the difference in penalties between medium and large firms becomes much smaller

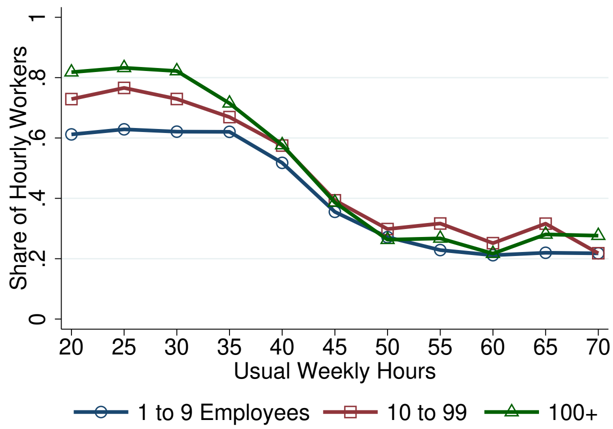


Figure B.4: Share of hourly workers by firm size and hours worked

Notes: The figure plots the share of workers that are paid by the hour, by firm size and usual hours worked. Data are from the outgoing rotation group (ORG) sub-sample in the pooled CPS sample. The ORG sub-sample makes up around 25% of the pooled CPS sample and contains information on whether respondents are paid by the hour.

when controlling for whether workers are paid by the hour—particularly for low levels of usual hours worked.

Taken together, this evidence suggests that differences in the composition of workers are not likely drivers of the differences in wage penalties observed across firm size bins.

Controlling for occupations, Our primary empirical analysis does not include controls for worker occupations. We make this choice to capture the idea that production involves the interaction of workers employed in different types of occupations. Specifically, the complementarity in hours may be particularly salient between occupations rather than within occupations. Having said this, here we show that our findings from Section 2 are robust to controlling for occupation.

First, we estimate Equation (1) by restricting the sample to a given occupation and show worker hours are increasing in firm size even within occupations. Panel (a) of Figure B.6 reports the estimated coefficient β_f when restricting the sample to one sample of each of one of 11 broad occupational categories. We construct these categories using 3-digit CPS occupation codes following Autor and Dorn (2013). Panel (a) shows that, with the exception of agricultural occupations, the estimated value of β_f is increasing in firm size for all other occupations. For example, compared to managers in firms with 1 to 9 employees, managers in firms with between 10 to 99 and over 100 employees work, respectively, 1.5 and 2.5 hours longer. Having said this, the difference in hours by firm size is not statistically significant for all occupations, including protective services, technicians, and food preparation and cleaning.

For completeness, Panel (b) of Figure B.6 reports the coefficient β_f when we estimate Equation (1) for workers in a single industry. As with occupations, the hours worked in all but one industry (agriculture) are increasing with firm size.

As further robustness, we also estimate Equations (1) and (2) while also including an additional regressor that includes dummies for 3-digit occupations as recorded in the IPUMS variable `occ90ly`. The first and second columns of Table B.3 report the coefficient β_f from

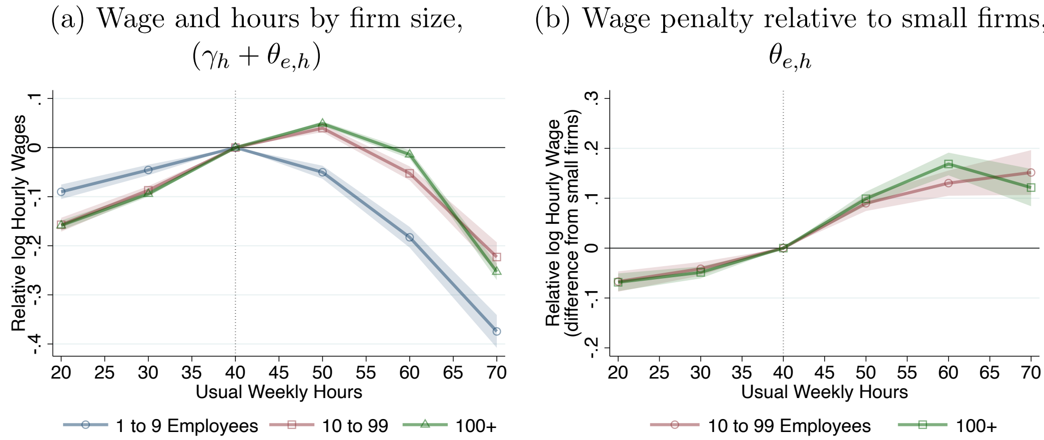


Figure B.5: Wage profiles by firm size and hours worked, controlling for hourly workers

Notes: The figure reports the coefficient $(\gamma_h + \theta_{e,h})$ in Panel (a) and $\theta_{e,h}$ in Panel (b) as estimated from Equation 2 with an additional indicator variable for whether or not a worker is paid by the hour. The reference group for usual hours worked in the regression is workers that work $40 - 49$ hours. The reference group for size is firms with 1 to 9 employees. The shaded regions are the 95% confidence intervals. Data are from the outgoing rotation group sub-sample in the pooled CPS sample.

estimating, respectively, a version of Equations (1) and (2) that controls for occupations. Controlling for occupations has little impact on the positive relationship between hours and firm size (first column) or on wages and firm size (second column).

Figure B.7 plots the relationship between hours and wages by firm size in the CPS as estimated from Equation (2) while also controlling for occupation. In particular, Panel (a) reports the sum of the coefficients γ_h and $\theta_{f,h}$, which captures the wage penalty of working outside of the 40-45 hours bin by firm size. Panel (b) reports the coefficient $\theta_{f,h}$ estimated from the same regression. The figure shows that controlling for occupation does not significantly alter the wage-hours relationship across firms. Panel (b) shows that the difference in relative penalties continues to be statistically significant.

Average hours and firm size, additional controls. As an additional robustness check on our finding that average weekly hours worked are increasing in firm size, we estimate Equation (1) using additional sets of controls and restrictions. Table B.4 reports the results of these exercises. The first column reports the coefficient β_f when we include a dummy variable indicating whether a worker works full time (i.e., at least 35 hours per week). Unsurprisingly, the coefficient on full-time status is larger, with full-time workers working over 17 hours longer per week. Importantly, the coefficient β_f is strictly positive and increasing in firm size despite controls for full-time worker status, suggesting that both full-time and part-time workers tend to work longer in larger firms.

The second column of Table B.4 estimates Equation (1) while also including a dummy variable for whether a worker is paid hourly or not. It shows that hourly workers tend to work longer, but even after controlling for workers that are paid hourly, the coefficient β_f is increasing in firm size.

Finally, the third column estimates β_f while restricting the sample to only those sets of

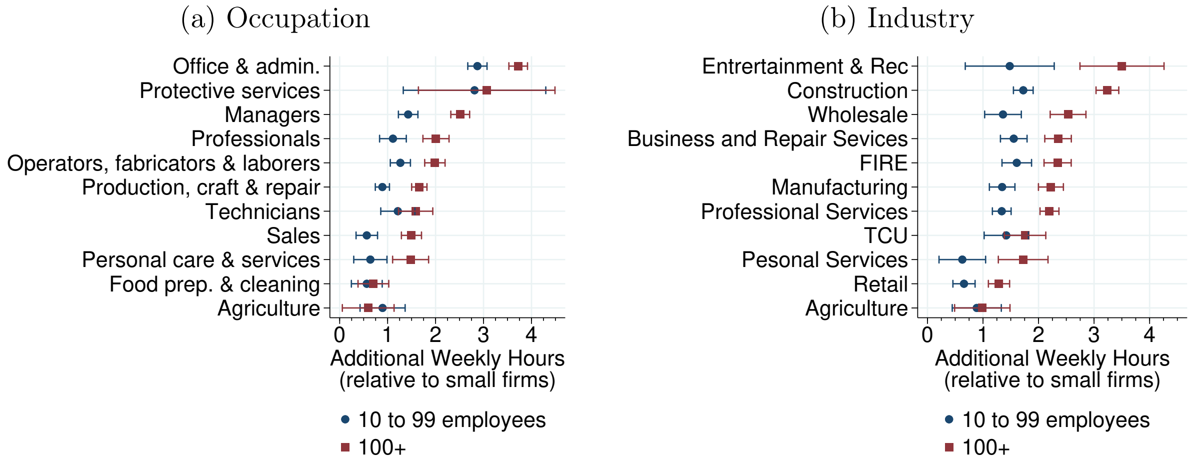


Figure B.6: Firm size and hours worked, by industry and occupation

Notes: The figure reports the coefficient β_f from estimating equation (1) on a sample of workers in a given occupation (Panel (a)) and workers in a given industry (Panel (b)). The reference group for firm size is firms with 1 to 9 employees. The vertical markers indicate the 95% confidence interval for the estimated coefficient. Occupations were grouped into broad categories by following Autor and Dorn (2013).

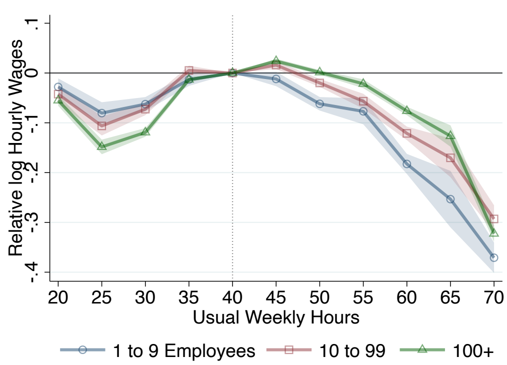
workers that work between 35 and 45 hours (inclusive). We find that even within this narrow window of hours worked, average weekly hours are increasing with firm size, with workers in mid-sized firms working 12 minutes longer and workers in larger firms working 18 minutes longer per week than workers in small firms.

Table B.3: The size-wage premium and the hours-size relationship

	Weekly Hours	log wages
10 to 99 Employees	1.195***	0.089***
	(0.039)	(0.003)
100+ Employees	1.994***	0.201***
	(0.039)	(0.003)
Year, State FE	Y	Y
Demographic Controls	Y	Y
4-digit Industry FE	Y	Y
3-digit Occupation FE	Y	Y
N	1,000,820	1,000,820
R^2	0.191	0.495

Notes: The first and second columns of the table report the coefficient β_f from estimating Equations (1) and (2), respectively, while also including controls for occupations. The reference size category is the smallest size firms—that is, firms with 1 to 9 employees. The reference hours bin is $40-44$ hours. Data are from the pooled CPS sample. Standard errors are reported in parentheses. *** indicates statistical significance at the 1% level.

(a) Wage-hours relationship by firm size



(b) Wage penalties relative to small firms

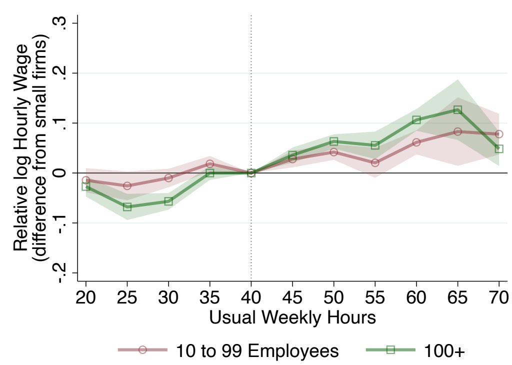


Figure B.7: The relationship between wages and hours, controlling for occupations

Note: Panel (a) reports the the sum of coefficients $(\gamma_h + \theta_{f,h})$ estimated from a version of Equation (2) that also includes controls for occupation. The reference group for usual hours worked in the regression is workers that work $40-44.9$ hours. The reference group for size is the smallest size category. Panel (b) reports the coefficient $\theta_{f,h}$ estimated from the same regression. The shaded regions are the 95% confidence intervals.

Table B.4: Firm size and hours worked, additional controls

	(1)	(2)	(3)
10 to 99 employees	0.078**	1.473***	0.206***
	(0.031)	(0.080)	(0.010)
100+	0.584***	2.317***	0.301***
	(0.030)	(0.079)	(0.010)
Full-Time Employee	17.255***	-	-
	(0.029)	-	-
Paid Hourly	-	3.438***	-
	-	(0.046)	-
Year, State FE	Y	Y	Y
Demographic Controls	Y	Y	Y
4-digit Industry FE	Y	Y	Y
Hours Range	All Hours	All Hours	[35,45]
N	1,000,819	240,552	717,962
R^2	0.406	0.161	0.068

Notes: The table reports the coefficient β_f estimated from Equation (1) where the reference size category is firms with 1 to 9 employees. The first column includes an additional indicator variable that is equal to 1 if workers are full-time workers and work at least 35 hours, and is 0 otherwise. The second column includes an indicator variable that is 1 if a worker is paid hourly and 0 otherwise. The last column estimates the equation by restricting attention to only those sets of workers that work between 35 and 45 hours (both inclusive). Standard errors are reported in parentheses. ** and *** indicates statistical significance at the 5% and 1% level, respectively.

B.2 Measurement error in hours

To address concerns of measurement error in reported hours in the CPS and their impact on our motivating facts relating to hours, we follow Bick et al. (2022), and merge the CPS data with data from the American Time Use Surveys (ATUS). The ATUS is a survey conducted since 2003 of a sub-sample of exiting CPS respondents and asks respondents to complete a time diary detailing how they spent their time over a 24-hour period. The ATUS data are extracted in IPUMS and detailed in Hofferth et al. (2020). Importantly, respondents of the ATUS are asked how long they spent working, which provides a high-quality measure of hours worked in a day. Then, by merging the CPS and ATUS, we can use firm size information from the CPS and the higher-quality measure of hours worked in the ATUS to re-evaluate the relationship between average hours worked and firm size. Further, by using the difference between hours reported in the CPS and the ATUS we can create alternative wage-hours profiles by firm size to evaluate the extent to which mismeasurement in hours may be driving the differences in wage-hours profiles across firms of different sizes.

Before describing our results, it is important to note that the ATUS is conducted 2 to 5 months after the CPS and it does not elicit information on firm size. So information on firm size is taken from the CPS, and to account for respondents switching employment across firm size categories between the CPS and ATUS, we restrict the ATUS sample to consider only those workers that are in the same industry and occupation as that reported in the CPS. We also remove all respondents that complete their time diary on a public holiday (such as New Year's Day, Easter, Memorial Day, 4th of July, Thanksgiving, and Christmas). All other sample restrictions are the same as in the CPS.

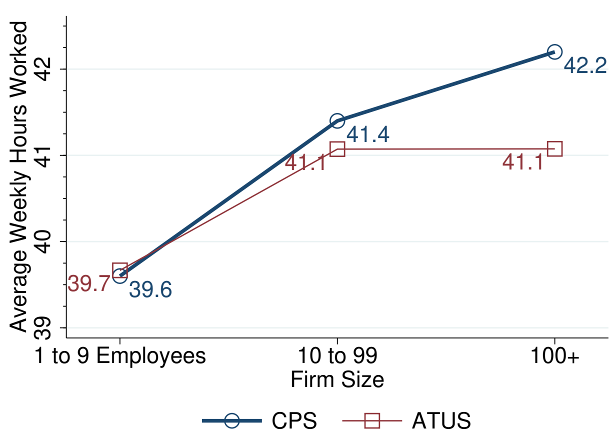


Figure B.8: Average weekly hours by firm size, CPS and ATUS

Notes: The figure compares the average usual weekly hours worked by firm size in the CPS and average reported hours in the ATUS. Average weekly hours in the ATUS are constructed by multiplying the average daily hours reported in weekdays by five and multiplying the average daily hours reported during weekends by two and summing these together.

Average hours by firm size, We begin by exploring whether average reported hours in the ATUS is increasing with firm size as in the CPS. Since the ATUS includes hours worked for different days of the week, we construct a measure of weekly hours in the ATUS by multiplying the average daily hours reported in weekdays by five and multiplying the average daily hours reported during weekends by two and summing these together. Figure B.8 compares the unconditional average weekly hours in the CPS and this measure of average weekly hours constructed using the ATUS. Compared to the CPS, the hours in the average ATUS are higher for small firms with under 10 employees and lower for larger firms with over 10 employees. Indeed, there is no difference in the (unconditional) average hours reported by employees in firms of 10 to 99 employees and firms with over 100 employees. This result is consistent with Bick et al. (2022), who show that respondents that report working shorter hours in the CPS tend to report longer hours in the ATUS, and those that report longer hours in the CPS report shorter hours in the ATUS. Given that larger firms tend to have a higher share of long-hour workers, a natural consequence of this is that average hours in large (small) firms will be lower (higher) in the ATUS.

We conduct a more rigorous analysis of the relationship between hours worked in the ATUS and firm size by estimating Equation (1) using daily hours reported in the ATUS. In addition to the controls described in the main text, we also control for whether respondents' time diaries were completed on a weekday or a weekend (i.e., Saturday or Sunday). Table B.5 reports the estimate of β_f, which indicates the additional hours worked *per day* by workers in a firm of size f relative to workers in firms with 1 to 9 employees.

Table B.5: Firm size and daily hours worked in ATUS

	(1)	(2)	(3)
10 to 99 Employees	0.207*	0.195*	0.197*
	[+1.4 weeks/yr]	[+1.4 weeks/yr]	[+1.4 weeks/yr]
	(0.110)	(0.109)	(0.108)
100+ Employees	0.218**	0.217**	0.220**
	[+1.8 weeks]	[+1.8 weeks/yr]	[+1.9 weeks/yr]
	(0.099)	(0.100)	(0.101)
Year, State FE	Y	Y	Y
Demographic Controls	N	Y	Y
4-digit Industry FE	N	N	Y
N	22,842	22,842	22,842
R^2	0.421	0.432	0.434

Notes: The table reports the coefficient β_f estimated from Equation (1) where the reference size category is firms with 1 to 9 employees and the dependent variable is daily hours worked in the ATUS-CPS merged sample. The brackets report the additional number of weeks worked per year implied by the estimated regression coefficient. For example, an additional, relative to small firms, hour worked per week over 5 working days and 52 working weeks implies an additional 52 hours worked per year. Given that the median workweek consists of 40 hours, this suggests an additional 1.3 (52/40) weeks worked per year. Standard errors are reported in parentheses. ** indicates statistical significance at the 5% level.

When including all demographic and industry controls, we find that workers in medium-sized firms work around 12 minutes longer per day, which, assuming workers work five days a week, amounts to an additional hour of work per week (or an additional 1.4 weeks per year assuming the modal hours worked of 40). Workers in large size firms (with over 100 employees) work around 13 minutes longer per day, which amounts to an additional hour and six minutes of work per week or almost 2 additional weeks worked per year. Unlike the unconditional averages, the conditional averages of the hours of workers in medium and large firms do indeed differ with longer hours in large firms, although the point estimates are not statistically significantly different from each other.

Figure B.8 and Table B.5 show that average hours, as reported in the ATUS, are increasing between small and medium-sized firms but are relatively similar between medium and large firms. To explore the robustness of these results, we also consider the average weekly hours worked, as reported in the ATUS, using the more detailed firm size categories available in the CPS. The first column of Table B.6 reports this average and shows that, with the exception of employees in firms with 500 to 999 employees, average weekly hours in the ATUS (weakly) increase over firm size. Indeed, workers in firms with 500 to 999 employees tend to work even shorter hours than employees in firms with 1 to 9 employees. The second and third columns of Table B.6 report daily average hours for weekdays and weekends, respectively, and show a similar finding. Workers in firms of size 500-999 tend to work significantly shorter hours regardless of whether they are interviewed during the weekdays or weekend. Finally, the last two columns report, respectively, the share of workers that completed their time diary for a weekend and the total number of observations in the merged CPS-ATUS sample. Although the ATUS is designed to elicit an equal share of responses on weekends and weekdays, once again, firms with between 500 and 999 employees are an exception in that the share of workers that complete their time use diary on a weekend is 53%, differing from the near equal split in all other firm size categories. While this could suggest that firms of this size tend to hire a greater share of workers that work on weekends, the significantly lower average hours worked during weekends suggests that employees in this firm size category that completed their survey for a weekday may be underrepresented in the merged CPS-ATUS sample.

Taken together, these results suggest that using higher-quality time use data shrinks the difference in average hours worked between medium- and large-sized firms, but there still exists a weakly increasing relationship between firm size and hours worked relative to small firm.

Wages and hours by firm size. Next, we revisit the relationship between wages and hours by firm size using time use data from ATUS. With the underlying assumption that working hours reported in the ATUS are reported without measurement error, we compare hours reported in the ATUS to hours reported in the CPS to quantify the degree of measurement error in the CPS. We then use this measure of measurement error to adjust the reported hours (and wages) of workers in the CPS and recompute wage-hours profiles by firm size.

Before conducting this exercise, it is important to note that classical measurement error, which would affect employees of all firms identically, is not likely to change our finding that wage-hours profiles differ by firm size. Instead, if measurement error is not classical and is, for example, correlated with firm size, then this might be a driver of the differential profiles we document in the main text. Bick et al. (2022) found that measurement error in hours, as

Table B.6: Average hours reported in ATUS by detailed firm size categories

	Avg. Weekly hours	Avg. Daily Hours		Share Weekend	N
		Weekdays	Weekends		
1-9 employees	39.7	7.27	1.67	0.50	2,418
10-24	39.9	7.36	1.55	0.51	1,352
25-99	41.1	7.70	1.31	0.49	1,971
100-499	41.1	7.69	1.33	0.49	3,577
500-999	39.6	7.44	1.19	0.53	1,472
1000+	41.3	7.62	1.60	0.50	9,982
Only available 2010-2017					
10-49	41.3	7.61	1.63	0.50	1,341
50-99	42.2	7.93	1.26	0.49	729

Notes: The table reports measures of average weekly and daily hours in the merged CPS-ATUS sample. The first column reports average weekly hours reported in the ATUS. This average is constructed by multiplying the average daily hours reported on weekdays by five and multiplying the average daily hours reported on weekends by two and summing these together. The second and third columns report the average daily hours reported for time use diaries covering weekdays and weekends, respectively. The fourth column reports the shares of respondents whose time use diary covers weekends, while the last column reports the total number of respondents by each firm size bin.

proxied by comparing ATUS and CPS hours, was correlated with reported CPS hours such that those that reported shorter (longer) hours in the CPS tended to report longer (shorter) hours in the ATUS. Given our finding that workers in larger firms tend to work longer, such correlations of measurement error with reported hours would also lead to correlated measurement errors with firm size.

Figure B.9 confirms this. In it, we report the difference between average weekly hours in the CPS with average weekly hours in the ATUS by hours worked bins and by firm size. Given the low number of observations in the merged CPS-ATUS sample when grouping by hours bin and firm size, we group together workers that work between 50 to 59 and above 60 hours into two large bins while all other hours bins are as in the main text.[1] As in Bick et al. (2022), we find that, across all firm sizes, workers that report shorter (longer) hours in the CPS report longer (shorter) hours in the ATUS. Importantly, the difference in ATUS and CPS hours does indeed differ by firm size, with larger differences, particularly for workers that work over 50 hours, in larger firms. We also observed that employees of medium-sized firms that report shorter hours in the CPS tend to understate their hours.

Taking the differences reported in Figure B.9 as a measure of the degree of measurement error in the CPS, we adjust the reported usual weekly hours of workers by adding in this measure to the reported hours worked. Since reported hours tend to be bunched around multiples of five, we round the difference between ATUS and CPS to the nearest multiple of 5. For example, a worker in a firm with 10 to 99 employees that reported working 65 hours in the CPS has their hours adjusted down by 15. Similarly, a worker in a firm with over 100 employees that reports working 45 (25) hours will have their hours adjusted down (up) by 5. With these adjusted measures of hours, we also recompute hourly wages by taking annual

[1]For example, there are only 8 respondents that work 65-69 hours in firms with 1 to 9 employees.

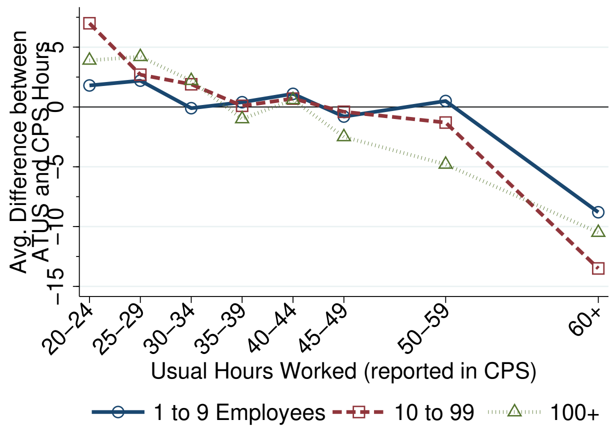


Figure B.9: Difference in average weekly hours by firm size, CPS and ATUS

Notes: The figure reports the average usual weekly hours worked by firm size using data from the CPS.

income and dividing it by the product of weeks worked and adjusted weekly hours.

With these adjusted measures of hours and wages, we re-estimate Equation 2. The resulting wage-hours profile by firm size is reported in Panel (a) of Figure B.10. Consistent with Bick et al. (2022), correcting for measurement error in hours in the CPS does not change the overall hump shape of the wage-hours profile. Further, we find that the adjusted wage penalties for working shorter hours (under 35 hours) are larger in medium and large firms, as in the baseline estimates in Figure 3 of the main text. Having said this, the penalties for working shorter hours tend to be more severe, particularly in larger firms, than those in the baseline estimates. This is because workers that report shorter hours in the CPS have their hours adjusted upward, which results in lower hourly wages.

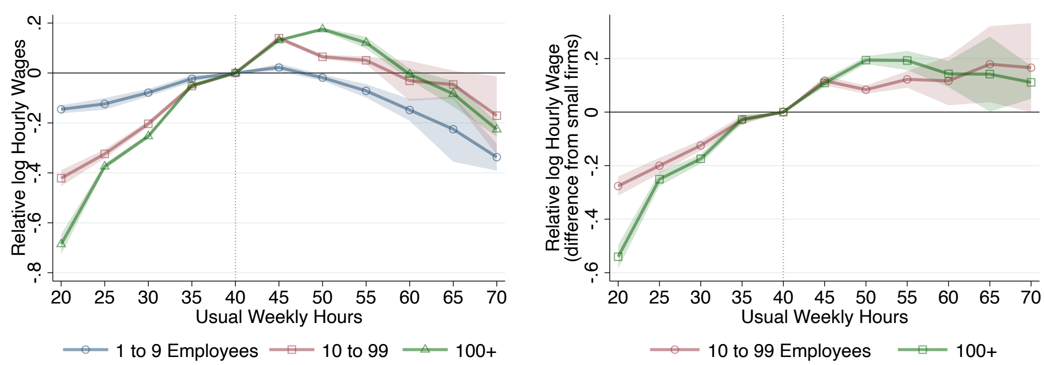


Figure B.10: Wage profiles by firm size and hours worked, adjusting for measurement error

Notes: The figure reports the coefficient $(\gamma_h + \theta_{e,h})$ in Panel (a) and $\theta_{e,h}$ in Panel (b) as estimated from Equation 2 by adjusted hours and wages reported in the CPS using the CPS-ATUS merged sample. The reference group for usual hours worked in the regression is workers that work $40 - 44$ hours. The reference group for size is firms with 1 to 9 employees. The shaded regions are the 95% confidence intervals.

Penalties for working longer hours continue to differ by firm size, particularly for workers working between 40 and 60 hours. However, for even longer hours worked, there is little difference between the long-hour wage penalties of workers in medium and large firms. Though, due to the nature of the adjustment of hours, there are much fewer workers that have adjusted hours above 60, and the estimated penalties for this region are much noisier for all firm sizes. Panel (b) shows this clearly by only plotting the wage penalties in medium and large firms, relative to small firms. Focusing on workers that work between 25 and 55 hours, the region of the hours distribution where the vast majority of workers work, we can see that adjusting for measurement error does not change our baseline empirical finding: penalties for working shorter (longer) hours are more severe in larger (smaller) firms.

Taken together, the results in this section confirm that measurement error in hours is not a likely driver of our empirical findings related to hours worked and firm size.

B.3 Evidence from the Canadian LFS

Our primary empirical analysis utilizes data from the United States. In this appendix, we document our three motivating facts by firm size using data from the Canadian Labor Force Survey (LFS) between 1998 and 2018. Similar to the CPS, the LFS is a nationally representative survey containing detailed information on respondents' economic activity for the month they are interviewed, such as hourly earnings, usual weekly hours worked, and firm size. Firm size is recorded in one of four bins, and for clarity we combine the larger two size bins into one and report size in three categories: i) small (under 20 employees), ii) medium (between 20 and 100 employees), and iii) large (over 100 employees). Our sample starts in 1998 as this is the first year that information on establishment size is available in the LFS. Our treatment of the LFS data remains identical to that of the CPS. In particular, we restrict attention to respondents aged 25 to 64 who worked for a single private employer during the reference month. We exclude workers who usually work fewer than 10 hours per week and those that earned less than half the minimum wage.[2] Since 1997, the LFS has also reported establishment size, and we conduct our empirical analysis at both the firm and establishment level.

Fact 1 Average hours increase with size.

We begin by showing that workers in larger firms work longer hours than workers in smaller firms. To do this, we estimate Equation (1) using LFS data and report the coefficient β_f in Table B.7, where f represents firm size (first three columns) and establishment size (last three columns). As with the CPS data, usual hours worked are longer in larger firms. Indeed, workers in larger establishments work around 4% longer than similar workers in smaller establishments. Relative to workers of firms with under 20 employees, workers in mid-sized firms work between 4% and 5% longer in larger firms.

Fact 2 Average wages increase with size.

Next, we estimate Equation (2) using LFS data and report the coefficient β_f, which captures the size-wage premium, in Table B.8. Consistent with the data from the US, wages in the largest firms are indeed higher than those in the smallest firms. The LFS data indicate a wage premium of around 19% for workers in firms or establishments with over 100 employees compared to those with under 20.

Fact 3 Long-hours (short-hours) penalty decreases (increases) with size.

Finally, we show that, consistent with data from the CPS, the short- and long-hours penalties also vary systematically by firm and establishment size in the LFS. Figure B.11 plots the relationship between hours and wages in the LFS as estimated from Equation (2). In particular, Panels (a) and (b) report the sum of the coefficients γ_h and $\theta_{f,h}$, which capture the wage penalty of working outside of the 40-44.9 hours bin by firm and establishment size, respectively. The panels show that for both firms and establishment size categories,

[2] The minimum wage in Canada is taken to be an employment-weighted average of the minimum wage across provinces as reported by Statistics Canada.

Table B.7: Size and hours worked

	Firm Size			Establishment Size		
	(1)	(2)	(3)	(4)	(5)	(6)
20 to 99 Employees	0.064***	0.051***	0.048***	0.049***	0.038***	0.034***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
100+ Employees	0.060***	0.047***	0.041***	0.072***	0.054***	0.039***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Year, Province FE	Y	Y	Y	Y	Y	Y
Demographic Controls	N	Y	Y	N	Y	Y
2-digit Industry FE	N	N	Y	N	N	Y
N	6,552,536	6,552,536	6,552,536	6,846,599	6,846,599	6,846,599
R^2	0.015	0.103	0.137	0.019	0.106	0.137

Notes: The table reports the coefficient β_f estimated from Equation (1) where the reference size category is the smallest size category. The first three columns report results where f represents firm size categories. The last three columns report results where f represents establishment size categories. All data are from the pooled LFS sample, and firm size data are available starting in 1998 while establishment size data are available starting in 1997. Standard errors are reported in parentheses. *** indicates statistical significance at the 1% level.

Table B.8: The size-wage premium

	Firm Size			Establishment Size		
	(1)	(2)	(3)	(4)	(5)	(6)
20 to 99 Employees	0.097***	0.078***	0.080***	0.093***	0.077***	0.081***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
100+ Employees	0.212***	0.176***	0.185***	0.241***	0.201***	0.191***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Year, Province FE	Y	Y	Y	Y	Y	Y
Demographic Controls	N	Y	Y	N	Y	Y
2-digit Industry FE	N	N	Y	N	N	Y
N	6,552,536	6,552,536	6,552,536	6,838,698	6,838,698	6,838,698
R^2	0.146	0.273	0.342	0.159	0.284	0.348

Notes: The table reports the coefficient β_f estimated from Equation 2 where the reference size category is the smallest size category. The reference hours bin is $40 - 44.9$ hours. The first three columns report results where f represents firm size categories. The last three columns report results where f represents establishment size categories. All data are from the pooled LFS sample, and firm size data are available starting in 1998 while establishment size data are available starting in 1997. Standard errors are reported in parentheses. *** indicates statistical significance at the 1% level.

there exist hump-shaped relationships between hours and wages. Importantly, as with the US data, the short-hours wage penalty in larger firms is much more severe than the penalty in smaller firms. For example, compared to small firms, working 25 hours in large firms (with over 100 employees) results in around 15% lower wages. The analogous measure for establishment size is around 10% lower wages. Conversely, the long-hours wage penalty is much more severe in smaller firms than in larger firms. For example, compared to large firms, working 60 hours in small firms (with under 20 employees) results in around 5% lower wages. The analogous measure for establishment size is around 3% lower wages.

(a) Firm Size (b) Establishment Size

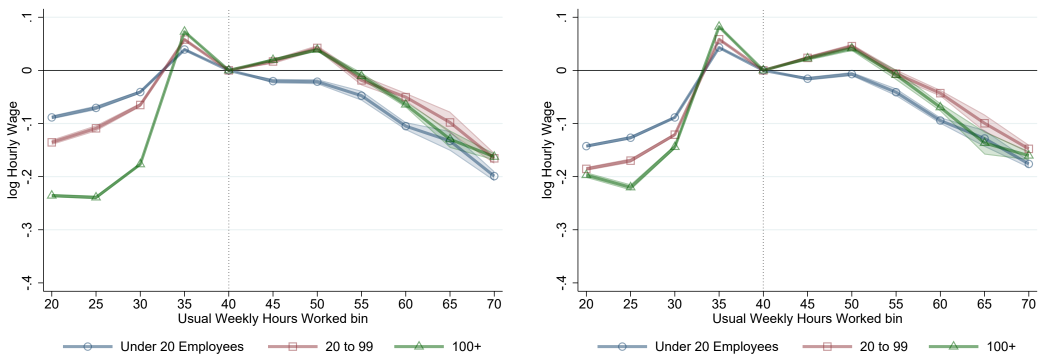


Figure B.11: The relationship between wages and hours, Canada

Note: Panels (a) and (b) report the the sum of coefficients $\left(\gamma_h + \theta_{f,h}\right)$ estimated from Equation (2) using LFS data where f represents, respectively, firm size and establishment size. The reference group for usual hours worked in the regression is workers that work $40 - 44.9$ hours. The reference group for size is the smallest size category—that is, firms or establishments with under 20 employees. The shaded regions are the 95% confidence intervals. Data are from the pooled LFS sample.

Taken together, the analysis with the LFS data is encouraging as is confirms that our main empirical findings are not simply an artefact of the US data. Additionally, replicating our motivating facts using Canadian data is encouraging as it suggests that the US and Canadian economies are similar and may share similar fundamentals, such as the substitution parameter ρ.

C Model appendix

In this appendix, we provide additional details and discussions related to the model and quantitative analysis.

C.1 Definition of the equilibrium in the extended model

In the main body of the paper, we provide the definition of the equilibrium for the baseline model. The extended model differs from the baseline in being dynamic and having additional heterogeneity in worker efficiency and wealth. Here we provide the definition of the stationary general equilibrium modified for the extended model.

Stationary general equilibrium of the extended model. A stationary general equilibrium consists of a set of policy functions: $\mu_j^*(l,x)$ for firms $j \in \{1,..,J\}$ and $\mathbf{l}(a,x,z_j,v)$, and $\mathbf{o}(a,x,z_j,v)$ for workers in firm group j, wage functions $w_j(l,x)$, and a time-invariant distribution of workers $\varphi(a,x,z_j,v)$ over the type of their employer (z_j), wealth (a), idiosyncratic efficiency x, and the value of leisure v, such that:

(i) The policy functions solve the problems of workers and firms.

(ii) Labor markets clear. The total measure of workers demanded by all firms for each level of firm productivity z_j, idiosyncratic efficiency x, and working hours $l \in [0,1]$ is equal to the corresponding labor supply:

$$\mu_j^*(l,x)F(z_j) = \int\limits_{a=0}\int\limits_{v\in B_v} \varphi(a,x,z_j,v)\mathbb{1}[\mathbf{l}(a,x,z_j,v)=l]dvda, \ \ \forall x \in B_x, \ j \in \{1,2,..,J\}$$

(iii) The evolution of the distribution across workers satisfies, for each $a \geq 0$ $x \in B_x$, $v \in B_v$, and $j \in \{1,2,..,J\}$:

$$\begin{aligned}\varphi(a,x,z_j,v) = &\int\int\int \Gamma_x(\tilde{x},x)\Gamma_v(\tilde{v},v)\mathbf{o}(a,x,z_j,v)\times \\ &\sum_{\tilde{j}=1}^{J}\varphi(\tilde{a},\tilde{x},z_{\tilde{j}},\tilde{v})\left(s+(1-s)\mathbb{1}[\tilde{j}=j]\right)\mathbb{1}[\mathbf{l}(\tilde{a},\tilde{x},z_{\tilde{j}},\tilde{v})=a]d\tilde{x}d\tilde{v}d\tilde{a}.\end{aligned}$$

C.2 The role of taste shocks

Described in Section 3, our model features shocks to the return of workers to working for firms of differing productivity. The computational advantage of these shocks is that they help "convexify" the occupational choice of workers by introducing additional randomness in their decision.

In particular, by assuming a Generalized Extreme Value Distribution for these shocks, the occupational choice of workers can be considered as a probability, which is given by the value obtained in each occupation—net of the ϵ-shocks, relative to the aggregation of values in all other firm productivity levels,

$$H(j;v) = \frac{\exp\left(V^G(z_z, v)\right)^{\frac{1}{\sigma_\epsilon}}}{\sum_{k=1}^{J} \exp\left(V^G(z_k, v)\right)^{\frac{1}{\sigma_\epsilon}}}.$$

Having a probability as the policy function, instead of a binary indicator of 0 or 1 for choosing each occupation, smooths out the value function of workers and helps with convergence.

Existing literature has used similar "tastes" in different models of discrete choice, such as McFadden (1978) for households' location choice and Wolpin (1984) in a model of fertility. The role of taste heterogeneity in shaping wage heterogeneity between employers has previously been highlighted and modeled in Card et al. (2018). Other papers that use similar shocks in the context of occupational choice of workers are Artuç et al. (2010) and Caliendo et al. (2019).

As with Card et al. (2018), these taste shocks play a crucial role in generating a size-wage premium in our model. We interpret these taste shocks as capturing a number of data features that affect individuals' sorting into firms of different productivity and size, which are not accounted for by wages. Below we discuss several features of the labor market that motivate introducing the taste shocks in the model.

Small and large firms differ in the multi-dimensional non-pecuniary benefits they offer to workers. Studies show that workers' heterogeneous preferences over these non-pecuniary characteristics are important in generating earning inequality (Rosen, 1986, Morchio and Moser, 2018, and Lamadon et al., 2022). Importantly, we emphasize that these non-pecuniary characteristics might differ across firm size groups. While small firms have a more friendly and less rigid work environment (Agell, 2004 and Idson, 1990), large firms might excel in some other dimensions, such as safety in the work environment (Oi, 1974).

Further, there may exist logistical and technological reasons why different workers do not find it equally feasible to work in larger firms. For example, several studies have documented firms in urban areas are more productive and larger than firms in rural areas (Headd, 2000 and Melo et al., 2009). If workers are tied to a specific location, moving costs or commuting costs could contribute to the taste shocks we build into our model. There also exists cross-sector differences in firm size, with smaller firms more likely to be in the construction, services, and agriculture sector and large firms more likely to be in the manufacturing, retail, transportation, and finance sector (Headd, 2000). In this regard, the taste shock could also be interpreted as the limited transferability of sector-specific human capital.

The taste shocks we introduce are a reduced-form representation of the real-world features we discuss here. Having said this, much of the worker side's heterogeneity can be argued to be persistent, yet our taste shocks are independently drawn each period. The reason for this is tractability and simplicity. We could generate similar implications with our model in a static model with static heterogeneity in workplace preferences.

Notice also that our taste shocks are over firms' productivity levels, even though the discussion above corresponds to the preferences or ability to work in firms of different sizes. However, firm size and productivity will be isomorphic in our model and are strongly positively correlated in the data (see, for example, Leung et al. 2008 and Bartelsman et al. 2013).

C.3 Showing the optimality of the symmetric solution

In our analysis, we focus on a symmetric equilibrium in which all the firms in a z_j-market absorb the same share of the supply $\mu_j^s(l,x)$ for all $l \in [\underline{l}_j, \bar{l}_j]$ and $x \in B_x$.[3] Accordingly, we need to assure that a demand schedule $\mu_{a,j}(l) = \mu_j^s(l,x)/F(z_j)$ solves the maximization problem in Equation (5) for each firm i with firm productivity z_j. To do so, we characterize a firm's optimization problem and show that, under fairly relaxed constraints on labor demand, firms do not benefit from deviating from the symmetric solution.

We assume that firms are subject to a natural upper bound, $\bar{\mu}_j$, on their labor demand from any hour and we parameterize this as

$$\bar{\mu}_j = X_\mu \times \max_{l\in[\underline{l}_j,\bar{l}_j],\ x\in B_x} \mu_{a,j}(l,x)$$

where $X_\mu \in (1,\infty)$. In words, we assume that the z_j-specific upper bound is high enough so that it does not bind for any level of hours and worker efficiency in any labor market z_j in the symmetric equilibrium.

The first-order conditions from firm i's problem implies that for all $l \in [\underline{l}_j, \bar{l}_j]$ and $x \in B_x$,

$$xz_j\theta L_i^{\theta-1}E_i(l^\rho)^{\frac{1}{\rho}}\left[\frac{1}{\rho}\frac{l^\rho}{E_i(l^\rho)}+1-\frac{1}{\rho}\right]-w_j(l,x)l-\lambda_i(l,x)+\chi_i(l,x)=0 \tag{C.1}$$

where we denote the Lagrange multipliers of the non-negativity constraint by $\chi(l,x)$ and the one for the upper bound restriction by $\lambda(l,x)$. Here, we maintain the notation that

$$E_i(l^\rho) \equiv \left(\int_{x\in B_x}\int_{\underline{l}_j}^{\bar{l}_j} x\mu_i(l,x)l^\rho dldx\right),$$

$$L_i \equiv \left[\left(\int_{x\in B_x}\int_{\underline{l}_j}^{\bar{l}_j} x\mu_i(l,x)l^\rho dldx\right)^{\frac{1}{\rho}}\left(\int_{x\in B_x}\int_{\underline{l}_j}^{\bar{l}_j} \mu_i(l,x)dldx\right)^{1-\frac{1}{\rho}}\right]$$

are the labor aggregation terms that correspond to a given demand schedule μ_i. $E_{a,j}(l^\rho)$ and $L_{a,j}$ are defined analogously.

In the symmetric equilibrium, the non-negativity and the upper bound restrictions do not bind, i.e., $\lambda_{a,j}(l,x) = \chi_{a,j}(l,x) = 0$ for all $l \in [\underline{l}_j, \bar{l}_j]$ and $x \in B_x$. Hence, the wage function in this equilibrium has to be equal to the marginal productivity of a worker from

[3] In equilibrium, all z_j-markets contain the full set of efficiency types (B_x). Without loss of generality, we proceed with the analysis by assuming that for all x-type of workers in the market z_j, there exists a common set of l, denoted by $[\underline{l}_j, \bar{l}_j]$, where there is a positive supply for worker type (l,x), i.e., $\mu_j^s(l,x) > 0$. Our analysis is still valid even if the set $\{l|\mu_j^s(l,x) > 0\}$ differs by x in the equilibrium.

each hour-efficiency pair for these firms:

$$w_j(l,x)l = xz_j\theta L_{a,j}^{\theta-1}E_{a,j}(l^\rho)^{\frac{1}{\rho}}\left[\frac{1}{\rho}\frac{l^\rho}{E_{a,j}(l^\rho)}+1-\frac{1}{\rho}\right],$$

for all $l \in [\underline{l}_j, \bar{l}_j]$ and $x \in B_x$.

Next, we pursue whether a given firm i finds it optimal to deviate from the symmetric solution by choosing a different demand schedule. Define:

$$\begin{aligned} g^i(l,x) &\equiv xz_jL_i^{\theta-1}E_i(l^\rho)^{\frac{1}{\rho}}\left[\frac{1}{\rho}\frac{l^\rho}{E_i(l^\rho)}+1-\frac{1}{\rho}\right]-w_j(l,x)l \\ &= xz_j\theta L_i^{\theta-1}E_i(l^\rho)^{\frac{1}{\rho}}\left[\frac{1}{\rho}\frac{l^\rho}{E_i(l^\rho)}+1-\frac{1}{\rho}\right]-xz_j\theta L_{a,j}^{\theta-1}E_{a,j}(l^\rho)^{\frac{1}{\rho}}\left[\frac{1}{\rho}\frac{l^\rho}{E_{a,j}(l^\rho)}+1-\frac{1}{\rho}\right] \end{aligned}$$

Implicit in the function g^i is firm i's demand schedule μ_i. First, notice that a choice with $\mu_i(l,x) = \mu_{a,j}(l,x)$ for all $l \in [\underline{l}_j, \bar{l}_j]$ satisfies the first-order conditions: it implies $g^i(l,x) = 0$, with $\lambda_i(l,x)$=0 and $\chi_i(l,x) = 0$ for all $l \in [\underline{l}_j, \bar{l}_j]$ and $x \in B_x$.

Suppose there exists a demand schedule $\mu_i(l,x) = \tilde{\mu}_i(l,x)$ that delivers a higher level of profits than $\mu_i(l,x) = \mu_{a,j}(l,x)$. In Claim 1 and 2, we provide an analytical characterization of such a demand schedule. Guided by these results, we then examine whether there exists such an alternative demand schedule in our calibrated model where firms are better off than under the symmetric solution.

Claim 1. Suppose there exists a demand schedule $\mu_i(l,x) = \tilde{\mu}_i(l,x)$ that delivers a higher level of profits than $\mu_i(l,x) = \mu_{a,j}(l,x)$. Then, for any given $x \in B_x$, the function $g_l^i(l,x)$ associated with this demand schedule is either i) strictly positive for all $l \in [\underline{l}_j, \bar{l}_j]$, or ii) strictly negative for all $l \in [\underline{l}_j, \bar{l}_j]$.

Taking the derivative of g^i with respect to l for a given x:

$$g_l^i(l,x) = xz_j\theta l^{\rho-1}\left[L_i^{\theta-1}E_i(l^\rho)^{\frac{1}{\rho}-1} - L_{a,j}^{\theta-1}E_{a,j}(l^\rho)^{\frac{1}{\rho}-1}\right].$$

The term in brackets is constant for all $l \in [\underline{l}_j, \bar{l}_j]$. Our objective is to show that for a schedule $\tilde{\mu}_i$ to achieve higher profits than $\mu_{a,j}$, the following condition has to hold:

$$A \equiv L_i^{\theta-1}E_i(l^\rho)^{\frac{1}{\rho}-1} - L_{a,j}^{\theta-1}E_{a,j}(l^\rho)^{\frac{1}{\rho}-1} \neq 0.$$

To see that, take some $\hat{x} \in B_x$. Suppose $A = 0$, which implies a constant $g^i(l,\hat{x})$ over l. If this constant value is negative, it implies that $g^i(l,x) < 0$ for all $l \in [\underline{l}_j, \bar{l}_j]$ and $x \in B_x$ because g^i is proportional to x. That gives $\tilde{\mu}_i(l,x) = 0$ for any $l \in [\underline{l}_j, \bar{l}_j]$ and $x \in B_x$, which implies zero output and zero profits. With the same token, if $g^i(l,\hat{x})$ over l is constant at a positive level, it implies that $g^i(l,x)$ for all $l \in [\underline{l}_j, \bar{l}_j]$ and $x \in B_x$. $\chi_i(l,x) > 0$ and $\tilde{\mu}_i(l,x) = \bar{\mu}_j$ for any $l \in [\underline{l}_j, \bar{l}_j]$. This solution implies negative profits for any $\bar{\mu} > \max\limits_{l \in [\underline{l}_j, \bar{l}_j]}(\mu_{a,j}(l,x))$ in our calibration. The only remaining possibility to evaluate is that $g^i(l,\hat{x}) = 0$ for all $l \in [\underline{l}_j, \bar{l}_j]$, which implies $g^i(l,x) = 0$ for all $l \in [\underline{l}_j, \bar{l}_j]$ and $x \in B_x$. For $A = 0$ and $g^i(l,x) = 0$ to hold simultaneously, it is required that $E_i(l^\rho) = E_{a,j}(l^\rho)$ and $L_i = L_{a,j}$. This implies that $Y_i = Y_{a,j}$—that is, the output levels are the same under schedules $\tilde{\mu}_i$ and $\mu_{a,j}$. Since profits are equal to a constant fraction of output when $g^i(l,x) = 0$, the profits under the two schedules are also the same. This shows that for an alternative schedule to strictly dominate $\mu_{a,j}$, it has to imply that $A \neq 0$; hence $g^i(l,x)$ is strictly monotone over l for each $x \in B_x$.

Claim 2. Suppose there exists a demand schedule $\mu_i(l,x) = \tilde{\mu}_i(l,x)$ that delivers a higher level of profits than $\mu_i(l,x) = \mu_{a,j}(l,x)$. Then, one of the following statements must be true: Either

(i) $\exists\ l_0 \in [\underline{l}_j, \bar{l}_j]$ **such that for any** $x \in B_x$**,** $\tilde{\mu}_i(l,x) = 0\ \forall\ l < l_0$**,** $\tilde{\mu}_i(l,x) = \bar{\mu}_j\ \forall\ l > l_0$**, and** $\tilde{\mu}(l_0,x) \in [0, \bar{\mu}_j]$**, or**

(ii) $\exists\ l_0 \in [\underline{l}_j, \bar{l}_j]$ **such that for any** $x \in B_x$**,** $\tilde{\mu}_i(l,x) = 0\ \forall\ l > l_0$**,** $\tilde{\mu}_i(l,x) = \bar{\mu}_j\ \forall\ l < l_0$**, and** $\tilde{\mu}(l_0,x) \in [0, \bar{\mu}_j]$**.**

We first show that $\exists\ l_0 \in [\underline{l}_j, \bar{l}_j]$ such that $g^i(l_0,x) = 0$ for all $x \in B_x$. Take an arbitrary $\hat{x} \in B_x$. We know that $g^i(l,\hat{x})$ is strictly increasing or strictly decreasing over l. If $g^i(l,\hat{x}) < 0$ $\forall\ l \in [\underline{l}_j, \bar{l}_j]$, then the proportionality of g^i with respect to x implies that $g^i(l,x) < 0$ $\forall\ l \in [\underline{l}_j, \bar{l}_j]$ and $x \in B_x$. This implies $\tilde{\mu}_i(l,x) = 0\ \forall\ l \in [\underline{l}_j, \bar{l}_j]$ and $x \in B_x$, which gives zero output and profits. Similarly, if $g^i(l,\hat{x}) > 0\ \ \forall\ l \in [\underline{l}_j, \bar{l}_j]$, it implies that $g^i(l,\hat{x}) > 0$ $\forall\ l \in [\underline{l}_j, \bar{l}_j]$ and $x \in B_x$. This implies $\tilde{\mu}_i(l,\hat{x}) = \bar{\mu}_j$. We have shown in Claim 1 that this schedule gives lower profits than the solution $\mu_{a,j}$. Hence, we can focus on the case where $g^i(l,\hat{x})$ intercept 0 at some $l_0 \in [\underline{l}_j, \bar{l}_j]$, i.e., $g^i(l_0,\hat{x}) = 0$. From the proportionality of g^i with respect to x, this gives $g^i(l_0,x) = 0$ for all $x \in B_x$. We next prove the claim by case.

Case (i): Suppose $g^i(l,x)$ is strictly increasing over l for each $x \in B_x$. Then for all $l \in [l_0, \bar{l}_j]$, we have $g^i(l,x) > 0$, hence $\lambda_i(l,x) > 0$ according to C.1. The complementary slackness condition implies that $\tilde{\mu}_i(l,x) = \bar{\mu}_j$. Similarly, for any $l < l_0$, $g^i(l,x) < 0$, which implies $\tilde{\mu}_i(l,x) = 0$.

Case (ii): Suppose $g^i(l,x)$ is strictly decreasing over l for each $x \in B_x$. Then, for all $l \in [\underline{l}_j, l_0]$, we have $g^i(l,x) > 0$, hence, $\lambda_i(l,x) > 0$, which implies $\tilde{\mu}_i(l,x) = \bar{\mu}_j$. Analogous to case (i), for any $l > l_0$, $g^i(l,x) < 0$, which implies $\tilde{\mu}_i(l,x) = 0$.

Lastly, an immediate implication from the above analysis is that for any $x \in B_x$ there exists a unique l such that $l = l_0$ and $0 < \tilde{\mu}_i(l,x) < \bar{\mu}_j$. For all other $l \in [\underline{l}_j, \bar{l}_j]$ and $l \neq l_0$, either $\tilde{\mu}_i(l,x) = \bar{\mu}_j$ or $\tilde{\mu}_i(l,x) = 0$.

Taking stock. In the light of the results above, for an alternative schedule $\tilde{\mu}_i$ to dominate the solution $\mu_{a,j}$, the only possibility is that it entails a demand equal to $\bar{\mu}$ for a compact set of hours that includes either the lowest or the highest hours in the choice set (i.e., a or b, respectively), and a demand of zero amount for the rest of the hours.

For illustration purposes, let c and d be the lower and the upper bound of the compact set that demand $\bar{\mu}$. The profits obtained by this case are:

$$\tilde{\pi}_j(c,d) = z_j \left[\left(\int\limits_{x\in B_x} \int\limits_c^d x\bar{\mu}_j l^\rho dldx \right)^{\frac{1}{\rho}} \left(\int\limits_{x\in B_x} \int\limits_c^d x\bar{\mu}_j dldx \right)^{1-\frac{1}{\rho}} \right]^\theta - \int\limits_{x\in B_x} \int\limits_c^d w_j(l,x)\bar{\mu}_j ldldx \quad \text{(C.2)}$$

It will be useful to go further in computing Equation (C.2) to get:

$$\tilde{\pi}_{c,d} = z_j \bar{\mu}_j^\theta \left(\frac{\bar{x}^2 - \underline{x}^2}{2} \right)^\theta \left\{ \left(\frac{d^{\rho+1} - c^{\rho+1}}{\rho+1} \right)^{\frac{1}{\rho}} (d-c)^{1-\frac{1}{\rho}} \right\}^\theta \quad \text{(C.3)}$$

$$-\bar{\mu}_j \left(\frac{\bar{x}^2 - \underline{x}^2}{2} \right) z\theta L_{a,j}^{\theta-1} E_{a,j}(l^\rho)^{\frac{1}{\rho}} \left[\frac{1}{\rho} \frac{d^{\rho+1} - c^{\rho+1}}{(\rho+1)E_{a,j}(l^\rho)} + (1-\frac{1}{\rho})(d-c) \right] \quad \text{(C.4)}$$

For any x, we can find l_0 by solving the equation $g^i(l_0,x) = 0$, in which both L_i and $E_i(l^\rho)$ can be written as a function of l_0 and $\bar{\mu}$. If $g^i(l,x)$ is strictly increasing over l, then the profit is given by $\tilde{\pi}_j(l_0,\bar{l}_j)$. On the other hand if $g^i(l,x)$ is strictly decreasing over l, then the profit is given by $\tilde{\pi}_j(\underline{l}_j,l_0)$.[4]

Next we evaluate the gains and losses for a firm that deviates from the symmetric solution $\mu_i(l,x) = \mu_{a,j}(l,x)$ to the alternative demand schedule $\mu_i(l,x) = \tilde{\mu}_i(l,x)$ that attains the highest profit highlighted above. In doing so, we compute the largest gain attained by any firm (of any productivity z_j) relative to the symmetric solution as:

$$\max_j \left[\frac{\max\{\tilde{\pi}_j(\underline{l}_j,l_0), \tilde{\pi}_j(l_0,\bar{l}_j)\} - \pi_{a,j}}{\pi_{a,j}} \right].$$

Notice that this formula takes the restriction parameter X_μ as given. Accordingly, we compute these profit gains and losses for each $X_\mu \in [1,20]$. We show the results in Figure C.1. The figure shows that for relatively relaxed restrictions on the labor demand, we can sustain our symmetric equilibrium. In particular, if we limit the labor demand for each hour-efficiency pair to a maximum of 15 times the labor supply of the largest group of hour-efficiency type per firm, then the symmetric solution is the optimal solution for each firm. Accordingly, in our analysis, we assume that X_μ falls within a range $(1,15]$ and focus on the symmetric equilibrium.

[4]In theory, where there is a continuum of l, there exists a cut-off type l_0 such that firms' demand for this type is positive but not necessarily equal to $\bar{\mu}_j$ (see Claim 2). However, in our quantitative exercise, this knife-edge condition does not hold for any hour-efficiency type in the equilibrium. That is, for any type with a positive demand in the alternative solution, their demand schedule hits the upper bound $\bar{\mu}_j$.

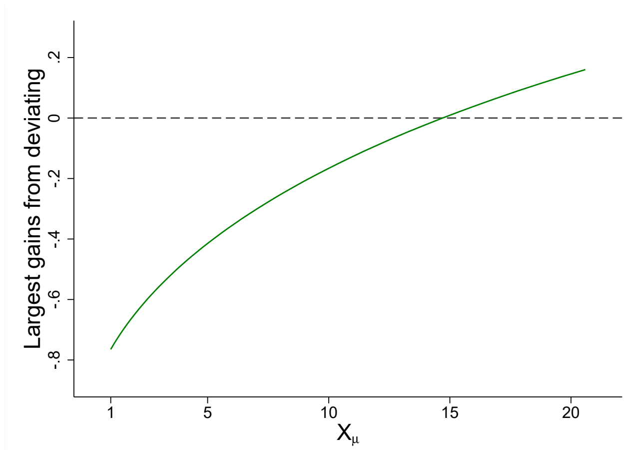


Figure C.1: Profits in the alternative solution relative to the symmetric solution

Notes: The model imposes the restriction on the demand for a given hour-efficiency pair of workers as $\mu(l,x) \leq \bar{\mu}_j$. We parameterize the upper bound as $X_\mu \times \max_{l\in[\underline{l}_j,\bar{l}_j],x\in B_x} \mu_{a,j}(l,x)$ in order to represent the tightness of the restriction with one parameter, X_μ, common across all firm productivity levels. The figure shows the rate of change in the objective function of the firms if they deviate from the symmetric solution in which all z_j-firms demand the same amount of (l,x) pair of workers.