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The Macroeconomic Effects of Debt Relief Policies During Recessions

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Abstract
I study debt relief as a stimulus policy using a dynamic stochastic general equilibrium model that captures the rich heterogeneity in households' balance sheets. In this environment, a large-scale mortgage principal reduction can amplify a recovery, support house prices and lower foreclosures. The nature of the intervention, in terms of its eligibility, liquidity and financing, shapes its macroeconomic impact. This impact rests on how resources are redistributed across households that vary in their marginal propensities to consume. The availability of bankruptcy on unsecured debt quantitatively changes the macroeconomic response to large-scale mortgage relief by reducing precautionary savings.

Topics: Business fluctuations and cycles, Debt management, Housing, Credit and credit aggregates

JEL codes: E21, E32, E6

Résumé
J'étudie le recours à l'allègement de la dette comme politique de relance. Pour ce faire, j'utilise un modèle dynamique stochastique d'équilibre général qui tient compte de la grande hétérogénéité des bilans des ménages. Dans ce contexte, une réduction à grande échelle du capital des prêts hypothécaires peut accentuer la reprise, soutenir la croissance des prix des logements et freiner les saisies immobilières. La nature de l'intervention – critères d'admissibilité, types de liquidités et modes de financement – détermine ses effets macroéconomiques. L'ampleur de ces effets dépend de la façon dont les ressources sont redistribuées aux différents ménages selon leur propension marginale à consommer. La possibilité de déclarer faillite pour radier des créances non garanties change de manière quantitative la réaction macroéconomique aux mesures d'allègement hypothécaire à grande échelle en réduisant l'épargne de précaution.

Sujets : Cycles et fluctuations économiques; Gestion de la dette, Logement, Crédit et agrégats du crédit
Codes JEL : E21, E32, E6

1 Introduction

What are the effects of introducing debt relief programs during recessions? If a severe downturn is associated with high levels of household leverage, does reducing households' debt burden help stabilize the economy? Since the Great Recession, a widely held view is that alleviating underwater borrowers' financial distress could have prevented the sharp rise in foreclosure and dampened the fall in house prices.[1] Moreover, preventing large initial declines in house prices might have reduced subsequent foreclosures, thereby supporting house prices at later dates and household spending over time. To date, however, there is little quantitative analysis of debt relief as a stimulus policy.

My goal is to quantitatively assess the effects of mortgage relief programs using a dynamic stochastic general equilibrium model. The model is designed to reproduce the heterogeneity in households' assets and liabilities. I find that in a recession that involves an unusually large drop in house prices, a large-scale mortgage principal reduction can lower foreclosure but does not mitigate the recession. Instead, it amplifies a recovery. I show that general equilibrium responses in prices play an important role in propagating the effects of the policy intervention over time. Additionally, the design of the policy, eligibility, the type of transfer and how it is financed shapes its macroeconomics consequences. The magnitude of these effects rests on the extent to which debt relief policies redistribute resources across households that vary in their marginal propensity to consume (MPC).

I develop a model that allows me to study the effects of mortgage-related programs without shortcuts. Households may borrow or save in liquid assets and can also hold illiquid assets and liabilities in the form of houses and mortgages. Housing provides service flows, which are valued alongside non-durable consumption. Households face idiosyncratic risks and have limited ability to insure themselves as markets are incomplete. Borrowers may declare bankruptcy on unsecured debt and foreclose on their mortgages.

[1]For example, see Eberly and Krishnamurthy (2014), Mian et al. (2015), Posner and Zingales (2009), and Agarwal et al. (2017).

The model has three distinctive features. First, it includes aggregate business cycle risk in the form of shocks to total factor productivity. As we consider debt relief as a macroeconomic policy tool to mitigate the severity of downturns, the model must have aggregate uncertainty. While it is costly to solve a stochastic equilibrium, the decisions of households under aggregate uncertainty may significantly differ from those under perfect foresight.

Second, all prices (interest rates, wages, house prices, and loan rates) are determined endogenously in my model. This allows me to carefully consider the general equilibrium responses to large-scale debt relief programs, both for the macroeconomy and individual households. For all exercises, I show how the responses of households to a policy intervention impact house prices, wages, and interest rates, and how these changes in prices feed back to household decisions and shape the paths of aggregate variables.

Third, the model allows a default on unsecured and secured debt separately. While Mitman (2016) and Li et al. (2011) show that there are significant interactions between these two types of debt, as well as their respective defaults, including both bankruptcy and foreclosure in a model is rare. I argue that when one tries to evaluate debt relief programs, the model should contain both types of defaults. In their absence, the effects of a temporary debt relief program can be overstated.

The distribution of households over assets and liabilities changes with the availability of default. For example, in the absence of short-term credit and bankruptcy, households increase holdings of liquid assets for consumption smoothing. This, in turn, lowers housing demand, house prices, as well as the debt burden of households. In this way, allowing for bankruptcy leads to important changes in the effects of policy.

My model is calibrated to match a large set of relevant features of the US economy. I then evaluate it along untargeted dimensions. The model successfully reproduces household wealth distribution, not only for net worth but also for key components: financial assets, housing wealth and mortgages. It also generates business cycle moments similar to the US data.

In a recession involving a large fall in house prices, I study an unanticipated intervention where households with loan-to-value (LTV) ratios above 95% have them reduced to this level via a one-time mortgage reduction. This is a large intervention that affects about 22.6% of the mortgagors and costs 5.2% of GDP. Crucially, the policy is debt financed and involves increases in future taxes.

I find that the mortgage reduction lowers foreclosures and bankruptcies but does not stimulate consumption and house prices immediately. However, it amplifies the recovery, raising output and consumption over the long run. These findings differ from those of Kaplan et al. (2020), who consider the same mortgage forgiveness program. In their model, the program reduces foreclosure rates significantly but has little effect on house prices or consumption. Among other things, the presence of capital and bankruptcy, how the policy is funded, and equilibrium responses in wages and interest rates in my model help explain this difference. A detailed analysis of a series of counterfactual exercises allows me to draw the following lessons.

First, general equilibrium effects matter. As the policy is debt financed, there is no net transfer to the economy. Instead, the principal reduction redistributes resources from low MPC households to high MPC households, as highly indebted households tend to have high MPCs. This drive rises the consumption of those who receive the principal reduction and lowers savings and capital during the recession. Later, as the economy moves into an expansion, the government raises tax rates to repay debt. These payments are made to low MPC households holding government debt. They, in turn, increase their investment in physical capital and raise capital stock. When the capital falls and grows, wages fluctuate as well. While the principal reduction directly benefits those who receive the transfer, changes in wages affect all households. These income effects build up over time, moving house prices and consumption beyond the direct effects of the mortgage principal reduction.

Second, how the government transfers resources to households matters. I compare the effects of the mortgage debt relief with those of a tax rebate of the same size. Mortgage forgiveness targets highly indebted homeowners while tax rebates are not a targeted policy. The former provides illiquid assets by increasing home equity while the other provides liquid assets. These differences lead to different distributional and aggregate implications.

Households' initial consumption responses following a tax rebate are characterized by a distribution of MPC; the MPC is larger for households with low liquid assets or large debt. In contrast, there is no monotone pattern between MPC and liquidity or indebtedness following a principal reduction. As it provides illiquid assets, a large response in household consumption accompanies a costly adjustment in mortgage or housing. In the aggregate, the tax rebate is more effective than the principal reduction at boosting consumption because it directly increases households' liquid wealth and distributes benefits to a larger group of high MPC households including those who do not own houses.

In addition to mortgage forgiveness, I consider a mortgage payment reduction and easier access to bankruptcy. For the payment reduction, I assume that per-period payments of principal are halved for four years. This policy effectively extends the duration of loans by implying slower amortization. The rise in households' disposable income initially leads to a slight increase in aggregate consumption. However, the slowdown in mortgage payments lowers savings available for investment, and capital falls. Larger interest payments from higher mortgage balances and lower wages coming from decreases in capital eventually depress consumption. In contrast, making bankruptcy less costly does not increase consumption immediately, but as more households use bankruptcy and discard their unsecured debt, both general equilibrium effects and lower household leverage contribute to a persistent rise in aggregate consumption.

Literature This work is part of the literature on household credit and default, in particular, the branch that studies debt relief as a stimulus policy tool. Since the Great Recession, a large body of work has shown that the high leverage of households can exacerbate an economic downturn or slow down a recovery.[2] Following these findings, a set of papers have estimated the effectiveness of mortgage debt relief programs. For example, Agarwal et al. (2017) show that although participation rates were low, the Home Affordable Mortgage Program

[2]Examples include Mian et al. (2013), Mian and Sufi (2014), Guerrieri and Lorenzoni (2017), Jones et al. (2011), and Verner and Gyöngyösi (2020).

(HAMP) reduced foreclosure and increased spending. Further examining HAMP, Ganong and Noel (2018) found that principal reduction was less effective than payment reduction in reducing default and increasing consumption.

The principal reduction I consider is comparable to Kaplan et al. (2020). In contrast to HAMP, it affects a larger population and provides more substantial debt reduction that, critically, does not leave borrowers underwater. Thus, Ganong and Noel's (2018) findings are not inconsistent with mine. Empirical evidence supporting the effectiveness of larger principal reductions can be found in Cespedes et al. (2021). Examining cramdowns that discharged the underwater portion of mortgages during Chapter 13 bankruptcy proceedings between 1989 and 1993, they find that foreclosure rates fell. Lastly, Piskorski and Seru (2021) show that alleviating frictions affecting debt relief (e.g. refinancing, loan renegotiation) could have reduced foreclosure rates and resulted in up to twice as fast a recovery in house prices, consumption, and employment during the Great Recession.

Two papers, both assuming nominal rigidities, are most closely related to my work. Examining unsecured loans, Auclert et al. (2019) find that debt forgiveness provided by the US consumer bankruptcy system increases consumption and this increased consumption helps to stabilize employment. Gete and Zecchetto (2018) argue that non-recourse mortgages in the US contributed to a faster recovery in the US compared to Europe, where most countries only have recourse mortgages. I complement their work by showing that debt relief programs, over and beyond existing bankruptcy and foreclosure arrangements, can have a lasting impact on the economy.

My work is broadly related to research on fiscal policy with heterogeneous agents and incomplete markets. Oh and Reis (2012) show that increases in transfers over the Great Recession exceeded that in goods and services. In a model with incomplete markets, Heathcote (2005) studies the effects of temporary tax changes and finds large real effects. Consistent with these findings, the stimulus policies I study have large real effects, and the policy effects depend on the distribution of households over assets and liabilities.

The model developed here is related to existing quantitative analyses of housing, mortgages, and foreclosures.[3] My model contains the same elements while also allowing for default on unsecured debt.[4] To my knowledge, Mitman (2016) is the only other paper exploring the interaction between bankruptcy and foreclosure. By allowing for both types of default, he reconciles the negative correlation between the generosity of bankruptcy laws and bankruptcy rates across the US states. While my model abstracts from rich heterogeneity in the bankruptcy system across states, I introduce aggregate uncertainty to quantify the role of debt relief policies during recessions.

The design of mortgage principal reduction follows the seminal contribution of Kaplan et al. (2020), discussed above. Although my model shares many elements with theirs, the introduction of capital and bankruptcy, and equilibrium movements in wages and interest rates lead to different aggregate effects of a debt relief program in my framework.

The rest of the paper is organized as follows. In Section 2, I describe the model economy. Section 3 explains the calibration and examines untargeted moments. I study the effects of debt relief policies in Section 4, and Section 5 concludes.

2 Model

2.1 Overview

The economy consists of a continuum of infinitely lived households, banks, non-durable goods producers and government. Households are indexed by their liquid assets, a, house, h, mortgage, b, labor productivity, ε, and credit history, o. They are subject to uninsurable idiosyncratic shocks to their labor productivity, which they supply inelastically to competitive firms. Households can save or borrow in a financial asset whose return is determined in equilibrium.

[3] See, for example, Jeske et al. (2013), Corbae and Quintin (2015), Hatchondo et al. (2015), Chatterjee and Eyigungor (2015), Kaplan et al. (2020), and Garriga and Hedlund (2020).

[4] In this sense, I integrate models with secured debt and models with consumer credit and bankruptcy. Examples of the latter include Athreya (2002), Li and Sarte (2006), Livshits et al. (2007), Chatterjee et al. (2007), and Nakajima and Ríos-Rull (2019).

They consume non-durable goods and service flows from their housing.

Housing services are derived from home ownership; however, households do not have to own a house. When a household buys a house, it can take out a mortgage to fund the purchase. Mortgages are long-term debt and subject to a loan-to-value (LTV) constraint at origination. Houses and mortgages are illiquid in the sense that costs are incurred when buying, selling, refinancing or prepaying. Both types of debt (unsecured debt and mortgages) are defaultable.

Fixed costs and indivisibilities in house size lead households to make discrete choices. Households' portfolios involve a large change when they i) buy or sell houses, ii) refinance or prepay mortgages, iii) default on unsecured debt, iv) default on a mortgage, or v) default on both debts. When households do not make a substantial change to their portfolio, they choose how much to consume.

There are a large number of identical firms that produce using capital, labor and a constant returns-to-scale technology. Their output is consumed or invested in physical capital. The supply of housing is fixed.

The financial sector is competitive. Banks price unsecured and secured debt, taking into account households' default risk. Perfect competition leads to zero expected profit on each loan. The government collects taxes from households, which funds transfers alongside government consumption.

The aggregate states of the economy are g, the distribution of households, and z, aggregate total factor productivity. For brevity, I summarize the household state as $\omega \equiv (a, b, \varepsilon, h, o)$ and set $\Omega \equiv (g, z)$ to represent the aggregate state.

2.2 Households

2.2.1 Environment

Labor productivity Each household's labor productivity follows a Poisson process. With frequency λ_ε, households receive a shock and draw new productivity from a time-invariant distribution.

Liquid assets Households can save or borrow using a liquid asset, a. When

a is negative, it represents unsecured debt, and households may default on such debt. When a household chooses to default, the debt is erased. However, the borrower has a record of bankruptcy in her credit history and pays a utility cost, ξ_a. The possibility of default by households means that the price of unsecured borrowing will depend on individual and aggregate states as these determine the probability of default. Borrowing costs include a unit cost of lending, $\iota(z)$.

Houses A house, h, is chosen from a discrete set, and buying or selling a house involves adjustment costs. Houses provide utility flows to households and incur maintenance costs. One component of these maintenance costs is a property tax, which is tax deductible. I assume the total supply of houses is fixed to $\overline{H}$. Importantly, house prices, $p(g, z)$, are determined in equilibrium and vary as a function of the aggregate state. Moreover, households do not need to hold this asset. When they do not have h, they do not pay maintenance costs but receive utility flow $\underline{h} < min\{h\}$.[5]

Mortgages House purchases can be funded using mortgages, b. These are distinct from unsecured debt. Thus, similarly to Mitman (2016) and differently from Kaplan et al. (2020), I allow for both unsecured credit and mortgages. This debt is refinanceable, long-term, secured, and defaultable. When choosing the size of their mortgage, households are subject to an LTV constraint: the choice of b' must be less than a fraction (γ) of the value of the collateral.

Mortgage loans are discounted by $q(a', b', \varepsilon, h, g, z)$ at the time of origination. That is, a household that takes debt with a face value of b' receives a loan, $q(\cdot)b'$, where $q(\cdot)$ reflects the probability of payment. Notice the loan discount rate depends not only on the choice of b' but on the full individual state and the aggregate states as these determine the probability of default.

Households can refinance a mortgage loan by paying a fixed cost, $\xi_{r_0} b' + \xi_{r_1}$. When refinancing, they first pay the remaining balance on the current loan and

[5] Given the complexity of the model, I abstract from a rental market while still allowing households to not own houses. While Favilukis et al. (2017), Kaplan et al. (2020), and Greenwald and Guren (2021) show modeling a rental market is crucial to explain effects of credit conditions on house prices, abstracting from it is unlikely to affect my results. First, in my model, aggregate dynamics are not driven by credit shocks. Second, as noted, households do not have to own houses.

then take out a new loan.[6] All mortgage interest rates are adjustable. Thus, refinancing is only used to extract equity or prepay the outstanding balance. While a household is holding a mortgage, it pays the interest rate as well as a fraction of the principal at each instant. The mortgage interest rate is equal to the return on saving plus a premium reflecting the unit cost of lending, $\iota(z)$.[7]

The fraction of the principal that is paid each instant is $\theta(b, \overline{p}h)b$, where $\theta(b, \overline{p}h) = \overline{\theta}\overline{p}h/b$ and $\overline{p}$ represents the long-run average price of housing. Actual mortgages involve the borrower paying a fraction of the original loan at the time of mortgage origination. However, this requires the introduction of an additional state variable for the original loan size. Commonly, housing literature avoids this problem by assuming that households pay a fraction of their outstanding mortgage. To better capture the nature of conventional mortgages, I assume that this fraction of the mortgage rises over time. This implies that a household pays a constant fraction of the value of the house, valued using a long-run average price that is different from the actual purchase price, p.

As b is long-term debt, there is no requirement that the size of this loan remains less than γ times the current value of a house. Thus, the LTV limit only applies at origination. If the price of houses decreases, a household could find itself with negative equity. However, as long as the household pays off the required amount of the outstanding balance of the loan at the moment, it is not forced to default or refinance.

When a household chooses to default, the remaining balance of debt is forgiven and a financial intermediary takes the house. I assume the financial intermediary suffers a loss when foreclosing, and the sale value is $(1 - \delta_h)ph$. The household will have a foreclosure recorded on their credit history, which precludes them from buying houses. Households that enter foreclosure incur a utility cost, ξ_b, at that moment.

Bankruptcy and foreclosure histories While a bankruptcy remains on a household's credit history, unsecured borrowing, refinancing a mortgage, new

[6] This implies that new loans are always subject to the same LTV limit and discounting.

[7] Allowing this cost of lending to vary with the aggregate shock is important to reproduce the procyclicality of credit.

origination of a mortgage, and purchasing a house are not allowed. I assume, for tractability, that the bankruptcy flag is removed stochastically with intensity λ_d. Bankrupt households' non-financial assets are fully protected, and they can enter foreclosure if they have a mortgage.

A household that has defaulted on a mortgage is unable to purchase a new house while its credit history has a foreclosure record. Since they cannot buy a house, such households are excluded from taking new mortgages. However, they can still take on unsecured debt and choose to default on any such unsecured debt they already have. If these households go bankrupt, their foreclosure flag will be replaced by a bankruptcy flag. For tractability, I assume that the foreclosure flag is removed stochastically with intensity λ_f.

2.2.2 Household Problem

Households receive flow utility from consuming non-durable goods and from the service flow from their houses. Their utility function, $u(c, h)$, is strictly increasing and strictly concave in c and h.

A household's problem is given by (1):

$$\begin{aligned} v(\omega_t) &= \max_{\{c_t\},\tau} \mathbf{E_0} \int_0^\tau e^{-\rho t} u(c_t, h_t) dt + \mathbf{E_0} e^{-\rho\tau} v^*(\omega_\tau) \\ \dot{a}_t &= w_t \varepsilon_t + r_{at}(\omega) a_t - (r_t + \theta(b, \overline{p}h)) b_t - c_t - T_t(b, \varepsilon, ph) - \xi_h p_t h_t \\ \dot{b}_t &= -\theta(b, \overline{p}h) b_t, \quad \omega_0 = \omega. \end{aligned} \tag{1}$$

Households choose non-durable consumption $\{c_t\}$ and their optimal stopping time τ. Stopping involves a household making a discrete choice that causes a large shift in its asset position or credit history. When a household chooses to stop, they do one of the following: i) buy or sell a house, ii) refinance their mortgage, iii) default on unsecured debt, iv) default on their mortgage or v) default on both unsecured debt and mortgage. These five choices are available to households without bankruptcy or foreclosure flags ($o = 0$). Households with bankruptcy or foreclosure flags have a subset of the above options. Households with a bankruptcy flag ($o = 1$) can choose to sell a house or default on a mortgage, and households with a foreclosure flag ($o = 2$) can choose to default on unsecured debt.

In the absence of such a discrete choice, households that have mortgages repay them at the rate $\theta(b, \overline{p}h)b$. A household's income tax is given by the function $T(b, \varepsilon, p(\Omega)h)$. Taxable income is a function of earnings, mortgage interest payments and property tax. Homeowners also need to pay the maintenance cost of their houses, $\xi_h ph$.

The Hamilton-Jacobi-Bellman (HJB) equation prior to stopping is

$$\begin{aligned}
\rho v(\omega, \Omega) = \max_c u(c,h) + \partial_a v(\omega, \Omega)\dot{a} + \partial_b v(\omega, \Omega)\dot{b} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(\omega^{\varepsilon_j}, \Omega) \\
+ \sum_{k=1}^{n_z} \lambda_{zz_k} v(\omega, \Omega^{z_k}) + \lambda_d (v(\omega, \Omega | o=0) - v(\omega, \Omega | o=1))|_{o=1} \\
+ \lambda_f (v(\omega, \Omega | o=0) - v(\omega, \Omega | o=2))|_{o=2} + \int \frac{\delta v(\omega, \Omega)}{\delta g(\omega)} \mathcal{K} g(\omega) d\omega \\
\dot{a} = \ w(\Omega)\varepsilon + (r_a(\omega, \Omega) + \iota(z)|_{a<0})a - (r(\Omega) + \iota(z) + \theta(b, \overline{p}h))b - c \\
- \xi_h p(\Omega)h - T(b, \varepsilon, p(\Omega)h), \\
\dot{b} = -\theta(b, \overline{p}h)b, \quad a > 0|_{o=1}, \quad b = 0|_{o=2}, \quad v(\omega, \Omega) \geq v^*(\omega, \Omega).
\end{aligned} \tag{2}$$

Above, $\lambda_{\varepsilon\varepsilon_j}$ describes the labor productivity process and $|_{o=j}$ means that a term in front of it only applies to the households with $o = j$.[8]

Aggregate productivity follows a stochastic process described by λ_{zz_k}. I define ω^{ε_j} as $(a, b, \varepsilon_j, h, o)$ and Ω^{z_k} as (g, z_k). $\mathcal{K}$ is a Kolmogorov forward operator that operates on the distributions of households, g_t, which evolves according to shocks and households' decisions, $\frac{dg_t(\omega)}{dt} = \mathcal{K} g_t(\omega)$.[9]

The stopping value $v^*(\omega_\tau, \Omega_\tau)$ is the maximum of the values listed below in (1)–(5).

[8]When it is the intensity of jumping to ε_j from ε_i, and $\varepsilon_i \neq \varepsilon_j$, $\lambda_{\varepsilon_i\varepsilon_j} > 0$. In contrast, the intensity of losing the current level of labor productivity is $\lambda_{\varepsilon_i\varepsilon_i} < 0$. We also have $\sum_j \lambda_{\varepsilon_i\varepsilon_j} = 0 \quad \forall i = 1, ..., n_\varepsilon$.

[9]Ahn et al. (2018) describe the recursive formulation of a model with aggregate uncertainty using the Kolmogorov forward operator.

1. Buying or selling a house

$$
\begin{aligned}
&v^m(a_\tau, b_\tau, \varepsilon, h_\tau, o, g, z) = \max_{h', b'} v(a', b', \varepsilon, h', o, g, z) \\
&a' = a_\tau - b_\tau + p(g,z)h_\tau - p(g,z)h' - \xi(p(g,z), h') + q(a', b', \varepsilon, h', o, g, z)b' \\
&b' \leq \gamma p(g,z)h', \;\; h' = 0 \;\; \text{if } o = 1
\end{aligned}
$$

In the second constraint, γ is the LTV limit. When changing the size of its house, a household chooses the optimal size (h') and the amount of the mortgage (b'). The remaining balance that is attached to the current house has to be repaid. The transaction cost is given by $\xi(p(g,z), h') = p(g,z)\xi_0 h' + \xi_1$. For households with a bankruptcy flag ($o = 1$), selling their house is the only available option.

2. Refinancing a mortgage

$$
\begin{aligned}
&v^r(a_\tau, b_\tau, \varepsilon, h, o, g, z) = \max_{b'} v(a', b', \varepsilon, h, o, g, z) \\
&a' = a_\tau - b_\tau + q(a', b', \varepsilon, h, o, g, z)b' - \xi_{r_0} b' - \xi_{r_1}, \;\; b' \leq \gamma p(g,z)h
\end{aligned}
$$

Households that hold a mortgage have the option to refinance by repaying the remaining balance and a fixed cost, $\xi_{r_0} b' + \xi_{r_1}$. They do not change their house size but simply originate a new loan, b', which is subject to the LTV limit.

3. Bankruptcy

$$
v^a(a_\tau, b, \varepsilon, h, o_\tau, g, z) = v(0, b, \varepsilon, h, 1, g, z) - \xi_a
$$

4. Foreclosure

$$
v^b(a, b_\tau, \varepsilon, h_\tau, o_\tau, g, z) = v(a, 0, \varepsilon, 0, 2, g, z) - \xi_b
$$

The utility costs associated with bankruptcy and foreclosure are ξ_a and ξ_b.

5. Defaulting on both mortgage and unsecured debt

$$v^{ab}(a_\tau, b_\tau, \varepsilon, h_\tau, o_\tau, g, z) = v(0, 0, \varepsilon, 0, 1, g, z) - \xi_a - \xi_b$$

I assume that if a household declares both forecloses and bankruptcy, it receives a bankruptcy flag in its credit history. When a household defaults on both types of debt, its debts held as a and b are erased and a financial intermediary takes over its house, h. Such households are not allowed to use any credit while they have a bankruptcy in their credit history.

The overall stopping value for a household is v^*, where

$$\begin{aligned} v^*(a, b, \varepsilon, h, 0, g, z) &= \max\{v^m, v^r, v^a, v^b, v^{ab}\}, \\ v^*(a, b, \varepsilon, h, 1, g, z) &= \max\{v^m, v^b\}, \\ v^*(a, b, \varepsilon, h, 2, g, z) &= v^a. \end{aligned} \tag{3}$$

As mentioned above, the available stopping options depend on households' credit history. Only households without default in their credit history can choose any option while households with bankruptcy flags can sell or foreclose their houses. Households with foreclosure flags can go bankrupt. The household's problem can be compactly written as an HJB variational inequality (HJBVI). See Appendix B for details.

2.3 Financial intermediaries

Financial intermediaries are perfectly competitive and risk neutral. These banks issue short-term deposits and loans, as well as mortgages, to households. They also lend capital to firms. The possibility of default leads banks to offer loan rates based on a household's portfolio and income. Bank loans have expected discounted profits equal to zero. While individual loans may generate a profit or a loss, ex post, banks will have zero profit on their total portfolio in the absence of aggregate shocks. However, there will be systematic profits and losses as a result of aggregate risk. I assume that the government absorbs any realized profits or losses using taxes or subsidies paid to intermediaries.[10]

Unsecured debt As shown in Bornstein (2018), the expected interest on lending in the region of no default, ($D_a(\omega, \Omega) = 0$), is a return minus a default probability. In our context, the expected return of an unsecured loan is the interest payment net of the expected loss from the borrower moving into the default region. Over an interval of duration dt, we then have

$$E[dr_a(\omega, \Omega)] = r_a(\omega, \Omega)dt - \lambda_z \sum_{z'} p_{zz'} \lambda_\varepsilon \sum_{\varepsilon'} p_{\varepsilon\varepsilon'} D_a(\omega^{\varepsilon'}, \Omega^{z'})dt,$$

where $p_{\varepsilon\varepsilon'}$ is the probability of moving from ε to ε' conditional on receiving a labor productivity shock. In the default region ($D_a(\omega, \Omega) = 1$), we set

$$r_a(\omega, \Omega) = \infty. \tag{4}$$

The zero profit condition in the region of no default implies that the return, $r_a(\omega, \Omega)$, should be equal to the risk free rate, $r(\Omega)$:

$$r_a(\omega, \Omega) = r(\Omega) + \lambda_z \sum_{z'} p_{zz'} \lambda_\varepsilon \sum_{\varepsilon'} p_{\varepsilon\varepsilon'} D_a(\omega^{\varepsilon'}, \Omega^{z'}). \tag{5}$$

When considering savings, because no household defaults on a in the region where a is positive, $r_a(\omega, \Omega)) = r(\Omega)$.

Mortgages As explained above, households holding a mortgage pay an interest rate, $r(\Omega) + \iota(z)$, and a fraction, $\theta(b, \overline{p}h))$, of the remaining balance b at each instant where $\iota(z)$ is a cost. Therefore, the flow income from a loan is $(r_t + \theta(b_t, \overline{p}h))b_t$. Banks discount the loan with a rate, $r_t + \theta_t$, as the loan matures at the rate θ_t. Recall that if a household defaults on its secured debt, the bank recovers the depreciated value of the house, $(1 - \delta_d)ph$.

Since banks expect zero profit on each loan, the discounted value of the loan at origination has to be equal to its expected cash flow. The price of the

[10] See Ozkan et al. (2017) for a similar assumption.

loan in the non-default region is given by

$$q_0(\omega,\Omega))b_0 = \mathbb{E}\Big[\mathbb{E}_\tau \int_0^\tau e^{-\int_0^s (r_s+\theta_s)ds}(r_t+\theta_t)b_0 dt + e^{-\int_0^\tau r_s ds} b(\omega_\tau,\Omega_\tau)\Big].$$

The scrap value $b(\omega_\tau,\Omega_\tau)$ at the stopping point depends on a household's discrete choice. In the case of foreclosure, $b(\omega_\tau,\Omega_\tau) = (1-\delta_d)p(\Omega)h$. When a household prepays the loan due to refinancing or a new house transaction, the scrap value is $b(\omega_\tau,\Omega_\tau) = e^{-\int_0^\tau \theta_s ds} b_0$.

Applying the Feynman-Kac formula, the above equations can be written as the following partial differential equation.[11] At $t \in [0,\tau)$,

$$\begin{aligned}(\theta(b,\overline{p}h) + r(\Omega))q(\omega,\Omega) &= \theta(b,\overline{p}h) + r(\Omega) + q_a(\omega,\Omega)\dot{a} + q_b(\omega,\Omega)\dot{b} \\ &+ \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} q(\omega^{\varepsilon_j},\Omega) + \sum_{k=1}^{n_z} \lambda_{zz_k} q(\omega,\Omega^{z_k}) + \int \frac{\delta q(\omega,\Omega)}{\delta g(\omega)} \mathcal{K} g(\omega) d\omega.\end{aligned} \tag{6}$$

At $t = \tau$,

$$q(\omega,\Omega) = \begin{cases} \frac{(1-\delta_d)p(\Omega)h}{b} & \text{if foreclose,} \\ 1 & \text{otherwise.} \end{cases} \tag{7}$$

Firms' problems and the government's budget constraint, as well as the definition of equilibrium, are described in Appendix A.

2.4 Computation of equilibrium

The aggregate state of the model contains distribution of households and is high dimensional. The solution algorithm I use is based on the finite difference method in Achdou et al. (2022) with several notable differences. First, there are multiple stopping choices including two types of default, the buying and selling of houses, refinancing and prepayment. Second, the model solution is

[11] The Feynman-Kac formula establishes a connection between partial differential equations and stochastic processes. See Hurtado et al. (2023) and Kaplan et al. (2018) for similar usages of the Feynman-Kac formula.

nonlinear in both the individual and aggregate state vectors. Importantly, I do not assume certainty equivalence but allow for aggregate uncertainty. I solve stochastic equilibrium following the approach in Krusell and Smith (1998). Technical Appendix A provides a detailed description of the computation.

3 Mapping Model to Data

I choose model parameters to match key cross-sectional features of the US economy in the early 2010s. A quantitative study of debt relief programs requires reproducing the distribution of assets and liabilities across households. Moreover, as changes in income drive changes in households' balance sheets and labor income shocks are the source of uninsurable risk, the calibration of the stochastic process for labor earnings must be consistent with the data.

A subset of parameters is assigned in advance of solving the model's stationary state. In addition, the earnings process is estimated outside of the model. Finally, 9 parameters are jointly calibrated in the steady state by targeting 12 moments. These are parameters specifying household preference (σ, κ, ρ), housing transaction cost (ξ_1), default costs (ξ_a, ξ_b), the grid of house sizes (h, $\underline{h}$), and the tax function (τ_0). Table 4 lists parameters and Table 3 reports targeted data moments and model moments. I associate targets with specific parameters, but this correspondence is only suggestive as the parameters are jointly determined. At the end of this section, I show untargeted moments to validate the model. My model captures the wealth distribution of households in data well in addition to hitting the targeted moments closely.

Earnings process I model the labor earnings process as a combination of two independent processes, $\varepsilon_{ij} = \varepsilon_i^p(1 + \varepsilon_j^t)$. In calibrating this earnings process, I target both cross-sectional inequality and individual risk. The former involves the earnings distribution described in Table 1, and the latter uses moments from the distribution of earnings growth listed in Table 2. Detailed information about the calibration of the earnings process and the labor productivity parameters in Table 4 can be found in Appendix C.1.

Housing The survey by Davis and Van Nieuwerburgh (2015) finds maintenance

Table 1: Earnings distribution

	Variance	Quintiles (%)					Top (%)		
		1q	2q	3q	4q	5q	90-95	95-99	99-100
Data	0.92	-0.1	4.2	11.7	20.8	63.5	11.7	16.6	18.7
Model	0.73	5.4	6.9	8.6	17.7	61.3	13.7	12.7	17.2

Data: Song et al. (2018), SCF (2007)

Table 2: Earnings dynamics

	Std.		Skewness		Kurtosis		$\mathbf{P}(\|\Delta y\|) < \mathbf{x}$			$\mathbf{P}(\|\Delta y\|) \in [\,\underline{x}, \overline{x}\,]$	
	1y	5y	1y	5y	1y	5y	x = 0.2	0.5	1.0	[0,0.25)	[0.25,1)
Data	0.51	0.78	-1.07	-1.25	14.93	9.51	0.67	0.83	0.93	0.31	0.16
Model	0.29	0.57	-0.58	-0.16	21.28	11.68	0.61	0.97	0.98	0.41	0.19

$|\Delta y|$: Absolute log earnings change. Data: Guvenen et al. (2015)

costs to be between 1% and 3% of the value of a house, so I set maintenance costs, ξ_h, to 2%. The parameter for the housing transactions cost function, ξ_0, is 0.07 to match the average transaction cost of 7% in Delcoure et al. (2002), and $\xi_1 = 0.01$ is jointly calibrated.[12] The fixed stock of houses, $\overline{H}$, is set to the value of housing wealth in SCF (2007). Normalizing this by average labor income, the corresponding ratio in the model implies $\overline{H}$ is 1.3.[13]

Mortgages The loan-to-value ratio, γ, is set to 1.05 based on the observations that mortgages are available with zero down payment and home equity lines of credit are available to households.[14] This is comparable to the two values, 0.95 and 1.1, in Kaplan et al. (2020). The amortization rate of mortgages, $\overline{\theta}$,

[12] Other estimates are similar to 7%. Smith et al. (1988) estimate the transaction costs of changing houses, and their estimate is approximately 8–10%. Martin (2003) shows that cost of buying a new home is 7–11% of the purchase price of a house.

[13] To convert the 2007 dollars to model consistent values, I use average labor income in SCF (2007) and the corresponding value in the model.

[14] An LTV limit above 1 is consistent with the following observations. First, 100% LTV loans are available. For example, the United States Department of Veterans Affairs and the United States Department of Agriculture guarantee purchase loans to 100%, and the Federal Housing Administration (FHA) insures purchase loans to 96.5%. Further, in the data, there are households with negative home equity. In the 1989–2013 waves of the SCF, the size of secured debt exceeded the value of non-financial assets among the poorest 20% of households.

is set to 0.025, which implies that the duration of a loan is approximately 40 years if a household fully finances the purchase of the house. The refinance cost, $\xi_{r_0} b' + \xi_{r_1}$, is a sum of 1% of the loan principal and a fixed cost. The latter is the equivalent of $1,500 (2007 dollars) in the model accounting for the sum of application, loan origination, attorney, insurance and inspection fees.[15]

Bankruptcy and foreclosure The utility costs ξ_a and ξ_b are calibrated to match the bankruptcy and foreclosure rates. The bankruptcy rate target is 1.06%, which is constructed using the number of Chapter 7 and Chapter 13 bankruptcy filings over the number of households (averaged over 2000–2017). The foreclosure rate target is 0.55% (average during the late 1990s). The intensities at which the bankruptcy and the foreclosure flags are removed are set to match the following. After filing for Chapter 7 bankruptcy, households cannot file again for 6 years. Households that file for Chapter 13 bankruptcy enter into repayment plans that last 3–5 years. Accordingly, I choose λ_d to 0.167 to match an average bankruptcy duration of 6 years. For foreclosure, Fair Issac reports that households' FICO scores can recover in as little as 2 years (see Mitman (2016)). Hence, λ_f is set to 0.5. The depreciation rate when foreclosing, δ_d, is 22% and is taken from Pennington-Cross (2006).

Preferences Households receive a utility flow from consuming non-durable goods and a service flow from their houses. Their utility function is

$$u(c,h) = \frac{c^{1-\sigma}-1}{1-\sigma} + \kappa log(h + \underline{h}).$$

The parameter $\underline{h} > 0$ allows for households without houses, and its value is chosen to match the share of homeowners. The curvature parameter $\sigma > 0$ and weight $\kappa > 0$, alongside the discount rate ρ, primarily affect households' asset and debt composition and the real interest rate. They are used to match the total debt to asset and the debt payment to income ratios across households. The annualized real interest rate is 6.5%.

Production The production technology is Cobb-Douglas and constant returns

[15] See `https://www.federalreserve.gov/pubs/refinancings/default.htm` for more information about refinancing costs. A fraction of a loan principal ranges from 0.0 to 1.5%, and I chose a value within that range.

Table 3: Targeted moments and model values

Moment	**Data**	**Model**	**Data Source**
Foreclosure rate	0.005	0.004	Mortgage Banker's Association
Bankruptcy rate	0.010	0.014	US Courts & Census
Tax revenue to output	0.16	0.16	CBO
30th pctl debt payment to income	0.00	0.00	SCF (2007)
50th pctl debt payment to income	0.12	0.05	SCF (2007)
70th pctl debt payment to income	0.22	0.27	SCF (2007)
30th pctl debt to asset	0.00	0.00	SCF (2007)
50th pctl debt to asset	0.16	0.24	SCF (2007)
70th pctl debt to asset	0.42	0.44	SCF (2007)
Net worth/Income	4.72	4.85	SCF (2007)
Share of homeowners	0.68	0.67	SCF (2007)
Households with mortgage	0.49	0.50	SCF (2007)

Note: Business assets and vehicles are excluded from non-financial assets. More details on the categorization of assets and debt can be found in Appendix C.2.

to scale. The labor share of output is taken from Giandrea and Sprague (2017).[16] I use the average value between 1989 and 2013, 60.5%; thus, α is 0.395. The depreciation rate δ is 0.069 (see Khan and Thomas (2013)).

Government The income tax function $T(y) = y - \tau_0 y^{1-\tau_1}$ is taken from Heathcote et al. (2017) where y is taxable income. Taxable income is labor income minus the tax deductible interest payments on mortgages and property taxes. Taxable income is $y = w(g,z)\varepsilon - r(g,z)\min(b,\bar{b}) - \min(\tau_h p(g,z)h, \overline{\tau_h})$. The property tax rate (τ_h) is set to 1%, which is the median tax rate across US states.[17] The IRS's rules imply that the maximum size of a mortgage ($\bar{b}$) for interest rate payment deduction is \$1,000,000 and the maximum value of tax deductible property tax ($\overline{\tau_h}$) is \$10,000.[18,19] The parameter τ_1,

[16] The labor share of output is the sum of employees' and proprietors' labor compensation. They calculate labor's share in the non-farm business sector from 1947 through 2016.

[17] See Kaplan et al. (2020), who reference data from the Tax Policy Center.

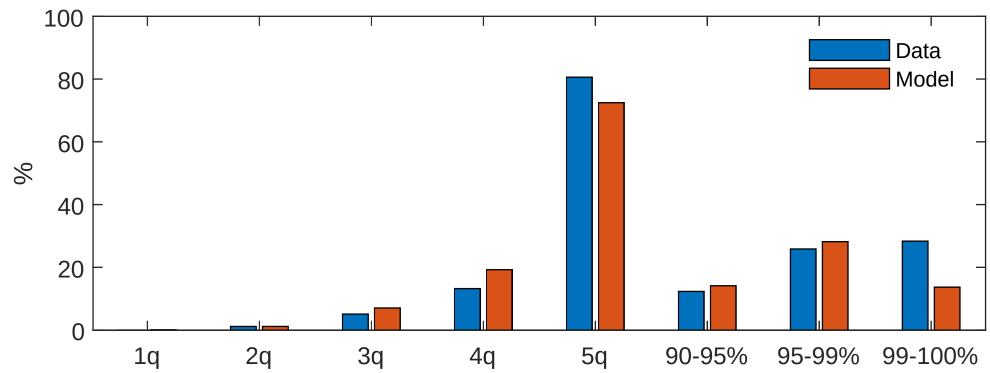
[18] When considering $\bar{b}$, in 2018 the deduction for home mortgage interest was limited to the first \$750,000 (\$375,000 if married filing separately) of debt. If a household was deducting mortgage interest incurred on or before December 15, 2017, the corresponding limits were \$1,000,000 (\$500,000 if married filing separately). Since I target 2007, I apply the limit before 2017.

[19] The parameter $\overline{\tau_h}$ matches the limit on deductions of state and local taxes. These include general sales taxes, real estate taxes and personal property taxes.

Table 4: Parameter values

Parameter	Value	Internal	Description
Preferences and production			
ρ	0.086	Y	Discount rate
σ	2.0	Y	Curvature of the utility function
κ	2.5	Y	Weight on durable good
α	0.395	N	Capital share
δ	0.069	N	Depreciation rate
Tax			
τ_0	0.585	Y	Tax rate
τ_1	0.181	N	Tax progressivity
τ_h	0.01	N	Property tax rate
$\overline{b}$	1,000,000	N	Max. debt to deduct interest payments
$\overline{\tau_h}$	10,000	N	Max. deduction on property tax
Labor productivity			
$\overline{\varepsilon}^p$	8.5	N	Upper bound of Pareto distribution
$\underline{\varepsilon}^p$	0.08	N	Lower bound of Pareto distribution
η^p_ε	1.526	N	Shape of Pareto distribution
$\eta_{\varepsilon^p_i}$	[1.9,1.5,1.3,0.6]	N	Shape of Pareto distribution
λ^p	0.048	N	Shock intensity
λ^t	1.260	N	Shock intensity
χ	0.239	N	Size of the ε^t shock
p^t	0.600	N	Probability of drawing negative ε^t
Assets and debts			
ξ_h	0.02	N	Depreciation rate of h
ξ_0	0.07	N	h transaction cost
ξ_1	0.01	Y	h transaction cost
$\overline{H}_s$	1.3	N	Supply of housing
h	[0.0,0.3,1.6 ,2.8,3.9,7.6]	Y	House sizes
$\underline{h}$	0.18	Y	Input to utility of non-homeowners
γ	1.05	N	Loan-to-value ratio
$\overline{\theta}$	0.025	N	Amortization rate of b
ξ_{r_0}	0.01	N	Refinancing cost
ξ_{r_1}	0.01	N	Refinancing cost
ξ_a	1.2	Y	Utility cost
ξ_b	0.5	Y	Utility cost
λ_d	0.16	N	Removal of bankruptcy flag
λ_f	0.5	N	Removal of foreclosure flag
δ_d	0.22	N	Depreciation due to foreclosure
Aggregate shock			
$[z_1, z_2]$	[0.96, 1.02]	Y	Level of total productivity
$[\lambda_1, \lambda_2]$	[0.364, 0.149]	N	Shock intensity
$[\iota(z_1), \iota(z_2)]$	[0.011, 0.0]	Y	loan rate premium

Figure 1: Net worth distribution



Note: Each bar shows a net worth share held by quintiles or the top 10% of households. Business assets and student loans are excluded. Data: SCF (2007)

determining the degree of progressivity of the tax system, is 0.181 as in Heathcote et al. (2017).[20] Next, τ_0 is set to 0.585 to match the tax revenue-output ratio 16.7%.[21]

Aggregate shocks Aggregate productivity z follows a two-state Poisson process, $z \in [z_1, z_2]$. The process jumps from state 1 to state 2 with intensity λ_1 and in the reverse direction with intensity λ_2. These two states represent a recession (z_1) and an expansion (z_2). The support of aggregate TFP, $[z1, z2]$, is $[0.96, 1.02]$ to match the standard deviation of US output given the shock intensities, $\lambda_2 = 0.1498$ and $\lambda_1 = 0.3636$. This implies the average duration of an expansion is 26.7 quarters and the average duration of a recession is 11 quarters. I choose these durations following Nakajima and Ríos-Rull (2019). The loan rate premium in recessions is set to 1.1% to replicate the procyclicality of credits. Table 6 reports the cyclical properties of the US economy from the data and the model.

Validation I compute non-targeted moments to confirm the model's validity. First, the model captures the household net worth distribution very well, as

[20] They estimate this parameter using the Panel Study of Income Dynamics (PSID) for survey years 2000, 2002, 2004, and 2006, in combination with the NBER's TAXSIM program.

[21] I use the value of tax revenue-output ratio (Congressional Budget Office) between 2000 and 2014.

Table 5: Share of assets and debt

		Asset		Debt	
		Non-financial	Financial	Secured	Unsecured
Data	Q1	0.59	0.16	2.22	14.71
	Q2	4.42	1.13	11.40	18.92
	Q3	11.14	3.47	20.52	21.83
	Q4	19.29	9.72	23.94	23.41
	Q5	64.58	85.53	41.97	21.13
Model	Q1	0.16	0.13	0.38	0.08
	Q2	1.35	1.78	3.15	0.13
	Q3	8.64	7.08	10.81	79.12
	Q4	27.02	15.32	27.35	20.34
	Q5	62.83	75.70	58.31	0.32

Note: Share of assets and debt by net worth quintiles. Data: SCF (2007)

Table 6: Cyclical properties

	Data		**Model**	
	std (%)	corr. with output	std (%)	corr. with output
Output	1.8	1.0	1.8	1.0
Consumption	1.3	0.9	0.6	0.8
Investment	6.9	0.8	4.3	0.8
Unsecured debt	5.8	0.4	3.0	0.0
Mortgage	4.4	0.2	3.9	0.7
Bankruptcy	11.1	0.1	7.7	0.1
Foreclosure	-	-	1.3	-0.7
House price	4.3	0.2	2.6	0.7

Note: Logs of the data are filtered using the HP filter with a smoothing parameter of 100. Output, consumption and investment data combine information from NIPA tables 1.1.5 and 2.3.5. from 1963 to 2007. Output: real GDP minus net export; consumption: real private consumption expenditures minus housing service; investment: real gross domestic investment; unsecured debt: consumer credit (flow of funds) deflated by GDP deflator; secured debt: home mortgages (flow of funds) deflated by GDP deflator; house price: Freddie Mac US house price index divided by durable good GDP deflator (NIPA table 1.1.4, line 4). Unlike other data, house price statistics are computed using 1975–2007 data. The standard deviation and the correlation with output of bankruptcy are taken from Nakajima and Ríos-Rull (2019). Number of bankruptcies (from US courts) is normalized by the number of households (from Census) using 1995–2004 data. The model series are simulated quarterly and aggregated to the annual frequency to compute the statistics.

shown in Figure 1. Moreover, my model allows me to further break down households' net worth. Table 5 shows the distribution of assets and debt. The share held by each net worth quintile of households of non-financial assets, financial assets and secured debt from the model is reasonably close to the data.[22] However, unsecured debt is mostly held by the middle class in the model while it is evenly distributed over net worth quintiles in the data. Despite the model's rich asset structure, households in the model cannot have unsecured debt and liquid savings at the same time. As a result, given that the data indicates that 9.6% of households have net negative financial assets, it is hard to match the distribution of financial assets and unsecured debt at the same time.

Second, the model generates rich heterogeneity in the MPC from change in liquid assets. This is consistent with evidence in Misra and Surico (2014) and Parker et al. (2013). For example, households with a share of liquid assets over a net worth less than 5% have an MPC of 18.2% while those with a share of liquid assets over a net worth higher than 50% have an MPC of 9.2%.

Third, the model captures the cyclical properties of the economy. Table 6 compares the aggregate statistics from the US data and my model. It shows that the model captures key properties of the data. In particular, both consumption and investment are strongly correlated with output, and the investment is more volatile than consumption. Furthermore, secured credit is positively correlated with output.

House prices are procyclical in the model as in the data, and the volatility of house prices is also consistent with the data. It is useful to mention that the model is not calibrated to generate an episode like the Great Recession nor to capture house price volatility. Therefore, not every recession causes large house price falls. House prices decrease significantly when a recession is exceptionally long or when many households are financially distressed at the beginning of a recession.

[22] Appendix C.2 provides detailed information about categorization of assets and debts.

4 Debt relief programs in recessions

Above, I have established that my model is consistent with salient empirical regularities characterizing the distribution of households (Section 3). Furthermore, my model generates business cycles that resemble the data along important margins. These make the model useful for the analysis of debt relief programs during recessions. In this section, I analyze such programs using a series of policy experiments.

For the policy experiments below, I chose a recession where, in the absence of intervention, house prices fall more than 8% from their peak.[23] Using this no-policy economy as the benchmark, I compare it to an alternative where the government intervenes when it observes house prices have fallen by 5%. In this intervention, all households with loan-to-value (LTV) ratios above 95% are eligible for a mortgage principal reduction. These households receive partial mortgage debt forgiveness, and their LTV ratios fall to 95%. I assume that this intervention is unanticipated.[24] While the policy intervention is unanticipated, it is important to note that household decisions are affected by aggregate uncertainty.

The mortgage debt relief studied here, larger than policies that were implemented during the Great Recession, is chosen to allow comparability with the principal reduction examined by Kaplan et al. (2020). (Appendix D.6 discusses mortgage relief policies in the Great Recession.) This program affects 22.6% of the mortgagors. The average size of the mortgage principal reduction for eligible households corresponds to roughly about $78,000 (2007 dollars), and

[23]As the total factor productivity shock is calibrated to capture the volatility of the output, most model recessions do not involve such large house price drops. The model is designed to study the effects of debt-related policies, not to explain the origins of housing crises. While in the data, house price declines may lead to a recession, falls in income cause house price declines in this model. The ordering of a fall in house prices and income is unlikely to be important to the results that follow, since interventions happen when both are relatively low. Furthermore, a house price drop before a recession can be generated by a shock to preferences or credit conditions.

[24]Before and after the shock, the simulation is based on forecasting functions estimated in an environment without the policy intervention. Thus, these policies are unanticipated. This is intended to capture the unusual nature of the Great Recession, whose severity was unexpected by most policy makers and market participants.

the total cost is 5.2% of GDP.

The government covers the cost of the policy by issuing debt, which is repaid by increasing taxes over subsequent expansions. Therefore, when the government intervenes, households expect future taxes to rise. Once the debt is repaid, the government returns to the pre-intervention tax regime. For tractability, I assume that households do not know exactly when the cost will be paid off but expect the procyclical tax regime will end with a 20% probability at any time. A full characterization of the procyclical tax rate economy can be found in Appendix D.1. Whether the policy is funded or not and how it is funded are critical in shaping the effects of the policy intervention. I show this by comparing the results of an unfunded program with the funded one.

I also compare the effects of the targeted mortgage debt relief program with those of a tax rebate, a cash transfer of equal amount to all households. While the tax rebate is not a direct debt relief policy, it is a common stimulus policy. There are important differences between these two policies. First, mortgage forgiveness targets highly indebted homeowners while tax rebates are not a targeted policy. Second, mortgage forgiveness provides illiquid assets but the tax rebate provides liquid assets.[25] For comparability, I set the size of the tax rebate to match the cost of the mortgage reduction. This implies each household receives a lump sum transfer equivalent to $5,300 (2007 dollars).[26] In addition, to see the role of liquidity of transfers, I discuss the results of a policy where the eligibility and size of transfers are the same as in the mortgage reduction but involve liquid assets (targeted tax rebate).

In Section 4.1, I show the effects of each policy on aggregate variables and then analyze consumption responses. Next, to highlight the importance of

[25]There is an indirect liquidity effect of the principal reduction program. Since households who have mortgages pay interest on their loans, a reduction in the outstanding loan increases liquidity by reducing interest rate payments.

[26]I show results from a recession in this section, but I can observe multiple episodes of large house-price drops over the simulation. The efficacy of a policy intervention can be state-dependent. As each episode follows a different history, the distributions of households are different when the government intervenes. Table 7 in Appendix E shows the range of variables summarizing these distributions, and Figures 9 and 10 in Appendix E show the aggregate responses to the principal reduction and tax rebate policies across these different economies.

allowing for bankruptcy, I study the principal reduction and the tax rebate in an economy with only a foreclosure option, in Section 4.2. Finally, I examine alternative debt relief programs in Section 4.3.

4.1 Mortgage forgiveness versus tax rebate

4.1.1 Aggregate responses

Figure 2 shows an economy initially in a recession transitioning to an expansion in the 16th quarter. Throughout all the exercises, the figures show variables as *percent deviations from the corresponding value in the economy without policy intervention.*[27] I begin by examining the response in the model economy to an unfunded mortgage principal reduction, as studied by Kaplan et al. (2020). As will be seen below, when the government borrows to fund a debt relief program, there is a large change in the timing of its impact. Studying the unfunded case allows me to highlight this effect.

The blue dashed lines in Figure 2 show that a one-time mortgage reduction has an immediate, strong and persistent stimulus effect on the economy. As the eligibility for the policy depends solely on a household's LTV, the recipients of the mortgage reduction vary in their net worth. Some are households with relatively large savings. However, more have low savings. These highly indebted households have high MPCs.

Among the recipients, households with a high MPC increase their consumption, and aggregate consumption rises by 0.7%. The policy intervention increases illiquid assets – home equity. As interest payments fall, income available for consumption rises. However, most program recipients' consumption responses occur when they refinance, withdraw home equity and increase liquid assets. As refinancing happens gradually, this mutes the initial response of aggregate consumption and adds persistence.

Financial assets in the model with an unfunded policy intervention are allocated to financing mortgages and investing in capital. The reduction in

[27] Aggregate variables in levels for the baseline economy can be found in Appendix E, Figure 8.

aggregate mortgage increases investment and physical capital rises. This leads to a protracted increase in GDP and real wages.[28] Higher wages and lower LTV ratios reduce foreclosures. The fall in foreclosures reduces the supply of available houses, and the rise in wealth increases the demand for houses. In equilibrium, house prices rise.

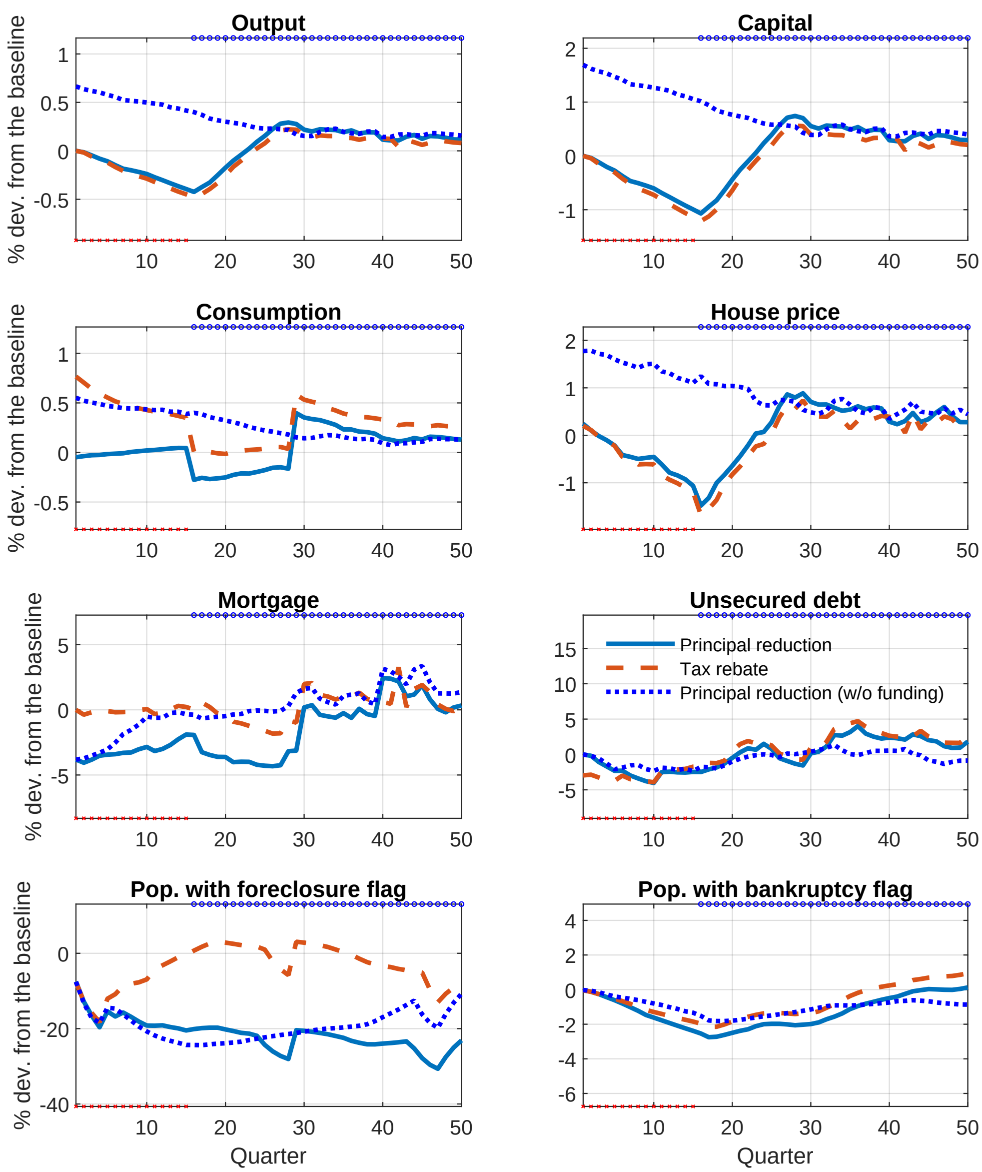
I now study the same debt relief program but assume that it must be fully funded. As above, the policy intervention is at $t = 0$ when the economy is in a recession. Now, however, it is financed by issuing government debt. The proceeds are immediately spent on mortgage principal reduction. All households know that at the onset of a recovery taxes will rise.

The private sector balance sheet sees no net change following the intervention. As the fall in mortgage debt is offset by a rise in government debt, total resources available to the economy do not change. In effect, the mortgage principal reduction tends to be a redistribution from low MPC households to heavily indebted households with higher MPCs. This raises consumption of those who receive the principal reduction. These changes are partly offset by their expectations of increases in future taxes. The fall in future disposable income implies a rise in current savings as households attempt to smooth their consumption. This dampens the rise in their consumption. Additionally, households that do not receive the principal reduction also reduce their consumption, so the overall consumption during the recession remains almost unchanged. Following the intervention, there is a gradual decrease in capital and output because the expenses incurred by the government debt used to finance the policy dominate the increase in household savings resulting from the anticipation of higher future taxes.

At the onset of a recovery, the government increases taxes and uses the additional revenue to begin repaying its debt. This debt is held by wealthier households with lower MPCs and high savings rates. As these households adjust their portfolios away from government bonds and into capital, aggregate

[28]While the model does not have a labor supply choice, the increase in wages and the positive wealth effect experienced by recipients of the mortgage reduction lead to higher consumption.

Figure 2: Response of aggregate variables



Note: Aggregate series are shown as percent deviations from the corresponding series in the economy without policy intervention. The dashed blue lines indicate responses to unfunded principal reduction, the solid blue lines indicate the funded principal reduction, and the dashed red lines indicate the funded tax rebate.

capital rises faster than in the no-policy benchmark.[29] This drives a more rapid recovery in GDP, and consumption and capital remain higher over extended periods than seen in the absence of policy.

Summarizing the effects on aggregate quantities and house prices, the anticipated rise in future taxes that accompanies a funded debt relief shifts the timing of responses to the stimulus. There is less effect on consumption and output during the recession, but growth in these series is amplified over the recovery. In this sense, the debt relief policy strengthens a recovery without changing its timing. This is a result of assuming that the government will begin to repay its debt, redistributing resources to low MPC households that invest in physical capital, only once a recovery begins.

While the stimulus effects of debt relief are delayed until the recovery, the impact of funding the debt relief program has little effect on debt. Changes in aggregate credit following the interventions are the result of decisions by highly indebted households with little liquid assets. The responses of these households to future tax changes is minimal, hence credit variables are largely unchanged by funding.

By reducing the number of financially distressed households, the interventions significantly reduce foreclosures.[30] Forgiving a fraction of eligible households' mortgages, this program reduces mortgages relative to the benchmark. Thereafter, mortgages rise over roughly 7 years in the unfunded cases while remaining below their level in the benchmark economy. Program recipients see a rise in their home equity and gradually refinance to increase consumption spending.

Lastly, unsecured credit and bankruptcy fall following the mortgage principal reduction. While many households who receive a principal reduction tend to have low savings and are likely to use unsecured credit, reducing their mortgage reduces their debt payment and lowers their need for unsecured debt.

Comparison to Kaplan et al. (2020) As discussed in Section 1, Kaplan

[29] Appendix D.5 discusses the accuracy of forecast rules over the debt policy.

[30] This result is consistent with the negative relationship between the amount of negative equity and mortgage default rates as in Haughwout et al. (2009) and Gerardi et al. (2017).

et al. (2020) perform a similar experiment. They also consider a policy that forgives a fraction of mortgages, leaving no households with an LTV ratio higher than 95%. The timing and the scale of the policy are also similar to mine. They assume the policy is implemented in a timely manner (two years into the bust), and it affects over a quarter of homeowners with mortgages.

Their results are very different from mine. They find a mortgage forgiveness program would not have prevented a sharp drop in house prices and aggregate expenditures but would have significantly dampened the rise in foreclosures. While my household environment shares several features in common with theirs, there are several important differences that lead to different policy outcomes. These include i) the presence of capital, ii) equilibrium wages and interest rates, iii) differences in the preferences for housing, and iv) the availability of bankruptcy. As Kaplan et al. (2020) examine an unfunded program, I focus on comparing their results to the effects of the unfunded principal reduction shown in Figure 2 (dashed blue lines). Both models predict that the principal reduction leads to a large decrease in foreclosures. In my model, there is also a substantial rise in house prices. Turning to real quantities, in contrast to their results, we see a large and persistent increase in aggregate consumption.

In the Kaplan et al. (2020) model, households' taste for housing varies over time, and changes in taste account for most of their house price dynamics. Leverage has a limited role in affecting house prices. In addition, only house prices are determined in equilibrium. In contrast, in my model, interest rates and wages are also determined in equilibrium. Figure 2 shows that with the intervention, capital is persistently higher than the benchmark economy. The resulting higher wage benefits all households, and the associated fall in interest rates helps net borrowers at the expense of net savers. Overall, the income and substitution effect arising from changes in aggregate capital help to support consumption and house prices. Higher house prices reduce subsequent foreclosures and loosen financial constraints, preventing further decline in house prices. In the absence of interest rate and wage responses, Kaplan et al.'s (2020) model delivers small responses in aggregate variables.

Comparison to a tax rebate We now compare the mortgage principal reduction program to a tax rebate. This transfers funds equally across households, directly adding to their liquid wealth. While the total cost of the tax rebate is the same as the principal reduction, the *equal* distribution of *liquid* wealth leads to different aggregate dynamics. The tax rebate is more effective than the principal reduction at boosting consumption. This is because liquid assets can be used for consumption without paying adjustment costs. Also, some households that receive the rebate and would not have been eligible for the principal reduction (for example, households without houses and mortgages) have very high MPCs. In Figure 2, as consumption increases more following a tax rebate, capital falls slightly faster when compared to a principal reduction during the recession. This leads to output and house prices following similar paths. The tax rebate is less effective in reducing foreclosure and supporting house prices because it does not target households that are likely to sell their houses or foreclose.

To isolate how the liquidity of transfers affects aggregate responses, I keep the eligibility and size of the subsidy of the principal reduction but give unchanged households liquid assets instead of a mortgage reduction (targeted tax rebate). Figure 11 in Appendix E shows that the transfer of liquid assets is more effective in raising output and consumption and supporting house prices, as households can use the subsidy without paying adjustment costs.

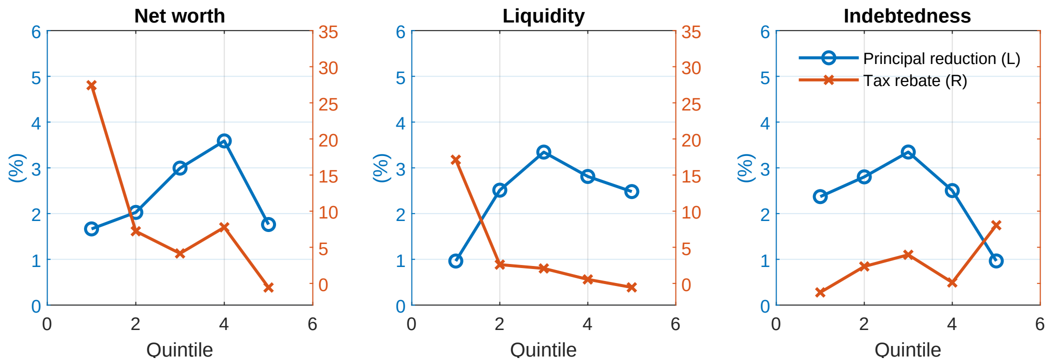
4.1.2 A closer look at non-durable consumption responses

Having explored aggregate responses in consumption, I now explore differences in households' consumption responses. Two main results arise. First, my model reproduces results from empirical work that uses tax rebates as natural experiments. The MPC is larger for households with high levels of debt and low shares of liquid assets. Second, this clear relation disappears when we consider mortgage principal reduction.

Figure 3 shows the mean of MPCs by quintiles. For example, each point in the upper left panel is the average MPC, to a policy intervention, of the quintiles of the net worth distribution.[31] In these figures, liquidity is the share

[31] After the mortgage reduction, the MPC is computed using a sample of eligible households.

Figure 3: MPC distribution



Note: I simulate a sample of 18,000 households. MPCs are defined as $\frac{\text{c(policy) - c(no policy)}}{\text{transfer size}}$. Liquidity is a ratio of liquid assets over net worth. Indebtedness is total debt over net worth. In the panel showing liquidity, households without houses are dropped. For the indebtedness panel, households without debt are dropped.

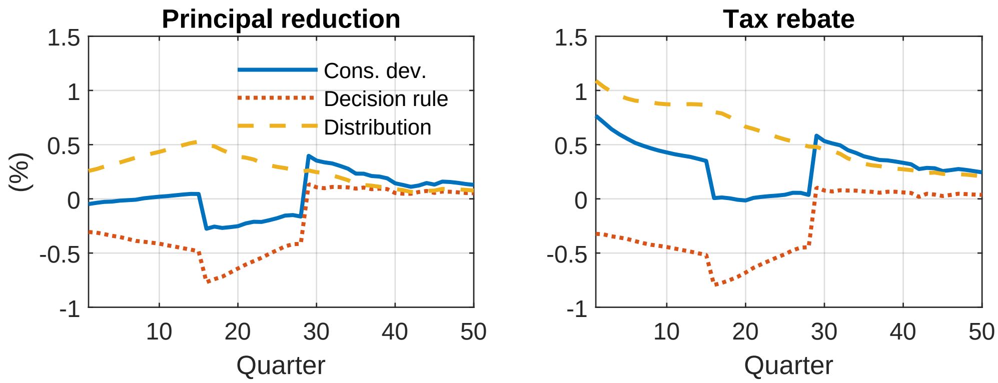
of the liquid assets over net worth and indebtedness is total debt over net worth.

Existing studies measuring the MPC from cash handouts find that liquidity and indebtedness are important determinants of differences in MPCs. (For example, see Misra and Surico (2014) and Fagereng et al. (2021).) Figure 3 shows that the distributions of MPCs to a tax rebate are consistent with these findings. As in Misra and Surico (2014), the MPC is larger for households with high levels of debt and low shares of liquid assets.

In contrast, there is no monotone pattern between MPCs and liquidity or indebtedness after the principal reduction. As debt relief provides illiquid assets, funds are not immediately available for households to spend. In general, a large response in non-durable consumption occurs after a discrete choice such as refinancing. This changes the distribution of consumption responses compared to the tax rebate. For example, in the middle panel of Figure 3, the MPC is highest for those in the middle of the distribution of liquidity. The principal reduction makes downsizing feasible for previously underwater households. As some of these households downsize, they achieve large increases

However, consumption of ineligible households may also differ from the benchmark because of differences in prices and tax rates.

Figure 4: Consumption response decomposition



Note: Counterfactual consumption is computed as follows. "Decision rule" is computed using the consumption decision rule of the policy economy and the distribution of the benchmark economy. "Distribution" is computed using the consumption decision rule of the benchmark economy and the distribution of the policy economy.

in consumption.

Long-term consumption responses While we see the initial responses in households' consumption in Figure 3, Figure 2 shows that the policy has a long-lasting effect on consumption. The response in consumption is, in part, the result of changes in the distribution of wealth across households. Also, it is the result of changes in households' decisions, at each wealth level, responding to changes in the tax regime and general equilibrium price movements. House prices and loan rates respond immediately to the policy intervention, while, given the initially small effect on aggregate capital, wages and interest rates see little change at first. Fifteen quarters after the intervention, as the economy enters a recovery, the government increases tax rates to reduce public debt. As public debt falls, private investment rises and aggregate capital starts to rise compared to its benchmark. As a result, the risk-free real interest rates fall and the wages rise. Higher wages benefit all households while lower interest rates benefit net borrowers. The intervention also boosts house prices in the recovery, benefiting sellers at the expense of buyers. For homeowners that do not buy or sell, higher house prices lower disposable income as they increase property taxes and maintenance costs.

I decompose the consumption response into "decision rule" and "distribution" in Figure 4 to see how general equilibrium and changes in wealth determine consumption responses. Let $c_t^p(\omega)$ and $g_t^p(\omega)$ be consumption decision rules and the distribution of households with the policy interventions, and $c_t^b(\omega)$ and $g_t^b(\omega)$ be those in the benchmark economy. The decomposition is as follows:

$$\begin{aligned}\Delta C_t &= \frac{\int c_t^p(\omega) g_t^p(\omega)\, d\omega - \int c_t^b(\omega) g_t^b(\omega)\, d\omega}{\int c_t^b(\omega) g_t^b(\omega)\, d\omega} \\ &= \underbrace{\frac{\int (c_t^p(\omega) - c_t^b(\omega)) g_t^p(\omega) d\omega}{\int c_t^b(\omega) g_t^b(\omega)\, d\omega}}_{\text{Decision rule}} + \underbrace{\frac{\int (g_t^p(\omega) - g_t^b(\omega)) c_t^b(\omega) d\omega}{\int c_t^b(\omega) g_t^b(\omega)\, d\omega}}_{\text{Distribution}}.\end{aligned}$$

ΔC is the consumption deviation from the benchmark, and the first term above, labelled "decision rule", captures general equilibrium effects. The second term, "distribution", shows the effect of changes in the distribution of households, initially the result of changes in wealth brought about by the policy intervention. In both the case of the principal reduction and the tax rebate, the distributional effect drives all of the rise in consumption until the debt from a policy intervention in paid off at $t = 28$.

At the policy intervention, future taxes are expected to rise. This happens at $t = 16$ as the economy enters an expansion. The "decision rule" effect on aggregate consumption is dominated by a strong negative wealth effect of higher taxes, which dampens the "distribution" effect. This becomes stronger when taxes actually rise and disappears when they return to normal. At that point increases in wages leads to "decision rule" boosting consumption.

The distribution effect is stronger with the tax rebate where it drives a larger consumption response and lower capital. In Figure 2, capital rises by slightly more after the principal reduction, which implies larger general equilibrium effects on consumption. Once the policy cost is paid off, the general equilibrium effects explain about 30% and 10% of consumption rise, the principal reduction and the tax rebate, respectively. Appendix D.7 provides additional results for the two policies, examining their fiscal multipliers and welfare.

4.2 The role of bankruptcy

Including an option to declare bankruptcy – default on unsecured debt – to a model with housing and mortgage is rare. However, to study the effects of extraordinary debt relief programs during crises, it is important to include bankruptcy. Bankruptcy, along with foreclosure, exists as partial consumption insurance for households that are unable to pay their debt. As such, additional debt relief may be unnecessary in ordinary times.[32] Thus, the policy effects may be overstated without the bankruptcy option in the short run. At the same time, a lack of bankruptcy may make households save more to better insure themselves, altering the distribution of households, which is a key determinant of the effects of policies. To understand cumulative policy effects over the long run, capital accumulation from household savings is an important element.

Here, I investigate the role of bankruptcy in determining the effects of debt relief and tax rebate policies by studying the response of an economy with foreclosure but without bankruptcy. To make it comparable, I set the cost of the policy at 5.2% of GDP, which is the cost of the policy in the benchmark.[33] An exogenous borrowing constraint on a is set at 0 when there is no bankruptcy option to prevent the feasible consumption set becoming empty.[34]

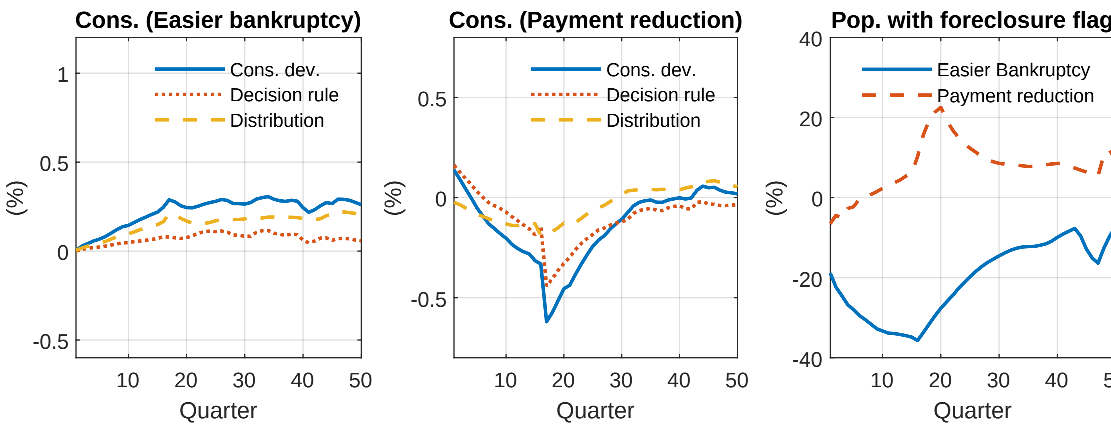
Both principal reduction and tax rebates show qualitatively similar responses to the benchmark responses: increasing consumption, output, and house prices (Figure 12 in Appendix E). Quantitatively, both policies are more effective

[32]Bankruptcy is one of the largest social insurances in the US affecting households' motives for saving. Auclert et al. (2019) show that the amount of unsecured credit discharged in Chapter 7 bankruptcies is as large as the total payments of the unemployment insurance system. Mitman (2016) shows that the presence of bankruptcy and foreclosure affect each other.

[33]To match the cost of the intervention, the policy design is different from the benchmark. Households need to have an LTV higher than 99.4% to be eligible and and their LTV ratios fall to 99.4% after receive partial mortgage forgiveness. The program affects 39% of mortgagors, and the average size of the mortgage reduction is about $32,200 (2007 dollars).

[34]It is possible to allow unsecured credit without providing a bankruptcy option. However, to do so, the exogenous borrowing limit has to be tight to ensure positive consumption for all households over the business cycle.

Figure 5: Consumption response decomposition



Note: "Decision rule" is computed using the consumption decision rule of the policy economy and the distribution of the benchmark economy. "Distribution" is computed using the consumption decision rule of the benchmark economy and the distribution of the policy economy.

when there is no bankruptcy option. The output multipliers of a principal reduction and a tax rebate are 2% point and 16% point higher than the benchmark, respectively. Furthermore, house prices are better supported than in the benchmark.[35] As mentioned above, the lack of bankruptcy and resulting changes in the distribution of households which affects the effectiveness of the policy. In the short run, the consumption responses of the two policies are similar to the benchmark. Afterward, as these policies involve redistribution from households with low MPCs to households with high MPCs, savings and thus capital falls. In the absence of bankruptcy, households tend to hold more liquid savings and the dispersion in MPCs seen in previous sections is reduced.[36] Therefore, in this environment, capital falls less during the recession, leading to a larger cumulative rise in output over the long run compared to the benchmark.

[35]The output multiplier is a cumulative change in output relative to the intervention cost. Appendix D.7 defines the multiplier.

[36]On average, the share of households with zero or less liquid assets is 11% in the model with foreclosure only, while it is 21% in the benchmark model.

4.3 Other policies

In addition to mortgage forgiveness, I consider a mortgage payment reduction and an easier bankruptcy. Neither policy requires government funding, and the tax regime does not change upon implementation.

Several papers compare policies that provide wealth (principal reduction) against those providing liquidity (payment reduction). Ganong and Noel (2018) find that debt relief is less effective at reducing default than payment reductions.[37] In contrast, examining cramdowns that discharged the underwater portion of mortgages during Chapter 13 bankruptcy, Cespedes et al. (2021) find that foreclosure rates fell. Due to the differences in implementation, these findings are not comparable to the results in my work.[38] However, they still suggest a potentially important role for payment reductions, and I explore that below. For this policy experiment, I assume that per period repayments of principal are reduced by half for 16 quarters. This policy effectively extends the duration of a loan by allowing slower amortization of debt.

Alternatively, we can consider alleviating unsecured debt to reduce foreclosure and mitigate recessions. Bankruptcy and foreclosure can be substitutes as distressed homeowners would have more money to pay their mortgages if they spent less on payments to unsecured lenders. If the cost of foreclosure is higher than the cost of bankruptcy, making it easier for borrowers to file for bankruptcy during mortgage crises can be a valuable policy.[39] For this experiment, I set the utility cost of bankruptcy (ξ_a) to zero for 16 quarters.

As the payment reduction increases households' disposable income, aggregate consumption rises for 3 quarters. Over the same period, a slowdown in

[37] Indarte (2019), who studies bankruptcy, also finds payment reduction is more effective at reducing default.

[38] Ganong and Noel (2018) use HAMP as a natural experiment. In contrast to the cramdowns in Cespedes et al. (2021) and the mortgage reduction policy I study, HAMP reduced mortgages to 115% of house value, still leaving households underwater. Moreover, the share of households that modified their mortgages via HAMP was far lower than the 15% who receive mortgage reductions in my experiment.

[39] White (2009) argues that the social cost of foreclosure is very high, and expanding the save-your-home feature of bankruptcy by allowing bankruptcy judges to modify residential mortgages can be a useful policy for solving a mortgage crisis.

mortgage payments lowers savings available for investment, thus capital falls. At the same time, household leverage rises, compared to the economy without the intervention. Larger interest payments due to higher mortgage balances and lower wages coming from lower capital depress consumption even as mortgagors continue to pay a lower fraction of their balance. The middle panel in Figure 5 shows the consumption decomposition introduced in Section 4.1.2. In contrast, making bankruptcy less costly does not increase consumption immediately, but as more households use bankruptcy and discard their unsecured debt, both general equilibrium effects (in "decision rule") and lower household leverage (in "distribution") contribute to a persistent rise in aggregate consumption.

The impact of the two policies on foreclosure are also different. Initially, reducing payments decreases foreclosure rates, but as households become more indebted and wages decline over time, the rates escalate rapidly. On the other hand, making bankruptcy more accessible results in lower foreclosure rates that persist for a longer time, but at the expense of a higher number of bankruptcy filings compared to payment reduction. Figure 13 in Appendix E shows responses of other aggregate variables.

5 Concluding remarks

I have quantitatively assessed the effects of debt relief programs during recessions. While a growing empirical literature studies effects of such policies on foreclosure and household consumption, there is little equilibrium analysis of the dynamic response in the aggregate economy to large-scale debt relief policies.

To understand the effects of debt relief programs better, I build a model with financial assets, unsecured debt, housing, and mortgages as well as the option to default on both forms of borrowing. The model successfully replicates the household distribution for overall net worth, as well as liquid assets, mortgages, and housing. My model captures key business cycle moments while reproducing households' default behavior.

I show that a large mortgage principal reduction has persistent effects on aggregate consumption and output as well as bankruptcy, foreclosure, and house prices. By comparing debt relief to a tax rebate, an untargeted, liquid income transfer, I find that the liquidity of transfers and the distribution of recipients' MPCs play a large role in determining the response in macroeconomic variables. Including the bankruptcy option matters in assessing large-scale debt relief policies as it provides partial consumption insurance to households, which may make additional debt relief unnecessary. The availability of bankruptcy also affects households' borrowings and savings decisions, altering the distribution of assets across households. This distribution is a key determinant of the effects of policies, and I found policies appear to be more effective when a bankruptcy option does not exist.

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Appendix For Online Publication

This Appendix is organized as follows. Section A provides additional details for Section 2 in the paper. Section B reformulates the household problem towards computation. Section C describes the estimation of the earnings process and categorization of assets and debt used in computing calibration targets. Section D describes the procyclical tax-rate economy that is used in experiments with funded policy interventions. It also provides additional discussion and results not included in Section 4. Lastly, Section E contains additional figures and tables.

A Firms, government and equilibrium

There are identical, competitive firms that produce non-durable consumption goods using a constant return to scale technology.[40] Firms rent capital from banks and employ labor to solve the following problem:

$$\max_{k,\ell} z f(k,\ell) - (r(\Omega) + \delta)k - w(\Omega)\ell. \tag{8}$$

The production function $f(k,\ell) = k^{\alpha}\ell^{1-\alpha}$ and the capital depreciation rate is δ. Let L and K represent the aggregate quantities of labor and capital. In equilibrium, firms' profit maximization implies that the equilibrium real interest rate satisfies $r(\Omega) = z\alpha\frac{L}{K}^{1-\alpha} - \delta$ while the wage rate is $w(\Omega) = z(1-\alpha)\frac{L}{K}^{-\alpha}$.

The government collects taxes from households, levied on the labor income net of deductions. Households can deduct the interest paid on their mortgage and property taxes. As already noted, the government also absorbs realized profits and losses from unsecured and secured lending by banks, arising from aggregate shocks, through taxes and subsidies. The remaining revenue is spent on government consumption of non-durable goods, which is not valued by households.

The government's budget constraint is

[40] I assume the stock of durable goods is given.

$$\int T(b, \varepsilon, p(\Omega)h)g(\omega)d\omega - G = 0, \tag{9}$$

where G is government consumption. In the definition of equilibrium in the following section, there is no government debt. However, when there is a policy intervention and the government issues bonds to finance debt relief or a tax rebate, the definition of equilibrium must be generalized. See Appendix D.1.

An equilibrium is a set of functions that satisfies the following:

1. Households solve their lifetime optimization problems. Given price functions $\{r_a, r, q, w, p\}$, v solves (1)–(3).
2. Firms maximize profits by solving (8).
3. The unsecured debt price function r_a is determined by (4) and (5).
4. The mortgage price function q is determined by (6)–(7).
5. Capital market clears: $\int (a - b)g(\omega)d\omega = K$.
6. Labor market clears: $\int \varepsilon g(\omega)d\omega = L$.
7. Housing market clears: $\int hg(\omega)d\omega = \overline{H}$.
8. The government budget constraint (9) holds.
9. The Kolmogorov forward operator, $\mathcal{K}$, that describes the change of density function g is generated by agents' optimal choices.

B Household problem reformulated

Household problems in Section 2 can be written as a HJB variational inequality (HJBVI). Households decide whether to continue or to stop and choose any of the stopping options listed above in Section 2. The value of the latter, which is the second choice below, is given by $v^*(\omega, \Omega)$ in (3), while the value of continuing, the first choice, is the HJB equation in (2):

$$
\begin{aligned}
&\min\Bigg\{\rho v(\omega,\Omega)-\max_c u(c,h)-\partial_a v(\omega,\Omega)\dot{a}-\partial_b v(\omega,\Omega)\dot{b}-\sum_{j=1}^{n_\varepsilon}\lambda_{\varepsilon\varepsilon_j}v(\omega^{\varepsilon_j},\Omega)\\
&-\lambda_d(v(\omega,\Omega|o=0)-v(\omega,\Omega|o=1))_{o=1}-\lambda_f(v(\omega,\Omega|o=0)-v(\omega,\Omega|o=2))_{o=2}\\
&-\sum_{k=1}^{n_z}\lambda_{zz_k}v(\omega,\Omega^{z_k})-\int\frac{\delta v(\omega,\Omega)}{\delta g(\omega)}\mathcal{K}g(\omega)d\omega,\;\; v(\omega,\Omega)-v^*(\omega,\Omega)\Bigg\}=0.
\end{aligned}
\tag{10}
$$

C Additional information on calibration

This section is organized as follows. Section C.1 explains the calibration of the earnings process. Next, Section C.2 describes the categorization of assets and debt used in computing calibration targets.

C.1 Earnings process

I model the labor earnings process as a combination of two independent processes:

$$\varepsilon_{ij}=\varepsilon_i^p(1+\varepsilon_j^t),$$

where each component follows a Poisson jump process. Jumps arrive at Poisson rate λ^p for ε^p and λ^t for ε^t. Conditional on a jump in ε^p, a new earnings state ε_k^p is drawn from a bounded Pareto distribution; conversely, when ε^t has a jump, it is drawn from the discrete set $\{-\chi,\chi\}$. Kaplan et al. (2018) explain why this type of process is useful for matching high frequency earnings dynamics. The size and frequency of each shock determine the shape of the earnings distribution. Large, infrequent shocks are likely to generate a more leptokurtic distribution, and small, frequent shocks are likely to generate a platykurtic distribution. Kaplan et al. (2018) model the earnings process as a sum of two jump-drift processes, representing a persistent and a transitory component of the earnings process.

The distribution of ε_i^p is determined by a choice of the upper bound, the lower bound, and a curvature parameter. I choose these parameters to match the variance as well as several additional moments of the earnings distribution: the share of earnings over quintiles and the shares in the top 5–10, 1–5 and 1 percentiles. As there is little effect on the earnings distribution from ε^t, in matching these moments I start by discretizing ε^p. As a first step in this discretization, I create a set of 4 points that are not linearly spaced. Instead, the 4 points aim to capture the bottom 41.0, 69.0 and 98.0 percentiles of the earnings distribution and the remaining top 2.0%.

The first three probabilities represent population shares of three educational attainment levels. These educational levels are high school graduate and below, some college or a bachelor's degree, and higher (averaged over 1992 to 2013 as reported by the BLS). The last point to compute the share of earnings held by the top 1.5% is to capture concentration at the top of the distribution.

The set of 4 points is found as follows. Let $\underline{\varepsilon}^p$, $\overline{\varepsilon}^p$ and η^p_ε be the lower and upper bounds and a curvature parameter, respectively, of the bounded Pareto distribution. The first and second points, x_1 and x_2, solve $f(x_1) = \frac{1-(\underline{\varepsilon}^p/x_1)^{\eta^p_\varepsilon}}{1-(\underline{\varepsilon}^p/\overline{\varepsilon}^p)^{\eta^p_\varepsilon}} = 0.41$ and $f(x_2) = \frac{1-(\underline{\varepsilon}^p/x_2)^{\eta^p_\varepsilon}}{1-(\underline{\varepsilon}^p/\overline{\varepsilon}^p)^{\eta^p_\varepsilon}} = 0.69$, where $f(x_i)$ is the CDF of the bounded Pareto distribution.

The next step is to use these points to determine transition probabilities for a discretized grid for ε^p. As is conventional when discretizing a continuous distribution, the support is chosen so that the vector ε^p has ε^p_1 as the midpoint between $\underline{\varepsilon}$ and x_1 and ε^p_2 is a midpoint between x_1 and x_2 and so forth.

Having chosen the values for ε^p_i, the probability of drawing a new value upon the arrival of the income shock can be defined. In the following, I assume that the probability of drawing ε^p_k depends on the current level of productivity, ε^p_i. This requires choosing bounded Pareto distributions over ε^p_k for each ε^p_i. Each of these distributions is bounded by the same $\underline{\varepsilon}^p$, $\overline{\varepsilon}^p$ described above. They are distinguished by curvature parameters, $\eta_{\varepsilon^p_i}$, $i = 1, .., 4$. Given the discretized support, the shape parameters $\eta_{\varepsilon^p_i}$ need to be estimated.

We use the above distributions, alongside the points x_i described above, to construct conditional probabilities. Conditional on a jump, let the probability of a change from ε^p_i to ε^p_k be $f(\varepsilon_{k|i}) - f(\varepsilon_{k-1|i})$, where $f(\varepsilon_{k|i}) = \frac{1-(\underline{\varepsilon}^p/x_k)^{\eta^p_{\varepsilon_i}}}{1-(\underline{\varepsilon}^p/\overline{\varepsilon}^p)^{\eta_{\varepsilon^p_i}}}$. Recall λ^p is the intensity for the arrival of an ε^p shock. The intensity of jumping from i to k is $\lambda^p_{ik} = \lambda^p(f(\varepsilon_{k+1|i}) - f(\varepsilon_{k|i}))$.

To set the curvature values $\eta_{\varepsilon^p_i}$, $i = 1, .., 4$, the shock intensities λ^p and λ^t, the size of the shock χ, and the probability of drawing a negative transitory component conditional on arrival of shock in ε^t, I estimate the earnings process using the Simulated Method of Moments to match the higher order moments of the earnings growth rate distribution reported in Guvenen et al. (2015), using Social Security Administration (SSA) data from 1994 to 2013.

I simulate the discretized earnings process to compute corresponding moments. The panel size is 4,000 and the simulation length is 5,000. The 800 periods of each simulated series are discarded before computing statistics. Increasing the panel size or the number of periods has little effect on the results. Since the data moments are computed using annual earnings, I simulate at a higher frequency and aggregate the results into annual earnings.

To summarize, the number of parameters specifying the earnings process is 11 and the number of targets is 20. The parameters include the 3 parameters that shape the bounded Pareto distribution for ε^p: $\overline{\varepsilon}^p$, $\underline{\varepsilon}^p, \eta^p_{\varepsilon}$. In addition, the 4 parameters that set the probability of drawing a new value for ε^p are $\eta^p_{\varepsilon_i}, i \in [1, .., 4]$. Next, the 2 parameters that set shock intensity are λ^p and λ^t, and χ is the size of the ε_t shock. Finally, p^t is the probability of drawing negative value upon the arrival of an ε^t shock.

Note that as λ_{ik} affects the ergodic distribution of households over labor productivity, the cumulative population shares by ε^p_i could be different from the 41.0, 69.0, 98.0 and 100 percentiles I set above. Therefore, an additional restriction to the Simulated Method of Moments is that the abcissa x_i must be consistent with the educational attainment earnings shares provided at the start of this section. Hence, the objective function minimized includes the percentiles of the earnings distribution that are used to space the grid for ε^p. Beyond this, there are 20 targets listed in Tables 1 and 2 in Section 3 of the text.

The estimated process implies that a shock to ε^p arrives on average once every 21 years. Upon the arrival of a shock, one's income level jumps to a different level. Turning to the other labor productivity shock, a shock to ε^t arrives on average once every 0.9 years. The infrequent component of the shock, ε^p can be interpreted as the persistent component and ε^t as the transitory component. Households do not experience a large shock often, but income fluctuates around their persistent component through frequent shocks, ε^t.

C.2 Categorization of assets and debts

Mapping the model to the data requires categorizing assets held by US households into financial assets, non-financial assets and secured debt. I target the asset and debt distribution reported in the 2007 SCF. In the SCF data, net worth comprises assets and debt, and total assets are the sum of financial assets and non-financial assets. Financial assets include transaction accounts, certificates of deposit, money market funds, stocks, cash, quasi-liquid retirement accounts and other financial assets. Non-financial assets are predominantly the value of vehicles and houses (primary and non-primary residential property, non-residential real estate) and the value of business. Debt comprises debt secured by residential properties, credit card loans, and installment loans (e.g., student loans, vehicle loans). When mapping the model to the data, I exclude the value of a business and vehicles from non-financial assets because my model does not have such assets. For debt, I exclude installment loans, which include student loans and vehicle loans, for the same reason. Student loans are not short-term, unsecured debt nor are they secured by collateral or dischargeable in bankruptcy. After excluding installment loans, credit card loans are considered as unsecured debt, and the remaining components of debt are assigned to secured debt.

D Stochastic results

D.1 Procyclical tax-rate economy

In Section 4, for policy interventions incurring explicit costs (mortgage principal reduction, tax rebate), the government finances its policy intervention by issuing bonds that are repaid by increasing taxes only during expansions. Therefore, when the government intervenes, households expect future taxes to rise in expansions. For tractability, I assume that households do not know exactly when the cost will be paid off but expect the procyclical tax rate regime will end with a 20% probability at any time. In this section, I present details for this procyclical tax-rate economy.

D.2 Household problem

As laid out in Section 2, a household may or may not have a bankruptcy or foreclosure flag and has different problems depending on these flags. However, all problems are stopping time problems and they can be compactly written as an HJBVI. There are two differences when tax is procyclical. First, the budget constraints become

$$\begin{aligned}\dot{a} = w(\Omega)\varepsilon + (r_a(\omega,\Omega) + \iota(z)|_{a<0})a - (r(\Omega) + \iota(z) + \theta(b,\overline{p}h))b \\ - c - \xi_h p(\Omega)h - T(b,\varepsilon,p(\Omega)h,z).\end{aligned} \tag{11}$$

The only difference here is the tax function, which becomes $T(b,\varepsilon,p(g,z)h,z)$ instead of $T(b,\varepsilon,p(g,z)h)$ because the tax rate now varies with total factor productivity. Specifically, the tax function is $T(y,z) = y - \tau_0(z)y^{1-\tau_1}$. While τ_1 is unchanged at 0.181, τ_0 during expansions is 0.575 and τ_0 during recessions is 0.585. This implies approximately 0.0–1.5 percentage points higher tax rates during expansions compared to the benchmark.

Second, households' problems become the following. Let $v^{tax}(\cdot)$ be the value function of the procyclical tax rate economy. The HJBVI for a household is shown below. Households decide whether to continue or to stop and choose any of the stopping options, which are computed the same way as in Equation (3).

The value of continuing, the first choice, is the HJB equation as in Equation (2), with the modified budget constraint in Equation (11) and with the extra term $\lambda_{tax}(v(\omega,\Omega) - v^{tax}(\omega,\Omega))$ to account for the possibility of moving back to the economy without the procyclical tax rate. The HJBVI is

$$\min \Bigg\{ \rho v^{tax}(\omega,\Omega) - \max_c u(c,h) - \partial_a v^{tax}(\omega,\Omega)\dot{a} - \partial_b v^{tax}(\omega,\Omega)\dot{b}$$
$$- \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v^{tax}(\omega^{\varepsilon_j}, \emptyset - \lambda_d(v^{tax}(\omega,\Omega|o=0) - v^{tax}(\omega,\Omega|o=1))_{o=1}$$
$$- \lambda_f(v^{tax}(\omega,\Omega|o=0) - v^{tax}(\omega,\Omega|o=2))_{o=2} - \sum_{k=1}^{n_z} \lambda_{zz_k} v^{tax}(\omega,\Omega^{z_k})$$
$$- \int \frac{\delta v^{tax}(\omega,\Omega)}{\delta g(\omega)} \mathcal{K} g(\omega) d\omega - \lambda_{tax}(v(\omega,\Omega) - v^{tax}(\omega,\Omega)),$$
$$v^{tax}(\omega,\Omega) - v^{tax*}(\omega,\Omega) \Bigg\} = 0.$$

D.3 Financial intermediaries

Loan price functions for short-term debt and mortgages are determined by the zero expected profit condition of competitive banks, as in Section 2. The only difference is that there is a possibility of moving back to the benchmark economy, and the loan price functions take this possibility into account.

Unsecured debt Let $r_a^{tax}(\omega,\Omega)$ be the short-term loan price function. In the default region ($D_a^{tax}(\omega,\Omega) = 1$), and we set

$$r_a^{tax}(\omega,\Omega) = \infty.$$

The zero profit condition in the region of no default implies that the return $r_a^{tax}(\omega,\Omega)$ should be equal to the risk free rate, $r(g,z)$,

$$r_a^{tax}(\omega,\Omega) = r(g,z) + (1-p_{tax})(\lambda_z \sum_{z'} p_{zz'} \lambda_\varepsilon \sum_{\varepsilon'} p_{\varepsilon\varepsilon'} D_a^{tax}(\omega^{\varepsilon'}, \Omega^{z'})) + p_{tax} D_a(\omega,\Omega).$$

Mortgages Since banks expect zero profit on each loan, the discounted value of the loan at origination has to equal its expected cash flow. The price of the loan in the no-default region is given by

$$q_0^{tax}(\omega,\Omega)b_0 = \mathbb{E}\Big[\mathbb{E}_\tau \int_0^\tau e^{-\int_0^s (r_s+\iota_t+\theta_s)ds}(r_t+\iota_t+\theta_t)b_0 dt + e^{-\int_0^\tau r_s ds} b(\omega_\tau,\Omega_\tau)\Big].$$

The scrap value $b(\omega_\tau,\Omega_\tau)$ at the stopping point depends on a household's discrete choice. In the case of a foreclosure, $b(\omega_\tau,\Omega_\tau) = (1-\delta_d)p(g,z)h$. When a household prepays its loan due to refinancing or a new house transaction, the scrap value is $e^{-\int_0^\tau \theta_s ds} b_0$.

Applying the Feynman-Kac formula, the above equations can be written as the following partial differential equation. At $t \in [0,\tau)$,

$$\begin{aligned}(\theta(b,\overline{p}h) + r(\Omega) + \iota(z))q^{tax}(\omega,\Omega) &= \theta(b,\overline{p}h) + r() + \iota(z) + q_a^{tax}(\omega,\Omega)\dot{a} + q_b^{tax}(\omega,\Omega)\dot{b} \\ &+ \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} q^{tax}(\omega^{\varepsilon_j},\Omega) + \sum_{k=1}^{n_z} \lambda_{zz_k} q^{tax}(\omega,\Omega^{z_k}) + \int \frac{\delta q^{tax}(\omega,\Omega)}{\delta g(\omega)} \mathcal{K} g(\omega) d\omega \\ &+ \lambda_{tax}(q(\omega,\Omega) - q^{tax}(\omega,\Omega)).\end{aligned}$$

When, at $t=\tau$, the stopping time decision is to foreclose,

$$q^{tax}(\omega,\Omega) = \frac{(1-\delta_d)p(\Omega)h}{b},$$

and, if instead, the stopping time decision involves prepayment, we have

$$q^{tax}(\omega,\Omega) = 1.$$

D.4 Government debt dynamics

When there is a policy intervention, the government issues bonds to finance debt relief or a tax rebate. This debt is repaid during expansions through higher tax rates. As before, government consumption is determined by pre-intervention tax rates. Given the tax function before the intervention, $T(b,\varepsilon,p(g,z)h)$, government spending, G, is given by

$$\int T(b, \varepsilon, p(\Omega)h) g(\omega) d\omega - G = 0.$$

Thus, there is no change in the government consumption function compared to the pre-intervention economy. However, now the government has debt, B, which it repays during expansions using extra tax revenue generated as the difference between the acylical tax function above and the procyclical function described in D.2. From the time the government intervenes to the beginning of an expansion, government debt evolves as

$$\dot{B}_t = r_t B_t.$$

Once the economy transitions to an expansion, the government starts to repay its debt and it evolves as

$$\dot{B}_t = r_t B_t - \int (T(b, \varepsilon, p(\Omega)h, z) - T(b, \varepsilon, p(\Omega)h)) g(\omega) d\omega,$$

where $T(b, \varepsilon, p(\Omega)h, z)$ is the procyclical tax function. Therefore, the government uses the increase in its tax revenue to repay debt, leaving consumption as it would have been if there had been no change in tax rates for a given distribution of households.

D.5 Error in the forecasting function

The households' and financial intermediaries' problems described in this appendix involve a high-dimensional object in the state vector, the distribution of households. Solving their problems requires knowing how the distribution of households changes. The solution method used is the same as that applied for the model described in Section 2 of the main text. The algorithm is a version of that in Krusell and Smith (1998) and is described in Section A.3.

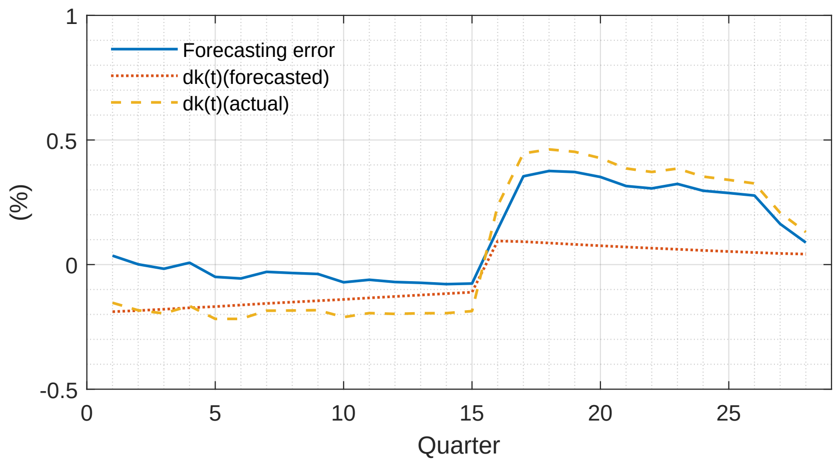
The solution algorithm uses forecasting rules, which are used to describe the change in the approximate aggregate state. I apply it to solve the economy described here as follows. First I solve the no-intervention model with an acyclical tax function, described in Section 2. The forecasts for this model are drawn from the stationary distribution of the stochastic economy. This is also the long-run model for the policy intervention economy, once debt is repaid and the tax policy returns to its original form. Until the post-intervention public debt is paid, taxes are procyclical and households expect to eventually transition to the long-run model. I solve this model using the forecast rules derived from an economy with a procylical tax policy but no policy intervention or government debt. This model has the same constant probability of permanently transitioning to the long-run model, in households' and firms' expectations, as the policy intervention model. Forecasting rules for this case are derived from a simulation of the economy before a transition.

Once the government issues debt, the aggregate capital has an additional term, B_t, and it becomes $K_t = \int (a-b) g_t d\omega - B_t$. Since there is no government debt in the simulation step, the forecast function for the procyclical tax economy will not fully capture the path of aggregate capital after the policy interventions. For instance, after the intervention and before the recession is over, aggregate capital in the intervention model falls faster than the forecasting function predicts as the government debt grows at the rate $\dot{B}_t = r_t B_t$. However, the forecasting function is estimated assuming $B_t = 0$. Likewise, once the economy transitions to an expansion, the aggregate capital stock grows faster than the speed that the forecasting function predicts because $\dot{B}_t$ is negative, thanks to the government's repayment.

Forecasting errors might mitigate policy effects on consumption and house price. Households' savings decisions involve expectations of future wages and interest rates, and these are functions of the future capital stock. Thus, if there is a downward bias in the forecasting function during expansions, it will lead to higher savings as a lower expected $\dot{k}$ implies a higher return on savings.

Any such error is not likely to significantly change the results because the errors are small. Figure 6 shows the error. At $t = 16$ the economy transitions to an expansion. As explained above, the forecasting errors are negative during the recession and positive during the expansion. We see that the largest positive

Figure 6: Forecasting error of the aggregate capital



Note: $dk(t)$ is the change of capital, $\dot{k}_t$. Forecasting error is $\dot{k}_t$(actual) - $\dot{k}_t$(forecasted).

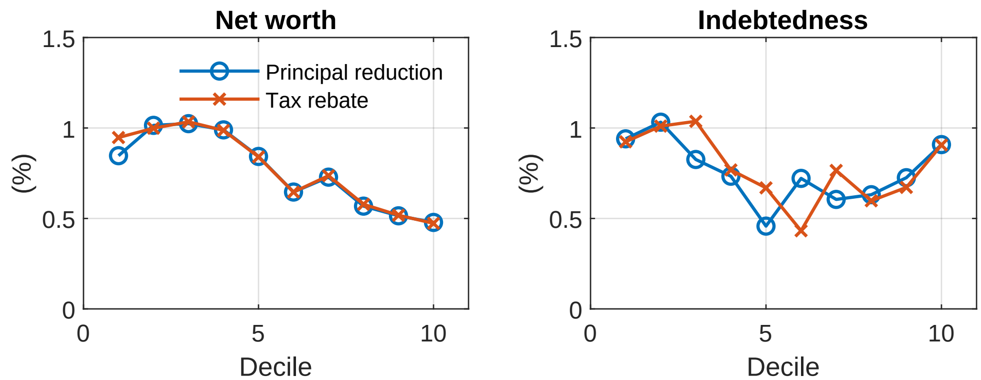
error is about 0.37%. This is slightly larger than the maximum error of the benchmark model, which is 0.24%. This model has no policy intervention or procyclical tax and therefore no related source of possible bias in its forecast errors.

D.6 Mortgage relief policies in the Great Recession

During the Great Recession, the US government intervened in mortgage markets through household debt relief policies. In particular, the government introduced principal reduction modifications in 2010 in the Home Affordable Modification Program (HAMP). This was a response to growing concerns that debt levels, not just debt repayments, were causing high foreclosure rates. Under this modification, mortgage borrowers' principals were forgiven until their new monthly payment fell below 31% of income or the LTV ratio dropped to 115%, whichever came first. Although participation rates were perceived to be low, Agarwal et al. (2017) show that the program was associated with reduced rates of foreclosure, consumer debt delinquencies and house price declines. I complement studies estimating causal relations, as in Agarwal et al.

(2017), by assessing the effects in aggregate quantities from resulting changes in the distribution of households and prices.

Figure 7: Consumption equivalent gain distribution



Note: Consumption equivalent gain over net worth and indebtedness deciles. Indebtedness is the sum of mortgages and unsecured credit over net worth, and the plotted numbers are average consumption equivalent gains in each decile.

D.7 Fiscal multiplier and welfare

In this section, I assess the effectiveness of each funded policy using two common measures. The first one is a fiscal multiplier. As is standard in the literature, I compute the cumulative fiscal multiplier through time T as the discounted cumulative change in output over government spending, that is,

$$M_T = \frac{\int_{t=0}^{T} e^{-r_s t}(y_t^p - y_t^b)dt}{\int_{t=0}^{T} e^{-r_s t} G_t dt},$$

where y_t^p and y_t^b are outputs of the policy economy and the benchmark economy, G_t is the cost of the intervention, and r_s is the average interest rate over time, following Mountford and Uhlig (2009).

The fiscal multiplier of the principal reduction and tax rebate policies are 0.08 and -0.04, respectively. The higher multiplier for the mortgage reduction policy stems from its stronger effect on capital accumulation, discussed above. Conversely, the fiscal multiplier on consumption is larger for the tax rebate (0.36) than the principal reduction (0.03).

The second measure of policy effectiveness is household welfare, which is calculated as the non-durable consumption-equivalent gain obtained by comparing economies with policy interventions and the benchmark economy. The welfare measure is

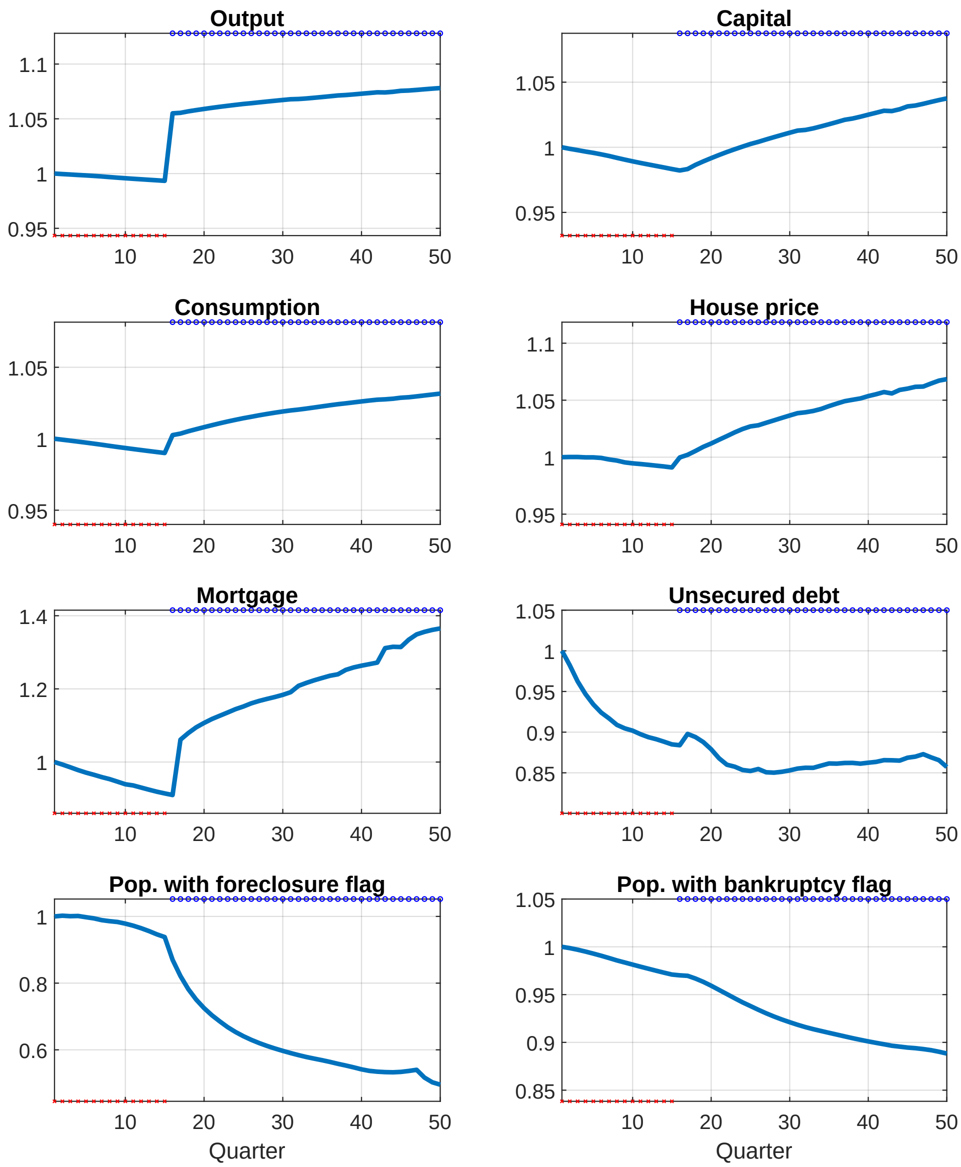
$$\mathcal{W}_i = \int g_i(\omega) x_i \, d\omega, \;\; x_i = \frac{[(1-\sigma)(\rho v_{benchmark} - \rho v_i + u(c_i))]^{\frac{1}{1-\sigma}}}{c_i} - 1,$$

where x_i is a consumption equivalent gain and $i \in$ {principal reduction, tax rebate}. Let x_i be the non-durable consumption difference that makes the value of the benchmark economy indifferent to the value of the economy with a principal reduction or tax rebate.

While not true for all households, there is a positive average welfare gain in each decline of the net worth and indebtedness distributions. Across both policies, the average consumption equivalent gain is 0.8%. Figure 7 shows, over deciles of net worth and indebtedness, who values the policy interventions more (or less). The principal reduction and the tax rebate show similar distributions of consumption equivalent gains; both policies are less favored by rich households and more favored by highly indebted households.

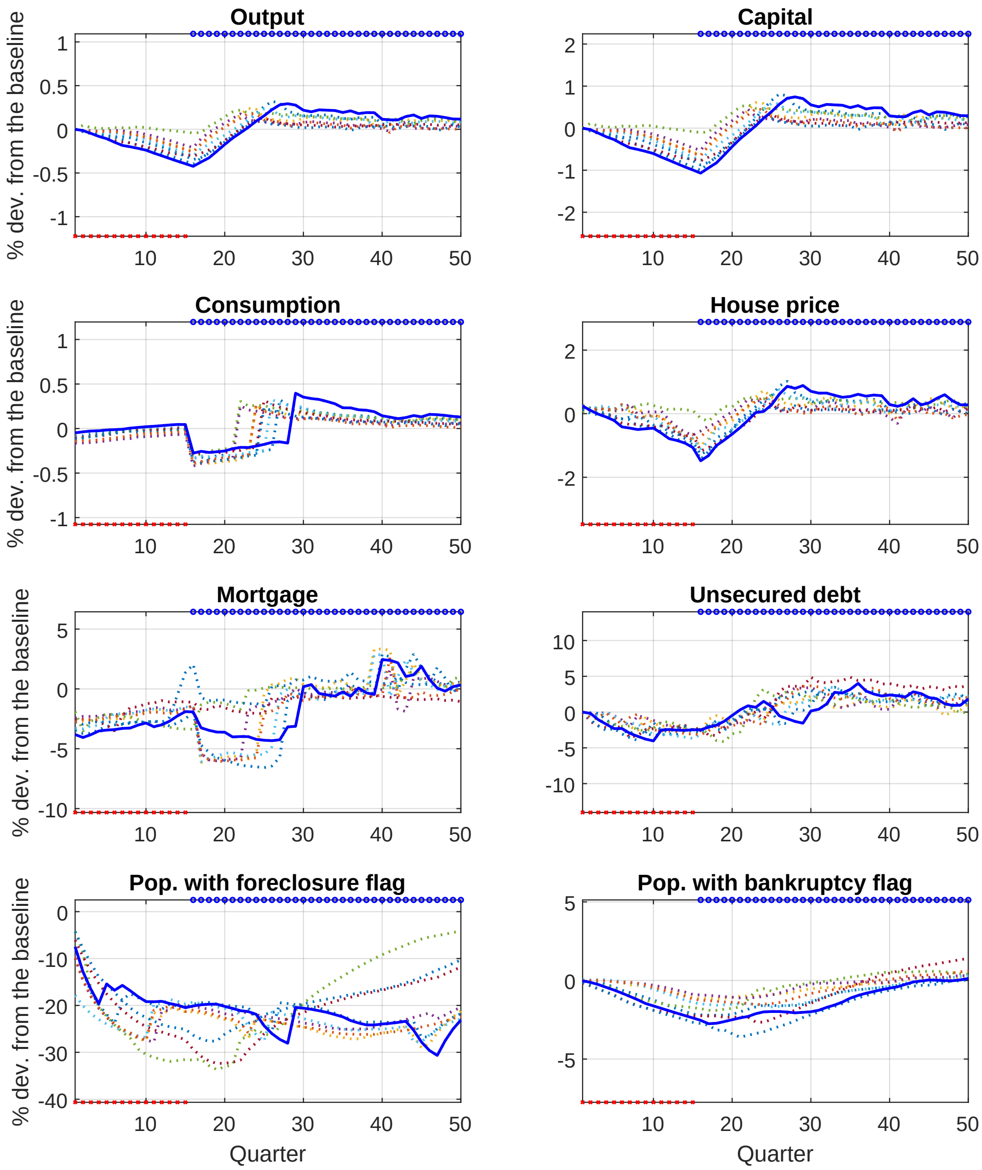
E Additional tables and figures

Figure 8: Aggregate variables



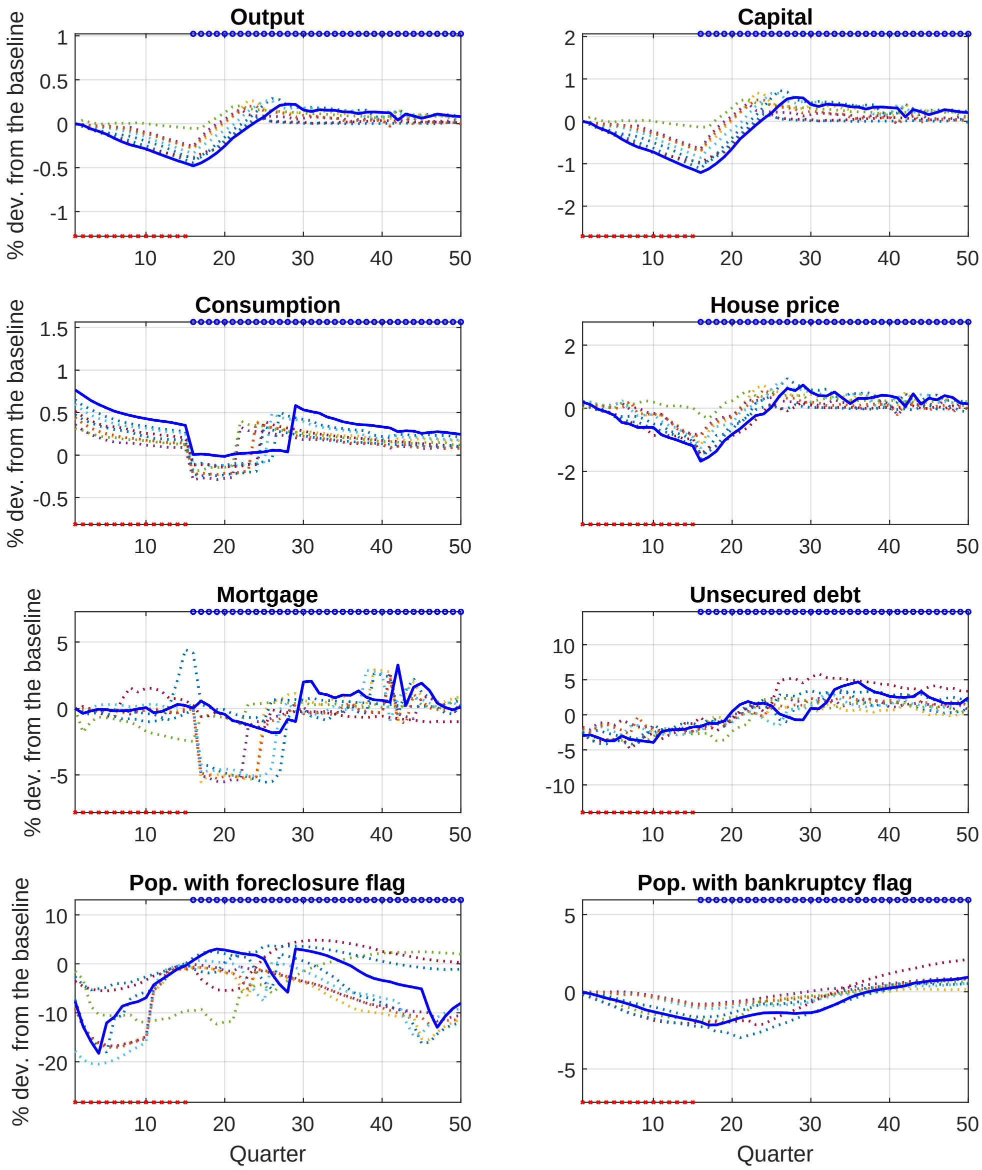
Note: Aggregate variable movements in the baseline economy without policy intervention. All series are normalized to 1 in the first period.

Figure 9: Aggregate variables: Principal reduction



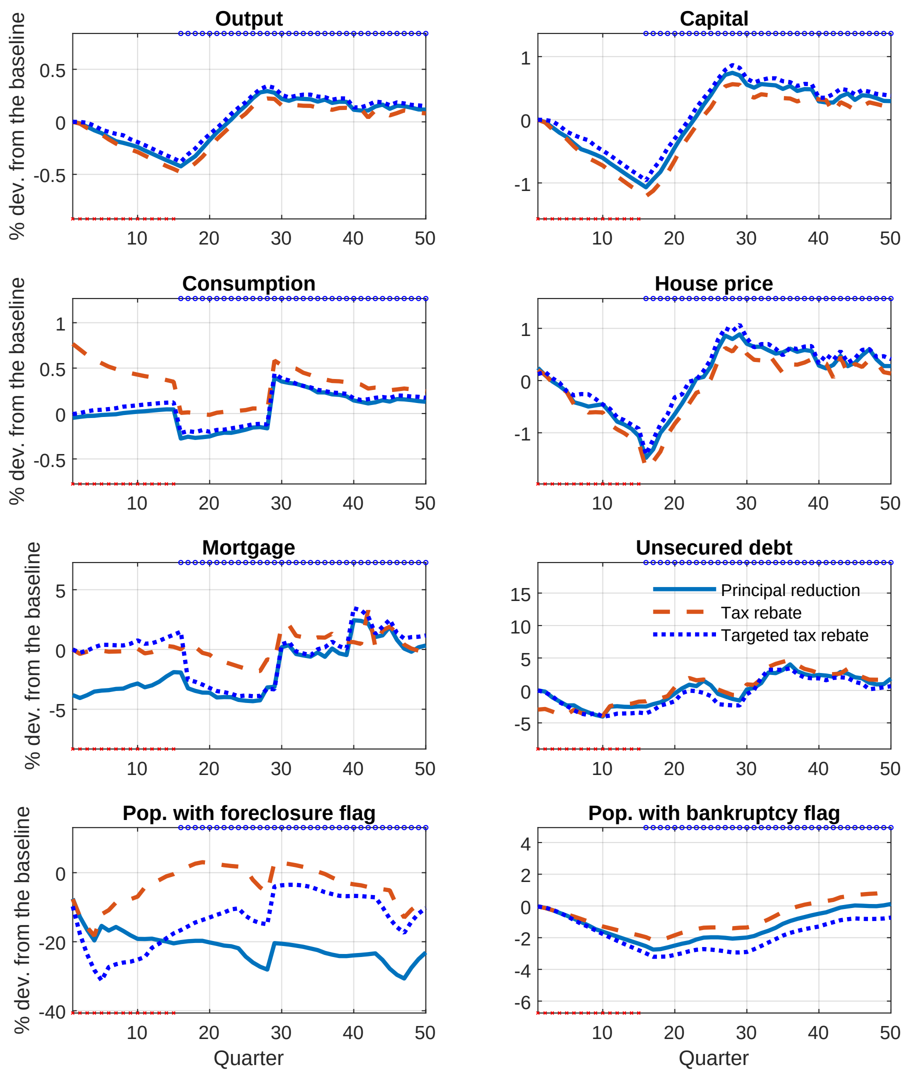
Note: Dashed lines show responses of aggregate variables with different initial distributions. For comparability, I set the path of total factor productivity from the intervention date onwards to be the same across all economies. The solid lines in Figures 9 and 10 correspond to the series in Figure 2.

Figure 10: Aggregate variables: Tax rebate



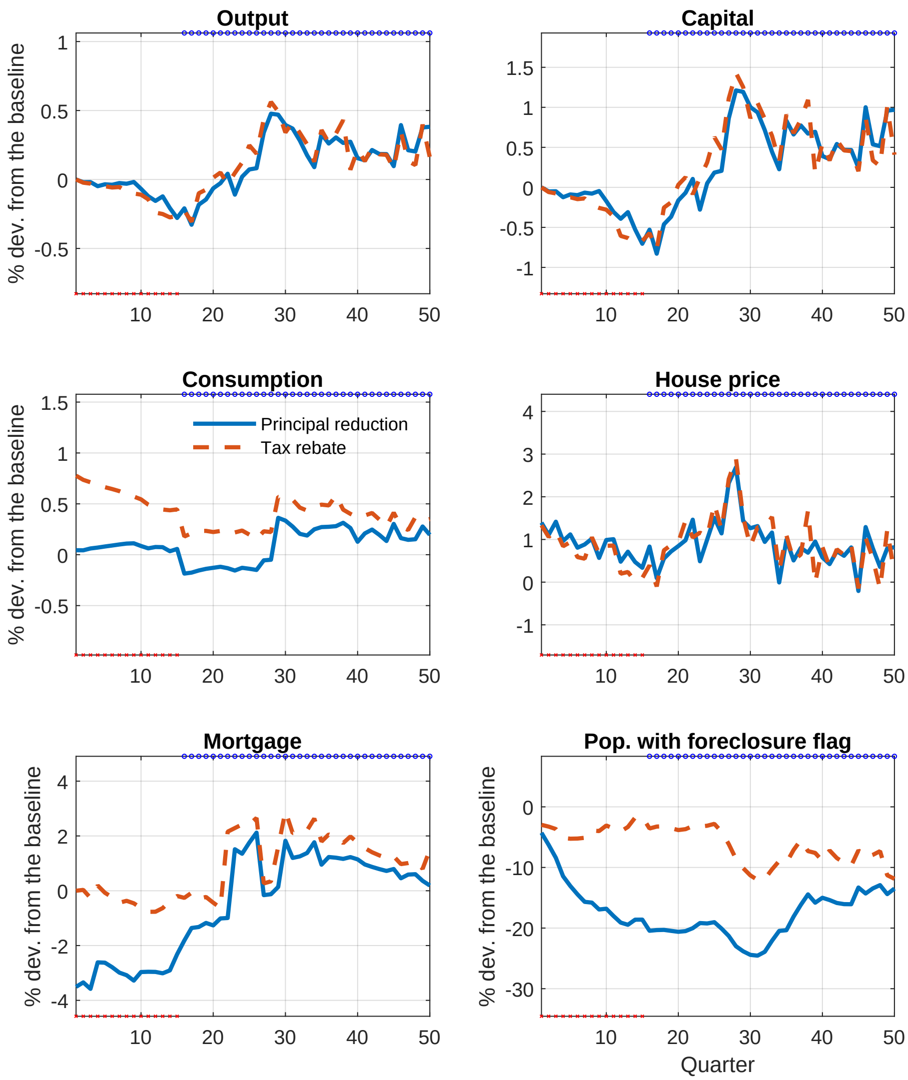
Note: Dashed lines show responses of aggregate variables with different initial distributions. For comparability, I set the path of total factor productivity from the intervention date onwards to be the same across all economies. The solid lines in Figures 9 and 10 correspond to the series in Figure 2.

Figure 11: Response of aggregate variables: Targeted tax rebate



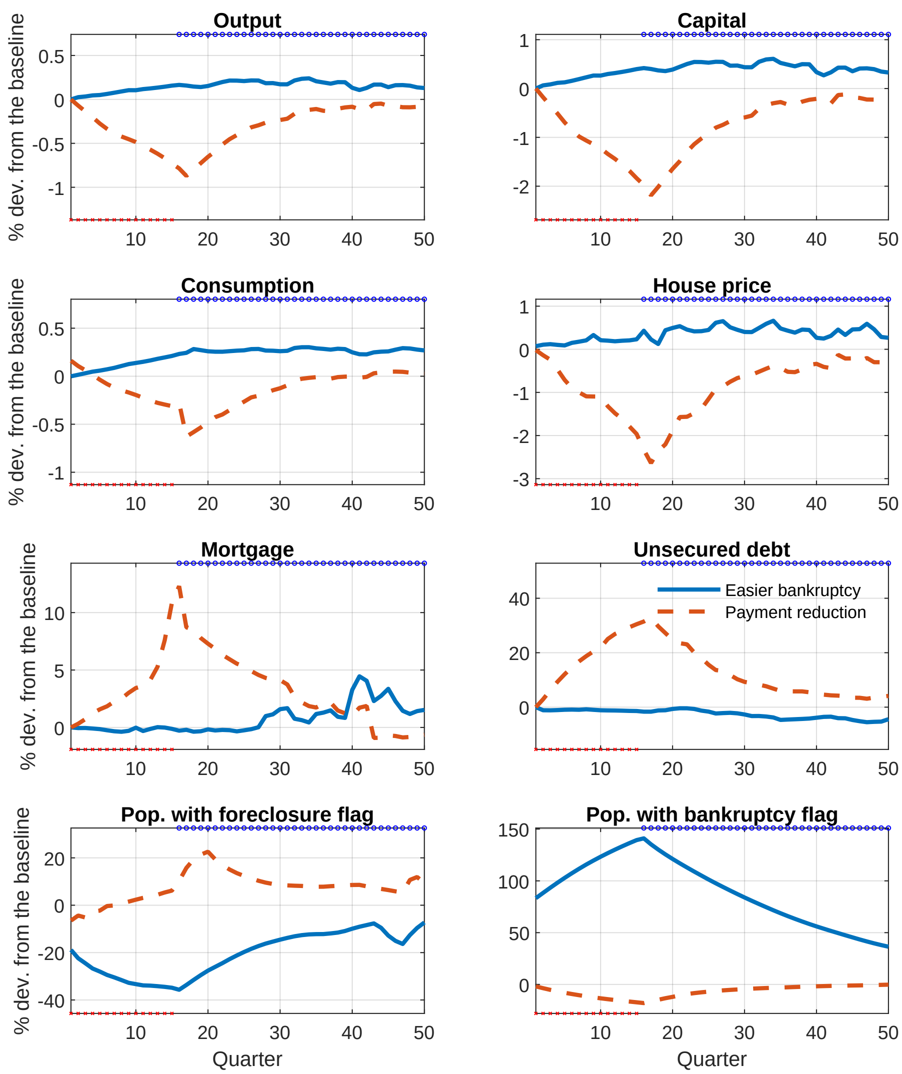
Note: Aggregate series are shown as percent deviations from the corresponding series in the economy without policy intervention.

Figure 12: Response of aggregate variables: Role of bankruptcy



Note: Aggregate series are shown as percent deviations from the corresponding series in the economy without policy intervention.

Figure 13: Response of aggregate variables: Other policies



Note: Aggregate series are shown as percent deviations from the corresponding series in the economy without policy intervention.

Table 7: Ranges for relevant variables

	Principal reduction	**Tax rebate**
Eligible mortgagors (%)	[11.0, 22.6]	-
Total cost/GDP (%)	[2.4, 5.2]	-
Fiscal multiplier	[-0.11, 0.23]	[-0.24, 0.29]
Avg. transfer amount/household	[67,000, 90,000]	[2,400, 5,300]

Note: Ranges in each row are derived from economies with different initial distributions. Details for computing the fiscal multiplier can be found in Appendix D.7.

Table 8: Portfolio composition

	Asset		**Debt**	
	Non-financial	Financial	Secured	Unsecured
Data				
Q1	-524.62	-112.52	581.57	155.57
Q2	257.43	52.01	-196.31	-13.14
Q3	147.40	36.29	-80.25	-3.44
Q4	98.34	39.16	-36.08	-1.42
Q5	54.03	56.56	-10.038	-0.21
Model				
Q1	117.65	107.44	-123.37	-1.72
Q2	70.33	100.66	-70.80	-0.20
Q3	91.38	81.62	-49.44	-23.56
Q4	86.26	53.29	-37.72	-1.83
Q5	52.30	68.68	-20.97	-0.01

Note: Average portfolio composition by net worth quintiles. Non-financial assets include "Housing and cars" and "Business and non-financial assets" in the SCF. Here, business assets and vehicles are excluded from non-financial assets. Installment loans are excluded. After excluding installment loans, credit card loans are considered as unsecured debt and the remaining compositions of debt are assigned to secured debt. Data: SCF (2007)

Table 9: Cyclical properties

	Benchmark		**Foreclosure only**	
	std (%)	corr. w/ Y	std (%)	corr. w/ Y
Output	1.8	1.0	1.8	1.0
Consumption	0.6	0.8	0.4	0.8
Investment	4.3	0.8	5.4	0.7
Unsecured debt	3.0	0.0	-	-
Mortgage	3.9	0.7	2.7	0.7
Bankruptcy	7.7	0.1	-	-
Foreclosure	1.3	-0.7	3.5	0.1
House price	2.6	0.7	2.5	0.6

Note: "Bankruptcy" only refers to an economy in which households cannot foreclose but can go bankrupt, and "Foreclosure" only refers to an economy with only foreclosures. None refers to an economy without any option to default. Y in "corr. w/ Y" is output.

Technical Appendix

A Numerical Solution Method

This section describes the computational method used to solve the model and a measure of accuracy. The existence and uniqueness of the viscosity solution of an HJB equation are shown by Crandall and Lions (1983). The existence of the viscosity solution of an HJBVI equation when a value function is not always differentiable, which corresponds to the problem in the paper, was proven in Øksendal and Sulem (2005) (See chapter 9, theorem 9.8).

The solution algorithm is based on the finite difference method in Achdou et al. (2022) with several important differences. First, there are multiple stopping choices including two types of default, the buying and selling of houses and refinancing and prepayment. Additionally, the model solution is nonlinear in both the individual and aggregate state vectors. Since the aggregate state vector is high-dimensional, I use state-space approximation, following the approach in Krusell and Smith (1998).

A.1 HJBVI as a Linear Complementarity Problem (LCP)

To solve the stopping time problems, for example the three HBJVI problems described in Appendix B and those in D.2, I transform each into a linear complementarity problem.[41] The HJBVI equation can be written as

$$\min\Big[\rho\mathbf{v}-\mathbf{u}-\mathbf{A}\mathbf{v},\ \ \mathbf{v}-\mathbf{v}^*\Big]=0, \tag{12}$$

where $\mathbf{A}$ summarizes changes caused by decisions and shocks. Below I describe how to construct $\mathbf{A}$. Equation (12) implies

$$\Big(\mathbf{v}-\mathbf{v}^*\Big)'\Big(\rho\mathbf{v}-\mathbf{u}-\mathbf{A}\mathbf{v}\Big)=0 \tag{13}$$

[41] See `http://www.princeton.edu/ moll/HACTproject/option-simple.pdf` for an example.

$$\rho \mathbf{v} - \mathbf{u} - \mathbf{A}\mathbf{v} \geq \mathbf{0}$$

$$\mathbf{v} \geq \mathbf{v}^*.$$

Let $\mathbf{z} = \mathbf{v} - \mathbf{v}^*$, $\mathbf{B} = \rho\mathbf{I} - \mathbf{A}$ and $\mathbf{q} = -\mathbf{u} + \mathbf{B}\mathbf{v}^*$. Then (13) is equivalent to (14).

$$\mathbf{z}'\big(\mathbf{B}\mathbf{z} + \mathbf{q}\big) = 0 \tag{14}$$

$$\mathbf{B}\mathbf{z} + \mathbf{q} \geq 0$$

$$\mathbf{z} \geq \mathbf{0}.$$

This is the standard form for a linear complementarity problem, and several numerical solvers are available.[42]

Construction of A I solve for optimal decisions over a discretized grid of the state space by iterating on the value function, v. Let Equation (13) be a matrix representation of Equation (10), where $\mathbf{v} = [v_1, v_2, .., v_N]$, $\mathbf{u} = [u_1, u_2, .., u_N]$ and N is the number of points in the value function. Thus, $\mathbf{A}$ describes $\dot{a}$, $\dot{b}$, and the shocks that households are exposed to. Here, I first describe the construction of $\mathbf{A}$ excluding terms related to the aggregate states (g, z), and Section A.3 contains a description of how to solve the model with aggregate uncertainty. Also, I describe the problem of households without default flags, $o = 0$, but the method applies to problems of households with default flags. As $\dot{b}$ is given by $\theta(b, ph)b$ and the earnings process is exogenously set, below I explain how to solve $\dot{a}$.

I choose a number of grid points $(n_a, n_b, n_\varepsilon, n_h)$ for the corresponding variables (a, b, ε, h). Let v_{ijkp} be the value function of a household without a bankruptcy or foreclosure flag, with liquid assets a_i, mortgage b_j, labor productivity ε_k and house h_p. The derivative with respect to a, v^a_{ijkp} is approximated

[42] I use the LCP solver at `https://www.mathworks.com/matlabcentral/fileexchange/20952-lcp-mcp-solver-newton-based`.

with either a forward or a backward first difference:

$$v_{ijkp}^{a} \approx \frac{v_{i+1jkp} - v_{ijkp}}{\Delta a} = v_{ijkp}^{a,F} \text{ or } v_{ijkp}^{a} \approx \frac{v_{ijkp} - v_{i-1jkp}}{\Delta a} = v_{ijkp}^{a,B}.$$

Likewise, the derivative with respect to b, v_{ijkp}^{b} can be approximated with a forward or backward difference.

Applying this method to the first argument in Equation (10), we have

$$\begin{aligned} \rho v_{ijkp} = u(c_{ijkp}, h_p) + v_{ijkp}^{a}\dot{a}_{ijkp} + v_{ijkp}^{b}\dot{b}_{ijkp} + \sum_{\varepsilon_k^{'}} \lambda(\varepsilon_k, \varepsilon_{k'})(v_{ijk'p} - v_{ijkp}) \\ \forall\, i = \{1, .., n_a\}, j = \{1, .., n_b\}, k = \{1, .., n_\varepsilon\}, p = \{1, .., n_h\}, \end{aligned} \tag{15}$$

where

$$\dot{a}_{ijkp} = w\varepsilon_k + r_{a,ijkp}a_i - (r + \theta(b_j, ph_p))b_j - c_{ijkp} - T(b_j, ph_p, \varepsilon_k) - \xi_h ph_p,$$

$$\dot{b}_{ijkp} = -\theta(b_j, ph_p)b_j.$$

The household choice of non-durable consumption can be solved from the FOC:

$$\begin{aligned} u^c(c_{ijkp}, h_p) - v_{ijkp}^{a} &= 0, \\ c_{ijkp} &= (v_{ijkp}^{a})^{\frac{1}{-\sigma}}. \end{aligned}$$

As the derivatives of the value function have two forms, forward and backward, c_{ijkp} is either $(v_{ijkp}^{a,F})^{\frac{1}{-\sigma}}$ or $(v_{ijkp}^{a,B})^{\frac{1}{-\sigma}}$.

To find the drift, $\dot{a}$, it is necessary to select which derivative to use. I follow Achdou et al.'s (2022) upwind scheme. The key idea is to use a forward derivative when the drift is positive and a backward derivative when it is negative. To ease notation, for variable x, let $x^+ = max(x, 0)$ and $x^- = min(x, 0)$. Also, let x^F be the value computed using a forward derivative and x^B be the value derived from the backward derivative. With this notation, savings, $s(= \dot{a})$, can be computed as below:

$$
\begin{aligned}
s^{c,F}_{ijkp} &= w\varepsilon_k + r_{a,ijkp}a_i - (r + \theta(b_j, ph_p))b_j - c^F_{ijkp} - T(b_j, ph_p, \varepsilon_k) - \xi_h ph_p \\
s^{c,B}_{ijkp} &= w\varepsilon_k + r_{a,ijkp}a_i - (r + \theta(b_j, ph_p))b_j - c^B_{ijkp} - T(b_j, ph_p, \varepsilon_k) - \xi_h ph_p.
\end{aligned}
\tag{16}
$$

The upwind scheme can be applied to all of the points in the state space except for the points at the boundaries. Clearly, only one of the forward or backward derivatives can be approximated using a finite difference at the boundaries. The way this is handled for an exogenous borrowing constraint in a one-asset model is explained in Achdou et al. (2022). In my model, there are three assets. Across the two loans, mortgages are secured but there may be an endogenous borrowing limit with respect to a as there is an option to default. Over levels of a where a household would default, there cannot be lending and an endogenous bound is imposed such that $\dot{a} \geq 0$. I explain the details of how I handle boundaries in the rest of this section.

I follow Bornstein (2018), who shows how to solve the problem of an endogenous borrowing limit with respect to a. For each $(b_j, \varepsilon_k, h_p)$, $\forall\, j = \{1, .., n_b\}, k = \{1, .., n_\varepsilon\}, p = \{1, .., n_h\}$, assume that we know the level of unsecured debt, $\underline{a}(b_j, \varepsilon_k, h_p)$, where a household chooses to default when a is below $a_{Djkp} = \underline{a}(b_j, \varepsilon_k, h_p)$. The value function below a_{Djkp} becomes flat over a, and the backward derivative does not exist at $(D, b_j, \varepsilon_k, h_p)$. I impose the endogenous borrowing constraint $\underline{a}(b_j, \varepsilon_k, h_p)$; below this point, I restrict consumption to equal income. One potential issue is consumption at the endogenous borrowing limit (or below the limit) can be negative. If this is the case, I assign a very low value to ensure that these are default points. Note that a will not fall below $\underline{a}(b_j, \varepsilon_k, h_p)$ as the household would have defaulted beforehand. Without loss of generality I set savings to be zero at these points.

Finally, we must know the set of default points, $i \leq D$, at each $(b_j, \varepsilon_k, h_p)$ to find $\underline{a}(b_j, \varepsilon_k, h_p)$. As I solve the value function iteratively, I used the default points identified by the last iteration to set $\underline{a}(b_j, \varepsilon_k, h_p)$. Stopping points including default points are given by LCP solution algorithms, as will be explained at the end of this subsection. Effectively, the endogenous borrowing limit is implemented at each iteration of the solution algorithm as follows. At iteration n, let D^A be a set of points where a household will default on a or on both a and b. If a point $[a_i, b_j, \varepsilon_k, h_p] \in D^A$, set $s^{c,F}_{ijkp} = s^{c,B}_{ijkp} = 0$ in Equation (17). These savings functions are the endogenous component of $\mathbf{A}$ in (13), which is used to solve v, as described next.

Before describing how to build $\mathbf{A}$, one last issue remains. At a_{n_a}, a forward derivative cannot be computed. However, if I set a_{n_a} large enough, savings at this point will be negative and the forward derivative will not be needed.

I now describe matrix $\mathbf{A}$ in (13). Using (16) in (15), the system of equations for the value function can be written as

$$\begin{aligned} \rho v_{ijkp} = u(c_{ijkp}, h_p) + x^a_{ijkp} v_{i-1jkp} + y_{ijkp} v_{ijkp} + z^a_{ijkp} v_{i+1jkp} \\ + x^b_{ijkp} v^{n+1}_{ij-1kp} + z^b_{ijkp} v_{ij+1kp} + \sum_{k'} \lambda_{kk'} v_{ijk'p}, \end{aligned} \tag{17}$$

$$\begin{aligned} x^a_{ijkp} &= -\frac{(s^{c,B})^-}{\Delta a}, \qquad x^b_{ijkp} = -\frac{(\theta(b,ph)b)^-}{\Delta b}, \\ y_{ijkp} &= -\frac{(s^{c,F})^+}{\Delta a} + \frac{(s^{c,B})^-}{\Delta a} - \frac{(\theta(b,ph)b)^+}{\Delta b} + \frac{(\theta(b,ph)b)^-}{\Delta b} + \lambda_{kk}, \\ z^a_{ijkp} &= \frac{(s^{c,F})^+}{\Delta a}, \qquad z^b_{ijkp} = \frac{(\theta(b,ph)b)^+}{\Delta b}. \end{aligned}$$

There are $n_a \times n_b \times n_\varepsilon \times n_h$ linear equations (17), one for each grid point. The system of equations can be written in matrix notation:

$$\rho \mathbf{v} = \mathbf{u} + \mathbf{A}\mathbf{v}. \tag{18}$$

Matrix $\mathbf{A}$ can be constructed step by step. First, I define a submatrix for a given level of housing. For each h_p, $\mathbf{A}_p, p \in \{1, 2, ..., n_h\}$ can be written as

$$\mathbf{A}_p = \begin{bmatrix} A_{11|p} & A_{12|p} & .. & A_{1n_\varepsilon|p} \\ A_{21|p} & A_{22|p} & .. & A_{2n_{\varepsilon|p}} \\ \vdots & \ddots & & \vdots \\ A_{n_\varepsilon 1|p} & A_{n_\varepsilon 2|p} & .. & A_{n_\varepsilon n_\varepsilon|p} \end{bmatrix},$$

where $A_{kk|p}$ is a matrix that is composed of x^a_{ijkp}, x^b_{ijkp}, y_{ijkp}, z^a_{ijkp} and z^b_{ijkp}, $k \in \{1, 2, ..., n_\varepsilon\}$. For example,

$$A_{11|2} = \begin{bmatrix} y_{1112} & z^a_{1112} & 0 & .. & 0 & z^b_{1112} & 0 & .. & 0 & 0 & 0 & .. & & 0 \\ x^a_{2112} & y_{2112} & z^a_{2112} & 0 & .. & 0 & z^b_{2112} & 0 & ... & 0 & 0 & 0 & .. & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & .. & 0 & x^a_{n_a 112} & y^a_{n_a 112} & 0 & .. & 0 & z^b_{n_a 112} & & 0 & ... & 0 & 0 \\ x^b_{1212} & .. & & & 0 & y_{1212} & z^a_{1212} & 0 & .. & 0 & z^b_{1212} & 0 & & 0 \\ \vdots & x^b_{2212} & \ddots & \ddots & \ddots & x^a_{2212} & y_{2212} & z^a_{2212} & \ddots & \ddots & \ddots & z^b_{2212} & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & .. & & & & & & x^a_{n_a n_b 12} & y^a_{n_a n_b 12} & 0 & .. & & 0 & z^b_{n_a n_b 12} \end{bmatrix}$$

and when $k \neq l$, $A_{kl|p}$ is a diagonal matrix with diagonal terms λ_{kl}. Using $\mathbf{A_p}$, $\mathbf{A}$ is a block diagonal matrix composed of $\mathbf{A}_1$,..,$\mathbf{A}_{n_h}$,

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & 0 & .. & & 0 \\ 0 & \mathbf{A}_2 & 0 & .. & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & .. & & 0 & \mathbf{A}_{n_h} \end{bmatrix}.$$

These steps have described how to create the system of equations for a household described by (10) with neither bankruptcy nor foreclosure in its credit record. There is a similar system of linear equations describing a household with a bankruptcy flag or foreclosure flag. Because $a \geq 0$ with the bankruptcy flag, values are not defined below $a = 0$ for households with a bankruptcy flag. Therefore, the number of equations is $n_{a+} \times n_b \times n_\varepsilon \times n_h$, where n_{a+} is the number of a grids where $a \geq 0$. When the household state includes a foreclosure flag, values are defined where $h = 0$ and $b = 0$, and the number of equations is $n_a \times n_\varepsilon$.

The systems of equations in (18) are converted into the form in (14), then solved iteratively using an LCP solver. Let $\mathbf{z} = \mathbf{v}^{n+1} - \mathbf{v}^{*,n}$, $\mathbf{B} = \frac{1}{\Delta} - \rho - \mathbf{A}$ and $\mathbf{q} = \mathbf{B}\mathbf{v}^{*,n} - \frac{\mathbf{v}^n}{\Delta} - \mathbf{u}^n$. Then

$$(\mathbf{v}^{n+1} - \mathbf{v}^{*,n})(\frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta} - \rho\mathbf{v}^{n+1} - \mathbf{u}^n - \mathbf{A}\mathbf{v}^{n+1}) = 0$$
$$\frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta} - \rho\mathbf{v}^{n+1} - \mathbf{u}^n - \mathbf{A}\mathbf{v}^{n+1} \geq 0$$
$$\mathbf{v}^{n+1} - \mathbf{v}^{*,n} \geq 0$$

is equivalent to

$$\mathbf{z}'(\mathbf{B}\mathbf{z} + \mathbf{q}) = 0$$

$$\mathbf{B}\mathbf{z} + \mathbf{q} \geq 0$$

$$\mathbf{z} \geq \mathbf{0},$$

which is the LCP problem from (14). As mentioned, I describe the value function solution algorithm below. The parameter Δ determines the speed of updating.

- Guess the value functions $\mathbf{v}^n$.
- Using the guessed value functions, construct $\mathbf{A}$.
- Compute $\mathbf{v}^{*,n}$ by solving Equation (3).
- Let $\mathbf{z} = \mathbf{v}^{n+1} - \mathbf{v}^{*,n}$, $\mathbf{B} = \frac{1}{\Delta} - \rho - \mathbf{A}$ and $\mathbf{q} = \mathbf{B}\mathbf{v}^{*,n} - \frac{\mathbf{v}^n}{\Delta} - \mathbf{u}^n$; solve the LCP.
- Using the solution to the LCP, set $\mathbf{v}^{n+1} = \mathbf{z} + \mathbf{v}^{*,n}$.
- With the updated value function $\mathbf{v}^{n+1}$, if $\max|\mathbf{v}^{n+1} - \mathbf{v}^n|$ is not small enough, return to the second step.

Above, $\mathbf{z}$ is the solution provided by the LCP solver, and, where $\mathbf{z} = 0$, households find it optimal to stop. Recall that a region of default on unsecured debt is necessary to construct $\mathbf{A}$ for the next iteration. This is found from $\mathbf{z}$ as follows. In every iteration, D^A is the set of points that satisfy $\{\mathbf{z} = 0\} \cap \{\mathbf{v}^{*,n} = \mathbf{v}^{a,n}\} \cup \{\mathbf{v}^{*,n} = \mathbf{v}^{ab,n}\}$. Finally, with many discrete choices, I find using a large value of Δ (for example, larger than 10) often leads to unstable updates.

A.2 Kolmogorov forward equation

After solving the value functions, I need to solve for the household density over their assets, mortgages, houses, labor productivity and default flags. In this section, I describe how to solve the density function.

Without stopping decisions, the Kolmogorov forward equation is

$$\partial_t g_{ijkp,t} = -\partial_a s^a_{ijkp} g_{ijkp,t} - \partial_b s^b_{ijkp} g_{ijkp,t} + \sum_{k'} \lambda_{k'k} g_{ijk'p,t},$$

where s^x_{ijkp} is shorthand notation for x decision rule at $(a_i, b_j, \varepsilon_k, h_p)$, $x \in \{a, b\}$ and $g_{ijkp,t}$ is a density function at time t. In my model, I need to account for i) movements due to housing transactions, refinancing, bankruptcy and foreclosure, ii) flows between a state without the default flag and a state with the bankruptcy flag and iii) flows between a state without the default flag and a state with the foreclosure flag.

The mathematical formulation of Kolmogorov forward equations with stopping choices is not straightforward.[43] Flows due to stopping decisions can be treated with the *intervention matrix*, M. First, let g_i be the i^{th} element of the density function where $i \in \{1, ...N\}$ and N is the total number of grid points:[44]

[43] See "Liquid and Illiquid Assets with Fixed Adjustment Costs" by Greg Kaplan, Peter Maxted and Benjamin Moll at `http://www.princeton.edu/ moll/HACTproject/liquid-illiquid-numerical.pdf`

[44] $N = n_a \times n_b \times n_\varepsilon \times n_h \times 2 + n_a \times n_\varepsilon$. $n_a \times n_b \times n_\varepsilon \times n_h$ is multiplied by 2 because there are points with and without a bankruptcy flag. The number of points in the state with the foreclosure flag is $n_a \times n_\varepsilon$.

$$M_{i,j} = \begin{cases} 1 & \text{if } i \in I \text{ and } i = j \\ 1 & \text{if } i \notin I \text{ and } j^*(i) = j \\ 0 & \text{otherwise,} \end{cases}$$

where I is the non-stopping region and $j^*(i)$ is the target point of point i. A target point is a point arrived at as a result of a stopping choice such as buying a house. For example, if a household with $(a_i, b_k, \varepsilon_j, h_p)$ decides to buy a house and ends up having $(a_{i1}, b_{k1}, \varepsilon_j, h_{p1})$ as a result of the transaction, the latter is the target point.

The flow from a state with bankruptcy and foreclosure flags to a state without a bankruptcy flag is a shock and can be expressed as below. Let nd represent "non-default" and d represent "default":

$$\partial_t g^{nd}_{ijkp,t} = -\partial_a s^{a,nd}_{ijkp} g^{nd}_{ijkp,t} - \partial_b s^{b,nd}_{ijkp} g^{nd}_{ijkp,t} + \sum_{k'} \lambda_{k'k} g^{nd}_{ijk'p,t} + \lambda_l g^{d}_{ijkp,t},$$

$$\partial_t g^{d}_{ijkp,t} = -\partial_a s^{a,d}_{ijkp} g^{d}_{ijkp,t} - \partial_b s^{b,d}_{ijkp} g^{d}_{ijkp,t} + \sum_{k'} \lambda_{k'k} g^{d}_{ijk'p,t} - \lambda_l g^{d}_{ijkp,t},$$

where $\lambda_l = \lambda_d$ for the bankruptcy flag state and $\lambda_l = \lambda_f$ for the foreclosure flag state. These flows can be treated with matrices A^d and A^f:

$$\mathcal{A}^d_{i,j} = \begin{cases} \lambda_d & \text{if } 1 \leq j \leq n_1 \text{ and } i = j + n_1 \\ -\lambda_d & \text{if } n_1 + 1 \leq j \leq n_1 \times 2 \text{ and } i = j \\ 0 & \text{otherwise,} \end{cases}$$

$$\mathcal{A}^f_{i,j} = \begin{cases} \lambda_f & \text{if } 1 \leq j \leq n_a \times n_\varepsilon \text{ and } i = j + n_1 \times 2 \\ -\lambda_f & \text{if } n_1 \times 2 + 1 \leq j \leq N \text{ and } i = j \\ 0 & \text{otherwise,} \end{cases}$$

where $n_1 = n_a \times n_b \times n_\varepsilon \times n_h$. The sizes of A^d and A^f are $N \times N$, and I stack points from the state without default flags, points from the state with the bankruptcy flag and points from the state with the foreclosure flag.[45] Finally, define B as below:

$$B = \mathcal{A} + \mathcal{A}^d + \mathcal{A}^f,$$

where $\mathcal{A}$ is a block diagonal matrix that is composed of $\mathbf{A}$, $\mathbf{A}^d$ and $\mathbf{A}^f$:

$$\mathcal{A} = \begin{bmatrix} \mathbf{A} & 0 & 0 \\ 0 & \mathbf{A}^d & 0 \\ 0 & 0 & \mathbf{A}^f \end{bmatrix}.$$

Given M and B, the density function can be solved by iterating over the following two steps until g converges:

$$g^{n+\frac{1}{2}} = M^T g^n,$$

$$\frac{g^{n+1} - g^{n+\frac{1}{2}}}{\Delta t} = (BM)^T g^{n+1}.$$

A.3 Stochastic model

The aggregate state contains an argument that has an infinite dimension, g, the distribution of households over $\omega = (a, b, \varepsilon, h, o)$. To make the computation feasible, this distribution needs to be approximated. I assume the households only use a finite set of moments from g to form their expectations, as in Krusell and Smith (1998).[46] Specifically, I assume that the households keep track of the aggregate capital stock, k.

I redefine the problem using the approximate state, (k, z). For example,

---

[45] n_1 is the number of points in the state without flags, and $n_1 \times 2$ is the sum of the number of points of the no-flag state and the bankruptcy flag state.

[46] Ahn et al. (2018) develop a method of solving continuous time heterogeneous agent models with aggregate uncertainty based on linearization and dimension reduction. Fernández-Villaverde et al. (2023) also present a method to solve such models. They assume the households only track a finite set of moments of the distribution to form their expectations as well, but use tools from machine learning to estimate the perceived law of motion of the households.

Equation (2) can be written as

$$\rho v(\omega, k, z) = \max_c u(c, h) + \partial_a v(\omega, k, z)\dot{a} + \partial_b v(\omega, k, z)\dot{b} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(\omega^{\varepsilon_j}, k, z)$$
$$+ \sum_{k=1}^{n_z} \lambda_{zz_k} v(\omega, k, z_k) + \lambda_d (v(\omega, k, z|o = 0) - v(\omega, k, z|o = 1))_{o=1}$$
$$+ \lambda_f (v(\omega, k, z|o = 0) - v(\omega, k, z|o = 2))_{o=2} + \partial_k v(\omega, k, z)\dot{k},$$
$$\dot{k}_t = \frac{\mathbb{E}[dk_t; k_t, z_t]}{dt} = f^k(k_t, z_t).$$

The last term in Equation (2), which captures the evolution of the distribution, is replaced with $\partial_k v(a, b, \varepsilon, h, k, z)\dot{k}$. I assume f has a log-linear form:

$$dlog(k_t)dt = \beta_z^0 + (\beta_z^1 - 1)log(k_t).$$

Given the approximate aggregated state, equilibrium wage rates and interest rates are marginal productivities of labor and capital. Since hours worked is not a choice, aggregate labor is fixed. Therefore, computing wage rates and interest rates at a given capital is trivial. However, in addition to wage rates and interest rates, house prices, $p(k, z)$, are necessary to solve the model. I also assume a log-linear form to estimate house prices:

$$log(p_t) = \phi_z^0 + \phi_z^1 log(k_t).$$

In Section A.1, I described the computational steps in the absence of an aggregate state (g, z). Having approximated the high-dimensional object g with k, now I include terms related to the aggregate state. Like other variables, I discretize k, and n_k is the number of grid points for k. From Section A.1, the only part needing to be changed is the construction of matrix $\mathbf{A}$, which describes $\dot{a}$, $\dot{b}$ and the shocks. Consider the $\mathbf{A}$ in Section A.1 as an $\mathbf{A}$ at a given (k_i, z_j), $\mathbf{A}^{ij}$. Then $\mathbf{A}$ should be replaced with $\mathbb{A}_a + \mathbb{A}_k + \mathbb{A}_z$, where

$$
\mathbb{A}_a = \begin{bmatrix}
\mathbf{A}^{11} & & & & & & \\
& \ddots & & & & & \\
& & \mathbf{A}^{n_k 1} & & & & \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
& & & & \mathbf{A}^{1 n_z} & & \\
& & & & & \ddots & \\
& & & & & & \mathbf{A}^{n_k n_z}
\end{bmatrix}
$$

$$
\mathbb{A}_k = \begin{bmatrix}
x_{11}\mathbf{I} & x_{11}^F\mathbf{I} & & & & & \\
x_{21}^B\mathbf{I} & x_{21}\mathbf{I} & x_{21}^F\mathbf{I} & & & & \\
& & & & & & \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
& & & & & & \\
& & & & & \ddots & \\
& & & & & x_{n_k n_z}^B\mathbf{I} & x_{n_k n_z}\mathbf{I}
\end{bmatrix}
$$

$$
\mathbb{A}_z = \begin{bmatrix}
-\lambda_{z1}\mathbf{II} & \lambda_{z1}\mathbf{II} \\
\lambda_{z2}\mathbf{II} & -\lambda_{z2}\mathbf{II}
\end{bmatrix}
$$

and $-\frac{\dot{k}_{ij}^+}{\Delta k} + \frac{\dot{k}_{ij}^-}{\Delta k} = x_{ij}$,$-\frac{\dot{k}_{ij}^-}{\Delta k} = x_{ij}^B$, $\frac{\dot{k}_{ij}^+}{\Delta k} = x_{ij}^F$, $\mathbf{I}$ is $N \times N$ identity matrix, and $\mathbf{II}$ is $Nn_k \times Nn_k$ identity matrix. I use $n_z = 2$ and $n_k = 10$.

Solution algorithm

1. Guess parameters of the forecasting functions. With the forecasting functions, $\dot{k}$ and the house price over (k, z) are known. Also, interest rates and wages over (k, z) can be computed using the firm's marginal conditions.
2. Solve the value function.
    - Guess the loan price functions, $r_a(\omega, k, z)$ and $q(\omega, k, z)$.
    - Guess the value function, $v^0(\cdot)$.
    - Update the value functions and the loan price schedules until they

converge.

- Save decision rules.

3. Simulate the model for n periods. Simulation gives the sequence of aggregate variables $\{z_t, k_t, p_t\}_{t=1}^n$.
   - Guess the initial distribution. The distribution in the steady state can be used as a good initial distribution.
   - At the beginning of each period, (k, z) are known. The risk free rate and wage can be computed.
   - Compute the loan price functions. To compute $r_a(\omega_t, k_t, z_t)$, interpolate the default decisions that are obtained from step 2. Using the default decisions and the risk free rate, $r_a(\omega_t, k_t, z_t)$ can be computed using Equation (5). To compute $q(\omega_t, k_t, z_t)$, interpolate $q(\omega, k, z)$ obtained from step 2, over k.
   - Guess the house price.
   - With the wage, the loan price schedules and the house price, solve the household problem.
   - Compute the aggregate demand for housing. If the aggregate demand is not close enough to the supply, adjust the house price to clear the housing market.
   - Once the housing market is cleared, move to the next period.
4. Using the sequence of aggregate variables $\{z_t, k_t, p_t\}_{t=1}^n$, update the forecasting functions.
5. Check the convergence of the simulated aggregate variables. If the distance between the $\{k_t\}_{t=1}^n$ from the current iteration and the previous iteration is less than a tolerance level, an approximate recursive equilibrium has been found. Otherwise, go back to step 2 with the updated forecasting functions.

Predictive power of the forecasting functions

Table 10 shows the R^2 of the forecasting functions. Since forecasting functions are conditional on the aggregate state, there are R^2s for both expansions and recessions. The R^2s are high for k'. Capital moves slowly and this makes

Table 10: R^2 of forecasting functions

	Benchmark		Foreclosure only	
	Exp.	Rec.	Exp.	Rec.
k'	0.999	0.999	0.999	0.999
p	0.988	0.972	0.966	0.948
Procyclical tax-rate				
k'	0.999	0.999	0.999	0.999
p	0.984	0.968	0.954	0.938

Note: Based on a simulation of 4,000 periods.

the forecasting function very accurate for any version of the model. R^2s for house prices are slightly lower and they are lower during recessions than expansions. This is probably because housing choices are discrete.