An Estimation of the Real Interest Rate Component of the Nominal Interest Rate for Long-term Industrial Bonds

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Prepared for:

Department of Regional Economic Expansion
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Introduction:

The experience of significant rates of inflation in North America during the past five years has created a great deal of interest in the analysis of the effect of inflation on financial markets. One important aspect of inflation is its impact on nominal interest rates as determined in the capital markets. This study will examine the determinants of the nominal interest rates paid on corporate bonds and will estimate for the period 1959 to 1974 the real rate of interest the borrowers expected to pay and lenders receive when these loans were transacted.

The nominal rate of interest reflects the opportunity cost of holding a security, which has a principal value fixed in nominal terms upon maturity, rather than investing in physical assets or stocks which yield a return made up of the increase or decrease in the current value of the assets as well as a stream of income received during the period the assets are held. This opportunity cost to holding a bond also includes the expected erosion in the real purchasing power of the principal repayments that is to occur because of inflation during the length of life of the bond. Thus, the nominal interest rate (i) that is paid to the holder of a bond contains two components; (a) the real rate of interest (r*) that was expected to prevail over the length of life of the security, and (b) the expected rate of change of the price level (gP*).

$$(1) \quad i = r^* + gP^*$$

The expected real rate of interest in the long-run is primarily determined by the marginal productivity of capital. In the short-run, changes in this variable are often initiated by changes in the supply of money, shifts in the demand for money, changes in the demand and supply of credit, and expectations of exchange rate changes.

The expectation that inflation will occur during the term of a loan causes lenders to be unwilling to buy nominally denominated securities—unless the nominal interest paid by the security is equal to the expected real return obtainable on other assets, plus the expected rate of decline in real value of the nominally denominated security due to inflation. If the borrowers who sell these securities and purchase physical capital items or equity with these funds have the same expectations with respect to inflation, they should be willing to pay a higher nominal interest rate to compensate for inflation because the nominal values of the assets they purchase will be increasing at approximately the rate of inflation.

In this study an interest rate equation will be estimated which attempts to explain the movements in real interest rate caused by changes in the money supply, the size of the government deficit and other macro-economic variables. In addition, an estimation of the expected rate of inflation component of the nominal interest rate will be made. From this equation the estimated values for the nominal interest rate $\hat{i}$ can be derived from the parameters and values of the variables in this equation. The estimated value of the real interest component of nominal interest rate is then calculated by subtracting the estimated values of the expected growth in prices $\hat{gp}^*$ from the estimated nominal interest rate as follows:

(2) $\hat{r}_t = \hat{i}_t - g\hat{p}^*$

Where "^" refers to the estimated value of the variable derived from the statistically fitted interest rate equation.

We now turn to a brief discussion of the theory of interest rate determination in order to choose which variables should be included in the estimated interest rate equation and the direction these variables are expected to influence interest rates.

To understand the adjustment of interest rates to changes in monetary policy we must have a more complete theory of the demand for money which considers variables other than just interest rates. Following the traditional formulation of the demand function for money, the variables which are important in determining the demand for money include: the level of real wealth (W) held by individuals, the ratio of human to non-human wealth (h), the level of current real income (Q), and price level (P). The demand for nominal money balances can be summarized as follows:

(3) $M_d = f(r^*, gP^*, Q, W, h)\ P$ 1/

The quantity of money demanded being inversely related to the expected real interest rate, and the expected rate of growth of the price level,

$$(\partial M_d / \partial r^*) < 0 \qquad\qquad (\partial M_d / \partial gP^*) < 0$$

positively related to the level of real income, real wealth, and the ratio of human to non-human wealth,

$$(\partial M_d / \partial Q) > 0 \qquad (\partial M_d / \partial W) > 0 \qquad (\partial M_d / \partial h) > 0$$

and proportional to the price level.

$$(\partial M_d / \partial P) = 1$$

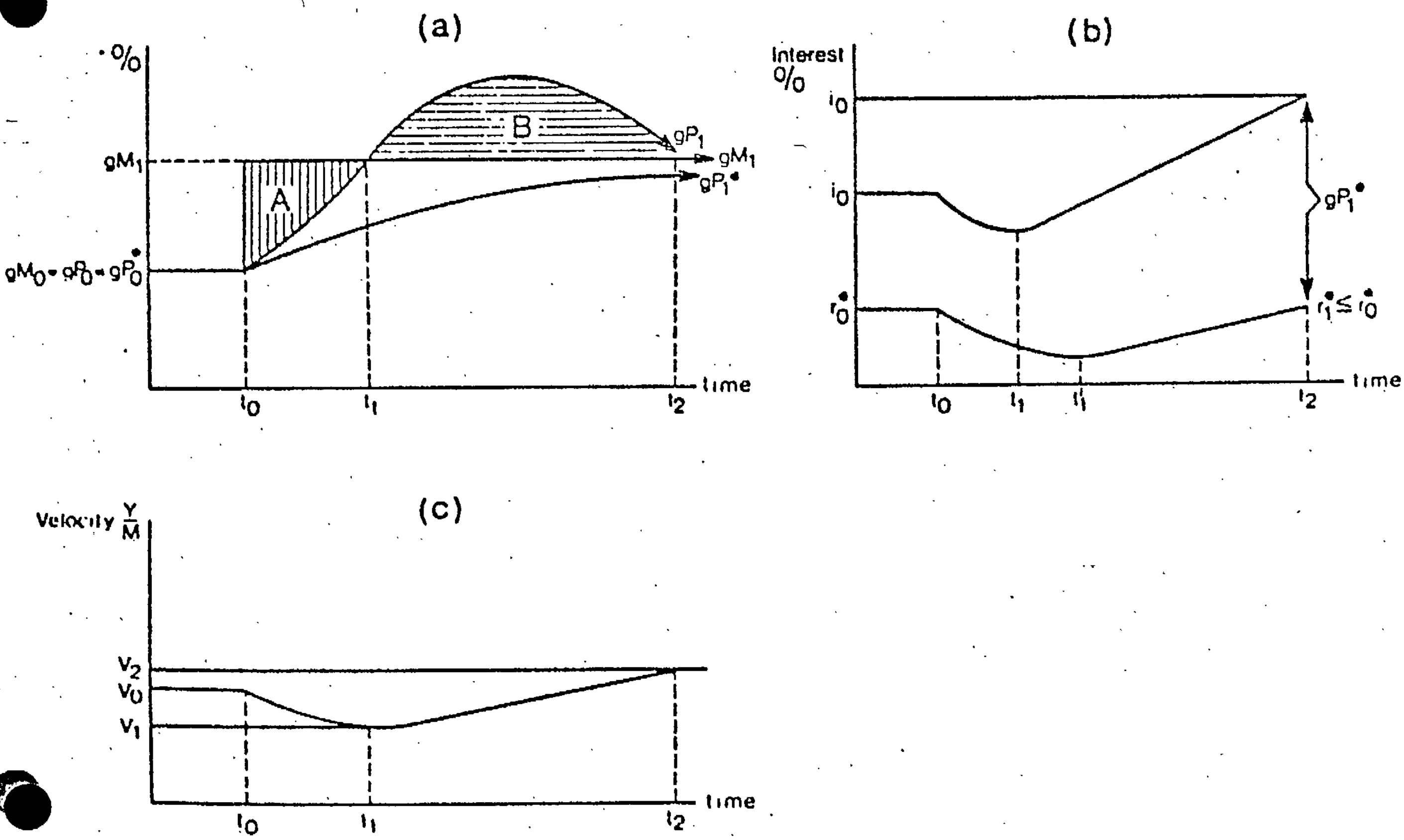
$$(4) \quad M_s = M_0$$

We initially assume that the nominal supply of money (M_s) is exogenously determined by the Canadian Central Bank.

The relationship through time between the growth of the money supply, the level of interest rates, the growth of prices, and the velocity of money is shown in Figure 1. Initially, we assume the economy is in equilibrium. Until period such as (t_0), the growth rate of the price level (gP_0) is equal to the growth rate of the nominal stock of money (gM_0), which is also equal to the expected growth rate of prices (gP^*_0). Real output is assumed to be constant; with these assumptions, the stock of real money balances must also be constant.

At time period (t_0), the growth rate of the nominal stock of money is exogenously increased to gM_1, which is greater than the growth in demand for the nominal money stock. Because it takes time for individuals to realize that actual real cash balances are greater than desired real cash balances, an economy normally would not respond immediately by equating the growth of the price level to gM_1, but usually prices for a period will grow more slowly than the new growth rate of nominal money stock and prices in Figure 1a, we see that at the time the growth in prices catches up to the growth in the stock of money (shown as period t_1), velocity (Figure 1c) is at its lowest value (V_1). If the growth rate of prices does not immediately adjust to the growth in the nominal money stock, then nominal interest rates must have fallen (Figure 1b) so that individuals are willing, at least temporarily, to hold the increased real cash balances. Also if velocity is to return to its initial value, the rate of growth of the price level must for some time exceed the growth rate of nominal monetary expansion.

Figure 1



In this model the expected growth of prices is assumed to adjust with a lag to the actual growth rate of the price level. Since the expected inflation is a cost to holding cash balances, people will demand fewer real cash balances causing the final equilibrium velocity (V_2) to be greater than the initial velocity (V_0). Because people expect a higher growth in prices, fewer cash balances are ultimately desired, the path of the growth in the price level (Figure 1a) must be such that the shaded area B is larger than the area A so that real cash balances are decreased below their previously desired level.

At time that velocity is at a minimum, nominal interest rates will also have fallen to their minimum level. The expected real rate of interest may reach its minimum at this point or at some later date depending on whether the price expectations have started to increase. If they have, then the demand for money decreases, forcing the level of the expected real interest rates to fall further. The liquidity effect of the change in monetary policy, as measured by the fall in the real interest rate, reaches a maximum at time period t_1, or if price expectations respond quickly at a slightly later date (t_1'). From period t_1 to t_2, the growth rate of prices is greater than the growth rate of nominal money, causing velocity to increase. The liquidity effect declines to zero and the expected real rate of interest r^* rises to a point such as r_1^*, slightly less than r_0^*.[2/] Finally, as the expectations concerning the growth of the price level increase, a gap emerges between the expected real rate of interest and the nominal interest rate equal to the size of the expected rate of inflation.

Interest rates will also react to some types of government expenditure policies. Government expenditures are usually financed either by increasing taxation, selling of bonds to the central bank (increasing nominal stock of money), or by borrowing from the public. We would expect the adjustment of interest rates to differ depending on the method by which the government expenditures are financed. An increase in government expenditure financed by borrowing from the public is expected to cause an increase in interest rates as it involves a direct bidding against private sector investors for the available savings in the economy.

In an economy where the elasticity of private consumption, investment expenditures and the demand for money are all negative and finite, a <u>change</u> in the level of government expenditures financed by borrowing leads to an adjustment of the expected real rate of interest in the same direction. For an expansion in government expenditure and borrowing, the increase in the real rate of interest comes about because the public require a higher rate of interest to induce them to give up other real investment, consumption, or private securities. Because of the magnitude of the government sector in the capital markets in Canada an equation to explain movements in market interest rates must take into consideration the rate of growth at which the Government sector is expanding the real value of its obligations.

As the demand for cash balances is positively related to changes in real output then we would expect to find that some of the movements of the real rate of interest can be explained by the growth of real output in Canada. This relationship would be reinforced if the demand for new investment in real assets is positively related to increases in real output. While economic theory would justify the inclusion of this variable in our final equation we find that the coefficients for this variable are either insignificant or do not have the expected signs.

Because of the close relationship between the capital markets in Canada and the United States we would expect to find that expectations of forthcoming changes in the exchange rates would cause capital flows of sufficient magnitudes to alter Canadian short term interest rates. When the Canadian exchange rate (defined as number of Canadian $ per U.S. $) is expected to increase then there would be a capital outflow from Canada causing Canadian rates to rise for the period that the exchange rate is expected to increase. The converse would hold for an expected decline in the exchange rate.

Because of the short time period over which expectations concerning changes in the exchange rate are formed we would expect that this variable would have little significance in explaining fluctuations of long term interest rates but would play a more important role in explaining movements of interest rates for short term securities.

Specification of Interest Rate Equation

To correctly measure the effect of monetary policies, we must construct an explanatory variable which will produce a zero liquidity effect once the monetary stock has grown for a long enough period at a constant rate, so that its total effect is reflected in the growth of real output and the expected growth in prices.3/ Also such a variable must not be constrained so that the liquidity effect appears to have its largest value immediately after the change in monetary policy.4/ As shown by Figures 1b and 1c, we would expect to find that the liquidity effect reaches its maximum value in a later period after the supply of cash balances has had a chance to either accumulate or be drawn down because of the change in monetary policy. A distributed lag function (almon lag see statistical appendix B) of current and past changes in the growth rate (acceleration) of the nominal stock of money is used as an explanatory variable of nominal interest rates. This variable identifies the liquidity effect of a change in monetary policy and allows the correct distribution of this effect over time to be determined statistically.

The impact of the growth in real output is measured as a lagged function of current and past quarterly growth rates of real gross national product in Canada. Similarly, the effect of government borrowing is estimated as a function of current and past values of the growth rate over each quarter of the real stock of government debt held by the public. The real stock of government debt held by the public is calculated by deflating the current value of the government debt by the implicit gross national expenditure deflator.

The expected change in the exchange rate is calculated by subtracting the 90 day forward exchange rate from the current period's spot exchange rate and this difference is then expressed as annual percentage of the current period's spot exchange rate. This variable is included twice in the interest rate operation. The second time multiplied by a dummy variable that has a value of 0 for the fixed exchange rate period 1961 IV to 1970 IV and 1 for all other periods when a flexible exchange rate policy was being followed in Canada.

Because of the difficulty of predicting the timing of the influence on the growth of prices from monetary and fiscal policies, we estimate the expected rate of change of prices directly as a function of past changes of the price level. Expectations of future changes in prices are assumed to be formed from observed price changes in an adaptive manner. The expected change in the price level is estimated from the cumulative values of the coefficients for the lagged quarterly growth rates of the implicit gross national expenditure deflator. If current expectations of future price changes are fully adaptive to past changes in prices the sum of the coefficients on the lagged price variables should sum to approximately one.

The quarterly values of the stock of money, nominal gross national product, nominal government debt and the implicit gross national expenditure deflator were seasonally adjusted. A more detailed discussion of the formulation and sources of the variables used in this equation is contained in Statistical Appendix

The form of the interest rate equation estimated is as follows:

$$(5) \quad i = C + \sum_{0}^{n} a_i DGM_{t-i} + \sum_{0}^{k} b_i GY_{t-i} + \sum_{0}^{m} d_i GDT_{t-i} + e_i EX + f_i DEX + \sum_{1}^{q} g_i GP_{t-i}$$

where:

i_t is nominal interest rate for the McLeod, Young, and Weir composite rate for 10 industrial long term bonds, published in Bank of Canada Review.

DGM_t is the change in the growth rate per quarter at annual rates of the seasonally adjusted nominal money supply of Canada, defined as total currency, demand, and time deposits.

GY_t is the growth rate per quarter at annual rates of the seasonally adjusted real gross national production in Canada.

GDBT is the growth rate per quarter at annual rates of the seasonally adjusted real government debt held by the public.

EX_t is the expected change in the exchange rate expressed as an annual rate and defined as follows:

$$EX_t = \frac{\text{spot exchange rate} - \text{90 day forward exchange rate}}{\text{spot exchange rate } t} \times 400$$

DEX_t is EX_t multiplied by a dummy variable which has a value of 0 from 1961 IV to 1970 IV and a value of 1 for all other periods.

GP_t is the growth rate per quarter at annual rates of the seasonally adjusted implicit gross national expenditure price index.

The econometric estimation of equation (5) is presented in Table 1 for the case of the long term industrial bond interest rate in Canada.

The overall explanatory power of equation (5) is quite good with the independent variables explaining over 88 percent of the total variation in the long term interest rate for industrial bonds. For the period 1956 I to 1973 IV the mean of this long term interest rate is 6.945 percent yet the standard error of estimate for the estimated equation is only .474 of a percent. Figure 2 contains a plot of the actual nominal interest rate the estimated nominal interest rate from equation (5) and the estimated real interest rate for the period 1959 III to 1973 IV. Several other specifications of equation (5) were estimated including a longer distributed lag function on the monetary policy variable and the price expectation variable. Also different degrees of polynominal for the distributed lag functions were used. The estimated real rate of interest differed insignificantly between any of the specifications. The equation in Table 1 represents one of the simplest forms and the one with the soundest theoretical justification for the choice of degree of polynominal in the distributed lags.

From the coefficients on the acceleration of the growth of the Canadian nominal money stock (DGM_t) we find that the liquidity effect of a change in monetary policy starts from its initial effect of lowering interest rates in the current quarter to produce its maximum effect three to four months after the change in monetary policy. For a 10 percent change in the growth rate of the nominal stock of money the maximum liquidity effect on the long term industrial bonds is .4 of one percent. The pattern of the coefficient would indicate that within 18 months after a change in monetary policy its liquidity effect on interest rates will have disappeared, even if the nominal money supply continues to grow at its new rate.

The growth of real output (GY col. 5 Table 1) has had a negative effect on interest rates as measured by the coefficients on this variable in equation (5) rather than a positive effect as would be predicted by economic theory. However, the close relationship between interest rates in the U.S. and Canada would lead us to expect that the growth in

TABLE 1

Estimation of Equation $i_t = C + \sum_{i=0}^{5} a_i DGM_{t-i} + \sum_{i=0}^{6} b_i GY_{t-i} + \sum_{i=0}^{6} d_i GDBT_{t-i} + e\, EX_t + f\, DEX_t + \sum_{i=1}^{13} g_i GP_{t-1}$

1 Interest Rate	2 Estimation Period	3 C	4 a_i	5 b_i	6 d_i	7 e	8 f	9 g_i
10 Industrial bond rates	1954 I to 1973 IV	4.534 (14.24)	a_0=-.0284 (-2.038)	b_0=-.0272 (-1.795)	d_0= .0104 (0.911)	0.0896 (0.7569)	0.1394 (0.8301)	e_1=.0356 (1.245)
			a_1=-.0366 (-3.090)	b_1=-.0261 (-2,646)	d_1= .0140 (1.980)			e_2=.0593 (2.966)
			a_2=-.0415 (-3.436)	b_2=-.0240 (-2.512)	d_2= .0159 (2.848)			e_3=.0786 (6.148)
			a_3=-.0416 (-3.655)	b_3=-.0211 (-1.913)	d_3= .0161 (2.678)			e_4=.0935 (12.536)
			a_4=-.0357 (-2.841)	b_4=-.0172 (-1.506)	d_4= .0146 (2.314)			e_5=.1040 (18.435)
			a_5=-.0222 (-2.019)	b_5=-.0124 (-1.255)	d_5= .0114 (2.051)			e_6=.1100 (14.833)
			Sum of a_i = -0.206	b_6=-.0066 (-1.092)	d_6= .0066 (1.873)			e_7=.1110 (11.405)
			Average Lag = 2.418	Sum of b_i = -0.134	Sum of d_i = 0.892			e_8=.1090 (9.459)
								e_9=.1010 (8.283)
			DOP = 3	Average lag = 2.287	Average lag = 2.794			e_{10}=.0903 (7.508)
				DOP = 2	DOP = 2			e_{11}=.0743 (6.963)
								e_{12}=.0540 (6.560)
								e_{13}=.0292 (6.251)
								Sum of e_i=1.0522
								Average lag=6.907
								DOP = 2

t values are in brackets, DOP = Degree of polynomial for Almon Lag function

R^2 (corrected) = .888 — Sample period = 1954 I to 1973 IV

S.E. = .474 — Fixed exchange rates from 1961 IV to 1970 IV, dummy variable D = 0 on EX variable for this period.

Canadian output might not be as important in explaining changes in real interest rates in Canada as the growth in output of Canada and the U.S. combined.

The growth in real government (GDBT Col 6, Table 1), has the expected positive effect on the long term industrial interest rate. A 10 percent growth in the real value of government debt for a quarter will alter the long term interest rate by approximately .15 of a percent point for between four and six quarters.

Expectations of future changes in the Canadian exchange rate (EX col. 7, Table 1, and DEX col. 8, Table 1) are found to not have a significant impact on long-term industrial interest rates in Canada. However, for short term interest rates these variables are significant and thus have been left in the long term interest rate equation for theoretical completeness even if they add very little explanatory power to the equation.

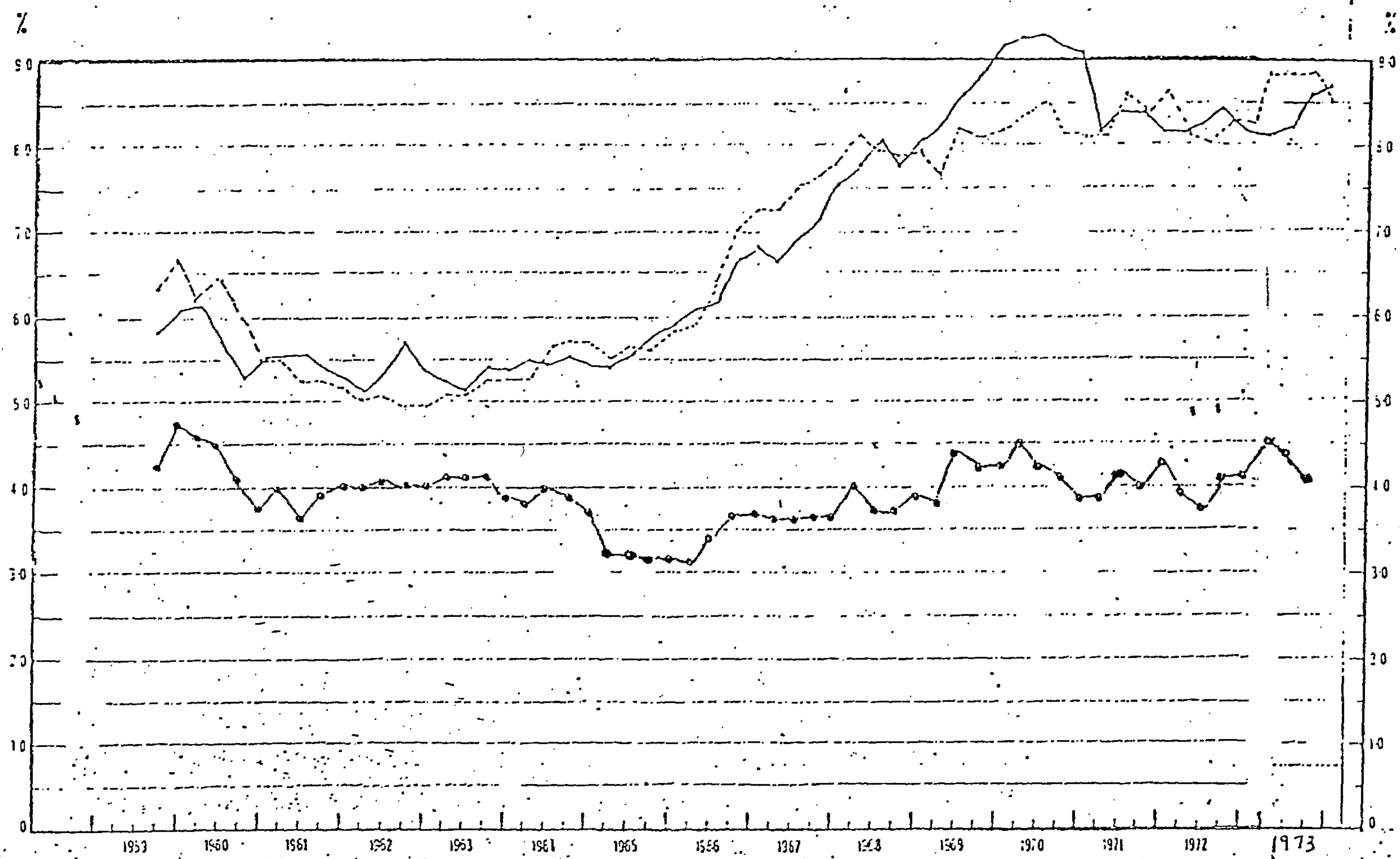
Past changes in prices are very significant in explaining the current level of the long term nominal interest rate on industrial bonds. We find that after 13 quarters the total effect of a change in the growth rate of prices is reflected in the long term nominal interest rate, i.e., the coefficients for lagged prices sum to approximately one. It is the coefficients of the lagged growth in prices that are used to estimate the component of the nominal interest rate that reflects the expected growth in the price level.

Using the coefficients for equation (5) that are presented in Table 1 the estimated nominal interest rate and the estimated expected growth in prices are calculated for each period. With these two values for each period and using equation (2) we are able to calculate the estimated real interest rate for long term individual bonds. The movement of this variable through time is shown in Figure 2.

Table 2 summarizes the relationship between the actual and estimated long term nominal interest rate and the estimated real rate of interest. For the entire period 1959 III to 1973 IV the average actual and estimated nominal interest rate was 6.9 percent while the estimated real interest rate is calculated to be 4.0 percent. For the five year sub period of 1969 I to 1973 IV the values for these three variables are 8.6, 8.4 and 4.1 percent, respectively. Similarly, for the period from 1964 I to 1968 IV the values for these variables are 6.5, 6.7 and 3.6 percent, and for the period 1959 III to 1963 IV they are 5.5, 5.6 and 4.2 percent.

Figure 2

Comparison of Actual and Estimated Nominal Interest Rate with Estimated Real Interest Rate for Long Term Industrial Bonds.



LEGEND

OBSERVED

CALCULATED

REAL RATE

Table 2

Actual and Estimated Nominal Interest Rate and the Real Interest Rate Component of Long Term Industrial Bonds

Period	Mean of Actual Nominal Rate	Mean of Estimated Nominal Rate	Mean of Estimated Real Rate
1959 III to 1973 IV	6.9	6.9	4.0
1969 I to 1973 IV	8.6	8.4	4.1
1964 I to 1968 IV	6.5	6.7	3.6
1959 III to 1963 IV	5.5	5.6	4.2

From this analysis we find that while the nominal interest rate has ranged considerably over time from an average of 5.5 percent in the 1959 to 1963 period to 8.6 percent in the 1969 to 1973 period the movement in the estimated real rate of interest has only been approximately .6 of a percentage point. Therefore, while the real rate of interest might move temporarily because of changes in monetary and fiscal policy it is not subject to the influence of expectations of price changes which is the the primary cause of the large movements in nominal interest rates through time.

In the derivation of the real financial cost of capital before considering taxation matters it is the real rate of interest of approximately 4 percent that is relevant not the nominal interest rate that is currently in the range of 10 percent. The difference between these two rates of interest is a compensation paid to lenders because of the decrease in purchasing power of the future loan repayment dollars that is expected because of this inflation. This compensation is essentially part of the repayment of the real principal value of the loan and should not be included in the real time value of money used for financial purposes.

FOOTNOTES

1. For a more complete discussion of the demand for money see: Milton Friedman, "The Quantity Theory of Money, A Restatement", "The Demand for Money: Some Theoretical and Empirical Results", and "The Supply of Money and Changes in Prices and Output", in Milton Friedman, The Optimum Quantity of Money and Other Essays, Aldine Publishing Company, 1969.

2. The real rate of interest may not return to its initial level because the rise in the expected rate of inflation causes people to demand more real assets, bidding the price up or alternatively lowering the real rate of interest. See Robert Mundell, "Inflation and Real Interest", Journal of Political Economy, June, 1963.

3. In some of the previous estimations of the interest rate equation (e.g., Gibson, 7) the rate of change of the nominal money stock or the level of the nominal money stock was used as the variable to explain the liquidity effect. However, the effect of this variable does not disappear when prices begin to grow at the same rate as the growth in money. These variables may produce a bias in the estimated interest rate when the rate of change of the money stock has remained constant for a longer than average period.

4. Carr and Smith (2) tried to measure the liquidity effect by constructing a variable which is the difference between the actual and expected growth in the nominal money stock. The expected growth of the nominal stock of money is made a function of past changes in the actual stock of money. This variable takes on its largest value in the initial period of the change in the growth of the monetary stock, and thus forces the liquidity effect to be at its maximum during this period.

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