Total Factor Productivity (TFP) in a Telecommunications Utility as a Measure of Efficiency and as a Regulatory Tool

A Note on Interfirm Comparisons

Presented to the third International Conference on
Analysis, Forecasting and Planning
for Public Utilities

Fontainebleau
25-29 June, 1980

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INTRODUCTION

Comparisons in the performance of countries, sectors or firms within a sector are a common occurence. The problem with most comparisons is that the framework within which such comparisons are meaningful is not properly specified. A recent example would be the conflicting claims made by the IREST in a 1978 bulletin and by Bell Canada in a recent newsletter to their customers. The IREST quoted a German source which ranked Canada eighth among ten Western nations in terms of cost of telephone services to the customers. Bell Canada used a United Nations (1978) source to indicate that Montreal was the cheapest of a whole list of major cities of the Western World.

Comparisons are particularly tempting when one is looking at productivity performance, however there, as much as anywhere else, one can find the temptation to make superficial comparisons. A recent example would be Caves, Christensen, and Swanson (1980)'s attempt to associate the observed productivity difference between U.S. and Canadian railroads with the regulatory process without either modelling or quantifying it.

It is the object of this paper to present a formal framework to analyse the difference in total factor productivity between two firms. The paper starts from the model developed by Jorgenson and Nishimizu (1978) and from the TFP analysis developed in Denny, Fuss and Everson (1979). It draws heavily from the first synthesis of those two approaches presented in Denny, Fontenay and Werner (1980). It utilizes Fuss and Waverman (1978) together with recent research done for the Department of Communications by Corbo *et al* (1978), Corbo *et al* (1979), Corbo and Smith (1979), Corbo, Breslaw and Smith (1979), Breslaw and Smith (1980) and finally by Breslaw (1980).

In the second chapter, total factor productivity is introduced and the underlying assumptions are analysed. In the next chapter, the Denny, Fuss and Everson TFP decomposition analysis is presented and expended to cover the short run problem. The striking feature of this approach is that it makes it possible, in principle, to decompose an observed productivity growth in terms of the contributions of scale effects, non-marginal pricing effects, rate of return regulation constraints and short run constraints. As noted by Gollop (1980), this analysis goes a long way toward bridging the gap between the Kendrick-Denison approach and the Jorgenson-Griliches (1967) one. Jorgenson and Nishimizu (1978) expanded the total factor productivity analysis to the comparative analysis of productivity and Denny, Fontenay and Werner (1980) generalized the approach by removing such constraints as constant return to scale (CRS),... The last chapter presents and develops these results and shows how the observed difference in productivity can also be decomposed in scale effects,... This analysis requires the use of econometrics to disentangle the various elements in productivity differences. Even though Fontenay (1980) indicates that index number methods may be properly applicable to processes which are not CRS, they cannot be used for TFP decomposition. This is a major problem in telecommunications since few adequate historical series are available to do any econometrics.

Last of all, it should be noted that we start from the assumption that the data have already been reconciled between the firms to be compared. Even though data reconciliation is not the object of this paper, it must be emphasized that a poor reconciliation would destroy the validity of the proposed approach and that the task of data reconciliation is certainly more formidable than the proposed analysis.

We do not address the problem raised by the comparison of more than two firms.

TOTAL FACTOR PRODUCTIVITY

Introduction

Total factor productivity (TFP) is the ratio of total output to total input, i.e. if total output is denoted by Y and total input by X, then

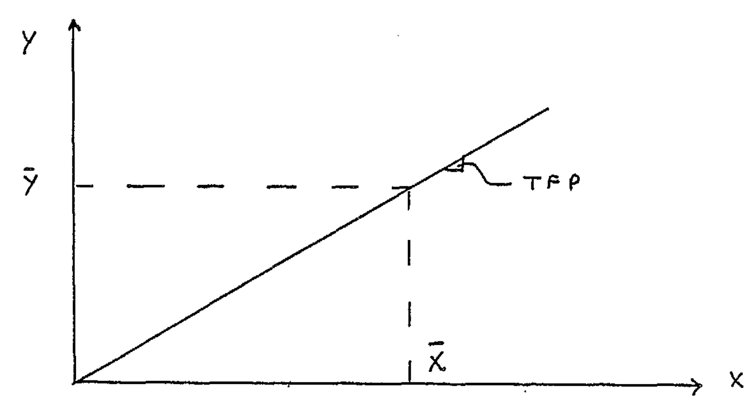
$$TFP = \frac{Y}{X}$$

The relationship between TFP, the output Y and the input X can be illustrated by a simple example. Let's assume that the only input is labour, i.e. X = x where x is a quantity of labour service, and that there is only one output, i.e. Y = y where y is the quantity of that output. Then to any level of input x will correspond a maximum level of output y beyond which one cannot produce on a continuous basis and, equivalently, to any level of output y will correspond a minimum quantity of labour x needed to produce it. The relationship between the output and the input is the production function:

$$y = f(x)$$

which is illustrated in Fig. 1, on the assumption of CRS.

Figure 1: Total Factor Productivity of a CRS Process



It should be noted that, as TFP corresponds to the slope of the production function in the example, it is constant and independent of the level of production. Furthermore, by a simple rescaling of either the output, or the input, or both, the TFP measure can be made to take any value. To avoid this problem, it is convenient to measure the inputs and outputs as index numbers. In addition, it is convenient to scale the input and the output to be equal in the base year.

In practice, we expect to have a production process which shifts through time due to the impact of technological change, i.e., we expect the relationship between Y and X not to be time-invariant, but dependent upon some technological variable t which could simply be time or, alternatively, some measure of technological development such as, say, DDD (direct distance dialing). Let's assume the production process is described by

$$Y = T(t)f(X)$$

where T(t) is some monotonic function of t, then, if the function exhibits CRS as in Figure 1,

$$TFP = k.T(t)$$

i.e., the TFP measure is proportional to the contribution of the technology.

In practice, more than one input will be needed to produce more than one output, and the first problem which arises is that of the aggregation of each of the inputs and of the outputs respectively to obtain X and Y. In fact, we shall see that there might not exist an aggregation which can be analysed in the economic context of the production function.

Hypotheses

In this section, we shall expand on the preceding introduction to investigate the assumptions which must be made to analyse within the context of production theory TFP measures. We shall, nevertheless, ignore for the time being, the fact that more than one output is being produced.

(i) The output y is produced at some time t from a set of inputs (x_j; $j = 1,2\ldots,m$). By this it is assumed both that given the set (x_j) of inputs, no more than y can be produced by the given firm at that given time, and that given y and given any input ℓ, $1 \leq \ell \leq m$, and the subset of input quantities (x_j; $j = 1,2,\ldots,m$; $j \neq \ell$), at present x_ℓ of input ℓ is necessary.

(ii) The relationship between the maximum output which can be produced, y, and the minimum quantity of inputs needed to produce it (x_j) is dependent upon a "technological" variable t which could be time or some other measure of technology such as DDD (direct distance dialing),...(Breslaw and Corbo, 1979).

(iii) The relationship between y, (x_j) and t is the production function:

$$F(y,x_1,x_2,\ldots,x_m,t) = 0$$

(iv) The production process is positive linear homogeneous (PLH) with respect to the inputs, i.e. given $t = t_0$

any level of technology, λ any positive constant, y_λ the level of output which results from an increase by a factor λ of all inputs, such that

$$F(y_\lambda, \lambda x_1, \lambda x_2, \ldots, \lambda x_m, t_0) = o$$

then

$$y_\lambda = \lambda y$$

i.e. the output increases in the same proportion λ as all the inputs. In other words, there is CRS.

(v) The production process is separable, i.e. that, as technological change occurs, there is an aggregate level of input X which is independent of the level of technology such that

$$y = T(t)X(x_j;\ j = 1,2,\ldots,m)$$

(vi) Producers are profit maximizers.

(vii) There is perfect competition.

It should be noted that, in the absence of institutional barriers to competition, the last hypothesis can be derived from the others since independently of the number of producers, given the optimizing behaviour as specified in (vi), PLH ensures that an entry is always possible, regardless of the size.

Now the function X is an aggregator function of the inputs x_j and it is a natural choice for the aggregate input. Since its scaling is arbitrary, in terms of the comments made in the previous section, it is natural to set $T(t_0) = 1$, and $X_0 = Y_0$. Then

$$TFP_t = \frac{Y_t/Y_0}{X_t/X_0}$$

Now, by PLH and Euler's theorem,

$$X(x_j;\ j=1,2,\ldots,m) = \sum_{j=1}^{m} X_j x_j$$

where $X_j = \partial X/\partial x_j$

Then, as a result of profit maximization and perfect competition,

$$T(t)X_j = w_j/p$$

where p is the price of the output y

w_j that of the jth input.

Assuming that $p_t = p_0$, since, then, $w_{j,t} = T(t)w_{j,0}$,

$$TFP_t = \frac{p_0 y_t / p_0 y_0}{\sum_{j=1}^{m} w_{j,0} x_{j,t} / \sum_{j=1}^{m} w_{j,0} x_{j,0}}$$

i.e. the TFP measure is obtained as a Laspeyres index of output divided by the Laspeyres index of inputs.

It is easy to see that the TFP measure thus introduced will not fluctuate very much with fluctuation in the production process. To emphasize the fluctuations, one usually looks at the total factor productivity growth.

If the rate of change in a variable z through time is denoted by $\dot{z}$, then

$$\dot{TFP} = \dot{Y} - \dot{X}$$

and, if we return to the production function, totally differentiating it, we obtain

$$\dot{y}_t = \dot{T}(t) + \sum_{j=1}^{m} s_{j,t}\,\dot{x}_{j,t}$$

where $s_{j,t}$ is the share of the expenditures spent on factor x_j, i.e.

$$s_{j,t} = (s_{j,t}x_{j,t}) / \sum_{j=1}^{m} (s_{j,t}x_{j,t})$$

In other words, a natural aggregator for the growth rate in the input, $\dot{x}_{j,t}$, given the earlier hypotheses, is the Divisia index of inputs:

$$\dot{X}_t = \sum_{j=1}^{m} s_{j,t}\dot{x}_{j,t}$$

Moreover, under the above set of hypotheses, the technological contribution T(t) and the TFP measure are one and the same. In most of the modern literature on total factor productivity, because of the possibility that the production function may be non-separable, hence that there does not exist an aggregate input and an aggregate output which are, in terms of production analysis, independent of one another, the attention has centered on the contribution of the technological variable t rather than on the ratio of output to input.

Multiple Output Processes

While, in the introduction, we talked of output and input aggregator functions respectively, in the preceding section, we considered only the single output production function. We may now generalize the approach to the multiple output production function.

Let (y_i; $i=1,2,\ldots,n$) be the set of outputs produced from the set of inputs (x_j; $j=1,2,\ldots,m$) such that, given any ℓ, $1 \leq \ell \leq n$, (y_i, $i=1,2,\ldots,n$, $i \neq \ell$) and (x_j), y_ℓ is the maximum quantity of the output ℓ which can be produced under the existing technology, and, given any k, $1 \leq k \leq m$, (y_i) and $(x_j, j=1,2,...,m, j \neq k)$, x_k is the minimum quantity of the kth input necessary for the associated production process under the existing technology. The production process can be represented by a production function:

$$F(y_i, x_j, t;\ i=1,2,...,n,\ j=1,2,...,m) = 0$$

The separability hypothesis implies that there exist functions Y, T, and X such that

$$Y(y_i;\ i=1,2,...,n) = T(t)X(x_j;\ j=1,2,...,m)$$

Scaling T(t) such that $T(t_0) = 1$, it is natural to take Y and X as outputs and inputs aggregator functions, hence to define total factor productivity as

$$TFP_t = \frac{Y_t/Y_0}{X_t/X_0}$$

CRS implies that $X(\lambda x_j;\ j=1,2,...,m) = \lambda X = \lambda Y$ and that $\lambda Y = Y(\lambda y_i;\ i=1,2,...,n)$ for all positive scalar λ. By Euler's theorem, then,

$$Y(y_i;\ i=1,2,...,n) = \sum_{i=1}^{n} Y_i y_i$$

where $Y_i \doteq \partial Y/\partial y_i$

Perfect competition and profit maximization, in this context, imply

$$p_i \equiv (Y_i/Y_1) \qquad i = 1,2,\ldots,n$$

$$r_j \equiv T(t)X_jY_1 \qquad j = 1,2,\ldots,m$$

The rate of change in TFP is given by

$$\dot{TFP} = \dot{Y} - \dot{X}$$

where $\dot{Y}$ and $\dot{X}$ are derived by totally differentiating the aggregator functions Y and X so that

$$\dot{Y} = \sum_{i=1}^{n} s_{i,t}\dot{y}_{i,t}$$

where $s_{i,t}$ is the share of the revenues received from the ith output.

DECOMPOSITION OF THE TFP MEASURE

Introduction

The theory of TFP measurement presented in the preceding section is not without serious problems when applied to most sectors, and certainly to the Canadian telecommunications sector.

Accumulated evidences force us to question and even reject assumptions of separability, of P.L.H., of perfect competition in the factors market,... Furthermore, most carriers, and, at least all federally regulated carriers are in principle regulated in terms of an allowed rate of return, even though it is not yet clear whether the rate of return (ROR) regulation is an effective and binding constraint (Breslaw, Corbo and Smith, 1979). Either in place or in addition to an ROR regulation, one can observe quasi-universally at least some price regulation; thus it seems reasonable to propose that the carrier sets its basic local rates so as to maximize profits. Rather one can propose that basic local rates are set by the regulators (Corbo et al, 1979). Finally, it is known that telecommunications carriers work in terms of construction plans defined over many years and that unless drastic changes appear - as the increase in the price of fuel for electric public utilities - the carriers have much less flexibility to modify their production process in the short run than in the long run.

Even when we accept the neo-classical theory of production as a tool to analyse TFP measures, all of the above factors contribute to the observed TFP measures. In other words, those measures are more than a residual since they include elements which can be explained and quantified in terms of specific factors which have nothing to do with the efficiency by which the firm is using its resources or with the technological contribution of recent innovations. Thus, other things being equal, if the quantity demanded increases over time, say simply through population growth, and if at the same time, there is a scale effect in the production process, a TFP growth will be observed which, in fact, results solely from the fact that a growth in output can be generated through a less than proportional growth in input.

In themselves, those factors do not invalidate the TFP measures, however, as the measure is typically developed for some well-defined purpose, a proper quantification of those factors will generally be essential. Hence, it will matter to the regulator whether, say, the observed higher productivity of a carrier is solely due to its scale.

In this chapter, various factors which can contribute to generating growth in TFP will be analysed and an efficiency measure which abstracts from these various factors to associate productivity growth to technological change will be developed.

Scale

In a previous section, Figure 1 had been introduced to present the concept of TFP. It can be reintroduced in a modified form here to illustrate the impact of economies and diseconomies of scale, i.e. that of the absence of PLH. In Figures 2 and 3, production functions are introduced which are similar to the one introduced in the section "Hypotheses", but for the presence of economies and diseconomies of scale respectively. They could be represented by functions such as

$$Y_1 = T_1(t)X_1^2$$

and $$Y_2 = T_2(t)X_2^{\frac{1}{2}}$$

The TFP measure had been associated with the slope of the production function, however it is more properly associated with its average, i.e. with the slope of the line which goes from the origin to a given point on the production function. Once the PLH assumption is removed, it is easy to see that the TFP measure is a function of the input level (hence of the output level). In Figure 3, for instance, we may consider two dictinct points on the production function, (Y_a, X_a) and (Y_b, X_b) such that $X_b > X_a$. Then it is easy to see that the TFP measure corresponding to (Y_a, X_a), TFP_a, will be greater than that which corresponds to (Y_b, X_b), namely TFP_b.

In fact

$$TFP_2 = Y_2/X_2$$

and, by substitution,

$$TFP_2 = T_2(t)X_2^{-\frac{1}{2}}$$

It is observed that the TFP level in Figure 3, TFP_2, is inversely related to the input level X_2.

Figure 4 provides a simple illustration of the importance of disentangling, within a TFP measure, the scale contribution

Figure 2: TFP and Increasing Return to Scale

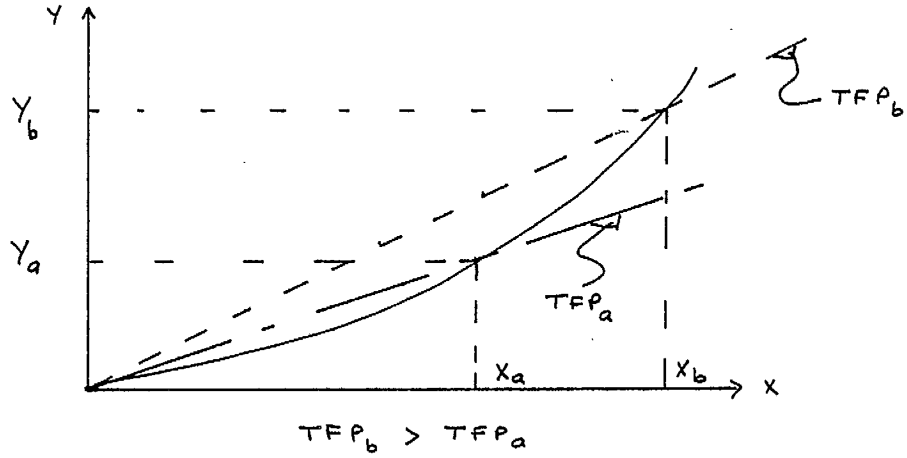


Figure 3: TFP and Decreasing Return to Scale

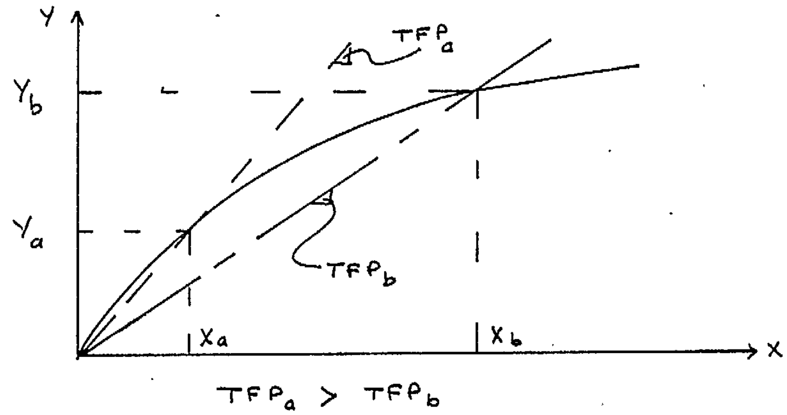
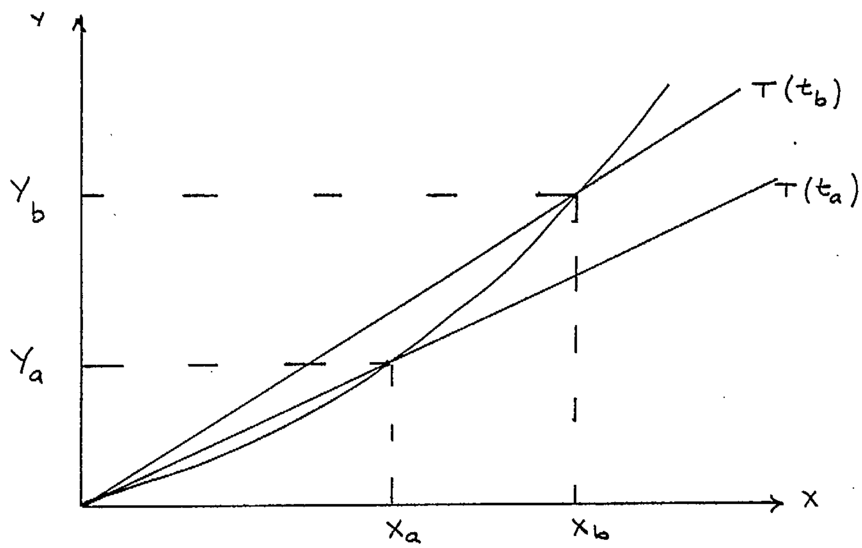


Figure 4: TFP, CRS and Increasing Return to Scale



associated with a movement along the production surface from the standard increase in efficiency associated with technological progress, hence with a shift in the production surface. In Figure 4, we consider a given level of technology t_1, two distinct production functions, one with PLH and one with economies of scale, and two distinct points on that last equation, (Y_a,X_a) and (Y_b,X_b) such that $X_b>X_a$, and such that (Y_a,X_a) is a point on the production function with PLH. It follows immediately that only with an increase in the technology from t_a to t_b would the PLH production function shift upwards and pass through (Y_b,X_b). Now the naive TFP measures associated with both points will be TFP_a and TFP_b respectively. However, while, if the process exhibits scale economies, the increase from TFP_a to TFP_b is not associated with any structural change but only the result of a higher output level, the passing from TFP_a to TFP_b is solely the result of a structural change in terms of technical progress which has increased the efficiency of the process, when that process is PLH.

Most recent results tend to confirm the presence of scale effects in the Canadian telecommunications sector. Thus Corbo and Smith (1979), Denny, Fuss and Everson (1979), Breslaw and Smith (1980), all find some increasing return to scale for Bell Canada while Bernstein (1980) obtains relatively similar results for British Columbia Telephone.

The relationship between the growth in TFP and the growth in technical development has been established by Denny, Fuss and Everson (1979) as

$$\dot{TFP} = \dot{T}(t) - (1 - \varepsilon)\dot{X}$$

where $\dot{X}$ is the Divisia index of the inputs, i.e.

$$\dot{X} = \sum_{j=1}^{m} s_j \dot{x}_j$$

$$s_j = w_j x_j / \sum_{j=1}^{m} w_j x_j$$

w_j is the price of the jth factor

ε is the scale elasticity which is none other than the inverse of the output elasticity of cost.

Given the production function in Figure 2,

$$\dot{TFP} = T_1(t) + \dot{X}_1$$

Multiple Outputs and Scale

In our analysis of scale effects, we had restricted ourselves to one output-one input processes. However, most if not all processes do utilize more than one input and produce more than one output; thus, the analysis will standardly differentiate between at least labour, capital and material. This feature has a crucial impact on the analysis of scale since, through time, those factors have not generally increased in the same proportion. This is in fact expected since, through time, such elements as the scale of production, the relative prices of the factors, the nature of technical change,... are likely to create an incentive for management to modify the mixture used in production. The net effect of these elements is that there will normally not exist a well defined input expansion path, a problem which did not exist when only one input was considered. The most standard approach consists in defining an arbitrary expansion path, and the simplest one consists in expanding all inputs in the same proportion. This was the approach considered in the preceding chapter. This approach also implies that the cost of production will increase by exactly the same proportion, as long as there is perfect competition in the factors market. Then we may measure the scale elasticity as the inverse of the output elasticity of cost. This definition, however, is more general since a proportional change in the total cost need not imply a corresponding proportional change in all inputs. In fact, if, given the production function

$$y = F(x_j, t;\ j=1,2,\ldots,n)$$

one starts from the cost function

$$C = g(y, w_j, t;\ j=1,2,\ldots,n)$$

which is defined as

$$g(y, w_j, t;\ j=1,2,\ldots,n) = \min_{x_j}\{w_j x_j;\ F(x_j, t;\ j=1,2,\ldots,n) \geqslant y\}$$

This alternative approach to scale implies that the optimal path in terms of the input prices, w_j, rather than the proportional path in the input space will be selected. This alternative approach is evidently more attractive for most applications since it is also the path which would be selected by the optimizing producer.

For many years now, researchers have considered multiple output production processes to analyse Canadian telecommunications (Corbo et al, 1978; Fuss and Waverman, 1978). In general, the production function will be of the form

$$F(y_i, x_j, t;\ i=1,2,\ldots,n,\ j=1,2,\ldots,m) = 0$$

Now to approach the scale problem, not only is it necessary to define the path in the input space in terms of which scale will be considered, but we must also define the path in the output space. One logical extension of the approach standardly adopted in the one output-many inputs case, and the one considered earlier, consists in defining paths in both the input and output spaces corresponding to proportional increases of all inputs and proportional increases of all outputs. Then the scale effect is simply measured as the ratio of the proportional increase in outputs with the proportional increase in inputs.

Evidently this approach to the scale problem suffers from the deficiency of the corresponding definition in the one output situation. However, we may observe that the scale measure in terms of the cost function can also be generalized. Let the cost function be

$$C = g(w_j, y_i, t;\ i=1,2,\ldots,n,\ j=1,2,\ldots,m)$$

and let us consider a proportional increase in all outputs for a given set of factor prices, then we may define the scale elasticity as the inverse of the sum of the output elasticity of cost, i.e. as

$$dC = \sum_{i=1}^{n} \frac{\partial g}{\partial y_i} . dy_i$$

$$d\ell nC = \sum_{i=1}^{n} \varepsilon_{c,i} d\ell n y_i$$

and as, by assumption, we have selected the output expansion path such that, given any two outputs, y_i and y_ℓ,

$$d\ell n y_i = d\ell n y_\ell = d\ell n y$$

$$\frac{d\ell n C}{d\ell n y} = \sum_{i=1}^{n} \varepsilon_{C,i}$$

An alternative approach, assuming a competitive output market would have been to start from the revenue function,

$$R = h(p_i, x_j, t;\ i=1,2,\ldots,n,\ j=1,2,\ldots,m)$$

Then the scale elasticity could be defined as the input elasticity of revenue, i.e. as $(d\ell n x/d\ell n R)$ where

$$\frac{d\ell n R}{d\ell n x} = \sum_{j=1}^{m} \varepsilon_{R,j}$$

$$d\ell n x_j = d\ell n x \qquad j=1,2,\ldots,m$$

This consists in selecting as a path in the input space, a proportional increase in all inputs. The path in the output space will be the one the revenue maximizer would select.

Logically, the next step would consist in going to the profit function. However the profit function implies profit maximization under perfect competition in both the output and the input markets, hence either a non-identified solution, given PLH, or a given optimal level of production. In either case it does not enable us to define an expansion path since at best only one point is defined under the technology t.

To return to the interdependence of scale and TFP in the context of the cost function, assuming that

$$C = g(w_j, y, t;\ j=1,2,\ldots,m)$$

Denny, Fuss and Everson (1979) have shown that, given perfect competition and profit maximization

$$\dot{T}(t) = -\varepsilon_{c,y}\dot{\Theta}(t)$$

where $\dot{\Theta}(t) = (\partial g/\partial t)/C$

and that

$$\dot{TFP} = -\dot{\Theta}(t) + (1 - \varepsilon_{c,y}^{-1})\dot{y}$$

Given the multiple outputs situation they have shown that

$$\dot{TFP} = -\dot{\Theta}(t) + \{(1 - \sum_{i=1}^{n} \varepsilon_{c,i})\dot{Y}\}$$

In terms of the revenue function, the equivalent results are

$$\dot{TFP} = \dot{\Gamma}(t) - \{(1 - \sum_{j=1}^{m} \varepsilon_{R,j})\}\dot{X}$$

where $R = h(p_i, x_j, t;\ i=1,2,\ldots,n,\ j=1,2,\ldots,m)$

$$\dot{\Gamma}(t) = (\partial h/\partial t)/R$$

The coefficients $(1 - \sum_{i=1}^{n} \varepsilon_{C,i})$ and $(1 - \sum_{j=1}^{m} \varepsilon_{R,j})$ will be zero whenever there is CRS. If, however, there are increasing (or decreasing) returns to scale, then $\dot{TFP}$ will include a component which is fully explained by the scale of production.

Marginal Cost Pricing

The results obtained in the previous section depend crucially upon the assumption of marginal cost pricing in either the input or the output market. Thus the relationship which was established between $\dot{TFP}$ and $-\dot{\Theta}(t)$ was based on the equality

$$\dot{Y}^* = \dot{Y}$$

where $\dot{Y}^* = \sum_{i=1}^{n} (\varepsilon_{C,i} / \sum_{i=1}^{n} \varepsilon_{C,i})\dot{y}_i$

i.e. on the equality, given $i=1,2,...,n$,

$$\frac{p_i y_i}{\sum_{i=1}^{n} p_i y_i} = \frac{\varepsilon_{C,i}}{\sum_{i=1}^{n} \varepsilon_{C,i}} .$$

Since $\varepsilon_{C,i} = \frac{\partial C}{\partial y_i} \cdot \frac{y_i}{C}$, we must have for some constant γ,

$$p_i = \gamma\left(\frac{\partial C}{\partial y_i}\right)$$

i.e. the price of every output must be proportional to the marginal cost of that output. As long as the producer faces no constraint while maximizing revenues, he will automatically fulfill this condition. However, available evidence would suggest that this is not the case in Canadian telecommunications, thus Corbo et al (1979), Breslaw and Smith (1980) and others have consistently confirmed a contention made by Bell Canada that the price elasticity of demand for local service is very low and that it is much lower than the marginal cost of that service, while the price of message toll is not very different from its marginal cost. It follows that, as far as many Canadian telecommunications carriers are concerned, the proper relationship betwee (t) and TFP would be

$$\dot{TFP} = -\dot{\theta}(t) - \{(1 - \sum_{i=1}^{n} \varepsilon_{C,i})\dot{Y}^*\} + (\dot{Y} - \dot{Y}^*)$$

Rate of Return Regulation

Again it is Denny, Fuss and Everson (1979) who have introduced the impact of the rate of return regulation in the TFP analysis. It can be modelled in terms of the Averch-Johnson model (1962) by defining the Lagrangian:

$$L = \sum_{i=1}^{n} p_i y_i - \sum_{j=1}^{m-1} w_j x_j - rK - \lambda\{\sum_{i=1}^{n} p_i y_i - \sum_{j=1}^{m-1} w_j x_j - sK\}$$

$$- \gamma\{F(y_i, x_j, t;\ i=1,2,\ldots,n,\ j=1,2,\ldots,m)\}$$

where $K = x_m$

Profit maximization yields the pseudo-prices

$$p_i^* = \frac{(1-\lambda)}{\gamma} p_i = F_i \qquad i=1,2,\ldots,m$$

$$w_j^* = \frac{(1-\lambda)}{\gamma} w_j' = F_j \qquad j=1,2,\ldots,n$$

$$r^* = \frac{r-\lambda s}{\gamma} = F_K$$

Fuss and Waverman (1978) have defined a cost function g^* in terms of those input pseudo-prices and the corresponding optimal output levels, y_i^*:

$$C^* = g^*(w_j^*, r^*, y_i^*, t)$$

The scaling of the production function is arbitrary and it can be selected such that $\gamma = 1$. Then, using the regulatory constraint,

$$C^* = C - \lambda \sum_{i=1}^{n} p_i y_i$$

C is the total cost in terms of the observed variable, and to C corresponds a new cost function in terms of observed variables:

$$C = g(w_j, r, s, y_i, t; \; i=1,2,\ldots,n, \; j=1,2,\ldots,m-1)$$

The cost function can be estimated directly, and Fuss and Waverman show that the following pseudo quantities are thus defined:

$$g_j = \left(\frac{1-\lambda}{\gamma}\right) x_j \qquad j=1,2,\ldots,m-1$$

$$g_r = K$$

$$g_s = -\lambda K$$

Defining the technological contraction of the cost function as $\Theta(t)$, then

$$-\dot{\Theta}(t) = \{\sum_{i=1}^{n} \varepsilon_{C,i}\dot{y}_i - \dot{X}\} - \lambda\{\sum_{j=1}^{m-1} s_j\dot{w}_j + (\frac{s_m K}{C})\dot{s}\}$$

$$\dot{TFP} = -\dot{\Theta}(t) \quad (1 - \sum_{i=1}^{n} \varepsilon_{C,i})\dot{Y}^* + (\dot{Y}-\dot{Y}^*)$$

$$- \lambda\{\sum_{j=1}^{m-1} s_j\dot{w}_j + (\frac{s_m K}{C})\dot{s}\}$$

Short Run TFP Analysis

Telecommunications carriers, like most public utilities, have construction programs which cover many years, and, unless some drastic shift in relative input prices occur as in the case of the oil crisis, they will not be able to adjust rapidly to changes in prices. It is reasonable to assume, therefore, that in the short run certain factors such as capital should be considered as fixed. To describe the short run behavior of the carriers, one should adopt the variable cost function,

$$V = k(w_j, y_i, K, t; \; j=1,2,\ldots,m-1, \; i=1,2,\ldots,n)$$

where V is the variable cost, i.e.

$$V = \sum_{j=1}^{m-1} w_j x_j$$

If this cost function is defined in terms of the pseudo-prices, as in the preceding section with the rate of return regulation, then it can be denoted by

$$V^* = k^*(w_j^*, y_i^*, K, t;\ j=1,2,\ldots,m-1,\ i=1,2,\ldots,n)$$

To it will correspond a cost function in terms of the observed prices

$$V = k(w_j, y_i, K, t;\ j=1,2,\ldots,m-1,\ i=1,2,\ldots,n)$$

with $V^* = (1-\lambda)V$

The technological contraction is now solely in terms of variable factors. It will be denoted by $\dot{\theta}(t)$. $\dot{\theta}(t)$ cannot be meaningfully related to the total cost function even though it can be trivially related to the total cost itself since the contribution of the fixed factors is independent of time:

$$\frac{\partial V}{\partial t} = \frac{\partial C}{\partial t}.$$

and

$$\dot{\theta}(t) = \frac{1}{V}\frac{\partial V}{\partial t}$$

$$= \left(\frac{C}{V}\right)\frac{\partial C}{\partial t}$$

In any case, if some factors are in fixed quantities, then we must distinguish between the short run and the long run TFP measures:

$$\dot{TFP}_L = \sum_{i=1}^{n} s_i \dot{y}_i - \sum_{j=1}^{m} s_j \dot{x}_j$$

$$\dot{TFP}_S = \sum_{i=1}^{n} s_i \dot{y}_i - \sum_{j=1}^{m-1} \tilde{s}_j \dot{x}_j$$

where $\tilde{s}_j$ are the variable cost shares.

Since $s_j = (\frac{V}{C})\tilde{s}_j$

then

$$\dot{TFP}_S = \dot{TFP}_L + (\frac{V-C}{C}) \sum_{j=1}^{m-1} \tilde{s}_j \dot{x}_j$$

The same general relationship will hold for $\dot{\theta}(t)$ as for $\dot{\Theta}(t)$ and between $\dot{TFP}_S$ and $\dot{\theta}(t)$ in the short run as between $\dot{TFP}_L$ and $\dot{\Theta}(t)$ in the long run, namely

$$-\dot{\theta}(t) = \sum_{i=1}^{n} \varepsilon_{V,i} \dot{y}_i - (1-\lambda)\dot{X}_S$$

where $\dot{X}_X = \sum_{j=1}^{m-1} \tilde{s}_j \dot{x}_j$

$$\dot{TFP}_S = -\dot{\theta}(t) + (1-\sum_{i=1}^{n} \varepsilon_{V,i})\dot{Y}^{+} + (\dot{Y}-\dot{Y}^{+}) + \lambda\dot{X}_S$$

where $\varepsilon_{V,i}$ is the ith output elasticity of the variable cost i.e.

$$\varepsilon_{V,i} = \frac{\partial \ell n V}{\partial \ell n y_i}$$

$$\dot{Y}^{+} = \sum_{i=1}^{n} (\varepsilon_{V,i} / \sum_{i=1}^{n} \varepsilon_{V,i})\dot{y}_i$$

COMPARATIVE ANALYSIS

Introduction

In the previous chapter, following the approach of Denny, Fuss and Everson (1979), the TFP measure was analysed and decomposed in terms of elements which could help explain it. The major element is evidently the scale effect; other elements are the impact of the price and/or the rate of return regulatory constraint and finally whether the analysis is a short run or a long run TFP analysis.

In this chapter, we will be concerned with the comparative analysis of productivity between two carriers. Immediately it becomes clear that the previous analysis is crucial. In the traditional productivity analysis of Kendrick and Denison, the emphasis was on the diversity of forces which contribute to the observed productivity growth; the limitation of the analysis is that the productivity measures developed are of limited use since very little can be said with respect to quantifying and explaining observed differences. In the modern analysis pioneered by Jorgenson and Griliches (1967) total factor productivity is associated with technical change in a production function context. The shortcoming of such an analysis follows from the kind of assumptions which are required. As we saw in the second chapter, two of these assumptions are perfect competition and positive linear homogeneity. The more recent trend has led to some sort of synthesis of those two approaches. While the production-cost function framework is maintained, hypotheses regarding scale effects, competition,...are tested in terms of a more general model and the observed TFP is explained in terms of a variety of factors in addition to technical change. This recent step has extremely important consequences for an inter-firm comparison since observed differences in measured TFP can be associated to factors other than the rate of technical change.

In this chapter, we shall present the Jordenson-Nishimizu (1978) model and expand it along the approach adopted by Denny, Fontenay and Werner (1980) to show how one could quantify the impact of various factors on the observed TFP of various firms. In practice, the TFP measure as such is not used since it implies a comparison of one given firm's performance between two periods of time, and in its place a DFP measure is introduced. The DFP measure is there to quantify the difference between two firms' performance at one given time period. Evidently, it is possible to express the change through time in the DFP measure in terms of the difference between the two firm's TFP growth:

$$DFP_t - DFP_{t-1} = \dot{TFP}_t^2 - \dot{TFP}_t^1 + \tfrac{1}{2}Z$$

where $DFP \equiv (\ell n y_t^2 - \ell n y_t^1) - \frac{1}{2}\sum_{j=1}^{m}(s_{j,t}^2 + s_{j,t}^1)(\ell n x_{j,t}^2 - \ell n x_{j,t}^1)$

as introduced in this chapter.

The superscript indicates the firm

$s_{j,t}^h$ denotes the cost share of input j in the hth firm in period t.

Z is a residual such that

$$Z \equiv \sum_{j=1}^{m} (s^1_{j,t-1} - s^2_{j,t})(\ell n x^2_{t-1} - \ell n x^1_t)$$
$$+ (s^2_{j,t-1} - s^1_{j,t})(\ell n x^2_t - \ell n x^1_{t-1})$$

however the residual Z does not appear to have any intuitive meaning.

The Difference in Total Factor Productivity (DFP)

It seems too restrictive to assume that various firms, even in a given sector, will follow the same technology. Hence, in Canada, while B.C. Telephone decided to bypass the cross-bar technology to go directly from step-by-step equipment to electronic equipment, Bell Canada has invested significantly in the cross-bar type of central offices. On the other hand, it seems unlikely that two firms which are in the same sector would not have a great degree of commonality in their technology.

It is convenient to approximate the true process of production of a carrier by a flexible functional form, which will be assumed, with little loss of generality, to be a trans log production function. Denoting the output of the carrier h by y^h, we have

$$\ell n y^h = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ell n x^h_i + \tfrac{1}{2} \sum_{i=1}^{n} \alpha_{i,i} (\ell n x^h_i)^2$$
$$+ \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} \alpha_{ij} \ell n x^h_i \ell n x^h_j + \alpha_t t^h + \tfrac{1}{2} \alpha_{t,t} t^2$$
$$+ \sum_{i=1}^{n} \alpha_{i,t} t^h \ell n x^h_i$$

The other carriers can be assumed to have processes which can also be approximated by translog functions. In the most general situation, it could be that the coefficients $\alpha_{k,\ell}$ and α_i are dependent upon the firm, i.e. that

$$\ell n y^h = \alpha_0^h + \sum_{i=1}^{n} \alpha_i^h \ell n x_i^h + \tfrac{1}{2} \sum_{i=1}^{n} \alpha_{i,i}^h (\ell n x_i^h)^2$$

$$+ \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} \alpha_{i,j}^h \ell n x_i^h \ell n x_j^h + \alpha_t^h t^h + \tfrac{1}{2}\alpha_{t,t}^h (t^h)^2$$

$$+ \sum_{i=1}^{n} \alpha_{i,t}^h t^h \ell n x_i^h$$

However those translog functional forms are second order approximations to the true form, and it seems reasonable to assume that the difference between firms will show up only in its main component, i.e. in the first order approximation "$\alpha_0 + \sum_{i=1}^{n} \alpha_i \ell n x_i + \alpha_t t$". Then we would write

$$\ell n y^h = (\alpha_0^h + \sum_{i=1}^{n} \alpha_i^h \ell n x_i^h + \alpha_t^h t^h) + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} \alpha_{i,j} \ell n x_i^h \ell n x_j^h$$

$$+ \tfrac{1}{2} \sum_{i=1}^{n} \alpha_{i,i} (\ell n x_i^h)^2 + \sum_{i=1}^{n} \alpha_{i,t} t^h \ell n x_i^h + \tfrac{1}{2}\alpha_{t,t} (t^h)^2$$

Evidently, to the extent that there are series sufficiently long and to the extent that processes are sufficiently well behaved, it would seem to be possible in principle to start from the most general form to test whether the firms differ solely in terms of the first order approximation. If the hypothesis were to be accepted, there appears to be no further problem. However, it is not clear that a rejection of the hypothesis by such a test would represent a valid test, especially if the two firms, as is generally the case, are in different regions of the production space. The inapplicability of such a test follows from the fact that (i) the forms are assumed to be local second order approximations of the unknown true forms and, if any statement is to be made with respect to both production processes at the same time, that (ii) the approximation must be made over a region which covers at least the firms being compared. In other and simpler words, if the various firms' production processes differ in all of their coefficients, as in the second example above, there is no regularity between them and hence comparisons are not meaningful.

To test the approximation which allows differing first order components, it is necessary to assume that at least one coefficient is independent of firms considered. That could be α_0, α_t, $\alpha_{t,t}$ or any of the $n\alpha_i$, the $\frac{n(n-1)}{2}\alpha_{i,j}$, or/and the $n\alpha_{i,t}$. The choice is arbitrary.

Hence, in the rest of our analysis, it will be assumed that the second-order part of the approximation is common to all carriers which are being compared, i.e. that the third form applies.

Without loss of generality, we can assume that there are only two carriers, i.e. h=1,2. Then, we may rewrite the coefficients α as

$$\alpha_0^1 = \alpha_0 \qquad \alpha_0^2 = \alpha_0 + \beta_0$$

$$\alpha_j^1 = \alpha_j \qquad \alpha_j^2 = \alpha_j + \beta_j \qquad j=1,2,\ldots,m$$

$$\alpha_t^1 = \alpha_t \qquad \alpha_t^2 = \alpha_t + \beta_t$$

or still that

$$\alpha_\ell^h = \alpha_\ell + \beta_\ell D \qquad \ell=0,1,2,\ldots,m,t;\; h=1,2$$

where D is a dummy variable taking the value zero with h=1 and one with h=2.

Then the translog can be rewritten as

$$\ln y^h = (\alpha_0 + \sum_{j=1}^{m} \alpha_j \ln x_j^h + \alpha_t t^h + \beta_0 D) + \tfrac{1}{2} \sum_{j=1}^{m} \alpha_{j,j} (\ln x_j^h)^2$$

$$+ \sum_{\substack{i=1 \\ i\neq j}}^{n} \sum_{j=1}^{n} \alpha_{i,j} \ln x_i^h \ln x_j^h + \sum_{j=1}^{n} \alpha_{j,t} t^h \ln x_j^h + \tfrac{1}{2}\alpha_{t,t} (t^h)^2$$

$$+ \sum_{i=1}^{n} \beta_j D \ln x_j^h$$

which corresponds to the second order approximation of the production function

$$y^h = f(x_j^h, t^h, D;\; j=1,2,\ldots,m)$$

with the constraint that $\beta_{0,0} = 0$.

This is the form hypothesized by Jorgenson and Nishimizu (1978) and by Denny, Fontenay and Werner.(1980).

In the Jorgenson-Nishimizu method, the difference in efficiency between any two carriers is estimated through index numbers, and it is shown that, if that difference is denoted by DFP,

$$DFP = (\ln y^2 - \ln y^1) - \tfrac{1}{2} \sum_{j=1}^{m} (s_j^1 + s_j^2)(\ln x_j^2 - \ln x_j^1)$$

where s_j^{ℓ} is the cost share of the jth input in the ℓth firm, $\ell=1,2$.

It is shown that

$$DFP = \tfrac{1}{2}\left(\frac{d\ell ny^2}{dD} + \frac{d\ell ny^1}{dD}\right)$$

$\left(\frac{d\ell ny^h}{dD}\right)$ measures the change in output, given production in firm h, when one marginally converts the production process to that of the other firm, the input level being held constant.

The interpretation of DFP, however, raises serious problems. First of all, there have been various approaches to the treatment of technology in the modern literature, the two main trends consisting of either taking the technical change to be a residual modelled in terms of a time trend or in attempting to model specific characteristics of the network as being that technical contribution. In telecommunications, the latter approach appears to be dominating (Corbo and Smith, 1979), whereas in most of the other domain, the time trend is the most common indicator of technology. It should be noted that, in fact, there is no conflict in using both simultaneously since even though variables such as DDD are likely to have had significant impact on the technological development of the telecommunications sector, a lot of other factors have also had their impact. The practical problem, in econometrics, is the constraint in terms of available degrees of freedom imposed by the number of available observations.

If t is a time trend, then, since one would normally compare the two firms at the same period, it would not show up in the comparison. However, if t stands for some other variable, such as DDD, it is unlikely that $t^2=t^1$ and it can be shown that DFP will also include the factor $\{\frac{1}{2}(\frac{d\ell n y^2}{dt^2} + \frac{d\ell n y^1}{dt^1})(t^2-t^1)\}$. The original model implies that, in general, even if the two firms have available to them the same level of technical development, i.e. $t^2=t^1$, they still are not on the same production hyperplane. This could be associated with such features as regional terrain,... Hence, even if various services were demanded in the same proportion in B.C. Telephone's territory, as in Bell's territory, the presence of the mountains in B.C. may imply that given today's technology, a different process is selected. On the other hand, if t is taken as an unspecified residual, since its scale is arbitrary, it can always be defined to take the value zero in one firm and one in the other. Then DFP becomes the simple measure of TFP growth when passing from the technology of firm 1 to that of firm 2. This approach, implicit to the Jorgenson-Nishimizu method, does not enable us to study the possibility that the technology available to one firm is not available to the other, since the two contributions cannot be disentangled.

The problem of measuring the efficiency levels can be approached from the dual side using the cost function, (Denny, de Fontenay and Werner). Under certain regularity conditions, given a cost minimizing firm, to the production $y = f(x_j, D, t;\ j=1,2,\ldots,m)$

corresponds a cost function g where

$$C = g(w_j, y, D, t;\ j=1,2,\dots,m)$$

Supposing that g is approximated by a translog cost function, a theorem proved by Diewert (1976) will yield:

$$\ell nC^2 - \ell nC^1 = \tfrac{1}{2}\sum_{j=1}^{m}(s_j^2+s_j^1)(\ell nw_j^2-\ell nw_j^1) + \tfrac{1}{2}(\varepsilon^2_{C,y}+\varepsilon^1_{C,y})(\ell ny^2-\ell ny^1)$$

$$+ \tfrac{1}{2}\left(\frac{\partial \ell nC^2}{\partial t} + \frac{\partial \ell nC^1}{\partial t}\right)(t^2-t^1) + \tfrac{1}{2}\left(\frac{\partial \ell nC^2}{\partial D^2}+\frac{\partial \ell nC^1}{\partial D^1}\right)(D^2-D^1)$$

A logical measure of the difference in efficiency would be (-H) where

$$H = \tfrac{1}{2}\left(\frac{\partial \ell nC^2}{\partial D^2} + \frac{\partial \ell nC^1}{\partial D^1}\right)$$

Then it can be seen that whenever the two firms face the same prices on the factor markets, then ($\ell nw_j^2 = \ell nw_j^1$) and the first set of RHS terms disappear. Similarly, if the two firms use the same technology, $t^2 = t^1$, and the third set of RHS terms also disappear. Then

$$(-H) = \tfrac{1}{2}(\varepsilon^2_{C,y}+\varepsilon^1_{C,y})(\ell ny^2-\ell ny^1) - (\ell nC^2 - \ell nC^1)$$

If, to be able to use the index number approach, we assume PL , then

$$(-H) = (\ell ny^2-\ell ny^1) - (\ell nC^2-\ell nC^1)$$

Hence, by following the cost function approach, i.e. by using the hypothesis of cost minimization, we are able to isolate in our new measure of the difference in technical efficiency, that part which is due to the difference in the factor prices:

$$\tfrac{1}{2}\sum_{j=1}^{m}(s_j^2+s_j^1)(\ell nw_j^2-\ell nw_j^1)$$

from that part which is due to the difference in technology:

$$\tfrac{1}{2}\left(\frac{\partial \ln C^2}{\partial t} + \frac{\partial \ln C^1}{t}\right)(t^2 - t^1)$$

from the difference in the ability or capacity to use the known technology:

$$(\ln y^2 - \ln y^1) - (\ln C^2 - \ln C^1)$$

It is now necessary to relate the new measure of productivity difference $(-\tilde{H})$, to Jorgenson and Nishimizu's DFP. The original model was

$$y = F(x_j, D, t;\ j=1,2,\ldots,n)$$

If, for instance, we take D to be a technical level variable scaled so as to take the values 1 with firm 2 and 0 with firm 1, then

$$\frac{dy}{dD} = \sum_{j=1}^{m} F_j \frac{dx_j}{dD} + F_D + F_t \frac{dt}{dD}$$

where $F_D = \partial F / \partial D$

Following the approach of Denny, Fuss and Everson, we may define

$$\tilde{E} = F_D / y$$

then $\tilde{E} = \tilde{y} - \varepsilon^{-1}_{C,y}\ \tilde{X}$

where $\tilde{X} = \sum_{j=1}^{m} s_j \tilde{x}_k$

$$\tilde{y} = (dy/dD)/y$$

$$\tilde{x}_j = (dx_j/dD)/x_j \qquad j=1,2,\ldots,m$$

and where we have assumed $(dt/dD)=0$.

Defining $\widetilde{DFP}$ such that DFP is its local approximation,

i.e. $\widetilde{DFP} = \tilde{y} - \tilde{x}$

it follows that

$$\widetilde{DFP} = \tilde{E} + (1-\varepsilon_{C,y}^{-1})\dot{\tilde{X}}$$

If now we start from

$$C = g(w_j, y, D, t;\ j=1,2,\ldots,m)$$

we can derive by the same procedure

$$\frac{dC}{dD} = \sum_{j=1}^{m} g_j \frac{dw_j}{dD} + \frac{\partial g}{\partial y}\cdot\frac{dy}{dD} + g_D + g_t \frac{dt}{dD}$$

Since

$$\frac{1}{C}\cdot\frac{dC}{dD} = \sum_{j=1}^{m} s_j \tilde{x}_j + \sum_{j=1}^{m} s_j \tilde{w}_j$$

if we define $(-\tilde{H})$ such that

$$\tilde{H} = (\partial g/\partial D)/C$$

then

$$(-\tilde{H}) = \varepsilon_{C,y}\tilde{y} - \tilde{X}$$

$$(-\tilde{H}) = \varepsilon_{C,y}\tilde{E}$$

i.e. that the (-H) derived from the cost function can be (i) estimated by index numbers just as E

$$(-H) = (\ell n\, y^2 - \ell n\, y^1) + (\ell n\, C^2 - \ell n\, C^1) - \tfrac{1}{2}\sum_{j=1}^{m}(s_j^2 + s_j^1)(\ell n w_j^2 - \ell n w_j^1)$$

(ii) compared to $\tilde{E}$ since

$$(-\tilde{H}) = \varepsilon_{C,y}\tilde{E}$$

while (iii) providing a further decomposition through its use of the cost minimization hypothesis.

Finally in a scenario where the firms not only face different technologies ($dt/dD \neq 0$) but where they also face a different physical environment ($\partial F/\partial D \neq 0$), following the Denny, Fuss and Everson approach, we can define

$$(-\tilde{\Theta}) = -(\partial g/\partial t)(dt/dD)/C$$

to obtain

$$(-\tilde{H}) = D\tilde{F}P - (1-\varepsilon_{C,y})\tilde{y} - (-\tilde{\Theta})$$

Given a scenario where the two firms operate on the same production hyperplane ($\partial F/\partial D=0$) while facing different factor market prices due to their different geographical locations

$$(-\tilde{H}) = D\tilde{F}P - (1-\varepsilon_{C,y})\tilde{y} - (-\tilde{\Theta})$$

i.e.

$$D\tilde{F}P = (1-\varepsilon_{C,y})\tilde{y} + (-\tilde{\Theta})$$

If indeed there is PLH, then we would expect

$$D\tilde{F}P = -\tilde{\Theta}$$

Finally, whenever the two firms face the same technology ($dt/dD=0$) but a different physical environment ($\partial F/\partial D\neq 0$), the last term ($-\tilde{\Theta}$) disappears and

$$(-\tilde{H}) = D\tilde{F}P - (1-\varepsilon_{C,y})\tilde{y}$$

Marginal Cost Pricing, Rate of Return Regulation and Short Run Analysis

It is an easy matter to expand the result of the previous chapter on the decomposition of TFP to the comparative analysis since D can be seen as a technological variable properly scaled. In fact, all the previous results can be transposed to this new problem.

First of all, the multiple output situation is a straightforward generalization, and, given

$$C = g(w_j, y_i, t, D;\ i=1,2,\ldots,n,\ j=1,2,\ldots,m)$$

$$(-\tilde{H}) = \sum_{j=1}^{m} \varepsilon_{C,j}\tilde{y}_j - \tilde{X} - (-\tilde{\Theta})$$

If we define $\tilde{Y}^*$ as $\{\sum_{j=1}^{m}(\varepsilon_{C,y}/\sum_{j=1}^{m}\varepsilon_{C,j})\tilde{y}_j\}$, then

$$(-\tilde{H}) = D\tilde{F}P - (\tilde{Y}-\tilde{Y}^*) + \{(1-\sum_{j=1}^{m}\varepsilon_{C,y})\tilde{Y}^*\} - (-\theta)$$

The analysis of the previous section still holds if there is marginal cost pricing, $(\tilde{Y}-\tilde{Y}^*)=0$, and if there is constant return to ray scale, $(1-\sum_{j=1}^{m}\varepsilon_{C,j})=0$.

Similarly the rate of return regulation is also a straightforward generalization with D being an argument of F in the Lagrangian, of g* and of g.

Following the same approach as presented in the last chapter, one obtains

$$D\tilde{F}P = (-\tilde{H}) + (\tilde{Y}-\tilde{Y}^*) + \{(1-\sum_{j=1}^{m}\varepsilon_{C,j})\tilde{Y}^*\} + (-\tilde{\theta})$$
$$+ \lambda\{\sum_{j=1}^{m-1} s_j\tilde{w}_j - (s_m K/C)\tilde{s}\}$$

where λ is the Lagrange multiplier of the rate of return constraint,

s_j is the cost share of the jth factor

$\tilde{w}_j = (\partial w_j/\partial D)/w_j$

$\tilde{s} = (\partial s/\partial D)/s$

Finally, fiven that in the short run, capital is a fixed factor, the variable cost functions k* and k will have D as an argument. We can define

$$\tilde{H}_s = \frac{1}{V}.\frac{\partial V}{\partial D}$$

$$\tilde{\theta} = \frac{1}{V}.\frac{\partial V}{\partial t}.\frac{dt}{dD}$$

Then it can be shown that

$$\tilde{DFP}_s = (-\tilde{H}_s) + (\tilde{Y}-\tilde{Y}^{+}) + \{(1-\sum_{i=1}^{n} \varepsilon_{V,i})\tilde{Y}^{+}\} + (-\tilde{\theta}) + \lambda\tilde{X}_s$$

where $\tilde{DFP}_s = \tilde{Y} - \tilde{X}_s$

$\tilde{X}_s = \sum_{j=1}^{m-1} \tilde{s}_j \tilde{x}_j$, $\tilde{s}_j$ being the variable cost shares,

$\tilde{Y}^{+} = \sum_{i=1}^{n} (\varepsilon_{V,i} / \sum_{i=1}^{n} \varepsilon_{V,i}) \tilde{y}_i$

$\varepsilon_{V,i}$ is the variable cost elasticity of the ith output.

Through the last two expressions, it only remains to analyse and quantify the impact of a set of specific factors which contribute to the observed difference in total factor productivity. Those factors are (i) difference in non-marginal cost pricing i.e. $(\tilde{Y}-\tilde{Y}^*)\neq 0$ and $(\tilde{Y}-\tilde{Y}^{+})\neq 0$, (ii) short and/or long run scale economies, i.e. $(\sum_{i=1}^{n} \varepsilon_{C,i}\neq 1)$ or $(\sum_{i=1}^{n} \varepsilon_{V,i}\neq 1)$, (iii) difference in the use of the available technology, i.e. $(-\tilde{\Theta})\neq 0$ and/or $(-\tilde{\theta})\neq 0$, and (iv) the rate of return regulatory constraint, $\lambda\neq 0$.

CONCLUSION

In this paper we have integrated the Denny, Fuss and Everson TFP decomposition analysis with the Jorgenson-Nishimizu method for interfirm comparisons of total factor productivity. That way, we have been able to show that the conventionally measured Jorgenson-Nishimizu measure of difference in total factor productivity can be decomposed and explained, either in a short run or in a long run context, in terms of (i) departure from marginal cost pricing, (ii) non-CRS, (iii) effective rate of return regulation and (iv) differing level of technical knowhow in addition to (v) the difference in the capacity and ability to use the existing technology because of the physical environment in which the firms operate.

It was also shown that the Jorgenson-Nishimizu measure need not to be solely one of different capacity and ability but that it could also be related to the fact that the firms do not face the same factor prices, even under perfect competition. Again, this could be due to geographical constraints.

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