PLANNING MODELS OF INTERNATIONAL COMMUNICATIONS NETWORKS

COMMUNICATIONS PLANNING BRANCH

COMMUNICATIONS CANADA

OTTAWA

FEBRUARY 1973

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CONTENTS

OVERVIEW WITH ACKNOWLEDGEMENTS

This work has been motivated by the desire to develop tools that permit rational planning and evaluation of large scale networks. We feel that this study clearly demonstrates our conviction that powerful computer based simulation tools exist based on topological, graph theoretic, queueing and certain heuristics and that furthermore, these tools produce valid, readily interpreted and in many cases, optimum solutions to large scale problems that previously had been simply "out of reach".

At a time of accelerated data network development, problems of transmission, message handling, adaptive routing, and numbering schemes, converge and rapidly focus on a network design situation. Nowhere is there more of a requirement for strategic planning in the international communication area, than for the determination of those, circuit expansion, message handling, routing and numbering schemes that will produce the maximum revenue situation for Canada and at the same time, enhance the position of its International Carrier.

Very simply then, this report is the first step in the development of a new class of interactive planning tools for generating optimum policies for planning networks.

Many people contributed directly or indirectly to this study and in particular, the authors are grateful to Don Robertson of the International Branch for his many insights and the "feel" that he gave them for the practical considerations.

The executive secretary of the Canadian Advisory Committee on New Data Networks, provided a valuable and continuing interface between this study and the relevant activities of the CCITT.

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OTTAWA
26 February, 1973

INTRODUCTION

This is the first report of the *World Data Network Modelling Program.* It contains the description and results of a generalized simulation model, that permits the analysis of the traffic handling characteristics of the International Telephone Network. This report also addresses the problems of routing plans and the number of circuits in the International Network. This preliminary model is now being further defined to be of assistance in generating routing and other traffic handling schemes that will produce the maximum benefit to Canada.

We feel that the work to date has clearly established the use of such models as a valuable planning tool for future communication networks. Furthermore, there are a number of urgent reasons why Canada should maintain its lead in these developments.

First, there is increasing attention being paid in CCITT studies to the development of techniques for international traffic handling using dynamic routing strategies. These studies will undoubtedly produce routing strategies that take into account the state of occupancy of the different routes available to set up a connection between an origin and a destination. Such routing strategies define in turn an international plan that will permit not only an improvement in traffic distribution and an increase in the traffic handling capabilities of the network, but will also utilize the international network in a more efficient manner by taking advantage of the non-coincidence of busy hours in different countries. Thus in order to protect Canada's interests, it is important to consider the ramifications of this international plan with respect to the Canadian traffic. In addition, Canadian contributions to CCITT studies in this area should promote the national interest in an optimum manner.

Secondly, a number of other nations, notably Australia and the U.K. are actively preparing alternative routing and other plans for proposed expansions of the International Network. It can be expected that these plans will not necessarily be the ones that will bring the most financial benefit to Canada.

This report is then concerned with the development of planning tools that will permit the

- *determination of those networking policies which are most advantageous to Canada;*

- *instantaneous evaluation of alternate schemes and the impact of possible compromises on Canadian plans.*

The approach taken in this report is to give an outline of the simulation model, then describe by an example its utility as a planning tool. A flow chart of the Simulation Model is given in Appendix A.

A mathematical formulation based on linear programming techniques is presented in Appendix B. It is also shown that these techniques can be used for small or moderate networks.

A short survey of possible capacity cost functions for telephone networks is given in Appendix C. Those cost functions could represent the amount of money a country has to pay (or looses) if a message is routed through a specific branch in the network.

PROBLEM FORMULATION

The international telephone network, now in operation, is electromechanically/manually controlled and offers limited possibilities as regards call routing. However, with the gradual introduction of stored program control in electronic switching machines, a great deal of flexibility will be introduced into the network. It is then well known that at the expense of a a small increase in memory size, elaborate strategies can be implemented to produce optimal traffic flow and to increase the network's reliability and efficiency.

The number of calls simultaneously in progress in a network is quite large. Consequently, the efficiency of any network will depend crucially on the routing procedure used. Other countries in the world are developing tools which will enable them to propose international network routing and numbering plans. It can be expected that these plans will result in the optimum use of their facilities and hence will bring them the greatest financial returns.

The immediate consequence of the above development is that in all likelihood, plans prepared by other ***nations*** *will represent a sub-optimum situation for Canada.*

The problem of finding an optimal routing strategy for Canada can therefore be stated as follows:

> *"For any proposed or existing network, and given traffic statistics; how can the network be utilized so that both, the total traffic handled, and Canadian revenues are maximized."*

CLASSIFICATION OF ROUTING STRATEGIES

The possible routing strategies in a telecommunication network can be divided into two main categories:

- *deterministic*
- *stochastic*

Deterministic strategies are those in which a path from origin to destination is predetermined. That is, the path is a priori chosen without taking into account the state of occupancy of the circuits at the time the message is being handled. The existing international network uses such a routing scheme, that is, an a priori defined route exists between all pairs of points in the network.

A routing strategy is *stochastic* if the choice of the route involves to some extent a probability distribution. The term *dynamic* or *adaptive* routing is used to denote a strategy where the choice of the route depends on the state of occupancy of the different groups of circuits at the time the message is being handled. In a store and forward computer network, for example in the ARPA network, adaptive routing schemes are used. These schemes route messages over the least delay path. In such networks, it is quite common that different messages with the same origin and destination will not always use the same path.

Furthermore, in international situations it may happen that busy hours of two different centres or countries in the network do not coincide. In this case, the routing is really time dependent. Thus in the design of international networks, savings in the number of circuits can be obtained by taking into account the non-coincidence of busy hours in different countries.

Up to the present time, only cost independent routing strategies have been considered. In a cost independent routing strategy the route chosen is one for which some criteria function is minimal (or maximal). The route chosen for handling a message may also depend on the transmission quality of the different sections involved in the possible routes. This could very well happen in a network having heterogeneous transmission lines (wires, radio links, satellite). According to study group XII of CCITT the tandem connexion of two or more satellite circuits should only be used either, in case of failure of a terrestrial circuit or, for data transmission. It was recommended by the study group that the route of minimum propagation time be used if several routes were available.

An important consideration in optimal strategy is to take into account the grade of service i.e. minimize the number of lost calls while maximizing the network traffic. The optimization criterion should be expressed in terms of cost. In other words, maximize the traffic handled by the network at minimum cost. If a penalty or price is associated with the loss of a call the grade of service will also be minimized.

Taking into consideration the strategies formulated so far, a linear programming formulation to solve the optimal routing problem is presented in Appendix B, which also discusses its limitations and difficulties. Due to the large number of constraints and variables, a solution to the problem using standard linear techniques was shown to be impossible for large networks.

SIMULATION MODELLING

Our studies have indicated that presently available analytical techniques are inadequate to cope with large scale network problems. Therefore, simulation was the tool used to analyse the traffic handling characteristics of the network considered.

The simulation model described here can be used to study the traffic flow in the international line switched network. For a given network configuration, this model will generate messages at each node in a random fashion. These nodes represent in actual fact a country generating messages. Each message is served upon arrival in a first-come first-served basis using a direct route to its destination. If such a route is busy, the message will be directed through unsaturated lines using alternate routing schemes. The message will occupy a voice line for a time randomly chosen. In the present simulation, the average time a message occupies a voice line was arbitrarily chosen as 8 minutes. This can however easily be varied in the model. If a message cannot be routed either through a direct route or through some alternate route, that message is considered to be *lost*.

The messages originating at each node are assumed to come from a Poisson traffic source. The average arrival rate of messages is also variable, which can be chosen as a function of the traffic intensity statistics of the countries involved.

EXAMPLE

The following simple example, illustrates some considerations followed in applying the simulation model to study the behaviour of the network in Figure 1.

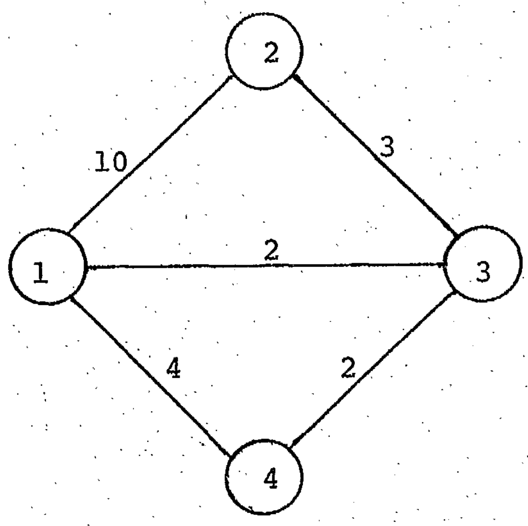


Figure 1

Figure 1 is a four node network with assigned voice line circuits for each node. (e.g. there are 10 voice circuits between nodes 1 and 2) Each node represents a switch.

In this example, node 2 sends messages to node 1 at a rate of 5 messages per minute. Node 2 also sends messages at a rate of one message per minute to node 3. Consequently, on the average, there are 5 times more messages generated for node 1 than for node 3. This gives therefore the possibility of controlling the level of traffic between countries (nodes).

The next step in applying the simulation model is to specify the routing scheme to be used.

ROUTING SCHEME

An essential and integral aspect of the simulation of any communication network is the assignment of the routes through the network by means of which messages can be sent from switching center to switching center. This choice of the routing algorithm between the different switching centers, that is, the hierarchy of direct and alternate routes, will have a significant effect upon the trunk capacity and on the probability of a message successfully reaching its destination.

In order to select the optimum route for a message the following information is required:

- *capacity of lines*
- *network topology*
- *location of nodes (switching centers).*

It is recognized that due to political considerations certain messages must be routed solely on the direct route. The simulation model caters to these situations by prohibiting alternate routing.

The progressive routing of a message is illustrated in the previous example (Figure 1), where a message originates at node 2 and terminates at node 1. The first choice route for the message is the direct route, node 1 to node 2. If this is fully occupied, then an alternate route is chosen. Hence, if 10 messages (which is the capacity of the direct route between nodes 1 and 2) occupy the route when a message is originated at node 2, then the message will be rerouted through node 3. This particular alternate route is chosen first, because it has a greater capacity and covers a shorter distance than the other possible route through node 4. The route through node 4 is only used if the direct route and the first alternate route are busy. When all routes are busy the message will be considered as *lost*.

In resumé, a message is routed through the network one switch at a time until it reaches its destination or a blocking situation occurs and it is *lost*.

The first problem at which we directed our efforts, was to become familiar with the complexities of the network and to study network behaviour under different traffic loads. To facilitate this, a predetermined message routing algorithm for each node was built into the simulation model.

The next stage of the development of the model will include additional steps. The first will, of course, be the selection of a set of reasonable routes from each originating country to Canada. These routes will be determined by considering the distance involved, the number of tandem switches and the conditions of the links relative to the capacity, that is, it will be a dynamic adaptive routing scheme. The second step will include the introduction of a control procedure which will limit the choice of routes available to a message. For example, it will be necessary to restrict messages so that they cannot pass through the same node twice. This will prohibit a particular message from travelling the same route more than once.

RESULTS OF THE SIMULATION MODEL

In order to establish the ground rules for the study of the International Network in a meaningful manner, a partial layout of the Commonwealth Telephone Network, shown schematically in Figure 2, was used.

Different levels of traffic were generated from each node so that a picture of the message routing could be viewed. For example, if it is assumed that only one circuit exists between Canada-India and that the first choice alternate route is via Australia and the second choice alternate route is via Hong-Kong, the following results were obtained; see Table 1.

		CANADA-INDIA	INDIA-CANADA
Messages	Routed Directly	122	100
"	through Australia	25	22
"	through Hong-Kong	7	8
Lost messages		0	

Table 1

These results show the breakdown of total messages per route for a 7 day week, 24 hours per day period. With the chosen traffic levels and alternate routing scheme no messages would be lost.

Another very important area of interest, is that of knowing the condition of the lines so as to see how close the network is to saturation. Table 2 is a representation of this type of information. It gives us a general picture of the network. For example, it shows that the average line utilization between India and Canada is 22.6% with the alternate routing scheme described previously. A detailed explanation of the terminology used in Table 2 is given below.

Line is an identifier of the circuits of the network.

Line Capacity indicates the number of available voice circuits on the route identified.

Average Content is the average number of messages per minute that existed on the circuit during the simulation.

Total Entries represent the total number of messages on that circuit. For example, in the case of the traffic between Canada and India, the total number of messages in both directions was 222.

Average Message Time is, as the title implies, the average time for the messages to be transmitted through the network.

Current Content indicates the number of messages currently in the network when the simulation was stopped.

Maximum Content refers to the maximum number of messages that exist at any one time on a given circuit. In our particular example i.e. for line 9, the maximum content was recorded as 1. This is quite reasonable since the total capacity of that route is one voice circuit.

Table 3 is a sample printout of the Simulation Model representing the number of messages and their routing in the network.

PARTIAL LAYOUT OF THE COMMONWEALTH TELEPHONE NETWORK

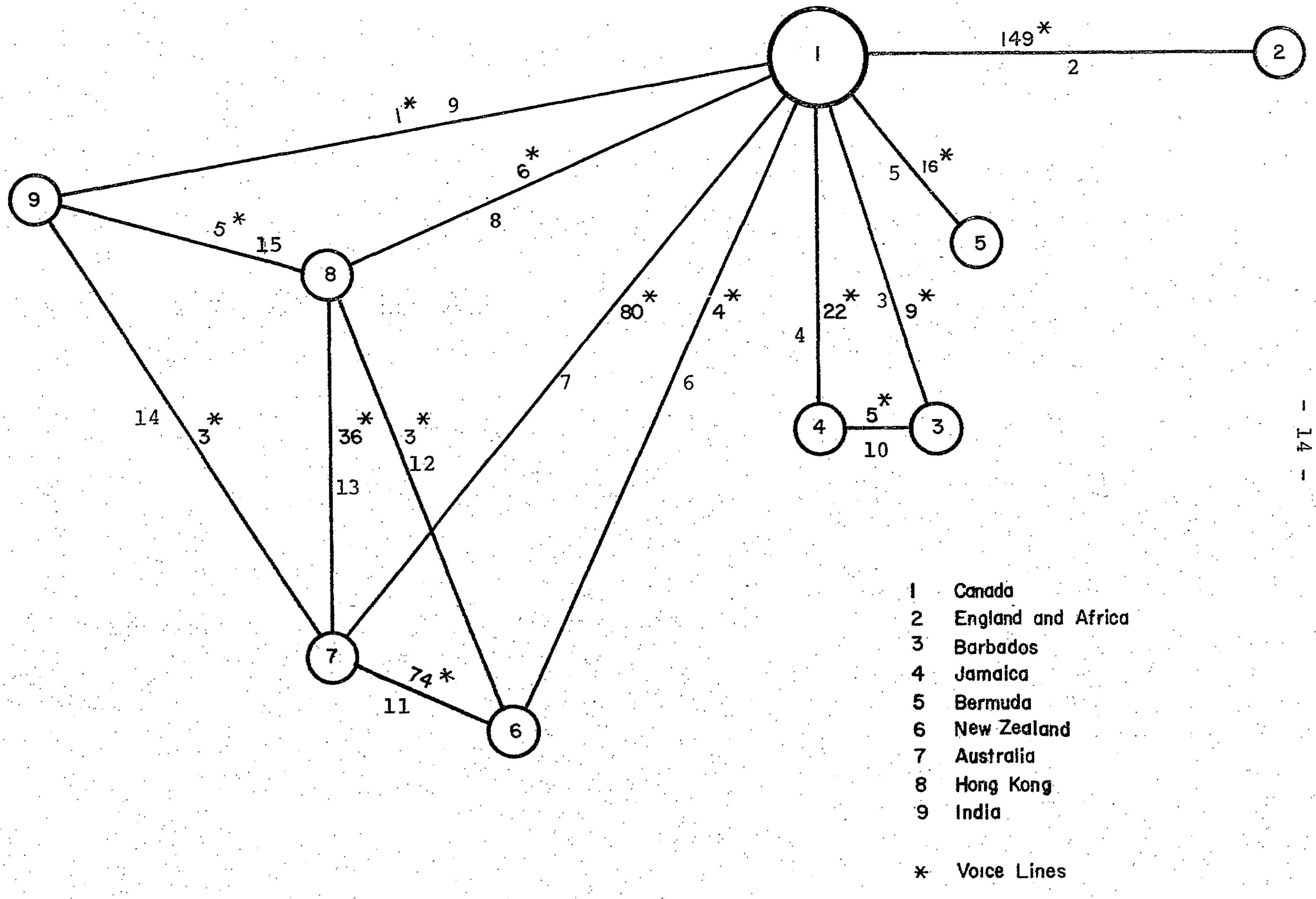


FIGURE 2

Table 2

LINE	LINE CAPACITY	AVERAGE CONTENT	AVERAGE UTILIZATION	TOTAL ENTRIES	AVERAGE MESSAGE TIME	CURRENT CONTENT	MAXIMUM CONTENT
2	149	10.048	0.067	10723	9.445	6	23
3	9	0.591	0.065	667	8.934	2	5
4	22	1.973	0.089	2008	9.526	1	9
5	16	0.420	0.026	444	9.540	0	4
6	4	1.437	0.359	1505	9.627	1	4
7	80	6.101	0.076	6459	9.520	6	17
8	6	0.432	0.072	480	9.079	0	5
9	1	0.226	0.226	222	10.288	0	1
10	5	1.919	0.383	2020	9.576	2	5
11	74	2.180	0.023	2259	9.729	2	9
12	3	1.515	0.505	1710	8.932	1	3
13	36	2.015	0.055	2131	9.534	6	10
14	3	1.582	0.527	1616	9.869	1	3
15	5	1.909	0.397	2084	9.621	5	5

Table 3

CANADA BRITAIN CANADA

NUMBER OF DIRECT MESSAGES	= 10395
LOST MESSAGES	= 0
TOTAL REVENUE	= 113810

CANADA AFRICA CANADA

NUMBER OF DIRECT MESSAGES	= 328
LOST MESSAGES	= 0
TOTAL REVENUE	= 10057

CANADA BERMUDA CANADA

NUMBER OF DIRECT MESSAGES	= 444
LOST MESSAGES	= 0
TOTAL REVENUE	= 4443

CANADA BARBADOS CANADA

	FROM CANADA	FROM BARBADOS
DIRECT MESSAGES -	338	327
THROUGH JAMAICA -	0	0
LOST MESSAGES = 0		
TOTAL REVENUE = 6334		

CANADA JAMAICA CANADA

	FROM CANADA	FROM BARBADOS
DIRECT MESSAGES -	1025	1062
THROUGH BARBADOS -	0	0
LOST MESSAGES = 0		
TOTAL REVENUE = 20965		

CANADA _ NEW ZEALAND _ CANADA

	FROM CANADA	FROM NEW ZEALAND
DIRECT MESSAGES -	1053	451
THROUGH AUSTRALIA -	75	21
THROUGH HONG-KONG -	0	0

LOST MESSAGES = 0
TOTAL REVENUE = 46072

CANADA _ AUSTRALIA _ CANADA

	FROM CANADA	FROM AUSTRALIA
DIRECT MESSAGES -	3163	3147
THROUGH NEW-ZEALAND -	0	0
THROUGH HONG-KONG -	0	0
THROUGH INDIA -	0	0

LOST MESSAGES = 0
TOTAL REVENUE = 126962

CANADA _ HONG-KONG _ CANADA

	FROM CANADA	FROM HONG-KONG
DIRECT MESSAGES -	246	219
THROUGH AUSTRALIA -	0	0
THROUGH NEW-ZEALAND -	0	0
THROUGH INDIA -	0	0

LOST MESSAGES = 0
TOTAL REVENUE = 17792

CANADA _ INDIA _ CANADA

	FROM CANADA	FROM INDIA
DIRECT MESSAGES -	122	100
THROUGH AUSTRALIA -	25	22
THROUGH HONG-KONG -	7	8

LOST MESSAGES = 0
TOTAL REVENUE = 12472

GENERAL OBSERVATIONS

The picture presented so far is for onlv a portion of the Commonwealth Telephone Network. It is by no means the complete picture since our main objective in this report is to show that simulation models can be used to study and analyse the International Network as it interests Canada.

However, it should be noted that the whole network can be represented in such a model and real system behaviour can be analysed and literally reproduced with existinq statistical data.

At the present time, there does not seem to be too much of an interest in costs related to transit traffic. The reason is a direct result of present policies since no cost sharing exist with the transit centers. That is, the revenue of the call is presently shared with the originating and destination countries only. In time, however, this policy may be changed. A center which transits a call, depending on its origination and destination could collect a certain percent of the total revenue of this call. This would directly affect the revenues of any particular country and would therefore force negotiation as to where a call may be routed to minimize the revenue distribution of the call.

The model developed so far will not give us the set of optimal alternate routes but will give us an appreciation of those routes which will benefit Canada when a choice is available.

It can therefore be seen that the next step in this project is to develop a mathematical model which will produce the set of optimal alternate routes for messages destined for Canada, taking into consideration the cost or revenues produced from transit messages.

INPUT/OUTPUT OF THE MODEL

Input to the Model

For summary the input required to the simulation model is:

1. mean time for messages sent from Canada
2. mean time for messages sent from other countries to Canada
3. mean time for messages sent between other countries
4. cost of the first 3 minutes for a message from each country plus
5. the cost of each additional minute
6. network configuration
7. capacity of the lines.

Output of the Model

For each message to Canada from any country and vice versa, the model will give:

1. the total number of direct messages from each country
2. the total number of messages through the first alternate route
3. the total number of messages through the second alternate route
4. the total number of messages through the n^{th} alternate route
5. the total number of lost messages
6. total Canadian revenue.

The output of the simulation model gives also a detailed analysis of line utilization for the simulated period of time.

For each line, the following is given:

- Assigned capacity of the line
- Average number of messages in the line during the simulated time
- Total number of messages during simulated time
- Average time per message
- Utilization of the line (%).

CONCLUSIONS & RECOMMENDATIONS

The Simulation Model described in this report will be in a state of perturbation and evolution for the next few months. During this time, statistical data will be gathered to evaluate the accuracy of the model related to the existing network. During this time, the model will also be expanded to include all the International Network.

There remains many problems which have not been solved or identified. However, we have attempted to create a model to help us analyse and make recommendations on the efficient use, for Canada, of the International Network.

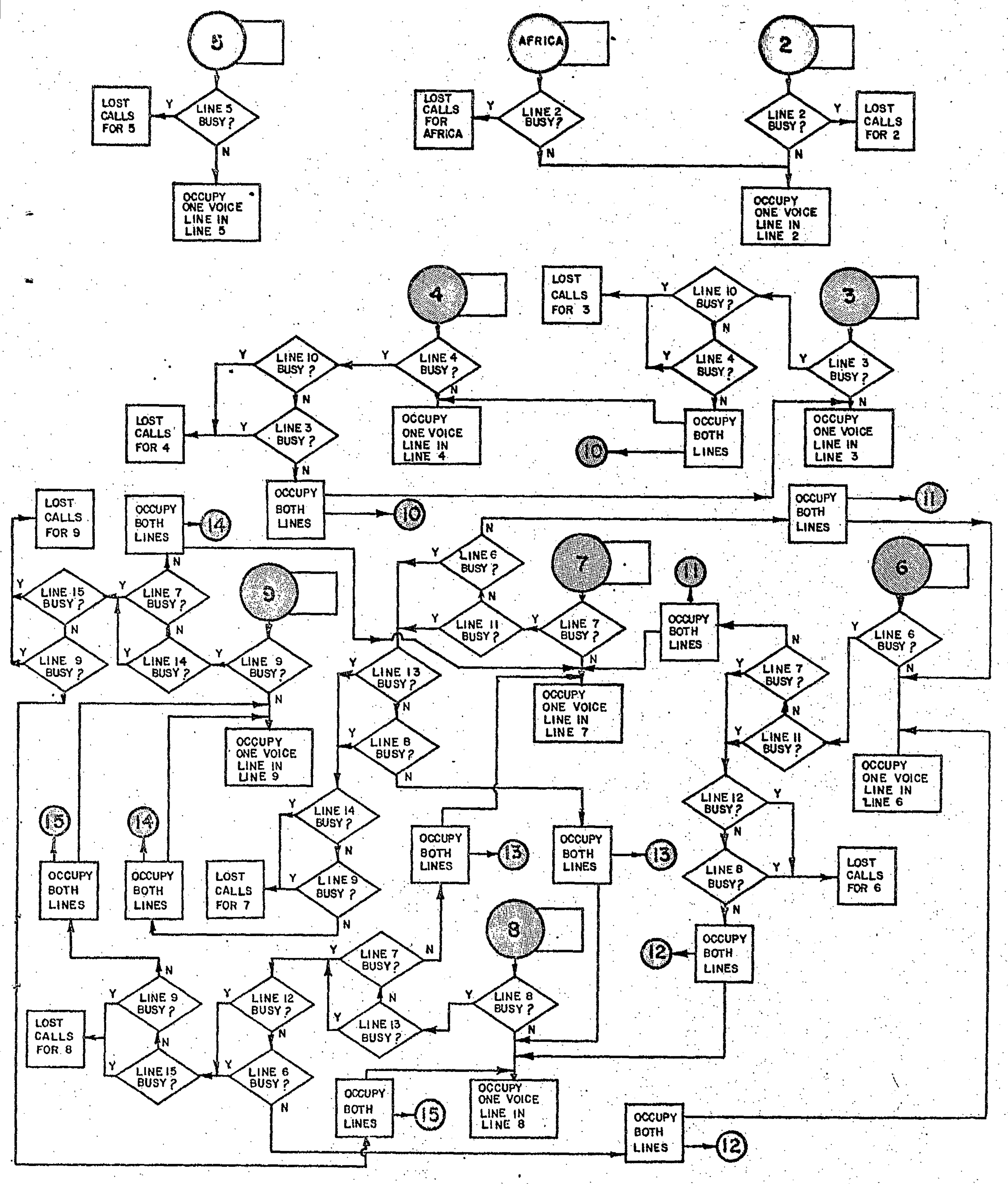
We recommend that this project continue and;

- study the impact on revenues that result in modifying the routing and numbering schemes in any proposed <u>International Data Network</u>;
- develop a model for optimizing the configuration of an expanding network. This will cater for the case of projected future traffic;
- maximize Canadian revenues by proposing optimum routing and numbering plans and minimizing the number of lost calls.

To ensure the value of this exercise for all concerned, we recommend that detailed discussions with COTC be held for the purpose of securing their agreement to assist in providing the detailed required information.

APPENDIX A

FLOW CHART OF THE SIMULATION MODEL


5
AFRICA
2
LOST CALLS FOR 5
LINE 5 BUSY ?
OCCUPY ONE VOICE LINE IN LINE 5
LOST CALLS FOR AFRICA
LINE 2 BUSY?
LINE 2 BUSY ?
LOST CALLS FOR 2
OCCUPY ONE VOICE LINE IN LINE 2
4
LINE 10 BUSY ?
LINE 4 BUSY ?
LOST CALLS FOR 4
LINE 3 BUSY ?
OCCUPY ONE VOICE LINE IN LINE 4
LOST CALLS FOR 3
LINE 10 BUSY ?
LINE 4 BUSY ?
OCCUPY BOTH LINES
3
LINE 3 BUSY ?
OCCUPY ONE VOICE LINE IN LINE 3
10
OCCUPY BOTH LINES
10
OCCUPY BOTH LINES
11
LOST CALLS FOR 9
OCCUPY BOTH LINES
14
LINE 6 BUSY ?
7
11
6
LINE 15 BUSY ?
LINE 7 BUSY ?
9
LINE 11 BUSY ?
LINE 7 BUSY ?
OCCUPY BOTH LINES
LINE 6 BUSY ?
LINE 9 BUSY ?
LINE 14 BUSY ?
LINE 9 BUSY ?
LINE 13 BUSY ?
OCCUPY ONE VOICE LINE IN LINE 7
LINE 7 BUSY ?
LINE 8 BUSY ?
OCCUPY ONE VOICE LINE IN LINE 9
LINE 11 BUSY ?
OCCUPY ONE VOICE LINE IN LINE 6
LINE 14 BUSY ?
LINE 12 BUSY ?
15
14
OCCUPY BOTH LINES
13
OCCUPY BOTH LINES
13
OCCUPY BOTH LINES
OCCUPY BOTH LINES
LOST CALLS FOR 7
LINE 9 BUSY ?
LINE 8 BUSY?
LOST CALLS FOR 6
8
OCCUPY BOTH LINES
12
LINE 7 BUSY ?
LINE 9 BUSY ?
LINE 12 BUSY ?
LINE 13 BUSY ?
LINE 8 BUSY ?
LOST CALLS FOR 8
LINE 15 BUSY ?
LINE 6 BUSY ?
OCCUPY BOTH LINES
15
OCCUPY ONE VOICE LINE IN LINE 8
OCCUPY BOTH LINES
12
Y
N

APPENDIX B

MATHEMATICAL FORMULATION OF THE OPTIMAL ROUTING PROGRAM

APPENDIX B

THE OPTIMAL ROUTING PROBLEM (static model)

"Given a telephone-type network and a traffic demand, we define the optimal routing problem as one that searches for the "cheapest" way to service the largest amount of the given traffic through the network".

A telephone network is an undirected network with the following relationship between the flows in branch $|i,j|$ and the capacity of the branch:

$$x(i,j) = c|i,j| - x(i,j) \qquad \ldots\ldots (1)$$

Where $x(i,j)$ is the flow from node i to node j and $c|i,j|$ is the capacity of the undirected branch $|i,j|$. Equation (1) shows that for a saturated branch the sum of the flows in the i,j direction with the flows in the j,i direction is equal to the total capacity (# of circuits) of the branch.

For a telephone network the measure of performance is the Grade of Service (G/S) defined as the ratio of the number of rejected messages to the total number of messages that requested service, averaged over the busy hour period.

OPTIMAL ROUTING MODEL

definition

$x^k(i,j)$ = the flow from node i to node j through the undirected branch $[i,j]$, that corresponds to the traffic demand (amount of messages) with origin s_k and destination t_k.

For a n node network the maximum number of all possible demands is given by:

$$M = n(n-1)$$

From those two definitions, the first constraint for this mathematical model, the "branch capacity constraint" given by:

$$\sum_{k=1}^{M} \left(x^k(i,j) + x^k(j,i)\right) \leq c|i,j| \quad \forall |i,j| \qquad \ldots\ldots (2)$$

Defining Q^k and x_k^r as follows:

Q^k = the requested number of messages with origin s_k and destination t_k

x_k^r = the number of rejected messages with origin s_k and destination t_k.

The second constraint for this model, the "conversation of flow constraint" is represented by:

$$\sum_{j=1}^{n} \left(x^k(i,j) - x^k(j,i)\right) = \begin{cases} Q^k - x_k^r & \text{for } i = s_k \\ 0 & \text{otherwise} \\ -Q^k + x_k^r & \text{for } i = t_k \end{cases} \qquad \ldots\ldots (3)$$

Given the constraints (2) and (3) we would like to find a flow pattern that minimizes the total cost of routing messages. Two cost functions are needed. The first one denoted by $q|i,j|$ represents the value or price (cost) associated with using one unit of capacity on branch $|i,j|$. The second one denoted by d_k represents the amount lost (penalty) for not satisfying one unit of requested flow with origin s_k and destination t_k. The total cost Z associated with the messages flowing and not flowing in the network will be given by:

$$Z = \sum_{\forall |i,j|} \sum_{k=1}^{m} q|i,j|\ x^k|i,j| + \sum_{k=1}^{m} d_k\ x_k^{\ r} \quad .....(4)$$

where:

$$x^k[i,j] = x^k(i,j) + x^k(j,i) \quad ....(5)$$

Two additional constraints are required to satisfy the feasibility of the flow patterns. These are:

$$\left.\begin{aligned} x_k^r &\geqq 0 \\ x^k(i,j) &\geqq 0 \end{aligned}\right\} \quad ....(6)$$

We also require that:

$$x^k(j,s_k) = x^k(t_k,j) = 0 \quad ....(7)$$

ANALYSIS OF THE OBJECTIVE FUNCTION Z

The problem now is to minimize the objective function Z, as given by equation (4). A close examination of equation (4) reveals that the first sum on the right hand side will optimize the routing on the basis of the relative path values. The second sum will minimize the grade of service (G/S) by routing through the network as large a portion of the demand as possible. To ensure the minimization of Z the values taken by d_k must be much higher than those taken for $q|i,j|$.

THE OPTIMAL ROUTING PROBLEM (Dynamic model)

In the preceeding pages a mathematical model was formulated to solve the optimal routing problem given a certain vector of demands constant in time. What happens in the real world is that the vector of demands fluctuates in time. We could therefore have a situation where at time t_i, the vector of demands is satisfied by the network i.e. no messages were rejected, while at time t_{i+1} the vector of demands cannot be satisfied by the network i.e. a certain amount of messages are rejected.

We have therefore to minimize the value of Z over a sequence of s time samples.

We will now propose an algorithm to solve the dynamic optimal routing problem.

ALGORITHM:

Step 1: At each time sample check if the given demand is feasible i.e. check if the corresponding grade of service is zero (all messages were accepted). The routing optimization can therefore be performed using a simplified objective function (including) only the first sum of equation (4).

Step 2: The demand is infeasible (the grade of service is > 0). Find the largest feasible demand by putting $q|i,j| = 0$ for all branches in the network and solve the problem with the following objective function:

$$Z = \sum_{k=1}^{m} d_k \, x_k^r$$

Subtract the elements of the vector $\bar{x}^r$ just found from the corresponding elements in the vector $\bar{Q}$ to get the vector of accepted message $\bar{Q}_a$.

Step 3: Find the solution of the model as in Step 1 with $\bar{Q}_a$ as the new request vector and the following objective function:

$$Z = \sum_{\forall|i,j|} \sum_{k=1}^{m} q|i,j| \, . \, x^k|i,j|$$

REMARKS:

The formulation of the presented optimal routing problem is a Linear Programming (L.P.) formulation and could therefore be solved using the Simplex Method. However, the number of equations and unknowns increases dramatically as the number of nodes increases (roughly as n^3) which means that for large scale networks no optimal solution can be found using L.P. techniques.

For moderate size networks (10 nodes, 50 links), it is possible to take advantage of the special form of the L.P. problem by using the Decomposition Principle of Dantzig-Wolfe. Therefore an optimal solution can be found for moderate size networks. If the linear programming techniques cannot be applied, one has to use simulation techniques or some form of heuristic solution to the problem.

APPENDIX C

CAPACITY COST FUNCTIONS FOR TELEPHONE NETWORKS

APPENDIX C

COST FUNCTIONS

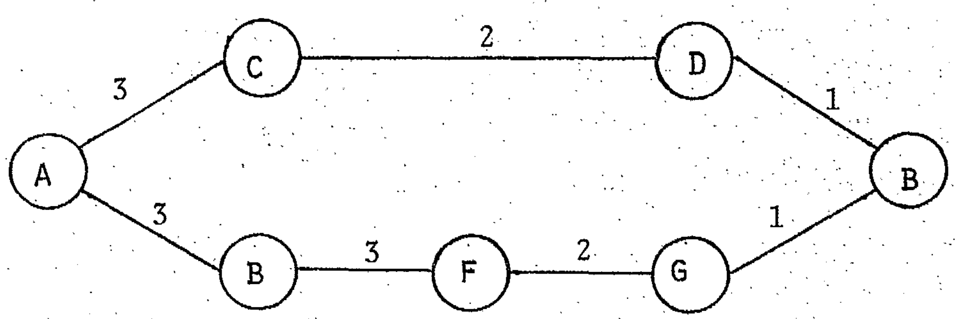
PROBLEM

Given a nonoriented network with N nodes and B branches or links, a set of origin destination nodes with known demands (number of messages and a cost function or price associated with each branch

WHAT ARE THE COST FUNCTIONS?

Associated with each branch in the network is a cost function. This cost function could represent the amount of money a country has to pay (or looses) if a message is routed through that specific branch.

EXAMPLE:



Suppose that the cost of transmitting a message from A to B is \$10 and that the cost of routing that message through the network are as shown above. If the message is routed from A to B through C and D, the revenue for B would be $|10 - (3+2+1)| = \$4$. If the message is routed through EFG, the revenue would be $(10-9) = \$1$. Hence the optimal route would be ACDB.

WHAT DIFFERENT COST FUNCTIONS SHOULD BE CONSIDERED?

Uniform Cost Function: - The price of routing a message through branch (i,j) would be constant i.e. independent of the number of messages already using that branch.

$$q(x_{|ij|}) = q_{|ij|}$$

Inverse-Harmonic Price Function - The price of routing a message through branch (i,j) would be a monotonic increasing function of the form:

$$q(x_{|ij|}) = q_{|ij|} \left[\rho + \frac{1}{C_{|ij|} - x_{|ij|} + 1} \right]$$

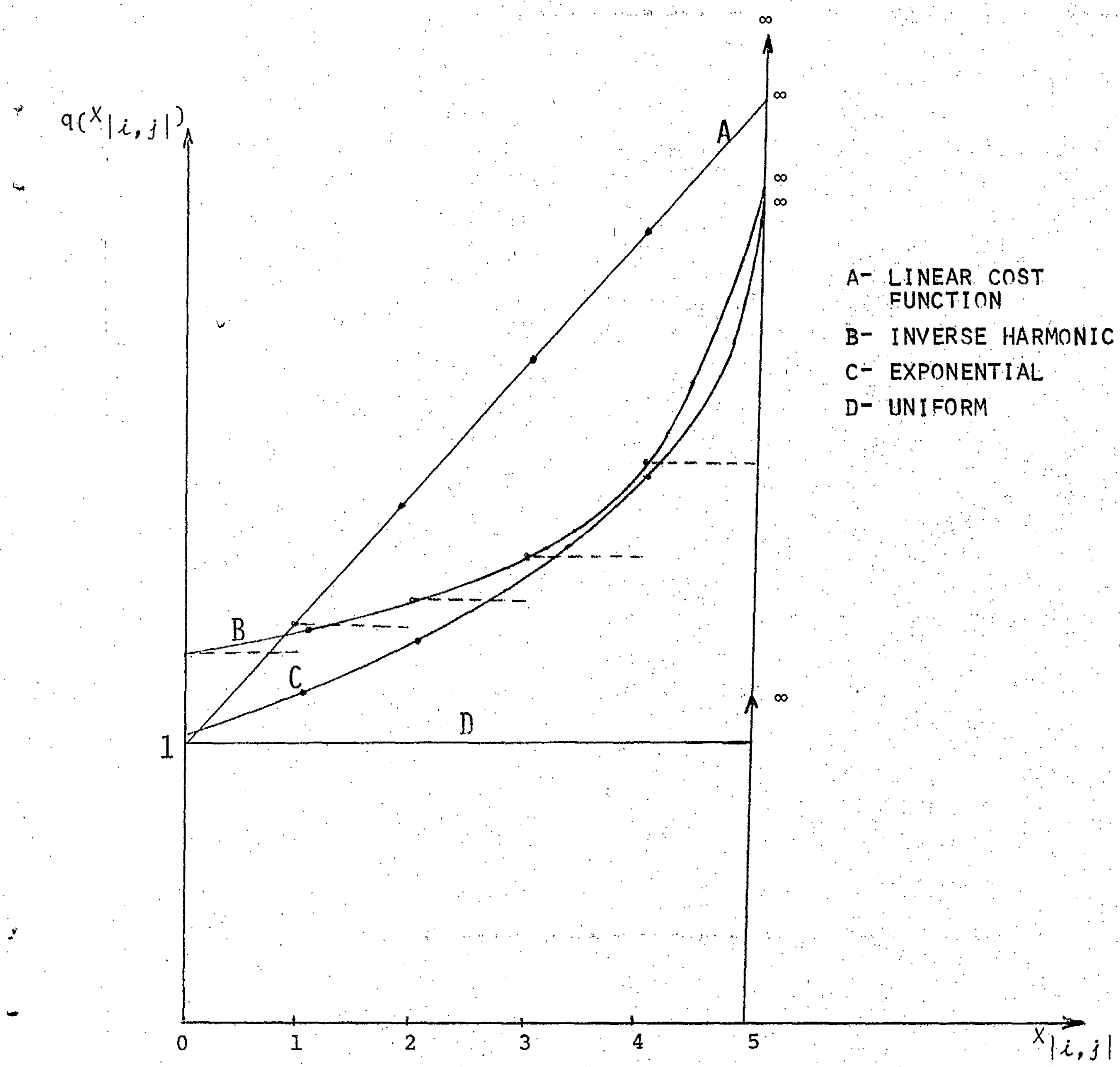
Exponential Price Function Also a monotonic increasing function of the form:

$$q(x_{|ij|}) = q_{|ij|} \left[\rho + \frac{1}{2 \exp (C_{|ij|} - x_{|ij|})} \right]$$

Linear Cost Function - also an increasing function (linear)

$$q(x_{|ij|}) = q_{|ij|} \left[\rho + \frac{x_{|ij|}}{C_{|ij|}} \right]$$

COST OF SENDING ONE ADDITIONAL UNIT OF FLOW, GIVEN THAT $X_{|i,j|}$ ARE ALREADY IN THE CIRCUIT



$C_{|i,j|} = 5$

$\rho = 1$

$q|i,j| = 1$

NOTATION

$x_{|ij|}$ = total number of messages on branch $|ij|$ from i to j and from j to i.

$C_{|ij|}$ = total capacity (total number of circuits) of branch $|ij|$.

$q_{|ij|}$ = weighting factor for the branch $|ij|$

ρ = constant.

CONSTRAINTS

The constraints can be formulated as follows:

(1) $$\sum_{k=1}^{M} \left(x_{ij}^{k} + x_{ji}^{k} + x_{|i,j|}^{s} \right) = C_{|ij|} \qquad \text{for } (i,j) \in B$$

This is a branch capacity constraint, where $x_{|i,j|}^{s}$ represents the "slack" flow on branch i,j.

(2) $$\sum_{\substack{j \\ |i,j| \in B}} \left(x_{ij}^{k} - x_{ji}^{k} \right) = \begin{cases} Q^k - x_k^r & i = s_k \\ 0 & \text{otherwise} \\ -Q^k + x_k^r & i = t_k \end{cases}$$

this constraint is simply the conservation of flow constraint.

(3) $x_{j,s_k}^{k} = 0 \quad ; \quad x_{t_k,j}^{k} = 0$

$x_k^r \geq 0$ (number of rejected messages cannot be negative)

$x_{ij}^{k} \geq 0$

$x_{|i,j|}^{s} \geq 0$

OBJECTIVE FUNCTION

Our purpose is to minimize the cost associated with routing the messages.

We can:

a) *minimize the cost of routing over the entire network*

$$\min Z = \sum_{(ij)\varepsilon B} q(x_{|ij|}) + \sum_k d_k x_k^r \qquad \text{(for all k)}$$

where;

d_k = penalty for not satisfying one message of the requested k^{th} demand

x_k^r = number of messages rejected in the k^{th} demand

or b) *or minimize the cost of routing for only the messages addressed to Canada.*

$$\min Z = \sum_{(ij)\varepsilon B} q(x_{|ij|}) + \sum_k d_k x_k^r \qquad \text{(for messages routed to Canada)}$$

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PLANNING MODELS OF INTERNATIONAL COMMUNICATIONS NETWORKS