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ABSTRACT

Dimensional analysis shows that the commonly used dimensionless transverse dispersion coefficient, e_z/u_*h, should depend on both the friction factor and the width to depth ratio of the flow. Inspection of published data fails to resolve this dependence and experiments were performed to conduct a systematic investigation. Results for e_z lead to the conclusion that secondary circulation, and not turbulent fluctuations, is the dominant transport mechanism. These results help to explain the failure to find consistent trends in the variation of e_z/u_*h and suggest that u_*h is not a good representation for e_z. An alternate dimensionless dispersion coefficient, e_z/uW, is introduced and appears to account for secondary circulation effects more properly. The results can be correlated with width to depth ratio and friction factor quite systematically and e_z/uW is suggested as the dimensionless dispersion coefficient which should be used for transverse spreading.

"EFFET DU FACTEUR DE FROTTEMENT ET COEFFICIENT D'ORIENTATION SUR LA DISPERSION TRANSVERSALE DANS LES CANAUX RECTANGULAIRES"

Y.L. Lau et B.G. Krishnappan

RESUME

L'analyse dimensionnelle indique que le coëfficient de dispersion transversale sans dimension utilisé le plus fréquemment, e_z/u_*h, devrait être fonction du facteur de frottement et du rapport largeur/profondeur de l'écoulement. L'étude des données publiées ne résout pas cette dépendance et des expériences ont été faites pour procéder à une étude systématique. Les résultats e_z mènent à la conclusion que la circulation secondaire, et non les fluctuations turbulentes, constitue le mécanisme de transport dominant. Ces résultats permettent d'expliquer pourquoi on n'est pas parvenu à trouver des tendances uniformes dans la variation de e_z/u_*h et proposent que u_*h ne constitue pas une bonne représentation pour e_z. Un autre coëfficient de dispersion sans dimension, e_z/uW, est introduit et semble mieux justifier les effets de la circulation secondaire. On peut mettre les résultats en corrélation de façon assez systématique aven le rapport largeur/profondeur et le facteur de frottement et on propose que e_z/uW soit le coëfficient sans dimension à utiliser pour la dispersion transversale.

INTRODUCTION

The calculation of the spreading rate of materials and their concentration in natural rivers is usually based on solutions of the mass conservation equation. Analytical solutions exist for uniform flows in straight channels while more complicated stream geometries or flow conditions can be handled by numerical methods. However, the accuracy of these solutions depends on having correct values for the turbulent mixing coefficients. For lateral spreading, it is customary to assume that the lateral mixing coefficient is proportional to the product of shear velocity and average depth. The proportionality constant is called the dimensionless transverse dispersion coefficient, K, and has been measured in the laboratory as well as in the field. Unfortunately, the value for K has shown considerable variations, even for straight laboratory channels. Some of the variation was thought to be the effect of the channel geometry and Okoye (12) has proposed that K increased with the width to depth ratio of the flow. Channel roughness seems to be another logical parameter but has received relatively little attention. Prych (14) concluded from his experiments that the friction factor f had no effect on K. However, several others (Engmann (4) Jobson (5), Lau and Krishnappan (9) have shown that K increases with increased roughness.

This paper presents the results of some experiments conducted to investigate the variation of the transverse dispersion coefficient and in particular, its dependence on friction factor and width to depth ratio in straight rectangular channels.

BACKGROUND

Mass Conservation Equation: For transverse mixing problems in rivers, concentration variations in the vertical direction are often negligible and the depth averaged mass conservation equation is generally used. The equation is written as

$$\frac{\partial (hc)}{\partial t} + \frac{\partial (huc)}{\partial x} + \frac{\partial (hwc)}{\partial z} = \frac{\partial}{\partial x}\left(he_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial z}\left(he_z \frac{\partial c}{\partial z}\right) \quad \ldots\ldots\ldots (1)$$

where c = depth averaged concentration;
u,w = depth averaged velocities in the x and z directions, respectively;
h = mean flow depth;
e_x,e_z = depth averaged longitudinal and transverse dispersion coefficient, respectively;
x,z = longitudinal and transverse co-ordinate, respectively;
t = time.

The derivation of Equation (1) and the assumptions involved have been discussed by Holley (5). In this equation, the dispersion coefficients e_x and e_z are defined as follows:

$$e_x = -\frac{(\overline{u^y c^y} + \overline{T_x})}{\partial c/dx} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

$$e_z = -\frac{(\overline{w^y c^y} + \overline{T_z})}{\partial c/dz} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

where $\overline{T_x}$, $\overline{T_z}$ = depth averaged transport caused by turbulent fluctuations in the x and z directions, respectively;
$\overline{u^y c^y}$ = depth averaged transport due to differential convection in the x direction;
$\overline{w^y c^y}$ = depth averaged transport due to differential convection in the z direction;

and the overbar denotes depth average and the superscript y denotes deviations from the depth averaged value.

It can be seen that the dispersion coefficients e_x and e_z represent transport by the turbulent fluctuations as well as the transport arising from vertical shear in the case of e_x and from secondary circulation in the case of e_z.

Using Equation (1), Holley (5) derived a generalized change of moment method for evaluating e_z from measured concentration distributions, considering steady state conditions and neglecting longitudinal dispersion. For uniform flow in straight rectangular channels, the depth h is constant and the depth averaged transverse velocity w is zero. In this case, e_z is given by the usual formula

$$e_z = \frac{u}{2} \frac{d\sigma^2}{dx} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where σ^2 is the second moment of the concentration distribution, given by the relation

$$\sigma^2 = \frac{\int_{-w/2}^{w/2} z^2 \, c \, dz}{\int_{-w/2}^{w/2} c \, dz} \qquad \ldots\ldots\ldots\ldots\ldots (5)$$

where W is the width of the channel.

It should be pointed out that even though the depth averaged velocity w is zero, the deviation w^y is not, and therefore, even for a straight rectangular channel, e_z represents the rate of spread of the concentration distribution wiich is caused by turbulent fluctuations as well as the secondary circulation in the cross section.

Dimensional Analysis: Since a theoretical relationship for e_z does not exist, it ought to be useful to perform a dimensional analysis and examine the variables which may be important. In straight rectangular open channels, the parameters affecting the lateral dispersion coefficient are the mean velocity u, the depth h, the width W, the bottom shear stress or shear velocity u_*, the density ρ and the viscosity μ. Therefore,

$$e_z = \Phi\left(u, h, W, u_*, \rho, \mu \right) \qquad \ldots\ldots\ldots (6)$$

Dimensional analysis then gives

$$\frac{e_z}{u_* h} = \Phi\left(\frac{u_*}{u}, \frac{W}{h}, \frac{\rho u_* h}{\mu} \right) \qquad \ldots\ldots\ldots (7)$$

where $\frac{e_z}{u_* h}$ is the dimensionless transverse dispersion coefficient K.

For highly turbulent flows with rough boundaries, the effect of viscosity can be neglected and one can write

$$\frac{e_z}{u_* h} = \phi\left(f, \frac{W}{h} \right) \qquad \ldots\ldots\ldots (8)$$

where $f = 8\,(u_*/u)^2$ is the Darcy-Weisbach friction factor. Therefore, it can be seen that the usual dimensionless dispersion coefficient is a function of both the friction factor and the width to depth ratio. This is to be expected since the friction factor indicates the bottom shear which generates the turbulent fluctuations in the flow and the width to depth ratio affects the secondary circulation in the channel.

Previous results: An inspection of the previously published data on the transverse dispersion coefficient K in straight open channels can be made in the light of the results of dimensional analysis. Data are available from nine different publications. The laboratory flumes which were used varied from 36 cm to 238 cm in width, with either smooth or artificially roughened bottoms. In most instances, no more than three or four data points are available from each study and these cover only very small variations in f and W/h. Therefore, it is necessary to include data from all pertinent sources. All the data are tabulated in Table III-1 in Appendix III.

Figure 1 is a plot of e_z/u_*h versus W/h using all the data points. The value of f is written beside each data point. It is evident that e_z/u_*h is not a constant even for straight rectangular channels. The values in Figure 1 varied from 0.08 to 0.24. It should be pointed out that some published values for natural streams are as high as 0.73. However, natural streams usually have non-uniform cross sections and meanders which introduce advective transport by the transverse velocity w. This transport has not been separated out but has been lumped into e_z in the published reports. Therefore, the reported field values are likely over-estimated and are bound to be higher than flume values.

Figure 1 does not give a clear indication of how e_z/u_*h varies with W/h at constant values of f or how much effect changes in friction factor have on e_z/u_*h. Certainly the effect of friction factor is evident in some cases. For example, one can detect a consistent increase of e_z/u_*h with f from the data of Miller and Richardson (11). This effect has been pointed out by Engmann (4), Jobson (6) and Lau and Krishnappan (9) in their discussions of the work of Miller and Richardson.

Figure 2, which is a plot of e_z/u_*h versus f for points all having W/h ratio close to five, shows this effect rather clearly. However, the rest of the data do not exhibit such clear trends. It can be seen in Figure 1 that there are points at the same W/h ratio but with vastly different friction factors which have the same values of e_z/u_*h. For example, at W/h of about 16, a point from Prych with f = 0.018 (smooth bottom) gives e_z/u_*h equal to 0.145, whereas, a point from Okoye with f = 0.16 (stone roughness) gives e_z/u_*h equal to 0.14.

The manner in which e_z/u_*h ought to vary with W/h is also not very clear. The only published work so far which tries to account for the effect of width to depth ratio is by Okoye (12), who suggested that e_z/u_*h should increase with the width to depth ratio and indeed, his own data appears to behave in such a manner. However, when one takes into consideration all the other data, there is no indication that such a variation applies in all cases. Okoye suggested that as the width to depth ratio increased, the turbulence scale was increased, giving a larger spreading rate. However, careful inspection of the data reveals some contradictions to this reasoning. Fig.3 shows a plot of e_z versus h for some of the data of Okoye, Prych and Engmann. To investigate the effect of depth on e_z, points with constant width and velocity have to be chosen and not many points can be found for each case. (The straight lines in Fig. 3 are used just to indicate data with same u and W and do not suggest linear variation of e_z with h). At constant u and W, as the depth is decreased, the friction factor and the shear velocity increase and the turbulent fluctuations should increase. However, it can be seen that in every case, e_z, which is a measure of the spreading rate, decreased as the depth became smaller. Therefore, as the width to depth ratio increased, the spreading rate actually decreased, contary to what Okoye had suggested.

It is evident that further investigations are needed in order to clearly define the dependence of the transverse dispersion coefficient on friction factor and width to depth ratio. For such an investigation, experiments should be conducted by varying only one of the independent parameters while the other is kept constant so that the functional relationship indicated by Equation 8 can be sought.

EXPERIMENTAL SET-UP AND PROCEDURE

Experiments were conducted in a flume 30.7 metres long and 60 cm wide. Extra sidewalls could be installed to convert the flow channel to 45 cm or 30 cm in width. The flume slope could be adjusted by a set of motorized screwjacks. Discharge measurements were made using a weir box at the downstream end.

The flume bed roughness was varied by using sands of different sizes. First the flume bottom was covered with 'Mactac' self-adhesive vinyl covering. Then a thin layer of varnish was brushed on and the sand was sprinkled on top so that the 'Mactac' was completely covered. After drying overnight, the excess sand was washed off and a uniform layer of sand was left on the flume bed. To change over to another roughness, the 'Mactac' was stripped off and the process was repeated. Three different sizes of sand were used, with mean diameters of 0.27 cm, 0.20 cm and 0.04 cm, respectively. In addition, some runs were made using only the smooth vinyl covering on the flume bottom.

For a given bed roughness, dispersion experiments were conducted for flows at several depths, with flume width at 60 cm. Then the flume was narrowed and the runs were repeated, giving a variation in width to depth ratio with friction factors the same as the previous runs. For a new roughness, flow depths were varied so that runs could be made at the same values of friction factor as the previous set. Usually, either the largest or the smallest friction factor values could not be repeated because of limitations in flow depths which could be used. However, the intermediate values of f were all the same. The flow depths varied between 1.3 cm and 5.0 cm. Mean flow velocities were kept approximately constant at 20 cm/s for most of the runs. The hydraulic data for the 22 experimental runs are summarized in Table 1.

Dispersion measurements were made using salt solution as the tracer. The salt solution was mixed with methanol to make it neutrally buoyant. The concentration of salt in the tracer solution was 62.5 g/ℓ. The solution was discharged continuously from a constant head injection apparatus into the middle of the flume at approximately mid-depth. The injection rate was adjusted so that the tracer was issued from the discharge nozzle at the same speed as the mean flow velocity in the flume. The tracer concentrations were measured at mid-depth using a single electrode conductivity probe of the type used by McQuivey and Keefer (10). The output was recorded on a Hewlett Packard model 7100BM strip chart recorder. The probe construction, bridge circuit, calibration and operational details are explained in a laboratory report by Dunnett (1).

Traverses with the conductivity probe were made at eight or more stations downstream from the injection point to obtain cross-stream concentration distributions. The measuring stations were chosen so that vertical mixing was completed before the first station. Great care was taken to obtain the correct concentration values at the edges of the plume by measuring the background concentration without tracer injection either immediately before or after measurements in the plume. This was done at every station.

RESULTS AND DISCUSSION

The concentration distributions were obtained from the strip chart recordings and the variances of the distributions were calculated from Equation (5). The dispersion coefficient e_z was calculated using Equation (4). The values of e_z, as well as the non-dimensionalized dispersion coefficients are listed in Table 1 together with the hydraulic data.

Figure 4 shows a plot of e_z versus h for the five different flume width and bed roughness combinations, with velocity held constant at about 20 cm/s in each case. The results indicate the same behaviour of e_z with h as those of the previous workers given in Figure 3, i.e. e_z or the spreading rate, increased as the depth increased, with width and velocity kept constant. Also, given the same bed roughness and flow depth, e_z also increased as the width was reduced. This increase in spreading rate, as flume width was reduced, happened in every case. In one instance, when the flume was narrowed from 45 cm to 30 cm, the increased spreading caused the plume to reach the sidewalls right from the first station. The concentration readings were quite high at the walls and because of this effect, the results of that particular test were not used. These results all confirm that as the width to depth ratio is decreased, the spreading rate is increased and vice versa. Therefore, as the flow depth becomes larger, the spreading rate increased, even though the bottom shear stress was smaller and the turbulent fluctuations ought to be less intense. Recalling that e_z represents the sum of the transports caused by the turbulent fluctuations which are generated by the bottom shear, and by the secondary circulation which is largely affected by the width to depth ratio in the channel, one is led to the conclusion that as the width to depth ratio was reduced, the transport due to the secondary circulation increased and this increase was more than sufficient to compensate for the decrease in transport by the turbulent fluctuations, resulting in a net increase in spreading rate. This also suggests that as far as transverse spreading is concerned, the dominant mechanism is the secondary circulation and not the fluctuations caused by turbulence.

The secondary circulations which are responsible for the lateral transport is caused by the uneven distribution of shear stress along the wetted perimeter of the channel and has been termed secondary currents of the second kind by Prandtl (13). Our understanding of this secondary circulation is still rather incomplete and there is no means of predicting its intensity. Kartha and Leutheusser (8) did make measurements of the tractive force distribution for rectangular channels of various width to depth ratios. Their results showed that variations in shear stress along the bed, given by the ratio between the maximum stress and the average stress, was a maximum for W/h of about 3 and decreased as W/h increased. This ratio also decreased for W/h less than 3 but such small W/h ratios are not of much practical significance. An analytical solution based on the assumption of laminar flow was also presented and although it cannot be expected to be correct in value, it also showed the same trend. The same behaviour was found for the shear stress variation along the side walls. Since the secondary circulation is driven by the transverse variation in shear stress, it is reasonable to expect that for the smaller width to depth ratio, when the variation in shear is larger, the secondary circulation will be stronger. It follows then that the transport due to the secondary circulation should be larger for the smaller W/h ratio and this is exactly what the experimental results indicate.

Comparisons between the data for the 2.7 mm sand and the 0.4 mm sand also shows that for the same width and depth, e_z, as expected, is larger for the flows with the rougher bed. This is the effect of the increase in bottom shear and turbulent fluctuations.

Figure 5 shows the variation of the dimensionless dispersion coefficient e_z/u_*h with width to depth ratio W/h. The other parameter affecting e_z/u_*h is, of course, the friction factor f and the values of f are shown beside each data point. The dashed lines in Figure 5 are used to show the trends in the variations of e_z/u_*h at constant values of f. It can be seen that with f held constant, e_z/u_*h decreased with W/h for most of the cases, contrary to what Okoye (12) proposed. For W/h larger than about 15, there seems to be some organized behaviour, with e_z/u_*h decreasing with W/h and increasing with increased f. However, the picture is much more confusing for the data with smaller width to depth ratios. The data for f = .07 appears to reverse in trend, with $e_z/u\ h$ decreasing as W/h becomes smaller. More disturbing is the fact that the data for the smooth wall tests, with much smaller values of friction factor, have $e_z u_* h$ values just as large or larger than the rest. It appears that the organized picture shown by the dashed lines would not persist for all values of the friction factor.

Some understanding of this confusion may be obtained by reviewing the behaviour of e_z. The data presented in Figures 3 and 4 have shown that, with all other parameters kept constant, e_z decreased as the width W was increased, and this appears to be the effect of secondary circulation which is governed by the W/h ratio. When the width to depth ratio becomes large, the channel is essentially two-dimensional and the spreading rate should no longer be affected and e_z should approach a constant value. If the bed roughness is increased, with u and h constant so that f is larger, it has been shown that e_z increases. However, u_* has also increased. In the present experiments, the increases in e_z were larger than the increases in u_*, resulting in increases of e_z/u_*h with f. This may not necessarily be so for all cases and may be different for other ranges of friction factor. For smaller width to depth ratios, the spreading rate increased as the W/h ratio decreased and the results strongly indicate that the transport due to the secondary circulation is dominant over that due to turbulent fluctuations. Therefore, for the same W, h and u, the spreading rates would not be much different whether the flow is over a rough or smooth bed. Since the rougher flow has larger values of u_*, it is entirely possible for e_z/u_*h to be larger for the flows with smaller f.

In the light of the above discussion, the confused picture shown by the data in Figure 3 is really not surprising and it may be virtually impossible to obtain an organized set of curves indicating the variation of e_z/u_*h with W/h. This points to the fact that (u_*h) is not a good parameter for the representation of e_z because it emphasizes the bed shear which is actually not the dominant transport mechanism.

An Alternate Dimensionless Dispersion Coefficient

Considering that secondary circulation appears to be the more important factor governing transverse spreading, it seems more appropriate to use the channel width as the characteristic length scale. Since the effect of u_* is already included in the parameter f, it may be simpler to use u instead of u_* for the characteristic velocity. If the dispersion coefficient e_z is assumed to be proportional to uW, Equation (8) can be rewritten as

$$\frac{e_z}{uW} = \phi \left(f, \frac{W}{h}\right) \quad \ldots\ldots\ldots\ldots\ldots (9)$$

The dimensionless dispersion coefficient e_z/uW is also a function of the friction factor and the width to depth ratio. Values for e_z/uW were calculated and are also listed in Table 1. Figure 6 shows the plot of e_z/uW versus W/h with friction factor as the third parameter. It can be seen that the data fall very nicely into lines of constant f. At constant width to depth ratios, when the secondary circulation should have the same effect, the rougher flows should have larger values of dispersion coefficient and this is found to hold true for all the points including the smooth flows. Only two of the data points did not fall on the correct line for their f value, but this can be attributed to experimental scatter. Comparison of Figure 5 and 6 clearly indicates that uW is a better representation of e_z than u_*h.

The previously published data of e_z/u_*h which are shown in Figure 1 are now replotted in the form e_z/uW in Figure 7. All the data are included except for a few from Miller and Richardson (11) which had values too large to be included in the figure. It can be seen that although the correlation is not quite as good as in Figure 6, there is some semblance of order as opposed to the picture of confusion presented in Figure 1. All the smooth bed flows with f approximately equal to 0.022 lie along a lower curve, with increasing values of e_z/uW for larger values of f. The trends shown are the same as those in Figure 6. A fair amount of scatter along lines of constant f values can be expected from such a wide assortment of data. The concentration measurement methods were different. Various types of roughnesses were used including expanded metal lath, stones, and staggered wooden blocks. The calculations of blockage by the roughnesses and the methods of determining the effective flow depths were different. Some studies made corrections for the side wall shear, whereas others did not. All these factors contributed towards the variation of f values. Three of the curves from Figure 6 are shown as dashed lines in Figure 7. The agreement can be considered reasonable in light of the above discussion. Figure 7 supports the argument that $e_z \sim uW$ is a more appropriate assumption than $e_z \sim u_* h$ and e_z/uW is the dimensionless variable which should be used.

Table 1 - SUMMARY OF HYDRAULIC DATA AND RESULTS

Bed roughness (1)	W, in centimeters (2)	h, in Centimeters (3)	u, in Centimeters per second (4)	u_*, in Centimeters per second (5)	f (6)	e_z, in Centimeters squared per second (7)	$\frac{e_z}{u_* h}$ (8)	$\frac{W}{h}$ (9)	$\frac{e_z}{uW}$ (10)
2.7 mm sand	60	3.92	20.41	1.88	.068	1.105	0.150	15.31	9.02x10
"	"	3.32	19.58	2.03	.086	0.878	0.130	18.07	7.47 "
"	"	1.74	20.08	2.39	.114	0.592	0.142	34.48	4.91 "
"	"	1.43	20.83	2.80	.145	0.601	0.150	41.96	4.81 "
"	45	3.75	19.50	1.82	.070	0.929	0.136	12.00	10.59 "
"	"	3.22	20.15	2.09	.086	1.162	0.173	13.97	12.81 "
"	"	1.80	20.00	2.39	.114	0.85	0.197	25.00	9.44 "
"	"	1.27	19.95	2.69	.145	0.883	0.259	35.43	9.84 "
0.4 mm sand	60	3.06	19.66	1.54	.049	0.615	0.131	19.61	5.21 "
"	"	2.30	20.36	1.78	.061	0.502	0.123	26.09	4.11 "
"	"	1.79	20.02	1.87	.070	0.363	0.108	33.52	3.02 "
"	"	1.40	20.12	2.09	.086	0.335	0.114	42.86	2.78 "
"	45	4.0	19.94	1.58	.050	0.88	0.139	11.25	9.81 "
"	"	2.78	19.82	1.76	.063	0.67	0.137	16.19	7.51 "
2.0 mm sand	30	3.38	20.41	1.91	.070	0.884	0.137	8.88	14.43 "
"	"	2.52	20.10	2.08	.086	0.915	0.175	11.90	15.17 "
"	"	1.55	20.00	2.39	.114	0.74	0.199	19.35	12.34 "
Smooth	60	4.15	33.73	1.96	.027	1.399	0.172	14.46	6.91 "
"	"	3.90	30.77	1.82	.028	1.144	0.161	15.38	6.19 "
"	"	4.20	17.90	1.096	.030	0.74	0.161	14.29	6.89 "
"	"	4.85	15.46	0.93	.029	0.907	0.201	12.37	9.78 "
"	"	4.96	30.24	1.72	.026	1.41	0.165	12.10	7.77 "

SUMMARY

Results from the present experiments showed that, at constant velocity, the value of e_z increased when the flow depth for a given channel was increased. Similar results were found for other published data. Because the spreading due to turbulent fluctuations ought to have decreased when the flow depth is larger, the increase in spreading rate must then reflect an increase of the spreading due to secondary circulation. Increases in e_z were also found when the depth was constant but the channel width was reduced. These results lead to the conclusion that the dominant mechanism in transverse spreading is the secondary circulation which is driven by the variations in transverse shear, which in turn is governed by the width to depth ratio in a rectangular channel. This conclusion offers some explanation why no consistent patterns of variation can be found for the published values of the commonly used dispersion coefficient e_z/u_*h. The assumption that e_z is proportional to u_*h is not a good one because u_*h reflects the turbulent shear and not the secondary circulation which is more important. Predictions of e_z/u_*h for different flows are pratically impossible. It was reasoned that $e_z \sim uW$ might be a better assumption and plots of e_z/uW versus W/h did show much more consistent variations with e_z/uW decreasing with W/h and increasing with f. This supports the argument that e_z/uW is a better dimensionless dispersion coefficient to use than e_z/u_*h.

More data are needed for W/h ratios larger than 50, although it can be expected that for very large W/h ratio, e_z should approach a constant for constant u, and therefore, curves for constant f should vary as $1/W$. It is also expected that e_z/uW will be different for

for other cross-sectional shapes such as trapezoidal channels because the secondary circulation may be quite different. Whether the difference will be very significant can only be determined by experiments.

ACKNOWLEDGEMENT

The writers wish to acknowledge the assistance of R. Toni and F. Dunnett in the conducting of the experiments and the analysis of data.

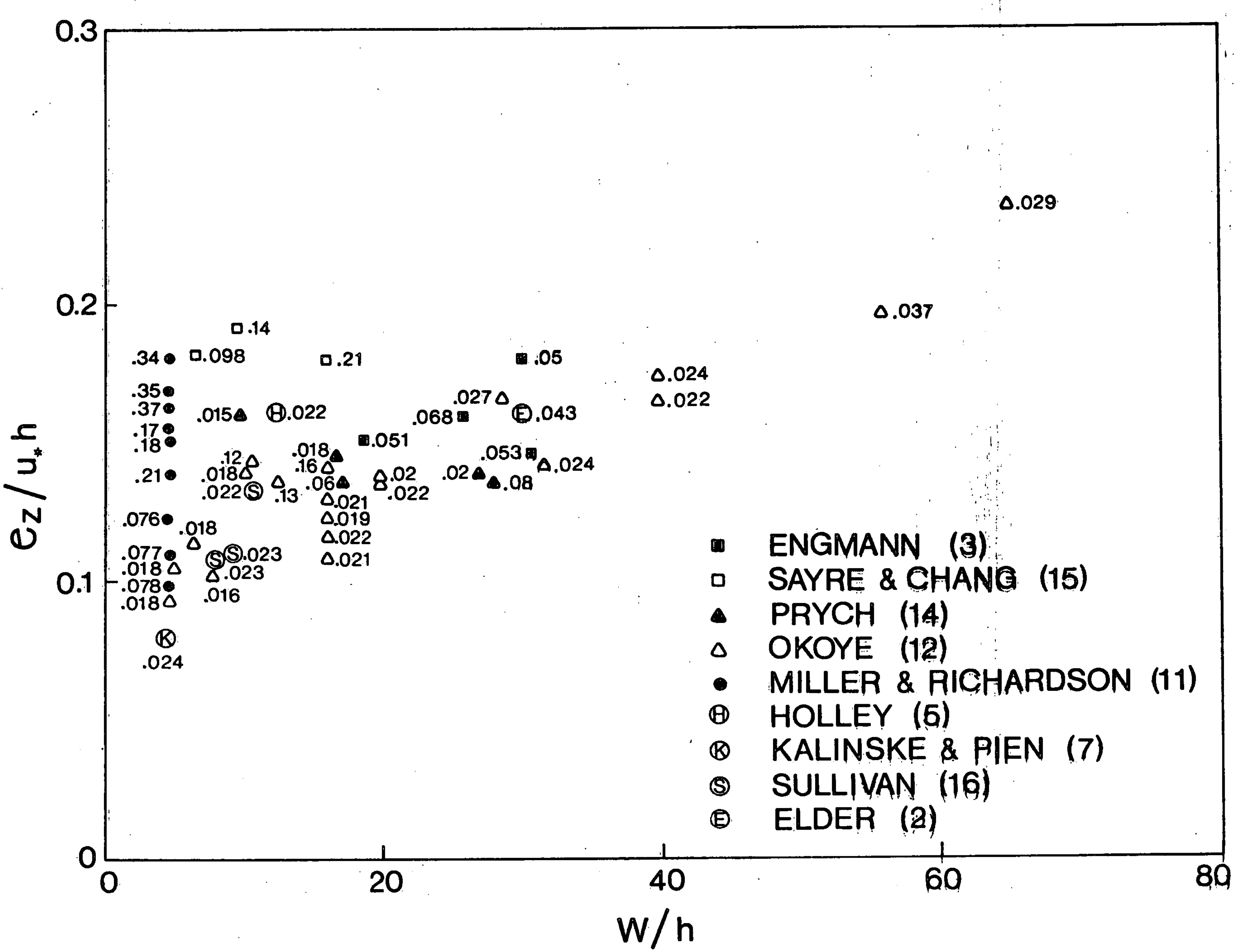

Figure 1. Published Data of e_z/u_*h for rectangular channels. (The values of f are written beside each point)

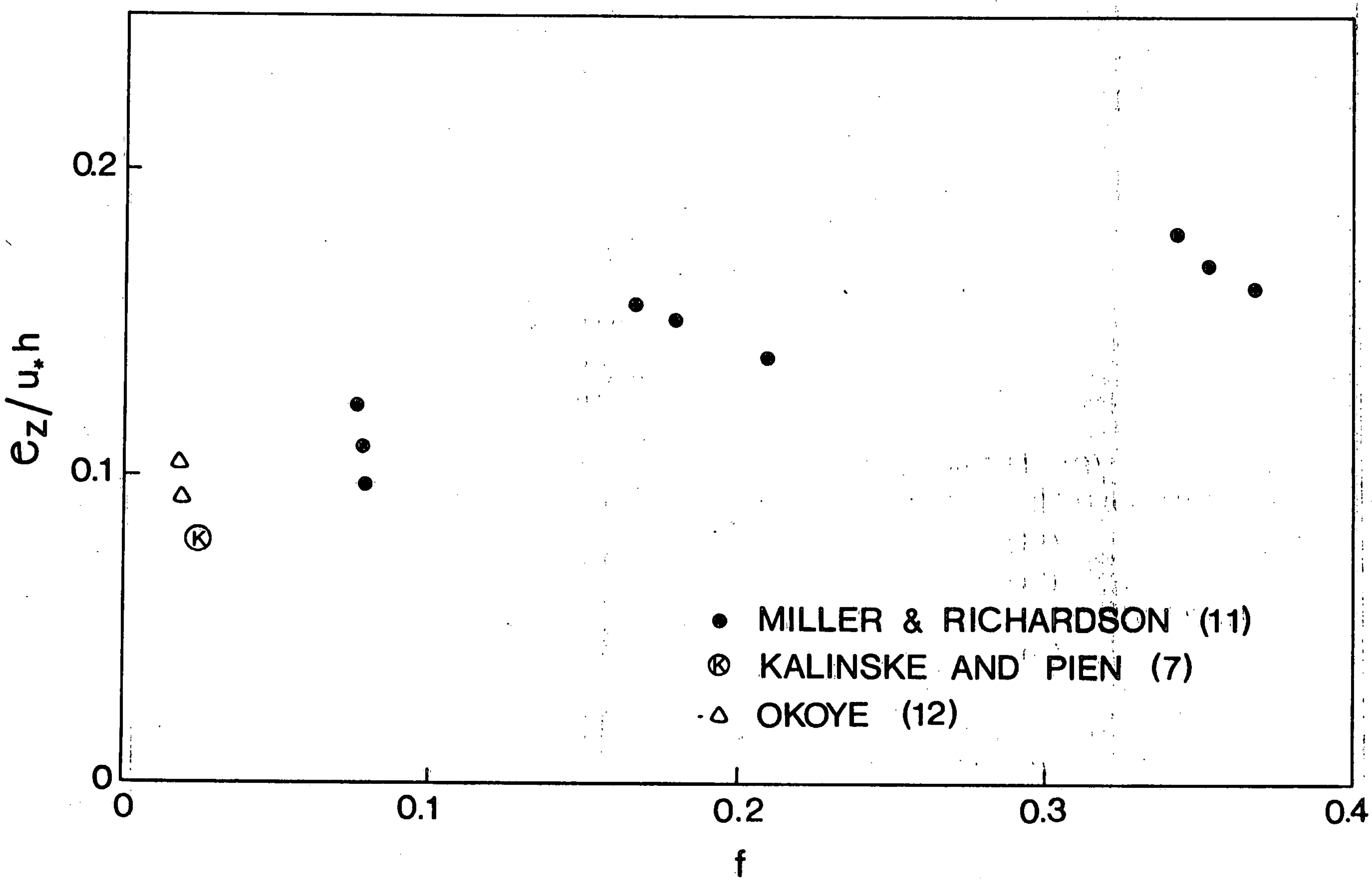

Figure 2. Variation of e_z/u_*h with friction factor at constant width to depth ratio (from published data)

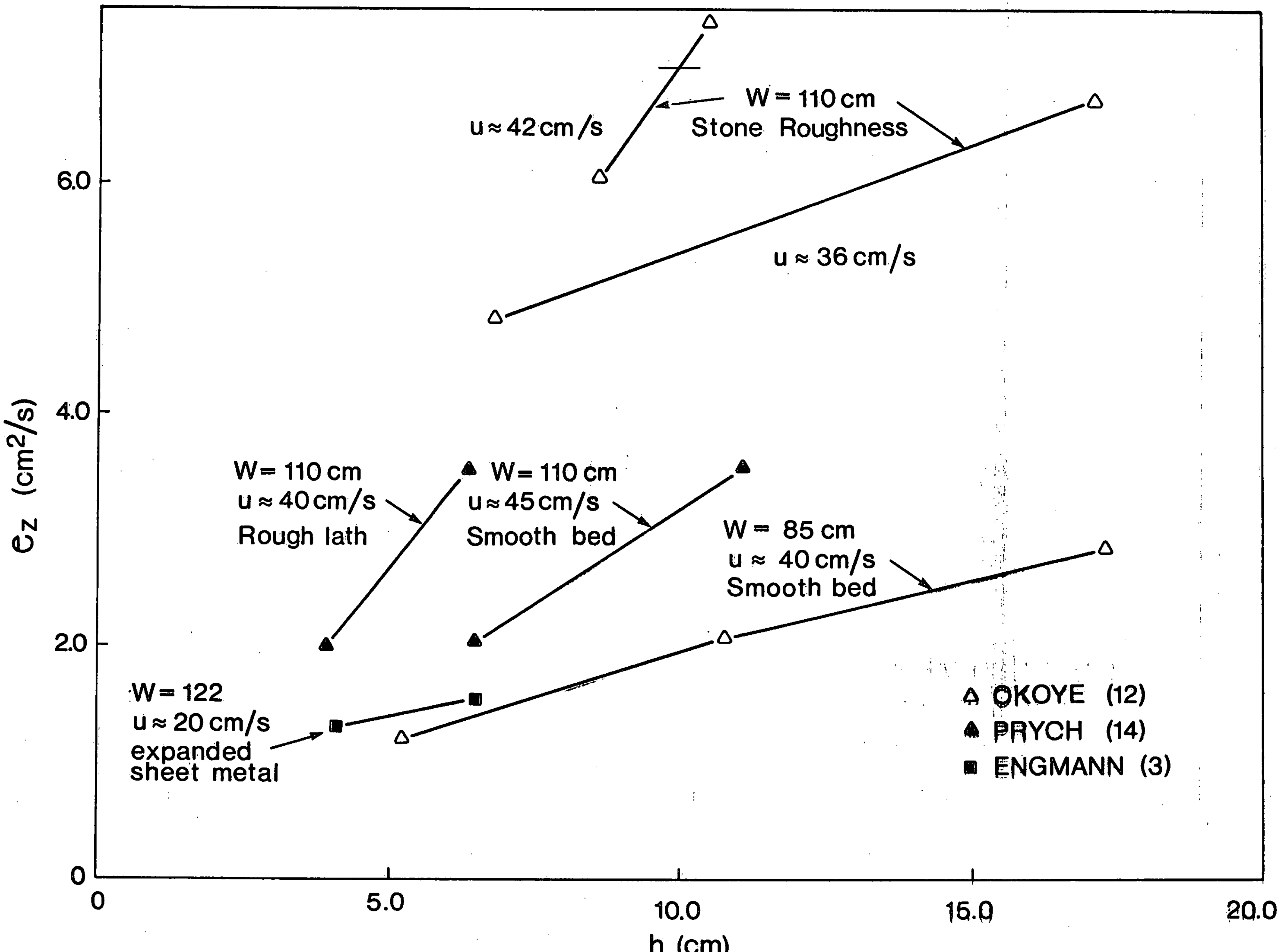

Figure 3. Variation of e_z with h at constant widths and velocities. (from published data)

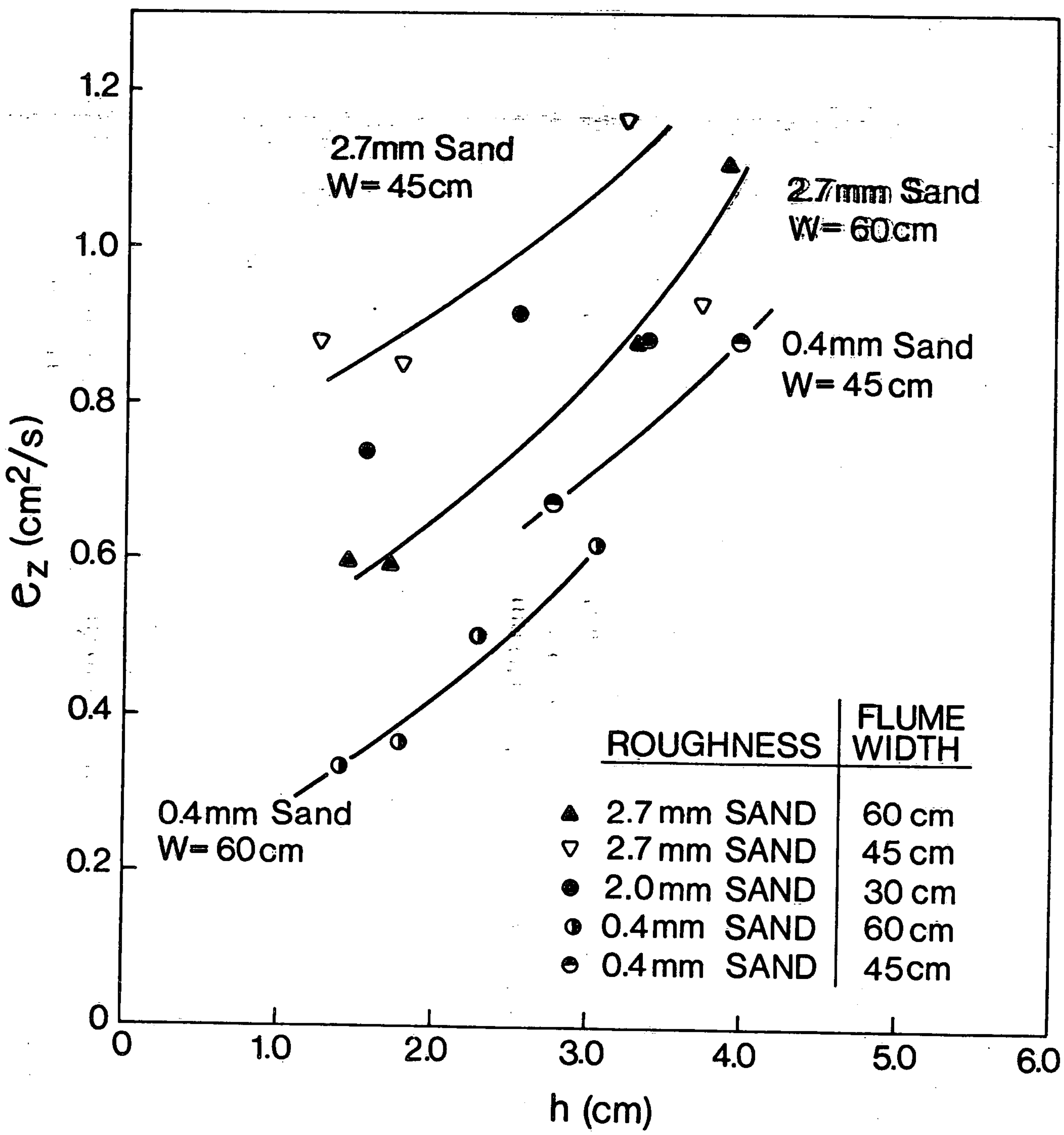

Figure 4. Variation of e_z with h for channels of different widths and roughnesses - velocity was constant at 20 cm/s.

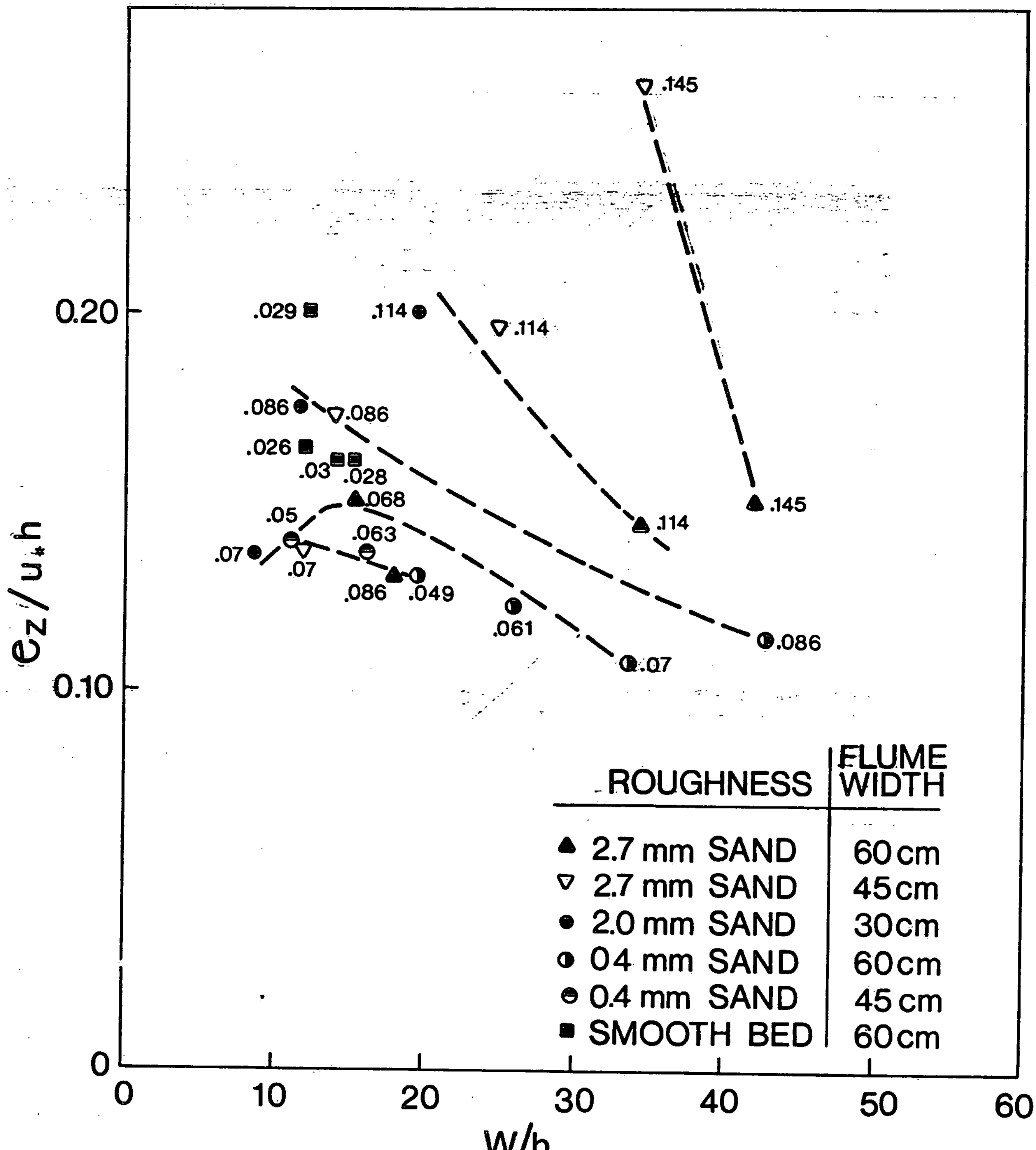

Figure 5. Variation of e_z/u_*h with W/h and f for the writers' data.

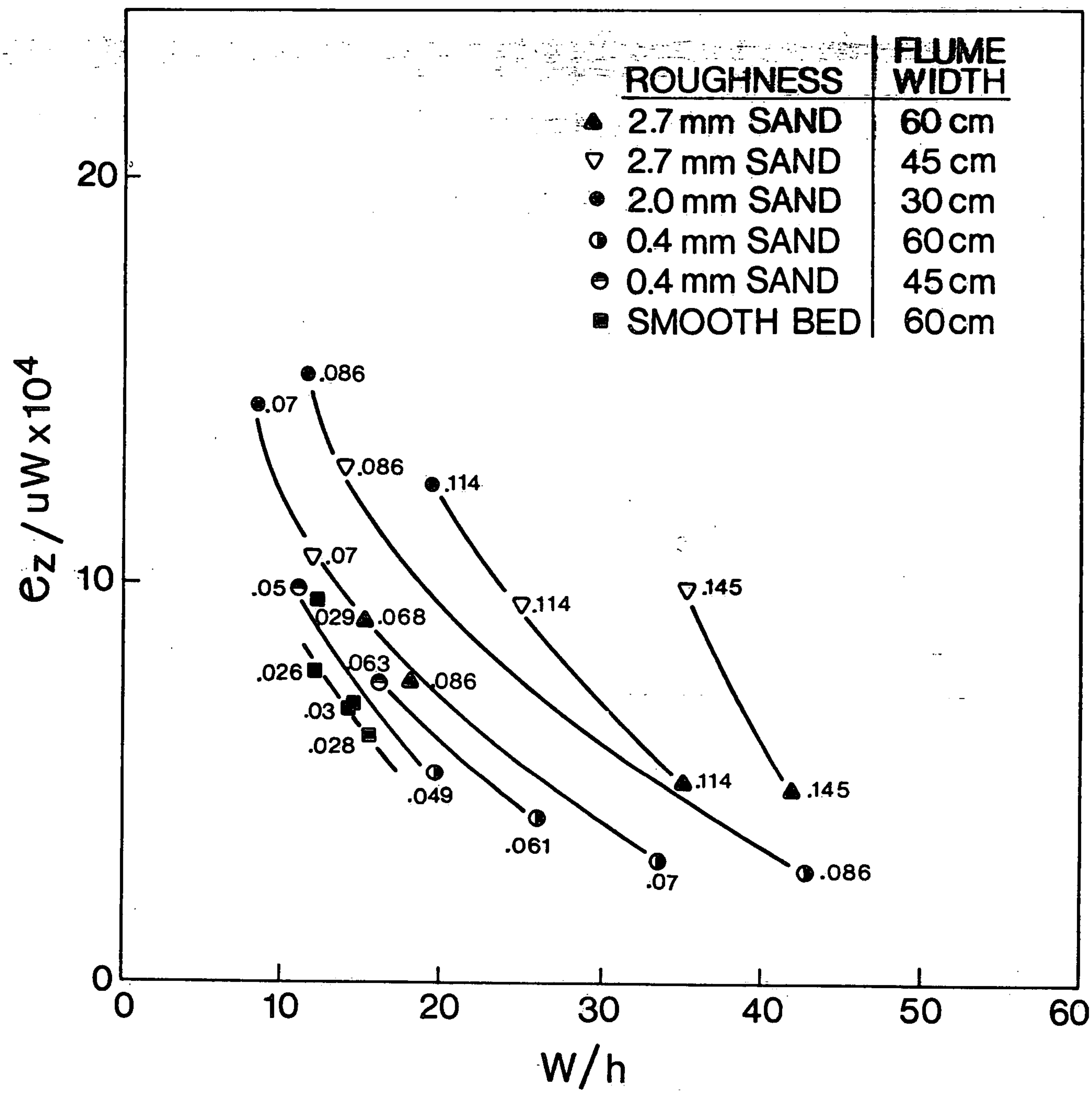

Figure 6. Variation of e_z/uW with W/h and f for the writers' data.

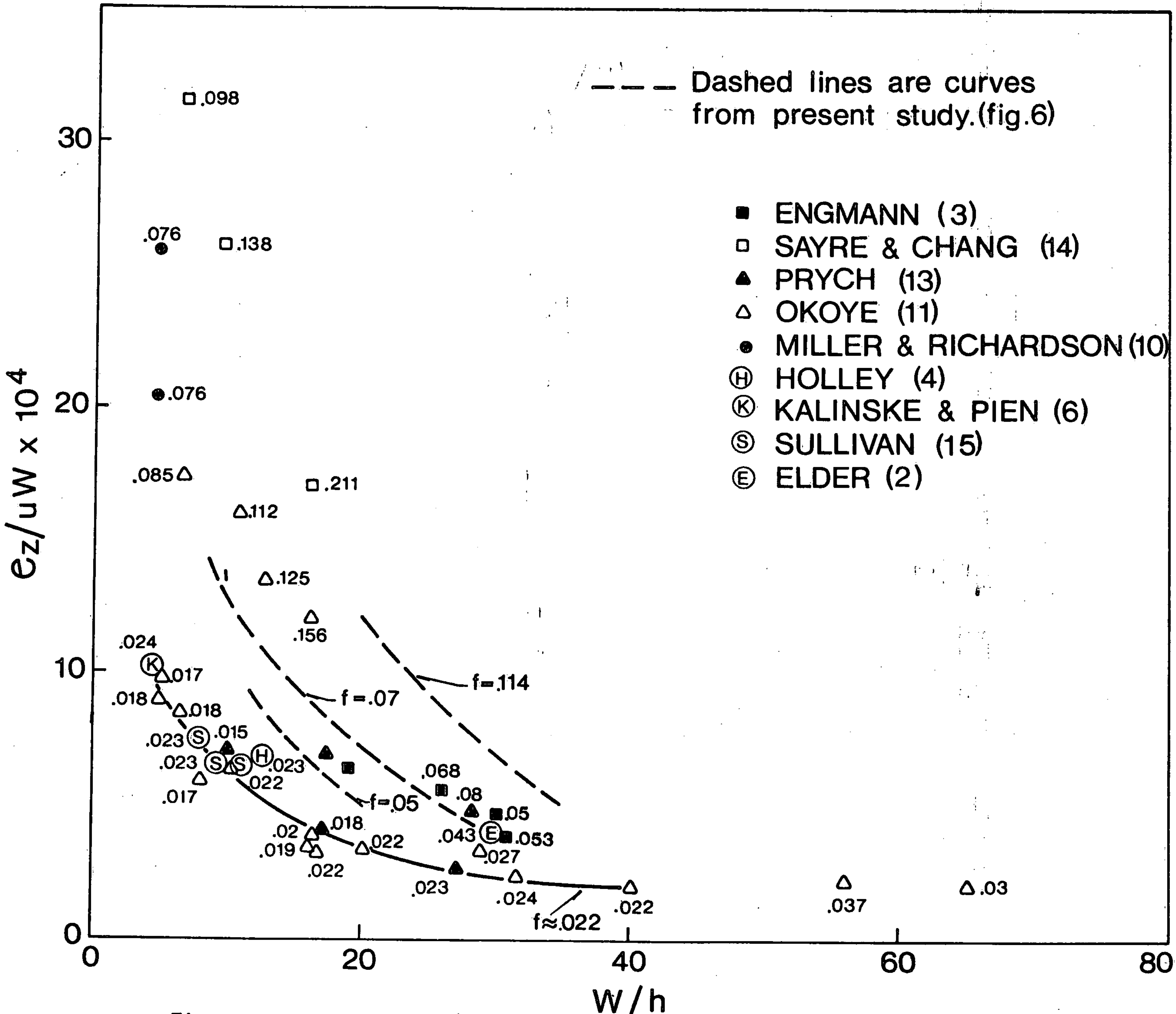

Figure 7. Variation of e_z/uW with W/h and f for the previously published data.

APPENDIX I - REFERENCES

1. Dunnett, F., "Concentration Measurement System for Diffusion Experiments in Laboratory Flumes," Hydraulics Research Division Unpublished Report, Canada Centre for Inland Waters, Burlington, Ontario, 1976.

2. Elder, J. W., "The Dispersion of Marked Fluid in Turbulent Shear Flow," Journal of Fluid Mechanics, Vol. 5, 1959, pp. 544-560.

3. Engmann, E. O., "Transverse Mixing Characteristics of Open and Ice-Covered Channel Flows," thesis presented to the University of Alberta, Edmonton, Alberta in 1974, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

4. Engmann, E. O., Discussion of "Diffusion and Dispersion in Open Channel Flow," by A. C. Miller and E. V. Richardson, Journal of the Hydraulics Division, ASCE, Vol. 100, No. HY12, Dec., 1974, pp. 1850-1851.

5. Holley, E. R., "Transverse Mixing in Rivers," Report No. S132, Delft Hydraulics Laboratory, Delft, The Netherlands, 1971.

6. Jobson, H. E., discussion of "Diffusion and Dispersion in Open Channel Flow," by A. C. Miller and E. V. Richardson, Journal of the Hydraulics Division, ASCE, Vol. 100, No. HY 12, Dec., 1974, pp. 1851-1853.

7. Kalinske, A. A., and Pien, C. L., "Eddy Diffusion," Industrial and Engineering Chemistry, Vol. 36, 1944, pp. 220-223.

8. Kartha, V. C., and Leutheusser, H. J., "Distribution of Tractive Force in open Channels," Journal of the Hydraulics Division, ASCE, Vol. 96, No. HY 7, Proc. Paper 7415, July, 1970, pp. 1469-1483.

9. Lau, Y. L. and Krishnappen, B. G., discussion of ""Diffusion and Dispersion in Open Channel Flow," by A. C. Miller and E. V. Richardson, Journal of the Hydraulics Division, ASCE, Vol. 101, No. HY 3, Mar., 1975, pp. 547-549.

10. McQuivey, R. S. and Keefer, T. N., "Measurement of Velocity - Concentration co-variance, " Journal of the Hydraulics Division, ASCE, Vol. 98 No. HY 9, 1972.

11. Miller, A. C. and Richardson, E. V., "Diffusion and Dispersion in open Channel Flow," Journal of the Hydraulics Division, ASCE, Vol. 100, No. HY 1, Proc. Paper 10298, Jan., 1974, pp. 159-171.

12. Okoye, J. K., "Characteristics of Transverse Mixing in Open Channel Flows, "Report No. KH-R-23, W. M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology, Pasadena, Calif., 1970, 269 pp.

13. Prandtl, L., Essentials of Fluid Dynamics, Blackie and Son Ltd., Glasgow, Scotland, 1952.

14. Prych, E. A., "Effects of Density Differences on Lateral Mixing in Open Channel Flows," Report No. KH-R-21, W. M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology, Pasadena, Calif. 1970, 225 pp.

15. Sayre, W. W., and Chang, F. M., "A Laboratory Investigation of the Open Channel Dispersion Process for Dissolved, Suspended and Floating Dispersants," U. S. Geological Survey Professional Paper 433-E, 1968, 71 pp.

16. Sullivan, P. J., "Dispersion in a Turbulent Shear Flow," thesis presented to Churchill College, University of Cambridge, at Cambridge, England, in 1968, in partial fulfillment of the requirements for the degree of Doctor of Philosphy.

APPENDIX II - NOTATION

The following symbols are used in this paper:

c = depth averaged concentration;

c^y = deviation from depth averaged concentration;

e_x, e_z = depth averaged longitudinal and transverse dispersion coefficient respectively;

f = Darcy - Weisbach friction factor

h = mean flow depth;

K = dimensionless dispersion coefficient e_z/u_*h;

t = time

$\overline{T}_x, \overline{T}_z$, = depth averaged transport caused by turbulent fluctuations in the x and z directions, respectively;

u, w = depth averaged velocities in the x and z directions, respectively;

u^y, w^y = deviation of velocity from the depth averaged values;

u_* = shear velocity;

W = width of channel

x, z = longitudinal and transverse co-ordinate respectively;

ρ = fluid density;

μ = dynamic viscosity;

ϕ = a function;

σ^2 = second moment of the concentration distribution

APPENDIX III - PREVIOUS DATA

Table III - 1. - SUMMARY OF PREVIOUS PUBLISHED DATA ON TRANSVERSE DISPERSION IN RECTANGULAR CHANNELS

Reference (1)	Bed roughness (2)	W, in Centimeters (3)	h, in Centimeters (4)	u, in Centimeters per second (5)	$\frac{W}{h}$ (6)	f (7)	e_z, in Centimeters squared per second (8)	$\frac{e_z}{u_* h}$ (9)	$\frac{e_z}{uW}$ (10)
Elder (2)	Smooth	36.0	1.2	21.6	30.0	0.043	0.305	0.16	3.92x10
Engmann (3)	Metal lath	121.92	3.96	26.4	30.76	0.053	1.24	0.146	3.87x10
	"	"	4.05	22.89	30.07	0.050	1.32	0.18	4.73 "
	"	"	4.72	12.31	25.81	0.068	0.86	0.159	5.69 "
	"	"	6.49	19.66	18.78	0.051	1.55	0.152	6.47 "
Holley (5)	Concrete	120.0	9.7	11.2	12.37	0.023	0.924	0.16	6.87x10
Kalinske and Pien (7)	Smooth	69.0	15.8	94.5	4.37	0.024	6.56	0.08	10.06x10
Miller and	Rectangular	59.74	12.65	30.48	4.72	0.078	3.72	0.098	20.4x10^{-4}
Richardson(11)	Blocks	"	12.65	53.34	4.72	0.076	8.27	0.123	25.9 "
	"	"	12.50	81.38	4.78	0.077	18.87	0.110	38.8 "
	"	"	12.47	31.09	4.79	0.210	9.38	0.139	50.5 "
	"	"	12.62	53.34	4.73	0.180	15.52	0.151	48.7 "
	"	"	13.20	76.81	4.53	0.166	22.76	0.155	49.6 "
	"	"	12.71	31.39	4.70	0.343	14.77	0.180	78.8 "
	"	"	12.62	53.34	4.73	0.368	23.78	0.163	74.6 "
	"	"	13.11	77.42	4.56	0.353	36.24	0.169	78.3 "
Okoye (12)	Smooth	85.0	1.52	31.2	55.92	0.037	0.635	0.197	2.39x10
	"	"	2.95	27.1	28.80	0.027	0.790	0.166	3.34 "
	"	"	5.25	42.6	16.19	0.022	1.33	0.112	3.67 "
	"	"	5.25	42.8	16.19	0.019	1.33	0.122	3.66 "
	"	"	5.25	42.5	16.19	0.021	1.43	0.126	3.99 "
	"	"	5.26	41.7	16.09	0.021	1.21	0.108	3.41 "
	"	"	5.26	42.4	16.16	0.020	1.45	0.130	4.00 "
	"	"	10.70	41.8	7.94	0.017	2.09	0.103	5.88 "
	"	"	17.34	36.9	4.90	0.018	2.86	0.094	9.09 "

Table III - 1 - CONTINUED

Reference (1)	Bed roughness (2)	W, in Centimeters (3)	h, in Centimeters (4)	u, in Centimeters per second (5)	$\frac{W}{h}$ (6)	f (7)	e_z, in Centimeters squared per second (8)	$\frac{e_z}{u_* h}$ (9)	$\frac{e_z}{uW}$ (10)
koye (12)	Smooth	110	1.69	32.8	65.1	0.029	0.79	0.235	2.19x10^{-}
	"	"	2.74	50.4	40.15	0.022	1.19	0.164	2.15 "
	"	"	2.75	30.0	40.0	0.024	0.79	0.174	2.39 "
	"	"	3.46	32.0	31.79	0.024	0.87	0.142	2.46 "
	"	"	5.41	43.5	20.33	0.020	1.62	0.137	3.39 "
	"	"	5.53	42.0	19.89	0.022	1.65	0.135	3.55 "
	"	"	10.81	39.2	10.17	0.018	2.78	0.138	6.42 "
	"	"	17.31	34.9	6.35	0.018	3.25	0.113	8.47 "
	"	"	21.97	30.5	5.0	0.017	3.26	0.107	9.81 "
	Stones	"	6.81	35.9	16.15	0.156	4.81	0.141	12.18 "
	"	"	8.66	41.0	12.70	0.125	6.03	0.136	13.37 "
	"	"	10.36	42.8	10.62	0.112	7.48	0.143	15.89 "
	"	"	17.07	35.3	6.44	0.085	6.74	0.108	17.28 "
Prych (14)	Smooth	110	4.05	35.4	27.16	0.023	1.06	0.138	2.72x10^{-}
	"	"	6.55	45.2	16.79	0.018	2.04	0.145	4.10 "
	"	"	11.10	46.0	9.9	0.015	3.56	0.162	7.03 "
	Metal lath	"	3.90	37.3	28.2	0.080	1.98	0.136	4.82 "
	"	"	6.40	45.9	17.18	0.061	3.52	0.137	6.97 "
ayre and	Wooden	238.65	14.81	23.47	16.11	0.211	9.57	0.179	17.08x10^{-}
hang (15)	Blocks	"	24.81	34.14	9.62	0.138	21.37	0.192	26.22 "
	"	"	37.09	47.55	6.43	0.098	35.76	0.183	31.52 "
ullivan (16)	Smooth	80.0	7.32	22.90	10.93	0.022	1.18	0.133	6.43x10^{-}
	"	"	8.95	18.50	8.94	0.023	0.97	0.110	6.58 "
	"	"	10.20	15.30	7.84	0.023	0.91	0.108	7.43 "