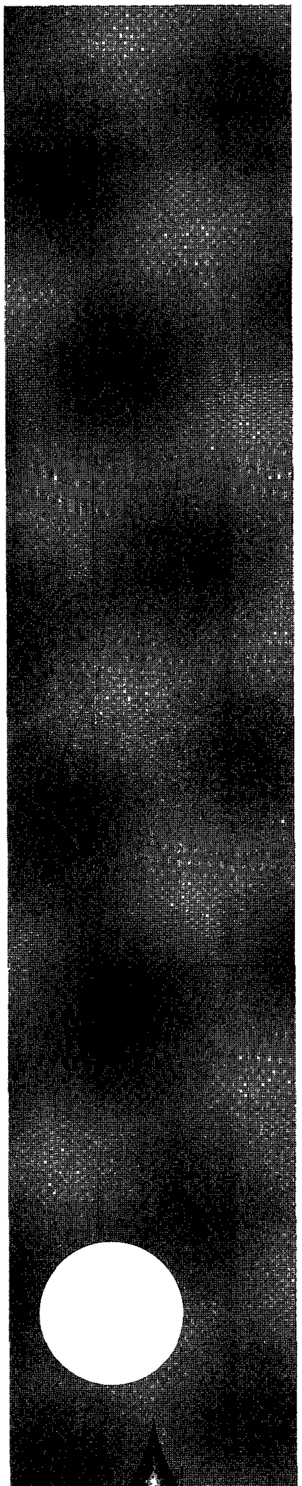


AN ANALYTICAL METHOD FOR LOCATING A DOMESTIC WASTE WATER DISTRIBUTION FIELD WITH REFERENCE TO A PUMPING WELL IN A SLOPING NATURAL GROUNDWATER FLOW SYSTEM

December 1994

Canada – New Brunswick
Water/Economy Agreement

Entente Canada – Nouveau–Brunswick
Eau/Économie

New Nouveau Brunswick
Environment / Environnement

An Analytical Method for Locating a Domestic Waste Water Distribution Field with Reference to a Pumping Well in a Sloping Natural Groundwater Flow System

M. M. Uddin and N. J. Anglin

Groundwater Studies Group
Department of Civil Engineering
University of New Brunswick
Fredericton, N.B

For: Environmental Planning and Sciences Branches
New Brunswick Department of the Environment
Fredericton, New Brunswick

December 1994

ABSTRACT

The main objective of this study is to obtain points beyond which input from a domestic waste water distribution field will not enter a single pumping well. The study also compares the effects of both steady and unsteady flow situations for various hydrogeological settings. The results show that the analytical approach is fruitful and can serve as a basis for evaluating situations related to multiple pumping well and distribution fields.

INTRODUCTION

Many of the residential wells in New Brunswick are in fractured rock where a natural groundwater slope has to be considered. Some recent studies have been done to characterize typical fracture properties from a hydrogeological perspective (Anglin, 1994a/1994b). It is important to note that drawdown characteristics are directly related to the fracture properties of the storage areas. The areas that are highly fractured and hence have a high storage coefficient, S, are characterized by a relatively small drawdown compared to areas that are less fractured.

In New Brunswick, it is a common practice to place a waste water disposal field down gradient from the drinking water well to avoid contamination. However, contamination from the disposal field may result if the disposal field is not outside the capture zone associated with the pumping well.

The study defines the capture zones by using a simple analytical approach. The method uses the principle of superposition involving a uniform one-dimensional flow and a horizontal radial flow toward the well. Using this theoretical approach, it is possible to define a practical guideline to locate the disposal fields such that the contaminated water will not enter a single pumping well.

The main concern of this study is contamination of the water supply from disposal fields. The objective is to provide a simple guideline to protect drinking water wells from such contamination based on certain assumptions concerning typical storativity and transmissivity values found in the fractured bedrock in New Brunswick. Carr (1968) provides a range of storativity from $1x10^{-2}$ to $1x10^{-6}$ and transmissivity from $1x10^{-2}$ m^2/sec to $1x10^{-5}$ m^2/sec for typical New Brunswick bedrock wells.

Figure 1 illustrates a pumping well in a natural gradient flow. From the conceptual diagram the general shape of the drawdown curve is illustrated. The location of two possible disposal fields are shown. One is outside the capture zone while the other is within it.

Assumptions

The following assumptions concerning the aquifer, the pumping rate and the natural gradient have been made for this investigation:

(i) single isolated well and single waste water distribution field,
(ii) steady state conditions at large time,
(iii) fracture frequency and aperture size uniform over the depth,
(iv) infinite and confined flow field, and
(v) aquifer thickness, B, is constant

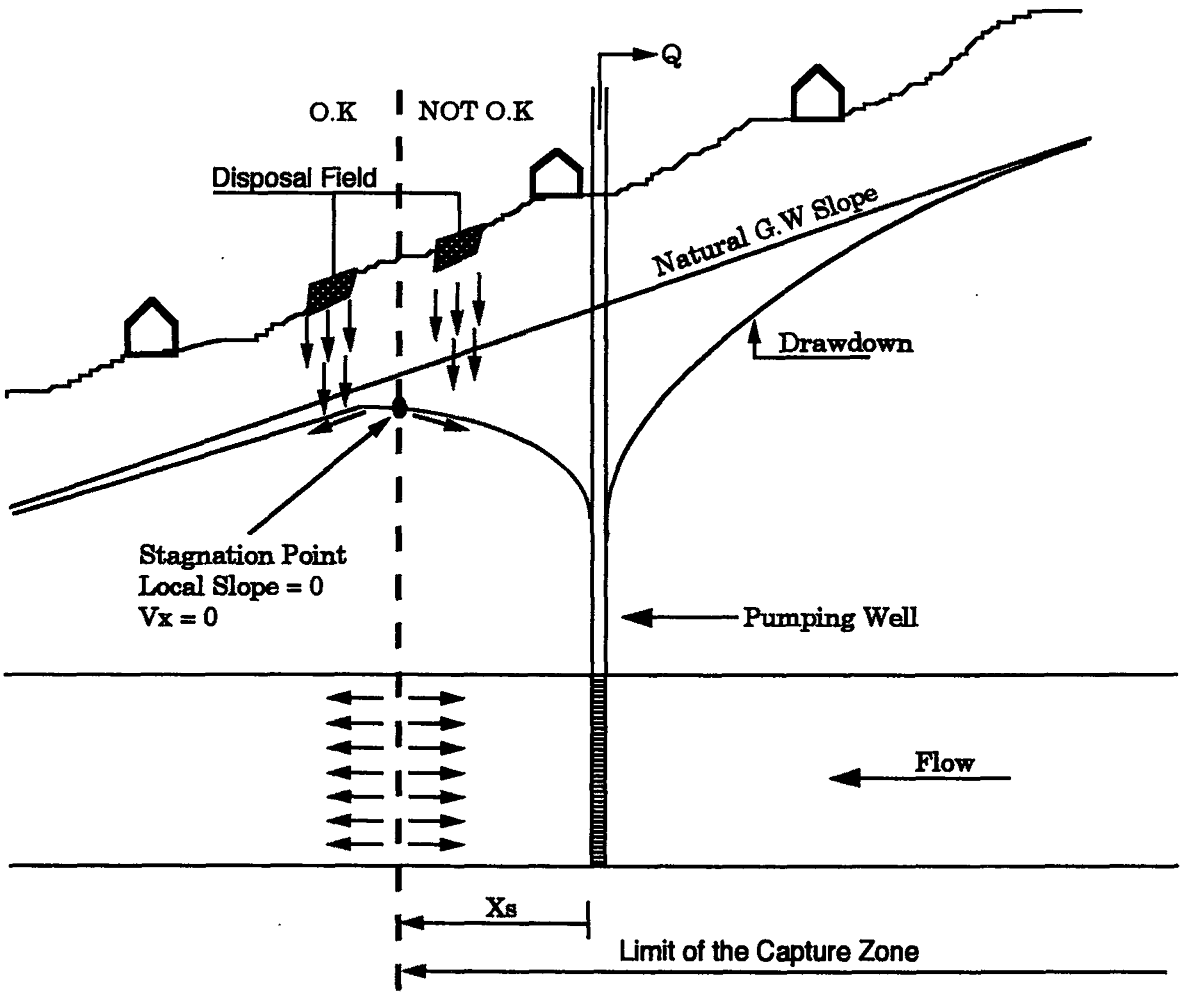


Figure 1 : Pumping well in a natural gradient flow with two probable disposal fields

Although these assumptions may seem to be restrictive they will provide reasonable results for deep unconfined aquifers in fractured rock where the drawdown, s, is small relative to the aquifer thickness ($s/B < 0.1$) (Bouwer, 1978). The definition sketch in Figure 1 is based on the assumptions listed above, however for practical purposes the following analysis can be used for the more realistic unconfined flow case.

Zone of Capture

Figure 2 shows a plan and a section view of the flow toward a pumping well in a uniform flow field as well as the streamline defining the groundwater divide. The point at which the streamline forming the groundwater divide crosses the x-axis downstream of the well is called a stagnation point, X_s, because the Darcy velocity is zero at this location.

The main objective is to locate, X_s, the distance from the pumping well to the line of zero velocity or the stagnation point. A derivation of the following equation for the distance, X_s, is presented in Appendix A for large pumping time.

$$X_s = \pm \frac{Q}{2\pi KBi} \qquad (1)$$

where, Q is the pumping rate (L^3/T), B is the average thickness of the aquifer (L), K is the hydraulic conductivity (L/T) and i is the natural groundwater slope. Note that i may be positive or negative depending upon the coordinate system selected. It is recognized that T = KB, where T is the transmissivity.

The development of Equation 1 indicates that X_s is greatest for steady state conditions. Hence the study considers the steady state situation to determine the capture zone.

The maximum width, W, of the capture zone upstream of the pumping well is given by

$$W = \pm \frac{Q}{KBi} \qquad (2)$$

where, W is twice the magnitude of the maximum value of y_s in Figure 2.

The resultant drawdown has been computed by using the following equation

$$s = ix + \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-u}}{u}du \qquad (3)$$

Equation 1 indicates that the location of the stagnation point moves further downstream from the pumping well with an increase in the pumping rate. Based on this theoretical concept, an example depicting a confined aquifer with T = 0.001 m^2/s, S = 0.001, in a uniform flow field

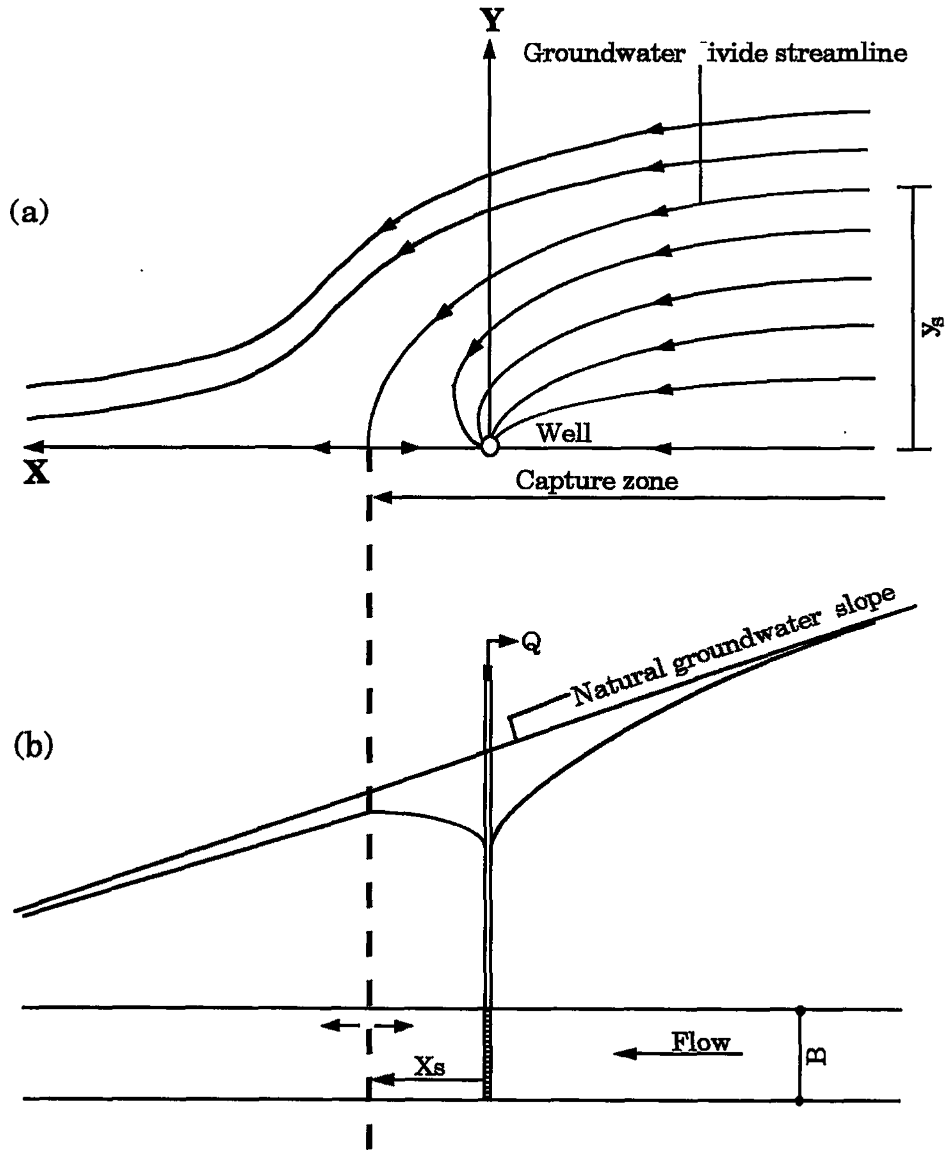


Figure 2 : Streamlines for uniform flow with a well (a) half section plan (b) vertical section through the pumping well

having a natural gradient of i =0.001 is presented in detail. The typical long time pumping rate for domestic homes has been assumed to range from 0.1 to 0.5 L/s.

RESULTS FOR SOME TYPICAL CASES FOR A SINGLE WELL

The results presented in this section show the effect of pumping rate and length of pumping time on the piezometric surface along a line through the pumping well.

Figure 3 shows drawdown vs. radial distance for pumping rates of 0.1 L/s, 0.5 L/s and 1 L/s at the end of 6 hrs. This figure also shows the location of the groundwater divide for the three discharge rates. It further demonstrates that the drawdown cone of depression increases very rapidly in the vicinity of the pumping well. It is also possible to define influence zones which can be associated with a specified drawdown, s, for different pumping rates.

The resultant drawdown equation as derived from the principle of superposition, predicts that the cone of drawdown around the well develops instantaneously and extends to an infinite distance. This phenomenon is demonstrated from the drawdown curves at the end of 1 day, 10 days and 100 days under constant pumping rate 0.5 L/s (Figure 4). The figure shows long term pumping effects on the groundwater table and clearly indicates that the dewatering process continues with the radial distance resulting in a decline in the groundwater table. It is also shown that the effect of drawdown expands rapidly at the initial period and more slowly as the time elapses.

Equation 1 can be used to estimate the distance of the stagnation point, X_s, for a specified pumping rate, natural groundwater slope, saturated thickness of the aquifer, and hydraulic conductivity of the fracture rock aquifer. For example if pumping rate , Q = 0.0008 m^3/s, natural groundwater slope, i = 0.008, thickness of the aquifer, B = 50 m, and hydraulic conductivity, K = 1.2×10^{-5} m/s, then the estimated value of X_s is 27 m for steady state conditions.

Figure 5 shows the location of the stagnation point where Darcian velocity is zero as a function of pumping rate and natural groundwater slope for specific values of saturated aquifer thickness, B, and hydraulic conductivity, K. This simple linear relationship, based on Equation 1 can be used as a preliminary guideline to locate the distribution field in relation to the pumping water well for the conditions stated. The distance of the stagnation point for any practical natural groundwater slope and pumping rate can be interpolated from this simplified relationship. One example problem is shown in Figure 5.

It is recognized that several assumptions have been made in the development of this method and hence the assumptions should be evaluated when applying the technique.

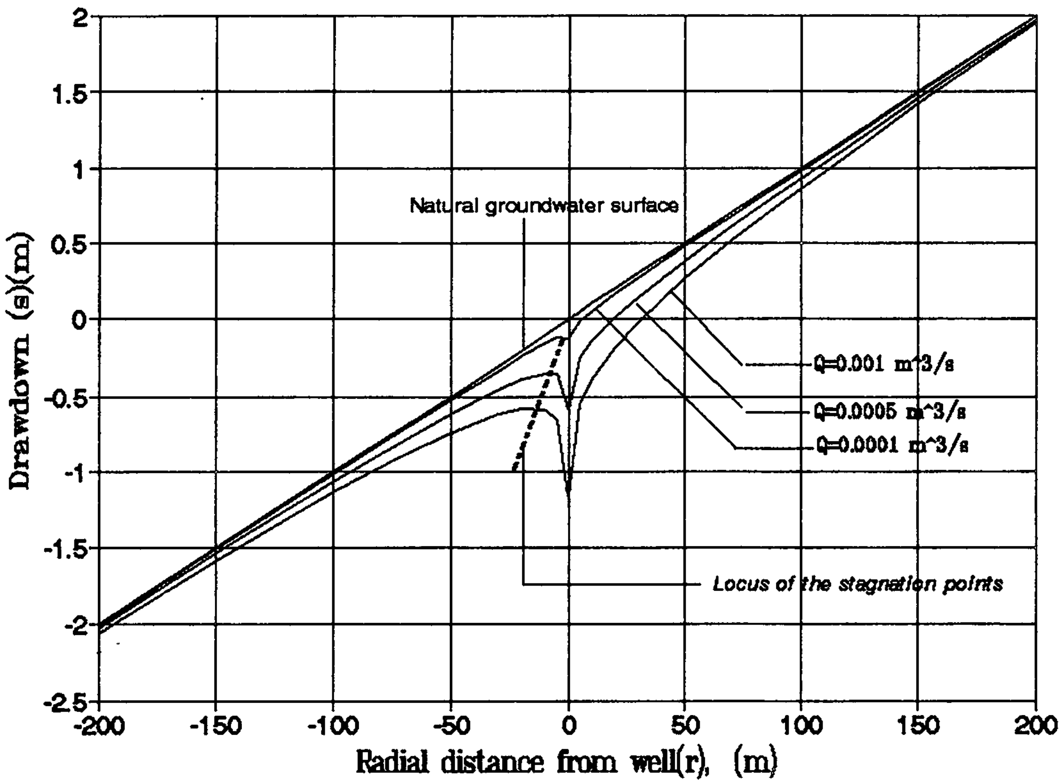


Figure 3 : Drawdown curves for different pumping rates (Q)
(S=0.001, T=0.001 m²/s, t=6 hrs)

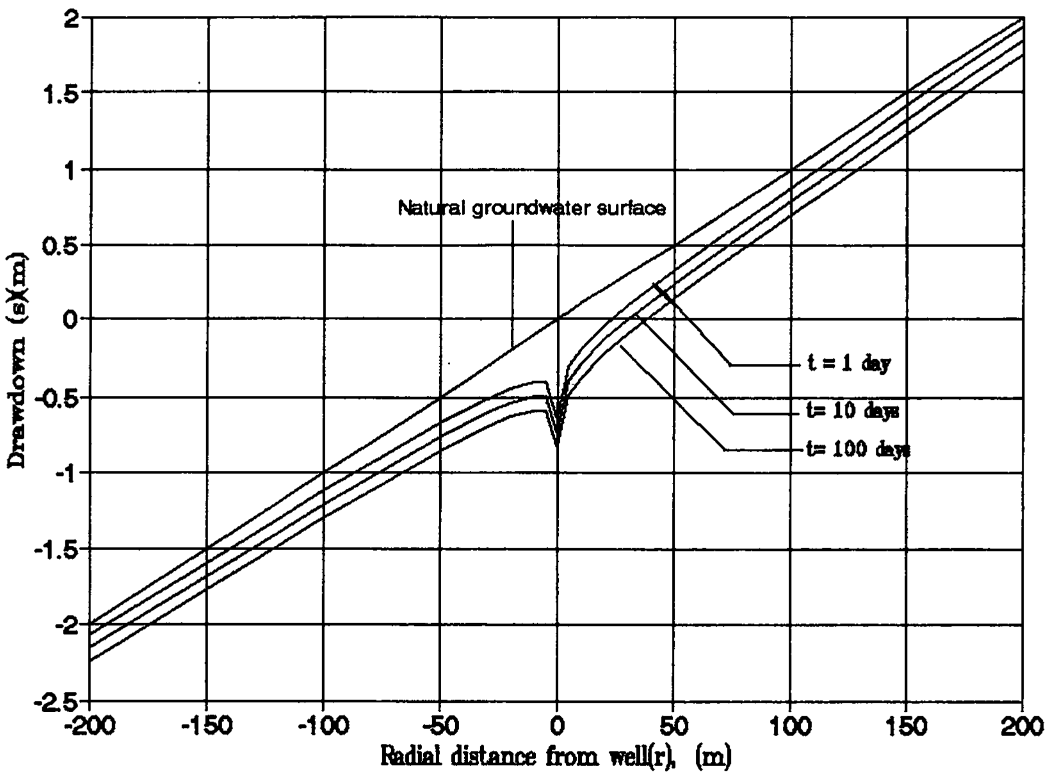
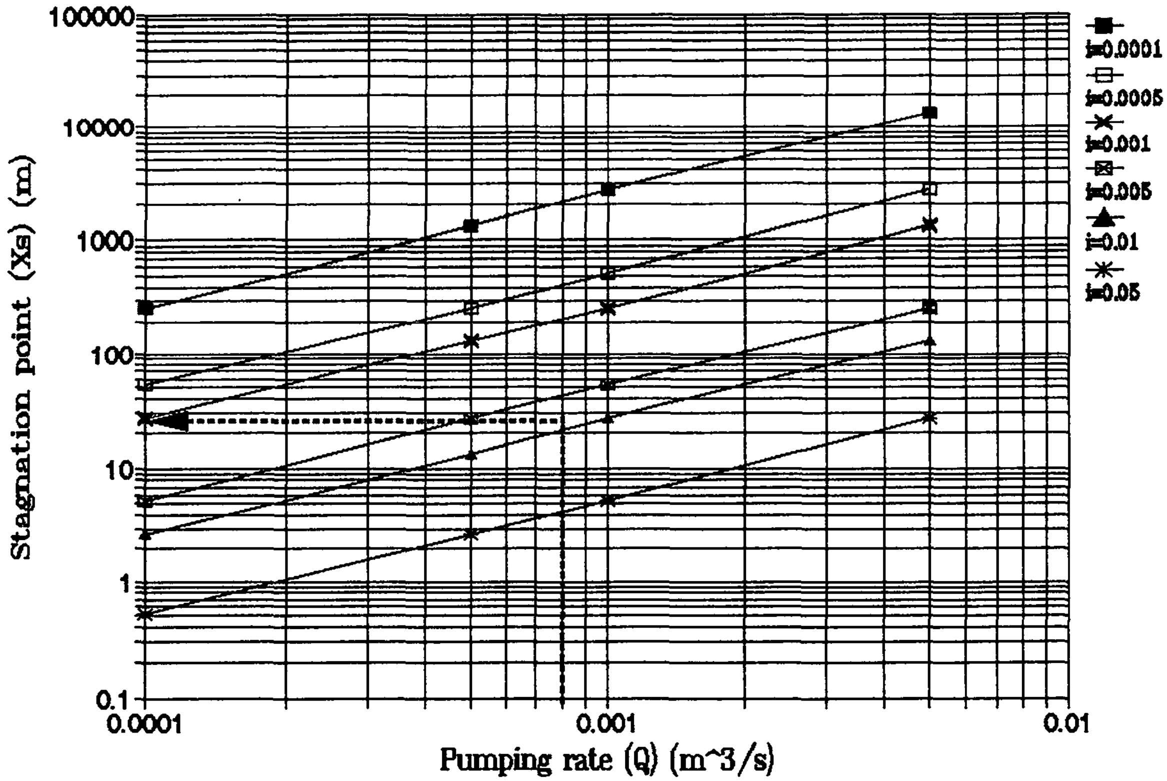


Figure 4 : Drawdown curves at the end of different pumping times (t)
(S=0.001, T=0.001 m^2/s, Q=0.0005 m^3/s)



If B=50 m, K=1.2x10^{-5}, and if i=0.008 and Q=0.0008 m^3/s then X_s is determined to be 27 m.

Figure 5 : Location of stagnation points (X_s) with pumping rates (Q) and natural groundwater slope (i) (B=50 m, K=1.2x10^{-5} m/s)

The guideline presented in this report for the location of waste water distribution fields under more practical situations could be tested through extensive field tests. The analytical approach could be extended for multiple pumping wells. For more complex situations, numerical methods using finite difference or finite element models could be adopted.

CONCLUSIONS

(i) With regard to groundwater contamination from a waste water fields, the steady state flow situation is the worst case for a given pumping rate and it is not necessary to evaluate the unsteady flow conditions.

(ii) The theoretical approach can be used as a preliminary quideline to locate domestic waste water distribution fields in relation to a pumping well if the underlying assumptions are not significantly violated.

(iii) The principle of superposition could possibly be used for multiple distribution fields and pumping wells to define the zones of influence and the stagnation point for each well.

References

Anglin, N.J., 1994a. An evaluation of the bedrock types of New Brunswick and their corresponding fracture patterns from a hydrogeologic perspective. A report prepared by the Groundwater Studies Group, University of New Brunswick, Fredericton, New Brunswick, for: Environmental Planning and Sciences Branch, New Brunswick Department of the Environment, Fredericton, New Brunswick, August.

Anglin, N.J., 1994b. An illustrative look at domestic water systems in the fractured bedrock typical of that found in New Brunswick. A report prepared by the Groundwater Studies Group, University of New Brunswick, Fredericton, New Brunswick, for: Environmental Planning and Sciences Branch, New Brunswick Department of the Environment, Fredericton, New Brunswick, September.

Bouwer, H, 1978. Groundwater Hydrology, McGraw-Hill Series in Water Resources and Environmental Engineering, New York, NY, pp.200-210.

Carr, P.A., 1968. Hydrogeology of the Moncton Map-Area, New Brunswick. Inland Waters Branch, Department of Energy, Mines and Resources. Scientific Series No. 1, Ottawa, ON.

Kashef, A.I, 1986. Groundwater Engineering, McGraw-Hill, Inc., New York, NY, pp.358-361.

McWhorter, D. B. and D. K. Sunada, 1977. Ground-Water Hydrology and Hydrauics, Water Resources Publications, Littleton, CO, pp.131-133.

Strack, O.D.L., 1989. Groundwater Mechanics, Prentice-Hall, Inc., Englewood Cliffs, NJ, pp.225-255.

Appendix A

Derivation of location of groundwater divide or the stagnation point

The derivation presented here are based, among others, on the book by McWhorter, D.B. and D.K. Sunada (pp.131-133).

The stream function is obtained by superposition of the expressions derived for uniform flow and for flow toward a well. The stream function for one-dimensional, uniform flow in the x-direction is

$$\psi = -q_x y + Constant = -Kiy + Constant \tag{A.1}$$

where, q_x is the darcian flux in the x direction, K is the hydraulic conductivity and i is the natural groundwater slope.

Next, radial flow toward a single isolated well in a two dimensional flow field will be considered. For this case, the equation for drawdown, s, is

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-u}}{u}du \tag{A.2}$$

where, Q is the pumping rate (L^3/T), T is the transmissivity (L^2/T) and u is dimensionless parameter defined as $u = r^2S/(4Tt)$, t is the pumping time (T) and storage coefficient (dimensionless).

The magnitude of the radial velocity due to the well effect can be determined as

$$\frac{\partial s}{\partial r} = \frac{\partial s}{\partial u}\frac{\partial u}{\partial r} = \frac{e^{-u}}{u}\frac{Q}{4\pi T}\frac{\partial u}{\partial r} \tag{A.3a}$$

but

$$\frac{\partial u}{\partial r} = \frac{\partial}{\partial r}\left(\frac{r^2 S}{4Tt}\right) = \frac{rS}{2Tt} \tag{A.3b}$$

By substitution in Eq. (A.3a)

$$\frac{\partial s}{\partial r} = \frac{Qe^{-u}}{2\pi rT} \tag{A.3c}$$

At any cylindrical surface of radius **r** the rate of flow, $\mathbf{Q}_{r,t}$, at time **t** is

$$Q_{r,t} = 2\pi rBK\frac{\partial s}{\partial r} = 2\pi rT\frac{\partial s}{\partial r} \tag{A.4}$$

By combining Equations (A.3c) and (A.4), we can relate the pumping rate, **Q**, with the instantaneous flow rate, $Q_{r,t}$

$$Q_{r,t} = Qe^{-u} \tag{A.5}$$

The radial velocity potential, $q_{r,t}$, due to the well effect can be expressed as

$$q_{r,t} = \frac{Qe^{-u}}{2\pi rB} \tag{A.6}$$

In the polar coordinates with radius, r, and angle, δ, the velocity potential and the stream function are related by

$$q_r = -\frac{\partial\phi}{\partial r} = -\frac{1}{r}\frac{\partial\psi}{\partial\delta} \tag{A.7a}$$

and

$$q_\delta = -\frac{1}{r}\frac{\partial\phi}{\partial\delta} = \frac{\partial\psi}{\partial r} \tag{A.7b}$$

from which the stream function can be defined by using Eq. (A.7a)

$$\psi = -\int\frac{Qe^{-u}}{2\pi B}\partial\delta + Constant \tag{A.8}$$

For two extreme cases, (i) when t is very large then e^{-u} becomes 1 and the corresponding stream function is same as the steady flow condition. (ii) when t is very small then e^{-u} becomes 0.

Therefore the stagnation point is further downstream for the steady flow condition compared to the unsteady flow condition. Hence, we have considered only the steady state case since it becomes the worst situation in that the groundwater divide is farthest downstream from the pumping well for a given rate, Q.

Therefore, we can approximate the stream functions for the situation when t is very large as

$$\psi = -\frac{Q}{2\pi B}\delta + Constant \tag{A.9}$$

Finally, the resultant stream function for steady flow can be defined by using the principle of superposition from Eqs. (A.1) and (A.9) as

$$\psi = -Kiy - \frac{Q}{2\pi B}\delta + Constant \tag{A.10}$$

The streamline $\Psi=0$ separates flow that eventually contributes to well discharge from flow that bypasses the well this streamline is called the groundwater divide (Fig. A1). The relationship between the x and y coordinates of all points on the groundwater divide is

$$Kiy = -\frac{Q}{2\pi}\delta \tag{A.11}$$

where $\delta = \tan^{-1}(y/x)$ for $x \geq 0$ and $\delta = \pi - \tan^{-1}(|y/x|)$ for $x \leq 0$. For the coordinate system shown in Fig. A1, the slope i of the piezometric surface in uniform flow is negative.

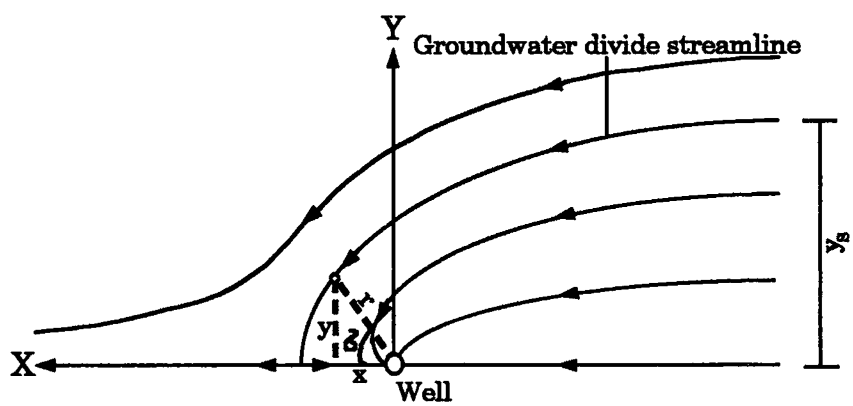


Figure A1 : Streamlines for uniform two dimensional flow field

The coordinate x_s of the stagnation point is obtained from

$$x_s = \left[\frac{y}{\tan\left(-\frac{2\pi Tiy}{Q}\right)}\right] = -\frac{Q}{2\pi Ti} = -\frac{Q}{2\pi KBi} \tag{A.12}$$

When $x \to \infty$ then δ becomes π, therefore at a point far upstream of the well Eq. A.11 yields

$$y_s = \pm \frac{Q}{2Ti} \tag{A.13}$$

which is the half-width of that portion of the aquifer in which flow contributes to the well discharge. Hence, the maximum width of the capture zone upstream of the pumping well will be

$$W = 2y_s = \pm \frac{Q}{BKi} \tag{A.14}$$

The resultant drawdown from the principle of superposition of uniform and radial flow toward the well is given as

$$s = ix + \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-u}}{u} du \tag{A.15}$$