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Technical Memoranda

CROSS CONTOUR FLOW IN JET STREAMS

by

H.P. WILSON

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ENVIRONMENT CANADA - ATMOSPHERIC ENVIRONMENT SERVICE
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ABSTRACT

A relatively simple treatment of cross-contour motion in jet streams is presented which may have some value for operational meteorologists as well as from the point of view of theory.

LES COURANTS TRANSVERSAUX DANS LES JETS

par

H.P. Wilson

RÉSUMÉ

L'auteur présente une étude relativement simple du mouvement des courants transversaux dans les jets qui peut être utile aux météorologistes professionnels et servir à des études théoriques.

CROSS-CONTOUR FLOW IN JET STREAMS

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H.P. Wilson

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Introduction

The subject of cross-contour flow in jet streams has been dealt with several times previously in Canadian Meteorological literature. Godson (1) has contributed a detailed and rigorous treatment as part of a study of dynamic instability. Simla (2) developed a formula for the angle between streamline and geopotential contour which has been useful in analysis work at the Central Analysis Office in Montreal. Wilson (3) showed that the variation of the angle with height is important with respect to the vertical structure of jet streams. Recently, Treidl (4) has demonstrated that the cross-contour component of ageostrophic motion depends mainly on the forward and transverse components of wind shear. The present offering has been developed from ideas gleaned from these earlier works and also partly from Bjerknes (5) and Haltiner and Martin (6).

The aim has been to find a derivation that would be as simple as possible but which would be complete enough to be valid as a first approximation.

The Angle Between Streamline and Contour

For convenience, it is assumed that the flow through the jet maximum is from the west and that the winds are mainly from the west in the confluence zone. The flow is assumed to be isentropic and non-turbulent. Then

$$du/dt = (v - V)f, \quad \text{and} \qquad (1)$$

$$dv/dt = (U - u)f \qquad (2)$$

where u and v are the west and south wind components,
U and V, the corresponding geostrophic components, and
f is the Coriolis parameter.

Considering only the part of the confluence zone that is within 200 miles or so of the axis, v and V are assumed to be significantly smaller than u and U, say one-fifth or less. However, u - U and v - V are assumed to be comparable.

A relationship involving ψ, the angle between a streamline and a contour may be obtained as follows. From (2)

$$d^2v/dt^2 = (dU/dt - du/dt)f + (U - u)\,df/dt$$

The second term on the right is small and may be neglected. If the pattern is steady so that $\partial U/\partial t = 0$,

$$d^2v/dt^2 = (u\,\partial U/\partial x + v\,\partial U/\partial y - (v-V)f)f$$

$$= (\partial U/\partial x - (f - \partial U/\partial y))\tan\psi + Vf/u)fu \qquad (3)$$

where $\tan\psi = v/u$

or,

$$\tan\psi = \frac{\partial U/\partial x + fV/u - (d^2v/dt^2)/fu}{f - \partial U/\partial y} \qquad (4)$$

A connection between ψ and dynamic instability may be seen in these four relationships. If $\partial U/\partial y = f$, ψ is indeterminate in (4) and v and u can increase or decrease at increasing rates.

The Term d^2v/dt^2

The term d^2v/dt^2 makes (4) rather awkward. Actually this term is likely to vary around zero in confluence zone. This may be seen by considering that the jet axis usually has a sinusoidal shape with respect to the contours. It can easily be shown that if the axis is represented by the positive half of a cosine cycle of y, $v = dy/dt$ completes one cycle, dv/dt one and one-half cycles, and $d^2v/dt2$ two cycles, one in the entrance and the other in the exit region. Then d^2v/dt^2 has an average value close to zero for the confluence as a whole and as an approximation,

$$\tan\psi = \frac{(\partial U/\partial x)_\theta + fV/u}{f - (\partial U/\partial y)_\theta} \qquad (5)$$

The subscript θ has been added to denote flow in an isentropic surface.

The Term $(\partial U/\partial y)_\theta$

It is preferable to modify (5) to show the relationship with respect to a pressure surface.

If the isentropic surfaces intersect pressure surfaces along lines that are parallel to contours, $(\partial U/\partial x)_\theta = (\partial U/\partial x)_p$. Such a condition corresponds to the absence of thermal advection and also to the assumption of steady state.

The term, $(\partial U/\partial y)_\theta$ may differ considerably from $(\partial U/\partial y)_p$ in confluence zones where the winds and thermal gradients are strong.

Let α be the angle between an isentropic and a pressure surface. Δy, a distance northward from the intersection, and Δz, the vertical separation of the two surfaces at the distance Δy north of the intersection. Then very nearly

$$\Delta y \left(\frac{\partial U}{\partial y}\right)_p + \Delta z \left(\frac{\partial U}{\partial z}\right) = \Delta y \left(\frac{\partial U}{\partial y}\right)_\theta \qquad \text{hence}$$

$$\tan\alpha = \frac{\Delta z}{\Delta y} = \frac{(\partial U/\partial y)_\theta - (\partial U/\partial y)_p}{\partial U/\partial z}$$

As α $\tan\alpha = -(\partial T/\partial y)_p/(\Gamma + \partial T/\partial z)$, and, using the Thermal Wind relation $(\partial T/\partial y)_p = -fT(\partial U/\partial z)/g$, and setting

$$hi = g\,(\Gamma + \partial T/\partial z)\,/\,T(\partial U/\partial z)^2$$

(assuming $\partial U/\partial z = \partial u/\partial z$), we end up with

$$f - (\partial U/\partial y)_\theta = f - (\partial U/\partial y)_p - f/Ri \tag{6}$$

The right-hand side of (6) is Godson's parameter b except for a curvature term:

$$b = 1 + \frac{1}{f}\left[\frac{U}{R} - \left(\frac{\partial U}{\partial y}\right)_p - \frac{f}{Ri}\right]$$

where R is the radius of curvature.

The Term fV/u

As V is assumed to be one fifth or less of u or U, f V/u may be approximated by ± 0.2f. However, for a confluence zone in which the flow is cyclonic to the north and anticyclonic to the south, along a central contour V is zero. In the following section, it is assumed that the central contour intersects the jet axis near the middle of the entrance and of the exit regions.

The Variation of ψ Near the Axis and Central Contour

From (5) and (6) and dropping subscripts,

$$\tan\psi = \frac{\partial U/\partial x}{f - \partial U/\partial y - f/Ri} \tag{7}$$

This relationship is applicable as an approximation within 150 miles or so of the axis or central contour. Representative values for the terms in (7) are shown in the following table.

Table 1 Representative Values

Entrance	$\partial U \ \partial x$	$\partial U \ \partial y$	f/Ri	tan ψ	ψ (Degrees)
N of axis	0.5f	- 3f	f	0.17	10
Along	0.5f	0	0.25f	0.67	31
S of axis	0.5f	0.7f	0.25f	10	84
Exit					
N of axis	-0.5f	- 3f	f	-0.17	-10
Along	-0.5f	0	0.25f	-0.67	-31
S of axis	-0.5f	0.7f	0.25f	-10	-84

Comments

Using Ri = 4 along the axis, as well as to the south, is questionable because $\partial u / \partial z = 0$ and f/Ri = 0 at the level of maximum wind. However, just below and above, Ri decreases northward roughly from 4 to unity. Then, in the entrance region ψ = 26 degrees at the level of maximum wind and 31 degrees just above and below, which seems improbable. Thus, the adoption of ψ = 31 degrees involves the assumption that the level of maximum wind does not represent a layer with significant thickness.

The 84 degree departures south of the axis are practically impossible because they would require v/u to have a value of 10 instead of around 0.2. Apparently such extreme transverse motions are precluded because the field of flow cannot adjust to them. This demonstrates that a simple parcel theory is of limited value in dealing with this sort of problem.

However, the table appears to be valid with regard to the sign of the departures from geostrophic directions of flow and qualitatively with regard to magnitudes. The streamlines converge or diverge toward the axis more than indicated by the contours. In other words there is ageostrophic velocity convergence along the axis in the entrance and divergence in the exit region.

It may be noted that if the flow outside of the confluence zone may be assumed to be geostrophic, the table indicates divergence in the southwest and northeast, and convergence in the northwest and southeast quadrants.

Summary

A simple treatment of ageostrophic motion near jet streams has been presented which may provide some fresh insight into the structure of airflow in confluence zones. It may be noted that formulae given are on the whole compatible with the results of Godson, Simla, and Treidl.

APPROVED

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Atmospheric Environment Service.

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