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A Critical Examination of the Significance of the Diabatic Influence Functions and the von Karman Constant

by

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Abstract

In studies of flux-profile relationships for wind velocity and temperature, the Monin-Obukhov similarity theory for the constant flux layer is frequently used. There have been a number of widely accepted forms for the diabatic influence functions proposed in the literature. In many cases different values for the von Karman constant, k, were used to achieve agreement with experimental data.

This paper examines the differences in fluxes compiled from the same profile data but using different flux-profile relationships. It is felt that, at least partially, these differences are due to experimental errors and the deviations of the field sites from the ideal conditions that were assumed in the derivation of the Monin-Obukhov Similarity Theory.

1. Introduction

The important dimensional parameters for a constant flux layer are the friction velocity, $u_* = \sqrt{-\tau/_\rho}$, the scaling virtual temperature, $\theta_* = -H/\rho C_p u_*$ and the Monin-Obukhov length, $L = -\frac{c_p \rho \theta_o u_*^3}{kgH} = \frac{\theta_o u_*^2}{\theta_* kg}$, where H is the virtual heat flux, c_p the specific heat at constant pressure, ρ the air density, θ_o the reference temperature, g the gravitational constant and k the von Karman constant. The von Karman constant plays a very important role in relating u_*, θ_* and L. The determination of these parameters is mainly based on observational data using either the profile gradient method or direct flux measurements. Either approach applies the Monin-Obukhov similarity theory. The von Karman constant is determined from the best fit of the experimental data with respect to the flux profile relationships. Its values vary widely, from .35 to .43 and sometimes even beyond that range. The argument over the value of k has been a long time controversy within the field of turbulence and micrometeorology.

2. Flux-Profile Relationships

A rather comprehensive survey of the existing flux-profile relationships has been given by Yaglom (1977). The present study considers only the three most commonly accepted forms proposed by Businger, Wyngaard, Izumi and Bradley (1971), by Dyer and Hicks (1970), and by Pruitt, Morgan and Lourence (1973).

Hereafter called BWIB, DH and PML models respectively. A review and discussions of these three cases has been given by McBean (1976). Therefore only a brief summary will be given here.

For unstable stratification, both BWIB and DH used the following forms of flux-profile relationships:

$$\phi_m = \left(1-\alpha\frac{z}{L}\right)^{-\frac{1}{4}}$$

$$\phi_H = A\left[1-\gamma\left(\frac{z}{L}\right)\right]^{-\frac{1}{2}}$$

Then by definition that

$$\psi_m = \int_o^{z/L} \frac{1-\phi_m}{z/L} d\left(z/L\right), \text{ we have}$$

$$\psi_m = 2 \ln\left[(1+x)/2\right] + \ln\ (1+x^2)/2\Big] - \tan^{-1} x+\frac{\pi}{2}$$

similarly $$\psi_H = 2A\ln\left[(1+y^2)/2\right]$$

where $$x = \left(1-\alpha\frac{z}{L}\right)^{\frac{1}{4}}$$

and $$y = \left(1-\gamma\frac{z}{L}\right)^{\frac{1}{4}}$$

For stable stratification, the same formula

$$\phi_m + 1 + \sigma\frac{z}{L} \qquad\qquad \phi_H = A + \sigma\frac{z}{L}$$

are again used by both BWIB and DH. The difference between BWIC and DH is that BWIB used $\alpha = 15$, $\gamma = 9$, $\sigma = 4.7$ and $A = .74$ and DH used $\alpha = \gamma = 16$, $\sigma = 5$ and $A = 1$. In analyzing the Kansas experiment data, BWIB determined a value of .35 for k. DH, however, determined a value of .41 from their experimental data.

PML used a slightly different approach by suggesting a more direct relationship between the diabatic influence function and the Richardson number, i.e.,

$$\phi_m(Ri) = (1-16\ Ri)^{-1/3} \quad \text{and} \quad \phi_H = .86\ \phi_m$$

for unstable stratification,

and $$\phi_m(Ri) = (1+16\ Ri)^{1/3} \quad \text{and} \quad \phi_H = .8\ \phi_m$$

for stable stratification.

Based on six studies conducted over a grass turf at Davis, California, PML concluded that a mean value of $k = .42$ is more appropriate. The above three studies by BWIB, DH and PML are all based on reliable data. The resulting discrepancy in the values of the von Karman constant, however, amount to about twenty per cent.

In attempting to provide explanations for the discrepancies and its significance, the present study made applications of these three flux-profile relationships to the same data collected in the Kansas experiment.

3. <u>Numerical Procedures for Flux Calculations</u>

Based on Monin-Obukhov similarity theory, the velocity and potential temperature profiles are:

$$U = \frac{u_*}{k}\left[\ln\left(\frac{z}{z_o}\right) - \psi_m\left(\frac{z}{L}\right)\right] \quad \ldots (1)$$

and

$$\theta - \theta_o = \frac{\theta_*}{k}\left[\ln\left(\frac{z}{z_T}\right) - \psi_H\left(\frac{z}{L}\right)\right] \quad \ldots (2)$$

Equations (1) and (2) can be rearranged into a more convenient form as follows:

$$\ln z - \psi_m\left(\frac{z}{L}\right) = \frac{k}{u_*} u + \ln z_o \quad \ldots (1)$$

and

$$\ln\left(\frac{z}{z_T}\right) - \psi_H\left(\frac{z}{L}\right) = \frac{k\theta}{\theta_*} - \frac{k\theta_o}{\theta_*} \quad \ldots (2)$$

where z_T is the height at which $\theta = \theta_o$. For simplicity, its value is taken to be the same as that of z_o.

Using the known wind velocities at different height levels z, and assuming that functional form of $\psi_m\left(\frac{z}{L}\right)$ is known, u_* and z_o can be estimated from (1). In the same manner, θ_* and θ_o can be estimated from (2). In the actual process of evaluating ψ_m and ψ_T, the Monin-Obukhov length, L, has to be specified. Since there is no direct relationship between L and the profile gradient, the following approximate forms proposed by Binkowski (1975) were used, i.e.,

$$L = \frac{Ri}{A}\left(\frac{1-\gamma Ri}{1-\alpha Ri}\right)^{\frac{1}{2}} \quad \text{for } Ri < 0 \text{ and}$$

$$L = \frac{Ri/A}{1-\beta Ri} \quad \text{for } Ri > 0$$

where $A = .74$, $\beta = 4.7$ and $\gamma = 15$ for BWIB case

$A = 1$, $\beta = 5$, $\gamma = 16$ for DH case.

For the PML case, the diabatic influence function is directly evaluated from the Richardson number. The calculations for u_* and θ_* are therefore much simpler. They are obtained from the profile gradient relationships

$$\frac{\partial u}{\partial z} = \frac{u_*}{k}\frac{\phi_m}{z} \text{ and } \frac{\partial \theta}{\theta z} = -\frac{\theta_*}{k}\frac{\phi_H}{z} \quad \ldots (3)$$

An alternative approach to equation (3), used by PML, is to evaluate ψ_m and ψ_h as functions of diabatic influence functions ϕ_m and ϕ_H as follows:

$$\psi_m = \frac{\pi}{2} - 1 + \ln 8 + \phi_m + 3\ln\phi_m - 2\ln(\phi_m+1) - \ln(1+\phi_m{}^2) - 2\tan^{-1}\phi_m$$

$$\phi_H = \frac{2}{3}(\phi^{3/2}-1) + 3\ln\phi - \ln(\phi^2+1) - 2\ln(\phi+1) + 2\tan^{-1}\sqrt{\phi} - \frac{\pi}{2} - \frac{1}{\sqrt{2}}\ln\left(\frac{\phi-\sqrt{2\phi}+1}{\phi+\sqrt{2\phi}+1}\right) + \frac{1}{\sqrt{2}}\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) + \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\phi}}{\phi-1}\right) - \pi/\sqrt{2} + \ln 8$$

The derivations of these two formulae have been given by Lo (in press). With ψ_m and ψ_H known, we can then use (1) and (2) to evaluate the fluxes. The advantage of this alternate approach is the elimination of any additional discrepancies due to using gradient relations (3) instead of equations for the profile itself such as (1) and (2).

4. Results and Discussions

Flux calculations using the Kansas experiment data based on flux-profile relationships proposed by BWIB, DH and PML are carried out and presented in graphical form.

The lower half of Fig. 1 gives the friction velocities obtained from the three different cases. Although their values deviate from each other, a similar distribution trend is observed within the unstable and near neutral regime.

It is to be noted that the von Karman constant k has values of .35, .41, and .42 for the BWIB, DH and PML models respectively. In the upper half of Fig. 1, the same results were plotted in the form of u_*/k. We immediately noticed that the BWIB case becomes almost indistinguishable from the DH case.

Figure 2 gives the results for the scalimg temperature $\theta_* = \theta_*/k$, and again the BWIB and DH model results agree very closely. Figure 3 is the comparison of u_*, obtained in the present study, with u_* determined from flux measurements (Izumi, 1971). Due to the different k values used in each case, discrepancies are obvious. As before, these results were replotted in Fig. 4, using u_*/k indicating that both the BWIB and DH model agree well with that based on direct flux measurements. Figure 5 shows similar agreement for the scaling temperature θ_*.

As a by-product of flux calculations, values of roughness length, z_o, are also obtained. It is well known that in the constant flux layer, the roughness length plays an important role in determining the profile shape especially near the surface. It has been pointed out by Lo (in press) that the interaction of the atmospheric boundary layer with its underlying surface not only affects the development of the wind field but also alters the dynamic characteristics of the underlying surfaces. It is therefore expected that z_o should not only depend on the physical structure of the surface but also reflect the nature of the wind field over the surface. Figures 6 and 7 are the results for z_o under various wind speeds and stability conditions.

The values of z_o versus wind speed at a reference height of 16 meters are plotted in Fig. 6. At a first glance, the result seems to have a lot of scatter. Closer examination, however, reveals a very slight trend indicating that the roughness length tends to increase with wind speed. In Fig. 7, the dependence of z_o on stability is examined. It can be seen that z_o undergoes very little change within the unstable regime. The dependence of z_o on stability becomes more obvious under stable conditions. The values of z_o decrease in a rather speedy fashion as the stability is increased. The explanation for this phenomenon is that there are far more eddies engaging active mixing under unstable conditions than that under stable conditions. Such situation analogous to turbulence above an aerodynamically rough surface as compared to that above a smoother surface.

Figures 8 and 9 compare errors in fluxes caused by experimental errors to that due to using different flux-profile relations. Numerical experiments on flux discrepancies by imposing artificial errors in velocity profiles were performed. Distorted velocity profile-1 is obtained by decreasing the values in the odd levels by 5% and increasing the even levels by 5%. Distorted velocity profile-2 is obtained by increasing the upper half profile data by 5% and decreasing the lower half profile data by the same amount. The resulting errors due to such artificial distortions are then compared with the discrepancy between results from BWIB and DH models. As shown in Figs. 8 and 9, errors or discrepancies in each case remain relatively constant within the unstable regime before rapidly diverging as the stratification becomes more stable. The important evidence here is that the deviation in flux values caused by errors in profile measurement is comparable to that of the composite effect of using different diabatic influence functions and von Karman constants. In other words, a 5%

error in profile measurements is significant enough to alter the outcomes of the flux and the value of the von Karman constant.

In almost all the results presented, deviations within the stable regime are substantially higher than that under unstable conditions. Our interpretation of this is that turbulence is greatly suppressed under stable conditions and flux fluctuations under stable conditions are not well understood. In fact the validity of the Monin-Obukhov similarity theory under stable conditions is not fully established.

5. Conclusions

Based on the evidence obtained in this study, we believe that there are at least two sources that might contribute to errors in fluxes and hence in the related von Karman constant. The fundamental one is that Monin-Obukhov's similarity theory is based on an ideal model of steady wind flow over a horizontally homogeneous terrain. In the real world, however, experimental sites may deviate considerably from such an idealized condition. The second source is instrument errors, which is often underestimated. Independent studies such as the BWIB, DH and PML cases considered here, were carried out at different sites using different instruments. Therefore, it is futile to attempt to argue that the von Karman constant proposed in any one of them is more correct than the other.

Since we believe that the von Karman constant bears no physical meaning, it does not really matter as to what value it is assigned to so long as the individual using it is aware of the possible inherent error due to its source.

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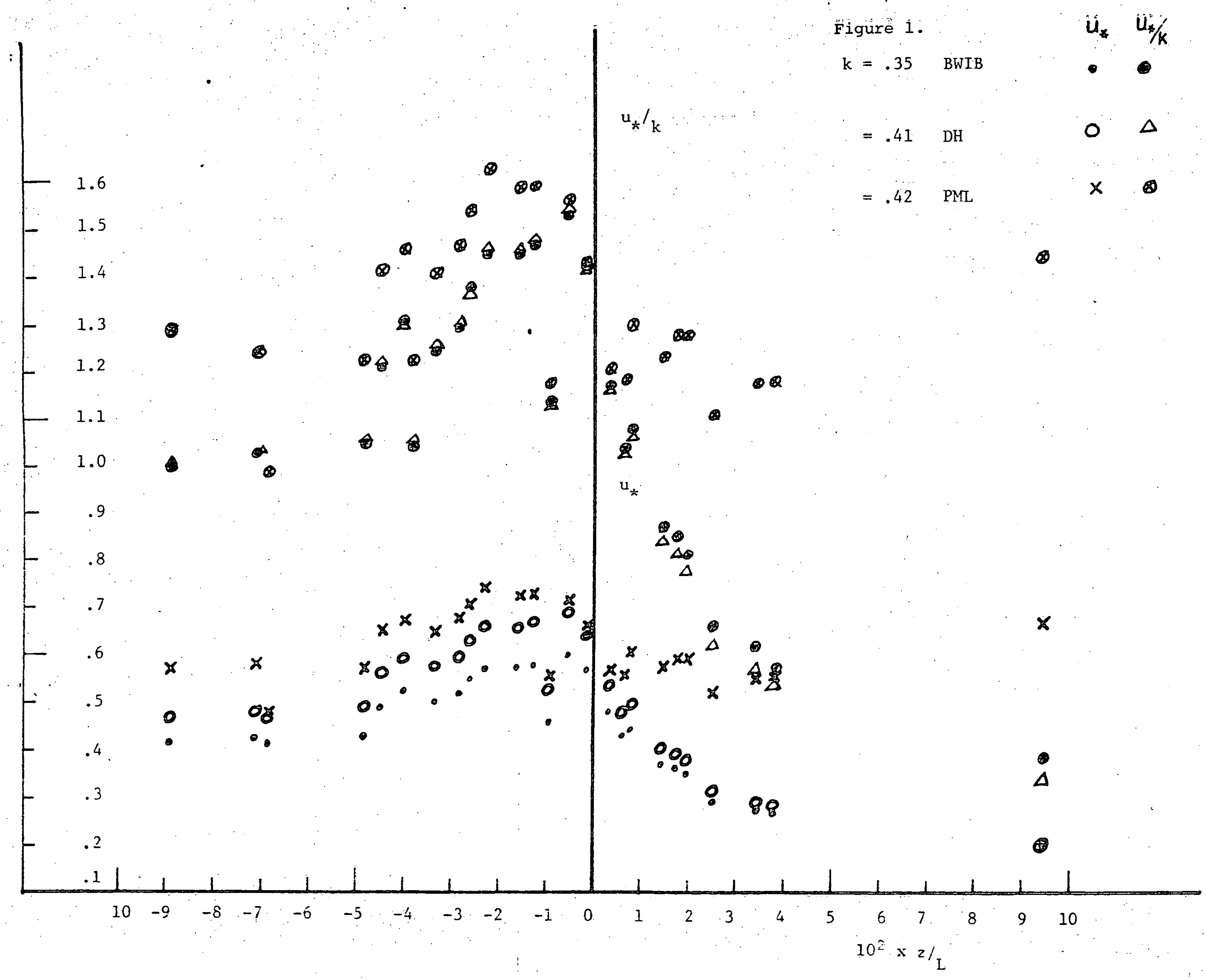
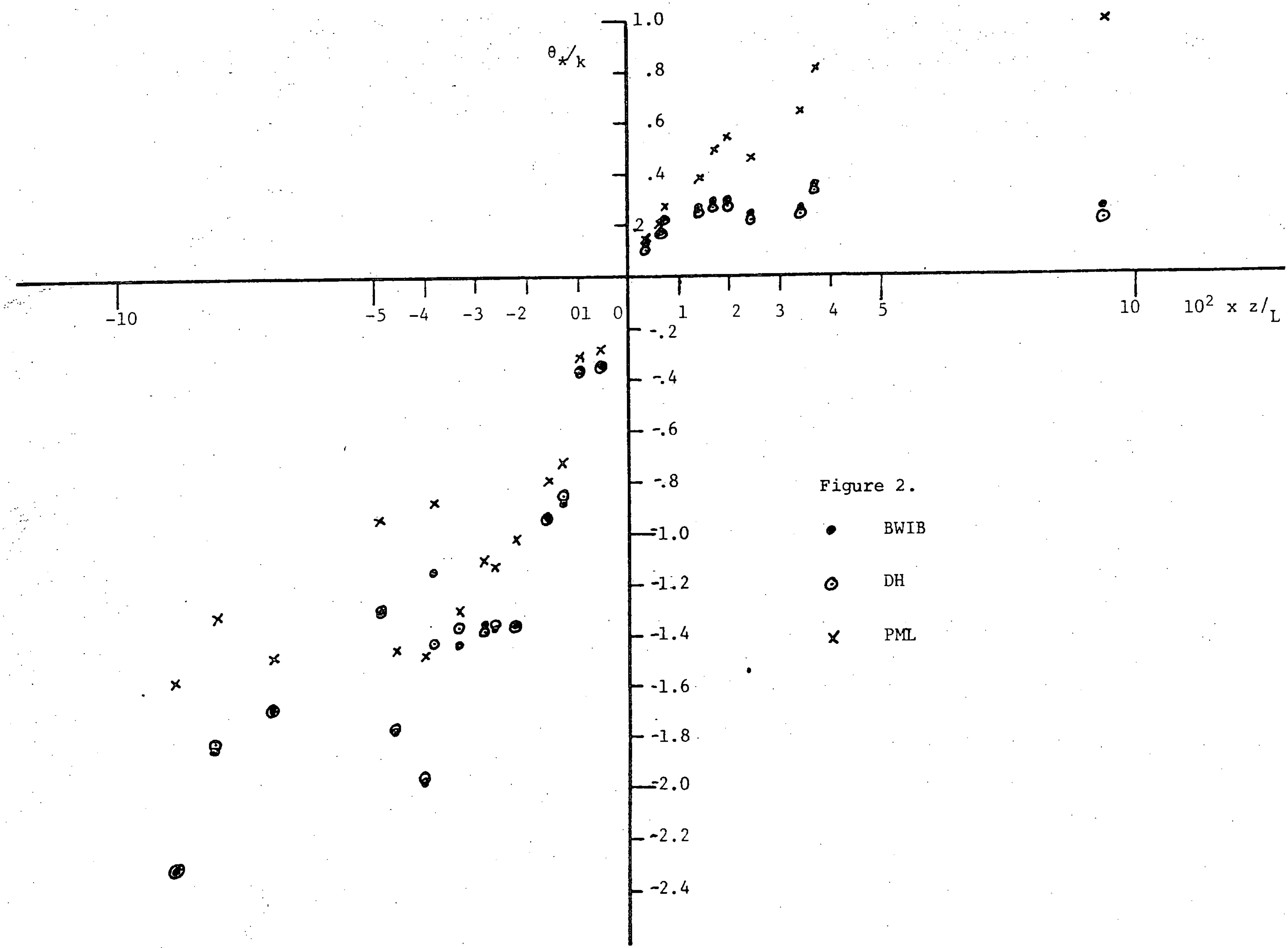


Figure 1.
k = .35 BWIB
= .41 DH
= .42 PML
u_* u_*/k
u_*/k
u_*
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
$10^2 \times z/L$



θ*/k
1.0
.8
.6
.4
.2
-.2
-.4
-.6
-.8
-1.0
-1.2
-1.4
-1.6
-1.8
-2.0
-2.2
-2.4
-10
-5
-4
-3
-2
-1
0
1
2
3
4
5
10
10² x z/L
BWIB
DH
PML

Figure 2.

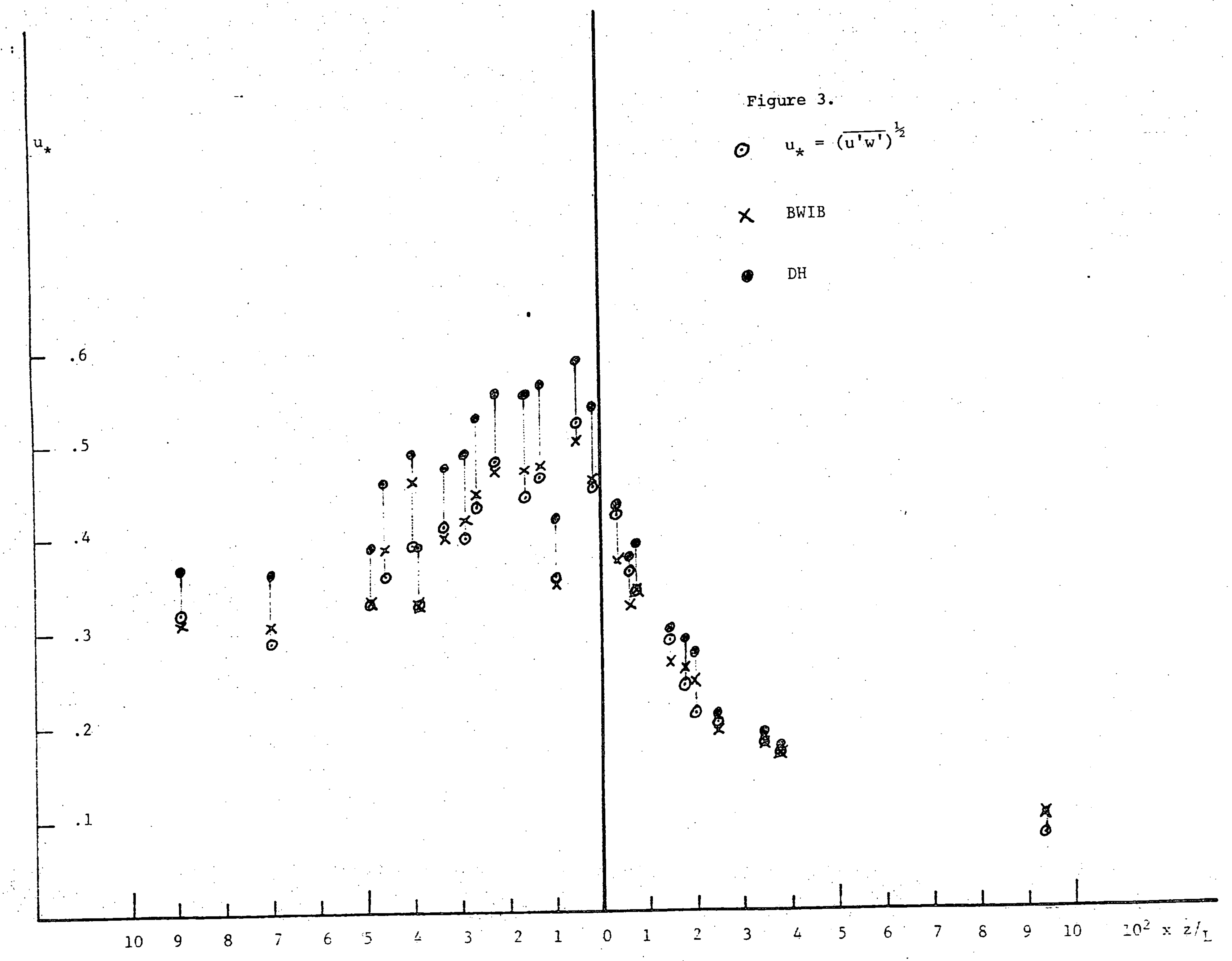
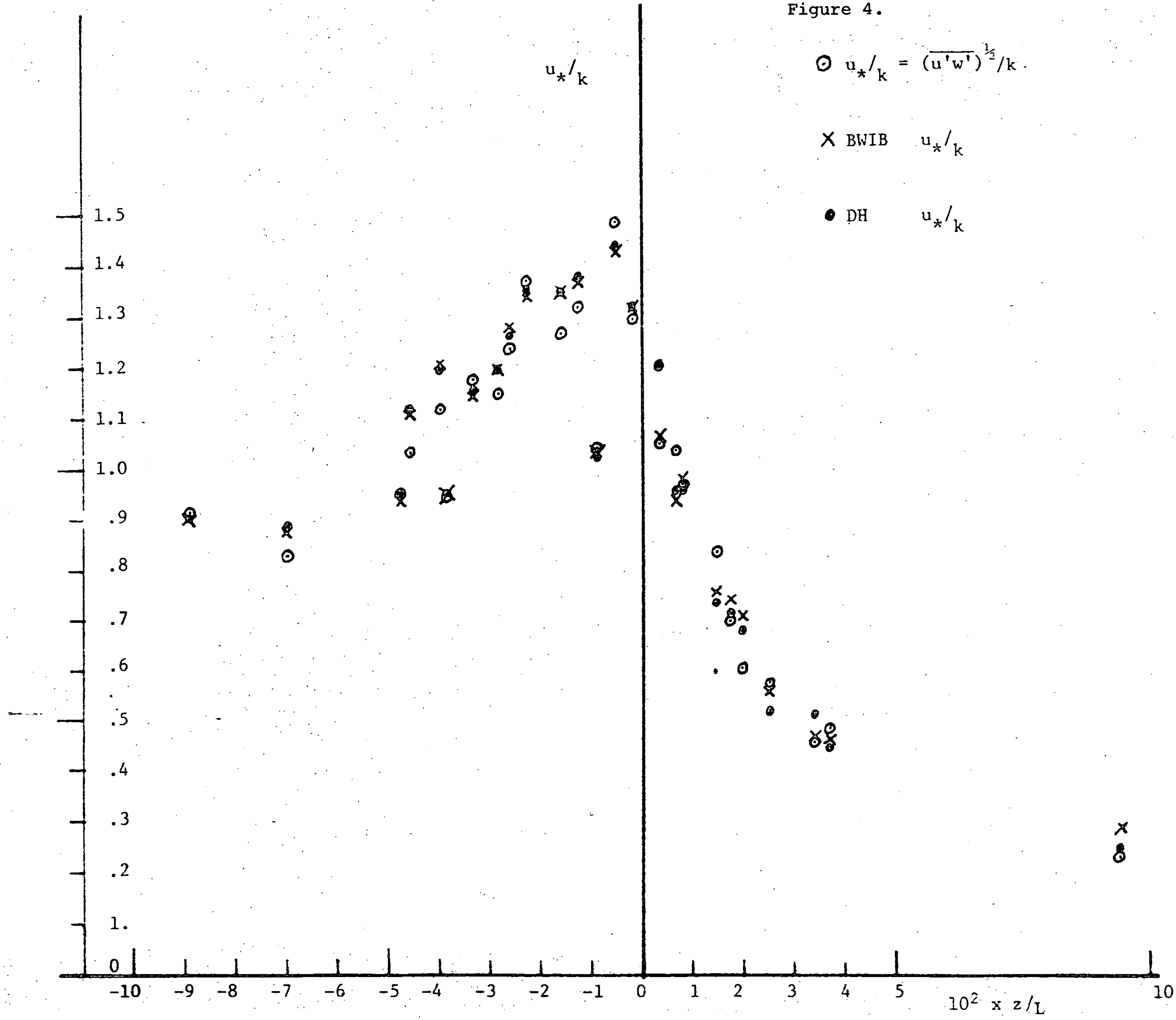


Figure 3.
$u_* = (\overline{u'w'})^{1/2}$
BWIB
DH
u_*
.1
.2
.3
.4
.5
.6
10
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
$10^2 \times z/L$

Figure 4.



⊙ u*/k = (u'w')½/k
× BWIB u*/k
● DH u*/k
u*/k
0
1.
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
-10
-9
-8
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-4
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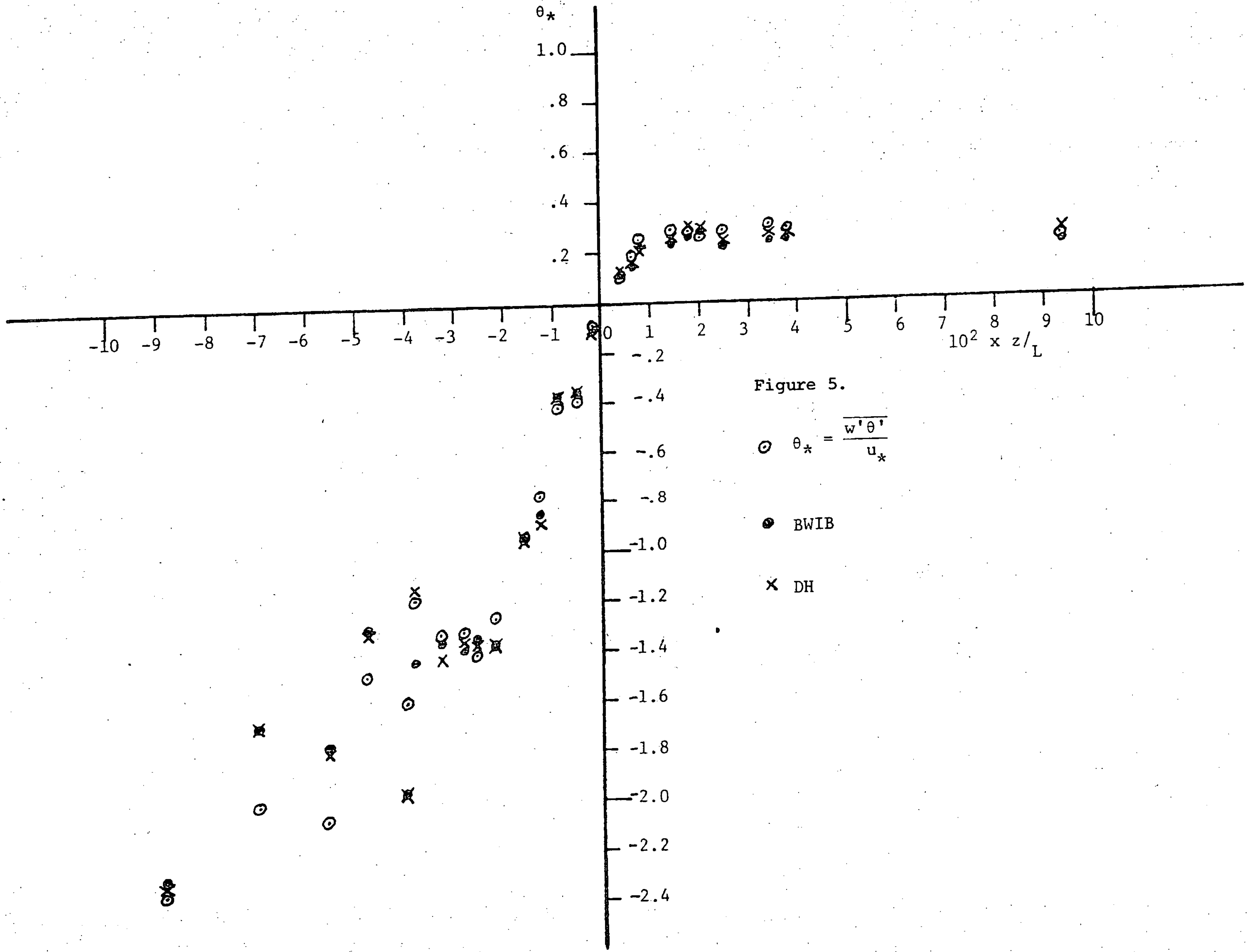
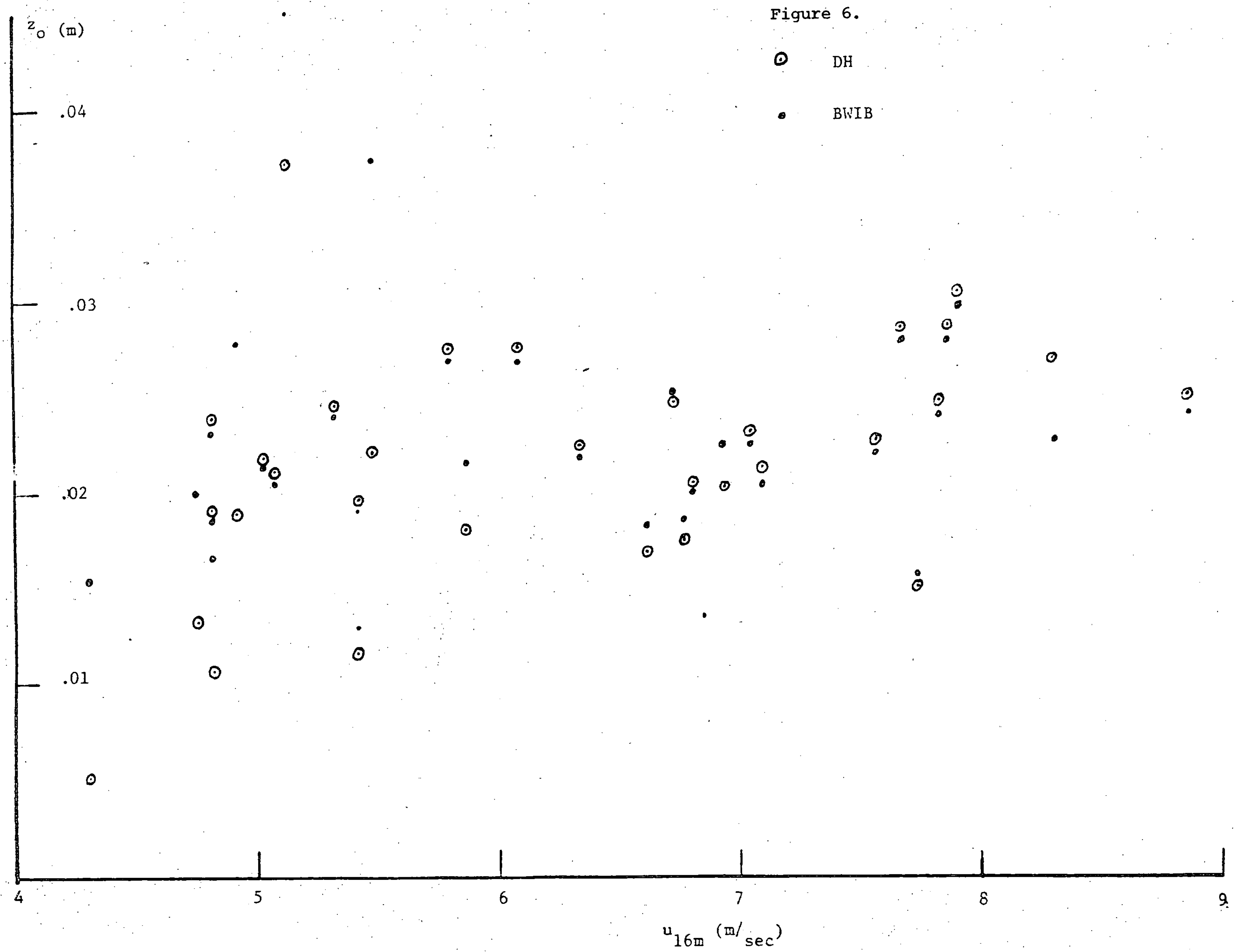
-2
-1
0
1
2
3
4
5
10
10² x z/L



θ*
1.0
.8
.6
.4
.2
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10
10² x z/L
-.2
-.4
-.6
-.8
-1.0
-1.2
-1.4
-1.6
-1.8
-2.0
-2.2
-2.4
θ* = w'θ'/u*
BWIB
DH

Figure 5.

Figure 6.



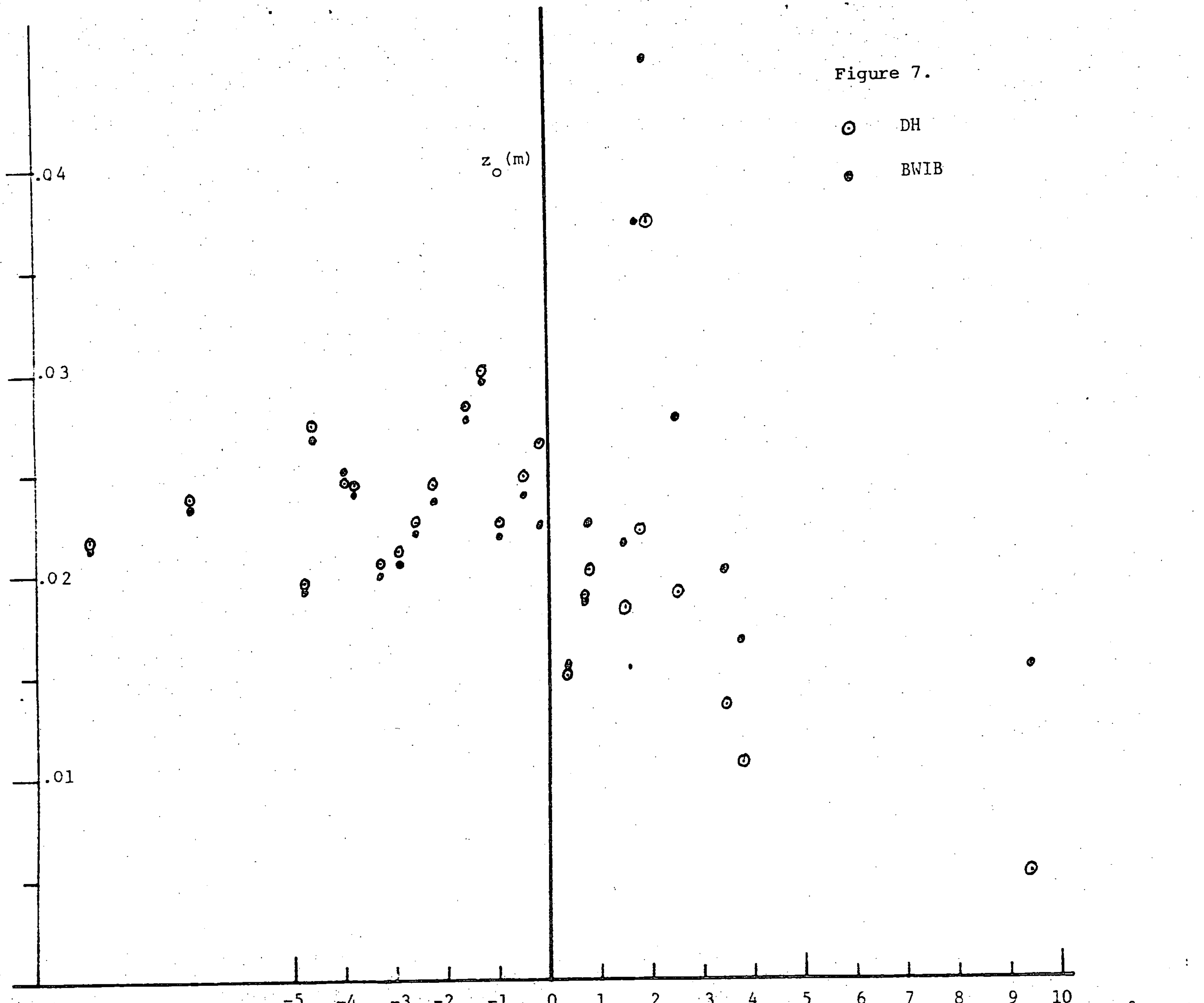


Figure 7.

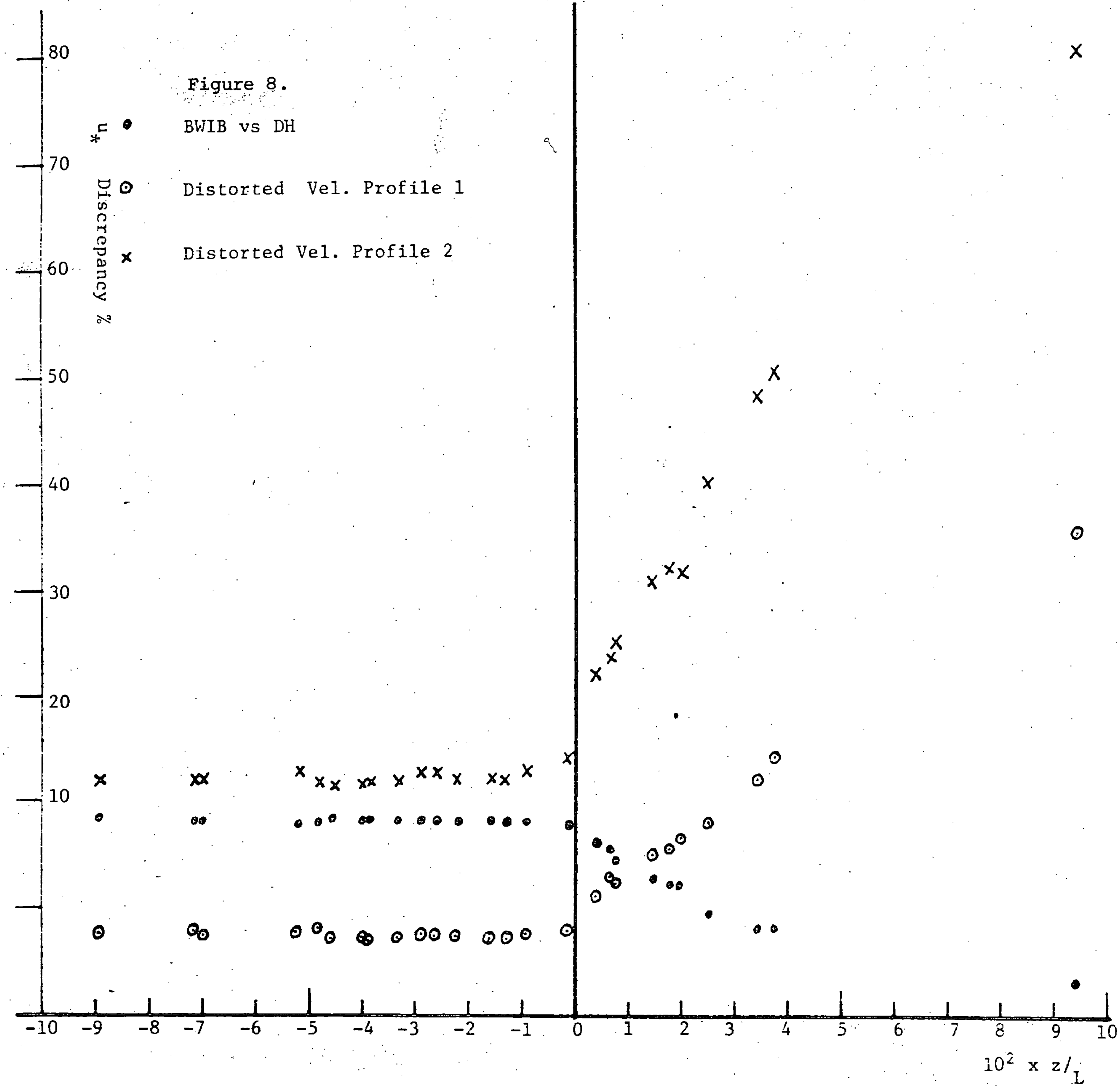
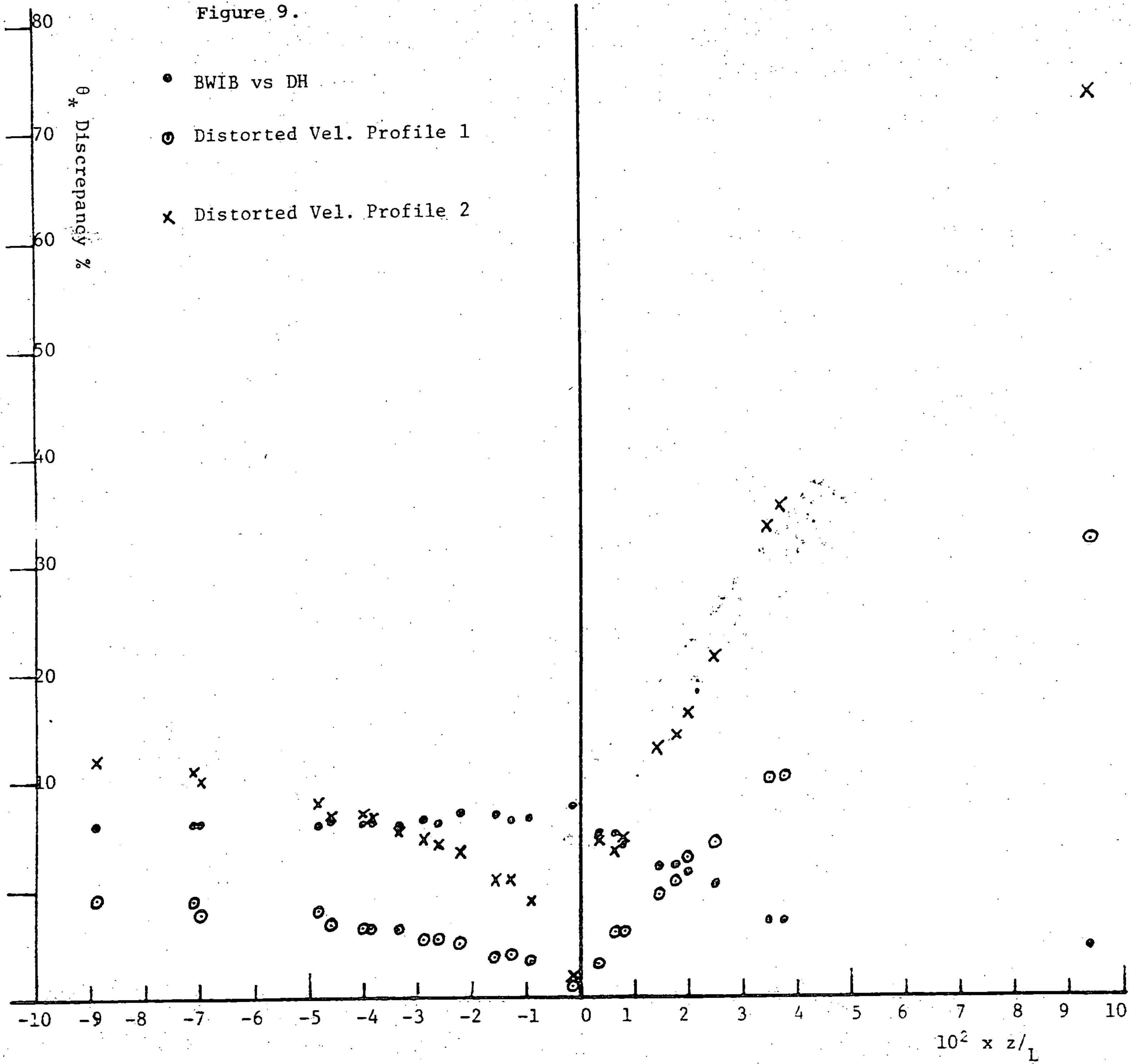


Figure 8.
BWIB vs DH
Distorted Vel. Profile 1
Distorted Vel. Profile 2
u_* Discrepancy %
80
70
60
50
40
30
20
10
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10
$10^2 \times z/_L$

Figure 9.



BWIB vs DH
Distorted Vel. Profile 1
Distorted Vel. Profile 2
θ_* Discrepancy %
80
70
60
50
40
30
20
10
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
$10^2 \times z/_L$