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Volume of liquid contained in elliptical tanks.

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DEPARTMENT OF TRADE AND COMMERCE
CANADA

STANDARDS LABORATORY

TECHNICAL MEMORANDUM
NO. 2

VOLUME OF LIQUID CONTAINED IN ELLIPTICAL TANKS – FILLED, PARTIALLY FILLED AND INCLINED

by
G. E. Anderson

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STANDARDS BRANCH

TECHNICAL MEMORANDUM NO. 2

VOLUME OF LIQUID CONTAINED IN ELLIPTICAL TANKS - FILLED, PARTIALLY FILLED AND INCLINED

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INTRODUCTION:

This memorandum is intended to serve four main purposes:

(i) to derive formulae for establishing the volume of liquid contained in elliptical tanks (with plane ends) of various sizes and shapes -
  - (a) when the liquid completely fills the tank
  - (b) when the liquid only partially fills the tank
    - (α) if the longitudinal axis of the tank is horizontal
    - (β) if the longitudinal axis of the tank is inclined to the horizontal;

(ii) to derive tables from these formulae for typical cases, from which close approximations to the volume of contained liquid can be made in actual cases, using simple interpolation;

(iii) to establish the percentage error which might be expected if a tank-truck is calibrated to a fixed-marker setting, when the tank is not truly horizontal in the longitudinal (fore-and-aft) direction; and

(iv) to serve as the basis for recommendations regarding closer control of the permissible "% offset" of calibration-markers from the mid-plane of tanks.

DEFINITIONS:

Throughout this memorandum, the term "elliptical cylinder" will be used to describe a right cylinder of constant elliptic cross-section and will include as a particular case a right circular cylinder. Note that the ends of the elliptical cylinder will be assumed to be plane in all cases, not "dished" (for rigidity) as is usually the case in practice.

Also, by the term "longitudinal axis" is meant the line joining the centres of all the elliptic sections, and by the term "longitudinal plane" is meant the plane containing the major axes of all the elliptic sections. (Since the shapes of the tanks can in practice be expected to vary widely,

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the formulae derived will cover cases where the vertical diameter of the ellipse, denoted by 2b, will be greater than the horizontal diameter, denoted by 2a. However, even in these cases, the horizontal axis will be referred to as the "major axis" and the vertical axis will be referred to as the "minor axis".)

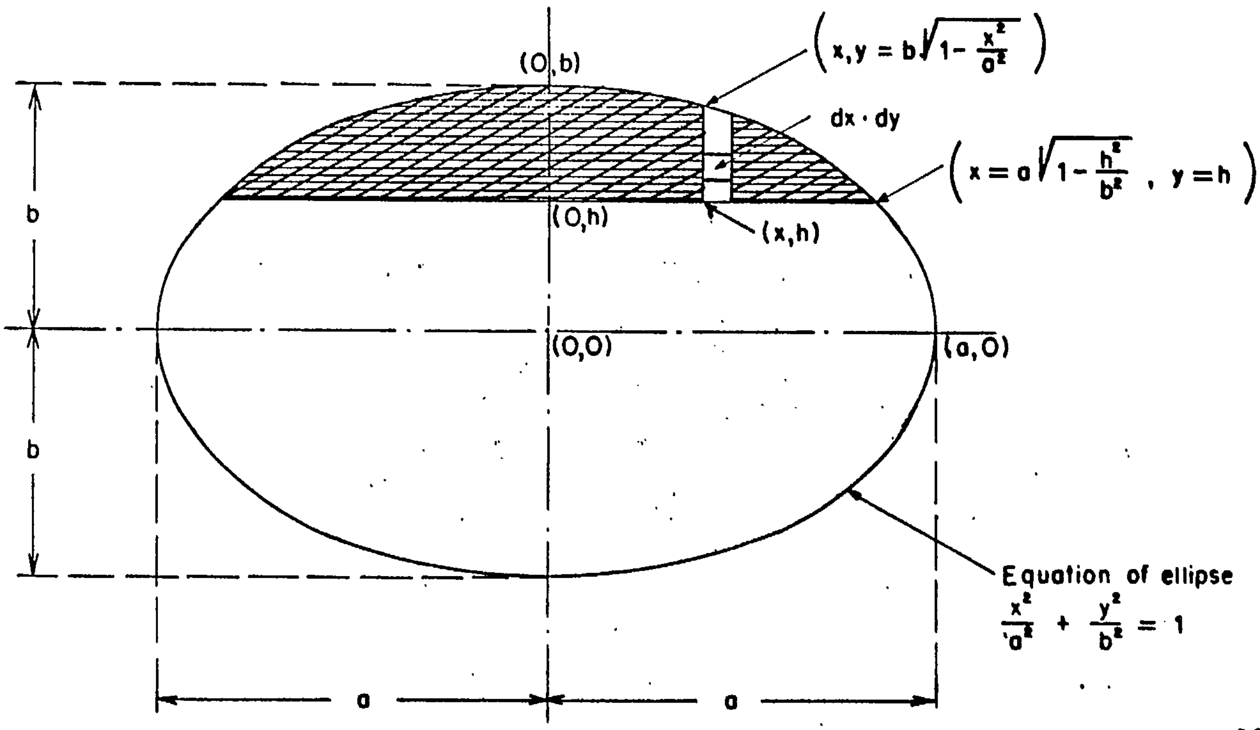
Further, when the longitudinal axis is stated to be inclined to the horizontal, it will be assumed that "pitching" only is involved about some major axis and that there is no "rolling" about the longitudinal axis. Hence it is assumed that inclination of the longitudinal axis to the horizontal continues to leave the major axes of the various cross-sections horizontal.

Thus, when an elliptical tank partially filled with liquid is "inclined to the horizontal", the liquid-level at one plane end of the tank will be at a greater distance from the bottom of the tank than will be the liquid-level at the other plane end of the tank -- but the liquid will stand at the same distance from the bottom on each curved side of the same elliptic cross-section.

The use of the words "complete ungula" and "partial ungula" should be noted. By a "complete ungula" we mean a wedge-shaped slice of the cylinder bounded by the curved surface of the cylinder and only two planes - the "cutting plane" (or air-liquid interface) and one plane end of the cylinder. By a "partial ungula" we mean a wedge-shaped slice bounded by the curved surface and three planes - the "cutting plane" and the two plane ends of the cylinder.

Other terms, such as "fractional height", "fractional length", "offset", "% offset", "k-factor", etc. are defined below in the course of this memorandum.

Area of a segment of an ellipse



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The area of the segment of an ellipse, which is shown cross-hatched in the preceding diagram, is given by the equation:

$$\text{Area} = A = 2\int_{x=0}^{x=a\sqrt{1-\frac{h^2}{b^2}}} dx \cdot \int_{y=h}^{y=b\sqrt{1-\frac{x^2}{a^2}}} dy$$

$$\text{Using} \int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$$

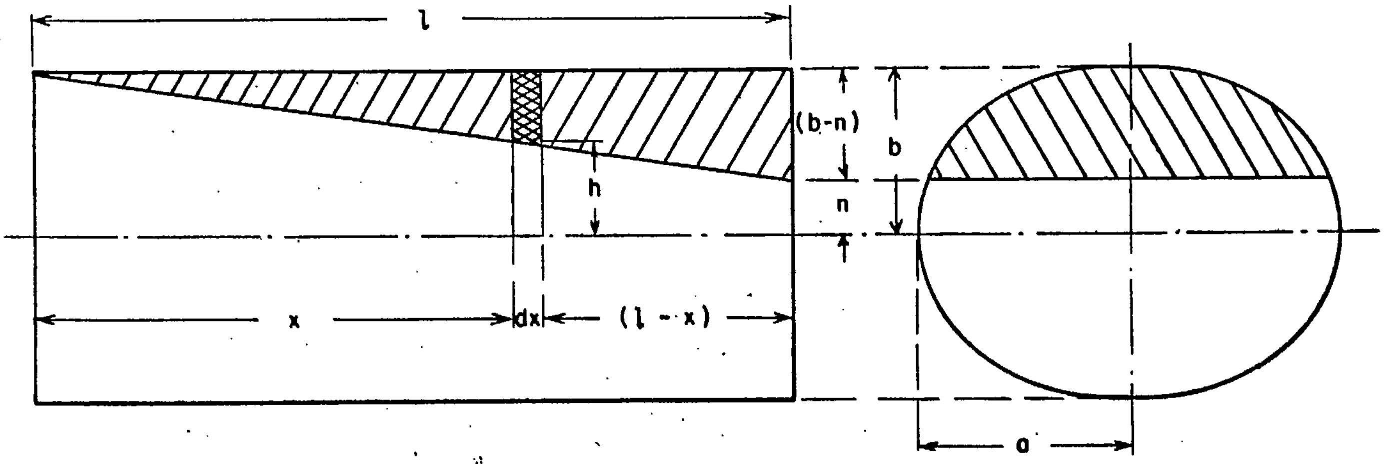
$$\text{we obtain} \quad A = ab\sin^{-1}\sqrt{1-\frac{h^2}{b^2}} - ah\sqrt{1-\frac{h^2}{b^2}} \quad \ldots\ldots\ldots\ldots\ldots (A).$$

Volume of a "complete" ungula of an elliptical cylinder:

Let us consider a complete ungula of an elliptical cylinder of length "l", with major (horizontal) semi-diameter "a", and with minor (vertical) semi-diameter "b".

Assume that the "trace" of the longitudinal or "cutting" plane of the ungula on one of the plane ends of the cylinder is parallel to the major (horizontal) axis, at a height "n" above this axis.

The diagram below illustrates the complete ungula as described above and which is shown shaded.



Consider an elementary slab of thickness "dx", shown double-hatched above.

Let the vertical distance of this slab from the longitudinal axis of the cylinder be "h" when the horizontal distance from the left-hand end is "x".

Then $h(x) = \frac{1}{l}\left\{bl - (b-n)\,x\right\}$

$$\therefore \sqrt{1 - \frac{h^2}{b^2}} = \frac{(b-n)}{bl}\sqrt{\frac{2blx}{b-n} - x^2}$$

and $h\sqrt{1-\frac{h^2}{b^2}} = \frac{(b-n)}{bl^2}\left\{bl - (b-n)\,x\right\}\sqrt{\frac{2blx}{b-n} - x^2}$

Now volume of complete ungula

$$= V = \int_{x=o}^{x=l}\left\{ab\sin^{-1}\sqrt{1-\frac{h^2}{b^2}} - ah\sqrt{1-\frac{h^2}{b^2}}\right\}dx$$

$$\therefore V = ab\int_o^l \sin^{-1}\sqrt{1-\frac{h^2}{b^2}}\,.dx - a\int_o^l h\sqrt{1-\frac{h^2}{b^2}}\,.dx$$

$$= ab\int_o^l \sin^{-1}\left\{\left(\frac{b-n}{bl}\right)\sqrt{\frac{2blx}{b-n} - x^2}\right\}.dx - \frac{a(b-n)}{bl^2}\int_o^l\left\{bl-(b-n)x\right\}\sqrt{\frac{2blx}{b-n} - x^2}.dx$$

Then first integral on R.H.S. is:

$$I_1 = \int_o^l \sin^{-1}\left\{\frac{(b-n)}{bl}\sqrt{\frac{2blx}{b-n} - x^2}\right\}dx$$

Put $y = \frac{(b-n)}{bl}\sqrt{\frac{2blx}{b-n} - x^2}$

$$\therefore \left(\frac{b-n}{bl}\right)^2.x^2 - 2\left(\frac{b-n}{bl}\right).x + y^2 = o$$

$$\therefore x = \frac{bl}{b-n} \pm \frac{bl}{b-n}\sqrt{1-y^2},$$

whence we obtain

$dx = \pm\left(\frac{bl}{b-n}\right).\frac{y\,dy}{\sqrt{1-y^2}}$ (use + sign only)

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If $x = o$, $y = o$

and if $x = \ell$, $y = \left(\frac{b-n}{b}\right)\sqrt{\frac{b+n}{b-n}}$

$$\text{Hence, } I_1 = \int_{y=o}^{y=\left(\frac{b-n}{b}\right)\sqrt{\frac{b+n}{b-n}}} \frac{bl}{b-n}\frac{y}{\sqrt{1-y^2}} \cdot \sin^{-1} y . dy$$

$$\therefore I_1 = \frac{\ell}{b-n}\sqrt{b^2 - n^2} - \frac{\ell n}{b-n} \sin^{-1}\left\{\frac{1}{b}\sqrt{b^2 - n^2}\right\}$$

The second integral on R.H.S. of equation for V is:

$$I_2 = \int_o^\ell \left\{bl - (b - n)\, x\right\}\sqrt{\frac{2blx}{b-n} - x^2} . dx$$

$$= \frac{bl}{b-n}\int_o^\ell \sqrt{2bl\,(b-n)\,x - (b-n)^2x^2} . dx - \int_o^\ell \sqrt{2bl\,(b-n)x-(b-n)^2x^2} . x . dx$$

$$\text{Now} \int_o^\ell \sqrt{2bl(b-n)x-(b-n)^2x^2} . x . dx$$

$$= \frac{-1}{3(b-n)^2}\left[\left\{2bl(b-n)x-(b-n)^2x^2\right\}^{3/2}\right]_o^\ell + \frac{bl}{b-n}\int_o^\ell \sqrt{2bl(b-n)x-(b-n)^2x^2} . dx$$

(cf. G. Petit-Bois, "Table of Indefinite Integrals" p. 57.)

$$\therefore I_2 = \frac{1}{3(b-n)^2}\left\{2bl^2(b-n) - (b-n)^2\ell^2\right\}^{3/2}$$

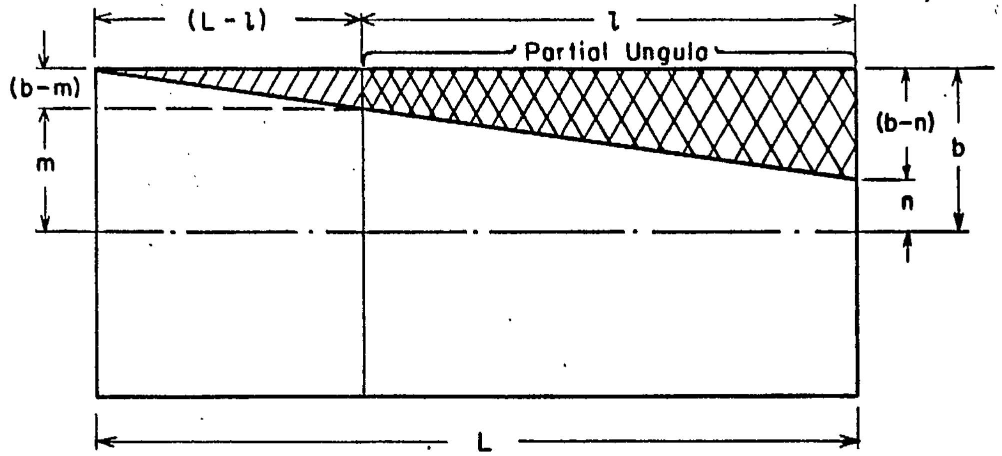
$$= \frac{\ell^3}{3(b-n)^2} \cdot \left\{b^2 - n^2\right\}^{3/2}$$

$$\therefore V = ab.I_1 - \frac{a\,(b-n)}{bl^2} \cdot I_2$$

$$\therefore V = \frac{abl}{b-n}\sqrt{b^2-n^2} - \frac{abln}{b-n} . \sin^{-1}\left\{\frac{1}{b}\sqrt{b^2-n^2}\right\} - \frac{al}{3b(b-n)}\left\{b^2-n^2\right\}^{3/2}$$

Volume of a "partial" ungula of an elliptical cylinder

Consider the case of a partial ungula cut from an elliptical cylinder, which is shown cross-hatched in the illustration below:



Note that the "trace" of the longitudinal or "cutting" plane of the partial ungula on the plane ends of cylinder is at a height "m" above the axis of cylinder at one end and at a height "n" at the other end.

We have: $L = \left(\frac{b-n}{m-n}\right)\ell$

$$\therefore L - \ell = \left(\frac{b-m}{m-n}\right)\ell$$

Let volume of partial ungula (of length "ℓ") be V_1

Let volume of complete ungula (of length L) be V_2

Let volume of complete ungula (of length $L-\ell$) be V_3

Then $V_1 = V_2 - V_3$

But from equation given above for volume of a complete ungula, we have:

$$V_2 = \frac{abL}{b-n}\sqrt{b^2-n^2} - \frac{aL}{3b(b-n)}\left(b^2-n^2\right)^{3/2} - \frac{abLn}{b-n}\sin^{-1}\left\{\frac{\sqrt{b^2-n^2}}{b}\right\}$$

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and $V_3 = \frac{ab(L-\ell)}{b-m}\sqrt{b^2-m^2} - \frac{a(L-\ell)}{3b(b-m)}(b^2-m^2)^{3/2} - \frac{ab(L-\ell)m}{b-m}\sin^{-1}\left\{\frac{\sqrt{b^2-m^2}}{b}\right\}$

Hence, volume of partial ungula (of length ℓ)

$$= V_1 = \frac{abl}{m-n}\left[\left\{\sqrt{b^2-n^2} - \sqrt{b^2-m^2}\right\} - \frac{1}{3b^2}\left\{(b^2-n^2)^{3/2} - (b^2-m^2)^{3/2}\right\}\right.$$

$$\left. - \left\{n \sin^{-1}\left(\frac{\sqrt{b^2-n^2}}{b}\right) - m \sin^{-1}\left(\frac{\sqrt{b^2-m^2}}{b}\right)\right\}\right]$$

Therefore, the volume of the portion of the cylinder exclusive of the ungula (i.e. beneath the longitudinal plane of the ungula)

$$= Vc_T = \pi abl - V_1$$

We will call this volume, Vc_T, the "volume of the contents - tilted", since it is the volume of the liquid in the tank, if the tank is tilted to the horizontal in a fore-and-aft direction so that the liquid level stands at one end at a height "m" above the longitudinal axis of the cylinder and at a height "n" at the other end.

Let us express b, m and n as fractions of a, b and m respectively,

i.e. let $b = pa$ $\qquad (0 < p)$
$m = qb = pqa$ $\qquad (0 < q \leq 1)$
$n = rm = pqra$ $\qquad (0 < r < 1)$

Then we have for the "volume of the contents - tilted",

$$Vc_T = \pi a^2 lp - a^2 lp\left\{\frac{1}{q(1-r)}\right\}\left[\left\{(1-q^2r^2)^{1/2} - (1-q^2)^{1/2}\right\} - \frac{1}{3}\left\{(1-q^2r^2)^{3/2} - (1-q^2)^{3/2}\right\}\right.$$

$$\left. - q\left\{r \sin^{-1}\left(\sqrt{1-q^2r^2}\right) - \sin^{-1}\left(\sqrt{1-q^2}\right)\right\}\right]$$

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For the purposes of electronic data processing, let us use the relation:

$$\sin^{-1} x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$\therefore$ Volume of contents - tilted

$$= Vc_T = \pi abl\left\{1 - \frac{1}{\pi q(1-r)}\left[\left\{(1-q^2r^2)^{1/2} - (1-q^2)^{1/2}\right\} - \frac{1}{3}\left\{(1-q^2r^2)^{3/2} - (1-q^2)^{3/2}\right\} - q\left\{r\tan^{-1}\left(\frac{\sqrt{1-q^2r^2}}{qr}\right) - \tan^{-1}\left(\frac{\sqrt{1-q^2}}{q}\right)\right\}\right]\right\}$$

If we express the volume of the contents as a fraction of the total volume (πabl) of the cylinder, we have:

$$\text{Fractional volume of tilted tank} = \frac{\text{Volume of contents - tilted}}{\text{Total volume of tank}}$$

$$= FV_T = \frac{1}{\pi\, abl} \cdot Vc_T$$

$$\therefore FV_T = 1 - \frac{1}{\pi q(1-r)}\left[\left\{(1-q^2r^2)^{1/2} - (1-q^2)^{1/2}\right\} - \frac{1}{3}\left\{(1-q^2r^2)^{3/2} - (1-q^2)^{3/2}\right\} - q\left\{r\tan^{-1}\left(\frac{\sqrt{1-q^2r^2}}{qr}\right) - \tan^{-1}\left(\frac{\sqrt{1-q^2}}{q}\right)\right\}\right]$$

------ Equation (A)

It will be noted that the preceding equations for the volume of a partial ungula, volume of contents and fractional volume become indeterminate when $r=1$ or $m=n$, i.e. when the tank is horizontal, so that the level of the liquid stands at the same height at each end of the tank.

However, in this case we can easily derive the equations for volume and fractional volume, using the formula for the area of a segment of an ellipse.

Put $b = pa$ $(0 < p)$

$m = n = qb = pqa$ $(0 < q \leq 1)$

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We obtain the following expression for the "volume of the contents - horizontal"

$$Vc_H = a^2 lp \left\{ \pi + q\sqrt{1-q^2} - \sin^{-1}\left(\sqrt{1-q^2}\right) \right\}$$

As before, set $\sin^{-1}x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$\therefore$ Volume of contents - horizontal

$$= Vc_H = \pi abl \left\{ 1 + \frac{q}{\pi}\sqrt{1-q^2} - \frac{1}{\pi} \tan^{-1}\left(\frac{\sqrt{1-q^2}}{q}\right) \right\}$$

$\therefore$ Fractional volume of horizontal tank = $\frac{\text{Volume of contents-horizontal}}{\text{Total volume of tank}}$

$$= FV_H = \frac{1}{\pi abl} \cdot Vc_H$$

$$\therefore \quad FV_H = 1 + \frac{q}{\pi}\sqrt{1-q^2} - \frac{1}{\pi} \tan^{-1}\left(\frac{\sqrt{1-q^2}}{q}\right)$$

-------Equation (B)

Calculations and Graphs 1 and 2:

A FORTRAN II programme (given in an appendix)was set up to calculate values of FV corresponding to various values of $r = \frac{n}{m}$ from r = 0.0 to 1.0. for each value of $q = \frac{m}{b}$ from q = 0.05 to 1.0.

This table of fractional volumes (FV) is given in the appendix as Table I.

Using the values of FV from Table I, the family of curves of FV versus r or (1-r) was drawn, each individual curve being given for a constant value of q. These curves are shown in Graph No. 1 below.

By cross-plotting from Graph No. 1, pairs of values of q and r were obtained for values of constant FV. The family of curves of q versus r and (1-r) was drawn, each individual curve being given for a constant value of FV. These curves are shown in Graph No. 2, below.

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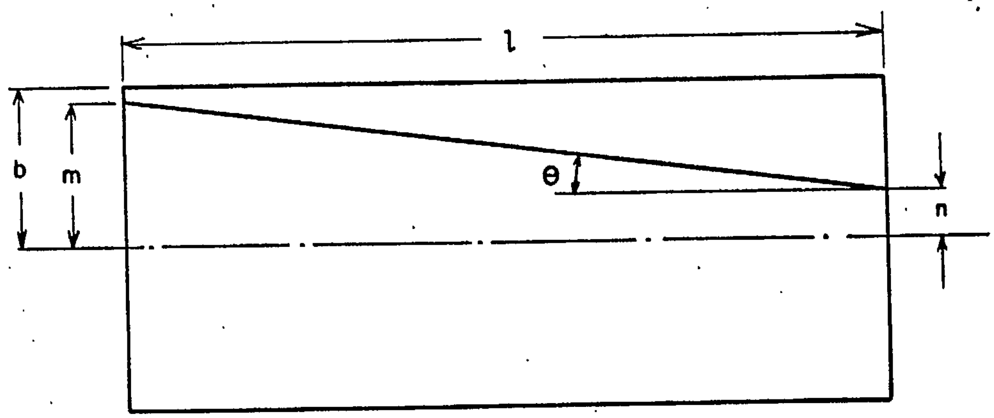
Also on Graph No. 2 is drawn a family of hyperbolae. Each curve of this family has the equation:

$$q\ (1 - r) = k, \quad (\text{ a constant})$$

The family is obtained by setting in turn k = 0.025,0.05,...0.90.

The significance of the k-factor (or "tilt factor") can be seen by reference to the diagram below. This diagram represents a tilted tank in which the liquid surface is at a height "m" above the longitudinal axis at one end and at a height "n" at the other end.



When the liquid stands at the heights m and n as shown, the tank will be tilted to the horizontal (in the fore-and-aft direction) through an angle θ, such that

$$\tan\theta = \frac{m-n}{\ell}.$$

$$\text{Now } k = q\ (1-r) = \frac{m}{b}\ (1-\frac{n}{m}) = \frac{m-n}{b} = \frac{\ell}{b} \cdot \left(\frac{m-n}{\ell}\right)$$

$$\therefore\ k = q\ (1-r) = \frac{\ell}{b} \cdot \tan\theta \qquad \text{..............Equation (C.)}$$

Hence the curves of constant value of "k" on Graph No. 2 represent constant values of $\frac{\ell}{b} \cdot \tan\theta$.

GRAPH No. 3

By making use of data derived from Graph No. 2, we can obtain Graph No. 3 which is useful in estimating the effect of "tilt" on the error of calibration, when a tank is calibrated to a fixed marker setting.

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Suppose the tank is horizontal and a definite volume of liquid is measured into the tank. This volume of liquid will be a definite fraction of the total capacity of the tank, say FV= Z. Since the tank is horizontal, the liquid will stand at the same height at each end and r = 1.0. The intercept of the curve of FV= Z. with the axis of "q" on Graph No. 2 (for which r = 1.0) gives a definite value of q.

Suppose now the tank is tilted through some angle θ, so that $\frac{\ell}{b} \tan\theta = k$ has a definite value, say k = K. Now the liquid surface will remain horizontal in space, but will have a different position relative to the tank, and the liquid surface will stand at different heights above the longitudinal plane at the two ends of the tank. Since the fractional volume in the tank will remain as before, FV = Z, the new value of q will be given by the intercept of FV = Z and k = K. This intercept defines an unique pair of values of q and r, whence we can obtain the unique pair of values of $q = \frac{m}{b}$ and of $rq = \frac{n}{b}$.

We shall call $q = \frac{m}{b}$, and $rq = \frac{n}{b}$ the "fractional heights" at the two ends of the tank.

Let us construct a figure in which the abscissa represents the "fractional length" from the end of the tank and the ordinate represents the "fractional height" above the longitudinal plane of the tank.

The "fractional length" will be defined as the ratio

$$\frac{\text{distance from left-hand end of tank}}{\text{total length of tank}} = \frac{x}{\ell}.$$

If a straight line is drawn to join q and rq, then this line will represent the surface of the liquid in the tank, for the chosen values of the k-factor and of FV. By choosing different values of k for the fixed value of FV, we obtain a "pencil" of lines. Again, by choosing different values of FV (with a series of values of k for each value of FV), we obtain a family of "pencils", in non-dimensional form.

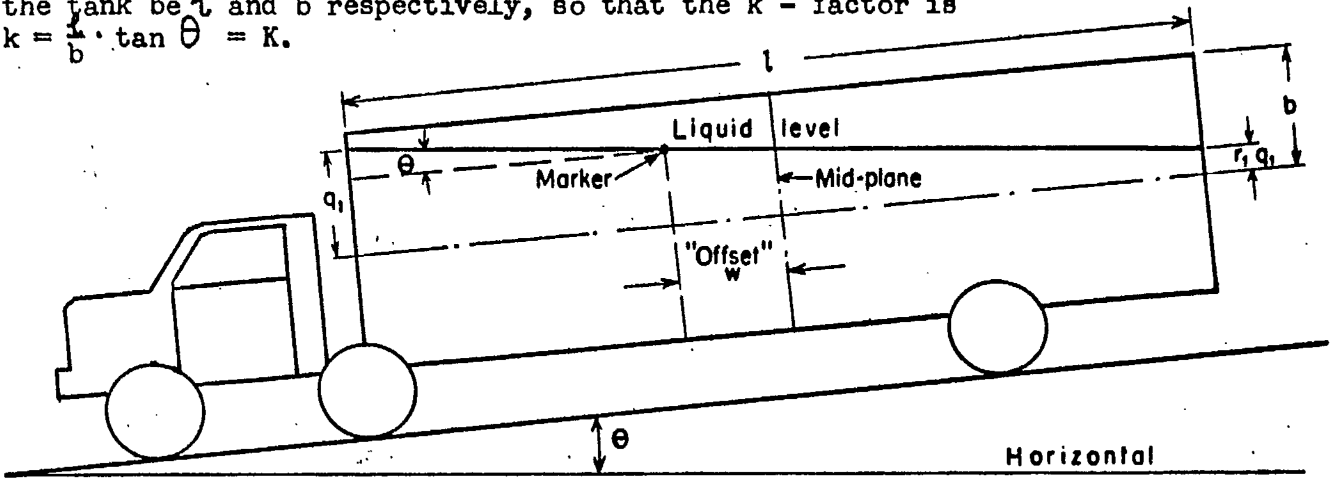
This is shown in Graph No. 3 (which is given for illustrative purposes only) where three "pencils" have been drawn for FV = 0.75, FV = 0.95, and FV = 0.99.

NOTE:

It will be seen from Graph No. 3 that the use of the word "pencil" is not strictly correct, for the lines representing the surface levels for a constant value of FV and various values of k do not all pass exactly through one point. This is particularly apparent in the case of FV = 0.99.

Estimate of "Double Error" Due to Tilt of a Tank-Truck

Suppose a tank-truck is standing upon a sloping roadway during initial calibration, with its front-end down. Let the angle of slope of the roadway be θ, let the length and minor semi-diameter of the tank be ℓ and b respectively, so that the k - factor is $k = \frac{\ell}{b} \cdot \tan\theta = K$.

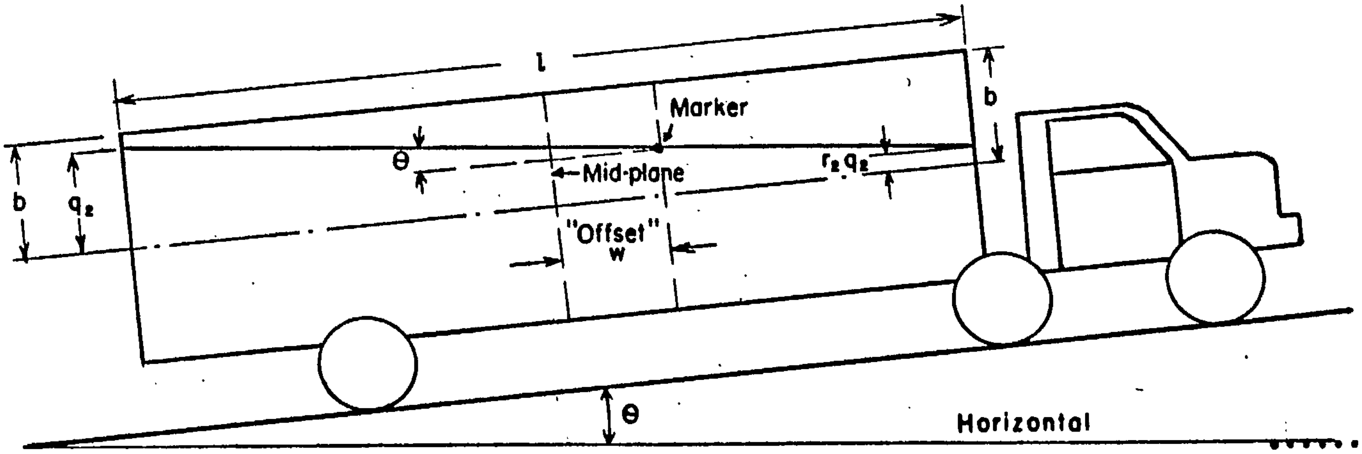


If the tank is filled to a definite fractional volume, $(FV)_1$ the fractional liquid levels will be q_1 and $r_1 q_1$

Suppose a marker which is offset by a distance "w" from the mid-plane of the tank (the mid-plane being the elliptical section which is mid-way between the ends of the tank) is so positioned as to touch the surface of the liquid.

Now if on each subsequent occasion on which the tank-truck is filled to the level indicated by the marker, the truck stands front-end down on a slope having the same angle of slope, θ, then the marker will indicate the same fractional volume $(FV)_1$, as during the initial calibration.

However, if the tank is filled to the level indicated by the marker when the truck is standing front-end up on the same slope (with angle θ), the marker will not indicate the same value of FV as before.



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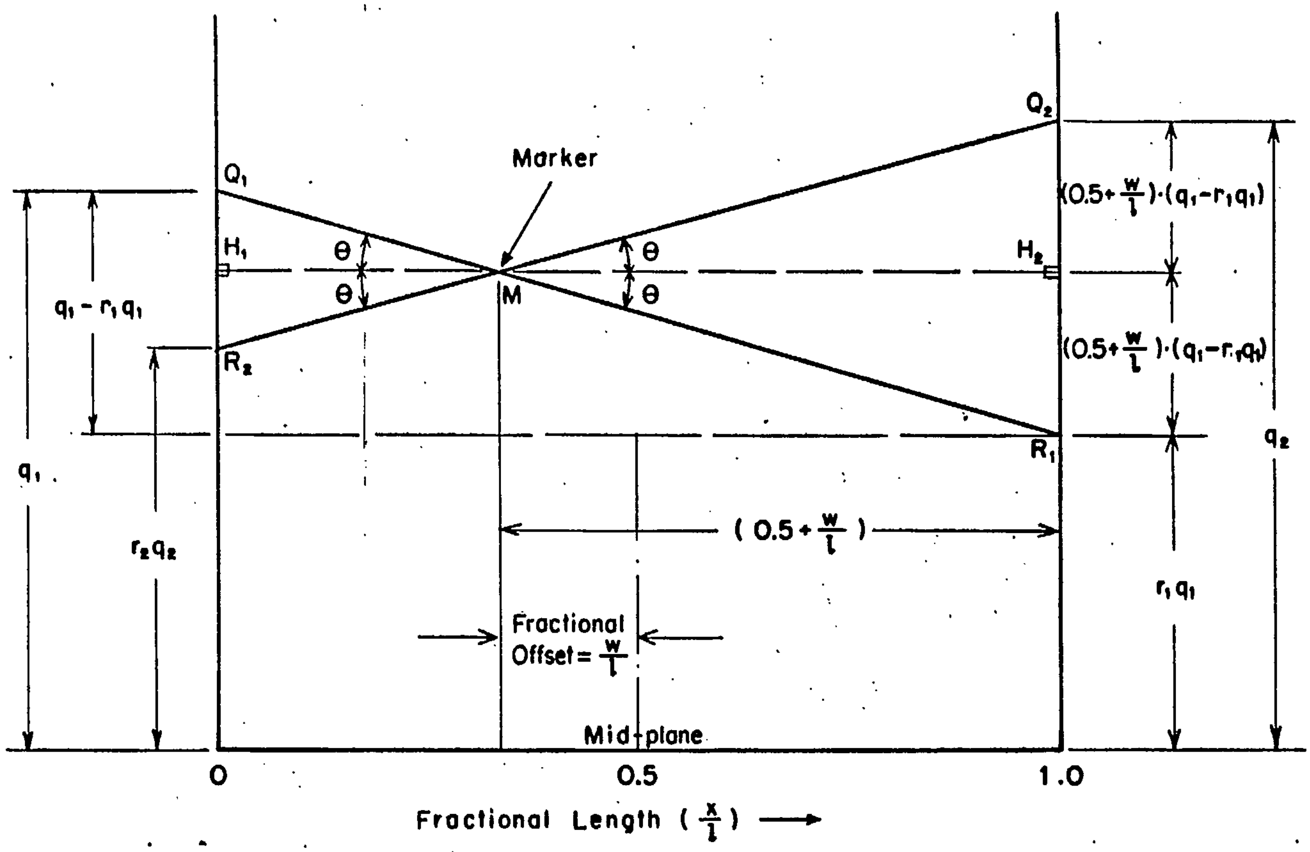
In this case, the fractional heights of the liquid at the two ends will be q_2 and r_2q_2 which will not be the same as q_1 and $r_1 q_1$ (for $w \neq o$).

The level of the liquid relative to the tank for these two cases can be illustrated as below:



In this diagram, the line Q_1MR_1 represents the liquid level (relative to the tank) when the tank is calibrated front-end down. The line Q_2MR_2 represents the liquid level (relative to the tank) when the tank is filled to the same marker setting, on the same slope, front-end up.

Since the angle θ is not altered and the dimensions ℓ and b are unchanged, then the k - factor is k = K as before.

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For given values of $(FV)_1$ and k=K, the initial values of q_1 and r_1 (and hence of r_1q_1) can be read from Graph No 2. Note that line-segment MR_1 lies at an angle θ <u>below</u> "horizontal".

When the tank is loaded front-end up on the same slope, the position of q_2 is determined by drawing a line MQ_2 through the marker-position so that the line-segment MQ_2 lies at an equal angle θ <u>above</u> the "horizontal."

Hence, $H_2Q_2 = H_2R_1$

$$\therefore \; q_2 = r_1q_1 + (1+2\frac{w}{\ell}).(q_1 - r_1q_1)$$

where w/ℓ is the "fractional offset" = $\frac{\text{distance w of marker from mid-plane}}{\text{total length of tank.}}$

(N.B. w/ℓ is considered "positive" if the marker is offset towards the "high end" of the tank and "negative" if offset towards the "low end".)

In the above equation, q_1 and r_1 areobtained from a knowledge of $(FV)_1$ and k, using Graph No. 2 and therefore q_2 can be obtained. Now for constant k, it can be shown graphically that to a close approximation FV is linear with q.

Thus if the values $(FV)_1$, $(FV)_2$, $(FV)'$, $(FV)''$ correspond respectively to q_1, q_2, q' and q'' (all for the same k),

then % "double-error" = $\frac{(FV)_2 - (FV)_1}{(FV)_1} \times 100$

$$= \frac{(FV)' - (FV)_1}{(FV)_1} \times \frac{q_2 - q_1}{q' - q_1} \times 100. \quad \text{(for positive offsets)}$$

$$\text{or} \quad = \frac{(FV)_1 - (FV)''}{(FV)_1} \times \frac{q_1 - q_2}{q_1 - q''} \times 100 \quad \text{(for negative offsets)}$$

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Under Canadian Regulations, the lowest marker cannot be set below the 75% capacity level. Accordingly, calculations were made for a marker setting at $(FV) = 0.75$, with $(FV)' = 0.80$ and $(FV)'' = 0.70$ and corresponding values of q' and q'' being determined from Graph No. 2 for a series of values of k.

The results of these calculations are shown in Graph No. 4, for both positive and negative offsets.

(Note: If instead of elliptical tanks, we had a tank of rectangular section (ie. a right-rectangular prism), then the error due to an offset of the marker from the mid-plane can be shown to be less than one-half of the corresponding error for an elliptical tank. Thus, a calculation for a rectangular tank, with a marker at the 75% level, offset by 10% from the mid-plane and with k = 0.1, would have a "double-error" of only 0.67% instead of over 1.5% for the elliptical tank.)

It will be noted that a horizontal line has been drawn at 0.25% error, which represents the tolerance in Canada for the calibration of tank-trucks with markers.

From Graph No. 4, it will be seen that if the dimensions ℓ and b of the tank and the angle θ of the slope are such that the k-factor is 0.05, then an offset of the marker of 3% could be permitted. Again, if the k-factor is 0.4, then the offset should not exceed 0.3%.

Application of Analysis to Typical Tank-Trucks:

To estimate the magnitude of these "double errors" in actual practice, a quick survey of the dimensions of typical tank-trucks was made.

It was found that in many cases, the tanks were approximately elliptical in cross-section, with a width of 8 ft., so that a = 4 ft. Typically, the height of the tank was 6 ft., so that b = 3 ft. The lengths of tanks varied from 3 ft. (for a single compartment of 700 gallons), to 30 ft. (for a single compartment of 7,000 gallons).

Using the formula, volume = $\pi ab\ell$ cu. ft. = $6.229\,\pi ab\ell$ Canadian gallons, we have the following data for typical tanks:

a = 4 ft.,
b = 3 ft.,
ℓ = 3 ft., for 705 gals.
ℓ = 5 ft., for 1175 gals.
ℓ = 10 ft., for 2350 gals.
and ℓ = 30 ft., for 7050 gals.

....../16

If we assume that a roadway slope of 1 in 20 (or 3°) might be considered by the average inspector or truck operator as being "horizontal" for all practical purposes, then the following k-factors might be expected:

$$\text{For a 700 gal. tank, } k = \frac{\ell}{b} .\tan 3^\circ = 0.05$$

$$\text{For a 7000 gal.tank, } k = \frac{\ell}{b} .\tan 3^\circ = 0.4$$

By referring to Graph No. 4 and limiting the "double error" to the tolerance of 0.25%, we see that for a 700 gal. tank, with k = 0.05, the % offset should not exceed 3%, or approximately 1 inch on a length of 36 inches, in the case of a marker set at the 75% capacity position.

Again, for a 7000 gal. tank, with k = 0.4, the % offset should not exceed 0.3%, or approximately 1 inch on a length of 360 inches, for a 75% marker.

Official Regulations and Specifications:

It is of interest now to refer to the various official regulations governing the use of markers in tank-trucks: -

The present Canadian Weights and Measures Regulations (passed in 1952), Schedule V, Section 7, state that the marker shall be located adjacent to the inspection opening of each compartment, and that such opening "shall be located approximately mid-way between the ends of the compartment". However, the significance of the word "approximately" is nowhere defined.

The recommended practice for the United States, as defined in N.B.S. Handbook 44 (1965 edition) is more specific. Section S.2.4 (p. 109) under "Vehicle Tanks" reads as follows: -

"An indicator shall be positioned as nearly as practicable

(a) midway between the sides of its compartment and
(b) midway between the ends of its compartment.

----- In no case shall an indicator (marker) be offset from a position midway between the ends of the compartment by more than 10% of the compartment length."

By referring to Graph 4, it can be seen that for a marker set at 75% of capacity, the "double error" would be approximately 0.75% on a 700 gal. compartment and approximately 6.0% on a 7000 gal. compartment, if the 10% offset were permitted.

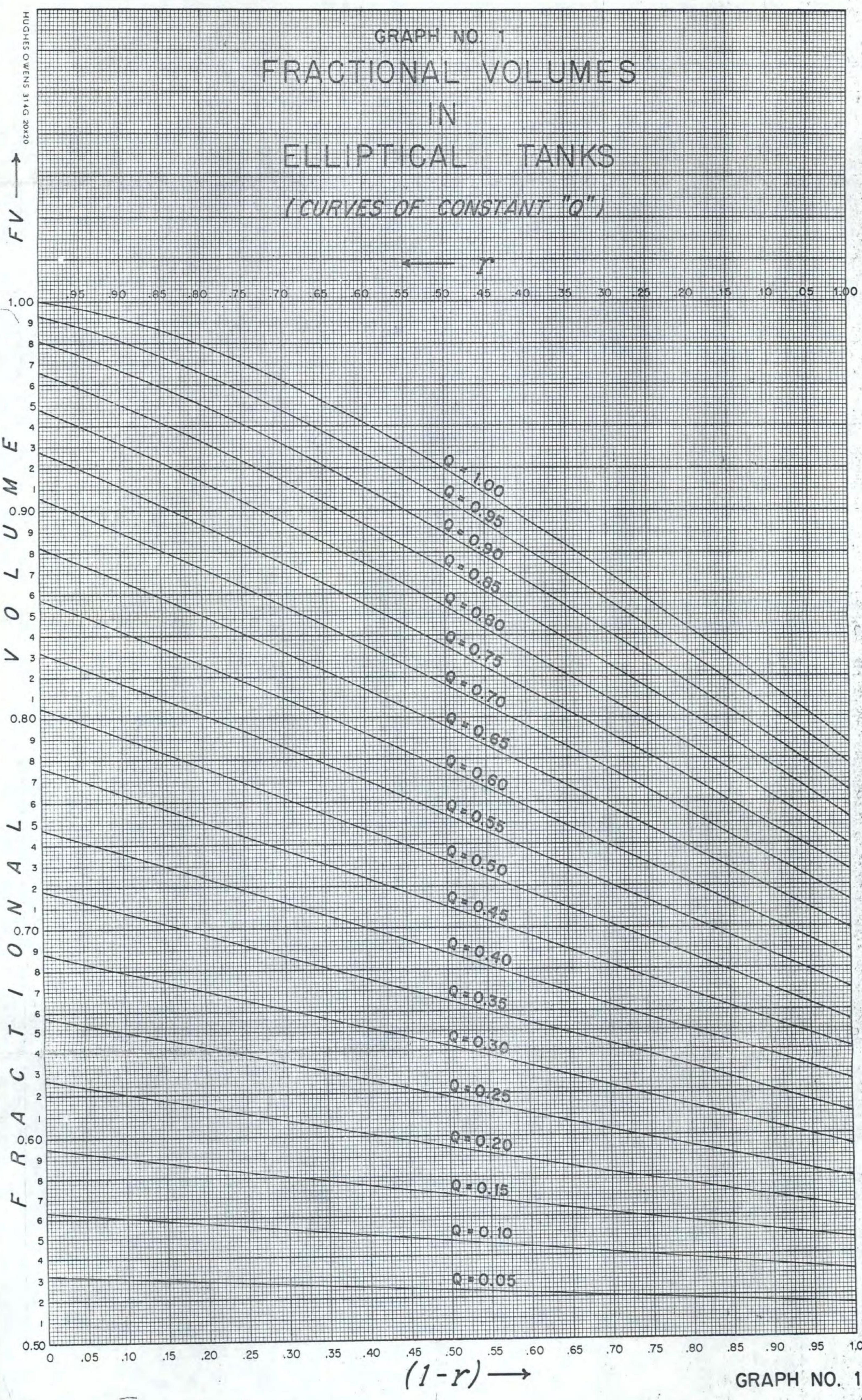
....../17

Now it is possible to exercise some control over conditions prevailing at the time a tank-truck is calibrated, in so far as permissible slope of road surface is concerned, but no such control is possible over the slope of the road surface at loading racks in daily service. Again, with a multi-compartment truck, it is possible that the slope of the truck during service loading will differ from that which existed during calibration, due to the fact that the order of loading of compartments may not always be the same (i.e. the front compartment may be the first one loaded on one occasion and the last one loaded on another). This may cause the front-end to be low at one time and the back-end to be low at another time. Further, the springs of the vehicle may gradually weaken in service and thus alter the slope of the tank relative to the ground.

Recommendations:

For the foregoing reasons it would seem advisable to tighten the Canadian Regulations so that markers may not be offset from the mid-plane by more than a small amount - and it is suggested that this be limited to an offset (w) of ± 1 inch.

It will be noted in the foregoing analysis of the problem that it has been implicitly assumed that the marker is positioned midway between the sides of the tank (as is the recommended practice in the United States). It is felt that this requirement should be rigorously enforced, since it should not pose any particular problem in service. The case with respect to tight control of offset may not be as easy to enforce, but if the claim of ± 0.25% error on initial calibration is to have any real significance, then it would appear necessary to control the permissible offset much more closely than is done at the present time.


HUGHES OWENS 314G 20x20
GRAPH NO. 1
FRACTIONAL VOLUMES
IN
ELLIPTICAL TANKS
(CURVES OF CONSTANT "Q")
← r
FV →
FRACTIONAL VOLUME
Q = 1.00
Q = 0.95
Q = 0.90
Q = 0.85
Q = 0.80
Q = 0.75
Q = 0.70
Q = 0.65
Q = 0.60
Q = 0.55
Q = 0.50
Q = 0.45
Q = 0.40
Q = 0.35
Q = 0.30
Q = 0.25
Q = 0.20
Q = 0.15
Q = 0.10
Q = 0.05
(1-r) →

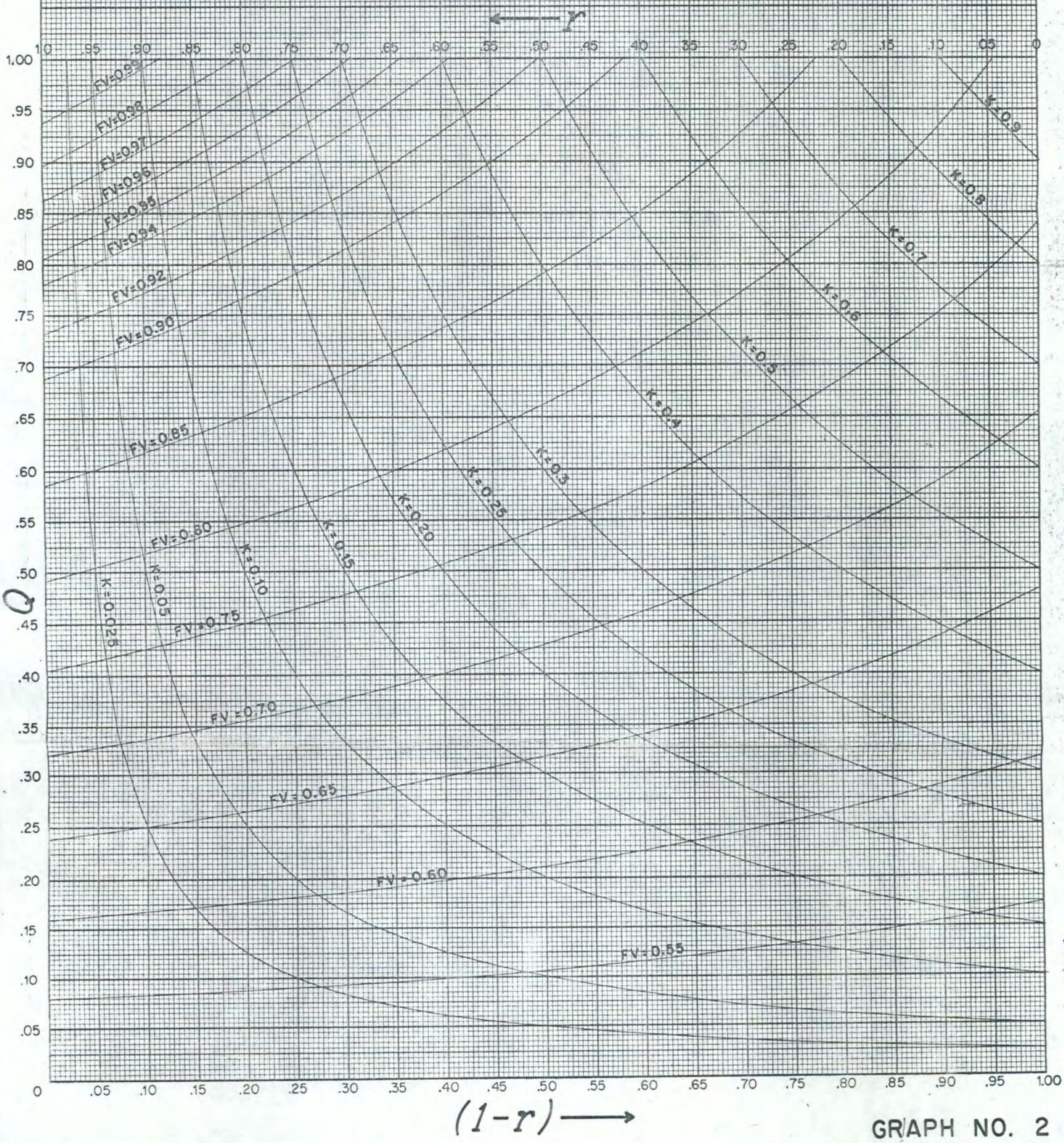
GRAPH NO. 1

HUGHES OWENS 314G 20x20

GRAPH NO. 2

FRACTIONAL VOLUMES IN ELLIPTICAL TANKS

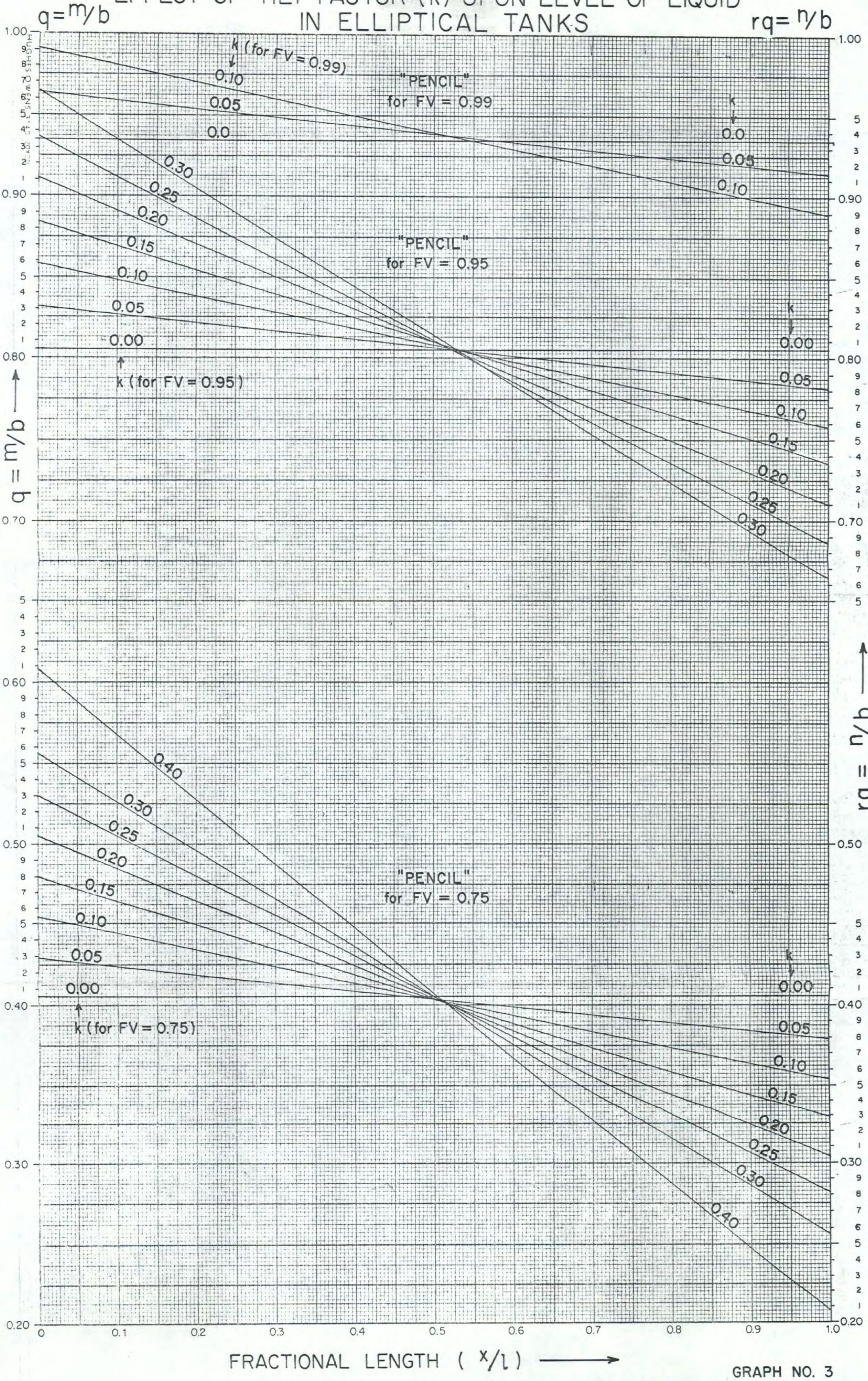
(CURVES OF CONSTANT "FV" AND CONSTANT "K")



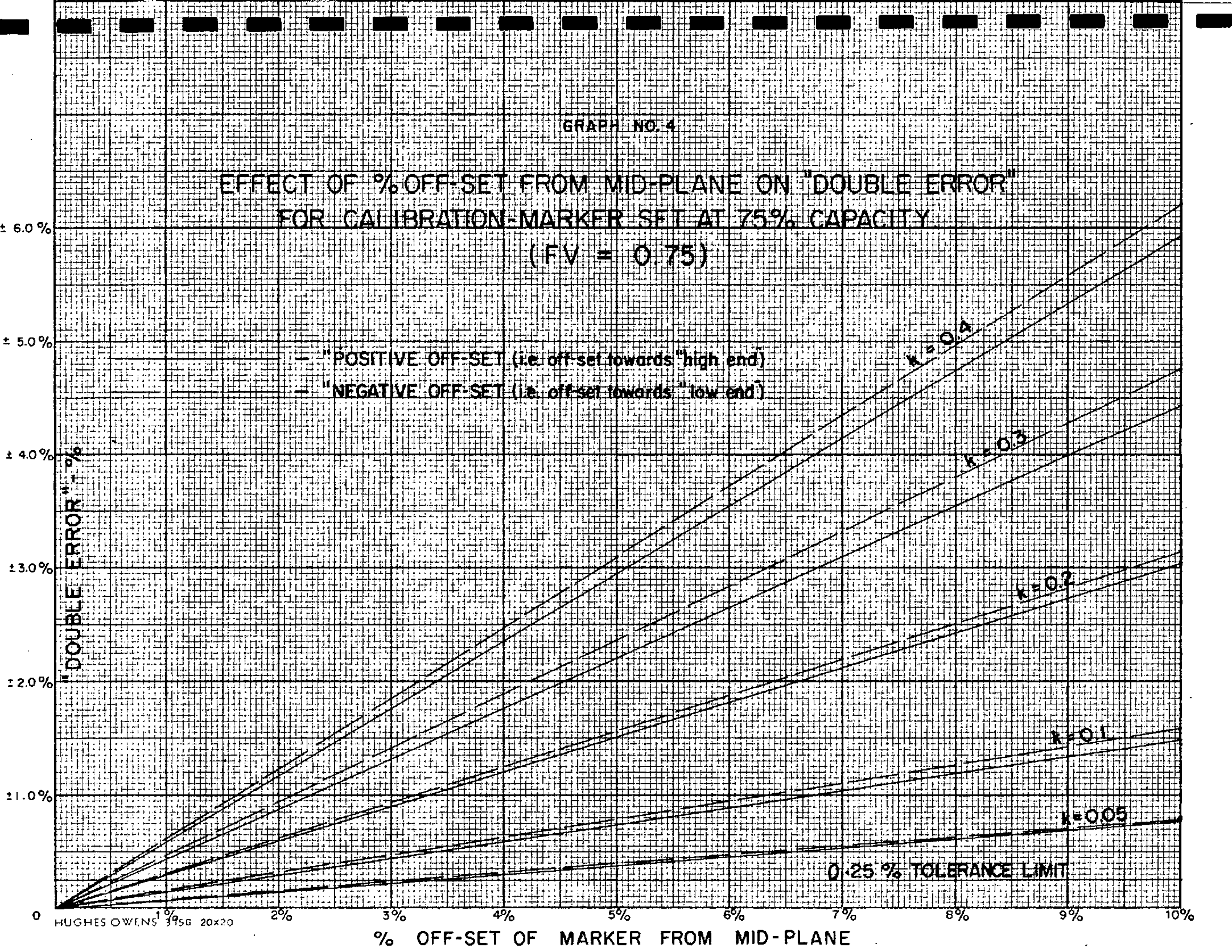
GRAPH NO. 2

GRAPH NO. 3

EFFECT OF TILT FACTOR (k) UPON LEVEL OF LIQUID IN ELLIPTICAL TANKS



GRAPH NO. 3



GRAPH NO. 4
EFFECT OF % OFF-SET FROM MID-PLANE ON "DOUBLE ERROR"
FOR CALIBRATION-MARKER SET AT 75% CAPACITY
(FV = 0.75)
"POSITIVE OFF-SET (i.e. off-set towards "high end")
"NEGATIVE OFF-SET (i.e. off-set towards "low end")
k = 0.4
k = 0.3
k = 0.2
k = 0.1
k = 0.05
0.25 % TOLERANCE LIMIT
"DOUBLE ERROR" - %
± 6.0 %
± 5.0 %
± 4.0 %
± 3.0 %
± 2.0 %
± 1.0 %
0
1%
2%
3%
4%
5%
6%
7%
8%
9%
10%
% OFF-SET OF MARKER FROM MID-PLANE
HUGHES OWENS 315G 20x20

TABLE OF FRACTIONAL VOLUMES

FV versus Q
for various values of R

Top line in each entry for Q

R=1.0	.95	.90	.85	.80	.75	.70	.65	.60	.55	.50

Bottom line in each entry for Q

R=.45	.40	.35	.30	.25	.20	.15	.10	.05	0	

Q											
Q = 1.0	1.0000	.9973	.9924	.9862	.9789	.9708	.9618	.9521	.9419	.9311	.9198
	.9080	.8959	.8833	.8705	.8573	.8438	.8301	.8162	.8021	.7877	
Q = .95	.9933	.9879	.9814	.9741	.9659	.9571	.9477	.9378	.9273	.9165	.9052
	.8936	.8816	.8693	.8567	.8439	.8308	.8176	.8041	.7905	.7767	
Q = .90	.9813	.9746	.9672	.9591	.9505	.9413	.9317	.9216	.9112	.9004	.8893
	.8778	.8661	.8542	.8420	.8296	.8170	.8042	.7913	.7782	.7650	
Q = .85	.9659	.9585	.9505	.9420	.9331	.9238	.9141	.9041	.8938	.8832	.8723
	.8611	.8498	.8382	.8264	.8145	.8024	.7902	.7778	.7653	.7527	
Q = .80	.9479	.9400	.9318	.9231	.9141	.9048	.8952	.8854	.8753	.8649	.8543
	.8436	.8326	.8215	.8102	.7988	.7872	.7756	.7638	.7519	.7399	
Q = .75	.9278	.9197	.9114	.9027	.8938	.8846	.8752	.8656	.8558	.8458	.8356
	.8252	.8148	.8041	.7934	.7825	.7715	.7604	.7493	.7380	.7267	
= .70	.9059	.8978	.8895	.8810	.8722	.8633	.8541	.8449	.8354	.8258	.8161
	.8062	.7963	.7862	.7760	.7657	.7553	.7449	.7344	.7238	.7131	

TABLE OF FRACTIONAL VOLUMES
FV versus Q
for various values of R

Top line in each entry for Q

R=1.0	.95	.90	.85	.80	.75	.70	.65	.60	.55	.50

Bottom line in each entry for Q

R=.45	.40	.35	.30	.25	.20	.15	.10	.05	0	

Q = .65										
.8824	.8745	.8663	.8580	.8496	.8409	.8322	.8233	.8143	.8052	.7960
.7866	.7772	.7677	.7581	.7484	.7387	.7289	.7191	.7092	.6992	
Q = .60										
.8576	.8499	.8420	.8340	.8260	.8177	.8094	.8010	.7925	.7839	.7752
.7665	.7576	.7487	.7398	.7308	.7217	.7126	.7034	.6942	.6850	
Q = .55										
.8315	.8242	.8167	.8092	.8015	.7938	.7860	.7781	.7701	.7621	.7540
.7458	.7376	.7294	.7211	.7127	.7043	.6959	.6875	.6790	.6705	
Q = .50										
.8044	.7975	.7905	.7834	.7763	.7691	.7619	.7545	.7472	.7398	.7323
.7248	.7172	.7096	.7020	.6944	.6867	.6790	.6712	.6635	.6557	
Q = .45										
.7764	.7700	.7635	.7570	.7505	.7438	.7372	.7305	.7237	.7170	.7102
.7033	.6965	.6896	.6827	.6757	.6688	.6618	.6548	.6478	.6407	
Q = .40										
.7476	.7418	.7359	.7300	.7240	.7180	.7120	.7060	.6999	.6938	.6877
.6816	.6754	.6692	.6630	.6568	.6506	.6443	.6381	.6318	.6255	
Q = .35										
.7181	.7129	.7076	.7024	.6971	.6918	.6864	.6811	.6757	.6703	.6649
.6595	.6541	.6486	.6431	.6377	.6322	.6267	.6212	.6157	.6102	

TABLE OF FRACTIONAL VOLUMES
FV versus Q
for various values of R

Top line in each entry for Q

R=1.0	.95	.90	.85	.80	.75	.70	.65	.60	.55	.50

Bottom line in each entry for Q

R=.45	.40	.35	.30	.25	.20	.15	.10	.05	0	

Q = .30											
	.6880	.6835	.6789	.6743	.6697	.6651	.6605	.6558	.6512	.6465	.6418
	.6372	.6325	.6278	.6231	.6184	.6136	.6089	.6042	.5995	.5947	
Q = .25											
	.6574	.6536	.6497	.6458	.6420	.6381	.6342	.6303	.6264	.6225	.6185
	.6146	.6107	.6067	.6028	.5989	.5949	.5910	.5870	.5831	.5791	
Q = .20											
	.6264	.6233	.6202	.6170	.6139	.6108	.6076	.6045	.6013	.5982	.5950
	.5919	.5887	.5856	.5824	.5792	.5761	.5729	.5697	.5666	.5634	
Q = .15											
	.5951	.5927	.5904	.5880	.5856	.5833	.5809	.5785	.5761	.5738	.5714
	.5690	.5666	.5643	.5619	.5595	.5571	.5548	.5524	.5500	.5476	
Q = .10											
	.5635	.5619	.5603	.5588	.5572	.5556	.5540	.5524	.5508	.5492	.5476
	.5461	.5445	.5429	.5413	.5397	.5381	.5365	.5349	.5333	.5318	
Q = .05											
	.5318	.5310	.5302	.5294	.5286	.5278	.5270	.5262	.5254	.5246	.5238
	.5230	.5222	.5214	.5206	.5198	.5190	.5182	.5175	.5167	.5159	

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C     STANDARDS BRANCH PROGRAM.9012-01-03
C     VOLUME OF LIQUID CONTAINED IN ELLIPTICAL TANKS
C         FILLED, PARTIALLY FILLED AND INCLINED (AT LEAST HALF FILLED)
C     TABULATED DATA GIVE  FRACTIONAL CONTENTS  (FILLED EQALS UNITY)
C     WHERE SUCCESSIVE ROWS GIVE  FRACTION FOR SUCCESSIVE
C     VALUES OF Q = M/B
C     AND SUCCESSIVE COLUMNS (LEFT TO RIGHT) GIVE FRACTION FOR
C     SUCCESSIVE VALUES OF R = N/M
      DIMENSION  FV(21)
      Q1=1.0
      Q=Q1
      R1=1.0
      PI=3.141593
      DO 8 I=1,21
      R=R1
      DO 3 J=1,21
      S=SQRTF(1.-Q**2)
      T=SQRTF(1.-(Q**2)*(R**2))
      V=R*ATANF(T/(Q*R))
      W=ATANF(S/Q)
      IF(R-1.) 2,1,10
    1 FV(J) =1.+(Q*S)/PI-(W/PI)
      GO TO 3
    2 FV(J) =1.-((T-S)-(T**3-S**3)/3.-Q*(V-W))/(PI*(Q*(1.-R)))
    3 R=R-.05
    4 PRINT 6,FV
    6 FORMAT (11F7.4 ///10F7.4////)
    8 Q=Q-.05
    9 PAUSE
   10 STOP
      END
```

INDUSTRY CANADA/INDUSTRIE CANADA
115743

QUEEN QC 104 .A54 1968
Anderson, G. E.
Volume of liquid contained i

DATE DUE
DATE DE RETOUR

CARR McLEAN 38-296