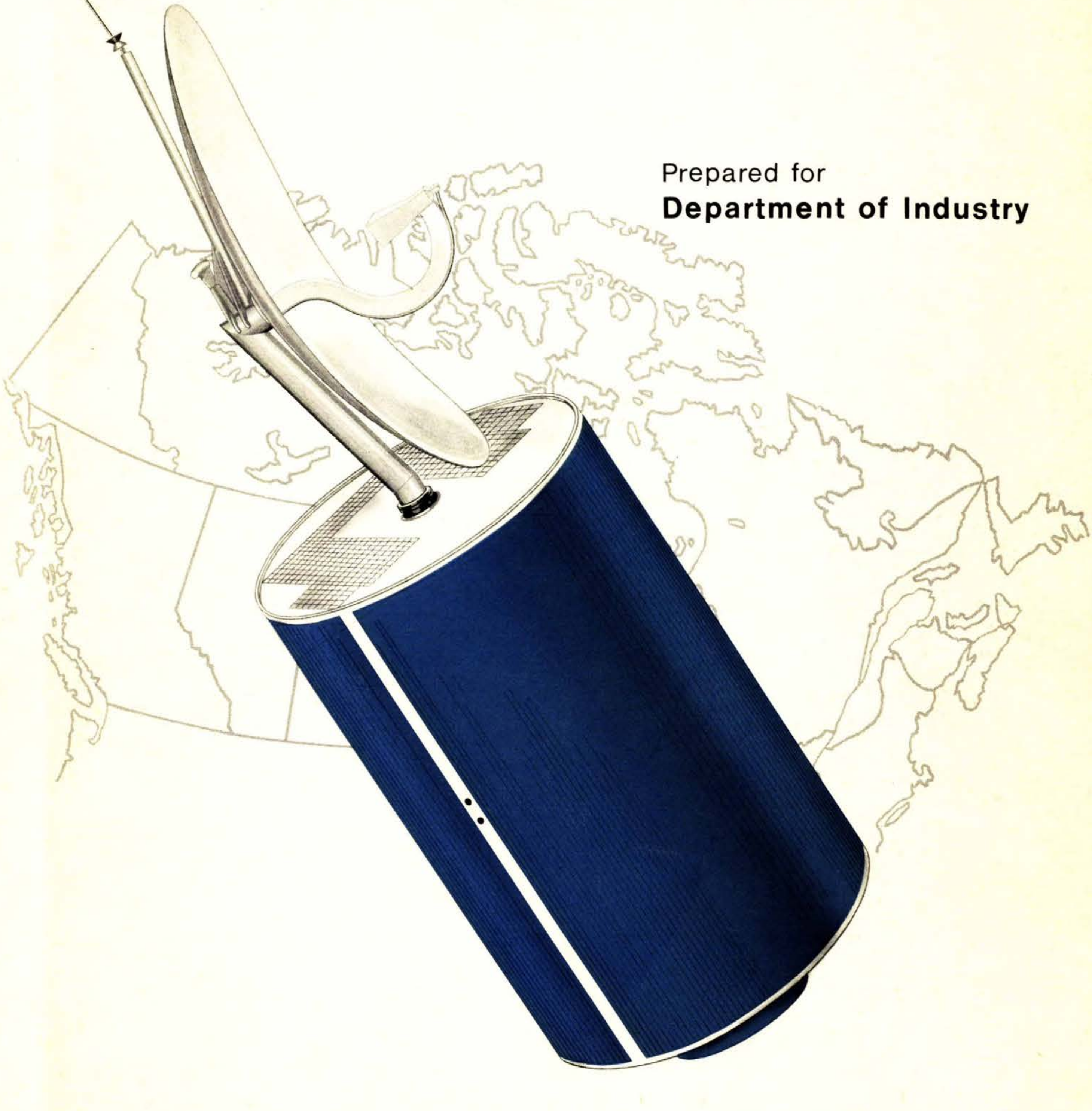
Study Program for

Canadian Domestic Communications Satellite



Prepared for
Department of Industry

Final Report
Volume Three - Technical Appendices

Space Systems

Copy No. 46

STUDY PROGRAM

for the

DESIGN, DEVELOPMENT AND SUPPLY

of a

DOMESTIC SATELLITE COMMUNICATIONS SYSTEM

FINAL REPORT
VOLUME 3, TECHNICAL APPENDICES

Prepared for

DEPARTMENT OF INDUSTRY

by

RCA LIMITED, Space Systems
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PROPOSITION No. 2220-I

NOVEMBER 25th, 1968

PREFACE

This report is submitted by RCA Limited to the Department of Industry in compliance with Section 4.2 of the Statement of Work forming part of D.O.I. Contract, File No. IRA. 9122-03-4.

The report is in six volumes, namely:

Volume 1	Design Considerations
Volume 2(a)	Spacecraft Design - Electrical
Volume 2(b)	Spacecraft Design - Mechanical
Volume 3	Technical Appendices
Volume 4	Program Plan
Volume 5	Program Costs

The information contained in the report is supplied to Her Majesty for use solely in connection with the design, development, manufacture, operation, repair, maintenance and testing of a Canadian Domestic Satellite Communication System.

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VOL. 1 APPENDICE

APPENDICES - Vol. 1

1. ANTENNA GAIN REDUCTION DUE TO SATELLITE DRIFT

The satellite longitude drift and the orbit inclination can be taken as ± 0.1°. The apparent antenna pointing error due to satellite drift can be taken as 1.1 x drift for St. Johns, Newfoundland, located near the edge of the beam with the satellite located 105° west longitude. (If the satellite is located at the same longitude for a station like Winnipeg, the pointing error is 1.12 x drift).

Resultant drift for longitude and inclination,

$$d = \sqrt{l^2 + i^2} \qquad \text{-----------(1)}$$

where $l = \pm$ longitude drift

$i = \pm$ inclination drift

Therefore, pointing error, $\phi = 1.1 \sqrt{l^2 + i^2}$ -----------(2)

Since the shape of the earth station antenna can be taken as parabolic up to 3 db point, the gain reduction can be taken as:

$$R = \frac{3\phi^2}{\phi^2_{3\ \text{db}}} \ \text{db} \qquad \text{-----------(3)}$$

where ϕ = angle off beam axis or pointing error

$\phi_{3\ \text{db}}$ = one half of full 3 db beamwidth

$$R = \frac{3}{\phi^2_{3\ \text{db}}} \times \left(1.1 \sqrt{l^2 + i^2}\right)^2$$

$$= \frac{3.63}{\phi^2_{3\ \text{db}}} (l^2 + i^2) \qquad \text{-----------(4)}$$

If 1 is taken as ± 0.1° and Ø is taken as ± 0.285° for a 30-ft. dia. antenna,

$$R = 44.75 \left(0.01 + i^2\right)$$

$$= 0.4475 + 44.74\ i^2 \qquad \text{----------(5)}$$

$$R = 0.895 \text{ db for } i = \pm 0.1^{\circ}$$

R is plotted for a 25, 30 and 40 ft. diameter antenna for a fixed longitude drift of ± 0.1° in Figure A-1. If the longitude drift is decreased to ± 0.05°, the gain reduction is decreased to 0.56 db for a 30 ft. dia. antenna.

Increasing G/T by increasing antenna diameter has the side effect of narrowing the beam and hence causing a greater loss for a fixed angle off-axis. The following calculations are for a 5° elevation angle and 4.20 GHz. A noise temperature of 150°K is used. The noise temperature will vary somewhat with size, however the greates contribution does not depend on antenna size.

$$T = 150^{\circ}\text{K at } 5^{\circ} \text{ elevation, } 4 \text{ GHz}$$

$$G = 19.6 + 20 \log_{10} D \qquad \text{----------(6)}$$

$$G/T = 20 \log_{10} D - 2.16 \text{ db} \qquad \text{----------(7)}$$

Losses due to fixed angle off-axis from (3)

$$R = \frac{3\phi^2}{\phi^2_{3\ db}}$$

The 3 db beamwidth $\phi_{3\ db}$ can be taken as $\phi_{3\ db} = 8.2/D$ ----------(8)

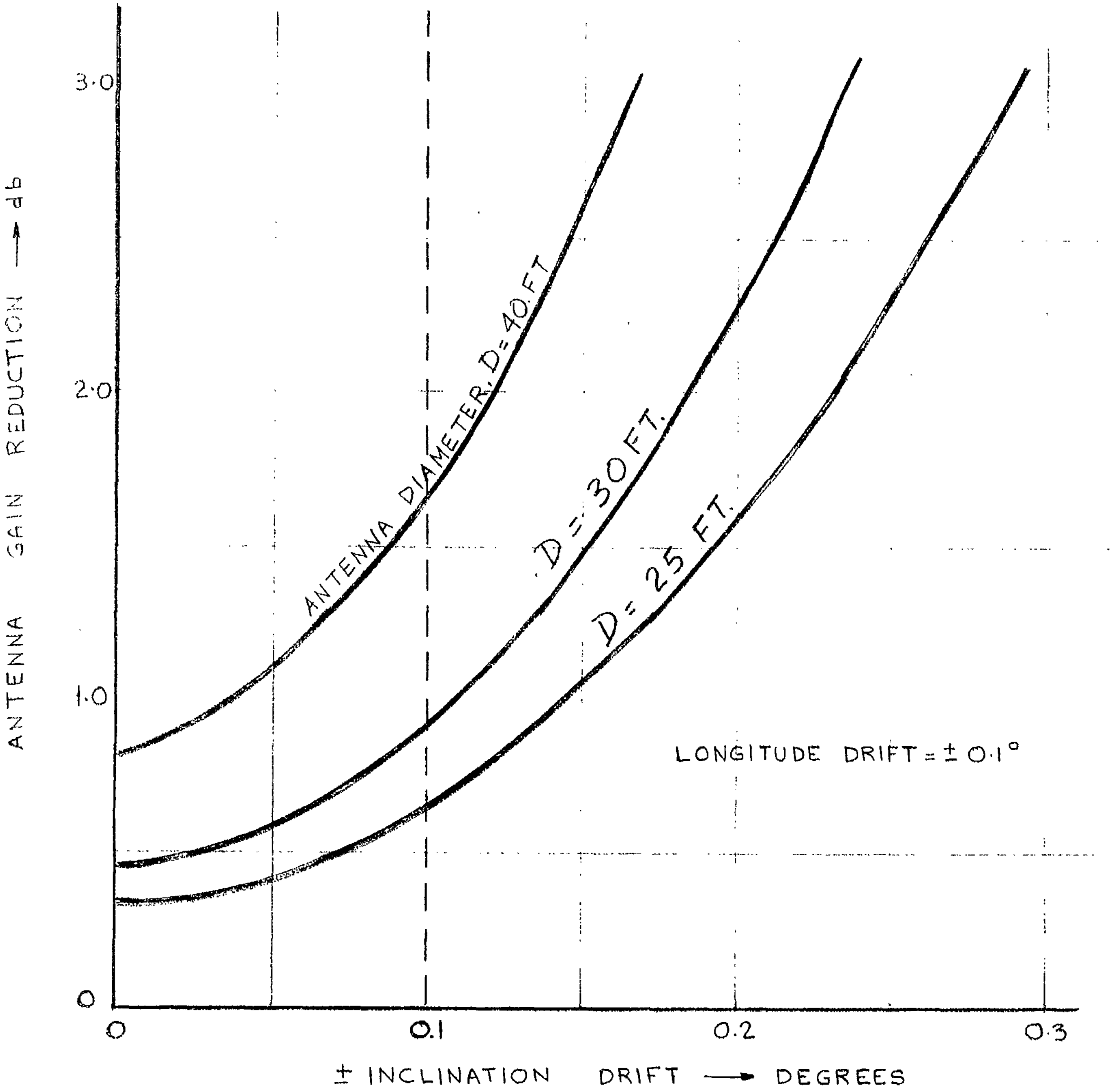
If ϕ is taken as 0.156° (0.1° station keeping, 0.1° inclination, antenna at 50° latitude),

then $R = 0.00109\ D^2$.

Therefore, $G/T = 20 \log_{10} D - 2.16 - 0.00109\ D^2$ -----------(9)

This equation is plotted in figure A-2, and it can be seen that maximum G/T is reached at $D \simeq 66'$. This is not precisely true, in that the above relationships of db reduction due to off-axis angles do not follow a square law after the 3 db point. This means that the optimum size is larger than indicated.

FIG. A-1 THE EFFECT OF SATELLITE INCLINATION DRIFT ON LOSS IN EARTH STATION ANTENNA GAIN.



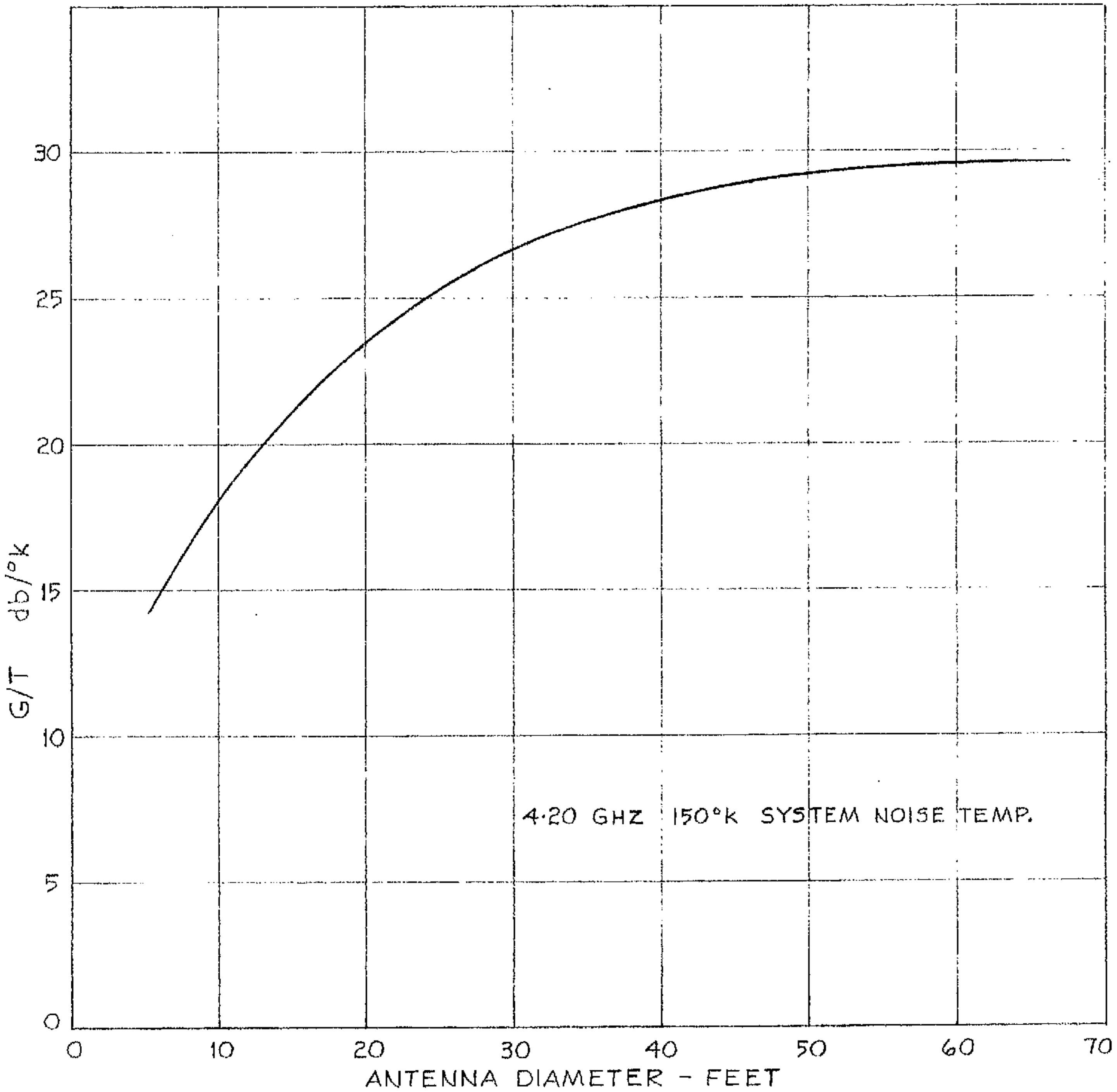


FIG. A-2

MINIMUM EFFECTIVE G/T FOR NON-TRACKING ANTENNA VS. SIZE.

SATELLITE STATION KEEPING ± 0·1 DEGREE IN DRIFT AND INCLINATION.

2. POINTING ACCURACY OF 30 FT. DIA. ANTENNA

A station with 3 30-ft. diameter antenna can be considerably simplified if it can be shown that a servo-controlled antenna is not required

Discussions with antenna manufacturers indicate that the pointing error of a 30' antenna would be approximately 0.02 degs (.34 mr) due to dead load and wind load. Given that the pointing error in winds of 45 mph gusting to 60 mph for a 95' antenna is 0.7 degs, and that the wind torque is approximately proportional to the third power of the diameter, the figure of 0.02 degs appears to be conservative although it is not known what cost/error trade-offs could be made.

To the antenna pointing error must be added the pointing error due to non-standard propogations which is estimated as ± 0.4 mr, for a 5° elevation angle. This will be reduced considerably for greater elevation angles, hence ± .4 mr is a conservative estimate.

Antenna pointing error	$= .02^{\circ}$	=	.34 mr
Non-standard propogation		=	.4 mr
TOTAL		=	.526 mr

Non standard propogation only affects elevation, azimuth error will be .34 mr.

Total pointing error = .626 mr

= $.036^{\circ}$

This peak error (3σ) will result in a loss in antenna gain of 0.048 db.

No other error sources are included, as, if a non-steerable antenna is chosen, the alignment would be done by peaking the received signal. It might be necessary to provide an angular read-out, but this would read incremental rather than absolute angle to facilitate finding the centre of the received beam. Hence, alignment errors do not contribute to pointing error and the servo and encoder errors are eliminated.

Total estimated gain reduction due to pointing is then 0.048 db.

One firm indicated that the RMS surface accuracy under winds of 45 mph gusting to 60 mph would be approximately .060 ins.

Ruze's formula gives a gain reduction of .32 db due to this at 4.2 GHz.

A gain reduction at a 3 sigma pointing error coincident with maximum satellite drift of 0.048 db is predicted. To this must be added further 0.32 db due to surface inaccuracy, which results in a total loss of 0.368 db for about 0.03%. Because of the large contribution of refraction to the pointing error there seems to be little point in reducing antenna pointing errors below those predicted. Cost of a suitable antenna appears to be apprximately $50K in the presently envisaged quantities.

3. VARIATION OF PATH LOSS WITH ELEVATION ANGLE

Two components that contribute to total path loss are free space attenuation and atmospheric attenuation, both of which are a function of elevation angle.

The path loss can be calculated from the path length. From the geometry of Figure A-3, the following equation can be written for path length between a point on earth and the satellite.

$$R' = \sqrt{r^2 \sin^2\theta + R^2 + 2rR} - r \text{ Sin } \theta \qquad \text{----------(1)}$$

where the symbols are defined in figure 1. The path loss due to path length can be calculated from:

$$Ldb = 36.58 + 20 \log f + 20 \log R' \qquad \text{----------(2)}$$

where f = frequency in Megahertz

R = path length in statute miles

Atmospheric attenuation depends on the atmospheric model used. For the purposes of this study, the following equation will be sufficiently accurate:

$$A \text{ db} = \frac{0.038}{\sin \emptyset} \quad \text{valid for } \emptyset > 5^{\circ}. \qquad \text{----------(3)}$$

where $\emptyset$ is the elevation angle.

From spherical trigonometry the following relation holds on the surface of a sphere.

$$\text{Cos } \theta = \text{Cos } x \text{ Cos } y \qquad \text{----------(4)}$$

where x = site latitude

y = difference between site longitude and satellite subpoint longitude

θ = great circle distance between site and satellite subpoint.

From figure A-3, the relationship between great circle distance and elevation is:

$$\frac{r}{r + R} = \frac{\sin (90 - (\emptyset + \theta))}{\sin (90 + \emptyset)}$$

This is a tedious equation to solve for θ in terms of $\emptyset$. An approximation, which is accurate for sites in Canada is therefore made of the form $Y = M - k\theta$. The best fit values are $M = 85^{\circ}$, $k = 1.05$.

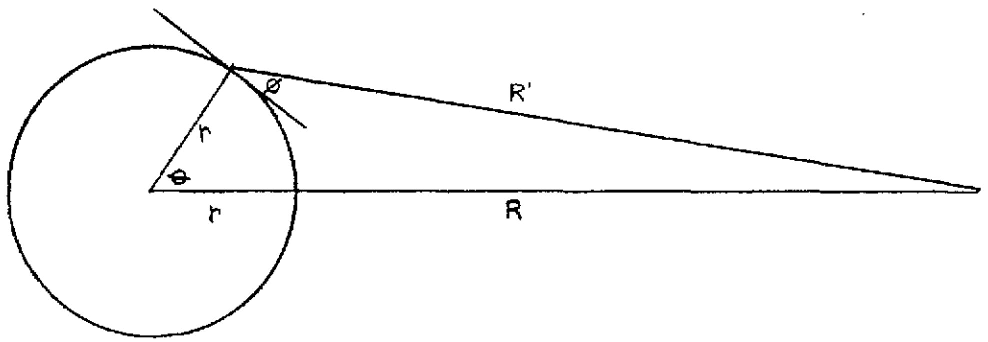
Therefore:

$$\emptyset = 85 - 1.05\theta \qquad \text{----------(5)}$$

Using (4) and (5), elevation angles can be accurately computed for sites in Canada.

The results are shown in figure A-4.

FIG. A-3 - CROSS SECTION THROUGH SATELLITE SUBPOINT MERIDIAN



r = EARTH DIAMETER

R = SATELLITE ALTITUDE

R' = SLANT RANGE

ϕ = ELEVATION ANGLE

θ = LATITUDE (GREAT CIRCLE DISTANCE)

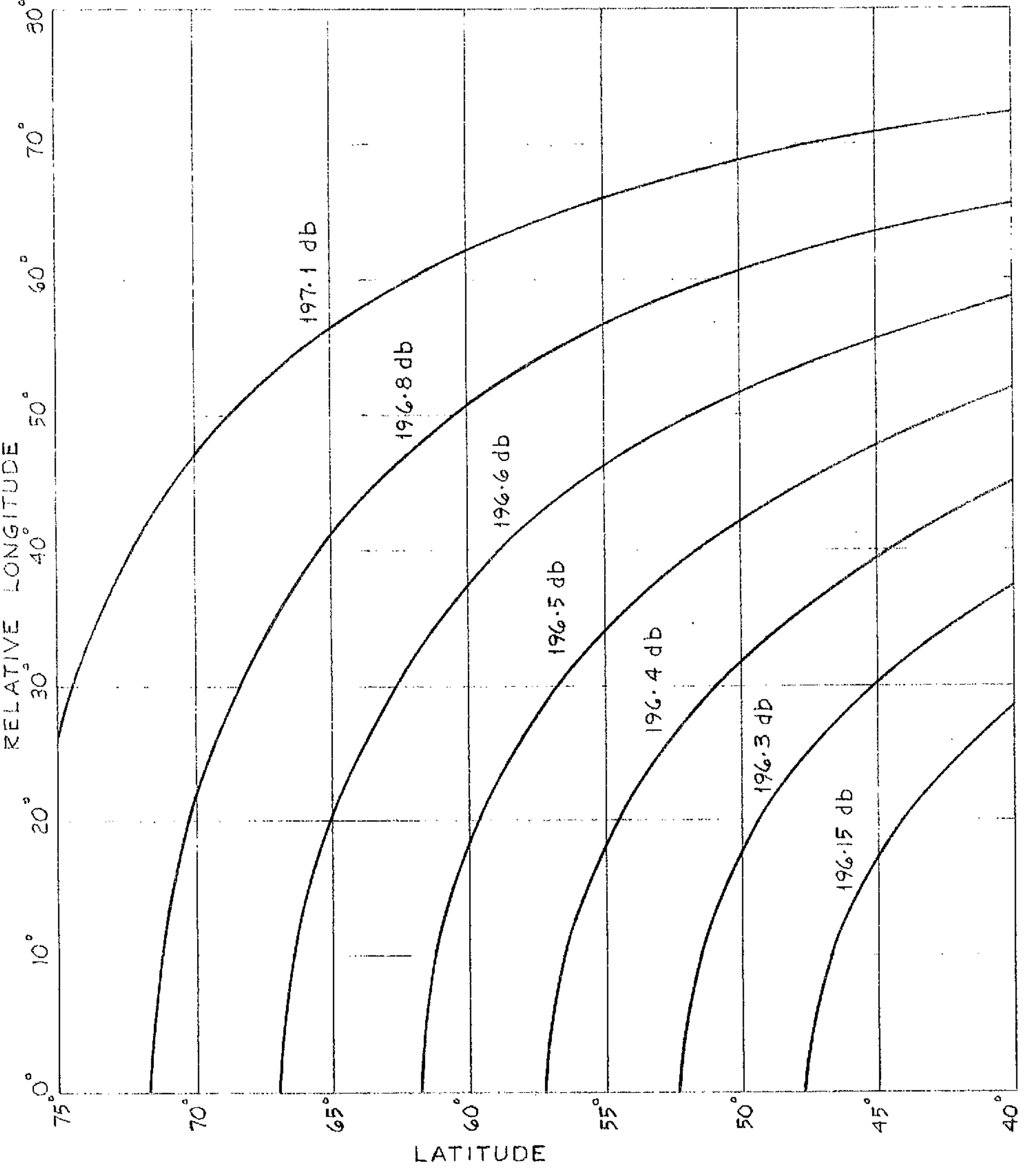


FIG. A-4

TOTAL PATH LOSS IN db VS. SITE LOCATION

4.0 GC.

4. FADING DUE TO WEATHER

The primary fade contribution is due to rainfall. Wind fade with these relatively small diameter antennas is negligible and fade due to snowfall and ice accumulation is expected to be small. Rainfall data for Canada shows that the region of maximum annual rainfall occurs along the B.C. coastline and maximum short term rainfall intensity (5 mins. or 1 hour) occurs in Southern Ontario. Central Alberta and the Atlantic coastline also have relatively high short term rainfall rates. For southern Canadian stations, we will use rainfall distribution data available for Mill Village, N.S., as typical and we shall also calculate possible maximum instantaneous attenuations for southern Ontario for a 5-minute interval.

Assume the following:-

1. Rainfield width vs. rainfall rate follows Hannaford's(1) formula, i.e., rainfall width, km = 41.4 - 23.5 log R_r, where R_r = rainfall rate - mm/hr.

   Further assume minimum depth = 10 km.

2. Rainfall attenuation = .00125 db/km/mm/hr at 4.2 GHz.

Using the preceding data and rainfall distribution curves given in the COTC specification(2), we can plot the rainfield attenuation curves. This is shown in figure A-5a.

(1) D.A. Hannaford, "Meteorological Factors in Space Communications Systems", International Conference on Satellite Communications, London, U.K., November 1962.

(2) Specification for a Communication Satellite Earth Station for Commercial Trans Atlantic Communications by COTC, October 1966.

When a rainfield exists in front of an antenna the effect on system noise temperature is to reduce the atmospheric and galactic noise temperature, T_A, by the attenuation factor of the rainfield, and to introduce an additional noise temperature component which is approximately equal to placing an equivalent attenuation, at 290°K, in front of the receiver.

Thus, under rainfall, the actual system noise temperature becomes:

$$T_S = \frac{T_A}{L} + \frac{L - 1}{L} \cdot 290 + T_G + T_F + T_{RX}$$

where T_A = atmospheric and galactic noise temp.

T_G = ground contribution via side lobes

T_F = feed and waveguide losses

T_{RX} = paramp noise temperature

L = rainfield attenuation

For 4.2 GHz and 5^o elevation angle, we will assume the following values:

$T_A = 24^oK$

$T_G = 18^oK$

$T_F = 8^oK$

$T_{RX} = 100^oK$

Thus in clear weather, $T_S = 150^oK$.

It is now necessary to estimate the amount of interference power due to up-link and satellite noise. Communications system performance is calculated on the basis that up-link thermal noise degrades the overall receive C/T by 1 db and adjacent satellite interference and terrestrial interference will effectively degrade the receive C/T by a further 1 db (0.5 db each source). We can thus say that the effective overall system noise temperature in clear weather is 150°K plus 2 db = 235°K.

During rainfall the down-link signal plus the up-link noise and adjacent satellite noise will all be attenuated by the rainfield. In the worst case, however, the terrestrial interference will not be attenuated. Thus the effective system noise temperature becomes:

$$T_S' = \frac{T_A + T_{up} + T_{AdjSat}}{L} + \frac{L - 1}{L} \cdot 290 + T_G + T_F + T_{RX} + T_{Terr}$$

As a matter of convenience the effective system noise temperature T_S' can be further modified to compensate for signal attenuation through the rainfield.

Now $T_{overall} = L \cdot T_S'$

The overall system fade during rainfall is directly proportional to $T_{overall}$. Figure A-5 shows station fade on a percentage of time basis for a typical southern Canada site. As stated previously, fade due to wind is negligible and will not affect the fade distribution shown for rainfall.

The maximum short-term fade can be predicted from DOT Meteorological branch rainfall intensity maps. Areas in Southern Ontario may be subjected to rainfall rates of 6"/hr for 5 minute periods at a 5 year return period and 3½"/hr for 15 minute periods on a 5 year return period.

Station fade for these conditions are:

Rainfall Rate	Fade
3½"/hr	0.8 db
6" /hr	1.15 db

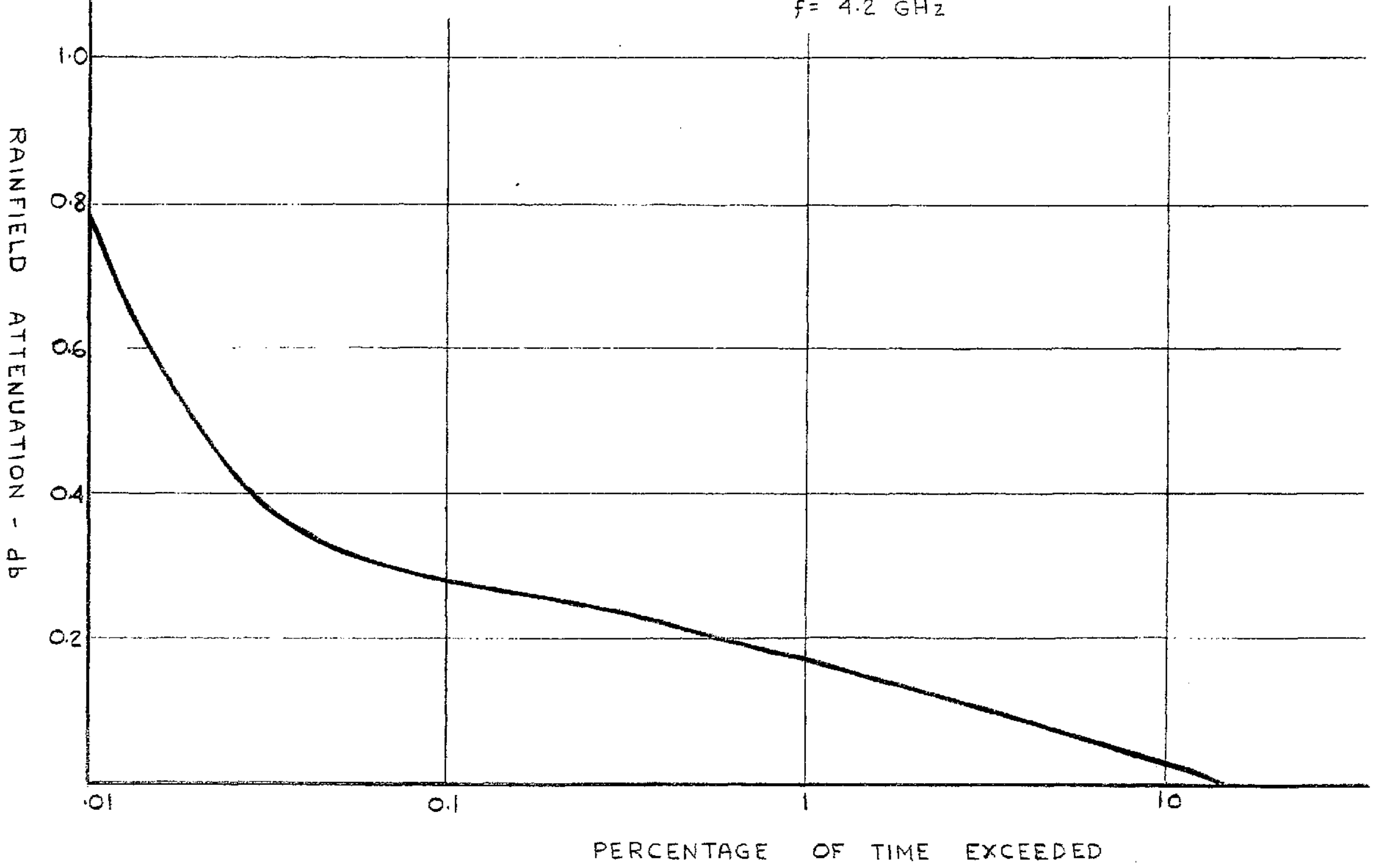
It is evident that station performance degradation due to rainfall will be insignificant.

Ice and Snow

Ice or snow accumulation on the antenna will cause some degradation in station G/T but since the dielectric constant and dissipation "lossiness" of frozen precipitation is relatively low, the electrical effects of this accumulation is not too serious. Our study indicates that less than 0.5 db loss in gain will be experienced for a 90-foot diameter antenna completely covered with 1¼ inches of ice or 6 inches of snow. Most of this error is due to reflector deformation. If the ice or snow deposit is non uniform there could be additional degradation due to the resultant phase errors.

With a 30-foot antenna the effects of ice and snow should be less critical than 90-ft., in terms of dish and structure deformation and the resultant pointing error. However, a fixed thickness of ice or snow on the reflector surface will have a greater defocusing effect for a small dish so this will increase the antenna gain degradation. We must consider whether deicing will be required for any stations in the system. Deicing in any form; electrical heating, oil fired heaters, anti-freeze or deicing solutions would be relatively expensive and should be avoided if at all possible. Further study will be required before a detailed Earth Station specification can be generated, but it is believed that complex deicing can be avoided for most sites. Snow or ice accumulation will be heavy in some locations, but periodic snow and ice removal should be sufficient to hold degradation within acceptable limits. On this basis, it is suggested that we assume a 0.5 db degradation in station G/T during winter months for performance calculations.

FIG. A-5a TYPICAL RAINFALL ATTENUATION FOR SOUTHERN CANADA
(BASED ON MILL VILLAGE DATAS WORST MONTH PER YEAR)
$f = 4.2$ GHz



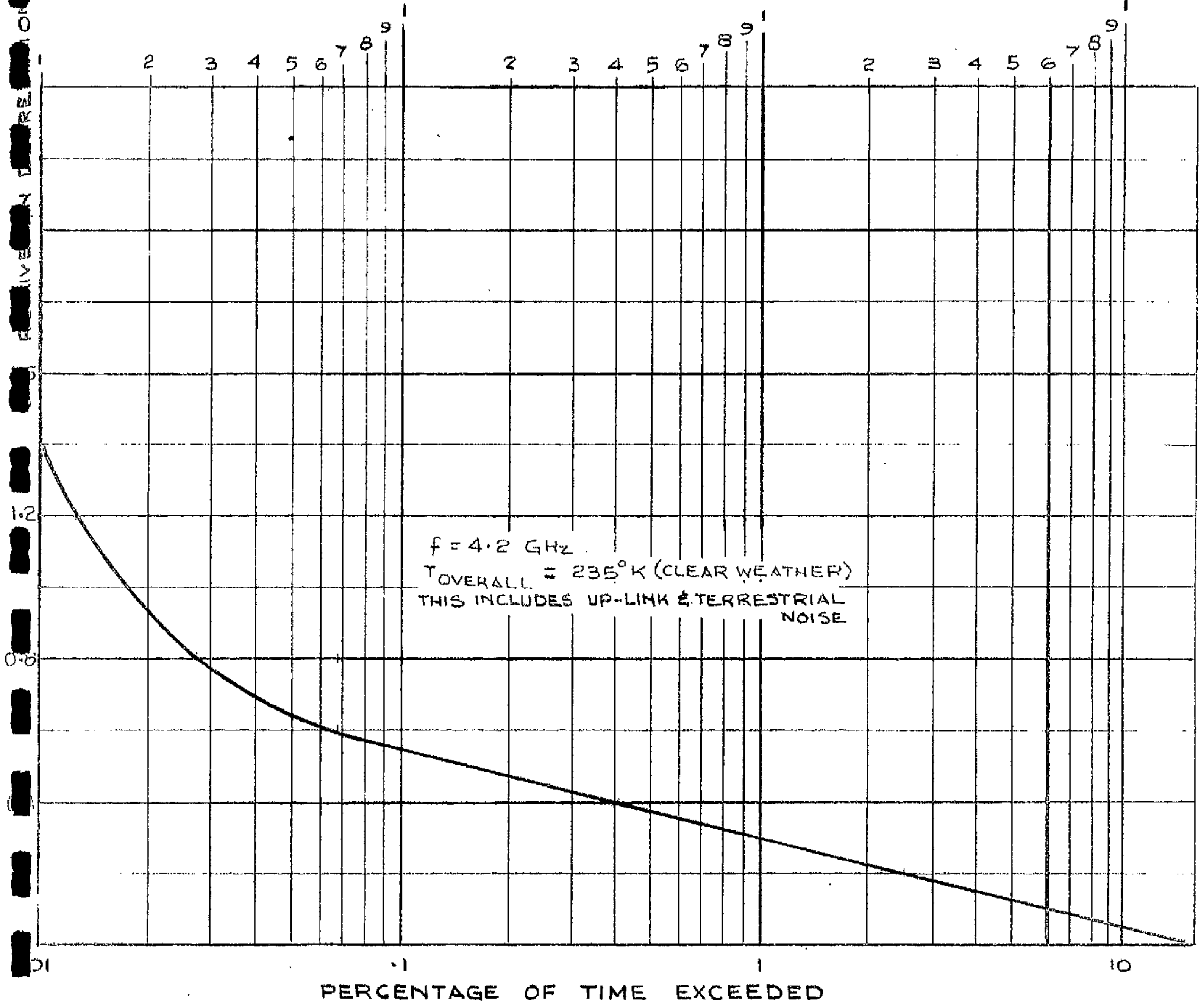


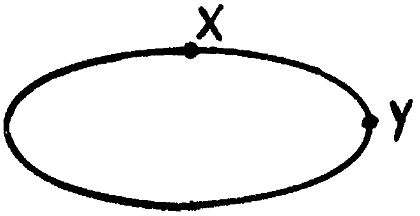
FIG. A5

TYPICAL EARTH STATION FADE FOR SOUTHERN CANADA

5. FADING DUE TO SATELLITE ANTENNA POINTING INACCURACIES

The computer calculated beam coverage of Canada shows that the entire country can be covered with an $8\frac{1}{2}^{\circ} \times 3\frac{1}{4}^{\circ}$ beam, with $\pm\ 0.5^{\circ}$ satellite antenna pointing inaccuracies, E-W and N-S included. If it is assumed that this $\pm 0.5^{\circ}$ motion is a 3 sigma occurance and that the error distribution is gaussian, we can calculate the received signal strength, vs. percentage of time.

Consider an elliptical beam and a point (X) on its minor axis.



$8\frac{1}{2}^{\circ} \times 3\frac{1}{4}^{\circ}$ (4.3 db beam)

If the beam centre is now subjected to $\pm\ 0.5^{\circ}$ motion along each axis we note the following. Motion in the E-W direction will result in very small level changes at point X while motion in the N-S direction will result in a large signal level change at point X. Thus for this location we can ignore the E-W beam motion and calculate the change in received signal strength for $\pm\ 0.5^{\circ}$, 3 sigma gaussian motion along the N-S axis. Similarly a station at point Y will be affected primarily by E-W motion of the beam and much less by N-S motion. In this particular case, the assumption that cross-axis motion can be ignored is less valid, but the mathematics required to solve this case exactly is rather tedious and does not seem warranted at this time. The results obtained by the simplified approach will be slightly optimistic.

The results are plotted on figure A-6, It is assumed that the EIRP for the worst case beam edge is 34 dbW. From the curves, it can be seen that the EIRP is well above 34 dbW for the majority of the time; in the case of a station located along the minor axis, the level will rise more than 3 db.

The relatively large values of EIRP for the majority of the time are due to the assumed gaussian distribution on the pointing error. Figure A-7 shows the variations in EIRP assuming that all positions in the $\pm 0.5^{o}$ control range are equiprobable. This is perhaps more realistic.

FIG. A6 - EIRP TOWARDS EARTH STATION WITH SATELLITE ANTENNA MOVEMENT

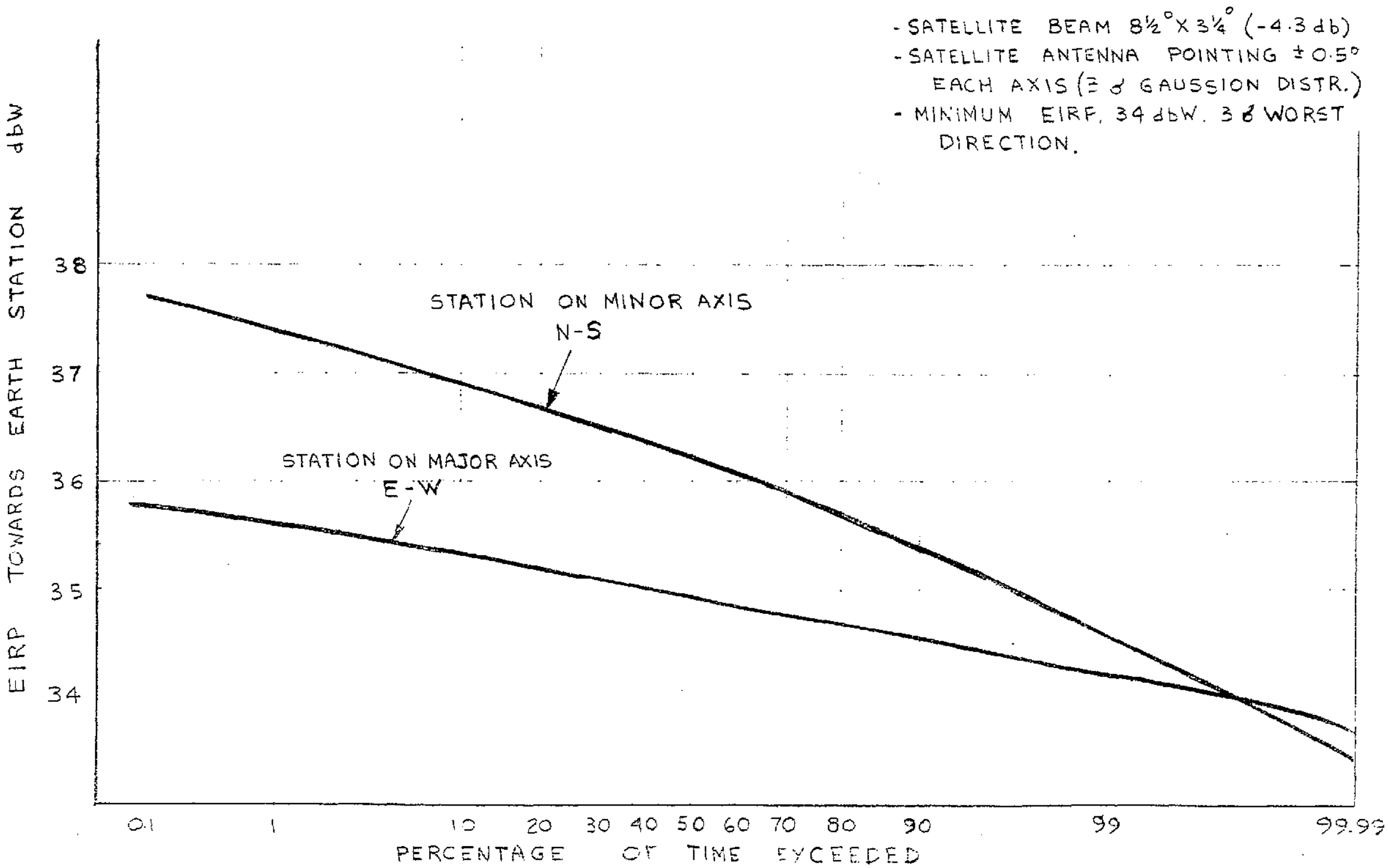
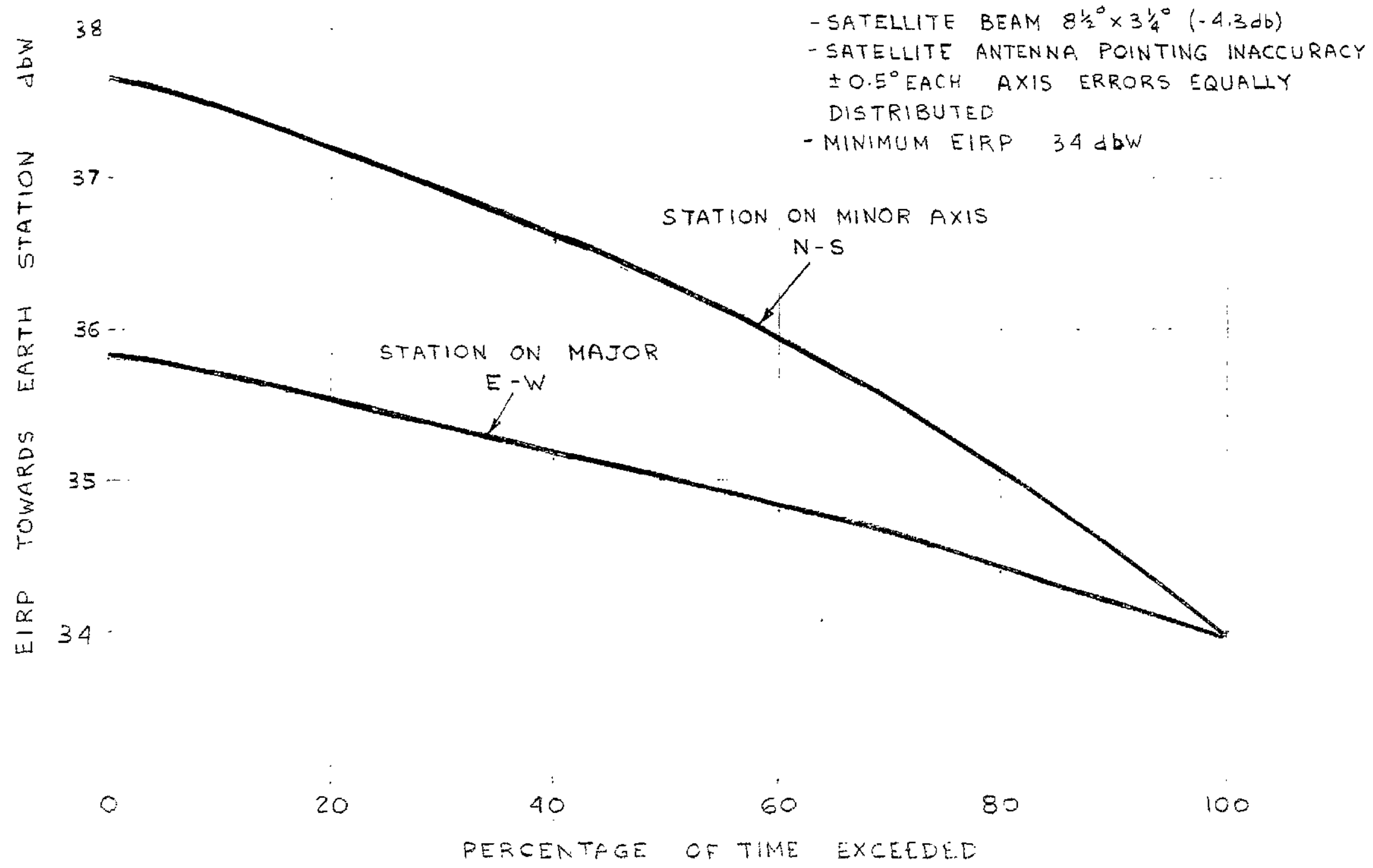

- SATELLITE BEAM 8½° X 3¼° (-4.3 db)
- SATELLITE ANTENNA POINTING ± 0.5° EACH AXIS (3 σ GAUSSION DISTR.)
- MINIMUM EIRP, 34 dbW, 3 σ WORST DIRECTION.

FIG. A-7 - EIRP TOWARDS EARTH STATION WITH SATELLITE ANTENNA MOVEMENT.



6. ESTIMATED SYSTEM NOISE TEMPERATURE BUDGET

The estimated system noise temperature budget at 5° and 20° elevation angle is shown in the following table for a 30-foot diameter antenna.

	3.70 GHz		4.00 GHz		4.20 GHz	
	5° El.	20° El.	5° El.	20° El.	5° El.	20° El.
GALACTIC	1.05°K	1.05°K	0.96°K	0.96°K	0.91°K	0.91°K
ATMOSPHERIC	24.00	14.00	24.40	14.40	23.60	13.60
GROUND	14.65	8.65	11.73	5.73	17.31	11.31
FEED LOSS	5.30	5.30	6.80	6.80	6.20	6.20
CONNECTING LINE LOSS	.94	.94	.91	.91	.90	.90
SWITCH LOSS	1.00	1.00	1.00	1.00	1.00	1.00
PARAMP & RECEIVER	100.00	100.00	100.00	100.00	100.00	100.00
TOTAL	146.94	130.94	145.80	129.80	149.92	133.92

Note that we have based the above temperatures on using a paramp with a noise temperature of 100°K rather than 150°K which we had previously thought was feasible. However after many discussions with three paramp suppliers we believe that 100°K can be easily achieved.

Summary of Discussions with Paramp Suppliers

AIL, TRG and Comtech are all willing to quote fixed price on a 500 MHz bandwidth room temperature parametric amplifier. Following is a brief breakdown of expected performance, price and delivery of their initially proposed system.

	AIL	TRG	Comtech
Bandwidth MHz	500	500	500
T_e °K	100-120	90	65-85
Mount	balanced	single	single
Pump Freq. GHz	24	37	27
Budgetary price* per unit (qty. 10)	< $25,000.	< $25,000.	< $25,000.
Delivery months	4 - 5	4 - 5	4 - 5

*Price is considerably less for a unit with a 50 MHz bandwidth.

Both TRG and Comtech seem to have a slight advantage over AIL since they have already incorporated the single ended mount into their amplifier operating at a substantially higher pump frequency with the inherently lower noise temperature. TRG has done some preliminary performance evaluation on the room temperature paramp by optimising one of the production (500 MHz, 18°K cooled) amplifiers to operate at ambient room temperature. They have already actually measured noise temperatures in the range below 100°K using two stages of amplification. Comtech, initially in their program, decided to design their system with a somewhat lower pump frequency than used at TRG (i.e., 27 GHz vs. 37 GHz), since the transition to a solid state pump source would be more easily done. The pump power requirement at the diode is approximately 25 m watts and it is presently possible to attain this power level using an 100 mw avalanche diode operating at approximately 15 GHz followed by a single doubler stage.

Comtech state that in order to meet the present gain and phase stability requirements stipulated for Intelsat III, the parametric amplifier would need to be kept operating within a temperature controlled environment with a minimum ambient temperature variation. This could be done by either slightly heating above or cooling below the paramp enclosure. This could be done by either keeping the paramp enclosure slightly above ambient temperature (i.e., by heating), or keeping the enclosure below ambient temperature (i.e., by cooling). Comtech maintain that the solution which looks most promising at this time is to make use of the thermal electric cooling effect of varactor junctions, thereby cooling the enclosure to approximately $32^{o}F$. The thermal electric cooling device is of rugged construction and involves no moving parts, consequently resulting in very high MTBF's. The equivalent noise temperature advantage if we compare operation at 80 - $90^{o}F$ and $32^{o}F$ is expected to be in the order of $20^{o}K$ (i.e., $T_e = 65^{o}K$, compared to $T_e = 85^{o}K$, respectively.

7. DEFINING SYSTEM CONSIDERATIONS FOR CANADIAN DOMESTIC SATELLITES

In 1963, the Extraordinary Radio Administrative Conference (EARC) of the International Telecommunication Union (ITU) in Geneva authorized communications satellite systems to share frequencies with terrestrial systems, provided that certain power limits are observed. This historic event paved the way for the use of satellites for communications and since that time, INTELSAT I and several INTELSAT II satellites have been placed in operation carrying commercial traffic.

The importance of Canada's role in satellite communications can be seen from the fact that all traffic between Europe and North America via INTELSAT I is handled by Canada's experimental station at Mill Village, Nova Scotia.(1) Towards the end of 1968 INTELSAT III, the global system is expected to be placed into orbit and Canada's first commercial station will be carrying traffic from the new system. The INTELSAT IV satellites are already planned. Since all these satellites are of the synchronous type, one can probably conclude that synchronous satellites for international use are here to stay.

Great interest has recently been shown by various nations(2-5) including Canada(6-9) in the use of synchronous satellites for domestic use. As a result there appears to be a need for more technical agreements to insure that domestic satellites placed in synchronous orbit make efficient use of the radio spectrum. The synchronous orbit, like the radio frequency spectrum, is an important natural resource, and it is vital that no effort be spared to ensure its efficient use.

It must be protected from a proliferation of unnecessary satellites, but on the other hand, it is important for Canada to place some satellites for domestic use in this belt visible to Canada between about 85° and 120°W longitude.

Technical agreements between nations will have to be established and perhaps new radio regulations or CCIR recommendations will have to be written to insure that all communication satellite systems using the same frequency bands can exist side by side without undue interference and technical constraints. At the moment, these problems are difficult to solve because the common carrier bands from 1 to 10 GHz have to be shared with the terrestrial services. In the future, when hardware becomes available, it is to be hoped that satellite systems can operate on exclusive frequency allocations in the higher frequency bands; the problem of sharing frequencies will then be greatly eased. For the present, and probably for the next ten or perhaps twenty years, the frequency bands below 10 GHz will be of greatest concern for communication satellite services. Domestic satellite systems will therefore have to share frequencies with terrestrial systems, as well as with other satellite systems and, furthermore, satellites will have to share space on the unique synchronous belt.

All these constraints lead to a big problem - one that requires prudent study of many considerations which can affect not only technical agreements, but also the design of satellites and earth stations.

SATELLITE ERP IS PRIME CONSIDERATION

It is highly desirable for satellites to radiate the maximum allowable effective isotropic radiated power (ERP) in order that they may carry as many communication channels as possible, or to permit the use of small earth station antennas.

The EARC(10) permits satellites to radiate a maximum power flux density at the surface of the earth of -149 dbW/m^2 in a 4 kHz bandwidth and a total power flux density of -130 dbW/m^2 for systems using wide-angle frequency modulation based on recommendation of the CCIR Plenary Xth Assembly (Geneva, 1963). At the XIth Plenary Assembly of the CCIR, (Oslo, 1966), the maximum flux limit was changed to $-152\ dbW/m^2/4\ kHz + \theta/15$ via Rec. 358(11) and it is expected that the EARC will also make a revision accordingly. This limit, equivalent to a satellite ERP of 12 dbW/ 4 kHz for low angles of arrival, was derived from CCIR Rec. 357 which states the allowable noise in a telephone channel caused by interference from the "aggregate" of all communications satellite systems into the radio-relay hypothetical reference circuit. CCIR Rec. 357 states an allowable mean value in any hour, or the value not to be exceeded for more than 20% of any month, of 1000 pWp, and a value of 50,000 pWp not to be exceeded for more than 0.01% of any month.

Since the derivation of the flux limit in Rec. 358 was based on a single exposure of the main beam of a radio-relay, the flux limit or satellite ERP must be reduced by the number of satellites causing interference. It should be noted that the allowable noise could also be exceeded by a large number of exposures into the antenna sidelobes of the radio-relay system.

The maximum allowable satellite ERP depends on the amount of energy dispersal that can be achieved practically for systems using frequency modulation. Energy dispersal is a technique by which the carrier is artificially modulated to spread its power over the transmission bandwidth, thereby minimising the power in any 4 kHz bandwidth. Theoretically, the amount of energy dispersal that can be achieved for a 36 MHz bandwidth is 39.5 db; in practice, however, this value is found be be closer to 30 db without paying too high a penalty in extra bandwidth or earth station gain-to-temperature (G/T) ratio to accommodate the extra artificial frequency modulation. The amount of energy dispersal that can be achieved is shown in Figure A-8 which is based on the use of a triangular waveform; Figure 1 also shows the satellite ERP.

A satellite designed for 42 dbW ERP requires a peak-to-peak frequency deviation of the dispersal waveform of 4 MHz, or 11% of a 36 MHz bandwidth. Increasing the dispersal beyond about 10 MHz, which provides 34 db of dispersal, does not appear to pay because the loss in signal-to-noise ratio due to reduced bandwidth begins to outweigh the permitted increase in satellite ERP. Also, it is doubtful whether dispersal of this order could be applied without causing distortion problems, because the dispersal deviation is becoming a significant portion of the television picture deviation. A more likely value of acceptable energy dispersal is about 31 db and therefore a maximum ERP of 43 dbW is permitted, assuming that the allowable interference into a radio-relay HRC is not exceeded.

If the allowable interference is exceeded (say by 6 db) then the permitted ERP must be reduced (to 37 dbW), otherwise the spacing between satellites in the stationary orbit might have to be increased. In order to achieve efficient utilization of the stationary orbit, it is necessary to keep satellite spacing to a minimum; therefore, the trade-off might be to reduce the satellite ERP.

Another consideration is earth station interference to or from adjacent satellites. A difference of 7.5 db between the ERP of adjacent satellites can double the required satellite spacing. To allow minimum spacing between satellites, all satellites in the synchronous belt over the United States and Canada should therefore have nearly the same ERP. It is recognized that it may not be feasible for domestic and global satellites to have equal ERP and therefore the spacing between these different systems may have to be much greater.

ALLOWABLE INTERFERENCE BETWEEN SATELLITE SYSTEMS

Interference between satellite systems is one of the major controlling factors affecting the efficient use of the stationary orbit. The CCIR has recommended the values of allowable noise due to interference between radio-relay systems and communication-satellite systems. For a communication satellite system, CCIR Rec. 356 states the noise power in a telephone channel due to interference caused by radio-relays should not exceed:

1000 pWp	mean value in any hour
1000 pWp	one-minute mean power for more than 20% of any month
50,000 pWp	one-minute mean power for more than 0.03% of any month

CCIR Rec. 357 (in the reverse direction - from satellite systems into radio-relay systems) is similar to Rec. 356, except that the 50,000 pWp value should not be exceeded for more than 0.01% of any month.

There are as yet no allowable limits for noise between one communication satellite system and another, although the CCIR is expected to be studying this aspect. For interference into a satellite system from all other satellite systems, a recommendation with the same values as Rec. 356 would probably not be unreasonable and therefore is assumed here. Taking the noise power lower than, say, 1000 pWp value appears to increase satellite spacing unnecessarily. On the other hand, raising this value appears to cause some undue hardship on communication satellite systems which must already accept 1000 pWp of interference from radio-relays; moreover, increasing the value above 1000 pWp does not appear to permit efficient use of the stationary orbit. For the purpose of determining interference, the baseband noise power budget for the communication satellite system shown in Table is is used. The total noise is chosen to be the same as that for the radio-relay hypothetical reference circuit.

TABLE 1

Contributions	Noise Power in a Telephone Channel
Interference from radio-relay systems	1000 pWp
Interference from all other satellite systems	1000 pWp
Uplink fluctuation noise	1000 pWp
Downlink fluctuation noise	4000 pWp
Intermodulation noise	500 pWp
	7500 pWp*

---

* Reference 5 has used a value of 5600 pWp which does not appear to include any interference noise.

In order to discuss interference between satellite systems, it is convenient to refer this to the receiver input of the interfered system in terms of the carrier-to-interference ratio (C/I). The interference between communication satellite systems will be primarily caused by adjacent synchronous satellites radiating into earth station receivers, and by earth station transmitters beamed at adjacent synchnchronous satellites, where unwanted power falls into the satellite receiver via the sidelobes of the transmitting antenna.

C/I is related to the receiver output noise as follows:

$$C/I = -B-N, \text{ db} \qquad \text{----------(1)}$$

where N = unweighted noise power at a point of zero relative level, dBm0

B = interference reduction factor, db[12]

The B-factor has been calculated for the various cases of telephony based on the parameters shown in Figure A-9.

The practical satellite ERP has been taken to be 37 dbW. Assuming that the earth station is located at the satellite antenna beam edge, and that an earth station G/T ratio of 40.7 db (95 ft. diameter antenna is used), the maximum channel capacity is about 1500 without exceeding 36 MHz bandwidth.

The interference between video carriers, or between video and telephony carriers using FM, is still under study. Requirements for 58 db picture-to-rms interfering tone in the baseband have been stated. Using energy dispersal, a high modulation index and pre-emphasis will tend to eliminate any high concentration of energy at any particular frequency and will tend to make the FM spectrum Gaussian. Interference of this type will not produce an interfering tone in the baseband but will appear in the baseband as noise with an approximate triangular spectrum. The video signal-to-interference noise ratio, S/I, can be determined by convolution of the wanted and unwanted RF spectra, as for the telephony case, and a value of 34 dB was obtained for colour 525-line video having a subcarrier with 75% saturation. If the interference noise is taken as 10 dB below the CCIR recommended signal-to-noise ratio, as in the previous case for telephony, the required S/I is 65 db; the C/I must be 31 db. The subjective effects of this type noise on the television viewer have not yet been completely studied and the results of these effects might increase the required C/I. The various C/T for some typical channel capacities and antenna sizes are shown in Table 2.

Interleaving frequencies by 20 MHz between adjacent satellites will reduce interference and an improvement of about 15 db is estimated by this means.

TABLE 2

od. Index RMS M	N Channels	Earth Station G/T (db)	Earth Station Ant. Dia. (feet)	Maximum Satellite ERP (dbW)	* Required Co-Channel C/I (db)
	1800	43.1	125	37	42
	1500	40.7	95	37	40.5
	1200	36.7	60	37	35
	960	26.5	30	37	30
	120	40.7	95	10	42
	120	26.5	30	24.2	27.8
	TV	40.7	95	37	31
	TV	26.5	30	37	31
	TV	23.8	22	37	31

* Corrected to account for any difference from maximum satellite ERP of 37 dbW.

FREQUENCY PLANNING

It appears that, in order to have efficient use of the stationary orbit and the frequency spectrum, the channel frequencies of alternate satellites should be interleaved in a manner similar to that in Annex of CCIR Rec. 382-1. An example of a channelling plan is shown in Figure A-10 with adjacent satellites interleaved. Every alternate satellite will use the same frequency plan. Such interleaving of frequencies between adjacent satellites can reduce the required spacing by almost one half.

Interleaving could make the satellite system somewhat less flexible, but it appears that interleaving frequencies, which allows more efficient utilization of the stationary orbit, outweighs the other considerations.

EARTH STATION ERP VERSUS ANTENNA SIZE

At the XIth Plenary of the CCIR, it was recommended, via Rec. 406, that all future radio relay stations which share frequencies with the satellite up-link should not have their antennas aimed at the stationary orbit where feasible. It is believed that many common carriers have now written this restriction into their practices. This recommendations will, in future, protect the stationary orbit(14) from interference caused by the radiation of radio-relays. Rec. 406 also recommends that the radio-relay transmitter power should not exceed 13 dbW, which demands that radio relay systems employ relatively large, high-gain antennas. These will have narrow antenna beams and therefore the probability of exposure to a satellite is minimized.

It would appear that consideration should also be given to minimizing the space between synchronous satellites by minimizing sidelobe radiation from the antennas of earth stations aimed at the stationary orbit. In order to accomplish small spacings it would be necessary to specify a minimum antenna size for transmitting earth stations.

It can probably be assumed that all transmitting earth stations will have a uniform ERP for a particular channel capacity or for television; this ERP can be achieved with high power transmitters and small antennas or with lower-power transmitters and large antennas. In order to minimize interference to neighboring satellites it will be necessary to insist that radiating earth stations employ reasonably large antennas. It will also be necessary to agree on the minimum antenna size for earth stations receiving high-quality television. If it is desired to use an antenna smaller than the minimum size specified for receiving TV, that station would suffer more interference, but possible the quality might still be acceptable. For example, if satellite spacing is based on a minimum antenna diameter of 30 ft., and a 22 ft. diameter antenna is used to receive TV, the station would be subjected to only 2.7 db more interference noise. In any case the station using the small antenna is only receiving and therefore cannot cause additional interference to adjacent satellites. There is also a need to establish recommendations for earth station ERP, because earth stations in a different system radiating more power for the same channel capacity will also increase the required satellite spacing.

FACTORS AFFECTING SATELLITE SPACING

The spacing of satellites is of great importance, for it will ultimately determine the number of satellites that can be placed in the stationary orbit for particular frequency bands. The 6 GHz up-band and the 4 GHz down-band are of immediate interest.

The spacing of satellites is dependent on twelve factors which govern interference between communication satellite systems for any given frequency bands. These factors are:

- Difference in ERP between satellites
- Allowable interference
- Antenna size of earth stations
- Difference in ERP between earth stations
- Type of modulation and signal processing
- Modulation index (for FM systems)
- Receiver Interference Reduction factor
- Amount of energy dispersal
- Sidelobe level of the earth station antennas
- Frequency discrimination
- Satellite antenna discrimination for very narrow beams
- Polarization discrimination

For determining satellite spacing, no difference in transmitted ERP is assumed other than that discussed earlier. Also, no discrimination is assumed from frequency, satellite antenna beam and antenna polarization.

Polarization discrimination is very difficult to achieve over 500 MHz bandwidth at angles far off the main beam. Using the CCIR antenna gain off the main beam for the earth station, which is given by:

$$g = 32 - 25 \log \theta \text{ (db)} \qquad \text{----------(2)}$$

(where θ is the angle off the main beam center in degrees)

and also using the 4 GHz main-beam gain given by:

$$G = 19.6 + 20 \log D \text{ (db)} \qquad \text{----------(3)}$$

(where D is earth station diameter in feet)

the interference in the down path can be established. The geometry for this interference is shown in Figure A-11.Choosing six interfering satellites on each side of the "wanted" satellite, the required carrier-to-interference ratio is given by:

$$C/I = G - g - 4.14 \text{ db} \qquad \text{----------(4)}$$

Assuming that the same earth station antenna size is used for transmitting, the interference due to the up-path is 4 db less than for the down-path. This assumption is independent of the number of earth stations transmitting. It is only important that the 500 MHz band be illuminated (say 12 RF carriers); whether this is accomplished by one earth station or by a dozen does not matter. It is assumed that the minimum antenna size for transmitting is the same as thatused for receiving. This assumption is conservative because it is likely that many transmitting earth stations will have antennas larger than those of receiving earth stations. The required C/I is therefore increased by 1.44 db due to the up-path contribution of interference, ie.,

$$C/I = 20 \log D + 25 \log \theta - 18 \text{ db} \qquad \text{----------(5)}$$

The satellite spacing is assumed to be the same between satellites and the angular spacing is assumed to be equal to θ. This assumption makes the interference about 1 db less for earth stations located near the Canadian-United States border, looking at satellites due south, but the assumption is almost correct for earth stations which operate with satellites at low elevation angles. Obviously, all stations in the far north fall in this category.

The required C/I is shown as a function of satellite spacing for various antenna sizes in Figure A-12.The required co-channel C/I is 31 db, using a 30 ft. diameter antenna from Table 2. This requires a satellite spacing of 6° which would mean that about seven satellites could be placed in the stationary orbit visible to Canada. Interleaving frequencies will almost double the number of satellites. If satellite spacing is based on a much smaller antenna size, such as 15 ft. diameter, the spacing between satellites will have to be almost doubled. However, it must be recognized that the use of antenna feed with lower sidelobe levels than that assumed by the CCIR, or the use of PCM in the future, could significantly increase the number of satellites in this 85° to 120°W longitude belt. Although the spacing is based on a minimum antenna diameter of 30 ft., for receiving high quality television, this choice does not preclude the use of a smaller antenna as mentioned previously.

Studies done by the Canadian CCIR (dealing with frequency sharing) has indicated that a minimum satellite spacing of 3 to 6° is required to meet the CCIR requirements of interference from satellites into line-of-sight radio relays sharing the same frequency bands depending on the degree of "avoidance" of the stationary orbit. The satellites were assumed to be radiating the maximum flux values permitted by the CCIR. Assuming that in practice they may be radiating somewhat less it appears that the satellite spacing could not be reduced below 3°. This appears to be the ultimate spacing when frequencies are shared with terrestrial services. In the more distant future, if the higher frequency bands above 10 GHz are used, it appears that satellite spacing can be greatly reduced.

Efficient use of the stationary orbit is of vital interest to Canada whose visible stationary belt is only about 35° wide in contrast to the United States proper (80°) and South America (105°). Although these belts overlap each other, it appears that the optimum portion of the United States belt including Alaska is somewhat west to that for Canada.

Much work is required before technical guidelines or agreements can be reached by the ITU and also between Canada and her neighbors, for positioning synchronous satellites over the 85° to 120°W belt. Since it is estimated that North America will run out of useful orbital space in the 4 and 6 GHz bands before 1980, there appears to be an immediate need to arrive at agreements. Some of these agreements might include:

- Satellite ERP
- Allowable Interference
- Frequency Planning
- Earth Station ERP and Minimum Antenna Size
- Satellite Spacing
- Satellite Slot Allocation

This paper has looked at only some of the considerations connected with the controlling of interference between neighbouring satellite systems sharing frequencies, and with making efficient use of the synchronous orbit; it is recognized that there are many others. It appears that some price will have to be paid by the satellites and by earth stations, but then there is only one stationary orbit and it is priceless.

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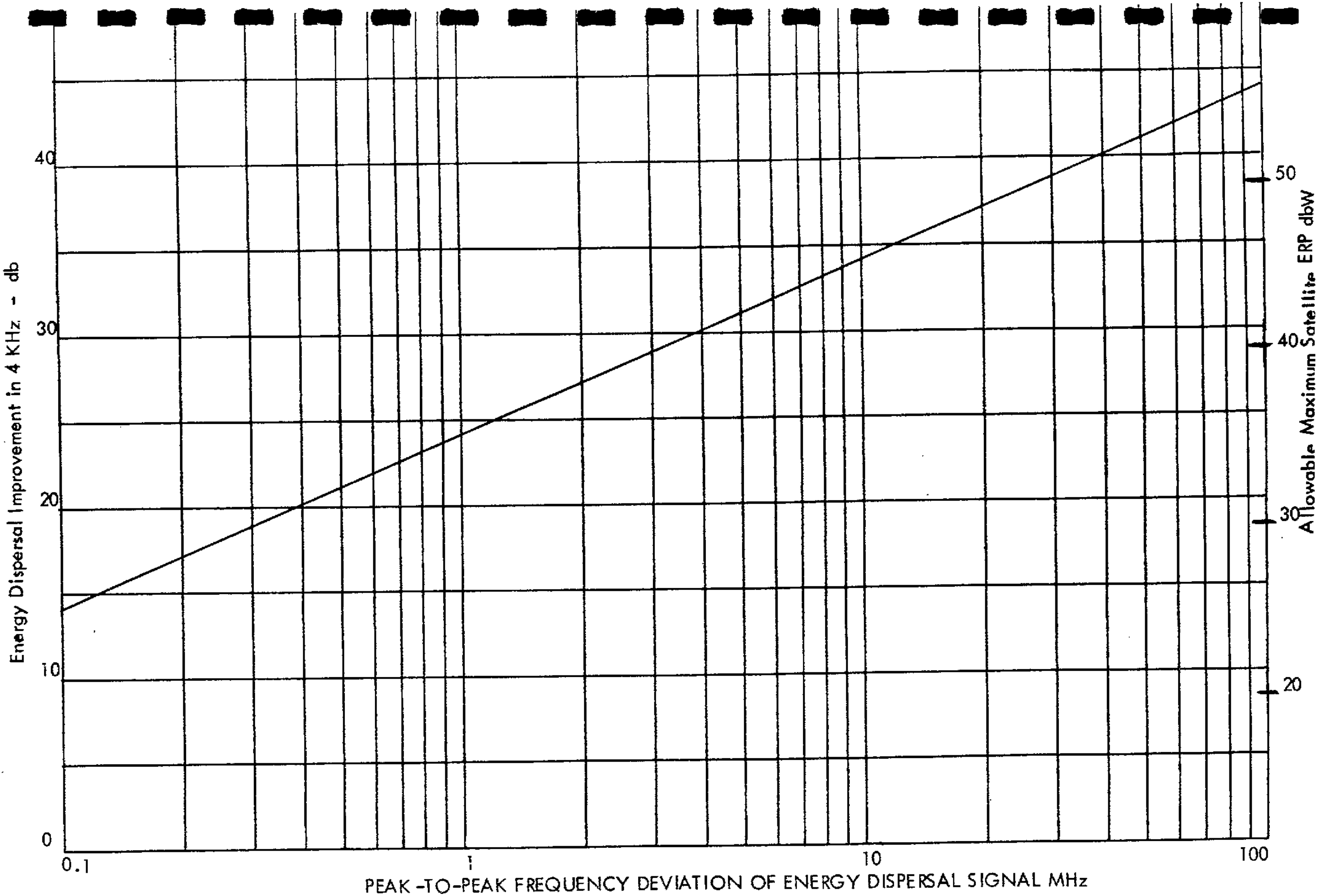


FIGURE A-8 THE EFFECT OF ENERGY DISPERSAL

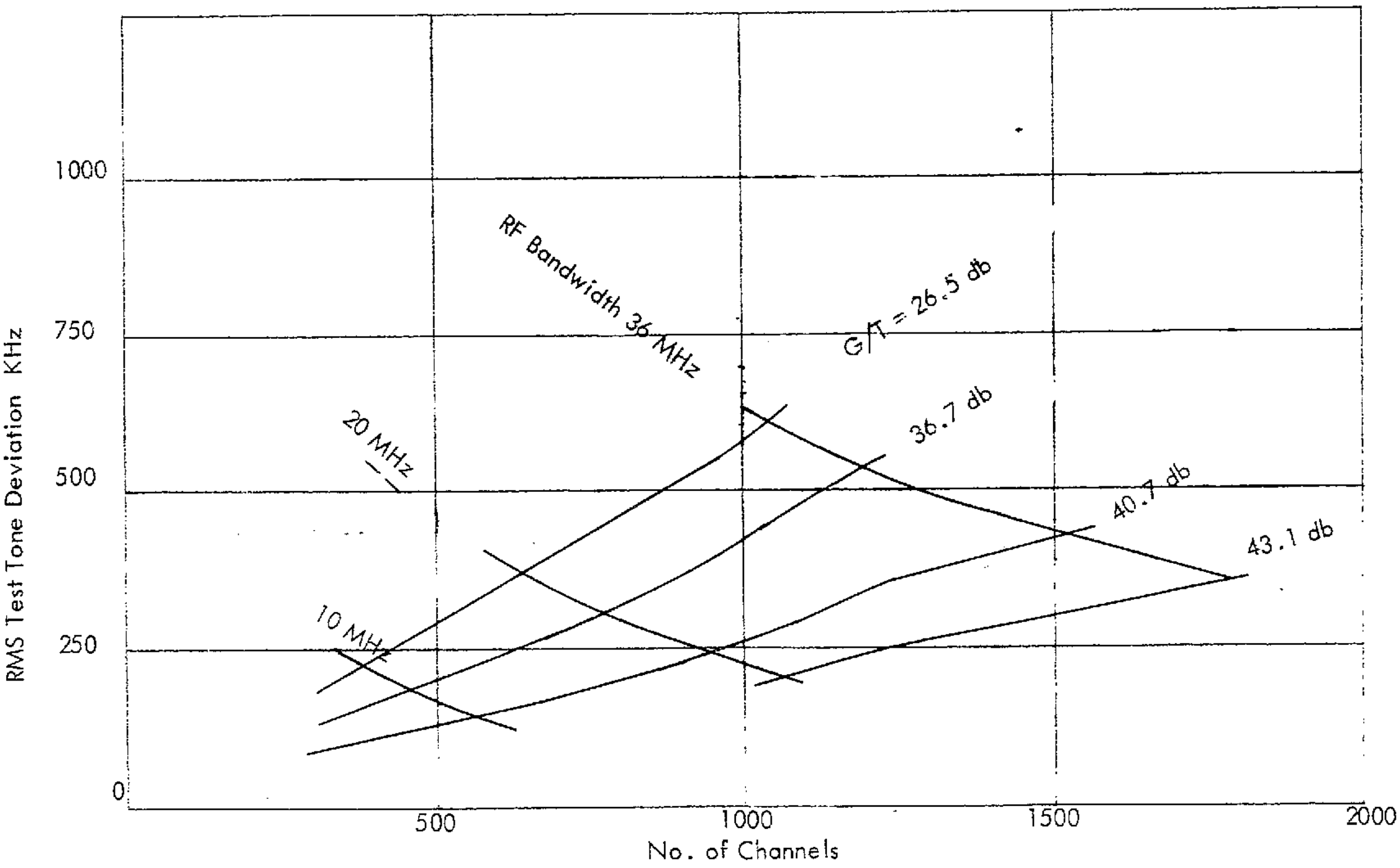


FIGURE A-9 COMMUNICATION CHARACTERISTICS

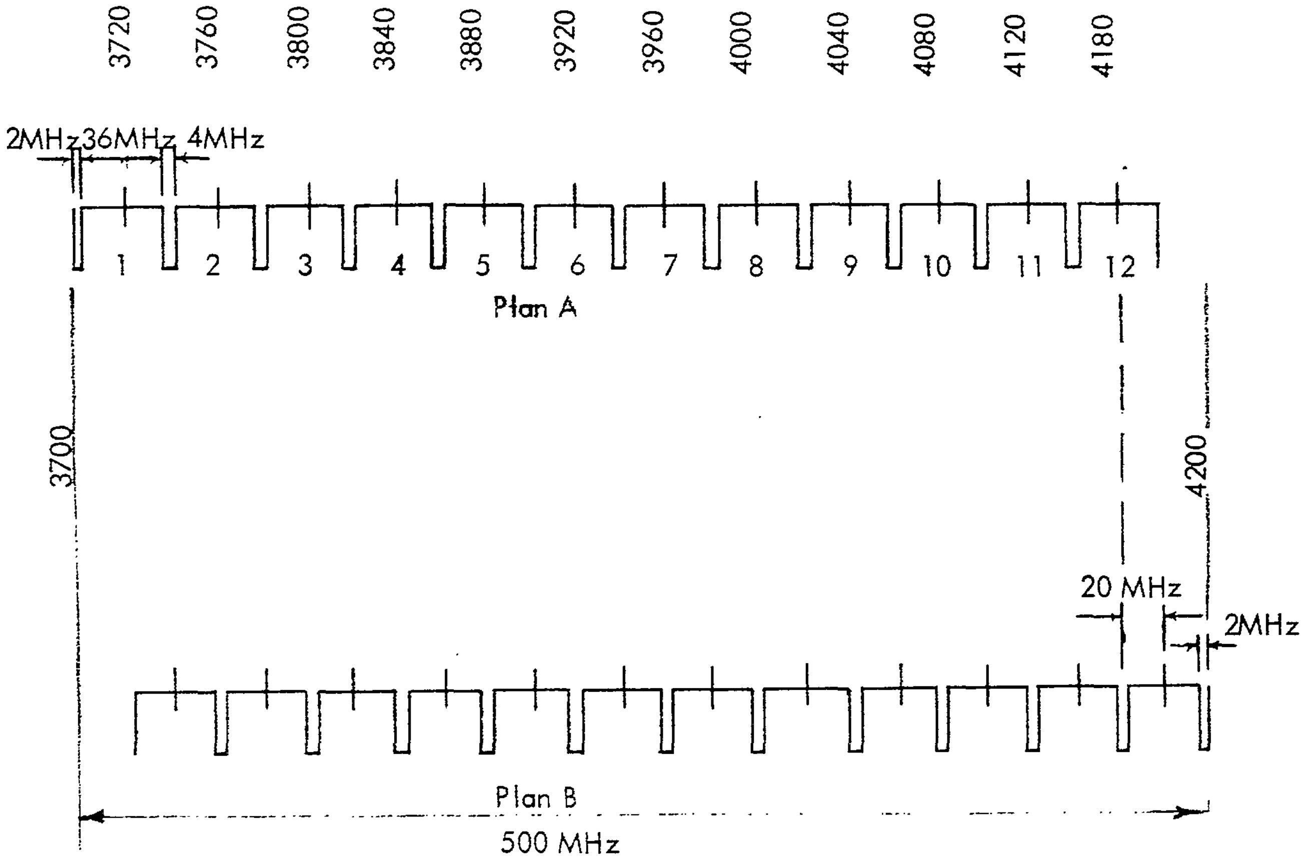


FIGURE A-10 FREQUENCY PLAN

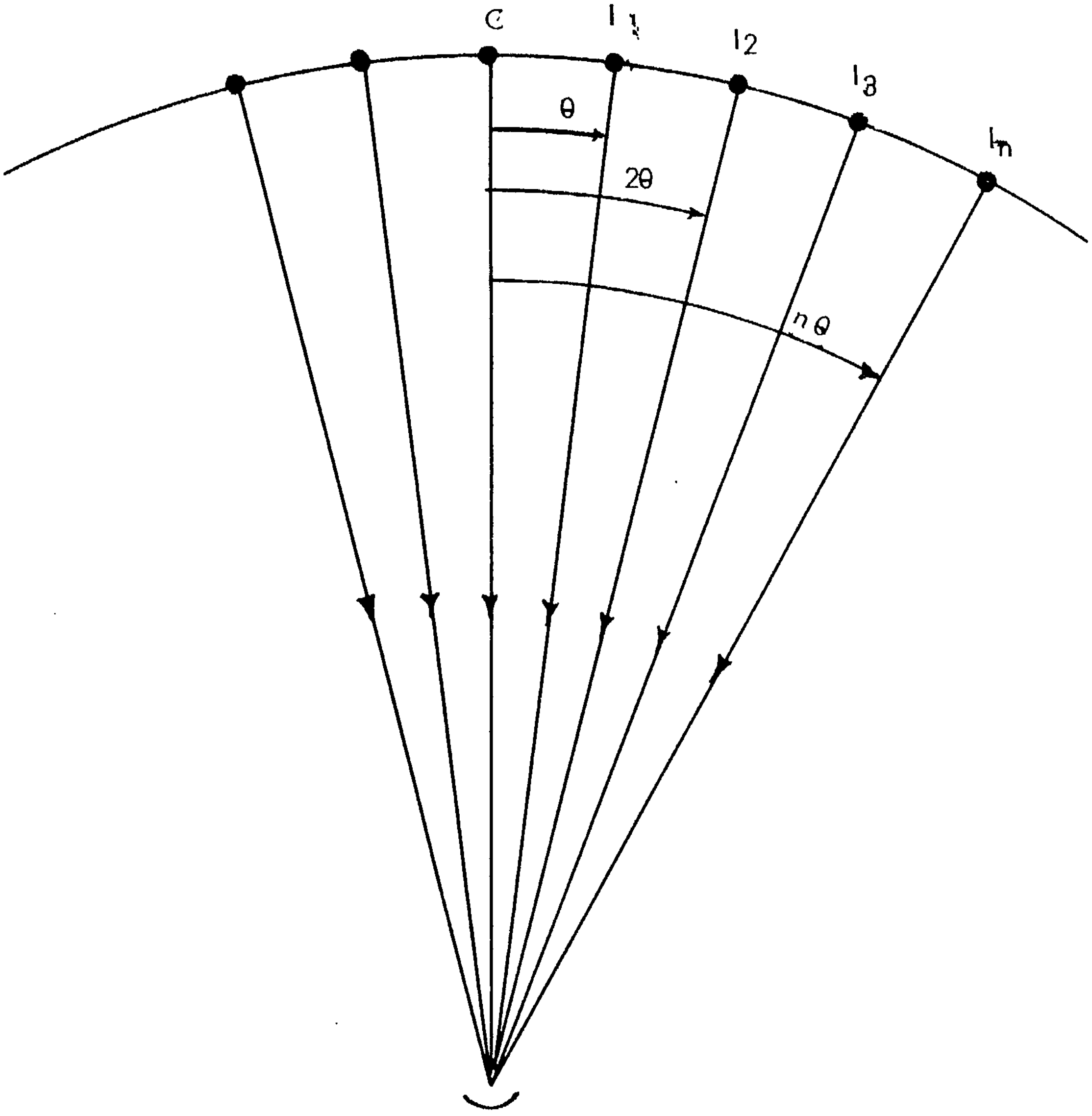


FIGURE A-11 GEOMETRY FOR SATELLITE SYSTEM INTERFERENCE WITH MANY SATELLITES IN SYNCHRONOUS ORBIT

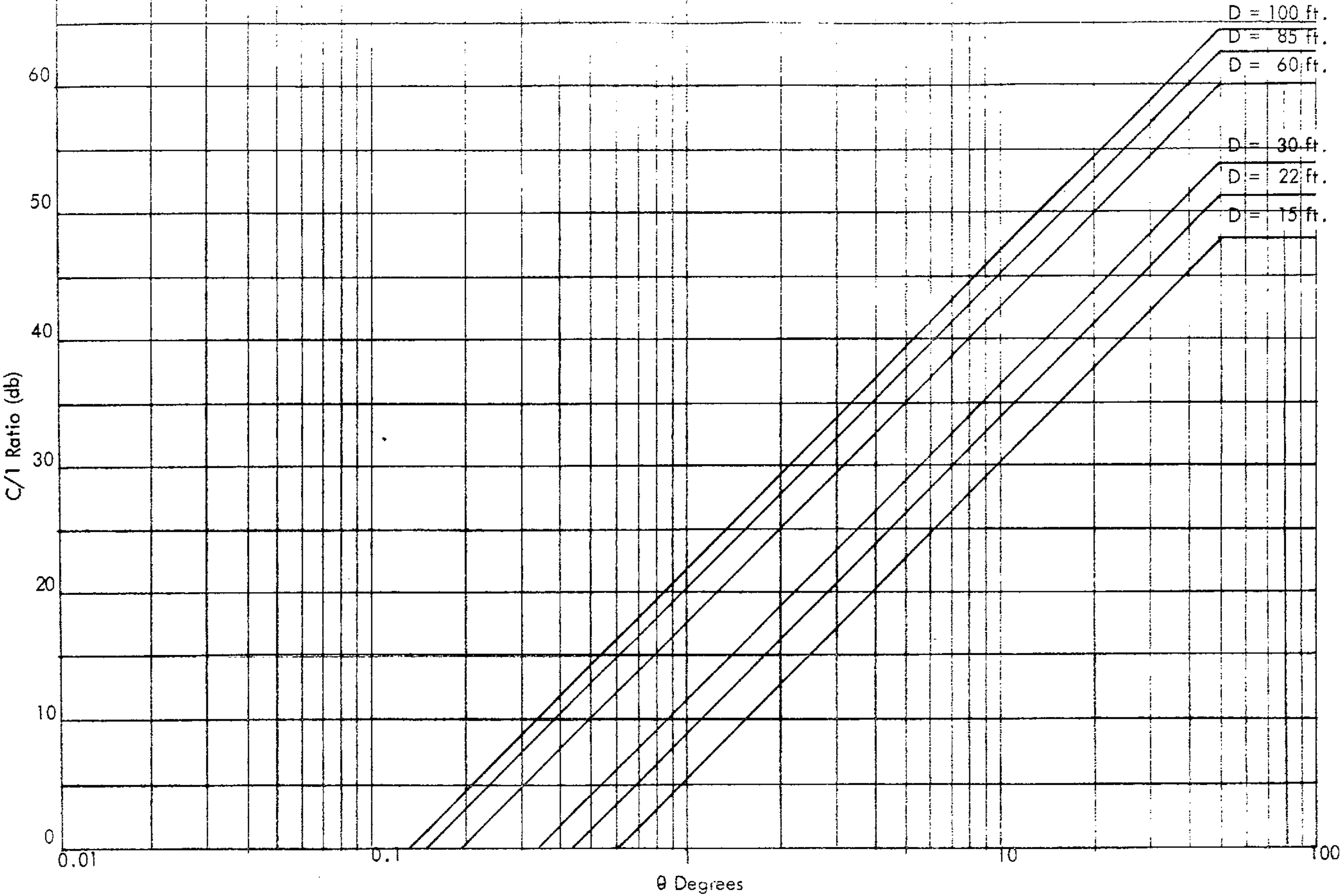


FIGURE A-12 SYNCHRONOUS SATELLITE SPACING FOR VARIOUS EARTH STATION ANTENNA SIZES

8. RECEIVER INTERFERENCE REDUCTION FACTORS (B-FACTORS)

Interference Reduction factors have been calculated for the following cases:

1. a 120 channel carrier interfering with a 120 channel carrier
2. a 120 channel carrier interfering with a 1500 channel carrier
3. a 1500 channel carrier interfering with a 120 channel carrier

The 120 channel carrier parameters were derived from the assumption that there would be 4 carriers per 36 MHz transponder or 9 MHz per carrier. For 1500 channels, it was assumed that the whole 36 MHz bandwidth is utilized. The calculations are based on Medhurst's formula* assuming a gaussian spectrum distribution and a carrier to interference ratio of 0 db.

1. 120 channels into 120 channels

Bandwidth = 9 MHz

Top Frequency f = 504 kHz

Peak Factor = 13 db

Loading Factor = 7.32 db

From Carson's bandwidth formula, we have:

Loaded rms deviation ΔF = 0,895 MHz

RMS test tone deviation Δf = 0.386 MHz

---

* R.G. Medhurst et al, "FM Interfering Carrier Distortion: General Formula, Proc. IEE 109B, 149, 1962".

Using Medhurst's formula, the B-factor can be calculated:

$$B = -10 \log \frac{f\ b}{P(\Delta f)^2\ 2f_s \sqrt{2\pi}} \left[\exp \frac{-(f_o - f)^2}{2f_s^2} + \exp \frac{-(f_o + f)^2}{2f_s^2} \right]$$

Where:

B = Interference factor for C/I = 0

f = top baseband frequency

b = channel bandwidth 3.1 kHz

f = rms test tone deviation

P = pre-emphasis improvement factor 4 db

$f_s = \sqrt{\Delta F_1^2 + \Delta F_2^2}$

ΔF_1 = loaded rms deviation of wanted carrier

ΔF_2 = loaded rms deviation of unwanted carrier

f_o = spacing between carriers.

For example, for zero frequency spacing the above formula yields a B-factor of 32.1 db. This means that if 1000 pw psophometrically weighted is allowed for interference noise (i.e., S/N = 60 db), then the C/I required is 60 - 32.1 + 2.5 = 25.4 db, where 2.5 db is the psophometric weighting factor. This C/I is that required at the input to the receiver.

2. 120 Channels into 1500 Channels

Bandwidth = 36 MHz

Top frequency f = 6.89 MHz

Peak factor = 10 db

Loading factor = 16.76

From Carson's bandwidth formula, we have:

Loaded rms deviation ΔF = 3.52 MHz

RMS test tone deviation Δf = 0.511 MHz

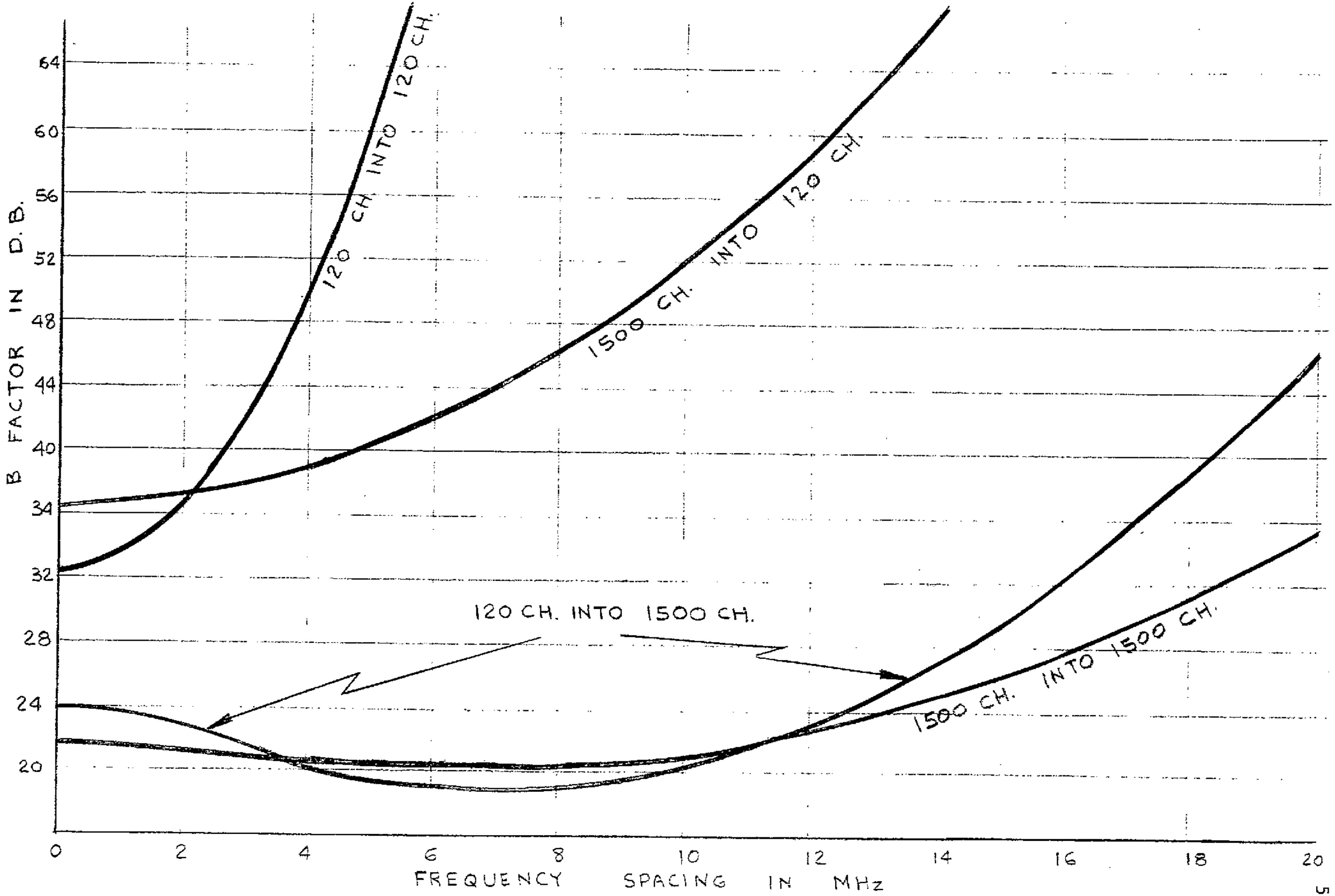
For zero frequency spacing we have B = 23.8 db.

3. 1500 Channels into 120 Channels

For zero frequency spacing, the calculations yield a B-factor of 36.4 db.

Figure A-13 shows B-factor vs. frequency spacing for the three cases considered.

FIG. A-13 INTERFERENCE REDUCTION OR B-FACTORS VS. FREQUENCY SPACING



III 2(a)

VOL. 2 APPENDICES

APPENDIX A

RANGE SYSTEM CALCULATIONS

Introduction

In ranging, tones are transmitted from a ground station to a satellite, through a satellite transponder, and are returned to the ground station. The time delay of the returned tone with respect to the transmitted tone produces a phase shift. Measurement of the phase shift thus provides information from which range can be calculated.

In this appendix, consideration is given to the choice of the tone frequencies. Taking into account the limit of accuracy for measurement of phase, a range of frequencies has been established for ranging operations. The block diagram of a typical ranging system is given in Figure 3-2.

Consider a tone frequency f_a. Given that the velocity of electromagnetic radiations is C then the ranges at which the compared phases are the same occur where $2R = C/f_a$, or at half wavelength intervals. A lower frequency thus results in a greater unambiguous range interval.

On the other hand the accuracy of the range measurement depends upon the accuracy to which the phase can be measured. If the phase measuring accuracy is the same regardless of frequency, then high frequencies favor accuracy.

Assume we wish to know the spacecraft range to an accuracy of, say, ± 150 meters. An accuracy limit is established by

a) uncertainties in the "turn about" time of the tones in the satellite transponder

b) noise introduced in the up and down paths

c) uncertainties in the delay time in the ground equipment

d) uncertainties in the measurement of phase.

Of these four, uncertainties (c) can probably be largely calibrated out. Uncertainties (a) and (b) result from spacecraft operations while uncertainty (d) results from ground operations.

It is usual to assign most of the uncertainty allowance to the spacecraft portion of the system. Therefore, of the ± 150 meters total allowance, we will assign ± 116.7 meters to the spacecraft and only ± 33.3 meters to the phase measurement error contributions.

If we assume that we can measure phase to 2 degrees, then we can derive the appropriate "high" frequency value using the equation:

$$f_h = \frac{2}{360} \cdot \frac{C}{|\Delta_p|} \qquad (A.1)$$

where Δ_p = error in range due to phase
C = velocity of light
= 3×10^8 meters/second
f_h = "high" tone frequency.

hence

$$f_h = \frac{2}{360} \times \frac{3 \times 10^8}{33.3}$$

$$= 50 \text{ kHz}$$

One wavelength at this frequency corresponds to a range interval of about 3000 meters.

Now, of the ± 116.7 meters alloted to the spacecraft, we will assume that ± 66.7 meters are the result of noise contributions. The phase error corresponding to ± 66.7 meters error in 3000 meters is ± .1398 radians. This will be taken as the 3Δ error from which the rms phase error Δ becomes .0466 radians. The signal-to-noise ratio required to provide this maximum error is calculated from

$$S/N = (2\Delta^2)^{-1} \qquad (A.2)$$

therefore

$$S/N = (2 \times .0466^2)^{-1}$$

$$= 230$$

and, expressing the result in logarithmic form

$$(S/N) = 23.6 \text{ dB}$$

The remaining uncertainty ϵ of ± 50 meters is assigned to the variations in turn-around time, or "group delay time", in the spacecraft. The group delay time Δt_d is given by

$$\begin{aligned} \Delta t_d &= 2\,\epsilon/C \qquad (A.3) \\ &= 2(\pm 50)/3 \times 10^8 \\ &= \pm .33 \quad \text{seconds} \end{aligned}$$

This group delay characteristic is not too difficult to achieve in the command receiver.

Thus far we have considered the high frequency. Now, we must consider the lower frequencies. For complete freedom from ambiguity, the lowest frequency should be such that $\lambda/2$ is equal to the maximum range. For an orbit with an apogee of about 22,300 nautical miles, this frequency would be about 7.3 Hz.

In practice, however, we know much about the orbit which will help us to remove some of the ambiguity. Simple orbital observations via the VHF beacon emissions will permit the ranges to be resolved quite rapidly to the order of hundreds of kilometers. If, say, we pessimistically assume that the range ambiguity amounts to a distance of 1000 kilometers, then the lowest frequency that we need transmit is

$$f_a = 3 \times 10^8/2 \times 10^6$$

$$= 150 \text{ Hz} .$$

For safety sake, then, we will choose

$$f_a = 100 \text{ Hz} .$$

If we assume that the measurement errors for 100 Hz is the same as that for 50 kHz tones, then the total error due to the noise at the 3 σ level and the measurement error is 0.140 radians + 0.035 radians (2°) respectively, totalling 0.175 radians. If the next tone frequency is f_k, K times 100 Hz, the total phase error at 100 Hz corresponds to .175K radians at f_k. We require .175K be less than $\pi/2$ radians at f_k from which

$$K < 9 .$$

All frequency ratios between the tones must be K, therefore, for N tones intervals

$$K^N = \frac{50,000}{100}$$

and, for N = 4,

$$K = 4.73$$

while for N = 3,

$$K = 7.94$$

hence we can take an N of 3 yielding 4 tones at a frequency ratio 7.94 times. We may round off for convenience to these frequencies:

f_{r1} = 100 Hz
f_{r2} = 800 Hz
f_{r3} = 6,400 Hz
f_{r4} = 50,000 Hz

During transmission, however, the two lowest tones would be too close to the carrier thereby making recovery difficult. Therefore we up-convert the 100 Hz and the 800 Hz tone by mixing them with the 6,400 Hz tones and, by selecting the lower sidebands, produce 6300 Hz and 5600 Hz respectively.

This mixing action, however, degrades the signal-to-noise ratio characteristic by 3 dB. Therefore, we require 26.6 dB rather than 23.3 dB to meet the permissible tolerance.

To complete the analysis we will add a further equipment degradation allowance of 3 dB requiring that the minimum S/N ratio requirements be 29.6 dB. This level is for both up path and down path contributions. We will assume, to conserve spacecraft transmitter power, that only 10% of the noise contribution comes from the up path, therefore

$$(S/N)_{down} = 30.0 \text{ dB}$$

$$(S/N)_{up} = 39.1 \text{ dB}$$

are the required signal-to-noise ratios for the ranging tones in the down and up paths respectively.

APPENDIX B

COMMAND SIGNAL PROVISIONAL LISTING

A provisional listing of commands is given in this Appendix. They are arranged under the following headings:

1.	Command and Telemetry	Table B.1
2.	Spacecraft Control Systems	Table B.2
3.	Communications System	Table B.3
4.	Power System	Table B.4

The commands are classified as Type S, R, A or C according to their functional requirements. A detailed discussion of these classifications is given in section 3.2.6, but briefly we may summarize the results as follows:

Type S	Commands Set the spacecraft into a specific configuration.
Type R	Commands are commands which are sent in Real time and for these to be accepted a Type S command must first be transmitted.
Type A	Commands are reserved for Adjustment of the attenuator settings of the output TWT's of the communication system.
Type C	Commands are for transmitting Artificial Earth Pulses Continuously for long periods.

Table B.1

Command and Telemetry Subsystem Commands

Command	Type	Function	Notes
Select Encoder 1	S	Selects PCM Encoder 1	
Select Encoder 2	S	Selects PCM Encoder 2	
Telemetry Transmitter 1 Off	S	Turns off TX1, Turns on TX2	
Telemetry Transmitter 2 Off	S	Turns off TX2, Turns on TX1	
Telemetry Transmitters on	S	Turns on both transmitters	
Reset	S	Resets Decoder and Stored commands	1
Decoder 1 Standby	S	Places Decoder 1 on Standby	
Decoder 2 Standby	S	Places Decoder 2 on Standby	
136 MHz Beacon Off	S	Turns off 136 MHz Beacon	
136 MHz Beacon On	S	Turns on 136 MHz Beacon	

Command Totals: Type S - 10

Note: 1 Reset Command does not require separate execution.

TABLE B.2

Spacecraft Control Subsystems Commands

Command	Type	Function	Notes
Earth Sensor One Select	S	Selects Earth Sensor No. 1 for Single Earth sensor operation	
Earth Sensor Two Select	S	Selects Earth Sensor No. 2 for Single earth sensor operation	
Control Logic Normal	S	Normal mode dual earth sensor operation	
Despin Control channel change	S	Selects alternate channel electronics in MDA unit	
Artificial Earth Pulse Select	S	Replaces Earth Sensor pulses to CLA by Artificial Earth Pulses	1
Aritificial Earth Pulses	C	Real Time Earth Pulses	1
Select Valve Driver Set One	S	Applies main bus voltage to No. 1 bank of thruster coils	
Select Valve Driver Set Two	S	Applies main bus voltage to No. 1 bank of thruster coils	
Axial Thruster A	R	Operates axial thruster A solenoid valve	2,3
Axial Thruster B	R	Operates axial thruster B solenoid valve	2,3
Radial Thruster A	R	Operates radial thruster A solenoid valve	2,3
Radial Thruster B	R	Operates radial thruster B solenoid valve	1
Valve Drivers Off	S	Removes main bus voltage from thruster coil banks	
Activate Propellant System Cross Connect	S	Fires cross-connect squibs to interconnect tanks	
Apogee Motor Ignition	S	Fires Apogee Motor (also turns off T.WT high voltage)	
Ordnance Safe/Arm(back-up)	S	Energizes safe arm relay	

TABLE B.2 (Cont'd)

Command	Type	Function	Notes
Despin System Zero Bias Ready	S	Readies Despin System for bias adjustment	
Despin System Zero Bias Adjust	A	Adjusts zero bias despin system	

Notes: (1) Artificial Earth Pulses should be being transmitted to spacecraft before sending command "Artificial Earth Pulse Select".

(2) Requires prior transmission of Valve Drivers selection by type S command.

(3) Separate RESET command required after execution.

Command Totals:
Type R = 4
Type S = 12
Type C = 1
Type A = 1

TABLE B.3

Communications Subsystem Commands			
Command	Type	Function	Notes
TWT 1 On	S	Applies power to TWT 1	
TWT 1 Gain Adjust Ready	S	Readies TWT 1 Attenuator for Gain Adjustment	
TWT 1 Off	S	Removes power from TWT 1	
TWT 2 On	S	Applies power to TWT 2	
TWT 2 Gain Adjust Ready	S	Readies TWT 2 attenuator for Gain Adjustment	
TWT 2 Off	S	Removes power from TWT 2	
TWT 3 On	S	Applies power to TWT 3	
TWT 3 Gain Adjust Ready	S	Readies TWT 3 attenuator for Gain Adjustment	
TWT 3 Off	S	Removes power from TWT 3	
TWT 4 On	S	Applies power to TWT 4	
TWT 4 Gain Adjust Ready	S	Readies TWT 4 attenuator for Gain Adjustment	
TWT 4 Off	S	Removes power from TWT 4	
TWT 5 On	S	Applies power to TWT 5	
TWT 5 Gain Adjust Ready	S	Readies TWT 5 attenuator for Gain Adjustment	
TWT 5 Off	S	Removes power from TWT 5	
TWT 6 On	S	Applies power to TWT 6	
TWT 6 Gain Adjust Ready	S	Readies TWT 6 attenuator for Gain Adjustment	
TWT 6 Off	S	Removes power from TWT 6	
TWT Gain Adjust - dB	A	Adjusts TWT Gain by dB	
TWT Gain Adjust - dB	A	Adjusts TWT Gain by dB	
TWT Gain Adjust - dB	A	Adjusts TWT Gain by dB	
TWT Gain Adjust - dB	A	Adjusts TWT Gain by dB	
TWT Gain Adjust - dB	A	Adjusts TWT Gain by dB	
TWT 1 Standby-Select TWT 7	S	Replaces TWT 1 by TWT 7	
TWT 2 Standby-select TWT 7	S	Replaces TWT 2 by TWT 7	
TWT 3 Standby-select TWT 8	S	Replaces TWT 3 by TWT 8	
TWT 4 standby-select TWT 8	S	Replaces TWT 4 by TWT 8	
TWT 5 standby-select TWT 9	S	Replaces TWT 5 by TWT 9	
TWT 6 standby-select TWT 9	S	Replaces TWT 6 by TWT 9	

TABLE B.3 (Cont'd)

Command	Type	Function	Notes
Driver TWT 1 On	S	Applies power to Driver TWT 1	
Driver TWT 1 Off	S	Removes power for Driver TWT 1	
Driver TWT 2 On	S	Applies power to Driver TWT 2	
Driver TWT 2 Off	S	Removes power for Driver TWT 2	
Select Driver TWT 1	S	Selects Driver TWT 1	
Select Driver TWT 2	S	Selects Driver TWT 2	
TDA 1 On, TDA 2 Off (Preamp)	S	Applies voltage to Preamp TDA 1 Selects TDA 1	
TDA 2 On, TDA 2 Off (preamp)	S	Applies voltage to Preamp TDA 1 Selects TDA 2	
LO 1 On, LO 2 Off	S	Applies power to LO 1, selects LO 1	
LO 2 On, LO 1 Off	S	Applies power to LO 2, selects LO 2	
TDA 1 On, TDA 2 Off (driver)	S	Applies power to Driver TDA 1, Selects TDA 1	
TDA 2 On, TDA 1 Off (driver)	S	Applies power to Driver TDA 2, Selects TDA 2	
Receiver - Off	S	Removes power from communications systems, excluding TWT's	
TWT HV Timer Override	S	Bypasses Timer Delay circuit in TWT High Voltage supply in case of delay circuit failure	

Command Totals: Type S - 38
Type A - 5

TABLE B.4

Power Subsystem Commands

Command	Type	Function	Notes
Battery Off - Recondition/Charge	S	Removes Battery Off Bus - Places on Charge	
Battery Off - Recondition/Discharge	S	Removes Battery Off Bus - Places on Discharge	
Battery On	S	Places Battery on Charge	
Select Converter 1	S	Selects Converter 1	
Select Converter 2	S	Selects Converter 2	
Current Monitors On	S	Applies Power to current monitors	
Current Monitors Off	S	Removes power from current monitors	
Undervoltage Control Normal	S	Allows undervoltage control to remove selected loads during low primary bus voltage conditions	
Undervoltage Control Override	S	Inhibits Normal Condition	

Command Totals: Type S - 9

APPENDIX C

COMPARISON OF PDM AND FSK TONE DIGITAL SYSTEMS

The Pulse Duration Modulation (PDM) and the Frequency Shift Key System (FSK) techniques for the tone digital system are considered in detail to demonstrate the advantages of the latter technique for the command system.

General Considerations

The optimum signal power, for a certain probability rate will first be calculated for both techniques, assuming that the digital message is transmitted by on-off bursts of a tone (sine wave), and is received by an envelope detector.

In our case, the required bit error rate probability has been established as 1 x 10^{-6}, and it is assumed that the noise at the receiver input is gaussian.

Error rates will depend on how the decision is made at the envelope detector output. In our case we consider that a simple threshold detector is set for a threshold level of E_c. Thus when the envelope exceeds the value 'E_c', it is judged to be a mark, otherwise it is called a space. Hence two types of errors can occur

Type a) Send 'Mark' and receive 'Space'
Type b) Send 'Space' and receive 'Mark'

Bennet (Ref.C.1) shows that the conditional probabilities for the two kinds of errors can be given by

$$P_a = \exp\left(-\frac{A^2 + E_c^2}{2 \quad 2}\right) \sum_{n=1}^{\infty} \left(\frac{E_c}{A}\right)^n I_n \left(\frac{A^2}{\Delta^2}\right) \qquad (C.1)$$

$$P_b = \exp\left(-\frac{E_c^2}{2\Delta^2}\right) \qquad (C.2)$$

P_a = Probability of error type a

P_b = Probability of error type b

A = Amplitude of 'Mark'

E_c = Threshold level

Δ = Mean noise amplitude

$I(m)$ = mth order modified Bessel Function

If the numbers of 'Marks' and 'Spaces' are equal, and if $E_c = A/2$, then the probability of type 'a' errors will be less than that of type 'b'. However the two types of errors can be made equally probable by setting the threshold level at slightly more than half the peak carrier amplitude. This has been done by Bennet (Ref. C.1).and the results appropriate for our requirements are reproduced here in Figure C.1.

b) F.S.K System

In this system, "1"s and "0"s are distinguished by using two separate tone frequencies. Further the tone burst representing the 'Mark' is only present for half of the bitwidth. A simplified block diagram of this system is given in Figure C-2.

If the two tone frequencies are 'f_1' and 'f_2' and the bit rate is 'F' then the filter bandwidths required are $f_1 \pm 3F$ and $f_2 \pm 3F$ in order to pass both the fundamental and the third harmonic of the tones.

We will consider a return to zero signal with a 50% mark to space ratio and with equal probability of "1"s and "0"s. Then a bit error probability of 1×10^{-6} at the decoder is realized for a bit error probability of 2.5×10^{-7} for each of the command tone pulses, with the detector set for 52% of the peak signal amplitude. From Figure C-1 this corresponds to a 'on state' signal-to-noise power ratio of 17.45 dB, or, for an average signal energy per bit to average noise energy per bit, of 14.45 dB where the filter bandwidth is 6F.

c) P.D.M. System

This system requires only one tone to transmit both '1' and '0', however, the duration of the tone burst is varied to distinguish between the two. We will consider the case of return to zero signal where the tone burst for a '1' is half of a bit width and for the '0' it is one quarter of a bit width. A simplified block diagram for this system is shown in Figure C-3.

For comparison purpose, we assume that the bit detector is ideal, hence does not introduce additional errors.

To achieve a bit error probability of 1×10^{-6} at the decoder the required error probability of the '0' signal is 2.5×10^{-7}. From the graph (Figure C-1) the 'on state' signal-to-noise required is 17.45 dB, or the average signal energy per bit to average noise ratio required is 11.45 dB with a filter bandwidth of 12F.

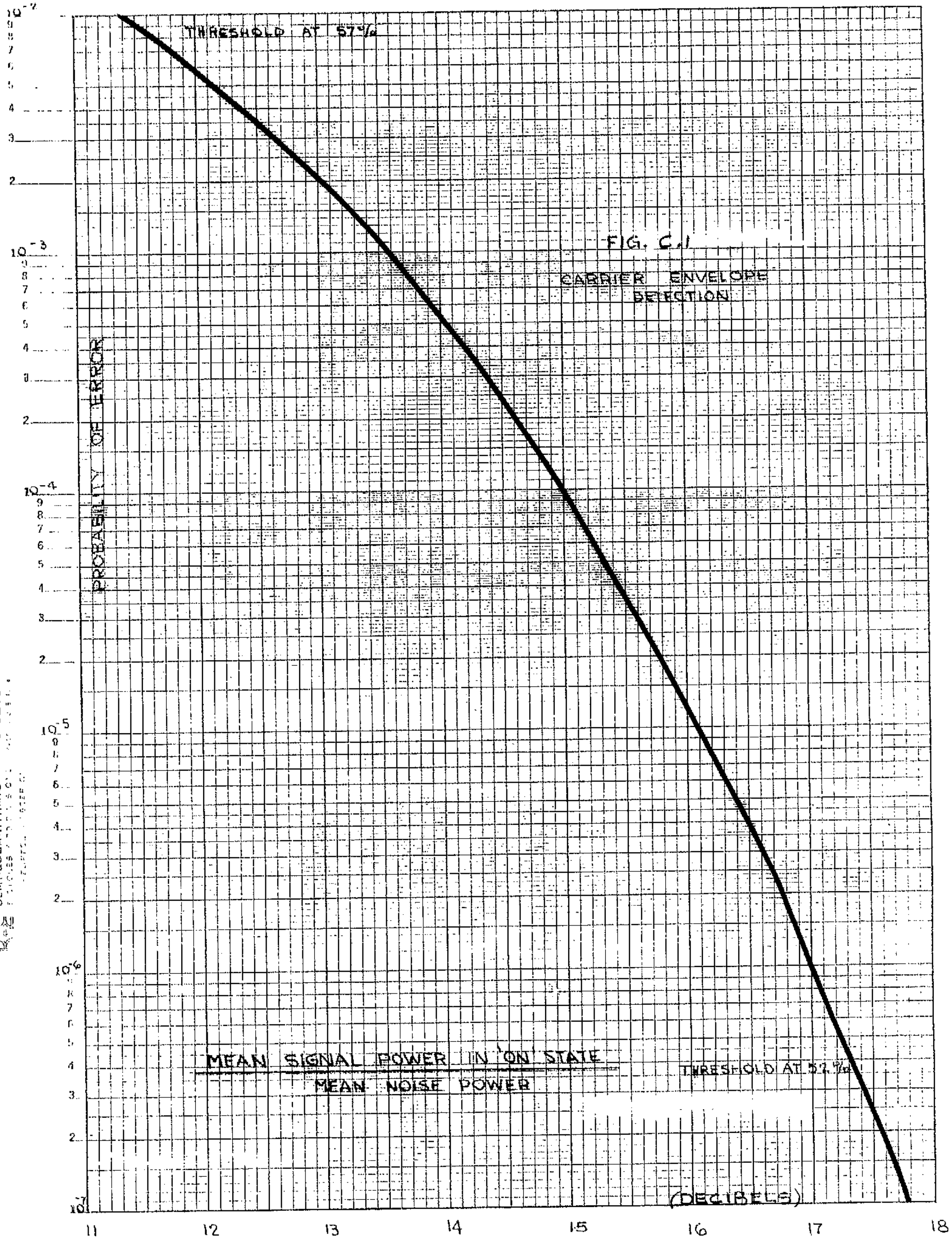


FIG. C.1

CARRIER ENVELOPE DETECTION

FIG. C.2

BLOCK DIAGRAM FSK SYSTEM

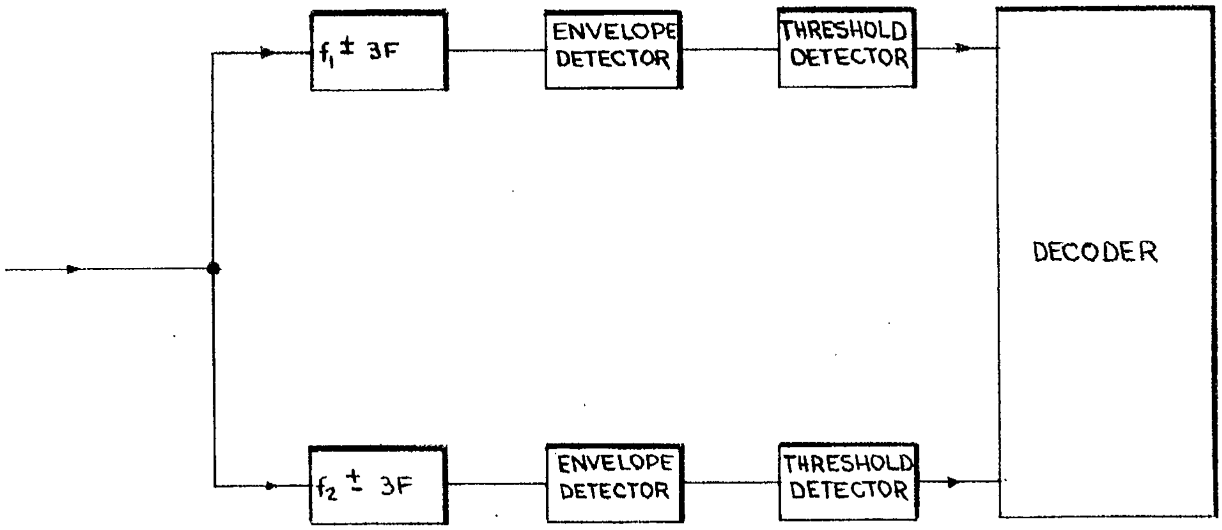


FIG. C.3

BLOCK DIAGRAM OF PDM SYSTEM

The required error probability of the '1' signal is 0.5×10^{-7} to achieve a bit error probability of 1×10^{-6}. Thus from the graph (Fig. C-1)the 'on state' signal to noise ratio required is 17.2 dB, or the average signal energy per bit to average noise ratio requires is 14.2 dB with a bandwidth of 12F.

Thus in the P.D.M. System to achieve 1×10^{-6} bit error probability the minimum average signal energy per bit to average noise ratio required is 14.2 dB with a filter bandwidth of 12F and an ideal '1', '0' detector.

d) Comparison of F.S.K. and P.D.M. Systems

We note the following:

1) If 'No' is the average noise power in a unit bandwidth of 'F' Hz then the minimum signal to noise requirement is,

for FSK, $S/No = 14.5 + 7.8 = 22.3$ dB

for PDM, $S/No = 14.2 + 10.8 = 25.0$ dB

2) The P.D.M. System requires a pulse width detector to distinguish between '1' and '0' where as in F.S.K this is achieved by using two tones.

The above two reasons favor the F.S.K. technique; hence we choose F.S.K. for the command system.

Reference

C.1 Bennett, W.R.: "Electrical Noise" McGraw-Hill Book Company, Inc. New York, 1960

APPENDIX D

1.. COMMAND SIGNAL BANDWIDTH OPTIMIZATION

Introduction

In this appendix the optimum frequency deviations for the operational commands, the ranging tones and the subcarrier are derived and used to calculate the receiver noise bandwidth. The latter portion deals with the Command System RF link. The various parameters of the RF equation are defined and the command transmitter power is calculated.

We begin with a discussion of the requirements of each signal and then combine them optimumly.

a) Operational Commands

Since only one of three tones is present at one time we shall consider only the highest frequency tone of 17 kHz, the assumed bit rate of 128 bits/sec and the required average signal energy per bit to noise ratio required is 14.5 dB in a bandwidth of 6 times the bit rate (from Appendix E).

Over a 5 year period a certain degradation in the system due to components and filter drifts can be expected. We will allow 6 dB for equipment degradation and 2 dB for filter drift.

We want the commands to be accepted with reasonable accuracy 5 dB below the receiver threshold. Below the receiver threshold the loss in signal power is of the order of 3 dB per dB loss in carrier power. Adding up all the allowances leads to a minimum S/N requirement at 10 dB C/N of 37.5 dB. We shall round this off to 40 dB for convenience.

Now the standard FM equation is

$$S_t/N = (C/N)\ M_t^2\ \frac{B}{2b_t} \qquad (D.1)$$

where M_t = modulation index for command tones
= $\Delta ft/ft$
f_t = highest frequency command tone
= 17 kHz
$\Delta\ f_t$ = Peak deviation due to command tone
B = receiver noise bandwidth
b_t = command signal bandwidth
= 6 x bit rate = 850 Hz

If Δf_p is the peak carrier deviation due to all the signals present in the baseband, then by Carson's rule

$$B = 2(\Delta f_p + f_m)$$

where f_m is the highest baseband frequency of 50 kHz.

Then $$B = 2(\Delta f_p + 50 \times 10^3). \qquad (D.2)$$

Substituting the values in the FM equation and assuming a 10 dB C/N ratio we find that

$$(\Delta f_t)^2 \ (B/2) = 2.46 \times 10^{14} \qquad (D.3)$$

b) Ranging Tones

The minimum signal to noise ratio required for these is 40 dB in 1 Hz output bandwidth. From the standard F.M. equation

$$S_r/N = C/N \ M_r^2 \ B/2b_r \qquad (D.4)$$

where M_r = Modulation index for the highest range tone $= \dfrac{\Delta f_r}{f_r}$

f_r = highest range tone

= 50 kHz

Δf_r = Peak deviation due to range tone

b_r = range tone bandwidth
= 1 Hz

By substituting the known values we find

$$(\Delta f_r)^2 \ (B/2) = 2.5 \times 10^{12} \qquad (D.5)$$

c) Sub-carrier modulated by Artificial Earth Pulse (A.E.P)

The bandwidth allowed for these pulses is 250 Hz and the minimum signal to noise ratio is 37 dB.

For optimum condition the receiver threshold should occur simultaneously with that of the S.C.O. discriminator. Then for the S.C.O. discriminator alone we can write

$$S_c/N = \frac{3S_c}{N} \quad M_a \quad \frac{B_s}{2b_a} \qquad (D.6)$$

where

S_c/N = signal to noise ratio at the input of the discriminator
= 11 dB at threshold

M_a = Modulation index for the A.E.P.

B_1 = Discriminator noise bandwidth

= $250\,(M_1 + 1) \times 2$ by Carson's rule

b_a = signal bandwidth

= 250 Hz.

Solving for M_a we have

$M_1 = 4.8.$

For the complete system we write

$$S_a/N = \frac{3}{2}\,(C/N)\;M_s^2\;M_a^2\;\frac{B}{2b_a} \qquad (D.7)$$

where M_s = modulation index due to S.C.O.
= $\Delta f_s/ f_s$

f_s = S.C.O. frequency
= 22 kHz

Δf_s = peak deviation due to S.C.O.

By substituting the known values we get

$$(\Delta f_s)^2 \quad \frac{B}{2} = 1.4 \times 10^{12} \qquad (D.8)$$

d) Bandwidth and Frequency Deviations

Solving the equations (D.2),(D.3), (D.5), and (D.8) we find

$\Delta f_r = 2.12$ kHz
$\Delta f_s = 1.6$ kHz
$\Delta f_t = 21.0$ kHz
$\Delta f_p = 24.72$ kHz

Substituting the value of Δf_p in (D.2) we get

$$B = 2(24.72 + 50) = 149.44 \text{ kHz}$$

In Section 3.4.3, the receiver study shows that a minimum of 220 kHz bandwidth tolerance is required to allow for the stability and drift over a period of 5 years. Therefore,

Bandwidth for signal spectrum	= 149.44 kHz
Bandwidth tolerance for drift	= 220.0 kHz
Manufacturing tolerance 5%	= 22.0 kHz
Thus total 3 dB Bandwidth	= 391.44 kHz.

Therefore the receiver noise bandwidth = 1 .1 x 3 dB bandwidth

= 431 kHz

2. RF LINK

General

The basic Command System RF link is given in Figure 3-6. The command transmitter output power P_c is defined at the antenna terminal and is thus assumed to include the effects of transmission line losses. The command antenna gain G_c is referred to an isotropic antenna.

RF Equation

The required transmitter power P_c at the earth antenna terminal is given by the following equation:

$$P_c = P_r - G_c - G_r + L_p + P_a \qquad \text{(D.9)}$$

where

P_r = Minimum carrier power required

G_c = Transmitting Antenna Gain (60 ft parabola)

G_r = Receiving Antenna Gain

L_p = Path Loss

P_a = Atmospheric Loss

Required Carrier Power at Receiver (P_r)

All our earlier calculations are based on a carrier to noise power ratio of 10 dB at threshold. In view of the change in path losses due to rain and other atmospheric conditions, and small antenna pointing errors we will allow 10 dB for fading.

Equivalent Noise power at the input of receiver = KTBF

where K = 1.38×10^{-23} Boltzman Constant

T = 290°K Absolute Temperature

B = 431 kHz Receiver noise bandwidth

F = 15 dB Receiver system noise figure (see receiver system)

Therefore the equivalent receiver input noise power = -132.7 dBw

Therefore, the carrier power required for 10 dB carrier to noise ratio is

$$\begin{aligned} C &= (-132.7 + 10) \text{ dBw} \\ &= -112.7 \text{ dbw} \end{aligned}$$

Earth Antenna Gain (G_c)

The earth station antenna is assumed to be a 60 feet parabolic dish. For the command frequency of 6.425 GHz the gain of the antenna is 59.0 dB for 50% efficiency.

Receiving Antenna Gain (G_r)

The satellite antenna is presently visualized to be of biconical design. This antenna radiates a "doughnut" shaped pattern about the axis of symmetry. Since the antenna is located on the spacecraft so that its axis of symmetry coincides with the axis of rotation of the spacecraft, the radiated pattern is strongest about the satellite's equatorial plane. Thus, when the satellite is "on station" the command antenna will be most efficient in receiving the signals.

Whilst the antenna gain in the direction of the earth is slightly greater than isotropic, we will ignore the difference. This will permit accommodation of possible small "nulls" in the antennas radiation pattern, and will also provide a safety factor. Thus, we shall assume that

$$G_r = 0 \text{ dB}$$

Path Loss

The path Loss L_p is obtained by standard definition. The equation for path loss calculations is

$$L_p = 20 \log \frac{4\pi S}{\lambda}$$

where

L_p = path loss expressed in decibels

S = slant range of satellite S from ground station G
= 22,300 n miles

λ = signal wavelength

Therefore $L_p = 200.7$ dB

Atmospheric Loss (P_a)

The radiated command signal suffers a slight attenuation whilst passing through the atmosphere. This is quite small, but in recognition of this we shall assume that 1 dB of signal is lost. Note that this is under standard weather conditions. The effects of rain or other atmospheric effects will be considered via "fading".

Transmitter Power Required

Substituting the values in the RF equation (D.9) we have

$$P_c = -112.7 - 59.0 - 0 + 200.7 + 1 \text{ dBw}$$

$$\doteq 30 \text{ dBw}$$

$$= 1 \text{ kW}$$

APPENDIX E

TELEMETRY CODING SELECTION

Pulse Amplitude Modulation (PAM), Pulse Duration Modulation (PDM) and Pulse Coding Modulation (PCM) techniques for multiplexing of the sampled telemetry data appear most promising. We shall in this appendix consider each technique separately and then make a decision as to which is the most appropriate for our purposes. We begin with PAM.

a) Pulse Amplitude Modulation Considerations

With PAM, monitor signals are sampled over a short interval of time and the value of the signal more or less averaged over the sample interval is represented by the amplitude of the pulse. Basic sampling theory shows that the sampling rate must exceed twice the bandwidth for proper sampling.

Analogue samples in a PAM system may have any amplitude within the measurement range. Flags, on the other hand, will only have either the minimum or maximum amplitude. The simplest way to transmit both analogue samples and flags over a given PAM link is to provide equal sample weightings so that a PAM sample will be either an analogue sample or a flag. Although simple, this method is quite wasteful of spectrum utility since the information carried by an analogue sample is much greater than that carried by a flag, but the spectrum requirements for each are the same.

More efficient use of the spectrum could be obtained by multiplexing several flags into one PAM sample. Typically, several flag signals with geometrically adjusted amplitudes are added together so that the resultant amplitude is an unambiguous function of the number and distribution of the flag states. As an example, if four flags are to be multiplexed into a single PAM sample, then their amplitudes may be adjusted in the ratio 1:2:4:8 giving a possible 16 discrete amplitude levels (including the zero level) covering all possible flag combinations.

The multiplexing technique, while being more efficient in spectrum usage, is more complex and requires additional (although simple in concept) multiplexing in the encoder section of the telemetry link, and must be processed further in the decoder section before display. One of the desirable features of the flags will thus be lost: simplicity.

b) Pulse Duration Modulation Considerations

With Pulse Duration Modulation (PDM) the width of a pulse is proportional to the sample value of an analogue signal. But PCM is inferior to PAM in efficiency (information rate vs transmitter power) and therefore will increase telemetry transmitter power requirements. With current multiplexing and demultiplexing techniques, PAM can be handled as easily as PDM although this was not the case in the past. (See ref. E.1). We see little to recommend PDM over PAM.

c) Pulse Code Modulation System

In a Pulse Code Modulation (PCM) system, the magnitude of an analogue sample to be transmitted is first adjusted, or "quantized" to the closest level of a set of discrete values. This set of discrete values is then transmitted in binary form. For an accuracy of 2% of full scale, we would require 6 bits of information per sample.

Considering flags, we note that these may have only one of two possible values. Therefore, we require only one bit to transmit a flag sample.

PCM thus can be easily optimized for maximum efficiency since the bit assignments can be adjusted in accordance with the signal requirements.

d) Sample Rates

The choice of coding is between PAM and PCM. PAM is ordinarily favored for low capacity systems whilst PCM is favored for high capacity systems. Since we have a medium capacity system, either PAM or PCM appears practicable. Let us therefore proceed one step further and determine what sample rates are practical or tolerable from the sampled signals. We start with the housekeeping signals.

The housekeeping signals are used to telemeter parameter values such as battery voltage, etc. Ordinarily, the values do not change very rapidly therefore the sample rates can be relatively slow. The only possible problem is in measuring transients, if indeed measuring of transients is considered desirable. Transients occurring on voltage or current lines are usually relatively sharp therefore monitoring of these becomes quite prohibitive in a sampled system. We are led therefore, to consider sample rates of analogue systems on the basis of normal monitoring only. Assuming an interval based on 2^n times one second, we feel that a 32 second sample rate for analogue signals is reasonable.

Let us now consider flags. When we change the spacecraft from one operational state to another, it is the change of flag status that confirms it. Operationally, therefore, we would like to know the flag status relatively soon after a command is sent and accepted.

The exact delay tolerance depends on the individual but a 4 second (maximum) lag between acceptance of a command by the spacecraft, and verification on the ground by flag seems reasonable.

We also find it convenient to allow that the spacecraft signature be telemetered once every four seconds. This will help to simplify the encoder requirements.

Insofar as the command verification is required, we assume that of all the telemetered signals, this is the one that is required with the least delay. We shall therefore assume that the rate of transmission of the verified command is in 1 second intervals. On the average, then, we would receive verification of command in 1/2 second plus the turn around time, or slightly less than 1 second after the execute command has been sent.

e) Number of Sampled Channels

With the sampling rates established we may now proceed to develop the telemetry formats by determining the numbers of analogue, flag, etc. signals and to weigh their influence on the signal in accordance with the sample rates. A provisional listing of telemetry channels is given in Appendix F. From this listing we can convert the requirements as appropriate for both PAM and PCM.

f) Coding Selection

We will first determine the possible frame configuration and bandwidth requirement for a simple PAM scheme, then for a 6 bit PCM scheme, and will select the most appropriate of the two.

Case 1 - PAM

If we consider a thirty-two second interval, the number of analogue samples within that interval totals 88, while the number of flags, including verification and signature, totals $(73 + 5) \times 8 + 14 \times 32$ or 1072. Since both analogue samples and flags have equal weightings, the minimum number of PAM samples is 1160 resulting in a sample rate of about 36 per second. Since it is convenient for sample rates to correspond to multiples of 2, and to provide for expansion (since this is still early in the development of the system) we shall assume a sample rate of 40 per second.

All data will have been transmitted once every 32 seconds so the total number of samples per main frame is 32 x 42 or 1280. We thus have a reserve of 120 samples per main frame.

From the telemetry channel requirements we see that many flags will be transmitted every 4 seconds. This rate suggests itself as a reasonable sub-frame rate. Thus there will be 8 sub-frames per main frame, and 160 PAM samples per sub-frame.

The 14 command verification signals are scheduled to be sampled once per second. To fit this schedule into a sub-frame, these samples can be super-commutated 4 times thus utilizing 56 of the 160 sub-frame samples. Of the remaining 104 sub-frame samples, 78 are required for the 4 second interval flags and signature, leaving 26 for analogue samples, synchronization and spares. The 88 analogue samples will be subcommutated so that only 11 sub-frame samples are required. The synchronizing requirements will be two PAM sub-frame samples whilst the sub-frame identification will require third. Thus we utilize 14 more sub-frame samples, leaving 12 as spares.

The spare samples can now be apportioned. We increase the capacity of the 4 second flags from 73 to 80 and the analogue samples from 11 to 16 or 48 of the available spare samples. We will not increase the capacity for verification or signature signals.

A typical 160 sample PAM sub-frame is shown in Figure E-1, arranged into a 16 x 10 format for convenience.

Case 2 - PCM

For our purposes, we shall consider a basic PCM system using 6 bits per analogue channel and 1 bit per flag channel.

From the channel listings, we require 88 analogue samples in a 32 second interval; therefore the number of bits is 88 x 6 or 528 in a 32 second interval.

The required number of bits for flags, verification and signature is 1072 in a 32 second interval. Therefore the total number of bits is 1600 per 32 seconds so that the average basic bit rate R is 50 bits per second.

For practical purposes, and to allow for expansion, synchronization and so forth,we can assume that

$$R = 72 \text{ bits/second}$$

giving rise to a word rate of 12 words/second.

SAMPLE RATE 40 SAMPLES/SECOND

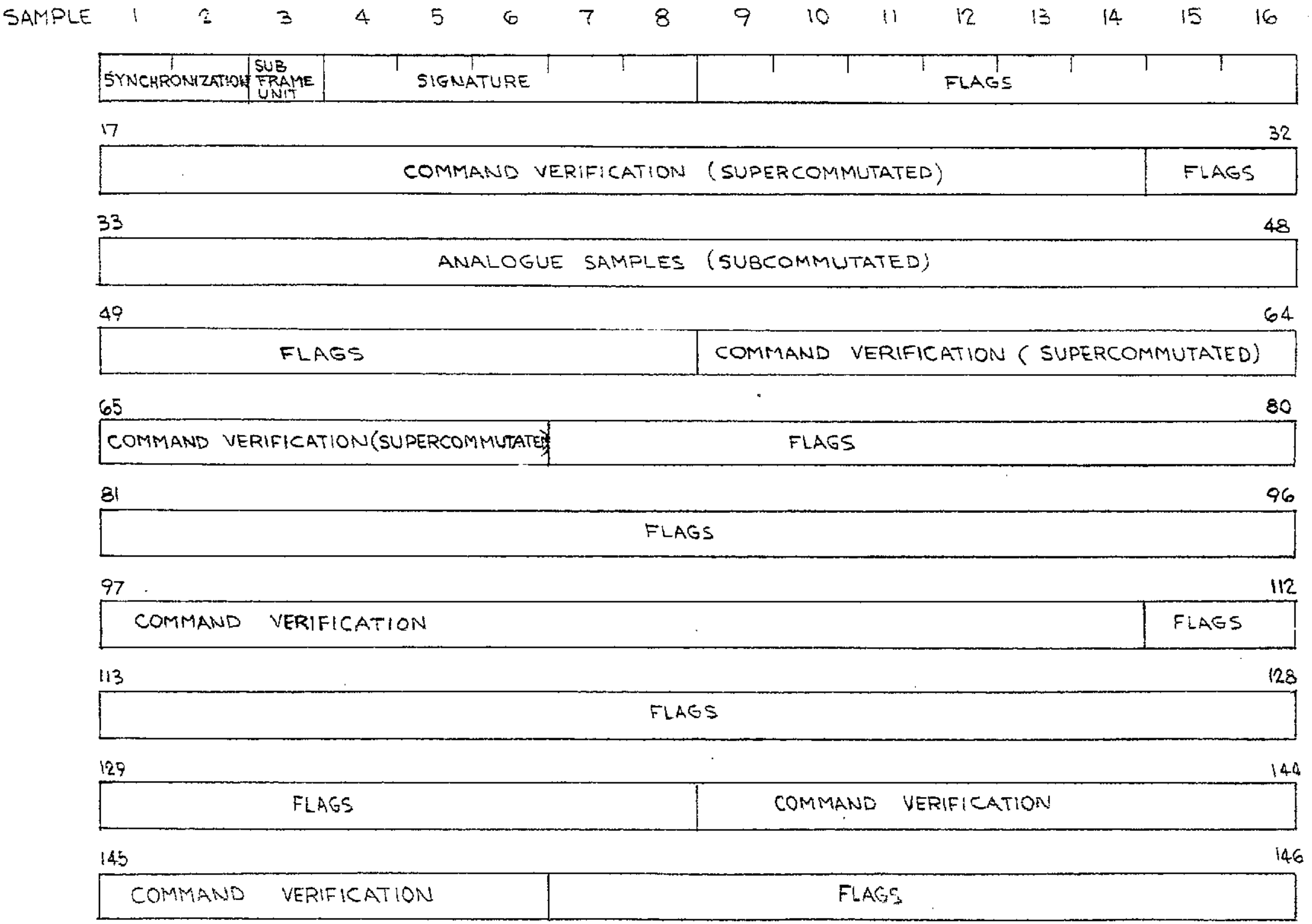


FIG. E.1
TYPICAL PAM SUBFRAME FORMAT

Since the maximum sample interval is 32 seconds, that will also be the main frame interval. The basic flag sample interval is 4 seconds which we choose for sub-frame interval, hence there will be 8 sub-frames per main frame. The 1 second intervals can be achieved by super-commutating four times within a sub-frame.

Therefore

No. of words per sub-frame = 48

No. of words per major frame = 384

Now, we add synchronizing and frame identification. In each sub-frame the first two PCM words are used for synch, and the third for sub-frame identification. (Only 3 bits of the 6 available are required to identify 8 sub-frames hence the remaining 3 bits are available as spares).

Each sub-frame now comprises, by "rounding off" for command verification and spacecraft signature*

Synch + sub-frame identification	= 3 words
Verification (3 x 4)	= 12 words
Spacecraft Signature	= 1 word
4 sec. Flag samples	= 12 words
32 sec. Analogue samples	= 11 words
Spare words	= 9 words
Total	48 words

A PCM sub-frame format based on this distribution is shown in Figure 3-7 of section 3.3.4. The words are arranged into an 8 x 6 matrix for convenience.

d) Comparison

Nichols and Rauch (ref. E.1) have shown that for PAM a channel bandwidth about 2.7 times the sample rate is required if crosstalk is to be held to 40 dB or better. Since our assumed sample rate is 40 per second, we would require a channel bandwidth of about 108 Hz.

---

* 14 bits are required for command verification. We shall assume that 3 PCM words will be used, the remaining 4 bits are available as spares. In a similar manner we assume 1 PCM word is used for the 5 spacecraft signature bit.

In PCM, however, the channel bandwidth need not be much greater than the sample rate, and since we assume 72 samples per second, we require a channel bandwidth slightly more than 72 Hz. Thus the channel bandwidth requirements are greater for PAM.

For a 6 bit PCM system, the required S/N ratio for an error of 1 in 10^5 bits is 13.3 dB, using non coherent detection. For a PAM system with an error of 1.6% RMS (roughly equivalent to the above 6 bit PCM system) a S/N ratio of about 23.5 dB is required. Thus, the PAM system would require about 9 dB more transmitter power than would the PCM system.

Thus we conclude that the PCM system is more preferable to the PAM on S/N considerations alone. When we add the considerations that much of the data is binary in any case, then the advantage of PCM is very clear, indeed.

References

E.1 Nichols and Rauch: "Telemetry Systems" John Wiley and Sons, Inc., New York, 1954.

APPENDIX F

PROVISIONAL SAMPLED TELEMETRY CHANNEL LISTING

A provisional listing of sampled Telemetry Channel Listing is given in this Appendix. These are arranged under the following headings

1) Command and Telemetry - Table F.1

2) Spacecraft Control - Table F.2

3) Communication System - Table F.3

4) Power Subsystem - Table F.4

The commands are classified as Analogue, Flag, Signature or Verify. A detailed description of these classifications is given in section 3.3.2 but briefly we may summarize them as follows

Analogue:	These signals are general housekeeping signals and carry information as to the value of the parameter telemetered (such as battery voltages, etc.)
Flag:	These signals indicate on/off or connected/disconnected status of units by means of a single binary digit.
Signature:	These signals, consisting of binary number sequence are used to identify a particular spacecraft.
Verify:	These signals permit the operator to verify that the command has been understood correctly by the spacecraft's decoder.

TABLE F.1

PROVISIONAL SAMPLED TELEMETRY CHANNEL LISTING for the COMMAND AND TELEMETRY SUBSYSTEM

Parameters	Sampling Interval	No. of Measure-Ments	Signal	Units	Range
Command Receiver 1 AGC	32 sec.	1	Analogue	Volts	0 to 5
Command Receiver 2 AGC	32	1	Analogue	Volts	0 to 5
Receiver Temperature	32	1	Analogue	°F	-30 to +150
Transmitter 1 RF Power	32	1	Analogue	MW	0 to 600
Transmitter 2 RF Power	32	1	Analogue	MW	0 to 600
Transmitter Temperature	32	1	Analogue	°F	-30 to +150
Encoder 1	4	1	Flag	-	-
Encoder 2	4	1	Flag	-	-
Transmitter 1	4	1	Flag	-	-
Transmitter 2	4	1	Flag	-	-
Command Verification Encoder 1	1	7	Flag	-	-
Command Verification Encoder 2	1	7	Flag	-	-
Full Scale Calibration	32	1	Analogue	Volts	-
Zero Scale Calibration	32	1	Analogue	Volts	-
Spacecraft Signature	4	5	Flag	-	-
Command Decoder 1 Standby	4	1	Flag	-	-
Command Decoder 2 Standby	4	1	Flag	-	-

Telemetry Sample Totals: -			
	8 Analogue	@	32 seconds
	6 Flags	@	4 seconds
	14 Verify	@	1 second
	5 Signature	@	4 seconds

TABLE F.2

PROVISIONAL SAMPLED TELEMETRY CHANNEL LISTING FOR THE SPACECRAFT CONTROL SUBSYSTEM

Parameter	Sampling Interval	No. of Measure-Ments	Signals	Units	Range
Motor Current	32 sec.	1	Analogue	Amperes	0 to 2
Motor Temperature	32	1	Analogue	°F	0 to 200
Control Channel Select Status	4	2	Flag	-	-
Despin Motor Center Bias	4	7	Flag	volts	
Thruster Temperature	32	4	Analogue	°F	40 to 1000
Propellant System A Tank Pressure	32	1	Analogue	PS1	0 to 600
Propellant System B Tank Pressure	32	1	Analogue	PS1	0 to 600
Apogee Motor Temperature	32	1	Analogue	°F	40 to 1000
Heat Shield Temperature	32	1	Analogue	°F	-300 to +600
Booster Separation	4	1	Flag	-	-

Telemetry Sample Totals:			
	10 Analogue	@	32 second intervals
	10 Flag	@	4 second intervals

TABLE F.3

PROVISIONAL SAMPLED TELEMETRY CHANNEL LISTING FOR THE COMMUNICATIONS SUBSYSTEM

Parameters	Sampling Interval	No. of Measure-Ments	Signal	Units	Range
Preamp TDA 1 Bias Voltage	32 sec.	1	Analogue	mv	0 to 250
Preamp TDA 2 Bias Voltage	32	1	Analogue	mv	0 to 250
LO 1 Power Output	32	1	Analogue	mw	
LO 2 Power Output	32	1	Analogue	mw	
Driver TDA 1 Bias Voltage	32	1	Analogue	mv	0 to 250
Driver TDA 2 Bias Voltage	32	1	Analogue	mv	0 to 250
Power and Drive TWT -					
Helix Current	32	11	Analogue	ma	0 to 500
Heater Current	32	11	Analogue	ma	0 to 500
Collector Current	32	11	Analogue	ma	0 to 500
RF Power Output (Relative)	32	11	Analogue	dbw	20
Temperature	32	11	Analogue	°F	-30 to +200
Preamp TDA 1 On	4	1	Flag	-	-
Preamp TDA 2 On	4	1	Flag	-	-
LO 1 On	4	1	Flag	-	-
LO 2 On	4	1	Flag	-	-
Driver TDA 1 On	4	1	Flag	-	-
Driver TDA 2 On	4	1	Flag	-	-
Driver TWT 1 On	4	1	Flag	-	-
Driver TWT 2 On	4	1	Flag	-	-
Main TWT's On	4	6	Flag	-	-
Standby TWT's On	4	3	Flag	-	-
TWT 1 Gain Setting	4	7	Flag	-	-
TWT 2 Gain Setting	4	7	Flag	-	-
TWT 3 Gain Setting	4	7	Flag	-	-
TWT 4 Gain Setting	4	7	Flag	-	-
TWT 5 Gain Setting	4	7	Flag	-	-
TWT 6 Gain Setting	4	7	Flag	-	-

Telemetry Sample Totals: 61 Analogue @ 32 second intervals

59 Flags @ 4 second intervals

TABLE F.4

PROVISIONAL SAMPLED TELEMETRY CHANNEL LISTING FOR THE POWER SUBSYSTEM

Parameters	Sampling Interval	No. of Measure-Ments	Signal	Units	Range
Battery Temperature	32 sec.	1	Analogue	°F	-30 to +150
Battery Voltage	32	1	Analogue	volts	16 to 33
Shunt Voltage Monitor	32	1	Analogue	volts	0 to 20
Main Bus Voltage	32	1	Analogue	volts	16 to 35
Main Bus Current	32	1	Analogue	amps	0 to 20
Converter 1 Volts	32	1	Analogue	Volts	0 to 32
Converter 2 Volts	32	1	Analogue	Volts	0 to 32
Solar Array Temperature	32	1	Analogue	°F	-175 to +100
Battery Current	32	1	Analogue	amps	-30 to +10
Converter 1 Status	4	1	Flag		
Converter 2 Status	4	1	Flag		
Battery on Charge	4	1	Flag		
Battery on Discharge	4	1	Flag		
Battery Current Sign	4	1	Flag		

Telemetry Sample Totals: 9 Analogue Channels @ 32 second intervals
5 Flags @ 4 second intervals

APPENDIX G

VHF BEACON PERFORMANCE

A Beacon transmission in the 136 MHz band is recommended to ensure a relatively simple and quick acquisition following launch, and continuous tracking in the parking or transfer orbits.

For the purpose of establishing the transmitter power, we can assume that either the NASA Minitrack slot array or a 9 element Yagi antenna will be used. Using the data available (Space Tracking and Data Acquisition Network Facilities Report, Dec. 1965) we list in table G.1 the relevant parameters for the two antenna systems.

Table G.1

Tracking Antenna Gains and Polarization Losses

	Slot Array	9 Element Yagi
Antenna Gain, G_R	16.3 dB	19.2 dB
Polarization alignment loss	3.0 dB	-
Polarization loss	-	3.0 dB

Transmitter Power Required using Slot Array for Tracking:-

Transmitter power requirements are calculated using equation G.1

$$P_R = P_T + G_T + G_R - P_L - P_P - P_A - P_L \qquad (G.1)$$

where P_L = Polarization loss

P_R = Power required at the receiver

P_T = Required Transmitter power

P_L = Free space path attenuation

P_P = Passive losses in the spacecraft

G_T = Transmitting Antenna Gain

G_R = Receiving Antenna Gain

P_A = Polarization alignment loss

The parameter values assumed are

$P_R = -120$ dBm

$P_L = 167$ dB

$P_P = 2.7$ dB

$G_R = 16.2$ dB

$P_A = 3.0$ dB

$G_T = 0$ dB

Substituting in equation G.1,

$$P_T = P_R - G_T - G_R + P_L + P_P + P_A + P_L$$

$$= -120 - 0 - 16.3 + 167.3 + 2.7 + 3.0 + 0$$

$$P_T = +36.7 \text{ dBm}$$

Transmitter Power Required using 9 element Yagi Antenna for Tracking:-

In this case there is no polarization alignment loss but there is a 3 dB polarization loss.

Thus using the equation G.1

$$P_T = -120 - 0 - 19.2 + 167.3 + 2.7 + 0 + 3.0$$

$$P_T = -140.5 + 173.0 \text{ dBm}$$

$$P_T = +33.8 \text{ dBm}$$

A 5 watt beacon transmitter operating in the 136 MHz band is thus recommended.

APPENDIX H

TELEMETRY CALCULATIONS

General

The telemetered signals from the spacecraft is recovered at the earth station. For this study, we assume that a phase locked receiver is used., The baseband signal is complex and comprises one subcarrier modulated by coded telemetry signals three subcarriers modulated by real time sensor signals, one execute monitor tone, and four ranging tones.

In a general case of 'N' baseband signals, the recoverable power in the 1st order sidebands P is given by

$$P = \frac{2J_1^2(X_n)}{J_0^2(X_n)} \prod_{i=1}^{N} J_0^2(X_i) \; P_R \qquad \text{(H.1)}$$

and the remaining carrier power P_c is given by

$$P_c = P_R \prod_{i=1}^{N} J_0^2(X_i) \qquad \text{(H.2)}$$

where P_R = the total receiver power

X_i = phase deviation due to the ith base band signal

$J_0(X_i)$ = zero order Bessel function of argument X_i

$J_1(X_i)$ = 1st order Bessel function of argument X_i

Modulation Loss

We shall use the concept of modulation loss 'ML' defined as follows:

ML = the ratio of the total received power to that recovered in the first order sidebands of the n^{th} signal.

$$= P_R/P_n \qquad \text{(H.3)}$$

and ML_c = the ratio of the total received power to that remaining in the carrier

$$= P_R/P_c \qquad \text{(H.4)}$$

Now we can write

$$(ML_n)^{-1} = \frac{2J_1^2(X_n)}{J_0^2\,(X_n\;)} \prod_{i=1}^{N} J_0^2(X_i) \tag{H.5}$$

and

$$(ML_c)^{-1} = \prod_{i=1}^{N} J_0^2(X_i) \tag{H.6}$$

Baseband

The baseband signal listed in table H.1 consists of four sub-carriers and five tones.

Table H.1 Baseband Signals

Signal	Use	Frequency
1) Sub carrier 1	telemetry signal	8.64 kHz
2) Sub carrier 2	real time sensor 1	22 kHz
3) Sub carrier 3	real time sensor 2	40 kHz
4) Sub carrier 4	real time solar sensor	30 kHz
5) Ranging tone f_{r1}	ranging	100 Hz
6) Ranging tone f_{r2}	ranging	800 Hz
7) Ranging tone f_{r3}	ranging	6.4 kHz
8) Ranging tone f_{r4}	ranging	50 kHz
9) Execute monitor tone	Monitor	14.5 kHz

Telemetry Signal

The telemetry signal is a PCM signal phase modulating a sub-carrier. If ϵ is the error rate then the signal to noise ratio for a coherent system is

ϵ = 1 in 10^6 for S/N = 10.3 dB

ϵ = 1 in 10^5 for S/N = 9.4 dB

ϵ = 1 in 10^4 for S/N = 8.2 dB

Whereas for a noncoherent system we have

ϵ = 1 in 10^6 for S/N = 14.3 dB

ϵ = 1 in 10^5 for S/N = 13.3 dB

ϵ = 1 in 10^4 for S/N = 12.1 dB

We shall assume a minimum error rate of 1 in 10^5 for a non coherent system, thus if we use a coherent system it will give us additional safety margin. Thus for an error rate of 1 in 10^5 we require that

$$S/N = 13.3 \text{ dB}$$

where the signal bandwidth is 72 Hz.

Real Time Sensor Signals

The real time signals require a bandwidth of 250 Hz and a Speak/N of 30 dB. Thus on rms basis

$$S/N = 27 \text{ dB}$$

The relation between S/N, C/N and modulation index M is given by

$$S/N = 3\, C/N\, M^2 \frac{B}{2b} \qquad \text{(H.7)}$$

where B = receiver bandwidth
b = baseband signal bandwidth

By Carson's rule we have

$$B = 2(M + 1)b \tag{H.8}$$

Therefore

$$S/N = \frac{3C}{N} M^2(M + 1) \tag{H.9}$$

Therefore assuming C/N is 10 dB for the threshold condition we have

$$10 \log M^2(M + 1) = 10 \log \frac{S/N}{3C/N} = (27.0 - 10 - 4.8) \text{ dB}$$

$$= 12.2 \text{ dB}$$

$$\text{or } M^2(M + 1) = 16.6 \tag{H.10}$$

Solution of the above cubic equation gives

$$M = 2.26 \tag{H.11}$$

The subcarrier oscillator bandwidth required using Carson's rule is given by

$$B = 2(M + 1) 250$$

$$= 1630 \text{ Hz} \tag{H.12}$$

Execute Monitor Tone

The up-link calculations for a F.S.K. system with an envelop detector indicate that for a bit error rate of 1 in 10^6 we have

$$\frac{\text{Mean signal power in 'ON' state}}{\text{Mean noise power}} = 17.2 \text{ dB} \tag{H.13}$$

The execute tone is also used to command the operation of thrusters, where minimum on-time required is 20 msecs. A bandwidth of 150 Hz is sufficient to pass both the fundamental and third harmonic components of this execute signal.

Ranging Tones

From section 3.1.7 and Appendix A, the downlink requirements for the ranging tone is a 1 Hz bandwidth and a 30 dB signal to noise ratio.

Summary

A summary of the performance requirements of the various baseband signals is tabulated in Table H.2, where 'No' is the noise density per unit (1 Hz) bandwidth.

Table H.2
Baseband S/No Summary

Signal	Function	S/N dB	B.W. Hz	S/No dB/Hz
1	Telemetry	13.3	72	31.9
2	Real Time 1	10.0	1630	42.1
3	Real Time 2	10.0	1630	42.1
4	Real Time solar sensor	10.0	1630	42.1
5	Ranging Tone 1	30.0	1	30.0
6	Ranging Tone 2	30.0	1	30.0
7	Ranging Tone 3	30.0	1	30.0
8	Ranging Tone 4	30.0	1	30.0
9	Execute Tone	17.2	150	39.0

Calculations

For convenience of calculation, let us assume that ranging tones also require a S/No of 31.9 dB and the Execute tone 42.1 dB. We can now divide the above nine baseband signals in two groups, one of four and the other of five, where each signal in any group phase-deviates the carrier by equal amount as the others.

If Xa = phase deviation for a performance of 42.1 dB/Hz
and Xb = phase deviation for a performance of 31.9 dB/Hz

then the modulation loss from equation H.5 for one of the group 'a' signals is given by

$$(MLa)^{-1} = 2\, J_1^{\,2}(Xa)\; J_0^{\,6}(Xa)\; J_0^{\,10}(Xb) \qquad \text{(H.14)}$$

and that for one of the group 'b' signals

$$(ML_b)^{-1} = 2J_1^{\,2}(X_b)\ J_0^{\,8}(Xa)\ J_0^{\,8}(X_b) \qquad (H.15)$$

For optimum performance, given the above equations, we will evaluate 'X_a' and 'X_b' to minimize modulation loss.

$$\text{Let } A = \frac{(S/No)a}{(S/No)b} \qquad A = 10 \log \frac{(S/No)a}{(S/No)b} \qquad (H.16)$$

$$= (42.1 - 31.9) \text{ dB}$$
$$= 10.2 \text{ dB}.$$

Thus

$$\frac{ML_b}{ML_a} = \frac{2J_1^{\,2}(X_a)\ J_0^{\,2}(X_b)}{2J_1^{\,2}(X_b)\ J_0^{\,2}(X_a)} \qquad (H.17)$$

hence

$$\frac{J_1^{\,2}(X_a)}{J_0^{\,2}(X_a)} = \frac{J_1^{\,2}(X_b)}{J_0^{\,2}(Xb)} \text{ antilog } A \qquad (H.18)$$

Thus for every value of 'X_a' there is a corresponding value of 'X_b' as tabulated in Table H.3.

Now using equations H.14 and H.15 from Table 1 the modulation losses MLa and MLb are computed. They are also tabulated in Table H.3

Table H.3
Deviations due to Baseband Signals

X_a rad	X_b rad	$(ML_a)^{-1}$ dB	$(ML_b)^{-1}$ dB
0.32	0.1	-13.7986	-24.0032
0.40	0.127	-12.3719	-22.7790
0.50	0.13	-11.1421	-23.1030
0.60	0.19	-10.1344	-20.9893
0.70	0.23	-10.5179	-20.6884
0.80	0.275	-10.8267	-20.7658

This yields as a minimum

$$ML_a = 10.5 \text{ dB} \qquad (H.17)$$

for $X_a = 0.7$ radians

and $ML_b = 20.7$ dB (H.18)

for $X_b = 0.23$ radians

Transmitter Power Calculation

Consider

$$S/No = 42.1 \text{ dB}$$

where $No = KT$ (H.19)

Assuming that the noise temperature for a typical earth station is 200°K we have

$$No = 1.38 \times 10^{-23} \times 200 \text{ watts/Hz}$$

$$= 2.76 \times 10^{-21} \text{ watts/Hz}$$

$$= -205.6 \text{ dBw/Hz} \qquad (H.20)$$

Therefore

$$S = -205.6 + 42.1$$

$$= -163.5 \text{ dBw} \qquad (H.21)$$

and the receiver power P_R may be given by

$$P_R = -163.5 + ML_a$$

$$= -163.5 + 10.5$$

$$= -153.0 \text{ dBw} \qquad (H.22)$$

This as yet does not allow any margin. We shall assume a 6 dB margin, hence the nominal received power required is

$$P_R = -147.0 \text{ dBw} \qquad (H.23)$$

Assuming a 60' parabola, with a 50% efficiency, the ground antenna gain at 4200 MHz is 55.6 dB. Taking a path loss of 197.3 dB, the required EIRP of the spacecraft is

$$EIRP = P_R - G_R + L_P$$

$$= -147 - 55.6 + 197.3$$

$$= -5.3 \text{ dBw} \qquad (H.24)$$

In the antenna and harness section of the spacecraft the calculated power loss is 2.6 dB (Section 3.4.2), hence the required telemetry transmitter power is

$$P_T = (-5.3 + 2.6) \text{ dBw}$$

$$= -2.7 \text{ dBw} \qquad (H.25)$$

$$= 537 \text{ milliwatts}$$

Finally, we will check that for above deviations there is sufficient carrier power for tracking purposes. From equation H.2

$$(ML_a)^{-1} = J_0^{8}(X_a)\ J_0^{10}(X_b)$$

$$= J_0^{8}(0.7)\ J_0^{10}(0.23)$$

$$= -4.3940 - 0.5765 \text{ dB}$$

$$= -4.9705 \text{ dB} \qquad (H.26)$$

Therefore 30% of the carrier power will be available for tracking purposes.

Tracking Performance Requirements

The telemetry transmitter will also be used for tracking. In the previous section we have shown that 30% of the received signal is available for tracking. Here we shall check to see if the carrier-to-noise ratio is adequate.

We will assume that for acquisition the required phase locked loop bandwidth is 3 kHz, while tracking is done with a bandwidth of 300 Hz.

If earth station noise temperature = 200°K
then system noise density K_T = -205.6 dBw/Hz

Noise power in 3 kHz bandwidth = -170.8 dBw
30% of carrier power (See transmitter power calculation) = -158.0 dBw
Carrier-to-noise ratio available for acquisition = 22.8 dB
Carrier-to-noise ratio available for tracking = 12.8 dB

The minimum carrier to noise ratio required for acquisition and tracking can be as low as 3 dB. Hence we have minimum of 9.8 dB safety margin which is adequate.

III 2(b)
VOL. 2 APPENDICES

APPENDIX A

PRELIMINARY STRUCTURAL ANALYSIS

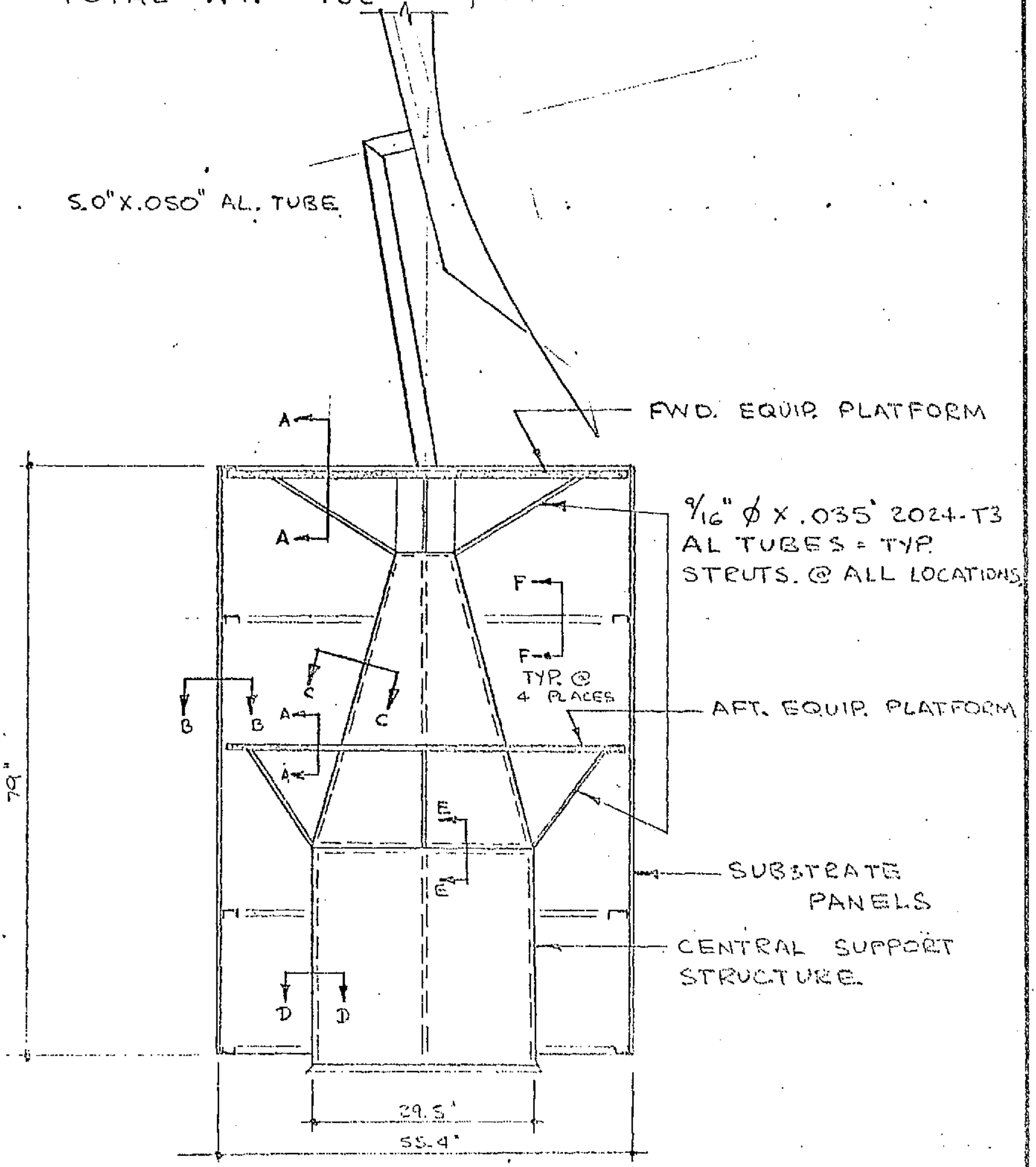
PREPARED R. BUCKINGHAM 9-26-68
CHECKED N. ROSEN 9-26-68
MODEL
REPORT NO.

TRW
SYSTEMS GROUP
ONE SPACE PARK • REDONDO BEACH, CALIFORNIA

PREPARED R. BUCKINGHAM AUG 14' 68
CHECKED N. ROSEN 9-23-68
MODEL
REPORT NO.
PAGE 1

CANADIAN INTELSAT - SUMMARY SHEET

BASED ON A PRELIMINARY DESIGN ANALYSIS, THE FOLLOWING STRUCTURE IS RECOMMENDED.
TOTAL WT. = 932#

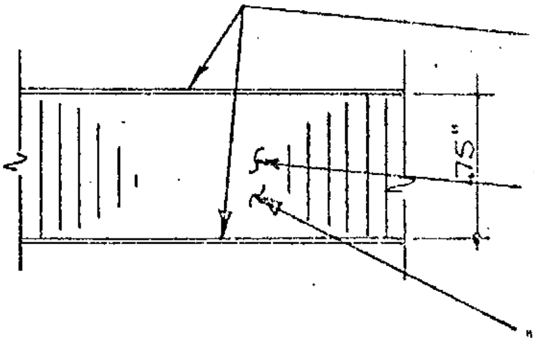


PREPARED R. BUCKINGHAM 8-14-68

REPORT NO.

CHECKED N. ROSEN 9-29-68

MODEL

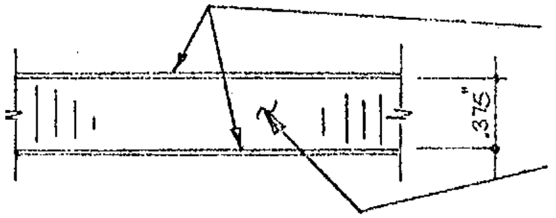


.012" THICK 2024-T3 AL SHEETS BONDED @ TOP & BOTTOM.

"HEXCELL" 1/4" CELLS WITH .0010" WALL THICKNESS 5052 AL. AT UPPER EQUIP. PLATFORM

"HEXCELL 3/16" CELLS WITH .0010" WALL THICKNESS 5052 AL., AT CENTRAL PLATFORM.

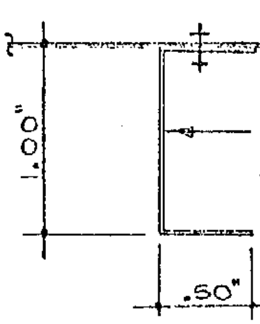
SECT. AA (FULL SCALE)



.010" FIBERGLASS SHEETS BONDED @ TOP & BOTTOM. MIL R-9500 (120 CLOTH)

CORE:
1.6 - 1/4 - .7P 5052 AL CORE.

SECT. BB FULL SCALE

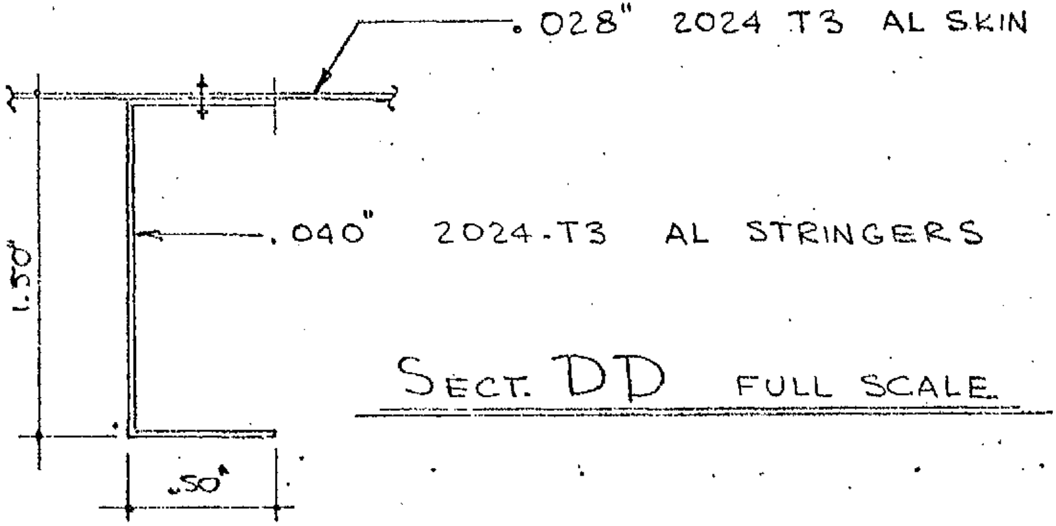


.020" 2024 T3 AL SKIN

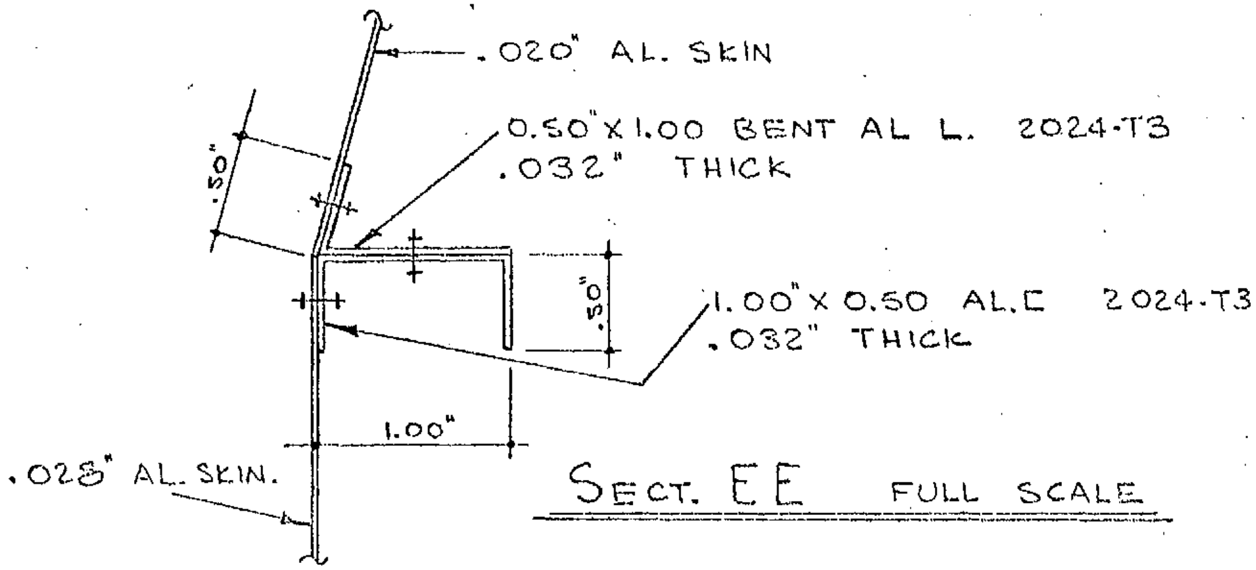
.030" 2024 AL STRINGERS

SECT. CC FULL SCALE

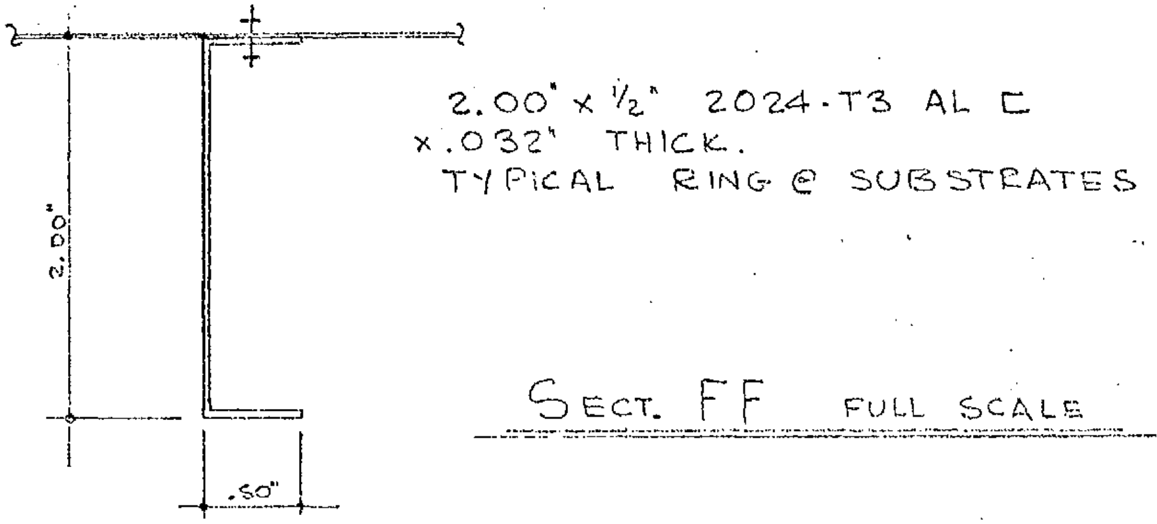
PREPARED E. B 8-14-68
CHECKED N. ROSEN 9-29-68
MODEL
REPORT NO.



SECT. DD FULL SCALE



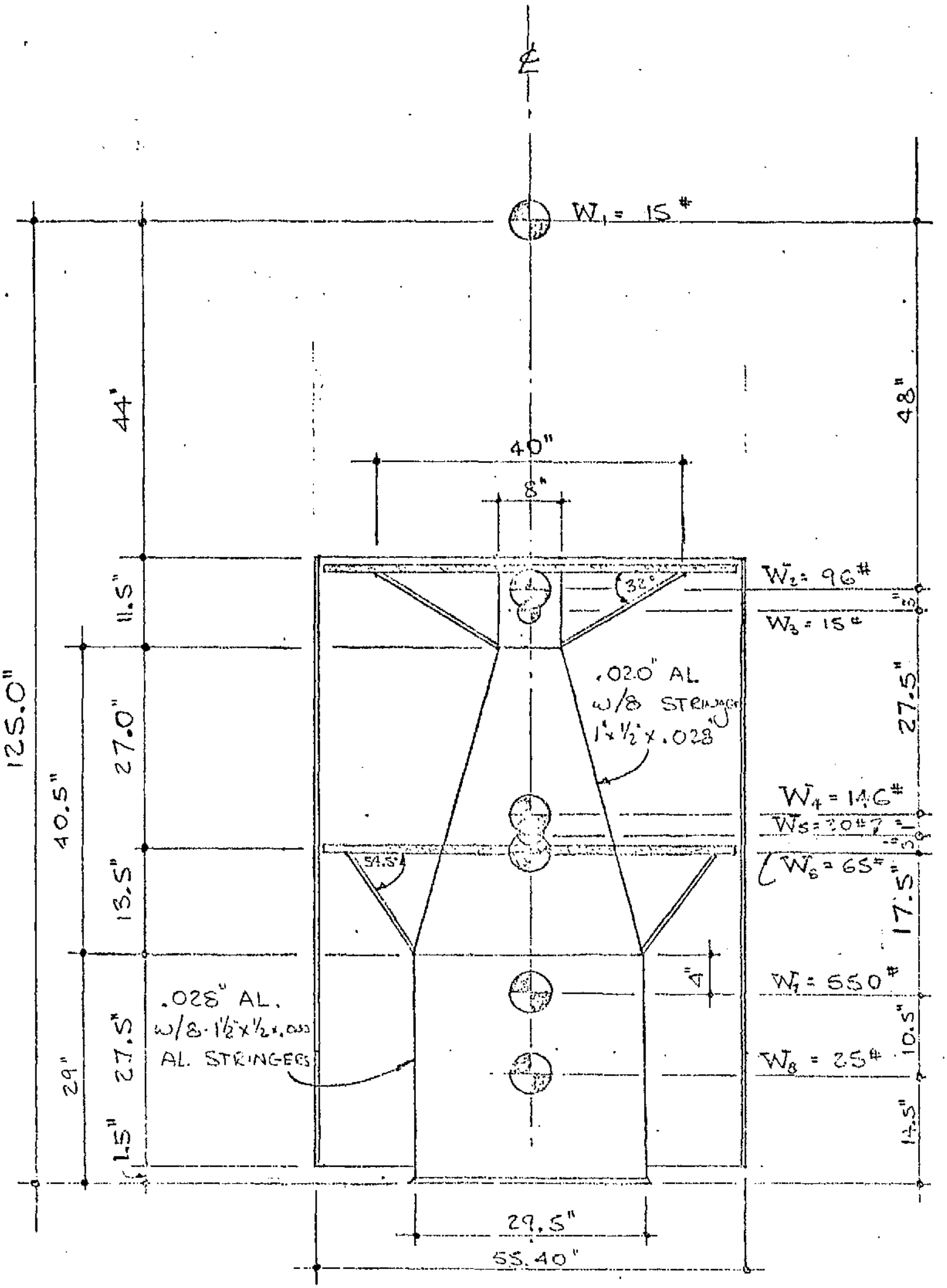
SECT. EE FULL SCALE



SECT. FF FULL SCALE

PREPARED R. BUCKINGHAM 8-14-68 REPORT NO. PAGE 4
CHECKED N. ROSEN 9-29-68
MODEL

DESIGN AND ANALYSIS OF S/C CENTRAL CYLINDER



SCALE 1/20" = 1"

PREPARED R. BUCKINGHAM 8-14-68
CHECKED N. ROSEN 9-29-68
MODEL
REPORT NO.

S/C PRIMARY STRUCTURE

WEIGHT BREAKDOWN

W_1	=	ANTENA + SUPPORT STRUCTURE	15#
W_2	=	UPPER PLATFORM + LOADS	96#
W_3	=	WEIGHT OF DESPIN MECH.	15#
W_4	=	LOWER EQUIP. PLATFORM	146
W_5	=	CONICAL SECT. OF CENT. STRUCT.	20
W_6	=	SUBSTRATES AS ATTACHED TO PLATFORM	65#
W_7	=	FUEL TANK	550#
W_8	=	CYL. SECTION OF CENT. STRUCT.	25#
			932#

COMMENTS

1. CG OF $W_2 = \sim 2"$ BELOW FWD. EQUIP. PLATFORM
2. CG OF $W_4 = \sim 4"$ FWD. OF CENT. EQUIP. PLATFORM
3. CG OF $W_7 = \sim 3'$ FWD. OF FUEL TANK ATTACH POINTS TO CENT. STRUCTURE.

DESIGN CRITERIA (FOR CENT. SUPPORT STRUCT.)

COND. I 21 g AXIAL + 0 g LAT.
COND. II 12 g AXIAL + 3 g LAT.
} ULT. LOADING.

∴ AT INTERFACE w/ ADAPTER

COND I: $\Sigma F_{Ax} = 21g \times 932\# \approx 19{,}600\#$

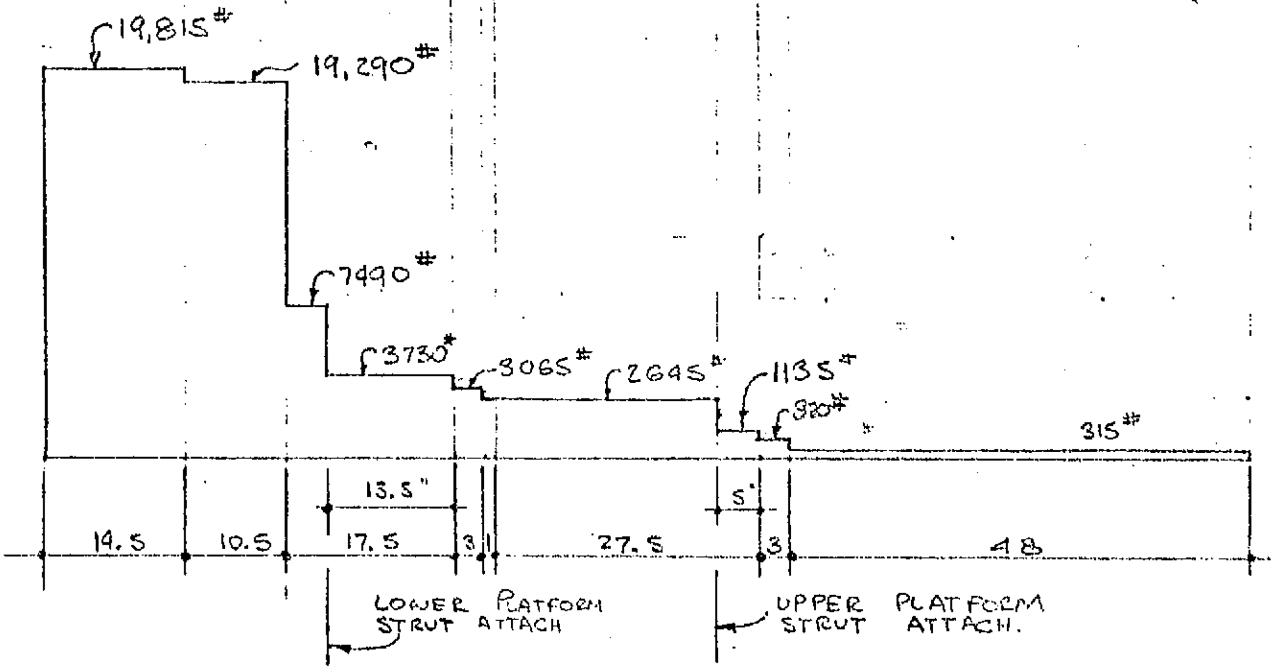
COND II: $\Sigma F_z = 12g \times 932\# = 10{,}900\#$

$\Sigma F_x = 3g \times 932\# = 2{,}800\#$

PREPARED RB 8-14-68
CHECKED NR 9-29-68
MODEL
REPORT NO.

CENTRAL STRUCTURE

LOAD COND. I



AXIAL LOAD DIST. TO CENTRAL STRUCTURE

VERT. SCALE 1/10" = 1000# ; HORIZ 1/20" = 1"

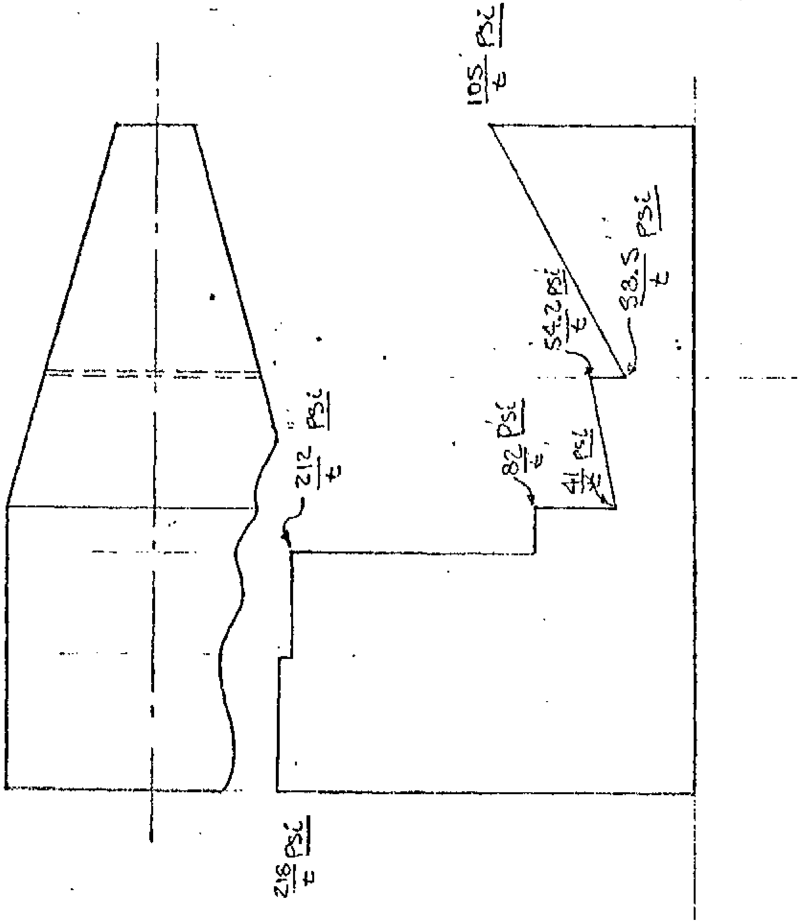
PREPARED R. B. 8-16-68 REPORT NO.

CHECKED N. R. 9-29-68

MODEL

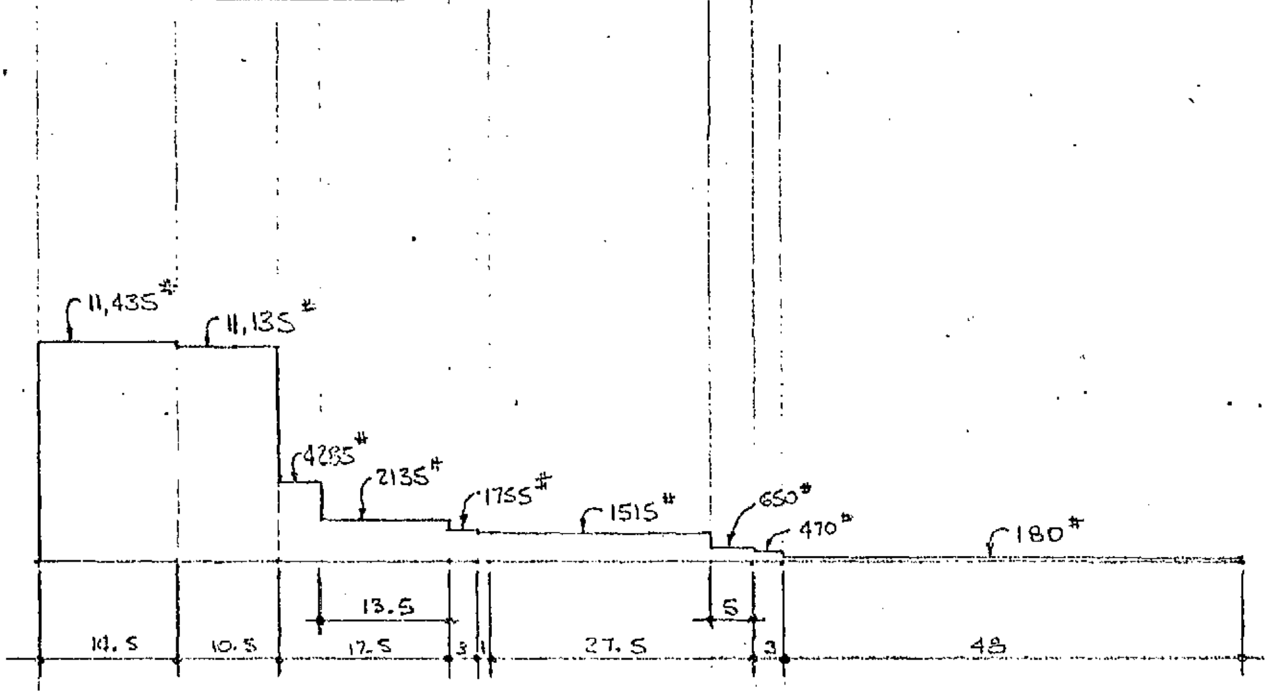
LOAD COND. I, STRESS DISTRIBUTION

(MONOCOQUE CONFIG.)



PREPARED RB 8-16-68
CHECKED NJR 9-29-68
MODEL
REPORT NO.

LOAD COND II

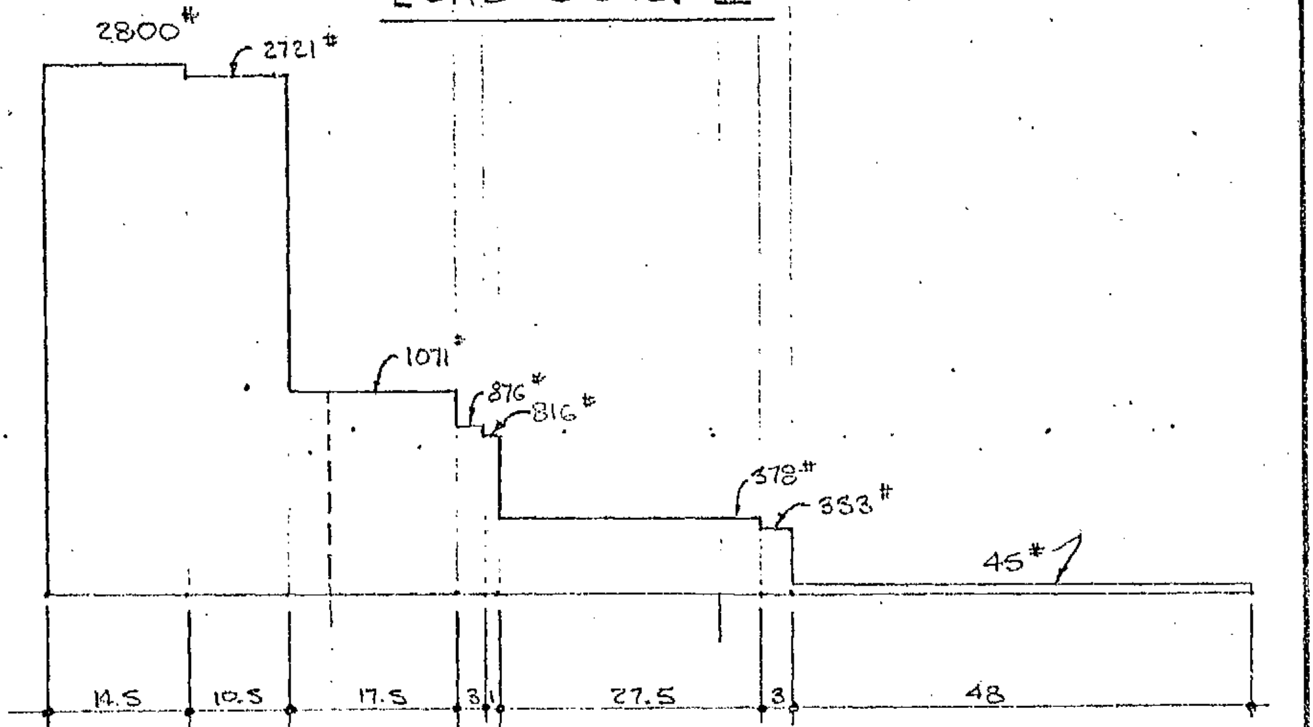


AXIAL LOAD DIST. TO CENTRAL STRUCTURE

TRW SYSTEMS GROUP
ONE SPACE PARK · REDONDO BEACH, CALIFORNIA

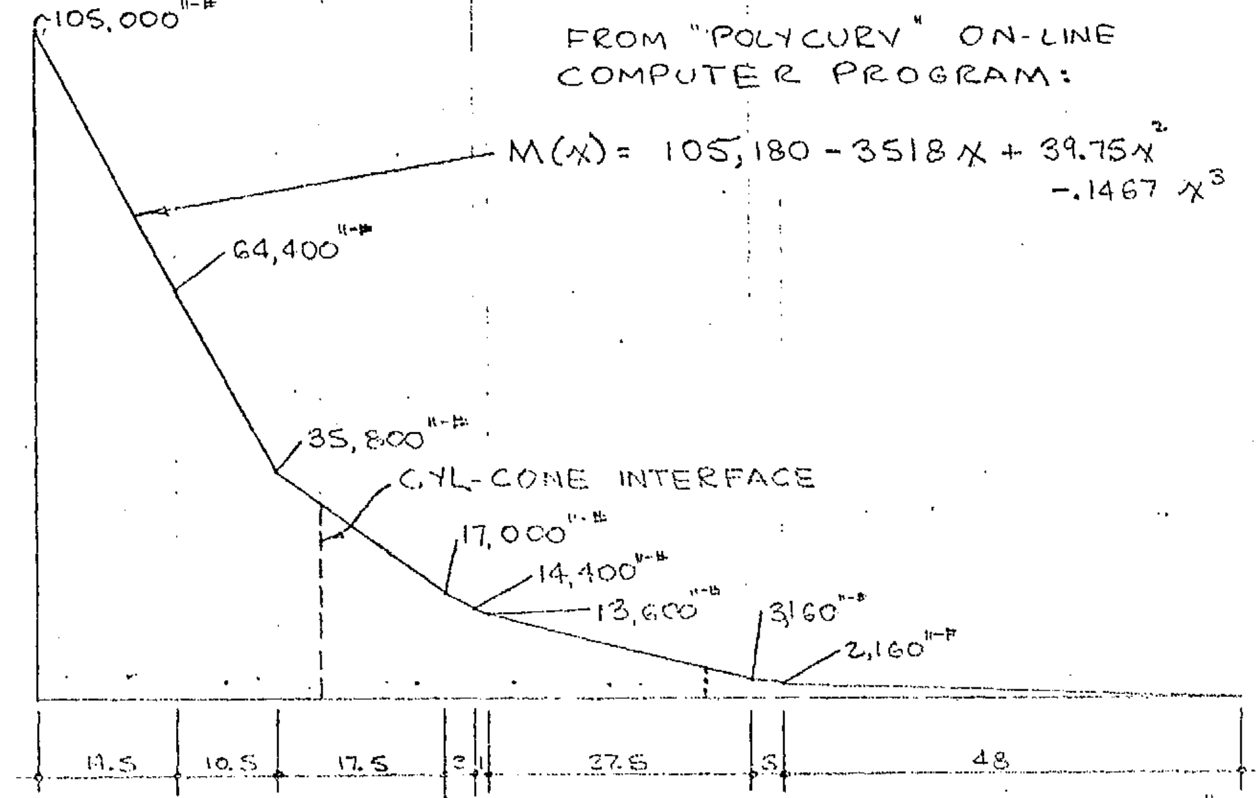
PREPARED R. B. 8-16-68
CHECKED NR 9-29-68
MODEL
REPORT NO.

LOAD COND. II



LATERAL LOAD DIST. TO CENT. STRUCT. (SHEAR DIAG.)

VERT. SCALE 1/10" = 100#, HORIZ. 1/20" = 1"



FROM "POLYCURV" ON-LINE COMPUTER PROGRAM:

$$M(x) = 105,180 - 3518x + 39.75x^2 - .1467x^3$$

MOMENT DIAGRAM VERT. SCALE 1/30" = 1000"-#

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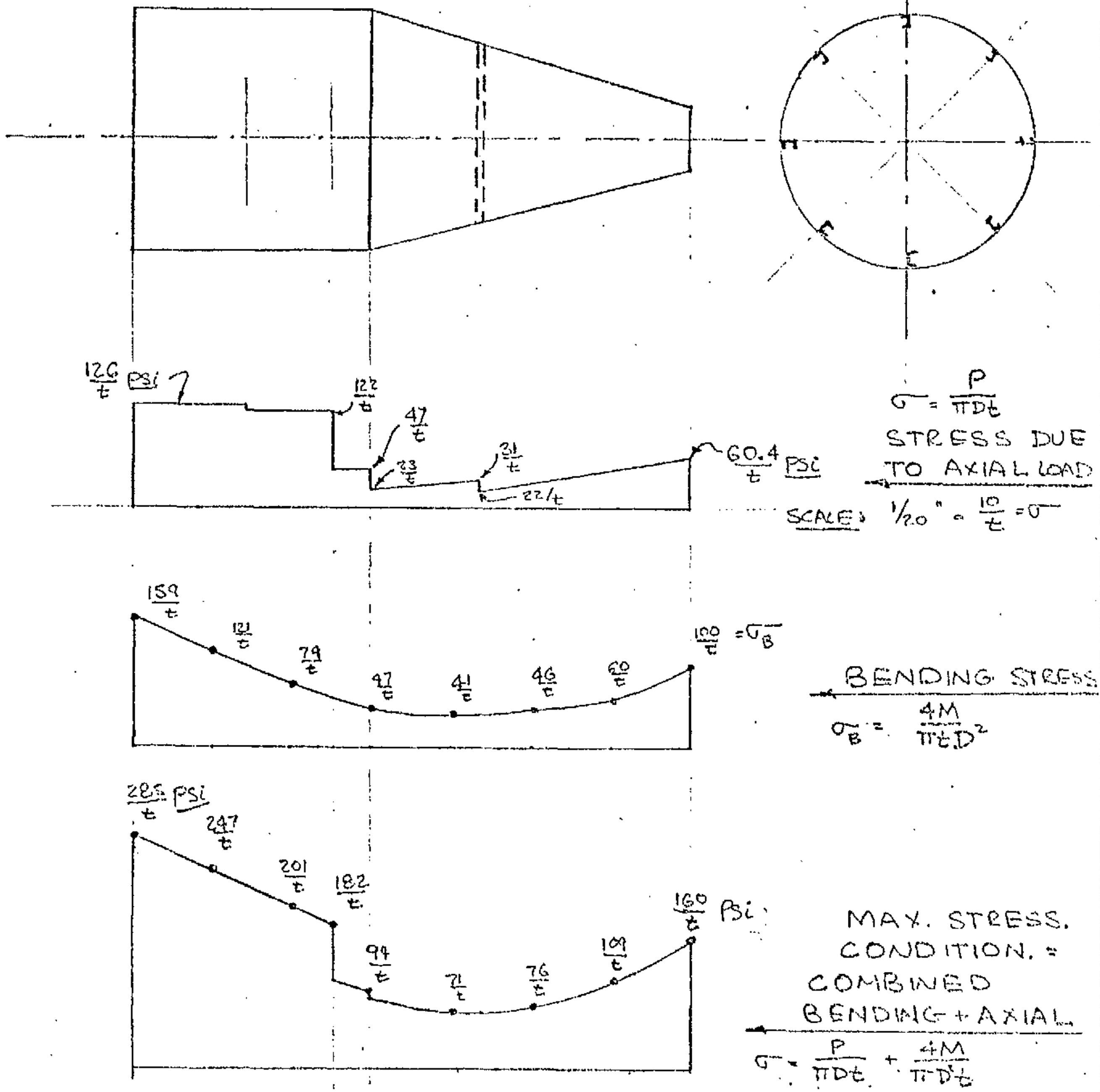
LOAD COND. II, STRESS DISTRIBUTION

(MONOCOQUE CONFIG.)



$\sigma = \frac{P}{\pi D t}$

STRESS DUE TO AXIAL LOAD

SCALE: $1/20" = \frac{10}{t} = \sigma$

BENDING STRESS

$\sigma_B = \frac{4M}{\pi t D^2}$

MAX. STRESS. CONDITION. = COMBINED BENDING + AXIAL

$\sigma = \frac{P}{\pi D t} + \frac{4M}{\pi D^2 t}$

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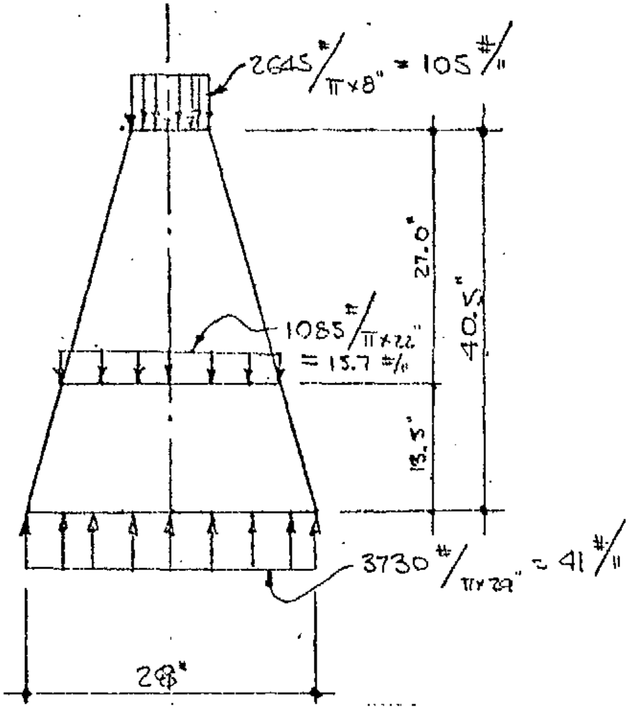
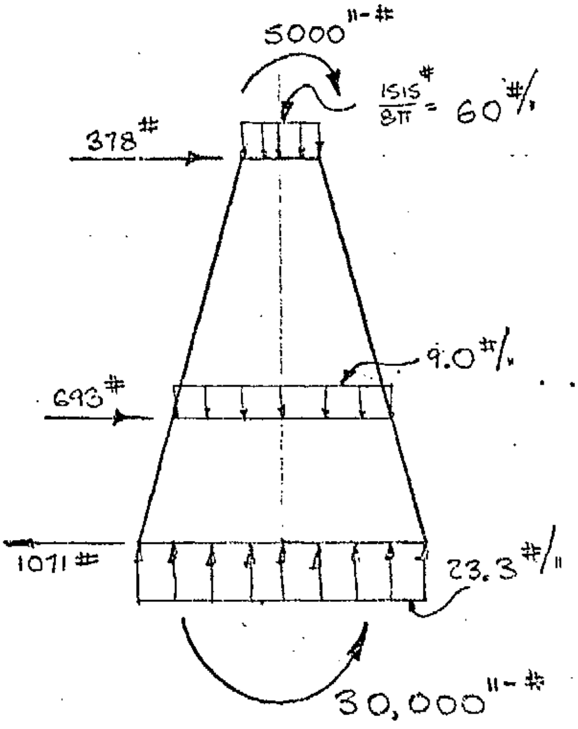
CONICAL SECTION DESIGN



LOAD COND. I



LOAD COND. II

DESIGN CONDITIONS

1. TRUNCATED CONE IS CONSIDERED TO BE SIMPLY SUPPORTED AT TOP, BOTTOM, AND AT INTERMEDIATE PLANE WHERE PLATFORM INTERSECTS CONE.
2. 8 - EQUALLY SPACED LONGERONS ARE USED.
3. MAT'L = ALUMINUM THROUGHOUT.

DESIGN PROCEDURE (MONOCOQUE)

1. AXIAL STRESS IS DISTRIBUTED IN TERMS OF "t"

$$\sigma_z = \frac{P}{A} = \frac{P(z)}{\pi D t} \quad @ \text{ ANY STATION}$$

2. BENDING STRESS @ ANY STATION

$$\sigma = \frac{M(z)\,R}{I} \qquad \text{FOR } R >> t \longrightarrow I = \pi t R_o^3$$

$$\therefore \sigma = \frac{M(z)}{\pi t R^2} = \frac{4\,M(z)}{\pi t D^2}$$

PREPARED RB 8-16-68
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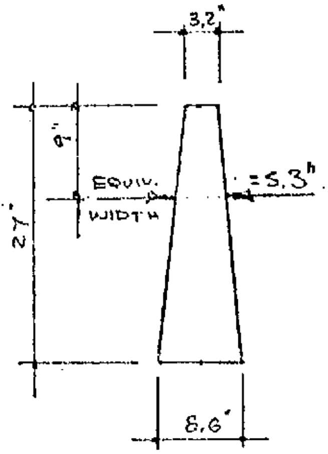
CONICAL PANEL DESIGN (CONT.)

CONSIDER THAT PANEL = FLAT

TOP = $8 \times \pi / 8 = 3.14''$ $\quad L = 27''$

BOTTOM = $22 \times \pi / 8 = 8.6''$



PANEL = SS ON ALL EDGES.
CONSIDER THAT, DUE TO STRESS DIST., BUCKLING MOST LIKELY TO OCCUR IN UPPER PORTION.

∴ USE EQUIV. WIDTH = ≈5.3"

FROM ROURK: PG. 348

$$\sigma_{CRIT} = \frac{KE}{1-\nu^2}\left(\frac{t}{b}\right)^2$$

$$a/b = 27/5.3 = 5.10 \rightarrow K = 3.29$$

$$\therefore \sigma_{CRIT} = \frac{3.29 \times 10 \times 10^6}{1-(0.3)^2}\left[\frac{t}{5.3}\right]^2 = 1.29 \times 10^6 \, t^2$$

FROM, LOAD COND II, THE MAX. STRESS AT A POINT 9" FROM TOP OF CONE IS

$$\sigma_{MAX} = \approx 120/t \text{ PSI}$$

$$\therefore \quad \frac{120}{t_c} = 1.29 \times 10^6 \, t_c^2 \qquad \text{FOR } t_{CRITICAL} = t_c$$

$$\therefore \quad t_c = \left[\frac{120}{1.29 \times 10^6}\right]^{1/3} = \left[93 \times 10^{-6}\right]^{1/3} = 4.55 \times 10^{-2}$$

$$\longrightarrow \quad t_c = .0455''$$

CONSIDERING CURVED PANEL, ROURK PG. 350

$b = 5.3''$ $\quad a = 27''$ $\quad r = \approx 8''$

$$\sigma_{CRIT} = \frac{E}{6(1-\nu^2)}\left\{\left(\frac{\pi t}{b}\right)^2 + \sqrt{12(1-\nu^2)\left(\frac{t}{r}\right)^2 + \left(\frac{\pi t}{b}\right)^4}\right\}$$

$$= \frac{10 \times 10^6}{6(.91)}\left\{\left(\frac{\pi}{5.3}\right)^2 t^2 + \sqrt{\frac{12(.91)\,t^2}{64} + \left(\frac{\pi}{5.3}\right)^4 t^4}\right\}$$

$$1.84 \times 10^6 \left\{.350 t^2 + \sqrt{.171 \, t^2 + .122 t^4}\right\}$$

FOR $\sigma_{crit} = \dfrac{120}{t_c}$

$$\frac{120}{t_c} = 1.84 \times 10^{6} \left[.350 t^{2} + \sqrt{.171 t^{2} + .122 t^{4}} \right]$$

$$65.2 \times 10^{-6} = .350 t_c^{3} + \left[.171 t_c^{4} + .122 t_c^{6} \right]^{1/2}$$

FROM TRIAL & ERROR SOLUTION:

$$t_c = 1.3 \times 10^{-2} \text{ INCHES.}$$

$$\therefore \sigma_{CRIT} = 120/t = 120/1.3 \times 10^{-2} = 92 \times 10^{2} = 9200 \text{ psi}$$

MAKE ALUMINUM FROM .020" THICK PL.

$$\sigma = 120/2.0 \times 10^{-2} = 6000 \text{ psi}$$

$$MS = 9200/6000 - 1.0 = +.53 \; ^{*}$$

(* BASED ON APPROX. ANALYSIS)

PREPARED RB 8-16-68
CHECKED NR 9-26-68
MODEL
REPORT NO.

CYLINDRICAL SECTION, PANEL DESIGN

a = 29"   r = 14.5"   b = 11.4"

$$\sigma_{crit} = \frac{10\times10^{6}}{6\,(1-.3^{2})}\left[\left(\frac{\pi}{11.4}\right)^{2} t^{2} + \sqrt{\frac{12\,(1-.3^{2})\,t^{2}}{(14.5)^{2}} + \left(\frac{\pi}{11.4}\right)^{4} t^{4}}\;\right]$$

$$= 1.83\times10^{6}\left[.076\,t^{2} + \sqrt{.052\,t^{2} + .00577\,t^{4}}\;\right]$$

$$\sigma_{MAX} = 247/t$$

$$t$$

$$135\times10^{-6} = .076\,t^{3} + \sqrt{.052\,t^{4} + .00577\,t^{6}}$$

FROM TRIAL & ERROR SOLUTION

$$t_{CRIT} = .024"$$

∴ USE .028" THICK AL. Pℓ

$$\sigma_{MAX} = 285/.028 = 10{,}150 \text{ PSI}$$ @ ADAPTER

$$\sigma_{CRIT_{BUCKLE}} = 285/.024 = 11{,}900 \text{ PSI}$$

M.S. = + 0.18

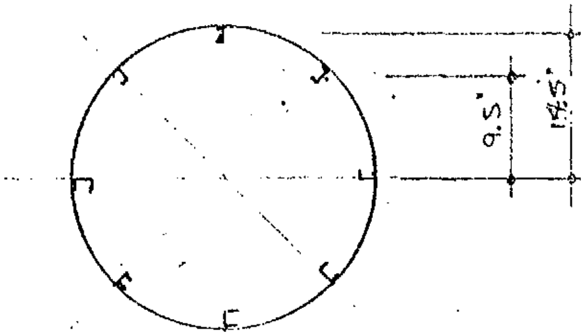
WT. = $\pi\times29"\times.028"\times29"\times.10\ \#/in^3 = 7.4\ \#$

PREPARED RB 8-16-68 REPORT NO.
CHECKED NR 9-26-68
MODEL

LONGERON DESIGN

1. CONSIDER LONGERONS TO TAKE ALL BENDING + COMPRESSIVE STRESS. [NOTE THIS IS CONSERVATIVE]
2. TOTAL OF 8 - LONGERONS.

CONICAL PORTION



CONSIDER EFFECTIVE SKIN @ LONGERON

$= 2.0''$

$\therefore A = (2.0 \times t) + A_{LONGERON}$

$I = \Sigma A R^2$

$= 2[A(13.5)^2 + 2A(9.5)^2]$

$= 365A + 361A = 726A$

FOR $f_c = 10{,}000$ psi

$$\sigma = \frac{MR}{I} + \frac{P}{8A}$$

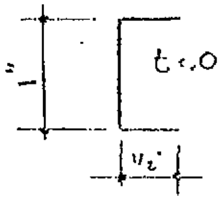
$$= \frac{30{,}000''\# \times 14.5''}{726A} + \frac{2135\#}{8A}$$

$$= 600/A + 267/A = 867/A$$

FOR $\sigma = 10{,}000$ psi $\rightarrow A_{REQ'D} = 867/10{,}000 = .0867''$

FOR .020" SKIN — $A_{SKIN} = 2'' \times .02'' = .04''$

∴ CONSIDER LONGERONS OF .028 MAT'L



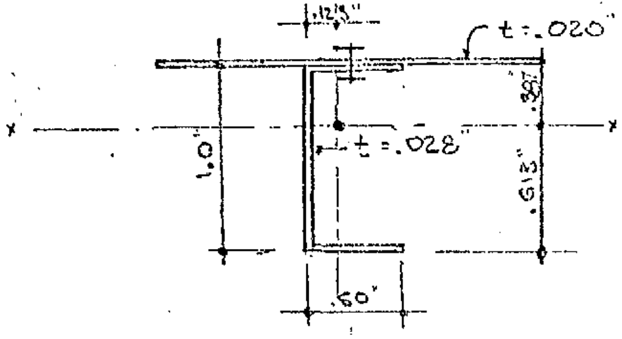
$A_L = 2'' \times .028 = .048''$ = OK!

$\Sigma A = .048'' + 2(.020) = .088\ in^2$

$\sigma = 867/.088 = 9{,}900$ psi

PREPARED RB 8-16-68 REPORT NO.
CHECKED NR 9-26-68
MODEL

LONGERON BUCKLING



$$NA_{xx} = \frac{1.0(.028)(.50) + 2.0 \times 1.0 \times .020}{(.028)(2.0) + .020(2.0)} = \frac{.014 + .040}{.048 + .040}$$

$$= .613''$$

$$NA_{yy} = \frac{2(.50)(.028)(.25)}{2(.028)} = .125''$$

$$I_{xx} = .50(.613)^2(.028) + \frac{.028(.613)^3}{3} + \frac{.028(.387)^3}{3} + .028(.50)(.387)^2$$

$$+ .020(2.0)(.387)^2$$

$$= .0053 + .00215 + .00047 + .0021 + .0015$$

$$I_{xx} = .00959 \text{ in}^4$$

$$A = .088 \text{ in}^2$$

$$r_{xy} = \sqrt{I/A} = \sqrt{\frac{.959 \times 10^{-2}}{.088}} = 3.3 \times 10^{-1} = 0.33''$$

FOR $L = 27''$ $L/R = 27/0.33 = 80 > 120$ = OK!

$\sigma_{CR} = 15.9$ KSI

$MS = 15.9/9.9 - 1 = +.60$

CHECK FLANGE BUCKLING

FROM "GERARD" - PG. 271

$\frac{b_f}{b_w} = \frac{0.5}{1.0} = 0.5$ $\frac{t_w}{t_f} = 1.0$ ∴ $K_w = 2.2$ FROM CURVE

$$\sigma_{cr} = K_w E \left(\frac{t_w}{b_w}\right)^2 = 2.2 \times 10 \times 10^6 \left[\frac{.028}{1.00}\right]^2 =$$

$$2.2 \times 10^7 \times 7.85 \times 10^{-4} = 17.3 \times 10^3$$

∴ $\sigma_{cR} = 17{,}300$ psi = OK!

$MS = 17.3/9.9 - 1 = +.76$

PREPARED RB 8-16-68 REPORT NO.
CHECKED NR 9-26-68
MODEL

CYLINDRICAL PORTION - LONGERON DESIGN

$I = 726\ A$

$\sigma = \dfrac{MR}{I} + \dfrac{P}{8A}$ $\quad M = 105{,}000^{\text{''-\#}}$ $\quad P = 2800\ \#$

$\therefore\ \sigma_{MAX} = \dfrac{105{,}000 \times 14.5}{726\ A} + \dfrac{11{,}435}{8\ A}$

$= 2100/A + 1430/A = 3530/A$

FOR $\sigma \approx 15{,}000$ psi

$A_{REQ'D} = 3530/15000 = .235\ \text{in}^2$

FOR .028" SKIN.

$A_{REQ'D} = .235\text{in}^2 - 2(.028) = .179\ \text{in}^2$ FOR Longeron.

$\therefore$ CONSIDER LONGERON OF .040" THICK $1\frac{1}{2}$" w/ $\frac{1}{2}$" WEB.

$A = .040''(2.5'') = .100\ \text{in}^2$

$A_{SKIN} = .028\ (2'') = \underline{.056}$

$.156\ \text{in}^2$

$\sigma_{MAX} = 3530/.156\text{in}^2 = 22{,}600$ psi

CHECK FLANGE BUCKLING, [GERARD P. 271]

$b_F/b_W = 0.5/1.5 = 0.333$ $\quad \dfrac{t_W}{t_F} = 1.0 \rightarrow k_W = 3.8$

$\sigma_{CR} = k_W E \left(\dfrac{t_W}{b_W}\right)^2 = 3.8 \times 10 \times 10^6 \left[\dfrac{.040}{1.5}\right]^2$

$= 3.8 \times 10^7 \times 7.04 \times 10^{-4} = 26{,}800$ psi

$M.S. = 26.8/22.6 - 1 = +.19$

CHECK EULER BUCKLING

$I \approx \dfrac{.04 \times (1.50)^3}{12} + 2(.04)(.50)(.75)^2 + .028 \times 2 \times (.75)^2$

$= .012 + .0225 + .0315 = .067\ \text{in}^4$

$R = \sqrt{\dfrac{I}{A}} = \sqrt{\dfrac{.067}{.156}} = .656$ in

$L/R = 15''/.656 = 23.0$

σ_{BUCKLE} = VERY HIGH

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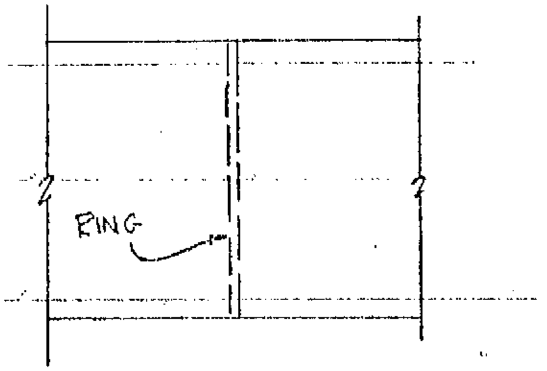
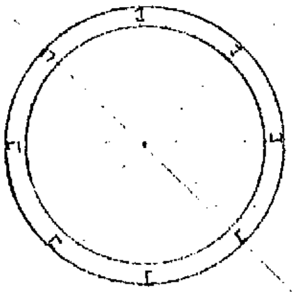
MODEL

RING @ MOTOR LOCATION

DIAM = ~ 20"
WT. OF MOTOR = ~ 550 #, SUPPORTED CONTINUOUSLY BY RING.



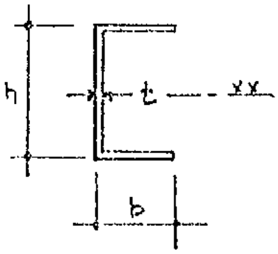


1. CONSIDER RING SUPPORTED BY 8 - LONGERONS.
2. AXIAL LOAD = 550# × 21g = 11,550#

"BEAM" LENGTH = $\pi \times 29"/8$ = 11.4"
LOAD / BEAM = $11{,}550"/8$ = 1440 #

$M = wL^2/12 = WL/12 = 1440^{\#} \times 11.4"/12 = 1370$ "-#

CONSIDER σ_{CRIPP} = ~ 2.5 KSI FOR SECTION



$$I_{xx} = \frac{ht^3}{12} + 2bt\left(\frac{h}{2}\right)^2 = \frac{ht^3}{12} + \frac{bth^2}{2}$$

$$I = \sim bth^2/1.85$$

$$SM = I/_{h/2} = \frac{2I}{h} = 1.08\,thb$$

$$\sigma = M/_{SM}$$

$$\therefore 25{,}000\,psi = 1370^{"-\#}/1.08\,thb$$

$$\rightarrow bth = .0508$$

∴ 2.0" X .75" X .040" [WILL SATISFY THIS.

CHECK BUCKLING STRESS (GERARD PG. 171)

$b_f/b_w = .75/2.0" = .375$ $\qquad t_w/t_f = 1.0 \rightarrow K_w = 5.0$

$$\sigma_{ce} = 5.0 \times 10 \times 10^6 \left[\frac{.040}{2.0}\right]^2 = 20 \times 10^{-4} \times 10^7 = 20{,}000\,psi$$

∴ 2" X .75" X .040" [= NG!

CONSIDER $\sigma_{cr} = 20$ ksi → $bth = .0638$

TRY 2" × .75" × .050" [

$b_f/b_w = .375$ $t_w/t_f = 1.0$ $k_w = 5.0$

$\sigma_{cr} = 5.0 \times 10^7 \left[\frac{.050}{2.0} \right]^2 = 31,200$ psi

$SM = 1.08\, thb = .081\, in^3$

$\sigma = Mc/I = \frac{1370^{in-\#}}{.081} = 16,900$ psi

M.S. $= 31.2/16.9 - 1.0 = +0.85$

$A = 3.50" \times .050" = 0.175\, in^2$

$WT = \pi \times 29 \times 0.175\, in^2 \times .10\, \#/in^3 = 1.60$ LB'S.

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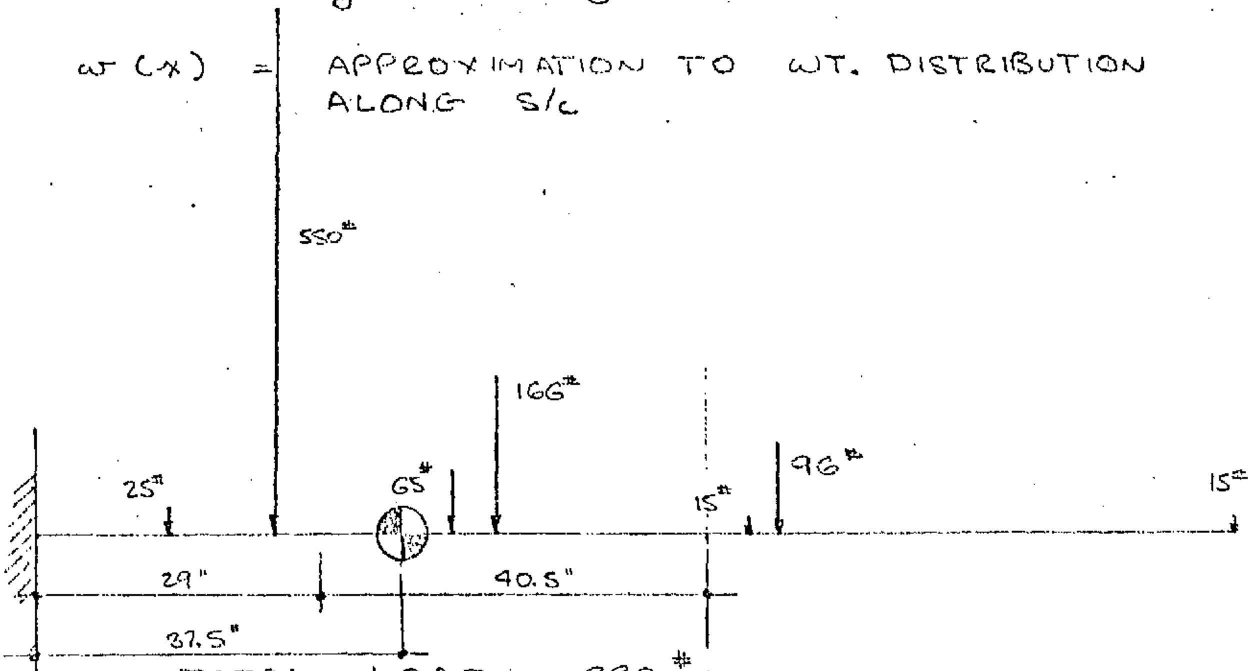
MODEL

CENTRAL STRUCTURE DYNAMIC ANALYSIS - RAYLEIGH APPROXIMATION

$$\omega^2 = \frac{\int_0^L EI(x)\left[\frac{d^2y}{dx^2}\right]^2 dx}{\frac{1}{g}\int_0^L w(x)\; y^2\; dx}$$

$w(x)$ = APPROXIMATION TO WT. DISTRIBUTION ALONG S/C



TOTAL LOAD = 932#

$$\text{CG OF LOAD} = \frac{105{,}000''\text{-}\# \times 1/3}{932\#} = 37.5''$$

∴ ASSUMED CONT. LOAD DISTRIBUTION IS:
UNIFORM ($CG_{UNIF} = 34.75''$ ∴ $\Delta_{cg} = 2.75''$ = ERROR)

∴ $w = 932\#/69.5'' = 13.3\ \#/''$

PREPARED R. BUCKINGHAM 8-16-68 REPORT NO.
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MODEL

PAGE

DEFLECTION ANALYSIS, OF CENT. CYLINDER

FOR CONE: $I = 4.00\,A R^2(2)$

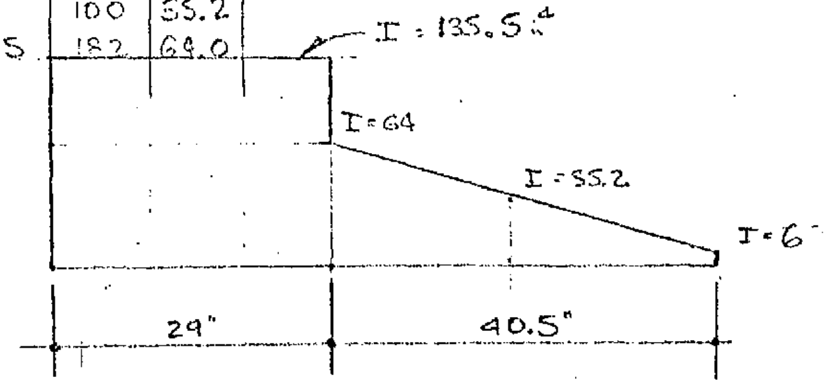
$A = .088 \text{ in}^2$

$\therefore I = .352\,R^2$

R	R^2	I
4	16	5.6
10	100	55.2
13.5	182	64.0

FOR CYL:

$I = 4.00 \times .186 \times 13.5^2 = 135.5 \text{ in}^4$

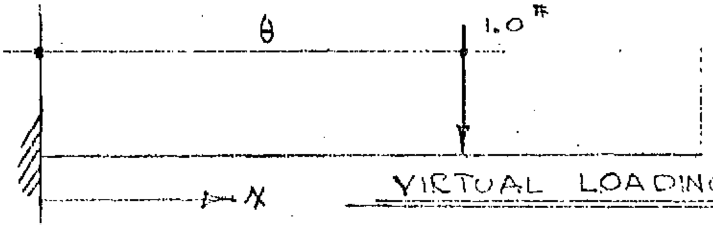


MOMENT OF INERTIA DIST.

$0 \leq x \leq 29$ $\quad I = 113.5 \text{ in}^4$

$29 \leq x \leq 69.5"$ $\quad I = 64 - \frac{58}{40.5}(x - 29)$

$I = 105.5 - 1.431\,x$



VIRTUAL LOADING DIAGRAM

$m(x) = 0 \quad \text{FOR } x > \theta$

$m(x) = 1.0[\theta - x] \quad \text{FOR } x \leq \theta$

$$\therefore \delta = \int_0^L \frac{M(x)\,m(x)\,dx}{E\,I(x)}$$

$M(x) = 105{,}180 - 3518\,x + 39.75\,x^2 - .1467\,x^3$

(FOR 3g. LATERAL LOADING)

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$$- \delta_\theta = \int_0^{29^*} \frac{[105{,}180 - 3518x + 39.75x^2 - .1467x^3][\theta - x]}{135.5 \times 10^7}\,dx$$

$$+ \int_{29}^{\theta} \frac{[105{,}180 - 3518x + 39.75x^2 - .1467x^3][\theta - x]}{10^7\,(105.5 - 1.431x)}\,dx$$

*NOTE WHEN $\theta \leq 29$ THE SECOND INTEGRAL $= 0$, AND THE UPPER LIMIT OF THE FIRST INTEGRAL BECOMES θ, RATHER THAN 29.

θ	$\int_0^\theta$	$\int_{29}^\theta = 0$	δ_θ
0	0	0	0
10	.00347	0	.00347
20	.01244	0	.01244
29	.013079	0	.01308
	$\int_0^{29}$	$\int_{29}^\theta$	δ_θ
40	+.03887	.002863	.04173
50	+.052656	.009542	.062198
60	+.066438	.03142	.09786
69.5	+.07954	+.03142	.11096

= MAX DEFLECTION UNDER 3g. LAT. LOAD.

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CHECKED NR 9-26-68
MODEL

WRITING THE DEFLECTION EQUATION AS AN ANALYTICAL FUNCTION, FOR 3g LATERAL LOADING, USING "POLYCURV" ON-LINE PROGRAM:

$$\delta(x) = -.01379x + .003179x^2 - .0002608x^3 \quad \text{ETC.}$$

$$y(x) = \delta(x) = A_0 + A_1 x + Ax^2 + Bx^3 + Cx^4 + Dx^5 + Ex^6 + Fx^7$$

$$dy/dx = A_1 + 2Ax + 3Bx^2 + 4cx^3 + 5Dx^4 + 6Ex^5 + 7Fx^6$$

$$d^2y/dx^2 = 2A + 6Bx + 12Cx^2 + 20Dx^3 + 30Ex^4 + 42Fx^5$$

$$\int_0^{69.5} EI(x)\left[{d^2y}/{dx^2}\right]^2 dx = 9{,}014.601 + 15{,}820.28 = \boxed{24{,}834.881}$$

$$= \int_0^{29} \frac{10^7 \times 113.5}{9}\left[.635\times10^{-2} - 1.565\times10^{-3}x + 1.242\times10^{-4}x^2 - 4.29\times10^{-6}x^3 + 6.64\times10^{-8}x^4 - 3.79\times10^{-10}x^5\right]^2 dx$$

$$= 9{,}614.601$$

$$+ \int_{29}^{69.5} \frac{10^7}{9}\left[105.5 - 1.431x\right]\left[.635\times10^{-2} - 1.565\times10^{-3}x + 1.242\times10^{-4}x^2 - 4.29\times10^{-6}x^3 + 6.64\times10^{-8}x^4 - 3.79\times10^{-10}x^5\right]^2 dx$$

$$= 15{,}820.28$$

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$$\int_0^L \frac{w(x)}{g} y^2 \, dx =$$

$$\int_0^{69.5} \frac{13.3\,x}{386 \times 9} \Big[-.1379 \times 10^{-1} x + .3179 \times 10^{-2} x^2 - .2608 \times 10^{-3} x^3 + .1037 \times 10^{-4} x^4$$
$$- .2141 \times 10^{-6} x^5 + .2216 \times 10^{-8} x^6 - .0908 \times 10^{-10} x^7 \Big]^2 dx$$

$$= .1574$$

$$\omega^2 = \frac{24{,}834}{.1574}$$

$$= 158 \times 10^3$$

$$\omega^2 = 15.8 \times 10^4$$

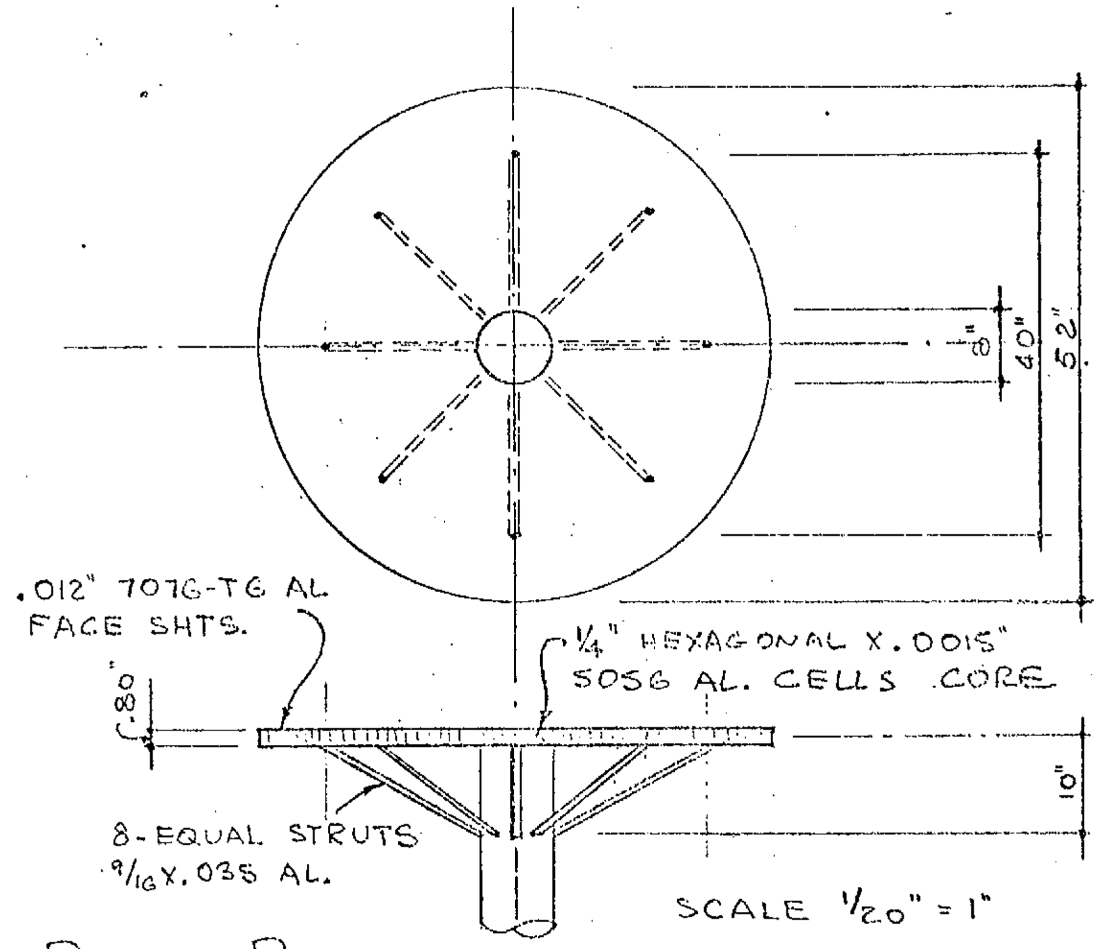
$$\omega = 3.975 \times 10^2 = 397.5$$

$$\boxed{f_N = \frac{\omega}{2\pi} = 63 \text{ CPS.}}$$

NOTE THAT ONLY BENDING DEFLECTION HAS BEEN CONSIDERED. y DUE TO SHEAR WILL LOWER f_N SOMEWHAT.

PREPARED R. BUCKINGHAM JULY 23 '68 REPORT NO.
CHECKED MR 9-26-68
MODEL

PRELIMINARY DESIGN OF UPPER EQUIP. PLATFORM



SCALE 1/20" = 1"

DESIGN PARAMETERS

UPPER PLATFORM

	WT OF	EQUIP	= 71	LB'S
	" "	HARNASSING	= 10	LB'S
EST.	" "	STRUCTURE	= 15	LB'S

TOTAL DYN. MASS OF THB = 96 LB'S.
CONSIDER ULT. DYN. AXIAL LOAD = 25 g'S.
∴ ULTIMATE LOAD = 25g'S X 96 LB'S = 2,400 LB'S

THIS LOAD IS DISTRIBUTED EVENLY THROUGHOUT THE PLATFORM ∴ AREA LOADING:

$$A_{PLATFORM} = \frac{\pi}{4}\left[52^2 - 8^2\right] = 2060 \text{ in}^2$$

$$w_{ULT} = 2400^{\#}/2060 \text{ in}^2 = 1.16 \ ^{\#}/\text{in}^2$$

CONSIDER A FACTOR OF 13% FOR CONC. LOADING:

$$\boxed{\therefore \ w_{ULT} = 1.30 \ ^{\#}/\text{in}^2}$$

PREPARED RB 8-16-68 REPORT NO.
CHECKED NR 9-26-68
MODEL

UPPER EQUIP. PLATFORM

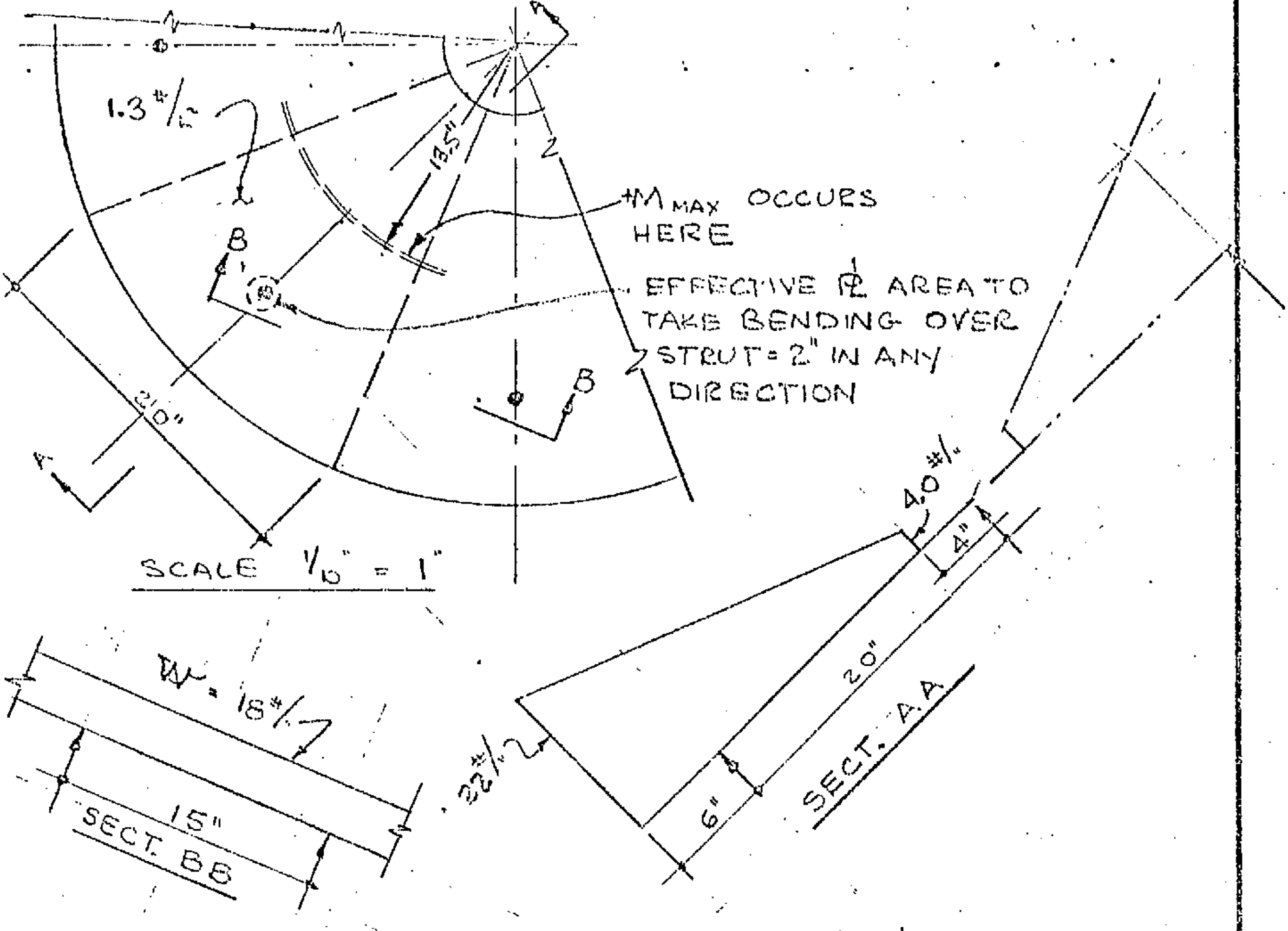
TYPICAL LOADING DISTRIBUTION:

STIFFNESS CRITERIA:

CONSIDER THAT THE LOWEST AXIAL FREQ. MODE OF THE SYSTEM OR ANY OF ITS COMPONENTS:

$$f_n \text{ (LOWEST ALLOW)} = 30 \text{ cps.}$$

ANALYSIS WILL BE PERFORMED FOR 1/8 OF SURFACE AREA, DUE TO SYMETRY.



$$\text{TOTAL LOAD}/\tfrac{1}{8}\text{ AREA} = \frac{1.3\#/in^2 \times \pi\,[26^2 - 4^2]}{8} = 337\#/_{SECTOR}$$

PREPARED RB 8-16-68 REPORT NO.

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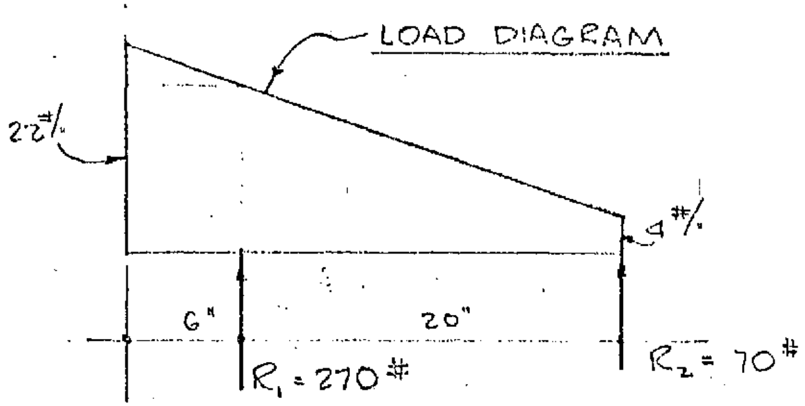
MODEL

UPPER EQUIP. PLATFORM

LOAD DISTRIBUTION TO MEMBERS



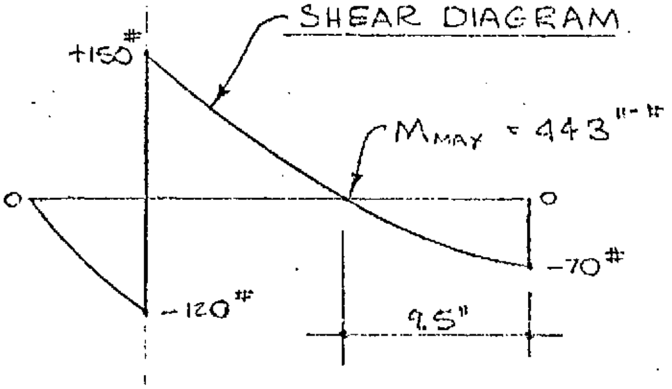
$R_1 + R_2 = 4(26) + \frac{1}{2}(22)(22) = 340$ LB'S.

$\Sigma M_{R2} \rightarrow \quad 4(26)(13) + 18 \times 26 \times \frac{1}{2} \times 26\left(\frac{2}{3}\right) = 20\,R_1$

$1350 + 4050 = 20\,R_1$

$R_1 = 270$ LB'S.

$R_2 = 70$ LB'S



$M_{MAX} \doteq \frac{2}{3}(9.5'' \times 70^{\#}) = 443$ in.#

BY SCALING PLAN VIEW - SECTOR = 10" WIDE @ POINT OF M_{MAX}

$\therefore +M_{MAX}/_{INCH} = 443''^{-\#}/10'' = 44.3\,''^{-\#}$

$-M(MAX) = 120^{\#} \times 6'' \times \frac{2}{3} = 480\,''^{-\#}$

CONSIDER AN EFFECTIVE WIDTH @ STRUT = 2"

$\therefore -M_{MAX}/_{INCH} = 480''^{-\#}/2'' = 240\,''^{-\#}$

PREPARED RB 8-16-68 REPORT NO.
CHECKED NR 9-26-68
MODEL

UPPER EQUIP. PLATFORM

SECTION BB LOAD DISTRIBUTION

CONSIDER THE "BEAM" AS BUILT-IN @ EACH END.
STRUT REACTIONS WERE CALCULATED @ $270^{\#}/_{STRUT}$

$\therefore W/_{BEAM} = 270^{\#} = 18^{\#}/_{''}$ OF BM. LENGTH

$$M = \frac{wL^2}{12} = \frac{WL}{12} = \frac{270^{\#} \times 15''}{12''} = 337^{''-\#}/_{BEAM}$$

CONSIDER EFFECTIVE WIDTH OVER
THE STRUT = 2" WIDE.

$$\therefore M/_{INCH} = 337^{''-\#}/2'' = 169^{''-\#}/_{1''\,STRIP\ @\ STRUT.}$$

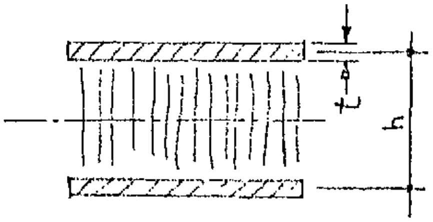
DESIGN OF PLATFORM FOR BENDING MOMENT

THE MOST CRITICAL CALCULATED MOMENT IS NEGATIVE BM. BENDING @ THE CANTILEVER PORTION.

$$-M = 240^{''-\#}/_{INCH}$$

CONSIDER USING 7075-T6 AL. FOR FACE SHEETS.

$F_{TU} = 74$ KSI $F_{SU} = 45$ $F_{BRgU} = 140$
$F_{TY} = 63$ KSI $F_{SY} = 36$ $F_{BRY} = 103$



CONSIDERING 1" STRIP:

$$I = 2(1 \times t)\left[\frac{h}{2}\right]^2 = \frac{2th^2}{4} = \frac{th^2}{2}$$

$$SM = I/_{h/2} = \frac{2I}{h} = \left(\frac{2}{h}\right)\left(\frac{th^2}{2}\right) = th$$

$$\sigma = M/_{SM} = \frac{M}{th}$$

$$\therefore th = \frac{M}{\sigma} = \frac{240^{''-\#}}{74{,}000\,psi} = .00324$$

$\therefore$ THE DESIGN EQUATION IS:

$$h = \frac{.00324}{t}$$

$\therefore$ FOR $t = .010''$ AL ℞

h REQ'D = .324"

$\therefore$ 3/8" THICK HONEYCOMB ℞ w .010" 7075-T6 SHTS = OK!

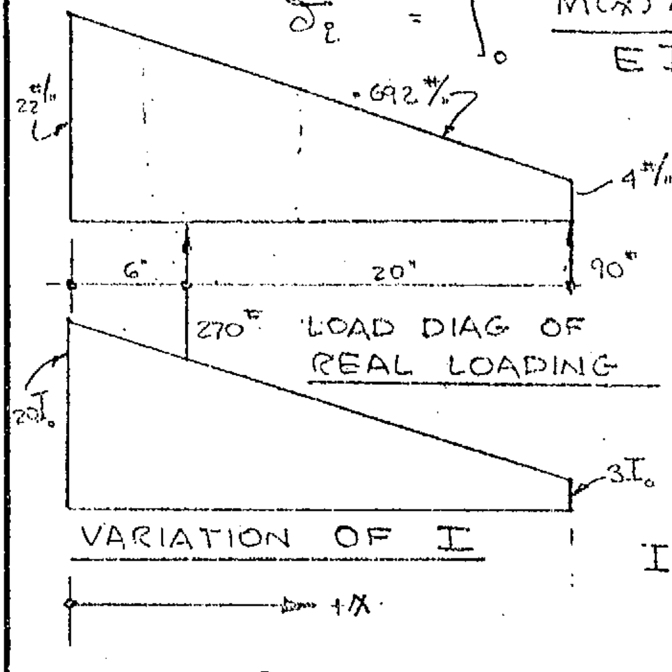
PREPARED RB 8-16-68 REPORT NO.
CHECKED NR 9-26-68
MODEL

UPPER EQUIP. PLATFORM

DEFLECTION OF SECTOR - CONSIDERED AS A BEAM

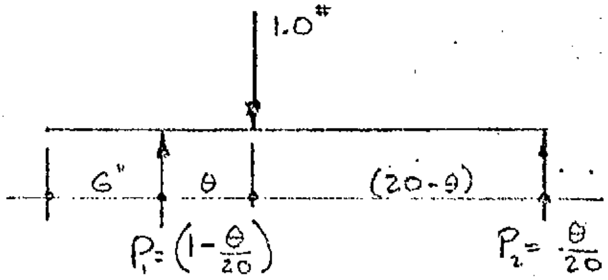
CONSIDER BEAM DEFLECTION BY THE ENERGY METHOD, CONSIDERING A VARIABLE "I".

$$\delta_i = \int_0^L \frac{M(x)\, m(x)\, dx}{E I(x)}$$



LOAD DIAG OF REAL LOADING

VARIATION OF I



$P_1 = \left(1 - \frac{\theta}{20}\right)$ $P_2 = \frac{\theta}{20}$

MOM. DIAG. OF VIRTUAL LOADING

$$I(x) = 20 I_0 - .654\, I_0\, x$$

$[0 \leq x \leq 6]$:

$$M(x) = \frac{22 x^2}{2} - \frac{.692 x^2}{2} \cdot x \left(\frac{1}{3}\right) \qquad m(x) = 0$$

$$= 11 x^2 - .115 x^3$$

$[6 \leq x \leq (6+\theta)]$

$$M(x) = 11x^2 - .115x^3 - 270(x-6)$$

$$M(x) = -.115x^3 + 11x^2 - 270x + 1620$$

$$m(x) = -\left[1 - \frac{\theta}{20}\right]\left[x - 6\right] = \left[\frac{\theta}{20} - 1\right]\left[x - 6\right]$$

$(6+\theta) \leq x \leq 26$

$$M(x) = 11x^2 - .115x^3 - 270x + 1620$$

$$m(x) = \left[\frac{\theta}{20} - 1\right]\left[x - 6\right] + 1.0\left[x - (6+\theta)\right]$$

$$m(x) = \left[\frac{\theta}{20} - 1\right]\left[x - 6\right] + x - 6 - \theta = \frac{\theta}{20}\left[x - 26\right]$$

DEFLECTION BY ENERGY METHOD (CONT)

$6 \leq x \leq \theta$:

$$EI_0 \delta_\theta' = \int_6^{6+\theta} \frac{\left[11x^2 - .115x^3 - 270x + 1620\right]\left[\frac{\theta}{20} - 1\right]\left[x - 6\right]}{20 - .654x}\,dx$$

θ = 5., 10, 15, 20

$EI_0 \delta_\theta'$ = 20.7; 536.4; 923.9 0

PREPARED R B 8-16-68 REPORT NO.

CHECKED NR 9-26-68

MODEL

$(6+\theta) \leq x \leq 26$

$$EI_0 \delta_\theta^{\prime} = \int_{6+\theta}^{26} \frac{\left[11x^2 - .115x^3 - 270x + 1620\right]\left[\frac{\theta}{20}\right]\cdot\left[x - 26\right]}{20 - .654x}\, dx$$

FOR VALUES OF θ = 5; 10, 15, 20

INTEGRAL = 898.3; 992; 320.9; 0

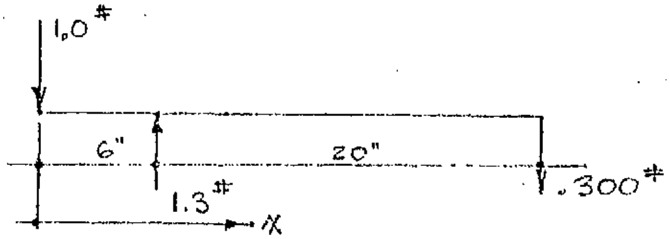
DEFLECTION CURVE OF $EI\, y_\theta(x) = EI\left[\delta_\theta^{\prime} + \delta_e^{\prime}\right]$

PREPARED RB 8-16-68 REPORT NO.

CHECKED HR 9-26-68

MODEL

DEFLECTION @ TIP OF CANTILEVER



$[0 \leq x \leq 6]$

$$m(x) = 1.0\,x = x$$
$$M(x) = 11x^2 - .115x^3$$

$6 \leq x \leq 26$

$$m(x) = x - 1.3^{\#}(x-6) = x - 1.3x + 7.8 =$$
$$\rightarrow m(x) = 7.8 - 0.3x$$
$$M(x) = 11x^2 - .115x^3 - 270x + 1620$$

$$\therefore EI_o\, \delta'_{TIP} = \int_0^6 \frac{x\left[11x^2 - .115x^3\right]}{20 - .654x}\, dx$$

15.346

$$EI_o\, \delta''_{TIP} = \int_6^{26} \frac{\left[7.8 - 0.3x\right]\left[11x^2 - .115x^3 - 270x + 1620\right]}{20 - .654x}\, dx$$

FROM COMPUTER EVALUATION,
INTEGRAL = −758

PREPARED RB	8-16-68	REPORT NO.
CHECKED NR.	9-26-68	
MODEL		UPPER EQUIP. PLATFORM

DYNAMIC ANALYSIS & DESIGN CRITERIA

CONSIDER THE SECTOR + LOAD AS THE DYNAMIC MASS, BEHAVING LIKE A BEAM.
THIS SIMULATES THE FIRST OR "UMBRELLA" MODE OF THE PLATFORM.
IT IS REQUIRED THAT $f_N \geq 30$ CPS.

CONSIDER THAT ACTUAL PLATE IS 10% STIFFER THAN THAT SHOWN IN TREATING IT AS A BEAM, DUE TO THE STIFFENING EFFECTS DESCRIBED BY POISSON'S RATIO:

IE: $\lambda = 1/[1-\mu^2] \quad 1/[1-.30^2] \approx 1.11$
= ABOUT 10% FACTOR.

THE NATURAL FREQUENCY WILL BE COMPUTED BY A RAYLEIGH APPROXIMATION, ONCE $y(x)$ HAS BEEN DETERMINED.

$$\omega^2 = \frac{\int_0^L EI(x)\left[\frac{d^2y}{dx^2}\right]^2 dx}{\frac{1}{g}\int_0^L w(x)\, y^2\, dx}$$

CONSIDER $y(x)_{STAT}$ for STATIC LOAD = $1/_{25 \times 1.13}\, y(x)_{28g}$.
CALCULATED FOR 25 g ULT. LOADING.

$w(x) = 1/_{28}$ THE ULT LOAD. DIAGRAM.

$$w(x) = \frac{1}{28}\left\{ 22x - .692x \cdot x \cdot \tfrac{1}{2} \right\}$$

$$w(x) = 0.79x - .0124x^2$$

CHECK → $0.79(26) - .0124(26)^2 = 20.5 - 8.4 = 12.1^{\#}$
$12.1^{\#}$/PANEL × 8 = $96.7^{\#}$ TOTAL = OK!

PREPARED RB 8-16-68
CHECKED NR 9-26-68
MODEL
REPORT NO.

UPPER EQUIP. PLATFORM

$$\omega^2 = \frac{EI_o \int_0^{26} [20 - .654x] \left[\frac{d^2y}{dx^2}\right]^2 \left(\frac{1}{EI_o}\right) dx}{\frac{1}{g} \int_0^{26} [0.79x - .0124x^2] \frac{y^2}{(EI_o)^2} dx}$$

FOR $f_N = 30$ CPS → $\omega = 2\pi \times 30 = 189$ RAD/SEC.

$\omega^2 = (1.89 \times 10^2)^2 = 3.57 \times 10^4$

$E = 10 \times 10^6$ PSI FOR ALUMINUM

$g = 386.$ in/sec^2

$$gE/\omega^2 = \frac{3.86 \times 10^9}{3.57 \times 10^4} = 1.08 \times 10^5$$

["POLYCURV" PROGRAM]

FROM COMPUTER PROGRAM FOR 25g. LOADING:

$$EI_o y_{20g}(x) = -758 + 75.4x + 7.21x^2 + .572x^3 - .067x^4 + .001x^5$$

$$EI_o y_{STATIC}(x) = -27.0 + 2.69x + .258x^2 + .0206x^3 - .0024x^4 + 3.56 \times 10^{-5} x^5$$

$$EI_o \frac{dy}{dx} = +2.69 + .515x + .0618x^2 - .0096x^3 + 1.78 \times 10^{-4} x^4$$

$$EI_o \frac{d^2y}{dx^2} = .515 + .124x - .0288x^2 + 7.12 \times 10^{-4} x^3$$

∴ FROM COMPUTER RESULTS: OF INTEGRATION.

$$\frac{1}{I_o} = \frac{gE}{\omega^2} \times \frac{\text{INTEGRAL IN NUMERATOR}}{\text{INTEGRAL IN DENOMINATOR}} =$$

$$\frac{1}{I_o} = 1.08 \times 10^5 \; \frac{6.238 \times 10^2}{2.833 \times 10^5} = \frac{2.38 \times 10^7}{10^5} = 2.38 \times 10^2$$

∴ I_o REQ'D FOR 30CPS = f = 4.2×10^{-3} in^4

$$\int_0^{26} \left[20 - .654x\right]\left[.515 + .124x - .0288x^2 + 7.12\times10^{-4}x^3\right]^2 \, dx$$

$$= \underline{623.825}$$

$$\int_0^{26} \left[0.79x - .0124x^2\right]\left[-27.0 + 2.69x + .258x^2 + .0206x^3 - .0024x^4 + 3.56\times10^{-5}x^5\right]^2 dx$$

$$\underline{\underline{283,278.15}}$$

PREPARED RB 8-16-68 REPORT NO.
CHECKED NR. 9-26-68
MODEL

DESIGN FOR STIFFNESS

CONSIDER THAT THE DYNAMIC ANALYSIS BASED ON A BEAM WAS TOO CONSERVATIVE AND, SINCE A REAL PLATE IS STIFFER:

$$I_o \text{ (REQ'D) for 30 cps} = f_n] = [4.2 \times 10^{-3} \text{ in}^4][1-\mu^2]$$

$$= \approx 4.2 \times 10^{-3} \times .91 = 3.82 \times 10^{-3} \text{ in}^4$$

CONSIDERING A 1" SEGMENT OF PLATFORM

$$I = \frac{t h^2}{2}$$

$$\rightarrow h = \sqrt{\frac{2I}{t}} = \frac{8.75 \times 10^{-2}}{\sqrt{t}} = h_{\text{REQ'D}}$$

t	$\sqrt{t}$	h
.010	.10	.875
.016	.1265	.692
.020	.1415	.620
.022	.1482	.590
.028	.1675	.522
.012	.1095	.800

.012 SHTS. WEIGH $2060 \text{ in}^2 \times 2 \times .012" \times .100 \#/\text{in}^3 = 4.9\#$

1/4" HEXCELL CORE WEIGHS $2060 \text{ in}^2 \times .80" \times 1.33 \times 10^{-3} \#/\text{in}^3 = 2.2\#$

UPPER PLATFORM WT. = 7.1#

PREPARED R. BUCKINGHAM 8-16-68 REPORT NO.

CHECKED NR 9-26-68

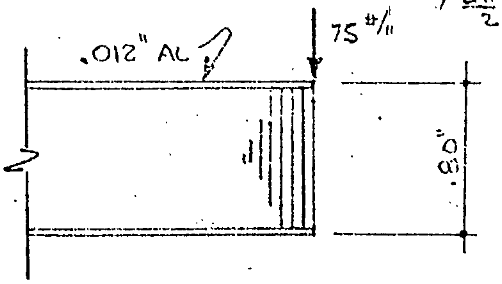
MODEL

HONEYCOMB CORE DESIGN

MAX SHEAR OCCURS AT THE HARD POINTS WHERE STRUTS ATTACH.

CONSIDER A 2"Ø EFFECTIVE CIRCLE TO CARRY SHEAR. FROM SHEAR DIAG.

$\bar{V}_{ULT} = +150^{\#} / \tfrac{2\pi}{2}$ STRIP $= 48^{\#}/''$ VERTICAL



$$Q_{MAX} = \frac{V}{Ib}\int y\,dA$$

$V = 48^{\#}/''$

$I = 3.82 \times 10^{-3}\ in^4/''$

$b = 1''$

$\int y\,dA = .40'' \times .012'' \times 1'' = .0048\ in^3/''$

$$\therefore Q_{MAX} = \frac{48 \times 4.8 \times 10^{-3}}{3.82 \times 10^{-3}} = 60.5\ ^{PSI}/'' = \underline{\text{MAX. HORIZ SHEAR}}$$

CONSIDERING CATALOG OF "HEXCEL" HONEYCOMB CORPORATION OF AMERICA

NOTE THAT THESE VALUES ARE <u>NOT</u> ULT. VALUES IN HEXCEL CATALOG.

CONSIDER 1/4 HEX. CELLS w/.0010 5052 AL. CELLS.

TRANSVERSE SHEAR = 65 PSI = !

$MS = 65/60.5 - 1.0 =$

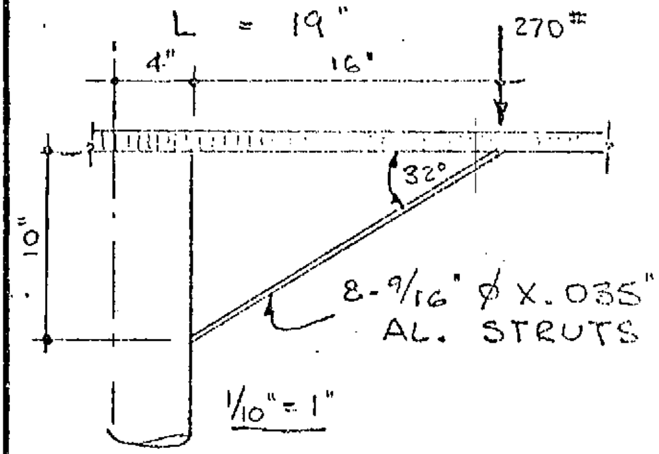
$WT = 2.3\ ^{\#}/_{FT^3} = 1.33 \times 10^{-3}\ ^{\#}/_{in^3}$

PREPARED RB 8-16-68 REPORT NO.
CHECKED NR 9-26-68
MODEL

UPPER EQUIP PLATFORM

DESIGN OF STRUTS

EACH STRUT TAKES 270# / ULT. LOAD.



$P_{ULT} = 270\#/\sin 32° = 510\#$

FOR $L/R = 120 \rightarrow R_{REQ'D} \geq 0.158$

~~TRY 3/8" ϕ X .035" AL. LEGS:~~

~~$L/R = 19/.1208 = 157$~~

~~$f_{BUCKLE} = 4.2$ KSI~~

~~$f_{ULT} = .510^{k}/.0374 = 13.7$ KSI > 4.2 = NG!~~

~~TRY 7/16 X .028:~~

~~$L/R = 19/.145 = 131$~~

~~$f_c = 6.0$ KSI~~

~~$f_{ULT} = .510^{k}/.0362$ = NG!~~

~~TRY 1/2" ϕ X .035:~~

~~$L/R = 19/.1649 = 115$~~

~~$f_c = 7.7$ KSI~~

~~$f_{ULT} = .510^{k}/.05113 = 10^{ksi} > 7.7^{ksi}$ = NG!~~

~~TRY 1/2" ϕ X .049":~~

~~$L/R = 19/.1604 = 119$~~

~~$f_c = 7.2$ KSI~~

~~$f_{ULT} = 7.4$ KSI > 7.2 KSI = NG!~~

PREPARED RB 8-16-68
CHECKED NR 9-26-68
MODEL
REPORT NO.

UPPER EQUIP PLATFORM

~~TRY 9/16 X .028 LEG~~

~~$L/R = 100$~~

~~$f_c = 10.2$ ksi~~

~~$f_{ULT} = .510^k / .04702 = 10.8$ ksi = NG!~~

TRY 9/16" ⌀ X .035 STRUT

$L/R = 102$

$f_c = 10.0$ ksi

$f_{ULT} = .510^k / .05800 = 8.8$ ksi < 10.0 ksi = OK!

$$MS\ (ULT) = \frac{10.0}{8.8} - 1.0 = +0.14$$

∴ USE 9/16 X .035 AL. STRUTS.

$WT/LEG = .59^{\#}\left(19''/100''\right) = 0.112\ \#/LEG$

TOTAL OF 8 LEGS WEIGH $8 \times 0.112 = 0.900$ LB'S

$A/LEG = .05800\ in^2$

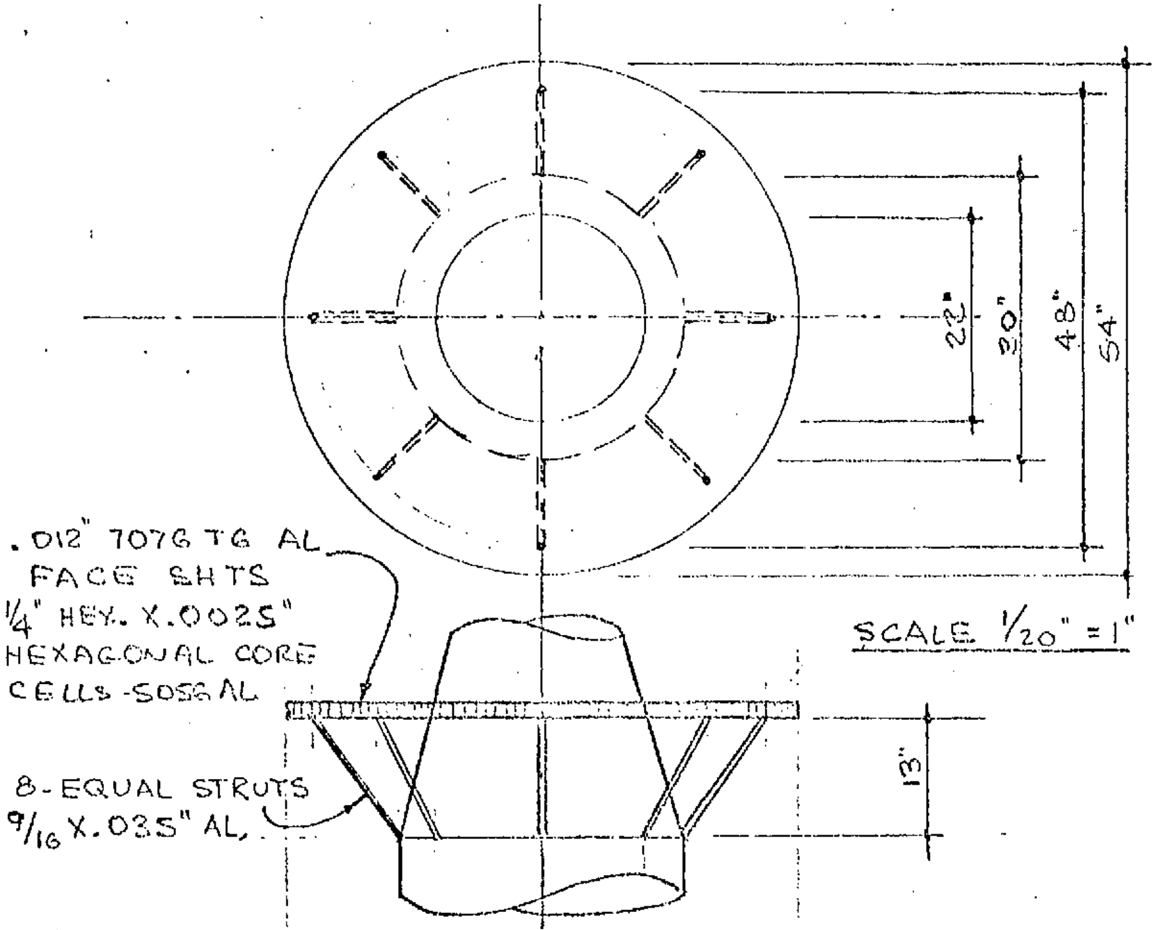
AXIAL STIFFNESS: $= \frac{P}{\delta} = \frac{AE}{L} = \frac{.058 in^2 \times 10^4\ ksi}{19} = 30.5\ ^k/''$
(AXIS OF LEG)

EQUIV. STIFFNESS/LEG OF S/C AXIS

$= 30.5\ ^k/'' \times \sin 32° = 16.1\ ^k/''$

PREPARED RB 8-16-68 REPORT NO.
CHECKED NR 9-23-68
MODEL

PRELIM. DESIGN FOR LOWER EQUIP. PLATFORM



DESIGN PARAMETERS

Σ WT OF SOLAR PANEL = 52.5# @ EDGE OF PLATFORM
(WET) Σ WT. OF FUEL TANKS = 4 X 18 LB'S = 72 LB'S
BATTERY = 23 LB'S
MISC. EQUIP = 31 LB'S
HARNASSING = 5
STRUCTURE = 15 LB'S

TOTAL WT. = 72 LB'S (TANKS) + 74# = 146#

CONSIDER FUEL TANKS, DUE TO MOUNTING, TRANSFER ONLY 2/3 OF LOAD TO PLATFORM $\therefore P_{EFF} = 2/3 \times 72\# = 48\#$

AREA = $\frac{\pi}{4}[54^2 - 22^2] = 1910 \text{ in}^2$
ULT. LOAD = 20 g'S $\times$ [74# + 48#] = 2440 LB'S.
$\therefore w'_{ULT} = 2440/1910 = 1.27 \ \#/\text{in}^2$

$\therefore$ CONSIDER w_{ULT} + 20% FACTOR FOR UNEVEN LOAD DIST.

$$\boxed{\therefore w_{ULT} = 1.50 \text{ LB'S}/\text{in}^2}$$

PREPARED RB 8-16-68
CHECKED NR 9-26-68
MODEL
REPORT NO.

ULTIMATE LOAD OF SOLAR PANELS IS SPREAD OUT AROUND CIRCUMFERENCE OF PLATFORM.

CONSIDER THAT PANELS SEE 30 g'S. ULTIMATE

$P_{\text{SOLAR PANELS}}$ (ULT) = 30 × 52.5# = 1522 LB'S.

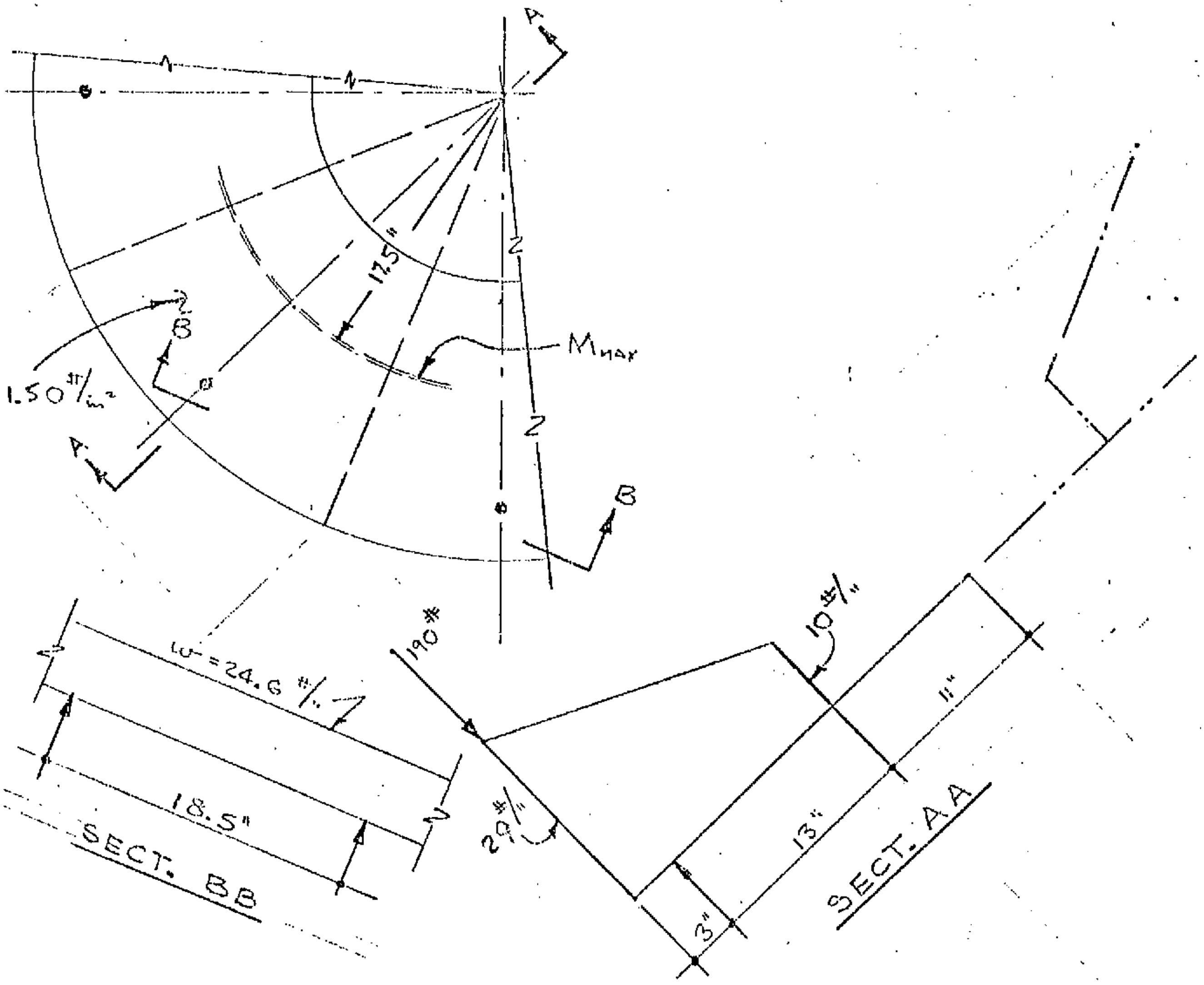
∴ EACH SECTOR SEES 1522#/8 = 190 LB'S AS A POINT LOAD.

PREPARED RB 8-16-68 REPORT NO. PAGE 40
CHECKED NR 9-26-68
MODEL

LOWER EQUIP PLATFORM

TYPICAL LOADING DISTRIBUTION

STIFFNESS CRITERIA : f_N (ALLOW) ≥ 30 cps ,



$$\text{TOTAL LOAD}/_{1/8}\ \text{SECTOR} = \frac{1.50\,\#/in^2 \times \pi\,[27^2 - 11^2]}{8} = 390\,\#/_{\text{SECTOR}}$$

PREPARED RB 8-16-68
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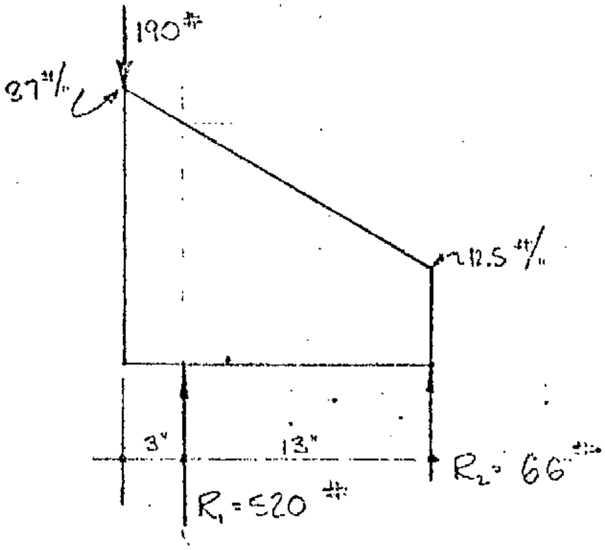
MODEL
REPORT NO.

LOWER EQUIP. PLATFORM

LOAD DISTRIBUTION TO MEMBERS



$R_1 + R_2 = 190 + 16(12.5) + \frac{1}{2}(24.5)(16) = 586^{\#}$

$\Sigma M_{R_2} \rightarrow \quad 16'' \times 12.5^{\#} \times 8'' \quad / = 1600$

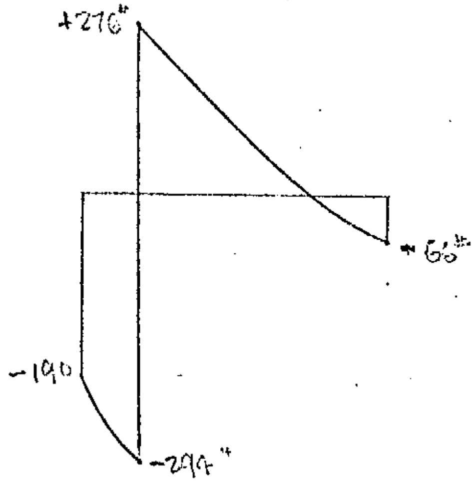
$24.5 \times 16 \times \frac{1}{2} \times 16 \times \frac{2}{3} = 2090$

$190^{\#} \times 16'' = 3040''\text{-}\#$

$6730''\text{-}\#$

$R_1 = 6730''\text{-}\# / 13' = 520^{\#}$

$R_2 = 520^{\#} - 586^{\#} = 66^{\#}$



M (MAX) = THE CANTILEVER

$= 190 \times 3'' = 570$

$+ 104 \times 3'' \times \frac{2}{3} = 208$

$-M \text{ (MAX} = 778''\text{-}\#$

SECT. BB LOAD DISTRIBUTION

$-M = \frac{WL}{12} = \frac{570'' \times 18.5}{12} = 880''\text{-}\#$

PREPARED RB. 8-16-68
CHECKED NR 9-26-68
MODEL
REPORT NO.

LOWER EQUIP. PLATFORM

DESIGN OF CORE

USE: .012" FACE SHTS, SEPARATED 0.80"
SIMILAR TO UPPER PLATFORM.

SHEAR LOAD IS SPREAD OVER 1/2 CIRCUMF. = $\frac{\pi D}{2} = 3.14"$

$\therefore V_{MAX} / \text{INCH} = 294^{\#}/_{3.14} = 94^{\#}/"$

$$Q_{MAX} = \frac{V}{Ib}\int^{1} y\,dA = \frac{1.94^{\#/"} \times .0048\ \text{in}^3/"}{3.82 \times 10^{-3}\ \text{in}^4/"} = 119 \underset{\text{ULT.}}{\text{psi}}$$

FROM "HEXCELL" CATALOGUE, 5052 AL. ALLOY
USE 3/16" CELLS X .0010" WALL, 5052 AL. ALLOY
CAN TAKE 110 psi = OK! (= AVG. VALUE
WT = 3.1 #/ft³ = 1.80×10^{-3} #/in³

TOTAL WT.

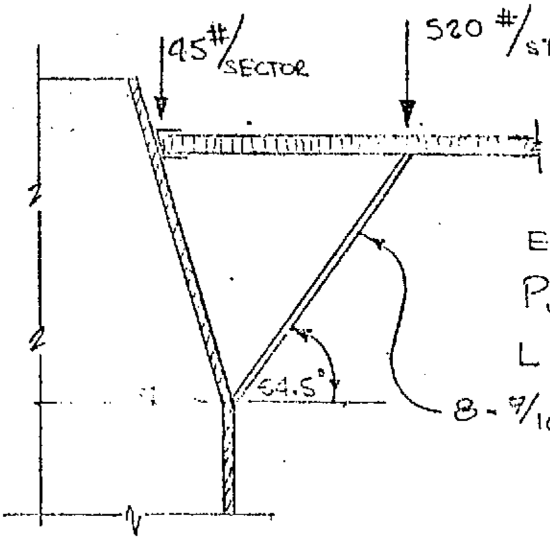
.012 SHTS = .012" X 1910 in² X 2 X .1 #/in³ = 4.6#
CORE = .80" X 1910 in² X 1.80×10^{-3} = 2.8#

Σ PLATFORM WT = 7.4#

PREPARED RB 8-16-68 REPORT NO.
CHECKED NR 9-26-68
MODEL

LOWER EQUIP. PLATFORM

DESIGN OF STRUTS



EACH STRUT CARRIES 457#/ = VERT. ULT. LOAD

$P_{ULT} = 457^{\#} / \sin 54.5^\circ = 560^{\#}/_{STRUT}$

$L = 16"$

TRY: 1/2" Ø X .035 AL. STRUT

$L/R = 16/.1649 = 97$

$f_c = 10.9$ KSI

$f_{ULT} = 560^{\#}/.05113 \text{ in}^2 = 10.9$ KSI = 10.9 KSI = ok!

$WT./_{STRUT} = .62^{\#} \left(16"/100" \right) = .100^{\#}/_{STRUT}$

Σ WT. OF 8-STRUTS = $8 \times .100^{\#} = .80^{\#}$

STIFFNESS ALONG AXIS OF STRUT = $\frac{.051 \text{ in}^2 \times 10^4}{16} = 31.9$ K/"

STIFFNESS/$_{STRUT}$ IN S/C AXIAL DIR. = $31.9 \sin 54.5^\circ = 25.9$ K/"

M.S. (ULT.) = 0

IF WE USE 9/16 X .035 TUBE

$f_c = 13.8$ KSI

$f_{ULT} = 520/.058 \text{ in}^2 = 9.0$ KSI

MS = +0.56

TOTAL WT = $.90^{\#}$

∴ USE 9/16 X .035 TUBES!

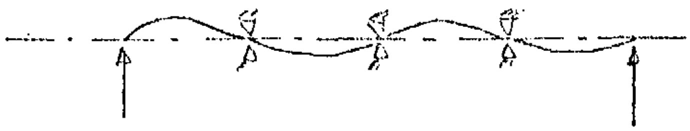
AXIAL STIFFNESS (S/C)/$_{TUB}$ = 29.5 K/"

PREPARED R. BUCKINGHAM 8-16-68
CHECKED NR 9-26-68
MODEL
REPORT NO.

SUBSTRATES

DESIGN SUBSTRATES FOR DYNAMIC CHARACTERISTICS

REF: CREDE & HARRIS, SHOCK & VIB. H'DBK, PG. 1-14



$$\omega_N = A\sqrt{\frac{EJ}{\mu L^4}} \longrightarrow I = \frac{\omega^2 \mu L^4}{A^2 E}$$

A = 158. (FROM TABLE)
E = (FIBERGLASS) = 3×10^6 PSI
μ = MASS/UNIT LENGTH

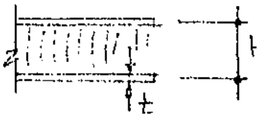
TOTAL WT. = 52.5#/8 PANELS = 6.57#/PANEL
MASS/PANEL = $6.57\#/386\ \text{in/sec}^2 = 1.70\times10^{-2}\ \frac{\#\ \text{SEC}^2}{\text{IN}}$
μ = MASS/LENGTH = $1.70\times10^{-2}/79'' = 2.15\times10^{-4}\ \frac{\#\ \text{SEC}^2}{\text{IN}^2}$
$\omega_N = 2\pi\times50$ CPS = 314 RAD/SEC

$$I_{\text{REQ'D}}\ (\text{FOR 50 CPS}) = \frac{[3.14\times10^2]^2\ 2.15\times10^{-4}\times(7.9)^4\times10^4}{[1.58\times10^2]^2\times3\times10^6}$$

$$= \frac{21.2\times3910\times10^7}{7.49\times10^{10}} = 11.05\times10^{-3}$$

$\therefore I_{\text{REQ'D}} = .01105\ \text{in}^4$ FOR $f_{N_{\text{PANEL}}} = 50$ CPS

CONSIDER HONEYCOMB PANEL W/FIBERGLASS FACE SHEETS:



$$I = 2A_{PL}\times\left(\frac{h}{2}\right)^2 = \frac{Ah^2}{2} = \frac{bth^2}{2}$$

FOR $b = \frac{\pi D}{8} = \frac{55.4\times\pi}{8} = 21.8''$ = PANEL WIDTH

$$\rightarrow h = \sqrt{\frac{2I}{bt}} = \sqrt{\frac{2\times1.105\times10^{-2}}{21.8\ t}} = \frac{3.17\times10^{-2}}{\sqrt{t}}$$

t	$\sqrt{t}$	h
.010	.10	.317"
.012	.11	.288"
.015	.123	.210"
.017	.131	.186"

PREPARED R. BUCKINGHAM 8-14-68
CHECKED NR 9-26-68
MODEL
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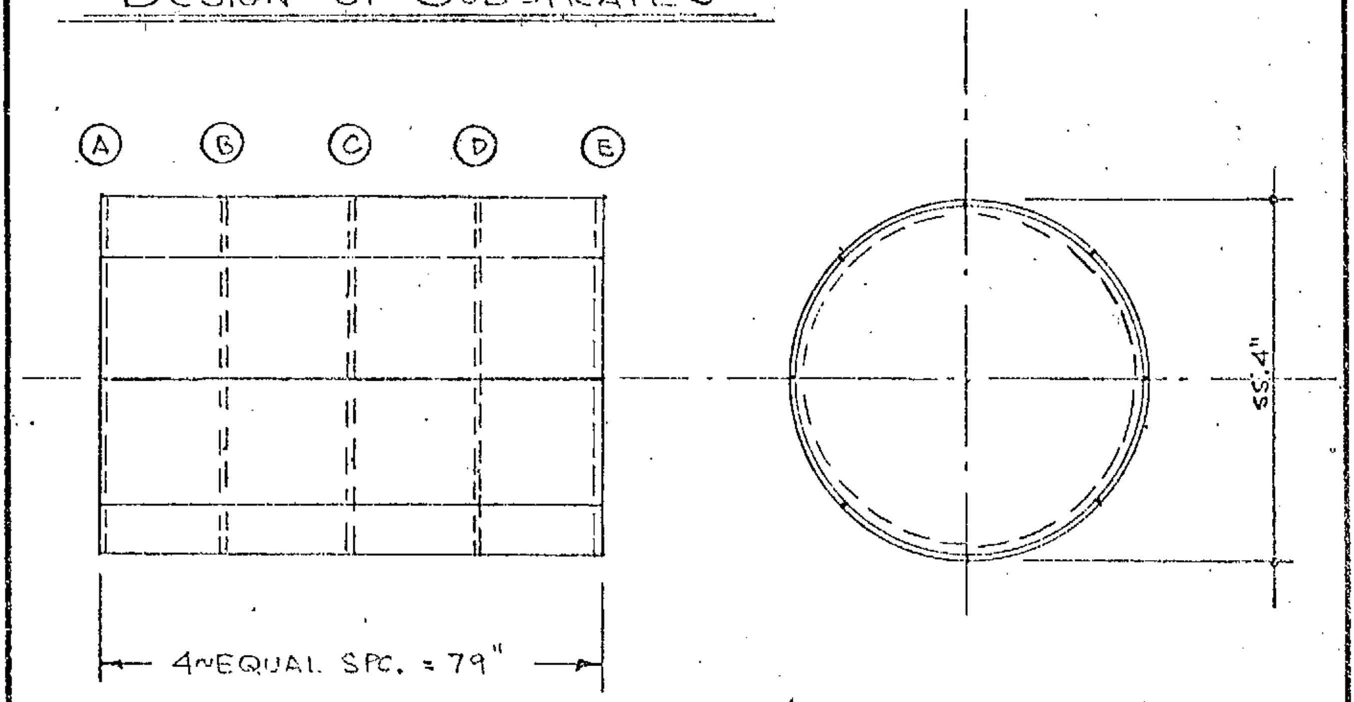
ANALYSIS OF SUBSTRATES.

DESIGN OF SUBSTRATES



SCALE 1/30" = 1"

DESIGN CRITERIA

1. SUBSTRATES ARE HONEYCOMB CORE w/FIBERGLASS FACE SHEETS.
2. WT. OF SOLAR ARRAYS = 52.5 LB'S. (TOTAL)
3. LATERAL LOAD (ULT.) = 40 g'S. / PANEL ASSY. [75_g FOR PANEL ALONE]
4. EACH SUBSTRATE IS INDEPENDENT AND MUST HAVE $f_N \geq 50$ CPS.
5. SUBSTRATES ARE SUPPORTED @ PLATFORM Ⓒ WITH RINGS @ Ⓑ & Ⓓ TO PREVENT BUCKLING.
6. THE RINGS ACT AS DISTRIBUTION MEMBERS, SO THAT LOAD NORMAL TO A PANEL → RING → ADJACENT SIDE PANELS → CARRIES LOAD IN SHEAR TO PLATFORM SUPPORT POINT.

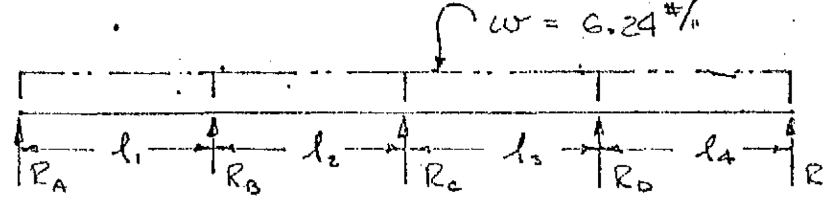
PREPARED R. BUCKINGHAM 8-16-68 REPORT NO.
CHECKED NR 9-26-68
MODEL

SUBSTRATES

DESIGN SUBSTRATES FOR STRENGTH CHARACTERISTICS

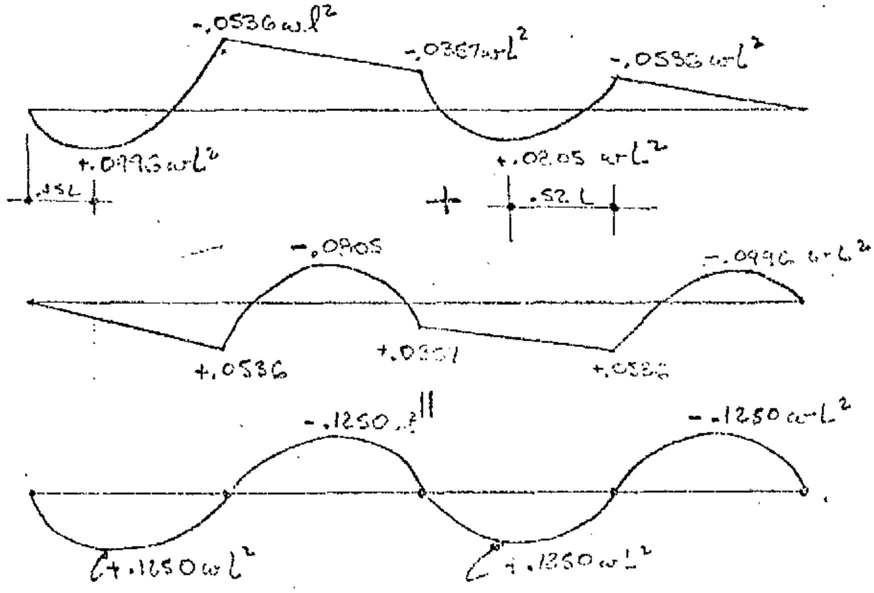
1. EACH PANEL, BY ITSELF, MUST BE CAPABLE OF RESISTING 75g (ULT.) LOADING, NORMAL TO THE SURFACE. THIS CONDITION WOULD OCCUR DURING QUAL. VIBRATION TESTING.

2. CONSIDER PANEL AS CONT. BEAM W/4 EQUAL SPANS.
WT. / PANEL = 6.57 LB'S.
DYN. LOAD / INCH = 6.57# × 75g × 1/79" = 6.24 #/INCH (ULT.)



REFERENCE: "A.I.S.C. STEEL CONST. HANDBOOK" SIXTH ED. PG. 2-133, BEAM FORMULA.

a) MAX. NEG. MOMENT OCCURS WHEN PANEL IS IN ITS VIBRATION MODE, SUCH THAT THE UNIFORM LOADS ON l_1 & l_3 ARE "DOWNWARD", AND LOADS ON l_2 & l_4 ARE "UPWARD".



∴ M (MAX) = .1250 × 6.24 × $(20)^2$ = 312 "-#

V (MAX) OCCURS WHEN ALL PANELS ARE LOADED & OCCURS @ LHS OF Ⓑ & RHS OF Ⓓ & = .607 ωL

V (MAX) = .607 × 6.24 × 20 = 75.5 #/PANEL

PREPARED E. BUCKINGHAM 8-16-68 REPORT NO.

CHECKED N.E. 9-22-68

MODEL

SUBSTRATES

SUBSTRATE DESIGN

$$M/_{\text{INCH}} = 312^{\text{"-\#}}/21.8^{\text{"}} = 14.3^{\text{"-\#}}/\text{"}$$

F_{TU} OF FIBERGLASS = 22,800 PSI

CONSIDER USING 3/8" HONEYCOMB X .010" FACE SHTS

$$I = bth^2/_2 = \frac{21.8\times10^{-2}\times(.375)^2}{2} = 1.53\times10^{-2}\ \text{in}^4$$

$$f = \frac{Mc}{I} = \frac{312^{\text{"-\#}}\times .375^{\text{"}}/2}{1.53\times10^{-2}} = 38.2\times10^{2} = 3820\ \text{psi}$$

$$MS = 22.8/_{3.82} - 1.0 = \approx 5.0 = \underline{OK}!$$

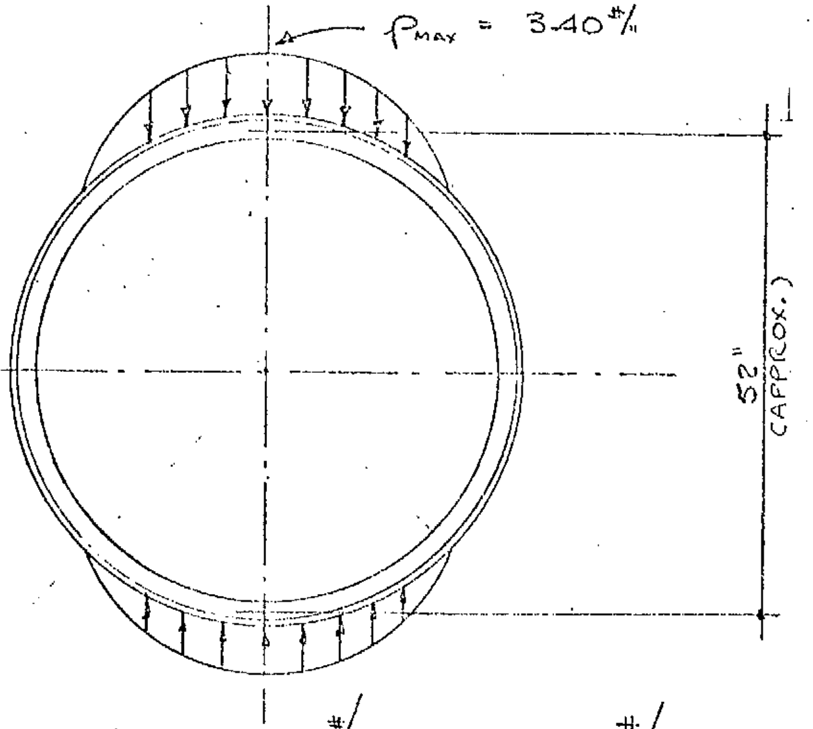
∴ USE 3/8" HONEYCOMB IN 1.6-1/4-.7P 5052
HONEYCOMB CORE W 0.10" FIBERGLASS
(MIL R-9500) FINE SHEETS (120 CLOTH)

$$f_N = 50\ \text{cps}\sqrt{\frac{1.53\times10^{-2}}{1.105\times10^{-2}}} = 59\ \text{CPS} = \text{OK}!$$

PREPARED RB 8-16-68
REPORT NO.
CHECKED MC 9-26-68
MODEL

RING DESIGN (FOR SUBSTRATES)

1. MAX SHEAR / RING OCCURS @ $R_B = R_D = 1.143\,\omega L$
2. CONSIDER: RING TAKES 40 "g" LOAD
3. MOST CRITICAL COND. = "BREATHING" MODE



LOAD / PANEL = 40 g × 6.57 #/PANEL = 263 #/PANEL

$\omega = 263\,\#/79" = 3.23\,\#/"/\text{PANEL}$

$\therefore$ MAX REACTION = 1.143 × 3.23 #/" × 20" = 74 #/PANEL

SPREADING THIS OUT ACROSS PANEL

$$\rho_{MAX} = \frac{74\,\#/\text{PANEL}}{21.8"} = 3.40\,\#/"$$

$$M\,(MAX) = -[.11392 + .06632]\,\rho_{MAX}\,r^2$$
$$= -.18024 \times 3.40\,\#/" \times (26)^2 = 415\ "\text{-}\#$$

PREPARED R. BUCKINGHAM 8-16-68 REPORT NO.

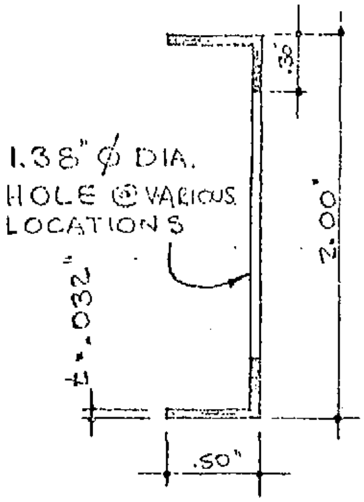
CHECKED MR 9-26-68

MODEL

SUBSTRATE ASSY.

RING DESIGN (CONT)

INVESTIGATE RING USED ON INTELSAT III :



$I = .0423 \text{ in}^4$

$f_B = MC/I = \dfrac{415^{\text{"-\#}} \times 1.0"}{.0423 \text{ in}^4} = 9800 \text{ psi}$

$F_{TU} = 64 \text{ ksi}$

MAT'L = 2024-T62 AL. SHT.

BUCKLING STRESS OF RING

USING METHOD OF INTELSAT III

$b/t = .50"/.032" = 15.6$

$KL/R = 5.13 \; b/t = 80.4$

$F_c = 102{,}000/(80.4)^2 = 15.8 \text{ ksi}$

$f_c = 9.8 \text{ ksi}$

$MS \text{ (ULT)} = 15.8/9.8 - 1.0 = \underline{+0.61 = OK!}$

PREPARED R. BUCKINGHAM 8-68
CHECKED N. ROSEN 9-68
MODEL
REPORT NO.

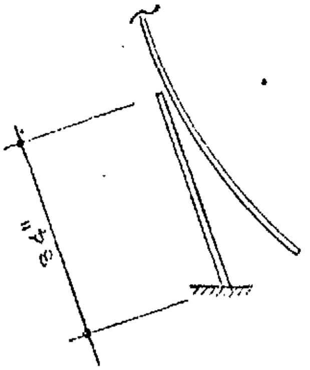
ANTENNA SUPPORT STRUCTURE

REQ'D THAT f_N (LONG) $\geq$ 40 CPS

f_N (LAT.) $\geq$ 15 CPS.

WT. OF ANTENNA = 13.3 LB'S.

L OF CANTILEVER = 34"



CONSIDER 5" DIAM X .050" AL TUBE.

$$WT = \pi \times 5.0 \times 34'' \times .050'' \times .10^{\#}/_{in^3} = 2.67^{\#}$$

WT. OF ENTIRE ASSY = 16#

$$I = \pi t R^3 = \pi \times .050 \times (2.5)^3 = 2.45\ in^4$$

$$K_{LATERAL} = 3EI/L^3 = \frac{3\times10^7 \times 2.45}{(3.4)^3\times10^3} = .189\times10^4\ ^{\#}/_{in}$$

$$K_{AXIAL} = AE/L = \frac{.786\ in^2 \times 10^7}{34''} = 2.31\times10^5\ ^{\#}/_{in}$$

$$m = 2.67^{\#}/_{386} = .692\times10^{-2}$$

$$M = 13.3^{\#}/_{386} = 3.45\times10^{-2}$$

$$f_N\ (AXIAL) = \frac{1}{2\pi}\sqrt{\frac{2.31\times10^5}{3.45\times10^{-2} + 0.33\times.692\times10^{-2}}} =$$

$$= \frac{1}{2\pi}\sqrt{\frac{.630\times10^5}{10^{-2}}} = \frac{2.51\times10^3}{2\pi} = 402\ cps.$$

$$f_N\ (LATERAL) = \frac{1}{2\pi}\sqrt{\frac{.189\times10^4}{3.45\times10^{-2} + .23\times.692\times10^{-2}}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{5.23\times10^2}{10^{-2}}} = \frac{229}{2\pi} = 36.5\ cps.$$

REF: DEN HARTOG: "MECH. VIBRATIONS" PG. 430
4 TH. EDITION.

PREPARED R. BUCKINGHAM 8-68 REPORT NO.

CHECKED N. ROSEN 9-68

MODEL

ANTENNA SUPPORT STRUCTURE

CONSIDER g (ULT) FOR ANTENNA IN LAT. DIRECTION:
FROM "DYNAMICS" OF ATLAS II

LATERAL @ $f_n = 36.5$ cps , $Q = 5$ $\longrightarrow$ .006 $g^2/_{cps}$ = RANDOM LEVEL

$$\bar{g}_{RMS} = \sqrt{\frac{\pi}{2} \times 5 \times 36.5 \times .6 \times 10^{-2}} = \sqrt{1.73} = 1.315\ g_{RMS}$$

$$\therefore g_{ULT} = 1.50 \times 3 g_{RMS} = 6.0\ g\text{'s}$$ LATERAL @ MAX Q

CONSIDER AXIAL LOAD @ MAX Q = 12.0 g.

$$\therefore M_{MAX} = 6.0g \times 16^{\#} \times 34'' = 3270^{''-\#}$$

$$P_{MAX} = 16^{\#} \times 12g = 192^{\#}$$

$$\sigma = \frac{Mc}{I} + \frac{P}{A} = \frac{3270 \times 2.5}{2.45} + \frac{192}{.786}$$

$$\sigma_{ULT} = 3350 + 244 = 3594 \text{ PSI}$$

$$R = \sqrt{I/A} = \left[2.45/.786\right]^{1/2} = 1.765$$

$$L/R = 34''/1.765'' = 19.2 = \text{OK!}$$

CHECK LOCAL BUCKLING OF TUBE:

$$R/t = 2.5/.050 = 50 \dashrightarrow \text{EQUIV } \frac{KL}{R} = 34.0$$

$$\sigma_{BUCKLE} @ KL/R = 34 = 83 \text{ PSI} = \text{OK!}$$

$$\therefore \text{M.S. OF TUBE} = \frac{64 \text{ KSI}}{3.6} - 1.0 = \approx +17.0$$

FOR 2024-T3 ALUMINUM.

CHECK LATERAL δ @ MAX Q.

$$\delta = P_{ULT}/K_{LAT} = 96^{\#}/1890^{\#}/'' = 5.1 \times 10^{-2} = .051''$$

APPENDIX B

DYNAMIC EQUATIONS OF MOTION

B. Dynamic Equations of Motion for the Spacecraft

A linearized dynamic model of the spacecraft was developed to study the performance of the despin control system in the normal cruise mode. The spacecraft was modelled as three rigid bodies; the spinning rotor, the despun antenna, and a pendulum damper located on the despun antenna. Damping due to fuel slosh on the rotor was neglected. Inertia cross-products on the despun antenna together with the antenna center of mass offset were the major spacecraft anomalies included in the model.

Figure B-1 illustrates the coordinate systems employed in the derivation of the dynamic equations of motion. The reference coordinate frame $(\bar{x}, \bar{y}, \bar{z})$ is a right-handed orthogonal frame with origin at the spacecraft center of mass, the $\bar{x}$ axis along the spacecraft velocity vector, and the $\bar{z}$ axis pointed down along the local vertical. Fixed in body 1 (the despun antenna) is an orthogonal frame $(\bar{x}_1, \bar{y}_1, \bar{z}_1)$ with the origin at the actual center of mass of body 1. Parallel to this coordinate frame is a similar frame $(\bar{x}', \bar{y}', \bar{z}')$ with origin $(0')$ at the nominal mass center of body 1. Body 2 (the rotor) also contains a body-fixed coordinate frame $(\bar{x}_2, \bar{y}_2, \bar{z}_2)$ with the origin at the mass center.

Define the angular position of body 1 by the transformation equation;

$$\begin{Bmatrix} \bar{x}_1 \\ \bar{y}_1 \\ \bar{z}_1 \end{Bmatrix} = \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix} \begin{Bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{Bmatrix}$$

where ϕ, θ, and ψ are the roll, pitch, and yaw errors of the despun antenna. Similarly the position of body 2 relative to body 1 is defined by;

$$\begin{Bmatrix} \bar{x}_2 \\ \bar{y}_2 \\ \bar{z}_2 \end{Bmatrix} = \begin{bmatrix} \cos \nu & 0 & -\sin \nu \\ 0 & 1 & 0 \\ \sin \nu & 0 & \cos \nu \end{bmatrix} \begin{Bmatrix} \bar{x}_1 \\ \bar{y}_1 \\ \bar{z}_1 \end{Bmatrix}$$

where ν is the relative angular displacement. The position vectors of bodies 1, 2, 3 (the damper mass) relative to $0'$ are;

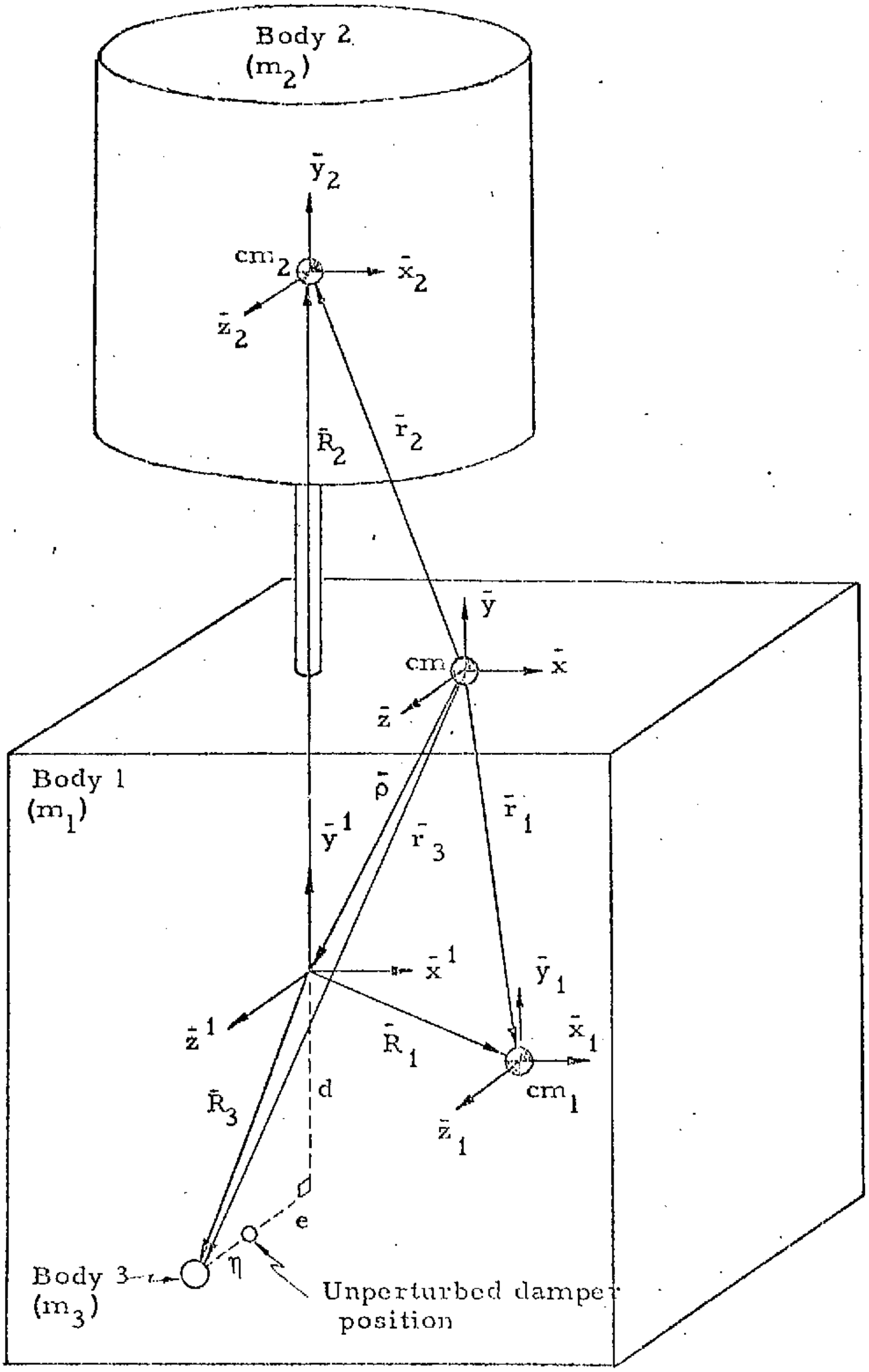


Figure B-1 COORDINATE SYSTEMS

$$\bar{R}_1 = a\,\bar{x}_1 + b\,\bar{z}_1$$

$$\bar{R}_2 = c\,\bar{y}_1$$

$$\bar{R}_3 = -d\,\bar{y}_1 + (e + \eta)\,\bar{z}_1$$

and the position vector of O' relative to the reference origin is $\bar{\rho}$. Neglecting orbit rate, the linearized body rates are:

$$\bar{\omega}_{B1} = \dot{\phi}\,\bar{x}_1 + \dot{\theta}\,\bar{y}_1 + \dot{\psi}\,\bar{z}_1$$

$$\bar{\omega}_{B2} = \dot{\phi}\,\bar{x}_1 + (\dot{\nu} + \dot{\theta})\,\bar{y}_1 + \dot{\psi}\,\bar{z}_1$$

In operation, the rotor relative speed $(\dot{\nu})$ will vary about some nominal value by a small amount. Let

$$\dot{\nu} = \Omega_s + q$$

where (Ω_s) is the rotor nominal relative spin speed (a constant).

The rotational equation of motion for body 1 can be derived as;

$$\bar{T}_o{}' = M\,\bar{R} \times \frac{d^2\bar{\rho}}{dt^2} + \frac{d}{dt}\left\{\bar{\bar{\Phi}}_2 \cdot \bar{\omega}_2 + m_3\,\bar{R}_3 \times \dot{\bar{R}}_3\right\} + \frac{d}{dt}\left\{\bar{\bar{\Phi}} \cdot \bar{\omega}_{B1}\right\} \qquad (1)$$

where:

$\bar{T}_o{}'$ = external torque on body 1

$M = m_1 + m_2 + m_3$

$\bar{R} = \frac{1}{M}\left(m_1\,\bar{R}_1 + m_2\,\bar{R}_2 + m_3\,\bar{R}_3\right)$

$\bar{\bar{\Phi}}_2$ = inertia dyadic of body 2

$\bar{\omega}_2$ = relative velocity of body 2 relative to body 1

$\bar{\bar{\Phi}} = \bar{\bar{\Phi}}_o + \bar{\bar{\Phi}}_r$

$\bar{\bar{\Phi}}_o$ = inertia dyadic about O' due to the subsystem masses

$$\bar{\bar{\Phi}}_r = \bar{\bar{\Phi}}_1 + \bar{\bar{\Phi}}_2 + \bar{\bar{\Phi}}_3$$

The inertia dyadics were defined as;

$$\bar{\bar{\Phi}}_1 = \begin{bmatrix} I_{11}\,\bar{x}_1\,\bar{x}_1 & -I_{12}\,\bar{x}_1\,\bar{y}_1 & -I_{13}\,\bar{x}_1\,\bar{z}_1 \\ -I_{12}\,\bar{y}_1\,\bar{x}_1 & I_{22}\,\bar{y}_1\,\bar{y}_1 & -I_{23}\,\bar{y}_1\,\bar{z}_1 \\ -I_{13}\,\bar{z}_1\,\bar{x}_1 & -I_{23}\,\bar{z}_1\,\bar{y}_1 & I_{33}\,\bar{z}_1\,\bar{z}_1 \end{bmatrix}$$

$$\bar{\bar{\Phi}}_2 = \begin{bmatrix} I_T\,\bar{x}_2\,\bar{x}_2 & 0 & 0 \\ 0 & I_R\,\bar{y}_2\,\bar{y}_2 & 0 \\ 0 & 0 & I_T\,\bar{z}_2\,\bar{z}_2 \end{bmatrix}$$

$$\bar{\bar{\Phi}}_3 = 0$$

and the inertia dyadic $\bar{\bar{\Phi}}_o$ was evaluated as:

$$\bar{\bar{\Phi}}_o = \begin{bmatrix} \left[m_1 b^2 + m_2 c^2 + m_3\left\{d^2 + (e+\eta)^2\right\}\right]\bar{x}_1\,\bar{x}_1 & 0 & -m_1 ab\,\bar{x}_1\,\bar{z}_1 \\ 0 & \left[m_1(a^2+b^2)+m_3(e+\eta)^2\right]\bar{y}_1\,\bar{y}_1 & m_3 d(e+\eta)\,\bar{y}_1\,\bar{z}_1 \\ -m_1\,ab\,\bar{z}_1\,\bar{x}_1 & m_3 d(e+\eta)\,\bar{z}_1\,\bar{y}_1 & \left[m_1 a^2+m_2 c^2+m_3 d^2\right]\bar{z}_1\,\bar{z}_1 \end{bmatrix}$$

In Equation (1), the term $\dot{\bar{R}}_3$ is the time derivative of $\bar{R}_3$ relative to the $(\bar{x}_1, \bar{y}_1, \bar{z}_1)$ coordinate frame. The operator $\frac{d}{dt}$ represents time differentiation relative to the reference frame.

Since the damper mass was assumed inertialess, the damper equation of motion may be obtained directly by Newton's second law as

$$\bar{F}_3 = m_3 \frac{d^2 \bar{r}_3}{dt^2} \qquad (2)$$

where

$$\bar{F}_3 = -(K \eta + B \dot{\eta}) \bar{z}$$

and (η) is the damper degree of freedom (parallel to the $\bar{z}_1$ axis), and (K) and (B) are the damper spring and damping constants respectively.

In general, body 1 (the despun antenna) is controlled in pitch such that the $\bar{z}_1$ axis points along a given reference axis (i.e. local vertical). Control is obtained via a pitch error sensor, a motor torquer, and the associated control electronics. Let the torque applied by the motor to the despun body along $\bar{y}_1$ be represented by τ_M. Clearly an equal and opposite torque is applied to the rotor (assuming no external disturbance torques on the vehicle) and hence the magnitude of the control torque is

$$\tau_M = - I_R (\dot{q} + \ddot{\theta}) \qquad (3)$$

Assuming no external forces acting on the spacecraft permits derivation of the vehicle translational equation as:

$$M \frac{d^2 \bar{\rho}}{dt^2} + m_1 \frac{d^2 \bar{R}_1}{dt^2} + m_2 \frac{d^2 \bar{R}_2}{dt^2} + m_3 \frac{d^2 \bar{R}_3}{dt^2} = 0 \qquad (4)$$

Upon solving for $(\bar{\rho})$ in Equation (4) and substituting into Equations (1) and (2), the rotational equation of motion for body 1 was derived as

$$\begin{bmatrix} L_{11} & -L_{12} & -L_{13} & -L_{14} \\ -L_{12} & L_{22} & -L_{23} & -L_{24} \\ -L_{13} & -L_{23} & L_{33} & -L_{34} \\ -L_{14} & -L_{24} & -L_{34} & L_{44} \end{bmatrix} \begin{Bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \\ \ddot{\eta} \end{Bmatrix} + \begin{Bmatrix} -I_R\,\Omega_s\,\dot{\psi} \\ 0 \\ I_R\,\Omega_s\,\dot{\phi} \\ B\,\dot{\eta} + K\,\eta \end{Bmatrix} = \begin{Bmatrix} 0 \\ \tau_M \\ 0 \\ 0 \end{Bmatrix} \qquad (5)$$

where:

$$L_{11} = I_{11} + I_T + m_1 b^2 + m_2 c^2 + m_3(d^2 + e^2) - \frac{1}{M}\left[(m_2 c - m_3 d)^2 + (m_1 b + m_3 e)^2\right]$$

$$L_{12} = I_{12} + \frac{m_1 a}{M}(m_3 d - m_2 c)$$

$$L_{13} = I_{13} + m_1 ab - \frac{m_1 a}{M}(m_1 b + m_3 e)$$

$$L_{14} = m_3 d + \frac{m_3}{M}(m_2 c - m_3 d)$$

$$L_{22} = I_{22} + m_1(a^2 + b^2) + m_3 e^2 - \frac{1}{M}\left[m_1^2 a^2 + (m_1 b + m_3 e)^2\right]$$

$$L_{23} = I_{23} - m_3 de + \frac{1}{M}\left[m_3^2 de + m_1 m_3 db - m_1 m_2 bc - m_2 m_3 ec\right]$$

$$L_{24} = -\frac{m_1 m_3 a}{M}$$

$$L_{33} = I_{33} + I_T + m_1 a^2 + m_2 c^2 + m_3 d^2 - \frac{1}{M}\left[m_1^2 a^2 + (m_2 c - m_3 d)^2\right]$$

$$L_{34} = 0$$

$$L_{44} = m_3\left(1 - \frac{m_3}{M}\right)$$

Finally the equation of motion may be transformed into the complex frequency domain and written in the more convenient form

$$\begin{bmatrix} L_{11}s^2 & -L_{12}s^2 & (-L_{13}s^2 - I_R\Omega_s s) & -L_{14}s^2 \\ -L_{12}s^2 & L_{22}s^2 & -L_{23}s^2 & -L_{24}s^2 \\ (-L_{13}s^2 + I_R\Omega_s s) & -L_{23}s^2 & L_{33}s^2 & 0 \\ -L_{14}s^2 & -L_{24}s^2 & 0 & (L_{44}s^2 + Bs + K) \end{bmatrix} \begin{Bmatrix} \phi(s) \\ \theta(s) \\ \psi(s) \\ \eta(s) \end{Bmatrix} = \begin{Bmatrix} 0 \\ \tau_M(s) \\ 0 \\ 0 \end{Bmatrix} \qquad (6)$$

where (s) is the complex frequency.