OPTIMIZING THE EXPANSION OF A LARGE TELECOMMUNICATIONS NETWORK WITH STEP TOTAL COST FUNCTIONS*

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P L A N

Résumé

The objective is a procedure to find a minimum cost capacity expansion program of an interurban telecommunications network containing a large number of nodes and in the presence of locally decreasing average costs and joint costs due to the existence of important indivisibilities and resulting in total cost functions, defined for individual elements or groups of elements of the network, being step functions.

The optimal capacity expansion program itself is found as a solution of a mixed integer programming problem. Each chain joining a pair of demand points of the network gives rise to an activity of this problem. Even for networks of moderate size, the number of possible chains may be enormous. This article is chiefly concerned with drastically reducing the number of chains to be submitted to the mixed integer programming problem with the help of the complementary notions of dominated and admissible chains. The algorithm described may be viewed as a generalization to the case of step functions of the well-known minimum cost chain algorithm for non-directed graphs and is likewise inspired by Bellman's "Principle of Optimality" of dynamic programming.

The appropriate software has been developed and made operational.

1. INTRODUCTION

The procedure described here was originally devised in the framework of a research project having to do with the Canadian interurban telecommunications network and conducted jointly by the National Telecommunications Branch of the Department of Communications, the firm Sorès Inc., of Montréal and the Laboratoire d'économétrie, Université Laval. [1]

A model had to be constructed whose function would be to find minimum cost capacity expansion programs designed to satisfy specified increases in the demand for telecommunication facilities for one, or more frequently, for several pairs of demand points, or nodes. It will be noted that the presence of joint costs or more generally of indivisibilities makes for the non-additivity of optimal solutions. Except by accident, a piecemeal approach consisting in considering one by one the pairs of nodes for which demand increases are specified will not lead to an optimal overall solution and a simultaneous approach becomes inevitable. It is also important to note that even if only one pair of demand points is considered, the traditional piecemeal approach consisting in considering small subsets of the network at a time is also inacceptable. It is the shape of the cost functions and not the number of pairs of demand points for which demand increases are specified that constitutes the fundamental characteristic of the problem.

The problem was also characterized by the large size of the networks to be considered: up to 100 nodes and more. The extensions of the original model, mentioned briefly in the last paragraphs of this paper and some of which have already been implemented, call in most cases for even larger networks.

The shape of the cost functions precludes the applications of the, now traditional, "proportional cost" procedure [2], [3]. The problem is also fundamentally different from that of non-proportional but piece-wise linear and non-decreasing average cost problem. This, together with the large size of the networks to be considered represented the challenge to which the proposed procedure is a response. The originality of this procedure lies in its containing a generalization to the case of step total cost functions (i.e. locally decreasing average costs) of the minimum cost chain algorithm. This generalization yields a subset of admissible chains, subset of the set of possible chains for one or several pairs of points of a non-directed graph. A chain which is not admissible is called dominated. These terms are defined in Section 3. below. Intuitively, a chain is dominated if it cannot form part of the optimal solution because it is more costly than some other chain or chains, whatever the allocation among the elements of the network of the demand increases to be considered. The admissible chain subset is thus seen to be the natural extension of the minimum cost chain concept. The subset of admissible chains only is submitted to the mixed integer programming algorithm which then chooses the optimal chain or chains.

2. THE MAIN PROBLEM

An interurban telecommunications network can always be represented by a non-directed simple graph. [2] Connected graphs only will be considered. Some or all the elements of the network, nodes and links, can have their capacities increased from known initial capacities. Elements which are contemplated but not yet in existence have zero initial capacities. Total cost functions of capacity expansion are given and are step functions. We assume here uniform capacity expansion steps. The relaxation of this hypothesis does not invalidate the method proposed here, but would call for an additional software effort. The problem has to do with demand for facilities and not with traffic and hence it is postulated that there is no unused capacity [3]. However, unused capacity can be taken into account, with consequent changes in the corresponding cost functions. It is required to satisfy exogeneous increases of demand for facilities for one or more pairs of nodes called demand points. The problem consists in finding a capacity expansion configuration which minimizes the total cost.

Let us define a few symbols:

- j for the typical node of the set of nodes N,
- l for the typical link in the set of links L,
- k for the typical element (node or link) of the set of elements K for which capacity expansion is possible,
- i for the typical pair of demand points of the set D of pairs of demand points.

The mixed integer programming problem distinguishes two types of activities (variables):

Capacity expansion activities: their levels are natural numbers

For each $k \in K$ a total cost function of capacity expansion is defined from the initial capacity to an upper bound. These bounds are sufficiently high for the demand increases considered to ensure the existence of at least one feasible solution. These total cost functions are step functions (see Figures 1., 2. and 3.). It is with reference to these functions that capacity expansion activities are defined. There is an ordered set of these activities for each element of the network for which expansion is possible.

Let t stand for the typical activity of such a set denoted by T(k).

The first activity represents successive discrete capacity expansions from the initial state and giving rise to the same total cost increments. As soon as the cost increments change, a second activity has to be defined, and so on. Thus the "steps" cost increments of the step functions do not have to be uniform. It will be noted that an activity can take a positive value only if the preceding activity is at its upper bound. Thus one has to introduce sequencing constraints.

Let y(k;t) denote the level of the expansion activity t for the element k, and c(k;t) the corresponding total cost increment.

Facility assignment activities: their levels are non-negative real numbers

For each pair of demand points $i \in D$, there exists a set of chains. A chain is a sequence of links without cycles joining the two points of the pair.

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Let R(i) denote such a set of chains and r its typical member.

A facility assignment activity is defined for each chain r. Its level is a non-negative real number x(i;r) and indicates the additional capacity assigned on each of the elements of r to satisfy the whole or part of the demand increase for the pair of points concerned. [4] It will be noted that given the presence of joint costs and other indivisibilities, capacity expansion costs cannot in general be defined for facility assignment chains. The corresponding variables enter the mixed integer programming problem with zero costs: costs are defined for the elements of the network.

Objective function

The objective function, which is to be minimized, is the total cost of the capacity expansion program:

$$\underset{x,y}{[\mathrm{MIN}]} Z = \sum_{k \in K} \sum_{t \in T(k)} c(k;t)y(k;t).$$

There are four types of constraints in the main problem, other than the non-negativity and the integrality (wherever applicable) constraints.

Demand constraints

Let d(i) denote the increase in the demand for facilities for the pair of nodes i. Then the additionnal capacity installed on the chain, or chains serving the two nodes concerned must be at least equal to d(i):

$$\sum_{r\in R(i)} x(i;r) \geq d(i) \quad , \quad i\in D.$$

Capacity constraints

The additional capacity installed on an element of the network must be at least equal to the sum of the levels of facility assignment activities (chains) passing through this element.

$$\sum_{i\in D} \sum_{r\in R(i)} \delta(r;k)x(i;r) - \sum_{t\in T(k)} y(k;t) \leq 0, k\in K,$$

where $\delta(r;k)$ takes the value 1 if the chain r passes through the element k, and 0 otherwise.

Sequencing constraints

The role of these constraints is to make sure that the capacity expansion activities on each element of the network respect the proper order of precedence: an activity cannot take a positive value unless the preceding activity is at its upper bound.

These are conditional, that is disjunctive, constraints of the type often encountered in mixed integer programming problems. They are of the form:

$$\text{either:} \quad y(k;t-1) < \bar{y}(k;t-1) \text{ and } y(k;t)=0$$

$$\text{or} \quad y(k;t-1) = \bar{y}(k;t-1) \text{ and } y(k;t)\geq 0,$$

$$k\in K \quad \text{and} \quad t\in T(k).$$

Upper bound constraints

Each capacity expansion activity y(k;t) has an upper bound $\bar{y}(k,t)$:

$$y(k;t) \leq \bar{y}(k;t),\ k \in K \text{ and } t \in T(k).$$

Non-negativity and integrality constraints

$$x(i;r) \geq 0,\quad i \in D \text{ and } r \in R(i);$$

$$y(k;t) \in I,\quad k \in K \text{ and } t \in T(k);$$

where I is the set of natural numbers (non-negative integers).

3. REDUCING THE SIZE OF THE MAIN PROBLEM

As stated above, the main problem can in principle be solved in a finite number of steps. In fact, however, even for a relatively small network the size of this problem may be enormous and direct solution utterly impractical. For instance, in a simplified version of the Canadian interurban telecommunications network containing about 60 nodes, the identification and numbering of possible facility assignment chains between Montreal and Vancouver reached the astonishing figure of 300,000 at which stage the enumeration was stopped as it was giving no sign of coming to an end by itself. It will be remembered that facility assignment chains are among the activities, that is variables, of the mixed integer programming problem. The procedure described below reduces drastically the number of activities in the main problem without running any risk of leaving out the optimal solution, or solutions.

This procedure defines and makes operational a sufficient condition for a facility assignment chain to be absent from the minimum cost capacity expansion program. If this condition is satisfied, the corresponding variable can be safely left out of the main problem. The search for such a condition was strongly inspired by Bellman's "Principle of Optimality" of dynamic programming whose main message in the present context is that the knowledge of the return functions makes it possible to find the optimal strategy (or strategies) without having to evaluate all possible strategies. More specifically, the proposed procedure is a generalization of the minimum cost chain algorithm to the case of step total cost functions, that is to the case of locally diminishing average costs. However, the outcome is not the minimum cost chain, or chains, but a subset of admissible chains. The final choice within this subset is made by the main problem.

Reducing the number of capacity expansion activities

Although the reduction of the size of the main problem is chiefly achieved by eliminating dominated facility assignment activities, a preliminary step relates to capacity expansion activities and leads to a reduction in their number.

It is clear that in any given problem a single element of the network will at most be called upon to handle the total of the specified demand increases for all the pairs of nodes, that is pairs of demand points, concerned. One can safely ignore capacity expansions going beyond this total.

Consequently the "maximum contemplated demand increase" is defined as:

$$mcd = \sum_{i \in D} d(i).$$

The set T(k) of capacity expansion activities to be considered becomes such that:

$$\sum_{t \in T(k)} \bar{y}(k;t) = mcd \quad , \quad k \in K.$$

This, of course, amounts to truncating the domains of all the cost functions according to the problem in hand.

Identifying dominated facility assignment chains

Consider the capacity expansion total cost function for a typical element, k, of the network. It is a non-decreasing step function defined on the interval [0,mcd].

$$\text{Define:} \quad cmax(k) = [MAX]\ c(k;t) \quad , \quad t \in T(k)$$

$$\text{and} \quad cmin(k) = [MIN]\ c(k;t) \quad , \quad t \in T(k)$$

that is respectively the largest and the smallest steps of the total cost function. They will also be referred to as the upper bound and the lower bound unit costs. It is to be noted that if the total cost function over the whole of the relevant interval is a constant function, then the lower bound is necessarily zero.

Let minup(i) designate the unit cost of the minimum cost chain for the pair of demand points i, the costs being fixed at their upper bounds.

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In other words, it is the cost of increasing by one unit the capacity of all the elements k, on the chain connecting the two points of the pair i, for which the sum of upper bound unit capacity expansion costs is the lowest.

$$\text{minup}(i) = \underset{r \in R(i)}{[\text{MIN}]} \sum_{k \in K} \delta(k;r)\text{cmax}(k),$$

where $\delta(k;r) = 1$ if the element k belongs to the chain r, and $\delta(k;r) = 0$ otherwise.

Let smin(i;r) designate cost of the minimum cost chain r joining the pair of demand points i with the costs of all the elements at their respective lower bounds.

$$\text{smin}(i;r) = \sum_{k \in K} \delta(k;r)\text{cmin}(k).$$

Lemma: For any chain connecting the two points of the pair of demand points i, the total cost of expanding the capacity by n units lies between the sum of the unit costs at their lower bounds of all the elements involved, multiplied by n and the sum of the unit costs at their upper bounds of all the elements involved, multiplied by n.

In effect, whatever the capacity increase on element k $n = \sum_{t \in T(k)} y(k;t)$, with $0 \le n \le \text{mcd}$, we have:

$$\text{cmin}(k) \le c(k;t) \le \text{cmax}(k).$$

It follows that, for a given chain r, with the capacity increase n on all the elements, the following relation holds:

$$\sum_k \sum_t \delta(k;r)\text{cmin}(k)y(k;t) \le \sum_k \sum_t \delta(k;r)c(k;t)y(k;t) \le \sum_k \sum_t \delta(k;r)\text{cmax}(k)y(k;t).$$

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Using the expressions for n and smin(i;r) given previously, this is equivalent to:

$$smin(i;r)n \leq \sum_k \sum_t \delta(k;r)c(k;t)y(k;t) \leq \sum_k \delta(k;r)cmax(k)n.$$

Before proceding further, we shall recall the definition of sub-chain.

Definition 1: Consider the chain r, connecting the two nodes of the pair i. Let i' be a pair of nodes belonging to this chain. Then a sub-chain s of the chain r is a chain connecting the two points of the pair i' and involving solely the elements of the original chain r. One, or both nodes of the pair i' may, of course, be the same as those of the pair i. It will be noted that for a given chain r, the sub-chain connecting the points of the pair i' is unique.

Proposition 1: Consider the chain r for the pair of demand points i, with capacity expansion of n units planned for all the elements of this chain. Suppose there exists a sub-chain s of the chain r, for the pair i' such that: minup(i') < smin(i';s), then it is possible to transfer n units of capacity expansion from the chain r to the new chain, which is the same as r except that between the points of the pair i' the sub-chain s is replaced by the chain corresponding to minup(i').

The possibility of such a transfer follows from the upper bounds on capacity expansion activities on all the elements of the network being large enough to accommodate the "maximum contemplated demand increase".

Proposition 2: The transfer of capacity expansion referred to in Proposition 1 necessarily reduces the cost of the capacity expansion program. The facility assignment activity corresponding to the old chain, r, is necessarily zero in the optimal solution of the main problem. This follows from the construction of minup(i'),from the Lemma and from Proposition 1, given above. Let the non-common parts of the old chain r, and the new chain which replaces it, be s and, say, s', where s' is the chain corresponding to minup(i') we then have:

$$\sum_k\sum_t \delta(k;s')c(k;t)y(k;t) \leq \text{minup}(i')n < \text{smin}(i';s)n \leq \sum_k\sum_t \delta(k;s)c(k;t)y(k;t),$$

with $n = \sum_t y(k;t)$ for both s and s'.

It is clear that if the transfer referred to in Proposition 1 is not carried out, then the total cost is larger than it could be and the corresponding solution cannot be optimal.

Definition 2: An admissible chain is a chain which contains no sub-chain, say s, for a pair of nodes i', such that: minup(i') < smin(i';s).

Definition 3: A chain which is not admissible is dominated.

It will be noted that whether a chain is admissible or dominated depends, inter alia, on the "maximum contemplated demand increase" (or the "maximum relevant demand increase" defined below).

A capacity expansion problem to be considered may concern one or more pairs of demand points for which demand increases are specified. A large number of facility assignment chains will in general connect the points of each of these pairs.

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Definition 4: A node is dominated for a given pair, or pairs, of demand points if all possible facility assignment chains relating to this pair, or pairs, and going through this node are dominated.

Definition 5: A node which is not dominated is admissible.

It is to be noted that in any given problem the status of every node can be established without explicit consideration of facility assignment chains. In fact, the dominated nodes are eliminated first, as well as all the links involving these nodes. This implicitly eliminates all the dominated facility assignment chains going through one or more dominated nodes. Once this is done, the remaining chains are identified and tested for admissibility. It will be noted that chains are identified step by step, that is element by element. As soon as an incomplete chain contains a dominated sub-chain it is abandoned: the corresponding chain will necessarily be dominated.

Apart from the role it plays in eliminating dominated facility assignment chains, the concepts of dominated and admissible nodes lead to an interesting by-product namely a non-arbitrary delineation of the "relevant region". In any given problem, the partial sub-graph of the original network consisting of the admissible nodes and links connecting them constitute this relevant region. It will be noted that the delineation of this region depends not only on the pair, or pairs of demand points for which demand increases are specified, but also on the "maximum contemplated demand increase" (or the "maximum relevant demand increase" - see below). In planning the optimal capacity expansion, only the part of the original network corresponding to the relevant region has to be taken into account. However, the larger the specified demand increase, or increases, the larger will, in general, be this relevant region.

4. OPERATIONAL PROCEDURE FOR ELIMINATING DOMINATED FACILITY ASSIGNMENT CHAINS

The structure of the main problem having been given in Section 2, the description which follows is concerned only with the procedure for eliminating dominated facility assignment chains which drastically reduces the number of variables and constraints to be submitted to the main problem which, it will be recalled, is a mixed integer programming problem.

The procedure begins by adding up all the demand increases specified in the problem to be solved. This total, called the "maximum contemplated demand increase", is then used to identify the upper bounds on capacity expansions of all the elements of the network and hence also of the lower and upper bounds on the unit cost of capacity expansion for every element.

Whereas the upper bounds on capacity expansions are, at this stage, the same for all the elements of the network, when counted from the initial state (one admits the possibility that all the demand increases wherever specified might all pass, at the same time, through any given element of the network), the lower and upper bounds on unit costs of capacity expansion will not, in general be the same for all the elements of the network, since this depends on where on the cost functions are the initial states, that is the installed and supposedly used capacity, for any given element of the network. These lower and upper bounds on unit costs of capacity expansion, original and then revised, cmin(k) and cmax(k) respectively, for all k, are calculated by the subroutine BØRNE.

The subroutine DØMINØ computes two tables, which take the form of symmetric matrices whose dimensions are equal to the number of nodes in the network.

- The cost of the minimum cost chain, costs being set at their upper bounds, i.e. minup(i), for every element involved, from every node to every other node of the network;

- The cost of the minimum cost chain, costs being set at their lower bounds, i.e. minlo(i), from every node to every other node in the network.

The costs appearing in the two above tables are unit capacity expansion costs.

It is to be noted that it is the cost of minimum cost chains that are calculated at this stage: the corresponding chains themselves are not yet identified, and most of them never will be. This is akin to the "marking" procedure often used in economic applications of the graph theory, especially those involving the use of Bellman's "Principle of Optimality".

With the help of the above tables and for each pair of demand points, typically: NØRG-NDEST, all the dominated nodes are eliminated. It is clear that for a different pair NØRG-NDEST a different set of nodes will be dominated. For each node a comparison is made between the two scalars:

1. The sum of the cost of the two minimum cost chains connecting this particular node to NØRG and to NDEST respectively, costs being set at their lower bounds.

2. The cost of the minimum cost chain connecting NØRG with NDEST, costs being set at their upper bounds.

If the first term of this comparison is greater than the second term, then the node concerned is eliminated as a dominated node, for this particular pair NØRG-NDEST. In that case, all the links connecting this node with any other node, dominated or not, are also eliminated.

Using the DØMINØ tables and the theory presented in Proposition 2, for each pair NØRG-NDEST, the admissible facility assignment chains are identified as follows:

- From the origin NØRG, chains of length one are constructed and dominated chains are immediately eliminated. It will be noted that a chain of length one may be dominated by a chain of length two or more.

- Then chains of length two from the origin NØRG are constructed by using all possible adjacent links to extend the chains of length one identified as admissible in the preceding step. However, before checking the admissibility of these length two chains, the links to be added are tested, the test consisting of checking whether the next adjacent node of the incomplete chain is dominated with respect to the previous node and the NDEST node. If this is the case, the links in question are eliminated. Then the admissibility of each remaining incomplete chain of length two is checked with the help of Proposition 2. of Section 3. and of the numerical information contained in two DØMINØ tables.

- This procedure is repeated for all the chains in the network between NØRG and NDEST, their maximum length being finite, since the least cost of a chain from a node to itself is necessarily zero.

- Having started from NØRG, once NDEST is reached, the set of admissible facility chains, and they will normally be more than one, is stored, another pair of demand points appearing in the problem is taken up and the procedure repeated.

If the problem involves only one pair of demand points, the stage of identifying the admissible chains is now completed. If there are two or more pairs of demand points the procedure of eliminating the dominated chains may be tightened with the corresponding reduction of the burden incumbing on the main problem. This tightening is related to the narrowing of the bounds on unit costs resulting from the fact that not all the demand increases concerning all the pairs of points involved could conceivably pass through every element of the network.

Once the elimination of the dominated chains has been done for all the pairs of demand points, a sorting is done of the elements of the network. For each element of the network those pairs of demand points are identified whose admissible chains (one or more) pass through this element and the corresponding demand increases added up giving what is called the "maximum relevant demand increase". This "maximum relevant demand increase" replaces now the concept of "maximum contemplated demand increase" used in earlier calculations. Unlike this last concept, the "maximum relevant demand increase" is not necessarily the same for all elements of the network. Those "maximum relevant demand increases" are then used to recalculate the lower and upper bounds on incremental capacity expansion costs and hence set in motion a new round of identifying dominated nodes and admissible chains. As a by-product one obtains also the upper bounds on admissible capacity expansion increments. The sorting out of the elements of the network, the calculation of the "maximum relevant demand increases" and the tightening of the bounds is done by the subroutine BØRNE. The number of iterations is evidently finite. The procedure stops as soon as no further tightening of the bounds is possible.

Once the admissible chains have been identified and upper bounds on capacity expansion increments have been set with the help of the subroutines CADUCEE and BØRNE, the main problem is taken up with the help of the subroutine TRANCHE. That module establishes an optimal, that is a minimum cost, capacity expansion program which satisfies the specified demands. The subroutine TRANCHE uses a standard branch-and-bound algorithm for mixed integer programming.

5. NUMERICAL EXAMPLES

Suppose we have a network with 30 nodes and 55 existing or contemplated links as shown in Figure 1. We have three types of links. The "light route" type corresponds to the cost function of Figure 2; the "heavy route" corresponds to Figure 3; finally the "heavy routes with priority costs" are specific variations from the "heavy route"type. We will consider three problems. The first one consists of a 3 unit demand increase between the nodes 10 and 27. The second involves a 2 unit demand increase between the nodes 9 and 23. The last problem is the simultaneous consideration of the two preceding ones.

Figures 1., 2. and 3.

For problems 1, 2, 3, the Figures 4, 5, 6 show respectively, the admissible nodes, the links belonging to the admissible chains and the links being part of the optimum solutions with the number of capacity units that should be added. The reader will note the relevant region delineated for each problem by the set of admissible nodes. The total expansion costs are respectively 2090, 1650 and 3580 value units. The solutions of problems 1 and 2 are clearly non additive since 3580 is smaller than 2090 + 1650; the economy of scale amounts to 160 value units.

Figures 4., 5. and 6.

The main point of this article is exemplified as follows: there exist perhaps several tens of thousands of possible chains and hence of facility assignment activities. The number of admissible chains, that is chains actually submitted to the main problem, is drastically less since there are 15 such chains for the first problem, 6 for the second one. As far as the third problem is concerned, we have, at the first iteration with a "maximum contemplated demand increase" of 5, 37 admissible chains for the pair (10-27) and 61 for the other pair (9-23); at the second iteration, the concept of "maximum relevant demand increase" plays its role and the number becomes 37 for the pair (10-27), that is no change, and 45 for the pair (9-23).

6. EXTENSIONS

The procedure described here is intented to be used within the framework of a larger model. It is in fact a preliminary procedure to reduce the number of variables to be submitted to the main mixed integer programming model. This main model itself is capable of farther extensions, most of which have already become operational in the framework of the research project conducted jointly by le Laboratoire d'économétrie de l'Université Laval, the firm Sorès Inc. and the Department of Communications [1].

To handle situation where there are more than one type of transmission facilities between any given pair of adjacent nodes and where there exist more than one carrier on any given link (so that in fact there are more than one link between two adjacent nodes and the network is represented by a multigraph) there is available what is called the "enlarged network" approach which makes use of the concepts of dummy nodes and dummy links. Thereby the original problem is formally reduced to a problem involving at most one link between any pair of nodes for which the conceptualization and the corresponding software are already available. In practice this increases substantially the number of nodes and links in the simple graph to be handled and calls for a more efficient software, which has been developed, although it is not reported on here.

The problem as it stands is essentially a static formulation. Planning capacity expansion over time gives rise to a host of additional and extremely difficult problems. Dynamic programming techniques do not appear practicable and simulation procedures aiming at sub-optimal solutions are being developed.

The main problem being a mixed integer programming problem, the introduction of additional constraints, whether they be simultaneous or conditional gives rise to no fundamental difficulties, although in practice, they cause computational difficulties. Among the possible additional constraints, those having to do with the survivability have been given particular attention and appropriate procedures developed. The additional computational effort is quite heavy owing to the combinatorial nature of the usual formulation of survivability constraints.

Although this paper is restricted to the case of cost functions which are step functions in contrast to the proportionate cost functions of the traditional approach to be found in the current literature on network analysis, procedures have been conceptualized, though not yet made operational, to handle problems where cost functions are the result of adding up step and proportionate cost functions. This may be of considerable practical importance in the handling of dynamic problems (the planning of capacity expansion over time) where both capital and operating costs have to be considered.

It is to be noted that the problem discussed here deals with the demand for telecommunication facilities and not with traffic. A model has been developed and made operational, which does not start with the demand for facilities, but with traffic forecasts and translates them into demand for facilities, before proceding on the lines described above. This model introduces a distinction between the so called switching network on the one hand and the facilities network on the other. The present paper is, of course concerned with the facilities network only.

Finally, an important extension of the models developed in this field will take account of the highly complex relations between the peak traffic demand and average traffic demand. Although some exploratory work in that direction has been done, we have no concrete progress to report on here.

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List of Figures

Footnotes

* This article is based on certain results of a research project carried out with the collaboration of the Federal Department of Communications in 1971-1972. The opinions expressed are those of the authors and do not necessarily reflect the views of the Department of Communications. The actual drafting of this article, including the preparation of numerical examples, benefitted from financial assistance of the Quebec Department of Education.

1 Apart from the authors, the following members of the Laboratoire d'économétrie contributed to the research the results of which form the basis of this article: L. Blais-Morin, M. Hupé, B. Paquet and J. Poirier, the following specialists of the firm Sorès Inc.: D. Geller, R. Riendeau and D.B. Webber and also of J.A. Guérin, J.W. Halina and E.E. King of the Department of Communications. Their help is hereby gratefully acknowledged. The computations for the numerical examples were made by Sorès Inc.

2 The coexistence of alternative transmission facilities for certain pairs of adjacent nodes means the presence of multiple links in the

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network. Such a network can however be reduced to a simple graph by the use of "dummy nodes": see the description of "enlarged network" in Section 6 below.

3 See however Section 6 below.

4 In the examples given here the units of these activities are uniform throughout the network and are also the same as the discrete steps of capacity expansions. These units correspond to groups of circuits. Having non-uniform units would not affect the nature and in particular the computability of the approach proposed.

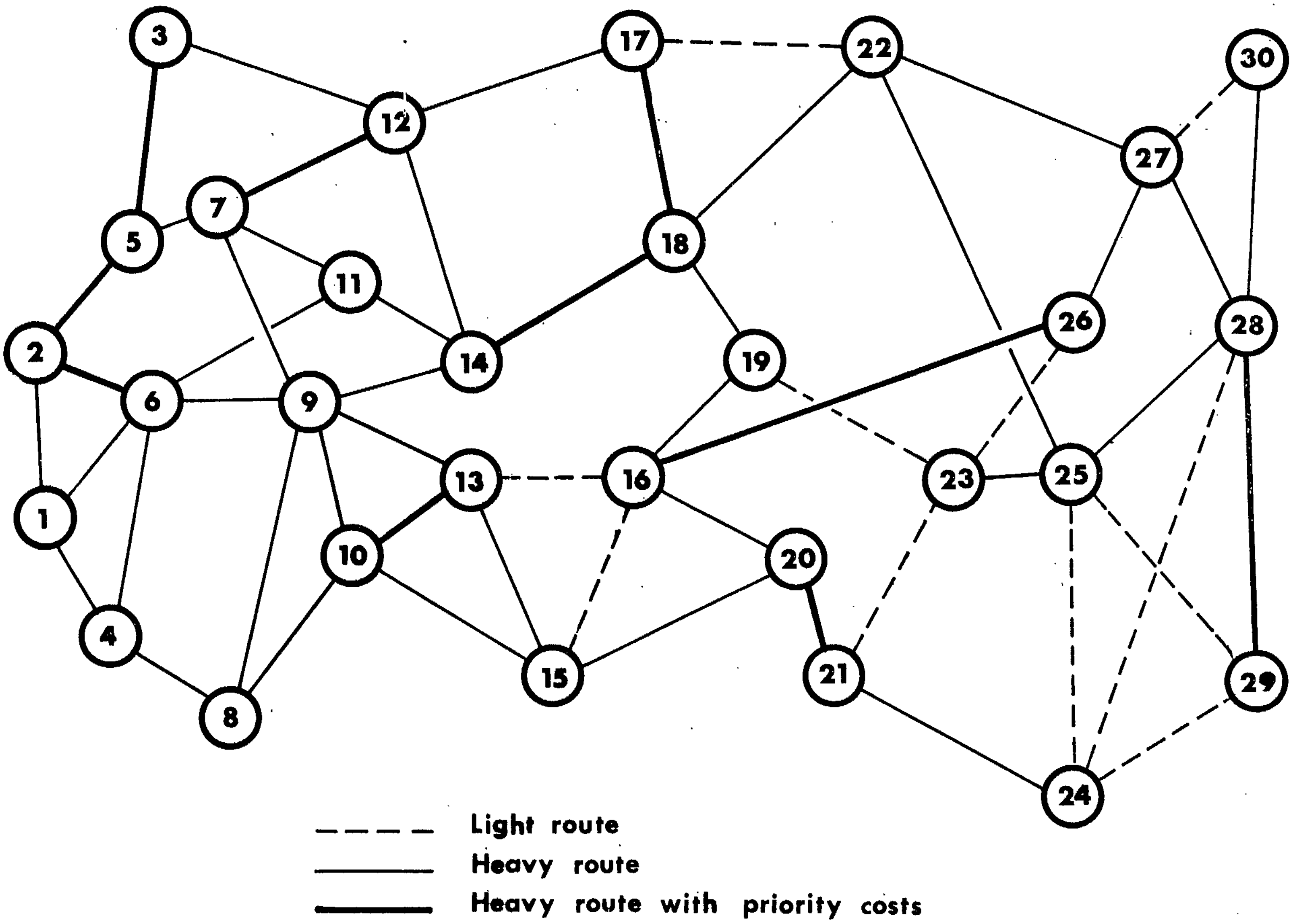

Figure 1: Transmission facilities network

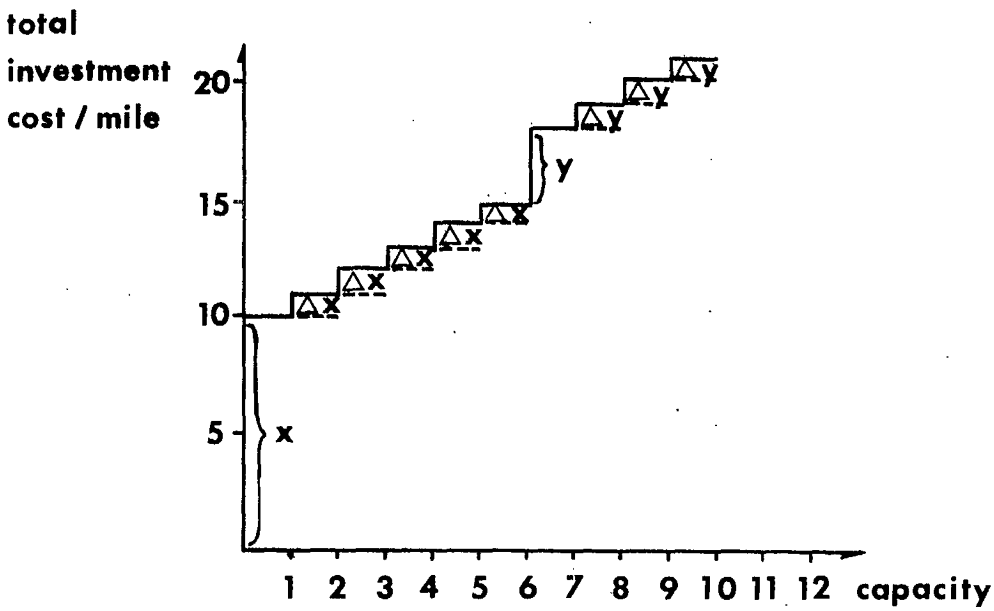

Figure 2: Cost function for heavy route

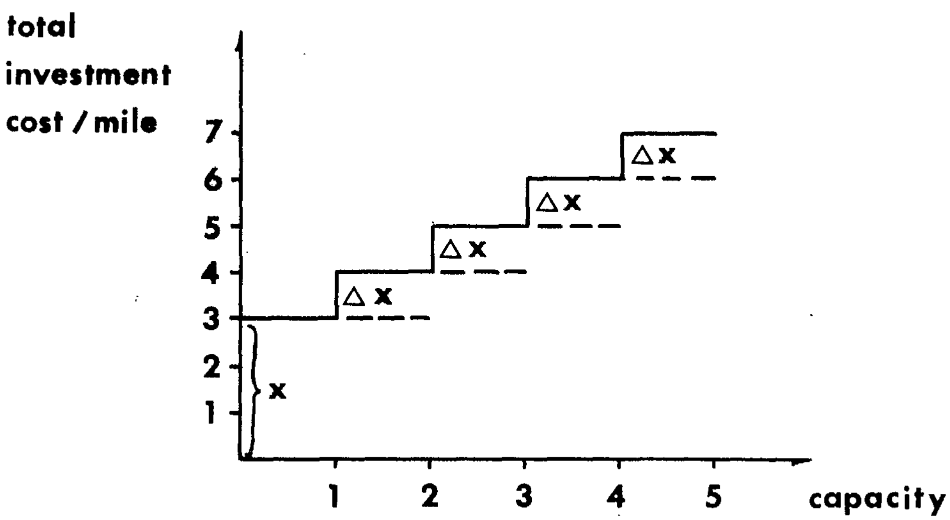

Figure 3: Cost function for light route

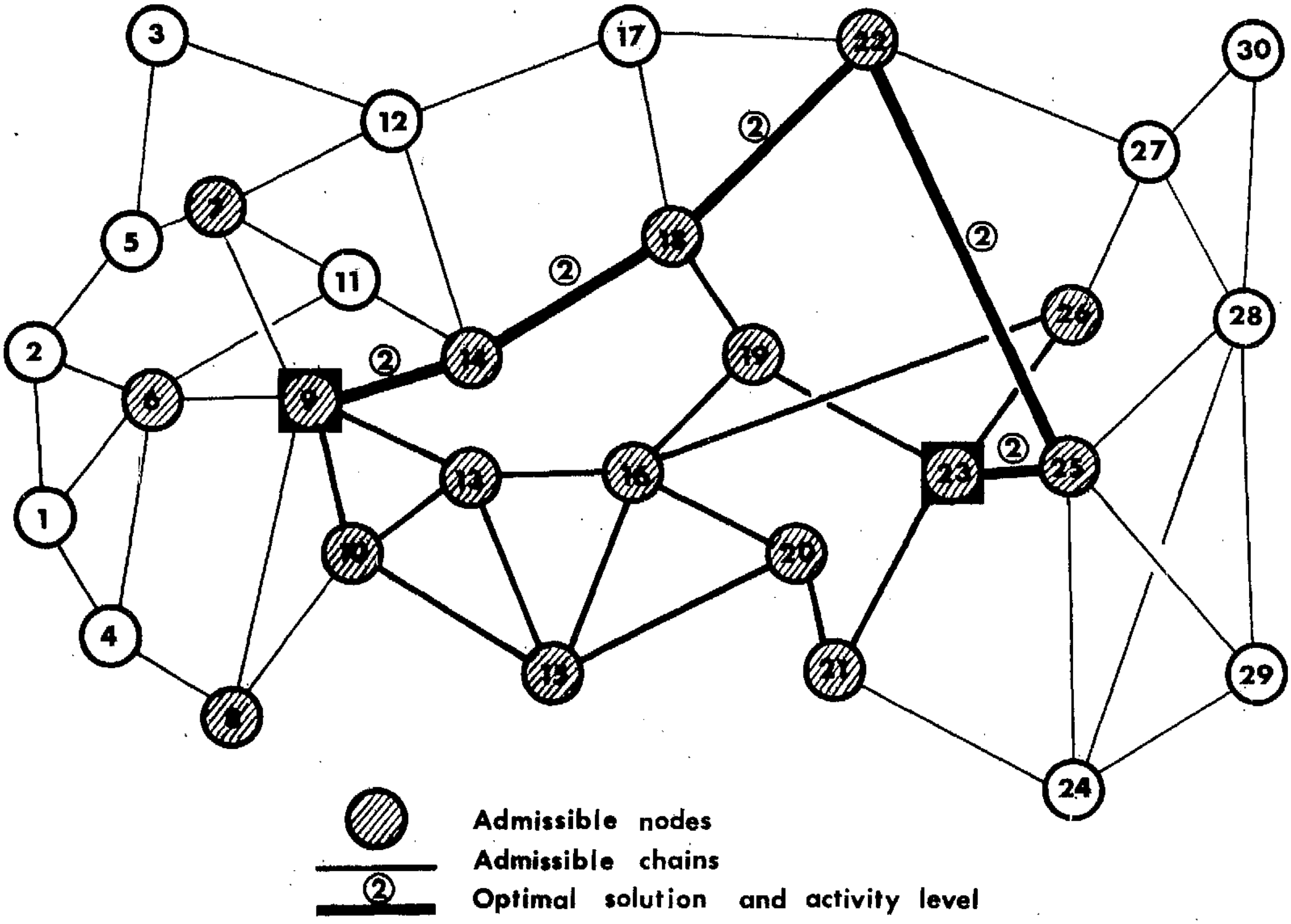

Figure 4: Solution for the demand pair 9-23

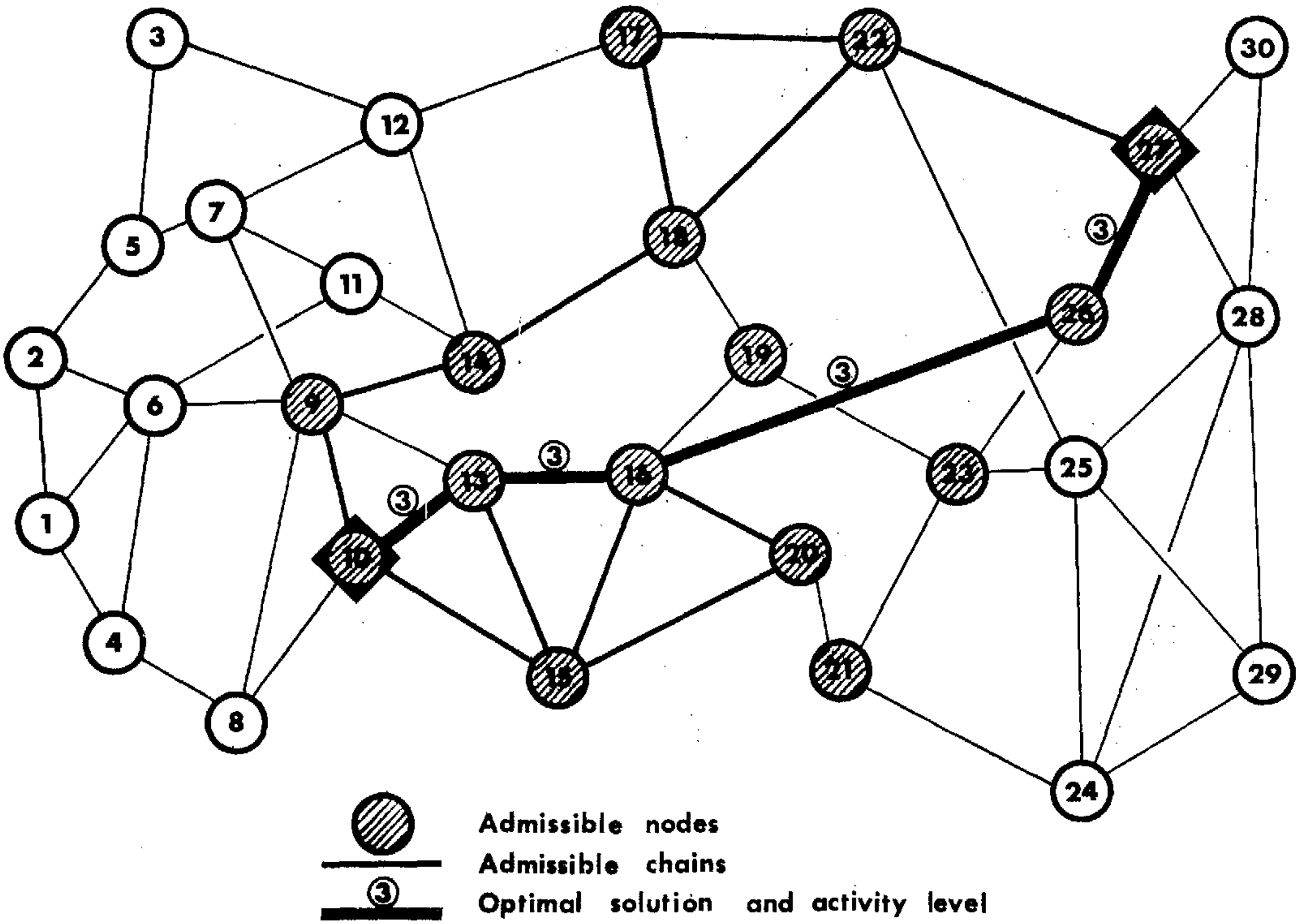

Figure 5: Solution for the demand pair 10-27

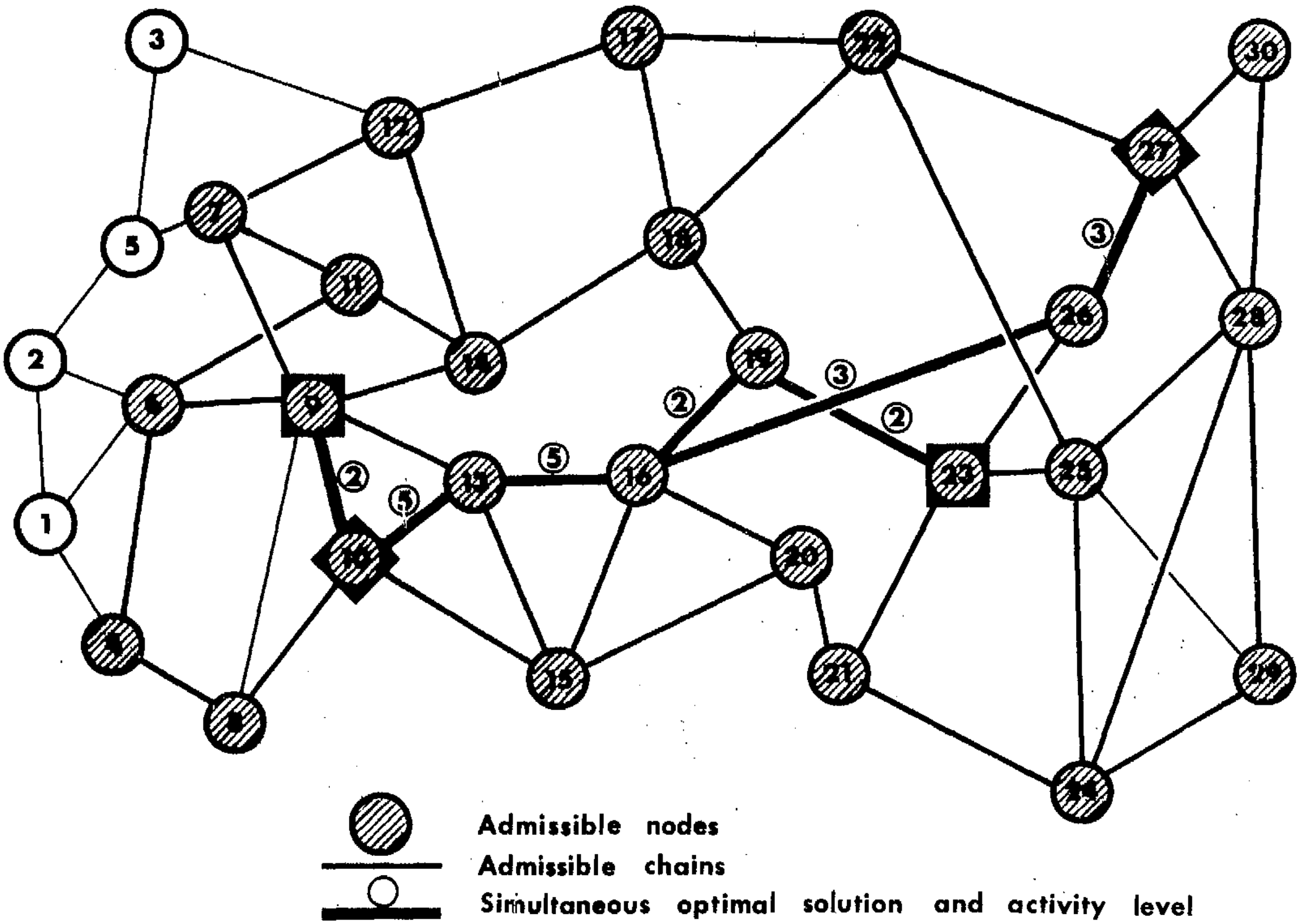

Figure 6: Simultaneous solution for the demand pairs 9-23 and 10-27