Regional-national growth rate differential decomposed by weight and growth effects by sectors - a suggested method and an empirical test

by

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Draft only.
For your comments.

REGIONAL-NATIONAL GROWTH RATE DIFFERENTIAL DECOMPOSED BY WEIGHT AND GROWTH EFFECTS BY SECTORS -- A Suggested Method and an Empirical Test

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INTRODUCTION

The purpose of this paper is to suggest a method by which inter-regional or regional-national aggregate growth rate differentials, be it for employment or value-added or any other variable, can be decomposed by sectoral growth rates and weight differentials. So far normalization procedures for decomposition for inter-regional or regional-national comparisons refer only to a particular point of time and have not been adapted to growth rates and weights simultaneously. Share-Shift analysis in a sense covers growth rates but neglects the role of weights whether these are for the initial year or the final year. The result is that it has not been possible to identify the contributions of weights and growth rates by sectors separately to the aggregate growth rate differential between two regions or between a region and the nation. This identification seems necessary not only to know, for any particular economy, which sector has contributed most to the economy's growth rate but also to assess how a sector's contribution to the aggregate growth rate can be decomposed into its own growth rate and its own share or weight in the initial year. In inter-regional or regional-national comparisons of aggregate growth rate differentials this issue has assumed considerable importance in recent times with particular reference to the choice of weights of sectors. Thus, to give an example, a recent attempt[1] at playing about with geometric mean of weights (from initial year

1. See H.W. Davis, R.T. Newsom and D.E. O'Neill, "Rate-Weight Analysis: A Suggested Technique for Examining Regional-National Growth Rate Differentials", The Annals of Regional Science, Vol. V, No. 2, December, 1971.

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and final year) by sectors has aggravated the problem further. It renders the geometric mean to be biased[1] in the sense that the sum of weights (geometric mean) do not add up to unity and that greater the disaggregation by sectors, greater becomes the deviation from unity.

In the following we shall present a simple algebraic formulation to formalize (a) the sectoral contributions to each economy's aggregate growth rate and (b) the contributions of sectoral growth rate differentials and weight differentials between any two economies to the aggregate growth rate differentials between these two economies. An empirical test will then be performed with two regions, Atlantic Provinces and Ontario vis-à-vis Canada for two time periods, 1961 and 1971, with employment by 11 sectors. An appendix at the end of the paper is added to separate out some mathematical derivations as well as to explain the notations used.

1. Decomposition of Weight (Size) and Growth (Rate) Effects by Sectors in the Overall Growth Rate in a Region and in the Overall Regional-National Growth Rate Differentials

Consider the following definitional equations:

$$\sum_{i=1}^{n} S_{it}^{A} = S_{t}^{A} \qquad (1)$$

$$\sum_{i=1}^{n} S_{it} = S_{t} \qquad (2)$$

$$w_{it} = \frac{S_{it}^{A}}{S_{t}^{A}} \qquad (3)$$

$$W_{it} = \frac{S_{it}}{S_{t}} \qquad (4)$$

1. A verification of this phenomenon can be obtained from the author on request.

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$$\sum_{i=1}^{n} w_{it} = 1 \qquad (5)$$

$$\sum_{i=1}^{n} W_{it} = 1 \qquad (6)$$

(See the Appendix for the notations).

Then from the above it can be shown[1] that

$$r = \left[\sqrt[t]{\sum_{i=1}^{n} w_{io}\,(1+r_i)^t}\right] - 1 \qquad (7)$$

$$R = \left[\sqrt[t]{\sum_{i=1}^{n} W_{io}\,(1+R_i)^t}\right] - 1 \qquad (8)$$

The implications of (7) and (8) are that over-all growth rate (annual compound rate) is expressed as a function of weights of the sectors in the initial period and the sectoral annual growth rates. But then they present a little involved relationship in the sense that there is a root factor for t time intervals and hence they are not amenable to a clear decomposition as an algebraic sum. To get around it, we shall dispense with the use of annual compound rates of growth by sectors i.e., r_i and R_i, and instead use the sectoral growth periodic rates over the whole time period T, $0 \leqslant t \leqslant T$.

Writing now

$$1 + r^* = (1+r)^t \qquad (9)$$

$$1 + r_i^* = (1+r_i)^t \qquad (10)$$

$$1 + R^* = (1+R)^t \qquad (11)$$

$$1 + R_i^* = (1+R_i)^t \qquad (12)$$

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1. See the Appendix.

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it can be shown[1] that:

$$1+r^* = \sum_{i=1}^{n} w_{io}\,(1+r_i^*) \qquad (13)$$

$$1+R^* = \sum_{i=1}^{n} W_{io}\,(1+R_i^*) \qquad (14)$$

which resolve into (15) and (16) respectively:

$$r^* = \sum_{i=1}^{n} w_{io}\, r_i^* \qquad (15)$$

$$R^* = \sum_{i=1}^{n} W_{io}\, R_i^* \qquad (16)$$

since by definition, viz, equations (5) and (6)

$\sum w_{io} = 1$ and $\sum W_{io} = 1$.

The formulations (15) and (16) are the crucial conditions of the over-all growth rate by size (weights) and growth rates by sectors. A little further manipulation of (15) and (16) results[1] in the decomposition of the over-all growth rate differential between the region and the nation by the following effects:

$$r^*-R^* = \sum_{i=1}^{n} w_{io}\, r_i^* - \sum_{i=1}^{n} W_{io} R_i^*$$

$$= \sum_{i=1}^{n} r_i^* (w_{io}-W_{io}) + \sum_{i=1}^{n} w_{io}\,(r_i^*-R_i^*)$$

$$- \sum_{i=1}^{n} (r_i^*-R_i^*)\,(w_{io}-W_{io}) \qquad (17)$$

1. See the Appendix.

where, the effects bear the following nomenclature --

$$a = \sum_{i=1}^{n} r_i^* (w_{io} - W_{io}) \longrightarrow \text{total weight effects from all sectors.} \quad (18)$$

$$b = \sum_{i=1}^{n} w_{io} (r_i^* - R_i^*) \longrightarrow \text{total growth or rate effects from all sectors.} \quad (19)$$

$$c = \sum_{i=1}^{n} (r_i^* - R_i^*) (w_{io} - W_{io}) \longrightarrow \text{total interaction effects from all sectors.} \quad (20)$$

imp.

Before we go into interpretations of the above expressions, we shall introduce a particular technique to isolate specific sectoral effects. This is achieved by forming a new variable, X_i, which is defined by

$$X_i = w_{io} r_i^* - W_{io} R_i^* \quad (21)$$

X_i can be christened as the "weighted growth rate differential between the region and the nation for any Sector i," where the weights are distinct for the region and the nation. The formulation (21) is a straight-forward approach to decomposition of the aggregate growth rate differential, $r^* - R^*$, by specific sectors as it can be shown from (15), (16) and (21) that

$$r^* - R^* = \sum_{i=1}^{n} X_i \quad (22)$$

Obviously then sectors can be ranked according to the descending or ascending values of X_i's and sectoral contributions to the aggregate growth differential between the region and the nation are evaluated in terms of X_i's. We shall label this decomposition formula as our Variant 1 formulation.

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One particular problem with (22) is that the weights are distinct by their origin, i.e., regional growth rates of sectors are multiplied by regional weights and national growth rates by national weights. In such a case standardization is lacking and comparisons seem to be odious. Hence to overcome this we introduce the following standardization:

Write

$$X_i = a_i + b_i + c_i \qquad (23)$$

where, $a_i = r_i^* (w_{io} - W_{io})$ (24)

$$b_i = w_{io}^* (r_i^* - R_i^*) \qquad (25)$$

$$c_i = -(r_i^* - R_i^*) (w_{io} - W_{io}) \qquad (26)$$

It can be shown that (23) corresponds exactly to (21) and that in the present version there is only a uniform system of weights by origin, i.e. weights are all regional. The nomenclature for a_i, b_i, c_i have the same connotations as in (18), (19) and (20) except in that they refer now to sectors only instead of the economy. The formulations given by (22) and (23) jointly will be christened as our Variant 2 formulation.

We shall now offer some interpretations of 'weight effects', 'growth or rate effects' and 'interaction effects' as these are formed by (24), (25) and (26). Primarily we are interested in signs of these effects for proper identification. Here we notice the following:

1. A positive sign arising from (24) cannot be regarded as a positive weight effect since one can get

a positive sign either from a combined result of negative r_i^* and negative weight differential, $w_{io} - W_{io}$, or from that of positive r_i^* and a positive $w_{io} - W_{io}$. Hence the weight effect is not properly identified by signs whenever there is a negative r_i^*. Barring this case, weight effects by sectors are all identified. We shall, therefore, check this constraint i.e., $r_i^* > 0$, for identification of signs in our empirical illustration.

2. Growth or rate effects are all identified by signs, since $w_{io} > 0$ and $W_{io} > 0$ by definition.

3. Interaction effects cause some trouble of interpretation by signs; there is also an additional negative sign before the product of the growth rate and weight differentials. However, we suggest two particular ways of minimizing this problem of interpretation. In the first place, we look for particular values of c_i. If any value of c_i is small which is possible when growth differentials and weight differentials are relatively small our task of interpretation becomes minimal. Secondly, for each sector i we try to locate which of the two effects, a_i or b_i cancel (more or less) the value of c_i leaving the other to be dominant in its influence on X_i. For most practical purposes either of two approaches turns out to be satisfactory.

In the empirical test that follows we shall observe the above rules for identification of separate effects.

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2. An Empirical Test of the Decomposition Formulae

In this section we utilize data of employment[1] by 11 sectors for two regions[2], namely, the Atlantic Provinces and Ontario, as against Canada separately for two time units, 1961 and 1971 to exemplify the formulation in the previous section. One can, however, generate further

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1. We have chosen employment rather than value added because data for value added by regions and sectors are not available. Moreover, we have restricted ourselves to 11 sectors as further breakdown would require additional time and resources for reasonable accuracy. One must note here an additional problem with respect to employment data which is at the moment bothering many researchers. There are mainly two important sources of employment estimates by sectors and by regions (particularly the non-agricultural segments of the regional economies) in Canada. These are: (a) Labour force estimates based on household surveys (cf. Statistics Canada Catalogue No. 71-001) and (b) Estimates of employees by Province and Industry based on establishment surveys (cf Statistics Canada Catalogue No. 72-008). These estimates vary widely not only in terms of absolute levels but also in terms of changes over time. Recently some attempts at reconciling these estimates have been made but without considerable success. (See for example, "Comparison of Labour Force Survey and Estimates of Employees Paid Worker Series", Working Paper No. 8005-512, Statistics Canada, August 1970). In our example, we have used the Labour Force Survey estimates for the sake of illustration rather than for making particular policy formulations.

2. The Atlantic Provinces include four provinces, namely New Brunswick, Nova Scotia, Prince Edward Island and Newfoundland. In our illustration the four Provinces constitute one region which we shall call the 'Atlantic' region. Historically this region has been a relatively depressed region whereas Ontario has been a prosperous one which makes such comparisons by sectors interesting.

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enquiries with shifting initial and final time periods but for our purpose this task has been avoided since we are only interested in making the illustrations clear-cut and implications logically spelled out. Three tables at the end of the text show (1) the original data by sectors and regions with accompanying growth rates and weights (Table 1), (2) the application of Variant 1 formulation in which the sectors are ranked according to whether values are higher or lower in terms of X_i's (Table 2 and (3) the validation of Variant 2 formulation where the implications of different effects, rate, weight and interaction, are brought out to explain the total differential, $r^* - R^*$ (Table 3). We shall discuss these tables one by one.

Table 1 is purely descriptive. Notice that:

1. The overall regional-national growth rate differentials for the period 1961-1971 are -0.1154 and +0.0227 for the Atlantic Region and Ontario respectively.
2. The largest net changes in weights and growth rates (independent of signs) took place in the following sectors for the regions and Canada:

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	Sectors with large weight changes	Sectors with high growth rates	Sector with greatest weight change	Sector with greatest growth rate
Atlantic	1,2,5,10	1,2,5,9,10	1	9
Ontario	1,5,10	2,4,6,9,10	10	10*
Canada	1,5,10	4,9,10,11	10	10
Common to all	1,5,10	9,10	-	-

NB: To determine the large changes we have chosen two cut-off points arbitrarily: for weight changes, we have taken a cut-off point as anything more than 2 percentage points, and for growth rate, the cut-off point has been taken to be anything exceeding 40 percentage points.

* This sector has been chosen vis-à-vis Sector 3 because the latter gives a somewhat redundant estimate as employment in the fishing industry has dropped to almost zero (in actual figures it is less than 1,000 and hence taken to be more or less nil) in the Ontario region. This requires an economic interpretation which is difficult to formalize and hence ignored.

In Table 2 the reader will find that the overall regional-national growth rate differentials are decomposed by sectors and have been ranked in terms of descending values of X_i's. The differences between the actual and the computed overall growth rate differentials are of the

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order[1] of -0.0053 (-0.1154+0.1101) and +0.0007 (+0.0227 -0.0220) for the Atlantic-Canada and Ontario-Canada comparisons respectively. These differences are largely due to rounding and the initial handicap of starting off with the sum of weights which fall a little short of unity, viz, Col. (8) in Table 1. However, these errors are very small and, therefore, can be ignored. In the evaluation of particular sectors in the two regions, Atlantic and Ontario, with reference to Table 2, the most dominating influence has been exercised by manufacturing for Atlantic and by agriculture for Ontario vis-a-vis Canada. For other sectors, interpretations are similarly easy to provide.

In Table 3, X_i's (from Table 2) are decomposed by separate effects which serve to highlight the principal aspects of our analysis:

1. For Atlantic-Canada, the overall growth rate differential has been largely determined by the total of negative growth (or rate effects (-0.1121) rather than weight effects since the total weight effects (-0.0485) are more or less swamped by the total of interaction effects (+0.0505) by sectors. For Ontario-Canada the situation has reversed: the positive weight effect dominates the overall growth-rate differential whereas the total of growth-rate effects is more or less neutralized by the total of interaction effects.

2. When sectors are analyzed one by one, i.e. row-wise, dominant effects are further identified. Almost all the sectors are dominated by a negative rate or growth effect for Atlantic-Canada. The situation is a little different for Ontario-Canada where one notices particularly the effect of positive weight in the manufacturing sector which is clear

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1. In our analysis we have not used the growth rates in percentage forms since the latter can be expressed simply by multiplication of 100 e.g. -0.1154 means -11.54%, etc.

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since the weight effect is large and it is also identified. Other sectors do not bear out large dominant effects.

3. Identification of weight effects is described in the table itself and, therefore, it merits no repetition.

Table 1

Composition of Employment by Sectors, 1961-1971 for Atlantic-Canada-Ontario: Weights (and their Changes) and Growth Rates

Sector Number	Sectors (1)	Atlantic 1961 in (000's) (2)	Canada 1961 (in 000's) (3)	Ontario 1961 (in 000's) (4)	Atlantic 1971 (in 000's) (5)	Canada 1971 (in 000's) (6)	Ontario 1971 (in 000's) (7)
1	Agriculture	55	681	162	23	510	134
2	Forestry	20	82	7	12	72	10
3	Fishing	13	18	1	15	22	0
4	Mining	12	78	29	11	129	47
5	Manufacturing	65	1456	680	96	1795	840
6	Construction	38	378	135	48	495	191
7	Transportation	56	560	188	68	702	231
8	Trade	92	1021	373	114	1330	484
9	Finance	12	239	103	20	385	163
10	Services	105	1178	443	164	2118	775
11	Administration	36	358	147	47	520	203
	Total	507	6055	2269	618	8079	3079

Sector Number	Sectors (1)	w_{io}^{A} (8)	W_{io} (9)	w_{10}^{B} (10)	r_i^{*A} (11)	R_i^{*} (12)	r_i^{*B} (13)	$w_{it}^{A}-w_{io}^{A}$ (14)	$W_{it}-W_{io}$ (15)	$w_{it}^{B}-w_{io}^{B}$ (16)
1	Agriculture	0.1085	0.1125	0.0714	-0.5818	-0.2511	-0.1728	-.0713	-.0494	-.0279
2	Forestry	0.0394	0.0135	0.0031	-0.4000	-0.1220	0.4286	-.0200	-.0046	+.0001
3	Fishing	0.0256	0.0030	0.0004	0.1538	0.2222	-1.0000	-.0013	-.0003	-.0004
4	Mining	0.0237	0.0129	0.0128	-0.0833	0.6538	0.6207	-.0059	+.0031	+.0025
5	Manufacturing	0.1282	0.2405	0.2997	0.4769	0.2328	0.2353	.0271	-.0183	-.0269
6	Construction	0.0750	0.0624	0.0595	0.2632	0.3095	0.4148	.0027	-.0011	+.0025
7	Transportation	0.1105	0.0925	0.0829	0.2143	0.2536	0.2287	-.0005	-.0056	-.0079
8	Trade	0.1815	0.1686	0.1644	0.2391	0.3026	0.2976	.0030	-.0040	-.0072
9	Finance	0.0237	0.0395	0.0454	0.6667	0.6109	0.5825	.0087	+.0082	+.0075
10	Services	0.2071	0.1945	0.1952	0.5619	0.7980	0.7494	.0583	+.0677	+.0565
11	Administration	0.0710	0.0591	0.0648	0.3056	0.4525	0.3810	.0051	+.0053	+.0011
	Total	0.9941	0.9990	0.9996	N.A.	N.A.	N.A.	.0059	.0010	.0001

Source: (1) Seasonally Adjusted Labour Force Statistics, **Statistics Canada, Catalogue No. 71-201** (Labour Force Survey Monthly Data).

(2) Canadian Statistical Review, Statistics Canada Publication No. 11-003, Vol. 47, No. 1, January 1972.

(3) The Labour Force, Statistics Canada, Catalogue No. 71-001.

(4) Special tabulation by the Economic Analysis Branch, Department of Regional Economic Expansion, Ottawa.

NOTE: (a) Totals in columns (2) through (7) are, wherever necessary, computed as an average of 12 monthly data from sources 1 and 2 above. The breakdown by sectors has been supplied to us through the courtesy of Statistics Canada. It should be noted that the sum of employment by sectors does not always add up to the totals given e.g. columns (2), (3), (4), (6) and (7). We did not attempt to distribute the residual error which is, however, small. Consequently sums of weights do not always exactly add up to one.

(b) Employment data have been rounded off to nearest thousands.

(c) Weights (and their changes) and growth rates have been worked out in the first instance on a six-decimal basis and then rounded off to four decimals.

(d) As regards the validity of employment data shown in columns (2) through (7), the reader is cautioned to read notes in the text.

(e) N.A. denotes 'not applicable' suggesting that the totals do not make sense.

(f) From the first part of the table the over-all growth rates of the regions and Canada are:

$$r^{*A} = 0.2189, \quad r^{*B} = 0.3570 \quad \text{and} \quad R^{*} = 0.3343$$

(g) Superscripts A and B stand for Atlantic and Ontario regions respectively.

Table 2

Regional-National Weighted Growth Rate Differentials by Sectors and Ranks of Sectors as Per Variant 1 Formulation

Sector Number	Sectors (1)	$r_i^{*A} w_{io}^{A}$ (2)	$R_i^{*} W_{io}$ (3)	$r_i^{*B} w_{io}^{B}$ (4)	$r_i^{*A} w_{io} - R_i^{*} W_{io} = X_i^{A}$ (5)	$r_i^{*B} w_{io}^{B} - R_i^{*} W_{io} = X_i^{B}$ (6)	Atlantic Ranks of Sectors by X_i^{A} Values (7)	Ontario Ranks of Sectors by X_i^{B} Values (8)
1.	Agriculture	-0.0631	-0.0282	+0.0123	-0.0349	+0.0159	10	1
2.	Forestry	-0.0158	-0.0016	+0.0014	-0.0142	+0.0030	9	4
3.	Fishing	+0.0039	+0.0007	-0.0004	+0.0032	-0.0011	2	7
4.	Mining	-0.0019	+0.0084	+0.0079	-0.0103	-0.0005	6	6
5.	Manufacturing	+0.0611	+0.0559	+0.0704	+0.0052	+0.0145	1	2
6.	Construction	+0.0197	+0.0193	+0.0247	+0.0004	+0.0054	3	3
7.	Transportation	+0.0237	+0.0234	+0.0189	+0.0003	-0.0045	4	10
8.	Trade	+0.0434	+0.0510	+0.0489	-0.0076	-0.0021	7	8
9.	Finance	+0.0158	+0.0241	+0.0265	-0.0083	+0.0024	8	5
10.	Services	+0.1164	+0.1552	+0.1462	-0.0388	-0.0090	11	11
11.	Administration	+0.0217	+0.0268	+0.0248	-0.0051	-0.0020	5	9
	Total	+0.2249	+0.3350	+0.3570	-0.1101 $= r^{*A} - R^{*}$	+0.0220 $= r^{*B} - R^{*}$	N.A.	N.A.

Source: Table 1.

NB: Computations have been worked out in the first instance on a six-decimal basis and then rounded off to four decimals. Any discrepancies that may arise could be attributed to rounding. The symbol N.A. stands for 'not applicable'. The rationale for the Table is to be found in the derivation of the formula in equation (21) in the text. Superscripts A and B stand for Atlantic and Ontario regions respectively.

Table 3

REGIONAL-NATIONAL DIFFERENTIALS IN GROWTH RATE, WEIGHT (SIZE) AND INTERACTION EFFECTS BY SECTORS: ATLANTIC-CANADA-ONTARIO COMPARISONS, 1961-1971

		A T L A N T I C - C A N A D A					
Sector Number	Sectors (1)	Weight Effect $r_i^{*A}(w_{io}^{A}-W_{io})$ (2)	Growth/Rate Effect $w_{io}^{A}(r_i^{*A}-R_i^{*})$ (3)	Interaction Effect $-(r_i^{*A}-R_i^{*})(w_{io}^{A}-W_{io})$ (4)	Total Effects ②+③+④ = (5)	Dominant Effect After Cancellation of Interaction Effect (6)	Identification of Weight Effect (7)
1.	Agriculture	0.0023	-0.0359	-0.0013	-0.0349	Negative Growth	No
2.	Forestry	-0.0104	-0.0110	+0.0072	-0.0142	Negative Growth	No
3.	Fishing	0.0035	-0.0018	+0.0015	0.0032	Positive Weight	Yes
4.	Mining	-0.0009	-0.0174	+0.0080	-0.0103	Negative Growth	No
5.	Manufacturing	-0.0535	0.0313	+0.0274	0.0052	Neither	Yes
6.	Construction	0.0033	-0.0035	+0.0006	0.0004	Neither	Yes
7.	Transportation	-0.0039	-0.0043	+0.0007	0.0003	Neither	Yes
8.	Trade	0.0031	-0.0115	+0.0008	-0.0076	Negative Growth	Yes
9.	Finance	-0.0105	0.0013	+0.0009	-0.0083	Negative Weight	Yes
10.	Services	0.0071	-0.0489	+0.0030	-0.0388	Negative Growth	Yes
11.	Administration	0.0036	-0.0104	+0.0017	-0.0051	Negative Growth	Yes
	TOTAL	-0.0485	-0.1121	+0.0505	-0.1101	Negative Growth	N.A.

		O N T A R I O - C A N A D A					
Sector Number	Sectors (1)	Weight Effect $r_i^{*B}(w_{io}^{B}-W_{io})$ (2)	Growth/Rate Effect $w_{io}^{B}(r_i^{*B}-R_i^{*})$ (3)	Interation Effect $-(r_i^{*B}-R_i^{*})(w_{io}^{B}-W_{io})$ (4)	Total Effects ②+③+④ = (5)	Dominant Effect After Cancellation of Interaction Effect (6)	Identification of Weight Effect (7)
1.	Agriculture	0.0071	0.0056	+0.0032	0.0159	Neither	No
2.	Forestry	-0.0045	0.0017	+0.0058	0.0030	Neither	Yes
3.	Fishing	0.0025	-0.0005	-0.0031	-0.0011	Neither	No
4.	Mining	-0.0001	-0.0004	-0.0000	-0.0005	Neither	Yes
5.	Manufacturing	0.0139	0.0007	-0.0001	0.0145	Positive Weight	Yes
6.	Construction	-0.0012	0.0063	+0.0003	0.0054	Positive Growth	Yes
7.	Transportation	-0.0022	-0.0021	-0.0002	-0.0045	Neither	Yes
8.	Trade	-0.0013	-0.0008	-0.0000	-0.0021	Neither	Yes
9.	Finance	0.0035	-0.0013	+0.0002	0.0024	Positive Weight	Yes
10.	Services	0.0005	-0.0095	+0.0000	-0.0090	Negative Growth	Yes
11.	Administration	0.0022	-0.0046	+0.0004	-0.0020	Negative Growth	Yes
	TOTAL	0.0204	-0.0049	+0.0065	0.0220	Positive Weight	N.A.

Source: Table 1.

NB:

1. Identification of weight effect is made by observing the signs of r_i^*. If r_i^* is negative, weight effect is termed un-identified or simply written 'no'. For $r_i^* > 0$ we have written 'yes' to say that the effect is identified by signs.
2. In column (6) the word 'neither' is purported to mean that neither of the two effects, weight and growth/rate, becomes dominant after cancellation of the interaction effect i.e. the end result is small.
3. In evaluating the detailed interpretations of this table compare equation (17) and the discussion following equations (22) through (26) in the text in Section 2.
4. N.A. stands for 'not applicable'.

APPENDIX - 1

1. List of Variables

r_i = annual compound rate of growth of Sector i for any chosen variable viz, value added or employment in the Region, A;

R_i = annual compound rate of growth of Sector i in the nation;

w_{it} = weight (or share) of Sector i in the total regional value of the variable in time, t;

W_{it} = weight (or share) of Sector i in the total national value of the variable in time, t;

S^A_{it} = value of the variable chosen in the Region A in Sector i in time, t;

S_{it} = value of the variable chosen in the nation in Sector i in time, t;

S^A_t = total value of the variable chosen from all sectors in the Region A in time, t;

S_t = total value of the variable chosen from all sectors in the nation in time, t;

r = annual average compound rate of growth from all sectors in the Region, A;

R = annual average compound rate of growth from all sectors in the nation;

r^* = aggregate growth rate of the Region, A over the entire time period;

R^* = aggregate growth rate of the nation over the entire time period;

r^*_i = growth rate of the region for Sector i over the entire time period;

R^*_i = growth rate of the nation for Sector i over the entire time period;

X_i = weighted growth rate differential between the region and the nation in Sector i;

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a_i = growth or rate effect in Sector i;

b_i = weight effect in Sector i;

c_i = interaction effect in Sector i;

t = time measured in years; t = 0 refers to the initial period.

NB: Superscripts A or B stand for Regions A or B. Whenever superscripts are not used, lower case letters will stand for regional variables. All capital letters without superscripts stand for national variables.

2. Derivation of Formulae (7) and (8)

We write:

$$(1+r)^t = \frac{S^A_{it}}{S^A_o} = \frac{\Sigma S^A_{it}}{S_o} = \frac{\Sigma S^A_{io}\ (1+r_i)^t}{S_o}$$

From (5) with t = 0, we then obtain:

$$(1+r)^t = \sum_{i=1}^{n} w_{io}\ (1+r_i)^t$$

Hence, $$r = \left[\sqrt[t]{\Sigma w_{io}\ (1+r_i)^t}\right] - 1$$

Similarly (8) can be derived.

3. Formulation (17)

From (15) and (16) we obtain:

$$r^* - R^* = \sum_{i=1}^{n} r_i^* w_{io} - \sum_{i=1}^{n} R_i^* W_{io}$$

or $$r^* - R^* = \sum_{i=1}^{n} \left[r_i^* w_{io} - r_i^* W_{io} + r_i^* w_{io} - R_i^* w_{io} - r_i w_{io} + r_i^* W_{io} + R_i^* w_{io} - R_i^* W_{io} \right]$$

This results in equation (17) in the text.