## WORKING PAPER



## ECONOMIC ANALYSIS BRANCH PLANNING DIVISION

r th REGIONAL ECONOMIC EXPANSION CANADA EXPANSION ÉCONOMIQUE RÉGIONALE CANADA

# IDENTIFICATION OF INDUSTRIAL COMPLEXES <br> FROM THE INPUT-OUTPUT TABLES OF CANADA \& THE U.S.A. 



B.K. Lodh

Economic Analysis Branch
Planning Division,
Department of Regional Economic Expansion.

Ottawa
February 6, 1974

# OBJECTIVE OF THE STUDY \& CONCLUSIONS <br> (A Brief Summary) 

The purpose of this study has been to identify industrial complexes from the Input-Output Tables of Canada, 1961 and 1966 and the U.S.A. 1963. The main idea behind the relevance of industrial complexes has been the notion that in stimulating growth centres for regional development, certain specific technical interdependencies among industries qualifying for a complex become a sine-qua-non. The latter has been captured by a criterion of maximal interdependence of industries from the national input-output tables which for Canada (1961 and 1966) were supplied by the Input-Output Division, Statistics Canada and for the U.S.A. (1963) was obtained from the Survey of Current Business, November, 1969. The study highlights the following important features hitherto unknown or loosely couched in general terms:
(1) As a methodological device of isolating industries having maximal interdependence this study has proposed a unique approach which is directly related to the problem and which respects the original backward and forward linkages derived from input-output tables. The factor analytic approaches available in the literature on the identification of complexes distort these linkages.
(2) 100 complexes for both Canada \& the U.S.A. are reported ${ }^{1}$ 1. See Tables A, B, C, pp 23-25.
here of which some twenty major complexes are further identified to be the dominant ones in both the economies. In Canada the major complexes are found to be in the nature of Steel Mills, Construction, Food and Beverage and Agriculture etc whereas in the U.S.A. the latter two types are also dominant (but Steel Mills is not).
(3) The structure of complexes in Canada has not significantly changed between 1961 and 1966.
(4) The study concentrates only on 165 goods producing industries in Canada and on 64 goods producing industries of the U.S.A. to the exclusion of 45 service industries of Canada and 23 similar industries of the U.S.A. Various experiments including service industries suggested only very round-about complexes which often begged interpretations and are, therefore, ignored.
(5) Special industrial complexes starting with any given industries, christened as "Island Industry Complexes"; are also an additional attraction of the study. Very often when questions like "what industries are associated with, say, breweries" are raised, for example in a feasibility study, our method helps to identify these industries both as súppliers and receivers.
(6) Never before the identification of industrial complexes
has been so exhaustive both in search as well as in detail for large disaggregated input-output tables.

The policy implications of industrial complex analysis in the context of regional development are significant. DREE's policy formulation in terms of grants to specific firms or industries may also be considerably tempered by these considerations. These are:
(i) If industry $A$ (or firm A) is considered to be eligible for grants by some criteria of financial and/or commercial viability of A,then A's viability cannot be presumed to be judged by its own performance only but must share its performance in some proportions to the linkages it maintains with other industries (or firms therefrom). In other words the whole industrial selection procedure and the exploration of economic opportunities should have to be cast in terms of discovering a group of industries rather than individual industries at íeast insofaras they are technically related which is what the industrial complexes reveal.
(ii) If a particular region has specific resources, say gypsum, the industrial complex approach helps to render the best block of industries, directly and indirectly linked with it, that is suited to its technical viability. This block of industries can then be looked for in the region in terms of its domestic
production capabilities failing which the costs of imports may be calculated to evaluate the commercial viability of gypsum production in the region.

The above implications remain valid despite the fact industrial complexes are identified only from the national inputoutput tables since. (a)... regional input-output tables donot exist in the same detail one would like to have for meaningful results, and (b) regional input-output tables tell generally very little about technical linkages between industries which are better revealed by the national input-output tables.

The results of this study owe its origin and initial development to the Input-Output Division of Statistics Canada from whom subsequently the Economic Analysis Branch, Planning Division, Department of Regional Economic Expansion, took over the task. In the present form of this study, the Branch is grateful to the special services of Mr. J.S. Lewis of Regional Statistics, Research and Integration Division of Statistics Canada for providing various algorithms for testing complexes.
Pages
Introduction ..... 1- 3
Methods in Complex Analysis ..... 3-10
The Method ..... 11-1.5
Island Industrial Complex ..... 15-16
The Data ..... 16-19
The Empirical Results \& Interpretations ..... 19-33
References ..... 33-36
Tables 1,2 and 3(100 Industrial Complexes for Canada \& the U.S.A.)
Appendices 1,2 and 3.

## IDENTIFICATION OF INDUSTRIAL COMPLEXES

FROM THE INPUT-OUTPUT TABLES
OF CANADA AND THE U.S.A.

## Introduction

Studies in the structure of production following the input-output methodology have been for a long time related primarily to the question of the inter-industrial dependence or hierarchy of industries or sectors. The latter has evolved into some approaches to 'triangulation' following the attempts by Leontief (1963), Chenery and Watanabe (1958) and later those by Helmstader (1964), Lamel, Richter and Teufelsbauer (1972) and Korte and Oberhoffer (1969, 1971). However, large disaggregated input-output tables rarely conform to the idealism of the triangularized hierarchy.. There are sets of industries mutually related by backward and forward linkages to an extent that they represent coherent groups such that they remain relatively unrelated to the remaining industries in terms of
transactions taking place in the input-output (I-0) tables. Such a possibility was recognised implicitly by the 'balanced growth' protagonists like Nurske (1953), Rosenstein Rodan (1943, 1957), Scitovsky (1954) and Lewis (1956) for industrial development ${ }^{1}$ at large and explicitly by Isard, Schooler and Vietorisz (1959), Simpson and Tsukui (1965), Streit (1969) and Czamanski (1972). It has been suggested that external economies exist and they arise out of technical interdependence of industries or sectors; this idea has also been used to explain, agglomeration economies (Hoover, 1948, Hirschman, 1964, Isard, 1956, Lösch, 1952, Ullman, 1964, Richter, 1969). However, this paper will be addressed particularly to the first question, namely, how to identify sets or groups of industries from disaggregated I-0 tables that have certain properties of maximal interdependence. In this vein an industrial complex in this paper will be defined as a group of industries that maximises a total linkage criterion, to be defined later, based on the backward and forward linkages derived from given I-0 tables. An outcome of this exercise is to show the existence ${ }^{2}$ of different types of complexes

1. The first two authors stress balance in demand whereas the latter two stress balance in supply. See Hirschman (1964, pp 50-51).
2. This has been also noted by Simpson and Tsukui (19) where decomposability i.e. existence of separable submatrices (which are almost akin to our definition of complexes) has been their prime concern. However their procedure of decomposition does not seem to follow any optimization criterion, howsoever postulated. Moreover their approach is based only on the technical coefficients, aij's, and not on ailj's which we have encompassed in our subsequent discussions.
that are likely to arise in large I-0 tables which contrast with the conventional results of 'triangulation'. The basic data of our study refer to the input-output ( $I-0$ ) tables of Canada, 1961 and 1966, and the I-0 table of the U.S.A. 1963. Section 1 of this paper deals with a review of the existing procedures together with a skeleton of the methodology used in the study; section 2 provides the data with their implications; Section 3 offers various interpretations of the derived complexes.

## 1. Methods in Complex Analysis

Generally the procedures for deriving industrial complexes from 1 -0 tables stem from the following four coefficients as defined by (1) and (2) that pertain to any pair of industries, i:and j.

$$
\begin{array}{ll}
a_{i j}=\frac{A_{i j}}{V_{j}}, & a_{j i}=\frac{A_{j i}}{V_{i}} \\
a_{i j}^{*}=\frac{A_{i j}}{V_{i}}, & a_{j i}^{*}=\frac{A_{j i}}{V_{j}} \quad \ldots \tag{2}
\end{array}
$$

where $\quad A_{i j}=\begin{aligned} & \text { dollar sales of industry } i \text { to } \\ & \text { industry } j \text {, }\end{aligned}$
i,j $=1,2,3, \ldots N, \ldots$.
$F_{i}=$ final demand of industry i. /
$V_{i}=$ gross dollar output of industry i.
and $\quad V_{i}=\sum_{j=1}^{P} A_{i j}+F_{i} \quad \cdots$
Coefficients given by (1) are the usual Leontief backward linkages and those by (2) are the forward linkages.

There are, however, other alternative ways of defining linkages. For example, instead of having a denominator of $V_{i}$ and $V_{j}$ in ( 1 ) and (2) one can use $V_{i}^{*}$ and $V_{j}^{*}$ where $V_{i}^{*}=$ $\sum_{j=1}^{N} A_{i j}$ and $V_{j}^{*}=\sum_{i=1}^{N} A_{i j}$. The coefficients then derived will be different from those in (1) and (2) as final demands are left out. If final demands are considered important as in Leontief schemes this procedure seems somewhet inadequate to reflect linkages in the whole economy. Finally, analysis may be conducted with special reference to a subset of industries, say $N(N<P)$, instead of having the whole set of industries, P. This depends on the assumptions one may hold with respect to the importance of $N$ industries vis-a-vis $P-N$ industries which are left out of account ${ }^{1}$.

Against this background it may be useful to have a short review of the existing procedures of identifying complexes which fall primarily in three categories:
(a) a specified industrial complex obtained from engineering information as developed by Isard - Schooler Vietorisz in the case of petro-chemicals complex for Puerto-Rico;

1. In the actual experiments with the I-0 tables, for example, we have worked on the $N$ material-goods industries and have left out $P-N$ service industries. A similar procedure is followed by streit (12).
(b) selection of combinations of pairs of industries that satisfy both the spatial and the economic linkages between industries where the economic linkages are derived from the national $I-0$ tables (Streit);
(c) the use of multivariate analysis e.g. the method of principal components, in the identification of industrial complexes from any given I-0 table (Czamanski) whether the latter refers to any particular region or nation.

As for (a) the question posed is specific, namely, what specific industrial products can be added or related to petro-chemicals that can render Puerto-Rico a comparative advantage in costs and/or revenues vis-a-vis an identical set of pro-duct-producing agents in the mainland U.S.A. This type of problem is initialized with a certain product or group of products and then further products are added or linked to form a complex. In terms of detailed examination the Isard - Schooler - Vietorisz study is a classic of its type, but it requires much more information than can be derived from the I-0 tables at the national or regional levels. Such a study can be viewed as a necessary subsequent development and evaluation of complexes derived from more generalized information of $I-0$ tables. As an initial ap-. proach we offer a procedure of isolating a complex starting with any given industry (christened as an 'Island industry') in a
given I-0 table. The details of the procedure will be pursued subsequently.

In (b) the procedure is basically subjective insofar as it is not clear whether the linkages, economic (technical)or spatial, between any pairs of industries can be additive and if so whether there exists any maximum (even if local) for the total linkages forming a complex. Secondly, Streit's method of averaging the four coefficients given by (1) and (2), i.e. summing the coefficients and dividing by four, makes the matrix symmetric which loses the propriety of an essential dichotomy between a supplying industry and a receiving industry. Also a particular weakness of Streit's procedure is that it is dependent on the small size ${ }^{1}$ of the matrix of linkages which perhaps has facilitated his search for complexes. In a nutshell, neither the objective of maximization of total linkages nor the search procedure are clearly delineated in Streit's work which deprives it of any analytical rigour.

The factor analytic approaches are exemplified by (c). Given a ( $\mathrm{N} \times \mathrm{N}$ ) I-0 data matrix of inter-industrial transactions and a vector of N gross outputs, the adaptation of multivariate analysis to the complex analysis requires to fulfill one major condition, namely, the conversion of I-0.table into a sort of

1. Streit's I-0 tables for West Germany and France refer only to 26 X 26 matrices for production-oriented goods. See Streit (12).
( $\mathrm{N} \times \mathrm{N}$ ) correlation matrix. The latter may be formed by (i) constructing a spatial correlation ${ }^{l}$ between any pair of industries, (ii) postulating an average linkage ${ }^{2}$ between any pair of industries from the four primary coefficients defined by (1) and (2) and treating this linkage as a surrogate correlation, and (iii) choosing the strongest correlation coefficient' from among the four correlation coefficients that can be obtained from the four primary coefficients (Czamanski) ${ }^{3}$ with respect to any pair of industries. The upshot of all these
2. A spatial correlation between any two industries may be obtained with respect to employment, value added or shipments data for these two industries over some defined spatial units. An approach of this sort has been made by Streit (12) who uses employment data.
3. Streit's procedure of dividing the sum of four coefficients, defined by (1) and (2), by four illustrates such possibilities.
4. Czamanski's procedure may be stated in a nutshell here. Taking any pair of industries, $k$ and $l$, pairwise sets of data such as (1) aik's and ail's, (2) akj's and alj's, (3) aij's and $a_{l i}^{*}{ }^{\prime \prime} s$ and (4) $a_{k i}^{*}{ }^{*}$ s and ail's can be arranged to render the four correlation coefficients. Thus a high r(aik, $a_{i l}$ ) is supposed to show a strong relationship between $k$ and $I$ in sofaras it draws heavily upon supplies from the same industries, i ranging over all industries. Similarly a high $r\left(a_{k j}, a_{l j}\right)$ means that industries $k$ and $l$ are supplying to a similar set of users, j ranging over all industries. Further, a high r(aik, $\left.a_{1 i}^{*}\right)$ implies that the supplies of $k$ industry are users of the products of 1 , and $a$ high $r\left(a_{k i}^{k}, a_{i 1}\right)$ signifies a reverse relationship between $k$ and $l$, namely the users of $k$ are supplies of 1 . Czamanski then picks up the highest of all four correlation coefficients between $k$ and 1 and similarly for all pairs of industries to obtain an intercorrelation matrix (symmetric). The major defect of this procedure, apart from the more damaging ones related to the application of a correlation matrix (these are reported in the text), is that in large disaggregated I-0 tables correlation coefficient.may be low anyway. Moreover, a low correlation coefficient, say $r\left(a_{i k}, a_{i l}\right)$, should not necessarily preclude considerations of a complex formation involving high values of original coefficients, aik's and ail's, whereby industries $i, k$ and $l$ can be considered to be members of a complex.
devices or short-cuts is that one mostly ends up with some biased linkages or associations standing for correlation coefficients. Thus spatial correlations are usually subject to the arbitrary definition of space and they may not at all reflect technical linkages that perhaps interest a researcher of complex analysis. Moreover data requirements over space may be difficult to fulfill. The shortcomings of the other approaches have been already noted and these perhaps merit no additional attention. However, in the application of correlation matrix to multivariate analysis by means of any sort of factor analytic devices for industrial complex identification, the following major deficiencies deserve particular attention:
A. An intercorrelation matrix (which is symmetric) invariably Joses the essential dichotomy between a supplying industry and a receiving (using) industry which industrial complex analysis should ultimately reveal. Any tinkering with the original four coefficients that distorts this asymmetry or dichotomy should be usually suspect.
B. All factor analytic approaches involve a progressive reduction of the matrix as the complexes (in the present context) are isolated and removed from the system. This is an undesirable feature since each complex obtained subsequently is determined by the context of those obtained and
removed earlier. Thus a linkage absorbed, at least partially, in one complex assumes a reduced stature relative to any other complex following it. That is, a linkage cannot be properly reflected in more than one complex.
C. The condition of orthogonalization used in
factor analytic approaches seems irrelevant to the complex analysis except in the trivial case where submatrices exist in a block-diagonal sense which in reality is never so.
Before we proceed to the primary analytical thrust of this paper a final comment seems in order. There exists some other procedures of decomposability of any data matrix into submatrices (conforming to our notion of complexes), namely the method of singular decomposition ${ }^{l}$, which particularly can take
5. See Good (1969). The basic procedure here is to decompose a given $m$ X $n$ matrix $A$ as:

$$
\begin{equation*}
A=e_{1} S_{1} R_{1}^{\prime}+e_{2} S_{2} R_{2}^{B}+\ldots \tag{a}
\end{equation*}
$$

where each term on the right is an $m X n$ matrix of rank one, the $S_{i}, R_{i}$ are normalized vectors of orders $m, n$ and the $e_{i}$ 's are positive scalars, the singular values (if A is symmetric, they are the eigen values). The vectors' are developed by direct iteration based on the relations $S^{\prime} A=e R^{\prime}$ and $A R=e S$. The orthogonality occurs as RíRj $=S_{i j}=S_{i} S_{j}$. The matrix is reduced by each decomposition before extracting the next (in effect subtracting terms of the right side successively from each side of (a)).

This procedure was applied to the 1963 U.S. I/O matrix with the following variation. The orthogonality was relinquished by suppressing components below a specified threshold of the $R$ and $s$. vectors. This leads to vectors approximating dominant submatrices of $A$; constituting "complexes". The matrix reduction was prevented from developing negative entries by arbitrarily replacing then with zeros. The complexes obtained were satisfactory until the submatrices involved began to overlap i.e. to incorporate elements which had been included in a prior complex and hence subjected to reduction. Furthermore as a final objection, this procedure and the other factor analytic approaches require prodigeous computation with large disaggregated I/0 matrices.
care of 'asymmetry' noted in (A) above. Unfortunately it does not enable one to overcome the limitations of (B) and (C). In some experiments worked out for large matrices of I-0 tables for Canada and the U.S.A. the results often begged interpretations, and we had no other alternative than to surrender it.

The Method
The method used to identify complexes in this study primarily hinges on the construction of an objective function, E , defined as follows:

$$
\begin{aligned}
& E=\frac{\sum_{i \varepsilon S}^{\sum} \sum_{j \varepsilon_{R}} b_{i j}}{n+k} \\
& b_{i j}=\frac{a_{i j}+a_{i j}^{*}}{2} \\
& \mathrm{n}=\text { total number of cells belonging to } \\
& \text { the complex. } \\
& k=\text { size control parameter, } k>1 \text {. } \\
& i_{\varepsilon S} \text { refer to industries } i \text { belonging to the } \\
& \text { whole set of suppliers, } S \text {. } \\
& j_{\varepsilon R} \text { refer to industries } j \text { belonging to the } \\
& \text { whole set of receivers, R. }
\end{aligned}
$$

The procedure of identifying complexes can be described in a nutshell as below:
(i) A matrix of $N$ dimensions ( $N \leqslant p$ ) is constructed with all $b_{i j}$ 's.
(ii) Starting with a maximum of $b_{i j}$ 's (call it $\hat{b}_{i j}$ ), keep adding and dropping $b_{i j}$ 's which are connected directly or indirectly with $\hat{\mathrm{b}}_{\mathrm{ij}}$ (without dropping $\hat{\mathrm{b}}_{\mathrm{ij}}$ ) in sofaras. E can be maximised ${ }^{1}$. Note that each time a

1. See the formal treatment in terms of an algorithm in Appendix 1.
supplier and/or a receiver industry is taken in (or out), $n$ keeps rising (or falling). To exemplify $n$, three supplying industries and two receiving industries will make n equal to six. k , the size control para-. meter, determines the size of the complex i.e. number of suppliers and receivers. A higher $k$ will increase the size and lower $k$ will diminish it. For practical purposes $k$ can be assumed to have positive integer values only, and a.final selection of $k$ requires different experiments with $k$ for satisfactory results.
(iii) Once the first complex is derived by maximization of $E$, the next highest $b_{i j}$ is chosen as a starter while keeping all $b_{i j}$ 's as they are and the process is repeated as per (ii) to obtain the second complex, and so on. Note that as more and more complexes are extracted only starting values of $b_{i j}$ 's change while all $b_{i j} ' s$ are kept in full play so that any linkage, say $b_{k l}$, can be found in more than one complex. obviously one can come across in this scheme a repetition of the same complex with different starters/r $b_{i j}$ 's, which may reasonably justify the uniqueness of that complex ${ }^{1}$.
2. Apart from exact repetition of complexes, one can arrive at 'overlapping' and 'nested' types too, whereby in the first case a subset of complex $A$ is also a subset of complex $B$, and in the second, complex $A$ is a complete subset of complex $B$.

The above procedure can be christened as 'forward step search' resulting in the formation of free complexes with specific statters, and adding or dropping takes place only in one step i.e. one row or column can be added or dropped. This scheme is followed because multistep additions or delections (many rows or. columns) is computationally unmanageable when maximization of the objective function, $E$, is also a concurrent aim. It seems also plausible to have a 'backward step' search starting with the whole matrix of $b_{i j}$ 's and maximising $E$, subject to a given $k$, to arrive at the first complex. But then the search for the second and subsquent complexes create additional difficulties as to the choice of $b_{i j}$ 's that needs to be dropped to effect such a program. This difficulty has partly prevented the authors from following the 'backward step approach' despite the general appeal of the backward search traditionally allowed in any reductionist procedure as in multivariate analyses.

The algorithm underlying the method suggested above is outlined in Appendix $1 \& 2$ with a list of statistical indicators that call for specific explanations of the findings of the study. Some final points seem in order. In deriving $b_{i j}$ 's, where ith industry is taken to be identified as a supplier and jth industry as a receiver with $b_{i j}$ 's signifying economic or technical linkages, we have followed a simple arithmetic mean (A.M) as a criterion rather than any other criteria of averaging,
namely for example, geometric mean (G.M.). The choice between A.M or G.M would largely depend on the pratical assessment of the data and on the considerations as to how stringent ${ }^{1}$ one would like to be with respect to the joint relationship between a supplier and a receiver, given the fact that A.M. $\geqslant$ G.M.and the difference between the two increasing with increasing asymmetry between $a_{i j}$ and $a_{i j}^{*}$. Secondly, the purpose of $k$, size control parameter, is directed towards obtaining a variety of complexes of different sizes while maximising $E$. Obviously with $k=\propto$ one finds the whole matrix of $b_{i j}$ 's, that is to say the whole transactions matrix under analysis, becoming the one and only one complex. Conversely with $k=$ zero only the highest $\mathrm{b}_{i j}$ makes a complex of one supplier and one receiver. Both cases are trivial. Generally an experiment with different values of $k$, say $k=2$ and $k=5$, will suggest that the dominant linkages with the first complex under alower value of $k$ will also be contained in the first complex under a larger value of k. In large matrices, however, subsequent complexes obtained under different values of $k$ appear to change the structure of their membership. Finally, this study does not pretend to lay any claim to a 'global maximum' for any choice of a complex since this requires a multi-step additions or delections of rows

1. Pratical considerations may lead to the choice of A.M. since I-0 data cannot be assumed to be perfect, nor nearly perfect. A.M. may also avoid some uncertainty in the pair-wise relations of the data much more effectively than G.M.
and columns which is not computationally feasible as the number of permutations of rows and/or columns for such an objective becomes astronomical with large matrices. Consequently our procedure is geared to discovering 'local Maxima' only.

Island Industrial Complex

Very often questions relating to the development of a particular industry plague researchers to look for a bunch of other industries that are directly or indirectly related (but closely) to the primary one. Unfortunately available methods incorporating the I-0 tables cannot effectively answer such questions since there exists no workable criterion to select such a bunch. Moreover for large I-0 tables eyeball search becomes inefficient and cumbersome. We have developed, therefore, the following method to meet this objective:
(1) Starting with any industry, say.m, inflate the $b_{m j}$ 's and $b_{i m}$ 's by a weight factor, say $\mathrm{w}=5$ or 10. Keep now all other $\mathrm{b}_{\mathrm{ij}}$ 's as they were before. (The weight factor is applied to give dominance to the direct relationships of industry.m as a supplier as well, as a receiver).
(2) Maximize $E$ now subject to a given $k$ under the new matrix of $\mathrm{b}_{\mathrm{ij}}$ 's including industry m .

The complex that is now so obtained with reference to a specific industry, $m$, is called for our purposes an 'island industry complex'.
2. The Data

The data for the Canadian I-0 Tables are obtained on tape from the Input-Output Division of the Statistics Canada. The tables have been made available to us for two separate years, 1961 and 1966, and are both in producers' prices and are of 210 dimensions i.e. in square matrix forms. Gross outputs in current dollars for 210 industries are also obtained from the same source for the two years. Incidentally the I-0 Table for 1961 which we have obtained from the Statistics Canada is revised version of an earlier 1961 table to accomodate changes in the industrial classification as well as in the national accounts of 1961 (as well as earlier and later years) that took place in the beginning of 1973. The classification of the first production-oriented 165 industries (of the total of 210 industries) actually used in this study is listed in Appendix 3. The remaining 45 service industries
is excluded from our analysis for reasons that will be explained shortly.

The I-0 table (expressed in producers' prices) of the U.S.A. refers to 1963 and is of 87 dimensions and is taken from the Survey of Current Business, November, 1969. The classification of the first 64 production-oriented industries from these is also listed in Appendix 3. The remaining 23 services industries are excluded for reasons that will be explained shortly.

The Canadian and American classifications of industries donot agree with each other and no attempt has been made to put them on a comparable basis since in most cases (as in the past) comparisons remain odious despite the appeal of comparability, unless considerable aggregation of industries is deliberately chosen. The latter alternative is, of course, repugnant to the very objective of our search for complexes in large disaggregated tables, and hence not pursued.

The data of the I-0 tables actually used in this study refer to the first 165 industries (of the total of 210 industries) of Canada and to the first 64 industries (of the total 87 industries) of the U.S.A. This has been necessitated by considerations bearing on the interdependence of productionoriented material goods only to the exclusion of goods of the
service type. The rationale for this choice is, first, based on the presumption that services are subordinate ${ }^{l}$ to the production relations and, second, that the inclusion of service type of goods often brings in some indirect networks or relationships of industries for which meaningful interpretations are difficult to offer.

Finally, a word needs to be said about the intraindustrial transactions of the I-0 tables. For the purposes of this study all intra-industrial transactions have been set to zero ${ }^{2}$ even though for some few industries these transactions are considerable judging from their shares in gross outputs either as suppliers or receivers. The major reasons for following this step are: (1) we are interested in inter-industrial relations rather than intra-industrial relations, and (2) some attempts at capturing complexes with intra-industrial transactions (following our criterion of maximizing $E$ ) occasionally have shown some complexes which are much too much indirectly linked. It is conceded

1. It is not intended to imply that the production of material goods is always feasible without essential service inputs. What is implied is that the service inputs by virtue of their non-material nature require that their demands are conditional upon the existence of demands for material goods. It is, however, conceded that today much of this distinction between the material and the non-material characterization of inputs is open to question and the matter is far from being unanimous in terms of its propriety.
2. It is of some inportance to note here that while we zero the intra-industrial transactions we use the intra-industrial cells in the specification of $n$ while maximizing $E$ only if some suppliers and receivers belong to the same industries. This does not appear to be a major restriction since in actual cases such events occur only rarely.
that for small matrices i.e. where industrial classifications are aggregated, the right procedure would be to include the intra-industrial transactions.
3. The Empirical Results and Interpretations.

The results of the study with respect to the free complexes are reported in Tables 1,2 and 3 at the end of the text where the first two tables refer to Canada, 1961 and 1966 and the last one to the U.S.A., 1963. The computer program has been set to obtain the first 100 complexes for both Canada and the U.S.A. as it is considered that further extractions would make interpretations difficult for complexes so obtained with gradually diminishing starting values of linkages since according to our method of extraction starting values cannot be dropped while search is being made for one-step maximization of E. Moreover, 100 complexes exhaust about $60 \%$ of the total dollar transactions, and about 47\% for the total linkage coefficients ( $b_{i j}$ 's) for Canada, 1961 and 1966. Similarly, for the U.S.A., 100 complexes exhaust about $68 \%$ of the total dollar transactions and about $59 \%$ of the total linkage coefficients. These reduction (or exhaustion) estimates, however, refer to, the inter-industrial transactions only of the industries we have chosen (See Appendix 3). It is considered that the information contained in these tables is sufficient to explore many
interesting facets of inter-industrial relations hitherto uncovered. To read any table some explanations appear to be necessary to bring the picture in sharp relief.

Consider the complexes 2 and 4 in Table 1. The initial starting elements ${ }^{1}$ (nuclei) of the two complexes are .7990 (with industry 85 as a supplier and industry 86 as a receiver) and . 6582 (with industry 4 as a supplier and industry 92 as a receiver) respectively. With the restriction that initial starting elements cannot be dropped in our program of optimization the composition of the two complexes has slightly changed. Whereas in complex 4 industry 4 has replaced industry 85 of complex 2 as a supplier and industry 92 (of complex 4) has replaced industry 6 (of complex 2) as a receiver, the other adjoining industries, namely $82,84,87$ and 130 , have remained the same core of suppliers. The result is that complex 4 overlaps complex 2 in four 'intersecting'elements which is also shown in the table. The types of intersection have been classified as 'independent', 'overlapping', 'nested' and 'repetitive!. Between any two complexes $A$ and B, the 'overlapping' case arises when a subset of complex $A$ is also subset of complex $B$, whereas the 'nested' case arises when complex $A$ is a complete subset of complex $B$ or vice-versa. The 'independent' and 'repetitive' cases are the polar cases which merit no further explanation.

1. These are marked with an asterisk sign.

Now in the above example how is it that complexes 2 and 4 with all the common suppliers like $82,84,87$ and 130 could not collapse into one single complex? The explanation lies mainly in the choice of $k$ (in our example $k=10$ ). A higher $k$ (say, $k=15$ ) perhaps could have done the trick but it may also bring in elements with lower values of linkages which may not be very welcome if compactness (as roughly measured by $C V$, coefficient of variation) of a complex is also a desirable objective. In very many experiments with varying $k$ we have observed that $k=10$ does appropriately measure up to our requirements, namely medium sized complexes, relatively low coefficient of variation and relatively few linkages of lower values. Hence for our purposes it is reasonable to suppose that the existence of 'overlapping' complexes should cause no concern. Finally for the tables the last column demonstrates the one-step selection of industries as they are added (-1) and dropped (1) to maximize E and it gives the sequential values of $E$ in this process until it cannot be increased anymore.

The results may now be summarized as follows:
(1) A large number of repetitive complexes appear in all three tables which makes it easier to evaluate only the 'independent', 'overlapping' and 'nested' types. The number of repetitions for Canada 1961 is 51 , ${ }^{\prime}$ for Canada 1966 it is 42 and for the U.S.A. it is 37. As noted before a
complex which is repeated many times is supposed to gain in credence as it becomes a unique complex insofaras it cannot be dismembered whatever may be the starting values.
(2) Disregarding the 'repetitive' types, the remaining complexes still remain quite large in number which may require individual attention of interpretation or which can be collapsed into some ranking order such that the problem of search with a view to comparing between any two or many complexes can be minimized. The latter has been simplified by postulating a numerical indicator with a multiplicative factor ${ }^{1}$ of $E$ and $g$ which is then organized in descending values (in absolute values as well as in rank). The first 20 complexes are then chosen and shown in Tables $A, B$ and $C$. These are also given some appropriate names particularly from the point of view of receiving industries. Now in all these tables one further notices that overlapping occurs as between complexes having similar titles. Thus in Table A complexes 2, 18, 94, 89, 4, 10 and 45 overlap in varying degrees and similarly for others. Occasionally, as.in.Table $B$, one gets complexes having multiple ties over and above overlapping.

1. One can suggest also alternative formulations, namely E. $\mathrm{m}_{1}$ instead of E.g. Since our objective is more directed toward linkage coefficients (which are reflected in g) rather than in total dollar transactions (embodied in $m_{1}$ ) we have thought that the inclusion of $g$ is more appropriate.

Table A

SELECTED INDUSTRIAL COMPLEXES BY RANK FROM THE 100 COMPLEXES EXTRACTED FROM THE INPUT-OUTPUT TABLE, CANADA, 1961.

| Rank | Complex No. | E.g | Name of the Complex |
| :---: | :---: | :---: | :---: |
| 1 | 2 | . 5501 | Steel \& Rolling Mills |
| 2 | 31 | . 5405 | Residential \& Non-Residential Construction |
| 3 | 18.\% | . 5136 | Sinter Plant \& Blast Furnaces \& Steel Mills |
| 4 | 63 | . 4720 | Residential, Non-Residential \& other Engineering Construction |
| 5 | 94 | . 4559 | Coke Oven, Steel Mills \& other Steel |
| 6 | 89 | . 4539 | Steel Mills \& other Smelting \& Refining |
| 7 | 4 | . 4478 | Steel Mills \& other Smelting \& Refining |
| 8 | 77 | . 4118 | Food \& Beverage \& Agriculture |
| 9 | 99 | . 4008 | Food \& Beverage \& Agriculture |
| 10 | 51 | . 3951 | Food \& Beverage \& Agriculture |
| 11 | 10 | . 3922 | Steel Mills \& Steel Pipe \& Tube Mills |
| 12 | 21 | . 3789 | Food \& Beverage \& Agriculture |
| 13 | 7 | . 3788 | Pulp and Paper |
| 14 | 70 | . 3763 | Sawmills, Veneer \& Plywood \& Pulp \& Paper |
| 15 | 67 | . 3662 | Clothing \& textiles |
| 16 | 12 | . 3634 | Clothing \& textiles |
| 17 | 45 | . 3590 | Steel Mills, Ferro Alloy \& Iron \& Steel |
| 18 | 37 | . 3546 | Sawmills, Wood Pulp, Pulp \& Paper |
| 19 | 76 | . 3502 | Residential, Non-Residential Construction \& Sash and Door |
| 20 | 84 | . 3141 | Clothing \& textiles and Fur Dressing |

Source: Table 1.
N.B.: Complexes are arranged here in a descending ranking order based on the values of E.g and some abbreviated names have been given to complexes for easy recognition. Repetitive complexes are excluded. For a complete description, see Table 1.

## Table B

SELECTED INDUSTRIAL COMPLEXES BY RANK FROM THE 100 COMPLEXES EXTRACTED FROM THE INPUT-OUTPUT TABLE, CANADA, 1966.

| Rank | Complex No. | E. 9 | Name of the Complex. |
| :---: | :---: | :---: | :---: |
| 1 | 1 | . 5961 | Steel \& Rolling Mills |
| 2 | 88 | . 4941 | Sinter Plant, Blast Furnaces \& Steel Mills |
| 3 | 6 | . 4902 | Steel Mills \& Other Smelting \& Refining |
| 3 | 28 | . 4902 | Sinter Plant, Blast Furnaces \& Steel Mills |
| 4 | 41 | . 4654 | Sinter Plant, Blast Furnaces, Ferro Alloy \& Iron \& Steel |
| 5 | 73 | . 4522 | Steel Mills \& Non-Residential Construction |
| 5 | 74 | . 4522 | Steel Mills \& Non-Residential Construction |
| 5 | 81 | . 4522 | Steel Mills \& Non-Residential Construction |
| 6 | 76 | . 4505 | Steel Mills \& Non-Residential Construction |
| 7 | 83 | . 4498 | Steel Mills \& Non-Residential Construction |
| 8 | 90 | . 4429 | Steel Mills \& Non-Residential Construction |
| 9 | 94 | . 4425 | Steel Mills \& Non-Residential Construction |
| 10 | 95 | . 4406 | Steel Mills \& Non-Residential Construction |
| 11 | 33 | . 4397 | Steel Mills \& Non-Residential Construction |
| 12 | 20 | . 4376 | Sinter Plant \& Blast Furnaces \& Steel Mills \& Iron \& Steel |
| 13 | 27 | . 4346 | Steel Mills, Non-Residential Construction \& Gas \& Oil Facility |
| 14 | 14 | . 4338 | Steel Mills \& Iron \& Steel |
| 15 | 38 | . 4326 | Saw Mills, Wood Pulp, Pulp \& Paper |
| 16 | 42 | . 4264 | Construction, Residential \& Non-Residential |
| 17 | 35 | . 3909 | Wood Pulp and Pulp \& Paper |
| 18 | 67 | . 3894 | Veneer \& Plywood, Wood Pulp, Pulp \& Paper |
| 19 | 4 | . 3512 | Steel Mills |
| 19 | 11 | . 3512 | Steel Mills |
| 20 | 7 | . 3454 | Wood Pulp, Pulp \& Paper |

Source: Table 2.
N.B. : Complexes are arranged here in a descending ranking order based on the values of E.g and some abbreviated names have been given to complexes for easy recognition. Repetitive complexes are excluded. For a complete description, see Table 2.

## Table C

SELECTED INDUSTRIAL COMPLEXES BY RANK FROM THE 100 COMPLEXES EXTRACTED FROM THE INPUT-OUTPUT TABI.E, the U.S.A., 1963.

| Rank | Complex No. | E.g | Name of the Complex |
| ---: | :---: | :---: | :--- |
| 1 | 40 | .5765 | Agriculture - Food - Containers |
| 2 | 21 | .5260 | Construction |
| 3 | 11 | .5138 | Agriculture - Food |
| 4 | 12 | .4469 | Agriculture - Food - Wood Products |
| 5 | 65 | .4396 | Agri products - Food |
| 6 | 59 | .4289 | Construction - Motor Vehicles - Equipments |
| 7 | 17 | .4246 | Construction - Stone \& Clay |
| 8 | 56 | .4203 | Construction |
| 9 | 31 | .4178 | Food |
| 10 | 74 | .4090 | Construction |
| 11 | 76 | .4077 | Construction |
| 12 | 14 | .4067 | Agriculture - Food |
| 13 | 83 | .4059 | Construction |
| 14 | 93 | .4022 | Construction |
| 15 | 15 | .4020 | Agriculture - Food - Paper Containers |
| 16 | 49 | .4015 | Agriuclture - Forestry \& Fishing |
| 17 | 100 | .3996 | Construction |
| 18 | 88 | .3950 | Agriculture - Forestry \& Fishing - Paper |
| 19 | 69 | .3942 | Construction |
| 20 | .3931 | Agricultual Products - Food |  |
|  |  |  |  |

Source: Table 3.
N.B.: Complexes are arranged here in a descending ranking order based on the values of E.g and some abbreviated names have been given to complexes for easy recognition. Repetitive complexes are excluded. For a complete description, see Table 3.
(3) Tables A, B and C speak unequivocally for some dominant complexes existing in the Canadian and the United States economies. In both cases the common complexes are of the nature of construction and food types despite the asymmetry of classification of industries. The steel mills complex which is dominant in Canada (both for 1961 and 1966) is conspicuous by its absence in the U.S.A. One further notices from the detailed Tables 1 and 2 that the membership structure of the selected complexes of Tables $A$ and $B$ has not significantly changed in Canada between 1961 and 1966.
(4) There are some complexes where a particular industry appears both as a supplier and as a receiver independent of destination or origin. Examples in Table l are complexes 2, 7, 18, 51, 56 and 84; in Table 2 these are complexes 1, 7, 16, 20, 28, 38, 41, 47, 50 and 84; in Table 3 these are complexes $3,8,10,16,20,28,65,68,73,80,87$ and 96. But there is hardly any complex where one can find that suppliers and receivers have interchanged their roles i.e. an industry $M$ supplying to industry $K$ is reciprocated by industry $K$ supplying to industry $M$. This asymmetry/is very well pervasive and casts doubts on the findings of Simpson and Tsukui (1965) where exchanges within a block of industries are deemed to be in the nature of a two-way traffic.

Finally, 'island' industry complexes with given initial industries are reported in Tables $D$ and $E$ for Canada and the U.S.A. The weight factor chosen for these exercises is taken to be equal to ten to render sufficient strength to the initial industry both as a supplier and a receiver. The full display of the ensuing values of different indicators such as $E_{p} \cdot g, m_{1}$ and the sequential maximization of $E$ as they are shown in Tables 1,2 and 3 are not reported here to save space. Tables $D$ and $E$ are only illustrations without posing comparisons between complexes since the latter does not appear to be strictly relevant. Further, it is interesting to note that the structure of the island industry complexes has hardly changed in Canada for the two time periods, 1961 and 1966, except slightly for the initial industry 74.

## TABLE D

## ISLAND INDUSTRY COMPLEXES ARISING OUT OF SELECTED INITIAL INDUSTRIES, CANADA, 1961 AND 1966

| Initial Industry No. | Structur Comp 19 | of the lex 61 | Structur Comp 19 | ```e of the lex 6``` |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Süpplier | Receiver | Supplier | Receiver |
|  | 1 | 16 | 1 | 16 |
|  | 1 | 17 | 1 | 17 |
|  | 1 | 18 | 1 | 18 |
|  | 1 | 22 | 1 | 23 |
|  | 1 | 23 | 1 | 29 |
|  | 1 | 29 | 1 | 35 |
|  | 1 | 35 |  |  |
| 16 | 1 | 16 | 1 | 16 |
|  | 1 | 40 | 1 | 40 |
|  | 16 | 40 | 16 | 40 |
|  | 29 | 16 | 29 | 16 |
|  | 29 | 40 | 29 | 40 |
| 33 | 30 | 33 | 30 | 33 |
|  | 78 | 33 | 78 | 33 |
|  | 99 | 33 | 99 | 33 |
|  | 134 | 33 | 134 | 33 |
| 47 | 47 | 44 | 47 | 44 / |
|  | 47 | 45 | 47 | 48 |
|  | 47 | 51 | 47 | 50. |
|  | 47 | 53 | 47 | 51 |
| - | 47 | 59 | 47 | 53 |
|  | 47 | 60 | 47 | 58 |
|  | 47 | 61 | 47 | 59 |
|  |  |  | 47 | 60 |
|  |  |  | 47 | 61 |


| Initial Industry No. | Structure of the Complex 1961 |  | Structure of the Complex 1966 |  |
| :---: | :---: | :---: | :---: | :---: |
| 73 | Supplier | Receiver | Supplier | Receiver |
|  | 2 | 73 | 2 | 73 |
|  | 2 | 74 | 2 | 74 |
|  | 72 | 73 | 72 | 73 |
|  | 72 | 74 | 72 | 74 |
|  | - 73 | 74 | 73 | 74 |
| 74 | 72 | 74 | 72 | 74 |
|  | 72 | 75 | 72 | 75 |
|  | 73 | 74 | 72 | 78 |
|  | 73 | 75 | 73 | 74 |
|  | 74 | 75 | 73 | 75 |
|  |  |  | 73 | 78 |
|  |  |  | 74 | 75 |
|  |  |  | 74 | 78 |
| 85 | 82 | - 85 | 82 | 85 |
|  | 82 | 86 | 82 | 86 |
|  | 84 | 85 | 84 | 85 |
|  | 84 | 86 | 84 | 86 |
|  | 85 | 86 | 85 | 86 |
|  | 87 | 85 | 87 | 85 |
|  | 87 | 86 | 87 | 86 |
|  | 130 | 85 | 130 | 85 |
|  | 130 | 86 | 130 | 86 |
| 110 | 38 | 110 | 38 | 110 |
|  | 48 | 110 | 58 | 110 |
|  | 112 | 110 | 112 | 110 |
|  | 134 | 110 | 134 | 110 |
| 137 | 9 | 137 | 9 | 137 |
| 146 | 146 | 38 | 146 | 1 |
|  | 146 | 39 | 146 | 38 |
|  | 146 | 47 | 146 | 39 |
|  | 146 | 55 | 146 | 47 |
|  | 146 | 73 | 146 | 73 |
|  |  |  |  |  |


| Initial Industry No. | Structure of the Complex 1961 |  | Structure of the Complex 1961 |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Supplier | Receiver | Supplier | Receiver |
| 146 | 146 | 137 | 146 | 137 |
| (contd) | 146 | 139 | 146 | 139 |
|  | 146 | 140 | 146 | 140 |
|  | 146 | 141 | 146 | 141 |
|  | 146 | 143 | 146 | 143 |
|  | 146 | 144 | 146 | 144 |
|  | 14.6 | 147 | 146 | 147 |
|  | 146 | 0 | 146 | 0 |

## TABLE E

 INDUSTRIES, the USA, 1963Initial Industry No.



REFERENCES .

1. ..H.B. Chenery and T. Watanabe: "International Comparisons of the Structure of Production", Econometrica, vol. 26 (1958), pp 487-521.
2. W.W. Leontief: "The Structure of Development", Scientific American, vol. 209, no. 3 (1963), pp 148-167.
3. P.N. Rosenstein-Rodan: "Problems of Industrialization of Eastern and South-Eastern Europe", Economic Journal, vol. 53, June-Sept, 1943.
4. P.N. Rosenstein-Rodan: "Notes on the theory of the Big Push", Economic Development for Latin America edited by H.S. Ellis, International Economic Association Proceedings, London, MacMillan \& Company, 1957.
5. R. Nurkse: "Problems of Capital Formation in Underdeveloped Countries", oxford, 1953.
6. T. Scitovsky: "Two Concepts of External Economies", Journal of Political Economy, vol. 62, April 1954.
7. W.A. Lewis: "Theory of Economic Growth"r George Allen \& Unwin, London, 1956.
8. A.O. Hirschman: "The Strategy of Economic Development", Yale University Press, reprinted, 1964.
9. A. Iösch: "The Economies of Location", John Wiley and Sons Inc, New-York, 1952.
10. W. Isard: "The Location of Space Economy", Cambridge, Mass, the MIT Press, 1956.
11. W. Isard, E.N. Schooler and T. Vietorisz: "Industrial Complex Analysis and Regional Development", John Willey \& Sons, New-York, 1959.
12. M.E. Streit: "Spatial Associations and Economic Linkages Between Industries", Journal of Regional Science, vol. 9, no. 2, August 1969.
13. C.E. Richter: "The Impact of Industrial Linkages on Geographic Association", Journal of Regional Science, vol. 9, no. 1, 1969.
14. E.M. Hoover: The Location of Economic Activity", McGrawHill, New-York, 1948, 19
15. E.L. Ullman: "Regional Development and the Geography of Concentration", in J. Friedman and W. Alonso, eds., Regional Development and Planning, Cambridge, Mass, the M.I.T. Press, 1964.
16. E. Helmstader: "The Triangular Form of the Input-Output Matrix and Its Possible Changes During the Process of Growth" (in German), in F. Newmark, ed., Strukturor and lungen einer Wachsenden Wirtschaft, Berlin, 1964.
17. B. Korte and W. Oberhoffer: "Triangularizing InputOutput Matrices and the Structure of Production", European Eco. Review, vol. 11, no. 4, Summer, 1971.
18. J. Lamel, J. Richter and W. Teufelsbauer: "Patterns of Industrial Structure and Economic Development Triangularization of Input-Output Tables of ECE Countries", European Economic Review, vol. 3, 1972.
19. D. Simpson and J. Tsukui: "The Fundamental Structure of Input-Output Tables, An International Comparison", the Review of Economics and Statistics, vol. 47, 1965.
20. I.J. Good: "Some Applications of the Singular Decomposition of a Matrix", Techonometrics, vol. 11, no. 4, November 1969.
21. S. Czamanski: "Regional Science Techniques in Practice", D.C. Heath and Company, Toronto, 1972, ch. 5.
22. National Economics Division: "Input-Output Structure of the United States Economy: 1.963", Survey of Current Business, vol. 49, pp 30-35, November 1969; also reprinted in J.W. Kendrick: Economic Accounts and Their Uses, McGraw-Hill Book Company, 1972, pp 62-73.

## TABLE 1

## Characterization of Industrial complexes from I-0 Table, Canada 1961.

$$
(k=20)
$$

Inter-industrial Linkages ( $b_{i f}$ 's)


* Starting element
** This colum is intended to describe the relevance of the present complex to the complexes obtalned previously i.e. whether the present complex is independent of the earlier ones or whether it resembles previously i.e. Whether the present complex is independent of the earlier on
d Starting element characterized by an asterik is not shown here since the element has to be retained anyway by the method we have followed.
N.B. For the notation See Appendix 2.


$\qquad$



Table 1 (cont ${ }^{\circ}$ d)






$.-12-$


- 13 -














Table 2 (cont'd)

Inter-industrial Iinkages (bij's)




Table 1 (cont'd)

Inter-industrial Linkages ( $\mathrm{b}_{\mathrm{ij}}$ 's)

- $29^{\circ}$ -


- 31 -






## TABLE 2

Characterization of Industrial Complexes Erom I-0 Table, canada 1966.
( $k=10$ )


* Starting element
** This column is intended to describe the relevance of the present complex to the complexes obtained ipreviously i.e. Whether the present complex is independent of the earlier ones or whether it resembles 'them by characteristics like 'overlapping'. 'nested' or 'repetitive' types.
- Starting element characterized by an asterik is not shown here since the element has to be retained anyway by the method we have followed.
K.B. Por the notation See Appendix 2.


















- 21 -.






[^0]

$$
\text { : Table } 2 \text { (cont'd) }
$$







TABLE 3

Characterizetion of Industrial Complexes from I-0 Table,; U.S.A., 1963.
$(k=10)$


* Starting element
** This colum is intended to describe the relevance of the present complex to the complexes obtained previously i.e. whether the present complex is independent of the earlier ones or whether it resembles them by characteristics like 'overlapplng', 'nested' or 'repetitive' types.
- Starting element characterized by an asterik is not shown here since the element has to be retained anyway by the method we have followed.
N.B. EOI the notation See Appendix 2.


Table 3 (cont'd)







Table 3 (cont'd)



- 11 -





Table 3 (cont'd)




- 18 -




$$
-21-
$$



- 22 -

- 23 -


Table 3 (cont'd)








- 31 -




## SEARCH ALGORITHM TO MAXIMIZE 'E' AND RELATED STATISTICAL INDICATORS

Search Algorithm ${ }^{1}$
Het"S"and $R$ "be the sets of all $N$ industries ( $N \leqslant P$ ) as suppliers and receivers respectively and let $S^{*}$ and $R^{*}$ be the sets of respective suppliers and receivers constituting any artificial initial complex, $E_{p, \prime}$ not maximizing E.

$$
\frac{\sum_{i \varepsilon S^{*}} \sum_{j \varepsilon R^{*}} b_{i j}}{n_{S^{*}} n_{R^{*}}+k} \cdots(A .1)
$$

where $n_{S *}, n_{R *}$ are the number of industries in sets $S^{*}$ and R*, and $k$ is a positive integer.

Obviously in conformity with the eqn (4) in the text,

$$
n=n_{S} * n_{R *} \quad \cdots \cdots(A, 2)
$$

Note that the numerator in (A.l) can start with only one element, say $b_{m l}$ (which is presumably the highest in value of all $b_{i j}$ 's) where $m$ is the only one supplying industry and 1 is the only receiving industry with the result that $n=1$. Now industries are added to (or dropped from) sets $S^{*}$ and $R^{*}$ of (A.l) in single steps as long as the objective function, E, can be increased. The move at each step is determined by
(1) A computer program with respect to the alogorithm is written in FORTRAN IV and is available on request. •
the following:
(1) Evaluate the $N$ quantities from the suppliers' side, $E_{S}^{\prime}$, and the $N$ quantities from the receivers' side, $E_{r}^{\prime}$ which are given by

$$
\begin{align*}
& E_{S}^{\prime}=\frac{E_{p} m \pm \frac{\sum_{j \varepsilon R^{*}} b_{S j}}{m \pm n_{R^{*}}}}{E_{r}^{\prime}=\frac{E_{p m}^{m \pm i \varepsilon S^{*}} b_{i r}}{m \pm n_{S *}}} \tag{A.3}
\end{align*}
$$

$m=n_{S *} n_{R^{*}}+k$
and

$$
\begin{aligned}
\pm & =-i f s \varepsilon S^{*} & & \left(\text { or } r \varepsilon R^{*}\right) \\
& =+i f s \notin S^{*} & & \text { (or } \left.r \notin R^{*}\right)
\end{aligned}
$$

(2) Let $E^{\prime}$ be the maximum among all the $E_{S}^{\prime}$ and $E_{r}^{\prime}$ ( $N$ of each).

If $E^{\prime} \leqslant E_{p}$ the process is stopped: the complex is considered formed. If $E^{\prime}>E_{p}$ the indicated move is taken yielding $E^{\prime}$ as the value of the objective function for the modified complex i.e. adding or dropping a supplier or receiver to the initial complex. The new $E_{S}^{\prime \prime}$ and $E_{x}^{\prime}$ values are obtained and the process is repeated until the function, $E$, cannot be further increased.

## Appendix 2

## Statistical Indicators

## Free Complexes:

The following indicators and symbols are used to highlight the quantitative findings in Tables 1,2 and 3.

$$
\begin{aligned}
\mathrm{E}_{\mathrm{x}}= & \text { optimal average linkage value of complex } \mathrm{x} . \\
\mathrm{x}= & 1,2, \cdot \cdot, \mathrm{z} . \\
= & \text { serial number of complexes as they arise in the } \\
& \text { process of extraction by the search procedure. }
\end{aligned}
$$

$g_{x}=\left[\sum_{\text {complex }} \sum_{i j} b_{i j} / \sum_{i=1}^{N} \sum_{j=1}^{N} b_{i j}\right] \times 100$ = $\begin{aligned} & \text { percentage reduc- } \\ & \text { tion by complex } .\end{aligned}$
$h_{x}=\sum_{\text {complex }} \sum_{i} b_{i j}=$ sum of total linkages or coefficients.
of complex $x$.
$C V_{x}=\sqrt{\sum_{\text {complex }} \sum_{i j}\left(b_{i j}\right)^{2 /\left(h_{x}\right)^{2}-1}}$
$=$ coefficient of variation of complex $x$.
$m_{1 x}=\left[\operatorname{complex~x~}^{A_{i j}} / \sum_{i=1, j}^{N} \sum_{j=1}^{N} A_{i j}\right] \quad x 100$
$=$ percentage reduction in dollar terms by complex $x$ in relation to the total dollar transactions of the $\mathrm{N} \times \mathrm{N}$ matrix.
$m_{2 x}=\left[\sum_{x=1}^{x} A_{x} / \sum_{i=1}^{N}, \sum_{j=1}^{N} A_{i j}\right] \quad x 100$
$=$ cumulative percentage reduction in dollar terms for all complexes preceding and upto xth complex with respect to the total dollar transactions of $N \times N$ matrix (here $A_{i j}$ 's in each $A_{x}$ are counted only once i.e. mul-

$$
\begin{aligned}
& \text { tiple counting of the same } A_{i j} \text { in various complex } \\
& \text { is avoided to adjust } \left.\sum_{x=1}^{z} A_{x}=1\right) . \\
m_{3 x}= & \sum_{\text {complex }} \sum_{i j} A_{i j} / \sum_{j=1}^{N} V_{j} \\
= & \text { percentage reduction by complex } x \text { in respect of } \\
& \text { total gross outputs of } N \text { industries. }
\end{aligned}
$$

```
Input-Output
Industry No. . Input-Output Industry Title
Input-Output Industry Title
```

Agriculture Forestry
Fishing, Hunting \& Trapping
Base Metal \& other Metal Mines
Uranium Mines
Iron Mines
Gold Mines
Coal Mines
Petroleum \& Gas Wells
Asbestos Mines
Gypsum Mines
Salt Mines
Other Non-Metal Mines
Quarries \& Sand Pits
Services Incidental to Mining
Slaughtering \& Meat Processors
Poultry Processors
Dairy Factories
Process Cheese Mfgrs.
Fish Products Industry
Fruit \& Vegetable Canners
Feed Mfgrs.
Flour Mills
Breakfast Cereal Mfgrs. /
Biscuit Mfgrs.
Bakeries
Confectionery Mfgrs.
Sugar Refineries

Vegetable Oil Mills
Miscellaneous Food Industries
Soft Drink Mfgrs.
Distilleries
Breweries
Wineries
Leaf Tobacco Processing
Tobacco Products Mfgrs.
Rubber Footwear Mfgrs.
Tire \& Tube Mfgrs.
Other Rubber Industries
Leather Tanneries
Shoe Factories

- Leather Glove Factories

Small Leather Goods Mfgrs.
Cotton Yarn \& Cloth Mills
Wool Yarn Mills
Wool Cloth Mills
Synthetic Textile Mills
Fibre Preparing Mills
Thread Mills
Cordage \& Twine Industry
Narrow Fabric Mills
Pressed \& Punched Felt Mills
Carpet, Mat \& Rug Industry
Textile Dyeing \& Finishing
Linoleum \& Coated Fabrics Inc.
Canvas Products Industry
Cotton \& Jute Bag Industry
Miscellaneous Textile Inc.
Hosiery Mills
Other Knitting Mills
Clothing Industries

Sawmills
Veneer \& Plywood Mills
Sash \& Door \& Planing Mills
Wooden Box Factories
Coffin \& Casket Industry
Miscellaneous Wood Industries
Household Furniture Industry
Office Furniture Industry
Other Furniture Industries
Electric Lamp \& Shade Industry
Pulp \& Paper Dummy Ind.
Wood Pulp
Paper Producing
Paper Converting
Pulp \& Paper Other Activities
Asphalt Roofing Mfgrs.
Paper Box \& Bag Mfgrs.
Other Paper Converters
Printing \& Publishing
Engraving, Stereotyping Ind.
Iron \& Steel Dummy Inc.
Coke Ovens
Sinter Plant \& Blast Furnaces
Steel Mills
Rolling Mills
Ferro Alloy Producers
Iron \& Steel Other Activities Steel Pipe \& Tube Mills Iron Foundries
Aluminum Smelting and Refining Other Smelting and Refining Aluminum Rolling \& Extruding Copper \& Alloy Rolling

Metal Casting \& Extruding NES Boiler \& Plate Works Fabricated Struct. Metal Ind. Ornamental \& Arch. Metal Ind. Metal Stamp. Press. \& Coat. Ind. Wire \& Wire Products Mfgrs. Hardware Tool \& Cutlery Mfgrs. Heating Equipment Mfgrs. Machine Shops. Misc. Metal Fabricating Ind. Agricultural Implement Ind. Misc. Machinery \& Equip. Mfgrs Comm. Refrig \& Air Cond. Mfgrs Office \& Store Machinery Mfgrs Aircraft \& Parts Mfgrs. Motor Vehicle Mfgrs. Truck Body \& Trailer Mfgrs. Motor Veh. Pts \& Access. Mfgrs Railroad Rolling stock Inc. Shipbuilding \& Repair Misc. Transp. Equip. Ind. Small Electrical Appliances Major Appliances Elect. \& Non: Radio \& Television Receivers Communications Equipment Mfgrs. Mfgrs of Elect. Inc. Equip. Battery Mfgrs. Mfgrs of Electric Wire \& Cable Mfgrs of Misc. Elect. Products Cement Mfgrs
Lime Mfgrs.
Gypsum Products Mfgrs.
Concrete Products Mfgrs.

Ready-Mix Concrete Mfgrs.
Clay Products Mfgrs. Refractories Mfgrs. Stone Products Mfgrs Mineral Wool Mfgrs. Asbestos Products Mfgrs. Glass \& Glass Products Mfgrs. Abrasives Mfgrs. Other Non-Metallic Prods. Ind. Petroleum Refineries Other Petrol \& Coal Prod. Ind. Explosives \& Ammunition Mfgrs. Mfgrs. of Mixed Fertilizers Mfgrs. of Plast. \& Synth. Res. Mfgrs. of Pharm. \& Medicines Paint \& Varnish Mfgrs. Mfgrs. of Soap \& Cleaning Comp Mfgrs. of Toilet Preparations Mfgrs. of Industrial Chemicals Other Chemical Industries Scient. \& Prof. Equip. Mfgrs. Jewelry \& Silverware Mfgrs. Broom Brush \& Mop Industry Venetian Blind Mfgrs. Plastic Fabricators, Nes. Sporting Goods \& Toy Industry Fur Dressing \& Dying Industry Signs \& Displays Industry Misc. Manufacturing Ind. Nes' Repair Construction Residential Construction Non-Residential Construction Road Highway Airstrip Const.

161
162
163
164
165

Gas and Oil Facility Const. Dams and Irrigation Projects Railway Telephone Telegraph Con. Other Engineering Construction Construction other Activities.

CLASSIFICATION OF 64 INPUT-OUTPUT INDUSTRIES OF THE U.S.A.

## Input-Output

 Industry No.1
2
3
4

5
6
7
8
9
10
11
12
13
14
15
16

Input-Output Industry Title

Livestock \& Livestock Products Other Agricultural Products Forestry \& Fishery Products Agricultural, Forestry \& Fishery Services
Iron \& Ferroalloy Ores Mining Nonferrous Metal Ores Mining Coal Mining
Crude Petroleum \& Natural Gas Stone and Clay Mining and Quarrying Chemical \& Fertilizer Mineral Kining New Construction
Maintenance \& Repair Construction
Ordnance \& Accessories
Food \& Kindred Products
Tobacco Manufactures
Broad \& Narrow Fabrics, Yarn \& Thread Mills

Miscellaneous Textile Goods \& Floor Coverings

## Apparel

Miscellaneous Fabricated Textile Products
Lumber \& Wood Products; Except Containers
Wooden Containers
Household Furniture
Other Furniture \& Fixtures
Paper \& Allied Products, Except Containers
Paperboard Containers \& Boxes
Printing \& Publishing
Chemicals \& Selected Chemical: Products
Plastics \& Synthetic Materials
Drugs, Cleaning \& Toilet Preparations
Paints \& Allied Products
Petroleum Refining \& Related Industries
Rubber \& Miscellaneous Plastics Products
Leather Tanning \& Industrial Leather Products
Footwear \& Other Leather Products
Glass \& Glass Products
Stone \& Clay Products
Primary Iron \& Steel Manufacturing
Primary Nonferrous Metal Manufacturing
Metal Containers
Heating, Plumbing \& Structural Metal Products
Stampings, Screw Machine Products \& Bolts
Other Fabricated Metal products

Engines \& Turbines
Farm Machinery \& Equipment
Construction, Mining \& Oil Field Machinery
Materials Handiing Machinery \& Equipment
Metalworking Machinery \& Equipment Special Industry Machinery \& Equipment
General Industrial Machinery \& Equipment
Machine Shop Products
Office, Computing \& Accounting
Machines
Service Industry Machines
Electric Industrial Equipment \& Apparatus
Household Appliances
Electric Lighting \& Wiring Equipment
Radio, Television \& Communication Equipment
Electronic Components \& Accessories
Miscellaneous Electrical Machinery, Equipment \& Supplies
Motor Vehicles \& Equipment
Aircraft \& Parts
Other Transportation Equipment
Scientific \& Controlling Instruments
Optical, Ophthalmic \& Photographic Equipment
Miscellaneous Manufacturing


[^0]:    - 26 -

