WORKING PAPER

SOME TESTS ON THE INTERPROVINCIAL CONVERGENCE OR DIVERGENCE BY SELECTED ECONOMIC INDICATORS IN CANADA

ECONOMIC ANALYSIS BRANCH
PLANNING DIVISION

REGIONAL ECONOMIC EXPANSION CANADA
EXPANSION ÉCONOMIQUE RÉGIONALE CANADA

HC
115
L6

HC
115
L6

DEPT. OF REGIONAL ECONOMIC EXPANSION
LIBRARY
APR 12 1974
OTTAWA
BIBLIOTHEQUE
MIN. DE L'EXPANSION ECONOMIQUE REGIONALE

SOME TESTS ON THE INTERPROVINCIAL CONVERGENCE OR DIVERGENCE BY SELECTED ECONOMIC INDICATORS IN CANADA

by

B. K. Lodh

Economic Analysis Branch
Department of Regional
Economic Expansion

June 2, 1972

INTRODUCTION

In recent times few subjects have generated so much concern as regional disparities in Canada. But rigorous statistical tests on the convergence or divergence of interregional welfare by selected indicators have been performed by only a limited few. Chernick (1), McInnis (3) and Firestone (2) are the solitary examples.* The method chosen to identify convergence or divergence by all of them rests on the use of 'coefficient variation'[1] which is undoubtedly ligitimate in this context. The studies show that in general for the entire period 1950-1970 the speed of convergence for some indicators, namely, per capita personal income and per capita earned income, appears to be low, although this speed[2] has not been very well estimated. A particular drawback of this method is that one does not know whether in fact

---

1. Coefficient of variation measured in each time unit over spatial units represents the amount of spatial divergence in any given indicator and it is a number which is independent of any unit chosen. The latter renders easy comparability with other indicators. The range of coefficient of variation lies between zero and one where zero value indicates complete uniformity and a value of one complete dispersion. A declining value of interregional coefficient of variation for any indicator over time shows that interregional dispersion is falling.

2. Estimation of the speed, we presume, is very important since as we shall see later a particular region or province may catch up with another one or with the national average over a period of 3,000 years depending on their growth rates and positions on the intercepts. But definitely 3,000 years are too long for convergence.

* All references to this paper are given at the end of the text. The bracketed numbers following the authors indicate the numbers in the references.

the poorer regions are pulling themselves upward or the richer ones pulling downward in order to account for a slowly declining coefficient of variation. The present paper purports to present for the first time some numerical estimates of the speed of convergence or divergence in selected inter-provincial indicators of welfare, namely per capita personal income and per capita earned income, and would also provide some quantitative measures of provincial pull factors. A corollary of this exercise is the pattern of provincial welfare indicators over time (from 1970 onwards) and correspondingly estimation of the coefficient of variation for the future years in Canada.

A second thrust of this paper is addressed to the validity of the procedure of computing a time series of region (or province) by nation ratios of any given indicator. We emphasize that this procedure is deficient in two important aspects: (a) the region or province is already included in the nation and hence the ratio is biased except when the regional or provincial weight in the nation is very small; (b) the procedure assumes a certain growth pattern of all other regions or provinces which comprise the nation although this growth pattern is never posed. To overcome these drawbacks we offer a simple formulation of national average of any indicator as a weighted sum of the regional (provincial) ones.

Last but not the least, we test a further hypothesis as to whether provincial migration does contribute to the reduction of provincial disparities in per capita incomes under some simplified assumptions.

...3

1. METHODOLOGY AND ANALYSIS

The methods and analyses adopted in this paper are presented in the following format:

(a) Time-Trend Equations of Particular Indicators by Provinces in Canada, 1961-70.

(b) Provincial-National Decomposition Formula with Population Weights.

(c) Net Provincial Migration, Potential Population and Formulation of Hypothetical Time-Trend Equations of Particular Indicators by Provinces in Canada, 1961-70.

(d) Convergence or Divergence Exercises: Projections based on (a), (b) and (c) above.

It is to be noted that the main basis of all these exercises is to be found in (a)with which we deal now. As we proceed, we shall explain both methodological and statistical implications simultaneously.

1.1 Time-Trend Equations of Particular Indicators by Provinces in Canada, 1961-70

In the first instance for two particular indicators in each province we experimented with various forms of regression equations with time as an independent variable. These forms are linear, double-log, second-order polynominal and exponential. The two indicators chosen are: per capita personal income and per capita earned income for 11[2] provinces and Canada over the period 1961-70[1]. The regression equation which performs best by R^2, t-statistics etc., is of the exponential type and is given by the following form:

---

1. Data are taken from the "Major Economic Indicators - Canada and Provinces", Economic Analysis Branch, DREE, 1972.

2. The 11 provinces referred to in this study include the Yukon and Northwest Territories, although the latter is not considered a separate province in Canada. For our analysis this distinction is, however, trivial.

...4

$$\hat{Y}_{it} = a_i \, b_i^{\,t} \tag{1}$$

where, $$b_i = 1 + r_i \tag{2}$$

$\hat{Y}_{it}$ = calculated value of the indicator in province i in time, t;

r_i = annual compound rate of growth of indicator in province, i;

and a_i, b_i are the constants arrived at by regression fitting.

The second-order polynominal[1] equation also performs well but for the sake of easier manipulation we have retained the exponential form. The regression coefficients of the latter are given in Table 1 with their necessary statistics for the two indicators. Most of the coefficients are statistically significant at 1% level of significance.

TABLE 1

REGRESSION COEFFICIENTS OF EXPONENTIAL EQUATIONS FOR PER CAPITA PERSONAL INCOME AND PER CAPITA EARNED INCOME BY PROVINCES AND CANADA, 1961-70

Provinces and Canada	Per Capita Personal Income						Per Capita Earned Income					
	Intercept (a_i)	Rank	Growth Rate (b_i)	Rank	R^2	t-Statistic for R	Intercept (a_i)	Rank	Growth Rate (b_i)	Rank	R^2	t-Statistic of R
NFLD.	$ 816	12	1.0784	7.5	0.9656	15.0	$ 656	12	1.0752	9	0.9732	17.1
P.E.I.	$ 850	11	1.0865	2	0.9711	16.4	$ 656	11	1.0845	2	0.9767	18.3
N.S.	$1,097	9	1.0826	3	0.9569	13.3	$ 893	9	1.0809	3	0.9545	12.9
N.B.	$ 961	10	1.0882	1	0.9681	15.6	$ 774	10	1.0869	1	0.9742	17.4
QUE.	$1,295	6	1.0799	4	0.9751	17.7	$1,079	7	1.0786	4	0.9814	20.5
ONT.	$1,715	1	1.0754	11	0.9744	17.4	$1,457	1	1.0727	11	0.9782	18.9
MAN.	$1,405	5	1.0789	6	0.9674	15.4	$1,168	5	1.0767	7	0.9711	16.4
SASK.	$1,274	7	1,0764	10	0.8971	8.3	$1,050	8	1,0730	10	0.8146	5.9
ALTA.	$1,426	4	1.0794	5	0.9622	14.3	$1,199	4	1.0774	6	0.9621	14.2
B.C.	$1,679	2	1,0694	12	0.9831	21.6	$1,374	2	1.0689	12	0.9856	23.4
Y&NWT	$1,217	8	1.0784	7.5	0.8713	7.4	$1,155	6	1.0775	5	0.8805	7.7
CANADA	$1,455	3	1.0779	9	0.9768	18.4	$1,218	3	1.0759	8	0.9816	20.7

Source: Major Economic Indicators, Canada and Provinces, Economic Analysis Branch, DREE, 1972.

NB: All R's are statistically significant at 1% level of significance.

---

1. The fitted coefficients in this form are all found to be positive and strengthen the hypothesis of some exponential character underlying the time series.

The values a_i's in Table 1 are to be interpreted as equal to values, $\hat{y}_{io}$ in time, $t = o$, and, therefore, they refer to the computed initial values of the indicator in 1960 since the original regression has been worked out on the assumption of $t = 1$ in 1961. The same table shows also the ranking by the Provinces and Canada by a_i's and b_i's. The important features characterizing Table 1 may now be summarized as follows:

1. The growth rates, b_i's vary inversely with the intercepts a_i's over the regions. This is true for both per capita personal income and per capita earned income. A Spearman's coefficient of rank correlation between a_i's and b_i's works out to be -0.64 for personal income and -0.59 for earned income which show a marked inverse relationship not elsewhere reported. Two notable exceptions violating this rule are Newfoundland and Saskatchewan whose prospects, as we shall see later on, are very bleak by poor performance on both b_i's and a_i's.

2. The so-called poor provinces i.e. Atlantic Provinces do all have the lowest a_i's but with the sole exception of Newfoundland they all have also the highest b_i's i.e., growth rates[1].

---

1. In Table 1 and in subsequent tables we have used b_i's as equivalents of growth rates although strictly speaking the latter refers to r_i's, c.f. equation (2).

...6

This gives some hope[1] for the poorer provinces to catch up with the richer ones by virtue of higher growth rates, but in a subsequent analysis concerning the speed of convergence we shall see that things are not as bright as they may appear in the first sight.

1.2 Provincial-National Decomposition Formula

Given the equations of the form given by equation (1) and regression coefficients by provinces in Table 1, we now derive and test the consistency[2] of the national coefficients of the same form which are shown in the last row of Table 1. It is to be reiterated that this test is very vital for any national-provincial or national-regional comparisons of any selected indicator although unfortunately this is hardly pursued in the available literature.

First, we write out some definitional equations of the following types:

$$Y_{it} = y_{it} P_{it} \tag{3}$$

$$\sum_{i=1}^{11} Y_{it} = Y_t \tag{4}$$

$$\sum_{i=1}^{11} P_{it} = P_t \tag{5}$$

$$\sum_{i=1}^{11} W_{it} = 1 \tag{6}$$

---

1. Of course, this finding is confined only to the two particular indicators we have chosen in our analysis. It is quite possible that for other indicators the situation may be different. See Firestone (2). However, as we have stated before, Firestone's study says very little about the speed of convergence or about the particular provincial pulls.

2. One may skip this sub-section if one likes to avoid technical details of consistency tests without losing perhaps substantive issues in the latter part of the paper.

where, $W_{it} = \frac{P_{it}}{P_t}$ (7)

(The variables are explained in the Appendix)

Assuming that the variable y_i stands for per capita personal income in province i, the equations (3) through (7) become self-explanatory. Nevertheless we shall repeat the interpretations. Equations (3) and (4) represent provincial total personal income and national total personal income respectively for any specified time, t. Equation (5) shows the total population of the nation as a sum of the provincial populations whereas the expression (6) represents the same story with provincial population weights, W_{it}. Combining (3) through (7) and (1), one then obtains the national per capita form as given by:

$$\hat{y}_t = \sum_{i=1}^{11} a_i \, b_i^{\,t} \, W_{it} \qquad (8)$$

The problem now is to discover the correspondence of form (8) with a form which is similar to equation (1) for the national counterpart.

Thus writing

$$\hat{y}_t = ab^{\,t} \qquad (9)$$

with t = o, one obtains from (8) and (9)

$$a = \sum_{i=1}^{11} a_i \, W_{io} \qquad (10)$$

and with t = 1, one further obtains

$$b = \sum_{i=1}^{11} \frac{a_i}{a} \, b_i \, W_{i1} \qquad (11)$$

The consistency tests consists in checking computed values of a and b for Canada from Table 1 with those represented by the right-hand-side expressions of (10) and (11). We pursued these checks with population weights, W_{io} (for 1960) and W_{i1} (for 1961) and found that errors are very small. An important feature of this exercise is the impossibility of isolating any one province from the nation since weights become the clear constraints on any decomposition. That is to say, it becomes a superficial exercise to divide a region or province by the nation for any indicator, if weights of all provinces are not carefully assumed or determined. We, therefore, conclude that the values of a_i and b_i for the provinces and Canada in Table 1 are all consistent and in the final analysis we shall use these values for convergence tests.

1.3 Population, Net Provincial Migration and Potential Population: Time-Trend Equations of Population, Actual and Potential, by Provinces and Canada, 1961-70

The problem of population weights and growth factors by provinces in the evaluation of indicators on a per capita basis has already been noted in Section 1.2. We shall introduce now one particular variable, namely, provincial net migration, which apparently may affect per capita estimates of any indicator for the provinces. However, first of all we test the type of regression equation that fits best for the actual population of the provinces and Canada over the period 1961-70 without adjusting for net provincial migration each year. Fortunately again, we find the exponential type performing the best which is given by the following expression.

$$\hat{P}_{it}=\hat{P}_i \, g_i^{\,t} \tag{12}$$

The coefficients, $\hat{P}_i$ and g_i and other statistics are shown on the left side of Table 2 which is self-explanatory. Obviously, $\hat{P}_{it}$ denotes here the computed population of province i in time, t, and the computed national population, $\hat{P}_t$ is given by -

$$\hat{P}_t = \sum_{i=1}^{11} \hat{P}_{it} \tag{13}$$

Like the formulations in the Section 1.3 we write -

$$\hat{P}_t = \hat{P}g^t \tag{14}$$

whereby we obtain

$$\text{for } t = o, \quad \hat{P} = \sum_{i=1}^{11} \hat{P}_i \tag{15}$$

$$\text{and for } t = 1, \quad g = \frac{1}{P} \sum_{i=1}^{11} \hat{P}_i \, g_i \tag{16}$$

$$= \sum_{i=1}^{11} \hat{W}_i \, g_i \tag{16a}$$

$$\text{where, } \hat{W}_i = \frac{\hat{P}_i}{\hat{P}} \tag{17}$$

Consistency checks on (15) and (16) revealed only small errors which, however, can be ignored for our purposes here.

We may now bring in a new construct that we shall label as 'potential population' by provinces. Potential population in a province is defined in our context as that population of the province if there are no net migration from that province. The formulation is simply stated thus:

$$P^*_{it} = P_{it} - M_{it} \tag{18}$$

$$M_t = \sum_{i=1}^{11} M_{it} \tag{19}$$

where,

P^*_{it} = potential population of province i, in year, t;

P_{it} = actual population of province i, in year, t;

M_{it} = net migration from province i, in year, t;

M_t = net migration in Canada in year, t.

If M_{it} is negative in any province, potential population is arrived at by adding that M_{it} to the actual population, P_{it}. The converse is also easy to see. Regression coefficients $\hat{P}^*_i$ and g^*_i are then similarly computed on the basis of the exponential form (20) and are reported on the right-hand-side of Table 2.

$$\hat{P}^*_{it} = \hat{P}^*_i \, g^{*\,t}_i \tag{20}$$

TABLE 2

REGRESSION COEFFICIENTS OF EXPONENTIAL EQUATIONS FOR ACTUAL AND POTENTIAL POPULATION BY PROVINCES AND CANADA OVER THE PERIOD 1961-70

(Potential Population Assumes Zero Net Provincial Migration)

Provinces and Canada	Coefficients for Actual Population: Intercept ($\hat{P}_i$) in 000's	Rank	Growth Rate (g_i)	Rank	R^2	t-statistic for R	Coefficients for Potential Population: Intercept ($\hat{P}^*_i$) in 000's	Rank	Growth Rate (g^*_i)	Rank	R^2	t-statistic for R
NFLD.	455	9	1.0134	7	0.9933	34.4	461	9	1.0137	7	0.9761	18.1
P.E.I.	106	10	1.0043	10	0.7814	5.3	107	10	1.0047	10	0.5392	3.1
N.S.	740	7	1.0035	11.5	0.8849	7.8	750	7	1.0034	12	0.6728	4.1
N.B.	599	8	1.0047	9	0.9464	11.9	608	8	1.0044	11	0.7523	4.9
QUE.	5,231	2	1.0154	6	0.9788	19.2	5,214	2	1.0167	6	0.9832	21.6
ONT.	6,066	1	1.0233	3	0.9970	60.0	6,054	1	1.0225	3	0.9944	37.8
MAN.	927	5.5	1.0061	8	0.8878	7.9	933	6	1.0067	8	0.7695	5.2
SASK.	927	5.5	1.0035	11.5	0.5977	3.4	934	5	1.0054	9	0.8163	6.0
ALTA.	1,316	4	1.0191	4	0.9878	25.4	1,320	4	1.0185	4	0.9723	16.7
B.C.	1,555	3	1.0319	1	0.9879	25.6	1,541	3	1.0307	1	0.9722	16.7
Y&NWT	37	11	1.0271	2	0.9431	11.5	38	11	1.0227	2	0.6983	4.3
CANADA	17,948	N.A.	1.0180	5	0.9995	123.4	17,948	N.A.	1.0180	5	0.9995	123.4

Source: (1) Population (Actual) from Statistics Canada, Cat. No. 91-201.

(2) Special computation of net migration by Provinces, Economic Analysis Branch, DREE, 1972 from the Vital Statistics, Statistics Canada.

N.A. denotes 'not applicable'.

The implications of Table 2 can be briefly summarized as below:

(a) Ranks by intercepts, $\hat{P}_i$ and $\hat{P}_i^*$, show virtually no changes whereas ranks by growth rates, g_i and g_i^*, show some slight changes.

(b) Absolute changes in the intercepts and growth rates are obviously due to the impact of provincial net migration. Notice particularly the impacts due to net migration on the differential between two growth rates, g_i and g_i^*, in Saskatchewan, Yukon and the Northwest Territories.

(c) Forecasting worsens by values of R^2 when one goes by computation of coefficients for potential population although t-statistics for R are still significant at 1% level of significance. A notable reversal of this phenomenon is to be found in Saskatchewan which suggests that for this province one can do the job of projections better for potential population than for actual population which includes net migration.

1.4 Convergence Tests

The methods used to test interprovincial convergence by personal income and earned income (both on a per capita basis) may now be clearly stated. First, we pose the general formulation as below.

Combining $\hat{y}_{it}$ and $\hat{P}_{it}$ from equations (1) and (12), one obtains:

$$\hat{Y}_{it} = \hat{P}_i g_i^t \; a_i \; b_i^t$$

$$\text{or} \quad \hat{Y}_{it} = \hat{P}_i \; a_i \; (b_i \; g_i)^t \qquad (21)$$

If $\hat{Y}_{it}$ is given by the following expression

$$\hat{Y}_{it} = \hat{Y}_i \, k_i^{\,t} \qquad (22)$$

then from (21) and (22) with $t = o$, one obtains

$$\hat{Y}_i = \hat{P}_i \, a_i \qquad (23)$$

and for any $t > o$, $k_i = b_i g_i$ (24)

$$\text{or} \quad b_i = \frac{k_i}{g_i} \qquad (25)$$

The simplicity of the expression (25) is that for any indicator, say personal income, provincial per capita growth rate (b_i) is expressed as a ratio of the growth rate of total personal income (k_i) to that of the total population in the province (g_i). The implication of this is that any desired change in b_i can be accomplished by a change in either k_i or g_i or both. Since convergence in a planning sense implies that b_i's have to be controlled one needs to control either population growth rate (g_i) or total personal income (k_i) or both. Generally, control of g_i's implies inter-regional migration policy whereas that of k_i's implies interregional expenditure policy, be it for transfer payments or any other expenditure flows. However, it is interesting to test how far provincial net migration flows may have contributed to the changes in b_i's that result from net migration, with and without, in each province since available data permit us to do so. A corollary of this exercise consists in the measurement of time taken, i.e. number of years, by each province to catch up with any other province or Canada in absolute or relative terms. The latter will be explained shortly.

1.4.1 Potential Per Capita Incomes by Provinces and Differences of the Estimated Coefficients Due to Net Provincial Migration

Given Y_{it} from (3) and P^*_{it} from (18) we first construct:

$$y^*_{it} = \frac{Y_{it}}{P^*_{it}} \quad (26)$$

Here y^*_{it} symbolises the potential per capita income by province under the assumption of zero net provincial migration. On the basis of (26) we compute a^*_i's and b^*_i's for each province where the estimated equation is of the following form -

$$\hat{y}^*_{it} = a^*_i \, b^{*\,t}_i \quad (27)$$

The coefficients a^*_i's and b^*_i's and their differences from a_i's and b_i's computed in (1) (reported in Table 1) are summarized in Tables 3 and 4 below for per capita personal income and per capita earned income.

TABLE 3

REGRESSION COEFFICIENTS OF EXPONENTIAL EQUATIONS FOR POTENTIAL PER CAPITA PERSONAL INCOME BY PROVINCES IN CANADA, 1961-70 (POTENTIAL ESTIMATES ARE MADE UNDER THE ASSUMPTION OF ZERO NET PROVINCIAL MIGRATION) AND DIFFERENCES FROM THE VERSION IN TABLE 1 WHICH INCLUDES PROVINCIAL NET MIGRATION

	Per Capita Personal Income						Migration Impacts by Differences from the Version in Table 1			
Provinces and Canada	Intercept (a^*_i) in $	Rank	Growth Rate (b^*_i)	Rank	R^2	t-Statistic of R	$a_i - a^*_i$ in $	Rank Differences for a_i and a^*_i	$b_i - b^*_i$	Rank Difference ($b^*_i - b_i$)
NFLD.	807	12	1.0781	8	0.9613	14.1	+9	Nil	+0.0003	+0.5
P.E.I.	843	11	1.0861	2	0.9654	14.9	+7	Nil	+0.0004	Nil
N.S.	1,082	9	1.0826	4	0.9519	12.6	+15	Nil	Nil	+1
N.B.	947	10	1.0886	1	0.9639	14.6	+14	Nil	-0.0004	Nil
QUE.	1,300	6	1.0786	6	0.9754	17.8	-5	Nil	+0.0013	+2
ONT.	1,719	1	1.0762	10	0.9750	17.6	-4	Nil	-0.0008	-1
MAN.	1,396	5	1.0783	7	0.9609	14.0	+9	Nil	+0.0006	+1
SASK.	1,265	7	1.0744	11	0.8895	8.0	+9	Nil	+0.0020	+1
ALTA.	1,421	4	1.0800	5	0.9557	13.1	+5	Nil	-0.0006	Nil
B.C.	1.695	2	1.0706	12	0.9864	24.1	-16	Nil	-0.0012	Nil
Y&NWT	1.196	8	1.0831	3	0.8332	6.3	+21	Nil	-0.0047	-4.5
CANADA	1,455	3	1.0779	9	0.9768	18.4	Nil	Nil	Nil	Nil

Source: Tables 1 and 2.

NB: 1. All positive signs show the provincial gains due to net migration; all negative signs show the provincial losses due to net migration for the columns under Migration Impacts.

2. All t-statistics for R are statistically significant at 1% level of significance.

...14

TABLE 4

REGRESSION COEFFICIENTS OF EXPONENTIAL EQUATIONS FOR POTENTIAL PER CAPITA EARNED INCOME BY PROVINCES IN CANADA, 1961-70 (POTENTIAL ESTIMATES ARE MADE UNDER THE ASSUMPTION OF ZERO NET PROVINCIAL MIGRATION) AND DIFFERENCES FROM THE VERSION IN TABLE 1 WHICH INCLUDES PROVINCIAL NET MIGRATION

	Per Capita Earned Income						Migration Impacts by Differences from the Version in Table 1			
Provinces and Canada	Intercept (a_i^*) in $	Rank	Growth Rate (b_i^*)	Rank	R^2	t-Statistic of R	$a_i - a_i^*$ in $	Rank Differences for a_i and a_i^*	$b_i - b_i^*$	Rank Difference ($b_i^* - b_i$)
NFLD.	649	12	1.0748	9	0.9695	15.9	+7	Nil	+0.0004	Nil
P.E.I.	650	11	1.0840	2	0.9712	16.4	+6	Nil	+0.0005	Nil
N.S.	881	9	1.0810	4	0.9494	12.2	+12	Nil	+0.0001	+1
N.B.	763	10	1.0872	1	0.9727	16.1	+11	Nil	-0.0003	Nil
QUE.	1,082	7	1.0772	6	0.9818	20.7	-3	Nil	+0.0014	+2
ONT.	1,460	1	1.0735	10	0.9787	19.2	-3	Nil	-0.0008	-1
MAN.	1,160	5	1.0761	7	0.9649	14.8	+8	Nil	+0.0006	Nil
SASK.	1,042	8	1,0710	11	0.8004	5.7	+8	Nil	+0.0020	+1
ALTA.	1,195	4	1.0781	5	0.9556	13.1	+4	Nil	-0.0007	-1
B.C.	1,387	2	1,0702	12	0.9880	25.6	-7	Nil	-0.0013	Nil
Y&NWT	1,135	6	1,0822	3	0.8431	6.6	+20	Nil	-0.0047	-2
CANADA	1,218	3	1.0759	8	0.9816	20.7	Nil	Nil	Nil	Nil

Source: Tables 1 and 2.

NB: 1. All positive signs show the provincial gains due to net migration; all negative signs show the provincial losses due to net migration for the columns under Migration Impacts.

2. All t-statistics for R are statistically significant at 1% level of significance.

The above tables are self-explanatory. However, some few comments may still be useful. In the first place, the right-side columns under migration impacts bear out in general the provincial gains and losses due to differences in $a_i - a_i^*$ and $b_i - b_i^*$ as caused by net migration. The changes in the ranking order of provinces for the intercepts are found to be nil whereas some ranking changes do occur for the growth rates. Since Canada is not affected by interprovincial migration all values in the Canada row for personal income and earned income, per capita, remain unchanged. Secondly, absolute changes i.e., $a_i - a_i^*$, due to net migration do not appear to be very large although one notices some important changes in growth rates, par-

...15

ticularly for Quebec and Saskatchewan showing gains, and British Columbia and Y&NWT showing some losses. How significant are these differentials with respect to convergence or divergence? To answer this question very precisely one needs to know the absolute differentials resulting from the time paths of provincial personal income and earned income (on a per capita basis) both with and without net migration. However, since the time paths are already known such differentials are not difficult to work out. This is what we set out to do now after noting some very specific assumptions under which the time paths are supposed to be valid. The following fundamental assumptions are involved in the construction of y^*_{it} (See equation (26)) and the consequent regression equation of the form (27). These are:

(1) that net migration from any province whenever it is adjusted to arrive at potential population, P^*_{it}, does not affect Y_{it}'s of either the province of origin or that of destination;

(2) that the proposition (1) is justified on the assumption that out-migration from a poor province takes place from the unemployed or underemployed pool of labour force without disrupting Y_{it} of the poor province;

(3) that the in-migration into a rich province does not also affect Y_{it} of that province which is tenable under the assumption that migrants from a poor province move to a rich province not under conditions of a sustained demand for labour in the rich province but under conditions of "money illusion".

Whereas assumption (2) can be generally accepted, assumption (3) is open to testing although it is found that in recent times the migrants into the so-called rich metropolitan areas do not necessarily increase the productivity of those areas. To be sure, however, one would expect that even if there is a demand for labour in the rich provinces, the effect of this phenonomenon with rising Y_{it} would be perhaps cancelled by in-migrants who diminish per capita productivity leaving the resulting Y_{it} unaffected.

1.4.2 Absolute and Relative Convergence Measures

The logic of our presentation here can be simplified by an illustration. Assume that two provinces, A and B, start in period 1970 with \$100 and \$200 as their per capita incomes with annual compound rates of growth of 8% and 5% respectively. We shall notice that in the first few years the absolute differential[1] in per capita incomes will first go on increasing and then start declining until the year 1995 when the difference vanishes. It is in the year 1995 that absolute convergence takes place whereas approximately from 1978-79 (on the average) the absolute differential starts declining. The latter we shall christen as 'relative convergence'. Thus in this illustration the time differential

---

1. The time series of absolute differentials and ratios based on the example here chosen is given as follows (for some selected years):

Time	1970	1971	1972	1975	1977	1978	1979	1980	1981	1985	1990	1995
(\$) Absolute Differential: (B-A)	100	102	104	108	110	110	110	110	109	99	65	Nil
Ratio (A/B) x 100	50	51	53	58	61	63	65	66	68	76	88	100

for absolute convergence is 25 years and for relative convergence it is 8-9 years. We maintain that the traditional methods of convergence i.e., the method of dividing one region or province by another for any given indicator, cannot bring out the issue of relative convergence at all[1]; nor can they measure the time differential to measure the speed of convergence. In our context, the speed of convergence is measured by the time differentials for absolute and relative convergence.

The methods of calculating time differentials for absolute and relative can now be formalized.

In the first instance, write from equation (1) the following identity for absolute convergence for two separate provinces, i and j, as follows:

$$\hat{y}_{it} = \hat{y}_{jt} = a_i \, b_i^{\,t} = a_j \, b_j^{\,t}, \quad i \neq j$$

From this it can be shown[2] that required $t = T$ is given by the following expression:

$$T = \frac{\log a_i - \log a_j}{\log b_j - \log b_i} \qquad (28)$$

1. See the time series of ratios in the footnote on page 16. The ratios show that they are increasing over time only and do not show any signs of the speed of convergence unless one perhaps applies the method of first or second differences.

2. The proof follows very easily from the identity equation. Nevertheless, interested readers can obtain it from the author on request.

where, T refers to the time differential for absolute convergence in per capita incomes of any two provinces chosen at a time.

Similarly it can be shown from equation (27) that the time differential for absolute convergence when per capita incomes for provinces are adjusted on the basis of potential population (with zero net provincial migration), can be given by the formula -

$$T^{*} = \frac{\log a_i^{*} - \log a_j^{*}}{\log b_j - \log b_i} \tag{29}$$

Time differentials for relative convergence, T^1 and T^{*1}, for estimated per capita income ($\hat{Y}it$) with adjustment for provincial net migration, are computed by the following formulae[1]:

$$T^1 = \frac{\log Z_{ji} + \log a_j - \log a_i}{\log b_i - \log b_j} \tag{30}$$

$$\text{where,} \quad Z_{ji} = \frac{\log b_j}{\log b_i} \tag{30a}$$

and,

$$T^{*1} = \frac{\log Z_{ji} + \log a_j^{*} - \log a_i^{*}}{\log b_i^{*} - \log b_j^{*}} \tag{31}$$

$$\text{with} \quad Z_{ji}^{*} = \frac{\log b_j^{*}}{\log b_i^{*}} \tag{31a}$$

In the empirical computations for T, T^*, T^1 and T^{*1} we have used these formulae subject to one slight change. All values have been shifted to the period 1970 which involved deductions of ten years from the computed results of (28), (29), (30) and (31). This is because the latter give

---

1. See the Appendix for the derivation.

measures from 1960 as a_i's and a_i^*'s belong to the year, 1960. The results are presented in Table 5 for per capita personal income and Table 6 for per capita earned income. The major observations on Tables 5 and 6 are listed as below.

1. The entries in the matrices (except the last column) should be read both row-wise and column-wise for any interprovincial or province-Canada comparisons. In this context Tables 1, 3 and 4 are particularly useful. For example, when one compares the convergence problems of Saskatchewan vis-a-vis Newfoundland in personal per capita income, Table 5 becomes relevant and one would then look at the Saskatchewan column and Newfoundland row. There are four entries then which tell the respective stories. T value is 221 years from 1970 which means that in the year 2191 Newfoundland will have absolute convergence with Saskatchewan. Here Newfoundland will catch up with Saskatchewan since a_i value of Newfoundland is less than that of Saskatchewan but b_i values show that Newfoundland is greater than Saskatchewan (See Table 1). Hence convergence is possible. If between two provinces, i and j, $a_i > a_j$ and $b_i > b_j$, then convergence is never possible and, therefore, we write the expression 'never'. Other interpretations are easy to follow and are given in the Tables.

TABLE 5

MATRICES OF INTER-PROVINCIAL AND PROVINCIAL-NATIONAL TIME DIFFERENTIALS (MEASURED IN NUMBER OF YEARS STARTING FROM 1970) IN CANADA FOR ABSOLUTE AND RELATIVE CONVERGENCE IN PER CAPITA PERSONAL INCOME WITH AND WITHOUT NET PROVINCIAL MIGRATION

Provinces and Canada		NFLD.	P.E.I.	N.S.	N.B.	QUE.	ONT.	MAN.	SASK.	ALTA.	B.C.	Y&NWT	CANADA	PROVINCE-CANADA $T^* - T$
NFLD:	T	-	Never	Never	Never	Never	255	Never	221	Never	75	Never	1,162	
	(T^*)	-	(Never)	(Never)	(Never)	(Never)	(425)	(Never)	(121)	(Never)	(97)	(Never)	(3,665)	+2,503
	T^1	-	Never	Never	Never	Never	242	Never	208	Never	61	Never	1,149	
	(T^{*1})	-	(Never)	(Never)	(Never)	(Never)	(412)	(Never)	(107)	(Never)	(83)	(Never)	(3,651)	
PEI:	T		-	59	Never	60	58	61	33	68	33	38	57	
	(T^*)			(68)	(Never)	(53)	(68)	(60)	(27)	(84)	(39)	(119)	(62)	+5
	T^1		-	46	Never	47	45	49	20	55	19	25	44	
	(T^{*1})		-	(56)	(Never)	(40)	(55)	(47)	(14)	(71)	(25)	(106)	(50)	
N.S.:	T			-	15	61	58	55	16	80	25	17	56	
	(T^*)			-	(14)	(39)	(68)	(53)	(10)	(105)	(30)	(Never)	(58)	+2
	T^1			-	3	48	45	32	3	67	11	14	43	
	(T^{*1})			-	(2)	(26)	(54)	(40)	(Passed)	(92)	(16)	(Never)	(45)	
N.B.:	T				-	29	39	34	16	38	22	16	33	
	(T^*)				-	(24)	(42)	(30)	(12)	(41)	(25)	(36)	(33)	Nil
	T^1				-	17	26	22	3	26	9	3	21	
	(T^{*1})				-	(12)	(29)	(28)	(Passed)	(29)	(11)	(24)	(21)	
QUE.:	T					-	56	72	Never	158	16	Never	50	
	(T^*)					-	(114)	(206)	(Never)	(Never)	(25)	(9)	(159)	+109
	T^1					-	43	59	Never	145	2	Never	37	
	(T^{*1})					-	(101)	(193)	(Never)	(Never)	(11)	(Passed)	(146)	
ONT.:	T						-	52	331	40	Never	112	61	
	(T^*)						-	(98)	(Never)	(43)	(Never)	(46)	(95)	+34
	T^1						-	38	317	27	Never	99	48	
	(T^{*1})						-	(85)	(Never)	(30)	(Never)	(33)	(82)	
MAN.:	T							-	Never	Never	10	Never	27	
	(T^*)							-	(Never)	(Never)	(17)	(24)	(113)	+86
	T^1							-	Never	Never	Passed	Never	14	
	(T^{*1})							-	(Never)	(Never)	(3)	(11)	(100)	
SASK.:	T								-	Never	32	14	Never	
	(T^*)								-	(Never)	(74)	(Passed)	(Never)	Nil
	T^1								-	Never	18	Passed	Never	
	(T^{*1})								-	(Never)	(60)	(Passed)	(Never)	
ALTA.:	T									-	7	Never	5	
	(T^*)									-	(10)	(50)	(2)	-3
	T^1									-	Passed	Never	Passed	
	(T^{*1})									-	(Passed)	(37)	(Passed)	
BC:	T										-	28	8	
	(T^*)										-	(20)	(12)	+4
	T^1										-	14	Passed	
	(T^{*1})										-	(6)	(Passed)	
Y&NWT:	T											-	341	
	(T^*)											-	(31)	-310
	T^1											-	338	
	(T^{*1})											-	(18)	
CANADA:	T												-	
	(T^*)												-	-
	T^1												-	
	(T^{*1})												-	

Source: Tables 1 and 3 and Equations (28) through (31).

NOTES:

1. The expression 'never' implies no convergence is ever possible, whereas 'passed' refers to the situation where convergence, relative or absolute, has already taken.
2. We did not work out the time differential due to $T^{*1} - T^1$ since it is more or less the same as $T^* - T$ shown in the last column.
3. Last column positive entires (by signs) signify the number of years reduced by the actual net provincial migration for each province vis-a-vis Canada for complete or absolute convergence. A reverse interpretation applies to negative entries.

TABLE 6

MATRICES OF INTER-PROVINCIAL AND PROVINCIAL-NATIONAL TIME DIFFERENTIALS (MEASURED IN NUMBER OF YEARS STARTING FROM 1970) IN CANADA FOR ABSOLUTE AND RELATIVE CONVERGENCE IN PER CAPITA EARNED INCOME WITH AND WITHOUT NET PROVINCIAL MIGRATION

Provinces and Canada		NFLD.	P.E.I.	N.S.	N.B.	QUE.	ONT.	MAN.	SASK.	ALTA.	B.C.	Y&NWT	CANADA	PROVINCE–CANADA $T^* - T$
NFLD:	T	-	Passed	Never	Never	Never	331	Never	221	Never	117	Never	Never	
	(T^*)	-	(Never)	(Never)	(Never)	(Never)	(625)	(Never)	(124)	(Never)	(166)	(Never)	(Never)	Nil
	T^1	-	Passed	Never	Never	Never	317	Never	207	Never	102	Never	Never	
	(T^{*1})	-	(Never)	(Never)	(Never)	(Never)	(611)	(Never)	(109)	(Never)	(152)	(Never)	(Never)	
PEI:	T		-	83	Never	81	63	70	34	83	41	78	68	
	(T^*)		-	(99)	(Never)	(71)	(72)	(69)	(29)	(101)	(49)	(317)	(73)	+5
	T^1		-	70	Never	68	50	57	21	70	28	65	55	
	(T^{*1})		-	(86)	(Never)	(58)	(59)	(56)	(16)	(89)	(36)	(304)	(60)	
N.S.:	T			-	16	79	54	60	12	82	29	71	57	
	(T^*)			-	(15)	(49)	(62)	(50)	(8)	(104)	(35)	(Never)	(58)	+1
	T^1			-	3	66	41	47	Passed	69	16	58	44	
	(T^{*1})			-	(3)	(36)	(59)	(37)	(Passed)	(91)	(21)	(Never)	(45)	
N.B.:	T				-	33	38	34	14	40	24	36	34	
	(T^*)				-	(28)	(41)	(31)	(11)	(43)	(29)	(75)	(35)	+1
	T^1				-	20	25	21	1	27	11	23	21	
	(T^{*1})				-	(15)	(28)	(18)	(Passed)	(31)	(15)	(63)	(22)	
QUE.:	T					-	44	36	Never	87	17	57	38	
	(T^*)					-	(75)	(55)	(Never)	(Never)	(28)	(Never)	(84)	+46
	T^1					-	31	23	Never	74	4	44	25	
	(T^{*1})					-	(61)	(41)	(Never)	(Never)	(14)	(Never)	(70)	
ONT.:	T						-	48	1,047	34	Never	42	50	
	(T^*)						-	(84)	(Never)	(36)	(Never)	(21)	(70)	+20
	T^1						-	35	1,033	20	Never	28	36	
	(T^{*1})						-	(70)	(Never)	(22)	(Never)	(8)	(56)	
MAN.:	T							-	Never	Never	12	6	43	
	(T^*)							-	(Never)	(Never)	(23)	(Passed)	(250)	+207
	T^1							-	Never	Never	Passed	Passed	29	
	(T^{*1})							-	(Never)	(Never)	(8)	(Passed)	(236)	
SASK.:	T								-	Never	61	Never	Never	
	(T^*)								-	(Never)	(366)	(Never)	(Never)	Nil
	T^1								-	Never	46	Never	Never	
	(T^{*1})								-	(Never)	(352)	(Never)	(Never)	
ALTA.:	T									-	7	676	Passed	
	(T^*)									-	(10)	(4)	(Passed)	Nil
	T^1									-	Passed	662	Passed	
	(T^{*1})									-	(Passed)	(Passed)	(Passed)	
B.C.	T										-	12	8	
	(T^*)										-	(8)	(15)	+7
	T^1										-	Passed	Passed	
	(T^{*1})										-	(Passed)	(Passed)	
Y&NWT	T											-	25	
	(T^*)											-	(2)	-23
	T^1											-	12	
	(T^{*1})											-	(Passed)	
CANADA:	T												-	
	(T^*)												-	-
	T^1												-	
	(T^{*1})												-	

Source: Tables 1 and 4 and Equations (28) through (31).

NB: For interpretations see notes on Table 5 and the text in Section 1.4.

2. The difference between absolute and relative convergence i.e., $T-T^1$ and T^*-T^{*1} works out to be about 13 years on the average for both personal income and earned income. This means that the time lost in terms of number of years between the absolute and relative convergence is not that large.

3. The differences between T^* and T or between T^{*1} and T^1 are more or less the same. In the last column we have shown the difference between T^* and T for each province vis-a-vis Canada. Some interesting results merit our attention.

   (a) Newfoundland, for example, has saved 2,503 years (Table 5) for both absolute and relative convergence in personal income (per capita) by the sheer fact that people have moved out of the province. However, out-migration did not help in any convergence for Newfoundland vis-a-vis Canada for earned income (per capita). (See Table 6).

   (b) Manitoba and Quebec on the contrary have gained on both counts, i.e., personal and earned income, due to net migration.

   (c) Saskatchewan has not gained anything since the situation with net migration has not at all improved relatively and convergence with Canada remains impossible in the light of the history of the period, 1961-70.

(d) Other cases which are worth attention are Ontario, British Columbia, Yukon and Northwest Territories. For Ontario-Canada comparisons, one notices that with migration between provinces, Canada gains 34 and 20 years in absolute (and relative) convergence in (per capita) personal and earned incomes respectively. For British Columbia-Canada, migration has helped convergence very little whereas for the Yukon-Northwest Territories-Canada, migration has deterred convergence.

Similar other interpretations are equally possible for any interprovincial comparisons with respect to the gains or losses due to provincial net migration.

4. As to the possibilities of convergence (either absolute or relative) the worst situations are invariably represented by Newfoundland and Saskatchewan followed partly by the Yukon and Northwest Territories. Other provinces have varying degrees of success.

1.4.3 Some Other Measures of Convergence On An Aggregate Basis for the Sample Period (1961-70) and Projections for the Future

In the preceding discussions we have demonstrated the possibilities of convergence between any two provinces taken at a time or between any province and Canada. These are, of course, very detailed results and are not elsewhere reported. In this Section the traditional method of coefficient of variation will be used to get a summary measure of convergence. However, one particular difference from the general use of this method may be noted. In the past the method has been applied only to the actual data for any given period, viz, Chernick (1) and Firestone (2), and a time series of coefficients obtained for that period only. In the present version we measure the coefficients not only for the sample period (1961-70) but also for future years. This is made possible by the use of time trend equations vide equations (1) and (27) for personal income and earned income (both on a per capita basis). The results are shown in the following Table 7.

TABLE 7

COEFFICIENTS OF VARIATION IN PER CAPITA PERSONAL INCOME AND PER CAPITA EARNED INCOME FOR THE PROVINCES OF CANADA, ACTUAL AND SIMULATED VALUES FOR THE PERIOD 1961-70 AND FOR SOME SELECTED YEARS IN THE FUTURE

Years	Per Capita Personal Income			Per Capita Earned Income			Col.(3)-Col.(2) / Col.(3) = % Reduction by Net Migration	Col.(6)-Col.(5) / Col.(6) = % Reduction by Net Migration
	Actual (x_1)	Simulated (x_2)	Simulated (x_2^*)	Actual (x_1)	Simulated (x_2)	Simulated (x_2^*)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1961	0.244	0.241	0.247	0.264	0.254	0.259	2.4	1.9
1962	0.236	0.239	0.245	0.249	0.251	0.257	2.4	2.3
1963	0.236	0.236	0.242	0.249	0.249	0.255	2.5	2.4
1964	0.235	0.233	0.240	0.245	0.247	0.253	2.9	2.4
1965	0.238	0.231	0.237	0.247	0.244	0.250	2.5	2.4
1966	0.242	0.228	0.235	0.252	0.241	0.248	3.0	2.8
1967	0.223	0.226	0.232	0.241	0.240	0.246	2.6	2.4
1968	0.217	0.224	0.230	0.233	0.237	0.244	2.6	2.9
1969	0.220	0.221	0.228	0.237	0.235	0.242	3.1	2.9
1970	0.221	0.219	0.225	0.237	0.233	0.240	2.7	2.9
1975	N.A.	0.208	0.214	N.A.	0.222	0.231	2.8	3.9
1980	N.A.	0.197	0.204	N.A.	0.212	0.223	3.4	4.9
1985	N.A.	0.188	0.195	N.A.	0.204	0.216	3.6	5.6
1990	N.A.	0.181	0.187	N.A.	0.197	0.211	3.7	6.6

Source: (a) Tables 1, 3 and 4 and Equations (1) and (27).
(b) Actual (x_1) is derived from the data from Major Economic Indicators, Provinces and Canada, DREE, 1972.

NOTES: (1) x's stand for coefficients of variation whereby x_1 refers to the actual data, x_2 refers to estimated per capita incomes generated by equation (1) and Table 1, and x_2^* to estimated per capita potential incomes (under assumptions of zero net provincial migration) generated by equation (27) and Tables 3 and 4.
(2) N.A. stands for 'not applicable'.

The following conclusions seem pertinent with respect to Table 7.

1. That all the coefficients of variation x_1, x_2, and x_2^* show a steady decline which suggests that the interprovincial dispersion is definitely declining for personal income and earned income both in the sample period and the period projected thereafter.

2. That the effects of migration on the differential between x_2^* and x_2 are only limited to a few percentage points i.e., 3% to 4% - a result which is very similar to the ones presented in Tables 5 and 6.

3. That the annual decline in the simulated coefficients of variation (x_2) over time is on the average of the order of 0.003 which is definitely very slow and which corrobates the intuition and some findings of Chernick (1). The annual decline in x_2^* is only a little lower i.e. 0.002 - a result which substantiates the low migration impacts as a whole.

2. OTHER POSSIBLE APPROACHES TO REDUCE INTERPROVINCIAL TIME DIFFERENTIALS IN CONVERGENCE FOR SELECTED INDICATORS

As previous analyses point to the overall inadequacy[1] of current migration flows in the rapid reduction of provincial disparities in per capita personal income and per capita earned income, it is worthwhile to discuss some options that may still remain open from the policy point of view for a sustained and rapid reduction in disparities.

---

1. Except for Newfoundland and Manitoba, as we have seen in Table 5 with respect to personal income. However, the benefit of migration disappears for Newfoundland with respect to earned income. See Table 6.

Obviously there are two options in the present framework of our analysis. These are:

(a) Perhaps the historical net migration flows from the provinces to bring about a rapid reduction in disparities were not large enough. Hence a much more vigorous policy of provincial migration is necessary.

(b) An expenditure policy to boost up total personal or earned incomes in the poor provinces. This policy relies on transfer payments for personal income boosting and investment policy for earned income boosting in the first round.

The implications of these two options for the per capita growth rates have been already simplified in equation (25). It is contended that the trade-offs between migration policy vis-a-vis expenditure policy to bring about desired reductions in the interprovincial disparities in person and earned incomes (per capita) can be very well worked out within our framework. The only thing one has to do is to stipulate ex-ante desired growth rates in per capita personal and earned incomes whereas the migration and the expenditure impacts are assumed to take different values (of course, within some constraints) vide equation (25).

One last point seems to be in order. In this study we have not enquired into why the growth rates differ in the different provinces in Canada. Only one aspect of 'why growth rates differ' has been touched upon, namely, the impact of net migration (provincial) on the growth rates of population in the different provinces, and correspondingly its consequences on the differentials in growth rates of per capita incomes. Chaudhury (4) has attempted to explain the differences in per capita income differentials in the provinces by standardizing for age-sex composition, education, participation rate and unemployment. Chaudhury claims to have reduced 50% of the income differentials in the provinces by the above four factors although his data refer only to 1961. Similar other claims are made by Coelho and Ghali (5) and Scully (6) for the interstate or interregional wage differentials in the United States. The main problem with all these approaches lies in the fact that the so-called standardizing factors or causes are invariably interrelated and, therefore, explanations as to 'why growth rates differ' are not justified.

3. CONCLUSION

The objective of this study has been to measure the speed of interprovincial speed of convergence by selected economic indicators on the basis of data for the period, 1961-70. The indicators chosen are: per capita personal income and per capita earned income by provinces in Canada where the Yukon and Northwest Territories are treated as a separate province. While working on this basic theme, the different methods followed in the current literature on the convergence tests are subjected to a critical examination and new methods are proposed and results evaluated. The major findings of the study are summarized as below.

1. Time paths of each indicator for each province and Canada are statistically tested and they are found to be significant. On this basis interprovincial and province-Canada time differentials for convergence are worked out by which

it is shown that Newfoundland and Saskatchewan do not leave any room for convergence whereas other provinces show among themselves some chances of convergence within a reasonable period of time.

2. The impact of provincial net migration flows during the period 1961-70 on the reduction of interprovincial and province-Canada convergence has been very limited with the exceptions of Manitoba and Quebec.

3. The overall interprovincial convergence ratings as shown by the use of coefficient of variation, both actual and simulated, over the sample period 1961-70 and for some selected years until 1995, show that the rate of decline in the coefficient is very slow. This shows that the prospects of rapid interprovincial convergence are very limited indeed.

4. Two policy options are finally suggested to initiate rapid convergence of the poorer provinces with the richer ones. These are:
   (a) Greater out-migration from the poor provinces.
   (b) Heavier expenditure policy to boost total personal and earned incomes in these provinces.

   Some trade-offs do exist between these policies by the very nature of our analytical framework, although their precise quantitative relations are not fully spelled out in this study. To

...29

implement it, one needs some desired rates of growth in the personal and earned incomes (on a per capita basis) in all the provinces for the future. The latter invites some speculations on other issues which call for compatibility with the desired growth rates.

LIST OF VARIABLES AND COEFFICIENTS

Variables

y_{it} = Actual value of indicator (either per capita personal income or per capita earned income) in province, i, in year, t.

$\hat{y}_{it}$ = Computed value of y_{it}.

Y_{it} = Actual total value of indicator (either personal income or earned income) in province, i, in year, t.

$\hat{Y}_{it}$ = Computed value of Y_{it}.

W_{it} = Weight of actual population of province, i, in the nation, in year, t.

P_{it} = Actual population of province, i, in year, t.

$\hat{P}_{it}$ = Computed value of P_{it}.

P_t = Actual population of the nation in year, t.

$\hat{P}_t$ = Computed value of P_t.

Y_t = Actual total value of indicator (either personal income or earned income) in the nation, in year, t.

y_t = Actual value of indicator (either per capita personal income or per capita earned income) in the nation in year, t.

$\hat{y}_t$ = Computed value of y_t.

P^*_{it} = Potential population of province, i, in year, t.

$\hat{P}^*_{it}$ = Computed value of P^*_{it}.

M_{it} = Actual net migration of province i, in year, t.

M_t = Actual net migration in the nation in year, t.

Variables (Cont'd)

y^*_{it} = Potential per capita value of the indicator (personal income or earned income) in province i in year, t.

$\hat{y}^*_{it}$ = Computed value of y^*_{it}.

T = Time differential for absolute convergence (on the basis of actual population) between any province i and any other province j or Canada for any indicator (per capita personal income or per capita earned income).

T^* = Time differential for absolute convergence (under assumptions of potential population i.e. with zero net provincial migration) between any province i and any other province j or Canada for any indicator (per capita personal income or earned income).

T^1 = Time differential for relative convergence i.e. the time taken to record the first decline in the interprovincial or province-Canada absolute differential in any given indicator (either per capita personal income or earned income) on the basis of actual population.

T^{*1} = The same interpretation as in T^1 subject to the assumption of zero net provincial migration or potential provincial population.

t = time, measured in discrete units, with $t = 1$ for 1961.

Coefficients

a_i = Intercept for $\hat{y}_{it}$ and refers to the provincial value of the indicator in year, 1960.

a = Intercept for $\hat{y}_t$ and refers to the national value of the indicator in year, 1960.

a^*_i = Intercept for $\hat{y}^*_{it}$ and refers to the year, 1960.

Coefficients (Cont'd)

r_i = Annual compound rate of growth (estimated) for any indicator in province i (on the basis of $\hat{y}_{it}$).

b_i = $1 + r_i$ = generally referred to as the growth rate of any indicator in province i.

b_i^* = Growth rate of province i on the basis of $\hat{y}_{it}^*$.

b = Growth rate for $\hat{y}_t$ (computed per capita personal income or per capita earned income) for Canada or nation.

g_i = Growth rate of population in province i.

$\hat{P}_i$ = Intercept for $\hat{P}_{it}$.

$\hat{P}$ = Intercept for $\hat{P}_t$.

$\hat{P}_i^*$ = Intercept for $\hat{P}_{it}^*$.

g_i^* = Growth rate for $\hat{P}_{it}^*$.

k_i = Growth rate of total value of the indicator i.e. total personal income or total earned income, in province i.

x_1 = Coefficient of variation in interprovincial personal incomes or earned incomes (both on a per capita basis) based on actual data.

x_2 = Coefficient of variation in interprovincial personal incomes or earned incomes (both on a per capita basis) based on the estimated values of $\hat{y}_{it}$.

x_2^* = Coefficient of variation in interprovincial personal incomes or earned incomes (both on a per capita basis) based on the estimated values of $\hat{y}_{it}^*$.

DERIVATION OF FORMULAE (30) AND (31)

From equation (1) for two provinces i and j one can write the first derivatives as below:

$$\frac{d\hat{y}_{it}}{dt} = a_i \, b_i{}^t \log b_i \qquad (A.1)$$

and

$$\frac{d\hat{y}_{jt}}{dt} = a_j \, b_j{}^t \log b_j \qquad (A.2)$$

The procedure then is to reduce the difference between (A.1) and (A.2) to zero which renders -

$$a_i \, b_i{}^t \log b_i = a_j \, b_j{}^t \log b_j \qquad (A.3)$$

or

$$\left[\frac{b_i}{b_j}\right]^t = \frac{\log b_j}{\log b_i} \times \frac{a_j}{a_i} \qquad (A.4)$$

Write now, $Z_{ji} = \frac{\log b_j}{\log b_i}$ (A.5)

(A.4) and (A.5) in combination then give -

$$t = \frac{\log Z_{ji} + \log a_j - \log a_i}{\log b_i - \log b_j} \qquad (A.6)$$

Put then $t = T^1$ i.e. the time taken to record the relative convergence with respect to values of $\hat{y}_{it}$.

A similar exercise is performed for $t = T^{*1}$ with respect to the values of $\hat{y}^*_{it}$ given by equation (27) which then results in the formulation (31).

...34

REFERENCES

1. S.E. Chernick: Interregional Disparities in Income; Economic Council of Canada, Staff Study No. 14, 1966.

2. O.J. Firestone: Regional Economic and Social Disparity; Regional Economic Development Conference, Ottawa, March 2-4, 1972.

3. M. McInnis: The Trend of Regional Income Differentials in Canada; Canadian Journal of Economics, Vol. 1, No. 2, May 1968.

4. M.D. Chaudhury: Economic Distance Among Regions: A Statistical Analysis; Economic Development and Cultural Change, Vol. 19, No. 4, July 1971.

5. P.R.P. Coelho & M.A. Ghali : The End of the North-South Wage Differential; The American Economic Review, Vol. LXI, No. 5, December 1971.

6. G.W. Scully: Interstate Wage Differentials: A Cross Section Analysis; the American Economic Review, December 1969.

7. DREE, Economic Analysis Branch: Major Economic Indicators - Canada and Provinces, 1972.

8. Statistics Canada : Catalogue No. 91-201 and various other issues.