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SARP C7 OPTICAL SIGHT TRIAL ANALYTICAL RESULTS

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ABSTRACT

This report summarizes the analyses carried out on comparative hit-miss data for the C7 rifle trial with optical and iron sights. The particular aspects of the trial that were analysed were the accuracies of the Leitz Elcan optical sight under low light level conditions and under daylight conditions. The technique used was that of minimum discrimination information. The report includes some background to the trial, the set up of the trial, a summary of the analysis performed plus some general conclusions.

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INTRODUCTION

1. This report summarizes the analytical work performed on comparative hit-miss data from a trial of the Leitz Elcan optical sight on the C7 rifle. The data were obtained from a user trial conducted at CFB Petawawa from 21-30 April 1986, under the control of trial officer Captain G.D.P. Von Einsiedel (See Reference 1).

2. This trial was part of a series of continuing trials being carried out on small arms by the Small Arms Replacement Project (SARP) office. In this trial, two of the aspects that were considered were the accuracy of the C7 rifle (using optical and iron sights) under daylight conditions and its accuracy under conditions of low light levels.

BACKGROUND

3. Ernst Leitz Canada Limited was requested by the Department of National Defence (DND) to produce a sight for small arms which would improve the ability of gunners to engage targets under low light level conditions. The Leitz Elcan sight was developed and various prototypes have been tested on the FNC1, C7 and the C9. It is now undergoing a series of tests on the C7 rifle. (See References 1 to 3).

AIM

4. The aim of the trial was to evaluate the optical sight on the C7 rifles to determine what further development would be required. This report only assesses the sight under two of the tests specified in the trial directive (Reference 1).

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a. Test No. 4 - Accuracy under Low Light Conditions: A comparison of the optical and iron sights under low light level conditions is made.

b. Test No. 5 - Accuracy under Daylight Condition. A comparison of the optical and iron sights under daylight conditions is made.

TEST DESIGN FOR DAYTIME FIRINGS

5. There were ten gunners each assigned to one of two groups, giving two groups of five gunners. Each gunner was assigned a weapon with an optical sight and another one with an iron sight. The gunners used the same two weapons throughout the trials. There were seven days of firing; each day was divided into two sessions, giving fourteen sessions in total. Group 1 fired in sessions 1, 3, 7, 8, 12, 13 and 14; group 2 fired in the other seven sessions.

6. The sessions consisted of deliberate and snap shooting with both the iron and optical sights. The rifles were fired at four ranges: 100m, 200m, 300m and 400m. There were ten rounds fired for each combination of type of shoot (deliberate or snap), sight type (iron or optical) and range. The number of hits was noted in each case.

TEST DESIGN FOR LOW LIGHT CONDITIONS

7. The gunners fired exactly the same weapons that they had used under daylight conditions. Group 1 gunners fired on two nights at a range of 100m. Each night had eight firing sessions, in which the

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gunners fired 10 shots from each of their rifles at the targets. Group 2 gunners fired on one night at a range of 200m. They had 6 firing sessions. In the first four, they fired 10 shots from each of their rifles; in the last two sessions they fired 10 shots using only their optical sights. These firing sessions took place over a period of about an hour therefore later firing sessions generally took place under conditions of increased darkness.

ANALYSIS

8. The technique used was the principle of minimum discrimination information, as discussed in References 4-7. The data consist of the number of hits and misses for each combination of factors (e.g. sight type, type of shoot, gunner, day, range, etc.). The data are set up in contingency tables, each dimension of the contingency table corresponding to a factor. These factors are then analysed so that they can be used to predict the hit probabilities under various conditions, using log linear models (see Annex A).

9. The test results from the trial are presented in Appendix 1 to Annex A, arranged in sets as follows:

SET A - group 2 daylight conditions
SET B - group 1 daylight conditions
SET C - group 1 low light conditions
SET D - group 2 low light conditions

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INITIAL DATA INSPECTION

10. The data were submitted for analysis in the form of the number of hits attained out of every 10 rounds fired by a gunner for each sight type, by type of shoot by range by day combination. (For the low light levels the combinations consisted of a gunner by sight type by day by firing session.)

11. The original data were separated into the four separate sub-groups, SET A to SET D, and each sub-group was analysed individually. The factors for each set are shown in Tables I-A, I-B and I-C.

12. Hit probabilities averaged over groups and days are given in Tables II-A to II-C. The hit probability was defined to be the number of hits divided by the number of shots fired. The general trends are towards lower probabilities as the ranges go from 100 to 400m, and as the light levels decrease under low light level conditions.

TABLE I-A

Factors for Sets A & B

NAME	VARIABLE	INDICES						
		1	2	3	4	5	6	7
Sight Type	g	Optical	Iron					
Type of Shoot	h	Deliberate	Snap					
Day	i	1	2	3	4	5	6	7
Gunner	j	1	2	3	4	5		
Range	k	100	200	300	400			
Score	ℓ	hit	miss					

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TABLE I-B

Factors for Set C (Range = 100m)

NAME	VARIABLE	INDICES							
		1	2	3	4	5	6	7	8
Sight Type	g	Optical	Iron						
Day	i	1	2						
Gunner	j	1	2	3	4	5			
Firing Session	k	1	2	3	4	5	6	7	8
Score	ℓ	hit	miss						

TABLE I-C

Factors for Set D (Range = 200m)

NAME	VARIABLE	INDICES					
		1	2	3	4	5	6
Sight Type	g	Optical	Iron				
Gunner	j	1	2	3	4	5	
Firing Session	k	1	2	3	4	5	6
Score	ℓ	hit	miss				

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TABLE II-A

AVERAGE HIT PROBABILITIES FROM RAW DATA FOR SET A

RANGE(m)	OPTICAL-DELIBERATE	OPTICAL-SNAP	IRON-DELIBERATE	IRON-SNAP
100	0.99	0.97	0.99	0.98
200	0.95	0.91	0.92	0.88
300	0.77	0.69	0.64	0.69
400	0.48	0.46	0.36	0.45

TABLE II-B

AVERAGE HIT PROBABILITIES FROM RAW DATA FOR SET B

RANGE(m)	OPTICAL-DELIBERATE	OPTICAL-SNAP	IRON-DELIBERATE	IRON-SNAP
100	0.99	0.95	0.97	0.97
200	0.90	0.81	0.84	0.84
300	0.65	0.66	0.69	0.63
400	0.45	0.43	0.45	0.35

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TABLE II-C

AVERAGE HIT PROBABILITIES FROM RAW DATA FOR SETS C AND D

	SET C*		SET D**	
FIRING SESSION	OPTICAL	IRON	OPTICAL	IRON
1	0.98	0.85	0.82	0.52
2	0.98	0.82	0.78	0.52
3	0.81	0.54	0.58	0.12
4	0.80	0.63	0.52	0.04
5	0.95	0.15	0.18	
6	0.92	0.07	0.44	
7	0.86	0.18		
8	0.77	0.25		

* averaged over two nights

** one nights' worth of data

COMPUTER PROGRAMS

13. The computer programs used to analyse the trial data were CONTAB and NEWKULL. CONTAB was used for an initial investigation of the data. The base model assumes that all the scores are independent, in fact equal. This base model was used to test each factor to determine which ones were significant. Those factors which were significant were included in a new base model used to test the significance of interactions of two factors.

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14. The results of the CONTAB program were studied in more detail using NEWKULL. With NEWKULL significant factors and interactions found in CONTAB were added to the model one at a time, and the amount of variability in the score explained by the model and the number of outliers examined.

MODEL SELECTION CRITERIA

15. The criteria used for model selection were:

a. the simplicity of the model (the more complicated the model, the fewer the number of degrees of freedom)

b. The amount of variation explained (the percentage decrease in the information statistic from the previous model)

c. the number of outliers associated with the model.

16. There is an obvious tradeoff between criteria a and b. As more factors are added, the model explains more of the variation in the data, but the model becomes more complex. A point has to be chosen where a reasonable amount of the variation in the data is explained, but the model is still simple enough to be easily interpreted.

17. Data which cannot be adequately fitted by a model are called outliers. They contribute much more than their fair share to the unexplained variation between the observed data and the fitted values of the model. NEWKULL automatically lists those cells which contribute

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more than a threshold amount to the variation. All else being equal the model with the fewest number of outliers should be chosen. When possible models are chosen, their outliers can be constrained to their actual values to determine how much they are contributing to the remaining unexplained variation.

RESULTS AND DISCUSSION

18. Tables 2-A to 2-D in Appendix 2 to Annex A summarize the results of the first stage of analysis using CONTAB. The first column on the left gives the factors that are being considered at each stage. The information statistic column gives a measure of the difference between the model and the actual data (the larger the value of the information statistic, the worse the fit of the model). The probability column reflects the probability that the information statistic for that model is different from the information statistic for the base model. Only factors with probabilities greater than 0.95 were considered for the model.

19. Tables 3-A to 3-D in Appendix 3 to Annex A are analysis of information tables for the four sets of data. The first column on the left lists the important factors chosen as per paragraph 18. The second column from the left lists the amount of decrease in the information statistic caused by that factor. The third column from the left gives the change in the information statistic due to that factor as a percentage of the information statistic for the base model (in which the only constraint is that the scores are equal). The next column gives the number of degrees of freedom lost when that factor is added to the model. The final column gives the probability that the decision to accept a factor as contributing to the model is in error.

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20. Tables 4-A to 4-D in Appendix 4 to Annex A show the results of the analysis using NEWKULL. Here the factors were added to the model one at a time. As each factor was added, the information statistic and the number of degrees of freedom decreased and the number of constraints increased. Possible final models were rerun constraining the outliers. These results are found at the bottom of the table. The third columns from the right gives the percentage of the unexplained variation in the model attributable to the outliers.

SET A

21. The significant main effects were range, day and sight type. As can be seen in Table 3-A in Appendix 3 to Annex A, the analysis of information table for SET A, the range factor explained about 58% of the variability in the data. In other words the most important determinant of a gunner's probability of hitting the target, is how close he is to the target. There was some day-to-day variation in the hit probability, with gunners in SET A as a whole doing worse in their second session. The gunners did slightly better with the optical sight than with the iron sight.

22. Of the significant interactions, only day by range, day by gunner and sight type by gunner interactions were kept in the model, to keep it as simple as possible. The day by range effect suggests that the range effect was not constant over the seven days - for example the best hit probability averaged over the five gunners, did not occur at 100 m for all seven sessions. The day by gunner effect shows that the gunners scores varied somewhat from day to day. The gunner by sight type interaction suggests that some gunners found the optical sight easier to use than other gunners did.

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23. The model chosen is a6 shown in Table 4-A in Appendix 4 to Annex A. It includes a constant term τ^{ℓ}_{1}, a range effect $\tau^{k\ell}_{k1}$, a day effect $\tau^{i\ell}_{i1}$, a sight type effect $\tau^{g\ell}_{g1}$, a day by range interaction effect $\tau^{ik\ell}_{ik1}$, a day by gunner interaction effect $\tau^{ij\ell}_{ij1}$, and a sight type by gunner interaction effect $\tau^{gj\ell}_{gj1}$, giving as a model:

$$\ln \frac{x^*(ghijk1)}{x^*(ghijk2)} = \tau^{\ell}_{,1} + \tau^{k\ell}_{k1} + \tau^{i\ell}_{i1} + \tau^{g\ell}_{g1} + \tau^{ik\ell}_{ik1} + \tau^{ij\ell}_{ij1} + \tau^{gj\ell}_{gj1}$$

24. The model explains 69.4% of the variation in the data, and has eighteen outliers. To check how much the outliers contribute to the variability, a6 was run with the outliers constrained. With the outliers constrained, this model explains 78.3% of the variation; 29% of model a6's variation can be attributed to its outliers.

SET B

25. In SET B all the factors have a significant effect in explaining the variation as can be seen in Table 3-B of Appendix 3 to Annex A. Once again the range factor explained over half of the variation. The day factor contributed approximately the same percentage of explanation in SET A and SET B. The increase in significance in SET B of the gunner factor suggests that there was more variation of ability in the gunners in group 1 than in the gunners in group 2. The gunners in SET B did slightly better at deliberate shooting than snap shooting. The same interactions were important in SET A and SET B: day by gunner, day by range and sight type by gunner.

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26. The model in NEWKULL which includes all these factors and interactions is b8. Model b6 was also considered and both models were run with their outliers constrained. In the end model b8 was chosen since it explained a higher percentage of the variation and had fewer outliers. Model b8 explains 67.4% of the variation and has 13 outliers. With the outliers constrained, setb8 explains 74.4% of the variation; 22% of the unexplained variation in model b8 is attributable to the thirteen outliers.

27. The model b8 includes a constant term τ_{l}^{ℓ}, a range effect $\tau_{kl}^{k\ell}$, a day effect $\tau_{il}^{i\ell}$, a gunner effect $\tau_{jl}^{j\ell}$, a type of shoot effect $\tau_{hl}^{h\ell}$, a sight type effect $\tau_{gl}^{g\ell}$, a day by gunner interaction effect $\tau_{ijl}^{ij\ell}$, a day by range interaction effect $\tau_{ikl}^{ik\ell}$, and a sight type by gunner interaction effect $\tau_{gjl}^{gj\ell}$. The model has the following form:

$$\ln \frac{x^*(ghijkl)}{x^*(ghijk2)} = \tau_{l}^{\ell} + \tau_{kl}^{k\ell} + \tau_{il}^{i\ell} + \tau_{jl}^{j\ell} + \tau_{hl}^{h\ell} + \tau_{gl}^{g\ell} + \tau_{ijl}^{ij\ell} + \tau_{ikl}^{ik\ell} + \tau_{gjl}^{gj\ell}$$

SET C

28. All the main factors were significant for SET C. Sight type was the most significant factor, contributing approximately 33% of the explanation of the variation of the data, as can be seen in Table 3-C of

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Appendix 3 to Annex A. The use of the optical sight improved the hit probability considerably over the use of the iron sight. As the darkness increased, (corresponding to increasing numbers for the firing session), the hit probabilities decreased. The gunners in SET C (Group 1) showed some variation in their ability to hit the target. As a whole, the hit probabilities were better on the first night.

29. The important interactions were seen to be sight type by shoot order, gunner by firing session and sight type by day. The sight type by firing session interaction implies that the optical and iron sights were affected differently by changing light levels, and in fact the optical sight did not seem to be affected as much by decreasing light levels. The gunner by firing session interaction suggests that different gunners were affected to greater or lesser amounts by the changing light levels. The sight type by day interaction suggests that there was some difference between the two nights. A letter from K. Sutton of DCIEM (Reference 7 - paper in progress) stated that there was a higher degree of illumination on the second night.

30. Both models c7 and c5 were examined with their outliers constrained. Model c7 was chosen because it explains a high pecentage of the variation (79.5%) and has only eight outliers. With the outliers constrained, it explains 88.0% of the variation; 50% of the variation unexplained by model c7 is attributable to its eight outliers.

31. The model c7 includes a constant term τ^{ℓ}_{1}, a sight type effect $\tau^{g\ell}_{gl}$, a firing session effect $\tau^{k\ell}_{kl}$, a gunner effect $\tau^{j\ell}_{jl}$, a day effect $\tau^{i\ell}_{il}$, a sight type by firing session interaction effect $\tau^{gk\ell}_{gkl}$, a gunner by firing session interaction effect $\tau^{jk\ell}_{jkl}$, and a sight type by day interaction effect $\tau^{gi\ell}_{gil}$. This gives the model the form:

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$$\ln \frac{x^*(gijk1)}{x^*(gijk2)} = \tau^{\ell}_{1} + \tau^{g\ell}_{g1} + \tau^{k\ell}_{k1} + \tau^{j\ell}_{j1} + \tau^{i\ell}_{i1} + \tau^{gk\ell}_{gk1} + \tau^{jk\ell}_{jk1} +$$

$$\tau^{gi\ell}_{gi1}$$

SET D

32. All the main factors were significant for SET D. The darkness level (firing session) was the most important factor, contributing approximately 16% to the explanation of the variability in the data, as can be seen in Table 3-D of Appendix 3 to Annex A. There was a large variation in the ability of the gunners. In general, the gunners had higher hit probabilities using the optical sight.

33. The gunner by firing session interaction implies that the gunners in group 2 were affected differently by changing light levels. The gunner by sight type interaction implies that some gunner found the optical (or iron) sight easier to use than the others did.

34. The final model, d5, explained 85.9% of the variation in the data, and had one outlier. When this one outlier was constrained, 93.0% of the variation was explained (i.e. half of the remaining unexplained variation for the final model was due to this one outlier). The model includes a constant term τ^{ℓ}_{1}, a firing session effect $\tau^{k\ell}_{k1}$, a gunner effect $\tau^{j\ell}_{j1}$, a sight type effect $\tau^{g\ell}_{g1}$, a gunner by firing session interaction effect $\tau^{jk\ell}_{jk1}$, and a gunner by sight type interaction effect $\tau^{gj\ell}_{gj1}$. The model has the following form:

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$$\ln \frac{x^*(gjk1)}{x^*(gjk2)} = \tau^{\ell}_{1} + \tau^{k\ell}_{k1} + \tau^{j\ell}_{j1} + \tau^{g\ell}_{g1} + \tau^{jk\ell}_{jk1} + \tau^{gj\ell}_{gj1}$$

35. A summary of the main effects and interactions in the models for the four sets can be found in Table III.

THE RELATIVE ODDS OF OPTICAL AND IRON SIGHTS

36. The output from NEWKULL also gives the tau values, which can be used to calculate the relative odds of hitting the target. The calculations for determining the relative odds for the optical and iron sights under different conditions for the four sets are shown in Annex B. Table IV-A shows the relative odds for the optical sight compared to the iron sight for the five different gunners in SET A when fired at the same range, on the same day, and for the same type of shoot (deliberate or snap). For the different gunners the optical sight varies in its odds of hitting the target from 1.04 to 2.84 times that of the iron sight. The variances in the taus were not available, so that the significance of these odds could not be calculated.

37. The relative odds of the optical sight compared to the iron sight were also calculated for SET B for the five gunners. The relative odds varied from 0.80 to 2.56; however the only statistically significant values were 1.66 for gunner 2 and 2.56 for gunner 5, as can be seen in Table IV-B. Tables IV-A and IV-B suggest that the optical sight shows some improvement over the iron sight even under daylight conditions.

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TABLE III

SUMMARY TABLE OF MODELS

	MAIN EFFECTS						INTERACTIONS						
	range	gunner	day	sight type	type of shoot	firing session	day by range	day by gunner	sight type by gunner	firing session by sight type	firing session by gunner	sight type by day	sight type by gunner
SET A	X		X	X		*	X	X	X	*	*		
SET B	X	X	X	X	X	*	X	X	X	*	*		
SET C	*	X	X	X	*	X				X	X	X	
SET D	*	X	*	X	*	X	*	*			X	*	X

*not applicable

TABLE IV-A

RELATIVE ODDS OF OPTICAL TO IRON SIGHTS FOR THE FIVE GUNNERS IN SET A

GUNNER	RELATIVE ODDS
1	1.47
2	1.04
3	1.75
4	2.84
5	1.21

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TABLE IV-B

RELATIVE ODDS OF OPTICAL TO IRON SIGHTS FOR THE FIVE GUNNERS IN SET B

GUNNER	RELATIVE ODDS
1	1.32
2	1.66*
3	0.80
4	0.84
5	2.56*

*significant at 5% level or better

TABLE IV-C-1

RELATIVE ODDS OF OPTICAL TO IRON SIGHTS FOR THE TWO NIGHTS IN SET C

NIGHT	RELATIVE ODDS
1	4.56*
2	30.5*

*significant at 5% level or better

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TABLE IV-C-2

RELATIVE ODDS OF OPTICAL TO IRON SIGHTS
FOR THE DIFFERENT SHOOTS IN SET C

FIRING SESSION		RELATIVE ODDS
increasing darkness ↓	1	26.2*
	2	32.8*
	3	3.48*
	4	6.75*
	5	500.*
	6	7.45*
	7	7.43*
	8	30.5*

*significant at 5% level or better

TABLE IV-D

RELATIVE ODDS OF OPTICAL TO IRON SIGHTS
FOR THE FIVE GUNNERS IN SET D

GUNNER	RELATIVE ODDS
1	0.095*
2	66.7*
3	189×10^3
4	99.2×10^3
5	416*

*significant at 5% level or better

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38. For SET C there were two interactions to consider, sight type with day and sight type with firing session. Table IV-C-1 shows the relative odds of the optical sight compared to the iron sight for each day when fired by the same gunner at close to the same time. Table IV-C-2 gives the relative odds for the different firing sessions in order, when fired by the same gunner on the same evening. The optic sight was significantly better than the iron sight in all cases, with relative odds for the optic sight varying from 3.48 to 500 times the ability to hit the target.

39. For SET D the relative odds of the optical sight compared to the iron sight were calculated for the five gunners. Fewer data were available for this set, so fewer results were statistically significant. The only statistically significant results were that gunners 2 and 5 did 66.7 and 416 times better respectively with the optical sight than with the iron sight, and that gunner 1 did 0.095 times as well with the optical sight as with the iron sight.

MODEL ANALYSIS RESULTS

40. The probabilities of hits calculated from the models for the sets A-D are shown in Tables V-A to V-C.

TABLE V-A

AVERAGE HIT PROBABILITIES USING MODEL FOR SET A

RANGE(m)	OPTICAL-DELIBERATE	OPTICAL-SNAP	IRON-DELIBERATE	IRON-SNAP
100	0.98	0.98	0.99	0.99
200	0.94	0.91	0.91	0.91
300	0.75	0.71	0.66	0.68
400	0.48	0.48	0.39	0.40

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TABLE V-B

AVERAGE HIT PROBABILITIES USING MODEL FOR SET B

RANGE(m)	OPTICAL-DELIBERATE	OPTICAL-SNAP	IRON-DELIBERATE	IRON-SNAP
100	0.98	0.97	0.97	0.97
200	0.88	0.83	0.85	0.81
300	0.66	0.67	0.68	0.62
400	0.46	0.41	0.38	0.36

TABLE V-C

AVERAGE HIT PROBABILITIES USING MODELS FOR SETS C AND D

	SET C		SET D	
	OPTICAL	IRON	OPTICAL	IRON
1	0.98	0.85	0.81	0.53
2	0.98	0.82	0.81	0.49
3	0.83	0.54	0.55	0.15
4	0.80	0.63	0.53	0.03
5	0.95	0.15	0.18	
6	0.92	0.07	0.44	
7	0.86	0.18		
8	0.77	0.25		

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CONCLUSIONS

41. During daylight conditions there seemed to be a slight increase in the probability of hitting a target when using the optical sight compared to the iron sight. This increase varied somewhat from gunner to gunner.

42. During daylight conditions range is the dominant factor in determining the probability of hitting a target. This is true for both iron and optical sights.

43. In low light level conditions, the light levels and the sight type are both important factors. The difference in effectiveness of the two sight types is much greater during low light levels than in daytime. The iron sight seems to be less effective in general and to drop off more rapidly as light levels decrease than the optical sight.

44. This is only an analysis of tests 4 and 5 of the trial. These results will have to be combined with other results from the same trial and the results of other trials to draw any firm conclusions.

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ANNEX A
TO ORAE PROJECT REPORT PR430
DATED JULY 1987

MINIMUM DISCRIMINATION INFORMATION (MDI) ANALYSIS

1. This method of analysis is non-parametric and makes no assumptions about the underlying probability distribution. The data are arranged in a contingency table so that (in this case) the number of hits and misses for each combination of factors is recorded. The object of the analysis is to try and fit a model to the data, so that the variation in the data is explained using a small number of the different factors, and combinations of factors.

2. If we use $x(w)$, where w is a vector of indices of the factors, to represent the actual contingency table values, and $x^*(w)$ to represent the predicted values using the model, the measure of deviation between the real and modelled values used in MDI analysis is:

$$I(x:x^*) = \sum_w x(w) \ln \frac{x(w)}{x^*(w)}$$

$I(x:x^*)$ is zero in the case of perfect fit, and becomes larger the worse the fit.

3. Estimates $x_a^*(w)$ are found, subject to a series of marginal constraints. The first step assumes that all the scores are independent, in fact equal. This leads to the following base model, as expressed using the log-odds representation:

$$\ln \left(\frac{\text{hits}}{\text{misses}} \right) = \text{constant} = \tau_{\text{base}}$$

Normalization is provided by the constraint that the estimated hits and misses add up to the total number of shots fired i.e.

$$\sum_a x^*_a(w) = \text{No. of shots fired.}$$

As factors are added one at a time, the model takes on the form:

$$\ln\left(\frac{\text{hits}}{\text{misses}}\right) = \tau_{\text{base}} + \sum_{\substack{\text{factors} \\ \text{included}}} \tau_{\text{factor}}$$

After all the important factors are included, any strong interactions are included one at a time, giving a final model of the form

$$\ln\left(\frac{\text{hits}}{\text{misses}}\right) = \tau_{\text{base}} + \sum_{\substack{\text{factors} \\ \text{included}}} \tau_{\text{factor}} + \sum_{\substack{\text{interactions} \\ \text{included}}} \tau_{\text{interactions}}$$

The analyst stops including factors when a reasonable amount of the variation in the data is explained, trying to keep the model as simple as possible.

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4. The τ's are written with superscripts and subscripts to distinguish them. The superscripts show the factors and interactions that are being considered. The subscripts identify the relevant category of factor or interaction. To ensure linear independence, some of the τ's must be set to zero. By convention the last cell in the contingency table (the cell with the highest indices) is used as a reference cell, and as a matter of convenience, the τ's with the largest subscripts are set equal to zero.

5. The actual data used in the analysis are found in Appendix 1. The columns give the number of hits and misses (starting with the number of hits) for each range (for SET A and SET B) or for each firing session (for SET C and SET D). The rows cycle through the gunners (in order of gunner 1 to gunner 5). In SET A and SET B the rows then cycle through the days (day 1 to day 7), followed by the type of shoot (deliberate then snap) then the sight type (first optical, then iron). In SET C the rows cycle through the gunner first, then the day (1 or 2), then the sight type. In SET D the rows cycle through the gunners followed by the sight type.

GROUP 2 - DAY - SET A

SIGHT TYPE	TYPE OF SHOOT	DAY	RANGE / GUNNER	100 H	100 M	200 H	200 M	300 H	300 M	400 H	400 M
Optical 1	Deliberate 1	1	1	10	0	9	1	10	0	5	5
			2	8	2	8	2	8	2	1	9
			3	10	0	10	0	6	4	5	5
			4	10	0	8	2	10	0	5	5
			5	10	0	10	0	10	0	0	10
		2	1	10	0	10	0	7	3	2	8
			2	10	0	10	0	7	1	1	9
			3	10	0	10	0	10	0	3	7
			4	10	0	10	0	7	3	0	10
			5	10	0	9	1	8	2	5	5
		3	1	10	0	10	0	4	6	7	3
			2	10	0	10	0	8	2	6	4
			3	10	0	10	0	5	5	4	6
			4	10	0	10	0	10	0	8	2
			5	10	0	10	0	10	0	7	3
		4	1	10	0	10	0	7	3	8	2
			2	10	0	10	0	10	0	10	0
			3	10	0	10	0	9	1	3	7
			4	10	0	10	0	10	0	5	5
			5	10	0	10	0	9	1	5	5
		5	1	8	2	10	0	10	0	6	4
			2	10	0	10	0	7	3	4	6
			3	10	0	9	1	8	2	7	3
			4	10	0	9	1	8	2	5	5
			5	10	0	8	2	8	2	3	7
		6	1	10	0	9	1	6	4	4	6
			2	10	0	9	1	10	0	7	3
			3	10	0	10	0	9	1	9	1
			4	10	0	10	0	6	4	8	2
			5	10	0	9	1	6	4	8	2
		7	1	10	0	8	2	4	6	3	7
			2	10	0	10	0	0	10	3	7
			3	10	0	10	0	7	3	4	6
			4	10	0	10	0	10	0	6	4
			5	10	0	7	3	7	3	1	9
	Snap 2	1	1	10	0	10	0	8	2	4	6
			2	10	0	7	3	7	3	2	8
			3	10	0	10	0	6	4	6	4
			4	7	3	9	1	5	5	7	3
			5	10	0	10	0	6	4	5	5

SIGHT TYPE	TYPE OF SHOOT	DAY	RANGE / GUNNER	100 H	100 M	200 H	200 M	300 H	300 M	400 H	400 M
Optical 1	Snap 2	2	1	10	0	10	0	8	2	3	7
			2	9	1	8	2	1	9	0	10
			3	10	0	10	0	7	3	2	8
			4	10	0	10	0	6	4	1	9
			5	9	1	9	1	7	3	1	9
		3	1	10	0	10	0	9	1	7	3
			2	10	0	10	0	8	2	3	7
			3	10	0	10	0	8	2	8	2
			4	10	0	9	1	10	0	8	2
			5	10	0	10	0	6	4	7	3
		4	1	10	0	7	3	10	0	7	3
			2	10	0	10	0	10	0	7	3
			3	10	0	10	0	9	1	3	7
			4	10	0	10	0	8	2	5	5
			5	10	0	10	0	9	1	5	5
		5	1	9	1	10	0	4	6	3	7
			2	10	0	10	0	8	2	4	6
			3	10	0	7	3	6	4	7	3
			4	10	0	8	2	7	3	6	4
			5	10	0	10	0	9	1	0	10
		6	1	9	1	10	0	5	5	3	7
			2	9	1	10	0	7	3	5	5
			3	10	0	10	0	9	1	9	1
			4	10	0	10	0	7	3	8	2
			5	10	0	9	1	7	3	1	9
		7	1	10	0	8	2	4	6	3	7
			2	10	0	2	8	0	10	6	4
			3	10	0	9	1	7	3	4	6
			4	10	0	7	3	10	0	8	2
			5	9	1	8	2	4	6	3	7
Iron 2	Deliberate 1	1	1	10	0	9	1	3	7	1	9
			2	9	1	7	3	8	2	5	5
			3	10	0	8	2	5	5	4	6
			4	9	1	9	1	10	0	6	4
			5	10	0	9	1	8	2	5	5
		2	1	10	0	8	2	4	6	1	9
			2	10	0	10	0	8	2	0	10
			3	10	0	10	0	7	3	2	8
			4	9	1	8	2	1	9	8	2
			5	10	0	10	0	9	1	2	8

SIGHT TYPE	TYPE OF SHOOT	DAY	RANGE / GUNNER	100 H	100 M	200 H	200 M	300 H	300 M	400 H	400 M
Iron 2	Deliberate 1	3	1	10	0	10	0	7	3	3	7
			2	10	0	10	0	8	2	3	7
			3	10	0	10	0	8	2	8	2
			4	10	0	10	0	10	0	8	2
			5	10	0	9	1	4	6	2	8
		4	1	10	0	10	0	10	0	7	3
			2	10	0	10	0	7	3	5	5
			3	10	0	10	0	8	2	3	7
			4	10	0	8	2	9	1	7	3
			5	10	0	10	0	4	6	3	7
		5	1	10	0	9	1	6	4	3	7
			2	10	0	10	0	8	2	6	4
			3	10	0	9	1	9	1	2	8
			4	10	0	7	3	5	5	0	10
			5	10	0	10	0	6	4	3	7
		6	1	10	0	9	1	8	2	6	4
			2	10	0	10	0	5	5	7	3
			3	10	0	10	0	5	5	4	6
			4	10	0	8	2	3	7	1	9
			5	10	0	10	0	5	5	3	7
		7	1	10	0	10	0	5	5	4	6
			2	10	0	10	0	6	4	7	3
			3	10	0	8	2	4	6	0	10
			4	10	0	8	2	4	6	0	10
			5	10	0	9	1	6	4	3	7
	Snap 2	1	1	9	1	8	2	9	1	0	10
			2	7	3	6	4	8	2	1	9
			3	10	0	7	3	7	3	6	4
			4	9	1	7	3	9	1	3	7
			5	10	0	9	1	9	1	3	7
		2	1	10	0	8	2	5	5	2	8
			2	10	0	9	1	8	2	8	2
			3	10	0	9	1	10	0	5	5
			4	10	0	8	2	8	2	1	9
			5	10	0	10	0	4	6	7	3
		3	1	10	0	10	0	4	6	4	6
			2	10	0	10	0	4	6	4	6
			3	9	1	8	2	6	4	7	3
			4	10	0	9	1	7	3	9	1
			5	10	0	10	0	5	5	3	7

.../A1-4

SIGHT TYPE	TYPE OF SHOOT	DAY	RANGE / GUNNER	100 H	100 M	200 H	200 M	300 H	300 M	400 H	400 M
Iron 2	Snap 2	4	1	10	0	10	0	6	4	6	4
			2	10	0	10	0	8	2	7	3
			3	10	0	9	1	8	2	4	6
			4	10	0	9	1	9	1	7	3
			5	10	0	8	2	4	6	6	4
		5	1	10	0	10	0	5	5	4	6
			2	10	0	10	0	8	2	4	6
			3	10	0	9	1	8	2	2	8
			4	10	0	8	2	6	4	4	6
			5	10	0	10	0	7	3	4	6
		6	1	10	0	9	1	9	1	3	7
			2	10	0	10	0	7	3	8	2
			3	10	0	9	1	7	3	2	8
			4	10	0	9	1	7	3	2	8
			5	10	0	9	1	8	2	6	4
		7	1	10	0	7	3	6	4	4	6
			2	10	0	10	0	7	3	7	3
			3	10	0	7	3	6	4	4	6
			4	10	0	7	3	6	4	3	7
			5	10	0	10	0	8	2	6	4

GROUP 1 - DAY - SET B

SIGHT TYPE	TYPE OF SHOOT	DAY	RANGE / GUNNER	100 H	100 M	200 H	200 M	300 H	300 M	400 H	400 M
Optical 1	Deliberate 1	1	1	10	0	8	2	9	1	3	7
			2	10	0	10	0	8	2	4	6
			3	10	0	10	0	8	2	5	5
			4	10	0	10	0	10	0	8	2
			5	10	0	10	0	10	0	5	5
		2	1	10	0	10	0	0	10	8	2
			2	9	1	10	0	10	0	3	7
			3	10	0	9	1	4	6	6	4
			4	10	0	10	0	8	2	6	4
			5	10	0	10	0	0	10	8	2
		3	1	10	0	10	0	10	0	6	4
			2	10	0	10	0	10	0	3	7
			3	10	0	10	0	6	4	5	5
			4	10	0	7	3	3	7	0	10
			5	10	0	10	0	10	0	8	2

SIGHT TYPE	TYPE OF SHOOT	DAY	RANGE / GUNNER	100 H	100 M	200 H	200 M	300 H	300 M	400 H	400 M
Optical 1	Deliberate 1	4	1	10	0	10	0	7	3	5	5
			2	10	0	10	0	2	8	5	5
			3	10	0	10	0	8	2	6	4
			4	10	0	8	2	6	4	2	8
			5	10	0	10	0	9	1	7	3
		5	1	10	0	9	1	6	4	6	4
			2	10	0	7	3	6	4	2	8
			3	10	0	8	2	5	5	3	7
			4	10	0	10	0	6	4	2	8
			5	10	0	10	0	8	2	9	1
		6	1	10	0	8	2	9	1	5	5
			2	10	0	8	2	5	5	1	9
			3	10	0	8	2	5	5	3	7
			4	9	1	7	3	6	4	2	8
			5	10	0	7	3	5	5	7	3
		7	1	10	0	8	2	8	2	4	6
			2	9	1	8	2	2	8	0	10
			3	10	0	6	4	6	4	7	3
			4	9	1	9	1	5	5	2	8
			5	10	0	9	1	6	4	3	7
	Snap 2	1	1	10	0	8	2	7	3	4	6
			2	10	0	10	0	7	3	3	7
			3	10	0	9	1	8	2	5	5
			4	9	1	9	1	4	6	2	8
			5	10	0	10	0	10	0	6	4
		2	1	10	0	6	4	7	3	8	2
			2	7	3	10	0	10	0	5	5
			3	10	0	3	7	4	6	2	8
			4	10	0	6	4	7	3	8	2
			5	10	0	8	2	7	3	6	4
		3	1	10	0	10	0	6	4	5	5
			2	10	0	10	0	10	0	4	6
			3	10	0	8	2	4	6	4	6
			4	10	0	9	1	4	6	2	8
			5	10	0	10	0	8	2	6	4
		4	1	10	0	8	2	8	2	6	4
			2	10	0	10	0	10	0	9	1
			3	10	0	9	1	8	2	6	4
			4	8	2	8	2	0	10	4	6
			5	10	0	10	0	9	1	7	3

SIGHT TYPE	TYPE OF SHOOT	DAY	RANGE / GUNNER	100 H	100 M	200 H	200 M	300 H	300 M	400 H	400 M
Optical 1	Snap 2	5	1	10	0	9	1	5	5	3	7
			2	10	0	7	3	9	1	1	9
			3	10	0	7	3	4	6	0	10
			4	9	1	10	0	6	4	6	4
			5	10	0	7	3	10	0	8	2
		6	1	10	0	8	2	9	1	7	3
			2	10	0	5	5	5	5	4	6
			3	9	1	7	3	6	4	2	8
			4	8	2	7	3	6	4	2	8
			5	10	0	9	1	8	2	3	7
		7	1	8	2	8	2	5	5	0	10
			2	9	1	8	2	4	6	2	8
			3	9	1	9	1	7	3	6	4
			4	8	2	8	2	4	6	2	8
			5	10	0	4	6	6	4	1	9
Iron 2	Deliberate 1	1	1	10	0	8	2	9	1	4	6
			2	9	1	10	0	7	3	0	10
			3	10	0	10	0	7	3	8	2
			4	10	0	9	1	6	4	4	6
			5	10	0	10	0	9	1	5	5
		2	1	8	2	10	0	5	5	5	5
			2	10	0	10	0	8	2	3	7
			3	10	0	8	2	8	2	5	5
			4	9	1	10	0	8	2	5	5
			5	8	2	10	0	5	5	5	5
		3	1	10	0	8	2	9	1	3	7
			2	10	0	10	0	7	3	3	7
			3	10	0	7	3	7	3	0	10
			4	10	0	7	3	5	5	1	9
			5	10	0	10	0	4	6	1	9
		4	1	10	0	8	2	10	0	6	4
			2	10	0	10	0	9	1	5	5
			3	10	0	7	3	10	0	4	6
			4	10	0	5	5	8	2	4	6
			5	10	0	10	0	10	0	4	6
		5	1	10	0	9	1	4	6	7	3
			2	10	0	4	6	9	1	1	9
			3	10	0	10	0	8	2	6	4
			4	10	0	6	4	9	1	2	8
			5	9	1	9	1	7	3	1	9

SIGHT TYPE	TYPE OF SHOOT	DAY	RANGE / GUNNER	100 H	100 M	200 H	200 M	300 H	300 M	400 H	400 M
Iron 2	Deliberate 2	6	1	10	0	8	2	7	3	5	5
			2	9	1	6	4	1	9	2	8
			3	10	0	8	2	9	1	0	10
			4	10	0	10	0	4	6	7	3
			5	9	1	6	4	4	6	8	2
		7	1	10	0	10	0	4	6	3	7
			2	10	0	5	5	4	6	3	7
			3	9	1	8	2	5	5	5	5
			4	9	1	8	2	9	1	4	6
			5	10	0	9	1	5	5	2	8
Iron 2	Snap 2	1	1	10	0	10	0	8	2	1	9
			2	10	0	10	0	8	2	1	9
			3	10	0	10	0	9	1	4	6
			4	10	0	9	1	7	3	4	6
			5	10	0	10	0	7	3	7	3
		2	1	9	1	8	2	6	4	2	8
			2	10	0	10	0	8	2	1	9
			3	10	0	10	0	5	5	1	9
			4	9	1	6	4	5	5	3	7
			5	9	1	8	2	6	4	2	8
		3	1	10	0	9	1	4	6	5	5
			2	10	0	10	0	1	9	5	5
			3	9	1	9	1	8	2	6	4
			4	10	0	9	1	3	7	4	6
			5	10	0	9	1	7	3	6	4
		4	1	10	0	8	2	10	0	4	6
			2	10	0	7	3	5	5	3	7
			3	10	0	10	0	8	2	6	4
			4	10	0	8	2	7	3	3	7
			5	10	0	9	1	7	3	6	4
		5	1	10	0	10	0	5	5	4	6
			2	7	3	9	1	8	2	0	10
			3	10	0	9	1	6	4	4	6
			4	10	0	8	2	9	1	2	8
			5	10	0	6	4	2	3	4	6
		6	1	10	0	8	2	6	4	8	2
			2	10	0	6	4	3	7	4	6
			3	10	0	7	3	5	5	1	9
			4	9	1	6	4	4	6	8	2
			5	10	0	7	3	7	3	1	9

SIGHT TYPE	TYPE OF SHOOT	DAY	RANGE / GUNNER	100 H	100 M	200 H	200 M	300 H	300 M	400 H	400 M
Iron 2	Snap 2	7	1	10	0	8	2	6	4	1	9
			2	10	0	4	6	5	5	4	6
			3	9	1	9	1	4	6	1	9
			4	9	1	8	2	8	2	4	6
			5	10	0	10	0	7	3	2	8

GROUP 1 - NIGHT - SET C

SIGHT TYPE	DAY	FIRING SESSION / GUNNER	1 H	1 M	2 H	2 M	3 H	3 M	4 H	4 M	5 H	5 M	6 H	6 M	7 H	7 M	8 H	8 M
			INCREASING DARKNESS →															
Opti-cal	1	1	10	0	10	0	10	0	9	1	10	0	10	0	8	2	6	4
		2	10	0	10	0	10	0	10	0	8	2	9	1	7	3	8	2
		3	9	1	8	2	6	4	6	4	10	0	9	1	10	0	8	2
		4	9	1	10	0	6	4	6	4	10	0	10	0	8	2	6	4
		5	10	0	10	0	8	2	4	6	9	1	10	0	9	1	7	3
	2	1	10	0	10	0	10	0	10	0	10	0	10	0	10	0	10	0
		2	10	0	10	0	9	1	9	1	10	0	10	0	10	0	10	0
		3	10	0	10	0	7	3	10	0	10	0	10	0	7	3	8	2
		4	10	0	10	0	7	3	10	0	10	0	10	0	7	3	8	2
		5	10	0	10	0	8	2	6	4	8	2	4	6	10	0	6	4
Iron 2	1	1	10	0	10	0	10	0	10	0	5	5	0	10	2	8	4	6
		2	9	1	10	0	10	0	8	2	2	8	2	8	3	7	5	5
		3	10	0	7	3	10	0	7	3	2	8	1	9	4	6	1	9
		4	5	5	6	4	10	0	9	1	1	9	3	7	2	8	4	6
		5	9	1	10	0	0	10	3	7	1	9	0	10	0	10	2	8
	2	1	8	2	8	2	0	10	6	4	0	10	0	10	0	10	0	10
		2	10	0	9	1	0	10	5	5	0	10	0	10	0	10	1	9
		3	9	1	9	1	8	2	10	0	2	8	0	10	1	9	2	8
		4	7	3	5	5	0	10	0	10	1	9	1	9	1	9	3	7
		5	8	2	8	2	6	4	5	5	1	9	0	10	5	5	3	7

GROUP 2 - NIGHT - SET D

SIGHT TYPE	FIRING SESSION / GUNNER	1		2		3		4		5		6	
		H	M	H	M	H	M	H	M	H	M	H	M
Optical 1	1	2	8	1	9	1	9	0	10	0	10	2	8
	2	9	1	10	0	7	3	9	1	1	9	10	0
	3	10	0	8	2	1	9	5	5	5	5	0	10
	4	10	0	10	0	10	0	6	4	0	10	6	4
	5	10	0	10	0	10	0	6	4	3	7	4	6
Iron 2	1	3	7	10	0	2	8	1	9				
	2	10	0	10	0	0	10	0	10				
	3	0	10	0	10	4	6	0	10				
	4	6	4	6	4	0	10	0	10				
	5	7	3	0	10	0	10	1	9				

Increasing Darkness →

APPENDIX 2
TO ANNEX A
TO ORAE PROJECT REPORT PR430
DATED JULY 1987

TABLE 2-A

CONTAB RESULTS FOR SET A

SET A MODEL	DEGREES OF FREEDOM	INFORMATION STATISTIC	PROBABILITY
Base (Score only)	559	2620.3	-
Main effects			
1. score and range	556	1111.6	0.9999
2. score and day	553	2549.6	0.9999
3. score and sight type	558	2608.6	0.9994
4. score and gunner	555	2613.7	0.8415
5. score and type of shoot	558	2619.7	0.5747
Main effects: score and range, day and sight type	549	1001.6	-
Main effects and first order interactions			
1. day by range	531	923.0	0.9999
2. day by gunner	521	912.8	0.9999
3. sight type by gunner	541	962.2	0.9999
4. sight type by day	543	981.6	0.9973
5. sight type by type of shoot	547	989.9	0.9972
6. type of shoot by range	545	989.0	0.9867

TABLE 2-B

CONTAB RESULTS FOR SET B

SET B MODEL	DEGREES OF FREEDOM	INFORMATION STATISTIC	PROBABILITY
Base (Score only)	559	2587.6	-
Main effects			
1. score and range	556	1188.1	0.9999
2. score and day	553	2515.3	0.9999
3. score and gunner	555	2556.1	0.9999
4. score and type of shoot	558	2582.8	0.9721
5. score and sight type	558	2583.8	0.9501
Main effects: score and range, day, gunner, type of shoot and sight type	544	1040.7	-
Main effects and first order interactions			
1. day by gunner	520	929.5	0.9999
2. day by range	526	982.0	0.9999
3. sight type by gunner	540	1017.3	0.9999
4. gunner by range	532	1026.7	?
5. type of shoot by gunner	540	1033.4	0.8783
6. sight type by range	541	1035.8	0.8226
7. sight type by day	538	1031.9	0.8117
8. type of shoot by range	541	1037.5	0.6385
9. type of shoot by day	538	1034.7	0.5774
10. sight type by type of shoot	543	1040.3	0.4924

TABLE 2-C

CONTAB RESULTS FOR SET C

SET C MODEL	DEGREES OF FREEDOM	INFORMATION STATISTIC	PROBABILITY
Base (Score only)	159	1155.1	-
Main effects			
1. score and sight type	158	774.8	0.9999
2. score and firing session	152	948.1	0.9999
3. score and gunner	155	1138.6	0.9975
4. score and day	158	1149.7	0.9798
Main effects: score and sight type, firing session, gunner and day	146	474.2	-
Main effects and first order interactions			
1. sight type by firing session	139	376.6	0.9999
2. gunner by firing session	118	412.0	0.9999
3. sight type by day	145	447.5	0.9999
4. day by gunner	142	457.5	0.9978
5. sight type by gunner	142	459.4	0.9949
6. day by firing session	139	458.5	0.9720

.../A2-4

APPENDIX 2
TO ANNEX A
TO ORAE PROJECT REPORT PR430

TABLE 2-D

CONTAB RESULTS FOR SET D

SET D MODEL	DEGREES OF FREEDOM	INFORMATION STATISTIC	PROBABILITY
Base (Score only)	49	425.7	-
Main effects			
1. score and firing session	44	357.3	0.9999
2. score and sight type	48	394.0	0.9999
3. score and gunner	45	374.1	0.9999
Main effects: score and firing session, sight type and gunner	39	219.6	-
Main effects and first order interactions			
1. gunner by firing session	19	149.5	0.9999
2. gunner by sight type	35	153.6	0.9999
3. sight type by firing session	38	211.3	0.9961

APPENDIX 3
TO ANNEX A
TO ORAE PROJECT REPORT PR430
DATED JULY 1987

TABLE 3-A

SET A - ANALYSIS OF INFORMATION TABLE

Total Information = 2620.3

FACTOR	INFORMATION	INFORMATION %	DEGREES OF FREEDOM	SIGNIFICANCE PROBABILITY
range	1508.7	57.6	3	<.001
day	70.7	2.7	6	<.001
sight type	11.7	0.4	1	<.001
day*range	78.6	3.0	18	<.001
day*gunner	88.8	3.4	28	<.001
sight type*gunner	39.4	1.5	8	<.001
sight type*day	20.0	0.8	6	.001 - .005
sight type*type of shoot	11.7	0.4	2	.001 - .005
type of shoot*range	12.6	0.5	4	.01 - .02

TABLE 3-B

SET B - ANALYSIS OF INFORMATION TABLE

Total Information - 2587.6

FACTOR	INFORMATION	INFORMATION %	DEGREES OF FREEDOM	SIGNIFICANCE PROBABILITY
range	1399.5	54.1	3	<.001
day	72.3	2.8	6	<.001
gunner	31.5	1.2	4	<.001
type of shoot	4.8	0.2	1	≃.025 - .05
sight type	3.8	0.1	1	≃.05
day*gunner	111.2	4.3	24	<.001
day*range	58.7	2.3	18	<.001
sight type*gunner	23.4	0.9	4	<.001

TABLE 3-C

SET C - ANALYSIS OF INFORMATION TABLE

Total Information = 1155.1

FACTOR	INFORMATION	INFORMATION %	DEGREES OF FREEDOM	SIGNIFICANCE PROBABILITY
sight type	380.3	32.9	1	<.001
firing session	207.0	17.9	7	<.001
gunner	16.5	1.4	4	.001 - .005
day	5.4	0.5	1	≃.02
sight type*firing session	97.6	8.4	7	<.001
gunner*firing session	62.2	5.4	28	<.001
sight type*day	26.7	2.3	1	<.001
day*gunner	16.7	1.4	4	.001 - .005
day*firing session	15.7	1.4	7	.025 - .05
sight type*gunner	14.8	1.3	4	≃.005

TABLE 3-D

SET D - ANALYSIS OF INFORMATION TABLE

Total Information - 425.7

FACTOR	INFORMATION	INFORMATION %	DEGREES OF FREEDOM	SIGNIFICANCE PROBABILITY
firing session	68.4	16.1	5	<.001
gunner	51.6	12.1	4	<.001
sight type	31.7	7.4	1	<.001
gunner*firing session	70.1	16.5	20	<.001
gunner*sight type	66.0	15.5	4	<.001
sight type*firing session	8.3	1.9	1	.001 - .005

TABLE 4-A

NEWKULL RESULTS FOR SET A

MODEL	INFORMATION STATISTIC	% VARIATION EXPLAINED	OUTLIERS	NO. OF CONSTRAINTS	DEGREES OF FREEDOM
a (base)	2620.3		64	561	559
a1 (base+range)	1111.6	57.6	39	564	556
a2 (base+range+day)	1017.5	61.2	34	570	550
a3 (base+range+day +sight type)	1001.6	61.8	34	571	549
a3a (base+range+day+ day*range)	939.2	64.2	?	588	532
a4 (base+range+day+ sight type + day*range)	923.1	64.8	28	589	531
a5 (base+range+day+ sight type + day*range+ day*gunner)	832.8	68.2	21	617	503
a6 (base+range+day+ sight type+ day*range+ day*gunner+sight type*gunner)	800.8	69.4	18	621	499
OUTLIERS CONSTRAINED			% REMAINING UNEXPLAINED VARIATION ATTRIBUTABLE TO OUTLIERS		
a6	568.2	78.3	29%	639	481

Model $\ln \dfrac{x^*(ghijk1)}{x^*(ghijk2)} = \tau^{\ell}_{1} + \tau^{k\ell}_{k1} + \tau^{i\ell}_{i1} + \tau^{g\ell}_{g1} + \tau^{ik\ell}_{ik1} + \tau^{ij\ell}_{ij1} + \tau^{gj\ell}_{gj1}$

.../A4-2

TABLE 4-B

NEWKULL RESULTS FOR SET B

MODEL	INFORMATION STATISTIC	% VARIATION EXPLAINED	OUTLIERS	NO. OF CONSTRAINTS	DEGREES OF FREEDOM
b (base)	2587.6		70	561	559
b1 (base+range)	1188.1	54.1	38	564	556
b2 (base+range+day)	1094.1	57.7	33	570	550
b3 (base+range+day+ gunner)	1052.3	59.3	34	574	546
b4 (base+range+day+ gunner+type of shoot)	1045.8	59.6	34	575	545
b5 (base+range+day+ gunner+type of shoot+sight type)	1040.7	59.8	32	576	544
b6 (base+range+day+ gunner+type of shoot+sight type +day*gunner)	929.5	64.1	17	600	520
b7 (b6+day*range)	867.9	66.5	19	618	502
b8 (b6+day*range+ sight type* gunner)	843.7	67.4	13	622	498
OUTLIERS CONSTRAINED			% REMAINING UNEXPLAINED VARIATIONS ATTRIBUTABLE TO OUTLIERS		
b8	661.6	74.4	22%	635	485
b6	710.9	72.5	24%	617	503

Model $\ln \dfrac{x^*(ghijkl)}{x^*(ghijk2)} = \tau^{\ell}_{1} + \tau^{k\ell}_{k1} + \tau^{i\ell}_{il} + \tau^{j\ell}_{jl} + \tau^{h\ell}_{hl} + \tau^{g\ell}_{gl} + \tau^{ij\ell}_{ijl} + \tau^{ik\ell}_{ikl} + \tau^{gj\ell}_{gjl}$

.../A4-3

TABLE 4-C

NEWKULL RESULTS FOR SET C

MODEL	INFORMATION STATISTIC	% VARIATION EXPLAINED	OUTLIERS	NO. OF CONSTRAINTS	DEGREES OF FREEDOM
c (base)	1155.1			161	159
c1 (base+sight type)	774.8	32.9		162	158
c2 (base+sight type+firing session)	509.5	55.9	20	169	151
c3 (base+sight type +firing session+ gunner)	483.0	58.2	19	173	147
c4 (base+sight type +firing session+ gunner+day)	474.2	58.9	21	174	146
c5 (base+sight type +firing session+ gunner+day+firing session*sight type)	376.6	67.4	14	181	139
c6 (c5+gunner* firing session)	310.6	73.1	11	209	111
c7 (c5+gunner* firing session+ sight type*day)	278.3	75.9	8	210	110
OUTLIERS CONSTRAINED			% REMAINING UNEXPLAINED VARIATION ATTRIBUTABLE TO OUTLIERS		
c5	203.0	82.4	46	195	115
c7	138.8	88.0	50	218	102

Model $\ln \dfrac{x^*(gijkl)}{x^*(gijk2)} = \tau^{l}_{1} + \tau^{gl}_{gl} + \tau^{kl}_{kl} + \tau^{jl}_{jl} + \tau^{il}_{il} + \tau^{gkl}_{gkl} + \tau^{jkl}_{jkl} + \tau^{gil}_{gil}$

.../A4-4

TABLE 4-D

NEWKULL RESULTS FOR SET D

MODEL	INFORMATION STATISTIC	% VARIATION EXPLAINED	OUTLIERS	NO. OF CONSTRAINTS	DEGREES OF FREEDOM
d (base)	425.7	-	27	51	49
d1 (base+firing session)	357.3	16.1	22	56	44
d2 (base+firing session+sight type)	290.1	31.9	16	57	43
d3 (base+firing session+sight type+gunner)	219.6	48.4	10	61	39
d4 (base+firing session+sight type+gunner+ gunner*firing session)	149.7	64.8	3	81	19
d5 (d4+gunner*sight type)	60.2	85.9	1	85	15
OUTLIERS CONSTRAINED			% REMAINING UNEXPLAINED VARIATION ATTRIBUTABLE TO OUTLIERS		
d5	29.8	93.0	50.5	86	14

$$\text{Model } \ln \frac{x^*(gjkl)}{x^*(gjk2)} = \tau^{\ell}_{l} + \tau^{g\ell}_{gl} + \tau^{j\ell}_{jl} + \tau^{k\ell}_{kl} + \tau^{jk\ell}_{jkl} + \tau^{gj\ell}_{gjl}$$

OUTLIERS FROM NEWKULL RESULTS

a6	b6	b8	c5	c7	d5
ghijkℓ	ghijkℓ	ghijkℓ	gijkℓ	gijkℓ	gjkℓ
111541	112131	111441	11321	11561	2331
113531	112531	112131	12441	21431	
114241	114231	112531	12561	21531	
115111	122141	114231	21131	21571	
117231	122211	122211	21231	22131	
121411	122321	122321	21331	22231	
121431	124241	124431	21431	22341	
122231	124431	127521	21531	22341	
124121	125341	213341	22131		
125541	127521	216341	22231		
127221	211241	216541	22341		
127231	213341	223231	22431		
211131	213541	226441	22431		
212441	215541		22571		
217341	216541				
217441	223231				
221141	226441				
222241					

ANNEX B
TO ORAE PROJECT REPORT PR430
DATED JULY 1987

RELATIVE ODDS CALCULATIONS

1. The model found using MDI analysis can also be used to make comparisons between the optical and iron sights under various conditions. If the model for the optical sight is:

$$\ln\left(\frac{\text{hit*}}{\text{miss*}}\right)_{\text{optical}} = \text{base} + \sum_{\substack{\text{factors included} \\ \text{sight type = optical}}} \tau_{\text{factor}} + \sum_{\substack{\text{interactions included} \\ \text{sight type = optical}}} \tau_{\text{interaction}}$$

and the model for the iron sight is:

$$\ln\left(\frac{\text{hit*}}{\text{miss*}}\right)_{\text{iron}} = \text{base} + \sum_{\substack{\text{factors included} \\ \text{sight type = iron}}} \tau_{\text{factor}} + \sum_{\substack{\text{interactions included} \\ \text{sight type = iron}}} \tau_{\text{interaction}}$$

then the difference between these two values, X, is the log odds ratio of the optic and iron sights, and exp(X) is the odds ratio. Since $z = X/\sqrt{\text{var}(X)}$ is approximately normally distributed, the significance of the odds ratio can be evaluated. The τ's and their variances are found in the outputs from NEWKULL. The exp(τ) values for the various factors and interactions are given in Appendix 1 to Annex B.

CALCULATIONS OF RELATIVE ODDS

SET A

2. The computer program stopped before the covariance matrix for the τ's was calculated, so the statistical significance of the various combinations cannot be determined. There is a gunner by sight-type interaction.

a. If optic and iron sight are fired by gunner 1 at same range, at same type of shoot (deliberate or snap) and on same day.

$$X = \ln \frac{x^*(1hi1k1)}{x^*(1hi1k2)} - \ln \frac{x^*(2hi1k1)}{x^*(2hi1k2)}$$

$$= \underset{11}{\tau g\ell} + \underset{111}{\tau gj\ell} - \underset{21}{\tau g\ell} - \underset{211}{\tau gj\ell}$$

$$= 0.1879 + 0.2001 - 0 - 0$$

$$= 0.3880$$

The odds of hitting a target with the optical sight are exp(0.388) or 1.47 times better than those with the iron sight.

b. If optic and iron sight are fired by gunner 2 at same range, at same type of shoot (deliberate or snap) and on same day.

$$X = \ln \frac{x^*(1hi2k1)}{x^*(1hi2k2)} - \ln \frac{x^*(2hi2k1)}{x^*(2hi2k2)}$$

$$= \underset{11}{\tau g\ell} + \underset{121}{\tau gj\ell} - \underset{21}{\tau g\ell} - \underset{221}{\tau gj\ell}$$

$$= 0.1879 - 0.1496 - 0 - 0$$

$$= 0.0383$$

.../B-3

The odds of hitting a target with the optical sight are exp(0.0383) or 1.04 times better than with the iron sight.

c. If optic and iron sights are fired by gunner 3 at same range, at same type of shoot (deliberate or snap) and on same day.

$$X = \ln \frac{x^*(1hi3k1)}{x^*(1hi3k2)} - \ln \frac{x^*(2hi3k1)}{x^*(2hi3k2)}$$

$$= \tau g\ell_{11} + \tau gj\ell_{131} - \tau g\ell_{21} - \tau gj\ell_{231}$$

$$= 0.1879 + 0.3729$$

$$= 0.5608$$

The odds of hitting a target with the optical sight are exp(0.5608) or 1.75 times better than those with the iron sight.

d. If optic and iron sights are fired by gunner 4 at same range at same type of shoot (deliberate or snap) and on same day.

$$X = \ln \frac{x^*(1hi4k1)}{x^*(1hi4k2)} - \ln \frac{x^*(2hi4k1)}{x^*(2hi4k2)}$$

$$= \tau g\ell_{11} + \tau gj\ell_{141} - \tau g\ell_{21} - \tau gj\ell_{241}$$

$$= 0.1879 + 0.8572$$

$$= 1.0451$$

The odds of hitting a target with the optical sight are exp(1.0451) or 2.84 times better than those with the iron sight.

e. If optic and iron sights are fired by gunner 5 at same range, at same type of shoot (deliberate or snap) and on same day.

$$X = \ln \frac{x^*(1hi5k1)}{x^*(1hi5k2)} - \ln \frac{x^*(2hi5k1)}{x^*(2hi5k2)}$$

$$= \tau_{11}^{g\ell} + \tau_{151}^{gj\ell} - \tau_{21}^{g\ell} - \tau_{251}^{gj\ell}$$

$$= 0.1879 + 0 - 0 - 0$$

$$= 0.1879$$

The odds of hitting a target with the optical sight are exp(0.1879) or 1.21 times better than those with the iron sight.

These results are shown in Table IV-A of the main text (p. 16).

SET B

3. There is a gunner by sight type interaction.

a. If optic and iron sights are fired by gunner 1 at the same range, at the same type of shoot (deliberate or snap) and on the same day.

$$X = \ln \frac{x^*(1hi1k1)}{x^*(1hi1k2)} - \ln \frac{x^*(2hi1k1)}{x^*(2hi1k2)}$$

$$= \tau_{11}^{g\ell} + \tau_{111}^{gj\ell} - \tau_{21}^{g\ell} - \tau_{211}^{gj\ell}$$

$$= 0.9409 - 0.6634 - 0 - 0$$

$$= 0.2775$$

The odds of hitting a target with the optical sight are exp(0.2775) or 1.32 times better than those with the iron sight. To determine if this value is statistically significant calculate

$$\text{var}(X) = \text{var}(\tau_{11}^{g\ell} + \tau_{111}^{gj\ell} - \tau_{21}^{g\ell} - \tau_{211}^{gj\ell})$$

$$= \text{var}(\tau_{11}^{g\ell} + \text{var}(\tau_{111}^{gj\ell}) + \text{var}(\tau_{21}^{g\ell}) + \text{var}(\tau_{211}^{gj\ell}) +$$

$$2\text{cov}(\tau_{11}^{g\ell}\ \tau_{111}^{gj\ell}) + 2\text{cov}(\tau_{21}^{g\ell}\ \tau_{211}^{gj\ell}) -$$

$$2\text{cov}(\tau_{11}^{g\ell}\ \tau_{21}^{g\ell}) - 2\text{cov}(\tau_{11}^{g\ell}\tau_{211}^{gj\ell}) -$$

$$2\text{cov}(\tau_{111}^{gj\ell}\tau_{21}^{g\ell}) - 2\text{cov}(\tau_{111}^{gj\ell}\tau_{211}^{gj\ell})$$

$$\text{var}(\tau_{11}^{g\ell}) = 0.02987$$

$$\text{var}(\tau_{111}^{gj\ell}) = 0.05635$$

$$\text{var}(\tau_{21}^{g\ell}) = 0.00000$$

$$\text{var}(\tau_{211}^{gj\ell}) = 0.00000$$

$$\text{var}(\tau\text{'s}) = 0.08632$$

$$2\text{cov}(\tau_{11}^{g\ell}\tau_{111}^{gj\ell}) = 2(-0.02990) \text{ (other covariances are zero)}$$

$$\therefore \text{var}(X) = 0.08632 - 2(0.02980) = 0.02652$$

$$Z = \frac{0.2775}{\sqrt{0.02652}} = 1.70 \qquad \text{not significant}$$

b. If optic and iron sights are fired by gunner 2 at the same range, at the same type of shoot (deliberate or snap) and on the same day.

$X = 0.9409 - 0.4324 = 0.5085$

The odds of hitting a target with the optical sight are exp(0.5085) or 1.66 times better than those with the iron sight.

$var(X) = 0.02997 + 0.05562 - 2(0.02987) = 0.02585$

$$Z = \frac{0.5085}{\sqrt{0.02585}} = 3.16$$ significant at the 0.2% level

c. If optic and iron sights are fired by gunner 3 at the same range, at the same type of shoot (deliberate or snap) and on the same day.

$X = 0.9409 - 1.1640 = -0.2231$

The odds of hitting a target with the optical sight are exp(-0.2231) or 0.80 times as good as those with the iron sight.

$var(X) = 0.02997 + 0.05611 - 2(0.02996) = 0.02616$

$$Z = \frac{-0.2331}{\sqrt{0.02616}} = -1.44$$ not significant

d. If optic and iron sights are fired by gunner 4 at the same range, at the same type of shoot (deliberate or snap) and on the same day

$X = 0.9409 - 1.1095 = -0.1686$

The odds of hitting a target with the optical sight are exp(-0.1686) or 0.84 times as good as those with the iron sight.

var(X) = 0.02997 + 0.05479 - 2(0.02998) = 0.02480

$$Z = \frac{-0.1686}{\sqrt{0.05478}} = -0.7204$$ not significant

e. If optic and iron sights are fired by gunner 5 at the same range, at the same type of shoot (deliberate or snap) and on the same day

X = 0.9409

The odds of hitting a target with the optical sight are exp(0.9409) or 2.56 times better than those with the iron sight.

var(X) = 0.02997

$$Z = \frac{0.9409}{\sqrt{0.02997}} = 5.43$$ significant at better than .005% level

These results are shown in Table IV-B of the main text (P. 17).

SET C

4. There are two interactions to consider: sight type with day, and sight type with firing session.

a. Sight Type with Day

(1) If optic and iron sights are fired by the same gunner, at the same approximate time, on day 1

$$X = \ln \frac{x^*(11jk1)}{x^*(11jk2)} - \ln \frac{x^*(21jk1)}{x^*(21jk2)}$$

$$= \underset{11}{\tau g\ell} + \underset{111}{\tau gi\ell} - \underset{21}{\tau g\ell} - \underset{211}{\tau gj\ell}$$

$$= 3.4176 - 1.9006$$

$$= 1.5170$$

The odds of hitting a target with the optical sight are exp(1.5170) or 4.56 times better than with the iron sight.

$$\text{var}(X) = 0.1634 + 0.1165 - 2(0.0725) = 0.1349$$

$$Z = \frac{1.5170}{\sqrt{0.1349}} = 4.13$$ significant at .005% level

(2) If optic and iron sights are fired by the same gunner, at about the same time, on day 2

$$X = 3.4176$$

The odds of hitting a target with the optical sight are exp(3.4176) or 30.5 times better than with the iron sight.

$$\text{var}(X) = 0.1634$$

$$Z = \frac{3.4176}{\sqrt{0.1634}} = 8.45$$ significant at better than .005% level.

b. Sight type with firing session (later session numbers were fired later in the evening).

(1) If optic and iron sights are fired by the same gunner, at the first firing session, on the same day.

$$X = \ln \frac{x^*(1ij11)}{x^*(1ij12)} - \ln \frac{x^*(2ij11)}{x^*(2ij12)}$$

$$= \tau^{g\ell}_{11} + \tau^{gk\ell}_{111} - \tau^{g\ell}_{21} - \tau^{gk\ell}_{211}$$

$$= 3.4176 - 0.1526$$

$$= 3.2650$$

The odds of hitting a target with the optical sight are exp(3.2650) or 26.2 times better than with the iron sight.

$$\text{var}(X) = 0.1634 + 0.7506 - 2(0.1235) = 0.6670$$

$$Z = \frac{3.2650}{\sqrt{0.6670}} = 4.00 \qquad \text{significant at .01\% level}$$

(2) 2nd firing session

$$X = 3.4176 + .0736 = 3.4912$$

The odds of hitting a target with the optical sight are exp(3.4912) or 32.8 times better than with the iron sight.

$$\text{var}(X) = 0.1634 + 0.7316 - 2(0.1230) = 0.6490$$

$$Z = \frac{3.4912}{\sqrt{0.6490}} = 4.33 \qquad \text{significant at .005\% level}$$

(3) 3rd firing session

$X = 3.4176 - 2.1699 = 1.2477$

The odds of hitting a target with the optical sight are exp(1.2477) or 3.48 times better than with the iron sight.

$var(X) = 0.1634 + 0.3421 - 2(0.1340) = 0.2375$

$$Z = \frac{1.2477}{\sqrt{0.2375}} = 2.56$$ significant at 1% level

(4) 4th firing session

$X = 3.4176 - 1.5081 = 1.9095$

The odds of hitting a target with the optical sight are exp(1.9095) or 6.75 times better than with the iron sight.

$var(X) = 0.1634 + 0.2667 - 2(0.1182) = 0.1937$

$$Z = \frac{1.9095}{\sqrt{0.1937}} = 4.35$$ significant at .005% level

(5) 5th firing session

$X = 3.4176 + 2.7983 = 6.2159$

The odds of hitting a target with the optical sight are exp(6.2159) or 500 times better than with the iron sight.

$var(X) = 0.1634 + 0.5176 - 2(0.1338) = 0.4134$

$$Z = \frac{6.2159}{\sqrt{0.4134}} = 9.67$$ significant at better than .001% level

(6) 6th firing session

$X = 3.4176 + 4.2455 = 7.6631$

The odds of hitting a target with the optical sight are exp(7.6631) or 2130 times better with the iron sight.

$var(X) = 0.1634 + 1.0072 - 2(0.1128) = 0.9450$

$$Z = \frac{7.6631}{\sqrt{0.9450}} = 7.88$$ significant at better than .005% level

(7) 7th firing session

$X = 3.4176 + 0.9174 = 4.3350$

The odds of hitting a target with the optical sight are exp(4.3350) or 76.3 times better than with the iron sight.

$var(X) = 0.1634 + 0.2949 - 2(0.1175) = 0.2233$

$$Z = \frac{4.3350}{\sqrt{0.2233}} = 9.17$$ significant at better than .001% level

(8) 8th firing session $X = 3.4176$

$var(X) = 0.1634$

$$Z = \frac{3.4176}{\sqrt{0.1634}} = 8.45$$ significant at better than .005% level

These results are shown in Tables IV-C-1 and IV-C-2 of the main text (pp 17-18).

SET D

5. There is a gunner by sight type interaction.

a. If optic and iron sights are fired by gunner 1 at about the same time

$$X = \ln \frac{x^*(11k1)}{x^*(11k2)} - \ln \frac{x^*(21k1)}{x^*(21k2)}$$

$$= \tau^{g\ell}_{11} + \tau^{gj\ell}_{111} - \tau^{g\ell}_{21} - \tau^{gj\ell}_{211}$$

$$= 6.0294 - 8.3806$$

$$= -2.3512$$

The odds of hitting a target with the optical sight are exp(-2.3512) or 0.095 times as good as with the iron sight. To determine if this value is statistically significant, calculate

$$\text{var}(X) = \text{var}(\tau^{g\ell}_{11} + \tau^{gj\ell}_{111} - \tau^{g\ell}_{21} - \tau^{gj\ell}_{211})$$

$$\text{var}(\tau^{g\ell}_{11}) = 1.9656$$

$$\text{var}(\tau^{gj\ell}_{111}) = 2.5396$$

$$\text{cov}(\tau^{g\ell}_{11}\ \tau^{gj\ell}_{111}) = -1.9656$$

$$\text{var}(X) = 1.9656 + 2.5396 - 2(1.9656) = 0.5740$$

$$Z = \frac{-2.2882}{\sqrt{0.5740}} = -3.02 \qquad \text{significant at 0.3\% level}$$

.../B-13

b. If the optic and iron sights are fired by gunner 2 at about the same time

$$X = 6.0294 - 1.8293 = 4.2001$$

The odds of hitting a target with the optical sight are exp(4.2001) or 66.7 times better than with the iron sight. To determine if this value is statistically significant, calculate

$$\text{var}(X) = 1.9656 + 3.3737 - 2(1.9656) = 1.4081$$

$$Z = \frac{4.2001}{\sqrt{1.4081}} = 3.54 \qquad \text{significant at .04\% level}$$

c. If the optic and iron sights are fired by gunner 3 at about the same time

$$X = 6.0294 + 6.1216 = 12.1510$$

The odds of hitting a target with the optical sight are exp(12.1510) or 189×10^3 better than with the iron sight. To determine if this is statistically significant, calculate

$$\text{var}(X) = 1.9656 + 61.1144 - 2(1.9656) = 59.1488$$

$$Z = \frac{12.510}{\sqrt{59.1488}} = 1.63 \qquad \text{not significant}$$

d. If the optic and iron sights are fired by gunner 4 at about the same time

$$X = 6.0294 + 5.4752 = 11.5046$$

The odds of hitting a target with the optical sight are exp(11.5046) = 99.2 x 10^3 better than with the iron sight. To determine if this is statistically significant, calculate:

$$var(X) = 1.9656 + 45.0632 - 2(1.9656) = 43.0976$$

$$Z = \frac{11.5046}{\sqrt{45.0632}} = 1.75 \qquad \text{not significant}$$

e. If the optic and iron sights are fired by gunner 5 at about the same time

$$X = 6.0294$$

The odds of hitting a target with the optical sight are exp(6.0294) or 416 times better than with the iron sight. To determine if this is statistically significant, calculate:

$$var(X) = 1.9656$$

$$Z = \frac{6.0294}{\sqrt{1.9656}} = 4.30 \qquad \text{significant at .005\% level.}$$

These results are shown in Table IV-D of the main text (p.18).

ODDS CALCULATIONS

6. The model can also be used to calculate the odds for any combination of factors by using the profile tables which follow on pages B1-1 to B1-6. For example, for SET A, gunner 2, using the optic sight on day 3 at a range of 200 m would have odds of:

$$\frac{x^*(123221)}{x^*(123222)} = 0.5226 \times 1.2067 \times 1.4177 \times 9.0397 \times 3.4812$$

$$\times 1.3695 \times 0.8611$$

$$= 33.2$$

By selecting combinations of characteristics which have large values of exp(τ), the best odds for hitting a target can be found. Similarly by choosing the characteristics with the lowest values of exp(τ), the worst odds can be found. It is necessary to consider each factor and its interactions when doing the calculations.

APPENDIX 1
TO ANNEX B
TO ORAE PROJECT REPORT PR430
DATED JULY 1987

SET A

PROFILE OF FACTORS WHICH DETERMINE THE RELATIVE ODDS OF HITTING A TARGET

MAIN EFFECTS

BASE	SIGHT TYPE (g)		DAY (i)		RANGE(k) (METRES)	
0.5226*	Optic	1.2067	1	1.7143	100	296.7380*
	Iron	1.0000	2	0.6098	200	9.0397*
			3	1.4177	300	2.3862*
			4	1.0939	400	1.0000
			5	1.2237		
			6	1.5066		
			7	1.0000		

*significant at at least 5% level

INTERACTIONS

DAY BY RANGE

	100	200	300	400
1	0.1148	0.9944	2.7336*	1.0000
2	1.1678	7.5617*	4.4818*	1.0000
3	0.5434	3.4812*	0.6821	1.0000
4	21.1536	3.5125*	1.8736	1.0000
5	1.0308	1.8811	1.6019	1.0000
6	0.3374	1.9043	0.8468	1.0000
7	1.0000	1.0000	1.0000	1.0000

*significant at at least 5% level

DAY BY GUNNER

	1	2	3	4	5
1	0.7008	0.3369*	0.5413	0.7176	1.0000
2	0.4459*	0.5818	1.1414	0.2049*	1.0000
3	1.2116	1.3695	1.5184	4.1233*	1.0000
4	1.8877	3.0539*	0.9809	1.5375	1.0000
5	0.7714	1.5057	0.8632	0.4283*	1.0000
6	0.8670	2.0475*	1.5165	0.6088	1.0000
7	0.7408	2.5694*	0.9137	1.1062	1.0000

*significant at at least 5% level

SIGHT TYPE BY GUNNER

	1	2	3	4	5
Optic	1.2215	0.8611	1.4519	2.3566*	1.0000
Iron	1.0000	1.0000	1.0000	1.0000	1.0000

*significant at at least 5% level

SET B

PROFILE OF FACTORS WHICH DETERMINE THE RELATIVE ODDS OF HITTING A TARGET

MAIN EFFECTS

BASE	SIGHT TYPE (g)	TYPE OF SHOOT (h)	DAY (i)	GUNNER (j)	RANGE(K) (METRES)
0.2705*	Optic 2.5622*	Deliberate 1.2065*	1 3.1675*	1 1.0885	100 45.3134*
	Iron 1.0000	Snap 1.0000	2 1.8717	2 0.5322*	200 11.5805*
			3 2.2592*	3 1.8386	300 3.2861*
			4 3.8308*	4 1.5271	400 1.0000
			5 1.9068	5 1.0000	
			6 1.1990		
			7 1.0000		

*significant at at least 5% level

INTERACTIONS

DAY BY GUNNER

	1	2	3	4	5
1	0.3679*	0.6049	0.6248	0.2750*	1.0000
2	1.2067	3.0611*	0.6393	1.1039	1.0000
3	1.0115	2.2786	0.5287	0.2189*	1.0000
4	0.7734	1.5760	0.6526	0.2515*	1.0000
5	0.8657	0.8010	0.4925	0.7078	1.0000
6	3.0495*	1.2697	0.8967	0.9180	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000

*significant at at least 5% level

DAY BY RANGE

	100	200	300	400
1	4.1659	3.0970*	2.0690*	1.0000
2	0.5641	0.8283	0.7855	1.0000
3	8.6054*	1.6286	0.9652	1.0000
4	2.4859	0.6575	1.4860	1.0000
5	1.7490	0.7774	1.2772	1.0000
6	1.2263	0.4429*	0.7320	1.0000
7	1.0000	1.0000	1.0000	1.0000

*significant at at least 5% level

SIGHT TYPE BY GUNNER

	1	2	3	4	5
Optic	0.5151*	0.6490	0.3122*	0.3297*	1.0000
Iron	1.0000	1.0000	1.0000	1.0000	1.0000

*significant at at least 5% level

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SET C

PROFILE OF FACTORS WHICH DETERMINE THE RELATIVE ODDS OF HITTING A TARGET

MAIN EFFECTS

BASE	SIGHT TYPE (g)	DAY (i)	GUNNER (j)	FIRING SESSION (k)
0.1270*	Optic 30.4974*	1 2.8779*	1 1.3338	1 32.8721*
	Iron 1.0000	2 1.0000	2 2.3871	2 51.0495*
			3 1.1549	3 13.3113*
			4 1.5408	4 2.2870
			5 1.0000	5 0.2261
				6 0.002454*
				7 6.4788*
				8 1.0000

*significant at at least 5% level

INTERACTIONS

FIRING SESSION BY SIGHT TYPE

	OPTIC	IRON
1	0.8585	1.0000
2	1.0764	1.0000
3	0.1142*	1.0000
4	0.2213*	1.0000
5	16.4161*	1.0000
6	69.7924*	1.0000
7	2.5028	1.0000
8	1.0000	1.0000

*significant at at least 5% level

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FIRING SESSION BY GUNNER

	1	2	3	4	5
1	1.1895	1.4036	1.3737	0.1490*	1.0000
2	0.7497	0.8864	0.2258	0.0919*	1.0000
3	236.3555	3.1041	0.9597	0.1670*	1.0000
4	7.8165*	2.4058	2.6661	3.0676	1.0000
5	5.2299	0.6055	4.6631	1.9205	1.0000
6	48.3528*	52.6466*	55.8413*	292.5428*	1.0000
7	0.0972*	0.0543*	0.1630	0.0581*	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000

*significant at at least 5% level

SIGHT TYPE BY DAY

	1	2
OPTIC	0.1495*	1.0000
IRON	1.0000	1.0000

*significant at at least 5% level

SET D

PROFILE OF FACTORS WHICH DETERMINE THE RELATIVE ODDS OF HITTING A TARGET

MAIN EFFECTS

BASE	SIGHT TYPE (g)	GUNNER (j)	FIRING SESSION (k)
0.001605	Optic 415.477	1 1635.582	1 1461.298
	Iron 1.0000	2 16911.66	2 30.5750
		3 0 000002	3 30.5750
		4 0.009426	4 3.4109
		5 1.0000	5 0.6429
			6 1.0000

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INTERACTIONS

FIRING SESSION BY GUNNER

	1	2	3	4	5
1	0.000199	0.000231	538.743	67.8509	1.0000
2	0.05331	0.8215	236.986	3242.861	1.0000
3	0.004529	0.000037	6.5757	6.8667	1.0000
4	0.01113	0.000798	530.591	0.2933	1.0000
5	0.003439	0.000096	2814.742	0.000573	1.0000
6	1.0000	1.0000	1.0000	1.0000	1.0000

SIGHT TYPE BY GUNNER

	1	2	3	4	5
Optic	0.000229	0.1605	455.591	238.699	1.0000
Iron	1.0000	1.0000	1.0000	1.0000	1.0000